RIVER MEANDER MORPHOMETRICS

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A Thesis Presented to the Graduate
Faculty of the University of Virginia
in Candidacy for the Degree of
Master of Science

Department of Environmental Sciences

University of Virginia

January 1986

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DEDICATION

This thesis is dedicated to my parents,
Paul and Priscilla Hemberger

ACKNOWLEDGEMENTS

It is truly humbling to consider what would be a complete list of acknowledgements. The following is my best effort at what must necessarily be a compromise toward the goal of giving credit where credit is due. Order of appearance is arbitrary, except for the first and last entries.

I must begin by acknowledging three essential players without whose guidance the research effort would have been impossible. Bruce Hayden and Mark Hawley each generously shared his expertise and my concern for the project. I am deeply grateful for their uncompromising willingness to contribute to my personal and academic success. For gracious understanding and encouragement, at times in the face of apparent analytical naivety, I am indebted to my major advisor, Alan Howard. Given the success and overall significance of this study, words of thanks (which could only be understated) are perhaps better replaced with unconditional praise for his intellectual contribution to both my research, and to the discipline of fluvial geomorphology.

For both moral and financial support, and especially for their unshakeable faith in this and all my endeavors, I am profoundly grateful to my parents, Paul and Priscilla. The benefits derived from their inspiration and personal

concern can never begin to be repaid, except in a symbolic sense. To this end, each minute spent on the project, and hence, this document, is dedicated to them.

I owe Eddie Dean (night crew supervisor) for more than this expression of thanks for the many cigaretts and late-night hours shared in Clark Hall. He and his crew, for weeks on end, were virtually my only personal contact within the Department.

Brenda Morris typed the manuscript, and contributed enormously to the successful completion of the final phases of my work; her good nature and eagerness to help made my last weeks at the University more pleasant than I would have thought possible. (And thank you Sam Gregory for use of your couch during these same weeks.)

Finally, to Kris Macoskey, whose dedication to and discipline for hard work has influenced me greatly. I am reminded of the line from the Siegfried Sassoon poem "The Wisdom of the World", where we are told that "there is no other wisdom but to gulp what time can give". That this thirst may never be quenched, I continue to look to Kris as a model for personal integrity and achievement.

ABSTRACT

A procedure for sampling and measuring meander geometry is presented. The sampling procedure, developed using a mathematically defined meander ensemble with known statistical properties, minimizes operator bias and sampling noise inherent in the digital data collection process. The suite of twenty-eight morphometric variables describes local, intermediate-scale, and ensemble features of meander geometry. Many of these variables are introduced for the first time in this paper. Sample length (expressed in terms of the number of meander loops), as well as the data collection process, influences statistical results to a greater extent than has been previously acknowledged. Samples that include fewer than 50 whole-loops (a whole-loop is the channel path between alternate inflection points) are shown to yield unreliable parameter estimates. A data base of thirty-three rivers from the United States representing a wide range of meander forms is developed to examine the variability of the morphometric variables.

Frequency of meandering of each sample river is examined using spectral analysis of the curvature series. The results of the spectral analysis suggests that there are four general classes of meandering based on the number of dominant peaks in the curvature spectrum. The classes represent unimodal, bimodal, trimodal, and flat/multimodal meander patterns. Dimensionality of meander geometry is

described using principal components analysis. The first five principal components (dimensions) account for 90.8% of the total variance. These components are, in order of prominence, related to half-loop path length, half-loop sinuosity, loop asymmetry, large-scale wandering of the channel, and the spread of the dominant spectral peak.

The sampling procedure and descriptive variables can be used to study fluvial process-response mechanisms using correlations between environmental controls and channel form. Theoretical models of meandering can be tested by comparing statistics for simulated patterns to those derived from the data base of natural rivers.

Because width-normalized variables dominate the first five principal components, it appears that other scaling indices should be developed. Self-scaling using a more characteristic measure of the channel pattern, such as average half-loop path length or dominant wavelength, may provide a more reliable index upon which the sample rivers may be normalized. Tests of sample stationarity must also be developed and applied to meander geometry data.

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I. INTRODUCTION

This research focuses upon three related aspects of fluvial geomorphology. A suite of morphometric parameters is developed which measure both local and ensemble properties of meander geometry. The new variables quantify many aspects of meander shape that have until now been neglected. Systematic approaches to digitizing meander patterns (represented by the centerline of the channel) are devised which enhance data accuracy. The sampling methodology is evaluated and calibrated using a mathematically defined meander ensemble with known statistical properties. Finally, the defined statistics are used to examine the statistical properties of a broad range of natural meander forms using a sample of thirty-three rivers from the United States.

A. Rationale for Meander Studies

Theoretical studies of meander geometry employ analytical functions which mathematically emulate meander patterns. These analytic approaches to studies of channel geometry are not linked to physical meander processes and cannot describe meander evolution. Analytic models are instead applied to fluvial investigations in a descriptive or qualitative mode.

Other theoretical studies focus on river hydraulics and deterministically simulate the processes which drive fluvial systems. Meander development and migration are the central

topics of these investigations. Such research is more mathematically sophisticated than the relatively straightforward analytical studies.

Quantitative measurements of planform properties are needed to calibrate simulation or analytical models, and to evaluate their ability to duplicate natural meander patterns.

Practical reasons for studying channel processes and meander development include land management, environmental hazards, engineering applications and channel maintenance. Knowledge of river hydrodynamics, and its influence on channel morphology, is necessary for dealing with aspects of fluvial mechanics that affect land resources and in engineering structures. Statistical characterization of meander geometry should prove useful in the future for prediction of meander behavior.

B. Scope

If models of meander development are needed to explain river dynamics, then procedures are required to sample and numerically evaluate the resemblance of the simulated patterns to those observed in nature. The purpose of this thesis is to develop such procedures. Much effort is also devoted to developing a data base of natural meandering. Thirty-three rivers have been digitized to evaluate the statistical nature of the descriptive parameters. The variable set was developed in part as an outcome of the

statistical tests; additional parameters may be examined when other aspects of meander geometry are revealed as more streams are included in the data base.

1. Sampling Procedure Refinement

Quantitative descriptions of meander geometry have been hampered by subjective possibly biased sampling methods. In an effort to overcome these problems, a digital sampling procedure was developed to take advantage of electronic data acquisition and analysis techniques. The refined methodology represents an improvement over previous approaches because optimal procedures were determined for each step in the sampling process. The procedures to be optimized include sampling density of raw digitized data, number of replications to reduce sample error, and the analytical interval and degree of smoothing of the raw data used in the morphometric analysis. A mathematically defined meander generating function with known statistical properties was used as a control to test the accuracy and precision of the sampling procedures. In addition to providing centerline data having a high signal to noise ratio, and low bias, the optimization tests were used to establish minimum sample length requirements.

2. Parameterization of Meander Properties

Investigations of meander geometry have traditionally relied upon a small number of descriptive variables for quantifying channel centerline patterns. Characterization of meander forms has thus been relatively crude because little attention has been paid to identifying and quantifying subtle features of meander patterns. Recently, emphasis has been placed upon ensemble characteristics because the first-order meander models being tested define only rudimentary features of the overall meander pattern. Consequently, little progress has been made toward developing a comprehensive set of descriptive parameters of meandering.

Twenty-two variables are introduced in this study that measure both local and ensemble characteristics of river planform. Measures of curvature, loop asymmetry, and half-loop summary statistics, as well as sinuosity variables (measured at several scales) are included within the variable set. The variables are reduced using principal components analysis to determine the essential dimensions of meander morphology.

To summarize, the scope of this study is to:

- 1) define a wide range of variables to quantify meander properties
- 2) develop a consistent, objective and unbiased sampling procedure

- 3) develop a data base of natural meandering streams to evaluate the variation of the descriptive parameters for comparison with simulated meander patterns.

The topics addressed in the remainder of this paper are as follows. The variables are defined quantitatively and discussed qualitatively in the Review of Morphological Variables section; sample selection and application of the data collection procedure are outlined in the first part of that chapter. The influence of the data collection process upon the statistical results is discussed in the Sensitivity Analysis section. Finally, the results of the statistical analyses (spectral analysis and principal components analysis) are presented in the Discussion of Results. Relevance of the work, applications to other fluvial geomorphic studies and research needs are given in the Conclusion.

II. REVIEW OF MORPHOLOGICAL VARIABLES

This section presents a summary of the morphological variables that have been used in previous studies of meandering. The aim of the discussion is to 1) describe the significance of each variable, and 2) evaluate relative strengths and limitations both morphometrically and computationally. Because parameter selection depends to a large extent upon the hypothesis being tested, models of channel planform are reviewed before evaluating the utility of the descriptive parameters. The descriptors used in the present study are defined at the end of this section.

A. The Models

Models of meander shape may be grouped according to the mechanisms that are assumed to give rise to their shape. At one extreme are models based on the notion that meander shape is regular and, as such, may be described deterministically. According to this view (favored by early researchers), the basic planform shape is a regular curve such as a sinusoid or sine-generated curve. Several researchers (Dury, 1964; Leopold and Wolman, 1960; Leopold, et al., 1960; Carlson, 1965) have defined procedures for fitting such regular curves to specific meander loops.

The irregularity of form observed in most natural streams, however, indicates that channel shape is poorly described as an exact periodic function. This observation

led to distinctions between regular and irregular meandering (Schumm, 1963). Speight (1965, 1967) examined the spectral characteristics of Australian rivers and demonstrated quantitatively that more than a single modal wavelength may exist along a reach. The pioneering work of Speight suggested that meander wavelength should be estimated statistically to account for the presence of polymodal meander frequency.

At the other extreme are researchers who argue that meander patterns are generated by random direction changes. These authors maintain that a channel planform is simply a type of confined random-walk between two points along the river valley (Langbein and Leopold, 1966; Scheidegger, 1967; Thakur and Scheidegger, 1968 and 1970; Surkan and Van Kan, 1969). Langbein and Leopold's (1966) model is based upon von Schelling's model of most frequent random-walks, and the assumption that the distribution of curvature is Gaussian (von Schelling, 1951). Langbein and Leopold (1966) showed that for a specific pathlength ($\mu\lambda$) and angle of maximum deflection (w), the most probable random path is an arc with minimum variance of curvature. This most probable path is known as the sine-generated curve which may be defined mathematically as follows:

$$w \sin(2\pi/\mu\lambda)s = \theta \quad (1)$$

where: w = angle of maximum deflection
 μ = sinuosity
 λ = wavelength measured along the valley axis
 s = centerline distance

The sine-generated curve can be expressed as a differential equation:

$$-1/k \, d^2\theta/ds^2 = \theta \quad (2)$$

where: $k = 2\pi/\mu\lambda$ is the wavenumber of meandering. Continuous random-walk curves are approximated by discrete series of direction (θ) or direction change ($\Delta\theta$) angles at regular intervals of unit length, Δs . A stochastic forcing function is used to define the dependence of θ or $\Delta\theta$ on preceding parameter values.

The original model of Langbein and Leopold assumed independent random direction changes from segment to segment. Surkan and Van Kan (1969), found that a better approximation to natural meanders occurs when the direction changes are based on correlated random-walk deviations chosen from populations with direction biased standard deviations.

The random-walk model has been criticized as conflicting with experimental evidence of Schumm and Kahn (1972) that regular meanders can form under favorable conditions. The central problem with the random-walk

model, however, is that its probability density function is such that the likelihood of generating a path resembling natural meanders, such as a sine-generated curve, is near zero (Ferguson, 1976).

Ferguson believes that the key to meander shape is with the regularity of the fluvial environment (Ferguson, 1975, 1976). Ferguson treats the process of meandering as deterministic, but suggests that local controls vary randomly from place to place. It is assumed that the disturbance sequence is a realization of a stationary stochastic process. The model implies that while the overall nature of the ensemble does not vary, local direction depends on the detailed structure of the disturbances (noise).

The Ferguson model is based on the sine-generated curve. It includes a wavenumber parameter, (k) as before. The shape parameter, (h), however, is related to damping of individual disturbances (Equation 3).

$$\theta + \frac{2h}{k} \frac{d\theta}{ds} + \frac{1}{k^2} \frac{d^2\theta}{ds^2} = \epsilon \quad (3)$$

The first derivative component of Equation 3 accounts for downstream damping of both the random and periodic forcing, and ϵ is the random forcing assumed to be normal Gaussian noise (Ferguson, 1976). Equation 3 reconciles the random and deterministic approaches to meander geometry by allowing

for the effect of varying degree of random irregularity superimposed on a deterministic description of the process.

Note that meander shape is portrayed analytically for each of the above models; that is, theoretical functions are used to generate single realizations of probable meander paths. Thus, the models do not depict evolutionary changes in loop development. Except for the implication that they induce deviations from regularity, the physical laws that govern fluvial systems are ignored. Understanding the link between form and process is essential for characterizing natural meandering suggesting that the above models are inadequate for detailed studies of meander geometry.

To examine the process-response link, Howard and Knutson (1984) favor a simulation rather than a theoretical approach for the study of meander patterns. The model uses a set of differential equations that are based upon Ikeda and Parker's (1981) model of fluid flow and bank shear stresses to simulate bank erosion along the channel. The erosion rate laws are described in terms of local and upstream curvature at discrete spatial and temporal increments. The digital simulation model duplicates the following features of natural meandering that the previously discussed analytical models do not:

- 1) asymmetrical loops
- 2) channel migration rates as non-linear functions of curvature (Hickin and Nanson, 1975)

- 3) neck cutoffs
- 4) temporal changes in meander pattern.

The meander paths produced by this model are fairly realistic but are too regular when compared to natural meanders. The regularity of the meander patterns is due to the deterministic nature of the model which to date lacks an explicit "noise" component. The need to test the validity of this model provided the impetus for the present study.

B. The Variables

There are two ways in which meander patterns have been analyzed to date. The difference amounts to a distinction between local and ensemble attributes.

The first approach involves direct (usually manual) analysis of the reach on a loop-by-loop basis. Values of morphometric properties for a reach are inferred from the estimated frequency distributions of the measured variables. Information about spatial ordering of loops is not preserved in this form of analysis. Early research into stream channel morphometrics often focused upon loop rather than ensemble statistics. Consequently, loop statistics have traditionally been associated with manual data collection (such as the use of dividers and rulers).

Variables measured using the manual approach include sinuosity, radius of curvature, wavelength, and asymmetry. Ideal ("representative") reaches of regular wavelength and

sinuosity are often chosen for study which allows for direct measurement of these parameters. The radius of curvature statistic is derived by fitting circular arcs to natural meander loops (see Leopold and Wolman, 1960; Brice, 1973, 1974 for applications for the techniques).

Subjective interpretations of what constitutes a representative reach and arbitrary approaches to curve fitting may produce inconsistent and biased results. Due to this possible error, curve fitting and direct measurement of morphometric quantities have not been used in this study.

The recent research uses an alternative approach which is based upon more objective digital sampling techniques. This method treats digitized river data as a series of direction (θ) or curvature ($\Delta\theta$) values for successive short channel segments (Figure 1). Spectral analysis of the series yields information about sinuosity, degree of irregularity, and wavelength characteristics of the ensemble. Spectral analysis is described in detail in the Analytical section. Statistic for individual loops may be readily calculated by digital methods, but until this study no effort has been made to do this.

Spatial analysis of meander patterns using methods of time-series analysis have been performed by Speight (1965, 1967), Surkan and Van Kan (1969), Thakur and Scheidegger (1970) Chang and Toebe (1970), Ghosh and Scheidegger (1971), and Ferguson (1975, 1976, 1977b). Spatial analysis

of channel planform involves the transformation of the x, y coordinates of a continuous channel trace (generally multi-values at x) to a single-valued distance function. The one-dimensional spatial series of direction θ , or curvature $\Delta\theta$, is represented as a function of distance (s) along the channel centerline (Figure 1). The channel is treated as a discrete series $\{\theta\}$ of direction associated with successive path segments of constant length, Δs . The curvature (direction change) series is obtained by differencing $\{\theta\}$: $\{\theta_i - \theta_{i+1}\} = \{\Delta\theta_i\}$. Values for unit segment length used in previous investigations range from 0.5 to 2.0 channel widths.

Both the direction spectra and curvature spectra contain equivalent information about the frequency characteristics of the spatial series. Direction spectra have higher ordinate values relative to curvature spectra for a given frequency because of the greater variance of direction. Direction variance may be especially large when there are strong trends in the axis of the river valley. Because the direction series can be influenced more by regional geologic structure than by the meandering process, only the curvature series is examined in this study.

Figure 1. Digital representation of channel centerline, and the direction (θ) and curvature ($\Delta\theta$) series. Dots are irregularly-spaced raw digitized points and circles are interpolated regularly-spaced sample points. (adapted from Ferguson, 1975)

C. Variables Used in This Study

Applications of digital analysis to studies of meander geometry have centered on ensemble attributes of meander shape. Thus, features of local meandering have not been examined using such methods. At the same time, new morphometric parameters have not been developed to take full advantage of modern analytical techniques. For example, the curvature spectra has not been used to define statistical parameters that could be used to compare spatial characteristics of different meandering streams. Commonly used ensemble statistics based on the spectral density function are thus limited by the narrow range of detail they convey about overall meander patterns.

It appears that no previous attempts have been made to assemble a representative suite of statistics from a range of meandering streams; thus relatively small data bases and statistics of narrow overall scope typify most previous studies of channel morphometrics. The present study attempts to integrate both local and ensemble statistics into a unified package to yield a detailed description of meander geometry.

The aim of the following discussion is to describe the variables that have been included in this research. Many of these variables have been developed as part of the present study, and are introduced for the first time in this paper. Quantitative definitions are discussed first. This is

followed by a qualitative description of the significance of each variable. Channel morphometrics are then discussed in general terms by describing the different classes of morphometric variables. Finally, comparing the measures used here are compared to those used by other authors for the same aspect of morphometry.

1. Quantitative Definitions

Many of the statistical measures used in this study describe the shape and size of individual whole-loops and half-loops. A half-loop is a single bend which is defined as the river path between inflection points (points where the curvature changes sign). A whole-loop is composed of two half-loops with curvature of opposite sign (clockwise curvature is positive in this study).

An inflection point occurs when the sum of the curvatures at two adjacent nodes is a different sign from the sum of the curvatures of the previous two nodes (Figure 2). For example, an inflection point would be defined at node C in Figure 2 since $\text{sign}(\theta_1 + \theta_2) \neq \text{sign}(\theta_2 + \theta_3)$. The location and total number of inflection points is strongly affected by the digitizing process. Part of this sampling noise is removed by the smoothing operation as discussed elsewhere.

Figure 2. Definition of inflection points of the curvature series. Node C defines an inflection point since $\text{sign}(\theta_1 + \theta_2) \neq \text{sign}(\theta_2 + \theta_3)$.

a. Ensemble-Averaged Variables

1) Sinuosity

Channel sinuosity has been resolved into three components. Half-loop sinuosity is measured by the path length between adjacent inflection points, h_i , and the straight-line distance between the inflection points, x_i (Figure 3a). A whole-loop is the double half-loop between every other inflection point, and is measured by the straight-line distance, y_j , and the total path length $h_i + h_{i+1}$ (note that the j index is incremented for each two i indices). The straight-line distance between the first and last inflection point in the sample is Z (Figure 3b). The total number of half-loops, n_i , is constrained to be even so that total whole-loops, $n_j = n_i/2$. The three components of sinuosity are:

$$\text{Whole-loop sinuosity, } \mu_w = (\Sigma y_j) / Z$$

$$\text{Half-loop sinuosity, } \mu_h = (\Sigma x_i) / \Sigma y_j$$

$$\text{Residual sinuosity, } \mu_r = (\Sigma h_i) / \Sigma x_i$$

The product of these sinuosities is the total sinuosity: Total sinuosity, $\mu_t = \mu_w * \mu_h * \mu_r = (\Sigma h_i) / Z$

Figure 3a. Components of a half-loop, where x_i is the straight-line distance between adjacent inflection points (also known as the half-loop axis), h_i is the path length of the half-loop, and ϕ_i is the angle between adjacent half-loop axes.

Figure 3b. Idealized meander ensemble showing half-loop (x_i) and whole-loop (y_j) axes (represented by dashed and solid lines, respectively), and the straight-line distance between sample endpoints, z (represented by the dot-dash line).

2) Curvature

Curvature is defined as the angle between successive step intervals divided by step length, $\Delta\theta/\Delta s$. To standardize the samples, this relationship is modified to produce dimensionless quantities as follows. The change of angle at a node is multiplied by average channel width, and the product is divided by the sum of the segment lengths on either side of the node. The absolute value of dimensionless curvature is also examined in this study.

dimensionless curvature,

$$e_s = (\theta * \bar{w}) / (\Delta S_i + \Delta S_{i+1})$$

absolute dimensionless curvature,

$$e_u = |e_s|$$

Local meander trends are described by the orientations of whole-loop and half-loop axes. Whole-loop and half-loop axes are described above as the straight-line distances y_j and x_i . The absolute value of the angle between adjacent axes, ψ_j and θ_i for whole-loop and half-loop orientations, respectively, is multiplied by average channel width, and the quantity is divided by the sum of the lengths of the adjacent axes. Figure 3a shows the case for a half-loop axis,

but the idea is the same for whole-loops. The result is n_{j-1} and n_{i-1} whole and half-loop dimensionless curvature values for the sample.

Whole-loop dimensionless curvature,

$$W_c = (\psi_j * \bar{w}) / (y_j + y_{j+1})$$

Half-loop dimensionless curvature,

$$H_c = (\theta_i * \bar{w}) / (x_i + x_{i+1})$$

The first four moments (average, variance, skewness, and kurtosis) were calculated for all curvature measurements.

3) Half-loop Size

Half-loop size is calculated in terms of both path length (in width units), h_i , and ^{AXIS}~~Q_i~~ length, x_i , as defined above. The first four moments are calculated for half-loop paths and half-loop axes.

4) Power Spectrum

Two ensemble descriptors are derived from the spectral density function (SDF) of the curvature series. The spectral peak, SV_p , is the period, T_{max} (in width units) that accounts for maximum spectral variance of the SDF. An alternative approach would be to use the first high-period peak rather than the largest peak. This approach has not been used, however, because the first high-period peak may account

for a low portion of the total variance. The spectral period is read directly from the associated frequency band; that is, neither interpolation nor quadratic curve fitting about the peak have been used to resolve the period.

The second descriptor, spectral ratio, SV_r , is a measure of the peakedness of the SDF; it is determined by dividing maximum spectral variance, SV_p , by the spectral variance that occurs at half the peak period, $SV_{0.5}$. The two power spectrum variables are defined as follows:

$$\text{spectral peak, } SV_p = T_{\max}$$

$$\text{spectral ratio, } SV_r = SV_p / SV_{0.5p}$$

b. Half-Loop Summary Statistics

In addition to the ensemble averaged variables discussed above, a set of half-loop statistics have been developed to examine features of local meandering.

Half-loop statistics are calculated for individual loops that range in length from $L = 1$ to $L = 30$ (L in width units). To aid interpretation and minimize parameter variance, it is convenient to group the half-loop statistics into fewer than thirty size classes. Using weighted averages, half-loop statistics were summarized for the following length categories:

category 1: L = 1
category 2: L = 2 - 3
category 3: L = 4 - 6
category 4: L = 7 - 10
category 5: L = 11 - 20
category 6: L = 21 - 30

The divisions were selected to ensure approximately equal representation for each size class (except Category 6). Categories 4 and 5 were grouped together to represent median half-loop statistics for the sample and were used for the principal components analysis. Variables unique to local meander shape are defined as follows.

1) Sinuosity

Local half-loop sinuosity is determined by dividing the average half-loop path length for the loops in Categories 4 and 5, H_p , by the average half-loop axis for the same set of loops, H_c .

$$\text{average half-loop path, } H_p = (\sum h_i) / n_i$$

$$\text{average half-loop axis, } H_c = (\sum x_i) / n_i$$

$$\text{average local sinuosity, } \mu_i = H_p / H_c$$

2) Asymmetry

Two indices were developed to characterize loop skewness. The first index is based on the location of

the point of maximum curvature, L_m , along the loop path (Figure 4a). The limbs of the bisected half-loop (L_u and L_d for the upstream and downstream limbs, respectively) are differenced and divided by the path length of the loop, h_i (note that $h_i = L_u + L_d$). Values for this index range from a minimum of -1 for upstream skewed loops to a maximum of +1 for downstream skewed loops. Perfectly symmetrical loops have a value of zero for this index.

The second asymmetry parameter is a function of average rather than maximum curvature of the half-loop. The dimensionless curvature at each node is multiplied by the physical distance along the loop path. These products are summed over the length of the half-loop and the result is divided by total path length, h_i . The resulting value, L_a , represents the location of the average curvature as a function of distance along the half-loop (Figure 4b). The half-loop limbs L_u' and L_d' , determined by L_a , are expressed in the same fashion as before, and the parameter has an identical range of values and implications.

The second asymmetry index is less sensitive to sample noise than the first index because it represents an integrated rather than a single curvature value.

$$\text{asymmetry index 1, } A_m = (L_u - L_d)/h_i$$

$$\text{asymmetry index 2, } A_a = (L_u' - L_d')/h_i$$

Figure 4a. Location of the point of maximum curvature, L_m , along the half-loop path used to calculate asymmetry index 1. The upstream and downstream limbs of the half-loop are denoted by L_u and L_d , respectively. Note that $L_u + L_d = h_i$, the half-loop path length.

Figure 4b. Graphic representation of the location of average curvature, L_a , as a function of distance $L\Delta s_i$ along the half-loop used to calculate asymmetry index 2. The ordinate represents dimensionless curvature at each node. The point of maximum curvature is denoted on the graph by L_m .

3) Curvature

Average absolute curvature, e_a , is calculated by dividing the sum of individual absolute dimensionless curvatures, $|e_{si}|$, by the number of observations, n . The standard deviation of the absolute curvatures is also calculated. Absolute loop angle (in radians), e_h , is derived by multiplying average absolute curvature, e_a , by the length of the loop, L , in width units. Note the difference between ensemble dimensionless curvature, described earlier, which is calculated directly at each node along the centerline, and this derivative form of dimensionless curvature which is averaged over each half-loop. Furthermore, this half-loop curvature variable is non-dimensionalized by half-loop length rather than by the channel width. Curvature ratio, C_r , is found by dividing maximum curvature of the half-loop, $|\Delta\theta_{max}|$, by average loop curvature, e_a . The three half-loop curvature variables are thus:

$$\text{average absolute curvature, } e_a = (\sum |e_{si}|) / n$$

$$\text{average dimensionless curvature, } e_h = e_a * L$$

$$\text{curvature ratio, } C_r = |\Delta\theta_{max}| / e_a$$

4) Half-Loop Distribution

Half-loop size distribution is described by comparing the number of half-loops in different sizes

classes. Two ratio variables were used to describe overall half-loop distributions for each sample. The first ratio, R_1 , is determined by dividing the sum of loops occurring in categories 3 and 4 by the total number of loops in the sample. The second relation, R_2 , compares the number of small loops to the number of large loops. This ratio is and is derived by dividing the sum of loops in the first two categories by the sum of loops in the last two categories:

$$\text{loop ratio 1, } R_1 = n_4^{-10} / n_{1-30}$$

$$\text{loop ratio 2, } R_2 = n_{1-3} / n_{11-30}$$

Table 1 lists the variables that have been defined mathematically above.

2. Qualitative Description

This discussion explores the relationships between the measured channel variables and the basic categories of channel morphometry. The morphometric categories include sinuosity, degree of pattern irregularity, characteristic period or wavelength, and loop asymmetry. Comparisons are made between the present variables and those used by other authors.

a. Sinuosity

Sinuosity is the degree to which a river departs from a straight course, and is defined as the ratio of

TABLE 1

Summary of Morphometric Parameters Examined in This Study

Variable	Definition
A. Ensemble-Averaged Variables	
1) Sinuosity	
a. total:	$(\Sigma h_i)/Z$
b. whole loop:	$(\Sigma j_i)/Z$
c. half loop:	$(\Sigma x_i)/(\Sigma y_j)$
d. residual:	$(\Sigma h_i)/(\Sigma x_i)$
2) Curvature	
* a. dimensionless curvature:	$(\Delta \theta_i \cdot \bar{w})/(\Delta S_i + \Delta S_{i+1})$
** b. absolute value dimensionless curvature:	$ \text{dimensionless curvature} $
*** c. absolute whole-loop dimensionless curvature:	$(\psi_j \cdot \bar{w})/(y_j + y_{j+1})$
*** d. absolute half-loop dimensionless curvature:	$(\phi_i \cdot \bar{w})/(x_i + x_{i+1})$
3) Half-Loop Size	
** a. half-loop path length:	arc length between inflection point, h_i
*** b. half-loop axis:	chord length subtending the arc, x_i
4) Power Spectrum	
a. spectral peak:	period (in width units) of spectral peak, SV_p
b. spectral ratio:	$SV_p/SV_{0.5p}$
B. Half-Loop Summary Statistics (by size category; see text)	
1) Sinuosity	
a. average sinuosity:	$\bar{h}_i/\bar{x}_i$

TABLE 1 (cont.)

Summary of Morphometric Parameters Examined in This Study

Variable	Definition
2) Asymmetry	
a. asymmetry index 1:	$Lu-Ld/h_i$
b. asymmetry index 2:	$Lu'-Ld'/h_i$
3) Curvature	
a. average (and standard deviation) of absolute curvature	$(\sum e_{si})/n$
b. average absolute dimensionless curvature:	$(\sum e_{si} /n)*L$
c. curvature ratio:	$ \Delta\theta_{max} /e_a$
4) Half-Loop Distribution	
a. half-loop ratio 1:	n_{4-10}/n_{1-30}
b. half-loop ratio 2:	n_{1-3}/n_{11-30}
* first four moments calculated but only variance was used in analysis	
** first four moments were calculated and used in the factor analysis	
*** first four moments calculated but only average was used in analysis	

path length to the linear distance spanned by the reach; the linear distance may include a single loop or many loops. To characterize different modes of channel behavior, several scales of sinuosity are measured in this study.

Residual sinuosity and total sinuosity account for the smallest and largest scales of sinuosity resolvable for a given meander pattern. Residual sinuosity is the average sinuosity for the component half-loops of the sample. Total sinuosity describes the extent of channel meandering, and, thus, provides information about between-loop interactions.

The difference between the two sinuosities can be visualized by considering two sine waves of fixed frequency and amplitude. Although the character of the individual loops (i.e., residual sinuosity) is essentially the same for both, the overall pattern (i.e., total sinuosity) will be different if the first ensemble has a linear and the second an accurate trend (Figures 5a and 5b). Note that if no large-scale wandering is present, residual sinuosity equals total sinuosity.

Two parameters are used to examine intermediate scales of sinuosity. Whole-loop sinuosity describes the wandering of the axis of the ensemble. Half-loop sinuosity describes the deviations of half-loop axes

(5a.)

(5b.)

Figure 5. Diagram illustrating the difference in total sinuosity (path length divided by linear distance, Z) between two channels having the same residual sinuosity (half-loop path length divided by the linear distance between the inflection points of the half-loop). Note that residual sinuosity equals total sinuosity when the half-loop path is linear (Figure 5a.).

relative to whole-loop axes. Because opposed asymmetrical half-loops with different orientations show large values for half-loop sinuosity (Figure 6), it is anticipated that loop asymmetry and half-loop sinuosity will be highly correlated.

Previous studies of channel sinuosity have focused almost exclusively upon total sinuosity, either directly by centerline measurement or indirectly through the variance of the spectral density function (Ferguson 1977a; Thakur and Scheidegger, 1970; Speight, 1965).

b. Period and Wavelength

Wavelength as measured digitally refers to the path length between alternate inflection points. This differs from the classic definition where wavelength is defined as the linear distance between corresponding points along the path (e.g. crest to crest). Wavelength values have been normalized by the channel width to allow rivers of different sizes to be compared.

Figure 6. Example of opposed asymmetrical half-loops which yield a large value for half-loop sinuosity ($\sum x_i / \sum y_j$) and a low value for whole-loop sinuosity ($\sum y_j / Z$).

Period is determined directly from the SDF as the reciprocal of frequency. Distinct peaks indicate the presence of dominant periods of oscillation, or characteristic wavelengths for the series. A broad, irregular SDF means that many wavelengths accounting for approximately equal variance are present along the reach.

Polymodal spectra indicate the presence of either complex loop shapes, or the superposition of loops of different wavelengths (Figures 7a and b, respectively). In the first case, distinct peaks in the SDF are the result of harmonics associated with the non-sinusoidal nature of asymmetric loops; the spectral peaks account for progressively less variance at integer values of the dominant low-frequency peak (Figure 7a). A hypothetical example of a river having a true polymodal meander pattern is shown with the associated SDF in Figure 7b.

In this study, wavelengths (i.e., periods of meandering) are measured both by peaks in the SDF and by statistics related to the size of half-loops (e.g., average half-loop path length and number of half-loops of various sizes).

c. Irregularity

Channel pattern irregularity refers to deviations

from a sine-generated curve of fixed wavelength and amplitude. Planform irregularity may refer to overall channel wandering such as described by total and whole-loop sinuosity. Irregularity of form may also be related to the range of dominant wavelengths present along the reach as depicted by the SDF. Finally, meander irregularity may be described by local pattern variability.

As discussed above, whole-loop, half-loop, and residual sinuosity indicate local deviations from straight trends. An alternative approach is to examine angular rather than linear differences between adjacent loop axes. Angular analogues for whole- and half-loop sinuosity are dimensionless curvature. Small average angles indicate near-linearity of the valley axis, and large average angles indicate that local wandering between the meander loops is extensive.

d. Asymmetry

Asymmetry is a measure of half-loop geometry. Little progress has been made in defining variables to quantify asymmetry of meander loops since interest in asymmetry has only recently developed.

Carson and LaPoint (1983) investigated loop asymmetry and have developed an index of asymmetry, z . The index is expressed as the percentage of upstream

(7a.)

(7b.)

Figure 7. Polymodal curvature spectra derived from different meander patterns. a: complex (modulated) half-loop shapes, and b: superposition of loops of different wavelengths, where λ_1 = dominant wavelength determined by path length from point A to point B, and λ_2 = secondary wavelength determined by the path length from point A to point B'.

and downstream curvature that is traversed normal to the valley axis. Carson and LaPoint note that the application of the method to natural channels may be difficult because of the problem of defining the valley axis. Since valley direction must be chosen arbitrarily, the method appears to be inappropriate for application to digital analysis techniques. Except for the Carson and LaPoint asymmetry statistic, no indices of asymmetry have been described in the literature.

In this paper, asymmetry is quantified in terms of the distribution of curvature along the path of the half-loop. Loop curvature varies from a theoretical value of zero at inflection points to a maximum value at some intermediate location. Symmetrical loops result when curvature values are distributed evenly around the center of the loop. Most meander loops are asymmetrical to some degree, and there is a tendency for asymmetrical loops to be skewed in the upstream direction (Carson and LaPoint, 1983).

III. METHODS

A. Sample Selection

Thirty-three rivers from seventeen states are investigated. Most of the 2,078 miles of centerline data were collected for Midwestern rivers; several of these rivers have been described previously in the literature (Brice 1974); Schumm 1963). The river reaches that were sampled satisfied the following criteria:

1. freely meandering (less than 15% channel impingement against valley walls or high terraces)
2. unimpeded flow (no apparent major upstream flow regulation or diversion within the reach)
3. no major confluences (no large tributaries and less than 50% change in channel width through the reach)
4. banks free of channelization or artificial modification so far as could be determined from 7.5-minute topographic maps.
5. length greater than 50 wavelengths (as discussed below)
6. river width discernible and measurable on the topographic maps (average width greater than 0.1 cm)

7. river size such that a reasonable number of 7.5-minute maps would provide an acceptable long sample.

The selection constraints were chosen to ensure a homogeneous (statistically stationary) sample i.e., one free of controls which might result in abrupt changes in meander character. Samples were also selected to represent a broad range of meander forms in terms of sinuosity and regularity of meandering. A random selection of rivers would have been made representative of the population mean, but would have provided less information about the range of variability.

Within these constraints, the selection process for rivers proceeded along two lines. First, a review of the literature was conducted to identify potential sample rivers that had already been studied (Brice 1974). The use of such rivers permits comparison of data and conclusions with those of previous authors. Second, additional samples were selected for states in which the first river set was found. Rivers were sometimes included as separate samples below major confluences, and these samples are designated by reaches A and B for the upstream and downstream sections, respectively. The entering river was itself included for analysis when possible. Thus, in some cases, several samples are located in close geographic proximity to one another. The rivers examined in this study and their general physical characteristics are listed in Table 2.

Table 2.

Physical characteristics of the thirty-three sample reaches. Dominant wavelengths are estimated from curvature spectra, and number of whole-loops are inferred from the dominant wavelength of each sample.

CASE	SAMPLE	STATE	SAMPLE WIDTH (m)	PATH LENGTH (km)	TOTAL SINUOSITY	DOMINANT WAVELENGTH (width units)	WHOLE- LOOPS
1	Arkansas	KS	62	157.1	1.19	16.80	398
2	Blacks						
	Fork (A)	WY	33	71.2	2.58	20.42	276
3	Blacks						
	Fork (B)	WY	48	58.5	2.31	19.93	160
4	Boulder	MT	33	30.7	1.23	20.07	122
5	Chena	AK	55	66.3	2.10	19.65	162
6	Cimarron	KS-OK	68	64.1	1.43	30.24	82
7	Edisto	SC	58	66.2	1.93	10.28	292
8	Elkhorn	NB	62	94.3	1.54	31.09	129
9	Fox (A)	WI	36	38.1	2.50	8.64	326
10	Fox (B)	WI	56	47.4	1.42	16.47	135
11	Iowa	KS	76	64.6	1.59	23.04	97
12	Little						
	Pee Dee (A)	SC	25	44.0	1.69	8.36	555
13	Little						
	Pee Dee (B)	SC	60	39.4	1.56	11.30	154
14	Lumber	NC-SC	40	50.2	1.94	15.47	212
15	Meherrin	VA-NC	32	90.7	2.26	26.71	282
16	Milk	MT	32	191.8	3.17	62.33	257
17	Obion	TN	39	79.7	1.80	18.12	300
18	Powder	MT	56	67.8	1.35	34.41	93
19	Red (A)	MN-ND	35	154.5	2.30	37.88	309
20	Red (B)	ND-MN	72	230.8	1.97	28.04	303
21	Rock	IA-SD	32	41.4	1.64	27.74	125
22	Rogue	MI	22	38.4	2.80	18.40	248
23	Saline	KS	24	248.3	2.81	36.35	747
24	Savannah	SC-GA	106	282.4	1.88	10.28	684
25	Sioux (A)	IA	34	87.2	1.84	11.99	573
26	Sioux (B)	IA-SD	44	105.2	2.12	16.29	390
27	Smith Fork	WY	28	19.7	1.71	13.12	141
28	Smoky						
	Hill (A)	KS	28	164.8	3.47	61.02	254
29	Smoky						
	Hill (B)	KS	59	107.4	2.10	21.45	226
30	Solomon	KS	26	219.5	2.63	73.87	305
31	Suwannee	GA-FL	38	114.3	2.39	24.14	325
32	Tongue (A)	WY-MT	31	37.6	1.75	22.90	138
33	Tongue (B)	WY-MT	38	49.0	2.65	35.72	96
Sample Average:			45	97.7	2.05	25.23	227

B. Data Collection

Channel centerlines were traced from U.S.G.S. 7 1/2-minute topographic maps onto paper overlays. A single overlay generally spanned several maps and was annotated to indicate valley edge location (approximated as the contour 20 feet above water level), tributaries and distributaries, gaging stations, gradient, vegetation patterns, and latitude/longitude.

Channel width and centerline data were recorded from the overlays using Numonics digitizing equipment. Raw centerline data were collected using short sampling intervals and three replicate measurements as described below.

1. Width

Channel width has been the fundamental unit of measure in studies of meander planform. It is used as a scaling parameter for morphometric variables of meander geometry, and is often used to create derivative variables (the most common of which is the radius/width statistic). In the present study, all linear measurements were scaled by the channel width. Width measurements were not taken at fixed distances downstream for all samples, but were instead spaced according to bank regularity and meander scale. The justification of this approach is that rivers having relatively straight banks require fewer measurements to

define average channel width than do rivers displaying highly variable banks.

The use of width for scaling is common in meander studies and is justified on the basis of the apparent scale-independence of meander geometry when so normalized. However, channel width as portrayed on maps may be subject to bias during map making (particularly for narrow rivers), and the width shown for different rivers may not represent the same flow frequency. Most, however, are based upon aerial photographs taken with discharge near the modal discharge (a fairly low flow). Theoretical studies (Ikeda and Parker, 1981) suggest that channel depth may be more directly related to meander size and geometry than channel width. However, depth cannot be measured from maps. Another alternative is self-scaling of meander patterns, based upon some length property such as dominant wavelength of the curvature spectrum or average wavelength between curvature inflection points along the centerline. Width is used in this paper, although in later stages of the project self-scaling will be investigated.

2. Centerline

A rigorous sampling procedure was devised for collecting the centerline data. The development of this sampling procedure, which includes operations needed to prepare the raw data for statistical analysis, is a major

contribution of the study. Refined sampling methods are unnecessary for estimation of certain properties of channel geometry, such as the spectral density function, which appears to be relatively insensitive to operator variance. For example, Figure 8 shows that the same river (the control sample, in this case) digitized in three replicates displays insignificant variations in the spectral density function between replicates. Consequently, sampling modifications are not needed to enhance the reliability of the curvature spectrum. However, for other types of parameters previous studies have not utilized stringent data collection procedures; operator bias and statistical variability have not been examined, thus little is known about parameter confidence limits.

The morphometric parameters examined in this paper (many of which are introduced for the first time) require a robust methodology to ensure the statistical viability of their estimated values. A specialized sampling procedure is needed because of the range and level of detail these new variables define. The primary difference between this and previous sampling approaches is that each centerline was digitized three times and the replicates were merged and smoothed to produce an average centerline. The influence of replications on the statistical results are discussed in the Sensitivity Analysis section below.

Figure 8. Effects of replicate sampling on the shape of the spectral density function. The solid curve is the spectrum for the three merged and smoothed replicates, and the other curves are spectra for the individual replicates. The figure suggests that little is gained by using replicate data to define the curvature spectrum. The control sample was used for this analysis.

The philosophy behind the sampling design is that the analysis of planform geometry is best approached using objective, automated techniques. This means that sampling methods must lend themselves to consistent, efficient and unbiased parameter estimates. Furthermore, the intricate nature of planform geometry places a burden upon the researcher to construct a sufficient set of descriptive variables for characterizing meander geometry. In adhering to a principle of objectivity and completeness, the sampling procedure adopted for the present study emphasized the following:

- 1) digital collection of raw data
- 2) large sample length for high confidence
- 3) techniques for reducing sampling errors (noise)
- 4) introduction of new and subtle measures of meander planform geometry.

3. Digitizing Procedure

The overlays were digitized as several segments of arbitrary length (about 15-25 centimeters, depending on the scale of meandering). Digitizing the overlay a segment at a time facilitates the sampling process since errors could be corrected simply by re-sampling a segment rather than by having to re-digitize the entire reach. A further advantage is that the propensity to retain marginal data after having sampled a long path is eliminated.

Raw data were recorded at a rate of about four points per second. Raw point density was thus determined by the rate of cursor movement. Initial point density per unit channel width was roughly constant between samples because narrow channels with tight bends were digitized more slowly than were the wide, straight channels. Spacing of raw points was approximately $1/4$ to $1/2$ the ultimate sampling interval ($1.0 \bar{w}$) to minimize the interpolation error associated with generating the uniformly spaced centerline data.

C. Sensitivity Analysis

Manually digitized data is properly viewed as a pure signal masked by sampling noise. Meaningful analysis of the data may proceed only after this noise has been reduced to an acceptable level. This noise may arise either from the digitizing process or from errors in constructing the original topographic map. The latter could not be controlled, and have been assumed to be of small magnitude.

The purpose of the sensitivity analysis is to examine the signal enhancement/noise reduction problem associated with sampling river centerlines, and to develop procedures that minimize both operator bias and noise inherent in the sampling process. The influence of sample length, in this case, is expressed in terms of the number of whole-loops included for analysis. Before discussing sample length

Figure 9. Centerline trace of the control sample used for the sensitivity analysis. The simulated meander path was generated using a disturbed periodic model (Ferguson, 1976, modified by Howard, 1985), and represents a natural river with total sinuosity = 1.6, path length = 36.44 km, and average width = 46.33 m. The bar represents twenty channel widths, which is one nominal along-path wavelength.

loops), long sample lengths are required to ensure that parameter estimates converge towards their population value.

It appears that the results of past investigations of channel geometry have been interpreted without regard to the number of loops used in the analysis, and that sample lengths have been chosen arbitrarily according to unreported criteria. Ferguson (1975) provides sample length and dominant wavelength values (the latter are estimated from direction spectra) in his study of nineteen British rivers. The average channel length, however, is 7.2 km, yielding an average of only 5.6 whole-loops for each sample. Although this estimate is inferred from dominant wavelength values, and the exact number of wavelengths in each case is unknown, statistical stability is nonetheless questionable for such small sample length.

The convergence of estimated total sinuosity as a function of sample length (expressed in terms of the number of whole-loops) is shown in Figure 10. It is clear from Figure 10 that sample lengths which include fewer than about 50 whole-loops are associated with high variance and, thus, unreliable parameter estimates. Identical analyses were performed for the other statistical parameters, with similar results (Figure 11 shows the effect for extended half-loop path length). Note that the parameter variation may be even greater for natural rivers than for the simulated

Figure 10. Variance of estimated total sinuosity as a function of number of whole-loops. Horizontal line indicates estimated population mean, and the inclined lines delimit the 95% confidence interval as a function of the number of whole-loops. The horizontal scale is proportional to the inverse of the square root of the number of whole-loops. The squares represent independent control samples, each generated with identical parameter values but different sequences of random noise. The figure indicates that samples which include fewer than approximately 50 whole-loops are associated with high parameter variance.

Figure 11. Variance of estimated average half-loop path length as a function of number of whole-loops. See Figure 10 for explanation of symbols. As with estimated total sinuosity, samples which include fewer than about 50 whole-loops provide unreliable parameter estimates.

control sample due to the presence of asymmetrical loops and other channel pattern irregularities.

The problem of sample length selection is compounded, however, by the stationarity constraint required for the calculation of the curvature spectrum and the other statistical parameters. The stationary assumption requires meander character to be constant over the length of the sample, that is, physical controls such as discharge and geology must be uniform within the reach. Thus, long sample lengths (e.g. greater than 400 loops) are likely to be associated with large variations in environmental controls, and, therefore, non-stationary meander behavior. Generally, however, it is difficult to find a river with 50 loops that satisfies the selection criteria.

Meander pattern stationarity was examined qualitatively in this study through a visual analysis of each channel trace. Rivers displaying systematic changes in channel width or meander wavelength were not included for study. The rivers investigated this paper have an average path length of 97.7 kilometers, with an average of 227 whole-loops for each sample (based on dominant wavelengths derived from the curvature spectra). In future studies the question of stationarity will be addressed by iterative fitting of autoregressive parameters equivalent to the disturbed periodic model of Ferguson.

3. Centerline Data Integrity

Factors which affect the integrity of centerline sample data are raw point density, degree of smoothing of the raw data, and step length of the equally spaced data file. The first factor is related to how the data are collected. The remaining two factors are associated with transformations required for statistical analysis. This distinction suggests that two approaches are available for solving the noise reduction problem. The data collection process is discussed first.

a. Raw Data

The straightforward nature of the digitizing operation suggests that few options are available for significantly improving the quality of the raw data as it is recorded. Loops are adequately defined so long as average raw segment lengths are between one-quarter and one-half the ultimate point spacing. Loop definition suffers when segment length approaches that used to space the data, while high raw point densities present the problems of excessive storage requirements and increased computation time for subsequent manipulations.

An area of concern is the random error related to the human element in digitizing. Noise introduced during the digitizing process is unavoidable and cannot

There is a significant difference, however, between individual replicates for parameters that are defined on the basis of inflection points, such as average half-loop path length (Figure 12b, curves Raw1 and Raw2). Furthermore, unsmoothed replicate estimates are in poor agreement with those for the control centerline (curve Theory). Of the digitized control data that were examined, it appears that the triple replicates (curve Merge3) smoothed using an 11-point moving average provide the most reliable parameter estimates relative to the unsmoothed mathematically defined centerline. Note that while double replicates (curve Merge2) smoothed using a 6-point moving average provide acceptable results for estimated total sinuosity and estimated average half-loop path length, associated estimates for more sensitive variables such as curvature variance are less reliable.

b. Transformations

Two transformations are required before the statistical analysis can be performed. First, residual sampling noise must be attenuated by smoothing the raw data. Second, a file of equally spaced points must be interpolated from the second data set.

Figure 12a. Influence of replicate sampling on estimated total sinuosity. The three abscissa scales represent equivalent smoothing for the raw and merged data. The solid line (Theory) represents estimates derived from the mathematically defined control for different degrees of smoothing. The short dashed lines (Raw1 and Raw2) represent estimates for individually digitized replicates. The long dashed line and the dot-dashed line show parameter estimates for merged double (Merge2) and merged triple (Merge3) replicates, respectively. The figure suggests that stringent sampling procedures are not needed to obtain reliable estimates of total sinuosity since the variation between the five curves is insignificant for river samples containing less than about 400 whole-loops.

Figure 12b. Influence of replicate sampling on estimated average half-loop path length. See Figure 12a for an explanation of the legend. The variation between estimates for individual replicates (curves Raw1 and Raw2) indicates that replicate averaging is desirable, particularly for intermediate degrees of smoothing. Triple merged data (curve Merge3) smoothed using a moving average of approximately 12 points appears to provide optimum results.

c. Smoothing

The merging file of three superimposed traces is smoothed to produce an average centerline by using a simple moving average filter with uniform weighting. A series of test were conducted to evaluate frequency response characteristics of filters of different resolutions. Results of the resolution tests are summarized in Figure 13. The influence of sampling noise on the statistical estimates is demonstrated by high-frequency variability of the curve for the unfiltered data. Excessive filtering, on the other hand, produces distortion by removing useful high frequency information from the signal. Proper smoothing of the raw data thus requires optimizing variance reduction against excessive losses of high frequencies from the signal. It appears from Figures 12 and 13 that a moving average of between 7 and 15 points yields optimum results for three merged replicates, each with 2 to 4 points per ultimate spacing interval. That is, the length of averaging is approximately equal to the spacing interval. An 11-point moving average was applied to each of the merged data files that were generated in this study.

d. Spacing

Calculation of the power spectrum of the curvature

Figure 13. Diagram of curvature spectra for unsmoothed and smoothed raw data. Moving averages of 7, 11, 15, and 19 points were used to filter the three merged replicates. These results suggest that an 11-point moving average provides an acceptable compromise between high-frequency distortion and signal loss that result from too little or too much smoothing, respectively.

series assumes that observations are separated by intervals of constant length. Thus, the final step in preparing the data for statistical analysis involves structuring the smoothed data to ensure a regular sampling frequency. Equally spaced points are interpolated at a specified radial distance along the smoothed but irregularly-spaced centerline path. Because meander geometry is essentially independent of scale, the critical distance, Δs is normalized in terms of the size (width) of the river being studied.

The difference between arc and chord lengths increases rapidly with direction change, e.g. from 0.2% for $\Delta\theta = 12$ degrees to 4% for 57 degrees (Ferguson, 1975). This discrepancy causes sinuosity and curvature variance to be underestimated while average curvature tends to be overestimated. Large direction changes may be avoided by choosing a small Δs . However, as Δs decreases, the likelihood of retaining high-frequency sample errors increases. Thus, the selection of Δs involves a trade-off between information loss on the one hand and noise amplification on the other.

Step length of 0.5, 1.0, 1.5, and 2.0 times average river width were selected to evaluate the Δs optimization problem. The shape and location of spectral peaks are not strongly affected by the value of Δs (Ferguson 1975). This means that wavelength

estimates derived from the SDF are not significantly influenced by sampling interval over the range of values used.

Morphometric parameters, based on the location of inflection points, on the other hand, are highly sensitive to the choice of Δs because fewer inflection points are defined as Δs increases. The extent of bias as a function of step length (expressed in terms of the number of points defining a whole-loop) for total sinuosity and average half-loop path length is shown in Figure 14a and 14b, respectively.

Figure 14a suggests that values for total sinuosity are underestimated when fewer than 20 points are used to define a nominal whole-loop (i.e., when step lengths are greater than $1.0 \bar{w}$). Total sinuosity is more accurately estimated, however, when Δs is less than or equal to $1.0 \bar{w}$. Note that the range of variation in estimated sinuosity as a function of Δs is smaller than the 95% confidence interval for fewer than 200 sampled whole-loops (Figure 10).

The opposite effect is observed for half-loop path length, where small step lengths yield lower parameter estimates (Figure 14b). This occurs because more inflection points are defined as additional points are used to describe the channel centerline. While

Figure 14a. Estimated total sinuosity for different step lengths (expressed as a function of number of points per nominal whole-loop). The four points which define the individual curves represent step lengths of 2.0, 1.5, 1.0, and 0.5 $\bar{w}$. Sinuosity values are underestimated and vary significantly when step lengths are greater than 1.0 $\bar{w}$ (less than 20 points per whole-loop). More accurate and less variable estimates are found when step lengths are less than or equal to 1.0 $\bar{w}$.

Figure 14b. Effects of step length on estimated average half-loop path length. As in Figure 14a, step length is expressed in terms of the number of points per nominal whole-loop, and the points defining the curves represent similar step lengths. It appears from this figure that half-loop path lengths are underestimated when point densities exceed 20 points per whole-loop, corresponding to step lengths that are less than $1.0 \bar{w}$. This occurs because the number of inflection points increases with decreasing step lengths due to noise. To prevent high-frequency bias, no more than 20 points should be used to define a nominal whole-loop.

sampling noise increases for smaller values of Δs , the use of a relatively large Δs results in information loss as the centerline becomes more generalized. A sampling frequency of about 20 points per whole-loop (approximately equal to one channel width) lies in the region where half-loop length changes relatively slowly. This frequency represents a reasonable compromise between high noise and loss of high-frequency information about the meandering pattern.

Based on the results of Figures 14a and 14b, it appears that a step length of $1.0 \bar{w}$, yielding approximately 20 points per nominal whole-loop, provide optimum parameter estimates. Note that previous authors (Ferguson, 1975; Speight, 1965; Chang and Toebe, 1970) have used step lengths of $1.5 \bar{w}$ or $2.0 \bar{w}$.

IV. ANALYSIS

Operational procedures for collecting planform data have been described above. What follows is a discussion of the statistical tests used to examine the data. The statistical procedures include spectral analysis of the curvature series, and principal components analysis of the morphometric variables. Spectral variances, lag correlations, and statistical moments of the morphometric variables were calculated for each curvature series using a statistical package developed by Howard (1985 pers. comm.), and the present author. The principal components analysis was performed using an SPSS factor subroutine. Assumptions and limitations of the statistical methods are reviewed, and expected results are considered.

A. Spectral Analysis

The spectral density function (SDF) of the curvature series is estimated using spectral analysis. Spectral analysis of the series yields information about sinuosity, degree of meander irregularity, and wavelength characteristics of the ensemble. For the purpose of this study, spectral analysis is not used as an end in itself. The SDFs of the samples are needed to determine characteristic wavelengths of the sample rivers, and for qualitative information concerning the nature of the individual curvature series. The quantitative data related

to the shape and location of the spectral peak are used as input data for a principal component's analysis.

The underlying concept of spectral analysis is that a variable, viewed as a function over time or distance, may be accurately represented as the sum of pure sine wave with each component sine wave defined by a different frequency, amplitude, and phase. Spectral analysis proceeds by Fourier transforming the data to obtain the coefficients of the sinusoids for the different frequencies and grouping adjacent frequencies into frequency bands. The number of frequency bands, m , is a function of sample size: $m = 2n$ where: n = number of observations. Normalizing the number of frequency bands to the length of the series provides a means for standardizing spectral resolution between samples (Chatfield 1980). The spectral density of the series is estimated by computing the average signed amplitude of the sinusoids within a frequency band. The estimate of the spectral density (variance) is plotted as a function of frequency (that is, as a function of frequency band center and frequency band width).

The spectral density function describes how the variation of a time series is accounted for by cyclic components at different frequencies. Dominant frequencies which explain most of the oscillatory variance of the data are revealed as peaks in the spectrum. Similarly, a nearly flat spectrum indicates that many frequencies are accounting

for approximately equal portions of total variance. In this case, the data cannot be characterized by a single dominant frequency. For background reading in spectral analysis see Chatfield (1980); Rayner (1971); Jenkins and Watts (1968) which is the definitive reference for detailed theoretical considerations. See Chang and Toebe (1970; Ferguson (1975); Speight (1965) for applications to meander morphometrics.

Figure 15 shows that differences between spectra calculated for the same river at different Δs are slight for regular meander patterns (i.e., for the control sample). In other words, although total spectral variance increases with decreasing step length, the shape and location of the spectral peaks are relatively constant. The influence of Δs on the SDF is more pronounced for high frequencies (wavelengths less than about 5.0 times average channel width) because the step length represents a higher proportion of the approximated arc. Long wavelengths, on the other hand, represent greater distances relative to Δs , thus, step length must change dramatically for a difference in loop definition to be observed.

Estimates of dominant wavelength derived from the SDF of the curvature series are independent of segment length (within the aforementioned range of Δs) because characteristic periods are generally greater than 10 times the average channel width.

Figure 15. Spectra of the control river sampled using different step lengths. Though total spectral variance (area under a given curve) decreases with increasing step length, the overall character of the spectra is relatively uniform for this range of sampling intervals.

Assumptions upon which the Fourier transformation are based pertain only to mathematical considerations, and are not restrictive when applied to river planform analysis. The assumptions, as given by Rayner (1971) are: 1) $k(t)$ is single-valued and finite, 2) $x(t)$ is defined everywhere in the interval, 3) there are a finite number of maxima and minima and discontinuities in the interval. The only practical requirement is that observations must be equally spaced along the interval.

The complicated nature of the processes of meander formation is responsible for the observed variability between meander patterns. Noticeable differences in shape may be detected even between rivers formed under apparently similar conditions. It is assumed, therefore, that significant differences will exist between the SDFs of the thirty-three sample rivers that are included in the study. In fact, the success of the sample selection process in choosing a wide variety of meander forms is suggested by the range of variability represented by the collection of spectral density curves for these samples (see Appendix I). The results of the spectral analysis and examples of the various classes of meandering are discussed in the following chapter.

B. Principal Components Analysis

Principal components analysis (PCA) is a descriptive

technique for studying the correlation structure of multivariate samples. The idea is to interpret the variance-covariance structure of a set of data through a few uncorrelated linear combinations of the original set of variables (Johnson and Wichern, 1982). No a priori patterns of causality are assumed. If most of the total population variance, for a large variable set, m , can be attributed to the first k linear combination, then these "components" can replace the original m variables without much loss of information. The objectives of PCA are 1) data reduction and 2) interpretation.

The principal component transformation produces a set of m linearly independent factors, known as eigenvectors, which are extracted from the m original variables such that the first eigenvector explains as much of the original variance as possible, the next as much of the remaining variance as possible, and so on. The eigenvectors are associated with categories of related variables, and the eigenvalues (also called factor loadings) describe the amount of variance accounted for by each eigenvector. PCA is used to determine the number of linearly independent vectors that exist in the variance--covariance matrix, and thus allows one to measure redundancy in the original set of variables.

In this study, values of the factor loadings are based upon the Varimax axis rotation. The Varimax rotation

maximizes the variance of the factor loadings within the columns of the factor loading matrix (i.e., maximizes variances across variables). The rotation is used to amplify the loading pattern such that a minimum number of variables account for most of the variance associated with a given factor.

The principal components may be interpreted geometrically as follows. The elements of an $m \times m$ matrix define points lying on a m -dimensional ellipsoid. The principal axes of the ellipsoid are the eigenvectors, and the length of the vectors are the eigenvalues (characteristic roots). For the simple case where $m=2$, if the first axis (first principal component) accounts for a large portion of the total variance, then the ellipse will be elongated in the direction of the first axis (Figure 16a). Similarly, if the two principal components account for approximately equal variance, then the distribution of points will be practically circular with each axis explaining nearly equal amounts of the total variance (Figure 16b). Because the matrix is symmetrical, the axes are orthogonal, hence linearly independent.

All of the variables to be examined using PCA must be measured in the same units and measurements must be of the same order of magnitude (Joreskog et al. 1976). Since the variables used in this study are not directly comparable it

(16a.)

(16b.)

Figure 16. Geometric representation of principal components. a: ellipse defined by two principal components where the first vector explains over 99% of the total variance; b: ellipse defined by randomized data where the two principal components account for approximately equal variance.
(adapted from Davis, 1973)

is necessary to convert all of them to standardized form. Although standarization has a tremendous influence on the variance-covariance matrix (Davis 1973) the raw matrix of variances and covariances contains such an assortment of measurement units that logical interpretation of the factors is impossible if z-scores are not used.

Since the number of factors produced equals the number of original variables, m , there is the problem of deciding which k factor accounts for a satisfactory portion of the total variance. Selection rules do not exist for deciding how many components are necessary to sufficiently characterize the data. Joreskog et. al. 1976) present some useful guidelines for approaching this difficulty: the first step is to examine the cumulative percentage variance contribution obtained for the factors. When the cumulative contribution is sufficiently large (70-90%) then it is safe to assume that the combination of these k principal components adequately characterizes the data. The first k eigenvalues should be significantly positive, and the fall off in size from one to the next should be appreciable. The residual eigenvalues should be small and approximately equal (the SPSS subroutine uses eigenvalues < 1.0 as a cutoff). It should be clear that the final decision is largely subjective, and must be considered in light of the system being considered and nature of the data (degree of colinearity and redundancy in it).

One of the major difficulties associated with applications of PCA to systems characterized by a wide variety of different measures is interpretation of the components. The meaning of the vectors may be difficult to determine since each one represents a combination of the original set of variables. An intuitive feel for the system being examined, experience with the technique, and a knowledge of what each of the original variables represents is essential for the meaningful use of this method. If there is uncertainty concerning these fundamental aspects of the analysis then the use of PCA can hardly be justified.

In the present study, it was expected a priori that between 80% and 90% of the variance may be accounted for by the first three or four vectors. The vectors were expected to be related to the size (curvature), shape (local and overall sinuosity), irregularity, and asymmetry of the meander loops. This supposition is based upon the observation that a high degree of order is represented by meander patterns. That is, although meander geometry may be highly variable between samples, obvious similarities exist which distinguishes them as river meanders.

PCA is used to reveal these fundamental properties of meander geometry though it is naive to assume that the principal components will be clearcut variations of the input variables. The results of the principal components analysis are discussed in the following section.

V. DISCUSSION OF RESULTS

The results of the spectral analysis and principal components are presented below. Physical interpretations are emphasized in the discussion by relating a given statistical result to its apparent manifestation in the associated channel pattern. Two sets of channel centerline traces have been plotted for this purpose.

The first set of centerline plots shows the entire sampled reach for each of the study rivers. Because the meander pattern of the long samples is difficult to interpret when the total reach is plotted on standard-size paper, a second set of channel traces have been produced to show a shorter section of each sample. These partial channel plots have path lengths that are equal to 500 times the average channel width of the sample. In these plots, the width of the trace is approximately equal to the channel width. The partial channel tracers were taken from the upstream end of the total reach in most cases, but for some of the samples the sub-reach was selected from a more representative downstream section. The associated improvement in meander pattern resolution is especially necessary for interpreting the subtleties of loop geometry that characterize the principal component factor loadings. The total and partial centerline plots are shown in Appendix III.

A. Results of Spectral Analysis

Spectral density functions for the thirty-three curvature series are presented in Appendix I. Spectral variance and frequency (in width units) are represented by the ordinate and abscissa, respectively, for each plot. The spectral patterns shown in Appendix I suggest that the SDF's may be grouped into four general shape classes based on the modality of the spectral peaks. The shape classes are unimodal, bimodal, trimodal, and flat/multimodal spectra.

Ideal examples of each shape class are rare because of the inherent complexity of meander geometry, and in some cases a SDF represents an intermediate pattern between two class types. Although the spectral patterns are classified subjectively, each shape class appears to be equally represented by the set of sample rivers included for analysis. Examples of each spectral category are discussed below, and references are made to the channel trace for which the SDF was derived. The number in brackets following the river name refers to the case number of that sample.

1. Unimodal Spectra

Rivers displaying a unimodal spectrum are: The Boulder River [4], the Cimarron River [6], the Elkhorn River [8], the Milk River [16], the Red (B) River [20], the Saline River [23], Smiths Fork [27], and the Solomon River [30]. Of these nine rivers, the existence of a single dominant

meander wavelength is especially evident from the partial channel trace of the Boulder River, and from the total channel traces of the Red River (A) and (B), and the Milk River. The spectral peak for the Elkhorn R., while clearly unimodal, is broader than the above; this is also evident in the partial and total channel traces.

The other samples in this category are not as clearly unimodal in their spectrum. Additional wavelengths are revealed primarily as subsidiary peaks in the spectral plots, particularly at the higher frequencies (e.g. Cimarron River, Smiths Fork, and Solomon River). The Saline River displays both unimodal and bimodal spectral features, and the channel traces for this river suggest that this sample represents an intermediate meander form.

2. Bimodal Spectra

Nine of the sample rivers are characterized by bimodal meandering: Black Fork (B) [3], the Fox (A) River [9], the Iowa River [11], the Little Pee Dee (A) River [12], the Savannah River [24], the Big Sioux (B) River [26], the Smoky Hill (B) River [29], the Suwannee River [31], and the Tongue (B) River [33].

The rivers in this category vary in the degree to which the two dominant wavelengths are widely and clearly separated case of a bimodal spectrum. The separation between the spectral peaks of the Blacks Fork (B) River, for

example, is relatively small implying in this instance that both large and medium-scale wavelengths are dominant along this reach. It is difficult, however, to visually separate the two wavelengths in the total channel trace for the Blacks Fork (B) River because of the small difference in size between the dominant wavelengths.

The mean the Suwannee River and Tongue (B) River is also characterized by large and medium-scale meander loops, but the separation between the two dominant wavelengths is greater than it is for the Black Fork (B) River. Bimodal channel pattern behavior is evident from both the partial and total channel traces of these rivers. The spectrum of the Suwannee River displays several subsidiary, high-frequency peaks, and the presence of the small-wavelength meander loops responsible for these peaks can be observed on the total channel trace.

The Fox (A) River demonstrates a bimodal spectrum where the dominant wavelengths are described by large and small-scale meandering. The physical manifestation of this spectral pattern can be seen on the total channel trace, and, to a lesser degree, on the partial centerline plot. Although some medium-size and very small meander loops are present along this reach, their occurrence is restricted by the predominance of both the small and large wavelengths.

Spectral bimodality is less obvious for the Smoky Hill (B) and Iowa Rivers because the second peak (the higher-frequency peak in each case) is either relatively small (Smoky Hill (B)), or indistinct (Iowa). Furthermore, the spectral plots suggest that these rivers may be intermediate forms between the bimodal and trimodal shape classes because a shoulder at the high-frequency side of the second peak is present.

Multiple subsidiary peaks in the Little Dee Pee (A) River, Savannah River, and Big Sioux (B) River spectra suggest that a bimodal meander pattern may be difficult to detect from the channel traces of these rivers. The tendency of these meander patterns to exhibit subtle multimodal characteristics is revealed in the total channel traces suggesting that these samples could be grouped into either the bimodal or multimodal shape class.

3. Trimodal Spectra

Of the seven samples that possess three meander wavelengths, the Blacks Fork (A) [2] and the Little Pee Dee (B) [13] display particularly striking trimodal spectral patterns, that is, each of the three peaks are well-defined, and subsidiary peaks are essentially absent from the spectral density functions. Although the peaks accounts for progressively less spectral variance with increasing

frequency, large, medium, and small-scale meander loops are apparent on the total channel traces of these samples.

The Rogue River [22] and the Big Sioux (A) River are examples of where the three dominant wavelengths account for approximately equal spectra variance. It is interesting to note that of the thirty-three sample rivers, these two channels (and the bimodal Suwannee River [31] which displays several prominent high-frequency peaks) are associated with the largest degree of valley wandering. The Rogue River is included in the trimodal shape class in this study, but the presence of four high-frequency spectral peaks suggests that this river might also display multimodal meander behavior. Based on the plot of the total channel centerline, however, the trimodal classification appears to be justified.

The Fox (B) River [101] spectrum represents an intermediate form between trimodal and multimodal meandering in that the spectral density is relatively broad, with two shoulders exhibited at the higher frequencies. Total sinuosity for this river is low (1.4), and meander loops at all scales are poorly developed (based on the total channel trace) implying that this sample represents a degenerate meander form.

The final two samples included in the trimodal category, the Chena River [5] and the Powder River [18], display bimodal as well as trimodal spectral features. The

is characteristic of the overall channel pattern. Since only one of the thirty-three samples displays this type of spectrum, it is assumed that such meander forms are relatively rare.

The seven multimodal rivers include: the Edisto River [7], the Lumber River [14], the Meherrin River [15], the Obion River [17], the Rock River [21], the Smoky Hill (A) [28], and the Tongue (A) River [32]. An overall similarity between the seven curvature spectra is the presence of a single peak which accounts for the largest percentage of the total spectral variance. No spectrum displays greater than six peaks, and except for the Obion River, there are no low displays greater than six peaks, and except for the Obion River, there are no low-frequency subsidiary peaks observed in these spectra. Beyond this, the samples may be divided into two groups with respect to the peakedness of the spectrum.

The first group includes spectra that are characterized by successive indistinct high-frequency peaks or shoulders which appear to cascade from the large low-frequency peak. Samples identified with this group are the Meherrin River, the Obion River, the Smoky Hill (A) River, the Tongue River, and the Rock River, and each sample is associated with five or more spectral peaks. Total channel traces for these rivers are complex, with wavelengths of various sizes

occurring in an apparent random sequence within the sample reaches. Four of the five peaks for the Rock River are closely spaced suggesting that the dominant wavelengths of this meander pattern are approximately equal in size. The corresponding total channel trace thus appears relatively uniform in terms of loop size variability.

The second group of multimodal samples are characterized by well-defined spectral peaks. This group includes the Edisto River and the Lumber River, and both of these spectra display four dominant peaks. As before, the accompanying channel traces are complex, and it is often difficult to distinguish between the dominant wavelengths that are present along the channel reaches. Because of the intricate nature of these meander patterns, however, multimodal meander forms are generally recognizable from center-line traces.

The four classes of meandering that are discussed above appear to adequately accommodate all of the samples that were examined in this study. Because of the variable and often complex nature of meandering, the spectral density function of each curvature series was used to provide information about dominant wavelength characteristics of each sample. The thirty-three sample rivers are approximately evenly divided among the four spectral shape classes (unimodal, bimodal, trimodal, and flat/multimodal)

suggesting that these categories, though general, are useful for characterizing the behavior of natural meander patterns.

Because the curvature spectra are classified subjectively, and because some samples display spectral traits that are associated with two or more meander modes, there may be a question as to which class a given sample belongs to. The problem may be resolved in part by a visual analysis of the channel centerline trace. It appears that some samples, however, represent intermediate meander pattern forms and, thus, cannot be conveniently grouped into one category or another. Finally, it should be noted that meander wavelength modality is independent of total channel sinuosity. For example, total sinuosity for the unimodal Milk and Boulder rivers is 1.2 and 3.2, respectively, while total sinuosity for the trimodal Rogue and Big Sioux (A) rivers is 2.8 and 1.8, respectively. Thus, it appears that the presence of additional dominant wavelengths along the channel is unrelated to the wandering of individual loop paths, and, to a lesser degree, valley wandering.

B. Results of Principal Components Analysis

Statistical values for twenty-eight morphometric variables (including the first four moments of absolute dimensionless curvature and half-loop path length) were used for a principal components analysis. All half-loop variables included for the PCA were averaged over the

intermediate size classes of half-loop length. That is, weighted averages were calculated for size classes of 7 to 21 width units. Based on preliminary analyses, this averaging was necessary to assume stable statistics for half-loop properties within the range of expected dominant wavelengths. In addition, several compound variables (discussed below) were created to examine the distribution of half-loop size within each sample.

Sinuosity variables were scaled by subtracting 1.0 from each value so that the minimum sinuosity would be zero instead of one. The purpose of this step is to increase the discriminating power of the sinuosity parameters and to standardize these measures to the other statistics in the sample.

Logarithmic transformations were made on all variables whose values were always positive and where the range over the sample was large. This transformation was applied following initial runs when it became apparent that it would be beneficial to compress the range of these values to minimize distortion. Variables which were not transformed include: skewness of absolute dimensionless curvature and half-loop path length, spectral peak, the two asymmetry indices, and half-loop ratio 1.

Three derivative variables were included in the analysis. The first two parameters describe the distribution of half-loop sizes for the individual samples.

The first variables, R_1 , is a measure of the number of intermediate sized loops relative to the total number of loops for a given river: R_1 is calculated by dividing the number of half-loops in the size range 4 to 10 width units by the total number of all half-loops in the sample. The second variable, R_2 , is the ratio of the number of smallest half-loops to the number of largest half-loops. It is calculated by dividing the number of loops of 1 to 3 width units in length by the number of loops in the 11 to 30 size categories. These two ratios describe the distribution of half-loop size for a sample which is useful for evaluating ensemble irregularity since an ensemble with a broad range of loop lengths tends to be less regular than one with a relatively narrow distribution of half-loop sizes. These indices of loop size distribution are more statistically stable than if only a single size class is used.

A third variable was included for experimental purposes. This variable, "sincurv" in the SPSS deck, was calculated by dividing a scale-independent variable, residual sinuosity, by average absolute dimensionless curvature which is scaled by channel width. The idea is to examine the interaction between two more or less independent morphometric variables with different degrees of scale dependence. The SPSS program listing, the PCA data, the correlation matrix, and variable means and standard deviations are given in Appendix II.

1. The Factors

The first five principal components explain 90.8% of the variance of the original variables set. The remaining factors have small eigenvalues (<1.0) and will not be discussed in the review below. Factors loadings and case weightings are listed in Table 3 and Table 4, respectively. Eigenvalues, percentage variance, and cumulative percentage variance for the five prominent factors are as follows:

Factor	Eigenvalue	Percent Variance	Cumulative Percent
1	14.5	51.9	51.9
2	6.3	22.7	74.5
3	2.0	7.1	81.7
4	1.3	4.8	86.4
5	1.2	4.3	90.8

2. First Principal Component

The first principal component accounts for nearly 52% of the variance and is characterized by high weightings on parameters that are scaled directly or indirectly by the channel width, including half-loop path length (primarily average and variance) and curvature (signed and absolute). This component, therefore, primarily measures variations in the characteristic size of meandering as measured by curvature and loop size that are not accounted for by normalization by channel width. High weighting on this

Table 3.

Factor loadings for the twenty-eight variables examined in the principal components analysis (see program listing, Appendix IIa., for variable names).

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5
UT	.3274	.4918	-.07836	.75293	-.20141
UH	.3843	.6603	-.03292	.48966	-.13549
UW	.2340	.1960	-.05545	.87517	-.21898
UR	.3004	.9282	-.06711	.12279	-.00731
SDCV	-.6993	.6922	-.13300	-.00633	.08528
UDCA	-.6486	.7312	-.16393	-.05298	.05304
UDCV	-.7441	.6357	-.09531	.04721	.11949
UDCS	-.7907	.4939	.03893	.07070	-.09909
UDCK	-.8122	.4930	-.08758	.11333	.13345
HLUDCA	-.4175	.8698	-.17899	.05890	-.05071
WLUDCA	-.2822	.8704	-.12129	.16942	-.13932
HLPATHA	.9638	.0493	-.02770	.16286	.11171
HLPATHV	.9548	-.1855	.11657	.13781	-.06431
HLPATHS	.7835	-.1360	.20689	.10860	-.37630
HLPATHK	.9028	-.2587	.16277	.13147	-.07403
SLHLA	.9013	-.3168	.03612	.16464	.15024
SVPEAK	.6905	-.1901	.21235	.25398	-.51267
SVRATIO	.0890	.0927	-.07495	-.23155	.70895
SINCURV	.8751	.3994	.06468	.17898	-.05267
UA	-.1540	.9443	-.14106	.06318	.11829
ASYM1	.0863	-.1367	.93146	-.02779	-.09281
ASYM2	.3240	-.2983	.80588	-.09791	-.10781
CRVSD	-.5869	.7455	-.13235	.08808	.19944
CRVAV	-.5370	.7921	-.21160	-.00494	.12995
CRVRAT	-.2268	-.3391	.51095	.46767	.40076
DIMCRV	-.2201	.8602	-.15993	.08738	.17092
RAT1	-.7416	.2066	-.25679	.07557	.31195
RAT2	-.8564	.0479	.09722	-.17394	-.31721

Table 4.

Case weightings for the thirty-three sample reaches (see text for factor interpretations).

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
1	-1.042229	-3.004897	-1.136229	-1.372782	-.915027
2	.256795	.078232	-1.515834	.987636	1.367532
3	.105376	.535573	-1.208474	.287680	.416312
4	-.121988	-1.292073	.715080	-1.420177	1.484117
5	1.004294	1.129379	-.735157	-.560999	.529932
6	.209680	-1.551965	-.164269	-.930919	.100219
7	-1.418066	.827702	1.885035	.002346	-.130611
8	.557320	.128578	1.802313	-1.063219	.359050
9	-.892886	-.574049	.458521	2.403678	1.778534
10	-1.124955	-.782184	1.099807	-.154486	.148483
11	.429953	.385420	.086832	-.971506	.819465
12	-1.929486	.867507	1.938466	-.009074	.570007
13	-1.024648	.910972	.197487	-1.780433	-.799152
14	-.906882	1.418174	-.836195	-1.131513	-1.322172
15	-.554545	.439040	.686504	1.109011	-.053081
16	2.379517	.352464	.858710	.200263	-1.625892
17	.173542	-.514620	.207041	-.014364	.627830
18	.538047	-1.934549	.046419	-.753921	-.787385
19	1.072651	.903000	-.178555	.475344	.408910
20	2.039810	.800317	-.067816	-1.264545	.382940
21	.002350	-.319039	-.207425	-.302965	-.358729
22	-1.520603	1.003962	-.778467	1.027194	-2.426873
23	.812555	.585203	.081665	.576372	-.752691
24	.003290	.778957	-1.609313	-.535353	.963070
25	-.604588	-.979474	-1.430933	.761557	.685609
26	.183068	.261006	-.142593	.560864	1.259518
27	-1.383525	.978088	-1.859661	-.526957	-.596355
28	.914118	-.812254	-.661464	1.665470	-1.373131
29	.692739	.245795	-.063749	-.087834	.690042
30	.947745	-.790810	1.399850	.955930	-1.865338
31	-.459871	-.577779	.391380	1.249969	-.073000
32	.137569	.645253	.728426	-.552179	.397299
33	.523854	-.140929	.012600	1.169911	.090567

parameter suggests that channel width, at least as measured from maps, may not be best scaling parameter for meandering, and that channel depth, channel cross-section, or some other parameter may be better. As previously discussed, self-scaling would be an alternative approach by using a parameter such as average half-loop length to reduced the importance of this component to emphasize more subtle aspects of meandering behavior.

High positive case weightings are shown in Table 4 for the Milk River [16] and the Red (B) River [20], and, correspondingly, long loop paths relative to channel width are apparent from the partial and especially the total channel traces of these two unimodal samples.

Note that half-loop path length is independent of sinuosity, that is, a given loop path may be highly curved or essentially straight depending upon the nature of the process-response mechanisms acting within the channel. Thus, while the meander pattern of the Milk and Red (B) Rivers is highly sinuous, a low-sinuosity channel with long half-loop paths would also be expected to have a high positive case weighting on this factor.

The Little Pee Dee (A) River [12] and the Rogue River [22] (bimodal and trimodal, respectively) are associated with high negative case weightings on the first factor suggesting that these samples are characterized by small half-loop path lengths. This result is supported by both of

the Little Pee Dee (A) River channel traces, and by the partial channel trace of the Rogue River which indicate that path lengths between successive inflection points are small. The total channel trace of the Rogue River does not appear to typify small-scale meandering because many of the smaller loops cannot be resolved at the scale to which the total channel is portrayed (examination of the partial centerline trace for this river indicates that many of the loops are little more than short wiggles).

The high positive and high negative case weighting discussed above are in agreement with average half-loop path length values for these rivers. While the average half-loop path length for all samples is 6.6 width units, corresponding values for the Milk River and the Red (B) River are 10.24 and 9.50, respectively, which represent the longest average half-loop path lengths of the rivers that were studied. The lowest average half-loop path length for all samples (4.41 width units) is associated with the Little Pee Dee (A) River, while the value of this parameter for the Rogue River is 5.00 width units (the sixth lowest). The nearly unimodal spectrum for rivers with high positive weightings on this vector, and the multimodal character of rivers with negative weightings, indicates that this component measures not only variations in meanderize relative to width, but also, to some degree, the relative

amounts of high-frequency meandering superimposed upon the dominant wavelength. The use of self-scaling might allow these two effects to be distinguished from one another.

3. Second Principal Component

The second vector, explaining 22.7% of the total variance, is related to sinuosity of half-loops as indicated by the high factor loadings on residual sinuosity and on average sinuosity for medium-size loops (7-21 width units); whole and half-loop dimensionless curvatures are also highly weighted on the second vector. The physical connotation of this component is that it describes the wandering of the individual bends of the ensemble. A river with large local sinuosity (i.e., tight bends) would be expected to be highly weighted on this vector.

The channel centerline traces of the Chena River [5] and the Lumber River [14] are examples of rivers which possess high residual sinuosity. These two samples are associated with the highest positive case weightings on the second vector, and the third and fifth largest values of average residual sinuosity of all samples, (residual sinuosity equals 1.42 and 1.40, respectively, where average residual sinuosity for the thirty-three rivers equal 1.27).

The Arkansas River [1] and the Powder River [18], on the other hand, are associated with large negative case weightings and the lowest and fourth lowest average residual

sinuosity values of all study rivers (residual sinuosity equals 1.05 and 1.13, respectively).

There appears to be a link between residual sinuosity and wavelength modality since each of the four samples discussed above belong to a high modal class: both the Chena River and the Powder River display a trimodal meander pattern, and the Lumber River and the Arkansas River are associated with multimodal meander behavior. Thus, this channel property appears to be related to the complex meander patterns that are characteristic of high-order modal classes. Note that the Arkansas River and the Powder River are rivers of low total sinuosity, so that sinuosity also influences the case weightings on this component.

4. Third Principal Component

A large decrease in the magnitude of the eigenvalues occurs between the second and third principal components. Indeed, while the first two vectors together explain 74.5% of the total variance, the remaining three vectors collectively account for only 16.3%. It should be clear that successive vectors in a principal components analysis are related to progressively less dominant system properties. This is borne out by the third vector which, like the first two axes, describes the nature of the half-loop path. In this case, 7.1% of the variance is associated with the generally subtle feature of loop asymmetry or

two asymmetry indices are -0.0270 , and -0.0133). The high positive case weighting associated with the Little Pee Dee (A) River appears to be anomalous since average half-loop asymmetry values of the two indices are -0.0038 and -0.0422 , the latter being one of the highest negative values observed in the study. Because the Little Pee Dee (A) River is also highly weighted on the first two principal components, the cause of this discrepancy may be related to the nature of the case weightings calculation.

The Savannah River [24] and Smiths Fork [27] are characterized by large negative case weightings on the third principal component and, thus, half-loops for these samples are typically skewed in the upstream direction. As before, the effect is subtle and is generally difficult to detect on even the high-resolution partial channel traces, though several prominent upstream-skewed loops are observed for each of these samples. Average values of the two asymmetry indices are -0.1880 and -0.0466 for the Savannah River, and -0.1304 and -0.0467 for Smiths Fork. These are the largest negative asymmetry values to appear in the analysis.

Except for one multimodal sample (Edisto River), all of these samples belong to either the unimodal (Elkhorn River and Smiths Fork) or the bimodal (Little Pee Dee (A) River and Savannah River) meander class suggesting that loop asymmetry may be best exhibited in low-order meandering.

5. Fourth Principal Component

Rather than a half-loop characteristic, an ensemble feature is the dominant property of meander geometry for the fourth vector. This vector describes large-scale valley wandering as implied by the high positive factor loadings for total and whole-loop sinuosity. Large-scale wandering of the channel path, generally associated with regional structural controls, is observed in the total channel traces of the Fox (A) River [9] and the Smoky Hill (A) [28]. In addition to having the largest positive case weightings on this vector, total sinuosity for the Fox (A) River and the Smoky Hill (A) River (2.50 and 3.47, respectively) are well above the sample mean of 2.05, and account for the eighth largest and the largest of all of the total sinuosity values in the study. Similarly, these two rivers are associated with the fifth largest and the largest whole-loop sinuosity values (1.96 and 2.71, respectively) relative to the sample mean of 1.53.

The Boulder River [4] and the Little Pee Dee (B) River [13] are associated with relatively linear ensemble trends as indicated by the corresponding total channel traces and their large negative case weightings. Total and whole-loop sinuosity for the Boulder River are 1.234 and 1.086 (second lowest and lowest of all sample values), and 1.561 and 1.193 for the Little Pee Dee (B) River (seventh and fifth lowest of all the study samples).

Since several meander wavelengths are dominant within both the Fox (A) River and the Smoky Hill (A) River (the latter being multimodal and the former representing an intermediate meander form displaying bimodal and multimodal spectral and channel features), it appears that large-scale valley wandering is associated with complex meandering that is characteristic of multimodal channel patterns. The absence of significant valley wandering does not appear to be related to a given modal class since both unimodal (Boulder River) and trimodal (Little Pee Dee (B) River) samples are associated with this ensemble feature.

6. Fifth Principal Component

The fifth principal component is related to the spread of dominant wavelength of the meander pattern as measured by SV Ratio. It is, therefore, a measure of meander irregularity. Interestingly, this parameter is not strongly correlated with properties such as the variance of absolute curvature or the variance of the half-loop path length which might also be expected to measure meander irregularity.

As discussed in a previous section, high SV Ratio values suggest that the spectral density function is peaked relative to the dominant spectral peak, and that whole-loop path length variability within the reach is low. Conversely, low SV Ratio values associated with broad

curvature spectra suggest that a wide range of whole-loop path lengths exist within the sample reach.

Large positive case weightings on the fifth principal component are observed for the Boulder River [4] and the Fox (A) River [9]. Both samples are characterized by predominantly medium-length whole-loop paths of various sinuosities. In the case of the unimodal Boulder River, some small whole-loops are present at a downstream end of the channel, but these loops account for only a relatively small portion of the total spectral variance of the sample.

The curvature spectrum and the two channel traces of the Fox (A) River indicate that this sample is bimodal with long whole-loops accounting for slightly less spectral variance than the medium length whole-loops. Though meander wavelengths are fairly uniform within this reach, the Fox (A) River does not appear to be an ideal example of low-variability whole-loop length as suggested by the large positive case weighting. Because this positive SV Ratio case weighting is not supported by the direct evidence provided by the curvature spectrum and channel traces, it is suspected that the SV Ratio statistic is an unreliable measure of the spread of the dominant wavelength of the meander pattern, as further discussed below. Values of SV Ratio for the Boulder River and the Fox (A) River are 4.932 and 4.158, respectively, which represent the third and sixth largest values of the total sample (mean SV Ratio for all

samples equals 2.845).

Large negative case weightings for this vector suggest that variable whole-loop lengths are displayed by the Rogue River [22] and the Solomon River [30]. The partial and total channel traces, and the trimodal curvature spectrum, support the indication that the Rogue River displays a wide range of meander wavelengths. Furthermore, this sample is associated with the lowest SV Ratio value of all samples (1.039). The SV Ratio of the Solomon River is the seventh lowest (1.612) of the river samples that were examined.

Because the Solomon River is unimodal, however, the total channel trace of this river does not appear to display highly variable meander wavelengths. A prominent secondary peak exists in this spectrum in addition to a number of smaller high-frequency peaks, but the collective effect of this high-frequency meandering is not enough to adequately explain the high negative case weighting that is associated with this sample.

Because the case weightings on the fifth principal component is not strongly supported by other evidence (e.g. channel traces and curvature spectra), it may be concluded that the SV Ratio parameter is not sufficiently robust to provide reliable estimates of the spread of the dominant spectral peak. The reasons for this are as follows. Since only two points from the spectral density function are used

to calculate SV Ratio values (i.e., the period corresponding to the maximum spectral variance, and the period corresponding to one-half the maximum spectral variance), this statistic is sensitive to the location of the spectral peak and, in particular, to fluctuations in the high-frequency portion of the spectrum. The practice of using this parameter to characterize features of polymodal spectra is, thus, questionable. In later studies, a more robust measure of this channel property will be developed which will incorporate, for example, averaging of the spectral peaks.

Summarizing the results of the principal components analysis, the five significant dimensions of meander geometry are: half-loop length (relative to the channel width), sinuosity of half-loops, and half-loop asymmetry (skewness), large-scale winding of the meander path (the only ensemble-related quality of the meander pattern to be revealed in this analysis), and the range of dominant wavelengths of meandering.

High positive case weightings on the first principal component (long half-loop lengths) appear to be associated with unimodal meandering. The relationship between short half-loops and wavelength modality is less clear since the largest negative case weightings are divided between bimodal and trimodal meander patterns. The second principal component (half-loop sinuosity) is related to complex

meander patterns since both the highest and lowest case weightings are associated with trimodal and multimodal meandering.

The third vector corresponds to half-loop asymmetry where positive case weightings indicate downstream, and negative weightings upstream skewness. This meander property tends to be associated with unimodal and bimodal channel patterns, though a high positive weighting is also observed for a multimodal sample.

Large-scale channel wandering described by high positive weightings on the fourth principal component appears to be related to multimodal meandering, while the absence of large-scale meandering is not strongly associated with a given modal class.

The fifth principal component describes the spread of the dominant spectral peak (in terms of SV Ratio), but the reliability of the associated case weighting is questionable due to the nature of the SV Ratio parameter. The strong association expected between high positive weightings and unimodal meandering, and between high negative weightings and trimodal or multimodal meander patterns, has not been adequately demonstrated because the SV Ratio statistic is highly sensitive to the distribution of peaks in the spectral density function. A more robust measure of the shape of the dominant spectral peak is needed for reliable estimates of this channel property.

VI. CONCLUSION

The results of this research suggest that meander sampling procedures have a greater influence on the outcome of statistical analysis than has been previously acknowledged. The extent of this influence has been illustrated quantitatively through the use of a mathematically defined control sample. The statistical properties of the control were known which allowed tests of statistical sensitivity of the variables to be made; an optimum sampling procedure was derived on the basis of the sensitivity analysis. The results of this aspect of the study have led to the following guidelines for digital sampling of meander patterns:

1. The number of points per unit distance for raw digitized data should be about twice that of the sampling interval (step length) used for producing the curvature series.
2. Adequate definition of a meander trace is attained using a step length of $1.0 \bar{w}$ (based upon width measurements derived from U.S.G.S. 7 1/2-minute topographical maps).
3. Replicate sampling is an effective way to minimize high-frequency sampling error associated with manually digitized channel centerline data.

4. An 11-point moving average is sufficient for smoothing the merged file of the three replicates to produce an average centerline trace given the raw point density described above.
5. Long reach lengths are required to produce stable statistics.

The above sampling guidelines specification yield accurate, consistent and unbiased results, and thus describe a robust methodology for collecting channel centerline data. Such procedures are necessary for studies of channel morphometrics where subtle features of meander geometry are investigated.

A suite of morphometric variables has been introduced that appears to be more comprehensive than any that has been previously reported. The essential difference between this and other sets of descriptive statistics is that meander properties are quantified at a variety of scales. Local, medium-scale, and ensemble attributes of meander patterns have been defined to yield a more complete characterization of channel geometry than has been previously described by other researchers.

Sophisticated meander sampling and parameterization techniques allow fluvial process-response mechanisms to be rigorously examined. Correlations between environmental controls and meander form may be made with greater accuracy than has been possible in the past. Furthermore,

length, 2) half-loop sinuosity, 3) asymmetry of the meander loops, 4) large-scale valley wandering, and 5) spread of meander wavelengths (meander irregularity) based upon the SDF.

Several areas of future research have been identified as a result of this study. First, a wider range of river samples is needed to 1) further explore the nature of, and variation in, natural meandering channels, and 2) enhance the reliability of the statistical analyses. In particular, streams of low sinuosity and confined meandering streams are needed to examine meander features that are otherwise difficult to detect from the more "normal" samples. Also, very large and very small streams must be studied to investigate the scale dependence of the morphometric patterns. Finally, other natural meander forms such as those created by supraglacial streams, the Gulf Stream, and by water streams on hydrophobic surfaces (e.g. see Nakagawa and Scott, 1984) should be included for analysis to examine the full extent of natural meander phenomena.

Second, other quantitative measures of the curvature spectrum are needed to take full advantage of the information contained in the spectral density function. A more robust measure of the dominant wavelength is needed because the period corresponding to the maximum spectral variance does not necessarily characterize the nature of the dominant peak. Low-order curve fitting, or averaging of the

points which define the spectral peaks, may prove useful for this purpose. Measures of the peakedness of the dominant peak(s) are needed to further resolve channel pattern modality. Lastly, criteria for evaluating the degree of multimodality must be developed to allow multimodal streams of different character to be distinguished from one another. Because non-stationary data may produce multimodal or broad spectral peaks, quantitative tests of stationarity should be developed to assess the nature of sample multimodality.

Finally, self-scaling using average half-loop length or the dominant wavelength based on the spectral density function has emerged as a new aspect of meander geometry studies. Average channel width as determined from maps is typically used for scaling, but this parameter is not always representative of the ensemble pattern as a whole, and is subject to error associated with the map making process. The need for an improved scaling parameter is evident from the principal components analysis since several factors are related directly or indirectly to the different width-scaled variables. This result suggests that more subtle meander properties cannot be discerned because of the influence of channel width on these statistical results.

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APPENDICES

APPENDIX I

Spectral density functions of the thirty-three centerline curvature series. Ordinates represent spectral variance, and abscissas represent frequency in width units.

$\times 10^{-3}$

$\times 10^{-2}$

$\times 10^{-2}$

0.25

[3] BLACKS FORK (B)

0.20

0.15

0.10

0.05

0.00

0.00

0.05

0.10

0.15

0.20

0.25

$\times 10^{-2}$

$\times 10^{-3}$

$\times 10^{-2}$

$\times 10^{-3}$

$\times 10^{-2}$

$\times 10^{-2}$

$\times 10^{-3}$

$\times 10^{-3}$

0.8

[17] OBION RIVER

0.6

0.4

0.2

0.0

0.00

0.05

0.10

0.15

0.20

0.25

$\times 10^{-3}$

$\times 10^{-3}$

$\times 10^{-3}$

$\times 10^{-3}$

$\times 10^{-2}$

$\times 10^{-3}$

$\times 10^{-2}$

$\times 10^{-3}$

$\times 10^{-3}$

$\times 10^{-3}$

$\times 10^{-3}$

[30] SOLOMON RIVER

$\times 10^{-3}$

$\times 10^{-2}$

$\times 10^{-3}$

[33] TONGUE RIVER (B)

APPENDIX II

Principal Components Analysis

Appendix IIa. Program listing

Appendix IIb. Data

Appendix IIc. Means and standard deviations

Appendix IId. Correlation matrix

Appendix IIe. Factor matrix

APPENDIX IIa.

Listing for Principal Components Analysis Program

```

RUN NAME          SPSS PRINCIPAL COMPONENTS ANALYSIS
PAGESIZE          55
VARIABLE LIST     CASE,UT,UH,UW,UR,SDCV,UDCA,UDCV,UDCS,
                  UDCK,HLUDCA,WLUDCA,HLPATHA,HLPATHV,HLPATHS,HLPATHK,
                  SLHLA,SVPEAK,SVRATIO,UA7,ASYM17,ASYM27,CRVSD7,CRVAV7,
                  CRVRAT7,DIMCRV7,
                  UA11,ASYM111,ASYM211,CRVSD11,
                  CRVAV11,CRVRAT11,DIMCRV11,XN1,XN2,XN4,XN7,XN11,XN21
INPUT FORMAT      FREEFIELD
PRINT FORMAT      UT TO XN21 (5)
N OF CASES        33
INPUT MEDIUM      DISK
COMPUTE           UT=UT-1.
COMPUTE           UH=UH-1.
COMPUTE           UW=UW-1.
COMPUTE           UR=UR-1.
COMPUTE           SVRATIO=SVRATIO-1.
COMPUTE           XNT=XN2+XN4+XN7+XN11+XN1+XN21
COMPUTE           SINCURV = UR/UDCA
COMPUTE           UA=((UA7 * XN7 + UA11 * XN11)/(XN7 + XN11))-1.
COMPUTE           ASYM1=(ASYM17 * XN7 + ASYM111 * XN11)/(XN7 + XN11)
COMPUTE           ASYM2=(ASYM27 * XN7 + ASYM211 * XN11)/(XN7 + XN11)
COMPUTE           CRVSD=(CRVSD7 * XN7 + CRVSD11 * XN11)/(XN7 + XN11)
COMPUTE           CRVAV=(CRVAV7 * XN7 + CRVAV11 * XN11)/(XN7 + XN11)
COMPUTE           CRVRAT=(CRVRAT7 * XN7 + CRVRAT11 * XN11)/(XN7 + XN11)
COMPUTE           CRVRAT=CRVRAT-1.
COMPUTE           DIMCRV=(DIMCRV7 * XN7 + DIMCRV11 * XN11)/(XN7 + XN11)
COMPUTE           RAT1=(XN4+XN7)/XNT
COMPUTE           RAT2=(XN2+XN1)/(XN11+XN21)
COMPUTE           UT      = LG10 (   UT      )
COMPUTE           UH      = LG10 (   UH      )
COMPUTE           UW      = LG10 (   UW      )
COMPUTE           UR      = LG10 (   UR      )
COMPUTE           SDCV     = LG10 (   SDCV     )
COMPUTE           UDCA     = LG10 (   UDCA     )
COMPUTE           UDCV     = LG10 (   UDCV     )
COMPUTE           UDCK     = LG10 (   UDCK     )
COMPUTE           HLUDECA  = LG10 (   HLUDECA  )
COMPUTE           WLUDCA   = LG10 (   WLUDCA   )
COMPUTE           HLPATHA  = LG10 (   HLPATHA  )
COMPUTE           HLPATHV  = LG10 (   HLPATHV  )
COMPUTE           HLPATHK  = LG10 (   HLPATHK  )
COMPUTE           SVRATIO  = LG10 (   SVRATIO  )
COMPUTE           UA       = LG10 (   UA       )
COMPUTE           CRVSD     = LG10 (   CRVSD     )
COMPUTE           CRVAV     = LG10 (   CRVAV     )
COMPUTE           CRVRAT   = LG10 (   CRVRAT   )
COMPUTE           DIMCRV   = LG10 (   DIMCRV   )
COMPUTE           RAT2     = LG10 (   RAT2     )
COMPUTE           SINCURV  = LG10 (   SINCURV  )
COMPUTE           SLHLA    = LG10 (   SLHLA    )

```


VAR LABELS

UT TOTAL SINUOSITY/
 UH HALFLOOP SINUOSITY/
 UW WHOLELOOP SINUOSITY/
 UR RESIDUA7L SINUOSITY/
 SDCV SIGNED DIM.CURV VAR/
 UDCA UNSIGNED DIM.CURV AVG/
 UDCV UNSIGNED DIM.CURV VAR/
 UDCS UNSIGNED M.CURV SKEW/
 UDCK UNSIGNED DIM.CURV KURT/
 HLU DCA HALFLOOP UNSIGNED DIM.CURV AVG/
 WLUDCA WHOLELOOP UNSIGNED DIM.CURV AVG/
 HLPATHA HALFLOOP PATH AVG/
 HLPATHV HALFLOOP PATH VAR/
 HLPATHS HALFLOOP PATH SKEW/
 HLPATHK HALFLOOP PATH KURT/
 SLHLA STRAIGHT HALFLOOP AVG/
 SVPEAK SPECTRAL PEAK/
 SVRATIO PEAK:HALFPEAK VAR/
 UA AVG HALFLOOP SINUOSITY/
 ASYM1 ASYM INDEX1/
 ASYM2 ASYM INDEX2/
 CRVSD STD DEV CURV/
 CRVAV AVG HALFLOOP CURV/
 CRVRAT CURVATURE RATIO/
 DIMCRV DIMENSIONLESS CURV/
 VARIABLES = UT TO SVRATIO,
 SINCURV,UA,ASYM1,ASYM2,CRVSD,CRVAV,
 CRVRAT,DIMCRV,RAT1,RAT2/
 TYPE = PA1/
 ROTATE = VARIMAX/
 FACSCORE

FACTOR

OPTIONS

STATISTICS

READ INPUT DATA

FINISH

/EOR

5,6,7,8,11
 1,2,4,5,6,7

APPENDIX IIb.

Raw Data for Principal Components Analysis
(38 variables for 33 cases)

TOTAL, HALF-, WHOLE-LOOP, RESIDUAL SINUOSITY; VAR SIGNED DIM.CURV;
AVE, VAR, SKEW, KURT UNSIGNED DIM.CURV;

AVE HALF-LOOP UNSIGNED DIM.CURV;
AVE WHOLE-LOOP UNSIGNED DIM.CURV; AVE, VAR, SKEW, KURT HALF-LOOP PATH;
AVE STRAIGHT LINE HALF-LOOP; PERIOD SPECTRAL PEAK;

SV PEAK / HALF PEAK; L7-10 HALF-LOOP SUMMARY STATS: AVE SINUOSITY,
ASYM INDEX1, ASYM INDEX2, CURV STD.DEV., AVE CURV, CURV RATIO,
DIM.CURV;

L11-20 HALF-LOOP SUMMARY STATS: AVE SINUOSITY, ASYM INDEX1, ASYM INDEX2,
CURV STD.DEV., AVE CURV, CURV RATIO, DIM.CURV;

NUMBER OF LOOPS BY SIZE CATEGORY: L1, L2-3, L4-6, L7-10, L11-20, L21-30

```
=====
1 1.194 1.008 1.130 1.047 5.998E-3 5.579E-2 2.884E-3 3.559E-4 1.118E-4
  2.428E-2 1.931E-2 4.747 1.431E+1 8.433E+1 1.198E+3 4.533 16.799
  1.943 1.055 -.0083 -.0037 .0335 .0539 2.0023 0.4397
  1.082 -.1604 -.0381 .0270 .0425 2.1424 .5721
  82. 181. 136. 86. 45. 1.
2 2.580 1.049 1.892 1.299 1.877E-2 1.013E-1 8.496E-3 1.334E-3 4.921E-4
  4.720E-2 3.163E-2 7.400 2.124E+1 8.680E+1 1.610E+3 5.698 20.415
  5.657 1.325 -.0678 -.0285 .0660 .1082 1.9234 0.8994
  1.627 -.1511 -.0538 .0646 .0964 2.2418 1.2932
  15. 50. 76. 81. 61. 4.
3 2.314 1.076 1.605 1.340 2.143E-2 1.129E-1 8.664E-3 1.022E-3 3.801E-4
  5.642E-2 3.482E-2 6.438 1.781E+1 7.586E+1 1.421E+3 4.807 19.933
  3.161 1.357 -.0647 -.0338 .0739 .1188 1.9497 0.9402
  1.797 -.1051 -.0423 .0678 .1119 2.1303 1.4439
  21. 34. 51. 49. 31. 1.
4 1.234 1.014 1.086 1.129 8.869E-3 6.926E-2 4.066E-3 4.765E-4 1.356E-4
  3.057E-2 1.823E-2 6.066 2.128E+1 9.575E+1 1.670E+3 5.378 20.069
  4.932 1.134 .0251 .0155 .0511 .0754 2.1273 0.6346
  1.199 -.0042 -.0307 .0425 .0638 2.2958 .833
  23. 36. 30. 34. 26. 1.
5 2.097 1.060 1.391 1.423 1.922E-2 1.029E-1 8.638E-3 1.448E-3 5.396E-4
  5.982E-2 3.577E-2 8.439 3.900E+1 2.369E+2 5.303E+3 5.934 19.648
  2.525 1.650 -.0695 -.0334 .0862 .1329 1.9762 1.1247
  1.625 -.0428 -.0273 .0535 .0943 1.9856 1.3039
  10. 27. 30. 28. 38. 8.
6 1.429 1.007 1.233 1.151 6.982E-3 5.682E-2 3.752E-3 6.446E-4 2.149E-4
  2.292E-2 1.656E-2 6.403 2.984E+1 3.286E+2 7.639E+3 5.540 30.239
  2.064 1.104 -.0800 -.0219 .0340 .0532 1.9582 0.4440
  1.291 .0387 .0094 .0412 .0624 2.1729 .8625
  15. 36. 44. 22. 23. 3.
```

- 7 1.931 1.039 1.482 1.254 3.761E-2 1.351E-1 1.934E-2 4.539E-3 2.205E-3
 6.670E-2 4.656E-2 4.712 1.036E+1 5.419E+1 7.359E+2 3.760 10.280
 3.237 1.523 .0773 .0112 .0776 .1229 1.9874 0.9780
 1.470 .0418 -.0245 .0656 .0884 2.6473 1.301
 27. 73. 86. 43. 11. 0.
- 8 1.536 1.035 1.201 1.236 1.082E-2 7.477E-2 5.224E-3 7.383E-4 2.185E-4
 3.731E-2 2.661E-2 7.203 3.370E+1 2.250E+2 5.087E+3 5.832 31.091
 2.013 1.233 .1071 .0250 .0596 .0890 2.0374 0.7506
 1.346 .1089 .0030 .0431 .0723 2.1308 .9988
 37. 34. 41. 45. 47. 5.
- 9 2.498 1.064 1.959 1.199 2.013E-2 9.337E-2 1.141E-2 2.953E-3 1.375E-3
 4.080E-2 2.968E-2 5.927 1.896E+1 1.419E+2 2.331E+3 4.949 8.642
 4.158 1.272 .0190 .0116 .0752 .1009 2.0792 0.8027
 1.300 -.1765 -.0825 .0661 .0743 2.9305 1.0364
 11. 47. 59. 40. 18. 3.
- 10 1.418 1.033 1.219 1.126 1.515E-2 8.228E-2 8.376E-3 2.049E-3 9.026E-4
 3.851E-2 2.669E-2 5.073 1.262E+1 6.201E+1 8.935E+2 4.506 16.473
 1.774 1.273 -.0515 -.0253 .0680 .0940 2.1144 0.7388
 1.223 .2165 .0666 .0451 .0678 2.3273 .8413
 20. 46. 54. 30. 13. 1.
- 11 1.594 1.036 1.175 1.310 1.650E-2 9.653E-2 7.175E-3 8.412E-4 2.708E-4
 4.083E-2 2.742E-2 6.917 2.454E+1 1.867E+2 3.809E+3 5.300 23.035
 2.311 1.368 -.0526 -.0108 .0714 .1064 2.1212 0.8934
 1.702 .0022 -.0126 .0545 .0963 2.0165 1.2774
 6. 27. 37. 28. 23. 2.
- 12 1.690 1.027 1.284 1.281 5.630E-2 1.738E-1 2.608E-2 5.269E-3 2.835E-3
 7.397E-2 4.719E-2 4.409 6.728E+0 2.597E+1 3.186E+2 3.442 8.360
 4.607 1.508 -.0038 -.0422 .1018 .1431 2.1867 1.1223
 1.405 .2700 .0825 .0784 .1010 2.4958 1.1947
 31. 134. 165. 51. 14. 0.
- 13 1.561 1.027 1.193 1.274 4.332E-2 1.537E-1 1.966E-2 3.811E-3 1.935E-3
 6.653E-2 4.268E-2 4.724 9.836E+0 4.868E+1 7.035E+2 3.711 11.299
 3.169 1.507 -.1056 -.0185 .0737 .1332 1.8802 1.0397
 1.584 .1819 .0310 .0514 .0784 2.0901 1.0334
 19. 36. 53. 22. 8. 0.
- 14 1.941 1.053 1.314 1.403 5.148E-2 1.784E-1 1.961E-2 2.942E-3 1.490E-3
 8.073E-2 4.692E-2 5.057 8.830E+0 2.672E+1 3.117E+2 3.614 15.467
 1.727 1.666 -.0537 -.0303 .0916 .1567 1.8788 1.2811
 1.937 -.1087 -.0265 .0806 .1311 2.0055 1.6202
 17. 68. 96. 49. 13. 0.
- 15 2.256 1.059 1.628 1.309 2.433E-2 1.094E-1 1.236E-2 2.426E-3 9.904E-4
 5.509E-2 3.647E-2 6.184 1.767E+1 7.922E+1 1.420E+3 4.725 26.710
 1.258 1.462 -.0120 -.0023 .0758 .1099 2.0174 0.9094
 1.685 -.0311 .0029 .0714 .0974 2.435 1.2792
 61. 70. 149. 110. 67. 3.
- 16 3.174 1.107 2.035 1.408 5.160E-3 5.457E-2 2.181E-3 1.723E-4 3.587E-5
 3.545E-2 3.101E-2 10.24 1.093E+2 1.349E+3 4.363E+4 7.274 62.333
 2.328 1.083 .0120 .0419 .0317 .0503 1.9233 0.4343
 1.281 .0147 .0032 .0357 .0577 2.0777 .8702
 135. 106. 62. 60. 125. 68.
- 17 1.799 1.028 1.451 1.206 1.122E-2 6.968E-2 6.365E-3 1.250E-3 4.344E-4
 3.574E-2 2.652E-2 6.361 3.134E+1 2.831E+2 5.846E+3 5.281 18.120
 3.934 1.210 .0103 .0151 .0532 .0766 2.0146 0.6372
 1.373 -.1267 -.0192 .0464 .0713 2.2548 .9710
 55. 73. 68. 70. 45. 10.

18 1.346 1.028 1.156 1.133 4.905E-3 4.993E-2 2.410E-3 2.891E-4 8.090E-5
 2.414E-2 1.725E-2 7.089 4.302E+1 5.586E+2 1.571E+4 6.258 34.413
 1.368 1.089 -.0010 .0228 .0321 .0510 1.9460 0.4231
 1.176 -.0409 -.0161 .0341 .0519 2.297 .07547
 24. 36. 37. 37. 28. 5.
 19 2.302 1.097 1.431 1.467 1.370E-2 8.484E-2 6.499E-3 1.028E-3 3.695E-4
 5.500E-2 4.392E-2 8.252 4.690E+1 3.409E+2 8.498E+3 5.626 37.883
 2.246 1.347 -.0236 -.0187 .0668 .0942 2.0472 0.8000
 1.73 -.0958 -.0371 .0599 .0882 2.2389 1.2831
 86. 86. 93. 98. 139. 28.
 20 1.965 1.071 1.263 1.452 8.236E-3 6.937E-2 3.422E-3 2.860E-4 7.051E-5
 4.000E-2 3.069E-2 9.498 6.599E+1 5.581E+2 1.710E+4 6.537 28.038
 4.353 1.229 -.0235 -.0019 .0505 .0746 1.9599 0.6436
 1.541 -.0430 -.0171 .0475 .0773 2.0331 1.0967
 57. 60. 27. 54. 100. 31.
 21 1.643 1.035 1.318 1.205 1.290E-2 8.717E-2 5.297E-3 7.429E-4 3.048E-4
 4.438E-2 3.204E-2 6.137 2.417E+1 2.022E+2 4.338E+3 5.096 27.737
 1.536 1.168 -.0594 -.0214 .0533 .0932 1.8886 0.7439
 1.367 -.0089 .0020 .0559 .0844 2.2712 1.1335
 21. 48. 60. 38. 34. 2.
 22 2.797 1.061 1.929 1.366 4.855E-2 1.606E-1 2.274E-2 5.036E-3 2.594E-3
 7.992E-2 4.731E-2 4.996 9.704E+0 3.550E+1 4.288E+2 3.647 18.399
 1.039 1.733 -.0663 -.0407 .1033 .1563 2.0187 1.2581
 1.733 -.0663 -.0407 .1033 .1563 2.0187 1.2581
 27. 103. 120. 67. 21. 0.
 23 2.813 1.087 1.902 1.360 1.195E-2 8.160E-2 5.286E-3 6.679E-4 2.227E-4
 5.023E-2 4.084E-2 7.276 4.124E+1 3.324E+2 7.528E+3 5.351 36.347
 1.627 1.215 -.0494 -.0271 .0504 .0841 1.8964 0.7143
 1.616 .0255 -.0019 .0569 .0887 2.1826 1.2644
 267. 273. 244. 260. 308. 53.
 24 1.877 1.054 1.343 1.326 2.825E-2 1.209E-1 1.361E-2 2.654E-3 1.227E-3
 6.745E-2 4.269E-2 6.276 1.789E+1 1.348E+2 2.493E+3 4.733 10.280
 10.341 1.546 -.1880 -.0466 .0741 .1269 1.8372 1.0200
 1.621 -.0588 .0009 .0610 .0949 2.2366 1.2598
 25. 79. 162. 104. 49. 4.
 25 1.843 1.030 1.544 1.159 1.534E-2 8.735E-2 7.710E-3 1.476E-3 5.866E-4
 4.142E-2 3.012E-2 6.018 1.558E+1 5.733E+1 8.880E+2 5.192 11.990
 2.935 1.211 -.0024 -.0276 .0555 .0881 1.9853 0.7258
 1.289 -.2324 -.0704 .0522 .0785 2.3145 1.0175
 43. 89. 130. 110. 59. 0.
 26 2.123 1.055 1.566 1.285 1.738E-2 9.599E-2 8.168E-3 1.106E-3 3.559E-4
 4.807E-2 3.617E-2 6.976 2.272E+1 1.531E+2 3.147E+3 5.432 16.285
 3.727 1.378 -.0370 -.0061 .0707 .1081 2.0160 0.9157
 1.532 -.0673 -.0195 .0602 .0899 2.2689 1.2044
 26. 60. 103. 89. 59. 5.
 27 1.706 1.048 1.207 1.349 5.432E-2 1.756E-1 2.344E-2 4.543E-3 2.356E-3
 8.384E-2 5.447E-2 4.720 7.596E+0 2.293E+1 2.378E+2 3.497 13.116
 2.826 1.692 -.1304 -.0467 .1056 .1671 1.9984 1.3506
 1.461 -.3882 -.2052 .0758 .1028 2.3672 1.2786
 6. 52. 51. 29. 5. 0.
 28 3.467 1.063 2.708 1.204 6.654E-3 5.902E-2 3.171E-3 4.037E-4 1.154E-4
 3.618E-2 2.888E-2 8.017 5.113E+1 5.348E+2 1.412E+4 6.657 61.018
 1.482 1.119 -.0337 -.0150 .0393 .0631 1.9547 0.5330
 1.283 -.0227 -.0167 .0379 .0616 2.1375 .8869
 105. 147. 140. 136. 147. 46.

29 2.097 1.037 1.575 1.283 1.326E-2 8.348E-2 6.284E-3 8.569E-4 2.659E-4
 4.249E-2 3.156E-2 7.720 3.218E+1 2.644E+2 6.324E+3 6.019 21.452
 2.981 1.306 -.0682 -.0288 .0602 .1006 1.8859 0.8399
 1.487 .0687 .0132 .0517 .0838 2.1680 1.1521
 19. 41. 56. 60. 53. 6.
 30 2.628 1.065 2.058 1.199 4.907E-3 5.046E-2 2.361E-3 2.567E-4 6.275E-5
 3.018E-2 2.739E-2 6.861 5.505E+1 7.384E+2 2.080E+4 5.722 73.872
 1.612 1.074 .0403 .0151 .0286 .0465 1.9498 0.3875
 1.242 .0655 .0265 .0346 .0543 2.2595 .8336
 295. 300. 208. 150. 206. 66.
 31 2.389 1.036 1.929 1.196 1.545E-2 8.433E-2 8.338E-3 1.683E-3 6.711E-4
 3.747E-2 2.574E-2 6.242 2.090E+1 1.366E+2 2.423E+3 5.220 24.139
 1.463 1.225 -.0116 -.0110 .0537 .0840 1.9797 0.6916
 1.403 .0699 -.0036 .0540 .0792 2.3352 1.0650
 37. 121. 139. 101. 69. 7.
 32 1.748 1.057 1.260 1.313 1.796E-2 1.017E-1 7.612E-3 9.497E-4 3.372E-4
 5.091E-2 3.516E-2 6.867 1.895E+1 7.262E+1 1.162E+3 5.234 22.902
 1.691 1.321 -.0183 .0053 .0651 .1049 1.9075 0.8598
 1.714 .0681 .0107 .0686 .1020 2.2995 1.4086
 9. 34. 49. 49. 31. 1.
 33 2.652 1.051 2.007 1.258 1.135E-2 7.608E-2 5.564E-3 8.603E-4 2.831E-4
 4.547E-2 3.267E-2 7.582 3.679E+1 3.588E+2 8.411E+3 6.031 35.722
 1.853 1.201 .0052 -.0051 .0485 .0749 1.9603 0.6227
 1.501 -.0414 -.0066 .0511 .0774 2.2473 1.1623
 18. 26. 52. 35. 33. 5.

APPENDIX IIc.

Variable Means and Standard Deviations
(see program listing, Appendix IIa., for variable names)

VARIABLE	MEAN	STANDARD DEV	CASES
UT	2.0498	.5577	33
UH	1.0484	.0235	33
UW	1.5295	.3726	33
UR	1.2712	.1024	33
SDCV	.0200	.0151	33
UDCA	.0966	.0370	33
UDCV	.0093	.0067	33
UDCS	.0017	.0015	33
UDCK	.0007	.0008	33
HLUDCA	.0482	.0166	33
WLUDCA	.0334	.0095	33
HLPATHA	6.5705	1.3919	33
HLPATHV	28.3992	20.7649	33
HLPATHS	239.1579	269.2680	33
HLPATHK	5985.9333	8577.8380	33
SLHLA	5.1678	.9577	33
SVPEAK	25.2275	15.4670	33
SVRATIO	2.8448	1.7871	33
SINCURV	3.0014	1.3876	33
UA	1.3948	.1790	33
ASYM1	-.0270	.0577	33
ASYM2	-.0133	.0194	33
CRVSD	.0607	.0190	33
CRVAV	.0940	.0291	33
CRVRAT	2.0820	.0877	33
DIMCRV	.9378	.2364	33
RAT1	.4771	.1001	33
RAT2	3.1171	3.0124	33

APPENDIX IId.
Correlation Matrix

	UT	UH	UW	UR	SDCV	UDCA	UDCV	UDCS	UDCK	HLUDCA
UT	1.00000	.81598	.93379	.64648	.10570	.11387	.09469	.05917	.04341	.35190
UH	.81598	1.00000	.62435	.77487	.16810	.20431	.12330	.04287	.02395	.40154
UW	.93379	.62435	1.00000	.34511	-.03865	-.05481	-.01937	-.00902	-.00996	.13927
UR	.64648	.77487	.34511	1.00000	.44445	.49186	.38250	.24496	.23475	.68814
SDCV	.10570	.16810	-.03865	.44445	1.00000	.99256	.99046	.89039	.93826	.91050
UDCA	.11387	.20431	-.05481	.49186	.99256	1.00000	.96643	.85624	.89257	.93233
UDCV	.09469	.12330	-.01937	.38250	.99046	.96643	1.00000	.91439	.97289	.87065
UDCS	.05917	.04287	-.00902	.24496	.89039	.85624	.91439	1.00000	.91117	.76244
UDCK	.04341	.02395	-.00996	.23475	.93826	.89257	.97289	.91117	1.00000	.77563
HLUDCA	.35190	.40154	.13927	.68814	.91050	.93233	.87065	.76244	.77563	1.00000
HLUDCA	.49525	.61180	.26985	.72777	.79243	.81677	.75240	.67840	.65510	.95288
HLPATHA	.43321	.45982	.34287	.36025	-.62364	-.58340	-.65733	-.73404	-.71781	-.35266
HLPATHV	.32186	.30922	.30653	.12700	-.81753	-.79076	-.83293	-.80242	-.84932	-.56727
HLPATHS	.33809	.31596	.32330	.12891	-.69096	-.65932	-.71466	-.53653	-.75095	-.45169
HLPATHK	.26906	.21980	.26662	.04259	-.83430	-.81628	-.83703	-.77659	-.83178	-.62007
SLALA	.22719	.18536	.25844	-.01080	-.83755	-.81854	-.84326	-.87445	-.84338	-.65683
SVPEAK	.40233	.31916	.42364	.05576	-.69079	-.65636	-.71558	-.58080	-.73230	-.44142
SVRATIO	-.16979	-.09617	-.22925	.05910	.07880	.07652	.07882	-.01919	.04096	.04038
SINCURV	.61137	.67565	.42375	.67973	-.34988	-.30429	-.39564	-.45330	-.49508	-.03253
UA	.44091	.55572	.17547	.06128	.79038	.81070	.75330	.56120	.64242	.88792
ASYM1	-.09156	-.12315	-.02069	-.16400	-.28541	-.30496	-.25707	-.13938	-.24474	-.32440
ASYM2	-.13404	-.12717	-.04252	-.26115	-.54193	-.55994	-.51256	-.40461	-.48900	-.54396
CRVSD	.16724	.30004	.01247	.54644	.95748	.95275	.94502	.80493	.88316	.90337
CRVAV	.19014	.30908	-.00471	.58953	.96271	.97319	.93330	.77277	.85164	.93734
CRVRAT	-.07156	-.10775	.06442	-.32939	-.12489	-.19397	-.04396	.05937	.05530	-.27273
DIACRV	.41130	.45105	.19474	.72858	.77275	.79651	.73144	.52262	.62561	.85300
RAT1	-.12246	-.15495	-.10206	-.00212	.73535	.70136	.76065	.59620	.78683	.51297
RAT2	-.31635	-.32184	-.27505	-.23223	.59153	.56318	.61192	.79028	.64203	.39592

	HLUDCA	HLPATHA	HLPATHV	HLPATHS	HLPATHK	SLHLA	SVPEAK	SVRATIO	SINCURV	UA
UT	.49525	.43321	.32186	.33809	.26906	.22719	.40233	-.16979	.61137	.44091
UH	.61180	.45982	.30922	.31596	.21980	.18530	.31918	-.09617	.67565	.55572
UM	.28985	.34287	.30653	.32330	.28662	.25844	.42364	-.22925	.42375	.17547
UR	.72777	.36825	.12700	.12891	.04259	-.01080	.05576	.05910	.67973	.86128
SDCV	.79243	-.62364	-.81753	-.69096	-.83430	-.83755	-.69079	.07880	-.34988	.79038
UDCA	.81677	-.58340	-.79076	-.65932	-.81828	-.81854	-.65636	.07652	-.30429	.81870
UDCV	.75240	-.65733	-.83293	-.71486	-.83703	-.84328	-.71558	.07882	-.39564	.75330
UDCS	.67840	-.73404	-.80242	-.53653	-.77659	-.87445	-.58080	-.01919	-.45330	.56120
UDCK	.65510	-.71781	-.84932	-.75095	-.83178	-.84338	-.73230	.04096	-.49508	.64242
HLUDCA	.95288	-.35266	-.56727	-.45169	-.62007	-.65683	-.44142	.04038	-.03253	.88792
HLUDCA	1.00000	-.23353	-.40957	-.26892	-.46031	-.54855	-.25871	.03174	.10818	.83614
HLPATHA	-.23353	1.00000	.92454	.78806	.85556	.92272	.61442	.06597	.88560	-.06619
HLPATHV	-.40957	.92454	1.00000	.84363	.97626	.93551	.77280	-.02471	.80508	-.33548
HLPATHS	-.26892	.78806	.84363	1.00000	.84050	.69870	.82802	-.08031	.69645	-.36852
HLPATHK	-.46031	.85556	.97626	.84050	1.00000	.90420	.76329	-.01911	.73593	-.39600
SLHLA	-.54855	.92272	.93551	.69870	.90420	1.00000	.64037	.05324	.67772	-.40689
SVPEAK	-.25871	.61442	.77280	.82802	.76329	.64037	1.00000	-.34760	.61393	-.36858
SVRATIO	.03174	.06597	-.02471	-.08031	-.01911	.05324	-.34760	1.00000	.00020	.06065
SINCURV	.10818	.88560	.80508	.69645	.73593	.67772	.61393	.00020	1.00000	.25935
UA	.83614	-.06619	-.33548	-.36852	-.39600	-.40689	-.36858	.06065	.25935	1.00000
ASYM1	-.27458	.05440	.20395	.23722	.23998	.16786	.31261	-.19002	.05560	-.27873
ASYM2	-.46107	.24188	.44165	.46754	.50883	.39439	.45469	-.12985	.18598	-.44938
CRVSD	.79008	-.47407	-.71279	-.66797	-.75654	-.71707	-.66202	.06291	-.20477	.85508
CRVAV	.82194	-.44797	-.69728	-.64874	-.74352	-.70985	-.63781	.06518	-.17483	.88643
CRVRAT	-.24130	-.13167	-.03077	-.07725	-.00431	.02708	-.02859	.06457	-.19697	-.28757
DIRCURV	.81753	-.14255	-.40011	-.43238	-.48420	-.44386	-.37922	.15190	.12613	.89114
RAT1	.34386	-.63319	-.80863	-.80909	-.76816	-.64198	-.73831	.07016	-.59315	.42910
RAT2	.33303	-.92071	-.81234	-.44566	-.75581	-.90044	-.44802	-.07503	-.72850	.07199

	ASYM1	ASYM2	CRVSD	CRVAV	CRVRAT	DIMCRV	RAT1	RAT2
UT	-.09156	-.13404	.18724	.19014	-.07156	.41130	-.12246	-.31835
UH	-.12315	-.12717	.30004	.30908	-.10775	.45105	-.15499	-.32184
UW	-.02069	-.04252	.01247	-.00471	.06442	.19474	-.10206	-.27505
UR	-.18400	-.26115	.54644	.58953	-.32939	.72858	-.00212	-.23223
SDCV	-.28541	-.54193	.95748	.96271	-.12489	.77275	.73535	.59153
UDCA	-.30496	-.55994	.95275	.97319	-.19397	.79651	.70136	.56318
UDCV	-.25707	-.51256	.94502	.93330	-.04396	.73144	.76065	.61192
UDCS	-.13938	-.40461	.00493	.77277	.05937	.52262	.59620	.79028
UDCK	-.24474	-.48908	.08316	.05164	.05530	.62561	.76683	.64203
HLUDCA	-.32440	-.54396	.90337	.93734	-.27273	.85300	.51297	.39592
HLUDCA	-.27458	-.46107	.79008	.02194	-.24130	.01753	.34386	.33303
HLPATHA	.05440	.24188	-.47407	-.44797	-.13167	-.14255	-.63319	-.92071
HLPATHV	.20395	.44165	-.71279	-.69728	-.03877	-.40011	-.00663	-.01234
HLPATHS	.23722	.46754	-.66797	-.64874	-.07725	-.43238	-.00909	-.44566
HLPATHK	.23998	.50883	-.75654	-.74352	-.00431	-.48420	-.76816	-.75581
SLHLA	.16786	.39439	-.71707	-.70985	.02700	-.44386	-.64198	-.90044
SVPEAK	.31261	.45469	-.66202	-.63781	-.02059	-.37922	-.73831	-.44002
SVRATIO	-.19002	-.12985	.06291	.06518	.06457	.15190	.07016	-.07503
SINCURV	.05560	.18598	-.20477	-.17483	-.19697	.12613	-.59315	-.72850
UA	-.27873	-.44938	.85508	.88643	-.28757	.89114	.42910	.07199
ASYM1	1.00000	.02878	-.30001	-.34487	.36999	-.26260	-.33049	-.01398
ASYM2	.02878	1.00000	-.57859	-.59607	.24146	-.49701	-.42164	-.21158
CRVSD	-.30001	-.57859	1.00000	.98001	-.02555	.83246	.68799	.42766
CRVAV	-.34487	-.59607	.98001	1.00000	-.20549	.86055	.67918	.40576
CRVRAT	.36999	.24146	-.02555	-.20549	1.00000	-.22977	.01344	.07660
DIMCRV	-.26260	-.49701	.83246	.86055	-.22977	1.00000	.40287	.12165
RAT1	-.33049	-.42164	.68799	.67918	.01344	.40287	1.00000	.41288
RAT2	-.01398	-.21158	.42766	.40576	.07660	.12165	.41288	1.00000

APPENDIX IIe.

Factor Matrix

FACTOR	EIGENVALUE	PCT OF VAR	CUM PCT
1	14.52362	51.9	51.9
2	6.34923	22.7	74.5
3	1.99705	7.1	81.7
4	1.33456	4.8	86.4
5	1.20953	4.3	90.8
6	.84674	3.0	93.8
7	.51706	1.8	95.6
8	.33462	1.2	96.8
9	.26895	1.0	97.8
10	.17015	.6	98.4
11	.12316	.4	98.8
12	.09310	.3	99.2
13	.07293	.3	99.4
14	.04728	.2	99.6
15	.03340	.1	99.7
16	.02532	.1	99.8
17	.01724	.1	99.9
18	.01007	.0	99.9
19	.00762	.0	99.9
20	.00738	.0	100.0
21	.00600	.0	100.0
22	.00206	.0	100.0
23	.00168	.0	100.0
24	.00075	.0	100.0
25	.00031	.0	100.0
26	.00015	.0	100.0
27	.00000	.0	100.0
28	-.00000	-.0	100.0

APPENDIX III

Channel centerline traces. Downstream
is to the right in each trace.

Appendix IIIa. Total centerline traces
Bar represents approximately 5.0 km.

Appendix IIIb. Partial centerline traces
Bar represents one nominal whole-loop.

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APPENDIX IIIa.

[1] ARKANSAS RIVER, KS

[2] BLACKS RIVER, WY I

[3] BLACKS RIVER, WY II

[4] BOULDER RIVER, MT

———— [5] CHENA RIVER, AK

[6] CIMARRON RIVER, KS-OK

— [7] EDISTO RIVER, SC

[8] ELKHORN RIVER, NB

[9] FOX RIVER, WI I

— [10] FOX RIVER, WY II

[11] IOWA RIVER, KS

[12] LITTLE P.D. RIVER, SC I

[13] LITTLE P.D. RIVER, SC II

[14] LUMBER RIVER, NC-SC

[15] MEHERRIN RIVER, VA-NC

[16] MILK RIVER, MT 10f2

[16] MILK RIVER, MT 20f2

[17] OBION RIVER, TN

[18] POWDER RIVER, MT

_____ [19] RED RIVER, MN-ND I 10f2

_____ [19] RED RIVER, MN-ND I 2 of 2

[20] RED RIVER, ND-MN II

I

[21] ROCK RIVER, IA-SD

[22] ROGUE RIVER, MI

[23] SALINE RIVER, KS 10f4

[23] SALINE RIVER, KS 20f4

[23] SALINE RIVER, KS 30f4

[23] SALINE RIVER, KS 40f4

[24] SAVANNAH RIVER, SC-GA

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— [25] SIOUX RIVER, IA-SD I

[26] SIOUX RIVER, IA-SD II

[27] SMITH FORK, WY

[28] SMOKY HILL R., KS I 10f2

[28] SMOKY HILL R., KS I 20f2

[29] SMOKY HILL R., KS II

— [30] SOLOMON RIVER, KS 1 of 3

[30] SOLOMON RIVER, KS 20f3

[30] SOLOMON RIVER, KS 3 of 3

[31] SUWANNEE RIVER, GA-FL

[32] TONGUE RIVER, WY-MT I

[33] TONGUE RIVER, WY-MT II

APPENDIX IIb.

[1] ARKANSAS RIVER, KS

[2] BLACKS RIVER, WY I

[3] BLACKS RIVER, WY II

[4] BOULDER RIVER, .MT

[5] CHENA RIVER, AK

I

[6] CIMARRON RIVER, KS-OK

[7] EDISTO RIVER, SC

[8] ELKHORN RIVER, NB

I

[9] FOX RIVER, WI .I

[10] FOX RIVER, WI II

I

[11] IOWA RIVER, KS

I

[12] LITTLE P.D. RIVER, SC I

[13] LITTLE P.D. RIVER, SC II

I

[14] LUMBER RIVER, NC-SC

[15] MEHERRIN RIVER, VA-NC

[16] MILK RIVER, MT

[17] OBION RIVER, TN

[18] POWDER RIVER, MT

[19] RED RIVER, MN-ND I

[20] RED RIVER, ND-MN II

I

[21] ROCK RIVER, IA-SD

[22] ROGUE RIVER, MI

[23] SALINE RIVER, KS

[24] SAVANNAH RIVER, SC-GA

I

[25] SIOUX RIVER, IA-SD I

[26] SIOUX RIVER, IA-SD II

[27] SMITH FORK, WY

[28] SMOKY HILL R., KS I

[29] SMOKY HILL R., KS II

[30] SOLOMON RIVER, KS

[31] SUWANNEE RIVER, GA-FL

[32] TONGUE RIVER, WY-MT I

[33] TONGUE RIVER, WY-MT II