Production Networks and Aggregate Fluctuations: Evidence, Theory, and Policy

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A Dissertation presented to the Graduate Faculty of the University of
Virginia in Candidacy for the Degree of Doctor of Philosophy

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University of Virginia
April, 2017

Abstract

In this dissertation we study the sources of aggregate fluctuations with emphasis on the role of firms' linkages and non-linearities in production. In Chapters 1 and 2, we present a new set of empirical facts on i) the relationship between the structure of inter-sectoral linkages and aggregate fluctuations, and ii) the relationship between firms' elasticity of substitution in production and the spreads on corporate bonds during periods of high corporate debt and in recessions. We then construct a multisector model that is able to deliver the observed empirical patterns. In Chapter 3 we test the predictions of different dynamic multisector models regarding how well they predict the observed co-movement of sectoral output growth in the U.S. industrial sectors. Finally, in Chapter 4 we study the normative implications of combining the models in Chapters 1-3.

In Chapter 1, we study what features of the countries' domestic structure of sectoral linkages can help account for the observed differences in cross-country volatility of growth and the size of macroeconomic downturns. The empirical cross-country evidence, cross-sectional and panel data, suggests that unlike previous multisector models predict, the density of sectoral connections – number of sectoral connections in countries' input-output tables – is strongly correlated with aggregate volatility and the size of downturns. Therefore, we develop a multisector model that is able to deliver a role for the density of connections. The key implications of the model are that: i) the density of sectoral connections amplifies shocks – as observed for manufacturing oriented countries – if firms have low elasticity of substitution between labor and intermediates, while ii) density mitigates shocks – as observed for service oriented countries – if firms have high elasticity. In addition, iii) the density of connections amplifies the size of downturns – as observed for manufacturing oriented countries – if low elasticity firms face frictions in the use of intermediates, while iv) density mitigates downturns – as observed for service oriented countries – if high elasticity firms face frictions in the use of labor.

In Chapter 2, we study what are the sectors of the economy that have more troubles in financing inputs during recessions in the U.S. Using firm level and sector level data, we find that during times of high corporate debt i) manufacturing sectors with lower estimated elasticities of substitution between labor and intermediates pay higher spread on corporate bonds, while service sectors with larger elasticities pay higher spreads on corporate bonds. We also find that during recessions firms with low elasticity between labor and intermediates pay higher spreads. The model economy in Chapter 1, with occasionally binding

working capital constraints on labor or intermediate inputs, can deliver these observed empirical regularities. Intermediates become more expensive in recessions, therefore inflexible and intermediate-intensive manufacturing firms need to increase borrowing and eventually become (more) financially constrained. On the other hand, flexible and labor-intensive service firms reduce their relative demand for (more expensive) intermediates and eventually become constrained in financing labor input. The model also implies that during recessions service sector firms face much tighter credit conditions.

In Chapter 3, we contrast the predictions of different multisector DSGE models with the observed correlations of sectoral output growth in the U.S. industrial sectors. We study models that differ in the timing of the investment decision and in the technology used to build physical capital. To focus on the role of capital accumulation, we assume unitary elasticity of substitution in production and never-binding working capital constraints. We find that our model with a single type of physical capital that firms rent from households outperforms alternative models with sector specific capital and investment goods' linkages. Unlike models with sectoral specific capital accumulation, our model has a reduced state space, therefore it can be solved using global methods and occasionally-binding working capital sectoral constraints.

Finally, in Chapter 4 we study the policy implications of the models in Chapter 1 and 2, extended to have physical capital as the model in Chapter 3. We characterize a network pecuniary externality that arises when working capital constraints bind or are likely to bind. There is an intra-temporal externality when firms are constrained in the use of inputs. Firms in one sector do not internalize how their production decisions affect equilibrium prices and then affect the constraints of firms' in other sectors. There are inter-temporal externalities in which households do not internalize that i) by increasing investment in a sector today, can relax that sector constraint tomorrow, and ii) that by increasing aggregate investment today can reduce tomorrow's cost of capital and relax firms constraints.

Acknowledgements

I feel extremely lucky of having the possibility of working with Eric Young. Having him as a Professor and then as coauthor was key in my professional development. Two of the chapters of my dissertation are joint work with him. I learned a great deal with Eric. In particular, I learned to go straight to the point, say things for their name, learn from the data and build better model, have always in mind the policy implication of the problem at hand, and accept difficult challenges.

I wish to thank Toshihiko Mukoyama for his continuous support and for sharing his academic excellence. Being part of his advising group was key to keep making progress and being prepare to address his deep questions and concerns. Being his teaching assistant and student taught me a great deal of how being an effective and motivating professor. He was always available to give great suggestions and comments. For example, the interpretation of a key result in Chapter 1 is purely his intuition that I then confirmed with extra math.

Sophie Osotimehin was a fundamental part in my professional development. A great Professor and then a great advisor is the one with great questions. All her concerns, papers' suggestions, and questions became an essential part of this dissertation. Going over the details of my research with her was extremely important. I learned how important these details can be and how fruitful is not to rush on the research process (we could argue that I learned the no rush part).

I am very indebted to Pierre-Daniel Sarte. I also had the honor of having him as a Professor and then as an advisor. Having his course in my second year was a game changer. His encouragement, suggestions, and deep knowledge of the literature were instrumental in this dissertation. His ability of thinking several things, and from different perspectives, at the same time was motivating and extremely useful after each meeting. The time he devoted to our meetings was precious and extremely productive. From his passion about research, but research with an impact, I felt highly motivated and encouraged.

I have learned a great deal from working with Peter Debaere, Daniel Murphy and Kieran Walsh. They complemented the set of skills I learned in the program with their practical way of thinking of the world. I learned how to better deal with empirical work as well as better nailing down the research question at hand. I am very indebted to them.

I would like to thank Zach Bethune, Enghin Atalay, Nicholas Trachter, Felipe Schwartzman, Kartik Athreya, Huberto Ennis, Ana Fostel, James Harrigan, Kerem Cosar, and Karim Chalak. Finally, I want to thank my fellow graduate students for their comments.

...This dissertation is dedicated to Dana. Without your support, love, comments, ideas, and encouragement none of this would have been possible. I also want to dedicate this dissertation to my parents, Silvana and Jorge. Thanks for motivating us and supporting us in all respects. I also want to dedicate this dissertation to Abelardo, Maria Isabel, and Natalia. Your stay here was fundamental to accomplish this.

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Chapter 1: Sectoral Connectivity, Volatility, and Downturns: Lessons from Cross-Country Data

Abstract

We show that the domestic structure of countries input-output matters for the size of aggregate fluctuations. In a sample of 48 economies for the period 1984-2014, we find that i) service oriented countries with denser sectoral connections – higher number of links between sectors of the economy – display lower aggregate volatility and milder macroeconomic downturns, and that ii) manufacturing oriented countries with denser sectoral connections present higher volatility and sharper downturns. We show that extending an otherwise standard multisector model to allow for non-unitary elasticity of substitution between labor and intermediates can account for these facts. The model implies that in manufacturing oriented economies, where the elasticity of substitution tends to be smaller than one, the density of connections amplifies complementarities in production thereby increasing aggregate volatility. In service oriented economies, where the elasticity tends to be larger than one, the density of connections diversifies the effect of shocks and reduces volatility. In addition, to be consistent with the empirical relationship between density and downturns, the model implies that firms in manufacturing oriented countries face troubles in the financing of intermediates during recessions, while firms in service oriented countries face troubles in the financing of labor during recessions.

1.1 Introduction

The production process, e.g., production of Cell Phones in Korea, entails complex connections between firms in different sectors of the economy. Samsung not only uses chips, plastic, and financial services but also uses equipments, water, and gas. Every economy produces different goods and services, which then translates into a particular structure of linkages among producers. What is the relationship between the production network structure and aggregate fluctuations? Does an economy with more interconnected sectors absorb idiosyncratic shocks or amplify them?

We start by providing a new set of empirical facts on the relationship between the production network structure and aggregate fluctuations. Even after controlling for several country characteristics, we find that the domestic structure of sectoral linkages has a strong relationship with

aggregate fluctuations.¹ In particular, the countries' production network density – the number of connections between sectors of the economy – is strongly correlated with aggregate volatility and the size of downturns. Service oriented economies with denser production network display lower macroeconomic volatility and milder downturns. On the other hand, manufacturing oriented economies with denser production structures display higher volatility and sharper downturns.

These empirical facts are at odds with the predictions of existing multisector models. Thus, we extend the analysis of an otherwise standard multisector economy to show that the network density has an important role in the propagation of sectoral shocks. The key element of the model that generates this prediction is the elasticity of substitution between labor and the bundle of intermediate inputs. In economies with labor and intermediates being substitute inputs, a higher network density mitigates the effect of shocks. Nevertheless, if labor and intermediates are complement inputs, a higher network density amplifies complementarities and generates larger aggregate fluctuations.

In Koesler and M. (2015) and in Chapter 2 of this dissertation (Henceforth Miranda-Pinto and Young (2016)), it is found that the elasticity of substitution between labor and intermediates is higher in service sectors and lower in manufacturing sectors.² Hence, this additional evidence on sectoral elasticity of substitutions supports the model's prediction and conciliates the differential relationship between network density and volatility in the data. In service oriented economies, where firms are flexible in substituting labor and intermediates, denser connections mitigate shocks among more actors which reduces volatility. In manufacturing oriented countries, where firms are inflexible substituting labor and intermediates, denser connections amplify complementarities in production and increase volatility.

The model economy has inter-sectoral linkages (John B. Long and Plosser (1983)) and constant elasticity of substitution (CES) technologies (Atalay (2014)). Firms use labor and intermediate input in production. Firms buy intermediate inputs from other firms in the same sector or in other sectors. The composition of firms intermediate input bundle defines the production network of the economy. The network structure is assumed to be exogenous and we study how

¹We control for trade openness, size of the country, concentration in production, GDP growth, sectoral composition, frequency of financial crisis, and rule of law, among others.

²Using a panel of countries from the WIOD database, Koesler and M. (2015) find that the elasticity of service sectors is mostly larger than one, while in manufacturing sectors is smaller than one. Miranda-Pinto and Young (2016) estimate for the U.S. an average elasticity for service sectors of 1.2 and for manufacturing sectors 0.8.

different networks amplify shocks differently. The only source of fluctuations in the model is given by the existence of sectoral productivity shocks.

To study the relationship between the network structure and downturns, we add to the model working capital borrowing constraints (Bigio and La'O (2016)) that bind occasionally. Firms need to pay inputs in advance in order to produce and sell their output. Firms do not hold cash, therefore need external funds to pay for inputs, which can be obtained by pledging a fraction of total sales as collateral for loans. As in Miranda-Pinto and Young (2016) we assume that sectors are heterogeneous in the fraction of their labor an intermediate input cost to be paid in advance as well as in the fraction of sales they can pledge as collateral.³ We study whether the network density amplifies or mitigates the likelihood that sectoral constraints become binding.

To study the role of the density of connections in a tractable way, we compare two regular and symmetric network structures: a *dense network* where every sector provides and uses intermediates from all other sectors in the same proportion, and a *sparse network* in which sectors supply and buy intermediates from only one sector. The row sum and column sum of both networks are the same, implying a symmetric composition in the extent to which sectors provide and use intermediates to and from other sectors. To understand the relationship between density and volatility we abstract from the existence of working capital constraints. However, to understand the relationship between density and the size of downturns we study in which circumstances the density or sparsity of connections can trigger financial cascades in which many sectors become constrained due to a negative sectoral shock.

The key feature of the model that can account for the differential correlation between the density of connections and volatility in emerging and developed economies is the elasticity of substitution between labor and intermediates. When the elasticity is below one, the model can replicate the positive correlation between density and volatility for emerging economies. When the elasticity is above one, the model can deliver the negative correlation between density and volatility for developed economies. These predictions are consistent with the fact that developed economies specialize in sectors with high elasticity (service sectors) while emerging economies

³This assumption can be thought as a combination of two constraints. First, a technological constraint where in some sectors, only workers need to receive money before they produce, but in other sectors the problem is in the input supplier firms that require the payment of intermediates before production. Second, a limited enforcement problem where firms in some sectors have lower credibility of repayment than others. In the model this implies a smaller fraction of sales to be pledged but can be thought as sectors having less tangible assets to be seized or sectors with more idiosyncratic volatility that have larger probability of default.

specialize in sectors with low elasticity (manufacturing, mining, and agriculture sectors).

In the sparse and the dense network there is a trade-off between the number of sectors that are directly affected by a shock and the intensity in which that shock affects those sectors. In the sparse network, a sectoral shock has a large effect on the intermediate-input bundle price of one sector, while in the dense network the same sectoral shock has a small effect on the intermediate-input bundle price of all the sectors in the economy. With Cobb-Douglas technologies, the intermediate input shares do not change with relative prices, implying that the tension between the number of sectors affected and the intensity in which sectors are affected by a shock is the same in both network. In other words, with Cobb-Douglas technologies and symmetric networks, adding up small changes in production of several sectors (dense network) delivers the same aggregate effect of adding up large changes in production of few sectors (sparse network). However, when the elasticity of substitution between intermediates and labor is non-unitary, the relative demand for inputs is non-linear.

If the elasticity is smaller than one, the intermediate input demand is concave, implying that the direct effect of a shock in the dense network, where there are small changes in the intermediate-input bundle price of several sectors, is larger than the direct effect of the same shock in the sparse network, where the intermediate-input bundle price of few sectors is largely affected. If the elasticity is larger than one, the intermediate input demand is convex and the dense network mitigates the effect of shocks instead. The total network effect is the infinite sum of further indirect effects implied by the network connections of the affected sector. Thus, the density of connections amplifies the aggregate effect of sectoral shocks in inflexible economies while it mitigates the aggregate effect of sectoral shocks in flexible economies.

Regarding the relationship between the density of connections and the size of macroeconomic downturns, the model can deliver the facts when low elasticity economies (emerging economies) are constrained in financing intermediate inputs while high elasticity economies (developed economies) are constrained in financing labor. The dense input-output structure in a low elasticity and intermediate input constrained economy (emerging economy), needs a smaller sectoral shock to generate a cascade in which most sectors of the economy become financially constrained. The increase in the price of one sector's output, makes intermediates more expensive for every sector. As sectors are inflexible substituting intermediates for labor, their intermediate input cost share increases, which eventually makes them financially con-

strained.

When firms use intermediates and labor as substitute and are constrained in financing labor (developed economies), the sparse input-output structure needs a smaller sectoral shock to generate a financial cascade. In this case, an increase in the cost of intermediates increases the firms' relative demand for labor. In the sparse network, where individual sectors rely only on one other supplier (customer), there is a greater substitution from intermediates to labor which makes more likely that firms become constrained in the use of labor.

In the last section of this chapter we perform quantitative exercises and show that in a more general calibration, the model is able to deliver a correlation between network density and volatility similar to the one observed in the data.

To the best of my knowledge this is the first study in finding an empirical and a theoretical relationship between the density of sectoral connections and aggregate fluctuations.⁴ We contribute to the literature on the network origins of aggregate fluctuations (John B. Long and Plosser (1983), Horvath (1998), Horvath (2000), Dupor (1999), Acemoglu et al. (2012), Forster et al. (2011), and Atalay (2014)) by showing that the number of sectoral connections in an economy could amplify or mitigate the effect of sectoral shocks on aggregate volatility, depending on the degree of substitutability between inputs.

This paper also contributes to the literature of financial frictions in multisector model started with Bigio and La'O (2016) and extended by Luo (2015), Altinoglu (2015), and Miranda-Pinto and Young (2016). Unlike Bigio and La'O (2016), Luo (2015), and Altinoglu (2015), the model in this paper delivers financial cascades. A sectoral productivity or financial shock that causes one sector to become constrained can generate a domino effect where many other sectors become financially constrained. This mechanism depends on the sectors' flexibility in production, the density of sectoral connections, and which inputs are subject to constraints. The domino effect of financial constraints is consistent with the observed asymmetric relationship between the density of connections and the size of macroeconomic downturns in the data.

⁴Dupor (1999) noted two things. First, for a given number of sectors N , the density of sectoral connections does not affect aggregate volatility. Second, when the number of sectors converges to infinity, the aggregate volatility of any symmetric network that only differs in the density of connections approaches zero (aggregation theorem). Acemoglu et al. (2012) extended these results for balanced network structures with limited asymmetry of sectors' supplier importance.

1.2 Related Literature

This paper builds on past work regarding the sources of aggregate volatility in different countries. Koren and Tenreyro (2007) study why poorer countries are more volatile than richer countries. They find that poor countries specialize in fewer but intrinsically more volatile sectors.⁵ Unlike Koren and Tenreyro (2007), we explicitly take into account the linkages across sectors and find that relatively small but very connected and volatile sectors impose higher macroeconomic volatility.⁶ Another contribution of this paper with respect to Koren and Tenreyro (2007) is the finding that the density of sectoral connections is correlated with growth volatility, and that this correlation differs for emerging and developed economies. Furthermore, we build a multi-sector model that provides an economic explanation for the observed facts.

di Giovanni and Levchenko (2012) find empirical evidence that smaller and more open economies are more volatile. They conciliate this evidence with the granularity hypothesis of Gabaix (2011), in which economies with few disproportionately large firms display higher aggregate volatility as a result of idiosyncratic shocks to individual firms. Smaller countries and more open countries display this pattern in the distribution of firm size. Building a model of heterogeneous firms, as in Melitz (2003), and incorporating power law distribution of firm size, as in Gabaix (2011), the authors are able to rationalize the empirical findings. This study complements the findings in di Giovanni and Levchenko (2012) by emphasizing the role of the structure of inter-sectoral linkages in the propagation and amplification of sectoral shocks.⁷

Moro (2012), Moro (2015), Carvalho and Gabaix (2013) study how the structural change in the U.S. and other low and high income economies towards a more service-intensive economy reduces aggregate volatility. The cross-country analysis in this paper adds a new dimension in which aggregate volatility is smaller in a service-intensive economy. In service-intensive economies the density of the input-output matrix, in which sectors rely on many intermediate-input providers and customers, plays the role of shocks' diversification mechanism. Moreover, we highlight how service-intensive and densely connected economies are able to resist the ad-

⁵The authors use the Herfindahl index of sectoral concentration to proxy for sectoral specialization, which essentially captures the largest sectors of the economy.

⁶Not considering sectoral linkages biases the sectoral volatility decomposition in Koren and Tenreyro (2007). If a shock to one sector propagates through the input-output matrix, a statistical decomposition of aggregate and sectoral shocks identifies that shock as an aggregate shock. This issue is studied in detail in Foerster et al. (2011).

⁷In an attempt to account for the di Giovanni and Levchenko (2012) finding at the sector level, we consider the asymmetry in the share of sectors output that is exported to foreign countries as a measure of a country's dependence on foreign shocks.

verse effects of negative sectoral shocks by reducing the likelihood of financial cascades.⁸

This paper is also related to the literature that studies the relationship between the structure of inter-sectoral linkages and macroeconomic outcomes such as aggregate productivity and the level of income of a country (Jones (2011), Bartelme and Gorodnichenko (2015), and Fadinger et al. (2015)). Jones (2011) notes that the existence of intermediate inputs in production imposes an aggregate multiplier similar to the one implied by capital stock in the standard growth model. This multiplier implies that in the presence of sectoral frictions in the use of inputs, for example sales taxes, aggregate productivity losses are an increasing function of the economy's intermediates share in production.⁹

Motivated by Jones (2011), Bartelme and Gorodnichenko (2015) find empirical evidence that the degree of connectivity among sectors, as measured by the average sectoral multiplier,¹⁰ is positively correlated with output per worker and total factor productivity. The authors interpret this result as the gains from reducing the distortions in the sectors' use of inputs. The intuition is that a positive productivity shock spillover easily as long as sectors in the economy are close to each other and frictions in the use of inputs are minimized.

In the same literature, Fadinger et al. (2015) take advantage of the heterogeneity in sectoral multipliers across countries. The authors find that poor countries have few sectors with large multipliers, meaning few sectors are important input suppliers. One of their main results obtained, by using a multisector model with unitary elasticity of substitution between inputs, is that if poorer countries had a more symmetric sectoral composition, their income level would increase. This would be due to the fact that poor economies have larger frictions in the use of inputs that would be amplified by higher sectoral connectivity.

This paper is different from Jones (2011), Bartelme and Gorodnichenko (2015), and Fadinger et al. (2015) in that it studies countries' aggregate dynamics rather than aggregate productiv-

⁸Even though we take the input-output structure as given, without modeling or targeting the structural change, the results in this paper hold as long as developed economies are on average more service oriented than emerging economies along the period 1984-2014. This is indeed what is observed in the data in Moro (2015) when comparing middle income economies with high income economies.

⁹Jones (2011) also highlights the role of complementarities in production, meaning low elasticity of substitution between inputs. However, his numerical results show a small role of complementarities in aggregate productivity losses. In this paper, we show that complementarities can be important for aggregate fluctuations but in non-linear ways.

¹⁰Consider the input-output of the economy Γ . The Leontief inverse is $L = (I - \Gamma')^{-1}$. Then the vector of sectoral multipliers is simply $L\mathbf{1}_{n \times 1}$, where n is the number of sectors in the economy. These multipliers contain the total upstream connections of a sector, summarizing the total aggregate effect of a sector solely as input supplier.

ity. This paper also considers different network measures that separate the role of the density of linkages – number of sectoral connections – from the role of the asymmetry of linkages in which some sectors are disproportionately important intermediate input suppliers or users. In this regard, assuming a non-unitary elasticity of substitution between intermediate inputs and labor, unlike the assumption of Cobb-Douglas technologies, is crucial to understand the role of the density of linkages separate from the asymmetry of linkages.

This paper is also related to Acemoglu et al. (2015b). The authors study how the density of linkages in a network of banks can enhance the propagation of bank failures. Their finding suggests that when negative shocks are large, a highly connected network enhances a cascade of banking failures. In this paper, rather than firms exiting, firms become borrowing constrained. For a parametrization relevant for emerging economies, where firms use intermediate inputs and labor as complement inputs and intermediate inputs are constrained, a dense production network facilitates the propagation of large negative shocks. If firms in one sector become constrained, a dense network generates a cascade in which many sectors become constrained.

Lastly, Contreras and Fagiolo (2014) study how the structure of the input-output matrix in 22 European countries amplifies the effect of exogenous shocks. They investigate the diffusion and propagation of shocks in light of three non-agent based models with sectoral linkages. The authors find that countries with denser sectoral linkages are more likely to experience stronger economic crises. Unlike Contreras and Fagiolo (2014) we document a systematic relationship between the countries' production structure and their volatility of growth and declines in GDP growth. We do not confirm their results regarding the density of linkages and economic crisis, instead we find that for developed economies, a densely connected input-output structure smooths the effect of shocks, reducing the median decline in GDP growth during macroeconomic downturns. Moreover, guided by the empirical facts, we build an economic agent based model that explicitly accounts for the role of sectors technologies and frictions in shaping the propagation of sectoral shocks.

The rest of Chapter 1 is organized as follows. Section 2 presents the empirical facts that motivate the paper regarding the relationship between the density and asymmetry in the input-output matrix and the volatility of GDP growth. Section 3 presents the model and its main predictions. Section 4 presents Monte Carlo simulations of the model to quantitatively assess the effect of linkages on aggregate volatility and declines in growth. Finally, Section 5 presents

the conclusions.

1.3 Structure of Linkages and Aggregate Fluctuations

In this section, we empirically investigate the relationship between the countries' structure of inter-sectoral linkages and: 1) the standard deviation of GDP growth, and 2) the declines in GDP growth during years of negative growth. We start by describing the data to then analyze the cross-country heterogeneity in aggregate fluctuations. Then, we define the network measures that capture the number of connections and the intensity of connections between sectors. We then show the main univariate relationships to later estimate the conditional correlations through cross-sectional and panel regressions.

1.3.1 Data

The variables of interest are the volatility of growth, as measured by the standard deviation of annual real GDP growth for the period 1984-2014, and the magnitude of macroeconomic downturns, as measured by the median growth in years with negative growth. We use countries' real GDP at constant national prices in millions of 2011 US\$ dollars from the Penn World Tables 9.0 (*rgdpna*).

The input-output matrix data is collected from the OECD input-output database. For each country we have information for 33 sectors of the economy for the period 1995-2011. We keep in the sample the countries in the OECD database for which we have real GDP data since 1983, implying a final sample of 48 countries with 25 developed economies and 23 emerging economies.

We collect data on countries' characteristics from the IMF World Economic Outlook and the Worldwide Governance Indicators.¹¹ The characteristics are GDP per capita in current US\$ dollars adjusted by Purchasing Parity Power (PPP), the size of the country in the world economy, openness to trade, financial openness (Lane and Milesi-Ferreti 2007), and the Rule of Law index.¹² Other controls considered are the average growth of a country, the share of ser-

¹¹The Worldwide Governance Indicators (WGI) are managed by the Natural Resource Governance Institute, the Brookings Institution, and the World Bank. It is a research dataset that surveys a large number of enterprise, citizen and expert survey respondents in industrial and emerging countries.

¹²This index ranges from -2.5 (weak) to 2.5 (strong) and reflects how agents in a country perceive the rules of society in terms of: quality of contract enforcement, property rights, the police, and the courts, as well as the likelihood of crime and violence.

vice sectors' gross output over total gross output, the size of the financial sector in the economy, the size of the most volatile sectors according to Koren and Tenreyro (2007), the average intermediate input share of the economy, the skewness in the sectors' output exported abroad, the sectoral dominance measure in Acemoglu et al. (2016), the sectoral Herfindahl index, and the frequency of financial crisis in Reinhart and Rogoff (2009) or Laeven and Valencia (2013).¹³

We also use the countries' development classification in the World Economic Situation and Prospects report from the United Nations (UN).

1.3.2 Cross-Country Differences in Aggregate Fluctuations

There are substantial cross-country differences in the standard deviation of GDP growth for the period 1984-2014. Figure 1 displays the histogram of countries' standard deviation of growth. In the right tail of the distribution, emerging countries such as Argentina and Singapore present a standard deviation of growth larger than 4% while developed economies such as the U.S. and France have a standard deviation of growth below 2%. After accounting for the main determinants used in the literature to account for this heterogeneity, there still persists substantial differences in residual volatility. Figure 2 displays the residual from an OLS regression between countries' volatility and the determinant used in the literature.¹⁴

Countries also differ significantly in the size of their macroeconomic downturns. Figure 3 depicts the histogram of the absolute value countries' median negative growth. A fraction of this dispersion is accounted by countries' characteristics in Figure 4, which depicts the histogram of the residual size of downturns, controlling by the same countries' characteristics used in the regression for residual volatility. There are still important differences in the size of countries macroeconomic downturns. The range between the minimum and maximum residual downturn is 0.5% while in the raw data is about 0.8%.

¹³Acemoglu et al. (2016) define sectoral dominance as the ratio between the maximum sectoral Leontief multiplier over the variability in the sectoral Leontief multipliers. See footnote 11 for a definition on the Leontief multipliers.

¹⁴This is the residual of a cross sectional regression between the volatility of growth and its main determinants in the literature such as: the level of development (GDP per capita PPP adjusted), country size (GDP in \$US dollars), average growth rate, rule of law index, trade openness, frequency of financial crisis, and service sectors' share of the economy. (Koren and Tenreyro (2007), di Giovanni and Levchenko (2012), Moro (2015)).

1.3.3 Service Share and Technologies

Countries are heterogeneous in the sectors they specialize in. The literature of structural transformation (Moro (2012), Carvalho and Gabaix (2013), and Moro (2015)) documents that developed economies tend to specialize in service sectors while emerging economies specialize in manufacturing, agriculture or mining activities. It is also well known that service sectors tend to be less volatile than non-service sectors (Koren and Tenreyro (2007)). Figure 5 shows in the horizontal axis the logarithm of the average GDP per capita adjusted by PPP and in the vertical axis the service sectors' share over gross output in 2011. Developed economies are more service oriented, which reduces their aggregate volatility compared to emerging economies (Moro (2015)).

Besides the fact that service sectors are less volatile, they are also more flexible in their production process. Koesler and M. (2015) use a panel of countries-sectors from the World Input-Output database and find that most of service sectors use labor-capital and intermediates as substitute inputs while non-service sectors use them as complement inputs. Similar results are found in Miranda-Pinto and Young (2016) (Chapter 2) who document for the U.S. that the elasticity of substitution between inputs of production is larger in service sectors. In particular, the authors estimate an average elasticity of substitution between labor-capital and intermediate inputs of 1.2 for service sectors and of 0.8 for non service sectors. Assuming that the technology of Restaurants in the U.S. is the same as Restaurants in the rest of the world, service oriented economies are relatively more flexible in their production technology.¹⁵

We define the country k weighted elasticity of substitution between labor-capital and intermediates as

$$\tilde{\epsilon}_k = \epsilon_{usa}^s service_k + \epsilon_{usa}^{ns} (1 - service_k),$$

where ϵ_{usa}^s is the elasticity of substitution of service sectors in the U.S., ϵ_{usa}^{ns} is the elasticity of substitution of non-service sectors in the U.S., and $service_k$ is the service share over gross output of country k . We use $\epsilon_{usa}^s = 1.2$ and $\epsilon_{usa}^{ns} = 0.8$ and plot the countries' weighted elasticity

¹⁵Baccianti (2013) and Koesler and M. (2015) estimate sectoral elasticities using international input-output data from the World Input-Output database. Their non-linear LS estimates show that service sectors tend to be more flexible in substituting labor-capital with intermediates (or only energy). In particular, Koesler and M. (2015) finds that most service sector have an elasticity above one while all the manufacturing sectors have an elasticity smaller than one.

in Figure 6. Richer economies, on average, use labor-capital and intermediates as substitute inputs, implied by an elasticity above one, while most of low-middle income economies use labor-capital and intermediates as complement inputs, implied by an elasticity below one.

In the reminder of this section, we will study how the structure of the input-output matrix in different countries can account for the observed residual differences in aggregate volatility and the size of macroeconomic downturns. In that regard, considering the differences in sectoral technologies across countries will be crucial.

1.3.4 Network Measures

The production structure of the U.S. is such that most sectors are inter-connected (Figure 7), while in Greece (Figure 8) sectors are more isolated. In particular, the U.S. has 264 more input-output shares that are larger than 0.05% (not completely dark), which is equivalent to set 7 entire columns, out of 33 columns, of the input-output matrix of the U.S. to be almost zero. Regarding the intensity of connections, the U.S. and Greece have few sectors that supply intermediate inputs to all other sectors. This is expressed by few very bright rows of the input-output matrix in Figures 7 and 8.

The input-output structure of an economy is the weighted adjacency matrix of a directed graph with N vertex (sectors) and potentially N^2 edges. Let Ω represents the input-output matrix of the economy. An element ω_{ij} is an input-output share that represents how important intermediate inputs shipped from sector i are to sector's j total expenditure on intermediates. These input-output shares are not symmetric, it might be the case that sector i provides inputs to sector j but sector j does not supply intermediates to sector i . The column i sum of Ω equals 1, as total intermediate expenses must be allocated to some (all) sector(s) of the economy. The row j sum represents how important sector j is as an intermediate input supplier to the economy as a share of other sectors intermediates expenditures. The actual input-output table of a country weights how important are intermediates in each sectors' total sales. Thus, assuming a common share of total intermediates over sales $(1 - a)$, the input-output of a country is represented by $(1 - a) \cdot \Omega$.

We are interested in network measures that capture sectoral connectivity in two dimensions. First, a binary measure that counts the non-zero sectoral connections, where a sectoral connections is defined by a sector supplying or buying intermediate inputs to or from other sector. A

network in which every sector relies only on a few suppliers or customers might have a very different diffusion of shocks than a network in which sectors have a more diversified set of suppliers and customers. In the literature of financial networks there are several studies that investigate the relationship between the number of connections among banks and the stability of the financial system. For example, Acemoglu et al. (2015b) find that if the size of the liquidity shock affecting one bank is large enough, a banking network with several connections facilitates the extent of financial contagion.

The second class of network measures explicitly take into account the intensity of sectoral connections. Horvath (1998) and Acemoglu et al. (2012) state that the asymmetry in which sectors supply intermediate inputs to other sectors is important to understand the aggregate effect of sectoral shocks. The authors find that in economies where a few sectors supply most of the intermediate inputs in the economy, sectoral shocks have large effects on aggregate volatility.

1.3.5 Density of Connections

To capture the first class of network measures we use the network density. The density of sectoral connections counts the number of active edges over the maximum possible number of edges in the networks.¹⁶ Therefore, a production network with N sectors is dense if a large number of intermediate-input shares are positive over the maximum number of sectoral connections $N(N - 1)$.¹⁷ A production network is sparse if few intermediate-input shares are positive. The standard measure in network theory to describe the density of linkages is the following. Let $T = \sum_i^N \sum_{j=1}^N \mathbb{1}[\omega_{ij} \geq 0]$, the number input-output shares above zero. Then, the Density measure is defined as:

$$\text{Density} = \frac{T - N}{N(N - 1)}, \quad (1)$$

where a network with density of 1 is a fully dense network while a density 0 implies a fully sparse network where firms in a sector only trade with one other sector. Due to measurement errors and the fact that input-output tables are Harmonized to be comparable across countries,

¹⁶The term density is widely used in the literature of networks. In the literature of multisector models, Horvath (1998) used the term density when studying how having more zero rows in the input-output would alter the law of large numbers in the propagation of sectoral shocks. Nevertheless, as detailed in this section, the density of linkages considers not only the rows of a network (out-degrees) but every potential connection among sectors.

¹⁷We suppress the diagonal of the input-output to focus in the connectivity among different sectors. In fact, in most countries firms of the same sector also purchase and sell intermediates to firms in the same sector.

we count the input-output connections that are larger than a small threshold $\hat{\omega}$. We consider a range of values between 0.1% and 1%.

1.3.6 Asymmetry in the Intensity of Connections

The literature on production networks in Horvath (1998) and Acemoglu et al. (2012) puts emphasis on the role of the structure of the Leontief inverse in shaping the way sectoral productivity shocks are propagated and amplified. The j_{th} element of the Leontief inverse can be expressed as

$$L_j = (I - (1 - a)\Omega')_j^{-1} = \sum_{k=1}^{\infty} ((1 - a)\Omega')_j^k,$$

and it contains the direct and indirect effects of a shock to sector j . The parameter a represents the importance of labor in production and it could be heterogeneous across sectors. The value of k represents the higher order network effects. When $k = 1$, a shock to sector j has a direct effect on the sectors that use sector j 's output as input. When k is bigger than 1, the shock to sector j affects the sectors that are indirectly connected to sector j by being directly connected to sector j 's customers.

Horvath (1998) and Acemoglu et al. (2012) argue that the asymmetry in $\{L_j\}_{j=1}^N$ is the key for sectoral shocks to have aggregate implications on volatility. As shown by the authors, under Cobb-Douglas production functions the asymmetry in the sectors' Leontief elements has a one to one mapping with the asymmetry in which sectors supply intermediate input to other sectors. The importance of sectors as input suppliers is given by the weighted out-degree d_j^{out} :

$$d_i^{out} = \sum_{j=1}^N (1 - a_j)\omega_{ij}. \quad (2)$$

To measure the asymmetry in the out-degrees, we calculate the sample skewness of the sectoral distribution of out-degrees ($S_{dout,k}$) for each country k .

An alternative way to account for the asymmetry in sectors importance is by using the sectoral dominance statistic in Acemoglu et al. (2016). The authors define the influence of a sector by

$$v_j = \sum_{i=1}^N \beta_i L_{ji},$$

where β_j is the share of households consumption in sector j goods and L_{ji} is the (j, i) element

of the Leontief inverse matrix. The sectoral dominance index is defined as

$$\text{dominance} = \frac{v_{\max} \sqrt{N}}{(\sum_{i=1}^N v_i^2)^{1/2}},$$

and captures the existence of sectors that are extremely important in the network.

Finally, we also consider the asymmetry in the intensity in which a sectors use intermediate inputs. As noted by Acemoglu et al. (2015a) and Luo (2015), government spending shocks and financial shocks propagate mainly from intermediate input user sectors to intermediate input provider sectors. Similarly, Carvalho et al. (2016) find that when firms use intermediate inputs and labor as substitute inputs, a productivity shock to sector j not only affects the users of sector j 's output but it also affects the demand of firms in sectors that supply goods to firms in sector j .

The intermediate input intensity in total sales is given by the weighted in-degree d_j^{in} :

$$d_j^{\text{in}} = \sum_{i=1}^N (1 - a_j) \omega_{ij}, \quad (3)$$

To measure the asymmetry in sectors in-degrees, we consider the sample skewness of the sectoral distribution of in-degrees ($S_{\text{din},k}$) for each country k .¹⁸

1.3.7 Cross-Country Differences in the Input-Output Structure

Figure 9 depicts the histogram of the Density of linkages for the 48 countries in the sample using the input-output matrices in 2011 and a threshold of $\hat{\omega} = 0.1\%$.¹⁹ There is substantial heterogeneity in the extent to which sectors interact in different countries. For example, emerging economies such as Brunei, Mexico, and Costa Rica have more than 40% of intermediate-input shares below 0.1%. In the developed world, Ireland, Denmark, and Greece are countries with more than 30% of intermediate input shares below 0.1%. The opposite holds for China, Poland, Spain, and the U.S., where more than an 86% of the intermediate input shares are greater than 0.1%.

¹⁸The sample skewness S of a realization of $\{x\}_{i=1}^N$ is given by

$$S = \frac{\frac{1}{n} \sum (x_i - \bar{x})^3}{\frac{1}{n-1} (\sum (x_i - \bar{x})^2)^{3/2}}$$

¹⁹Using the input-output matrices of previous years yields very similar results.

Regarding the distribution of in-degrees, the international evidence also shows substantial heterogeneity (Figure 10). Countries such as Belgium and Italy have a negative skewness in the distribution in-degrees, meaning a mass of sectors with intermediate-input shares around the mean and few sectors with disproportionately small intermediate-input shares. Other countries such as Singapore and Bulgaria have few sectors that use a disproportionately high intermediate input share in production (star input customers).

Figure 11 depicts the skewness of sectors out-degrees. The data shows a global right tail in the distribution of sectoral out-degrees, evidenced by the fact that all countries have a positive skewness of out-degrees. There is also significant cross-country heterogeneity in the asymmetry of input suppliers' importance. For instance, the U.S. and China have a relatively symmetric distribution of out-degrees while in Luxembourg and Singapore 2 sectors are the main intermediate input providers of the economy.

1.3.8 Univariate Relations

We start by exploring univariate relationships without controlling for other country-characteristics. Figure 12 shows that in the group of developed countries, the sparser the connections between sectors the larger is the standard deviation of GDP growth. For example, Luxemburg, Greece, and Iceland have a standard deviation of growth larger than 3% and a density of linkages lower than 70% while Austria, Norway, and the U.S. have a standard deviation of growth lower than 1.8% and a density larger than 85%. The difference between a density of 70% and a density of 85% is 160 potential connections among sectors of the economy or equivalently 5 entire rows (columns) in the input-output table.

The opposite is observed for the group of emerging countries. Figure 13 shows that Argentina, Bulgaria, and Romania have a standard deviation of growth larger than 5% and a density of linkages larger than 75% while Israel, Mexico, and Philippines have a standard deviation of growth lower than 3.1% and a density smaller than 60%. The difference between a density of 60% and a density of 75% is also 160 potential connections among sectors of the economy or equivalently 5 entire rows (columns) in the input-output table. This relationship between density and aggregate volatility is not predicted by existing literature. In fact, Dupor (1999) and Acemoglu et al. (2012) find that two economies that only differ by their density of linkages depict the same aggregate volatility. In the next section, we show that their result is due to the

assumption of unitary elasticity between labor and intermediate inputs.

The data also shows a positive relationship between the asymmetry of sectors' in-degrees and aggregate volatility (Figure 14). This relationship holds for the whole sample of countries and subgroups by development. For example, in Singapore the Computer and Electronics sector uses an intermediate-input share that is almost two times the average intermediate input share of the economy. In Hungary, the Food Products and Beverages sector uses an intermediate input share that doubles the average material share in the economy. The volatility of Singapore and Hungary is above 3.7%. On the other hand, in Belgium and France there are not manufacturing industries using excessively large material share but there are service industries – Education and Health – that use a disproportionately low share of materials in production. These countries have a volatility below 1.6%.

There is no apparent correlation between the asymmetry of sectors out-degrees distribution and the volatility of growth (Figure 15). This figure is crucial as does not show support for the main prediction of existing multi-sector models that sectoral shocks propagate the most in economies with few sectors being important input suppliers, this is, in economies with high (positive) skewness of sectors' out-degrees distribution (Horvath (1998) and Acemoglu et al. (2012)).

With respect to the relationship between the input-output structure and the magnitude of macroeconomic downturns, Figures 16 and 17 show that in developed economies a denser input-output implies a smaller log median decline in growth. The opposite stands for emerging economies, where denser connections imply a larger median decline in growth. Interestingly, this relationship is not observed during years of positive growth. Therefore, there seems to exist an asymmetric relationship between how connected sectors are and how shocks diffuse and amplify during years of negative growth.

1.3.9 Regression Analysis

We proceed to estimate the conditional correlation between aspects of growth – volatility and the median negative growth – and the density and asymmetry network measures described earlier, controlling for other determinants of aggregate fluctuations used in the literature such as: level of development (GDP per capita of a country, service share of the economy, share of the country in world GDP, average growth rate, rule of law index, size of the government as a frac-

tion of GDP, countries' exposure to international trade and financial integration (ratio exports plus imports to GDP, ratio financial assets plus financial liabilities to GDP, skewness of the share of sectoral output exported), importance of intermediate inputs in the economy (average intermediate input share in the economy), sectoral composition (output share of the most volatile sectors according to Koren and Tenreyro (2007)), concentration of production (Herfindahl sectoral index), financial development (financial sector share in the economy), history of financial crisis (frequency of financial crises according to Reinhart and Rogoff 2009), and pre-1984 measures of heterogeneity in aggregate volatility and downturns (1970-1983 growth volatility and downturns for each country).²⁰

We estimate the following regressions

$$\log(\sigma_{T,k}) = \beta_0 + \beta_1 \log(\text{Density}_{T,k}) + \beta_2 S_{in,T,k} + \beta_3 S_{out,T,k} + \mathbf{X}'_T \gamma + \varepsilon_{T,k}, \quad (4)$$

$$\log(|\text{Decline}_{T,k}|) = \beta_0 + \beta_1 \log(\text{Density}_{T,k}) + \beta_2 S_{in,T,k} + \beta_3 S_{out,T,k} + \mathbf{X}'_T \gamma + \varepsilon_{T,k}, \quad (5)$$

where $\sigma_{T,k}$ represents the standard deviation of GDP growth for country k in period T , and $\text{Decline}_{T,k}$ is the median negative growth of country k in period T . On the right hand side of the equation we add the network measures and the vector of country characteristics.²¹ There are potential reverse causality and endogeneity problems in these regressions as i) volatility and the size of macroeconomic downturns could cause changes in the structure of inter-sectoral linkages and ii) there are country unobservables that can affect the network structures and the nature of aggregate fluctuations.²² Regarding the problem of reverse causality, given the level of disaggregation in this analysis (33 sectors), it is unlikely that new sectoral connections were

²⁰The literature on the country characteristics that affect aggregate fluctuations is extensive. See Acemoglu, Johnson, and Robinson (2004), Koren and Tenreyro (2007), and di Giovanni and Levchenko (2009,2012), among others.

²¹The cross-sectional results presented use countries' production structure in 2011 but similar results hold when using the input-output in previous years. We keep the countries with data on real GDP for the period 1984-2014. In addition, for each country we drop the household services sector, which implies a total of 33 sectors instead of the 34 that are originally in the database.

²²Oberfield (2013) and Carvalho and Voigtländer (2015) study endogenous formation of the input-output linkages. The latter paper empirically explores the evolution of sectoral connections finding that sectors that were closely (indirectly) connected in 1967 are more likely to be connected in the four subsequent decades. This result holds at a very disaggregated level (4-digit industry) for more than 300 sectors.

adopted or that some links disappeared. It is more reasonable to think that changes in aggregate fluctuations might affect the measures of asymmetry in the intensity of sectoral connections.²³

There is still a potential endogeneity problem. Unobserved country characteristics could be correlated with the network structure and the volatility (or the size of downturns). Our data suggests a slow moving production network. The cross-country correlation between the network density in 1995 and the network density in 2011 is larger than 0.92. Thus, adding as a regressor the pre-1984 cross-country differences in aggregate fluctuations can mitigate the problem. The pre-1984 differences capture in part how non-observed country characteristics affect the determination of the network.²⁴ In addition, the purpose of this study is to investigate how the network structure and countries business cycle dynamics are related. In this sense, it is reasonable to assume that at the business cycle frequency our network measures, especially the density measure, are exogenous. Ultimately, this section aims to find empirical relationships – not in a causal way – that help us account for cross-country differences in aggregate fluctuations, so we can guide the construction of a model that delivers the facts.

1.3.10 Cross-Sectional Evidence

Table 2 presents the results of estimating (4) by OLS. In column (1) we observe a negative and significant correlation between countries production network density and volatility. A 1% increase in the network density is associated with a 0.4% increase in volatility. Motivated by the findings in Moro (2015), in column (2) we add the service share as a regressor. As expected, countries with a larger service share are less volatile. A 1% increase in service share is associated with a 0.8% decline in aggregate volatility. Interestingly, the coefficient for density is much smaller and non-statistically different from zero. However, when we include the interaction between the network density and the service share (column 3) we observe that i) countries with denser production network display smaller macroeconomic volatility and that ii) countries with higher service share and denser production network present even smaller aggregate volatility. The total effect of density on volatility is negative for countries with a service share over gross output larger than 0.45, while it is positive for countries with a service share smaller than 0.45

²³To account for reverse causality we estimate (4) and (5) using network measures in 1995 and evaluate how those measure relate to after 1995 measures of volatility and downturns. The results are similar which indicate that reverse causality is not an issue.

²⁴We thank Leopoldo Fergusson for pointing out this.

(below the 10th percentile of the distribution of service share).

This result is robust to adding several cross-country controls in columns (4) to (6). We also find that countries are more volatile if: grow less (Ramey and Ramey (1995) and Luo et al. (2017)), have a smaller financial sector (Srivisal and Dabla-Norris (2013)), and have a larger skewness of sectors intermediate usage (in degrees asymmetry). The coefficient of other covariates, e.g., openness to trade, have the expected sign but are not statistically significant. Even adding all these controls, we find that countries with more interconnected sectors are less volatile and countries with larger service share and higher interconnectedness are even less volatile. Based on column (6), countries with a service share larger than 0.57 (above the 25th percentile of the distribution of service share) have an overall negative relationship between density and volatility, while countries with a service share smaller than 0.57 present a negative correlation between density and volatility. In Tables 3-4 we show that our results are robust to change the threshold that define a connection. In Table 3 we use a threshold of 0.05% of total intermediate expenses and in Table 4 we use 0.1%.²⁵

We now study the relationship between countries size of macroeconomic downturns. Each country presents few episodes of annual negative growth. We investigate how the sectoral interconnectedness amplifies or mitigate these times of macroeconomic distress. In Table 5 we estimate equation (5) by OLS. We observe that in general our covariates have few explanatory power. Only in columns 5 and 6 we find that service oriented countries with denser connections display milder downturns. Only the interaction is significant, meaning an overall positive correlation between density and downturns. Density amplifies downturns but it does so less in more service intensive countries. Using higher thresholds to define a connection (Table 6 and Table 7) yield similar results. Denser connections mitigate the size of downturns and this mitigation is stronger in service oriented economies.

1.3.11 Panel Evidence

We divide the sample in three sub-periods: 1994-1995, 1996-2005, and 2006-2014. For each sub-period we obtain the standard deviation of GDP growth, the median negative and positive growth, and the average growth. Given that the OECD input-output data covers the period 1995-

²⁵These results are also robust to adding other covariates such as: the size of government in the economy, degree of financial openness, frequency of financial crisis, rule of law index, and GDP per-capita in 1984.

2011, we use the end of period production network measures. To be consistent, we use the end of period value of the control variables.²⁶ All the regressions include time fixed-effect dummy and country-specific time trends. Our data has insufficient time variation and sample size to take advantage of panel fixed effects. Therefore, our results rely on cross-sectional variation and on the assumption of random effects, where the unobserved country characteristics are independent of the observed countries covariates.

Table 8 presents the results for volatility of estimating a pooled a panel regression assuming that the unobserved country characteristics are independent of the covariates (random effects, RE). The results in Table 8 confirm the cross-sectional findings. Countries with denser production structures present lower macroeconomic volatility. This relationship is stronger the larger the service share of a country. For countries with a service share below 0.53 the sign of the relationship reverses, implying that denser connections are associated with higher macroeconomic volatility. This results are robust to adding countries covariates and the choice of the threshold to define a connection (Table 9 and Table 10).

The panel RE results for the size of macroeconomic downturns are depicted in Table 11. The results are similar to the cross-sectional evidence. The coefficient for network density is negative and statistically significant in some specifications. The interaction between density and the service share is negative and always statistically different from zero. This latter finding is robust to adding more covariates and to the threshold used to define a connection (Table 12 and Table 13).

This section has documented that production complementarities in the input-output structure account for an important part of the cross-country differences in business cycle dynamics. This relationship is particularly strong when accounting for volatility differences across countries. We find that non-service oriented economies with denser sectoral connections are associated with higher aggregate volatility and sharper downturns. The opposite holds for service oriented economies, where the density of connections mitigates the effect of shocks. The next section aims to give economic interpretation to the cross-country facts by building an economic model with inter-sectoral linkages in the spirit of John B. Long and Plosser (1983). The main focus is to understand the relationship between density and aggregate fluctuations which is not predicted

²⁶For the last sub-period 2006-2014, we must use the production network measures in 2011. In addition, using the beginning of the period controls or the average provides similar results.

by existing models.

1.4 The Model Economy

The model has CES technologies as in Atalay (2014) and working capital constraints in the use of inputs as in Bigio and La'O (2016). As in Miranda-Pinto and Young (2016), the difference of this model with respect to Atalay (2014) is the value of the elasticity of substitution between inputs, while the difference from Bigio and La'O (2016) is that we allow sectors to be different regarding which inputs have financial constraints. As shown later, combining these models will have important implications in terms of delivering the observed facts in section 2.²⁷

1.4.1 Firms

There are N sectors, each sector has a continuum of homogeneous firms that behave competitively. The constant returns to scale technology of firms in sector j is

$$Q_j = Z_j \left[a_j^{\frac{1}{\varepsilon_j}} L_j^{\frac{\varepsilon_j-1}{\varepsilon_j}} + (1 - a_j)^{\frac{1}{\varepsilon_j}} M_j^{\frac{\varepsilon_j-1}{\varepsilon_j}} \right]^{\frac{\varepsilon_j}{\varepsilon_j-1}}, \quad (6)$$

where the material bundle is

$$M_j = \prod_{i=1}^N M_{ij}^{\omega_{ij}}. \quad (7)$$

The output of the representative firm in sector j is denoted by Q_j , Z_j is total factor productivity, L_j is labor, M_j is the material bundle of sector j , and M_{ij} is the amount of materials purchased by sector j from sector i . The parameter a_j represents how important is labor in the total value of production.

The elasticity of substitution between labor and materials is denoted by ε_j . The element ω_{ij} reflects the importance of sector i as an input supplier to sector j , as a fraction of sector j total material cost. We assume constant returns to scale in the material bundle technology, $\sum_{i=1}^N \omega_{ij} = 1$, and unitary elasticity of substitution among material varieties. This assumption implies that the sector j 's material payment to sector i is a constant fraction of the sector j material bundle cost.²⁸ However, as $\varepsilon \neq 1$, the material bundle cost is not a constant share of

²⁷The model does not provide a theory of network formation. We follow Horvath (1998), Dupor (1999), and Acemoglu et al. (2012) and take different network structures and analyze how each of those networks affect the way shocks propagate.

²⁸This assumption is not empirically supported as Atalay (2014) and Miranda-Pinto and Young (2016) estimate

sales. Therefore, a typical element of the input-output matrix (γ_{ij}) – payments from sector j to i as a fraction of sector j sales – is endogenous and depends on the sector j 's productivity and the relative price of labor and the material bundle in (7).

Firms face an intra-period working capital borrowing constraint that limits the input expenses that can be externally financed to a fraction of total sales. This modeling assumption is based on Bigio and La'O (2016) and Miranda-Pinto and Young (2016).²⁹ This assumption can be better explained with two constraints. First, firms need to pay in advance a fraction of the materials and labor expenses. This fraction differs across sectors and can be financed by borrowing b_j and by using internal funds A_j . Thus, the sectoral constraint is

$$\theta_j^l L_j + \theta_j^m \sum_{i=1}^N P_i M_{ij} \leq b_j + A_j, \quad (8)$$

where P_j is output price of sector j and the wage rate is normalized to 1. Firms in sector j need to externally finance a fraction θ_j^l of the wage bill L_j , and a fraction θ_j^m of the cost of materials purchased from all other sectors.³⁰ For simplicity, we assume $A_j = 0$ for all j . Firms are also constrained in the amount of borrowing they can obtain by

$$b_j \leq \eta_j P_j Q_j \quad (9)$$

in which different sectors can pledge a different fraction η_j of total sales as collateral. The variable μ_j denotes the Lagrange multiplier for the sectoral borrowing constraint in (9), which represents the firms' shadow cost of debt. For unconstrained firms, the shadow cost of debt is zero, however, for constrained firms μ_j is positive. Thus, μ_j represents how much firms in sector j value the additional external funds that allow them to produce at the optimal scale.³¹

elasticities of substitution among material varieties smaller than 1. However, the main mechanism of the model relies on ε being different from 1. Relaxing the assumption of unitary elasticity among materials will only have quantitative and not qualitative effects on the main results.

²⁹Miranda-Pinto and Young (2016) show that this model delivers occasionally binding borrowing constraints that fit the observed time-varying negative (positive) correlations between sectoral elasticities of substitution between materials and value added (among materials) and spreads on corporate bonds in the U.S.

³⁰There is no reason to think of different sectors having the same working capital constraints. It might well be the case that for some firms it is easier to pay workers at the end of the month, e.g. that an important fraction of the salary of agriculture workers depends on the end of the period production. On the other hand, it could be the case that agriculture firms do have to pay a large fraction of their intermediate input purchases before they use those intermediates for production.

³¹The micro foundation of this borrowing constraint is the following. Before production takes place, firms borrow from a competitive financial intermediary the amount of input expenses needed to produce. There is a limited commitment problem, since after sales firms can default on their debt without paying back to the intermediary.

1.4.2 Households

The representative household maximizes utility

$$U(C, L) = \frac{C^{1-\sigma} - 1}{1-\sigma} - \psi \frac{L^{1+\xi}}{1+\xi}, \quad (10)$$

subject to

$$wL + \sum_{j=1}^N \pi_j = PC \quad (11)$$

$$C = \prod_{j=1}^N C_j^{\beta_j},$$

where C is the consumption bundle and L is total labor supply. $\sigma \geq 0$ is, for a given wage, the income elasticity of labor supply, and $\xi > 0$ is the inverse Frish elasticity of labor supply. The parameter ψ is 0 for the tractable case of inelastic labor supply and is 1 when labor is elastically supplied. In the household budget constraint, π_j is profit from firms in sector j , w is the wage rate, and P is the ideal price index of the consumption bundle C . The consumption shares β_j satisfy $\sum_{j=1}^N \beta_j = 1$.

Definition 1 (*Competitive Equilibrium*) A decentralized competitive equilibrium is a set of prices $\{w, (P_j, P_j^M)_{j=1}^N\}$ and allocations $\{C, L, (C_j, Q_j, L_j)_{j=1}^N\}$, $\{(M_{ij})_{ij}^N\}$ such that, given prices,

- The representative consumer maximizes utility (51) subject to the budget constraint (11).
- Firms maximize profits subject to constraints (8) and (9)
- And, the goods and labor markets clear

$$Q_j = C_j + \sum_{i=1}^N M_{ji}, \quad (12)$$

$$L = \sum_{j=1}^N L_j. \quad (13)$$

Therefore, firms are required to pledge a fraction of sales as collateral. If a firm does not pay back, the financial intermediary seizes a fraction $1 - \eta_j$ of total sales. In an equilibrium without default on debt, the incentive compatibility constraint implies that firms can externally borrow up to a fraction η_j of total sales.

1.4.3 What Do We Know So Far in the Literature?

Before analyzing the main predictions of the model, we discuss what are the direct implications of existing models regarding the empirical facts presented in section 2. For the relationship between the asymmetry of sectors importance and aggregate volatility, Horvath (1998) and Acemoglu et al. (2012) show that networks with a few very important input suppliers display higher aggregate volatility from sectoral shocks. The first study finds that economies with more zero rows in the input-output matrix have larger aggregate volatility. In a similar way, the second study finds that economies with fatter Pareto tails of sectors out-degrees distribution have higher aggregate volatility.³² Acemoglu et al. (2015a) empirically validate the predictions in Acemoglu et al. (2012) for the U.S. manufacturing sectors. The authors find that sectoral productivity shocks propagate purely from input supplier (upstream) sectors to input customer sectors (downstream).

Regarding the relationship between the Density of linkages and aggregate volatility, Dupor (1999) finds that economies that only differ in the degree of connectivity among sectors, but have the same in-degrees and out-degrees distribution, have the same aggregate volatility, which goes to zero when the number of sectors goes to infinity. Acemoglu et al. (2012) generalize this finding for balanced networks with limited out degree asymmetry but differences in the Density of linkages. In both cases, the aggregate volatility of the two economies that only differ in terms of the Density of linkages is the same. This result is the irrelevance theorem in Dupor (1999).

All these studies assume unitary elasticity of substitution between labor and materials. Atalay (2014) allows for elasticities to be different than one and estimate elasticities smaller than one. The focus in Atalay (2014) is to understand how non-unitary elasticities of substitution change the results in Foerster et al. (2011) about the contribution of sectoral shocks to aggregate fluctuations. The focus in this paper is rather different. We assume that sectoral shocks are the only source of fluctuations and study how the relationship between the density of linkages and aggregate volatility changes with non-unitary elasticity of substitution between inputs.

Acemoglu et al. (2015a) and Luo (2015) study how other types of shock – demand type shocks and financial shocks, respectively – propagate through the input-output linkages. The first study finds for the U.S. that government spending shocks and trade shocks (Chinese im-

³²Acemoglu et al. (2012) simplify the model by assuming that each sector has the same material share (in-degree) in production.

ports) that increase domestic demand for sectoral output propagate mainly from downstream (intermediate-input customer sectors) sectors to upstream sectors (intermediate-input supplier sectors). Luo (2015) also finds that in the U.S. financial shocks propagate mainly to upstream sectors. Acemoglu et al. (2015a) and Luo (2015) results indicate that the asymmetry of sectors in-degrees, associated to upstream propagation, could play a role in amplifying sectoral shocks. Carvalho et al. (2016) demonstrate that there exists upstream propagation of purely productivity shocks as long as firms have non-unitary elasticity of substitution between labor and intermediates. Unlike the Cobb-Douglas technologies case, productivity shocks that increase the price of intermediates do change downstream sectors' sales and therefore their demand for upstream sectors' output.

The empirical results in the previous section are consistent with most of the prediction of existing models, except with the prediction in Dupor (1999) and Acemoglu et al. (2012) regarding the irrelevance of the density of linkages. Thus, in the reminder of the paper we illustrate how the density of sectoral connections can amplify or mitigate shocks in a way it is consistent with the empirical evidence.

1.4.4 Density of Linkages and Volatility

In order to have analytically tractable results, we define two symmetric networks that only differ in terms of the Density of linkages. In the *sparse* network each sector uses (provides) intermediate inputs only from (to) one other sector, while in the *dense* network every sector is connected to every other sector in the same intensity. In both economies, labor and materials have the same importance in the sectors' production, implying $a_j^{sparse} = a_j^{dense} = a$ for all j .

With CES technologies between labor and the material bundle, the material bundle share over sales and the labor share fluctuate with relative prices, however, the assumption of unitary elasticity of substitution among material varieties implies that the sector j 's material payment to sector i (ω_{ij}) is a constant fraction of the total material expenditure of sector j . Therefore, we proceed to define the dense and sparse network in terms of the (constant) shares of the material bundle cost:

$$\Omega^{sparse} = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \Omega^{dense} = \begin{bmatrix} \frac{1}{n} & \frac{1}{n} & \dots & \frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & \dots & \frac{1}{n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{n} & \frac{1}{n} & \dots & \frac{1}{n} \end{bmatrix}.$$

we assume $\sigma = 1$, $\psi = 0$, and a labor endowment $\bar{L} = 1$.³³

Proposition 1 *Assuming that the constraints in (9) are not binding ($\mu_j = 0$ for all j), and that $Z_j = 1$ for all j , then*

- (i) *If $\varepsilon = 1$, $\frac{\partial \log GDP}{\partial \log Z_j}$ is the same in the sparse and dense network for all j*
- (ii) *If $\varepsilon > 1$, $\frac{\partial \log GDP}{\partial \log Z_j}$ is larger in the sparse network for all j*
- (iii) *If $\varepsilon < 1$, $\frac{\partial \log GDP}{\partial \log Z_j}$ is larger in the dense network for all j*

Proof: See Appendix C.

Proposition 2 part (i) confirms the results in Dupor (1999). When $\varepsilon = 1$, the density of linkages does not affect the GDP elasticity of sectoral shocks in symmetric networks. Even though in the sparse network each sector is strongly tight to its only intermediate input trade partner, the propagation of a shock is not amplified compared to the dense network. This result is due to the fact that firms optimally choose a constant input share in production regardless of changes in relative prices. In this case, as noted by Acemoglu et al. (2015a), shocks only propagate downstream – from input providers to input users – due to the increase in the cost of intermediates. This downstream propagation of a shock is entirely determined by the Leontief inverse matrix, which accounts for all the direct and indirect effects that a shock on sector j has on its intermediate input users and the users of its users.

However, with non-unitary elasticity of substitution between labor and intermediates, the number of active links among sectors does play a role. The intuitive result of density diversifying shocks among more sectors is only applicable to the case with substitute inputs. If firms are flexible enough to reduce their demand for intermediates when they become expensive, then

³³This parametrization of the utility is convenient to obtain closed form solutions. In the simulations of the next section we show that the main results still hold with an upward sloping labor supply.

the economy as a whole is less volatile out of sectoral productivity shocks. On the other hand, when firms are inflexible in production and bear the cost of more expensive materials, a denser production structure generates larger aggregate volatility.

To understand the role of the density of connections with non-unitary elasticity of substitution is instructive to assume that $Z_j = 1$ for all j . In this case, we can study the aggregate effect of sectoral shocks locally around $Z_j = 1$. For the sparse and the dense network we obtain the following GDP elasticity,

$$\frac{\partial \log GDP^{sparse}}{\partial \log Z_j} = \beta_j \mathbf{1}' [I - (1-a)\Omega']_j^{-1} = \left(\frac{P_j Q_j}{PC}\right)^{sparse} \quad (14)$$

$$\frac{\partial \log GDP^{dense}}{\partial \log Z_j} = \beta_j \mathbf{1}' [I - N^{1-\varepsilon}(1-a)\Omega']_j^{-1} = \left(\frac{P_j Q_j}{PC}\right)^{dense} \quad (15)$$

The local effect of a sectoral shock in each network also depends crucially on the Leontief inverse matrix. However, in the dense network there is an extra *diversification* term $N^{1-\varepsilon}$. When $\varepsilon = 1$, the *diversification* term is one, meaning that both networks would display the same aggregate volatility from the same sequence of sectoral TFP shocks. In addition, as shown in the appendix, the centrality of a sector in the network is described by the sales of a sector. This is consistent with the result in Hulten (1978).

When the elasticity of substitution is different than one, the size of sectors changes differently in the dense and the sparse network. When $\varepsilon < 1$, sectors become relatively larger in the dense network compared to the sparse network. The opposite happens in the case where $\varepsilon > 1$. In the sparse and dense network there is a trade-off between the number of sectors affected by a shock and the intensity in which the shock affects those sectors. In the dense, a sectoral shock affects every sector by a little. However, in the sparse network a sectoral shock directly affects one sector in a large extent. With non-unitary elasticity of substitution between labor and intermediates, there is a non-linear demand for intermediate inputs. The nature of this non-linearity is key to break down the trade-off. If the wage rate is normalize to one and $Z_j = 1$ for all j , we have

$$\frac{\partial^2 P_j^j M_j / P_j Q_j}{\partial (P_j^M)^2} = \varepsilon(\varepsilon - 1)(P_j^M)^{-\varepsilon-1} P_j^{\varepsilon-1} (1-a).$$

If $\varepsilon < 1$, the relative demand for intermediates is concave. Therefore, several small changes in intermediates demand in the dense network have a larger aggregate effect than one large change in intermediates demand in the sparse network. When $\varepsilon > 1$, the relative demand for intermediates is convex. Thus, the total direct effect of a sectoral shock is smaller in the dense network.

To understand the total network effect it is convenient to express the GDP elasticity of a sector in (14) in terms of the infinite higher order network effects. The Leontief inverse element j in both networks can be expressed as follows

$$\frac{\partial \log GDP^{sparse}}{\partial \log Z_j} = \beta_j \sum_{k=1}^{\infty} (1-a)^k (\Omega')_j^k$$

$$\frac{\partial \log GDP^{dense}}{\partial \log Z_j} = \beta_j \sum_{k=1}^{\infty} N^{k(1-\varepsilon)} (1-a)^k (\Omega')_j^k,$$

where k is the higher order network effect. We can see that the differential direct effect of a sectoral shock implied by the concavity or convexity of the intermediate input demand is present at every higher order network effect. Therefore, the indirect effects will further amplify ($\varepsilon < 1$) or mitigate ($\varepsilon > 1$) the effect of shocks in the dense network.

The quantitative importance of the density mechanism depends on the value of ε and the level of disaggregation of the economy. If $0 < \varepsilon < 1$, for a fixed number of sectors N , the smaller the ε the higher the effect of sectoral shocks on GDP in the dense network compare to the sparse. If $\varepsilon > 1$, for a fixed number of sectors N , the larger the ε the smaller the aggregate effect of sectoral shocks in the dense network compare to the sparse.

An important result that is implied by Proposition 2 can be summarized in the following proposition

Proposition 2 Define $S = \sum_{j=1}^N s_j$ as total sales of the economy. Let $H = \left(\sum_{j=1}^N \left(\frac{s_j}{S} \right)^2 \right)^{1/2}$ the Herfindahl index of the economy. Assume $\varepsilon \neq 1$, the constraints in (9) are not binding ($\mu_j = 0$ for all j), and that $Z_j = 1$ for all j , then

(i) The dense and the sparse network have the same H

(ii) $\sigma_{GDP}^{dense} > \sigma_{GDP}^{sparse}$ if $\varepsilon < 1$

$$(iii) \sigma_{GDP}^{dense} < \sigma_{GDP}^{sparse} \text{ if } \varepsilon > 1$$

Proof: See Appendix C.

Therefore, the Herfindahl index of is not a sufficient statistic to determine how volatile is an economy. When $\varepsilon \neq 1$, the details of how sectors are connected, in particular the density of connections, play an important role in shaping the aggregate effect of shocks. This result contributes to the results in Gabaix (2011) where the asymmetry in sectoral sales is what drives aggregate volatility. Using Hulten's theorem, Gabaix argues that the underlying network structure might be important to determine the equilibrium sales (as argued by Acemoglu et al. (2012)) but it is ultimately the asymmetry in sectoral sales – Herfindahl index of an economy – what drives aggregate fluctuations.

1.4.5 Density of Linkages and Downturns

In the data there is an asymmetric relationship between the density of sectoral linkages and declines in GDP growth when growth is negative. First, we explain the main mechanism in the model that implies a relationship between the network structure and the likelihood of sectoral constraints to bind.

The interesting case of the model is when elasticities between materials and value added are different than one and $\theta_j^l \neq \theta_j^m$. In this situation, the borrowing constraints bind endogenously due to fluctuations in input prices, implying the possibility of endogenous financial cascades. A financial cascade is an event in which several sectors initially unconstrained become financially constrained due to a sectoral shock in other sector.

A bad shock in one sector can drive other sectors to becoming borrowing constrained. When a firm in sector j is hit by a negative productivity shock, it reduces output and its price increases in equilibrium. There is also an equilibrium effect on other sectors' prices. Firms that are downstream of j will face an increase in their material cost, while firms that are upstream of j will see the demand for their output decrease.

To illustrate the main mechanism of the model, we provide a numerical example for a three sector vertical economy. Sector 1 provides inputs to sector 2, sector 2 provides input to sector 3, and sector 3 provides input to sector 1. Each sector uses labor such that $a = 0.3$. We assume

$\varepsilon = 1.5$, $\eta = 0.12$, $\theta_l = 1$, and $\theta_m = 0$ for all sectors. This is, sectors are flexible substituting materials and labor and need to externally finance the totality of the wage bill.

The horizontal axis of Figure 18 displays the sector 2 relative price of materials and labor. As materials become relatively more expensive than labor – e.g, due to a negative productivity shocks in sector 1 – firms in sector 2, that use materials and labor as substitute inputs ($\varepsilon = 1.5$), will optimally shift toward more labor in their production plan. However, as firms in sector 2 use more labor they increase borrowing and eventually become borrowing constrained. In this example, firms cannot borrow more than a 12% of total sales.

Figure 19 displays a different case. Firms in sector 2 are now inflexible substituting labor and intermediates, and face a constraint in the use of intermediates. The negative shock in sector 1 increases the intermediate input cost of sector 2. As firms in sector 2 still need to use intermediates, the intermediate cost share increases, which eventually makes firms constrained. The external fund to pay for intermediates in advance cannot exceed a 41% of sales, implying an inefficiency in the firms' decision. There is a wedge as firms end up paying more and using less intermediates compared to the frictionless competitive equilibrium.

We now proceed to show the main proposition that links the network density with the occurrence of financial cascades. The following proposition describes the predictions of the model when sectors are inflexible substituting inputs and material costs have to be paid in advance

Proposition 3 *Assume $\varepsilon < 1$, $\theta_j^m = 1$, and $\theta_j^l = 0$ for all j . Let $\tilde{Z}_j^k$ be the shock to sector j in network k that drives sector $j + 1$ to become constrained, with $\mu_{j+1} = 0$, then*

- (i) *In the dense network, a shock $\tilde{Z}_j^{dense}$ drives the constraint of all other sectors to bind.*
- (ii) *In the sparse network, a shock $\tilde{Z}_j^{sparse}$ only drives the constraint of sector $j + 1$ to bind.*

Proof: See Appendix C.

In this case, $\tilde{Z}_j^k$ represents the productivity shock in network k that affects sector j and drives the constraint of sector $j + 1$ to become binding, when still $\mu_{j+1} = 0$. Proposition 5 part (i) implies that when a shock to sector j is sufficiently large to drive the constraint of sector $j + 1$ to bind, a financial cascade is originated in the dense network when firms are inflexible and materials have to be paid in advance. The intuition is that density hurts firms when i) the

input they have to externally finance is precisely the input (materials from other) that connects one firm to every other, and ii) firms are not flexible shifting away from materials to labor.

Therefore, the asymmetric relationship between the density of linkages and GDP declines in the data for manufacturing oriented countries can be delivered by Proposition 3. According to the model, in manufacturing oriented economies a wedge in the use of material inputs increases in size during economic downturns. This wedge is even larger in emerging economies with denser sectoral connections.

1.4.6 Sparsity and Downturns, Substitute Inputs

We proceed to show that a sparse network is more prone to financial cascades when labor is constrained and firms are flexible in production. In this case there is no analytic solution, therefore we use numerical results for different parameter combinations. The model's prediction are the following. Assume $\varepsilon > 1$, $\theta_j^m = 0$, and $\theta_j^l = 1$ for all j .

- (i) In the sparse network, a smaller productivity shock to sector j , $\tilde{Z}_j$, is needed to drive the constraint of sector $j + 1$ to bind.
- (ii) In the sparse network, a smaller financial shock to sector j , $\tilde{\eta}_j$, is needed to drive the constraint of sector $j + 1$ and $j + 2$ to bind.

Notice that in this case there is an incomplete cascade. The sparse network facilitates the propagation of constraints with smaller shocks, however, when the shock is large enough the dense network still delivers a complete financial cascade. These results suggest that dense linkages help mitigating the effect of mild shocks. In a sparse network, when firms are flexible, sectoral shocks have a larger effect on the relative price of inputs, therefore firms increase labor demand relatively more. As labor is the constrained input, in the sparse network, it is more likely that firms become borrowing constrained.

In Appendix C we show how the solution for this case boils down to a system of equations between the Lagrange multiplier of sector $j + 1$ and the threshold shock on sector j $\tilde{Z}_j$. Numerical results are presented in the last section of this Chapter.

Hence, in light of the empirical results in the previous section, the model implies that in service oriented economies a wedge in the use of labor increases in size during years of negative growth. The wedge is larger the sparser are the sectoral connections.

1.4.7 Discussion About Frictions

In this model economy, the size of the friction in the cost of inputs for a sector j is expressed by³⁴

$$\phi_j^m = \frac{(1-a_j)(1-\eta_j)}{a_j\eta_j} \geq 1,$$

if $\theta_j^m = 1$ and $\theta_j^l = 0$ and

$$\phi_j^l = \frac{a_j(1-\eta_j)}{(1-a_j)\eta_j} \geq 1,$$

if $\theta_j^m = 0$ and $\theta_j^l = 1$. When constraints are not binding, firms pay the competitive input prices and $\phi_j^m = \phi_j^l = 1$. When constraints bind, firms that are constrained in the use of materials pay $P_j^M \phi_j^m$ and firms constrained in the use of labor pay $w \phi_j^l$. For a given a_j , the smaller the fraction of sales that firms can pledge as collateral (η_j) the larger is the wedge in the competitive input prices. Moreover, if material costs have to be paid in advance, the more important are materials in production the larger the friction. Similarly when the constraint is in financing the labor costs, the more important is labor in production the larger the friction.

In the data, service sectors have an average value of a_j that is larger than non-service sectors. Moro (2015) uses KLEMS dataset in 2008 to show that manufacturing industries in all countries in the sample use a larger material share in production. The cross-country average is $a_j = 0.36$ for manufacturing and $a_j = 0.6$ for service sectors. These numbers make sense of the prediction of the model that developed (service intensive) countries have higher frictions in the use of labor, while emerging economies (non service intensive) economies have higher frictions in the use of intermediates.

In addition, these predictions are consistent with the results in Miranda-Pinto and Young (2016) using a different set of moments. The authors take advantage of the technological heterogeneity within the group of manufacturing sectors ($\varepsilon < 1$) and service sectors ($\varepsilon > 1$) and find for the U.S. that: i) within manufacturing, sectors with smaller elasticity of substitution between labor and materials pay a higher spread on corporate bonds during recessions, while ii) within services, sectors with larger elasticity of substitution between labor and materials pay higher spreads on corporate bonds. The model delivers these facts when a wedge in the use of materials (labor) increases in size during recessions in manufacturing (service) sectors.

³⁴See Miranda-Pinto and Young (2016) for a detailed derivation.

This section has characterized the model's predictions. The results imply that the model economy in this paper can make sense of the empirical regularities in section 2. In the next section, we calibrate a general version of the model to investigate the quantitative performance of the model in delivering the observed relationship between the network density and aggregate fluctuations.

1.5 Monte Carlo Simulations

This last section generalizes the model economy in Section 3. It allows for elastic labor supply and network structures with less extreme density of connections. The following networks aim to capture the role of the density of linkages:

$$\begin{aligned}\Omega^1 &= \begin{bmatrix} 0.01 & 0.98 & 0.01 \\ 0.01 & 0.01 & 0.98 \\ 0.98 & 0.01 & 0.01 \end{bmatrix} & \Omega^2 &= \begin{bmatrix} 0.33 & 0.66 & 0.01 \\ 0.01 & 0.01 & 0.98 \\ 0.66 & 0.33 & 0.01 \end{bmatrix} \\ \Omega^3 &= \begin{bmatrix} 0.33 & 0.34 & 0.33 \\ 0.01 & 0.33 & 0.66 \\ 0.66 & 0.33 & 0.01 \end{bmatrix} & \Omega^4 &= \begin{bmatrix} 0.33 & 0.34 & 0.33 \\ 0.33 & 0.33 & 0.34 \\ 0.34 & 0.33 & 0.33 \end{bmatrix}\end{aligned}$$

For each of these networks we assume $a = 0.4$. With CES technologies, the input-output matrix of network k fluctuates around. We use a threshold of $\hat{\omega} = 0.011$. Thus, the density index of these networks are $\text{Density}^1 = 0$, $\text{Density}^2 = \frac{1}{3}$, $\text{Density}^3 = \frac{2}{3}$, $\text{Density}^4 = 1$.

We assume that sectoral productivity follows an AR(1) process:

$$\log Z_t = \bar{z} + \rho_z \log Z_{t-1} + \kappa_t \tag{16}$$

where κ is normally distributed with mean zero and variance covariance-matrix Σ_κ . To calibrate the process for sectoral productivity, we use Horvath (2000) estimated values for the average 2 digit SIC sector of the U.S. The author estimates $\rho_z = 0.93$ and an average diagonal value of

$$\sqrt{\Sigma} = 0.022.^{35}$$

We set $\sigma = 2$ (income elasticity of labor supply), $\xi = 1$ (the inverse Frish elasticity of labor supply), and the weight of labor on utility to be $\psi = 1$. We assume that each good has the same importance in the consumption basket, therefore $\beta_j = 1/3$ for all three sectors. We assume that each sector starts unconstrained with an equilibrium borrowing that is equal to the average borrowing in the unconstrained simulations. We vary the elasticity (ε) and the fraction of inputs to be paid in advance (θ_j^l and θ_j^m). For each network we simulate the economy $M = 500$ times and for each simulation we have series of size $T = 2,000$ of real GDP. We obtain the median standard deviation and median growth below the percentile 8.5%. The choice of this value is based on the U.S. empirical distribution of growth. At the percentile 8.5%, growth in the U.S. zero.

The results in Table 14 show the volatility of the percent change in GDP from the four networks when the economy is hit by productivity shocks κ . As noted in Proposition 2, the models with unitary elasticity of substitution do not deliver a relationship between density of linkages and volatility. This implies that the sparse and the dense networks have exactly the same standard deviation of the log change of GDP. When the elasticity of substitution is smaller than one, as in Atalay (2014), there is a clear positive correlation between density and volatility. For the case when inputs are substitute ($\varepsilon = 1.2$), there is a clear negative correlation. These two results confirm the predictions of Proposition 2.

To study the relationship between macroeconomic downturns and the density of linkages, we study how large shocks to one sector of the economy can be amplified or mitigated by the density of connections. We assume sector 1 receives a sequence of shocks with a standard deviation 4 times larger than the previous exercise. Then compute the median growth of the left tail (i percentile 8.5) of the distribution of growth in the model.

The results in Table 15 show that, when inputs are complements, the median decline in growth is larger the denser are the sectoral connections. This effect is slightly amplified when materials have to be financed externally. On the other hand, when inputs are substitute, the fall in growth is larger the sparser are the connections. This effect is amplified when labor is the

³⁵The standard deviation is for sectoral productivity shocks to gross output. In order to gauge the plausibility of this parameter, Horvath (2000) states that these stochastic processes for sectoral productivity yield a standard deviation of aggregate TFP in line with what the literature in Real Business Cycle models assume, a value of 0.0078.

input that is facing the financial constraint. In Table 16 it is possible to see the frequency in which a financial cascade occurs (all sectors become financially constrained). On average, the frequency of a domino effect is maximized in the dense network when materials are constrained and inputs are substitute. There is no propagation of the constraints due to negative shocks when labor is constrained instead. When inputs are substitute, the most sparse network maximizes the frequency of financial cascades are. Similarly, there is not domino effect when materials are constrained and inputs are substitute. These results confirm the predictions of the special case.

1.6 Can the Model Deliver the Observed Cross-Country Correlations?

As exposed in previous sections, the density and volatility relationship depends on the number of sectors and on the exact value of the elasticity. The model is highly non-linear and calibrating and simulating the general model with 33 sectors for each country is numerically expensive. However, we provide a much simpler experiment that sheds light on the ability of the model to deliver the observed relationship between density and volatility. We set $N = 6$ and use the parameters of the previous section. We generate 15 economies that only differ in their density of connections. For each economy we simulate – out of sectoral shocks – M series of GDP and then calculate the percent change in GDP for each economy. We focus in the case of $\varepsilon = 0.6$ and then regress the following equation

$$\sigma_k = c + \beta \text{density}_k + v,$$

where σ_k is the standard deviation of growth of each economy and density_k is each economy's network density. The OLS coefficient for β is 0.05 and it is statistically different from 0. The coefficient implies that a 10 percentage points increase in density raises GDP growth volatility in 0.5 percentage points. Running the same regression for the group of manufacturing oriented countries and adding other cross-country controls, we obtain an estimate for β of 0.1. Thus, in this numerical example the model delivers a correlation that is half of the one observed in the data. Increasing the number of sectors or changing the value of the elasticity would change this result.

A quantitative exercise to study the relationship between density and downturns is more challenging as we need values for the parameters of the collateral constraint η – the fraction of

sales that can be pledged as collateral – and θ^y – the fraction of input y to be paid in advance –. We leave this exercise to future projects that use cross-country data on spreads or firms financial health indicators.

This section has exposed that the theoretical predictions in a tractable version of the model still hold with elastic labor supply and more realistic network structures. The general lessons from combining the empirical evidence with the predictions of the model are stated in Table 17. Manufacturing oriented economies are described as having low elasticity and frictions on materials. On the other hand, service oriented economies are described as having flexible technologies and frictions on labor.

1.7 Conclusions

The literature on the network origins of aggregate fluctuations (Horvath (1998) and Acemoglu et al. (2012)) has documented the potential of sectoral independent shocks to generate sizable aggregate fluctuations (Horvath (2000), Foerster et al. (2011) and Atalay (2014)). Recent literature has also noted the importance of the production network in amplifying the effect of sectoral frictions (Bigio and La'O (2016)). The common feature of the input-output structure that amplifies and propagates sectoral shocks in this literature is the existence of few sectors providing intermediate inputs to the whole economy (asymmetry of out-degrees).

This paper argues that other two element of the production structure play an important role in propagating and amplifying the effect of sectoral shocks: the density of sectoral connections – the ratio between the number of sectoral connections and the total number of possible connections – and the existence of few intermediate-input intensive sectors (asymmetry of in-degrees). We show that these two network measures account for an important fraction of the heterogeneity in cross-country aggregate fluctuations. In particular, developed economies with denser sectoral connection are less volatile and experience milder downturns. Also, emerging economies with denser sectoral connections display higher aggregate volatility and sharper macroeconomic downturns. Moreover, countries with few intermediate input-intensive sectors – asymmetry of in-degrees – are more volatile.

We build a multisector model – based on Atalay (2014), Bigio and La'O (2016), and Miranda-Pinto and Young (2016) – with CES technologies and sectoral working capital constraints that can account for these facts. We find that the density of sectoral connections has

a differential effect on volatility and the magnitude of macroeconomic downturns depending on the flexibility in production – elasticity of substitution between materials and labor – and the type of frictions – constraints in the use of labor or material inputs – faced by firms. The model implies that in developed economies, where technologies are more flexible, the density of connections reduces aggregate volatility. In addition, for the model to deliver the observed negative correlation between density and downturns in developed economies, it has to be the case that during downturns firms in developed economies have trouble financing labor. In this configuration of the model, a dense network reduces the extent of financial cascades, in which a negative shock to one sector drives other connected sectors to become constrained. In emerging economies, where technologies tend to be less flexible, denser sectoral connections increase volatility and the likelihood of financial cascades when firms have trouble financing intermediate inputs during downturns.

Chapter 2: Flexibility and Frictions in Multisector Models

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Abstract

We establish correlations between "flexibility" of production (in terms of the elasticity of substitution between intermediates and labor-capital) and sectoral bond premia. Bond premia are countercyclical. During times of high corporate debt – when is more likely that firms face financial problems – manufacturing sectors with flexibility between intermediates and labor-capital pay lower premia, while service sectors with flexibility between intermediates and value added pay higher premia. On the other hand, during recessions – when most firms face financial problems – high elasticity firms pay lower bond premia. We use these facts to guide construction of a multisector model with occasionally-binding working capital constraints; the facts reveal that high elasticity service sectors appear to have difficulty financing labor and capital during downturns while low elasticity manufacturing sectors have difficulty financing intermediates. The model and the evidence also suggest that service sector firms face much tighter credit conditions during recessions.

2.1 Introduction

The standard narrative of the Great Recession is one where financial frictions and interconnected sectors translated a small shock to a relatively unimportant sector – often argued to be an unexpectedly-large number of subprime mortgage defaults – into a large economy-wide decline in economic activity. Clearly, sectoral linkages are critical for this narrative; recent work has shown the potential for sectoral productivity shocks to be amplified and propagated strongly when intermediate inputs generate tight connections between sectors (Foerster et al. (2011); Atalay (2014). Bigio and La'O (2016) show how financial shocks can be transmitted through the network in the presence of working capital financing constraints, leading to declines in measured aggregate TFP and aggregate labor wedges.

To date, however, this literature has neglected significant sectoral heterogeneity along two dimensions. First, the elasticity of substitution in sectoral production functions differs significantly across sectors, both in terms of the elasticity of substitution between value added (labor

and capital) and intermediates, which we denote ε_Q , as well as between intermediates ε_M ; indeed, with the exception of Atalay (2014), these elasticities are generally constrained not only to be equal but also equal to one. Atalay (2014) finds that both elasticities are significantly different than one, but does not permit them to vary across sectors. We find substantial heterogeneity in both elasticities, with large differences in particular between manufacturing and service sectors, and a positive but low correlation between the elasticities for a given sector. We find that manufacturing sectors have lower elasticities than service sectors in general, that elasticities between value added and intermediates are larger than those within intermediates, and that service sectors have an average elasticity of substitution between value added and intermediates that exceeds one.

Second, the spread that sectors pay to borrow (over the T-bill rate) also varies substantially across sectors. Using the bond premium data from Gilchrist and Zakrajšek (2012), we find two key facts. First, spreads are countercyclical for most sectors, in particular aggregates for manufacturing and services (see Figure 21). Second, spreads are systematically related to our estimated elasticities. We find that, during expansions, there is no evidence that either flexible (high elasticity) or inflexible (low elasticity) sectors pay higher premia, no matter which elasticity we consider. During recessions, however, things change – sectors that have a relatively difficult time substituting between value added and intermediates (low ε_Q) will pay higher premia, which is probably not surprising to most readers, but sectors that have a relatively difficult time substituting between different intermediates (low ε_M) actually pay lower premia. We also find that when corporate debt is high service sector firms with high elasticity pay higher premia, while manufacturing firms with higher elasticity pay lower premia.

We use these results to guide the construction of a model built on Bigio and La'O (2016). We allow for sectors to have heterogeneous values for ε_Q and ε_M , and we explore different assumptions about which inputs are subject to the working capital requirement. To facilitate intuition, we first study a very simple two-sector model. Sector one uses only labor to produce an intermediate good for use in sector two. Sector two combines intermediates from both sectors with labor to produce a good that can be consumed or used as an intermediate in sector two. We consider environments where the firms in sector two are required to finance one or both types of inputs in advance of production via working capital loans, and these loans must

be collateralized by sales.³⁶

When the production function is Cobb-Douglas between material inputs ($\varepsilon_M = 1$), we can analytically characterize the Lagrange multiplier as a function of sectoral productivity levels, in the sense that we can sign the derivative with respect to ε_Q . This multiplier can be viewed as the shadow price of borrowing, which in turn therefore can be interpreted as a spread.³⁷ Our first result characterizes a key multiplicative "wedge" between the costs of labor and intermediates that depends on (i) the share of labor in the production function and (ii) the fraction of sales that can be credibly pledged as collateral. If this wedge exceeds one, then the particular input is more costly (in shadow units). We can then show that when the wedge is smaller than one, sectors that must finance intermediates in advance will be more frequently constrained during recessions if they are a low ε_Q sector, provided $\varepsilon_Q < 1$; that is, among the low elasticity sectors the less elastic are constrained more often. If, in addition, the price of intermediates is high, these sectors will also be "more constrained" in the sense of facing a higher shadow cost conditional on being constrained. In contrast, sectors that must finance labor in advance will be more frequently constrained during recessions if they are a high ε_Q sector provided $\varepsilon_Q > 1$; that is, among the high elasticity sectors the more elastic are constrained more often. If, in addition, the price of labor is high, these sectors will also be more constrained.

While these results are consistent with our empirical work, the assumption that $\varepsilon_M = 1$ is definitely not. We then study cases where both elasticities are free to deviate from one. We have been unable to find analytical results for these cases, but numerical examples have led us to conclude the following statements are representative of the behavior of our model. The model can reproduce all three facts – spreads can be countercyclical and systematically related to elasticities. To match these facts, we need high elasticity service sectors (where $\varepsilon_Q > 1$) to face working capital requirements that burden the purchase of value-added inputs, while low elasticity manufacturing sectors (where $\varepsilon_Q < 1$) must face constraints on their purchases of intermediates. We also find that to conciliate our facts with the data it has to be the case that in during the Great Recession firms face tighter credit conditions.

Some additional evidence in favor of our model comes from studying the costs of intermediates for manufacturing and labor for services. We find that manufacturing costs for interme-

³⁶Formally this arrangement is quite similar to 'Sudden Stop' models with flow constraints, as in Bianchi (2011) or Benigno *et al.* (2013).

³⁷Bigio and La'O (2016) derive an equivalence in their appendix.

diates are countercyclical and labor costs for services are procyclical, which fits our narrative. During the Great Recession it also appears that service and manufacturing firms experienced a distortion in the use of inputs consistent with our models' predictions.

2.2 Facts

Our goal in this section is to lay out the connections between the spread that a sector pays (the median, specifically) and the elasticities of substitution in production for that sector.

2.2.1 Estimation of Elasticity

Suppose that sectoral production uses an aggregate of capital and labor (value added V_j) and an aggregate of intermediates (material input M_j) to produce a final good:

$$Q_j = Z_j \left(a_j^{\frac{1}{\varepsilon_{Q,j}}} V_j^{\frac{\varepsilon_{Q,j}-1}{\varepsilon_{Q,j}}} + (1 - a_j)^{\frac{1}{\varepsilon_{Q,j}}} M_j^{\frac{\varepsilon_{Q,j}-1}{\varepsilon_{Q,j}}} \right)^{\frac{\varepsilon_{Q,j}}{\varepsilon_{Q,j}-1}}$$

where $\varepsilon_{Q,j}$ is the elasticity of substitution and is sector-specific. The material input bundle M_j is constructed using intermediates from other sectors:

$$M_j = \left(\sum_{i=1}^J \omega_{ij}^{\frac{1}{\varepsilon_{M,j}}} M_{ij}^{\frac{\varepsilon_{M,j}-1}{\varepsilon_{M,j}}} \right)^{\frac{\varepsilon_{M,j}}{\varepsilon_{M,j}-1}}$$

where $\varepsilon_{M,j}$ is the elasticity of substitution between different material inputs.

To estimate the elasticities we follow Atalay (2014), but we allow for the elasticities to differ across sectors. We use the BEA annual Input-Output data for the period 1997-2014.³⁸ The firms' cost minimizing conditions in competitive markets imply the following empirical specification:

$$\Delta \log \left(\frac{P_{it} M_{ijt}}{P_{jt} Q_{jt}} \right) = \sum_{j=1}^N \alpha_j D_j \Delta \log \left(\frac{P_{jt}^M}{P_{it}} \right) + \sum_{j=1}^N \beta_j D_j \Delta \log \left(\frac{P_{jt}}{P_{jt}^M} \right) + \eta_{ijt}, \quad (17)$$

where D_j are sectoral dummies, P_{it} and P_{jt} are sectoral output prices, and P_{jt}^M is the price of the sector j intermediate bundle. We also include buyer-seller, sector, and time fixed effects. We

³⁸For each sector we keep the top 20 intermediate goods' supplier sectors.

can obtain the elasticities as

$$\begin{aligned}\varepsilon_{Q,j} &= 1 + \beta_j \\ \varepsilon_{M,j} &= 1 + \alpha_j.\end{aligned}$$

As noted in Atalay (2014), there is a potential endogeneity problem in that relative prices will be correlated with unobserved sectoral productivity in η_{ijt} . We consider the instrument used in Acemoglu et al. (2015a) and Atalay (2014), namely sectoral military spending. Military spending is thought as an exogenous demand shifter. Higher military spending in sector j (direct and indirect through expenses on sector j 's intermediate input users) increases demand for sector j 's output and therefore increases its price. The assumption is that military spending is orthogonal to changes in sectoral productivity. We also use the lag of relative prices in the right hand side of equation 1. The assumption that changes in unobserved productivity at period t are orthogonal to the changes in relative prices at $t - 1$ is sensitive to the specific stochastic process followed by changes in productivity.

Table 18 reports the estimates assuming common elasticities across sectors. Column (1) shows that both elasticities are below one when using OLS. Using the instruments in columns (2) and (3) yields an estimate for ε_Q above one and an estimate for ε_M that is below one and even negative with military spending as an instrument. The military spending instrument is weak as expressed by a very low F test value.

Now, we proceed to estimate the sectoral elasticities. The instruments are weaker when disaggregating, therefore we estimate equation 1 using OLS. As discussed at the end of the article, Monte-Carlo experiments suggest that the endogeneity critically affects the level of the elasticities but not necessarily the rank of sectors, which is the key element of the paper.

We plot the sectoral elasticities in Figure (20); the average elasticities are $\bar{\varepsilon}_Q = 1.15$ and $\bar{\varepsilon}_M = 0.5$. Elasticities are positively but weakly correlated across sectors, so that it is not the case that a sector with flexibility in substituting different intermediates also can easily substitute between intermediates and value added. We also find that there is substantial difference between the elasticities in manufacturing and service sectors – the average elasticities for manufacturing are $\bar{\varepsilon}_Q = 0.8$ and $\bar{\varepsilon}_M = 0.31$ while those for services are $\bar{\varepsilon}_Q = 1.2$ and $\bar{\varepsilon}_M = 0.8$. These results are consistent with previous evidence in Baccianti (2013) and Koesler and M. (2015).

The authors find that service sectors are relatively more flexible in substituting value-added and intermediates. In particular, Koesler and M. (2015) estimate CES production functions – building on non-linear least squares – find that most service sectors have an ε_Q above one while all the manufacturing sectors have an ε_Q smaller than one.

Note that we are forced to drop all sectors with negative estimated elasticities; this requirement causes us to lose a large number of sectors. As a result, we end up with roughly the same number of sectors as in Atalay (2014), despite starting with many more.

We proceed to study the relationship between sectoral elasticities and the cost of borrowing. The proxy for firms' cost of borrowing is the spread on corporate bonds. As pointed out by Gilchrist and Zakrajsek (2012), this measure contains information about aggregate credit conditions (supply) and firm level default risk (demand). In any case, when firms are financially constrained one would expect an upward sloping debt schedule. The purpose of the next section is to analyze whether sectors with different technologies have systematically different cost of borrowing.

2.2.2 Flexibility and Spreads

Sectoral bond spreads are obtained from Gilchrist and Zakrajšek (2012) (GZ), defined as the median spread of all firms in sector j at time t ; the data cover the period 1973Q1-2016Q1. The GZ credit spread measures for each non-financial firm the arithmetic average of the difference between firm i bond yield and a hypothetical Treasury security of the same maturity, for all the unsecured bonds issued by firm i at quarter t . The average maturity of the corporate bonds in Gilchrist and Zakrajsek (2012) is 13 years. However, because of the cash flows generated by coupon payment, the average duration of these bonds is considerably shorter.

To study the relationship between firms' elasticities of substitution and spreads, we use COMPUSTAT data to control for sectoral covariates such as: sales, the value of tangible assets, leverage (total debt divided by sales), the value of property and plants, inventories, and working capital as a fraction of sales; all the controls are plausible reasons unconnected to the elasticity that a sector might pay a higher premium at a given point in time. Given that sectoral elasticities are assumed to be constant overtime, our identification relies on interacting the elasticities with time-varying variables. In this case, we are interested in how sectoral spreads differ in recessions or in times of high corporate debt for firms with different elasticities of substitution.

Thus, we interact the elasticities with a recession dummy and with the ratio sectoral debt over sales. we run the regression

$$r_{jt} = \alpha_j + \beta_1 D_R + \beta_2 L_{jt} + \beta_3 \varepsilon_{Qj} D_{Rt} + \beta_4 \varepsilon_{Mj} D_{Rt} + \beta_5 \varepsilon_{Qj} L_{jt} + \beta_6 \varepsilon_{Mj} L_{jt} + \gamma X_{jt} + v_{jt}, \quad (18)$$

where r_{jt} is the median credit spread for sector j in quarter t , D_{Rt} is a recession dummy (or a banking crisis dummy), L_{jt} is leverage measured by total debt divided by sales, and X_{jt} is the vector of controls from before. All our specifications include year and sectors' fixed effects.³⁹ Our result suffer from the generated regressor problem. At the end of this section we use bootstrap and show that the main results are robust to this issue.

In Table 19 we present our estimates (errors are clustered at the sector level); when all sectors all pulled together we do not observe a clear pattern. Using the amount of sectoral debt as a proxy for how leveraged firms are, we find interesting sectoral heterogeneity. For the group of low elasticity sectors, having high ε_Q leads to low spreads during times of high debt. Completely different result are observed for the group of service (high elasticity) sectors where high ε_Q are associated with high spreads in times of high debt instead. All these coefficient are significant at least at the 99% of confidence. We also find that high elasticity service firms paid lower premia.⁴⁰

We supplement our results using firm level data. We merge COMPUSTAT balance sheet data of public U.S. firms with TRACE data on corporate bond spreads. Accessing firms level data allows us to control for unobserved firm level characteristics that vary within a sector. Our data covers the period 2002Q2-2015Q4 and contains mainly manufacturing firms. The firms' spread is the median of the difference between the firm j bond yield and the 5 years Treasury bill yield for all bonds issued by firm j at time t . Our identification relies entirely on the Great Recession as our recession dummy variable and firms' debt over sales only capture the large shock of the Great Recession. Table 20 depicts the results (errors clustered at the sector level). Having a high elasticity ε_Q leads to low spreads during a recession, while having high ε_Q and ε_M leads to low spreads during times of high corporate debt. These results are similar to the sectoral

³⁹We include year instead of year-quarter fixed effect to better capture the effect of crisis on spreads. Our results regarding the elasticities are the same with either time fixed effect dummies.

⁴⁰If we restrict our sample to years 2001-2016, so the Great Recession is our main identification, we also find that manufacturing firms with high elasticity paid lower premia

results for manufacturing firms, which makes sense given that our firm level data contain mainly firms in manufacturing industries and the Great Recession is our only recession.

As a last point, we plot the bond yields for manufacturing and services over time overlaid with the NBER recession dates (Figure 21); in general, yields rise during recessions (they are countercyclical). A similar and even clearer pattern can be seen for corporate debt over sales in Figure 22. In every NBER recession, corporate debt over sales spikes.

To address the generated regressor problem we use the asymptotic distribution of the estimators of $\hat{\epsilon}_Q$ and $\hat{\epsilon}_M$, for each sector. Based on this distribution we draw $M = 500$ sequences of sectoral elasticities. We then run the spread equation M times for the group of high ϵ_Q and low ϵ_Q and obtain the confidence intervals for our coefficients. We find that the significance of β_3 and β_5 for the two samples is robust to this correction. However, the results for β_4 and β_6 are not significant. Thus, with confidence we can claim that during periods of high corporate debt sectors with high elasticity ϵ_Q – within the group of flexible sectors – pay higher spreads. On the other hand, during times of high debt sectors with high elasticity ϵ_Q – within the group of inflexible sectors – pay lower spreads. Also, during recessions firms with high ϵ_Q pay lower spread.⁴¹

2.3 A Multisector Model

In this section we provide some intuition for how we use the empirical facts to guide development of a quantitative model. We suppose there are only two sectors – the first sector produces using only labor, and the second sector produces using labor and intermediates from both sectors:

$$Q_1 = Z_1 L_1$$

$$Q_2 = Z_2 \left(a_2^{\frac{1}{\epsilon_{Q,2}}} L_2^{\frac{\epsilon_{Q,2}-1}{\epsilon_{Q,2}}} + (1 - a_2)^{\frac{1}{\epsilon_{Q,2}}} \left(\omega_{12}^{\frac{1}{\epsilon_{M,2}}} M_{12}^{\frac{\epsilon_{M,2}-1}{\epsilon_{M,2}}} + \omega_{22}^{\frac{1}{\epsilon_{M,2}}} M_{22}^{\frac{\epsilon_{M,2}-1}{\epsilon_{M,2}}} \right)^{\frac{\epsilon_{M,2}}{\epsilon_{M,2}-1} \frac{\epsilon_{Q,2}-1}{\epsilon_{Q,2}}} \right)^{\frac{\epsilon_{Q,2}}{\epsilon_{Q,2}-1}}.$$

⁴¹We find that within the percentiles 5 and 95, our estimates preserve the sign of the point estimate. Alternatively, we compute the standard deviation of the simulated β_k and re-calculate the t-statistics. The results are similar.

We suppose that each sector faces a collateral constraint on working capital:

$$wL_1 \leq \eta_1 p_1 Q_1$$

$$\theta_2^w wL_2 + \theta_{12}^m M_{12} + \theta_{22}^m p_2 M_{22} \leq \eta_2 p_2 Q_2.$$

The representative household maximizes

$$U(C, L) = \frac{C^{1-\sigma} - 1}{1-\sigma} - \psi \frac{L^{1+\xi}}{1+\xi}$$

subject to the budget constraint

$$wL + \Pi \geq PC + T.$$

In equilibrium, labor market clearing requires

$$L = L_1 + L_2,$$

and goods market clearing requires

$$M_{12} = Q_1$$

$$C + M_{22} = Q_2.$$

Note that, for simplicity of the resulting algebra, the output of sector one is not consumed. Adding capital would not change our results if value added is produced using a Cobb-Douglas aggregate of capital and labor, so again for ease of presentation we simply ignore it.

We develop intuition through a series of special cases. We vary the values of the elasticities and examine the relationship between the Lagrange multiplier μ_2 on the collateral constraint for sector 2 and the elasticity of interest. We set $\psi = 0$, $\sigma = 1$, and normalize $w = 1$. The total labor endowment is normalized to 1, and we set $\theta_{22}^m = 0$. Some configurations of parameters yield uninteresting results. For example, if we set $\varepsilon_Q = \varepsilon_M = 1$, we get essentially Bigio and La'O (2016), where firms are either permanently constrained or unconstrained (since shares of inputs are constant, the firm can either collateralize enough sales to make its required prepayments or it cannot, independent of productivity); it turns out that this argument goes through for $\varepsilon_Q = 1$ independent of ε_M , so we ignore both of these cases and focus on cases where $\varepsilon_Q \neq 1$.

Abandoning Cobb-Douglas means we can study (i) which sectors will become constrained more frequently and (ii) when sectors become constrained which sectors are "more constrained". In contrast, under Cobb-Douglas assumptions sectors are either constrained or unconstrained if we maintain the assumption of constant returns to scale (for example, a sector is constrained if $\eta < 1$ and $\theta = 1$ since the left-hand-side of the collateral constraint equals revenue at the unconstrained profit-maximizing point). To deal with this problem while maintaining Cobb-Douglas production functions, Bigio and La'O (2016) assume sector-specific decreasing returns to scale; this assumption seems difficult to calibrate empirically, and their strategy is indirect.

2.3.1 Case 1: $\varepsilon_Q \neq 1$ and $\varepsilon_M = 1$

Proposition 4 Suppose $\mu_2 > 0$. Then, if $\theta_{12}^m = 1$ and $\theta_2^w = 0$, we have $\frac{\partial \mu_2}{\partial \varepsilon_Q} < 0$ if $p_1 \phi_m > 1$ and $\frac{\partial \mu_2}{\partial \varepsilon_Q} > 0$ if $p_1 \phi_m < 1$, where

$$\phi_m = \frac{(1 - \eta_2)(1 - a_2)}{\eta_2 a_2}.$$

Also, if $\theta_{12}^m = 0$ and $\theta_2^w = 1$, we have $\frac{\partial \mu_2}{\partial \varepsilon_Q} < 0$ if $\frac{\phi_w}{p_1} > 1$ and $\frac{\partial \mu_2}{\partial \varepsilon_Q} > 0$ if $\frac{\phi_w}{p_1} < 1$, where

$$\phi_w = \frac{(1 - \eta_2) a_2}{(1 - a_2) \eta_2}.$$

Proof. First set $\theta_2^w = 0$ and $\theta_{12}^m = 1$, which implies $L_2 = 1 - \eta_2$ and $Q_1 = Z_1 \eta_2$. Using the production function for sector 2 and the first-order condition for L_2 we obtain

$$\mu_2 = \left(\frac{(1 - \eta_2)(1 - a_2)}{a_2 \eta_2} \right)^{1 - \rho_Q} Z_1^{\rho_Q} - 1,$$

where $\rho_Q = (\varepsilon_Q - 1) / \varepsilon_Q$. Therefore,

$$\frac{\partial \mu_2}{\partial \varepsilon_Q} = \frac{1}{\varepsilon_Q^2} Z_1^{\rho_Q} \phi_m^{1 - \rho_Q} \ln \left(\frac{Z_1}{\phi_m} \right).$$

If $\frac{Z_1}{\phi_m} = \frac{1}{p_1 \phi_m} < 1$ the derivative is negative, otherwise it is positive. Now set $\theta_2^w = 1$ and $\theta_{12}^m = 0$, which implies $L_2 = \eta_2$ and $Q_1 = Z_1 (1 - \eta_2)$. Again using the production function and the first-order condition for L_2 we obtain

$$\mu_2 = \left(\frac{(1 - \eta_2) a_2}{\eta_2 (1 - a_2)} \right)^{1 - \rho_Q} Z_1^{-\rho_Q} - 1,$$

which implies

$$\frac{\partial \mu_2}{\partial \varepsilon_Q} = -\frac{1}{\varepsilon_Q^2} Z_1^{-\rho_Q} \phi_w^{1-\rho_Q} \ln(Z_1 \phi_w);$$

if $Z_1 \phi_w = \frac{1}{\bar{p}_1} \phi_w > 1$ the derivative is negative, otherwise it is positive. ■

The terms $p_1 \phi_m$ and p_1/ϕ_w can be interpreted as friction-adjusted prices of intermediates relative to labor. If the constraint affects only intermediate purchases ($\theta_{12}^m = 1$ and $\theta_2^w = 0$), when intermediates are relatively expensive ($p_1 \phi_m > 1$) there is a negative relationship between the elasticity and the shadow price. In this case, low ε_Q firms will face a high shadow cost for working capital, as such firms will be unable to easily shift their production away from intermediates. However, high ε_Q firms will face low shadow cost of intermediates, as those firms can easily shift towards more (unconstrained) labor.

On the other hand, if intermediates are relatively cheap ($p_1 \phi_m < 1$) there is a positive relationship between the elasticity and the shadow cost of working capital. In this case, more flexible firms will choose a higher intermediate share, and therefore face a higher shadow price. Nevertheless, inflexible firms will reduce their use of intermediates, and therefore reduce their shadow cost of working capital. Similar intuition applies to the second part of Proposition 4.

The next proposition describes the extensive margin of sectoral frictions. This is, how likely is that unconstrained firms of different elasticities become constrained.

Proposition 5 *Let Z_1^* denote the threshold productivity in sector 1 that results in sector 2 being constrained. Then, if $\theta_{12}^m = 1$ and $\theta_2^w = 0$, we have $\frac{\partial Z_1^*}{\partial \varepsilon_Q} < 0$ if $\phi_m < 1$ and $\frac{\partial Z_1^*}{\partial \varepsilon_Q} > 0$ if $\phi_m > 1$. Also, if $\theta_{12}^m = 0$ and $\theta_2^w = 1$, we have $\frac{\partial Z_1^*}{\partial \varepsilon_Q} < 0$ if $\phi_w > 1$ and $\frac{\partial Z_1^*}{\partial \varepsilon_Q} > 0$ if $\phi_w < 1$.*

Proof. Using the solution for μ_2 from Proposition 2, when $\theta_{12}^m = 1$ and $\theta_2^w = 0$, we have

$$Z_1^* = \phi_m^{\frac{1}{1-\varepsilon_Q}},$$

so that

$$\frac{\partial Z_1^*}{\partial \varepsilon_Q} = \phi_m^{\frac{1}{1-\varepsilon_Q}} \frac{1}{(1-\varepsilon_Q)^2} \ln(\phi_m).$$

The sign depends on whether ε_Q and ϕ_m are larger or smaller than 1. On the other hand, when if $\theta_{12}^m = 0$ and $\theta_2^w = 1$, we have

$$Z_1^* = \phi_w^{\frac{1}{\varepsilon_Q-1}},$$

so that

$$\frac{\partial Z_1^*}{\partial \varepsilon_Q} = -\phi_w^{\frac{1}{\varepsilon_Q-1}} \frac{1}{(\varepsilon_Q-1)^2} \ln(\phi_w).$$

The sign also depends on whether ε_Q and ϕ_w are smaller or larger than 1. ■

The interpretation of Proposition 5 depends on the values of ϕ_m and ϕ_w , and whether labor and intermediates are complementary inputs ($\varepsilon_Q < 1$) or substitute inputs ($\varepsilon_Q > 1$). For example, if $\phi_m < 1$, it holds that $(1 - \eta_2)(1 - a_2) < a_2\eta_2$, implying that for a given average labor share a_2 , the firm can use a larger fraction, η_2 , of sales as collateral. In this case, if the constraint affects the intermediate input and $\varepsilon < 1$, relatively inelastic firms become constrained with a higher Z_1 , as suggested by $\frac{\partial Z_1^*}{\partial \varepsilon_Q} < 0$. Under this parameter values, it is also the case that increases in the intermediate input price will increase the intermediates cost share. Therefore, a relatively more inelastic firm, is more likely to become constrained when facing an increase in the price of intermediates (Figure (23)).

For the case when the constraint affects the financing of labor, if $\phi_w < 1$ and $\varepsilon_Q > 1$, relatively more elastic firms become constrained with a smaller increase in the intermediate bundle price (Figure (24)). A higher intermediate bundle price increases the labor cost share, raising the likelihood of a firm to become financially constrained. For a given increase in the intermediate price, the increase in labor cost share is larger for more elastic firms, therefore in events of high intermediate price is more likely to observe more elastic firms being financially constrained.

2.3.2 Case 2: $\varepsilon_Q \neq 1$ and $\varepsilon_M \neq 1$

For this case we have been unable to construct a special case amenable to analysis, so we proceed using numerical examples. Figures (25) - (27) plot the Lagrange multiplier for sector 2 as a function of the two sectoral productivity levels under a variety of different assumptions about (i) the value of the two elasticities and (ii) the value of θ_{12}^m and θ_2^w . When $\varepsilon_Q < 1$, $\theta_{12}^m < 1$ and $\theta_2^w < 1$ the working capital constraint only affects intermediates, a larger ε_M reduces the constrained region. However, when $\varepsilon_Q > 1$ and the working capital constraint affects the use of labor, a larger ε_M increases the constrained region. Hence, a non-unitary of elasticity between intermediates reinforces the results in Proposition 5. It is important to note that the model displays constraints that bind in good times (high Z_1) and bad times (low Z_1). Given that our

empirical results suggest countercyclical spreads, we are interested in the case in which firms become constrained at low Z_1 .

2.3.3 Discussion: Model vs. Data

The propositions above identify the connection between the elasticities in production (flexibility) and the multiplier on the working capital constraint, both in terms of the extensive and intensive margins. Based on Proposition 5 (extensive margin), when intermediate prices are high, a negative correlation between ε_Q and the spread is consistent with a working capital requirement for intermediates; in contrast, a positive correlation would indicate that labor is subject to the working capital requirement. Figure (28) plots the relative price of intermediates to labor for a number of manufacturing sectors; note that this cost spikes up at the beginning of NBER-dated recessions. Using our model, we would infer that spreads would rise on manufacturing firms, as the rise in the cost causes the LHS of the collateral constraint to rise while the RHS falls, leading these sectors to become constrained (or more constrained).

We also looked at the behavior of relative price of intermediates to labor for service sectors; see Figure (29). Consistent with our model, we find that labor becomes much cheaper in 2008, leading to a rise in the share of expenditures on labor. In this case, the LHS of the constraint falls less than the RHS, leading again to countercyclical spreads.

Additional evidence in Figures (30)-(31) suggests that before 2007 the ratios M/L and P^M/W co moved positively for manufacturing firms and negatively for service sectors (consistent with their elasticities). However, during 2008-2009 we observe a disruption in the use of inputs. Figures (30)-(31) show the average growth of the quantity index of intermediates over the number of employees for different manufacturing and service sectors. We plot this ratio against the average growth of the ratio between the intermediates price index and the average wage of sectors from the BEA. During the Great Recession in 2007-2009, quantities (horizontal axis) and prices (vertical axis) show a vertical relationship, this is, quantities did not adjust to prices as they did before 2007. This is more pronounced for service industries in Figure (31). During 2007-2009 there is no change in the ratio intermediates (M) to labor (L) while relative prices did experience changes. These results support the predictions of the model regarding distorted input choices.

We can complement our analysis with Proposition 4 (intensive margin) to learn more about

the parameters of the working capital constraint, in particular η , the fraction of sales that firms can pledge as collateral. We make use of the point estimates for ε_Q in previous sections. We find that manufacturing sectors present an average ε_Q that is smaller than 1, whereas service sectors have an average ε_Q that is bigger than 1. Thus, for the model to be consistent with the data, it has to be the case that during recessions manufacturing sectors are constrained financing intermediates and can pledge a large fraction of sales as collateral, so $\phi_m < 1$. On the other hand, to rationalize the negative correlation between ε_Q and spreads for service sectors during recessions – together with the positive correlation between ε_Q and spreads during times of high debt – it has to be the case that service sectors are constrained financing labor and during recessions experience a decline in the fraction of their sales to be pledged as collateral for debt (negative financial shock), so ϕ_w becomes larger than 1, reversing the correlation.

Finally, we study how the predictions of the model regarding the relationship between ε_M and the shadow cost of working capital line up with the empirical evidence. Table 21 contains the complete set of results that summarize Propositions 4 and 5 and the numerical results when $\varepsilon_M \neq 1$. In columns (1) and (2) we focus on inflexible firms ($\varepsilon_Q < 1$). Column 1 has the models' predictions when firms have constraints in the use of materials (M), while column (2) depicts the results when firms are only constrained in the use of labor. Bold symbols indicate a countercyclical shadow cost of working capital (μ_2). The baseline results assume that firms can pledge a large fraction of working capital $\phi_m < 1$ and $\phi_w < 1$, and the results in parenthesis assume $\phi_m > 1$ and $\phi_w > 1$.

Overall, the model is able to deliver the main empirical regularities in the data. A negative correlation between elasticities (ε_Q and ε_M) and the (countercyclical) shadow cost of debt is observed when $\varepsilon_Q < 1$ and firms face frictions in the use of intermediates. This is consistent with what we observe for manufacturing firms in the data. When $\varepsilon_Q > 1$ the model delivers a positive correlation between ε_Q and the shadow cost of debt when labor is cheap and constrained, while a negative correlation between ε_Q and the shadow price when labor is constrained and firms can pledge a small fraction of sales as collateral. This implies that service sector firms are mostly constrained in the use of labor and during recessions experience much tighter credit conditions.

2.4 How important is the Endogeneity in the Estimation of ε_{Q_j} ?

As discussed earlier, the estimation of elasticities is biased due to unobserved productivity shocks that are correlated with prices and the input choice. To evaluate how this bias can affect our results, and given that our IV approach did not succeed, we use the model as a guide. We simulate series of output, prices, and input demand to estimate the same OLS regressions as in section 2.3.⁴²

The model used for the experiment is a more general version of the two sector model in section 2.3.

Firms in the intermediate good sector produce according to

$$Q_1 = Z_1 \left[a_1^{\frac{1}{\varepsilon_{Q_1}}} L_1^{\frac{\varepsilon_{Q_1}-1}{\varepsilon_{Q_1}}} + (1-a_1)^{\frac{1}{\varepsilon_{Q_1}}} M_1^{\frac{\varepsilon_{Q_1}-1}{\varepsilon_{Q_1}}} \right]^{\frac{\varepsilon_{Q_1}}{\varepsilon_{Q_1}-1}}, \quad (19)$$

where $M_1 = M_{11}^{\omega_{11}} M_{21}^{1-\omega_{11}}$.

Final good firms produce according to

$$Q_2 = Z_2 \left[a_2^{\frac{1}{\varepsilon_{Q_2}}} L_2^{\frac{\varepsilon_{Q_2}-1}{\varepsilon_{Q_2}}} + (1-a_2)^{\frac{1}{\varepsilon_{Q_2}}} M_2^{\frac{\varepsilon_{Q_2}-1}{\varepsilon_{Q_2}}} \right]^{\frac{\varepsilon_{Q_2}}{\varepsilon_{Q_2}-1}}, \quad (20)$$

where $M_2 = M_{22}^{\omega_{22}} M_{12}^{1-\omega_{22}}$.

The working capital constraints are

$$\theta_1^w w L_1 + \theta_1^m (P_1 M_{11} + P_2 M_{21}) \leq \eta_1 P_1 Q_1 \quad (21)$$

$$\theta_2^w w L_2 + \theta_2^m (P_1 M_{12} + P_2 M_{22}) \leq \eta_2 P_2 Q_2. \quad (22)$$

The market clearing conditions are

$$Q_1 = M_{11} + M_{12}, \quad (23)$$

$$Q_2 = C + M_{21} + M_{22}. \quad (24)$$

⁴²Feenstra *et al.* (2014) perform a similar exercise to study how biased are the usual estimates of consumption elasticities that enter in the definition of trade elasticities.

Households solve the same problem as in Section 2.3.

To derive the model counterpart of Equation (17) we solve the cost minimization problem

$$\begin{aligned} \mathcal{L} = & P_j^M M_j + w L_j + \lambda^1 \left(Q_j - Z_j \left[a_j^{\frac{1}{\varepsilon_{Q_j}}} L_j^{\frac{\varepsilon_{Q_j}-1}{\varepsilon_{Q_j}}} + (1-a_j)^{\frac{1}{\varepsilon_{Q_j}}} M_j^{\frac{\varepsilon_{Q_j}-1}{\varepsilon_{Q_j}}} \right]^{\frac{\varepsilon_{Q_j}}{\varepsilon_{Q_j}-1}} \right) \\ & + \lambda^2 \left(M_j - M_{jj}^{\omega_{jj}} M_{ij}^{1-\omega_{jj}} \right) + \mu_j^C \left(\eta_j P_j Q_j - \theta_j^w w L_j - \theta_j^m (P_{we} M_{ij} + P_j M_{jj}) \right). \end{aligned}$$

The first-order necessary and sufficient conditions for M_j are

$$P_j^M - \lambda \frac{\partial Q_j}{\partial M_j} + \mu_j^C \eta_j P_j \frac{\partial Q_j}{\partial M_j} - \mu_j^C \theta_j^m P_j^M = 0. \quad (25)$$

Rearranging and using the fact that in competitive markets the marginal cost of production in sector j (λ^1) is the price of good P_j , we have

$$P_j^M = Z_j^{\frac{\varepsilon_{Q_j}-1}{\varepsilon_{Q_j}}} \left(\frac{a_j Q_j}{M_j} \right)^{\frac{1}{\varepsilon_{Q_j}}} P_j \frac{(1 - \mu_j^C \eta_j)}{(1 - \mu_j^C \theta_j)}. \quad (26)$$

Let $\bar{\mu}_j = \frac{(1 - \mu_j^C \eta_j)}{(1 - \mu_j^C \theta_j)}$. Raising the previous equation to the power of ε_{Q_j} , taking logs, and rearranging we obtain

$$\log \left(\frac{P_{jt}^M M_{jt}}{P_{jt} Q_{jt}} \right) = \log a_j + (1 - \varepsilon_{Q_j}) \log \left(\frac{P_{jt}^M}{P_{jt}} \right) + (\varepsilon_{Q_j} - 1) \log Z_j + \varepsilon_{Q_j} \bar{\mu}_j. \quad (27)$$

There are two unobserved variables for the econometrician in equation 27. The level of productivity of firms in sector j , Z_j , and the Lagrange multiplier of the working capital constraint in sector j , $\bar{\mu}_j$.

For the Monte-Carlo experiment, the TFP shocks in each sector are assumed to be either *iid* standard normal or having persistence by following an AR(1) process with persistence parameter 0.9. In the first simulation we assume $\varepsilon_{Q_1} = 1$, $a_1 = 1$, $a_2 = 0.3$, and $\omega_{22} = 0$. Here we explore the effects of the bias – on what features of the environment does it depend?

In the next experiment we assume $\omega_{11} = \omega_{22} = 0.3$, $\eta_1 = \eta_2 = 1$, $a_1 = a_2 = 0.4$, $\theta_j^w = 0$, and $\theta_j^m = 1$. In this experiment one can study if the rank in terms of production flexibility is

preserved. For example, for true pairs of elasticities like $(\varepsilon_{Q_1} = 0.3, \varepsilon_{Q_2} = 0.8)$, is the OLS estimation still preserving the fact that sector 2 is more flexible?

We summarize our results for the bias as (we) there is no bias if the sector under investigation does not experience shocks (only the other sector does); (ii) when estimating only one elasticity, estimates are biased toward 1, even if constraints are not binding, and are exactly equal to 1 if constraints are always binding; (iii) estimates are biased downward when trying to estimate two elasticities if both sectors experience shocks. We conclude from these exercises that endogeneity may be an issue. However, the bias in the estimation does not alter the rank of sectors, which is the crucial element of our analysis.

2.5 Conclusion

In this paper we have provided new empirical evidence on the relationship between firms' technologies – elasticities of substitution between inputs in production – and the spreads on corporate bonds during recessions or periods of high corporate debt. We use this evidence to guide the construction of multisector models with inter-sectoral linkages through intermediates and occasionally-binding working capital constraints in the use of labor and/or intermediates. The model delivers a relationship between the elasticities in production and the shadow cost of debt (working capital). To capture the dynamics of sectoral bond premia the model needs high elasticity service sectors to be occasionally constrained in their use of labor and low elasticity manufacturing sectors to be occasionally constrained in intermediates.

The results in this paper highlight the importance of considering the non-linearities in production when modeling an economy with multiple sectors and firms trading intermediates for production. In addition, this paper provides a way to infer the size and nature of firms' frictions based on the relationship between firms cost of debt and fundamental characteristics of firms: parameters of the production function.

Chapter 3: What the Data Tells Us about Multisector DSGE Models?

Coauthored with Professor Eric Young

Abstract

We study whether a multisector model with sectoral linkages and capital accumulation is able to match the observed dynamics in the output growth of U.S. industrial sectors. We propose a model where firms rent the same type of capital from households and where the reallocation capital stock across sectors can be costly. We compare the performance of our model with existing model with sector specific capital stock. The results of comparing different models' implied VAR impulse-response and sectoral spectral densities indicate that our model with single type of physical capital and costless adjustment of capital outperforms alternative models with sector specific capital. The advantage of our model is that the single-capital model can be solved easily with many sectors and occasionally binding sectoral financial constraints.

3.1 Introduction

The contribution of idiosyncratic sectoral shocks to business cycles fluctuations is quantitatively relevant. Foerster et al. (2011) and then Atalay (2014) find that between 50% and 80% of fluctuations in aggregate output growth are due to independent sectoral shocks. This finding has motivated the use of multisector models to answer questions such as, how are financial shocks amplified by sectoral linkages? (Bigio and La'O (2016)) Or what is the role of sector specific shocks in shaping the observed macroeconomics downturns? (Acemoglu et al 2016). In this paper, we ask whether existing multi-sector DSGE models with sectoral capital stock can account for the joint distribution of sectoral output growth observed in the U.S. industrial sectors.

We test existing models' outcomes using two metrics. First, we compare the sectoral spectral densities implied by the models with those observed in the data. Second, we compare the impulse response functions and a GMM criteria resulting from estimating a VAR(1) on the models' and data series. In this exercise, we include an alternative multi-sector model in which

sectors rather rent the only capital stock in the economy. We find that none of these models do a good job fitting the time series properties of sectoral output growth and the conditional correlations across sectors. Moreover, existing models with sector specific capital stock are generally outperformed by the alternative model that considers only one aggregate capital stock in the economy.

In models with sector specific capital stock most of the variance of sectoral output growth is concentrated in high frequencies, regardless of the sector at hand. The empirical evidence suggest that for capital good sectors, e.g. machinery, the variance of output growth is concentrated in the business cycle frequencies (low-medium), as opposed to consumption goods sectors, e.g. food, where most of the variance in output growth comes from medium-high frequencies. The model with one capital stock delivers a similar pattern except that a higher portion of the variance is explained by lower frequencies.

When comparing the VAR(1) impulse response of the models and the data, the model with sector specific capital stock and investment goods linkages generally exacerbates, and in the wrong direction, the observed sectoral responses to shocks. For example, a 1% standard deviation positive shock in machinery depresses power generation output by 7%, as opposed to the data where power generation grows by a 3%. The model in Carvalho (2007) and the model with one capital stock generate a positive response of power generation of 0.3% and 1%, respectively. Consistent with this result, the GMM criteria that measures the distance between the observed VAR(1) estimates and those implied by the models, is minimized by the model with one capital stock, followed by the model with sector specific capital with no investment goods linkages.

The first paper in studying how sectoral independent shocks drive business cycle is John B. Long and Plosser (1983). The authors modeled an economy without capital stock, in which sectors use labor and (lagged) materials from other sectors in production. Then Horvath (1998), Dupor (1999), and Carvalho (2007) use a model in which sectors use materials contemporaneously and build their own capital stock, using the same sector output as an input. More recently, Foerster et al. (2011) added an additional source of linkages in the production of investment goods. The authors make use of the BEA capital flows matrix⁴³ to model an economy in which

⁴³This matrix states the expenditure of sector w on sector j output in goods that will be used in production of sector w for more than 1 year.

sector specific capital stock is built using the output from all other industries. Finally, Atalay (2014) extended the Foerster et al. (2011) model by departing from the Cobb-Douglas assumption in technology and allowing for non-unitary elasticity of substitution between inputs in the production of sectoral output and investment.

The aggregate relevance of sectoral idiosyncratic shocks in each of these models varies in important ways. The investment goods' linkages in Foerster et al. (2011) induce an important internal propagation mechanism with respect to Carvalho (2007). Simply put, in an economy absent the investment goods linkages, where each sector produces capital using its own output, a shock to machinery sector will only affect wood production via the intermediate input linkages between them and their common trade partners. On the other hand, in an economy with investment good linkages, a bad shock to machinery will not only depress wood production through materials use but also directly via capital accumulation, as machinery is one of the main investment goods providers in wood capital stock production.⁴⁴

Atalay (2014) shows that when the elasticity of substitution between inputs is less than 1, sectoral co-movements are mainly explained by sectoral shocks. In this economy, wood can not easily substitute machinery inputs with others industries inputs, therefore it has to bear the higher cost for its materials. At the same time, machinery clients will not reduce machinery demand as much as in the Cobb Douglas case. The first mechanism is quantitatively more important in Atalay (2014).

The choice of the model matters for assessing the importance of idiosyncratic sectoral shocks in aggregate fluctuations. The evidence suggests that the more realistic the model, the higher the importance of sectoral shocks in driving business cycle fluctuations. Nevertheless, when interested in studying other types of questions in multi-sector economies, one needs a model capable of accounting for the observed sectoral dynamics.

3.2 Existing Models

The most general multi-sector model is Foerster et al. (2011), which has intermediate inputs linkages and investment goods linkages. This model embeds the model in Carvalho (2007) for a particular structure of the capital flows matrix. In this economy, there are N sectors of the economy indexed by $j = 1, \dots, N$. In each sector there is a continuum of homogeneous firms

⁴⁴For a detailed discussion refer to Foerster et al. (2011) and Atalay (2014).

that engage in perfect competition. The representative firm of sector j produces Y_{jt} units of good j at time t , using capital stock K_{jt} , labor L_{jt} , and materials M_{ijt} from all other sectors, with the following constant returns to scale Cobb-Douglas production function:

$$Y_j = Z_j K_j^{\alpha_j} \prod_{we=1}^N M_{ij}^{\gamma_{ij}} L_j^{\eta_j}, \quad (28)$$

where $\eta_j = 1 - \alpha_j - \sum_{we=1}^N \gamma_{ij}$, and Z_{jt} denotes the productivity level of sector j , which follows an AR(1) process

$$\log(Z_{jt}) = \xi \log(Z_{jt-1}) + \varepsilon_{jt}. \quad (29)$$

The error term ε_{jt} is normally distributed with zero mean and standard deviation σ_ε . The variance covariance matrix for sectoral shocks is denoted by $\Sigma_{\varepsilon\varepsilon}$. A diagonal covariance matrix indicates the existence of purely idiosyncratic shocks, while no zero off diagonal terms imply that sectoral output are driven by common aggregate shocks. This matrix is key to understand the relevance of sectoral shocks in aggregate fluctuations later in the paper we will expand about its measurement.

Sectors accumulate their own capital stock according to

$$K_{jt+1} = I_{jt} + (1 - \delta)K_{jt}. \quad (30)$$

New capital is built using investment goods from all sectors,

$$I_{jt} = \prod_{we=1}^N X_{ijt}^{\theta_{ij}}. \quad (31)$$

The extra source of linkages is represented by θ_{ij} , which measures how much investment goods sector j purchases from sector we in the production of sector j capital stock. These shares define the capital flows matrix Θ .

The representative household consumes the N goods of the economy and provides labor to all the sectors. The discounted expected utility of the household takes the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^N \left(\frac{C_{j,t}^{1-\sigma} - 1}{1-\sigma} - \psi L_{jt} \right), \quad (32)$$

where $L = \sum_j^N L_{jt}$ is the total labor supply of the representative household.

The resource constraints of sector j , with $j = 1, \dots, N$, is

$$Y_{jt} = C_{jt} + \sum_{we=1}^N M_{jit} + \sum_{we=1}^N X_{jit}. \quad (33)$$

The steady state of these models as well as the linearized solution of the dynamic system of equations can be found in Foerster et al. (2011) online appendix.

3.3 An Alternative Model

In this section, we take a model with one capital stock that is owned and produced by households and then rented to firms at the market rental rate. Households buy investment goods from all sectors according the observed average capital flow vector $\tilde{\Theta}$. The general model allows for an intra-temporal adjustment cost of capital across sectors. When the representative households wants to alter the distribution of capital stock in the economy, some capital is forgone. This assumption is a reduced form for the sectoral specificity of capital stock in previous models.

The model keeps the sectoral linkages through the intermediate input purchases but maintains the one sector growth model assumption regarding the capital stock ownership. The key difference with respect to previous models is the timing in the firms' capital decision. In previous models, firms decide capital stock before the productivity shock is realized, while firms in this model rent capital after the productivity is observed.

The representative household has the same preferences in . The key difference is the law of motion of capital which is described by:

$$K_{t+1}^A = I_t + (1 - \delta)K_t^A. \quad (34)$$

The representative household builds capital stock using investment goods from different sectors with the following constant returns to scale (CRS) technology:

$$I_t = \prod_{j=1}^N X_{jt}^{\theta_j}. \quad (35)$$

This production function for capital stock captures part of the investment linkages in Foerster et al. (2011). Particularly, this technology is equivalent to a homogeneous version of

sector specific investment in Foerster et al. (2011), in which every sector uses the same share of investment goods from all other sectors.⁴⁵

The resource constraint for sectors j is:

$$Y_{jt} = C_{jt} + \sum_{we=1}^N M_{ji} + X_{jt}. \quad (36)$$

Finally, the key equation describing the adjustment cost of capital is

$$K_t^A = \left(\sum_{j=1}^N K_{jt}^{-\rho} \right)^{-\frac{1}{\rho}}. \quad (37)$$

Equation (37) can be interpreted as reverse CES technology for the redistribution of physical capital across sectors. When $\rho = -1$, capital is perfectly substitutable across different sectors. If a positive productivity shock hits the automobile industry, the representative household can freely reallocate capital from less productive industries to the production of vehicles. When $\rho < -1$, moving machines from textiles to automobile industry is costly, meaning that for households to produce 10 more motor equipments, the equipments in textile sector have to be reduced in more than 10.

As mentioned earlier, this assumption is a reduced form that aims to account for the specificity in the capital stock in previous models. It should be noted that this model does not embed previous model. When $\rho = -\infty$, the capital stock distribution is fixed in the economy, which is not the case in previous models, as over time capital stock distribution can change through sectors investment decisions.

3.3.1 Planner's Problem

We make use of the first fundamental welfare theorem to solve the planner's problem for these frictionless economies.

For existing models, the social planner chooses sequences of $\{C_{jt}\}_{j,t}$, $\{L_{jt}\}_{j,t}$, $\{K_{jt+1}\}_t$, $\{M_{ijt}\}_{we,j,t}$ such that, given a initial condition for sectoral capital stock, K_{j0} , and a sequence of sectoral productivity levels, $\{Z_{jt}\}_t$, the following holds:

- $\{C_{jt}\}_{j,t}$, $\{L_{jt}\}$ maximizes (54),

⁴⁵Foerster et al (2011) follow McGrattan and Schmitz 1999 and add maintenance costs of capital to the diagonal elements of Θ . This is more of a practical issue as small diagonal terms are problematic when solving the model.

- the sectoral resource constraints in (33) hold,

For the alternative model, the social planner chooses sequences of $\{C_{jt}\}_{j,t}$, $\{L_{jt}\}_{j,t}$, $\{K_{t+1}\}_t$, $\{k_{jt}\}_{j,t}$, $\{M_{ijt}\}_{we,j,t}$ such that, given a initial condition for aggregate capital stock K_0 and a sequence of sectoral productivity levels $\{Z_{jt}\}_t$, the following holds

- $\{C_{jt}\}_{j,t}$, $\{L_{jt}\}_{j,t}$, $\{K_{t+1}\}_t$ maximizes (54).
- the sectoral resource constraints in (36) hold.
- capital market clears, so (37) holds.

3.3.2 Models' Filter

As in Foerster et al. (2011), we use the linearized models' solutions around the deterministic steady state to derive the time series representation of the vector of sectoral output (For details see Appendix C). The vector of sectoral output growth is described by

$$Y_{t+1} = \rho Y_t + \Xi Z_t + B_z Z_{t+1}, \quad (38)$$

Where ρ , Ξ , and B_z are matrices that depend on the models parameters, policy rules, and law of motion of states.

The details about these matrices for the alternative model are in the appendix. Assuming the logarithm of sectoral shocks in (29) follows a random walk process, sectoral output growth follow a VARMA(1,1) process,

$$\Delta Y_{t+1} = \rho \Delta Y_t + \Xi \varepsilon_t + B_z \varepsilon_{t+1}. \quad (39)$$

To evaluate each model's fit, we follow Foerster et al. (2011) and back out the sequence of productivity shocks, $\{\varepsilon_{jt}\}_{t=1}^T$, that originated the observed series of sectoral output growth in the U.S. industrial sectors, according to

$$\varepsilon_{t+1} = B_z^{-1} \Delta Y_{t+1} - B_z^{-1} \rho \Delta Y_t - B_z^{-1} \Xi \varepsilon_t. \quad (40)$$

We then obtain the implied variance covariance matrix of shocks $\Sigma_{\varepsilon\varepsilon}$. The diagonal terms are the sectoral variance of sectoral shocks and the off diagonal terms represent common shocks

that affect different sectors. We use the Cholesky transformation of this matrix to simulate series of the model and study the models' fit using different metrics.

It is important to note that by construction the models' simulated series using this covariance matrix will replicate the observed covariance matrix of sectoral output growth. Nevertheless, the models' fit of others moments is not targeted. The spectral density, for example, contains information not only about the variance of sectoral output, but is also the frequency domain representation of the covariance generating function. On the other hand, the conditional correlation across sectors implied by the impulse response functions depends on the model's map of state variables to choice variables.

3.3.3 Calibration

To compare this model with previous literature, we calibrate the materials shares (γ_j), labor shares (ϕ) and capital shares (α) using the 1998 use input output table for United States. The capital flow matrix, Θ , is obtained from the BEA and is adjusted as in Foerster et al. (2011) to account for maintenance costs. The vector $\tilde{\Theta}$ aggregates sectors in one large sector using investment goods and then calculates the aggregate sector shares of investment goods. The parameter ρ in the one sector model will be estimated to minimize a GMM criteria between the model and the data VAR(1) estimates.

3.3.4 Cross-Model Comparison

Before comparing the models' performance with the data, we compare the spectral densities implied by each model. When sectoral productivities follow a random walk process, there exists a closed form solution for the spectral density function of output growth:

$$S_{\Delta Y}(\omega) = \frac{[we + \Xi e^{-i\omega}]B_z' \Omega B_z [we + \Xi e^{i\omega}]'}{2\pi [we - \rho e^{-i\omega}][we - \rho e^{i\omega}]'}. \quad (41)$$

For a given sequence ω , $S_{\Delta Y}(\omega)$ is an $N \times N$ matrix whose diagonal element (j, j) contains the contribution of frequency ω to the total variance of sector j output growth. To compare the spectral density of aggregate output growth, we follow Dupor (1999) and have,

$$S_{\Delta \bar{Y}}(\omega) = H' S_{\Delta Y}(\omega) H, \quad (42)$$

where $\bar{Y}$ is aggregate output and H is a N by 1 matrix of sectoral shares. For this exercise, we use the actual sectoral shares in the 1997 input-output table.

The results in Figure 32 suggest that the model without intra-temporal adjustment costs delivers the smallest business cycle fluctuations among the three models. Business cycle occur around frequencies of $\omega \in [0.1, 0.5]$. The model in Foerster et al. (2011) delivers a highest portion of aggregate output growth concentrated in the business cycle frequency. This is another way of presenting the results in Foerster et al. (2011), which is that their model delivers higher propagation of sectoral shocks than a model as in Carvalho (2007).

To the differences across models, we solve Foerster et al. (2011) using an average $\bar{\Theta}$ matrix, in which every sector uses the average shares in $\bar{\Theta}$ (Figure 33). Then, in Figure 34 we set $\bar{\Theta}_N$ such that each sector uses the same share $\frac{1}{N}$ from each other sector. Much of the business cycle fluctuations in Foerster et al. (2011) disappears when shutting down the heterogeneity in investments goods technology. At this point, the only difference between the Foerster et al. (2011) model and the model with one capital stock without adjustment cost is the timing in which the capital decision is made.

After understanding the main source of difference between the models, we use our model to match the spectral density in Foerster et al. (2011). We can do so by setting a value of $\rho = -1.8$ (Figure 35). We can see that the spectral density of aggregate output growth in both models is quite similar. Either the closed form solution for unit root productivity or with the non-parametric Bartlett estimate with a stationary process for productivity. Therefore, our model can be viewed as a reduced form representation of the model in Foerster et al. (2011). By increasing the intra-temporal adjustment cost of capital of reallocating physical capital, our model can deliver similar amplification of sectoral shocks.

3.4 Models and Data

In this section, we evaluate the models' performance in replicating the empirical spectral densities of sectoral output growth and the empirical impulse response functions from a VAR(1) on sectoral output growth series. For this purpose, we solve the models and simulate artificial series of output growth out of sectoral shocks. We then calculate the models' implied spectral densities and models' implied VAR impulse responses. Regarding our model we only report the results of using $\rho = -1$ as it is when the model fits the data the best.

3.4.1 Spectral Densities

There is no closed form solution for the empirical spectral densities, so we estimate it using the non-parametric Bartlett estimate.⁴⁶ At the same time, we abandon the random walk assumption for sectoral productivities and assume log TFP is stationary with $\xi = 0.99$. We assume mean reversed process for productivities for two reasons. First, we do this to compare mean reversed impulse responses. As in short samples a very persistent AR(1) process and a random walk are practically identical, we make use of the covariance matrix of errors, $\Sigma_{\varepsilon\varepsilon}$, obtained in the previous section to simulate series of the models

The Bartlett estimate is

$$\hat{S}_{\Delta y}(\omega) = (2\pi)^{-1} \left[\hat{\Sigma}_0 + \sum_{l=1}^q \left[1 - \frac{l}{q+1} \right] [\hat{\Sigma}_l e^{-i\omega k} + \hat{\Sigma}'_l e^{i\omega k}] \right], \quad (43)$$

where $\Sigma_l = \frac{1}{T} \sum_{v=0}^T (\Delta y_t - \bar{\Delta y})(\Delta y_{t-v} - \bar{\Delta y})'$, the sample covariance matrix of order k .

The choice of the bandwidth q is not straightforward, therefore we use different values ranging from 10 to 20. The ones reported are with $q = 15$, but the results do not vary substantially when q changes.

The results in Figures 36-38 suggest that the models cannot account for the large fraction of output growth variance that lies in the business cycle frequency, $\omega \in [0.1, 0.5]$ with quarterly data. The model without adjustment costs is the one that delivers higher variance of output growth in these frequencies.⁴⁷

Another results is that none of the models can account for the heterogeneity in sectoral spectral densities. The empirical evidence suggests important differences in the time series properties of capital goods sectors, e.g machinery, and consumption goods sectors, e.g food. Nevertheless, these models deliver sectoral spectral densities that are very alike.

3.4.2 Impulse Response Functions

We simulate S series of sectoral output growth of size T . Then, we average the S estimates for the VAR(1) coefficients and variance covariance matrix. Using those average estimates, we

⁴⁶We alternatively estimate a VAR(1) on the data and calculate the implied closed form spectral densities. Both approaches yield similar results.

⁴⁷When the adjustment cost parameter is -1.8, the alternative model depicts spectral densities similar to models with sector specific capital stock, consistent with the findings in the previous section for the random walk case

compute the impulse response functions of the models.

As the key difference between models lies in the capital stock accumulation and technology, we hit machinery and automobile sectors with a 1 standard deviation shock. For this exercise, we use each model's Cholesky transformation of the variance covariance matrix, $\Sigma_{\varepsilon\varepsilon}$.

The results are depicted in Figures 39-42. The data suggest that a shock to machinery has negative implications in the automobile industry. Only the Foerster et al. (2011) model fits the direction of this response. On the other hand, the response of power generation to a shock in machinery is better fit with the model with one capital stock and adjustment costs of capital. None of the models can account for the response of machinery to a shock in the motor industry. Finally, only Carvalho (2007)'s model and the one capital model without adjustment cost of capital, can account for the positive response of power generation a shock in motor industry. The model in Foerster et al. (2011) delivers impulse responses that are more sensitive and generally in the wrong direction compared to the empirical evidence.

3.4.3 Pairwise Correlations

One of the key differences between the models is the technology used to produce physical capital. We analyze the observed pairwise sectoral correlations of one of the key investment goods sectors: Machinery. In Figure 43 we plot the empirical pairwise correlations in the data together with the one implied by the model in Foerster et al. (2011) and our model with $\rho = -1$.

The horizontal axis represents each sector while the vertical axis is the pairwise correlation with Machinery. Our model follows closely the empirical pairwise correlations in the data while the models in Foerster et al. (2011) and Carvalho (2007) cannot account for the correct sign of several correlations. This is particularly true when we study the pairwise correlation between Machinery and other investment goods sectors as Computer and Peripheral Equipment; Electrical equipments and components; Motor, Aerospace, transportation equipment; Furniture; Medical Equipments and supplies (sectors 20-24).

3.4.4 GMM criteria

Finally, we define a GMM criteria M as

$$M = (\tilde{\beta}(m_i) - \hat{\beta})' W (\tilde{\beta}(m_i) - \hat{\beta}), \quad (44)$$

where $\tilde{\beta}(m_i)$ is the vector of VAR(1) estimates implied from model we, and $\tilde{\beta}$ are the VAR(1) estimates from the data. To capture how well the models fit the implied impulse responses, we include in β the Cholesky variance covariance estimates implied by the VAR(1) for the models and the data. The weighting matrix W , is the inverse of the asymptotic covariance matrix of the VAR(1) estimates using the data. For the Cholesky estimates, the weighting matrix is the identity matrix.

Across models, Table 24 shows that the model without intra-temporal adjustment cost ($\rho = -1$) is the one with the lowest GMM criteria. This result confirms the result in the previous section as the model in Foerster et al. (2011) cannot account for the conditional correlations across sectors.

3.5 Conclusion

In this paper we extended the model in previous sections to allow for physical capital accumulation. Our model has a single type of capital stock that firms in different sectors rent from the households. We show that by adding an intra-temporal adjustment cost of relocating sectoral capital, our model can deliver an amplification of sectoral shocks that is similar to the one in existing models with sector-specific capital.

We then asked how well our model and existing models can account for the dynamics in sectoral output growth in the U.S. industrial sectors. We find that, under different metrics, our model with single type of physical capital outperforms existing multisector model with sector-specific capital stock. Hence, besides being able to match the data better, our model – with reduced state-space – can be solved with sectoral working capital constraints as in previous Chapters.

Chapter 4: Optimal Policy in Multisector Models with Financial Frictions

Abstract

We study the normative implications of a model economy with sectoral linkages and financial frictions. We characterize a network pecuniary externality in which firms do not internalize how they affect other sectors' ability to borrow. In a model without capital accumulation, when sectors are constrained the optimal policy reallocates sectoral inputs, through sectoral input subsidies or taxes, to influence the financial constraints. A model with capital accumulation features an additional trade-off between prudential policies that reduce the probability of key sectors becoming constrained and bailing out sectors when they become constrained.

4.1 Introduction

The microeconomic origins of aggregate fluctuations, from granularity to networks, are a new, but widely accepted, phenomenon in macroeconomics. It has been shown, theoretically (Horvath (1998), Gabaix (2011), Acemoglu et al. (2012), Baqaee (2015)) and empirically (Foerster et al. (2011) and Atalay (2014)), that disruptions in the production of a firm or industry can have significant aggregate effects by means of sectoral linkages and firms' size.

Despite this new macroeconomic consensus, there are no policy lessons to be drawn from this literature. For example, was it optimal to bailout the U.S. automobile industry during the Great Recession? Or which sectors should we monitor more closely to avoid an episode similar to the housing industry contagion in 2007? In this paper, we fill this gap by studying the normative aspects of multisector economies with financial frictions as in Bigio and La'O (2016) and Miranda-Pinto and Young (2016).

In the presence of sectoral borrowing constraints, there exist network pecuniary externalities at work that open the scope for policy intervention. Firms in the wholesale trade sector do not internalize that their decisions affect the equilibrium price of its intermediate-input trade partners, e.g, food and beverage stores. Thus, if frictions in the use of input affect firms in food and

beverage store sectors, firms in the wholesale trade sector could increase production and supply more and cheaper inputs to relax the constraints of firms in food and beverage store sectors. However, firms in whole sale sector only maximize their profit and do not internalize that they could raise aggregate GDP — thus the households’ welfare – by adjusting their decisions.

A benevolent planner that maximizes the households’ utility and faces the same frictions (constrained planner problem) does internalize the network externality and reallocate sectoral inputs to increase the collateral value (sectors’ sales) of the constrained firms. In a model with capital accumulation – where households build capital stock using a portfolio of investment goods from different sectors – there are additional externalities. The households’ do not internalize that by increasing today’s investment in sector j can relax the constraint in sector j tomorrow. Also, households do not internalize that by increasing total investment today they reduce tomorrow’s cost of capital which relaxes tomorrow’s constraints of constrained firms.

This is the first study in addressing the normative implications in economies with sectoral linkages and financial frictions at the business cycle frequency.⁴⁸ Luo (2015) studies an economy with intermediate inputs and credit linkages where firms face financial frictions in financing inputs. The author investigates which sectoral credit policies deliver higher aggregate benefits.⁴⁹ However, the author does not characterize the externalities at place and do not study the optimal policy.

The importance of monitoring and eventually bailing out some sectors of the economy has been widely discussed in the policy environment, especially after the Global Financial Crisis. For example, the Bank of England has implemented capital requirements on banks that vary depending on which sectors of the economy are the main clients of the financial institution. In the U.S, one of the most discussed sectoral interventions was the bail-out of the automobile industry, in particular Chrysler company, during the Great Recession. Studying how optimal

⁴⁸Contemporaneous to this paper, Liu (2017) studies the normative implications of a multisector economy with financial frictions with emphasis on sectoral development policies. Even though the model and the externalities studied are similar, the results in this paper add an additional dimension in understanding sectoral policies at the business cycle frequency. While Liu (2017) finds that the largest sector of the economy is the one with the largest externality, we show that with occasionally-binding sectoral constraints and non-unitary elasticities of substitution in production, a purely network measure called *harmonic distance* determines what is the sector with the largest externality. This sectors is the one that drives the largest number of sectors to become constrained.

⁴⁹Luo (2015) studies the role of financial shocks. Given that financial shocks propagate mainly from downstream sectors to upstream sectors, she finds that in the presence of financial frictions (activated by financial shocks) relaxing the constraint of downstream sectors raises GDP the most. Nevertheless, the analysis in the paper is no on how can reallocate resources given a constraint but it asks if we could eliminate a sectoral constraint which sector we would choose.

it is to bail-out one firm is beyond the scope of this paper, although we provide a theoretical framework to study the optimality of bailing out the whole automobile industry in 2007-2008.

We contribute to the literature of pecuniary externalities and sudden stops in Bianchi (2011), Benigno et al. (2013), Itskhoki and Moll (2015), and Bianchi (2015) by characterizing a network pecuniary externality and studying optimal sectoral policy. Itskhoki and Moll (2015) study optimal sectoral development policies in a model economy with financial frictions but without input-output linkages. We instead analyze sectoral business cycle stabilization policies by explicitly studying how the structure of the economy interacts with the sectors' frictions.

4.2 A Network Externality

In this section we characterize the network pecuniary externality that arises due to credit frictions in financing inputs of production. we solve the *constrained efficient* planner problem as in Kehoe and Levine (1993). The social planner faces the same financial constraints and maximizes the representative agent's utility by letting markets clear, and taking as given the competitive equilibrium price rules relevant for the collateral.

Definition 2 (*Constrained Efficiency*) Let $\{C, (C_j, Q_j, L_j)_j^N\}$, $\{(M_{ij})_{ij}^N\}$ be the competitive equilibrium allocations. The decentralized competitive equilibrium is constrained efficient if it yields the same allocation and welfare that a social planner would achieve facing the same credit constraints.

To expose the externality as clearly as possible, it is convenient to start with Cobb-Douglas technologies. we also assume that firms have to pay all the input expenses before production takes place. In this environment, firms are exogenously and permanently (un) constrained if the value of the collateral constraint parameter η_j is smaller (larger) than the degrees of scale, which in this case we assume to be 1.

We assume there are two sectors in the economy, and the input-output matrix is vertical. Therefore, there is an intermediate good sector that provides intermediates to the final good sector. Firms in the intermediate input sector produce according to

$$Q_1 = Z_1 L_1^{\alpha_1}, \quad (45)$$

while firms in the final good sector have the following technology

$$Q_2 = Z_2 L_2^{\alpha_2} M_2^{1-\alpha_2}. \quad (46)$$

The goods' market clearing conditions are

$$Q_1 = M_2, \quad (47)$$

and

$$Y = C = Q_2. \quad (48)$$

Firms in each sector face the following working capital constraint:

$$wL_1 \leq \eta_1 P_1 Q_1, \quad (49)$$

and

$$wL_2 + P_1 M_2 \leq \eta_2 P_2 Q_2. \quad (50)$$

As in Bigio and Lao (2013) firms need to pay input before production. The external funds that a firm can obtain are limited by a fraction η_j of total sales. This assumption is the result of an enforcement problem. Firms could run away with revenues without paying back the intra-period loan to the financial intermediary. In an equilibrium without default on debt, where the default probability is $(1 - \eta_j)$, the incentive compatibility constraint implies the constraints above.

The planner maximizes the representative household utility

$$\max_{\{C, L, P_j, M_j, w, \mu_j, \}} \frac{C^{1-\sigma} - 1}{1 - \sigma} - \psi \frac{L^{1+\xi}}{1 + \xi}, \quad (51)$$

subject to technologies, market clearing conditions, and the firms optimality conditions in (45)-(50). The planner's optimality conditions for the use of materials and the price of intermediates are: ⁵⁰

⁵⁰The variables λ_j and γ_j , with $j = 1, 2$, represent the Lagrange multiplier for technologies and firms' first order conditions, respectively.

$$M_2 : -\lambda_1 + \lambda_2(1 - \alpha_2) \frac{C}{M_2} - \mu_2 P_1 + \underbrace{\mu_1 \eta_1 P_1}_{\text{Externality Down}} - \gamma_1 P_1 + \gamma_3 \frac{P_1}{\psi_2 \beta_2} = 0 \quad (52)$$

$$P_1 : \underbrace{-\mu_2 M_2}_{\text{Externality Up}} + \mu_1 \eta_1 M_2 - \gamma_1 M_2 + \gamma_3 \frac{M_2}{\psi_2(1 - \alpha_2)} = 0, \quad (53)$$

where $\psi_j = \min(1, \eta_j)$. Whenever $\psi_j < 1$, sector j has a wedge in the use of inputs. When the working capital constraint binds firms pay an extra cost for inputs compared to what they would pay in a frictionless economy.

To understand how the externality works, look at the planner's optimality condition for intermediate input use M_2 . One can see the collateral constraint multiplier of the intermediate good sector, μ_1 , in the choice of intermediates for the final good sector. This term is absent in the private optimality decision for M_2 . The intuition behind this result is that, when firms in the intermediate good sector are credit constrained, there is a social gain of using more intermediates M_2 . The increased demand for intermediates would rise the relative price of intermediate good firms, which in turns relaxes their borrowing constraint.

On the other hand, if firms in the final good sector were constrained, $\mu_2 > 0$, the planner internalizes the existence of an extra social cost of increasing the price of intermediates. This extra cost is represented in equation (52) by the term $-\mu_2 M_2$ in the optimal choice of P_1 .

4.2.1 Decentralized Equilibrium

We decentralize the constrained planner solution by backing out the set of input subsidies or taxes that yield the constrained efficient solution. we assume that these subsidies are financed via lump-sum taxes to households. In this experiment we do not choose a subsidy on sales as a tool because that simply gets rid of the constraint by giving more collateral. Rather we set subsidies on labor and the intermediate input to different sectors to investigate how by changing the relative price between inputs, firms make decisions that are socially optimal.

We start by backing out the set of labor subsidies that restore the constrained efficient allocation. This simple environment allows me to find analytic solutions for the competitive equilibrium labor allocations. Firms in the intermediate good sector solve the following problem:

$$\max \Pi_1 = P_1 Z_1 l_1^{\alpha_1} - w(1 - s_1)L_1,$$

subject to

$$w(1 - s_1)L_1 \leq \eta_1 P_1 Z_1 l_1^{\alpha_1} \quad [\mu_1].$$

Firms in the final good sector solve,

$$\max \Pi_2 = P_2 Z_2 L_2^{\alpha_2} Q_1^{\beta_2} - w(1 - s_2)L_2 - P_1 Q_1,$$

subject to

$$w(1 - s_2)L_2 + P_1 Q_1 \leq \eta_2 P_2 Z_2 L_2^{\alpha_2} Q_1^{\beta_2} \quad [\mu_2],$$

where s_j represents the labor subsidy to firms in sector j and μ_j the Lagrange multiplier of the collateral constraints.

Proposition 6 *Suppose that firms in the final good sector are financially constrained. The, the decentralized equilibrium is constrained efficient. There exist labor subsidies $s_1 = s_2 = 1 - \psi_2$ such that the allocation of resources, inputs and output, coincide with the social planner problem.*

Proof: see Appendix C

Even though firms in the intermediate good sector are unconstrained, these firms do not internalize that by increasing their production, Q_1 , they relax the final good borrowing constraint through an increase in $\frac{P_2}{P_1}$.

If instead, the policy tool is a subsidy on the intermediate input cost, we can understand how the externality works in the other direction of the network. In this case, firms in the intermediate good sector solve:

$$\max \Pi_1 = P_1 Z_1 l_1^{\alpha_1} - wL_1,$$

subject to

$$wL_1 \leq \eta_1 P_1 Z_1 l_1^{\alpha_1} \quad [\mu_1].$$

Firms in the final good sector solve,

$$\max \Pi_2 = P_2 Z_2 L_2^{\alpha_2} Q_1^{\beta_2} - wL_2 - P_1(1 - s_m)Q_1,$$

subject to

$$wL_2 + P_1(1 - s_m)Q_1 \leq \eta_2 P_2 Z_2 L_2^{\alpha_2} Q_1^{\beta_2} \quad [\mu_2],$$

where s_m represents the intermediate input subsidy to firms in the final good sector.

Proposition 7 *Suppose that firms in the intermediate good sector are financially constrained. Then, the decentralized equilibrium is constrained efficient. There exists an intermediate input subsidy $s_m = 1 - \psi_1$ such that the inputs and output allocations coincide with the solution to the social planner problem.*

Proof: see Appendix C

Again, this subsidy alters the relative price between materials and labor, which increases the demand for intermediates. The higher demand for intermediates increases the relative price of the intermediate good sector which in turn increases the collateral of constrained firms and relaxes the borrowing constraint.

4.2.2 Flexibility and The Importance of the Externality

An important element in the design of the optimal policy is to understand the interaction between the elasticities of substitution between inputs and the size of the pecuniary externality. Elasticities are important for at least two reasons. First, a non-unitary elasticities of substitution implies that sectors are occasionally credit constrained due to changes in inputs prices (Miranda-Pinto and Young 2016). As stressed out by Miranda-Pinto (2016) this non-linearity interacts with the structure of the production network. When firms are inelastic and face constraints in the use of intermediates, a dense production network facilitates the spread out of financial constraints.

Moreover, the observed heterogeneity in the way different sectors substitute inputs (Miranda-Pinto and Young 2016) suggests that the optimal policy might significantly depend on sectors' elasticities. In the simple decentralized example of the previous section, if the final good firms were to have a low elasticity of substitution between materials and labor, think of perfect complements, giving a subsidy s_m to final good firms would not be an efficient policy to increase the demand for intermediates.

To focus on how the optimal subsidies or taxes depend on the elasticities, we use a three sector ring economy example. Sector 1 provides intermediates to sector 2, sector 2 provides intermediates to sector 3, while sector 3 provides intermediates to sector 1. The households consume the goods produced by each sectors. The planner solves the following problem

$$\max_{\{C, L, P_j, M_j, w, \mu_j, \}} \frac{C^{1-\sigma} - 1}{1-\sigma} - \psi \frac{L^{1+\xi}}{1+\xi}, \quad (54)$$

subject to

$$Q_{jt} = Z_{jt} \left[a_j^{\frac{1}{\varepsilon_{Q_j}}} L_{jt}^{\frac{\varepsilon_{Q_j}-1}{\varepsilon_{Q_j}}} + (1-a_j)^{\frac{1}{\varepsilon_{Q_j}}} M_{jt}^{\frac{\varepsilon_{Q_j}-1}{\varepsilon_{Q_j}}} \right]^{\frac{\varepsilon_{Q_j}}{\varepsilon_{Q_j}-1}},$$

where

$$M_j = \prod_{i=1}^N M_{ij}^{\gamma_{ij}}.$$

The elasticity of substitution between labor and intermediate is ε_{Q_j} . The importance of labor in production is denoted by a_j and γ_{ij} is the fraction of intermediates from sector i in the total intermediates expenditure of sector j .

The collateral constraint for sector j is

$$\theta_j^w w_t L_{jt} + \theta_j^m \sum_{i=1}^N P_{it} M_{ijt} \leq \eta_j P_{jt} Q_{jt},$$

where θ_j^w and θ_j^m are the fraction of labor cost and intermediates cost to be paid in advance.

We show that there exists a *relaxed planner's problem*, in which the planner does not use the private FONC for labor and materials, but arrives to the same allocation of the constrained efficient planner. We prove it by showing that the relaxed planner FONCs, in addition to a redefinition of the planner's Lagrange multiplier, yield the same decentralized FONCs for materials and labor.

Proposition 8 *The relaxed planner's problem that omits the FONC for labor and materials, is equivalent to the constrained efficient planner's problem.*

Proof: see Appendix C

We assume $\theta_j^w = 0$ and $\theta_j^m = 1$ for $j = 1, 2, 3$. The relaxed planner's optimal choice of sector's 1 output Q_1 is

$$\mu_1^P \eta_1 P_1 - \mu_2^P P_1 + U_{c_1}(C) - \lambda_1^P = 0 \quad (55)$$

while the private choice of Q_1 for firms in sector 1 is

$$\mu_1 \eta_1 P_1 + P_1 - \lambda = 0 \quad (56)$$

The planner internalizes that whenever $\mu_2^P > 0$ there is lower social cost of producing Q_1 . The reason is that increasing supply of Q_1 would relax the constrained of sector 2 through lower input cost.

Let's focus in the role of the elasticity. Assuming that only sector 2 has problems financing inputs, the planner's optimal choice of intermediate inputs for sector 2, M_{12} , is

$$\underbrace{\frac{\partial Q_2}{\partial M_1}(\varepsilon_Q) \mu_2^P \eta_2 P_2}_{\text{Externality and } \varepsilon_Q} + \frac{\partial Q_2}{\partial M_1} \lambda_2^P = \mu_2^P P_1 + \lambda_1^P \quad (57)$$

The first term shows how the size of externality depends on the elasticity of substitution between labor and materials. The following proposition describes the mechanism

Proposition 9 *In a ring production network, the social value of relaxing the constraint in highly productive sectors ($Z > 1$) is increasing in the elasticity of substitution between materials and value added of the constrained sector*

$$\frac{\partial \log(\frac{\partial Q_j}{\partial M_j})}{\partial \varepsilon_{Q_j}} = \log\left(\frac{Z_j M_j}{(1 - a_j) Q_j}\right) > 0 \quad \text{if } Z > 1$$

Proof: See Appendix C

Proposition 9 states that, keeping everything else constant, for highly productive it is more efficient to subsidize the more flexible firms as they can cheaply reallocate inputs of production to help relaxing the constraints. In other words, for the economy is more costly not to bail-out highly productive and highly flexible firms. This sheds lights about the optimal policy in an event in which most of the sectors become constrained. If there are no lump-sum taxes available and the government has limited resources, we better boost the high elasticity sector, which in the data are service sectors like Transportation and Telecommunications and Automobile dealers.

Another question of interest is whether the elasticities of other sectors influence the effect of the externality. The answer to this question depends on the particular network structure and what kind of instrument the government has available. In the ring network, if firms in sector 2 are constrained, the government could support firms in sector 1 to then decrease the input price that sector 2 pays. The efficiency of this policy depends on how flexible is the technology of sector 1. Another option is to subsidize the input cost of firms in sector 3, that use sector 2's output as an input. In this case, inflexibility in the technology of firms in sector 3 makes hard for them to switch to more demand for sector 2, making the policy inefficient.

4.3 General Network

So far, we have abstracted from the investment decision, one of the main driving forces of business cycles. As discussed in this section, the inter-temporal nature of the economy will have important implications in the optimal policy analysis. In particular, today's households' investment decisions do not consider tomorrow's effect on firms credit constraints.

The model economy is based on Foerster et al. (2011) and Atalay (2014). The main difference is the assumption about the timing in the capital stock accumulation. In this economy, households accumulate capital and then rent it to firms. This assumption implies that firms choose physical capital after observing the sectoral productivity shock, whereas in previous papers firms decide capital before the shock is realized. This assumption reduces the state space in Foerster et al. (2011) and allows me to solve the model using global methods.⁵¹

⁵¹ Miranda-Pinto and Young (2016b) study the implications of this assumption by performing over identifying restrictions tests to a frictionless Cobb-Douglas version of existing dynamic multi sector models and the model presented in this section. The model in this paper outperforms other models in fitting the dynamics of sectoral output growth.

4.3.1 Firms

In a perfectly competitive environment, firms in sector j produce according to

$$Q_{jt} = Z_{jt} \left[a_j^{\frac{1}{\varepsilon_{Q_j}}} \left(\left(\frac{K_{jt}}{\alpha_j} \right)^{\alpha_j} \left(\frac{L_{jt}}{1 - \alpha_j} \right)^{1 - \alpha_j} \right)^{\frac{\varepsilon_{Q_j} - 1}{\varepsilon_{Q_j}}} + (1 - a_j)^{\frac{1}{\varepsilon_{Q_j}}} M_{jt}^{\frac{\varepsilon_{Q_j} - 1}{\varepsilon_{Q_j}}} \right]^{\frac{\varepsilon_{Q_j}}{\varepsilon_{Q_j} - 1}}, \quad (58)$$

where

$$M_{jt} = \left(\sum_{i=1}^N \gamma_{ij}^{\frac{1}{\varepsilon_{M_j}}} M_{ijt}^{\frac{\varepsilon_{M_j} - 1}{\varepsilon_{M_j}}} \right)^{\frac{\varepsilon_{M_j}}{\varepsilon_{M_j} - 1}}. \quad (59)$$

The total factor productivity (TFP) of sector j is Z_j , L_j is labor, M_{ij} is materials that sector j purchased from sector i , and K_j is capital rented from households. The elasticity of substitution between material varieties is ε_M . The element γ_{ij} represents the share of intermediate inputs from sector i in the total cost of intermediates in sector j .

The collateral constraint for sector j is

$$\theta_j^w w_t L_{jt} + \theta_j^k r_t K_{jt} + \theta_j^m \sum_{i=1}^N P_{it} M_{ijt} \leq \eta_j P_{jt} Q_{jt}. \quad (60)$$

This constraint is a general version of the one in the previous section. Firms in different sectors pay in advance only a fraction θ_j^k of the cost of input k . The capital stock of the economy is owned by households and the firms rent it from them.

Productivity in each sector j follows an AR(1) process

$$\ln Z_{jt} = \xi \ln Z_{jt-1} + \varepsilon_{jt}, \quad \text{where } \varepsilon_{jt} \sim N(0, \Sigma_\varepsilon). \quad (61)$$

4.3.2 Households

Households maximize the expected present discount value of utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \psi \frac{L_t^{1+\xi}}{1 + \xi} \right), \quad (62)$$

subject to the budget constraint

$$\sum_{j=1}^N w_t L_{jt} + r_t K_t^A + \sum_{j=1}^N \pi_{jt} \geq \sum_{j=1}^N P_{jt} C_{jt} + \sum_{j=1}^N P_{jt} X_{jt}, \quad (63)$$

$$C_t = \prod_{j=1}^N C_{jt}^{\omega_j},$$

and the law of motion of capital stock

$$K_{t+1}^A = \left(\sum_{j=1}^N \phi_j^{\frac{1}{\varepsilon_X}} X_{jt}^{\frac{\varepsilon_X-1}{\varepsilon_X}} \right)^{\frac{\varepsilon_X}{\varepsilon_X-1}} + (1 - \delta) K_t^A, \quad (64)$$

where π_j are profits in sector j . The household allocates a fraction ω_j of its total consumption expenditure to buy goods from sector j . X_{jt} is the amount of investment goods the households buys from sector j . The parameters ϕ_j represent how important is industry j in providing investment goods to the rest of the economy. The elasticity between investment goods of different sectors is ε_X and δ is the depreciation rate of physical capital.

4.4 Market Clearing

The market clearing conditions for goods markets, labor market, and capital stock are

$$Q_{jt} = C_{jt} + X_{jt} + \sum_{i=1}^N M_{jit} \quad (65)$$

$$L_t = \sum_{j=1}^N L_{jt} \quad (66)$$

$$K_t^A = \sum_{j=1}^N K_{jt} \quad (67)$$

4.5 Investment Externality

In this set-up, we are able to study dynamic externalities. Due to the extent of financial cascades, where many sectors become constrained due to a sectoral shock, it can be optimal to subsidize key sectors ex-ante to avoid the social cost of a large downturn implied by domino effects implied by the sectoral connections. Thus, in this environment we can study the trade-off

between preventive and state contingent policies.

The planner's problem solves

$$\max_{\{C, Q_j, L_j, M_{ij}, \mu_j, P_j, w, r, X_j, K_j, K_{t+1}^A\}} E_t \sum_{t=0}^N \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{L_t^{1+\xi}}{1+\xi} \right],$$

subject to technologies, the law of motion of capital, and the working capital constraints.

Similar to the static model, one can find the constrained efficient allocation by solving a smaller problem. The planner's Euler equation is

$$1 = E_t \left[\beta \frac{\lambda_{jt+1}^P \partial X_{t+1} / \partial X_{jt+1} (1 - \delta)}{\lambda_{jt}^P \partial X_t / \partial X_{jt}} + \lambda_{jt+1} \frac{\partial Q_{t+1}}{\partial K_{jt+1}} + \underbrace{\mu_{jt+1}^P (\eta_j P_{jt+1} \frac{\partial Q_{t+1}}{\partial K_{jt+1}} + \theta_j^k r_{t+1})}_{\text{Investment Externality}} \right] \quad (68)$$

When comparing the reduced planner's Euler equation with the private Euler for households (in Appendix C) we claim that

Proposition 10 *The decentralized competitive equilibrium is not always constrained efficient. When the constraint binds in sector j*

- *Households do not internalize that by investing more in sector j today, can relax its constraint tomorrow. This is represented by the term $E_t [\mu_{jt+1}^P \eta_j P_{jt+1} \frac{\partial Q_{t+1}}{\partial K_{jt+1}}]$.*
- *Households do not internalize that by increasing overall investment today, can relax sector j constraint tomorrow through a reduction in the cost of capital, which is represented by $E_t [\theta_j^k r_{t+1}]$.*

Proof: see Appendix C

4.6 Sectors Centrality: A Numerical Example

We have theoretically characterized the network externalities in the model. Based on Proposition 8 and the results in Chapters 1 and 2, there is an interaction between the flexibility in production and the structure of the network that rises or decreases the likelihood of a sector j , and then potentially a sector $j + 1$ and more sectors, to become financially constrained. The im-

plementation of the sectoral stabilization policies studied so far require a significant amount of information: sectoral elasticities, sectoral collateral constraint parameters, and sectoral prices.

In this section we study what unique and observable feature of sectors can contain important information for the design of the optimal policy. This is similar in spirit to the sufficient statistic literature. We focus in the case with inflexible technologies and frictions in the use of intermediates. We also abstract from the investment decision and simplify the number of sectors to 3. This setup is relevant for emerging countries (Chapter 1) and for the non-service sectors in the U.S. (Chapter 2). The following statement describes the key quantitative result:

Assume $\varepsilon < 1$, $\theta_m = 1$, $\theta_l = 1$. The largest decline in GDP from a sectoral binding constraint comes from the sector with the smallest *harmonic distance* $h_j = \sum_{i=1}^N h_{ij}$, where

$$h_{ij} = 1 + \sum_{k \neq j} \hat{\gamma}_{ik} \cdot h_{kj},$$

assuming $h_{ii} = 0$ and $\hat{\gamma}_{ii} = 0$ for all i , where $\hat{\gamma}_{ik} = \gamma_{ik} \frac{P_k Q_k}{P_i Q_i} = \frac{P_i M_{ik}}{P_i Q_i}$ is a weighted measure of how close is sector i to sector j , considering how close to i are all j 's trade partners $k \neq j$. Unlike other closeness network measures that find the shortest path from one node (sector) of the network to another, the *harmonic distance* also considers how important are the links among sectors. This centrality measure is used by Acemoglu et al. (2015b) to identify *systemic* banks that if default are more likely to make other banks to default as well. Even though the environment in this paper is very different, it is the same notion of non-linear relationships between agents of the economy.

Let's study a simple example where Γ represents the production network, and the sales vector is the share of each sector sales on total sales.⁵²

$$\Gamma = \begin{bmatrix} 0.2 & 0.22 & 0.2 \\ 0.2 & 0.25 & 0.2 \\ 0.2 & 0.1 & 0.2 \end{bmatrix} \quad \text{Sales vector} = [0.40 \quad 0.42 \quad 0.36].$$

For this example we can analytically solve for the Harmonic distance matrix. The distance from sector i to sector 1 is

⁵²The sectors' sales are according to Acemoglu et al. (2012) a sufficient statistic to know the aggregate effect of a shock to a sector j .

$$\begin{bmatrix} h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} \psi_2 \\ \psi_3 \end{bmatrix} + \begin{bmatrix} \hat{\gamma}_{23} & 0 \\ 0 & \hat{\gamma}_{32} \end{bmatrix}' \begin{bmatrix} h_{21} \\ h_{31} \end{bmatrix}$$

implying

$$h_{21} = \frac{1 + \hat{\gamma}_{23}}{1 - \hat{\gamma}_{23}\hat{\gamma}_{32}} = 1.23 \quad h_{31} = \frac{1 + \hat{\gamma}_{32}}{1 - \hat{\gamma}_{32}\hat{\gamma}_{23}} = 1.12.$$

The implied harmonic distance matrix is

$$H = \begin{bmatrix} 0 & 1.25 & 1.28 \\ 1.23 & 0 & 1.26 \\ 1.12 & 1.25 & 0 \end{bmatrix},$$

The column sum of H represents how close is sector j to all other sectors. In this case, sector 1 is the closest but not the largest

$$H_c = \begin{bmatrix} 2.35 & 2.5 & 2.54 \end{bmatrix}$$

We solve the competitive equilibrium for the above network and assume $\varepsilon = 0.8$, $\theta_j^l = 0$, and $\theta_{ij}^m = 1$. I assume sectors are financially unconstrained with $\eta_j = \left(\frac{P_j^M M_j}{P_j Q_j}\right)^* (1 + \xi)$, meaning sectors start in equilibrium with leverage being ξ percent above the borrowing limit.

The simulation results are in Tables 25 and 26. If the financial shock on sector j is such that only sector j is constrained, sales of sector j are a sufficient statistic for the GDP effect the shock. In this case, sector 2 is the largest sector and also has the largest effect on GDP. Nevertheless, when the shock is below a threshold such that other sectors also become constrained, a shock to sector 1 generates the largest decline in GDP. A large negative shock in sector 1 increases the price of its output and decreases the demand of sector 1 for other sectors' output. Indeed, as shown in Table 26, the same large shock in all sectors generates a complete financial cascade when it hits the sector with the smallest harmonic distance measure, sector 2. In this case, as all sectors become constrained, the declined in GDP is maximized.

This result shows that in a frictionless economy, it is enough to know how big is a sector

to measure its aggregate importance. This result is not new and was formalized by Acemoglu et al. (2012) and Acemoglu et al. (2016). Liu (2017) shows that with Cobb-Douglas technologies even with frictions in the use of inputs, the size of a sector is informative not only on its aggregate effect but also on the size of the externality it generates.⁵³

The contribution of the harmonic distance result to the literature of networks in macroeconomics is that it provides a pure network measure to determine how central is a sector in the network. This network centrality is relevant when the frictions in the use of input activate, which is why it can be thought as a measure of systemic importance: what are the sectors that need to be monitored to avoid a large contagion from a sectoral shock?

The next section describes the study of optimal sectoral policy.

4.7 Optimal Ramsey Policy

The final section is advocated to study the design of optimal Ramsey policy. The government has tools available, in principle it can levy taxes or give subsidies to households or different sectors. The Ramsey problem looks for the best competitive equilibrium, given taxes and subsidies, that meets the government budget.

We endow the government with subsidies or taxes on inputs, investment, and consumption. The Ramsey problem looks for the sequence of prices, allocation and policy instruments that maximize the consumers' utility. It is then possible to find optimal to intervene when no constraints are binding (preventive), only when they bind (state contingent), or a combination of both. Formally, the government maximizes

$$\max_{\{s_j^y, s_{wj}, s_{mij}, \tau^x C, Q_j, L_j, M_{ij}, \mu_j, P_j, w, r, X_j, K_j, K_{t+1}^A\}} E_t \sum_{t=0}^N \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{L_t^{1+\xi}}{1+\xi} \right], \quad (69)$$

subject to the government budget

⁵³Liu (2017) is a contemporaneous study that similar to this studies optimal sectoral policy. The key differences between Liu (2017) and this Chapter are that: i) Liu assumes that sectors are always constrained in the use of intermediates and labor and that ii) Liu focuses on the development policies rather than in downturns mitigation policies as the present chapter. One advantage of focusing on stabilization policies at the business cycle frequency is that taking the network structure as given is less of an issue. However, in studying the design of long-run sectoral development policies, the endogenous evolution of the network is crucial.

$$\tau_t^l w_t L_t + \tau_t^k r_t K_t = \sum_{j=1}^N s_{wj} w_t L_{jt} + \sum_{j=1}^N s_{jt}^c P_{jt} C_{jt} + \sum_{j=1}^N \sum_{we=1}^N s_{mijt} P_{it} M_{ijt} + \sum_{j=1}^N s_{jt}^x P_{jt} X_{jt}, \quad (70)$$

where τ^x are proportional taxes levied to household for income from labor or capital. s_j^y are subsidies to sectoral consumption or sectoral investment. s_{jw} and s_{mij} are subsidies to the use of labor and use of materials.

The government also takes as given the firms' optimality conditions and the household optimality conditions for labor, capital, and consumption. The household budget constraint becomes

$$\sum_{j=1}^N w_t (1 - \tau_t^l) L_{jt} + r_t (1 - \tau_t^k) K_t^A + \sum_{j=1}^N \pi_{jt} \geq \sum_{j=1}^N P_{jt} (1 - s_j^c) C_{jt} + \sum_{j=1}^N P_{jt} (1 - s_t^x) X_{jt}. \quad (71)$$

The distinction between subsidies and taxes is arbitrary ex ante and in an extreme case it could be optimal to charge negative subsidies or give negative taxes.

Definition 3 *A regulated competitive equilibrium is a sequence of prices, allocations and government policies, such that (i) household optimization conditions and firm optimization conditions are satisfied; (ii) markets clear (65); and (iii) the government budget constraint is satisfied (70).*

Definition 4 *The Ramsey problem is to maximize (69) over regulated competitive equilibrium.*

We plan to numerically solve the Ramsey problem for a calibrated version of the model. In this exercise we will study how the optimal sectoral policy relates to sectors' elasticities and harmonic distance embedded in the input-output table.

4.8 Conclusions

This chapter studied the normative implications of the models in previous chapters. We extended the analysis of pecuniary externalities in models with financial frictions to a model with firms linkages. Linkages create additional pecuniary externalities that open the scope for sectoral-network policies. Once the sectoral constraints bind, the government can relocate sectoral inputs to relax the constraints of constrained sectors.

The design of sectoral policy depends on the flexibility in sectoral technologies and the structure of sectoral connections. The more flexible is sector the larger the social benefit of relaxing its constraint, as the larger would be the sectoral adjustment absent frictions. The more inter-connected is a sector (harmonic distance) the larger is the social benefit of relaxing its constraint to avoid financial cascades to other sectors.

References

- Acemoglu, D., Akcigit, U., and Kerr, W. (2015a). *Networks and the Macroeconomy: An Empirical Exploration*, pages 276–335. University of Chicago Press.
- Acemoglu, D., Carvalho, V. M., Ozdaglar, A., and Tahbaz-Salehi, A. (2012). The Network Origins of Aggregate Fluctuations. *Econometrica*, 80(5):1977–2016.
- Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. (2015b). Systemic Risk and Stability in Financial Networks. *American Economic Review*, 105(2):564–608.
- Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. (2016). Microeconomic origins of macroeconomic tail risks. *American Economic Review*, Working Paper Series(4).
- Altinoglu, L. (2015). The origins of aggregate fluctuations in a credit network economy. Manuscript, Boston University.
- Atalay, E. (2014). How Important Are Sectoral Shocks. Working Papers 14-31, Center for Economic Studies, U.S. Census Bureau.
- Baccianti, C. (2013). Estimation of Sectoral Elasticities of Substitution Along the International Technology Frontier. *ZEW Discussion Paper*, (Discussion Paper No. 13-092):391–409.
- Baqaei, D. (2015). Cascade failures in production networks. Manuscript, Harvard University.
- Bartelme, D. and Gorodnichenko, Y. (2015). Linkages and economic development. Working Paper 21251, National Bureau of Economic Research.
- Benigno, G., Chen, H., Otrok, C., Rebucci, A., and Young, E. R. (2013). Financial crises and macro-prudential policies. *Journal of International Economics*, 89(2):453–470.

- Bianchi, J. (2011). Overborrowing and Systemic Externalities in the Business Cycle. *American Economic Review*, 101(7):3400–3426.
- Bigio, S. and La'O, J. (2016). Financial frictions in production networks. Working Paper 22212, National Bureau of Economic Research.
- Carvalho, V. (2007). Aggregate fluctuations and the network structure of intersectoral trade. Economics Working Papers 1206, Department of Economics and Business, Universitat Pompeu Fabra.
- Carvalho, V. and Gabaix, X. (2013). The Great Diversification and Its Undoing. *American Economic Review*, 103(5):1697–1727.
- Carvalho, V. M., Nirei, M., Saito, Y. U., and Alireza Tahbaz-Salehi, A. (2016). Supply Chain Disruptions: Evidence from the Great East Japan Earthquake. Cambridge Working Papers in Economics 1670, Faculty of Economics, University of Cambridge.
- Carvalho, V. M. and Voigtländer, N. (2015). Input diffusion and the evolution of production networks s. Working Paper 20025, NBER.
- Contreras, M. G. A. and Fagiolo, G. (2014). Propagation of Economic Shocks in Input-Output Networks: A Cross-Country Analysis. Papers 1401.4704, arXiv.org.
- di Giovanni, J. and Levchenko, A. A. (2012). Country size, international trade, and aggregate fluctuations in granular economies. *Journal of Political Economy*, 120(6):1083–1132.
- Dupor, B. (1999). Aggregation and irrelevance in multi-sector models. *Journal of Monetary Economics*, 43(2):391–409.
- Fadinger, H., Ghiglini, C., and Teteryatnikova, M. (2015). Income Differences and Input-Output Structure. Vienna Economics Papers 1510, University of Vienna, Department of Economics.
- Foerster, Sarte, and Watson (2011). Sectoral versus Aggregate Shocks: A Structural Factor Analysis of Industrial Production. *Journal of Political Economy*, 119(1):1 – 38.
- Gabaix, X. (2011). The granular origins of aggregate fluctuations. *Econometrica*, 79(3):733–772.

- Horvath, M. (1998). Cyclicalities and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks. *Review of Economic Dynamics*, 1(4):781–808.
- Horvath, M. (2000). Sectoral shocks and aggregate fluctuations. *Journal of Monetary Economics*, 45(1):69–106.
- Hulten, C. R. (1978). Growth accounting with intermediate inputs. *Review of Economic Studies*, 45(3):511–518.
- John B. Long, J. and Plosser, C. I. (1983). Real business cycles. *Journal of Political Economy*, 91(1):39–69.
- Jones, C. I. (2011). Intermediate Goods and Weak Links in the Theory of Economic Development. *American Economic Journal: Macroeconomics*, 3(2):1–28.
- Kehoe, T. J. and Levine, D. K. (1993). Debt-Constrained Asset Markets. *Review of Economic Studies*, 60(4):865–88.
- Koesler, S. and M., S. (2015). Substitution Elasticities in a CES Production Framework: An Empirical Analysis on the Basis of Non-Linear Least Squares Estimations. *Economic Systems Research*, 27(1).
- Koren, M. and Tenreyro, S. (2007). Volatility and Development. *The Quarterly Journal of Economics*, 122(1):243–287.
- Laeven, L. and Valencia, F. (2013). Systemic banking crises database. *IMF Econ Rev*, 61(2):225–270.
- Luo, S. (2015). Propagation of financial shocks in an input-output economy with trade and financial linkages of firms. Manuscript, Columbia University.
- Luo, Y., Nie, J., and Young, E. R. (2017). The Negative Growth-Volatility Relationship and the Gains from Diversification. *Working Paper*.
- Miranda-Pinto, J. and Young, E. (2016). Flexibility and frictions in multisector models. Manuscript, University of Virginia.

- Moro, A. (2012). The Structural Transformation Between Manufacturing and Services and the Decline in the US GDP Volatility. *Review of Economic Dynamics*, 15(3):402–415.
- Moro, A. (2015). Structural Change, Growth, and Volatility. *American Economic Journal: Macroeconomics*, 7(3):259–94.
- Oberfield, E. (2013). Business networks, production chains and productivity: A theory of input-output architecture. Technical report.
- Ramey, G. and Ramey, V. A. (1995). Cross-Country Evidence on the Link between Volatility and Growth. *American Economic Review*, 85(5):1138–1151.
- Srivisai, N. and Dabla-Norris, E. (2013). Revisiting the Link Between Finance and Macroeconomic Volatility . *IMF working paper*, (13:29).
- sectionAppendix 4:

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Chapter 6: Appendix B: Tables

6.1 Tables Chapter 1

Table 1: Growth (G), Network Measures and Development

Country	Vol G	Med. G < 0	S_din	S_dout	Density	emerging	Service
Argentina	5.4%	-0.58%	-0.51	1.35	0.82	1	0.57
Australia	1.2%	-0.43%	-0.21	2.02	0.85	0	0.76
Austria	1.6%	-0.39%	-0.24	1.98	0.87	0	0.69
Belgium	1.5%	-0.60%	-1.19	2.35	0.85	0	0.70
Brazil	2.7%	-0.33%	-0.49	1.55	0.77	1	0.61
Brunei	3.3%	-0.20%	0.74	2.64	0.21	1	0.36
Bulgaria	5.5%	-0.62%	-0.14	1.49	0.76	1	0.59
Cambodia	4.0%	-0.26%	1.24	1.39	0.54	1	0.42
Canada	1.8%	-0.24%	0.53	2.54	0.75	0	0.70
Chile	2.8%	-0.45%	0.02	1.94	0.74	1	0.63
China	3.0%	-	-0.53	0.39	0.86	1	0.38
Colombia	2.1%	-0.43%	-0.45	1.16	0.63	1	0.62
Costa Rica	2.5%	-0.13%	-0.43	2.22	0.60	1	0.65
Cyprus	3.6%	-0.25%	0.27	2.89	0.58	1	0.84
Denmark	2.0%	-0.62%	-0.70	2.39	0.69	0	0.76
Finland	3.4%	-0.46%	0.63	1.89	0.84	0	0.65
France	1.5%	-0.46%	-0.81	2.84	0.81	0	0.74
Germany	2.1%	-0.71%	-0.83	2.17	0.85	0	0.62
Greece	3.9%	-0.39%	-0.33	2.19	0.65	0	0.76
Hungary	3.8%	-0.33%	0.72	1.78	0.84	0	0.55
Iceland	3.4%	-0.37%	1.22	1.52	0.64	0	0.71
India	2.1%	-	-0.05	1.19	0.63	1	0.52
Indonesia	4.0%	-0.15%	-0.80	1.21	0.68	1	0.50
Ireland	3.8%	-0.40%	0.61	0.85	0.70	0	0.62
Israel	2.3%	-0.68%	-0.20	2.21	0.57	1	0.74
Italy	2.0%	-0.37%	-1.04	2.49	0.85	0	0.68
Japan	2.5%	-0.34%	-0.16 ¹³¹	2.27	0.73	0	0.66

Country	Vol G	Med. G <0	S_din	S_dout	Density	Emerging	Service
Korea	3.7%	-0.56%	-0.44	0.93	0.81	1	0.44
Luxembourg	3.4%	-0.85%	1.67	3.62	0.49	0	0.91
Malaysia	3.9%	-0.18%	0.03	0.72	0.59	1	0.39
Malta	2.2%	-0.25%	0.22	1.90	0.57	1	0.81
Mexico	3.0%	-0.43%	0.10	2.15	0.58	1	0.57
Netherlands	1.9%	-0.38%	-0.76	2.38	0.83	0	0.69
New Zealand	2.0%	-0.22%	-0.17	1.42	0.76	0	0.69
Norway	1.6%	-0.17%	0.30	1.91	0.86	0	0.64
Philippines	3.0%	-0.58%	0.26	1.65	0.59	1	0.54
Poland	3.9%	-0.43%	-0.22	2.62	0.92	0	0.61
Portugal	2.8%	-0.29%	-0.56	2.09	0.82	0	0.71
Romania	5.4%	-0.58%	-0.32	1.73	0.77	1	0.63
Saudia Arabia	6.1%	-0.29%	-0.19	2.54	0.61	1	0.47
Singapore	4.1%	-0.67%	0.92	3.35	0.67	1	0.65
South Africa	2.1%	-0.16%	-0.24	1.66	0.69	1	0.64
Spain	2.4%	-0.17%	-0.31	1.78	0.90	0	0.71
Sweden	2.4%	-0.29%	0.23	2.09	0.91	0	0.70
Switzerland	1.6%	-0.33%	0.42	2.24	0.80	0	0.70
Turkey	4.4%	-0.53%	-0.24	1.34	0.84	1	0.52
United Kingdom	1.9%	-0.43%	0.02	2.44	0.84	0	0.80
United States	1.6%	-0.29%	0.07	2.04	0.88	0	0.75

Notes: Vol G is the standard deviation of real GDP growth between 1984-2014. Med. G < 0 is the median negative value of GDP growth in the sample. S_din and S_dout are the skewness of sectors in and out degrees in 2011 while density is the density measure in 2011. Emerging is a dummy that equals 1 when United Nations classifies a country as emerging and 0 if it is developed. Service is the share of service sectors in total gross output in 2011.

Table 2: Volatility: Cross-Section (threshold 1)

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$\log(\text{density})$	-0.406** (0.202)	-0.186 (0.269)	-1.568*** (0.388)	-1.476*** (0.447)	-2.352*** (0.534)	-2.200*** (0.582)
$\log(\text{service})$		-0.807*** (0.245)	-1.728*** (0.393)	-1.740*** (0.392)	-2.753*** (0.485)	-2.614*** (0.484)
$\log(\text{dens}) \cdot \log(\text{serv})$			-1.939*** (0.475)	-1.888*** (0.500)	-3.188*** (0.651)	-3.395*** (0.696)
$\log(\text{trade_gdp})$				0.0500 (0.0894)	0.102 (0.0846)	0.0797 (0.0877)
growth					-10.67** (3.968)	-9.477** (4.322)
$\log(\text{domin})$						-0.323 (0.844)
$\log(\text{avmatsh})$						-0.364 (0.351)
S_{din}						0.130* (0.0747)
$\log HH$						0.142 (0.171)
$\log(\text{fin_sector})$						-0.300*** (0.102)
$\log(\text{most_vol_sec})$						-0.0718 (0.0729)
Observations	48	48	48	48	48	48
R-squared	0.062	0.223	0.354	0.359	0.462	0.607

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 3: Volatility: Cross-Section (threshold 2)

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$\log(density)$	-0.430** (0.204)	-0.232 (0.282)	-1.513*** (0.354)	-1.432*** (0.411)	-2.331*** (0.522)	-2.360*** (0.635)
$\log(service)$		-0.803*** (0.237)	-1.864*** (0.437)	-1.870*** (0.439)	-3.036*** (0.567)	-2.864*** (0.545)
$\log(dens) \cdot \log(serv)$			-1.866*** (0.459)	-1.823*** (0.483)	-3.179*** (0.671)	-3.485*** (0.725)
$\log(trade_gdp)$				0.0458 (0.0889)	0.0976 (0.0841)	0.0948 (0.0864)
$growth$					-10.92** (4.086)	-9.534** (4.278)
$\log(domin)$						-0.314 (0.828)
$\log(avmatsh)$						-0.366 (0.361)
S_{din}						0.114 (0.0747)
$\log HH$						0.0685 (0.179)
$\log(fin_sector)$						-0.355*** (0.106)
$\log(most_vol_sec)$						-0.0733 (0.0740)
Observations	48	48	48	48	48	48
R-squared	0.065	0.229	0.352	0.356	0.461	0.613

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 4: Volatility: Cross-Section (threshold 3)

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$\log(density)$	-0.473** (0.197)	-0.364 (0.286)	-1.361*** (0.360)	-1.340*** (0.422)	-1.885*** (0.543)	-2.480*** (0.856)
$\log(service)$		-0.818*** (0.228)	-2.629*** (0.745)	-2.622*** (0.765)	-3.944*** (1.019)	-4.192*** (0.990)
$\log(dens) \cdot \log(serv)$			-1.586*** (0.547)	-1.574*** (0.572)	-2.476*** (0.780)	-3.155*** (0.837)
$\log(trade_gdp)$				0.0105 (0.0934)	0.0476 (0.0913)	0.0836 (0.0912)
$growth$					-8.196* (4.082)	-7.631* (4.103)
$\log(domin)$						-0.487 (0.878)
$\log(avmatsh)$						-0.166 (0.543)
S_{din}						0.116 (0.0790)
$\log HH$						-0.0967 (0.234)
$\log(fin_sector)$						-0.445*** (0.123)
$\log(most_vol_sec)$						-0.0557 (0.0697)
Observations	48	48	48	48	48	48
R-squared	0.073	0.254	0.350	0.350	0.415	0.592

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 5: Downturns: Cross-Section (threshold 1)

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$\log(density)$	0.326 (0.241)	0.197 (0.300)	-0.680 (0.773)	-0.616 (0.786)	-1.042 (0.832)	-1.334 (0.996)
$\log(service)$		0.423 (0.425)	-0.252 (0.570)	-0.248 (0.582)	-0.749 (0.687)	-1.343 (0.861)
$\log(dens) \cdot \log(serv)$			-1.275 (0.873)	-1.232 (0.879)	-1.878* (0.971)	-1.893* (1.080)
$\log(trade_gdp)$				0.0308 (0.114)	0.0754 (0.115)	-0.0642 (0.123)
$growth$					-6.118 (5.208)	-7.942 (7.433)
$\log(domin)$						-0.912 (1.343)
$\log(avmatsh)$						0.216 (0.653)
S_{din}						0.00703 (0.114)
$\log HH$						0.192 (0.295)
$\log(fin_sector)$						-0.0787 (0.155)
$\log(most_vol_sec)$						-0.270*** (0.0960)
Observations	46	46	46	46	46	46
R-squared	0.029	0.056	0.095	0.096	0.118	0.294

Robust standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 6: Downturns: Cross-Section (threshold 2)

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$\log(density)$	0.269 (0.293)	0.135 (0.327)	-0.886 (0.684)	-0.855 (0.704)	-1.378* (0.746)	-1.825* (0.950)
$\log(service)$		0.459 (0.421)	-0.520 (0.644)	-0.516 (0.654)	-1.209 (0.792)	-1.641* (0.877)
$\log(dens) \cdot \log(serv)$			-1.555* (0.823)	-1.535* (0.832)	-2.344** (0.933)	-2.268** (1.005)
$\log(trade_gdp)$				0.0155 (0.116)	0.0679 (0.118)	-0.0564 (0.125)
$growth$					-7.401 (5.406)	-8.958 (7.091)
$\log(domin)$						-0.992 (1.329)
$\log(avmatsh)$						0.347 (0.659)
S_{din}						-0.00647 (0.115)
$\log HH$						0.131 (0.275)
$\log(fin_sector)$						-0.138 (0.153)
$\log(most_vol_sec)$						-0.277*** (0.0944)
Observations	46	46	46	46	46	46
R-squared	0.019	0.051	0.107	0.108	0.139	0.318

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 7: Downturns: Cross-Section (threshold 3)

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$\log(density)$	0.0781 (0.392)	-0.0396 (0.344)	-1.124** (0.448)	-1.188** (0.484)	-1.577*** (0.517)	-3.050*** (0.975)
$\log(service)$		0.528 (0.392)	-1.774 (1.073)	-1.810* (1.058)	-2.797** (1.262)	-2.593** (1.225)
$\log(dens) \cdot \log(serv)$			-1.883*** (0.691)	-1.924*** (0.685)	-2.596*** (0.788)	-2.240** (0.919)
$\log(trade_gdp)$				-0.0291 (0.128)	0.0187 (0.129)	-0.0954 (0.115)
$growth$					-6.868 (5.300)	-8.769 (6.024)
$\log(domin)$						-1.565 (1.161)
$\log(avmatsh)$						1.188 (0.758)
S_{din}						-0.0104 (0.118)
$\log HH$						-0.189 (0.262)
$\log(fin_sector)$						-0.273 (0.168)
$\log(most_vol_sec)$						-0.266*** (0.0888)
Observations	46	46	46	46	46	46
R-squared	0.001	0.048	0.127	0.128	0.158	0.375

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 8: Volatility: Panel RE (threshold 1)

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$\log(density)$	-0.456** (0.220)	-0.330 (0.232)	-1.911*** (0.307)	-1.764*** (0.334)	-2.307*** (0.349)	-1.988*** (0.475)
$\log(service)$		-0.664** (0.259)	-1.876*** (0.390)	-1.860*** (0.378)	-2.483*** (0.308)	-2.500*** (0.441)
$\log(dens) \cdot \log(serv)$			-2.459*** (0.438)	-2.349*** (0.443)	-3.071*** (0.375)	-3.127*** (0.355)
$\log(trade_gdp)$				0.0930 (0.0921)	0.145 (0.0991)	0.0615 (0.160)
$growth$					-10.90*** (2.444)	-9.731*** (2.716)
$\log(domin)$						1.022 (0.723)
$\log(avmatsh)$						-0.236 (0.370)
S_{din}						-0.0643 (0.111)
$\log HH$						0.0642 (0.194)
$\log(fin_sector)$						-0.0935 (0.122)
$\log(most_vol_sec)$						-0.0167 (0.0802)
Observations	144	144	144	144	144	144
Number of cou_code	48	48	48	48	48	48

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 9: Volatility: Panel RE (threshold 2)

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$\log(density)$	-0.398** (0.192)	-0.303 (0.215)	-1.667*** (0.263)	-1.522*** (0.299)	-2.040*** (0.311)	-1.811*** (0.413)
$\log(service)$		-0.684*** (0.262)	-2.077*** (0.443)	-2.042*** (0.432)	-2.730*** (0.359)	-2.721*** (0.465)
$\log(dens) \cdot \log(serv)$			-2.188*** (0.410)	-2.078*** (0.419)	-2.757*** (0.367)	-2.854*** (0.335)
$\log(trade_gdp)$				0.0924 (0.0943)	0.140 (0.102)	0.0655 (0.161)
$growth$					-10.90*** (2.472)	-9.714*** (2.704)
$\log(domin)$						1.060 (0.724)
$\log(avmatsh)$						-0.225 (0.397)
S_{din}						-0.0702 (0.108)
$\log HH$						0.0277 (0.199)
$\log(fin_sector)$						-0.111 (0.125)
$\log(most_vol_sec)$						-0.0108 (0.0816)
Observations	144	144	144	144	144	144
Number of cou_code	48	48	48	48	48	48

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 10: Volatility: Panel RE (threshold 3)

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$\log(density)$	-0.317** (0.159)	-0.304 (0.196)	-1.347*** (0.282)	-1.213*** (0.336)	-1.589*** (0.388)	-1.625*** (0.449)
$\log(service)$		-0.739*** (0.269)	-2.997*** (0.750)	-2.889*** (0.757)	-3.714*** (0.690)	-3.883*** (0.629)
$\log(dens) \cdot \log(serv)$			-1.831*** (0.487)	-1.729*** (0.504)	-2.223*** (0.522)	-2.545*** (0.440)
$\log(trade_gdp)$				0.0723 (0.107)	0.111 (0.120)	0.0606 (0.162)
$growth$					-10.39*** (2.536)	-9.268*** (2.883)
$\log(domin)$						1.199* (0.716)
$\log(avmatsh)$						-0.143 (0.501)
S_{din}						-0.0900 (0.109)
$\log HH$						-0.0376 (0.233)
$\log(fin_sector)$						-0.158 (0.127)
$\log(most_vol_sec)$						0.00998 (0.0862)
Observations	144	144	144	144	144	144
Number of cou_code	48	48	48	48	48	48

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 11: Downturns: Panel RE (threshold 1)

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$\log(density)$	0.298 (0.400)	1.074** (0.429)	-1.532 (0.937)	-1.632 (1.071)	-2.330** (0.962)	-2.066 (1.376)
$\log(service)$		-2.889*** (0.781)	-5.123*** (1.119)	-5.181*** (1.153)	-6.144*** (1.130)	-5.681*** (1.831)
$\log(dens) \cdot \log(serv)$			-3.933*** (1.182)	-4.033*** (1.294)	-5.065*** (1.246)	-5.055*** (1.382)
$\log(trade_gdp)$				-0.0535 (0.233)	0.162 (0.208)	0.136 (0.403)
$growth$					-26.53*** (6.268)	-21.51*** (6.675)
$\log(domin)$						2.216 (1.964)
$\log(avmatsh)$						0.302 (0.810)
S_{din}						-0.488** (0.213)
$\log HH$						0.363 (0.442)
$\log(fin_sector)$						-0.0539 (0.288)
$\log(most_vol_sec)$						0.0223 (0.186)
Observations	89	89	89	89	89	89
Number of cou_code	46	46	46	46	46	46

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 12: Downturns: Panel RE (threshold 2)

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$\log(density)$	0.296 (0.404)	1.002** (0.423)	-1.449* (0.856)	-1.565 (1.010)	-2.194** (0.912)	-2.493* (1.403)
$\log(service)$		-2.834*** (0.774)	-5.564*** (1.236)	-5.648*** (1.287)	-6.663*** (1.271)	-6.103*** (1.865)
$\log(dens) \cdot \log(serv)$			-3.807*** (1.101)	-3.923*** (1.226)	-4.836*** (1.179)	-4.963*** (1.259)
$\log(trade_gdp)$				-0.0601 (0.239)	0.149 (0.214)	0.198 (0.395)
$growth$					-26.13*** (6.319)	-21.94*** (6.456)
$\log(domin)$						2.101 (1.981)
$\log(avmatsh)$						0.450 (0.838)
S_{din}						-0.489** (0.220)
$\log HH$						0.180 (0.454)
$\log(fin_sector)$						-0.121 (0.283)
$\log(most_vol_sec)$						0.0288 (0.185)
Observations	89	89	89	89	89	89
Number of cou_code	46	46	46	46	46	46

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 13: Downturns: Panel RE (threshold 3)

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$\log(density)$	0.445 (0.463)	0.889* (0.480)	-1.461* (0.781)	-1.651* (0.905)	-1.885** (0.892)	-3.383** (1.369)
$\log(service)$		-2.635*** (0.751)	-7.858*** (1.866)	-8.111*** (1.949)	-8.974*** (2.081)	-8.821*** (2.301)
$\log(dens) \cdot \log(serv)$			-3.955*** (1.073)	-4.146*** (1.162)	-4.523*** (1.242)	-5.127*** (1.092)
$\log(trade_gdp)$				-0.0791 (0.247)	0.131 (0.229)	0.224 (0.394)
$growth$					-24.72*** (6.367)	-21.08*** (6.578)
$\log(domin)$						1.912 (2.045)
$\log(avmatsh)$						0.923 (1.025)
S_{din}						-0.527** (0.236)
$\log HH$						-0.0892 (0.430)
$\log(fin_sector)$						-0.309 (0.294)
$\log(most_vol_sec)$						0.0401 (0.182)
Observations	89	89	89	89	89	89
Number of cou_code	46	46	46	46	46	46

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 14: Productivity Shocks Simulation: Volatility

Models	Acemoglu et al	Atalay ($\varepsilon = 0.8$)	$\varepsilon = 1.2$	$\varepsilon = 0.8; \theta^m = 1$	$\varepsilon = 1.2; \theta^l = 1$
$\hat{\sigma}_g^2$					
density = 0	0.0194	0.0156	0.0274	0.0156	0.0274
density = 1/3	0.0194	0.0170	0.0231	0.0170	0.0231
density = 2/3	0.0194	0.0184	0.0205	0.0184	0.0205
density = 1	0.0194	0.0200	0.0189	0.0200	0.0189

Table 15: Productivity Shocks Simulation: Downturns (Large Shocks Sector 1)

Models	$\varepsilon < 1; \theta^m = 1$	$\varepsilon < 1; \theta^l = 1$	$\varepsilon > 1; \theta^m = 1$	$\varepsilon > 1; \theta^l = 1$
Median Growth		Below Percentile 8.5		
density = 0	-0.0597	-0.0597	-0.0610	-0.0680
density = 1/3	-0.0653	-0.0652	-0.0512	-0.0571
density = 2/3	-0.0707	-0.0706	-0.0457	-0.0509
density = 1	-0.0765	-0.0764	-0.0420	-0.0468

Notes: The percentile 8.5 is calculates from the U.S. empirical distribution of growth between 1980-2014. Below the percentile 8.5 real GDP growth is negative.

Table 16: Productivity Shocks Simulation: Frequency of All Sectors Constrained (Large Shocks Sector 1)

Models	$\varepsilon < 1; \theta^m = 1$	$\varepsilon < 1; \theta^l = 1$	$\varepsilon > 1; \theta^m = 1$	$\varepsilon > 1; \theta^l = 1$
Median Growth		Below Percentile 8.5		
density = 0	0.045	0	0	0.035
density = 1/3	0.02	0	0	0.01
density = 2/3	0.06	0	0	0.01
density = 1	0.095	0	0	0

Table 17: General Lessons

Outcome	Manufacturing Oriented	Service Oriented
Volatility	$\varepsilon < 1$	$\varepsilon > 1$
Downturns	$\varepsilon < 1$ and $\theta^m > \theta^l$	$\varepsilon > 1$ and $\theta^l > \theta^m$

6.2 Tables Chapter 2

Table 18: Estimates Common Elasticities

	(1)	(2)	(3)
	OLS	$IV(Prices_{t-1})$	IV (Mil. Spend.)
VARIABLES	All sectors	All sectors	All sectors
$\hat{\varepsilon}_M$	0.398*** (0.0934)	0.471 (0.693)	-2.481** (1.532)
$\hat{\varepsilon}_Q$	0.69*** (0.106)	1.377 (1.161)	1.123 (2.114)
Observations	24,138	22,718	22,098
F test		44	4.5
Number of partner	1,420	1,420	1,300

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 19: GZ Spreads and Elasticities

VARIABLES	(1) All	(2) $\varepsilon_Q < 1$	(3) $\varepsilon_Q > 1$
D_R	0.478*** (0.035)	0.64*** (0.10)	0.531*** (0.09)
L	0.419** (0.155)	-0.101 (0.169)	-0.0230 (0.109)
$\varepsilon_Q L$	0.0787 (0.113)	-0.432*** (0.126)	0.656*** (0.117)
$\varepsilon_M L$	0.102 (0.103)	-0.0141 (0.0500)	0.0552 (0.0544)
$\varepsilon_Q D_R$	-0.0220 (0.0362)	0.0592 (0.0606)	-0.177** (0.0757)
$\varepsilon_M D_R$	0.0571*** (0.0187)	0.0859*** (0.0262)	0.0661** (0.0282)
Constant	0.327 (0.458)	0.563 (0.441)	0.373 (0.441)
Observations	2,092	937	1,155
Number of sector	32	15	17
R-squared	0.645	0.670	0.609
Year FE	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes

Standard errors clustered at the sector level

*** p<0.01, ** p<0.05, * p<0.1

Table 20: TRACE Spreads and Elasticities

VARIABLES	(1) Spread	(2) Spread
D_R	0.264*** (0.0728)	0.262*** (0.0728)
L	0.281*** (0.0372)	0.203*** (0.0585)
$\varepsilon_Q D_R$	-0.134*** (0.0350)	-0.138*** (0.0350)
$\varepsilon_M D_R$	-0.00494 (0.0401)	-0.00826 (0.0401)
$\varepsilon_Q L$		-0.0586* (0.0336)
$\varepsilon_M L$		-0.0626* (0.0365)
Observations	3,583	3,583
R-squared	0.209	0.211
Year FE	Yes	Yes
Firm FE	Yes	Yes

Standard errors clustered at the sector level

*** p<0.01, ** p<0.05, * p<0.1

Table 21: Results from the General Case

$$\phi_m < 1, \phi_w < 1$$

$$(\phi_m > 1, \phi_w > 1)$$

Bold Countercyclical Spreads

	$\varepsilon_Q < 1$		$\varepsilon_Q > 1$	
	$\mathbb{M}$	L	M	$\mathbb{L}$
$\text{corr}(\varepsilon_M, \mu_2)$	-(-)	$+(+)$	$+(+)$	-(-)
$\text{corr}(\varepsilon_Q, \mu_2)$	-(+)	$-(+)$	$+(-)$	+(+)

Table 22: OLS Bias

ε_Q	0.5	0.65	0.8	0.95	1.1	1.25	1.4
Only Z_1 , iid							
OLS uncon	0.5	0.65	0.8	0.95	1.1	1.25	1.4
OLS con	1	1	1	1	1	1	1
(Z_1, Z_2) , iid							
OLS uncon	0.93	0.95	0.97	0.99	1.01	1.03	1.05
OLS con	1	1	1	1	1	1	1
(Z_1, Z_2) , persistence 0.9							
OLS	0.96	0.969	0.984	0.994	1.011	1.021	1.032
Binding Freq	0.58	0.57	0.57	0	0.42	0.42	0.42

Table 23: OLS Bias

ε_{Q_1}	0.5	0.6	0.5	0.8
ε_{Q_2}	1.2	1.2	1	1
Only Z_1 , iid				
ε_{Q_1} OLS uncon	0.5	0.5	0.5	0.5
ε_{Q_2} OLS uncon	1.42	1.38	1.42	1.23
Only Z_2 , iid				
ε_{Q_1} OLS uncon	0.5	0.6	0.5	0.8
ε_{Q_2} OLS uncon	1.42	1.36	1.42	1.29
(Z_1, Z_2) , iid				
ε_{Q_1} OLS uncon	0.45	0.79	0.44	0.94
ε_{Q_2} OLS uncon	0.67	0.93	0.67	1.01

6.3 Tables Chapter 3

Table 24: GMM criteria value (M)

Models	W=Identity	W=Asymptotic
Carvalho 2007	39.4	2.5E+05
Foerster et al 2011	841.4	4.6E+06
One K model	27.6	2.2E+05

6.4 Tables Chapter 4

Table 25: % GDP drop (shock to sector j only)

Shock	Drop η_1	Drop η_2	Drop η_3
10%	25.01	25.08	24.95
40%	38.73	37.33	36.10
50%	47.18	45.13	43.22

Table 26: Number of sectors credit constrained

Shock	Drop η_1	Drop η_2	Drop η_3
10%	1	1	1
40%	2	1	1
50%	3	2	1

Chapter 7: Appendix C: Mathematical Appendix

7.1 Mathematical Appendix Chapter 3

7.1.1 Solution of the Model

To ease notation recall that $\rho = \frac{\varepsilon-1}{\varepsilon}$ and $\varepsilon = \frac{1}{\rho-1}$. Also, let $b_j = 1 - a_j$ the importance of materials in production. The firms Kuhn-Tucker conditions are

$$\begin{aligned}
 P_j Z^\rho \left(\frac{a_j Q_j}{L_j} \right)^{1-\rho} (1 + \eta_j \mu_j) &= w(1 + \theta_j^l \mu_j) \\
 P_j Z^\rho \left(\frac{b_j Q_j}{M_j} \right)^{1-\rho_j} (1 + \eta_j \mu_j) &= P_j^M (1 + \theta_j^m \mu_j) \\
 \max(-\psi_j, 0)^2 - (\eta_j P_j Q_j - \theta_j^m P_j^M M_j - \theta_j^l w L_j) &= 0,
 \end{aligned}$$

where $\mu_j = \max(\psi_j, 0)^2$.

From the household FONC we have

$$P_j C_j = \beta_j P C.$$

The price index for consumption goods is obtained from minimizing the households' expenditures. The FONC yields $P_j = P \beta_j C / C_j$. Replacing this on $C = \prod_{j=1}^N C_j^{\beta_j}$ we obtain $P = \prod_{j=1}^N \left(\frac{P_j}{\beta_j} \right)^{\beta_j}$. The same for $P_j^M = \prod_{we=1}^N \left(\frac{P_w}{\omega_{ij}} \right)^{\omega_{ij}}$.

The production function of firms in sector j can be expressed as,

$$Z_j^{-\rho} = a_j^{1-\rho} \left(\frac{L_j}{Q_j} \right)^{\rho} + b_j^{1-\rho} \left(\frac{M_j}{Q_j} \right)^{\rho},$$

which combined with the FONC gives

$$P_j^{1-\varepsilon} = Z_j^{\varepsilon-1} (1 + \eta_j \mu_j)^{\varepsilon} \left(a_j w^{1-\varepsilon} (1 + \theta_j^l \mu_j)^{1-\varepsilon} + b_j (P_j^M)^{1-\varepsilon} (1 + \theta_j^m \mu_j)^{1-\varepsilon} \right).$$

When constraints are not binding, we simply have

$$P_1^{1-\varepsilon} = Z_1^{\varepsilon-1} (a w^{1-\varepsilon} + b (P_1^M)^{1-\varepsilon}) \quad (72)$$

$$P_2^{1-\varepsilon} = Z_2^{\varepsilon-1} (a w^{1-\varepsilon} + b (P_2^M)^{1-\varepsilon}) \quad (73)$$

$$\dots\dots\dots \quad (74)$$

$$P_n^{1-\varepsilon} = Z_n^{\varepsilon-1} (a w^{1-\varepsilon} + b (P_n^M)^{1-\varepsilon}). \quad (75)$$

If every sector has the same initial level of productivity, $Z_j = Z$ for all j , then P_j is the same across sectors within a network. The difference between networks lies in the price of the intermediate input bundle. In the sparse network $P_j^M = P_{j-1}$, while in the dense network, from the intermediate input bundle cost minimization problem, we have $P^M = P_j^M = \prod_{we=1}^N \left(\frac{P_{we}}{1/N} \right)^{1/N} = N \cdot P_j$. Therefore, assuming the wage rate is numerarie ($w = 1$), the solutions for prices are

$$P_j = \left(\frac{1 - b Z_j^{\varepsilon-1}}{a Z_j^{\varepsilon-1}} \right)^{\frac{1}{\varepsilon-1}} \quad \text{for all } j \text{ in sparse network}$$

$$P_j = \left(\frac{1 - N^{(1-\varepsilon)} b Z_j^{\varepsilon-1}}{a Z_j^{\varepsilon-1}} \right)^{\frac{1}{\varepsilon-1}} \quad \text{for all } j \text{ in dense network,}$$

provided that $b Z^{\varepsilon-1} < 1$ and $N^{1-\varepsilon} b Z^{\varepsilon-1} < 1$, which ensures that prices are non-negative in the sparse and the dense network, respectively.

With inelastic labor supply $\bar{L} = 1$, zero profits from the firms ($\text{CRS} + \theta^m \neq \theta^l$), and using the households budget constraint we find that real GDP is

$$C = \frac{w}{P}.$$

Sometimes is convenient to define the consumption basket as the numeraire $P = 1$, therefore real GDP equals $w = C$. However, otherwise noted, the wage rate, w , is normalized to 1 and aggregate consumption (or aggregate labor supply with elastic labor) is the real GDP in this economy.

7.1.2 Proof of Proposition 1

With the solutions for prices at hand, we proceed to find real GDP of this economy. Using the households' budget constraint, $PC = w = 1$, and the households' cost minimizing condition over the consumption basket, $P_j C_j = \beta_j PC$, we have

$$C_j = \frac{\beta_j}{P_j},$$

implying that real GDP is

$$GDP = \prod_{j=1}^N \left(\frac{\beta_j}{P_j} \right)^{\beta_j}.$$

Replacing the solutions for prices

$$GDP^{sparse} = \prod_{j=1}^N \left(\beta_j \left(\frac{aZ_j^{\varepsilon-1}}{1 - (1-a)Z_j^{\varepsilon-1}} \right)^{\frac{1}{\varepsilon-1}} \right)^{\beta_j}$$

$$GDP^{dense} = \prod_{j=1}^N \left(\beta_j \left(\frac{aZ_j^{\varepsilon-1}}{1 - N^{1-\varepsilon}(1-a)Z_j^{\varepsilon-1}} \right)^{\frac{1}{\varepsilon-1}} \right)^{\beta_j}$$

$$\log GDP^{sparse} = \sum_{j=1}^N \beta_j \left(\log \beta_j + \frac{1}{\varepsilon-1} [\log(aZ_j^{\varepsilon-1}) - \log(1 - (1-a)Z_j^{\varepsilon-1})] \right)$$

$$\log GDP^{dense} = \sum_{j=1}^N \beta_j \left(\log \beta_j + \frac{1}{\varepsilon-1} [\log(aZ_j^{\varepsilon-1}) - \log(1 - N^{1-\varepsilon}(1-a)Z_j^{\varepsilon-1})] \right),$$

therefore

$$\frac{\partial \log GDP^{sparse}}{\partial \log Z_j} = \frac{\beta_j}{1 - (1-a)Z^{\varepsilon-1}}$$

$$\frac{\partial \log GDP^{dense}}{\partial \log Z_j} = \frac{\beta_j}{1 - N^{1-\varepsilon}(1-a)Z^{\varepsilon-1}}.$$

For prices to be positive recall we need $(1-a)Z^{\varepsilon-1} < 1$ and $N^{1-\varepsilon}(1-a)Z^{\varepsilon-1} < 1$ in the sparse and dense network, respectively. Therefore, when $\varepsilon = 1$, $N^{1-\varepsilon} = 1$, we have

$$\frac{\partial \log GDP^{sparse}}{\partial Z_j} = \frac{\partial \log GDP^{dense}}{\partial Z_j},$$

whenever $\varepsilon > 1$, $N^{1-\varepsilon} < 1$, we obtain

$$\frac{\partial \log GDP^{sparse}}{\partial \log Z_j} > \frac{\partial \log GDP^{dense}}{\partial \log Z_j},$$

while when $\varepsilon < 1$, $N^{1-\varepsilon} > 1$, we have

$$\frac{\partial \log GDP^{sparse}}{\partial Z_j} < \frac{\partial \log GDP^{dense}}{\partial Z_j},$$

which proves Proposition 1.

The link between this result and the result in Hulten (1978), where the size of a sector is the statistic to gauge its aggregate effect, works as follow. From the market clearing conditions of sectoral output we have

$$Q_j = C_j + \sum_i^N M_{ji},$$

multiplying each size of the equation by P_j , assuming $Z_j = 1$ for every j , using the household optimal consumption share for each good, using the firms FONC along with the price solutions, and defining sectoral sales as S_j , we obtain the following for the sparse network

$$S_j = \beta_j PC + \sum_i^N (1-a)\omega_{ji}S_i,$$

and the following for the dense network

$$S_j = \beta_j PC + \sum_i^N N^{1-\varepsilon} (1-a) \omega_{ji} S_i,$$

which matrix form gives the following for the sparse network

$$S = \beta \cdot PC \cdot \mathbf{1}' [I - (1-a)\Omega']^{-1},$$

and the following for the dense network

$$S = \beta \cdot PC \cdot \mathbf{1}' [I - N^{1-\varepsilon} (1-a)\Omega']^{-1}.$$

From these results it is easy to show that the ratio of sales of a sector j to nominal GDP (PC) is $s_j = \frac{P_j Q_j}{PC} = \frac{\beta_j}{a}$ in the sparse network and $s_j = \frac{P_j Q_j}{PC} = \frac{\beta_j}{1 - N^{1-\varepsilon}(1-a)}$ in the dense, which corresponds exactly to $\frac{\partial \log GDP^k}{\partial \log Z_j}$ in each network k , around the point where $Z_j = 1$ for all j .

7.1.3 Proof of Proposition 2

From the firms' FONC we can rewrite the inputs shares over sales as

$$\begin{aligned} \frac{wL_j}{P_j Q_j} &= \frac{P_j^{\varepsilon-1} Z_j^{\varepsilon-1} a}{w^{\varepsilon-1} (1 + \theta_j^l \mu_j)^\varepsilon} \\ \frac{P_j^M M_j}{P_j Q_j} &= \frac{P_j^{\varepsilon-1} Z_j^{\varepsilon-1} b}{(P_j^M)^{\varepsilon-1} (1 + \theta_j^m \mu_j)^\varepsilon}. \end{aligned}$$

we focus on two extreme cases, when we) $\theta_j^m = 1$ and $\theta_j^l = 0$ and when ii) $\theta_j^m = 0$ and $\theta_j^l = 1$. In case we) if a sector j becomes just constrained, so $\mu_j = 0$ yet, the threshold value of η such that a sector j becomes constrained is a function of the sectors productivity, the elasticity, the importance of intermediate inputs in production, and the number of sectors (only in the dense network). Thus, the threshold η for each network is

$$\begin{aligned} \hat{\eta}_j &= b Z_j^{\varepsilon-1} && \text{sparse network} \\ \hat{\eta}_j &= N^{1-\varepsilon} b Z_j^{\varepsilon-1} && \text{dense network.} \end{aligned}$$

The same holds for the case ii) $\theta_j^m = 0$ and $\theta_j^l = 1$ we have

$$\begin{aligned}\hat{\eta}_j &= 1 - bZ_j^{\varepsilon-1} && \text{sparse network} \\ \hat{\eta}_j &= 1 - N^{1-\varepsilon}bZ_j^{\varepsilon-1} && \text{dense network.}\end{aligned}$$

The assumption that each sector has the same initial $Z_j = Z$, implies that the threshold for the collateral constraint parameter $\hat{\eta}$ is the same for every sector j , within a network k .

We want to study how a shock to sector j that drives sector $j+1$ to become constrained – such that $\mu_{j+1} = 0$ yet – affects other sectors constraint. Hence, for the case we) sector $j+1$ starts unconstrained such that $\frac{P_{j+1}^M M_{j+1}}{P_{j+1} Q_{j+1}} < \hat{\eta}(1 + \chi)$, however, a shock to sector j alters P_{j+1}^M / P_{j+1} driving the sector $j+1$ constraint to bind

$$\frac{P_{j+1}^M M_{j+1}}{P_{j+1} Q_{j+1}} = \hat{\eta}(1 + \chi). \quad (76)$$

Combining the previous equation with the sector $j+1$ FONC for intermediate inputs, when $\mu_{j+1} = 0$ yet, and the value for $\hat{\eta}$, we have

$$\begin{aligned}\left(\frac{P_j}{P_{j+1}}\right)^{1-\varepsilon} &= 1 + \chi && \text{sparse network} \\ \left(\frac{P_{j+1}^M}{P_{j+1}}\right)^{1-\varepsilon} &= N^{1-\varepsilon}(1 + \chi) && \text{dense network.}\end{aligned}$$

Plugging in these equations in (72) we obtain

$$\begin{aligned}P_{j+1} &= \left(\frac{1 - (1 + \chi)Z^{\varepsilon-1}(1 - a)}{(1 + \chi)aZ^{\varepsilon-1}}\right)^{\frac{1}{\varepsilon-1}} && \text{sparse network} \\ P_{j+1}^M &= \left(\frac{1 - N^{1-\varepsilon}(1 + \chi)Z^{\varepsilon-1}(1 - a)}{N^{1-\varepsilon}(1 + \chi)aZ^{\varepsilon-1}}\right)^{\frac{1}{\varepsilon-1}} && \text{dense network.}\end{aligned}$$

Use the fact the in the dense network $P^M = P_1^M = P_2^M = P_3^M$ while in the sparse network

$P_j^M = P_{j-1}$ to find

$$\begin{aligned} P_{j+1} &= \left(\frac{1 - (1 + \chi)Z^{\varepsilon-1}(1-a)}{aZ^{\varepsilon-1}} \right)^{\frac{1}{\varepsilon-1}} && \text{sparse network} \\ P_{j+1} &= \left(\frac{1 - N^{1-\varepsilon}(1 + \chi)Z^{\varepsilon-1}(1-a)}{aZ^{\varepsilon-1}} \right)^{\frac{1}{\varepsilon-1}} && \text{dense network} \end{aligned}$$

$$P_{j+3} = \left(\frac{1 - (1 + \chi)Z^{\varepsilon-1}(1-a)}{Z^{\varepsilon-1}[a - Z^{\varepsilon-1}\chi a(1-a)]} \right)^{\frac{1}{\varepsilon-1}} \quad \text{sparse network.}$$

From the price equations for the dense network is easy to see that $P_{j+2} = P_{j+3}$, which implies that

$$P_j = \left(\frac{P^M}{3P_2^{2/3}} \right)^3 = \frac{((1 + \chi)^N a)^{\frac{1}{1-\varepsilon}}}{Z(1 - N^{1-\varepsilon}(1 + \chi)Z^{1-\varepsilon}b)^{\frac{1}{1-\varepsilon}}} \quad \text{dense network.}$$

Having solved for all prices we proceed to show that in the dense network, unlike the sparse network, every sector becomes constrained given a shock $\tilde{Z}_j$ to sector j that drives the constraint of sector $j+1$ to bind. By construction, we just found the equilibrium prices such that a shock in sector j drives the constraint in sector $j+1$ to bind. we now show that in the dense network all other sectors become constrained as well, while in the sparse network the shock does not amplify further and only affect sectors j and $j+1$.

From the FONC, assuming common elasticities, we have

$$\frac{P_{j+2}^M M_{j+2}}{P_{j+2} Q_{j+2}} = \frac{P_{j+2}^{\varepsilon-1} Z_{j+2}^{\varepsilon-1} b}{(P_{j+2}^M)^{\varepsilon-1}}$$

Replacing the solution for prices it is immediate that in the sparse network, sector 3 leverage is

$$\frac{P_{j+2}^M M_{j+2}}{P_{j+2} Q_{j+2}} < \hat{\eta}(1 + \chi),$$

while in the dense network

$$\frac{P_{j+2}^M M_{j+2}}{P_{j+2} Q_{j+2}} = \hat{\eta}(1 + \chi).$$

This proves that in the dense network amplifies the shock by making sector $j+2$ constrained

as well. Given the symmetric structure of the dense network and the structure of connections, it can be shown that all the other sectors in the dense network become constrained as well.

7.1.4 Density and Downturns Substitute Inputs

In this case, $\theta^l = 1$ and $\theta^m = 0$. Let w be 1. It can be shown that a smaller productivity shock to sector j is needed to make sector $j + 1$ constrained in the dense network. Also, with financial shocks, a smaller shock in sector j is needed in the sparse network to make sector $j + 1$ and $j + 2$ constrained.

we assume that each sector has a collateral constraint parameter equal to $\hat{\eta}^k$ (the threshold where sectors become constrained in network k). Using the FONC of firms and the binding constraint for sector j and $j + 1$, we have

$$\frac{L_j}{P_j Q_j} = P_j^{\varepsilon-1} Z_j^{\varepsilon-1} a = (1 + \mu_j)^\varepsilon [1 - Z_{j,0}^{\varepsilon-1} b] (1 + \chi) \quad \text{sparse network} \quad (77)$$

$$\frac{L_j}{P_j Q_j} = P_j^{\varepsilon-1} Z_j^{\varepsilon-1} a = (1 + \mu_j)^\varepsilon [1 - N^{1-\varepsilon} Z_{j,0}^{\varepsilon-1} b] (1 + \chi) \quad \text{dense network,} \quad (78)$$

$$(79)$$

where $Z_{j,0}$ is the productivity before the shock (common for all sectors). Unlike proposition 3, in this case we have to deal with the Lagrange multiplier of the collateral constraint in sector j . This is because even though in the sparse network smaller shocks are needed for the propagation of the constraints, sectors do not become constrained together. Using 72 we have

$$P_j^{1-\varepsilon} = Z_j^{\varepsilon-1} (1 + \hat{\eta}^{sparse} \mu_j)^\varepsilon (a(1 + \mu_j)^{1-\varepsilon} + b(P_{j-1})^{1-\varepsilon} (1 + \mu_j)^{1-\varepsilon}) \quad \text{sparse network} \quad (80)$$

$$P_j^{1-\varepsilon} = Z_j^{\varepsilon-1} (1 + \hat{\eta}^{dense} \mu_j)^\varepsilon (a(1 + \mu_j)^{1-\varepsilon} + b(P_j^M)^{1-\varepsilon} (1 + \mu_j)^{1-\varepsilon}) \quad \text{dense network.} \quad (81)$$

As sector $j + 1$ becomes constrained in the threshold we have that P_{j+1} is

$$\begin{aligned} P_{j+1}^{\varepsilon-1} &= \frac{[1 - Z_{j+1}^{\varepsilon-1}b](1 + \chi)}{Z_{j+1}^{\varepsilon-1}a} && \text{sparse network} \\ P_{j+1}^{\varepsilon-1} &= \frac{[1 - N^{1-\varepsilon}Z_{j+1}^{\varepsilon-1}b](1 + \chi)}{Z_{j+1}^{\varepsilon-1}a} && \text{dense network.} \end{aligned}$$

Then, we can find P_j in the sparse network,

$$P_{j+1}^{1-\varepsilon} = Z_{j+1}^{\varepsilon-1}(a + b(P_j)^{1-\varepsilon}),$$

$$P_j^{1-\varepsilon} = \frac{a}{b} \left(\frac{1 - (1 - Z_{j+1}^{\varepsilon-1})(1 + \chi)}{(1 - Z_{j+1}^{\varepsilon-1})(1 + \chi)} \right). \quad (82)$$

To find P_{j-1} we use the fact that sectors $j - 1$ and $j - 2$ are unconstrained and that $Z_{j-1} = Z_{j-2}$, therefore

$$P_{j-1} = \left(\frac{1 - bZ_{j-1}^{\varepsilon-1}}{aZ_{j-1}^{\varepsilon-1}} \right)^{\frac{1}{\varepsilon-1}}. \quad (83)$$

we assume that all Z_{we} with $we \neq j$ have the same value. Combining (21), (22), (23), and (24) we have a system of two equations to solve for Z_j and μ_j .

In the dense network, $P_j^M = P^M = N \cdot P_j^{1/n} \prod_{we \neq j}^N P_{we}^{1/n}$. As only $\mu_j > 0$, it holds that

$$P_i^{\varepsilon-1} = \frac{[1 - N^{1-\varepsilon}Z_{we}^{\varepsilon-1}b](1 + \chi)}{Z_{we}^{\varepsilon-1}a},$$

for $we \neq j$. Also from 72 we know

$$P_{j+1}^{1-\varepsilon} = \frac{Z_{we}^{\varepsilon-1} a}{[1 - N^{1-\varepsilon} Z_{we}^{\varepsilon-1} b](1 + \chi)} = Z_{j+1}^{\varepsilon-1} (a + b(P^M)^{1-\varepsilon}) \quad (84)$$

$$P_{j+2}^{1-\varepsilon} = \frac{Z_{we}^{\varepsilon-1} a}{[1 - N^{1-\varepsilon} Z_{we}^{\varepsilon-1} b](1 + \chi)} = Z_{j+2}^{\varepsilon-1} (a + b(P^M)^{1-\varepsilon}), \quad (85)$$

$$(86)$$

then

$$P^M = \frac{a(1 - (1 - N^{1-\varepsilon} Z_i^{\varepsilon-1} b)(1 + \chi))}{b(1 - N^{1-\varepsilon} Z_i^{\varepsilon-1} b)(1 + \chi)}.$$

Combining (21), (22) and (25) we have a system of equations for Z_j (the shock on sector j that drives the constraint of sector $j + 1$ to bind) and μ_j .

7.1.5 General Model in Matrices

Assumptions: $\psi = \varepsilon_d = \varepsilon_m = 1$. To simplify notation let P_C be the price index for the consumption bundle, $C_C = \prod C_j^{\beta_j}$ be the consumption bundle, L_L be total labor supply.

$$wC_C^{-\sigma} = P_C L_L^{\xi} \quad (87)$$

$$P_j C_j = \beta_j P_C C_C \quad (88)$$

$$P_j Z_j^{\rho} \left(\frac{a_j Q_j}{L_j} \right)^{1-\rho} (1 + \eta_j \mu_j) = w(1 + \theta_j^l \mu_j) \quad (89)$$

$$P_j Z_j^{\rho} \left(\frac{(1 - a_j) Q_j}{M_j} \right)^{1-\rho_j} (1 + \eta_j \mu_j) = P_j^M (1 + \theta_j^m \mu_j) \quad (90)$$

$$\max(-\psi_j, 0)^2 - (\eta_j P_j Q_j - \theta_j^m P_j^M M_j - \theta_j^l w L_j) = 0, \quad (91)$$

$$P_{we} M_{ij} = \omega_{ij} P_j^M M_j \quad (92)$$

$$Q_j = C_j + \sum_{we=1}^N M_{ji} \quad (93)$$

where $\mu_j = \max(\psi_j, 0)^2$, $C = \prod C_j^{\beta_j}$, $P_j M_{ij} = \omega_{ij} P_j^M M_j$, $P_j^M = \prod \left(\frac{P_{we}}{\omega_{ij}} \right)^{\omega_{ij}}$, $P = \prod \left(\frac{P_j}{\beta_j} \right)^{\beta_j}$.

Combining equations (94) and (95) yields

$$\log P + \log C - \log \beta + \mathbf{1} \frac{\xi}{\sigma} \log L_L - \mathbf{1} \log w - \frac{(\sigma - 1)}{\sigma} \mathbf{1} [\beta' \log P - \beta' \log \beta] = 0, \quad (94)$$

where $P = [P_1 P_2 \dots P_n]'$, $C = [C_1 C_2 \dots C_n]'$, $\beta = [\beta_1 \beta_2 \dots \beta_n]'$, $\log C_c = \beta' \log C$, and $P_C = \beta \log P - \beta \otimes \log \beta$

The N FONC conditions for labor demand can be written as (assume $w = 1$)

$$\log P + \rho \odot \log Z + (\mathbf{1} - \rho) \odot [\log a + \log Q - \log L] + \log(1 + \eta \odot \mu) - \log(1 + \theta^l \odot \mu) - \mathbf{1} \log w = 0, \quad (95)$$

where $a = [a_1 a_2 \dots a_n]'$, $\mu = [\mu_1 \mu_2 \dots \mu_n]'$, $Q = [Q_1 Q_2 \dots Q_n]'$, $\theta^l = [\theta_1^l \theta_2^l \dots \theta_n^l]'$, $\rho = [\frac{\varepsilon_1 - 1}{\varepsilon_1} \dots \frac{\varepsilon_n - 1}{\varepsilon_n}]'$, and $\odot$ represents the Hadamard entry-wise product.

The FONCs for materials become

$$\log P - \log P^M + \rho \odot \log Z + (\mathbf{1} - \rho) \odot [\log(\mathbf{1} - a) + \log Q - \log M] + \log[\mathbf{1} + \eta \odot \mu] - \log[\mathbf{1} + \theta^m \odot \mu] = 0, \quad (96)$$

where $M = [M_1 M_2 \dots M_n]$ and $\log P^M = \Omega' \log P - \text{diag}(\Omega' \cdot \log \Omega)$

The composition of each sector's total intermediates is determined by

$$M_{rep} \log P + \log MM - \text{vec}(\log \Omega') - M_p \log P^M - M_p \log M = 0, \quad (97)$$

with

$MM_{n^2 \times 1} = [M_{11} M_{12} \dots M_{1N} \dots M_{n1} M_{n2} \dots M_{nn}]'$, and $\text{Vec}(\Omega') = [\omega_{11} \omega_{12} \dots \omega_{1N} \dots \omega_{n1} \omega_{n2} \dots \omega_{nn}]'$ and $M_p = \mathbf{1}_{n \times 1} \otimes I_{n \times n}$ and $M_{rep} = I_{n \times n} \otimes \mathbf{1}$, where $\otimes$ is the Kronecker product.

The Kuhn Tucker conditions for the inequality constraints

$$\tilde{\mu} - [\eta \odot P \odot Q - \theta^l \odot w \odot L - \theta^m \odot P^M \odot M] = 0, \quad (98)$$

The market clearing condition for goods market

$$Q - C - M_s = 0, \quad (99)$$

where M_s is a vector of dimension $N \times 1$ obtained from MM. $M_s = [\sum_i^N m_{1i} \dots \sum_i^N m_{Ni}]$

where (94) are the N FONCs for consumption varieties for the representative household, (95) are the labor demand FONCs for firms in each sector, (96) are the N FONCs for total

material demand, (97) are the N^2 conditions for intermediates shares on total intermediate expenditures, (98) are the N conditions for the complementary slackness of the inequality borrowing constraints to hold, and (99) are the market clearing conditions for each sector price. Adding the labor market clearing condition and the production function for each sector we have $6N + N^2 + 1$ equations and unknowns $(C_j, L_j, M_j, M_{ij}, P_j, L_L, Q_j, \mu_j)_{j=1}^N$

The vector of Lagrange multipliers for the collateral constraint are $\mu = \max(\vec{0}, \psi)^2$ and $\tilde{\mu} = \max(\vec{0}, -\psi)^2$.

7.2 Mathematical Appendix Chapter 2

7.2.1 Proof of Proposition 3

7.2.2 Proof of Proposition 4

7.3 Mathematical Appendix Chapter 3

The first order and necessary conditions for this problem are:

$$C_{jt} : C_{jt}^{-\sigma} = \lambda_{jt} \quad (100)$$

$$L_{jt} : \psi = \lambda_{jt} \eta_j \frac{Y_{jt}}{L_{jt}} \quad (101)$$

$$M_{ijt} : \lambda_{jt} \gamma_j \frac{Y_{jt}}{M_{ijt}} = \lambda_{it} \quad (102)$$

$$X_{jt} : \lambda_{jt} = \mu_t \theta_j \frac{I_t}{X_{jt}} \quad (103)$$

$$K_{t+1} : \mu_t = E_t \beta [\mu_{t+1} (1 - \delta) + v_{t+1}] \quad (104)$$

$$k_{jt} : \lambda_{jt} \alpha_j Y_{jt} k_{jt}^\rho = v_t (K_t)^{1+\rho} \quad (105)$$

Implying

$$K_{t+1} : \mu_t = E_t \beta \left[\mu_{t+1}(1 - \delta) + \lambda_{jt+1} \alpha_j \frac{Y_{jt+1} k_{jt+1}^\rho}{K_{t+1}^{1+\rho}} \right] \quad (106)$$

7.3.1 Steady State

Imposing steady state and taking logs

$$\ln X_j = \ln \mu - \ln \lambda_j + \ln \theta_j + \ln I \quad (107)$$

$$\ln I = \sum_{j=1}^N \theta_j \ln X_j \quad (108)$$

$$\ln k_j = \frac{1}{\rho} \left[\ln \mu - \ln(Y_j) + \ln \frac{[1 - \beta(1 - \delta)]}{\beta \alpha_j} - \ln \lambda_j + (1 + \rho) \ln K \right] \quad (109)$$

$$\ln(L_j) = -\ln(\psi) + \ln(\lambda_j) + \ln(Y_j) + \ln(\eta_j) \quad (110)$$

$$\ln(M_{ij}) = \ln(\lambda_j) - \ln(\lambda_i) + \ln(\gamma_{ij}) + \ln(Y_j) \quad (111)$$

$$\ln(Y_j) = \ln(Z_j) + \alpha_j \ln(k_j) + \sum_{we=1}^N \gamma_{ij} \ln(X_{ij}) + \eta_j \ln(L_j) \quad (112)$$

In matrices, and combining (107) and (108)

$$\ln I = \theta' (1_{N \times 1} \ln \mu - \ln \lambda + \ln \theta + 1_{N \times 1} \ln I)$$

$$\ln I (1 - \theta' 1_{N \times 1}) = \theta' (1_{N \times 1} \ln \mu - \ln \lambda + \ln \theta)$$

By CRS, the left hand side is zero, so

$$\ln \mu = \theta' \ln \lambda - \theta' \ln \theta \quad (113)$$

Using (113) in (109)

$$\ln k = \frac{1}{\rho} \left[1_{Nx1} (\theta' \ln \lambda - \theta' \ln \theta) - \ln Y + \ln C_\beta - \ln \lambda + (1 + \rho) 1_{Nx1} \ln K \right]$$

$$\ln k = \frac{1}{\rho} \left[(1_{Nx1} \theta' - I_N) \ln \lambda - 1_{Nx1} \theta' \ln \theta - \ln Y + \ln C_\beta + (1 + \rho) 1_{Nx1} \ln K \right] \quad (114)$$

$$l = -\ln \psi + \ln \lambda + y + \ln \bar{\eta} \quad (115)$$

$$m = M_\lambda \ln \lambda + \text{vec}(\ln(\Gamma')) + M_y y \quad (116)$$

$$\ln Y = \ln Z + \alpha_d \ln k + \bar{\Gamma} \ln M + \eta \ln L \quad (117)$$

Where $\eta = I_n - \alpha_d - \Sigma_\gamma$, $C_\beta = \frac{1-\beta(1-\delta)}{\beta\alpha}$, $\bar{\eta} = \eta 1_{nx1}$, $M_\lambda = 1_{Nx1} \otimes I_N - I_N \otimes 1_{Nx1}$, $M_y = 1_{Nx1} \otimes I_N$, and m is a $N^2 \times 1$ vector that contains sales from sector 1 to all the other in the first n rows (M_{ji} for all we) and so on.

$$\bar{\Gamma}_{n \times n^2} = \begin{bmatrix} \gamma_{11} & 0 & \cdots & \gamma_{21} & 0 & \cdots \\ 0 & \gamma_{12} & 0 & \vdots & \gamma_{22} & 0 \\ \cdots & \cdots & \gamma_{1n} & 0 & \cdots & \gamma_{nn} \end{bmatrix}, \text{vec}(\ln(\Gamma'))_{n^2 \times 1} = \begin{bmatrix} \gamma_{11} \\ \gamma_{12} \\ \gamma_{13} \\ \vdots \end{bmatrix}, \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \end{bmatrix}$$

$$\alpha_d = \begin{bmatrix} \alpha_1 & \cdots & 0 \\ \vdots & \alpha_k & \vdots \\ 0 & \cdots & \alpha_n \end{bmatrix}, \alpha = \begin{bmatrix} \alpha_1 \\ \alpha_k \\ \alpha_n \end{bmatrix}, \Sigma_{\gamma_{n \times n}} = \begin{bmatrix} \sum_{we=1}^N \gamma_{i1} & \cdots & 0 \\ \vdots & \sum_{we=1}^N \gamma_{ik} & \vdots \\ 0 & \cdots & \sum_{we=1}^N \gamma_{in} \end{bmatrix}$$

So the $\bar{\Gamma}$ matrix gives every sectors uses of inputs from all other sectors (M_{ij} for all we).

7.3.2 Special Case: $\rho = -1$

Replacing optimal decisions for l, x, k in the production functions we get:

$$\ln Y = \ln Z + \alpha_d \left[\ln Y + (I_N - 1_{Nx1} \theta') \ln \lambda + 1_{Nx1} \theta' \ln \theta - \ln C_\beta \right] + \bar{\Gamma} (M_\lambda \ln \lambda + \text{vec}(\ln \Gamma^T) + M_y \ln Y)$$

$$+\eta(-\ln\psi + \ln\lambda + \ln Y + \ln\bar{\eta}).T$$

$$(I_N - \alpha_d - \bar{\Gamma}M_y - \eta)\ln Y = \ln Z + \alpha_d \left[(I_N - 1_{Nx1}\theta')\ln\lambda + 1_{Nx1}\theta'\ln\theta - \ln C_\beta \right] +$$

$$\bar{\Gamma}(M_\lambda \ln\lambda + \text{vec}(\ln\Gamma^T)) + \eta(-\ln\psi + \ln\lambda + \ln\bar{\eta}).$$

By CRS,

$$0_{n \times n} = \ln Z + \alpha_d \left[(I_N - 1_{Nx1}\theta')\ln\lambda + 1_{Nx1}\theta'\ln\theta - \ln C_\beta \right] + [\bar{\Gamma}M_\lambda + \eta]\ln\lambda + \bar{\Gamma}(\text{vec}(\ln\Gamma^T)) + \eta(-\ln\psi + \ln\bar{\eta}).$$

Obtaining a closed form solution for the vector of λ ,

$$\ln\lambda = -(\alpha_d(I_N - 1_{Nx1}\theta') + \bar{\Gamma}M_\lambda + \eta)^{-1} [\ln Z + \alpha_d(1_{Nx1}\theta'\ln\theta - \ln C_\beta) + \bar{\Gamma}(\text{vec}(\ln\Gamma^T)) + \eta(-\ln\psi + \ln\bar{\eta})].$$

To solve for Y, we use the sectoral resource constraints

$$Y = C + MM + X.$$

From the FOC of investment goods we have

$$X = M_{XI}I,$$

$$\text{where } M_{XI} = \begin{bmatrix} \theta_1 \frac{\mu}{\lambda_1} \\ \vdots \\ \theta_n \frac{\mu}{\lambda_n} \end{bmatrix}$$

$$Y = \lambda^{-1/\sigma} + M_{my}Y + M_{XI}I$$

In the steady state, the investment is

$$I = \delta K = \delta 1_{1 \times N}k = \delta 1_{1 \times N} \exp((I_N - 1_{Nx1}\theta')\ln\lambda + \ln Y + 1_{Nx1}\theta'\ln\theta' - \ln C_\beta) = \delta 1_{1 \times N} C_{exp} Y$$

Replacing this back in the resource constraint

$$Y = \lambda^{-1/\sigma} + M_{my}Y + \delta M_{XI} 1_{lxN} C_{exp} Y,$$

so

$$Y = (I_n - M_{my} - \delta M_{XI} 1_{lxN} C_{exp})^{-1} \lambda^{-1/\sigma}. \quad (118)$$

As in Foerster et al. (2011), the rest of the variables can be backed out easily using the rest of the optimality conditions.

7.3.3 General Case $\rho \neq 1$

Replacing optimal decisions for l, x, k we got:

$$\ln Y = \ln Z + \frac{\alpha_d}{\rho} \left[1_{Nx1} \ln \mu - \ln Y - \ln \lambda + (1 + \rho) \ln K_{nx1} + \ln C_\beta \right] +$$

$$\bar{\Gamma}(M_\lambda \ln \lambda + \text{vec}(\ln \Gamma^T) + M_y \ln Y) + \eta(-\ln \psi + \ln \lambda + \ln Y + \ln \bar{\eta})$$

$$B \ln Y = z + \frac{\alpha_d}{\rho} \left[(1_{Nx1} \theta' - I_N) \ln \lambda - 1_{Nx1} \theta' \ln \theta + \ln C_\beta + (1 + \rho) 1_{Nx1} \ln K \right] +$$

$$\bar{\Gamma}(M_\lambda \ln \lambda + \text{vec}(\ln \Gamma^T)) + \eta(-\ln \psi + \ln \lambda + \ln \bar{\eta})$$

$$B = (I + \frac{\alpha_d}{\rho} - \bar{\Gamma} M_y - \eta)$$

$$\ln Y = B^{-1} z + B^{-1} \frac{\alpha_d}{\rho} \left[(1_{Nx1} \theta' - I_N) \ln \lambda - 1_{Nx1} \theta' \ln \theta + \ln C_\beta + (1 + \rho) 1_{Nx1} \ln K \right] +$$

$$B^{-1} [\bar{\Gamma}(M_\lambda \ln \lambda + \text{vec}(\ln \Gamma^T)) + \eta(-\ln \psi + \ln \lambda + \ln \bar{\eta})]$$

The aggregate resource constraint is:

$$Y = \lambda^{-1/\sigma} + M_{my}Y + M_{XI}I,$$

where

$$I = \delta K,$$

$$Y = \lambda^{-1/\sigma} + M_{my}Y + \delta M_{XI}K,$$

$$Y = (I - M_{my})^{-1}[\lambda^{-1/\sigma} + \delta M_{XI}K]. \quad (119)$$

To find out aggregate capital stock we start by using the Euler equation that determines each sectors demand for capital.

$$k = \exp(\ln k) = \exp\left(\frac{1}{\rho} \left[(1_{Nx1} \theta' - I_N) \ln \lambda - 1_{Nx1} \theta' \ln \theta - \ln Y + \ln C_\beta + (1 + \rho) 1_{Nx1} \ln K \right] \right)$$

$$k = Y^{\frac{-1}{\rho}} (1_{Nx1} K)^{\frac{(1+\rho)}{\rho}} C_\beta^{\frac{1}{\rho}} \exp\left[\frac{1}{\rho} (1_{Nx1} \theta' - I_N) \ln \lambda - 1_{Nx1} \theta' \ln \theta\right]$$

Here $Y * K$ is understood as a multiplication element by element.

So aggregate capital stock is

$$K = \left[1_{1xN} k^{-\rho} \right]^{-1/\rho}$$

$$K = \left[1_{1xN} (Y^{\frac{-1}{\rho}} (1_{Nx1} K)^{\frac{(1+\rho)}{\rho}} C_\beta^{\frac{1}{\rho}} \exp\left[\frac{1}{\rho} (1_{Nx1} \theta' - I_N) \ln \lambda - 1_{Nx1} \theta' \ln \theta\right])^{-\rho} \right]^{-1/\rho}$$

$$K = \left[1_{1xN} Y (1_{Nx1} K)^{-(1+\rho)} C_\beta^{-1} (\exp\left[\frac{1}{\rho} (1_{Nx1} \theta' - I_N) \ln \lambda - 1_{Nx1} \theta' \ln \theta\right])^{-\rho} \right]^{-1/\rho}. \quad (120)$$

Plugging in the equation for Y from the Resource Constraint, we have a system of two matrix equations, one for N λ 's and the other for aggregate capital stock.

$$\begin{aligned} \ln \left[(I - M_{my})^{-1} [\lambda^{-1/\sigma} + \delta M_{XI} K] \right] &= B^{-1} z + B^{-1} \frac{\alpha_d}{\rho} \left[(1_{Nx1} \theta' - I_N) \ln \lambda - 1_{Nx1} \theta' \ln \theta + \ln C_\beta + (1 + \rho) 1_{Nx1} \ln K \right] \\ &+ B^{-1} [\bar{\Gamma}(M_\lambda \ln \lambda + \text{vec}(\ln \Gamma^T)) + \eta(-\ln \psi + \ln \lambda + \ln \bar{\eta})] \end{aligned} \quad (121)$$

$$K = \left[1_{1 \times N} (we - M_{my})^{-1} [\lambda^{-1/\sigma} + \delta M_{XI} K] (1_{Nx1} K)^{-(1+\rho)} C_\beta^{-1} (\exp[\frac{1}{\rho} (1_{Nx1} \theta' - I_N) \ln \lambda - 1_{Nx1} \theta' \ln \theta])^{-\rho} \right]^{-1/\rho} \quad (122)$$

N+1 equations solving for the vector of λ and aggregate capital stock K instead of solving for $4N + N^2 + 2$.

7.3.4 Linearized Model (Level Deviations)

$$\hat{Y}_{jt} = \hat{C}_{jt} + \sum_{we=1}^N \hat{M}_{jit} + \hat{X}_{jt} \quad (123)$$

$$\hat{Z}_{jt+1} = \xi \hat{Z}_{jt} + Z_{jss} \hat{\epsilon}_{jt+1} \quad (124)$$

$$\hat{\mu}_t = \beta(1 - \delta) \hat{\mu}_{t+1} + \beta \hat{v}_{t+1} \quad (125)$$

$$\hat{K}_t = (K^{ss})^{1+\rho} \left[\sum_{j=1}^N (k_j^{ss})^{-\rho-1} \hat{k}_{jt} \right] \quad (126)$$

$$\hat{Y}_{jt} = \frac{Y_{jss}}{Z_{ss}} \hat{Z}_{jt} + \alpha_j \frac{Y_{jss}}{k_{jss}} \hat{k}_{jt} + \phi_j \frac{Y_{jss}}{L_{jss}} \hat{L}_{jt} + \sum_{we=1}^N \gamma_{ij} \frac{Y_{jss}}{M_{ijss}} \hat{M}_{ijt} \quad (127)$$

$$\psi \hat{L}_{jt} = -\sigma C_{jss}^{-\sigma-1} \eta_j Y_{jss} \hat{C}_{jt} + C_{jss}^{-\sigma} \eta_j \hat{Y}_{jt} \quad (128)$$

$$-\sigma \frac{\hat{C}_{it}}{C_{iss}} = -\sigma \frac{\hat{C}_{jt}}{C_{jss}} + \frac{\hat{Y}_{jt}}{Y_{jss}} - \frac{\hat{M}_{ijt}}{M_{ijss}} \quad (129)$$

$$-\sigma \nu K_{ss}^{1+\rho} \frac{\hat{C}_{jt}}{C_{jss}} + \nu K_{ss}^{1+\rho} \frac{\hat{Y}_{jt}}{Y_{jss}} + \rho \nu K_{ss}^{1+\rho} \frac{\hat{k}_{jt}}{k_{jss}} = K_{ss}^{1+\rho} \hat{\nu}_t + (1+\rho) \nu_{ss} K_{ss}^\rho \hat{K}_t \quad (130)$$

$$-\sigma C_{jss}^{\sigma-1} \hat{C}_{jt} = \frac{\theta_j w e_{ss}}{X_{jss}} \hat{\mu}_t + \frac{\theta_j \mu_{ss}}{X_{jss}} \hat{w} e_t - \frac{\theta_j \mu_{ss}}{X_{jss}^2} \hat{X}_{jt} \quad (131)$$

$$\hat{I}_t = \sum_{j=1}^N \frac{\theta_j I_{ss}}{X_{jss}} \hat{X}_t \quad (132)$$

$$\hat{K}_{t+1} = \hat{I}_t + (1-\delta) \hat{K}_t \quad (133)$$

To express these equations in matrix terms, let us assume $L_t = (\hat{L}_{1t}, \hat{L}_{2t}, \dots, \hat{L}_{Nt})$, $Y_t = (\hat{Y}_{1t}, \hat{Y}_{2t}, \dots, \hat{Y}_{Nt})$, $k_t = (\hat{k}_{1t}, \hat{k}_{2t}, \dots, \hat{k}_{Nt})$, etc. Therefore, the system of equations to be solved using Klein's code is:

$$Y_t = C_t + S_m M_t + X_t \quad (134)$$

$$Z_{t+1} = QZ_t + Z_{ss} \epsilon_{t+1} \quad (135)$$

$$\mu_t = \beta(1-\delta)\mu_{t+1} + \beta \nu_{t+1} \quad (136)$$

$$K_t = \phi_{k_{ss}} k_t \quad (137)$$

$$Y_t = \frac{Y_{ss}}{Z_{ss}} Z_t + \alpha_d \frac{Y_{ss}}{k_{ss}} k_t + \frac{Y_{ss}}{L_{ss}} \phi L_t + \bar{\Gamma}_{lev} M_t \quad (138)$$

$$\psi L_t = -\sigma C_{ss}^{\sigma-1} \eta Y_{ss} C_t + C_{ss}^{\sigma-1} \eta Y_t \quad (139)$$

$$M_{mlev} M_t = M_{ylev} Y_t + M_{clev} C_t \quad (140)$$

$$-\sigma v K_{ss}^{1+\rho} \frac{C_t}{C_{ss}} + v K_{ss}^{1+\rho} \frac{Y_t}{Y_{ss}} + \rho v K_{ss}^{1+\rho} \frac{k_t}{k_{ss}} = K_{ss}^{1+\rho} v_t + (1+\rho) v_{ss} K_{ss}^\rho K_t \quad (141)$$

$$-\sigma C_{ss}^{-\sigma-1} C_t = \frac{\theta w e_{ss}}{X_{ss}} 1_{Nx1} \mu_t + \frac{\theta \mu_{ss}}{X_{ss}} 1_{Nx1} w e_t - \frac{\theta \mu_{ss}}{X_{ss}^2} X_t \quad (142)$$

$$I_t = \theta_{IX} X_t \quad (143)$$

$$K_{t+1} = I_t + (1 - \delta) K_t \quad (144)$$

Becoming

$$A \begin{bmatrix} K_{t+1} \\ Z_{t+1} \\ C_{t+1} \\ k_{t+1} \\ Y_{t+1} \\ L_{t+1} \\ M_{t+1} \\ X_{t+1} \\ I_{t+1} \\ \mu_{t+1} \\ v_{t+1} \end{bmatrix} = B \begin{bmatrix} K_t \\ Z_t \\ C_t \\ k_t \\ Y_t \\ L_t \\ M_t \\ X_t \\ I_t \\ \mu_t \\ v_t \end{bmatrix} \quad (145)$$

Where

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & & & & & & & & & \\ 0 & \frac{I_N}{Z_{ss}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta(1-\delta) \\ -\beta & & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & & & & & & & & & \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & & & & & & & & & \end{bmatrix} \quad (146)$$

$$B = \begin{bmatrix} 0 & 0 & Id_n & 0 & -I_N & 0 & S_m & Id_m & 0 & 0 \\ 0 & & & & & & & & & \\ 0 & \frac{Q}{Z_{ss}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & & & & & & & & & \\ -1 & 0 & 0 & \phi_{k_{ss}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & & & & & & & & \\ 0 & \frac{Y_{ss}}{Z_{ss}} & 0 & \alpha_d \frac{Y_{ss}}{k_{ss}} & -I_N & \phi \frac{Y_{ss}}{L_{ss}} & \bar{\Gamma}_{lev} & 0 & 0 & 0 \\ 0 & & & & & & & & & \\ 0 & 0 & \frac{-\sigma \eta Y_{ss}}{C_{ss}^{\sigma+1}} & 0 & \frac{\eta}{C_{ss}^{\sigma}} & -\psi & 0 & 0 & 0 & 0 \\ 0 & & & & & & & & & \\ 0 & 0 & M_{clev} & 0 & M_{ylev} & 0 & -M_{mlev} & 0 & 0 & 0 \\ 0 & & & & & & & & & \\ (1+\rho)v_{ss}K_{ss}^{\rho} & 0 & \frac{\sigma v_{ss}K_{ss}^{1+\rho}}{C_{ss}} & -\frac{\rho v_{ss}K_{ss}^{1+\rho}}{k_{ss}} & -\frac{v_{ss}K_{ss}^{1+\rho}}{Y_{ss}} & 0 & 0 & 0 & 0 & 0 \\ K_{ss}^{1+\rho} & & & & & & & & & \\ 0 & 0 & \sigma C_{ss}^{-\sigma-1} & 0 & 0 & 0 & 0 & -\frac{\theta \mu_{ss}}{X_{ss}^2} & \frac{\theta \mu_{ss} 1_{Nx1}}{X_{ss}} & \frac{\theta we_{ss} 1_{Nx1}}{X_{ss}} \\ 0 & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{IX} & -1 & 0 \\ 0 & & & & & & & & & \\ (1-\delta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & & & & & & & & & \end{bmatrix} \quad (147)$$

Where $M_{\sigma\delta} = \frac{M_{1c}\sigma\beta}{C_{Nss}^{\sigma+1}(1-\delta)^{-1}}$, $M_{\sigma c_n} = M_{1c}\sigma C_{Nss}^{-\sigma-1}$, $S_m = I_{NxN} \otimes 1'_{Nx1}$, $M_{clev} = -\sigma \text{diag}(M_{ss})M_{m\lambda} \text{diag}(C_{ss}^{-1})$ and $M_y = 1_{Nx1} \otimes I_N$ and $M_{Nc} = 1_{Nx1}[0, 0, 0, \dots, 1]$, $\phi_{k_{ss}} = K_{ss}^{1+\rho}[\phi_1 k_{1,ss}^{-\rho-1} \dots \phi_N k_{N,ss}^{-\rho-1}]$, and S_I is a N by 1 matrix full of zeros except the element (N,1) which is $\frac{I_N}{Y_N} = \frac{\delta K}{Y_N}$. Note that K_t and v_t are scalars and no vectors. Q has in its diagonal the persistence of sectors' productivity. M_{1c} zeros(1,N) with the element (1,N) being 1.

7.3.5 Times Series Representation of the Model

The policy functions from solving (146) are

$$C_t = \Pi_{ck}K_t + \Pi_{cz}Z_t \quad (148)$$

$$k_t = \Pi_{kk}K_t + \Pi_{kz}Z_t \quad (149)$$

$$K_{t+1} = m_k K_t + m_z Z_t. \quad (150)$$

Here m_k is a scalar, m_z a $1 \times N$ vector that maps today's sectoral productivity shocks to optimal tomorrow's aggregate capital stock. The Π_{ij} are also $N \times N$ matrices that map states to the optimal rule for consumption and capital in every sector (Π_{kk} and Π_{ck} are diagonal matrices).

Combining equations (138), (139), and (140),

$$Y_t = \alpha_d^{-1} Z_t + k_t + \alpha_d^{-1} Q_c C_t,$$

in which $Q_c = \tilde{\Gamma} M_c - \sigma \phi$.

Replacing policy rules for control variables we have,

$$Y_t = \alpha_d^{-1} Z_t + [\Pi_{kk}K_t + \Pi_{kz}Z_t] + \alpha_d^{-1} Q_c [\Pi_{ck}K_t + \Pi_{cz}Z_t]$$

$$Y_t = [\Pi_{kk} + \alpha_d^{-1} Q_c \Pi_{ck}] K_t + [\alpha_d^{-1} + \Pi_{kz} + \alpha_d^{-1} Q_c \Pi_{cz}] Z_t$$

$$Y_t = B_K K_t + B_z Z_t$$

$$K_t = B_K^{-1} Y_t - B_K^{-1} B_z Z_t$$

In t+1 we also have

$$Y_{t+1} = B_K K_{t+1} + B_z Z_{t+1}.$$

Then use the policy function for aggregate capital in matrix terms: $K_{t+1} = M_k K_t + M_z Z_t$. With K a $N \times 1$ vector with aggregate K repeated in every row, M_z is a $N \times N$ matrix with N rows of vector m_z in it, and M_k a diagonal $N \times N$ with scalar m_k in it.

$$Y_{t+1} = B_K [M_k K_t + M_z Z_t] + B_z Z_{t+1}$$

$$Y_{t+1} = B_K [M_k (B_K^{-1} Y_t - B_K^{-1} B_z Z_t) + M_z Z_t] + B_z Z_{t+1}$$

$$Y_{t+1} = B_K M_k B_K^{-1} Y_t + B_k [M_z - M_k B_K^{-1} B_z] Z_t + B_z Z_{t+1}$$

$$Y_{t+1} = \rho Y_t + \Xi Z_t + B_z Z_{t+1}. \quad (151)$$

When $\xi = 1$, change in output is

$$\Delta Y_{t+1} = \rho \Delta Y_t + \Xi \varepsilon_t + B_z \varepsilon_{t+1}. \quad (152)$$

7.4 Mathematical Appendix Chapter 4

Static Two Sector Cobb-Douglas

The Planner maximizes households utility subject to technologies, collateral constraints, and the FONC from the firms to pin down the value of collateral. The Kuhn tucker conditions are:

$$C : U'(C) - \lambda_2 + \mu_2 \eta_2 P - \gamma_2 P - \gamma_3 P = 0 \quad (153)$$

$$x_1 : -\lambda_1 + \lambda_2 \beta_2 \frac{C}{x_1} - \mu_2 p_1 + \mu_1 \eta_1 p_1 - \gamma_1 p_1 + \gamma_3 \frac{p_1}{\phi_2 \beta_2} = 0 \quad (154)$$

$$l_1 : -V'(L) + \lambda_1 \alpha_1 \frac{x_1}{l_1} - \mu_1 w + \gamma_1 \frac{w}{\phi_1 \alpha_1} = 0 \quad (155)$$

$$l_2 : -V'(L) + \lambda_2 \alpha_2 \frac{C}{l_2} - \mu_2 w + \gamma_2 \frac{w}{\phi_2 \alpha_2} = 0 \quad (156)$$

$$P : \mu_2 \eta_2 C - \gamma_2 C - \gamma_3 C = 0 \quad (157)$$

$$p_1 : -\mu_2 x_1 + \mu_1 \eta_1 x_1 - \gamma_1 x_1 + \gamma_3 \frac{x_1}{\phi_2 \beta_2} = 0 \quad (158)$$

$$\mu_2 [\eta_2 P C - w l_2 - p_1 x_1] = 0$$

$$\mu_1 [\eta_1 p_1 x_1 - w l_1] = 0$$

7.4.1 Proof of Proposition 5

Sector 1: $\mathcal{L} = P_1 Z_1 l_1^{\alpha_1} - w(1 - s_1)L_1 + \mu_1(\eta_1 P_1 Z_1 l_1^{\alpha_1} - w(1 - s_1)L_1)$

$$FOC : L_1 : \alpha_1 P_1 \frac{Q_1}{L_1} - w(1 - s_1) + \mu_1 \eta_1 \alpha_1 P_1 \frac{Q_1}{L_1} - \mu_1 w(1 - s_1) = 0$$

$$w(1-s_1)L_1(1+\mu_1) = \alpha_1 P_1 Q_1(1+\mu_1 \eta_1)$$

Sector 2: $\mathcal{L} = PZ_2 L_2^{\alpha_2} Q_1^{\beta_2} - w(1-s_2)L_2 - P_Q 1_Q + \mu_2(\eta_2 PZ_2 L_2^{\alpha_2} Q_1^{\beta_2} - w(1-s_2)L_2 - P_1 Q_1)$

$$FOC : L_2 : \alpha_2 P \frac{C}{L_2} - w(1-s_2) + \mu_2 \eta_2 \alpha_2 P \frac{C}{L_2} - \mu_2 w(1-s_2) = 0$$

$$Q_1 : \beta_2 P \frac{Q_2}{Q_1} - P_1 + \mu_2 \eta_2 \beta_2 P \frac{Q_2}{Q_1} - \mu_2 P_1 = 0$$

$$wL_2(1-s_2)(1+\mu_2) = \alpha_2 PC(1+\mu_2 \eta_2)$$

$$P_1 Q_1(1+\mu_2) = \beta_2 PC(1+\mu_2 \eta_2)$$

The optimality conditions for each sector imply:

$$w(1-s_1)L_1(1+\mu_1) = \alpha_1 P_1 Q_1(1+\mu_1 \eta_1) \quad (159)$$

$$wL_2(1-s_2)(1+\mu_2) = \alpha_2 PC(1+\mu_2 \eta_2) \quad (160)$$

$$P_1 Q_1(1+\mu_2) = \beta_2 PC(1+\mu_2 \eta_2) \quad (161)$$

The optimality conditions for each sector imply:

$$w(1-s_1)L_1 = \psi_1 \alpha_1 P_1 Q_1 \quad (162)$$

$$w(1-s_2)L_2 = \psi_2 \alpha_2 PC \quad (163)$$

$$P_1 Q_1 = \psi_2 \beta_2 PC \quad (164)$$

(4) with (6) give $w(1-s_1)Q_1 = \psi_1 \psi_2 \alpha_1 \beta_2 PY$

The representative consumer will optimize when $U'(C)/V'(L) = P/w$

Combining the firms' FOC and the HH equation the equilibrium is defined by:

$$\frac{\psi_1 \psi_2 \alpha_1 \beta_2 Y}{(1-s_1)L_1} = \frac{w}{P} = \frac{V'(L)}{U'(C)}$$

$$\frac{\psi_2 \alpha_2 Y}{(1-s_2)L_2} = \frac{w}{P} = \frac{V'(L)}{U'(C)}$$

This implies

$$L_1 = \frac{\psi_1 \psi_2 \alpha_1 \beta_2}{(1 - s_1)}$$

$$L_2 = \frac{\psi_2 \alpha_2}{(1 - s_2)}.$$

Therefore, if only firms in the final good sector are constrained, $\psi_1 = 1$, a subsidy $s_1 = s_2 = 1 - \psi_2$ implements the constrained efficient solution.

7.4.2 Proof of Proposition 6

Sector 1: $\mathcal{L} = P_1 Z_1 L_1^{\alpha_1} - w L_1 + \mu_1 (\eta_1 P_1 Z_1 l_1^{\alpha_1} - w L_1)$

$$FOC : l_1 : \alpha_1 P_1 \frac{Q_1}{L_1} - w + \mu_1 \eta_1 \alpha_1 P_1 \frac{Q_1}{L_1} - \mu_1 w = 0$$

$$w L_1 (1 + \mu_1) = \alpha_1 P_1 Q_1 (1 + \mu_1 \eta_1)$$

Sector 2: $\mathcal{L} = P Z_2 L_2^{\alpha_2} Q_1^{\beta_2} - w L_2 - P_1 (1 - s_m) Q_1 + \mu_2 (\eta_2 P Z_2 L_2^{\alpha_2} Q_1^{\beta_2} - w L_2 - P_1 (1 - s_m) Q_1)$

$$FOC : L_2 : \alpha_2 P \frac{C}{L_2} - w + \mu_2 \eta_2 \alpha_2 P \frac{C}{L_2} - \mu_2 w = 0$$

$$Q_1 : \beta_2 P \frac{Q_2}{Q_1} - P_1 (1 - s_m) + \mu_2 \eta_2 \beta_2 P \frac{Q_2}{Q_1} - \mu_2 P_1 (1 - s_m) = 0$$

$$w L_2 (1 + \mu_2) = \alpha_2 P C (1 + \mu_2 \eta_2)$$

$$P_1 (1 - s_m) Q_1 (1 + \mu_2) = \beta_2 P C (1 + \mu_2 \eta_2)$$

The optimality conditions for each sector imply:

$$w L_1 (1 + \mu_1) = \alpha_1 P_1 Q_1 (1 + \mu_1 \eta_1) \tag{165}$$

$$w Q_2 (1 + \mu_2) = \alpha_2 P C (1 + \mu_2 \eta_2) \tag{166}$$

$$P_1 (1 - s_m) Q_1 (1 + \mu_2) = \beta_2 P C (1 + \mu_2 \eta_2) \tag{167}$$

(4) with (6) give $w L_1 = \frac{\psi_1 \psi_2 \alpha_1 \beta_2 P Y}{1 - s_x}$

This implies

$$L_1 = \frac{\psi_1 \psi_2 \alpha_1 \beta_2}{(1 - s_m)}$$

$$L_2 = \psi_2 \alpha_2$$

Therefore, if only firms in the intermediate good are constrained, $\psi_2 = 1$, a subsidy $s_m = 1 - \psi_2$ implements the constrained efficient solution.

7.4.3 Proof of Proposition 7

The relaxed planner's optimal condition for labor in sector j is

$$L_j : \mu_j^P \eta_j P_j \frac{\partial Q_j}{\partial L_j} - \mu_j^P \theta_j^w w + \lambda_j^P \frac{\partial Q_j}{\partial L_j} - V'(L) = 0$$

From the relaxed planner's optimal choice of consumption and leisure, we have $PV'(L) = wU'(C)$. Then, defining $\mu_j^P = \frac{\mu_j U'(C)}{P}$ and using $\lambda_j^P = U'(C) \omega_j \frac{C}{C_j}$, and $\omega_j = \frac{PC}{P_j C_j}$, we have that decentralized FONC for labor is met

$$L_j : P_j \frac{\partial Q_j}{\partial L_j} (1 + \mu_j \eta_j) = w(1 + \mu_j \theta_j^w)$$

The relaxed planner's FOC for materials is

$$M_{ij} : \mu_j^P \eta_j P_j \frac{\partial Q_j}{\partial M_{ij}} - \mu_j^P \theta_{ij}^m P_i + \lambda_j^P \frac{\partial Q_j}{\partial M_{ij}} - \lambda_i^P = 0$$

Using $\mu_j^P = \frac{\mu_j U'(C)}{P}$, $\lambda_j^P = \frac{\lambda_j^P P_j}{P_i}$, and $\frac{\lambda_j^P}{P_i} = \frac{U'(C)}{P}$, we have

$$M_{ij} : P_j \frac{\partial Q_j}{\partial M_{ij}} (1 + \mu_j \eta_j) = P_i (1 + \mu_j \theta_{ij}^m)$$

which is the optimality condition for materials use in competitive equilibrium.

7.4.4 Proof of Proposition 8

From the materials FONC

$$P_1 Z^\rho \left(\frac{(1-a)Q_1}{M_1} \right)^{1-\rho} (1 + \eta \mu_1) = P_1^M (1 + \mu_1)$$

which implies that

$$\left(\frac{P_1}{P_3}\right)^\varepsilon (1-a) Z^{\varepsilon-1} \left(\frac{1+\eta\mu_1}{1+\mu_1}\right)^\varepsilon = \frac{M_1}{Q_1}$$

Replacing this into the relevant partial derivative

$$\frac{\partial \log(\frac{\partial Q_j}{\partial M_j})}{\partial \varepsilon_{Q_j}} = \log\left(\frac{Z_j M_j}{(1-a_j) Q_j}\right) = \log\left(Z_j^\varepsilon \left(\frac{P_j}{P_j^M}\right)^\varepsilon \left(\frac{1+\eta\mu_j}{1+\mu_j}\right)^\varepsilon\right)$$

$$\frac{\partial \log(\frac{\partial Q_j}{\partial M_j})}{\partial \varepsilon_{Q_j}} = \varepsilon \log\left(Z_j \left(\frac{P_j}{P_j^M}\right) \left(\frac{1+\eta\mu_j}{1+\mu_j}\right)\right)$$

Right in threshold where sector j becomes constrained, this term becomes

$$\frac{\partial \log(\frac{\partial Q_j}{\partial M_j})}{\partial \varepsilon_{Q_j}} = \varepsilon \log(Z_j) > 0$$

as $\log(Z_j) = 1$ and $\varepsilon > 0$. In a case where sector 1 is constrained such that μ_j , the relative price increases (?), and the wedge is smaller than 1, so $\log(\exp(1) < 1 > 1)$ still > 0 (triple check)

7.4.5 Proof of Proposition 9

Firms solve

$$\begin{aligned} \mathcal{L} = & P_j Q_j - w L_j - \sum_{i=1}^N P_i M_{ij} + \lambda_j \left(Q_j - Z_{jt} \left[a^{\frac{1}{\varepsilon_{Q_j}}} \left(\left(\frac{K_{jt}}{\alpha_j} \right)^{\alpha_j} \left(\frac{L_{jt}}{1-\alpha_j} \right)^{1-\alpha_j} \right)^{\frac{\varepsilon_{Q_j}-1}{\varepsilon_{Q_j}}} + (1-a)^{\frac{1}{\varepsilon_{Q_j}}} M_{jt}^{\frac{\varepsilon_{Q_j}-1}{\varepsilon_{Q_j}}} \right]^{\frac{\varepsilon_{Q_j}}{\varepsilon_{Q_j}-1}} \right) \\ & + \left(M_j - \left(\sum_{i=1}^N \Gamma^{\frac{1}{\varepsilon_{M_j}}} M_{ij}^{\frac{\varepsilon_{M_j}-1}{\varepsilon_{M_j}}} \right)^{\frac{\varepsilon_{M_j}}{\varepsilon_{M_j}-1}} \right) + \mu_j (\eta_j P_j Q_j - \theta_j^w w L_j - \theta_j^k r K_j - \sum_{i=1}^N \theta_{ij}^m P_i M_{ij}) \end{aligned}$$

The Kuhn-Tucker conditions for the problem are

$$\begin{aligned}
P_j Z_j^{\rho_j} \left(a_j Q_j \left(\frac{\alpha_j}{K_j} \right)^{\alpha_j} \left(\frac{1-\alpha_j}{L_j} \right)^{1-\alpha_j} \right)^{1-\rho_j} \left(\frac{L_j}{(1-\alpha_j)K_j} \right)^{1-\alpha_j} (1 + \eta_j \mu_j) &= r(1 + \theta_j^k \mu_j) \\
P_j Z_j^{\rho_j} \left(a_j Q_j \left(\frac{\alpha_j}{K_j} \right)^{\alpha_j} \left(\frac{1-\alpha_j}{L_j} \right)^{1-\alpha_j} \right)^{1-\rho_j} \left(\frac{K_j}{\alpha_j L_j} \right)^{\alpha_j} (1 + \eta_j \mu_j) &= w(1 + \theta_j^w \mu_j) \\
P_j Z_j^{\rho_j} \left(\frac{(1-a_j)Q_j}{M_j} \right)^{1-\rho_j} \left(\frac{M_j \gamma_{ij}}{M_{ij}} \right)^{1-\rho_j^m} (1 + \eta_j \mu_j) &= P_i(1 + \theta_{ij}^m \mu_j) \\
\max(-\psi_j, 0)^2 - (\eta_j P_j Q_j - \theta_j^w w L_j - \theta_j^k r K_j - \sum_{i=1}^N \theta_{ij}^m P_i M_{ij}) &= 0
\end{aligned}$$

for materials demand it is convenient to write

$$P_j Z_j^{\rho_j} \left(\frac{(1-a_j)Q_j}{M_j} \right)^{1-\rho_j} (1 + \eta_j \mu_j) = P_i(1 + \theta_{ij}^m \mu_j),$$

and $P_i M_{ij} = \gamma_{ij} P_j^M M_j$, $P_j^M = \prod_{i=1}^N \left(\frac{P_i}{\gamma_{ij}} \right)^{\gamma_{ij}}$

The Household problem becomes:

$$\begin{aligned}
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{L_t^{1+\xi}}{1+\xi} + \lambda_t (w_t L_t + r_t K_t^A + \Pi_t - P_t C_t - \sum_{j=1}^N P_{jt} X_{jt}) \right. \\
\left. + v_t ((1-\delta)K_t^A - K_{t+1}^A + \prod_{j=1}^N X_{jt}^{\phi_j}) + \mu_t (C_t - \prod_{j=1}^N C_{jt}^{\omega_j}) \right]
\end{aligned}$$

The FONC are

$$\begin{aligned}
w C^{-\sigma} &= \psi L^{\xi} P \\
\frac{1}{\omega_i} P_i C_i &= \frac{1}{\omega_j} P_j C_j \\
\lambda_t \frac{P_{jt} X_{jt}}{\phi_j X_t} &= \beta E_t [\lambda_{t+1} (r_{t+1} + (1-\delta) \frac{P_{jt+1} X_{jt+1}}{\phi_j X_{t+1}})]
\end{aligned}$$

Where the euler equation becomes

$$1 = E_t \left[\beta \frac{U'(C_{t+1})}{U'(C_t)} \cdot \left(\frac{r_{t+1} + (1-\delta) P_{t+1}^x}{P_t^x} \right) \right]$$