ESSAYS ON FINANCIAL MACROECONOMICS

Yan Chen

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ABSTRACT

Debt plays a crucial role in shaping macroeconomic dynamics, with borrowing constraints amplifying economic shocks, while the currency denomination of debt influences long-term debt issuance patterns and financial stability across global economies. The first chapter investigates the role of different types of borrowing constraints in generating amplification effects. Economies regularly experience episodes during which a significant fraction of agents are borrowing constrained. These constraints give rise to amplification effects, which occasionally generate aggregate demand shortages. I analyzes such amplification effects in a stylized model with both asset- and income-based borrowing constraints. Income-based borrowing amplifies shocks to net worth when there is an aggregate demand shortage, and asset-based borrowing amplifies shocks to asset prices.

The second chapter investigates how macroeconomic stabilization policies can redress the amplification effects. A tax on lenders to subsidize borrowers improves the welfare of borrowers and undermines that of lenders when there is no aggregate demand shortage, but can lead to a Pareto improvement when aggregate demand externalities are large. Liquidity operations can lead to a Pareto improvement independent of whether there is an aggregate demand shortage. If both types of borrowing constraints are present, taxing lenders to subsidize asset-constrained agents rather than

income-constrained agents can improve welfare more. With either type of borrowing constraint, a macroprudential tax on debt issuance, combined with a lump-sum transfer between borrowers and lenders, will result in constrained efficient allocations.

The international currency status of the dollar and the euro underwent significant changes after the Great Financial Crisis. The third chapter identifies the rise of the dollar and the fall of the euro in foreign currency debt issuance in international capital markets by countries whose sovereign currencies are neither the U.S. dollar nor the euro after the Great Financial Crisis. This overall trend is not observed in the evolution of short-term debt, but rather in long-term debt, and the widened gap between dollar debt and euro debt is most pronounced for debt issued by the financial sector and by Emerging Market Economies. A recursive VAR analysis indicates the rise of the dollar and the fall of the euro as a result of growing safe asset demand as the dollar appreciates; and an increase in the issuance of dollar debt by firms in Advanced Economies seeking lower financing cost as yield differential shrinks, which in turn reduced euro debt issuance by both the financial and non-financial sectors in Advanced Economies and Emerging Market Economies.

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Contents

List of Abbreviations

ABCs Asset-Based Borrowing Constraints

AEs Advanced Economies

EBITDA Earnings Before Interest, Taxes, Depreciation and Amortization

EMEs Emerging Market Economies

GFC Great Financial Crisis

IBCs Income-Based Borrowing Constraints

ZLB Zero Lower Bound

Chapter 1

Amplification with Income-Based Versus Asset-Based Borrowing Constraints

1.1 Introduction

The 2008 Great Recession originated from shocks to the financial system but transmitted to the economy as a whole via falling asset prices and declining aggregate demand, partly due to household deleveraging. This chapter studies how debt in the private sector may exacerbate an economic slump by triggering amplification effects: asset-based borrowing constraints (ABCs) and income-based borrowing constraints (IBCs).

ABCs are widely incorporated in macroeconomic models with financial frictions.[1] In these models, agents —either households, financial intermediaries, or firms —face a borrowing constraint that restricts the maximum amount they can borrow to a fraction of the liquidation value of their asset holdings. Small and temporary shocks can have large and persistent effects on real variables through asset price feedback

[1] Classic macroeconomic models with financial frictions, as in Bernanke and Gertler (1989); Bernanke, Gertler, and Gilchrist (1999); Kiyotaki and Moore (1997); Mendoza (2010).

loops.

Although asset-based borrowing constraints seem to play an important role in episodes of deleveraging, empirical evidence has shown that income-based borrowing constraints also play a major role and may at times be more important than asset-based borrowing constraints for macroeconomic dynamics. For example, recent studies find only about 20% of non-financial corporate debt in the US is secured by assets. 80% is borrowed against the value of cash flows from firms' continuing operations. Over 80% of cashflow-based borrowing includes income-based covenants in the contract (Lian and Ma, 2021).[2] Given the importance of IBCs, their implications for macroeconomic stabilization policy have not been well explored in the economic literature. An important question then concerns the different macroeconomic implications of the two types of borrowing constraints and the optimal policy responses when both are present during a deleveraging episode such as the GFC.

IBCs manifest themselves in legally binding income/earning-based covenants specified in debt contracts. Income/earning-based covenants circumscribe borrowers' debt capacity to not exceed a multiple of current earnings. In corporate borrowing, the most relevant measure of current earnings is EBITDA in the past twelve months. EBITDA is earnings before net interest payments, income taxes, depreciation of fixed capital, and amortization of intangible assets. It is the broadest measure of net cash generated by firms' operations to cover taxes and financing cost. In accounting, it is computed by subtracting sales revenue by cost of goods sold and selling, general and administrative expense. The two most frequently used forms of IBCs are based on

[2] Covenants are specified in debt contracts and are legally binding. They prevent borrowers' debt capacity from exceeding a multiple of current income, and covenant infringement will directly lead to technical default and negative debt growth. More details in Lian and Ma (2021).

debt-to-earnings ratio ϕ and interest coverage θ:

$$\text{Debt}_t \leq \phi \text{EBITDA}_t$$
$$r_t \text{Debt}_t \leq \frac{1}{\theta} \text{EBITDA}_t$$

EBITDA can also indirectly affect creditors' decision on debt limit due to its influence on firms' credit ratings. Using loan information from Dealscan and bond information from Fixed Income Securities Database (FISD), Lian and Ma (2021) find in earning-based covenants the median value of ϕ is about 3.5, and θ, 2.5. Chodorow-Reich and Falato (2017) study syndicated loans in the US using data from Shared National Credit Program (SNC). They find loan covenants prohibit the ratio of consolidated senior secured debt to a four-quarter trailing moving average of consolidated EBITDA from exceeding 3 and EBITDA to interest payments ratio from exceeding 4, indicating a ϕ of 3, and a θ of 4. Drechsel (2019) also uses Dealscan database and finds a value of 4.6 for the mode of ϕ in loan covenants.

In loan contracts, an earning-based constraint is often specified in a covenant monitored on a quarterly basis. Covenant infringement will directly lead to technical default, which gives rise to renegotiation or condition changes in existing loans, such as higher interest rates, and affect new loan issuance[3]. In corporate debt issuance, earning-based constraints can be directly binding through financial covenants explicitly written in contracts. Those can be relevant for firms issuing new bonds.

In this chapter, I build a theoretical model to analyze amplification effects with ABCs, with IBCs, and with both types of constraints on households. I capture the potential

[3] The effectiveness of earning-based debt limits in non-financial corporate borrowing has been studied. For example, Lian and Ma (2021) find after violations of earning-based covenants, debt growth becomes negative on average.

for aggregate demand shortages by introducing a zero lower bound (ZLB) on the nominal interest rate.[4] The analytical results of the model with IBCs demonstrate the amplification of shocks to wealth through aggregate demand when the debt limit of borrowers is determined by current income. A fall in income will tighten the borrowing constraint, which reduces the amount of debt borrowers can take on. When they are more constrained in borrowing, borrowers reduce consumption spending, which lowers aggregate demand and production. Therefore, income falls and tightens the borrowing constraint further.

I consider an economy that starts with loose credit conditions in which agents can easily borrow and accumulate debt. An exogenous constraint on borrowing that depends on either an individual's asset holdings or income then forces borrowers to deleverage, which reflects tightened credit conditions in a slump. Because borrowers' issuance of debt is constrained, the interest rate must fall to induce lenders to hold less debt. Deleveraging will have two countervailing effects on aggregate demand. First, it will directly lower borrowers' demand, thus depressing aggregate demand; second, the endogenous fall in the real interest rate will boost aggregate demand. As long as the economy is away from the ZLB, the fall in interest rate fully counteracts the negative effect of deleveraging on aggregate demand, and there is no aggregate demand shortage. Firms can produce output at the efficient level. Otherwise, if the interest rate hits the ZLB, there will be an aggregate demand shortage. Given the lack of demand, firms are forced to scale down production and wages decline. Since borrowers are constrained by their income, lower income tightens the borrowing constraint and further reduces demand, which results in a negative feedback loop. Borrowers do not take

[4] It is sufficient but not necessary to generate demand-driven recessions. An alternative approach is to build a Bewley type of heterogeneous agents with incomplete market model as in Guerrieri and Lorenzoni (2017), but at a cost of analytically tractable results of amplification.

into consideration the adverse effect of their behavior on aggregate demand, which lowers production and wages during deleveraging. This leads to aggregate demand externalities.

When there is no aggregate demand shortage in an IBC model, the fall in interest rates generates wealth redistribution between borrowers and lenders, which renders borrowers better off and lenders worse off, but it does not generate any inefficiencies in the economy. Allocation in an IBC economy when there is no aggregate demand shortage is therefore constrained efficient. In an ABC economy, however, amplification through asset price will cause inefficiencies when there is no aggregate demand shortage. Deleveraging by asset-based borrowers depresses asset prices, which tightens the borrowing constraint.[5] Borrowers are forced to further deleverage, which reduces consumption and depresses asset prices further. This amplification effect through asset price gives rise to pecuniary externalities. The allocation in an ABC economy when there is no aggregate demand shortage is constrained inefficient.[6]

When there is aggregate demand shortage, the IBC economy is constrained inefficient. The inefficiencies originate from the aggregate demand externalities that lower income and tighten the borrowing constraint. The effects of low income and tightened borrowing constraints reinforce each other, similar to the effects of low asset prices and tightened borrowing constraints in the ABC economy when there is an aggregate demand shortage. Asset prices fall as consumption decreases, which forces borrowers to further deleverage. Deleveraging worsens negative aggregate demand externalities. The resulting lower consumption and lower asset prices are caused by

[5]The effect of deleveraging on asset price when there is no AD shortage is ambiguous, since lower interest rate drives up asset price, but when the fraction of lenders is much larger than constrained asset-based borrowers in the economy, it tends to lower asset price.

[6]Similar results in Jeanne and Korinek (2010) in an open economy and endowment economy model environment.

both the pecuniary externalities and aggregate demand externalities.

Literature Review. This chapter builds on several strands of the literature. First, it contributes to the literature on macroeconomics with financial frictions. In their seminal work, Kiyotaki and Moore (1997) adopt a collateral constraint on borrowing due to incomplete contracts microfounded by Hart and Moore (1994). In their model, creditor payoff in default and debt capacity are determined by the liquidation value of assets. Amplification arises from fire sales of land from the more productive sector to the less productive sector due to adverse productivity shocks, which depresses land prices and feeds back to net worth, both within a period and dynamically to future asset prices. Other related work studies the pecuniary externalities from asset fire sales, as in Jeanne and Korinek (2010); Bianchi (2011); and Mendoza (2010). My work differs in two respects. First, creditor payoff in default and debt capacity are determined by current earnings instead of the liquidation value of assets; second, shocks are amplified through aggregate demand instead of asset prices.

Second, this chapter is closely related to works on aggregate demand-driven recessions. Mian, Rao, and Sufi (2013) and Mian and Sufi (2014) focus on the housing net worth channel through which the fall in the housing net worth of households reduced aggregate demand by direct wealth effects or by tightening households' capability to borrow through a fall in the collateral value. Chaney, Sraer, and Thesmar (2012) and Duchin, Ozbas, and Sensoy (2010) also study the reduction in corporate investment through the fall in collateral value in the Great Recession Theoretically, my work closely follows that of Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017) who emphasize that deleveraging by borrowers in the economy weighs down on aggregate demand, and Farhi and Werning (2016) and Korinek and Simsek (2016), who highlight the importance of macroprudential policy to address aggregate demand externalities. My work also differs from their papers because I impose an income-based borrowing constraint that generates amplification, rather than an exogenous debt limit.

Third, my work builds on a new strand of the literature that features the significance of an income-based debt limit. Empirical works include Chava and Roberts (2008) and Roberts and Sufi (2009), who study the effect of the violation of debt covenants on borrowers and how lenders will gain rights to influence the financing and investment decisions of the firms; Chodorow-Reich and Falato (2017), who study an earning-based debt limit in the syndicated loan market; and Sufi (2009), who examines the widespread use of cash flow-based financial covenants in bank lines of credit. Ivashina, Laeven, and Moral-Benito (2019) investigate types of commercial credit in general. My theoretical model builds heavily on the comprehensive empirical work of Lian and Ma (2021), who establish the prevalence of cashflow-based borrowing among nonfinancial corporations in the US.

My work is also related to theoretical models that use income-based borrowing constraints to study the macroeconomic effects of debt deleveraging. Goldberg (2010) models income-based borrowing constraint on the firm side, but focuses on the effect of idiosyncratic shocks in a Bewley-Huggett-Aiyagari type of framework. Corbae and Quintin (2015) and Greenwald (2018) both study the importance of a borrowing constraint based on payment-to-income ratio in driving housing prices. The most relevant theoretical work to my paper is by Drechsel (2019), who studies an income-based debt limit in the nonfinancial corporate sector, both empirically and theoretically; incorporates income-based debt limits on firms in a business cycle model; and focuses on firms' response of borrowing to investment shocks. Benigno et al. (2013) incorporate income–based borrowing constraints in open economy models. My work contributes to the literature by studying the interactions of income-based and asset-based borrowing, and the differences in their policy implications.

The rest of this chapter is organized as follows. Section 1.2 introduces the IBC and ABC model set-up. Section 1.3 characterizes the decentralized equilibrium of the two models and compares the amplification effects.

1.2 Model Set-Up

In this section, I will demonstrate and compare the amplification effect with asset- and income-based borrowing constraints on households in a three-period model. The model has an environment that closely follows Korinek and Simsek (2014, 2016), but provides a more generalized framework to incorporate one or more types of borrowing constraints. Moreover, unlike an exogenous debt limit in their paper, the model has an endogenous debt limit dependent on households' asset holdings or current income rather than an exogenous value.

1.2.1 Environment

There are three discrete time periods $t = 0, 1, 2$. The economy consists of households and firms. Households are of measure one. There are H types of households, indexed by $h \in \mathcal{H}$. In some of our applications, the set of households will consist of only two types, e.g. lenders and borrowers. There can be type a borrowers constrained by asset value when $\mathcal{H} = \{l, a\}$, or type i borrowers constrained by income when $\mathcal{H} = \{l, i\}$. But we will also consider cases with additional heterogeneity. Each type of households has a weight of α^h with $\sum_h \alpha^h = 1$. Borrowers are more impatient than lenders, with the discount factors $\beta^h < \beta^l = 1$, for $h = a, i$, such that in equilibrium borrowers will take on debt. Households own firms and will obtain profits from firm sales. There are two types of commodities in the economy, a final good for consumption and labor.

Preferences. Households preferences are inseparable, following Greenwood, Hercowitz, and Huffman (1988).[7]

$$U^h = u(c_0^h - v(n_0^h)) + \beta^h u(c_1^h - v(n_1^h)) + (\beta^h)^2 u(c_2^h - v(n_2^h)) \tag{1.1}$$

where $u'(\cdot) > 0$, $u(\cdot)$ strictly concave, $\lim_{c \to 0} u'(c) = \infty, 0 < v'(\cdot) \leq 1$, $v(\cdot)$ strictly convex, $v'(0) = 0, \lim_{n\to\infty} v'(n) = \infty$.

Technology. The final good is produced competitively by a final good sector using differentiated intermediate goods according to the Dixit-Stiglitz technology:

$$y_t \equiv (\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}} \tag{1.2}$$

with ϵ greater than one. $y_t(j)$ the quantity of the intermediate good j produced by a continuum of monopolistic firms indexed by $j \in [0, 1]$. Each firm uses an identical linear technology to produce a differentiated good:

$$y_t(j) = n_t(j) \tag{1.3}$$

where $n_t(j)$ is the aggregate level of labor supplied by all types of households to produce the good j. Firms take household demand and the aggregate price level as

[7] Unlike separable preferences consistent with balanced growth, GHH preference eliminates wealth effects on labor supply, so it will generate more amplification compared to separable preferences as households will not increase labor supply to pay off debt when income falls.

given to set prices in each period. The aggregate price level is defined as:

$$P_t \equiv (\int_0^1 P_t(j)^{1-\epsilon}\, dj)^{\frac{1}{1-\epsilon}}$$

Aggregate price dynamics. In the baseline model, instead of assuming the full staggering pricing dynamics as in Calvo (1983), we assume in the baseline model that none of the monopolistic firms can reset prices due to an infinite price adjustment cost in each time period. Thus, the final good price and the aggregate price level stay constant, $P_t(j) = P_t = P$.

Market structure. Households have equal shares of firms. In each period, they earn labor income at a competitive wage rate and collect profits from firms to consume. There is a credit market in which households can issue a one-period bond at the prevailing real interest rate r_{t+1}[8]. b^h_{t+1} denotes bonds outstanding in period t and needed to be repaid in period $t+1$. Households are also endowed with an asset that yield d^h_t dividend in every period. The dividend is subject to shocks in period 1, but deterministic in period 0 and 2 with $d^h_t = d$. Each household is endowed with $\theta^h_0 = 1$ unit of the asset at the beginning of period 0, and the asset can be traded at a price p_t only within the same type of households. There is no uncertainty in the model, and agents fully anticipate future shocks.

1.2.2 First-best solution

I characterizes the first-best allocation $\{c^h_t, n^h_t\}_{t=0,1,2}$ as the planner's solution when market imperfections are absent. It serves as a benchmark for the later welfare

[8] r_{t+1} can be pinned down in a model with infinite time horizon. At steady state with borrowers constrained, r_{t+1} is equal to $\frac{1}{\beta^l} - 1$ since lenders are always unconstrained.

analysis.

The planner maximizes a weighted sum of utilities subject to the resource constraints. Let γ^h be the Pareto weight of type h agents, with $\sum_h \gamma^h = 1$. The social planner's problem is then given by:

$$\begin{aligned} \max_{\{c_t^h, n_t^h\}_{t=0,1,2}} \quad & \sum_{h\in\mathcal{H}} \sum_t \alpha^h \gamma^h [(\beta^h)^t u(c_t^h - v(n_t^h))] \\ \text{s.t.} \quad & \sum_{h\in\mathcal{H}} \alpha^h c_t^h = y_t + \sum_{h\in\mathcal{H}} \alpha^h \theta_t^h d_t, \quad \forall t \end{aligned} \tag{1.4}$$

At the optimum, the planner will equate households' marginal rate of substitution in the three periods to the Pareto weights ratio. Denote $u(\tilde{c}_t^h) = u(c_t^h - v(n_t^h))$, for any $h, k \in \mathcal{H}$:

$$\frac{\gamma^h}{\gamma^k} = \frac{u'(\tilde{c}_0^k)}{u'(\tilde{c}_0^h)} = \frac{\beta^k u'(\tilde{c}_1^k)}{\beta^i u'(\tilde{c}_1^h)} = \frac{{\beta^k}^2 u'(\tilde{c}_2^k)}{{\beta^i}^2 u'(\tilde{c}_2^h)} \tag{1.5}$$

Define n^* as the efficient level of labor. Aggregate employment is given by $n_t = y_t$, and is distributed uniformly among households such that $n_t^h = n_t, \forall h$. The first-best allocation for labor is then given by:

$$n_t^h = n^* = v'^{-1}(1)$$

Combine the resource constraints, the efficient labor supply, and Equation 1.5 to obtain the optimal allocation of consumption as a function of the Pareto weights. The Pareto weights will be consistent with the wealth of the households in second-best allocations for them to be comparable. Define the optimal consumption allocation as $\{{c_t^h}^{FB}\}_{t=0,1,2}$, and the corresponding social welfare as U_0^{FB}.

Due to market imperfections from monopolistic competition, firms will exploit a markup of the marginal cost. It is well-known to impose a subsidy τ on firms to correct the distortions from the monopolistic markups. Suppose the monopolistic firms can choose prices to set for now as a frictionless benchmark without price rigidities, and they maximize profit as follows:

$$\max_{\{P_t(j), y_t(j), n_t(j)\}_{t=0,1,2}} \quad \frac{P_t(j)}{P_t} y_t(j) - w_t(1-\tau(n_t))n_t(j)$$
$$\text{s.t.} \quad y_t(j) = n_t(j) \leq (\frac{P_t(j)}{P_t})^{-\epsilon} y_t$$

The subsidy will be financed by a lump-sum tax $T_t = \tau w_t \int_0^1 n_t(j)dj$ to all households. In equilibrium, the monopolistic firms will set

$$\frac{P_t(j)}{P_t} = w_t \frac{\epsilon}{\epsilon - 1}(1-\tau) \tag{1.6}$$

where $\tau(n_t)$ is set to $\frac{1}{\epsilon}$ when aggregate employment n_t is lower than or equal to n^*, and zero when aggregate employment n_t is above n^*. As a result of linear production technology, each firm will set the same price for their goods. Define w^* as the efficient level of real wage. When firms can freely adjust price and are appropriately subsidized, w^* will be one. Without the subsidy, households' employment and labor income will be lower.

1.2.3 Market imperfections

There are two major market imperfections in the model, financial frictions and the lower bound constraint on the real interest rate. Households can borrow against their income and/or against their asset holdings. They face a borrowing constraint with an endogenously determined debt limit in period 1 when issuing bonds. The debt limit is restricted by a fraction of their current income and a fraction of the value of assets they hold. In the baseline model, I focus on either an income-based borrower whose debt limit is determined solely by income, or an asset-based borrower whose debt limit is determined solely by asset value. The extent to which they are constrained by their income or asset is captured by the parameters ϕ^{Ih} or ϕ^{Ah}:

$$b_2^h \geq -\phi^{Ih} e_1^h - \phi^{Ah} \theta_1 p_1, \tag{1.7}$$

where household income e_t^h consists of labor income and profits from the monopolistic firms net of a lump sum tax:

$$e_t^h = w_t n_t^h + \Pi_t - T_t, \tag{1.8}$$

where $\Pi_t = \int_0^1 \Pi_t(j)\, dj$ is profits from firms. This constraint resonates with the empirical findings on the prevalence of income-based and asset-based borrowing. It is also an incentive compatibility condition where it is never optimal for a debtor to default given that creditors can seize a fraction of his or her income, or asset in bankruptcy. In addition, we can define e^* as the efficient level of income using the previously derived n^* and w^*:

$$\begin{aligned} n^* &= v'^{-1}(1) \\ w^* &= 1 \\ e^* &= v'^{-1}(1) \end{aligned}$$

These conditions will serve as an efficient benchmark.

Second, the nominal interest rate will be bounded by a lower bound following Korinek and Simsek (2014). In order to simplify the analytical solution, the lower bound is normalized to zero. With aggregate price level being sticky, the real interest rate will also be bounded by zero.

$$r_{t+1} \geq 0, \quad t = 0, 1 \tag{1.9}$$

The zero lower bound on nominal interest rate is crucial for the result of amplification through aggregate demand in this model, as it will force income to be below the efficient level and determined by aggregate demand. The fall in aggregate demand due to household deleveraging will lower income, tightening the borrowing constraint, which will result in further reduction in aggregate demand and income. This result will still hold if I relax the assumption of price rigidity. Indeed, the result from relaxing this assumption will be in line with the “perverse” proposition brought up by Eggertsson and Krugman (2012) that increasing price flexibility makes the real effect of an adverse shock on net worth worse. Therefore, relaxing this assumption will only make amplification greater in the model. I assume an extreme level of price stickiness to simplify the model.

1.2.4 Strategies

Since firms cannot reset prices in each period, the aggregate price level is completely sticky. Given the preset good prices, the monopolistic firms choose how much to produce and how many workers to hire to maximize profit:

$$\begin{aligned} \max_{\{y_t(j), n_t(j)\}_{t=0,1,2}} \quad & \frac{P_t(j)}{P_t} y_t(j) - w_t(1 - \tau(n_t)) n_t(j) \\ \text{s.t.} \quad & y_t(j) = n_t(j) \leq (\frac{P_t(j)}{P_t})^{-\epsilon} y_t \end{aligned} \tag{1.10}$$

where $P_t = P$ is constant, and $\frac{P_t(j)}{P_t}$ is equal to one by symmetry. In equilibrium, the monopolistic firms will always choose to produce to meet the demand since the marginal product is strictly higher than the marginal cost. Therefore, $y_t(j) = n_t(j) = y_t$. The monopolistic firms' production is essentially determined by the aggregate demand for the final good, which is ultimately determined by the real interest rate. Since price is fixed, production is determined by monetary policy that sets the nominal interest rate. Let r^* be the real interest rate at which production and employment are at the frictionless benchmark level. A constrained efficient monetary policy is set according to[9]:

$$i_{t+1} = r_{t+1} = \max(0, r^*) \quad \forall t \tag{1.11}$$

Households' maximization problem is given by:

$$\begin{aligned}
\max_{\{c_t^h, n_t^h, \theta_t^h, b_1^h, b_2^h\}_{t=0,1,2}} \quad & u(c_0^h - v(n_0^h)) + \beta^h u(c_1^h - v(n_1^h)) + (\beta^h)^2 u(c_2^h - v(n_2^h)) \\
\text{s.t.} \quad & \frac{b_1^h}{1+r_1} + c_0^h = e_0^h + \theta_0^h d_0^h + (\theta_0^h - \theta_1^h)p_0 + b_0^h, \\
& \frac{b_2^h}{1+r_2} + c_1^h = e_1^h + \theta_0^h d_1^h + (\theta_1^h - \theta_2^h)p_1 + b_1^h, \\
& c_2^h = e_2^h + \theta_2^h d_2^h + b_2^h, \\
& b_2^h \geq -\phi^{Ih} e_1^h - \phi^{Ah} \theta_1 p_1.
\end{aligned} \tag{1.12}$$

with $e_t^h = w_t n_t^h + \Pi_t - T_t = w_t n_t^h + n_t - w_t n_t$. Note that profits of firms net of the lump-sum tax will be positive if the real wage is below the efficient level, and will be zero if it is at the efficient level.

Definition 1.1. A decentralized equilibrium is a set of prices $\{w_0, w_1, w_2, r_1, r_2\}$, real allocations $\{c_t^h, n_t^h, e_t^h, y_t\}_{t=0,1,2,h\in\{a,i,l\}}$, asset allocations $\{\theta_t^h\}_{t=0,1,2,h\in\{a,i\}}$, bond hold-

[9] There is a discussion of the constrained efficiency of the monetary policy with or without commitment power in Korinek and Simsek (2016).

ings $\{b_t^h\}_{t=0,1,2,h\in\{a,i,l\}}$, and profits and taxes $\{\Pi_t, T_t\}$ such that households maximize utility as in (1.12); the final good sector produces according to (1.2); intermediate goods are produced by monopolistic competitive firms that maximize profits according to (1.10) given fixed intermediate goods price; the interest rates are set according to (1.11), and all markets clear.

1.3 Solving the Decentralized Equilibrium

The decentralized equilibrium will depend on the type of borrowers in the economy. I will first consider the case when $H = 2$, $\mathcal{H} = \{l, i\}$, and $\phi^{Ai} = 0$, where borrowers are constrained by their income. Next I will consider when $H = 2$, $\mathcal{H} = \{l, a\}$, and $\phi^{Ia} = 0$, where borrowers are constrained by the value of their asset holdings. The borrowing constraints can be binding or not binding in equilibrium. I will focus on the equilibrium when they are binding, since it is more relevant for policy interventions.

1.3.1 The decentralized equilibrium with IBCs

The model can be solved via backward induction. Period 2 consumption and labor choices are intratemporal decisions given b_2^h at the beginning of period 2. Because assets can only be traded among the same type of households, both income-based borrowers and lenders in the economy will have no incentive to trade assets. They hold the one unit of asset endowed in peiod 0 in equilibrium. By market clearing condition, lenders' bond holdings will be $\alpha^l b_t^l = -\alpha^i b_t^i$, where $b_2^i = -\phi^{Ii} w_1 n_1^i$ when borrowers are constrained in equilibrium. Since monetary policy attempts to replicate the efficient level of employment for lenders, the real wage is one. Let net consumption be $\tilde{c}_t^h$, which is equal to $c_t^h - v(n_t^h)$; let λ^i be the Lagrangian multiplier associated with the IBCs; given b_1^i, the equilibrium is pinned down by:

$$u'(\tilde{c}_1^i) = \beta^i(1+r_2)u'(\tilde{c}_2^i) + \lambda^i(1+r_2) \tag{1.13}$$

$$u'(\tilde{c}_1^i)(w_1 - v'(n_1^i)) + \phi^{Ii}w_1\lambda^i = 0 \tag{1.14}$$

$$u'(\tilde{c}_1^l) = \beta^l(1+r_2)u'(\tilde{c}_2^l) \tag{1.15}$$

$$\alpha^l b_1^l = -\alpha^i b_1^i \tag{1.16}$$

The first Euler equation indicates that higher current consumption makes borrowers less tempted to borrow, so the IBCs will be less tight. The second labor supply decision equation of the borrowers implies that although working more can relax the IBCs, it reduces welfare due to disutility from working, and the marginal benefit of work needs to be balanced out by the marginal cost. By substitution using the bonds market clearing condition and the budget constraints, the decentralized equilibrium can be reduced to the labor supply choice of the borrowers and the Euler equation of the lenders as follows:

$$(w_1 + \frac{\phi^{Ii}w_1}{1+r_2})u'(\tilde{c}_1^i) = v'(n_1^i)u'(\tilde{c}_1^i) + \beta^i\phi^{Ii}w_1u'(\tilde{c}_2^i) \tag{1.17}$$

$$u'(\tilde{c}_1^l) = \beta^l(1+r_2)u'(\tilde{c}_2^l) \tag{1.18}$$

Note that since borrowers can and are willing to work more hours to relax the borrowing constraint, their labor supply in equilibrium will be higher than the "efficient" level n^*, i.e., they tend to overwork whenever they are constrained in borrowing. Equation (1.17) implies that the marginal benefit of working an additional hour should be matched with the marginal cost of working an additional hour. It is also a debt supply equation linking the borrowers' labor choice which determines the quantity of debt issuance, to the interest rate. Higher labor supply of the borrowers is associated with a lower interest rate when ϕ^{Ii} is relatively small. To see this, define X_b^{in} as:

$$X_b^{in} = -\frac{\phi^{Ii} w_1}{(1+r_2)^2}[1 + \frac{\beta^i \phi^{Ii} w_1 n_1^i}{(u'(\tilde{c}_1^i))^2} u''(\tilde{c}_1^i) u'(\tilde{c}_2^i)] < 0$$

where "in" denotes income-based borrowing and no AD shortage, and "b" denotes borrowers. This restriction can be approximated as:

$$\phi^{Ii} < \sigma \frac{\tilde{c}_1^i}{w_1 n_1^i}$$

where σ is the elasticity of intertemporal substitution[10]. The net consumption of the borrowers is always higher when they increase the labor supply when the interest rate falls. The intuition is in some way similar to the case where borrowers are unconstrained: lower interest rate induces borrowers to issue more debt which raises net consumption. This relation is demonstrated as the IB curve in Figure 1.1. Equation (A.7) can be viewed as a bond demand equation that indicates higher interest is associated with higher bond demand as higher interest rate discourage lenders from consuming today, which is shown from the AD curve in Figure 1.1.

Consider higher leveraging in period 1 that leads to a lower b_1^i. This corresponds to loose credit conditions during economic booms. If borrowers cannot work more hours, the interest rate has to rise such that they will consume less with higher debt

[10]Derivations are in Appendix A.1. The restriction on ϕ^{Ii} indicates that borrowers may increase labor supply when the interest rate increases if ϕ^{Ii} is too large. This anomaly originates from the assumption that borrowers are always constrained. If ϕ^{Ii} is large enough, the amount of debt borrowers carries assuming they are constrained might be greater than that of they being unconstrained, which is impractical. And if the interest rate rises when borrowers increase labor supply, their net consumption could decrease. Another interpretation of the restriction is to think of $\sigma\tilde{c}_1^i$ as the inverse of risk aversion. Borrowers need to be relatively less risk averse, or the curvature of their utility is small, to issue more debt as the interest rate falls.

repayments, whereas for lenders the interest rate will fall for them to consume more with higher debt payments (the effects are shown in Figure 1.1). As long as ϕ^{Ii} is small enough that borrowers are tightly constrained by the amount they can borrow, the interest rate will eventually fall with more labor supplied by the borrowers. If borrowers are highly leveraged, deleveraging in period 2 can make the interest rate fall to the zero lower bound. Since prices are fixed at the preset level, the real interest rate will determine the demand and therefore how much firms produce. When the real interest rate cannot fall further to boost demand and clear the goods market, aggregate demand falls, which lowers production. Firms' demand for labor is reduced and the real wage will fall, resulting in higher markups. Output, falling below the natural level, will be determined by the aggregate demand at the zero interest rate. This threshold level of b_1^i is defined as $\underline{b}_1^i$, and the derivation of $\underline{b}_1^i$ is in Appendix A.1.

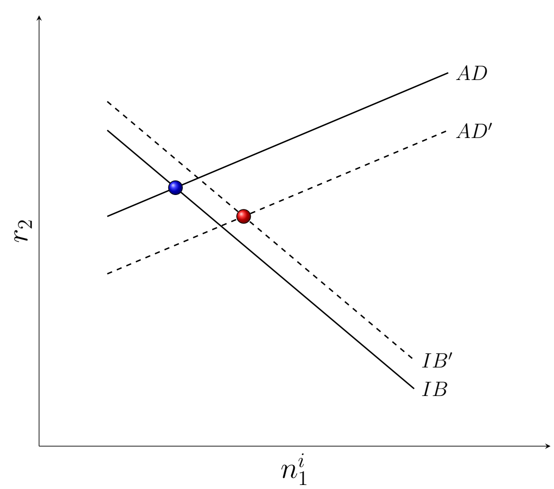

Figure 1.1: Effect of lower b_1^i on borrowers' employment and interest rate, no AD shortage

Lemma 1.2. *The decentralized equilibrium in period 1 given that borrowers are constrained is determined by* b_1^i*,*

- *when* $b_1^i \geq \underline{b}_1^i$*, the negative effect of deleveraging on aggregate demand is completely buffered by the fall in interest rate, and firms produce efficiently at* w^**, with lenders' employment* $n_1^l = n^*$ *and borrowers' employment* $n_1^i > n^*$*; there is no aggregate demand shortage;*

- *when* $b_1^i < \underline{b}_1^i$*, there is an aggregate demand shortage, since further fall in interest rate that could have recovered households' demand is circumscribed by the zero lower bound. Firms produce and earn an economic profit at* $w_1 < w^*$*, with lenders' employment* $n_1^l < n^*$*.*

When there is an aggregate demand shortage. If real interest rate is constrained by the lower bound when massive deleveraging triggers an aggregate demand shortage, wage will be below the efficient level. The decentralized equilibrium will be pinned down by the debt supply and demand equation at zero interest rate. Since lenders are unconstrained and their employment is given by $v'(n_1^l) = w_1$, which is an increasing transformation of the real wage, the two equations can be solved from either w_1 and n_1^i, or n_1^l and n_1^i. Note that the real wage will be below the efficient level and firms will earn positive profit with an aggregate demand shortage. I assume lenders and borrowers each obtain what they produce as their total income[11]. Thus households' income is given by $e_1^h = n_1^h$.

$$w_1 - v'(n_1^i) + \phi^{Ii} w_1 = \beta^i \phi^{Ii} \frac{u'(\tilde{c}_2^i)}{u'(\tilde{c}_1^i)} \tag{1.19}$$

$$u'(\tilde{c}_1^l) = \beta^l u'(\tilde{c}_2^l) \tag{1.20}$$

[11] This is an assumption that makes the decentralized equilibrium analytically tractable. The standard way is to compute total income as the sum of labor income and profits from firms.

and with $\beta^l = 1$, Equation (1.20) can be rewritten as:

$$n_1^l = 2\frac{\alpha^i}{\alpha^l}\phi^{Ii}n_1^i + v(n_1^l) + \frac{\alpha^i}{\alpha^l}b_1^i + (d_2^l - d_1^l) + (e^* - v(e^*)) \tag{1.21}$$

Since output is determined by aggregate demand, for borrowers, the tighter the borrowing constraint, the higher wage is to increase labor supply. Thus, the wage is increasing in borrowers' employment based on borrowers' labor supply decision (as in Equation (1.19) and the IB curve in Figure 1.2). The more hours borrowers work, the greater amount lenders will lend out today and get repaid tomorrow, which raises the marginal utility of consumption of today and decreases that of tomorrow. Since the interest rate is stuck at the lower bound, the wage will increase to induce lenders to work more so that lenders can increase their income and consumption. Thus, the wage is also increasing in borrowers' employment from the lenders' intertemporal consumption choice or bond demand (as in Equation (1.20) and the AD curve in Figure 1.2).[12]

Amplification. Next, consider a comparative static when borrowers take on more debt in period 0 (lower b_1^i). Since the economy is in a liquidity trap, higher leveraging will result in a greater demand shortage, which lowers the labor demand of the firms and dampens the real wage. From lenders' perspective, they will reduce labor supply. Since lenders get more debt repayments in period 1, and their consumption demand is fixed at the current interest rate, they need less labor income to consume (a rightward shift of the AD curve as in Figure 1.2). On the borrowers' side, accumulating more debt in period 0 worsens deleveraging in period 1, tightening the borrowing constraint and increasing borrowers' labor supply (a rightward shift of

[12]There is a reinforcing effect of wage on employment for Equation (1.19) and (1.20). For them to have a unique and well-defined solution, some restrictions need to be imposed. Derivations of the restrictions are in Appendix A.1.

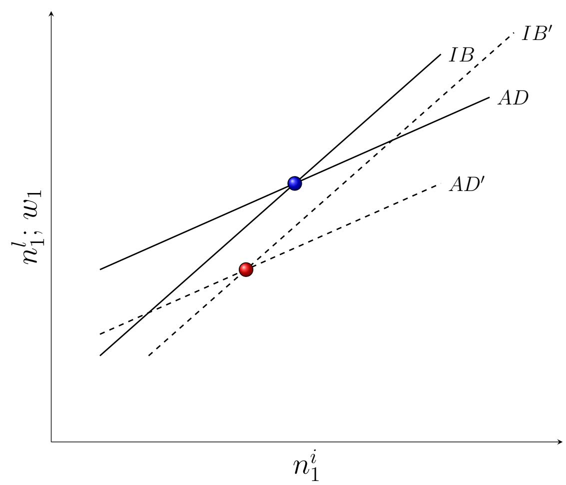

Figure 1.2: Effect of lower b_1^i on borrowers' employment and interest rate, with AD shortage

the IB curve). The new equilibrium wage and employment of all households will be lower if the borrowing constraint is sufficiently tight, i.e., ϕ^{Ii} is sufficiently small. As labor income falls, borrowers become more constrained in borrowing, which further lowers their consumption demand and reduces production. An initial small change in wealth can lead to a large change in wage and income by affecting aggregate demand. Borrowers do not take into consideration the negative effect of debt accumulation in the present on aggregate demand in the future, resulting in worse deleveraging and aggregate demand externalities.

Note that the requirement on ϕ^{Ii} is not critical in obtaining the amplification result. The key mechanism of amplification with IBCs hinges on aggregate demand instead of the individual labor supply decision of borrowers. I derive the decentralized equilibrium when borrowers are constrained by the aggregate income instead of the individual income in the Appendix. It better captures the amplification effect from aggregate demand and provides an analytically tractable solution of the multiplier. The tighter the borrowing constraint is, i.e., the smaller ϕ^{Ii} is, the greater amplification will be generated with IBCs.

Nevertheless, allocations from the decentralized equilibrium when there is no AD shortage are constrained efficient due to the individual labor supply decision of borrowers. Because borrowers are constrained in labor income, they will choose to work more to borrow more until they can consume at the optimal level. This leads to constrained efficient allocations. With AD shortages, consumption can no longer be optimal due to aggregate demand externalities. Although borrowers will still choose to work more, labor income and consumption are sub-optimal due to lower wages. The resulting allocations are inefficient.

1.3.2 The decentralized equilibrium with ABCs

Similar to the model with IBCs, the decentralized equilibrium can be solved backward. A symmetric equilibrium indicates $\theta_t^a = 1$ for all t. A general form of the asset pricing equation is given by:

$$p_1 = \frac{u'(\tilde{c}_2^a)}{u'(\tilde{c}_1^a)}\beta^a d_2^a$$

Asset price is determined by the present discounted value of future cash flows.[13] There also exists a threshold level of b_1^a such that:

Lemma 1.3. *The decentralized equilibrium in period 1 given that borrowers are constrained is determined by* b_1^a*,*

- *when* $b_1^a \geq \underline{b}_1^a$*, the negative effect of deleveraging on aggregate demand is completely buffered by the fall in interest rate, and firms produce efficiently at* w^**,*

[13]Due to the beginning-of-period asset sale, asset price in period 1 does not contain the Lagrangian multiplier associated with the borrowing constraint. This simplifies the derivations of the equilibrium and policy analysis in later sections, and does not affect the analytical results.

with lenders' employment $n_1^l = n^*$ *and borrowers' employment* $n_1^a > n^*$*; there is no aggregate demand shortage;*

- *when* $b_1^a < \underline{b}_1^a$*, there is an aggregate demand shortage, since a further fall in the interest rate that could have recovered households' demand is circumscribed by the zero lower bound. Firms produce and earn an economic profit at* $w_1 < w^*$*, with lenders' employment* $n_1^l < n^*$*.*

When borrowers are constrained, the interest rate must fall to induce lenders to hold less debt in equilibrium. Thus, the more borrowers are forced to deleverage in period 1, the lower the interest rate will be. As borrowers deleverage, the interest rate may hit the zero lower bound, which may lead to aggregate demand shortages.

When there is no aggregate demand shortage. The constrained equilibrium when $b_2^a = -\phi^{Aa} p_1$ and when there is no aggregate demand shortage is pinned down by the asset pricing equation and the Euler equation of the lenders:

$$p_1 = \frac{u'(e^* + d_2^a - \phi^{Aa} p_1 - v(e^*))}{u'(e^* + d_1^a + b_1^a + \frac{\phi^{Aa} p_1}{1+r_2} - v(e^*))} \beta^a d_2^a \tag{1.22}$$

$$u'(e^* + d_1^l + b_1^l - \frac{\alpha^a}{\alpha^l} \frac{\phi^{Aa} p_1}{1 + r_2} - v(e^*)) = \beta^l (1 + r_2) u'(e^* + d_2^l + \frac{\alpha^a}{\alpha^l} \phi^{Aa} p_1 - v(e^*)) \tag{1.23}$$

Assets in the model play two major roles: agents who hold the assets can get a dividend in the future which can increase consumption; assets can be used as collateral to borrow. The first role indicates that asset prices will be high when current consumption is high or expected future consumption is low. According to Equation (1.22), when the interest rate rises, asset prices fall because it lowers the value of bonds, which reduces the amount borrowers can borrow and thus current consumption. The inverse relation is captured by the AP curve in Figure 1.3.

Consider a comparative static with a fall in the net worth of the borrowers in period 1 will lead to lower consumption. If borrowers are constrained, it will depress asset prices as the demand for assets falls with lower current consumption and the higher marginal utility of current consumption. On the one hand, since borrowers are constrained, further deleveraging will induce a fall in the real interest rate r_2: $\frac{dr_2}{dp_1} \geq 0$, such that lenders are discouraged to hold debt, which tends to shift lenders' consumption to the current period.

On the other hand, lower asset prices will make borrowers more constrained, which further decreases consumption and lower asset prices, resulting in a feedback loop. The new decentralized equilibrium is shown in Figure 1.3, with lower interest rates and lower asset prices. Unlike in the model with an income-based borrowing constraint, this mechanism does not involve any fall in borrowers' or lenders' income as the income is at the efficient level. To have a unique equilibrium, the partial derivative of the right hand side of Equation (1.22) with respect to p_1 must be less than 1. This condition is satisfied if ϕ^{Aa} is small and satisfy:

$$Z_b^{an} = 1 + \frac{\phi^{Aa}\beta^a d_2^a}{(u'(\tilde{c}_1^a))^2}(u'(\tilde{c}_1^a)u''(\tilde{c}_2^a) + \frac{u''(\tilde{c}_1^a)u'(\tilde{c}_2^a)}{(1+r_2)}) > 0 \tag{1.24}$$

which simplifies to:

$$\phi^{Aa} < \sigma(\frac{\tilde{c}_1^a}{d_2^a} + \frac{\tilde{c}_2^a}{d_2^a}) \tag{1.25}$$

Note that since Z_b^{an} is less than one, a unit change in wealth of borrowers will cause $\frac{1}{1-Z_b^{an}}$ unit change in asset prices considering the partial equilibrium. Therefore, there is an amplification effect from the asset pricing equation.

When there is an aggregate demand shortage. The equilibrium will be pinned down by the asset pricing equation and the aggregate demand equation at the zero

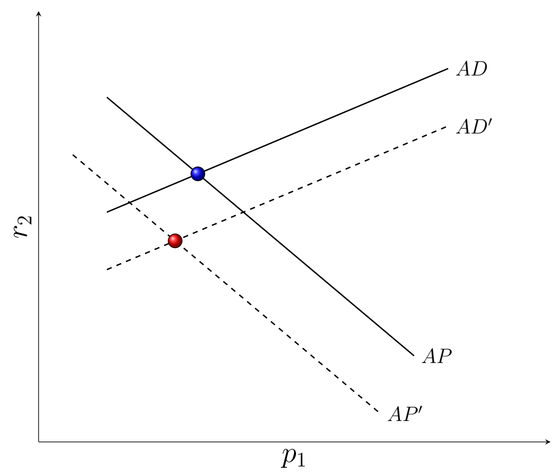

Figure 1.3: Effect of lower b_1^i on borrowers' employment and interest rate, no AD shortage

lower bound:

$$p_1 = \frac{u'(e^* + d_2^a - \phi^{Aa} p_1 - v(e^*))}{u'(e_1 + d_1^a + b_1^a + \phi^{Aa} p_1 - v(e_1))} \beta^a d_2^a \tag{1.26}$$

$$e_1 = 2\frac{\alpha^a}{\alpha^l}\phi^{Aa} p_1 + v(e_1) + \frac{\alpha^a}{\alpha^l} b_1^a + (d_2^l - d_1^l) + (e^* - v(e^*)) \tag{1.27}$$

For the asset pricing equation to have a unique and well-defined solution, it is necessary that $Z_b^{an} > 0$ at $r_2 = 0$. Let

$$X_b^{aa} = 1 - v'(e_1)$$

For the aggregate demand equation to have a unique and well-defined solution, X_b^{aa} needs to be less than one, which is equivalent to $v'(e_1) < 1$.[14] Decreasing the net worth of the borrowers now will not only depress asset prices through the feedback loop via the borrowing constraint, but also through the amplification mechanism by aggregate demand. That is, the lower consumption level that gives rise to falling asset prices is a result of both the asset-based borrowing constraint and the aggregate

[14] The first "a" in the notation "aa" denotes asset-based borrowing, and the second one denotes aggregate demand shortage.

demand externalities due to the lower bound on the interest rate. As in Figure 1.4, a reduced wealth of borrowers will shift the AP curve to the left as it depresses asset prices, and it will shift the AD curve to the right as it lowers income. As a result, both income and asset prices are lower in the new equilibrium. This result is in line with the literature on fire sales and amplification effects from asset-based borrowing.

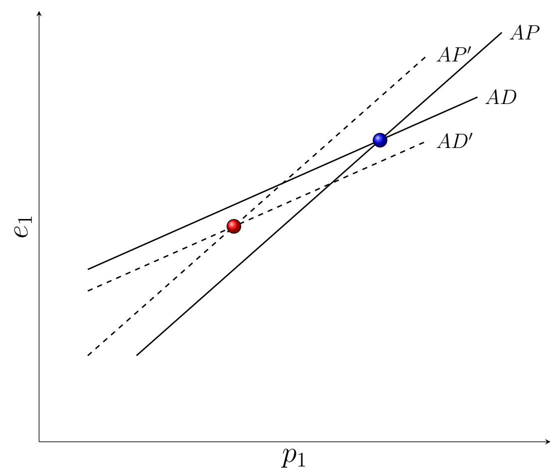

Figure 1.4: Effect of lower b_1^i on borrowers' employment and wage, with AD shortage

Chapter 2

Policy Responses with Income-Based Versus Asset-Based Borrowing Constraints

2.1 Introduction

This chapter analyzes the policy implications with the two types of borrowing respectively, and calibrates the model with both types of borrowing in one economy. It addresses two major questions: what are the differences in the effects of policy measures with the two types of constraints, and what is the optimal policy in a credit crunch under the two types of borrowing? I analyze the implications of two types of policies that I label fiscal policy and liquidity operations. I model fiscal policy as a transfer across agents during deleveraging. I model liquidity operations as a transfer across time, i.e., policymakers provide liquidity to borrowers in the period in which the constraint is binding, and they pay it back in the following period. This can also be interpreted as the government purchasing assets from borrowers during deleveraging and selling them back in the future.

Fiscal policy that taxes lenders and provides a transfer to borrowers in a crisis will improve the welfare of borrowers and undermine that of lenders when there is no aggregate demand shortage, in both the IBC and ABC economy. In the IBC economy, it also generates wealth redistribution by increasing the interest rate. In the ABC economy, it relaxes the borrowing constraint by boosting asset prices to improve the welfare of borrowers in addition to wealth redistribution due to changes in the interest rate. Lenders are always worse off due to the tax. When there is an aggregate demand shortage, fiscal policy that taxes lenders to provide transfers to borrowers in a crisis can improve the welfare of both borrowers and lenders. When aggregate demand externalities are large enough, such transfers can even lead to a Pareto improvement in both the IBC and ABC economy. Providing a transfer to ABC borrowers can improve welfare more than a transfer to IBC borrowers. The reason is that a lump-sum subsidy to IBC borrowers can reduce their labor supply, lower the amount they borrow, and depress aggregate demand when the interest rate cannot fall further. In contrast, a lump-sum subsidy to ABC borrowers raises asset prices, increases the amount they borrow, and boosts aggregate demand. As a result, income falls for IBC borrowers while it increases for ABC borrowers. And the welfare of ABC borrowers is improved more than that of IBC borrowers.

However, liquidity operations that transfer resources for the same agent across time, such as asset purchases during a deleveraging episode and sales after deleveraging can lead to a Pareto improvement independent of whether there is an aggregate demand shortage, in both the IBC and ABC economy. Since it involves a transfer across time, it improves borrowers' welfare by getting around the borrowing constraint when liquidity is most needed. For lenders, when there is no aggregate demand shortage, it improves their welfare by increasing the interest rate; when there is an aggregate demand shortage, it improves their welfare by increasing income.

The effectiveness of these ex post policies depends on the magnitude of amplification. In a model set-up with separable preferences of households and the wealth effect on labor supply, aggregate demand externalitities might not be large enough such that a fiscal policy as implemented in the previous section achieves such welfare improvements.[1] Therefore, it is important to understand how ex ante macroprudential policies, can be implemented to achieve an efficient outcome. I find that an optimal macroprudential policy can be implemented by either a quantity restriction on debt issuance of borrowers such that there will be no aggregate demand shortage, or a tax on any positive debt issuance, combined with lump-sum transfers between borrowers and lenders.

The outline of this chapter is as follows. Section 2.2 conducts comparative statics, and analyzes the implications of two ex post policies, fiscal policy and liquidity operations. Section 2.3 analyzes the optimal macroprudential policies. Section 2.4 introduces a numerical illustration of the model with both types of borrowing, and Section 2.5 concludes.

2.2 Comparative Statics and Ex Post Policies

In this section, I assume that households get a transfer of the final good t_t^h in every period. I will first consider the comparative statics of two marginal changes, a change in t_1^l and t_2^l to capture a shock on lenders' liquid wealth or a tax on lenders; and on t_1^i/t_1^a and t_2^i/t_2^a, to capture the shock on borrowers' liquid wealth, asset dividend, or a subsidy on borrowers. A complete list of results of the comparative statics are in the Appendix.

[1] For example, in Farhi and Werning (2016), the same type of fiscal policy will make lenders worse off.

Next, I will analyze the effect of two ex-post policies on welfare, fiscal policy, defined as a transfer across agents within period; and liquidity operations, defined as a transfer across time. I focus on households' welfare after deleveraging in period 1 and period 2, which is defined as the sum of the discounted utility of households in period 1 given by $V^h = u(\tilde{c}_1^h) + \beta^h u(\tilde{c}_2^h)$. The total welfare of all households is given by $V = \sum_h \frac{\alpha^h}{u'(\tilde{c}_1^h)} V^h$ with a normalization of the Pareto weights. I consider the welfare effects of two types of ex-post policies, fiscal policy and liquidity operations. The fiscal policy I focus on is defined as taxing lenders to subsidize borrowers in a lump-sum manner during the deleveraging period $t = 1$, and the government budget constraint is given by[2]:

$$\alpha^l t_1^l = \alpha^h t_1^h, \forall h \in \{a, i\}$$

Liquidity operation is defined as a lump-sum transfer financed by borrowing from lenders to purchase assets from borrowers in $t = 1$, and selling assets to the borrowers to pay back to lenders at $t = 2$. In practice, when the economy is in a liquidity trap, those liquidity provisions can be carried out at zero cost. Government budget constraints are given by:

$$\alpha^l t_1^l = \alpha^h t_1^h,$$
$$\alpha^l t_2^l = \alpha^h t_2^h \forall h \in \{a, i\}$$

where $t_1^h = t_2^h$. I will assume $\alpha^i = \alpha^a = 0.5$ in each economy for simplicity. The superscript notation denotes the type of borrowing "i" or "a" and whether there is an AD shortage: "n" for no AD shortage or "a" for AD shortage; the subscript notation denotes the type of agents: "b" for borrowers or "l" for lenders.

[2] I will also consider another type of fiscal policy that subsidizes labor income of the income-based borrowers by taxing lenders in later sections

Lemma 2.1. *A change in t_1^l and t_2^l has similar effects on an income-based borrowing economy and an asset-based borrowing economy when there is no aggregate demand shortage. An increase in t_1^l will improve welfare of both types of households: $\frac{\partial V^h}{\partial t_1^l} > 0$. In an income-based borrowing economy, it is achieved via a fall in the interest rate; in an asset-based borrowing economy, it is achieved through not only a fall in the interest rate, but also an increase in the asset price which affects the welfare of the borrowers, not lenders, and*

(a) the decrease in the interest rate generates a redistribution of wealth between borrower and lenders; however, it does not generate any inefficiencies;

(b) the increase in asset price alleviates the pecuniary externalities.

When there is no AD shortage in the asset-based economy, higher t_1^l or lower t_2^l to the lenders will increase lenders' demand for bonds, lowering the interest rate, and since lenders become more willing to hold debt, the collateral that the borrowers need for borrowing becomes more valuable, which boosts asset price. Therefore, the constraint on borrowers will be relaxed with higher collateral value. Both borrowers and lenders' income stay constant with production and wage at the efficient level. Households earn the same level of income, and there is no heterogeneity in income. The welfare of the borrowers is improved by higher asset price that relaxes their borrowing constraint and lower interest rate. Lenders, similar to lenders in the IBC economy with no AD shortage, are also better off due to the direct effect of higher consumption from greater wealth dominating the welfare loss from lower interest rate.

With a positive shock on wealth during deleveraging, the interest rate in both cases will fall as lenders' demand for bonds increases. In the IBC economy, the reduction in interest rate will induce borrowers to work more hours such that they can consume more; similarly, in the ABC economy, it drives up asset prices as higher collateral value enables borrowers to borrow more and consume more. The resulting higher labor supply of the borrowers does not affect welfare whereas higher asset prices can alleviate the pecuniary externalities from the asset price feedback loop when there is no AD shortage.

Lemma 2.2. *A change in t_1^l or t_2^l has an opposite impact on an income-based borrowing economy when there is no aggregate demand shortage and when there is an aggregate demand shortage. An increase in t_1^l or a decrease in t_2^l makes the households better off when the interest rate is above the lower bound $\frac{\partial V^h}{\partial t_1^l} > 0$, whereas it makes the households worse-off when the interest rate is stuck at the lower bound $\frac{\partial V^h}{\partial t_1^l} < 0$.*

When there is no aggregate demand shortage, both types of shocks will not have any impact on the real wage and production is at an efficient level. Lenders supply labor given the efficient level of wage. Borrowers, constrained in borrowing by their labor income, will increase labor supply if the demand for bonds is greater. t_1^l and t_2^l can indirectly affect welfare through the interest rate. Higher t_1^l or lower t_2^l of the lenders will induce them to save more and boost their demand for bonds, which lowers the interest rate. A lower interest rate improves the welfare of the borrowers. Borrowers will work more and thus have higher labor income, given a lower interest rate, but it does not affect their welfare since wage is constant[3]. Therefore, the welfare of both borrowers and lenders is affected through interest rate as in (B.3) and (B.4).

When there is an aggregate demand shortage, a positive shock on t_1^l has a similar effect as a negative shock on t_2^l: they both lower households' income. The decrease in income results from the binding constraint on the interest rate. A higher t_1^l or lower

[3]Also by the envelope theorem, changes in optimal labor supply does not directly affect welfare.

t_2^l makes lenders more willing to save, which should lower the interest rate. However, since the interest rate cannot fall further, the bonds market does not clear with an interest rate too high. In response, lenders save more than they should, which lowers demand. As a result, firms hire fewer workers, and scale down production, which decreases the wage rate. Falling income reduces borrowers' debt capacity, which reduces demand further, leading to a feedback loop[4]. With an AD shortage, the wage is below the efficient level, $w_1 = v'(n_1^l) < 1$, welfare of both borrowers and lenders is undermined due to lower income as in (B.6).

As the output is aggregate demand determined when prices are sticky, the interest rate will determine consumption demand and thus output. An increase in wealth will boost consumption of the lenders through a fall in the interest rate, leaving income at the optimal level when the interest rate is still flexible to move. The welfare of the borrowers is improved due to lower interest rate while that of the lenders is improved due to the direct effect of higher consumption dominating the adverse effect of lower interest rates. When the interest rate is at the lower bound, however, the demand shortage will be worsened by excessive savings of the lenders, which depresses production. The resulting lower wage and employment reduces income, further tightening the borrowing constraint when the debt limit is determined by income. The welfare of both types of households will be undermined as income decreases.

Lemma 2.3. *A change in t_1^a or t_1^i has different welfare implications for an income-based borrowing economy and an asset-based borrowing economy when there is no aggregate demand shortage. An increase in t_1^a or t_1^i will improve the welfare of borrowers: $\frac{\partial V^a}{\partial t_1^a} > 0$ and $\frac{\partial V^i}{\partial t_1^i} > 0$, and improve the welfare of lenders in the asset-*

[4] The GHH preference precludes the positive effect on labor supply when consumption falls and thus there is more amplification.

based economy but will undermine welfare of lenders in the income-based economy. The difference in welfare implications originates from the disparate effect on the interest rate:

(a) with IBC, interest rate falls due to less borrowing with lower labor supply;

(b) with ABC, interest rate rises due to more borrowing with higher asset prices.

For an income-based borrowing economy, when there is no aggregate demand shortage, an increase in t_1^i or a decrease in t_2^i will increase the consumption of the borrowers. Higher consumption makes borrowers less willing to borrow and therefore less incentivized to work so labor supply decreases, which decreases their debt with lower labor income. Interest rate falls in response to the lower supply of bonds. As with previous results when there is no AD shortage, changes in employment do not affect welfare. The welfare of the borrowers is improved through the direct effect of higher consumption and the reduction in interest rate, while the welfare of lenders is compromised due to lower interest rate. There is again a redistribution effect from interest rate changes, which does not generate any inefficiencies.

Consider a marginal increase in t_1^a or a decrease in t_2^a when there is no aggregate demand shortage in an ABC economy. An increase in asset dividends will make assets more valuable as it not only boosts the consumption by the borrowers in the current period directly, but relaxes the borrowing constraint as the price of the asset rises, which further increases consumption and inflates asset price. This is the canonical amplification mechanism with the asset-based borrowing constraint. Meanwhile, the interest rate must increase since the supply of bonds rises as the borrowers expand their debt capacity with more valuable collaterals. The welfare of borrowers is improved due to higher asset prices relaxing the borrowing constraint and the direct effect of higher consumption.

Fiscal policy with no AD shortage. In an IBC economy, the fiscal policy that transfers from lenders to borrowers will increase interest rate[5]. The increase in interest rate will have a redistribution effect in wealth from borrowers to lenders, but it does not generate any inefficiencies since the total welfare of all households is unchanged. Borrowers are better off and lenders are worse off due to the direct effect on consumption.

$$\begin{aligned} FP_b^{in} &= -\frac{\partial V_1^i}{\partial t_1^l} + \frac{\partial V_1^i}{\partial t_1^i}\frac{\alpha^l}{\alpha^i} \\ &= \frac{\alpha^l}{\alpha^i}u'(\tilde{c}_1^i) - \frac{\phi^{Ii} n_1^i}{(1+r_2)^2}u'(\tilde{c}_1^i))(\frac{\alpha^l}{\alpha^i}\underbrace{\frac{dr_2}{dt_1^i}}_{-} - \underbrace{\frac{dr_2}{dt_1^l}}_{-}) > 0 \end{aligned} \tag{2.1}$$

$$\begin{aligned} FP_l^{in} &= -\frac{\partial V_1^l}{\partial t_1^l} + \frac{\partial V_1^l}{\partial t_1^i}\frac{\alpha^l}{\alpha^i} \\ &= -u'(\tilde{c}_1^l) + \frac{\alpha^i}{\alpha^l}\frac{\phi^{Ii} n_1^i}{(1+r_2)^2}u'(\tilde{c}_1^l))(\frac{\alpha^l}{\alpha^i}\underbrace{\frac{dr_2}{dt_1^i}}_{-} - \underbrace{\frac{dr_2}{dt_1^l}}_{-}) < 0 \end{aligned} \tag{2.2}$$

$$\begin{aligned} FP^{in} &= \frac{\alpha^i}{u'(\tilde{c}_1^i)}FP_b^{in} + \frac{\alpha^l}{u'(\tilde{c}_1^l)}FP_l^{in} \\ &= 0 \end{aligned} \tag{2.3}$$

In an ABC economy, the impact of fiscal policy on interest rate is similar to that of income-based borrowing: interest rate will increase, which generates a wealth redistribution between borrowers and lenders. However, it has a positive impact on

[5] $|\frac{dr_2}{dt_1^i}| < |\frac{dr_2}{dt_1^l}|$ when ϕ^{Ii} is small.

asset prices in addition to the interest rate effect. Subsidizing borrowers and lenders both increase asset prices according to the comparative statics, but since a transfer of resources from lenders to borrowers relaxes the borrowing constraint, the positive effect on asset prices from a large purchase of asset can dominate the adverse effect on asset price from borrowing from lenders[6]. Lenders are worse off due to the direct effect of a reduction in consumption dominating the gain from the higher interest rate.

$$
\begin{aligned}
FP_b^{an} &= -\frac{\partial V_1^a}{\partial t_1^l} + \frac{\partial V_1^a}{\partial t_1^a}\frac{\alpha^l}{\alpha^a} \\
&= \frac{\alpha^l}{\alpha^a}u'(\tilde{c}_1^a) - u'(\tilde{c}_1^a)\frac{\phi^{Aa}p_1}{(1+r_2)^2}(\frac{\alpha^l}{\alpha^a}\underbrace{\frac{dr_2}{dt_1^a}}_{+} - \underbrace{\frac{dr_2}{dt_1^l}}_{-}) \\
&+ \frac{\phi^{Aa}}{1+r_2}[u'(\tilde{c}_1^a) - \beta^a(1+r_2)u'(\tilde{c}_2^a)](\frac{\alpha^l}{\alpha^a}\underbrace{\frac{dp_1}{dt_1^a}}_{+} - \underbrace{\frac{dp_1}{dt_1^l}}_{+}) > 0
\end{aligned}
\tag{2.4}
$$

$$
\begin{aligned}
FP_l^{an} &= -\frac{\partial V_1^l}{\partial t_1^l} + \frac{\partial V_1^l}{\partial t_1^a}\frac{\alpha^l}{\alpha^a} \\
&= -u'(\tilde{c}_1^l) + u'(\tilde{c}_1^l)\frac{\alpha^a}{\alpha^l}\frac{\phi^{Aa}p_1}{(1+r_2)^2}(\frac{\alpha^l}{\alpha^a}\frac{dr_2}{dt_1^a} - \frac{dr_2}{dt_1^l}) < 0
\end{aligned}
\tag{2.5}
$$

$$
\begin{aligned}
FP^{an} &= \frac{\alpha^a}{u'(\tilde{c}_1^a)}FP_b^{an} + \frac{\alpha^l}{u'(\tilde{c}_1^l)}FP_l^{an} \\
&= \frac{\phi^{Aa}}{1+r_2}[1 - \beta^a(1+r_2)\frac{u'(\tilde{c}_2^a)}{u'(\tilde{c}_1^a)}](\alpha^l\underbrace{\frac{dp_1}{dt_1^a}}_{+} - \alpha^a\underbrace{\frac{dp_1}{dt_1^l}}_{+}) > 0
\end{aligned}
\tag{2.6}
$$

Proposition 2.4. *A fiscal policy that taxes lenders to subsidize borrowers in a crisis will improve the welfare of the borrowers and undermine the welfare of the lenders*

[6]Proof of a net asset prices increase is in Appendix B.1

when there is no aggregate demand shortage, in both the IBC and ABC economy.

(a) In the IBC economy, it only generates a wealth redistribution by increasing the interest rate;

(b) In the ABC economy, it can relax the borrowing constraint by boosting asset prices to further improve the welfare of the borrowers in addition to a wealth redistribution.

Lemma 2.5. *A change in* t_1^l *and* t_2^l *has an opposing effect on an income-based borrowing economy when there is an aggregate demand shortage and an asset-based borrowing economy when there is no aggregate demand shortage. An increase in* t_1^l *undermines welfare with income-based borrowing* $(\frac{\partial V^h}{\partial t_1^l})^I_{AD} < 0$*, and improves welfare with asset-based borrowing* $(\frac{\partial V^h}{\partial t_1^l})^A_{NAD} > 0$.

Lemma 2.6. *A change in lenders' endowment* t_1^l *and* t_2^l *has similar effects on an income-based borrowing economy and an asset-based borrowing economy when there is an aggregated demand shortage. An increase in* t_1^l *or a decrease in* t_2^l *will lower income and undermine the welfare of both types of households:* $\frac{\partial V^h}{\partial t_1^l} < 0$*. In an asset-based borrowing economy, it affects the welfare of the borrowers through depressing asset prices and tightening the borrowing constraint in addition to the direct effect of lower wages and income; in an income-based economy, it affects the welfare of lenders through lowering income and tightening the borrowing constraint, and the direct effect of lower wage and income. Whether its impact is more pronounced will depend on the responsiveness of income to changes in the asset price* $\frac{Z_b^{aa}}{X_b^{aa}}$*:*

(a) If $\frac{Z_b^{aa}}{X_b^{aa}} > 1$*, the effect of changes in lenders' wealth will be greater in income than asset price for the ABC borrowers, and* $\frac{\partial V^a}{\partial t_1^l} > \frac{\partial V^i}{\partial t_1^l}$.

Consider a marginal increase in t_1^l and t_2^l when there is an aggregate demand shortage for an asset-based borrowing economy. As with an IBC economy with an AD shortage, higher t_1^l or lower t_2^l leads to excessive saving by lenders, and depresses demand and production. Wage is lower, resulting in lower income for all households. Lower income decreases asset prices, making it harder for borrowers to borrow. With a tighter constraint, borrowers reduce consumption further, which depresses demand and production further, leading to a feedback loop. Unlike in the IBC model, lower aggregate demand and lower asset price reinforce each other. In the IBC model, borrowers will increase working hours in response to lower consumption, which raises wages and tempers the negative effect on income.

An income-based borrowing economy with an AD shortage and an asset-based borrowing economy with no AD shortage can demonstrate the disparate transmission mechanisms of the two types of amplification. With income-based borrowing, shocks are transmitted through aggregate demand, and can be amplified only when wage falls. With asset-based borrowing, it is not necessary to have fluctuating income or wage for shocks to be amplified. Therefore, even when there is no AD shortage and wage is constant at the efficient level, amplification can occur through asset price changes. As t_1^l increases, it lowers income with income-based borrowing, but raises asset price with asset-based borrowing when aggregate demand externalities are absent. Thus, subsidizing lenders in the two economies will have an opposing impact on households' welfare.

Lemma 2.7. *A change in t_1^a or t_1^i has different welfare implications for an income-based borrowing economy and an asset-based borrowing economy when there is an aggregate demand shortage. An increase in t_1^a or t_1^i makes all households better off in an asset-based borrowing economy: $\frac{\partial V^h}{\partial t_1^a} > 0$, whereas it can make lenders worse* *off in an income-based borrowing economy when aggregate demand externalities are large. The difference in welfare implications originates from the disparate effect on aggregate demand:*

(a) with IBC, aggregate demand falls due to less borrowing with lower labor supply;

(b) with ABC, aggregate demand increases due to more borrowing with higher asset prices.

When there is an aggregate demand shortage and the interest rate is at the lower bound in an IBC economy, an increase in d_1^i or a decrease in d_2^i will increase the consumption of the borrowers. Higher consumption makes borrowers less willing to borrow and therefore less incentivized to work so labor supply decreases, which decreases their borrowing with lower labor income. Since the interest rate cannot fall to induce lenders to save less, the bonds market does not clear without adjustment of production and wage. Since lenders have excessive savings at the current interest rate, aggregate demand is lower, which decreases production. Firms will hire less and wages fall, reducing the income of households. The welfare of the lenders is undermined due to lower income. The welfare of the borrowers can still be improved by the direct effect of higher consumption.

A marginal increase in d_1^a or a decrease in d_2^a when there is an aggregate demand shortage in an ABC economy will increase the consumption of the borrowers. Higher current consumption boosts asset prices, enabling borrowing to take on more debt. Without adjustment of the interest rate, this boosts aggregate demand. Firms hire more labor and produce more, which raises income. Higher income further boosts consumption and asset prices. As a result, assets become more valuable and income is also higher. The welfare of both borrowers and lenders is improved. This result will hold if the asset-based borrowing constraint is in the form $b_1^a \geq \phi^{Aa}\theta_2 p_1$ instead of $b_1^a \geq \phi^{Aa}\theta_1 p_1$ as in the current model. Subsidizing the ABC borrowers to increase consumption will also make them less incentivized to borrow, which lowers asset price, but as long as $\phi^{Aa} is less than one$, the direct positive effect of higher current consumption on asset price will dominate. The smaller ϕ^{Aa} is, the greater asset price increases given the subsidy[7].

Fiscal policy with an AD shortage. When there is an aggregate demand shortage in an IBC economy, a fiscal policy that taxes the lenders to subsidize the borrowers during the deleveraging period at $t = 1$ will have an impact on households as follows:

$$\begin{aligned} FP_b^{ia} &= -\frac{\partial V_1^i}{\partial t_1^l} + \frac{\partial V_1^i}{\partial t_1^i}\frac{\alpha^l}{\alpha^i} \\ &= \frac{\alpha^l}{\alpha^i}u'(\tilde{c}_1^i) + (1 - v'(n_1^i))u'(\tilde{c}_1^i)(\frac{\alpha^l}{\alpha^i}\underbrace{\frac{de_1^i}{dt_1^i}}_{-} - \underbrace{\frac{de_1^i}{dt_1^l}}_{-}) + \phi^{Ii}[u'(\tilde{c}_1^i) - \beta^i u'(\tilde{c}_2^i)](\frac{\alpha^l}{\alpha^i}\underbrace{\frac{de_1^i}{dt_1^i}}_{-} - \underbrace{\frac{de_1^i}{dt_1^l}}_{-}) > 0 \end{aligned} \tag{2.7}$$

$$\begin{aligned} FP_l^{ia} &= -\frac{\partial V_1^l}{\partial t_1^l} + \frac{\partial V_1^l}{\partial t_1^i}\frac{\alpha^l}{\alpha^i} \\ &= -u'(\tilde{c}_1^l) + (1 - w_1)u'(\tilde{c}_1^l))(\frac{\alpha^l}{\alpha^i}\underbrace{\frac{de_1^l}{dt_1^i}}_{-} - \underbrace{\frac{de_1^l}{dt_1^l}}_{-}) > 0 \end{aligned} \tag{2.8}$$

The impact of fiscal policy on the income of lenders and borrowers is ambiguous since subsidizing the borrowers lowers income through aggregate demand as analyzed before. To have a positive net effect on income, first ϕ^{Ii} need to be small (to temper the negative effect of lower borrowing on aggregate demand and income) such that $\frac{J_{b1}^{ia}}{X_b^{ia}} < \frac{1}{X_l^{ia}}$ and thus $|\frac{de_1^i}{dt_1^i}| < |\frac{de_1^i}{dt_1^l}|$; second, the amount of lump-sum transfer to the IBC

[7]See proof in the Appendix.

borrowers need to be small if there are both ABC and IBC borrowers in the economy. Higher income will improve the welfare of the borrowers by directly boosting net consumption and relaxing the borrowing constraint. It can improve the welfare of the lenders by directly boosting net consumption. Note that this result will depend on the magnitude of the amplification effect as well. The multiplier effect on welfare from lower t_1^l is given by $\frac{1}{1-\frac{Z_l^{ia}}{X_l^{ia}}/\frac{Z_b^{ia}}{X_b^{ia}}} > 1$.

When there is an aggregate demand shortage in an ABC economy, a fiscal policy that taxes the lenders to subsidize the borrowers during the deleveraging period at $t = 1$ will have an impact on households as follows:

$$\begin{aligned}
FP_b^{aa} &= -\frac{\partial V_1^a}{\partial t_1^l} + \frac{\partial V_1^a}{\partial t_1^a}\frac{\alpha^l}{\alpha^a} \\
&= \frac{\alpha^l}{\alpha^a}u'(\tilde{c}_1^a) + (1 - v'(e_1))u'(\tilde{c}_1^a))(\frac{\alpha^l}{\alpha^a}\underbrace{\frac{de_1}{dt_1^a}}_{+} - \underbrace{\frac{de_1}{dt_1^l}}_{-}) \\
&+ \phi^{Aa}[u'(\tilde{c}_1^a) - \beta^a u'(\tilde{c}_2^a)](\frac{\alpha^l}{\alpha^a}\underbrace{\frac{dp_1}{dt_1^a}}_{+} - \underbrace{\frac{dp_1}{dt_1^l}}_{-}) > 0
\end{aligned} \tag{2.9}$$

$$\begin{aligned}
FP_l^{aa} &= -\frac{\partial V_1^l}{\partial t_1^l} + \frac{\partial V_1^l}{\partial t_1^a}\frac{\alpha^l}{\alpha^a} \\
&= -u'(\tilde{c}_1^l) + (1 - v'(e_1))u'(\tilde{c}_1^l))(\frac{\alpha^l}{\alpha^a}\frac{de_1}{dt_1^a} - \frac{de_1}{dt_1^l}) > 0
\end{aligned} \tag{2.10}$$

Unlike in the IBC model, subsidizing the ABC borrowers will increase asset prices, which reinforces the positive effect on aggregate demand and income. Therefore, a fiscal policy improves the welfare of the borrowers by boosting net consumption from higher income and relaxing the borrowing constraint with higher asset prices. It can also improve welfare of the lenders since the multiplier on income is greater than one and thus the positive effect on net consumption will dominate the negative effect from taxing the lenders.

Proposition 2.8. *A fiscal policy that taxes lenders to subsidize borrowers in a crisis will improve welfare of both borrowers and lenders when there is an aggregate demand shortage, in both the IBC and ABC economy. Subsidizing the ABC borrowers is more effective than subsidizing the IBC borrowers:*

(a) in the IBC economy, the sufficient condition for this result to hold is $\frac{1}{1-\frac{Z_l^{ia}}{X_l^{ia}}/\frac{Z_b^{ia}}{X_b^{ia}}} > 1$;

(b) in the ABC economy, the sufficient condition for this result to hold is $\frac{1}{1-\frac{Z_l^{aa}}{X_l^{aa}}/\frac{Z_b^{aa}}{X_b^{aa}}} > 1$;

(c) if $\frac{Z_b^{ia}}{X_b^{ia}} > \frac{Z_b^{aa}}{X_b^{aa}} > 1$*, fiscal policy improves the welfare of the ABC borrowers more than ABC borrowers.*

Liquidity operations with no AD shortage. In an IBC economy, liquidity operations have a similar impact on the interest rate as a tax on lenders, but it can make both borrowers and lenders better off. Since lenders are unconstrained, a transfer across time does not affect welfare directly through consumption. They are better off as a result of higher interest rate. Because borrowers are constrained, a transfer across time can improve welfare directly by relaxing the borrowing constraint.

$$\begin{aligned} LO_b^{in} &= -\frac{\partial V_1^i}{\partial t_1^l} + \frac{\partial V_1^i}{\partial t_1^i}\frac{\alpha^l}{\alpha^i} - \frac{\partial V_1^i}{\partial t_2^i}\frac{\alpha^l}{\alpha^i} + \frac{\partial V_1^i}{\partial t_2^l} \\ &= \frac{\alpha^l}{\alpha^i}(u'(\tilde{c}_1^i) - \beta^i u'(\tilde{c}_2^i)) - \frac{\phi^{Ii} n_1^i}{(1+r_2)^2} u'(\tilde{c}_1^i)[\frac{\alpha^l}{\alpha^i}(\frac{dr_2}{dt_1^i} - \frac{dr_2}{dt_2^i}) + (\frac{dr_2}{dt_2^l} - \frac{dr_2}{dt_1^l})] > 0 \end{aligned} \tag{2.11}$$

$$
\begin{aligned}
LO_l^{in} &= -\frac{\partial V_1^l}{\partial t_1^l} + \frac{\partial V_1^l}{\partial t_1^i}\frac{\alpha^l}{\alpha^i} - \frac{\partial V_1^l}{\partial t_2^i}\frac{\alpha^l}{\alpha^i} + \frac{\partial V_1^l}{\partial t_2^l} \\
&= \frac{\alpha^i}{\alpha^l}\frac{\phi^{Ii}n_1^i}{(1+r_2)^2}u'(\tilde{c}_1^l))[\frac{\alpha^l}{\alpha^i}(\frac{dr_2}{dt_1^i} - \frac{dr_2}{dt_2^i}) + (\frac{dr_2}{dt_2^l} - \frac{dr_2}{dt_1^l})] > 0
\end{aligned}
\tag{2.12}
$$

Similarly, liquidity operations in an ABC economy will improve the welfare of both borrowers and lenders in the asset-based borrowing economy as in the income-based borrowing economy.

$$
\begin{aligned}
LO_b^{an} &= -\frac{\partial V_1^a}{\partial t_1^l} + \frac{\partial V_1^a}{\partial t_1^a}\frac{\alpha^l}{\alpha^a} - \frac{\partial V_1^a}{\partial t_2^a}\frac{\alpha^l}{\alpha^a} + \frac{\partial V_1^a}{\partial t_2^l} \\
&= \frac{\alpha^l}{\alpha^a}(u'(\tilde{c}_1^a) - \beta^a u'(\tilde{c}_2^a)) - u'(\tilde{c}_1^a)\frac{\phi^{Aa}p_1}{(1+r_2)^2}[\frac{\alpha^l}{\alpha^a}(\frac{dr_2}{dt_1^a} - \frac{dr_2}{dt_2^a}) + (\frac{dr_2}{dt_2^l} - \frac{dr_2}{dt_1^l})] \\
&+ \frac{\phi^{Aa}}{1+r_2}[u'(\tilde{c}_1^a) - \beta^a(1+r_2)u'(\tilde{c}_2^a)][\frac{\alpha^l}{\alpha^a}(\frac{dp_1}{dt_1^a} - \frac{dp_1}{dt_2^a}) + (\frac{dp_1}{dt_1^l} - \frac{dp_1}{dt_2^l})] > 0
\end{aligned}
\tag{2.13}
$$

$$
\begin{aligned}
LO_l^{an} &= -\frac{\partial V_1^l}{\partial t_1^l} + \frac{\partial V_1^l}{\partial t_1^a}\frac{\alpha^l}{\alpha^a} - \frac{\partial V_1^l}{\partial t_2^a}\frac{\alpha^l}{\alpha^a} + \frac{\partial V_1^l}{\partial t_2^l} \\
&= u'(\tilde{c}_1^l)\frac{\alpha^a}{\alpha^l}\frac{\phi^{Aa}p_1}{(1+r_2)^2}[\frac{\alpha^l}{\alpha^a}(\frac{dr_2}{dt_1^a} - \frac{dr_2}{dt_2^a}) + (\frac{dr_2}{dt_2^l} - \frac{dr_2}{dt_1^l})] > 0
\end{aligned}
\tag{2.14}
$$

Liquidity operations with an AD shortage. Liquidity operations that borrow from lenders to purchase assets from income-based borrowers at $t = 1$, and sell assets to income-based borrowers to pay back to lenders at $t = 2$, will affect the welfare of the households:

$$\begin{aligned}LO_b^{ia} &= -\frac{\partial V_1^i}{\partial t_1^l} + \frac{\partial V_1^i}{\partial t_1^i}\frac{\alpha^l}{\alpha^i} - \frac{\partial V_1^i}{\partial t_2^i}\frac{\alpha^l}{\alpha^i} + \frac{\partial V_1^i}{\partial t_2^l} \\ &= \frac{\alpha^l}{\alpha^i}(u'(\tilde{c}_1^i) - \beta^i u'(\tilde{c}_2^i)) + (1 - v'(n_1^i))u'(\tilde{c}_1^i)[\frac{\alpha^l}{\alpha^i}(\frac{de_1^i}{dt_1^i} - \frac{de_1^i}{dt_2^i}) + (\frac{de_1^i}{dt_2^l} - \frac{de_1^i}{dt_1^l})] \\ &= \phi^{Ii}[u'(\tilde{c}_1^i) - \beta^i u'(\tilde{c}_1^i)][\frac{\alpha^l}{\alpha^i}(\frac{de_1^i}{dt_1^i} - \frac{de_1^i}{dt_2^i}) + (\frac{de_1^i}{dt_2^l} - \frac{de_1^i}{dt_1^l})] > 0\end{aligned} \tag{2.15}$$

$$\begin{aligned}LO_l^{ia} &= -\frac{\partial V_1^l}{\partial t_1^l} + \frac{\partial V_1^l}{\partial t_1^i}\frac{\alpha^l}{\alpha^i} - \frac{\partial V_1^l}{\partial t_2^i}\frac{\alpha^l}{\alpha^i} + \frac{\partial V_1^l}{\partial t_2^l} \\ &= (1 - w_1)u'(\tilde{c}_1^l))[\frac{\alpha^l}{\alpha^i}(\frac{de_1^l}{dt_1^i} - \frac{de_1^l}{dt_2^i}) + (\frac{de_1^i}{dt_2^l} - \frac{de_1^i}{dt_1^l})] > 0\end{aligned} \tag{2.16}$$

In an ABC economy, liquidity operations will affect the welfare of the households as follows:

$$\begin{aligned}LO_b^{aa} &= -\frac{\partial V_1^a}{\partial t_1^l} + \frac{\partial V_1^a}{\partial t_1^a}\frac{\alpha^l}{\alpha^a} - \frac{\partial V_1^a}{\partial t_2^a}\frac{\alpha^l}{\alpha^a} + \frac{\partial V_1^a}{\partial t_2^l} \\ &= \frac{\alpha^l}{\alpha^a}(u'(\tilde{c}_1^a) - \beta^a u'(\tilde{c}_2^a)) + (1 - v'(e_1))u'(\tilde{c}_1^a)[\frac{\alpha^l}{\alpha^a}(\frac{de_1}{dt_1^a} - \frac{de_1}{dt_2^a}) + (\frac{de_1}{dt_2^l} - \frac{de_1}{dt_1^l})] \\ &+ \phi^{Aa}[u'(\tilde{c}_1^a) - \beta^a u'(\tilde{c}_2^a)][\frac{\alpha^l}{\alpha^a}(\frac{dp_1}{dt_1^a} - \frac{dp_1}{dt_2^a}) + (\frac{dp_1}{dt_1^l} - \frac{dp_1}{dt_1^l})] > 0\end{aligned} \tag{2.17}$$

$$\begin{aligned}LO_l^{aa} &= -\frac{\partial V_1^l}{\partial t_1^l} + \frac{\partial V_1^l}{\partial t_1^a}\frac{\alpha^l}{\alpha^a} - \frac{\partial V_1^l}{\partial t_2^a}\frac{\alpha^l}{\alpha^a} + \frac{\partial V_1^l}{\partial t_2^l} \\ &= (1 - v'(e_1))u'(\tilde{c}_1^l))[\frac{\alpha^l}{\alpha^a}(\frac{de_1^l}{dt_1^a} - \frac{de_1^l}{dt_2^a}) + (\frac{de_1}{dt_2^l} - \frac{de_1}{dt_1^l})] > 0\end{aligned} \tag{2.18}$$

Liquidity operations will improve the welfare of both borrowers and lenders as previously.

Proposition 2.9. *Liquidity operations that borrow from lenders to purchase assets from borrowers in a crisis, and sell assets to borrowers to pay back to lenders in the future will improve the welfare of both borrowers and lenders when there is no aggregate demand shortage and when there is an aggregate demand shortage, in both the IBC and ABC economy.*

(a) when there is no aggregate demand shortage, it improves lenders' welfare by increasing interest rate;

(b) when there is an aggregate demand shortage, it improves lenders' welfare by increasing wages.

2.3 Macroprudential Policies

Ex post policies can lead to Pareto improvements when aggregate demand externalities are large. However, it depends on the magnitude of the amplification. In a model set-up with separable preferences of households and the wealth effect on labor supply, aggregate demand externalities might not be large enough such that a fiscal policy as implemented in the previous section achieves such welfare improvements. Therefore, it is important to understand how ex ante policies, such as macroprudential policies, can be implemented to achieve an efficient outcome. I analyze the problem of a constrained planner that faces the same borrowing constraints as households do in the decentralized optimization problem, choosing allocations during the debt accumulation stage.

Let B_{b1} be the aggregate level of debt in the $b \in \{a, i\}$ type of borrowing economy in period 1, and λ_h be the Lagrangian multiplier associated with the type h borrowers. The decentralized problem of the households in period one can be written as:

$$V^h(b_1^h, B_{b1}) = \max_{b_2^h, n_1^h}\{u(n_1^h(B_{b1}) + d_1^h + b_1^h - \frac{b_2^h}{1+r_2(B_{b1})} - v(n_1^h(B_{b1})))$$
$$+ \beta^h u(n_2^h + d_2^h + b_2^h - v(n_2^h)) + \lambda_h[b_2^h + \phi^{Ih} n_1^h(B_{b1}) + \phi^{Ah}\theta_1 p_1(B_{b1})]\} \quad (2.19)$$

where $n_1^l(B_{i1}) = 2\frac{\alpha^i}{\alpha^l}\phi^{Ii} n_1^i(b_{i1}) + v(n_1^l(B_{i1})) + \frac{B_{i1}}{\alpha^l} + (d_2^l - d_1^l) + (e^* - v(e^*))$ when there is an AD shortage; $\frac{dn_1^l}{dB_{i1}} = 0$ when there is no AD shortage. $\frac{dn_1^i}{dB_{i1}} > 0$ independent of AD shortage. And $p_1(B_{a1}) = \frac{u'(\tilde{c}_2^a)}{u'(\tilde{c}_1^a)}\beta^a d_2^a$. And $r_2(B_{b1}) = 0$ when there is an AD shortage; $r_2'(B_{b1}) > 0$ when there is no AD shortage. The first-order conditions are given by $u'(c_1^h) = (1+r_2)(\beta^h u'(c_2^h) + \lambda_h)$ and $u'(c_1^h)(1 - v'(n_1^h)) + \lambda_h \phi^{Ih} = 0$. The constrained planner takes into account the impact of aggregate debt on interest rate, aggregate demand, and asset price, so she chooses the aggregate level of debt in period 0 to:

$$\max_{\{c_0^h, n_0^h, B_{b1}\}} \quad \sum_{h\in\mathcal{H}} \alpha^h \gamma^h [u(c_0^h - v(n_0^h)) + \beta^h V^h(b_1^h, B_{b1})]$$
$$\text{s.t.} \quad \sum_{h\in\mathcal{H}} \alpha^h c_0^h = \sum_{h\in\mathcal{H}} \alpha^h (n_0^h + \theta_0^h d_0^h), \quad (2.20)$$
$$B_{i1} = \alpha^i b_1^i = -\alpha^l b_1^l, \quad \text{or} \quad B_{a1} = \alpha^a b_1^a = -\alpha^l b_1^l$$

The optimality conditions for the constrained planner's problem is given by:

$$v'(n_0^h) = 1 \quad (2.21)$$

$$\gamma^l u'(\tilde{c}_0^l) = \gamma^h u'(\tilde{c}_0^h) \quad \text{for} \quad h \in \{i, a\} \quad (2.22)$$

$$\sum_{h\in\mathcal{H}} \alpha^h \gamma^h \beta^h \frac{\partial V^h(b_1^h, B_{b1})}{\partial B_{h1}} = 0 \quad (2.23)$$

First consider an income-based borrowing economy, i.e., $b = i$. The optimality condition (2.23) can be written as:

$$\gamma^l\beta^l u'(\tilde{c}_1^l) = \gamma^i\beta^i u'(\tilde{c}_1^i) + \alpha^l\gamma^l\beta^l u'(\tilde{c}_1^l)(1-v'(n_1^l))\frac{dn_1^l}{dB_{i1}} + \alpha^i\gamma^i\beta^i u'(\tilde{c}_1^i)(1-v'(n_1^i))\frac{dn_1^i}{dB_{i1}}$$
$$+ [\alpha^l\gamma^l\beta^l u'(\tilde{c}_1^l)b_2^l + \alpha^i\gamma^i\beta^i u'(\tilde{c}_1^i)b_2^i]\frac{1}{(1+r_2)^2}\frac{dr_2}{dB_{i1}} + \alpha^i\gamma^i\beta^i\phi^{Ii}\frac{dn_1^i}{dB_{i1}}\lambda_i$$
$$= \gamma^i\beta^i u'(\tilde{c}_1^i) + \alpha^l\gamma^l\beta^l u'(\tilde{c}_1^l)(1-v'(n_1^l))\frac{dn_1^l}{dB_{i1}} + [\alpha^l\gamma^l\beta^l u'(\tilde{c}_1^l)b_2^l + \alpha^i\gamma^i\beta^i u'(\tilde{c}_1^i)b_2^i]\frac{1}{(1+r_2)^2}\frac{dr_2}{dB_{i1}} \tag{2.24}$$

Note that the planner will never choose a level of aggregate debt B_{i1} which leads to an aggregate demand shortage. The reason is that when there is an AD shortage, $\frac{dn_1^l}{dB_{i1}} = \frac{1+2\alpha^i\phi^{Ii}\frac{dn_1^i}{dB_{i1}}}{\alpha^l(1-v'(n_1^l))}$, which makes the optimality condition of the planner (2.24) impossible to hold with equality. Therefore, the constrained efficient allocations of the planner exist only when $b_1^i \geq \underline{b}_1^i$.

Proposition 2.10. *In both the IBC economy, a macroprudential policy can be implemented to achieve constrained efficient allocations in the decentralized equilibrium. The macroprudential policy can be implemented as a quantity restriction on any positive debt issuance such that $b_1^i \geq \underline{b}_1^i$ combined with a lump-sum transfer between borrowers and lenders.*

Next consider an asset-based borrowing economy, i.e., $b = a$. The optimality condition (2.23) can be written as:

$$\gamma^l\beta^l u'(\tilde{c}_1^l) = \gamma^a\beta^a u'(\tilde{c}_1^a) + \alpha^l\gamma^l\beta^l u'(\tilde{c}_1^l)(1-v'(n_1^l))\frac{dn_1^l}{dB_{a1}} + \alpha^a\gamma^a\beta^a u'(\tilde{c}_1^a)(1-v'(n_1^a))\frac{dn_1^a}{dB_{a1}}$$
$$+ [\alpha^l\gamma^l\beta^l u'(\tilde{c}_1^l)b_2^l + \alpha^a\gamma^a\beta^a u'(\tilde{c}_1^a)b_2^a]\frac{1}{(1+r_2)^2}\frac{dr_2}{dB_{a1}} + \alpha^a\gamma^a\beta^a\phi^{Aa}\frac{dp_1}{dB_{a1}}\lambda_a \tag{2.25}$$

Similarly, the planner will never choose a level of aggregate debt B_{a1} which leads to an aggregate demand shortage since when there is an AD shortage, $\frac{dn_1^l}{dB_{a1}} = \frac{1+2\alpha^a\phi^{Aa}\frac{dp_1}{dB_{a1}}}{\alpha^l(1-v'(n_1^l))}$, which makes the optimality condition of the planner (2.25) impossible to hold with equality. Therefore, the constrained efficient allocations of the planner exist only when $b_1^a \geq \underline{b}_1^a$.

Moreover, (2.25) implies the planner will distort the Euler equation of the households whenever the borrowers are constrained, i.e., $\lambda_a > 0$, such that $\frac{u'(\tilde{c}_1^l)}{u'(\tilde{c}_0^l)} > \frac{u'(\tilde{c}_1^a)}{u'(\tilde{c}_0^a)}$. The constrained efficient allocation can be implemented with a tax τ_0^a on bond issuance of the borrowers combined with a lump-sum transfer to the borrowers. Assume the Pareto weights are chosen such that $\frac{\gamma^l}{\gamma^a} = \frac{u'(\tilde{c}_1^a)}{u'(\tilde{c}_1^l)}$ for the equality of wealth distribution. The optimal macroprudential tax τ_0^a is then given by:

$$\tau_0^a = \frac{\alpha^a\beta^a\phi^{Aa}\frac{dp_1}{dB_{a1}}\lambda_a}{\beta^a u'(\tilde{c}_1^a) + \alpha^a\beta^a\phi^{Aa}\frac{dp_1}{dB_{a1}}\lambda_a} \tag{2.26}$$

Proposition 2.11. *In the ABC economy, a macroprudential policy can be implemented to achieve constrained efficient allocations in the decentralized equilibrium. The macroprudential policy can be implemented as:*

- *a quantity restriction on any positive debt issuance, or*

- *a tax τ_0^a given in (2.26) on any positive debt issuance which is rebated to households in a lump-sum manner,*

combined with a lump-sum transfer between borrowers and lenders.

2.4 An Economy with Two Types of Borrowers

In this section, I will consider the model with additional heterogeneity in which $\mathcal{H} = \{l, i, a\}$, and each type of households has a weight of α^h with $\sum_h \alpha^h = 1$. The model environment is the same as in the previous section. I restrict $\phi^{Ia} = \phi^{Ai} = 0$, and $\phi^{Ii} > 0, \phi^{Aa} > 0$. Firms and households optimization problem is given in (1.10) and (1.12). One important modification of the model in the numerical illustration is to have aggregate income, instead of individual income, in the income-based borrowing constraint. This modification enables the decentralized equilibrium at $t = 1, 2$ to be reduced to and pinned down by only two endogenous variables, interest rate and asset price when there is no aggregate demand shortage; and aggregate income and asset price when there is an aggregate demand shortage. Comparative statics of changes in t_1^l, t_1^i and t_1^a are similar to those of the model with individual income in the borrowing constraint. However, since borrowers no longer have the incentive to increase labor supply when consumption is low and to decrease labor supply when consumption is high, there will be no adverse impact on aggregate demand when t_1^i increases as seen in the model with individual income in the borrowing constraint when there is an aggregate demand shortage. Therefore, a transfer or subsidy to the IBC borrowers will improve the welfare of households more in the aggregate income model. All the derivations for the decentralized equilibrium and comparative statics are in the Appendix.

In the decentralized equilibrium, income- or asset-based borrowers can be the only type of households who are constrained in borrowing, but I will focus on the decentralized equilibrium in which both types of borrowers are borrowing constrained since it is more relevant for policy consideration. The bonds market clearing condition becomes $b_t^l = -\frac{\alpha^a}{\alpha^l} b_t^a - \frac{\alpha^i}{\alpha^l} b_t^i$.

When there is no aggregate demand shortage, the equilibrium is pinned down by:

$$u'(\tilde{c}_0^i) = \beta^i (1 + r_1) u'(\tilde{c}_1^i) \tag{2.27}$$

$$u'(\tilde{c}_0^a) = \beta^l (1 + r_1) u'(\tilde{c}_1^a) \tag{2.28}$$

$$u'(\tilde{c}_1^l) = \beta^l (1 + r_1) u'(\tilde{c}_1^l) \tag{2.29}$$

$$p_1 = \frac{u'(e^* + d_2^a - \phi^{Aa} p_1 - v(e^*))}{u'(e^* + d_1^a + b_1^a + \frac{\phi^{Aa} p_1}{1+r_2} - v(e^*))} \beta^a d_2^a \tag{2.30}$$

$$u'(e^* + d_1^l + b_1^l - \frac{1}{(1 + r_2)} (\frac{\alpha^a}{\alpha^l} \phi^{Aa} p_1 + \frac{\alpha^i}{\alpha^l} \phi^{Ii} e^*) - v(e^*)) \tag{2.31}$$

$$= \beta^l (1 + r_2) u'(e^* + d_2^l + \frac{\alpha^a}{\alpha^l} \phi^{Aa} p_1 + \frac{\alpha^i}{\alpha^l} \phi^{Ii} e^* - v(e^*)) \tag{2.32}$$

When there is an aggregate demand shortage, the equilibrium is pinned down by:

$$u'(\tilde{c}_0^i) = \beta^i (1 + r_1) u'(\tilde{c}_1^i) \tag{2.33}$$

$$u'(\tilde{c}_0^a) = \beta^l (1 + r_1) u'(\tilde{c}_1^a) \tag{2.34}$$

$$u'(\tilde{c}_1^l) = \beta^l (1 + r_1) u'(\tilde{c}_1^l) \tag{2.35}$$

$$p_1 = \frac{u'(e^* + d_2^a - \phi^{Aa} p_1 - v(e^*))}{u'(e_1 + d_1^a + b_1^a + \phi^{Aa} p_1 - v(e_1))} \beta^a d_2^a \tag{2.36}$$

$$e_1 = 2 \frac{\alpha^a}{\alpha^l} \phi^{Aa} p_1 + 2 \frac{\alpha^i}{\alpha^l} \phi^{Ii} e_1 + v(e_1) + \frac{\alpha^a}{\alpha^l} b_1^a + \frac{\alpha^i}{\alpha^l} b_1^i + (d_2^l - d_1^l) + (e^* - v(e^*)) \tag{2.37}$$

$$b_1^l = -\frac{\alpha^a}{\alpha^l} b_1^a - \frac{\alpha^i}{\alpha^l} b_1^i \tag{2.38}$$

$$b_2^l = \frac{\alpha^a}{\alpha^l} \phi^{Aa} p_1 - \frac{\alpha^i}{\alpha^l} \phi^{Ii} e_1 \tag{2.39}$$

Illustration: a numerical example. I assume the utility function takes the form of:

$$u(c_t^h, n_t^h) = \frac{1}{1-\frac{1}{\sigma}}(c_t^h - \chi \frac{{n_t^h}^{1+\xi}}{1+\xi})^{1-\frac{1}{\sigma}},$$

where σ is the intertemporal elasticity of substitution and ξ is the frisch elasticity of labor supply. Value of the parameters in the model is calibrated as in Table 2.1.

elasticity of substitution	σ	0.5	standard value
disutility parameter of labor	χ	1	
frisch elasticity of labor supply	ξ	1	
discount factor of asset-based borrowers	β^a	0.96	standard value
discount factor of income-based borrowers	β^i	0.96	standard value
discount factor of lenders	β^l	1	
fraction of asset-based borrowers	α^a	0.1	the share of borrowing households who have mortgage
fraction of income-based borrowers	α^i	0.15	
fraction of lenders	α^l	0.75	
tightness of the ABC	ϕ^{Aa}	0.3	mortgage debt service payments as a percentage of disposable income
tightness of the IBC	ϕ^{Ii}	0.1	credit card debt as a percentage of GDP
elasticity of substitution	ϵ	0.8	standard value
asset dividend	d_t^h	0.15	average of housing share of US GDP
initial bond holdings of asset-based borrowers	b_0^a	-0.2	household mortgage debt to GDP ratio
initial bond holdings of income-based borrowers	b_0^i	-0.2	household credit card debt to GDP ratio

Table 2.1: Assumptions on parameters

Following these assumptions on parameters, $e^* = n^* = 1$. The decentralized equilibria are characterized in Figure 2.1 when there is no AD shortage and in Figure 2.2 when there is an AD shortage. Both equilibria is unique and well-defined. When there is an AD shortage (given the initial debt of borrowers $b_0^i = -0.28$), there is an equilibrium at which aggregate income is above 1. This equilibrium is not sustainable since firms will earn negative profits if the wage is above one. When there is no AD shortage, a fiscal policy that taxes the lenders to transfer to the asset-based borrowers, will shift the AP and AD curve up, leading to higher asset prices and higher interest rate. When there is an AD shortage, it also shifts up both the AP and AD curve, leading to higher asset prices and aggregate income. With a transfer to the income-based borrowers, there will be no upward shift of the AP curve, and therefore, asset prices and income do not rise as much as subsidizing the asset-based borrowers, which results in a smaller welfare improvement.

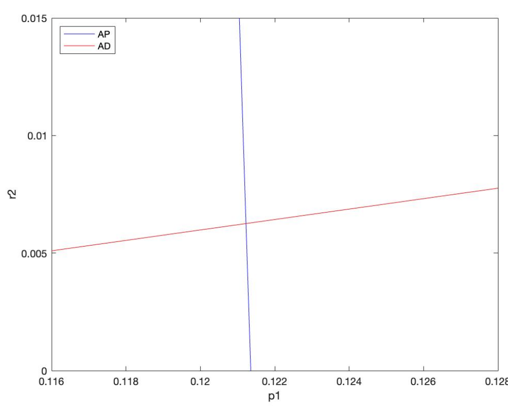

Figure 2.1: Equilibrium, No AD Shortage

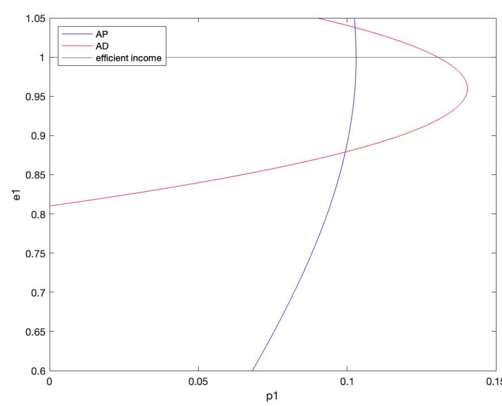

Figure 2.2: Equilibrium, AD Shortage

Figure 2.3 and 2.4 illustrate the marginal welfare gains from the fiscal policy. Fiscal policy does not lead to a Pareto improvement when there is no AD shortage. It incurs a welfare loss for the lenders due to a higher interest rate. However, it leads to a Pareto improvement when there is an AD shortage, since the income of both borrowers and lenders becomes higher, which improves their welfare. Moreover, as the fraction of subsidy given to the asset-based borrowers increases, the marginal gain in the welfare of both types of borrowers increases.

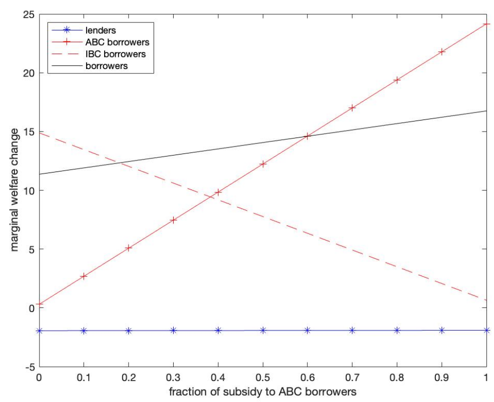

Figure 2.3: Welfare gains, No AD Shortage

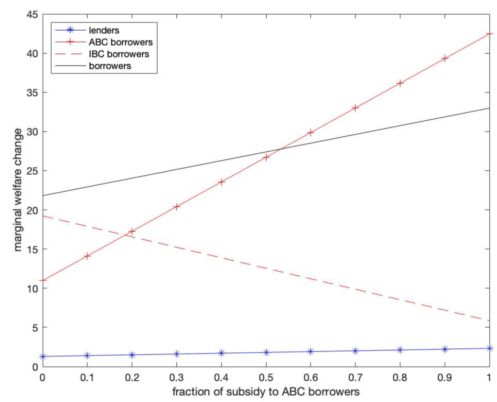

Figure 2.4: Welfare gains, AD Shortage

2.5 Conclusion

This chapter studies the amplification effects with income-based borrowing constraints versus asset-based borrowing constraints. The effects of shocks are amplified via the pecuniary externalities arising from falling asset prices with the asset-based constraints, whereas they are amplified via the aggregate demand externalities as a result of the binding lower bound on the interest rate with the income-based constraints. The differences in the transmission mechanism of shocks with these types of constraints have different policy implications.

A fiscal policy that taxes lenders to subsidize borrowers in a crisis will improve the welfare of the borrowers and undermine the welfare of the lenders when there is no aggregate demand shortage, in both the IBC and ABC economy. In the IBC economy, it only generates wealth redistribution by increasing the interest rate. In the ABC economy, it can relax the borrowing constraint by boosting asset prices to improve the welfare of the borrowers in addition to wealth redistribution. Lenders are always worse off due to the tax. A fiscal policy that taxes lenders to subsidize borrowers in a crisis can improve the welfare of both borrowers and lenders when there is an aggregate demand shortage, leading to a Pareto improvement when aggregate demand externalities are large in both the IBC and ABC economy. Subsidizing the ABC borrowers in a lump-sum form can improve welfare more than subsidizing the IBC borrowers.

Liquidity operations that borrow from lenders to carry out asset purchases during a deleveraging episode and sales after deleveraging to pay back to lenders can lead to a Pareto improvement independent of whether there is an aggregate demand shortage, in both the IBC and ABC economy. Since it involves a transfer across time, it improves borrowers' welfare by getting around the borrowing constraint. Since lenders are unconstrained, the effect of a current loss in wealth is completely offset by an increase in wealth in the future. When there is no aggregate demand shortage, it improves lenders' welfare by increasing interest rate; when there is an aggregate demand shortage, it improves lenders' welfare by increasing income.

A quantity restriction on debt issuance can achieve constrained efficiency with both IBCs and ABCs. A macroprudential tax on any positive debt issuance combined with a transfer between borrowers and lenders will lead to constrained efficient allocation with ABCs. Due to the form of preferences, it is not feasible to derive an analytical solution of the optimal macroprudential tax with IBCs, which opens up possibilities for future research.

Chapter 3

International Currency Status: A Perspective From Foreign Currency Debt Issuance

3.1 Introduction

International money has the same roles as money: medium of exchange, unit of account, and the store of value, and a currency's international role can reflect decisions made not only by private agents but also by official bodies, such as central banks. The U.S. dollar has been the predominant international currency since it overtook the British pound sterling in the late 1940s in the aftermath of the World War II. Its international presence can be seen from every aspect of the three functions of money. Krugman (1984) frames the dollar's role as a medium of exchange in private transaction as "vehicle", and in official transactions by central banks as an "intervention" currency; its role as a unit of account in trade contracts makes it an "invoice" currency and that in financial transactions dominates in international lending and borrowing, and additionally, some countries state the par values for exchange rates in terms of dollar, which makes it serve as a "peg"; lastly, agents hold liquid dollar-denominated assets, and central banks hold the bulk of non-gold reserves in dollars – its role as the store of value private, or the "banking" role.

Despite its current ascendancy among international currencies, the dollar did not overtake the sterling until over half a century after the U.S. economy surpassed the British economy in size in 1872 largely due to considerable inertia. (Chinn and Frankel, 2008) The landscape of international currencies has been characterized by slow and gradual changes, and the drivers of the changes are slow-moving variables. The literature on the determinants of currency internationalization are mainly focused on output and trade share, the size and depth of a country's financial market, the stability of the value of the currency, and network externalities. (Goldberg and Tille, 2008; Matsuyama et al., 1993; Rey, 2001; Trejos and Wright, 1995) A country with a large share in international output and trade and with free capital mobility and well-developed financial market has a natural advantage in making its currency internationalized. For the currency to be a good unit of account in trade invoicing and international borrowing, or store of value in denominating assets held by investors or reserves held by central banks, its value should not fluctuate widely. Moreover, a currency is used internationally because everyone else uses it, and if it is widely used in trade invoicing, it tends to be widely used in financial transactions and hence foreign exchange market, which generates a great degree of path dependency and inertia.

The dollar's status as a dominant international currency was challenged by the Japanese yen and the Deutsche mark in the 1980s as central bank foreign exchange reserve holdings of the two currencies gained steadily, but more recently by the euro with its official introduction in 1999. At its initial introduction, the euro was mainly used within the euro area, but its international role had increased gradually in particular during the first five years after its introduction, and had mostly advanced until 2003-05 and stabilized subsequently before the 2008 financial crisis. The share of euro-denominated instruments in international financial markets, including debt securities markets, derivatives markets, and foreign exchange markets, had increased substantially, and the same trend followed for the share of euro-denominated cross-border deposits and bank loans, the share of euro-denominated trade and the share of euro in official foreign exchange reserve holdings. (Bank for International Settlements, 2011; Hale and Spiegel, 2012) As the euro consolidated its role as the second international currency after the dollar and even rivaled against it in some domains of international currency usage, predictions about the euro surpassing the dollar in the future became popular. Chinn and Frankel (2008) estimate that the euro could overtake the dollar as early as 2015 if the dollar continues to depreciate and London becomes the financial center of the euro. However, such predictions did not realize as the euro lost momentum after the 2008 Great Financial Crisis (GFC) and in the subsequent European debt crisis.

This chapter tries to address why the euro has fallen into eclipse as the dollar has risen in the post-crisis era. While all domains of the international currency status are important, it focuses only on the store of value and unit of account role in debt securities in the international capital markets. The reasons are as follows. First, it is closely related with a phenomenon named by Eichengreen and Hausmann (1999) as "original sin" – that most countries find it difficult to borrow in international capital markets in their domestic currencies. Countries that need to borrow externally usually choose to issue debt denominated in international currencies, such as the dollar. Second, accumulation of foreign-currency debt can pose threat to financial stability, resulting in financial crisis or sovereign debt crisis. Emerging Market Economies (EMEs) stepped up offshore issuance after the 2008 financial crisis (as in Figure C.2a, C.2b).[1] The majority of bonds issued offshore are in foreign currencies, and firms tend to use offshore bond proceeds to increase holdings of short-term assets, which raises financial stability concerns. (Serena and Moreno, 2016) In addition, as private sector in EMEs continues to borrow externally in foreign currencies, government growingly borrows in domestic currency internationally; since it is too costly to depreciate for the private sector with currency mismatches, countries may prefer to default on sovereign debt than inflate the debt away. (Du and Schreger, 2016) Third, the medium of exchange, unit of account, and store of value roles of international currencies are interdependent among each other. If a currency is a good store of value, e.g., assets denominated in the currency are highly valued by investors, the costs of making markets against the currency are low, promoting the medium of exchange role of the currency. The medium of exchange role can in turn strengthen the unit of account role, such as trade invoicing and debt denomination. (Krugman, 1984) Although this chapter does not aim at a comprehensive analysis of roles of an international currency, understanding of one aspect of them will shed light on the others.

This chapter is laid out as follows. Section 3.2 gives an overview of the dollar and the euro's role as an international investment currency and an international financing currency in international capital markets. The former is associated with the demand for assets denominated in these currencies, and the latter is associated with the supply of them. Several possible explanations from both the demand and supply side are given for the changes in the dollar and the euro's role. Section 3.3 delves into the supply side and analyzes empirically the trends in dollar- and euro-denominated debt issuance by decomposing them by different sectors, country groups and maturities. In Section 3.4, I perform a recursive VAR to identify factors that drive the change in

[1] All figures are smoothed by a two-lag and two-lead moving average.

the supply of dollar and euro debt issuance. Section 3.5 concludes the chapter and provides plans for future study.

3.2 The International Financing Role of A Currency

A currency's role in international capital markets is associated with two questions: first, in which currencies an individual, a firm, or a government chooses to invest their wealth, or how they choose the currency composition of their portfolio – the investment role of a currency; second, in which currencies a firm or a government chooses to finance themselves – the financing role of a currency. (Detken and Hartmann, 2000) The investment role of a currency is related to the demand for bonds denominated in the currency, and the financing role is related to the supply of bonds denominated in the currency. While the demand and supply of bonds denominated in one currency must be matched in equilibrium, the investment and financing role of a currency in international capital markets might not be the same since the supply of bonds denominated in a currency can be met by domestic demand. Take the dollar as an example, agents in a country can issue dollar-denominated debt in domestic capital market and in international market. The amount of dollar-denominated debt issued by all countries in both markets must be equal to the amount of dollar debt securities in portfolios held by agents from the U.S. and by agents from other parts of the globe in equilibrium. However, since U.S. domestic demand for dollar debt is smaller than dollar debt issued by all countries in their domestic markets, dollar debt issued by all countries in international markets must be smaller than the international demand for dollar debt.

3.2.1 The international investment role

The international investment role of the dollar and the euro has changed after the Great Financial Crisis. Maggiori et al. (2018) document the rise of the dollar and the fall of the euro in the share of cross-border bond holdings in the post-crisis era: bonds denominated in euro in investors' portfolios fell sharply after the crisis while those in dollar grew steadily, whereas euro bonds accounted for a significant share of total global bonds held across borders until 2007; excluding government bond and other bonds, this trend holds for corporate bond as well.

A plausible explanation of foreign investors tilting their portfolio toward dollar-denominated bonds after the crisis might be an increasing demand for safe dollar assets. Investors value the liquidity and safety of the so-called "convenience assets", such as the Treasury bond, which drives up their prices and enables the investors to derive a convenience yield from holding these assets, and a decrease in the supply of the Treasury bond may widen the yield spread between the Treasury bonds and corporate bonds that offer less liquidity and safety as investors assign more value to the liquidity and safety attributes offered by the Treasury bond. (Krishnamurthy and Vissing-Jorgensen, 2012) To quantify the unobserved convenience yield, many studies in the literature use the Treasury basis proportional to the convenience yield as a proxy, and it is defined as the difference in yields between the dollar yield on short-term Treasury bonds and foreign government bonds hedged by forward contract into the dollar. A negative Treasury basis indicates the U.S. Treasury bond are expensive relative to foreign government bonds. Jiang et al. (2018) find that when the convenience yield increases or the Treasury basis becomes more negative, the dollar will immediately appreciate, lowering foreign investors' expected return on holding safe dollar assets, and negative Treasury basis was prevalent even before the

Great Financial Crisis because the U.S. Treasury chose not to expand supply in face of negative basis, and it plunged to the lowest level since 1993 in 2008, signifying pressing demand for dollar safety in the financial crisis.

While corporate bond is riskier and is attached less liquidity and safety by investors than the Treasury bond, the "flight-to-safety" argument might still be able to explain the fall of euro-denominated corporate debt and the rise of dollar-denominated corporate debt if investors can derive a certain level of convenience yield from investing in dollar corporate bond rather than invest in the highly risky euro particularly during the European debt crisis. Due to data source limitation, this chapter is not going to try to obtain a causal relationship between dollar safety and the fall of the euro, but more focus on the supple side of bonds – the international financing role of a currency.[2]

3.2.2 The international financing role

Firms can finance themselves through the usage of debt. They can borrow in local currency, or borrow in foreign currencies that essentially converted into local currency using foreign exchange derivatives, or borrow directly in foreign currencies, that is, debt left unhedged and subject to fluctuations in exchange rate. Issuing debt in foreign currencies incurs a payment of fixed cost, and engaging in the derivatives market is also costly, so why do firms issue foreign currency debt and what factors determine firms' decision to use local currency or foreign currency debt. There are several capital structure theories that make predictions about the decisions.

The static trade-off theory predicts that debt ratio increases with the benefits of

[2] To derive the Treasury basis, I would need data on the forward exchange rate, which is available from Datastream that I do not have access to.

debt, such as tax shields, and decreases with the cost of debt. (Frank and Goyal, 2008) If the level of interest rate in foreign markets is lower than that of domestic market, firms might be more willing to issue foreign debt, indicating local currency debt and foreign currency debt are used as substitutes.[3] Similarly, the cost of foreign exchange derivatives (reflected in bid-ask spread and trading volume) will affect the use of converted local currency debt. In addition, cost of financing may also arise from capital market imperfections, or agency cost. Legal barriers and information asymmetries will lead to high monitoring cost for investors, and the higher the cost, firms would borrow more locally.

The pecking order theory predicts that the preferred order of financing instruments by firms is internally generated cash flows, external debt, and external equity. (Myers and Majluf, 1984) Extending the pecking order to currency choice of debt, firms would prefer local currency, foreign currency, and lastly converted local currency debt; due to a lack of firm-level statistics, it is hard to differentiate between converted local currency debt and foreign currency debt, but previous studies find that converted local currency debt accounted for 41.7% while foreign currency debt, 10.7% of total debt issues by East Asian firms, indicating a likely predominantly larger share of foreign currency debt than converting local currency debt in general. (Allayannis et al., 2003) An implication of the pecking order theory is that local currency and foreign currency debt are used as complements.

The risk management theory suggests that for firms with foreign operations (e.g. multinationals) that generate cash flows in foreign currencies (operating motives), or those with foreign currency cash holdings, issuing foreign currency debt can be a

[3] A simplified assumption is made that debt issued in foreign market is denominated in foreign currency, and that issued in domestic market, denominated in local currency. The assumption is not perfect but empirical evidences show that it holds for a majority of debt issues.

natural hedge against exchange rate exposure. (Kedia and Mozumdar, 2003) Such hedging motives are typically associated with issuance of debt with short maturities rather than long-term corporate bonds. For firms with no foreign operations or foreign cash holdings but still issuing foreign currency debt, they might use derivatives to hedge against foreign exchange risk, that is, issuing converted local currency debt.

Another motive of firms issuing foreign currency debt is speculation. Standard uncovered interest rate parity does not hold empirically because the interest rate differential is negatively related with the subsequent change in exchange rate, which is dubbed as the "forward discount puzzle". Therefore, a relatively low interest rate currency tends to depreciate over time, which suggests that firms can borrow at a low cost or even make a profit from borrowing in low interest rate currencies and leaving it unhedged. Moreover, numerous studies have shown that deviations from covered interest rate parity are smaller in the short run than in the long run, creating arbitrage opportunities for firms to issue longer maturity debt in foreign currencies packaged with currency swaps. Using a sample of sovereign governments and non-financial corporations with operational cash flows only in domestic currencies, McBrady and Schill (2007) find that bond issuers issue a larger share of bonds in currencies with lower nominal interest rates, with lower interest rates even after accounting for the cost of hedging foreign exchange exposure using currency swaps, and with currencies that subsequently depreciate, suggesting the opportunistic motive of non-financial agents. More recently, Bruno and Shin (2017) investigate using firm-level balance sheet dataset dollar debt issuance by Advanced Economy (AE) and EME non-financial firms during 2002-2014 and find that dollar carry trade is prevalent for EME firms but not for AE firms. They show that EME firms tend to add the proceeds of dollar bond issuance as cash holdings and already cash-rich firms tend to issue dollar-denominated bonds, which is rarely the case for AE firms that are more conformed to predictions of the pecking order theory – to financing with debt only when internal funds are insufficient. The issuance of dollar debt is more common when dollar carry trade is more favorable in terms of a depreciation of dollar against local currency, widened bilateral interest rate differential, and low exchange rate volatility. In addition, they find such carry trade pattern does not carry over to the issuance of euro-denominated bond in a sense that issuers of euro debt tend to have lower level of cash holdings, indicating a special status of the dollar.

3.3 Empirical Analysis

To analyze the dollar and the euro's status in international debt markets, I use the quarterly debt securities statistics from the Bank for International Settlements, which distinguishes between debt securities issued in domestic and international markets.

The debt securities statistics of the Bank for International Settlements cover bills, bonds and asset-back securities issued by financial corporations, non-financial corporations, and general government. Financial corporations can be broken down into private corporations and public corporations, as shown in Figure C.1.[4] The general government sector can be broken down into central government, state government, local government and social security funds. The debt statistics distinguish between debt securities issued in domestic and international markets. The empirical part of this chapter considers international debt issuances.

[4]The financial corporations sector can also be broken down into the central bank, deposit-taking corporations, money market funds, non-MMF investment funds, securitization corporations, financial auxiliaries, captive financial institutions and money lenders, insurance corporations, pension funds and other financial intermediaries.

For debt securities issued in international markets, the BIS distinguishes the issuers by residence and by nationality. Nationality refers to the residence of the controlling parent, or the ultimate borrower, instead of the immediate borrowers on a residence basis. The nationality approach results in a reallocation of issuance from financial centers or tax havens to major economies. EME countries are known to use offshore affiliates as financing vehicles to transfer funds to the home country by direct lending so that the within-firm loan by the offshore affiliate will be classified as Foreign Direct Investment in the balance of payments, bypassing capital controls in many EME countries. Since a large proportion of countries in the sample are EMEs, data of issuers by nationality rather than by residence, are used in the analysis.

Other classifications of debt securities in the dataset include original maturity, remaining maturity, issue currency, and interest rate type. A debt security with a short-term maturity is defined as one that is payable on demand or in one year or less, while one with a long-term maturity is payable in more than one year or has no stated maturity. A debt security can have fixed interest rate, variable interest rate, and mixed interest rate.[5] The analysis in this chapter focuses on gross issues of debt securities denominated in dollar or in euro with all original maturities, all remaining maturities, and all interest rate types.

For the dataset used in the structural VAR, the GDP growth for the U.S. (A191RL1 Q225SBEA), the GDP growth for the Eurozone (CLVMNACSCAB1GQEA19), CPI of the U.S. (CPIAUCNS) and Eurozone (CP0000EZ19M086NEST), and effective federal funds rate (FEDFUNDS) are from the Federal Reserve Economic Data. The 5-year government bond yield in the Eurozone (YC.B.U2.EUR.4F.G_N_C.SV_C_YM.SR_5Y) is from the European Central Bank statistics. Since the data on government bond

[5] All data descriptions can be found in BIS Handbook on Securities Statistics.

yield in Eurozone are rates per annum by continuous compounding, the 5-year Treasury bond yield data are converted to annual rates using continuous compounding to make rates comparable. Because the 5-year government bond yield data are available from the ECB since 09/06/2004, I restrict my sample for VAR analysis from 2004Q4 to 2018Q4.

Since a majority of debt issued in domestic markets are denominated in local currencies, debt securities issued in the international markets are our main focus. The sample contains 25 countries, including 8 advanced economies (AEs) and 17 EMEs, which are listed in Table C.1. Excluded from this sample are the U.S., all European Union member countries, and financial centers, such as Singapore and Hong Kong.[6] The sample period covers between 1999 Q1, the official introduction of the euro, and 2018 Q4.

Debt securities denominated in dollar and euro together account for 80.3% on average of foreign currency debt issues over the sample period, reflecting the predominant international currency status of the dollar and the euro. Although dollar and euro debt issuance plunged to 150 billion of dollar during the Great Financial Crisis in 2008-09, it has gradually increased and reached around 400 billion of dollar in 2017 (Figure C.3a), with a pre-crisis average dollar and euro debt as a fraction of total foreign currency debt 76.8% compared to a post-crisis average of 84.2%. The increase has been driven by the rise of dollar debt issuance. As shown in Figure C.3b, dollar and euro debt have been acted as close substitutes; euro debt was on the upswing since 2001, challenging the dollar as euro debt accounted for 39% at its peak in 2005. Even at the onset of the Great Financial Crisis, the fraction of euro debt rose from

[6] Although only 19 out of the 28 EU member countries replaced their national currencies with the euro, those who did not still have close ties with other EU member countries, and are therefore excluded from the sample. 7 of them choose to use its own currencies instead of the euro, and Denmark and the U.K. opted out though they did not adopt the euro before opt-out.

23.7% in 2007Q4 to 38.7% in 2008Q3. However, its share dipped to 19.6% in the following quarter and fell further in the subsequent European debt crisis, and has never reached 30% ever since despite some signs of resurgence in 2013.

One drawback of using this dataset is that it is subject to valuation effects since the data on gross issues are calculated by converting flows in euro and other currencies into dollar amounts using period average exchange rates. To partially account for valuation effects, I use quarterly average of dollar-euro exchange rates to convert into euro debt issues denominated in euro but in unit of dollar, and compute the dollar value of them using an exchange rate fixed at 2018Q4. Unfortunately, without the knowledge of the composition and weight of currencies other than euro in total foreign currency debt issues, the numbers of total foreign currency debt are still subject to valuations effects. However, since dollar- and euro-denominated debt account for a predominant share of total foreign currency debt, the adjusted results can have some important interpretations.[7] In this section, trends in dollar debt and euro debt issued in international capital markets by different sectors, with various maturities, and by multiple country groups are presented and analyzed.

3.3.1 Trends in dollar debt and euro debt across sectors

Entities issuing foreign currency debt can be broken down into three categories: financial corporations, non-financial corporations, and general government.[8] Foreign currency debt issued by financial corporations makes up for the largest share among the three sectors (Figure C.4), with an average of 68.7% and the highest share of 86.1% in 2007Q3. Non-financial corporations and general government issues account

[7] The results are presented in Figure C.13-C.15, and only those sectors that have a majority of foreign currency debt denominated in dollar and in euro are shown.

[8] A decomposition of the financial corporation sector is in Figure C.1.

for only 21.5% and 9.8% on average respectively within the sample period. Therefore, the trend in the overall dollar and euro debt issuance over time must be largely driven by the pattern in the financial corporation sector, if there is any. A detailed currency decomposition of debt issues across sectors confirms this point.

Figure C.5-C.7 show respectively the evolution of dollar and euro debt in the financial corporation sector, the non-financial corporation sector, and the general government sector. As the overall trend in Figure C.3b, the share of euro debt issued by financial corporations gradually grew since 2001 and hiked in 2005 to reach the same level of share as dollar debt. In the earlier time of the Great Financial Crisis, the share of euro debt rose against that of the dollar to reach 43.7% in 2008Q3, but decreased subsequently and fell further in the European Debt Crisis (Figure C.5b) due to a faster increase of dollar debt issuance after the GFC (Figure C.5a). The falling trend of the euro has continued although there was a recovery in 2013 and 2014.

The evolution of euro debt in the non-financial corporation and the general government sector however, exhibits no downward trend in the post-crisis sample period (Figure C.6b, C.7b). First, there was no apparent boom period in the pre-crisis period, unlike euro debt issued by financial corporations during 2001-05. The share of euro debt issues by non-financial corporations was relatively stable and that issued by general government went through ups and downs before the Great Financial Crisis. Second, the share of euro debt in both sectors recovered, or even exceeded the pre-crisis level after 2012 while both exhibited similar rise and fall in the Great Financial Crisis and the European debt crisis as the share of euro debt in the financial sector. However, dollar debt's faster growth during the crisis period is seen in all three sectors despite a relative appreciation of the dollar against the euro, so is a recovery of euro debt around 2015, which coincided with an appreciation of the dollar against the euro starting in the second half of 2014. Noticeably, many EMEs suffered from a sharp depreciation of domestic currencies beginning in late 2014 while dollar debt issuance fell in the financial corporation sector and more so in the non-financial corporation sector (Figure C.5a, C.6a).

3.3.2 Trends in dollar debt and euro debt in AEs and EMEs

AEs and EMEs differ conspicuously in the pattern of choosing what currency to denominate debt. In general, EMEs issue a considerably higher fraction of dollar debt than AEs in all sectors as expected. Both country groups increased issuance of dollar debt compared to euro debt first in the GFC and even more in the subsequent European debt crisis, corroborating the "flight to safety" argument, but the EMEs responded more dramatically with the ratio of dollar debt to euro debt peaked over 600 during the euro crisis from less than 50 in the GFC (Figure C.8a, C.8b). In both country groups, the non-financial sector issues a higher fraction of dollar debt than the financial sector which seems relatively more balanced in currencies of financing.

3.3.3 Trends in dollar debt and euro debt with different maturities in FCs and NFCs

The maturity structure of foreign currency debt varies across sectors. Long-term debt accounts for a greater majority in the non-financial corporation and the general government sector than in the financial corporation sector (Figure C.9), although the share of long-term debt issuance is more than that of short-term debt in general, which is consistent with the previous findings that foreign currency borrowers tend to have a higher share of long-term debt than those who only borrow in domestic currencies.

Unlike the U.S. debt market, debt markets in many countries, especially the EMEs, are less liquid particularly for longer maturities, and therefore it is more likely for firms that want to tap the international capital market to issue longer-maturity debt if domestic market is too costly to fulfill its financing need. The fraction of long-term debt issued by financial corporations is 55.9% on average over the sample period, as compared to non-financial corporations' 79.8% with the maximum share of 95.9%, and to general government's 84.6% with the maximum of 99.4%. As a result, the trend in dollar and euro debt issues across sectors must be driven by long-term debt, which might be more pronounced in the non-financial corporation and the general government sector.

Figure C.10-C.12 present the evolution of the shares of dollar and euro debt in total foreign currency debt issues for debt with different maturities in three sectors. The rise of the dollar and the fall of the euro in the post-crisis period is most evident for long-term debt issued by financial corporations, whereas short-term debt either fluctuates with some traces of declining or even grows to a level unseen before for non-financial firms (Figure C.10a, C.11a).

To summarize, dollar- and euro-denominated debt issuance in international capital markets exhibited the following characteristics:

- the dollar's rise and the euro's fall during the GFC and the subsequent European debt crisis is mostly obvious seen in the financial sector and particularly for long-term debt;
- the non-financial sector issues a predominantly more share of dollar debt than the financial sector in EMEs while the gap is narrower in AEs;
- there was a recovery of euro debt in all sectors around 2015 when the dollar

appreciated sharply.

3.4 Recursive VAR Analysis

Given the theoretical predictions of firms' motives to issue debt and the empirical evidences on the heterogeneity of debt issuance across different sectors, country groups and maturities, I perform a recursive VAR with Cholesky decomposition to analyze what drives the trends in dollar debt and euro debt issuance. The variables of main interest are dollar debt issues and euro debt issues, nominal interest rate differential in the US and the Eurozone, and expected depreciation of the dollar against the euro. To control for firms' operational hedging motives, I exclude short-term debt and only use long-term debt issues in the sample since operational hedge is typically related with short-term debt. In addition, the rise of the dollar and the fall of the euro is seen prominently in long-term debt issuance but not so for short-term debt. The interest rate differential and the expected depreciation are a measure of relative borrowing cost associated with the static trade-off theory on the one hand, and are proxies for firms' speculative motives on the other hand. I do not combine the two terms to compute and include the deviations from the uncovered interest rate parity because first, firms could differentiate between the interest gain and gains from exchange rate movements in making decisions about the currency denomination of bond, and more importantly, the interest rate differential can be a gauge of demand factors (safety asset demand) that might be able to affect firms' decisions. Dollar debt and euro debt issues are taken natural logs. For interest rate differentials, I use the difference between 5-year Treasury bond yield, and 5-year government bond yield in the Eurozone. Expected depreciation is computed under the assumption that bond issuers form their expectation about future appreciation of currencies based on past movements in exchange rates. To simplify the expectation formation process, I include depreciation observed over the past four quarters.

Other variables included in the analysis are US GDP growth, US inflation and effective federal funds rate. I impose contemporaneous restrictions on the response of variables such that one variable can respond contemporaneously to variables preceding it but not to those ordered after it, that is, one ordered first is a slow-moving variable and one ordered last is a fast-moving variable. Thus, the order of the six variables is: GDP growth, inflation, debt issuance, yield differential, federal funds rate, and expected depreciation of the dollar. I assume debt issuance cannot respond to fluctuations in yield differential and expected depreciation of the dollar contemporaneously, but the exchange rate can respond to movements in the federal funds rate and government bond yield contemporaneously. The question of interest is how debt denominated in dollar and in euro will respond to movements in yield differential and expected depreciation of the dollar. Since without firm-level dataset that contains cash holding information and other important indicators, it is difficult to tell firms' motive to reduce financing cost apart from speculation when they increase debt issuance denominated in a currency that is depreciating and with a lower interest rate, I do not differentiate the two in the following analysis. Therefore, a positive response of dollar debt issues and a negative response of euro debt issues to a narrower yield differential and a larger expected depreciation of the dollar would lend support to the speculation motive in firms' choice of currency to issue debt. I use a two-lag VAR under AIC and BIC, with 95% confidence interval with the upper bound in yellow and the lower bound in red. The same recursive VAR for dollar debt and euro debt is performed separately. The impulse responses of dollar debt and euro debt issuance to a 1% expected depreciation of the dollar are presented in Figure C.16-C.17, and those of dollar debt and euro debt to a 1-basis point rise in yield differential are presented in Figure C.18-C.19. A qualitative description of the results is summarized in Table C.2 and C.3.

Dollar debt decreases immediately in response to an expected depreciation of the dollar regardless of country groups and sectors in general (Figure C.16), which is inconsistent with the prediction that a depreciation of dollar would boost dollar debt issuance should the speculative assumption hold for firms, but it increases after 2-6 quarters depending on sectors. The non-financial sector in EMEs has the largest rise among all. Moreover, firms in EMEs cut down issuance of dollar debt more than firms in AEs, and non-financial sector typically reduce dollar debt more greatly than the financial sector in both country groups.

Intuitively, expected dollar depreciation would reduce euro debt issuance, which is the case in Figure C.17c, C.17d, and C.17f where euro debt decreases immediately and rises after about 3 quarters, but it is not the case for the non-financial sector in AEs in Figure C.17e. Euro debt issued by firms in the non-financial sector in AEs rises in the first few quarters as dollar depreciates, suggesting their decision to issue debt is less out of speculative motive but more consistent with the pecking order theory.

A positive shock on yield differential (a higher yield in the US and a lower yield in the Eurozone) leads to starkly different responses of AE firms and EME firms' dollar debt issuance whereas the two country groups' euro debt issuance respond similarly to the same shock. An increase in yield differential will increase dollar debt issued by EME firms in both the financial and the non-financial sectors until the 6th quarter (Figure C.18d and C.18e) before it falls. Such an increase, however, will decrease dollar debt issued by AE firms (Figure C.18a, C.18c and C.18e), indicating dollar debt issuers in AEs are more concerned about borrowing cost while dollar debt issuers in EMEs might be driven by other factors, such as a high demand for dollar denominated assets. A rise in yield differential largely will increase euro debt issuance in both AEs and EMEs, which is consistent with firms' speculative motive, although the non-financial sector in EMEs tends to reduce euro debt issuance after five quarters (Figure C.19c, C.19d, C.19e and C.19f).

- expected dollar appreciation, which might be induced by an upsurge in safe dollar asset demand, will increase dollar debt issuance by all sectors in both AEs and EMEs, but in the longer horizon, the non-financial sector in EMEs is more likely to reduce dollar debt due to a higher financing cost or a smaller opportunistic gain. since dollar appreciated over the crisis period, the initial rise of dollar debt after the GFC might be largely driven by the rise in safe dollar asset demand;

- the effect of the yield differential is more ambiguous. An increase in yield differential will drive up dollar debt issuance in EMEs but not so in AEs. It shrank over the GFC, recovered a bit, and shrank in the European debt crisis and increased again afterwards (Figure C.20). Over the post-crisis period, foreign currency debt issued by AEs was higher than by EMEs despite the fast increase of foreign currency debt issued by EMEs and a higher proportion of dollar debt to euro debt issued by EMEs than by AEs. Thus, the rise of dollar debt might be a result of a dominant effect of the declining yield differential on firms in AEs, which tend to take advantage of a lower financing cost.

And euro debt issuance has the following characteristics:

- firms in the non-financial sector in AEs are the least opportunistic when deciding to issue euro-denominated bonds;

- the impact of expected appreciation of the dollar and rising yield differential by and large conforms to predictions of the trade-off theory or the speculation conjecture. Whenever the exchange rate and the interest rate are in favor of a lower borrowing cost, firms would issue more debt denominated in euro. Therefore, issuance of euro debt fell as the yield differential dropped during the GFC, and even more over the European debt crisis, indicating a high risk premium associated with bonds denominated in euro.

3.5 Conclusion

The international currency status of the dollar and the euro underwent significant changes in the post-crisis era. This chapter demonstrates the rise of the dollar and the fall of the euro in foreign currency debt issuance within international capital markets by countries whose sovereign currencies are neither the U.S. dollar nor the euro, following the Great Financial Crisis. This overall trend is not observed in the evolution of short-term debt, but rather in long-term debt, with the widened gap between dollar debt and euro debt being most pronounced for debt issued by the financial sector and by EMEs. A recursive VAR analysis suggests a starkly different pattern of responses to changes in yield differentials and expected dollar appreciation between dollar and euro debt issuance. In general, firms tend to issue more euro debt when costs are lower or speculative benefits are greater in terms of both interest rates and exchange rates, with the exception of the non-financial sector in AEs, which may be more aligned with standard financing behavior predicted by the pecking order theory. In contrast, dollar debt issuance in both sectors in AEs and EMEs will rise if the dollar appreciates, running counter to the speculative motive, and a larger yield differential tends to increase dollar debt issued by EMEs, conflicting with the speculative motive again, but not so for that issued by AEs. In conclusion, the rise of the dollar and the fall of the euro might be a result of growing safe asset demand as the dollar appreciates, accompanied by increased issuance of dollar debt by firms in AEs seeking lower financing costs as yield differentials shrink, which in turn reduces euro debt issuance by both sectors in AEs and EMEs.

That said, I understand these results are subject to bias. First, some long-term debt in the sample might be issued out of an operational hedging motive, even if I exclude short-term debt for the VAR analysis; a better approach would be to find firms with no foreign operations but still issuing foreign currency debt. Additionally, some of the dollar and euro debt in the sample might be hedged with currency swaps, thus becoming converted local currency debt (albeit in a small fraction), not foreign currency debt suitable for analysis. Second, I could include exchange rate volatility in the VAR analysis since it might also be an important factor firms consider when deciding which currencies to denominate bonds in; I could also divide the sample into pre-crisis and post-crisis periods and analyze them separately, although the sample size might be too small. The trend might be driven more by reduced financing costs and speculative motives in the post-crisis period, particularly after 2014. Third, dollar debt issued by EMEs goes up but not for AEs when yield differential widens, which I do not have a good explanation for so far. It might be related to deviations from uncovered interest rate parity and safe dollar asset demand.

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Appendices

Appendix A

Derivations and Proofs for Chapter 1

A.1 Solving the model

Conditions for deleveraging to occur. Borrowers need to be sufficiently more impatient than lenders so that they will choose a level of d_1 greater than $\bar{d}_1$. The Euler equations for households in the initial two time periods are given by:

$$1 + r_1 = \frac{u'(e^* - 1 - \frac{d_1}{1+r_1})}{\beta^l u'((1-\phi)e^* - 1 + d_1)} = \frac{u'(e^* - 1 + \frac{d_1}{1+r_1})}{\beta^b u'((1+\phi)e^* - 1 - d_1)} \tag{A.1}$$

Consider the LHS of Equation (A.1) when r_2 reaches 0. The LHS can be reduced to:

$$(\beta^l)^2(1 + r_1) = u'(e^* - 1 - \frac{\bar{d}_1}{1+r_1}) \tag{A.2}$$

Observe that r_1 is an increasing function of $\bar{d}_1$, and therefore, the upper bound on c_1^l, which is determined by β^l, determines the upper bound on d_1, $\bar{d}_1$, which defines an upper bound on r_1. Moreover, note that $\frac{d_{(d_1)}}{d_{(1+r_1)}} > 1$. Rewrite the RHS of (A.1):

$$\underline{\beta}^b = \beta^l \, \frac{u'(\bar{c}_1^l - 1)}{u'((1+\phi)e^* - 1 - \bar{d}_1)} \, \frac{u'(e^* - 1 + \frac{\bar{d}_1}{1+\bar{r}_1})}{u'(e^* - 1j - \frac{\bar{d}_1}{1+\bar{r}_1})} \tag{A.3}$$

A higher $\bar{d}_1$ indicates a lower β^l and a higher $\bar{c}_1^l$ due to the strict concavity of $u'(\cdot)$. This will render the first fraction on the RHS of (A.3) less than 1. Similarly, $\bar{r}_1$ increases, and with $\frac{d_{(d_1)}}{d_{(1+r_1)}} > 1$, the second fraction on the RHS of (A.3) will also be less than 1. Equation (A.3) then defines a lower bound for β^b. As long as $\beta^b < \underline{\beta}^b$, borrowers will choose a level of d_1 which is sufficiently high to trigger a demand-driven recession.

Restrictions on ϕ^{Ii} in the IBC model. To see why we need a restriction on ϕ^{Ii}, rewrite Equation (1.17) as:

$$w_1 - v'(n_1^i) + \frac{\phi^{Ii} w_1}{1 + r_2} = \beta^i \phi^{Ii} w_1 \frac{u'(\tilde{c}_2^i)}{u'(\tilde{c}_1^i)} > 0 \tag{A.4}$$

Take derivative with respect to n_1^i with Equation (A.6):

$$\begin{aligned} &- \frac{\phi^{Ii} w_1}{(1 + r_2)^2}[1 + \frac{\beta^i u''(\tilde{c}_1^i) u'(\tilde{c}_2^i)}{(u'(\tilde{c}_1^i))^2} \phi^{Ii} w_1 n_1^i] \frac{dr_2}{dn_1^i} = \\ &v''(n_1^i) - \frac{\phi^{Ii} w_1 \beta^i}{(u'(\tilde{c}_1^i))^2}\{-u'(\tilde{c}_1^i) u''(\tilde{c}_2^i) \phi^{Ii} w_1 - u''(\tilde{c}_1^i) u'(\tilde{c}_2^i)[w_1 - v'(n_1^i) + \frac{\phi^{Ii} w_1}{1 + r_2}]\} \end{aligned} \tag{A.5}$$

Since RHS is positive, if

$$1 + \frac{\beta^i u''(\tilde{c}_1^i) u'(\tilde{c}_2^i)}{(u'(\tilde{c}_1^i))^2} \phi^{Ii} w_1 n_1^i > 0,$$

the interest rate will be decreasing when employment of the borrowers increases. Approximate $\beta^i (1 + r_2) u'(\tilde{c}_2^i) \approx \beta^i u'(\tilde{c}_2^i) = u'(\tilde{c}_1^i)$, and the CRRA utility function with σ the elasticity of substitution, the inequality can be rewritten as:

$$\phi^{Ii} < \sigma \frac{\tilde{c}_1^i}{w_1 n_1^i}.$$

The threshold level of b_1^i in the IBC model. The threshold level of b_1^i can be derived from Equation (1.17) and (A.7) by setting the real interest rate to zero and the real wage to 1:

$$w_1 - v'(n_1^i) + \phi^{Ii} w_1 = \beta^i \phi^{Ii} w_1 \frac{u'(e^* + t_2^i + d_2^i - \phi^{Ii} w_1 n_1^i - v(n^*))}{u'(w_1 n_1^i + t_1^i + d_1^i + b_1^i + \phi^{Ii} w_1 n_1^i - v(n_1^i))} \tag{A.6}$$

$$u'(w_1 n_1^l + t_1^l + d_1^l - \frac{\alpha^i}{\alpha^l} b_1^i - \frac{\alpha^i}{\alpha^l} \phi^{Ii} w_1 n_1^i - v(n_1^l)) = \beta^l u'(e^* + t_2^l + d_2^l + \frac{\alpha^i}{\alpha^l} \phi^{Ii} w_1 n_1^i - v(n^*)) \tag{A.7}$$

With lower b_1^i or greater leverage, labor supply of the borrowers is increasing by both of the equations. Define the solution from the system of equations as $\underline{b}_1^i$. Therefore, ϕ^{Ii} has to be sufficiently small so that the interest rate will reach the zero lower bound before borrowers work more hours to be unconstrained by the borrowing limit.

Appendix B

Derivations and Proofs for Chapter 2

B.1 Comparative statics

A. a shock on t_1^l and t_2^l

Income-based borrowing with no AD shortage. When there is no aggregate demand shortage, both types of shocks will not have any impact on the real wage and production is at an efficient level. Lenders supply labor given the efficient level of wage. Borrowers, constrained in borrowing by their labor income, will increase labor supply if the demand for bonds is greater. t_1^l and t_2^l can indirectly affect welfare through the interest rate. Higher t_1^l or lower t_2^l of the lenders will induce them to save more and boost their demand for bonds, which lowers the interest rate. A lower interest rate improves the welfare of the borrowers. Borrowers will work more and thus have higher labor income, given a lower interest rate, but it does not affect their welfare since wage is constant[1]. Therefore, the welfare of both borrowers and lenders is affected through interest rate as in (B.3) and (B.4).

$$\frac{dn_1^i}{dt_1^l} = \frac{\frac{u''(\tilde{c}_1^l)}{X_l^{in}}}{\frac{Z_b^{in}}{X_b^{in}} - \frac{Z_l^{in}}{X_l^{in}}} > 0 \qquad \frac{dr_2}{dt_1^l} = \frac{\frac{u''(\tilde{c}_1^l)}{X_l^{in}}}{\frac{Z_b^{in}}{X_b^{in}} - \frac{Z_l^{in}}{X_l^{in}}} \frac{Z_b^{in}}{X_b^{in}} < 0 \tag{B.1}$$

$$\frac{dn_1^i}{dt_2^l} = \frac{-\frac{\beta^l(1+r_2)u''(\tilde{c}_2^l)}{X_l^{in}}}{\frac{Z_b^{in}}{X_b^{in}} - \frac{Z_l^{in}}{X_l^{in}}} < 0 \qquad \frac{dr_2}{dt_2^l} = \frac{-\frac{\beta^l(1+r_2)u''(\tilde{c}_2^l)}{X_l^{in}}}{\frac{Z_b^{in}}{X_b^{in}} - \frac{Z_l^{in}}{X_l^{in}}} \frac{Z_b^{in}}{X_b^{in}} > 0 \tag{B.2}$$

$$\frac{\partial V^i}{\partial t_1^l} = -u'(\tilde{c}_1^i)\frac{\phi^{Ii} w_1 n_1^i}{(1+r_2)^2}\frac{dr_2}{dt_1^l} > 0 \qquad \frac{\partial V^l}{\partial t_1^l} = u'(\tilde{c}_1^l)(1 + \frac{\alpha^i}{\alpha^l}\frac{\phi^{Ii} w_1 n_1^i}{(1+r_2)^2}\frac{dr_2}{dt_1^l}) > 0 \tag{B.3}$$

$$\frac{\partial V^i}{\partial t_2^l} = -u'(\tilde{c}_1^i)\frac{\phi^{Ii} w_1 n_1^i}{(1+r_2)^2}\frac{dr_2}{dt_2^l} < 0 \qquad \frac{\partial V^l}{\partial t_2^l} = u'(\tilde{c}_1^l)(1 + \frac{\alpha^i}{\alpha^l}\frac{\phi^{Ii} w_1 n_1^i}{(1+r_2)^2}\frac{dr_2}{dt_2^l}) > 0 \tag{B.4}$$

where $\frac{Z_b^{in}}{X_b^{in}}$ is the slope of borrowers' labor supply equation, and $\frac{Z_l^{in}}{X_l^{in}}$ is the aggregate

[1] Also by the envelope theorem, changes in optimal labor supply does not directly affect welfare.

demand equation with

$$Z_b^{in} = v''(n_1^i) - \frac{\beta^i \phi^{Ii} w_1}{(u'(\tilde{c}_1^i))^2}[\phi^{Ii} w_1 u'(\tilde{c}_1^i) u''(\tilde{c}_2^i) + (w_1 + \frac{\phi^{Ii} w_1}{1+r_2} - v'(n_1^i)) u''(\tilde{c}_1^i) u'(\tilde{c}_2^i)] > 0$$

$$X_b^{in} = -\frac{\phi^{Ii} w_1}{(1+r_2)^2}[1 + \frac{\beta^i \phi^{Ii} w_1 n_1^i}{(u'(\tilde{c}_1^i))^2} u''(\tilde{c}_1^i) u'(\tilde{c}_2^i)] < 0$$

$$Z_l^{in} = -\frac{\alpha^i}{\alpha^l} \phi^{Ii} w_1 [\frac{u''(\tilde{c}_1^l)}{1+r_2} + \beta^l (1+r_2) u''(\tilde{c}_2^l)] > 0$$

$$X_l^{in} = \beta^l u'(\tilde{c}_2^l) - \frac{\alpha^i}{\alpha^l} u''(\tilde{c}_1^l) \frac{\phi^{Ii} w_1 n_1^i}{(1+r_2)^2} > 0$$

Income-based borrowing with an AD shortage. When there is an aggregate demand shortage, a positive shock on t_1^l has a similar effect as a negative shock on t_2^l: they both lower households' income. The decrease in income results from the binding constraint on the interest rate. A higher t_1^l or lower t_2^l makes lenders more willing to save, which should lower the interest rate. However, since the interest rate cannot fall further, the bonds market does not clear with an interest rate too high. In response, lenders save more than they should, which lowers demand. As a result, firms hire fewer workers, and scale down production, which decreases the wage rate. Falling income reduces borrowers' debt capacity, which reduces demand further, leading to a feedback loop. With an AD shortage, the wage is below the efficient level, $w_1 = v'(n_1^l) < 1$, welfare of both borrowers and lenders is undermined due to lower income as in (B.6).

$$\frac{de_1^i}{dt_1^l} = -\frac{de_1^i}{dt_2^l} = -\frac{\frac{1}{X_l^{ia}}}{\frac{Z_b^{ia}}{X_b^{ia}} - \frac{Z_l^{ia}}{X_l^{ia}}} < 0 \qquad \frac{de_1^l}{dt_1^l} = -\frac{de_1^l}{dt_2^l} = -\frac{\frac{1}{X_l^{ia}}}{\frac{Z_b^{ia}}{X_b^{ia}} - \frac{Z_l^{ia}}{X_l^{ia}}} \frac{Z_b^{ia}}{X_b^{ia}} < 0 \tag{B.5}$$

$$\frac{\partial V_1^i}{\partial t_1^l} = \frac{v'(n_1^i)}{v'(n_1^l)}(1-w_1)u'(\tilde{c}_1^i))\frac{de_1^i}{dt_1^l} < 0 \quad \frac{\partial V_1^l}{\partial t_1^l} = u'(\tilde{c}_1^l) + (1-w_1)u'(\tilde{c}_1^l)\frac{de_1^l}{dt_1^l} < 0 \tag{B.6}$$

$$\frac{\partial V_1^i}{\partial t_2^l} = \frac{v'(n_1^i)}{v'(n_1^l)}(1-w_1)u'(\tilde{c}_1^i))\frac{de_1^i}{dt_2^l} > 0 \quad \frac{\partial V_1^l}{\partial t_2^l} = \beta^l u'(\tilde{c}_2^l) + (1-w_1)u'(\tilde{c}_1^l)\frac{de_1^l}{dt_2^l} > 0 \tag{B.7}$$

where $\frac{Z_b^{ia}}{X_b^{ia}}$ is the slope of borrowers' labor supply equation, and $\frac{Z_l^{ia}}{X_l^{ia}}$ is the aggregate demand equation with

$$Z_b^{ia} = v''(n_1^i) - \frac{\beta^i \phi^{Ii} w_1}{(u'(\tilde{c}_1^i))^2}[\phi^{Ii} u'(\tilde{c}_1^i) u''(\tilde{c}_2^i) + (1 + \phi^{Ii} - v'(n_1^i)) u''(\tilde{c}_1^i) u'(\tilde{c}_2^i)] > 0$$
$$X_b^{ia} = (1 + \phi^{Ii} - \beta^i \phi^{Ii} \frac{u'(\tilde{c}_2^i)}{u'(\tilde{c}_1^i)}) v''(n_1^l) > 0$$
$$Z_l^{ia} = 2\frac{\alpha^i}{\alpha^l}\phi^{Ii} > 0$$
$$X_l^{ia} = 1 - v'(n_1^l) > 0$$

To have a well-defined equilibrium, the slopes of the two equations are restricted such that $\frac{Z_l^{ia}}{X_l^{ia}} < \frac{Z_b^{ia}}{X_b^{ia}}$ (can be satisfied when ϕ^{Ii} is small). Note that the amplification effect is captured by the multiplier $(1 - w_1)\frac{\frac{1}{X_l^{ia}}}{\frac{Z_b^{ia}}{X_b^{ia}} - \frac{Z_l^{ia}}{X_l^{ia}}}\frac{Z_b^{ia}}{X_b^{ia}} = \frac{1}{1 - \frac{Z_l^{ia}}{X_l^{ia}} / \frac{Z_b^{ia}}{X_b^{ia}}} > 1$ with $\frac{Z_l^{ia}}{X_l^{ia}} < \frac{Z_b^{ia}}{X_b^{ia}}$ for the lenders. Moreover, the income of the lenders are affected more than the borrowers since borrowers will increase labor supply when consumption falls due to lower income, as they are constrained in borrowing by labor income, which counteracted the impact of higher t_1^l, that is $\frac{Z_b^{ia}}{X_b^{ia}} > 1$[2].

Asset-based borrowing with no AD shortage. When there is no AD shortage, higher t_1^l or lower t_2^l to the lenders will increase lenders' demand for bonds, lowering the interest rate, and since lenders become more willing to hold debt, the collateral that the borrowers need for borrowing becomes more valuable, which boosts asset price. Therefore, the constraint on borrowers will be relaxed with higher collateral value. Both borrowers and lenders' income stay constant with production and wage at the efficient level. Households earn the same level of income, and there is no heterogeneity in income. The welfare of the borrowers is improved by higher asset price that relaxes their borrowing constraint and lower interest rate. Lenders, similar to lenders in the IBC economy with no AD shortage, are also better off due to the direct effect of higher consumption from greater wealth dominating the welfare loss from lower interest rate. The marginal effects on the interest rate, asset price and

[2]See proof in the Appendix.

welfare are given by:

$$\frac{dp_1}{dt_1^l} = \frac{\frac{u''(\tilde{c}_1^l)}{X_l^{an}}}{\frac{Z_b^{an}}{X_b^{an}} - \frac{Z_l^{an}}{X_l^{an}}} > 0 \qquad \frac{dr_2}{dt_1^l} = \frac{\frac{u''(\tilde{c}_1^l)}{X_l^{an}}}{\frac{Z_b^{an}}{X_b^{an}} - \frac{Z_l^{an}}{X_l^{an}}}\frac{Z_b^{an}}{X_b^{an}} < 0 \quad \text{(B.8)}$$

$$\frac{dp_1}{dt_2^l} = \frac{-\frac{\beta^l(1+r_2)u''(\tilde{c}_2^l)}{X_l^{an}}}{\frac{Z_b^{an}}{X_b^{an}} - \frac{Z_l^{an}}{X_l^{an}}} < 0 \qquad \frac{dr_2}{dt_2^l} = \frac{-\frac{\beta^l(1+r_2)u''(\tilde{c}_2^l)}{X_l^{an}}}{\frac{Z_b^{an}}{X_b^{an}} - \frac{Z_l^{an}}{X_l^{an}}}\frac{Z_b^{an}}{X_b^{an}} > 0 \quad \text{(B.9)}$$

$$\frac{\partial V^a}{\partial t_1^l} = -u'(\tilde{c}_1^a)\frac{\phi^{Aa}p_1}{(1+r_2)^2}\frac{dr_2}{dt_1^l} \quad \text{(B.10)}$$

$$+ \frac{\phi^{Aa}}{1+r_2}\frac{dp_1}{dt_1^l}[u'(\tilde{c}_1^a) - \beta^a(1+r_2)u'(\tilde{c}_2^a)] > 0 \qquad \frac{\partial V^l}{\partial t_1^l} = u'(\tilde{c}_1^l)(1 + \frac{\alpha^a}{\alpha^l}\frac{\phi^{Aa}p_1}{(1+r_2)^2}\frac{dr_2}{dt_1^l}) > 0 \quad \text{(B.11)}$$

$$\frac{\partial V^a}{\partial t_2^l} = -u'(\tilde{c}_1^a)\frac{\phi^{Aa}p_1}{(1+r_2)^2}\frac{dr_2}{dt_2^l} \quad \text{(B.12)}$$

$$+ \frac{\phi^{Aa}}{1+r_2}\frac{dp_1}{dt_2^l}[u'(\tilde{c}_1^a) - \beta^a(1+r_2)u'(\tilde{c}_2^a)] < 0 \qquad \frac{\partial V^l}{\partial t_2^l} = u'(\tilde{c}_1^l)(1 + \frac{\alpha^a}{\alpha^l}\frac{\phi^{Aa}p_1}{(1+r_2)^2}\frac{dr_2}{dt_2^l}) > 0 \quad \text{(B.13)}$$

where where $\frac{Z_b^{an}}{X_b^{an}}$ is the slope of borrowers' labor supply equation, and $\frac{Z_l^{an}}{X_l^{an}}$ is the aggregate demand equation with

$$Z_b^{an} = 1 + \frac{\phi^{Aa}\beta^a d_2^a}{(u'(\tilde{c}_1^a))^2}(u'(\tilde{c}_1^a)u''(\tilde{c}_2^a) + \frac{u''(\tilde{c}_1^a)u'(\tilde{c}_2^a)}{(1+r_2)}) > 0$$

$$X_b^{an} = \frac{\phi^{Aa}p_1}{(1+r_2)^2}\frac{\beta^a d_2^a}{(u'(\tilde{c}_1^a))^2}u''(\tilde{c}_1^a)u'(\tilde{c}_2^a) < 0$$

$$Z_l^{an} = -\frac{\alpha^a}{\alpha^l}\phi^{Aa}[\frac{u''(\tilde{c}_1^l)}{1+r_2} + \beta^l(1+r_2)u''(\tilde{c}_2^l)] > 0$$

$$X_l^{an} = \beta^l u'(\tilde{c}_2^l) - \frac{\alpha^a}{\alpha^l}u''(\tilde{c}_1^l)\frac{\phi^{Aa}p_1}{(1+r_2)^2} > 0$$

Asset-based borrowing with AD shortage. Next, consider a marginal increase in t_1^l and t_2^l when there is an aggregate demand shortage for an asset-based borrowing economy. As with an IBC economy with an AD shortage, higher t_1^l or lower t_2^l leads to excessive saving by lenders, and depresses demand and production. Wage is lower, resulting in lower income for all households. Lower income decreases asset prices, making it harder for borrowers to borrow. With a tighter constraint, borrowers reduce consumption further, which depresses demand and production further, leading to a feedback loop. Unlike in the IBC model, lower aggregate demand and lower asset price reinforce each other. In the IBC model, borrowers will increase working hours in response to lower consumption, which raises wages and tempers the negative effect on income. The marginal effect on income, asset price and welfare are given by:

$$\frac{dp_1}{dt_1^l} = -\frac{\frac{1}{X_l^{aa}}}{\frac{Z_b^{aa}}{X_b^{aa}} - \frac{Z_l^{aa}}{X_l^{aa}}} < 0 \qquad \frac{de_1}{dt_1^l} = -\frac{\frac{1}{X_l^{aa}}}{\frac{Z_b^{aa}}{X_b^{aa}} - \frac{Z_l^{aa}}{X_l^{aa}}}\frac{Z_b^{aa}}{X_b^{aa}} < 0 \tag{B.14}$$

$$\frac{dp_1}{dt_2^l} = \frac{\frac{1}{X_l^{aa}}}{\frac{Z_b^{aa}}{X_b^{aa}} - \frac{Z_l^{aa}}{X_l^{aa}}} > 0 \qquad \frac{de_1}{dt_2^l} = \frac{\frac{1}{X_l^{aa}}}{\frac{Z_b^{aa}}{X_b^{aa}} - \frac{Z_l^{aa}}{X_l^{aa}}}\frac{Z_b^{aa}}{X_b^{aa}} > 0 \tag{B.15}$$

$$\frac{\partial V^a}{\partial t_1^l} = [(1 - v'(e_1))\frac{de_1}{dt_1^l} + \phi^{Aa}\frac{dp_1}{dt_1^l}]u'(\tilde{c}_1^a) \tag{B.16}$$

$$- \beta^a \phi^{Aa} u'(\tilde{c}_2^a)\frac{dp_1}{dt_1^l} < 0 \qquad \frac{\partial V^l}{\partial t_1^l} = u'(\tilde{c}_1^l) + [(1 - v'(e_1))]u'(\tilde{c}_1^l)\frac{de_1}{dt_1^l} < 0 \tag{B.17}$$

$$\frac{\partial V^a}{\partial t_2^l} = [(1 - v'(e_1))\frac{de_1}{dt_2^l} + \phi^{Aa}\frac{dp_1}{dt_2^l}]u'(\tilde{c}_1^a) \tag{B.18}$$

$$- \beta^a \phi^{Aa} u'(\tilde{c}_2^a)\frac{dp_1}{dt_2^l} > 0 \qquad \frac{\partial V^l}{\partial t_2^l} = u'(\tilde{c}_1^l) + [(1 - v'(e_1))]u'(\tilde{c}_1^l)\frac{de_1}{dt_2^l} > 0 \tag{B.19}$$

where $\frac{Z_b^{aa}}{X_b^{aa}}$ is the slope of the asset pricing equation, and $\frac{Z_l^{aa}}{X_l^{aa}}$ is the aggregate demand equation with

$$Z_b^{aa} = 1 + \frac{\phi^{Aa}\beta^a d_2^a}{(u'(\tilde{c}_1^a))^2}(u'(\tilde{c}_1^a)u''(\tilde{c}_2^a) + \frac{u''(\tilde{c}_1^a)u'(\tilde{c}_2^a)}{(1 + r_2)}) > 0$$

$$X_b^{aa} = -\frac{\beta^a d_2^a}{(u'(\tilde{c}_1^a))^2}(1 - v'(e_1))u''(\tilde{c}_1^a)u'(\tilde{c}_2^a) > 0$$

$$Z_l^{aa} = 2\frac{\alpha^a}{\alpha^l}\phi^{Aa} > 0$$

$$X_l^{aa} = 1 - v'(e_1) > 0$$

Z_b^{aa} is greater than zero under the previous restriction. I also restrict the slope of the asset equation and the aggregate demand equation in order to have a well-defined solution. That is, $\frac{Z_b^{aa}}{X_b^{aa}} > \frac{Z_l^{aa}}{X_l^{aa}}$. Note that the impact of one unit of increase in t_1^l on welfare through the channel of income will be amplified by $\frac{1}{1 - \frac{Z_l^{aa}}{X_l^{aa}} / \frac{Z_b^{aa}}{X_b^{aa}}} > 1$. To capture the reinforcing effect of asset price and aggregate demand, $\frac{Z_b^{ia}}{X_b^{ia}} > \frac{Z_b^{aa}}{X_b^{aa}}$ such that $|\frac{dp_1}{dt_1^l}| > |\frac{dn_1^i}{dt_1^l}|$.

B. a shock on borrowers' dividend t_1^i and t_2^i, or t_1^a and t_2^a

The effects of a shock on borrowers' dividend d_1^i and d_1^i, or d_1^a and d_2^a are equivalent to the effect of a change in t_1^i or t_1^a, so I will use the notation of the transfers instead of the dividends.

Income-based borrowing with no AD shortage. For an income-based borrowing economy, when there is no aggregate demand shortage, an increase in t_1^i or a decrease in t_2^i will increase the consumption of the borrowers. Higher consumption makes borrowers less willing to borrow and therefore less incentivized to work so labor supply decreases, which decreases their debt with lower labor income. Interest rate falls in response to the lower supply of bonds. As with previous results when there is no AD shortage, changes in employment do not affect welfare. The welfare of the borrowers is improved through the direct effect of higher consumption and the reduction in interest rate, while the welfare of lenders is compromised due to lower interest rate. There is again a redistribution effect from interest rate changes, which does not generate any inefficiencies.

$$\frac{dn_1^i}{dt_1^i} = \frac{\frac{J_{b1}^{in}}{X_l^{in}}}{\frac{Z_b^{in}}{X_b^{in}} - \frac{Z_l^{in}}{X_l^{in}}} < 0 \qquad \frac{dr_2}{dt_1^i} = \frac{\frac{J_{b1}^{in}}{X_l^{in}}}{\frac{Z_b^{in}}{X_b^{in}} - \frac{Z_l^{in}}{X_l^{in}}} \frac{Z_b^{in}}{X_b^{in}} < 0 \tag{B.20}$$

$$\frac{dn_1^i}{dt_2^i} = \frac{\frac{J_{b2}^{in}}{X_l^{in}}}{\frac{Z_b^{in}}{X_b^{in}} - \frac{Z_l^{in}}{X_l^{in}}} > 0 \qquad \frac{dr_2}{dt_2^i} = \frac{\frac{J_{b2}^{in}}{X_l^{in}}}{\frac{Z_b^{in}}{X_b^{in}} - \frac{Z_l^{in}}{X_l^{in}}} \frac{Z_b^{in}}{X_b^{in}} > 0 \tag{B.21}$$

$$\frac{\partial V^i}{\partial t_1^i} = u'(\tilde{c}_1^i)[1 - \frac{\phi^{Ii} w_1 n_1^i}{(1+r_2)^2} \frac{dr_2}{dt_1^i}] > 0 \qquad \frac{\partial V^l}{\partial t_1^i} = u'(\tilde{c}_1^l) \frac{\alpha^i}{\alpha^l} \frac{\phi^{Ii} w_1 n_1^i}{(1+r_2)^2} \frac{dr_2}{dt_1^i} < 0 \tag{B.22}$$

$$\frac{\partial V^i}{\partial t_2^i} = \beta^i(1+r_2)u'(\tilde{c}_2^i) - u'(\tilde{c}_1^i) \frac{\phi^{Ii} w_1 n_1^i}{(1+r_2)^2} \frac{dr_2}{dt_2^i} \qquad \frac{\partial V^l}{\partial t_2^i} = u'(\tilde{c}_1^l) \frac{\alpha^i}{\alpha^l} \frac{\phi^{Ii} w_1 n_1^i}{(1+r_2)^2} \frac{dr_2}{dt_2^i} > 0 \tag{B.23}$$

with $J_{b1}^{in} = -\frac{\beta^i \phi^{Ii} w_1}{(u'(\tilde{c}_1^i))^2} u''(\tilde{c}_1^i) u'(\tilde{c}_2^i) > 0$ and $J_{b2}^{in} = \frac{\beta^i \phi^{Ii} w_1}{(u'(\tilde{c}_1^i))^2} u'(\tilde{c}_1^i) u''(\tilde{c}_2^i) < 0$. By previous restriction, $0 < J_{b1}^{in} < 1$.

Income-based borrowing with an AD shortage. When there is an aggregate demand shortage and the interest rate is at the lower bound, an increase in d_1^i or a decrease in d_2^i will increase the consumption of the borrowers. Higher consumption makes borrowers less willing to borrow and therefore less incentivized to work so labor supply decreases, which decreases their borrowing with lower labor income. Since the interest rate cannot fall to induce lenders to save less, the bonds market does not clear without adjustment of production and wage. Since lenders have excessive savings at the current interest rate, aggregate demand is lower, which decreases production. Firms will hire less and wages fall, reducing the income of households. The welfare of the lenders is undermined due to lower income. The welfare of the borrowers can still be improved by the direct effect of higher consumption.

$$\frac{de_1^i}{dt_1^i} = -\frac{\frac{J_{b1}^{ia}}{X_b^{ia}}}{\frac{Z_b^{ia}}{X_b^{ia}} - \frac{Z_l^{ia}}{X_l^{ia}}} < 0 \qquad \frac{de_1^l}{dt_1^i} = -\frac{\frac{J_{b1}^{ia}}{X_b^{ia}}}{\frac{Z_b^{ia}}{X_b^{ia}} - \frac{Z_l^{ia}}{X_l^{ia}}}\frac{Z_l^{ia}}{X_l^{ia}} < 0 \tag{B.24}$$

$$\frac{de_1^i}{dt_2^i} = -\frac{\frac{J_{b2}^{ia}}{X_b^{ia}}}{\frac{Z_b^{ia}}{X_b^{ia}} - \frac{Z_l^{ia}}{X_l^{ia}}} > 0 \qquad \frac{de_1^l}{dt_2^i} = -\frac{\frac{J_{b2}^{ia}}{X_b^{ia}}}{\frac{Z_b^{ia}}{X_b^{ia}} - \frac{Z_l^{ia}}{X_l^{ia}}}\frac{Z_l^{ia}}{X_l^{ia}} > 0 \tag{B.25}$$

$$\frac{\partial V_1^i}{\partial t_1^i} = u'(\tilde{c}_1^i) + \frac{v'(n_1^i)}{v'(n_1^l)}(1-w_1)u'(\tilde{c}_1^i))\frac{de_1^i}{dt_1^i} > 0 \qquad \frac{\partial V_1^l}{\partial t_1^i} = (1-w_1)u'(\tilde{c}_1^l)\frac{de_1^l}{dt_1^i} < 0 \tag{B.26}$$

$$\frac{\partial V_1^i}{\partial t_2^i} = \beta^i u'(\tilde{c}_2^i) + \frac{v'(n_1^i)}{v'(n_1^l)}(1-w_1)u'(\tilde{c}_1^i))\frac{de_1^i}{dt_2^i} > 0 \qquad \frac{\partial V_1^l}{\partial t_2^i} = (1-w_1)u'(\tilde{c}_1^l)\frac{de_1^l}{dt_2^i} > 0 \tag{B.27}$$

with $J_{b1}^{ia} = -\frac{\beta^i\phi^{Ii}w_1}{(u'(\tilde{c}_1^i))^2}u''(\tilde{c}_1^i)u'(\tilde{c}_2^i) > 0$ and $J_{b2}^{ia} = \frac{\beta^i\phi^{Ii}w_1}{(u'(\tilde{c}_1^i))^2}u'(\tilde{c}_1^i)u''(\tilde{c}_2^i) < 0$. Note that $\frac{\frac{de_1^i}{dt_1^i}}{\frac{de_1^i}{dt_2^i}} = \frac{\frac{de_1^l}{dt_1^i}}{\frac{de_1^l}{dt_2^i}} = \frac{J_{b1}^{ia}}{J_{b2}^{ia}} = -\frac{\tilde{c}_2^i}{\tilde{c}_1^i} < -1$. In addition, J_{b1}^{ia} and J_{b2}^{ia} are relatively small when ϕ^{Ii} is small and both are less than one. Therefore, the effect on income is smaller compared to the case with a change in t_1^l or t_2^l.

Asset-based borrowing with no AD shortage. Consider a marginal increase in d_1^a or a decrease in d_2^a when there is no aggregate demand shortage. An increase in asset dividends will make assets more valuable as it not only boosts the consumption by the borrowers in the current period directly, but relaxes the borrowing constraint as the price of the asset rises, which further increases consumption and inflates asset price. This is the canonical amplification mechanism with the asset-based borrowing constraint. Meanwhile, the interest rate must increase since the supply of bonds rises as the borrowers expand their debt capacity with more valuable collaterals. The welfare of borrowers is improved due to higher asset prices relaxing the borrowing constraint and the direct effect of higher consumption. The welfare of lenders is also improved due to a higher interest rate.

$$\frac{dp_1}{dt_1^a} = \frac{\frac{J_{b1}^{an}}{X_b^{an}}}{\frac{Z_b^{an}}{X_b^{an}} - \frac{Z_l^{an}}{X_l^{an}}} > 0 \qquad \frac{dr_2}{dt_1^a} = \frac{\frac{J_{b1}^{an}}{X_b^{an}}}{\frac{Z_b^{an}}{X_b^{an}} - \frac{Z_l^{an}}{X_l^{an}}} \frac{Z_b^{an}}{X_b^{an}} > 0 \tag{B.28}$$

$$\frac{dp_1}{dt_2^a} = \frac{\frac{J_{b2}^{an}}{X_b^{an}}}{\frac{Z_b^{an}}{X_b^{an}} - \frac{Z_l^{an}}{X_l^{an}}} < 0 \qquad \frac{dr_2}{dt_2^a} = \frac{\frac{J_{b2}^{an}}{X_b^{an}}}{\frac{Z_b^{an}}{X_b^{an}} - \frac{Z_l^{an}}{X_l^{an}}} \frac{Z_b^{an}}{X_b^{an}} < 0 \tag{B.29}$$

$$\frac{\partial V^a}{\partial t_1^a} = u'(\tilde{c}_1^a) - u'(\tilde{c}_1^a) \frac{\phi^{Aa} p_1}{(1+r_2)^2} \frac{dr_2}{dt_1^a} \tag{B.30}$$

$$+ \frac{\phi^{Aa}}{1+r_2} \frac{dp_1}{dt_1^a} [u'(\tilde{c}_1^a) - \beta^a (1+r_2) u'(\tilde{c}_2^a)] > 0 \qquad \frac{\partial V^l}{\partial t_1^a} = u'(\tilde{c}_1^l) \frac{\alpha^a}{\alpha^l} \frac{\phi^{Aa} p_1}{(1+r_2)^2} \frac{dr_2}{dt_1^a} > 0 \tag{B.31}$$

$$\frac{\partial V^a}{\partial t_2^a} = \beta^a (1+r_2) u'(\tilde{c}_2^a) - u'(\tilde{c}_1^a) \frac{\phi^{Aa} p_1}{(1+r_2)^2} \frac{dr_2}{dt_2^a} \tag{B.32}$$

$$+ \frac{\phi^{Aa}}{1+r_2} \frac{dp_1}{dt_2^a} [u'(\tilde{c}_1^a) - \beta^a (1+r_2) u'(\tilde{c}_2^a)] \qquad \frac{\partial V^l}{\partial t_2^a} = u'(\tilde{c}_1^l) \frac{\alpha^a}{\alpha^l} \frac{\phi^{Aa} p_1}{(1+r_2)^2} \frac{dr_2}{dt_2^a} < 0 \tag{B.33}$$

with $J_{b1}^{an} = -\frac{\beta^a d_2^a}{(u'(\tilde{c}_1^a))^2} u''(\tilde{c}_1^a) u'(\tilde{c}_2^a) > 0$ and $J_{b2}^{an} = \frac{\beta^a d_2^a}{(u'(\tilde{c}_1^a))^2} u'(\tilde{c}_1^a) u''(\tilde{c}_2^a) < 0$.

Asset-based borrowing with an AD shortage. A marginal increase in d_1^a or a decrease in d_2^a when there is an aggregate demand shortage will increase the consumption of the borrowers. Higher current consumption boosts asset prices, enabling borrowing to take on more debt. Without adjustment of the interest rate, this boosts aggregate demand. Firms hire more labor and produce more, which raises income. Higher income further boosts consumption and asset prices. As a result, assets become more valuable and income is also higher. The welfare of both borrowers and lenders is improved.

$$\frac{dp_1}{dt_1^a} = -\frac{\frac{J_{b1}^{aa}}{X_l^{aa}}}{\frac{Z_b^{aa}}{X_b^{aa}} - \frac{Z_l^{aa}}{X_l^{aa}}} > 0 \qquad \frac{de_1}{dt_1^a} = -\frac{\frac{J_{b1}^{aa}}{X_l^{aa}}}{\frac{Z_b^{aa}}{X_b^{aa}} - \frac{Z_l^{aa}}{X_l^{aa}}}\frac{Z_l^{aa}}{X_l^{aa}} > 0 \tag{B.34}$$

$$\frac{dp_1}{dt_2^a} = \frac{\frac{J_{b2}^{aa}}{X_l^{aa}}}{\frac{Z_b^{aa}}{X_b^{aa}} - \frac{Z_l^{aa}}{X_l^{aa}}} < 0 \qquad \frac{de_1}{dt_2^a} = \frac{\frac{J_{b2}^{aa}}{X_l^{aa}}}{\frac{Z_b^{aa}}{X_b^{aa}} - \frac{Z_l^{aa}}{X_l^{aa}}}\frac{Z_l^{aa}}{X_l^{aa}} < 0 \tag{B.35}$$

$$\frac{\partial V^a}{\partial t_1^a} = [(1 - v'(e_1))\frac{de_1}{dt_1^a} + 1 + \phi^{Aa}\frac{dp_1}{dt_1^a}]u'(\tilde{c}_1^a) \tag{B.36}$$

$$- \beta^a\phi^{Aa}u'(\tilde{c}_2^a)\frac{dp_1}{dt_1^a} > 0 \qquad \frac{\partial V^l}{\partial t_1^a} = [(1 - v'(e_1))]u'(\tilde{c}_1^l)\frac{de_1}{dt_1^a} > 0 \tag{B.37}$$

$$\frac{\partial V^a}{\partial t_2^a} = [(1 - v'(e_1))\frac{de_1}{dt_2^a} + \phi^{Aa}\frac{dp_1}{dt_2^a}]u'(\tilde{c}_1^a) \tag{B.38}$$

$$- \beta^a\phi^{Aa}u'(\tilde{c}_2^a)\frac{dp_1}{dt_2^a} + \beta^a u'(\tilde{c}_2^a) \qquad \frac{\partial V^l}{\partial t_2^a} = [(1 - v'(e_1))]u'(\tilde{c}_1^l)\frac{de_1}{dt_2^a} < 0 \tag{B.39}$$

B.2 Aggregate income in the borrowing constraint

When the debt limit is determined by aggregate income with no asset-based borrowing households in the economy. Similarly, the model can be solved via backward induction. Period 2 consumption and labor choices are intratemporal decisions given b_2^h at the beginning of period 2. By market clearing condition, lenders' bond holdings will be $\alpha^l b_2^l = -\alpha^i b_2^i$. Let net consumption be $\tilde{c}_t^h$, which is equal to $c_t^h - v(n_t^h)$. With monetary policy replicating the first-best outcome in every period, the Euler equation of the lenders is then given by:

$$u'(\tilde{c}_1^l) = \beta^l(1 + r_2)u'(e^* + t_2^l + d_2^l - \frac{\alpha^i}{\alpha^l}b_2^i - v(n^*)) \tag{B.40}$$

For a given level of b_2^i that borrowers take on, as r_2 falls, net consumption of the lenders $\tilde{c}_1^l$ will increase. Since prices are fixed, the real interest rate will govern the demand and therefore how much firms produce. As borrowers accumulate debt, the IBC they face in period 1 may force them to deleverage. Deleveraging by the borrowers reduces consumption demand of the borrowers. The interest rate will have to fall to induce lenders to hold less bonds, which boosts lenders' consumption to an extent where firms produces optimally satisfying aggregate demand. However, if debt accumulation is beyond a threshold level, the real interest rate may not fall enough to clear the goods market. Since the intertemporal price cannot adjust, the intratemporal price, the wage rate will fall, reducing labor supply. Output, falling below the optimal, will be determined by the aggregate demand at the zero interest rate.

When there is no aggregate demand shortage. Consider the decentralized equilibrium when there is no demand shortage and all markets clear[3]. Due to the constraint on borrowers' debt, the maximum level of debt they can take on will be $\phi^{Ii}e^*$. This will define the corresponding upper bound on net consumption $\tilde{c}_1^l$, $\bar{\tilde{c}}_1^l$ when r_2 reaches the lower bound 0

$$\bar{\tilde{c}}_1^l = (1 + \frac{\alpha^i}{\alpha^l}\phi^{Ii})e^* + t_2^l + d_2^l - v(n^*)$$

Correspondingly, the upper bound on consumption of the lenders is given by:

$$\bar{c}_1^l = \bar{\tilde{c}}_1^l + v(n^*) = (1 + \frac{\alpha^i}{\alpha^l}\phi^{Ii})e^* + t_2^l + d_2^l \tag{B.41}$$

The upper bound on lenders' consumption in period 1 reflects that lenders' demand is constrained by the lower bound on the interest rate. Aggregate demand in period 1 can be written as:

$$\begin{aligned}\alpha^l c_1^l + \alpha^i c_1^i &= \alpha^l c_1^l + \alpha^i (e^* + t_1^i + d_1^i + \frac{\phi^{Ii}e^*}{1+r_2} + b_1^i) \\ &= e^* + \alpha^l(t_1^l + d_1^l) + \alpha^i(t_1^i + d_1^i)\end{aligned} \tag{B.42}$$

If real interest rate is above the lower bound, firms can always operate efficiently, and the efficient level of income is given by $e^* = n^*$, where $n^* = v'^{-1}(1)$, as in the first-best solution. The allocations are constrained efficient, with consumption of the households in period 1 given by:

$$c_1^h = e^* + t_1^h + d_1^h + b_1^h - \frac{\frac{\alpha^i}{\alpha^l}\phi^{Ii}e^*}{1+r_2}$$

When there is an aggregate demand shortage. If real interest rate is constrained by the lower bound, aggregate demand will be below the efficient level. This can be a result of large accumulation of debt in period 0 that triggers massive deleveraging in period 1 by the borrowers. The loss in demand by the borrowers need to be picked up by a fall in the interest rate, which will induce an increase in consumption demand by the lenders, as shown in Equation (B.42). If b_1^i exceeds a certain level, the interest rate will reach the zero lower bound. This threshold of debt is given by:

$$|\bar{b}_1^i| = 2\phi^{Ii}e^* + \frac{\alpha^l}{\alpha^i}(t_2^l + d_2^l - t_1^l - d_1^l) \tag{B.43}$$

[3] The sufficient conditions for the existence of such an equilibrium are in the Appendix.

Amplification. If $-b_1^i > |\bar{b}_1^i|$, deleveraging by borrowers will trigger a demand-driven recession when income becomes sub-optimal. Lenders' consumption demand cannot reach $\bar{c}_1^l$, but is still maximized at the zero interest rate. Note that since lenders and borrowers' labor supply $n_t^l = n_t^i$, they earn the same level of labor income. In addition, when wage is below the efficient level, firms will earn positive profits, and therefore $e_t^h = e_t$ and $e_1^h = w_1 n_1 + y_1 - w_1 n_1 = y_1 = n_1$ in equilibrium. Household income is then determined by aggregate demand at $r_2 = 0$ and is given by:

$$e_1 + \alpha^l(t_1^l + d_1^l) + \alpha^i(t_1^i + d_1^i) = \alpha^l c_1^l + \alpha^i c_1^i$$

$$e_1 = 2\frac{\alpha^i}{\alpha^l}\phi^{Ii}e_1 + v(e_1) + \frac{\alpha^i}{\alpha^l}b_1^i + (t_2^l + d_2^l - t_1^l - d_1^l) + (e^* - v(e^*)) \tag{B.44}$$

Equation (B.44) demonstrates the amplification of shocks through aggregate demand. A fall in borrowers' net worth will reduce borrowers' demand, leading to a fall in income. Lower income can dampen consumption demand by both the lenders and borrowers in period 1, which reduces income further. Equation (B.44) is equivalent to lenders' Euler equation at $r_2 = 0$:

$$u'(e_1 + t_1^l + d_1^l - \frac{\alpha^i}{\alpha^l}b_1^i - \frac{\frac{\alpha^i}{\alpha^l}\phi^{Ii}e_1}{1+r_2} - v(e_1)) = \beta^l(1+r_2)u'(e^* + t_2^l + d_2^l + \frac{\frac{\alpha^i}{\alpha^l}\phi^{Ii}e_1}{1+r_2} - v(e^*)). \tag{B.45}$$

When there is an aggregate demand shortage, the equilibrium is completely pinned down by lenders' Euler equation at $r_2 = 0$. This equation also shows how wage has to adjust when the intertemporal price the interest rate is fixed. To have a unique and well-defined equilibrium, it requires that $1 - 2\frac{\alpha^i}{\alpha^l}\phi^{Ii} - v'(e_1)$ to be greater than zero.

Figure B.1 illustrates this multiplier-effect result. One unit of decrease in borrowers' net worth can generate $(\frac{\alpha^i}{\alpha^l})\frac{1}{1-2\frac{\alpha^i}{\alpha^l}\phi^{Ii}-v'(e_1)}$ unit of fall in income.

A. a shock on lenders' endowment t_1^l

When there is no aggregate demand shortage, both types of shocks will not have any impact on the aggregate income. However, shocks on lenders' endowment can indirectly affect welfare through interest rate. More endowment of the lenders can boost their demand for bonds and will lower the interest rate, which benefits the borrowers while undermines the lenders. This result follows when the debt limit is determined by individual income: interest rate fall for the same reason, but borrowers will have

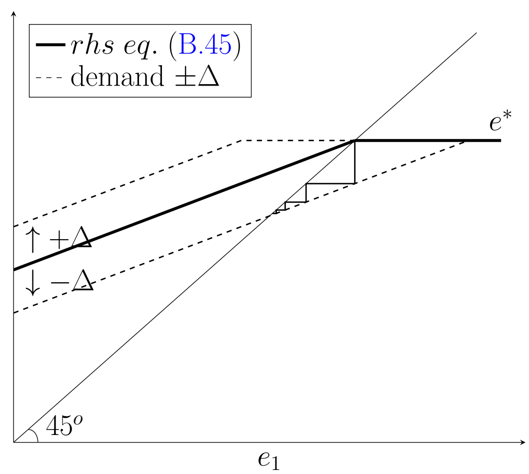


Figure B.1: Amplification Through Aggregate Demand

higher employment and thus higher individual income which further improves welfare.

$$\frac{de_1}{dt_1^l} = 0 \tag{B.46}$$

$$\frac{dr_2}{dt_1^l} = \frac{u''(\tilde{c}_1^l)}{\beta^l u'(\tilde{c}_2^l) - \frac{\phi^{Ii} e^*}{(1+r_2)^2} u''(\tilde{c}_1^l)} < 0 \tag{B.47}$$

$$\frac{\partial V^i}{\partial t_1^l} = -u'(\tilde{c}_1^i) \frac{\phi^{Ii} e^*}{(1+r_2)^2} (\frac{dr_2}{dt_1^l} > 0 \tag{B.48}$$

$$\frac{\partial V^l}{\partial t_1^l} = u'(\tilde{c}_1^l)(1 + \frac{\phi^{Ii} e^*}{(1+r_2)^2} (\frac{dr_2}{dt_1^l}) \tag{B.49}$$

$$= \frac{\beta^l u'(\tilde{c}_1^l) u'(\tilde{c}_2^l)}{\beta^l u'(\tilde{c}_2^l) - \frac{\phi^{Ii} e^*}{(1+r_2)^2} u''(\tilde{c}_1^l)} > 0 \tag{B.50}$$

When there is an aggregate demand shortage, a unit positive shock on lenders' endowment in period 1 has a similar effect as a negative shock on their endowment in period 2: they both lower households' income by $1 - 2\frac{\alpha^i}{\alpha^l}\phi^{Ii} - v'(e_1)$. The decrease in income results from the limit on lenders' demand. Higher endowment or transfer in period 1 makes lenders less willing to work as their demand is constrained by the lower bound on the interest rate; similarly, the consumption smoothing motive of the lenders prompts them to save more and consume less in period 1 when lower endowment (that is a decrease in t_2^l) increases the marginal utility of consumption in period 2[4]. The resulting lower labor supply decrease production and income, reducing borrowers' debt capacity, which reduces demand further. With individual income in

[4]The GHH preference precludes the positive effect on labor supply when consumption falls and thus there is more amplification.

the borrowing constraint, employment of both borrowers and lenders will decrease because of lower wage, which lowers utility.

$$\frac{de_1}{dt_1^l} = -(1 - 2\frac{\alpha^i}{\alpha^l}\phi^{Ii} - v'(e_1)) < 0 \tag{B.51}$$

$$\frac{dr_2}{dt_1^l} = 0 \tag{B.52}$$

$$\frac{\partial V_1^i}{\partial t_1^l} = [(1 - v'(e_1))u'(\tilde{c}_1^i) + \phi^{Ii}(u'(\tilde{c}_1^i) - \beta^i u'(\tilde{c}_2^i))]\frac{de_1}{dt_1^l} < 0 \tag{B.53}$$

$$\frac{\partial V_1^l}{\partial t_1^l} = u'(\tilde{c}_1^l) + (1 - v'(e_1))u'(\tilde{c}_1^l)\frac{de_1}{dt_1^l} \tag{B.54}$$

$$= u'(\tilde{c}_1^l)\frac{\phi^{Ii}}{\alpha^l}\frac{de_1}{dt_1^l} < 0 \tag{B.55}$$

A change in lenders' transfer t_1^l has an opposite impact on an income-based borrowing economy when there is no aggregate demand shortage and when there is an aggregate demand shortage. An increase in t_1^l makes the households better-off when interest rate is above the lower bound $\frac{\partial V^h}{\partial t_1^l} > 0$, whereas it makes the households worse-off when interest rate is stuck at the lower bound $\frac{\partial V^h}{\partial t_1^l} < 0$.

As output is aggregate-demand determined when prices are sticky, the interest rate governs the consumption demand and thus output. An increase in the endowment will boost consumption of the lenders through a fall in the interest rate, leaving income at the optimal level when the interest rate is still flexible to move. Welfare of the borrowers is improved due to lower interest rate while that of the lenders is improved due to the direct effect of higher endowment dominating the adverse of effect of lower interest rate. When the interest rate is at the lower bound, however, the demand shortage will be worsened by the increase in lenders' endowment since lenders do not need to earn that much income to consume the same amount. The resulting lower labor supply reduces income, further tightening the borrowing constraint. Welfare of both types of households will be undermined as income decreases.

Asset-based borrowing. when there is no aggregate demand shortage, a transfer to the lenders will increase lenders' demand for bonds, lower the interest rate, and since lenders become more willing to hold debt, the collateral that the borrowers need for borrowing becomes more valuable. Therefore, asset price will increase and the constraint on borrowers will be relaxed. The marginal increase in lenders' endowment will decrease the interest rate and increase asset price, though households' income stay unchanged as there is no aggregate demand shortage. The effect on welfare is similar to that with the income-based borrowing constraint. Define:

$$M = \frac{(1+r_2)\frac{dp_1}{dt_1^l} - p_1\frac{dr_2}{dt_1^l}}{(1+r_2)^2}$$

The marginal effect on income, interest rate, asset price and welfare is given by:

$$\frac{de_1}{dt_1^l} = 0 \tag{B.56}$$

$$\frac{dr_2}{dt_1^l} = \frac{u''(\tilde{c}_1^l)}{\beta^l u'(\tilde{c}_2^l) - \frac{\phi^{Aa}p_1}{(1+r_2)^2}u''(\tilde{c}_1^l) + \frac{\phi^{Aa}p_1}{X(1+r_2)^2}\frac{\phi^{Aa}\beta^a d_2^a}{(u'(\tilde{c}_1^a))^2}u''(\tilde{c}_1^a)u'(\tilde{c}_2^a)(\frac{u''(\tilde{c}_1^l)}{1+r_2} + \beta^l(1+r_2)u''(\tilde{c}_2^l))} < 0 \tag{B.57}$$

$$\frac{dp_1}{dt_1^l} = \frac{\phi^{Aa}\beta^a d_2^a p_1 u'(\tilde{c}_2^a)u''(\tilde{c}_1^a)}{X(1+r_2)^2(u'(\tilde{c}_1^a))^2}\frac{dr_2}{dt_1^l} > 0 \tag{B.58}$$

$$\frac{\partial V^a}{\partial t_1^l} = -u'(\tilde{c}_1^a)\frac{\phi^{Aa}p_1}{(1+r_2)^2}\frac{dr_2}{dt_1^l} + \frac{\phi^{Aa}}{1+r_2}\frac{dp_1}{dt_1^l}[u'(\tilde{c}_1^a) - \beta^a(1+r_2)u'(\tilde{c}_2^a)] > 0 \tag{B.59}$$

$$\frac{\partial V^l}{\partial t_1^l} = (1 + \frac{\phi^{Aa}p_1}{(1+r_2)^2}\frac{dr_2}{dt_1^l})u'(\tilde{c}_1^l) > 0 \tag{B.60}$$

Take partial derivative with respect to t_1^l to the asset pricing equation and the lenders' Euler equation to get:

$$M = -\frac{\frac{(u'(\tilde{c}_1^a))^2}{\phi^{Aa}\beta^a d_2^a} + u'(\tilde{c}_1^a)u''(\tilde{c}_2^a)}{u'(\tilde{c}_2^a)u''(\tilde{c}_1^a)}\frac{dp_1}{dt_1^l} \tag{B.61}$$

$$u''(\tilde{c}_1^l)(1-\phi^{Aa}M) = \beta^l(u'(\tilde{c}_2^l)\frac{dr_2}{dt_1^l} + \phi^{Aa}(1+r_2)u''(\tilde{c}_2^l)\frac{dp_1}{dt_1^l}) \tag{B.62}$$

Let $N = -\frac{\frac{(u'(\tilde{c}_1^a))^2}{\phi^{Aa}\beta^a d_2^a} + u'(\tilde{c}_1^a)u''(\tilde{c}_2^a)}{u'(\tilde{c}_2^a)u''(\tilde{c}_1^a)}$ such that $M = N\frac{dp_1}{dt_1^l}$. Equation (B.62) can be simplified to:

$$u''(\tilde{c}_1^l) = \beta^l u'(\tilde{c}_2^l)\frac{dr_2}{dt_1^l} + (\phi^{Aa}u''(\tilde{c}_1^l)N + \phi^{Aa}(1+r_2)u''(\tilde{c}_2^l))\frac{dp_1}{dt_1^l} \tag{B.63}$$

By the definition of M and (B.61),

$$\frac{dr_2}{dt_1^l} = \frac{1+r_2}{p_1}(1-(1+r_2)N)\frac{dp_1}{dt_1^l} \tag{B.64}$$

Plug N into (B.64) to get:

$$\frac{dp_1}{dt_1^l} = \frac{\phi^{Aa}\beta^a d_2^a p_1 u'(\tilde{c}_2^a)u''(\tilde{c}_1^a)}{X(1+r_2)^2(u'(\tilde{c}_1^a))^2}\frac{dr_2}{dt_1^l} \tag{B.65}$$

Since $X > 0$ by the previous assumption, $\frac{dr_2}{dt_1^l}$ and $\frac{dp_1}{dt_1^l}$ must be with opposite signs. Given (B.64), $1-(1+r_2)N < 0$ and thus $N > 0$. For (B.63) to be satisfied, $\frac{dr_2}{dt_1^l}$ has to be non-positive and $\frac{dp_1}{dt_1^l}$ has to be non-negative. Therefore, M is also non-negative. To solve for $\frac{dp_1}{dt_1^l}$ and $\frac{dr_2}{dt_1^l}$, plug (B.65) in (B.63).

Since the RHS of (B.62) is negative, $1-\phi^{Aa}M > 0$, which renders $\frac{\partial V^l}{\partial t_1^l} > 0$. And similarly as $1-(1+r_2)N < 0$, $\frac{\partial V^a}{\partial t_1^l} > 0$ is given by:

$$\begin{aligned}
\frac{\partial V^a}{\partial t_1^l} &= \phi^{Aa}Mu'(\tilde{c}_1^a) - \beta^a u'(\tilde{c}_2^a)\phi^{Aa}\frac{dp_1}{dt_1^l} \\
&= \phi^{Aa}\frac{dp_1}{dt_1^l}[Nu'(\tilde{c}_1^a) - \beta^a u'(\tilde{c}_2^a)] \\
&\geq \phi^{Aa}\frac{dp_1}{dt_1^l}[(1+r_2)N\beta^a u'(\tilde{c}_2^a) - \beta^a u'(\tilde{c}_2^a)] \\
&> 0
\end{aligned}$$

To further simplify the expression and to compare it with the welfare effect for the income-based borrowers when there is no aggregate demand shortage, we have:

$$\begin{aligned}
\frac{\partial V^a}{\partial t_1^l} &= \phi^{Aa}\frac{(1+r_2)\frac{dp_1}{dt_1^l} - p_1\frac{dr_2}{dt_1^l}}{(1+r_2)^2}u'(\tilde{c}_1^a) - \beta^a u'(\tilde{c}_2^a)\phi^{Aa}\frac{dp_1}{dt_1^l} \\
&= -u'(\tilde{c}_1^a)\frac{\phi^{Aa}p_1}{(1+r_2)^2}\frac{dr_2}{dt_1^l} + u'(\tilde{c}_1^a)\frac{\phi^{Aa}}{1+r_2}\frac{dp_1}{dt_1^l} - \phi^{Aa}\beta^a u'(\tilde{c}_2^a)\frac{dp_1}{dt_1^l} \\
&= -u'(\tilde{c}_1^a)\frac{\phi^{Aa}p_1}{(1+r_2)^2}\frac{dr_2}{dt_1^l} + \frac{\phi^{Aa}}{1+r_2}\frac{dp_1}{dt_1^l}[u'(\tilde{c}_1^a) - \beta^a(1+r_2)u'(\tilde{c}_2^a)]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial V^l}{\partial t_1^l} &= [1 - \phi^{Aa}\frac{(1+r_2)\frac{dp_1}{dt_1^l} - p_1\frac{dr_2}{dt_1^l}}{(1+r_2)^2}]u'(\tilde{c}_1^l) + \beta^l u'(\tilde{c}_2^l)\phi^{Aa}\frac{dp_1}{dt_1^l} \\
&= (1 + \frac{\phi^{Aa}p_1}{(1+r_2)^2}\frac{dr_2}{dt_1^l})u'(\tilde{c}_1^l) + \frac{\phi^{Aa}}{1+r_2}\frac{dp_1}{dt_1^l}[u'(\tilde{c}_1^l) - \beta^a(1+r_2)u'(\tilde{c}_2^l)] \\
&= (1 + \frac{\phi^{Aa}p_1}{(1+r_2)^2}\frac{dr_2}{dt_1^l})u'(\tilde{c}_1^l)
\end{aligned}$$

A change in lenders' endowment t_1^l has similar effects on an income-based borrowing economy and an asset-based borrowing economy when there is no aggregate demand shortage. An increase in t_1^l will improve welfare of both types of households: $\frac{\partial V^h}{\partial t_1^l} > 0$. In an income-based borrowing economy, it is achieved via a fall in the interest rate; in an asset-based borrowing economy, it is achieved through not only a fall in the interest rate, but also an increase in the asset price which affects welfare of the borrowers not lenders, and

(a) the decrease in the interest rate $(|\frac{dr_2}{dt_1^l}|)^{an} < (|\frac{dr_2}{dt_1^l}|)^{in}$;

(b) lenders' welfare increases $(\frac{\partial V^l}{\partial t_1^l})^{an} > (\frac{\partial V^l}{\partial t_1^l})^{in}$; welfare increases are ambivalent to compare between an asset-based borrower and an income-based borrower $(\frac{\partial V^a}{\partial t_1^l})^{an} \lesseqgtr (\frac{\partial V^i}{\partial t_1^l})^{in}$.

Next consider a marginal increase in t_1^l when there is an aggregate demand shortage for an asset-based borrower.

$$\frac{de_1}{dt_1^l} = -\frac{1 + \frac{\phi^{Aa}\beta^a d_2^a}{(u'(\tilde{c}_1^a))^2}[u'(\tilde{c}_1^a)u''(\tilde{c}_2^a) + u''(\tilde{c}_1^a)u'(\tilde{c}_2^a)]}{(1 - v'(e_1))[1 + \frac{\phi^{Aa}\beta^a d_2^a}{(u'(\tilde{c}_1^a))^2}(u'(\tilde{c}_1^a)u''(\tilde{c}_2^a) + (1 + \frac{1}{\alpha^l})u''(\tilde{c}_1^a)u'(\tilde{c}_2^a))]} < 0 \quad \text{(B.66)}$$

$$\frac{dr_2}{dt_1^l} = 0 \quad \text{(B.67)}$$

$$\frac{dp_1}{dt_1^l} = \frac{\beta^a d_2^a u'(\tilde{c}_2^a)u''(\tilde{c}_1^a)}{(u'(\tilde{c}_1^a))^2[1 + \frac{\phi^{Aa}\beta^a d_2^a}{(u'(\tilde{c}_1^a))^2}(u'(\tilde{c}_1^a)u''(\tilde{c}_2^a) + (1 + \frac{1}{\alpha^l})u''(\tilde{c}_1^a)u'(\tilde{c}_2^a))]} < 0 \quad \text{(B.68)}$$

$$\frac{\partial V^a}{\partial t_1^l} = [(1 - v'(e_1))\frac{de_1}{dt_1^l} + \phi^{Aa}\frac{dp_1}{dt_1^l}]u'(\tilde{c}_1^a) - \beta^a\phi^{Aa}u'(\tilde{c}_2^a)\frac{dp_1}{dt_1^l} < 0 \quad \text{(B.69)}$$

$$\frac{\partial V^l}{\partial t_1^l} = [(1 - v'(e_1))\frac{de_1}{dt_1^l} + 1 - \phi^{Aa}\frac{dp_1}{dt_1^l}]u'(\tilde{c}_1^l) + \beta^l\phi^{Aa}u'(\tilde{c}_2^l)\frac{dp_1}{dt_1^l} \quad \text{(B.70)}$$

$$= u'(\tilde{c}_1^l) + [(1 - v'(e_1))]u'(\tilde{c}_1^l)\frac{de_1}{dt_1^l} < 0 \quad \text{(B.71)}$$

Take the partial derivative with t_1^l to the asset pricing equation and the aggregate demand equation:

$$X\frac{dp_1}{dt_1^l} = -Z\frac{de_1}{dt_1^l} \quad \text{(B.72)}$$

$$Y\frac{de_1}{dt_1^l} = \phi^{Aa}\frac{dp_1}{dt_1^l} - \alpha^l \quad \text{(B.73)}$$

where $Z = \frac{\beta^a d_2^a(1-v'(e_1))}{(u'(\tilde{c}_1^a))^2}u'(\tilde{c}_2^a)u''(\tilde{c}_1^a) < 0$. Combine (B.72) and (B.73) to obtain:

$$\frac{dp_1}{dt_1^l} = \frac{\alpha^l Z}{XY + \phi^{Aa}Z} \quad \text{(B.74)}$$

$$\frac{de_1}{dt_1^l} = -\frac{\alpha^l X}{XY + \phi^{Aa} Z} \tag{B.75}$$

We restrict the slope of the asset equation and the aggregate demand equation in order to have a well-defined solution. That is, $\frac{de_1^{AP}}{dp_1} > \frac{de_1^{AD}}{dp_1}$, where

$$\frac{de_1^{AP}}{dp_1} = -\frac{X}{Z}$$
$$\frac{de_1^{AD}}{dp_1} = \frac{\phi^{Aa}}{Y}$$

With this restriction, $X + \frac{\phi^{Aa} Z}{Y} > 0$ and $\frac{dp_1}{dt_1^l} < 0$ and $\frac{dp_1}{dt_1^l} < 0$. Moreover, note that the slope of the AP equation and AD equation can be greater or less than one. We exclude the circumstance where both slopes are greater than one, as when $\frac{de_1^{AD}}{dp_1}$ is greater than one, $1 - \phi^{Aa} - \alpha^a - \alpha^l v'(e_1)$ will be negative, which contradicts with our assumptions for the income-based borrowing economy when there is an aggregate demand shortage if we set $\alpha^a = \alpha^i$ and $\phi^{Aa} = \phi^{Ii}$.

To compare the change in income and welfare with the income-based borrowing constraint, we redefine Y as $Y = 1 - \alpha^{i/a} - \alpha^l v'(e_1)$, and the marginal change in income with income-based borrowing Equation (B.51) can be written as $|\frac{de_1}{dt_1^l}| = \frac{\alpha^l}{Y - \phi^{Ii}}$. By Equation (B.74) and (B.74), we can rewrite $\frac{dp_1}{dt_1^l}$ and $\frac{de_1}{dt_1^l}$ as:

$$\begin{aligned} \frac{de_1}{dt_1^l} &= -\frac{\alpha^l X}{XY + \phi^{Aa} Z} \\ &= -\frac{\alpha^l X}{X(Y - \phi^{Aa}) + \phi^{Aa}(X + Z)} \\ &= -\frac{\alpha^l}{Y - \phi^{Aa}} \left(\frac{X}{X + \phi^{Aa} \frac{X+Z}{Y - \phi^{Aa}}} \right) \end{aligned} \tag{B.76}$$

$$\begin{aligned} \frac{dp_1}{dt_1^l} &= -\frac{Z}{X} \left(\frac{de_1}{dt_1^l} \right. \\ &= -\frac{\alpha^l}{Y - \phi^{Aa}} \left(\frac{-Z}{X + \phi^{Aa} \frac{X+Z}{Y - \phi^{Aa}}} \right) \end{aligned} \tag{B.77}$$

Consider first when $1 \geq \frac{de_1^{AP}}{dp_1} \geq \frac{de_1^{AD}}{dp_1}$, it renders $X \leq -Z$ and $Y > \phi^{Aa}$, and $\frac{-Z}{X + \phi^{Aa} \frac{X+Z}{Y - \phi^{Aa}}} \geq \frac{X}{X + \phi^{Aa} \frac{X+Z}{Y - \phi^{Aa}}} \geq 1$. Therefore, $|(\frac{dp_1}{dt_1^l})^{aa}| \geq |(\frac{de_1}{dt_1^l})^{aa}| \geq |(\frac{de_1}{dt_1^l})^{ia}|$. By Equation (B.53), (B.55), (B.69) and (B.71), we have $|(\frac{\partial V^a}{\partial t_1^l})^{aa}| \geq |(\frac{\partial V^i}{\partial t_1^l})^{ia}|$, and $|(\frac{\partial V^l}{\partial t_1^l})^{aa}|(\geq |\frac{\partial V^l}{\partial t_1^l})^{ia}|$.

when $\frac{de_1^{AP}}{dp_1} \geq 1 \geq \frac{de_1^{AD}}{dp_1}$, it renders $X \geq -Z$ and $Y > \phi^{Aa}$, and $\frac{-Z}{X+\phi^{Aa}\frac{X+Z}{Y-\phi^{Aa}}} \leq \frac{X}{X+\phi^{Aa}\frac{X+Z}{Y-\phi^{Aa}}} \leq 1$. Therefore, $|(\frac{dp_1}{dt_1^l})^{aa}| \leq |(\frac{de_1}{dt_1^l})^{aa}| \leq |(\frac{de_1}{dt_1^l})^{ia}|$. By Equation (B.53), (B.55), (B.69) and (B.71), we have $|(\frac{\partial V^a}{\partial t_1^l})^{aa}| \leq |(\frac{\partial V^i}{\partial t_1^l})^{ia}|$, and $|(\frac{\partial V^l}{\partial t_1^l})^{aa}| \leq |(\frac{\partial V^l}{\partial t_1^l})^{ia}|$. A change in lenders' endowment t_1^l has similar effects on an income-based borrowing economy and an asset-based borrowing economy when there is an aggregate demand shortage. An increase in t_1^l will lower income and undermine the welfare of both types of households: $\frac{\partial V^h}{\partial t_1^l} < 0$. In an asset-based borrowing economy, it affects welfare of the borrowers through depressing asset price in addition to lowering income as in an income-based economy. In both economies, it affects the welfare of lenders only through lowering income. Whether its impact is more pronounced will depend on the responsiveness of income to changes in the asset price:

(a) If $1 \geq \frac{de_1^{AP}}{dp_1} \geq \frac{de_1^{AD}}{dp_1}$,

  (i) $|\frac{dp_1}{dt_1^l})^{aa}| \geq |(\frac{de_1}{dt_1^l})^{aa}| \geq |(\frac{de_1}{dt_1^l})^{ia}|$;

  (ii) $|(\frac{\partial V^a}{\partial t_1^l})^{aa}| \geq |\frac{\partial V^i}{\partial t_1^l}|$, and $|\frac{\partial V^l}{\partial t_1^l})^{aa}| \geq |(\frac{\partial V^l}{\partial t_1^l})^{ia}|$.

(b) If $\frac{de_1^{AP}}{dp_1} \geq 1 \geq \frac{de_1^{AD}}{dp_1}$,

  (i) $|(\frac{dp_1}{dt_1^l})^{aa}| \leq |\frac{de_1}{dt_1^l})^{aa}| \leq |(\frac{de_1}{dt_1^l})^{ia}|$;

  (ii) $|(\frac{\partial V^a}{\partial t_1^l})^{aa}| \leq |(\frac{\partial V^i}{\partial t_1^l})^{ia}|$, and $|(\frac{\partial V^l}{\partial t_1^l})^{aa}| \leq |(\frac{\partial V^l}{\partial t_1^l})^{ia}|$.

B. a shock on borrowers' dividend d_1^i or d_1^a

Income-based borrowing. For an income-based borrowing economy, when there is no aggregate demand shortage, shocks on asset dividend do not even have any effect on the interest rate if borrowers are constrained. They only affect borrowers' welfare by direct wealth effect.

$$\frac{de_1}{dd_1^i} = 0 \tag{B.78}$$

$$\frac{dr_2}{dd_1^i} = 0 \tag{B.79}$$

$$\frac{\partial V^i}{\partial d_1^i} = u'(\tilde{c}_1^i) > 0 \tag{B.80}$$

$$\frac{\partial V^l}{\partial d_1^i} = 0 \tag{B.81}$$

Interest rate is unaffected because higher dividend boosts demand and thus income, which lowers interest rate as borrowers are less constrained by income. The reduction in interest rate is offset by a monetary policy that has to raise interest rate to maintain the optimal level of output and prevent an overheating economy.

when there is an aggregate demand shortage and the interest rate is at the lower bound, the shock on d_1^i does not influence income as in Equation (B.44), despite the negative effect on borrowers' demand. Income is left unaffected, and the welfare of the households similarly responds to the shock as with the case when there is no aggregate demand shortage.

$$\frac{de_1}{dd_1^i} = 0 \tag{B.82}$$
$$\frac{dr_2}{dd_1^i} = 0 \tag{B.83}$$
$$\frac{\partial V^i}{\partial d_1^i} = u'(\tilde{c}_1^i) > 0 \tag{B.84}$$
$$\frac{\partial V^l}{\partial d_1^i} = 0 \tag{B.85}$$

Asset-based borrowing. Next consider a marginal increase in d_1^a when there is no aggregate demand shortage. An increase in asset dividend will make asset more valuable as it not only boosts consumption by the borrowers in the current period directly, but relaxes the borrowing constraint as the price of the asset rises, which further increases consumption and inflates the asset price. This is the canonical amplification mechanism with the asset-based borrowing constraint. Meanwhile, the interest rate must increase since the supply of bonds rises as the borrowers expand their debt capacity with more valuable collaterals.

$$\frac{de_1}{dd_1^a} = 0 \tag{B.86}$$
$$\frac{dr_2}{dd_1^a} = Q\frac{dp_1}{dd_1^a} > 0 \tag{B.87}$$
$$\frac{dp_1}{dd_1^a} = \frac{1}{p_1 Q - (1+r_2) - \frac{(u'(\tilde{c}_1^a))^2}{\phi^{Aa}\beta^a d_2^a u'(\tilde{c}_2^a)u''(\tilde{c}_1^a)} - \frac{u'(\tilde{c}_1^a)u''(\tilde{c}_2^a)}{u'(\tilde{c}_2^a)u''(\tilde{c}_1^a)}} > 0 \tag{B.88}$$
$$\frac{\partial V^a}{\partial d_1^a} = u'(\tilde{c}_1^a)(1+\phi^{Aa}M) - \beta^a u'(\tilde{c}_2^a)\phi^{Aa}\frac{dp_1}{dd_1^a} > 0 \tag{B.89}$$
$$\frac{\partial V^l}{\partial d_1^a} = -\phi^{Aa}Mu'(\tilde{c}_1^l) + \beta^a u'(\tilde{c}_2^l)\phi^{Aa}\frac{dp_1}{dd_1^a} > 0 \tag{B.90}$$

Take partial derivative with respect to d_1^a to the asset pricing equation and the lenders'

Euler equation to get:

$$\frac{dp_1}{dd_1^a} = -\frac{\phi^{Aa}\beta^a d_2^a}{(u'(\tilde{c}_1^a))^2}[u'(\tilde{c}_1^a)u''(\tilde{c}_2^a)\frac{dp_1}{dd_1^a} + u'(\tilde{c}_2^a)u''(\tilde{c}_1^a)(1+M)] \tag{B.91}$$

$$-u''(\tilde{c}_1^l)\phi^{Aa}M = \beta^l(u'(\tilde{c}_2^l)\frac{dr_2}{dd_1^a} + \phi^{Aa}(1+r_2)u''(\tilde{c}_2^l)\frac{dp_1}{dd_1^a}) \tag{B.92}$$

Simplifying (B.92) to get an expression for $\frac{dp_1}{dd_1^a}$ and $\frac{dr_2}{dd_1^a}$:

$$[\frac{\phi^{Aa}p_1 u''(\tilde{c}_1^l)}{(1+r_2)^2} - \beta^l u'(\tilde{c}_2^l)]\frac{dr_2}{dd_1^a} = [\phi^{Aa}\beta^l(1+r_2)u''(\tilde{c}_2^l) + \frac{\phi^{Aa}u''(\tilde{c}_1^l)}{1+r_2}]\frac{dp_1}{dd_1^a} \tag{B.93}$$

according to which we can write $\frac{dr_2}{dd_1^a} = Q\frac{dp_1}{dd_1^a}$ where $Q = \frac{\phi^{Aa}\beta^l(1+r_2)u''(\tilde{c}_2^l) + \frac{\phi^{Aa}u''(\tilde{c}_1^l)}{1+r_2}}{\frac{\phi^{Aa}p_1u''(\tilde{c}_1^l)}{(1+r_2)^2} - \beta^l u'(\tilde{c}_2^l)} > 0$. Combine the definition of M and (B.91) to get

$$-X\frac{dp_1}{dd_1^a} = (1 - \frac{p_1\frac{dr_2}{dd_1^a}}{(1+r_2)^2})\frac{\phi^{Aa}\beta^a d_2^a u'(\tilde{c}_2^a)u''(\tilde{c}_1^a)}{(u'(\tilde{c}_1^a))^2} \tag{B.94}$$

Since $Q > 0$ and $X > 0$, $\frac{dp_1}{dd_1^a}$ and $\frac{dr_2}{dd_1^a}$ have to be both positive for (B.94) to be satisfied. Thus $1 - \frac{p_1\frac{dr_2}{dd_1^a}}{(1+r_2)^2} > 0$. Combine (B.94) and (B.93) to get:

$$\frac{dp_1}{dd_1^a} = \frac{\phi^{Aa}\beta^a d_2^a u'(\tilde{c}_2^a)u''(\tilde{c}_1^a)}{p_1Q\phi^{Aa}\beta^a d_2^a u'(\tilde{c}_2^a)u''(\tilde{c}_1^a) - (1+r_2)\phi^{Aa}\beta^a d_2^a u'(\tilde{c}_2^a)u''(\tilde{c}_1^a) - \phi^{Aa}\beta^a d_2^a u''(\tilde{c}_2^a)u'(\tilde{c}_1^a) - (u'(\tilde{c}_1^a))^2} \tag{B.95}$$

To see how welfare changes, note that $u'(\tilde{c}_1^a) > \beta^a(1+r_2)u'(\tilde{c}_2^a)$ and $1 - \frac{p_1\frac{dr_2}{dd_1^a}}{(1+r_2)^2} > 0$. ABC not clear. A marginal increase in d_1^a when there is an aggregate demand shortage.

$$\frac{de_1}{dd_1^a} = -\frac{\phi^{Aa}\beta^a d_2^a u'(\tilde{c}_2^a)u''(\tilde{c}_1^a)}{(u'(\tilde{c}_1^a))^2[1 + \frac{\phi^{Aa}\beta^a d_2^a}{(u'(\tilde{c}_1^a))^2}(u'(\tilde{c}_1^a)u''(\tilde{c}_2^a) + (1+\frac{1}{\alpha^l})u''(\tilde{c}_1^a)u'(\tilde{c}_2^a))]} > 0 \tag{B.96}$$

$$\frac{dr_2}{dd_1^a} = 0 \tag{B.97}$$

$$\frac{dp_1}{dd_1^a} = -\frac{(1-v'(e_1))u'(\tilde{c}_2^a)u''(\tilde{c}_1^a)}{(u'(\tilde{c}_1^a))^2[1 + \frac{\phi^{Aa}\beta^a d_2^a}{(u'(\tilde{c}_1^a))^2}(u'(\tilde{c}_1^a)u''(\tilde{c}_2^a) + (1+\frac{1}{\alpha^l})u''(\tilde{c}_1^a)u'(\tilde{c}_2^a))]} > 0 \tag{B.98}$$

$$\frac{\partial V^a}{\partial d_1^a} = [(1-v'(e_1))\frac{de_1}{dd_1^a} + 1 + \phi^{Aa}\frac{dp_1}{dd_1^a}]u'(\tilde{c}_1^a) - \beta^a\phi^{Aa}\frac{dp_1}{dd_1^a}u'(\tilde{c}_2^a) > 0 \tag{B.99}$$

$$\frac{\partial V^l}{\partial d_1^a} = [(1-v'(e_1))\frac{de_1}{dd_1^a} - \phi^{Aa}\frac{dp_1}{dd_1^a}]u'(\tilde{c}_1^l) + \beta^l\phi^{Aa}\frac{dp_1}{dd_1^a}u'(\tilde{c}_2^l) > 0 \tag{B.100}$$

Take the partial derivative with d_1^a to the asset pricing equation and the aggregate demand equation:

$$X\frac{dp_1}{dt_1^l} = -Z\frac{dr_2}{dt_1^l} - \frac{Z}{1-v'(e_1)} \tag{B.101}$$

$$Y\frac{dr_2}{dt_1^l} = \phi^{Aa}\frac{dp_1}{dt_1^l} \tag{B.102}$$

Combine (B.101) and (B.102) to obtain:

$$\frac{dp_1}{dt_1^l} = -\frac{YZ}{(1-v'(e_1))(XY+\phi^{Aa}Z)}$$

$$\frac{de_1}{dt_1^l} = -\frac{\phi^{Aa}Z}{(1-v'(e_1))(XY+\phi^{Aa}Z)}$$

Again, with the restrictions on the slope of the asset equation and the aggregate demand equation that $\frac{de_1^{AP}}{dp_1} > \frac{de_1^{AD}}{dp_1}$, $X + \frac{\phi^{Aa}Z}{Y} > 0$ and $\frac{dp_1}{dt_1^l} > 0$ and $\frac{dp_1}{dt_1^l} > 0$.

Appendix C

Tables and Figures for Chapter 3

C.1 Tables

Advanced Economies		Emerging Market Economies				
Australia	New Zealand	Argentina	Colombia	Mexico	South Africa	Vietnam
Canada	Norway	Brazil	India	Peru	Thailand	
Iceland	South Korea	Chile	Indonesia	Philippines	Turkey	
Japan	Switzerland	China	Malaysia	Russia	Ukraine	

Table C.1: Countries in the sample by country group

Country Group	Sector	1% depreciation of usd		1bp increase in yield differential	
		short horizon	long horizon	short horizon	long horizon
AEs	FC	- - -	+ +	- - -	- - -
AEs	NFC	- - -	+	- - -	+ - -
EMEs	FC	- - -	+	+ + +	- - -
EMEs	NFC	- - -	+ + +	+ + +	-

Table C.2: Response of dollar debt to shocks.

Country Group	Sector	1% depreciation of usd		1bp increase in yield differential	
		short horizon	long horizon	short horizon	long horizon
AEs	FC	- -	+ + +	+ + +	++
AEs	NFC	+ + +	-	+ + +	+
EMEs	FC	- -	+ + +	+ +	+
EMEs	NFC	- -	+	+ + +	- - -

Table C.3: Responses of euro debt to shocks

C.2 Figures

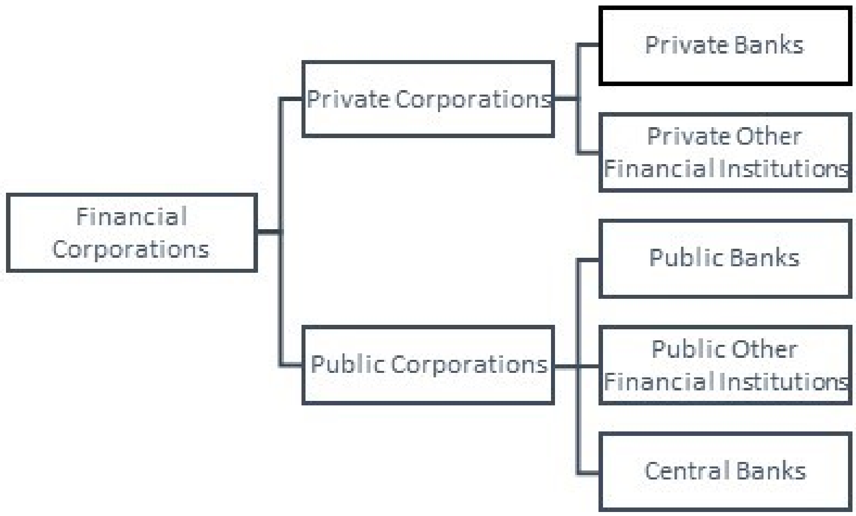
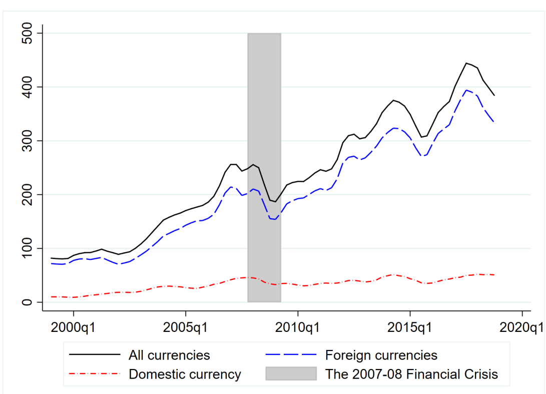
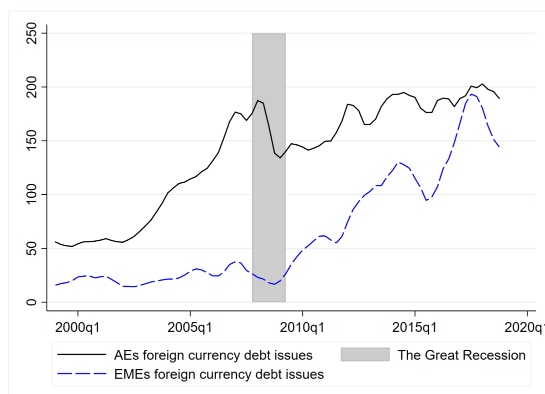


Figure C.1: Composition of the financial corporations sector.

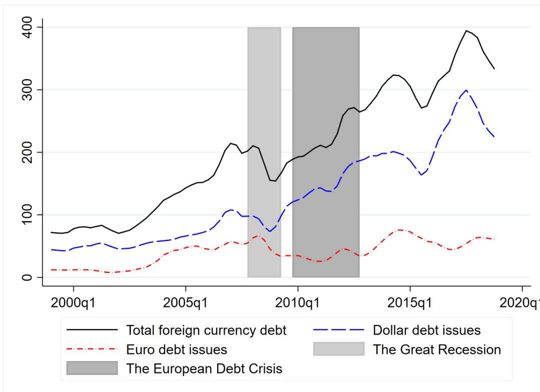


(a) Debt issues in domestic currency and foreign currencies.

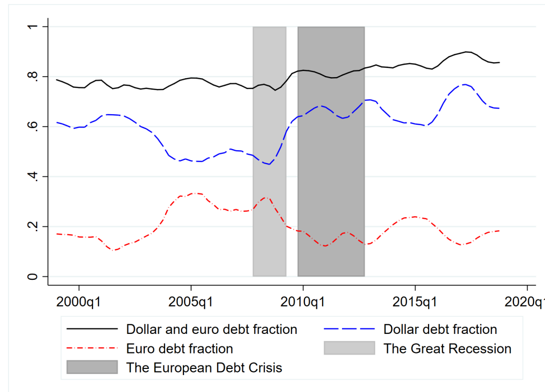


(b) Foreign currency debt issues by AEs and EMEs.

Figure C.2: Debt issues in international markets, in billions of dollars.

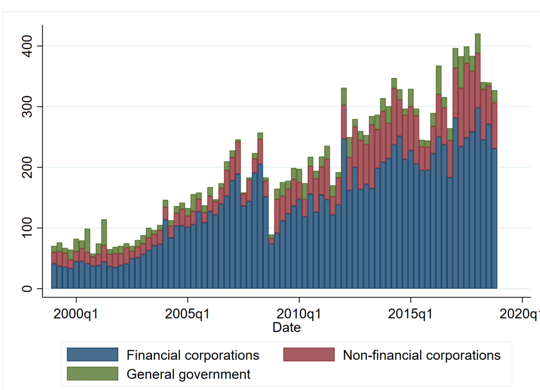


(a) Gross issues, in billions of dollars.

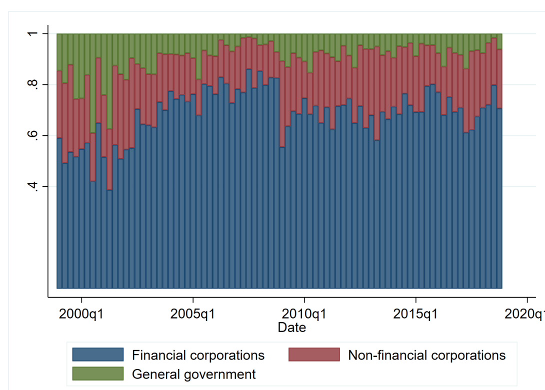


(b) Dollar and euro debt issues as a fraction of total foreign currency debt.

Figure C.3: Currency decomposition of foreign currency debt issues in all sectors.

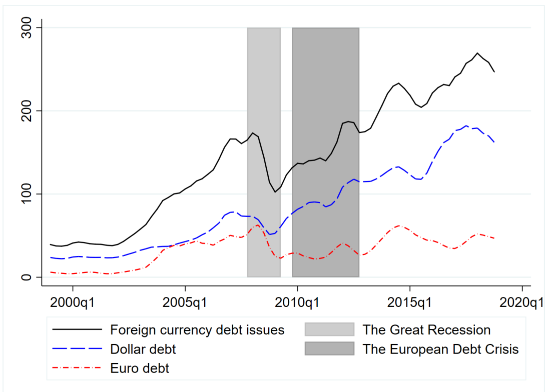


(a) Gross issues, in billions of dollars.

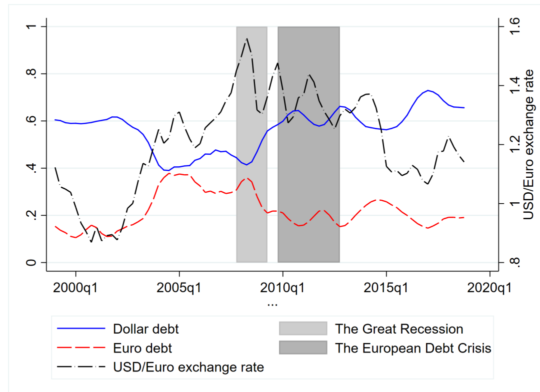


(b) Sector decomposition, in fractions.

Figure C.4: Foreign currency debt issues by sectors.

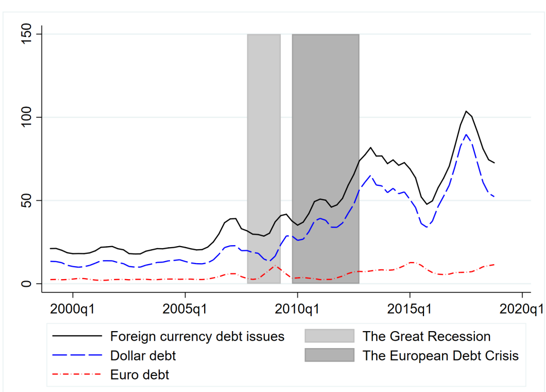


(a) Gross issues, in billions of dollars.

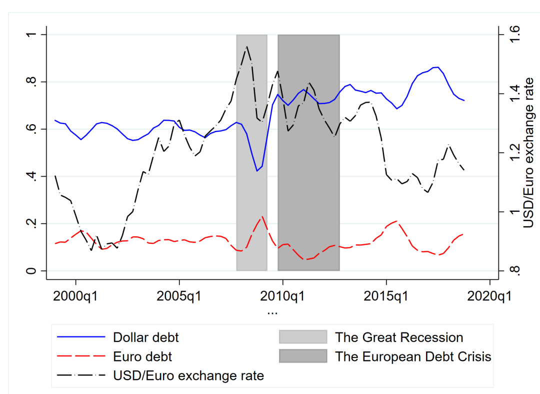


(b) Dollar and euro debt issues as a fraction of total foreign currency debt.

Figure C.5: Currency decomposition of foreign debt issues in the financial corporation sector.

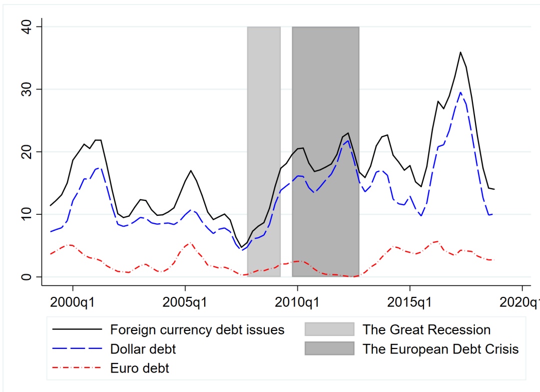


(a) Gross issues, in billions of dollars.

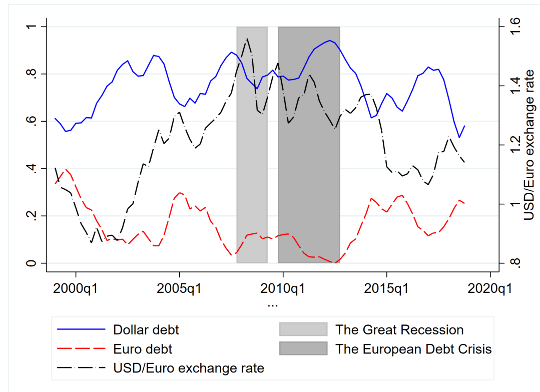


(b) Dollar and euro debt issues as a fraction of total foreign currency debt.

Figure C.6: Currency decomposition of foreign debt issues in the non-financial corporation sector.

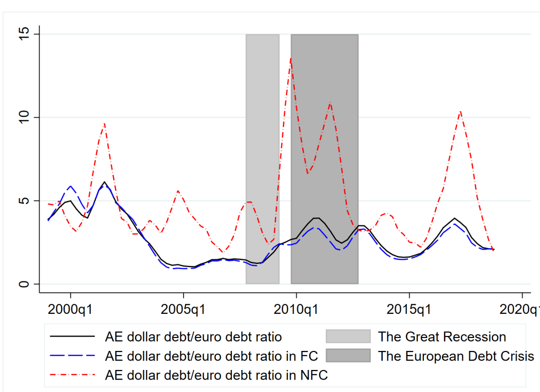


(a) Gross issues, in billions of dollars.

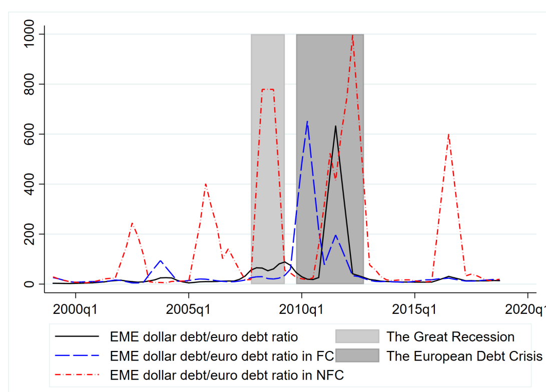


(b) Dollar and euro debt issues as a fraction of total foreign currency debt.

Figure C.7: Currency decomposition of foreign debt issues in the general government sector.

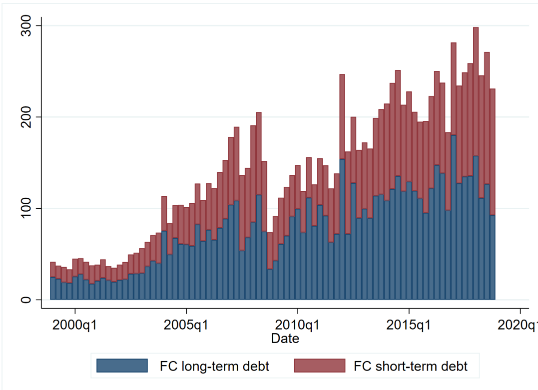


(a) Dollar and euro debt ratio in Advanced Economies.

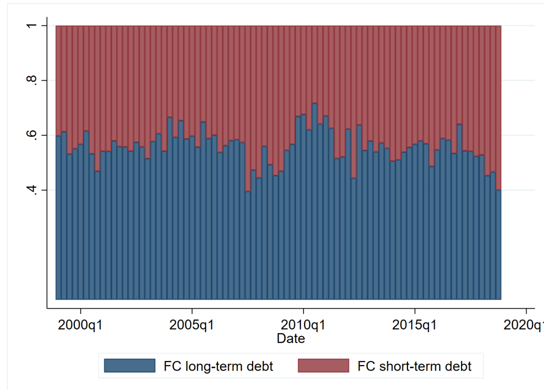


(b) Dollar and euro debt ratio in Emerging Market Economies.

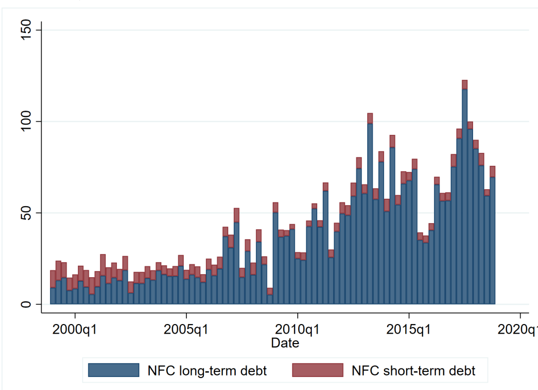
Figure C.8: Dollar and euro debt ratio by country group.



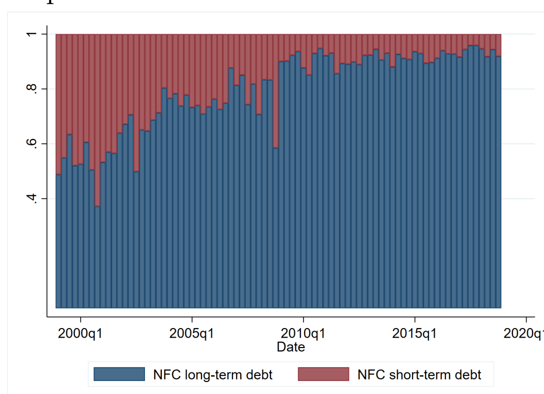
(a) Gross issues in the financial corporation sector, in billions of dollars.



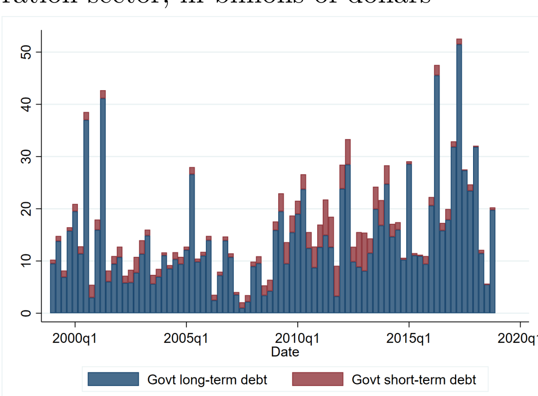
(b) fractions by maturities in the financial corporation sector.



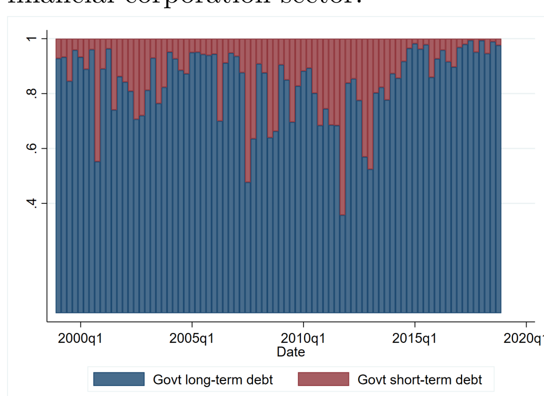
(c) Gross issues in the non-financial corporation sector, in billions of dollars



(d) fractions by maturities in the non-financial corporation sector.

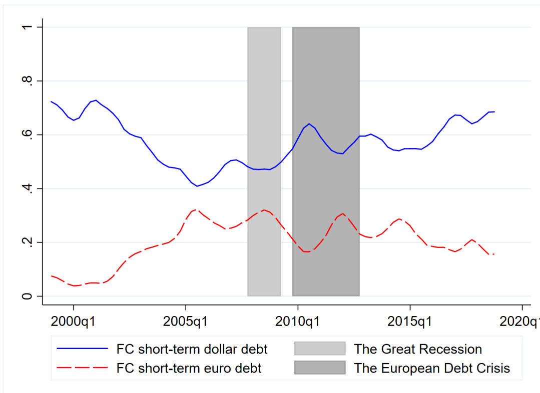


(e) Gross issues in the general government sector, in billions of dollars

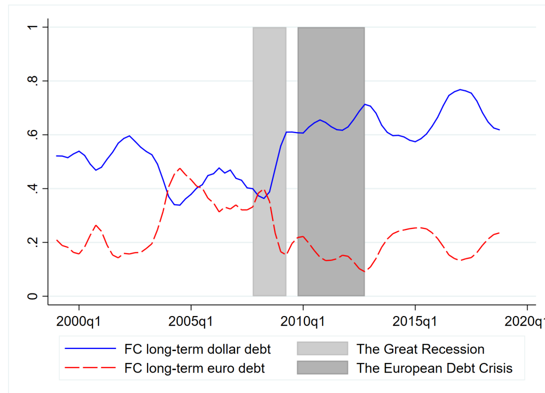


(f) fractions by maturities in the general government sector.

Figure C.9: Foreign currency debt issues in the financial, non-financial corporation and general government sector by maturities.

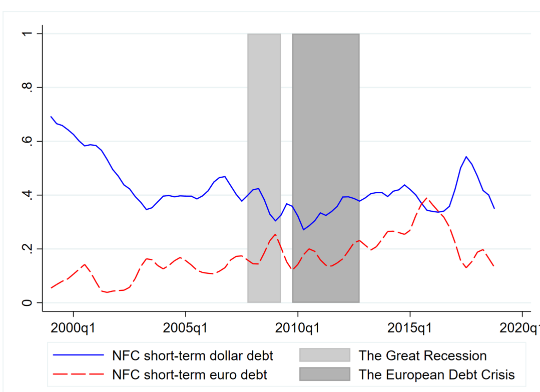


(a) Currency decomposition of short-term debt in the financial corporation sector.

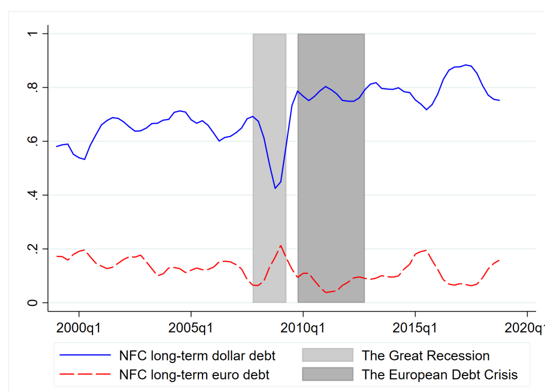


(b) Currency decomposition of long-term debt in the financial corporation sector.

Figure C.10: Currency decomposition of foreign currency debt issues by maturity in the financial corporation sector.

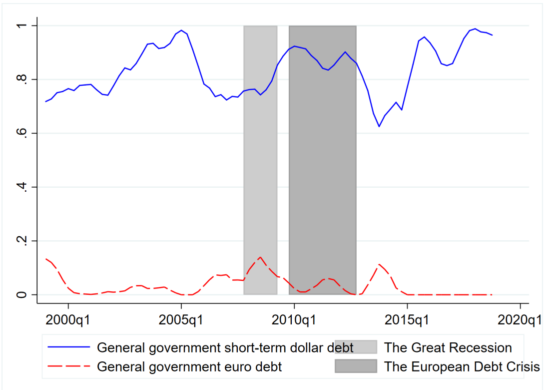


(a) Currency decomposition of short-term debt in the non-financial corporation sector.

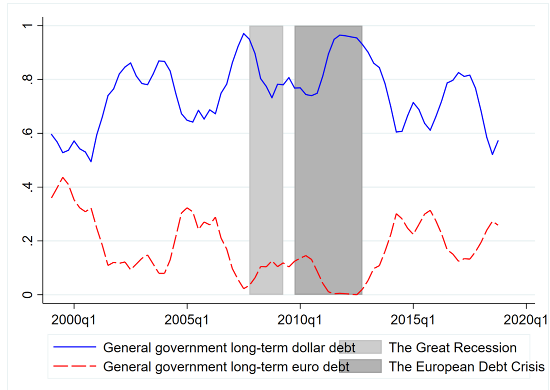


(b) Currency decomposition of long-term debt in the non-financial corporation sector.

Figure C.11: Currency decomposition of foreign currency debt issues by maturity in the non-financial corporation sector.



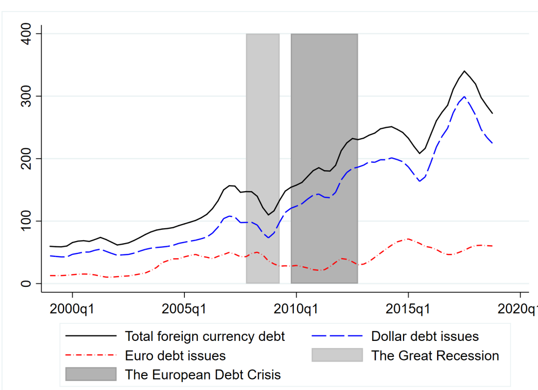
(a) Currency decomposition of short-term debt in the general government sector.



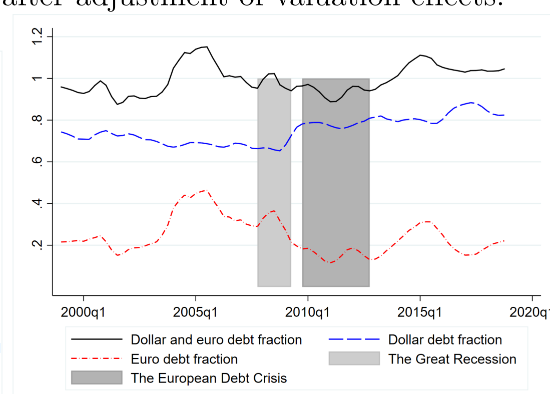
(b) Currency decomposition of long-term debt in the general government sector.

Figure C.12: Currency decomposition of foreign currency debt issues by maturity in the general government sector.

Note: Figure C.13-C.15 are plotted after adjustment of valuation effects.

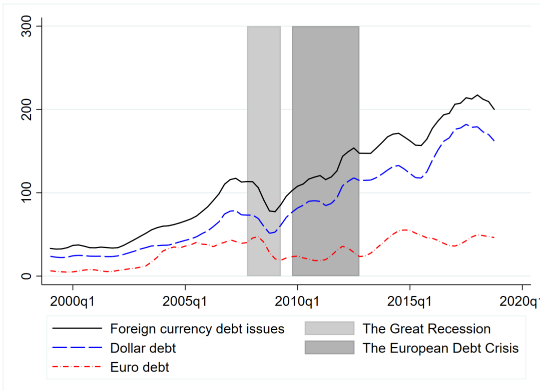


(a) Gross issues, in billions of dollars.

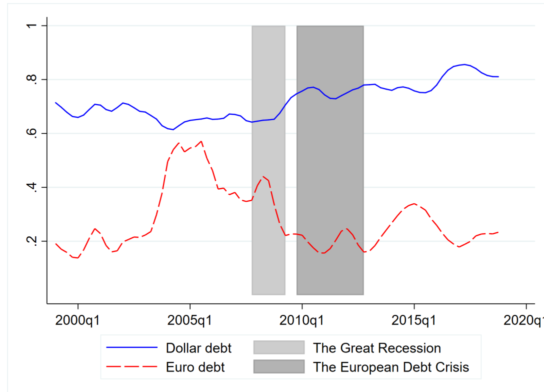


(b) Dollar and euro debt issues as a fraction of total foreign currency debt.

Figure C.13: Currency decomposition of foreign currency debt issues in all sectors.

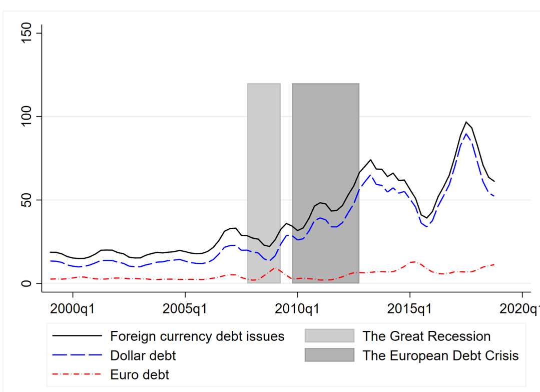


(a) Gross issues, in billions of dollars.

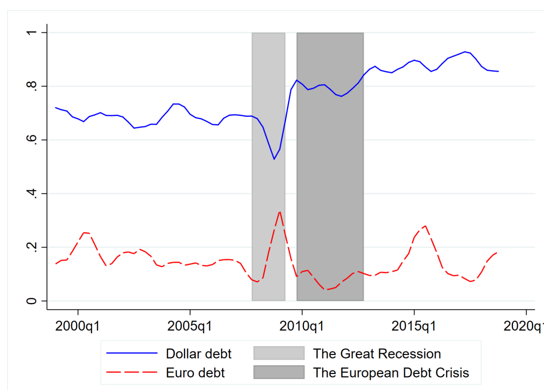


(b) Dollar and euro debt issues as a fraction of total foreign currency debt.

Figure C.14: Currency decomposition of foreign debt issues in the financial corporation sector.

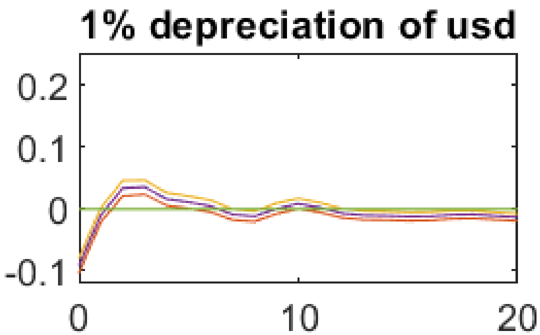


(a) Gross issues, in billions of dollars.

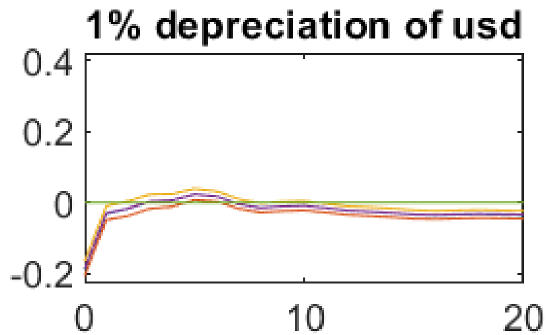


(b) Dollar and euro debt issues as a fraction of total foreign currency debt.

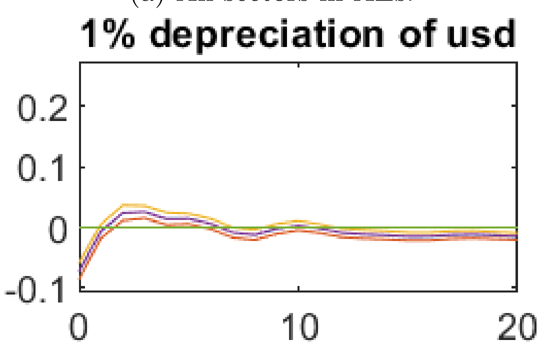
Figure C.15: Currency decomposition of foreign debt issues in the non-financial corporation sector.



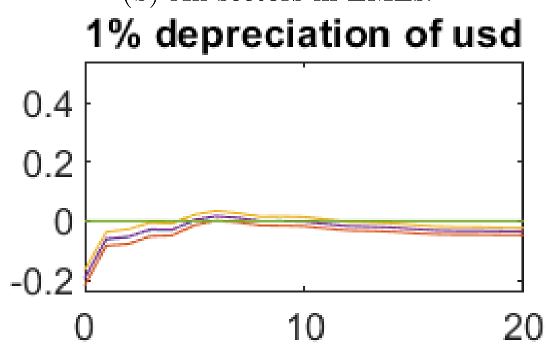
(a) All sectors in AEs.



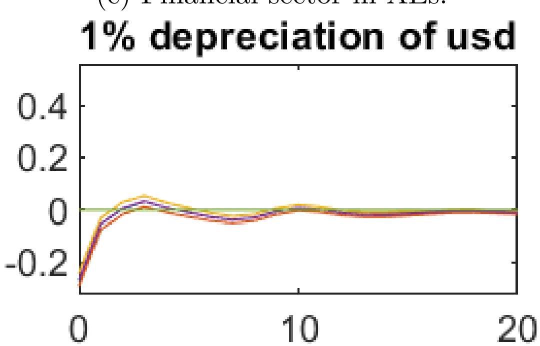
(b) All sectors in EMEs.



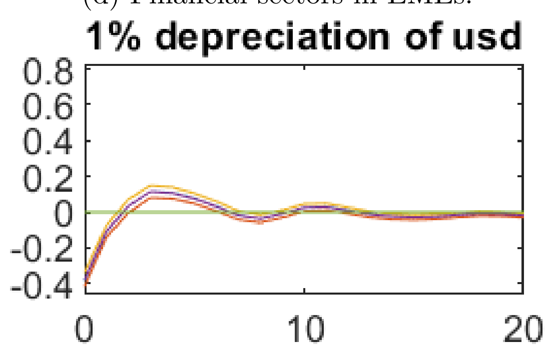
(c) Financial sector in AEs.



(d) Financial sectors in EMEs.

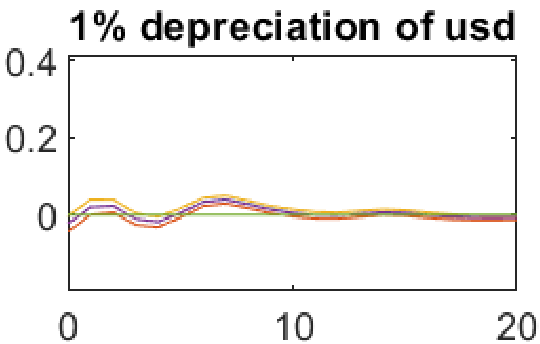


(e) Non-financial sector in AEs.

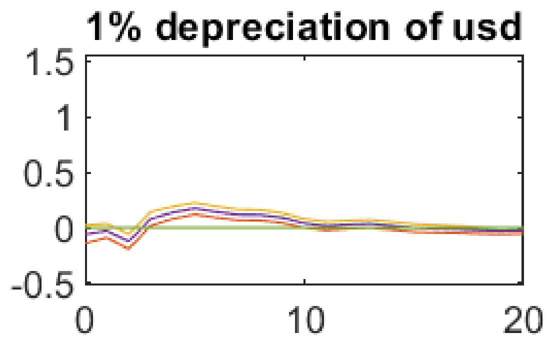


(f) Non-financial sectors in EMEs.

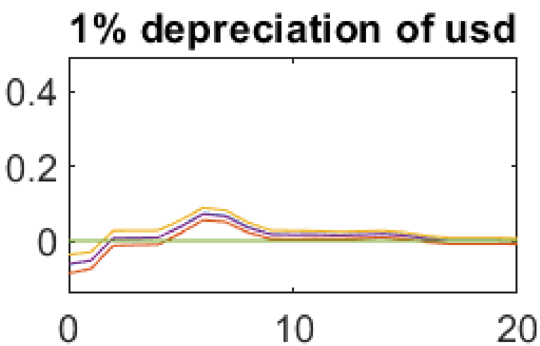
Figure C.16: Effect of a shock on the expected depreciation of the dollar on dollar debt issuance.



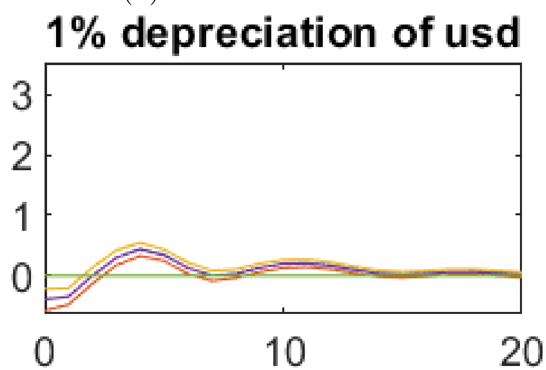
(a) All sectors in AEs.



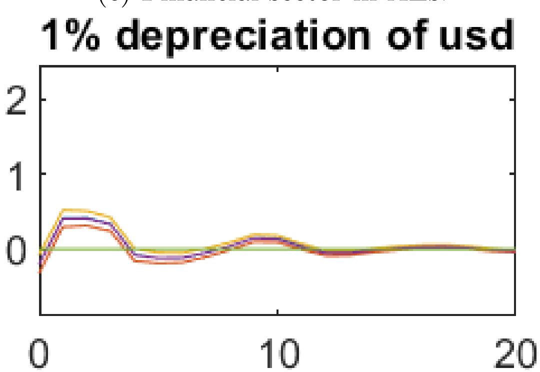
(b) All sectors in EMEs.



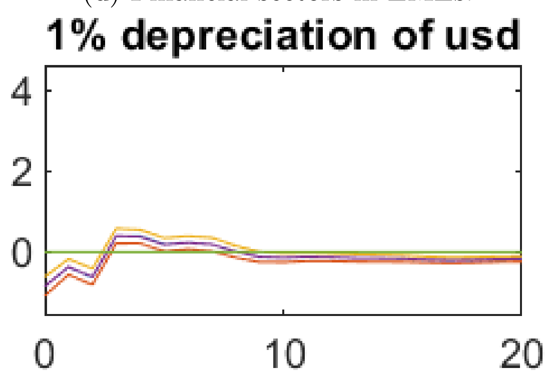
(c) Financial sector in AEs.



(d) Financial sectors in EMEs.

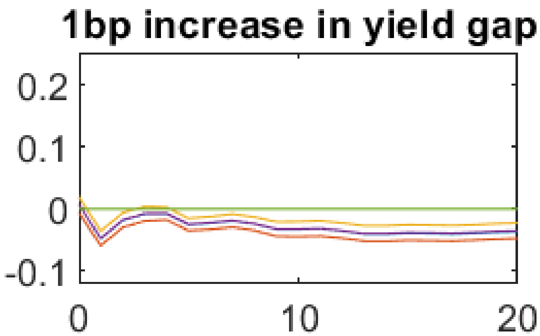


(e) Non-financial sector in AEs.

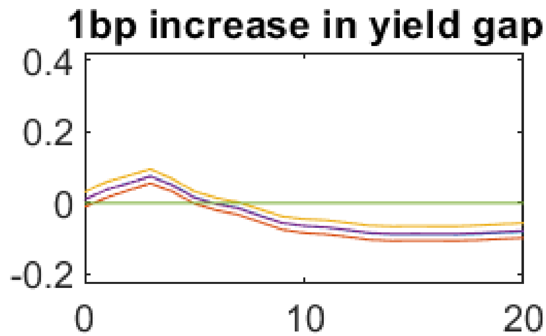


(f) Non-financial sectors in EMEs.

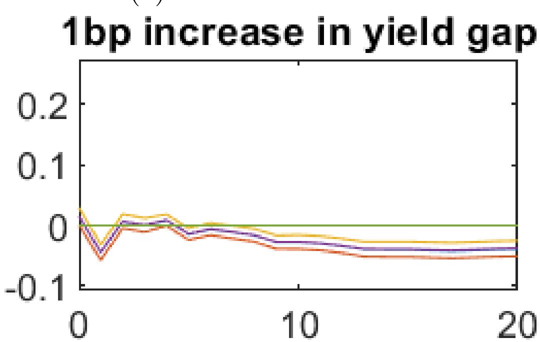
Figure C.17: Effect of a shock on the expected depreciation of the dollar on euro debt issuance.



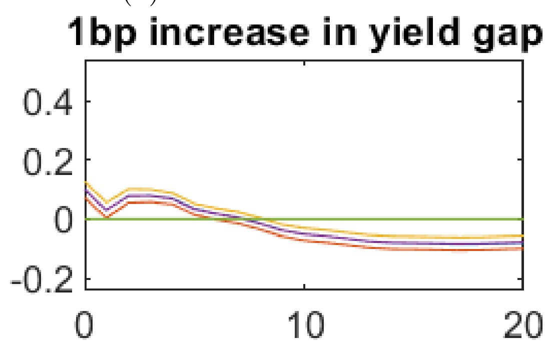
(a) All sectors in AEs.



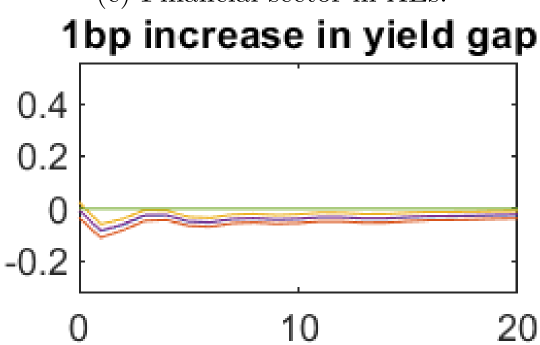
(b) All sectors in EMEs.



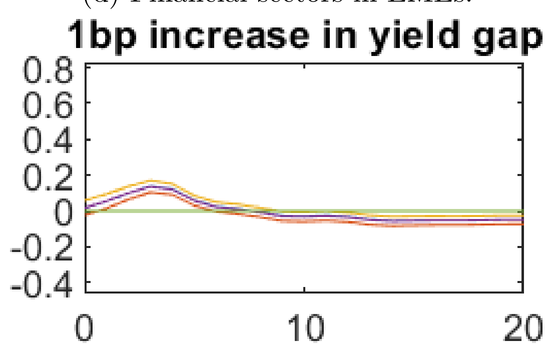
(c) Financial sector in AEs.



(d) Financial sectors in EMEs.

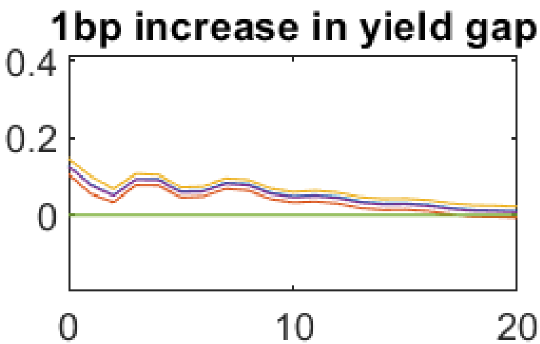


(e) Non-financial sector in AEs.

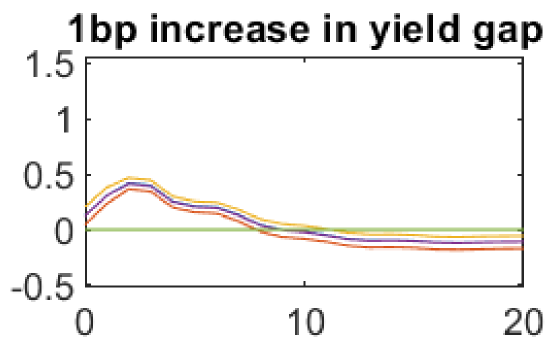


(f) Non-financial sectors in EMEs.

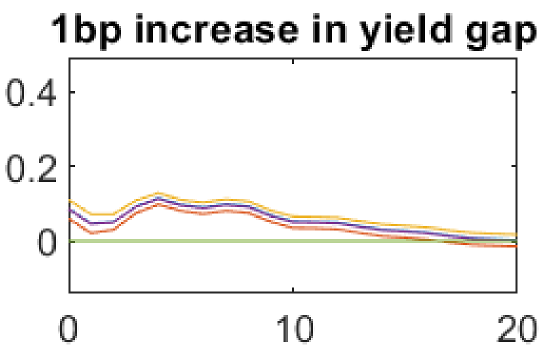
Figure C.18: Effect of a shock on the yield differential on dollar debt issuance.



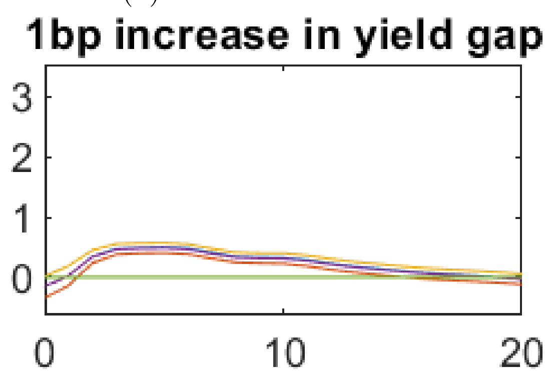
(a) All sectors in AEs.



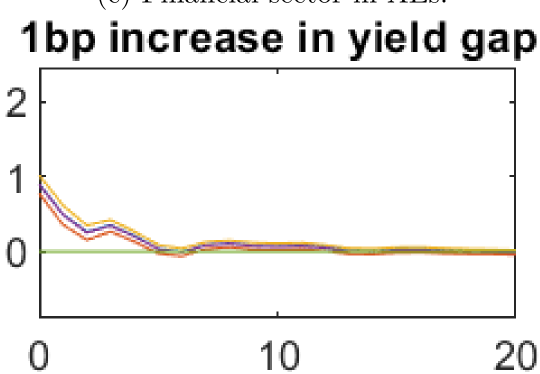
(b) All sectors in EMEs.



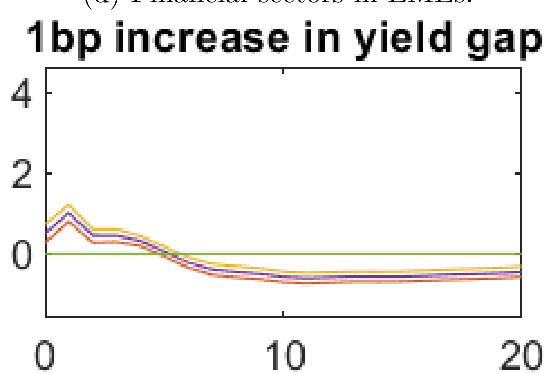
(c) Financial sector in AEs.



(d) Financial sectors in EMEs.



(e) Non-financial sector in AEs.



(f) Non-financial sectors in EMEs.

Figure C.19: Effect of a shock on the yield differential on euro debt issuance.

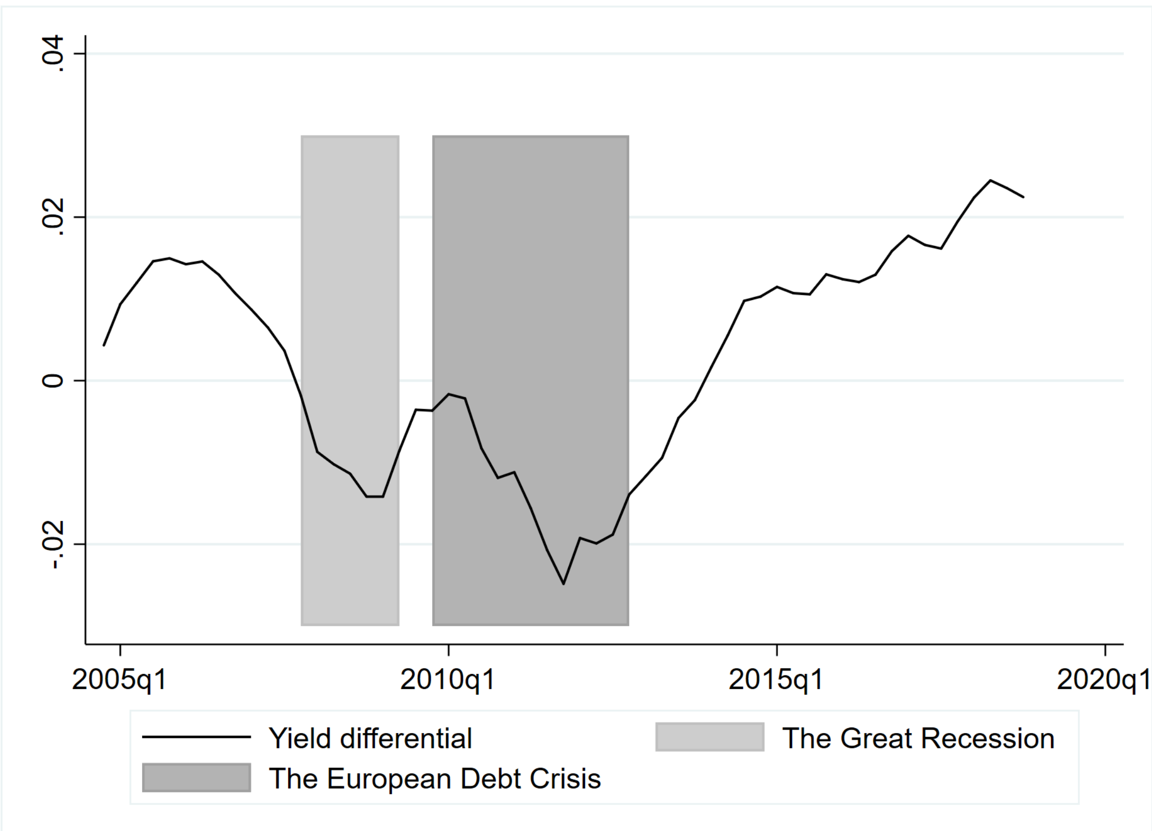


Figure C.20: Yield differential between the 5-year Treasury and 5-year government bond in the Eurozone.