Intersections in Actuated String Instrument Design, Performance, and Just Tuning Practices

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ABSTRACT

This dissertation examines the intersections between actuated string instrument design and the deployment of just tuning structures in composition and performance practice. Accompanying the written document, we present an original composition, *Artemisia*, as a tangible demonstration of these integrated practices. We begin by establishing a contextual definition for actuated instrumentation specific to those instruments, devices, and algorithms which employ a secondary or indirect method of inducing acoustic response from one or more strings. This trait distinguishes actuated devices from most other instrumentation, which generally employ or simulate some form of direct activation—usually applied by the performer's hands, digits, feet, breath, or other faculties. While physical properties may aid in transmitting acoustic vibrations, all instruments and devices discussed within the essay deploy some form of electronic mediation to achieve actuation. Methods for actuation include electronic transducers, sympathetic resonance, or impulses applied to a physical medium or model. To further delineate these methods, we divide electronically-mediated actuation into two sub-categories: electro-magnetic and electro-mechanical.

Chapter two provides a survey of ferromagnetic, Lorentz-Force, and tactile transduction techniques most applicable to actuated string instruments. Here, we discuss how pioneering research by Andrew McPherson, Per Bloland, Nicolas Collins, and others informed the design and construction of the author's own actuated string instruments: Rosebud I and Rosebud ('Louise') II. The following chapter introduces foundational principles in just intonation, focusing primarily upon Otonality, Utonality, Commas, Tonal Flux, Epimoric ratios and other practices relevant to composing and performing with actuated strings. With these principles in mind, we proceed to explicate the compositional structure of *Artemisia*.

In addition to defining the methods for actuation and tuning, we analyze emergent performance practices associated with actuated instrumentation. Preceding our research, Dan Overholt, Edgar Berdahl, and Robert Hamilton describe three categories of actuated instrument performance practice: "computer-mediated" electronic signals, "self-sustaining oscillation," and "third-party" audio streams. While all three categories reference the source for actuation, we propose a fourth category: disruptive preparation. This additional category extends Overholt, Berdahl, and Hamilton's source-based definition to include an exploration of performative interplay, acoustic artifacts, and other nonlinearities produced by interactions between actuated strings and foreign objects, external processing, or other interventions. In the final chapter, we focus upon historical precedents for these modes of performance, as well as those demonstrated in the composition and recording of *Artemisia*.

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The first performance and subsequent recording of *Artemisia* were made possible through the skill and careful attention of Loren Ludwig, Zoe Weiss, Kivie Cahn-Lipman, and Doug Balliett of Science Ficta. I thank Travis Thatcher for engineering this and many other sessions. Likewise, I acknowledge Jason Bennett for his assistance in constructing the instruments used in composing, performing, and recording this music. Though well beyond the scope of this dissertation, I offer my sincere thanks to Nels Cline, Ryan Winn, and Casey Poukkula for their help in elucidating the more arcane facets of instrument design.

Reflecting upon my own creative origins, I look to basement shows, four-track tapes, and punk rock experimentalism. I feel grateful to have shared in these formative experiences with Ryan Leeds, Mike Raven, Seth Spangler, Davis Hill, Kelly Fox, Kai Tantlinger, and BCA. In the same breath, I thank Jason Campbell and Andrew Senna for nearly twenty-five years of continuous collaboration and friendship. I am equally indebted to my stepfather, Bob Mott, for sharing his extensive record collection and love for music in all forms. As well as helping to raise me, I thank my grandparents for instilling intellectual curiosity and a passion for tinkering. Finally, to my mother, Laura Louise Luca: Your care and sacrifices made me who I am. As a single mom, you taught me to be 'tough-as-nails'—while offering love, humor, and generosity in equal measure. Though you left this world ten years ago, I will always feel pride in being your son.

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I. INTRODUCTION

This dissertation explores the intersections between actuated string instrument design and the deployment of just tuning structures in composition and performance practice. While a significant body of research has been conducted in the field of actuated instrumentation, none of these studies focus explicitly upon the application and creative implications of just tuning structures in actuated instrument design and performance. Likewise, comparably few existing compositions address the intersection of these practices. Through analysis of instruments, transduction techniques, and examples of creative works that engage respectively just tuning systems and actuated strings, we aim to contribute to a more substantive discussion on how these constructive practices inform one another. Moreover, our research entails the conception of new music, devices, and performance topologies. Accompanying the written document, we present an original composition, *Artemisia*, as a creative demonstration of these three integrated practices.

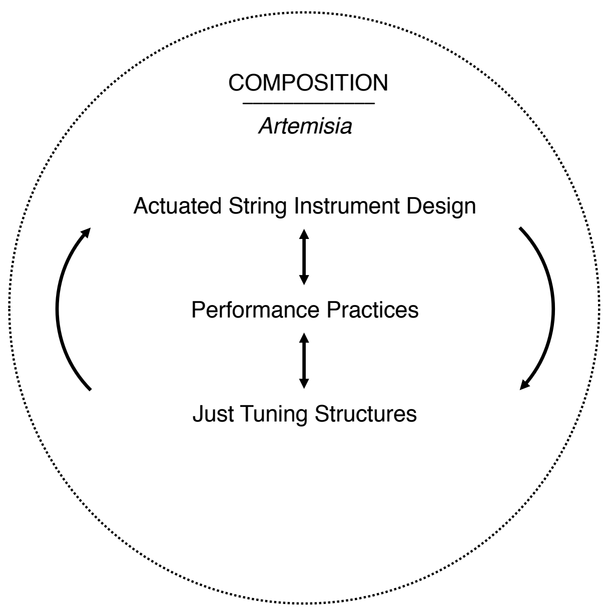

Figure 1. 1—Compositional Structure of *Artemisia* as an Original Field of Praxis

As integral components in our research and compositional process, we developed two new actuated string instruments: Rosebud I and Rosebud ('Louise') II. By documenting the design and construction of our own instruments and accompanying tuning systems, we offer tangible precedents for intersections between these fields within a unified, reciprocal practice. As well as a functional demonstration of actuated string techniques, established performance modalities, and relevant practices in just intonation, the accompanying score and recording of *Artemisia* frames the emergence of an original field of praxis. Therein, our work offers new insights on how features of actuated string instrument design, performance, and tuning mutually inform one another.

1.1 Definitional Features of Actuated String Instruments

We begin by establishing a contextual definition for actuated instrumentation, specific to those instruments which utilize one or more strings as a primary, sounding body. In seeking a contextual definition, one must first distinguish modes of actuation most relevant to musical practices from a more generalized, causal description. According to Merriam-Webster, the transitive verb "actuate" is broadly defined as "to put into mechanical action or motion" or "to move to action."[1] As musical instruments generally require some form of action—mechanical, electronic, or otherwise—to initiate sound production, this rather broad definition suggests that nearly *any* musical instrument may, in fact qualify as an *actuated* instrument. In light of the extensive research conducted on particular modes of actuation in the field of instrument design, we propose a comprehensive re-definition of actuated instrumentation exclusive to those instruments, devices, and algorithms which employ a secondary or indirect method of inducing acoustic response. This secondary method may take the form of an electronic transducer, sympathetic resonance, or impulse applied to a physical medium or model. Explicit use of a secondary method of acoustic activation distinguishes actuated instruments from most other instrumentation, which generally employ a direct—or *primary*—form of activation, usually applied by the performer's hands, digits, feet, breath, or other faculty upon the sounding body. For our purposes, we specify an instrument's string(s) as the sounding body.

To further delineate these techniques, we have developed a multi-tiered, taxonomical methodology for categorizing historical and emergent actuated instruments. At the lowest level, we define a single, broad family of electronically-mediated actuated instruments. Electronically-mediated actuation requires some form of electronic circuit or transducer (beyond amplification)

[1] Merriam-Webster Dictionary, "actuate," https://www.merriam-webster.com/dictionary/actuate, (accessed August 29, 2019).

to induce an audible response. In addition to electronic means, physical properties of the actuated instrument may also aid in transmitting acoustic vibrations to a secondary sounding body. For example, bridges, resonant chambers, and other integral components often induce secondary vibrations through sympathetic or modal resonances. In light of our stated emphasis upon the intersections between harmonic properties of just intonation and actuated instrument design, we limit further higher-level categorization of transduction methods to those most applicable to stringed instruments. As such, we divide electronically-mediated actuation into two sub-categories: electro-magnetic and electro-mechanical.

In addition to defining the methods for actuation, we will analyze emergent performance practices associated with actuated instrumentation. In their survey of contemporary actuated instrument design and implementation, "Advancements in Actuated Instruments," Dan Overholt, Edgar Berdahl, and Robert Hamilton describe three categories of actuated instrument performance practice. Based primarily upon the initial source or method of actuation, these categories include: "pre-recorded" or "computer-mediated" electronic signals, "self-sustaining oscillation" (e.g. recursive actuation), and so-called "third-party" audio streams from other instruments.[2] While all three categories reference the source(s) for actuation, we propose a fourth category of performance practice: disruptive preparation. This practice extends Overholt, Berdahl, and Hamilton's source-based definition to include an examination of the performative interplay and acoustic outcomes produced by interaction between actuated bodies—for our purposes, vibrating strings—and foreign objects, external processing, and other performer-based interventions to an instrument's sounding bodies or acoustic output. Here, we focus upon historical precedents for this mode of performance, as well as current examples from our own creative practice. Specifically, we discuss those practices and techniques demonstrated in the composition, design, and performance of *Artemisia* and actuated instruments, Rosebud I and Rosebud ('Louise') II.

[2] Dan Overholt, Edgar Berdahl, and Robert Hamilton, "Advancements in Actuated Instruments," *Organised Sound,* Vol. 16, Issue 2 (2011): 154-165, https://www.cambridge.org/core/journals/organised-sound/article/advancements-in-actuated-musical-instruments/AFBD83D9E53F8C0270492F06CD0F2380 (accessed August 5, 2019).

1.2 Distinguishing Actuation from Simulative Modes of Acoustic and Electronic Activation

In addition to the definitive distinctions for primary and secondary modes of performance, we can further refine our definition of actuated instrumentation to include those devices which also employ some novel means of activating acoustic response. Functionally, these methods remain distinct from manual means employed in standard performance practice or those which simulate human faculties. This feature distinguishes actuated instruments from musical robots and other simulative instruments, such as the player piano. These instruments utilize mechanisms overtly analogous to human hands, feet, digits, or breath to simulate familiar techniques and sonorities—albeit through electro-mechanical technologies. Besides the player piano and later iterations of musical robots, other early examples of simulative, mechanically-driven instruments include the inventions of Prussian-born acoustician and showman, Johann Baptist Schalkenbach (1824-1910). In 1861, Schalkenbach filed a patent for an instrument he called the 'Piano-Orchestre Électro-Moteur', followed in 1871 by the 'Orchestre-Militaire Électro-Moteur'. Both instruments utilized solenoids to activate a collection of "harmonium bellows, bells, triangles, drums, cymbals, and whistle-stops."[3] While certainly innovative by Victorian standards, neither instrument employs a truly novel means of inducing acoustic response, instead simulating traditional performative gestures, such as squeezing, blowing, or striking an instrument's primary, sounding body.

While, by our definition, actuated instruments require some novel means of activating acoustic response, the design of actuated instrumentation need not fully abandon the modalities and form of sounding bodies found in traditional or acoustic instruments. As we will see in later examples, electronically-mediated actuation often functions both in tandem, and as an extension of existing performance practices for traditional instruments. Overholt, Berdahl, and Hamilton illustrate this assertion in the following statement:

> "Actuated musical instruments inherit a pre-existing tradition of musical performance practice as well as pioneer a novel and extensible practice of extended and augmented performance technique. […] The classification of 'actuated musical instrument' thus

[3] Daniel Wilson, ""Electric Music" on the Victorian Stage: The Forgotten Work of J.B. Schalkenbach," *Leonardo Music Journal* 23 (2013): 79-85, http://www.jstor.org/stable/43832511. (accessed July 16, 2019)

> implies an idiomatic extension of performance technique, as the physical instrumental system itself serves as the starting point for technology-aided modification: the set of performance techniques and interaction methods traditionally associated with a given instrument are generally extended without reduction."[4]

Researchers and musicians alike often argue for a dual position in actuated instrumentation—one which links tradition and innovation. In regards to both physical design and performance practice, some assert the primacy of extant instruments (e.g. piano, fiddle, guitar) in assessing different modes of actuation. When linked to existing instrumentation, even the most technologically sophisticated advancements in electronically-mediated actuation still represent incremental additions to a centuries-old vocabulary of so-called, *extended techniques*. However, as to distinguish actuated instrumentation from what some researchers term "traditional (acoustic)" or "fully automated (robotic)" instruments, Overholt, Berdahl, and Hamilton further refine their criteria:

> "We define actuated musical instruments as those which produce sound via vibrating element(s) that are co-manipulated by humans and electromechanical systems. These instruments include the capability for control to be exerted by a simple or complex system of external agency in addition to existent methods of control through traditional instrumental performance technique with the instrument."[5]

Here, the researchers suggest a physically-embodied method of sound production, whose control structure is—to some extent—necessarily mediated by external or electronic forces. However, the group further refines the scope of electronic-mediation, defining an explicitly digital control structure in which physical instrumentation remains "[…] endowed with virtual qualities controlled by a computer in real-time, but which are nevertheless *tangible*."[6] While *tangibility* and physical re-embodiment of "virtual qualities" represent important facets of actuated instrument

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[4] Dan Overholt, Edgar Berdahl, and Robert Hamilton, "Advancements in Actuated Instruments," *Organised Sound,* Vol. 16, Issue 2 (2011): 154-165, https://www.cambridge.org/core/journals/organised-sound/article/advancements-in-actuated-musical-instruments/AFBD83D9E53F8C0270492F06CD0F2380 (accessed August 5, 2019).

[5] Ibid.

[6] Ibid.

research and performance practice, this essay seeks a more inclusive definition—one which also accounts for non-digital control structures. Indeed, Overholt, Berdahl, and Hamilton's delineation of virtuality as an exclusive product of the digital domain has multiple precedents in the development of many electronically-mediated actuated string instruments—the Electro-Magnetically Prepared Piano, Overtone Fiddle, and Feedback Lap Steel, to name a few.[7] [8] [9] However, this class of electronically-mediated actuation also includes instruments and devices which satisfy Overholt, Berdahl, and Hamilton's criteria for physical "tangibility," production of sound via "vibrating elements," and "co-manipulation" by human performers and "electro-mechanical systems," yet function *without* the aid of digital mediation. For example, in later chapters, we examine devices which employ analog feedback networks to induce physical vibration in strings. Moreover, in framing virtuality as the essence of some *secondary*—though not necessarily computer-generated—impulse, we may expand the field to include other acoustically-sympathetic modes of actuation. Thus, we continue to seek a more comprehensive definition.

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[7] Jiffer Harriman, "Feedback Lap Steel: Exploring Tactile Transducers as String Actuators," *Proceedings of the 2015 Conference on New Interfaces for Musical Expression (NIME 2015)*, 178-179. https://nime2015.lsu.edu/proceedings/152/0152-paper.pdf (accessed June 6, 2019).

[8] Per Bloland, "The Electromagnetically-Prepared Piano and its Compositional Implications," *Proceedings of the 2007 International Computer Music Conference,* http://www.perbloland.com/userfiles/file/EMPP-Comp-Implications.pdf (accessed August 5, 2019).

[9] Edgar Berdahl, Steven Backer, and Julius O. Smith III, "If I Had a Hammer: Design and Theory of an Electromagnetically Prepared Piano," *Proceedings of the 2005 International Computer Music Conference,* https://ccrma.stanford.edu/~eberdahl/Papers/ICMC2005.pdf (accessed August 5, 2019).

II. TECHNIQUES FOR ELECTRONICALLY-MEDIATED ACTUATION OF STRINGS

Moving forward, we proceed in defining proprietary features and techniques for two categories of electronically-mediated, actuated instrumentation. Namely, we focus upon those instruments which apply electro-magnetic or electro-mechanical transduction to generate sound from single or multiple sets of strings. In providing a survey of historical and contemporary instruments, designers, and practitioners, we aim to elucidate specific methods for actuation. This chapter includes brief explanations and examples of ferromagnetic actuation, Lorentz-Force actuation, and tactile transducers, as well as a discussion of specific affordances and creative implications for each mode.

2.1 Historic Origins and Developments in Electro-Magnetic Actuation

The origins of electro-magnetic actuation date back to at least 1886, when Richard Eisenmann of the firm *Electorphonisch Klavier* developed an actuated keyboard instrument capable of infinite sustain. As in later examples, such as the Electromagnetically Prepared Piano and Magnetic Resonator Piano, this early instrument utilized electromagnetic actuators positioned near sets of one or more strings.[10] However, earlier records of electro-magnetic actuation may precede the Electorphonisch Klavier by decades. By the 1840's, inventor August de la Rive had developed a means of electro-magnetically resonating the strings of a piano forte. His design involved driving a metal coil with a periodic pulse-train of variable current. Positioned in close proximity to the instrument's strings, this very early electro-magnetic actuator generated sympathetic vibrations at frequencies proportional to that of the input signal.[11] This pairing of input frequencies with those embodied by one or more resonant objects established a topology consistently found in later actuated instrumentation.

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[10] Per Bloland, "The Electromagnetically-Prepared Piano and its Compositional Implications," *Proceedings of the 2007 International Computer Music Conference,* http://www.perbloland.com/userfiles/file/EMPP-Comp-Implications.pdf (accessed August 5, 2019).

[11] Daniel Wilson, ""Electric Music" on the Victorian Stage: The Forgotten Work of J.B. Schalkenbach," *Leonardo Music Journal* 23 (2013): 79-85. http://www.jstor.org/stable/43832511. (accessed July 16, 2019)

Iterative developments for actuated instruments in the twentieth century reflect two universal concerns in design: efficiency for the method of actuation and performative features afforded by instrument's physical form. Notably, incremental developments and design considerations for Nicolas Collin's *Backwards Guitar* effectively trace this trajectory. Speaking to the first concern, Collins' earliest iterations of the instrument employed methods of actuation intrinsic to the un-modified electric guitar. Without substantially changing the instrument's physical design or internal wiring, the designer simply re-configures the guitar's electro-magnetic pickups to behave as input transducers—essentially reversing the directionality of signal flow. Here, the guitar's pickups no longer function as a means for capturing the vibrations of each string, nor amplifying the resultant output. Instead, the designer re-purposes the role of *pickup* to that of electro-magnetic *actuator*. In accomplishing this feat, Collins cites earlier methods employed by Ralph Jones of David Tudor's Composers Inside Electronics Ensemble.[12] As Jones suggests, one may feed the output from an external audio source, through a low wattage amplifier, and into a separate output transformer—wired in reverse. This simple circuit boosts the audio signal's amplitude (≈ 1-Volt "peak-to-peak") to ≈ 100 Volts at a comparably low current. Moreover, this modified circuit transforms the relatively low impedance of the amplifier output (≈ 8 Ohms) to a value of approximately 100 Ohms. As a result, these values closely match the impedance characteristics of the pickup's output. In *Handmade Electronic Music: The Art of Hardware Hacking,* Collin's describes a similar procedure for electro-mechanical actuation using a piezo-electric disc and an easily procurable Radio Shack (Model #273—1380) audio transformer.[13]

Despite the substantial boost in signal strength provided by the aforementioned circuit, Collins notes that the resultant string vibrations induced by these pre-purposed pickups appear quite low, relative to the volume of strummed notes. Consequently, even incidental brushes between the guitarist's hands and the instrument's strings generated significantly higher acoustic output than that induced by this form of actuator. To improve the efficiency for electro-magnetic

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[12] Nicolas Collins, "A Brief History of the 'Backwards Electric Guitar' (2009)," https://www.nicolascollins.com/texts/BackwardsElectricGuitar.pdf (Accessed August 9, 2019).

[13] Nicolas Collins, *Handmade Electronic Music: The Art of Hardware Hacking*. Second Edition, New York, NY: Routledge, 2009.

actuation, subsequent iterations of the Backwards Guitar replace the instrument's re-purposed pickups with modified relay coils, whose impedance properties more closely match those of standard audio amplifier outputs. These properties not only afford higher wattage from the amplifier and greater field strength to drive the strings into sympathetic vibration; but, also eliminate the need for an external transformer.

Both the shape and size for each iteration of the Backwards Guitar, as well as position of the actuator(s), inform how the performer interacts with these instruments. For example, spatial considerations and portability led Collins to augment a "Hawaiin [sic]" guitar in a manner similar to the first Backwards Guitar. Alternately dubbed "the Oahu" or Backwards Hawaiin Guitar, the shorter scale length and ability to play the instrument from a tabletop allows a single performer access to additional electronic processing in a manner less suited to the standing guitarist. The earliest performances with this instrument—including the piece, *Pet Sounds* (1987)—employed a single hand-held actuator. Later iterations of the Oahu integrated six smaller, electro-magnetic actuators positioned above each string.[14] The modular design of Collins' most recent iteration, the Level Guitar (2002), offers the performer the ability to adjust the position of individual actuators along the length of each string, thus emphasizing specific harmonic nodes.[15] Comparable design features allowing the performer to actuate each string or harmonic independently appear consistently in other actuated instrumentation. For example, Edgar Berdahl's Feedback Resonance Guitar utilizes two embedded electro-magnets to induce resonance from the six strings of a modified Fender Stratocaster. Here, multiple sources for actuation enable a specificity in control-structure generally reserved for digital instruments. Employing a topology akin to additive synthesis, performers interact with an iOS application to control the frequency and amplitude of five oscillators. Routed to actuators positioned below each of the six strings, these programmable

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[14] Nicolas Collins, "A Brief History of the 'Backwards Electric Guitar' (2009)," https://www.nicolascollins.com/texts/BackwardsElectricGuitar.pdf (accessed August 9, 2019).

[15] Ibid.

properties determine which strings become sympathetically activated and which harmonics sound for a given string.[16]

2.1.1 Discrete and Continuous Approaches for Pitch Modification

Returning to the Backwards Hawaiin Guitar, we shift focus from independent source assignment towards actuation topologies which also afford independent control over the output of each string. By integrating a commercially-manufactured bridge with separate piezo-electric pickups under each saddle, Collins allows for the parsing of output from each string, as well as diffusion of harmonic materials across stereo or other spatial fields. A clear emphasis on discrete assignment and subsequent diffusion of pitched elements is particularly evident in his piece, *It Was a Dark and Stormy Night* (1992). Composed for Backwards Hawaiin Guitar, spoken word, electronics, and small ensemble, specific instances of a given phrase become re-embodied within the spectra of individual strings.[17] In performance, each utterance activates one of four discrete tones: G, D, A, and E.[18]

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Phrase: "It was a dark and stormy night…"

		String Number (IV-I):	Pitch:
Instance #1	→	IV	'G'
Instance #2	→	III	'D'
Instance #3	→	II	'A'
Instance #4	→	I	'E'

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[16] Dan Overholt, Edgar Berdahl, and Robert Hamilton, "Advancements in Actuated Instruments." *Organised Sound,* Vol. 16, Issue 2 (2011): 154-165. https://www.cambridge.org/core/journals/organised-sound/article/advancements-in-actuated-musical-instruments/AFBD83D9E53F8C0270492F06CD0F2380 (accessed August 5, 2019).

[17] Nicolas Collins, *It Was a Dark and Stormy Night*, Trace Elements CD, 1992.

[18] Nicolas Collins, "A Brief History of the 'Backwards Electric Guitar' (2009)," https://www.nicolascollins.com/texts/BackwardsElectricGuitar.pdf (accessed August 9, 2019).

Building upon similar techniques, Martin Piñeyro's Electric Slide Organistrum enables the performer to active continuous changes in pitch, including sustained glissandi. Like earlier iterations of the Collin's Backwards Guitar, Piñeyro's instrument utilizes 12-Volt relay coils harvested from automotive applications, as well as a permanent magnet salvaged from a DC-motor. However, unlike Collins' instruments which access external signals for actuation, the Electric Slide Organistrum's actuation topology remains self-contained. Here, transduced vibrations from the instrument's single string act as the sole source for actuation. Accordingly, the instrument employs both a "driver" and "pickup" coil wound to eight and 50 Ohms impedance, respectively.[19] As defined by Dan Overholt, Edgar Berdahl, and Robert Hamilton, this method of electro-magnetic actuation embodies a "self-sustaining" or recursive performance modality.[20] Since the 1970's, similar recursive techniques have been implemented, with the Ebow (Direct String Synthesis™) garnering the widest usage amongst rock musicians.[21] Other, more recent electro-magnetically actuated devices for electric guitar include the Fernandes Sustainer™, Sustainiac (Stealth)™, and TC Electronic Aeon™.[22] [23] Building upon similar methodologies, Piñeyro describes his approach:

> "The movement of the string in presence of a magnetic field induces electrical current through the input coil, which is amplified by the audio amplifier and fed to the driver coil. This produces a varying magnetic field on the driving coil that drives the string at its

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[19] Martin Piñeyro, "Electric Slide Organistrum," *Proceedings of the 2012 Conference on New Interfaces for Musical Expression (NIME 2012)*. http://www.nime.org/proceedings/2012/nime2012_114.pdf (accessed March 2, 2017).

[20] Dan Overholt, Edgar Berdahl, and Robert Hamilton, "Advancements in Actuated Instruments." *Organised Sound,* Vol. 16, Issue 2 (2011): 154-165. https://www.cambridge.org/core/journals/organised-sound/article/advancements-in-actuated-musical-instruments/AFBD83D9E53F8C0270492F06CD0F2380 (accessed August 5, 2019).

[21] Gregory Heet,1978. String Instrument Vibration Initiator and Sustainer, US Patent 4,075,921.

[22] Floyd D. Rose, Steven M. Moore, and Richard W. Knotts, 1992, Musical Instrument Sustainers and Transducers, US Patent 5,123,324.

[23] Alan Hoover, 2000, Controls for Musical Instrument Sustainers, US Patent 6,034,316.

resonant frequency, and sustains the vibration. Because of the positive feedback, this system is able to induce the vibration from rest."[24]

Distinguishing itself from other electro-magnetically actuated instruments and devices, the Electric Slide Organistrum does not require electronic amplification. Instead, the pickup coil acts solely as a means for transmitting vibrations from the string to the actuator. Like other acoustic instruments, amplification is achieved by coupling vibrations from the bridge to a resonant wooden body. While employing decidedly 'low-tech' means for amplification, the simplicity of Piñeyro's design belies a more complex control structure. Integrating other interactive elements, an attached camera tracks movements from the performer's hands, translating physical gestures into continuous changes in pitch. As the performer alters their proximity to the instrument, a stepper motor controlled via an H-Bridge and Arduino microprocessor modifies the position of a metal "slide" along the length of the string. Therein, electronic-mediation enables discrete control over acoustic sonorities.

Concurrent applications involving automated pitch control for actuated instrumentation present similar contributions to the field. Developed by Shawn Trail and other researchers at the University of Victoria, the Self Tuning Auto-monochord Robot Instrument (STARI) also implements a stepper motor and embedded computing system to calculate and modify the pitch of a single string. However, instead of controlling pitch using a slide mechanism, the STARI institutes comparable changes in string tension by rotating a guitar-style machine tuner. Upon sensing the current pitch for the actuated string, an embedded computing system determines whether the stepper-motor (attached to the machine tuner) needs to "tune up", "tune down", or maintain steady string tension. According to Trail, this system is capable of affecting pitch-changes within 30 cents of a target frequency.[25] Though Piñeyro and other researchers have not

[24] Martin Piñeyro, "Electric Slide Organistrum," *Proceedings of the 2012 Conference on New Interfaces for Musical Expression (NIME 2012)*. http://www.nime.org/proceedings/2012/nime2012_114.pdf (accessed March 2, 2017).

[25] Shawn Trail, George Tzanetakis, Leonard Jenkins, Mantis Cheng, Duncan MacConnell and Peter Driessen, "STARI: A self-tuning auto-monochord robotic instrument," *2013 IEEE Pacific Rim Conference on Communications, Computers and Signal Processing (PACRIM)*. Victoria, BC, Canada, 2013, 405-409.

explicitly referenced the microtonal capabilities of automated tuning mechanisms, the potential for meaningful intersections of extended intonation and actuated performance practices abound.

2.2 Common Structural Features of Electro-Magnetic Actuation

As with other electronically-mediated forms of actuation, electro-magnetic actuation employs some form of transducer to convert fluctuations in electronic signals into acoustic vibrations. In the case of actuated string instruments, this method involves one or more electro-magnets, whose variable magnetic fields act upon ferrous-metal strings to induce sympathetic vibration. In recent years, a core group of researchers, including Andrew McPherson, Edgar Berdahl, Jeff Snyder, Per Bloland, and others, have sought to codify features common to electro-magnetically actuated instrumentation. As evident in McPherson's research, subsequent classification of electro-magnetic modes and instrumentation follow three separate physical topologies: "permanent magnet actuation," "Lorentz [Force] actuation," and "ferromagnetic actuation."[26] While specific applications vary, the majority of electro-magnetically actuated instruments retain similar structural components: namely, a solenoid electromagnet composed of a ferromagnetic core surrounded by multiple turns of wire. In each case, the magnetic flux density (B_a) of a solenoid actuator is proportional to the current (I) passing through the coiled wire. From the perspective of musical instrument design, the relationship between flux density (B_a) and the time-variant force (F) acting upon a sounding body or string determines the efficiency for a given actuator. Other variables, including the number of turns around the ferromagnetic core (N), permeability of the ferromagnetic core (μ), and total length of the solenoid (ℓ), are represented in the following expression:[27]

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[26] Andrew McPherson, Edgar Berdahl, Jeff Snyder, and Cameron Britt. Actuated Instruments Workshop (slides), *Presented May 20, 2012 at the Conference on New Interfaces for Musical Expression (NIME 2012),* Ann Arbor, Michigan.

[27] Andrew McPherson, "Techniques and Circuits for Electromagnetic Instrument Actuation," *Proceedings of the 2012 Conference on New Interfaces for Musical Expression (NIME 2012)*. http://www.nime.org/proceedings/2012/nime2012_117.pdf (accessed March 3, 2017).

(2 - 2)

$$Magnetic\ Flux\ Density = B_a(t) = \frac{\mu N}{\ell}\, I(t)$$

With this principle in mind, N. Cameron Britt, Snyder, and McPherson have proposed various applications of electro-magnetic actuation, in which a solenoid drives a permanent magnet attached to the body of an acoustic instrument. The practicality of so-called "permanent magnet actuation" is particularly evident with instruments whose sounding bodies are not constructed from ferromagnetic materials. For example, in the case of the EM-Vibe (Electro-magnetically Actuated Vibraphone), the designers affix permanent magnets to each aluminum bar on an existing vibraphone. Whereas, the aluminum bars of the vibraphone alone do not respond strongly to a magnetic field, the addition of a permanent magnet provides the necessary properties for actuation. Here, electro-magnetic actuators apply time-varying force upon the permanent magnetics, thus driving the attached bars into sympathetic vibration in accordance with frequency content present in the actuated signal.[28] While appropriate for instruments lacking a sounding body composed of ferromagnetic materials, this mode of actuation presents obvious design challenges when the intended sounding body is a freely-vibrating string. For reasons of weight alone, attaching permanent magnets to a string may not be feasible. Consequently, few applications of permanent magnet actuation appear relevant to our discussion of actuated string instruments.

2.3 Exceptional Features of Lorentz-Force Actuation

In contrast to other modes of electromagnetic actuation, Lorentz-Force actuation does not employ a solenoid. Instead, variable current passes directly through a length of metal string. In most applications, positive and negative leads from an audio amplifier connect to either end of a tightened string. Permanent magnets placed adjacent to the string generate a strong field, thus

[28] Cameron N. Britt, Jeff Snyder, and Andrew McPherson, "The EM-Vibe: An Electromagnetically Actuated Vibraphone," *Proceedings of the 2012 Conference on New Interfaces for Musical Expression (NIME 2012)*. http://vhosts.eecs.umich.edu/nime2012//Proceedings/papers/101_Final_Manuscript.pdf (accessed February 5, 2020).

inducing a force upon charged particles passing through the length of string. As with other actuation methods which employ electromagnetic induction, this force is proximally proportional to the periodic fluctuations in frequency and amplitude of the actuated input signal. Here, both attractive and repulsive forces induce vibration in the string. Notably, Lorentz-Force actuation presents a number of advantages over other modes of electro-magnetic actuation. For example, due to the relatively low inductance of metal strings, this technique affords greater linearity in amplitude response than solenoid-based actuation methods—particularly at high frequencies.[29] However, as confirmed by McPherson and others, Lorentz-Force retains a few substantial drawbacks, thus limiting its potential for wider application in actuated instrumentation. As conveyed in the equation below (2 - 3), force (F) acting upon the string is highly dependent upon current strength (I), the length of string (L), and field strength generated by the permanent magnets (B). Deficiencies in any combination of these factors greatly reduces force—thus, resulting in low amplitude vibrations.[30]

$$F = IL \cdot B \tag{2 - 3}$$

While these material limitations pose significant practical challenges, a few composers and sound artists have successfully implemented Lorentz-Force actuation within their creative practices. Most notably, Alvin Lucier's *Music on a Long Thin Wire* (1977) demonstrates a variant of Lorentz-Force actuation.[31] Here, Lucier stretches a single piano wire across two, widely-spaced bridges. With a large horseshoe-shaped magnet inducing a strong permanent field (B), the

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[29] Andrew McPherson, "Techniques and Circuits for Electromagnetic Instrument Actuation," *Proceedings of the 2012 Conference on New Interfaces for Musical Expression (NIME 2012)*. http://www.nime.org/proceedings/2012/nime2012_117.pdf (accessed March 3, 2017).

[30] Andrew McPherson, Edgar Berdahl, Jeff Snyder, and Cameron Britt, Actuated Instruments Workshop (slides), *Presented May 20, 2012 at the Conference on New Interfaces for Musical Expression (NIME 2012),* Ann Arbor, Michigan.

[31] Alvin Lucier, "Music for Piano with Magnetic Strings," Theme (Liner Notes). http://www.lovely.com/albumnotes/notes5011.html (accessed August 5, 2019).

composer employs sinusoidal generators to modulate current passing through the metal string.[32] Here, the rather extensive length of wire (L) contributes to increased force and higher resultant amplitude. Lucier attributes the earliest iterations of *Music on a Long Thin Wire* to a collaborative demonstration by physicist John Trefny. Remarkably, these initial experiments utilized an electro-magnet—as opposed to a permanent magnet, more typical of Lorentz-Force actuation. As the composer describes in the liner notes for a 1992 release on Lovely Records, “we [Alvin Lucier and John Trefny] extended a short metal wire across a laboratory table and placed an electromagnet over one end of it.”[33] Early experiments aside, later performances demonstrate methods more typical of Lorentz-Force actuation.

Nearly three decades on, Marielle V. Jakobsons’ installation *String TV* (2009) also employs a variant of Lorentz-Force actuation. Commissioned as part of the twenty-fifth anniversary exhibition for the LAB Gallery in San Francisco, the piece pays tribute to Bay-area sound artist and architect Scott Arford. As described by Jakobsons, elements of the piece directly reference Arford’s work with audio-visual feedback networks.[34] In the same spirit, *String TV* explores interactions between a self-oscillating string, visual representations of sound, and participatory interventions by the audience. In addition to audio analysis and signal processing software, the installation consists of three physically-embodied elements: a length of piano wire stretched between two floating bridges, a curved soundboard (fabricated by Max Allstadt), and a cathode-ray tube (CRT) display positioned above the instrument.[35] Consistent with Lucier’s approach to Lorentz-Force actuation, variable current passes directly through the length of wire. However, in contrast to the fixed orientation of *Music on a Long Thin Wire*, Jakobsons deploys two, moveable

[32] Per Bloland, “The Electromagnetically-Prepared Piano and its Compositional Implications,” *Proceedings of the 2007 International Computer Music Conference.* http://www.perbloland.com/userfiles/file/EMPP-Comp-Implications.pdf (accessed August 5, 2019).

[33] Alvin Lucier, Music on a Long Thin Wire, New York: Lovely Music, 1992.

[34] Marielle V. Jakobsons, “String TV Documentation,” http://mariellejakobsons.com/?p=267 (accessed February 17, 2020)

[35] Ibid.

permanent magnets for her instrument. By inviting the audience to change the physical position of these magnets, the artist (and luthier) affords the participants a tangible sense of agency in shaping the spectra of the vibrating string. As each magnet passes alongside the length of the actuated string, the balance of overtones shifts, emphasizing or attenuating certain frequencies in relation to the fundamental. Pressing upon the curved soundboard changes the position of the two, floating bridges—thus shifting the frequency of the entire spectrum.[36] In addressing broader implications for both autonomous and interactive elements within the piece, Jakobsons states:

> "The work is an autonomous feedback system: self-propogating, ever-changing, and highly responsive to space, people, and conditions around it. This work is very touchable—participants also touch and press on the body of the instrument, where they can feel the vibrations of the fundamental tone of the string and can alter the harmonic overtones. A small computer concealed inside the instrument body runs a custom program to monitor and stimulate the feedback. The TV monitor vibrations are fed back into the string, thus creating a sonic and visual feedback loop."[37]

In considering the role of intonation in both Lucier and Jakobsons' respective works, changes in frequency and spectra appear as a fluid, if not incidental features in composition. In either case, both artists have appeared to embrace this fluidity. As acknowledged by Lucier, "fatigue, air currents, heating and cooling, even human proximity could cause the wire to undergo enormous changes […] For example, visitors' footsteps on the Marley floor [in Kyoto] caused extremely slight shifts in the positions of the tables to which the wire was clamped, causing spectacular changes in the sound of the wire."[38] Furthermore, the large amounts of current passing through metal string tends to produce substantial heat. Anecdotal evidence by McPherson and others supports the notion that prolonged exposure to heat loosens the tension on steel strings,

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[36] Marielle V. Jakobsons, "Marielle Jakobsons' 'String TV' Demonstration," https://www.youtube.com/watch?v=OJRVVu9LbqY&feature=emb_title (Accessed January 18, 2020).

[37] Marielle V. Jakobsons, "String TV Documentation," http://mariellejakobsons.com/?p=267 (accessed February 17, 2020)

[38] Alvin Lucier, Music on a Long Thin Wire, New York: Lovely Music, 1992.

resulting in noticeable drops in pitch over time.[39] This latter property becomes crucial when considering Lorentz-Force actuation for applications where accurate reproduction of pitch is essential. For obvious reasons, this mode of actuation presents certain challenges for musicians and luthiers engaged with just intonation or other applications where precise pitch reproduction is prioritized. In the absence of automated pitch correction—mechanical or otherwise—such challenges may prove interminable where precise control over tuning and spectra is concerned.

2.4 Techniques for Ferromagnetic Actuation

Whereas application of a permanent magnet may not be feasible for most stringed instruments, McPherson, Berdahl, and others have proposed alternate modes of actuation which employ a similar, solenoid component to act upon non-polarized ferromagnetic objects. For our purposes, these ferrous objects may include steel strings found on guitars, pianos, and other extant or augmented instruments, as well as emergent instrumentation or devices. Notably, some of the most fruitful insights have been gleaned through practical research, design, and construction of devices intended to actuate strings of a grand piano. Thus, we shall center our examination of ferromagnetic actuation around techniques refined for these instruments. Notable developments include Andrew McPherson's Magnetic Resonator Piano (MRP), as well as earlier research conducted at Stanford University's Center for Computer Research in Music and Acoustics (CCRMA) and Instrumentation Lab at Miami University. Here, research conducted by a joint team consisting of Edgar Berdahl, Per Bloland, Steven Backer, and Julius O. Smith III, has culminated in multiple iterations of a modular actuated device, the Electromagnetically-Prepared Piano (EMPP).[40]

[39] Andrew McPherson, "Techniques and Circuits for Electromagnetic Instrument Actuation," *Proceedings of the 2012 Conference on New Interfaces for Musical Expression (NIME 2012)*. http://www.nime.org/proceedings/2012/nime2012_117.pdf (accessed March 3, 2017).

[40] Edgar Berdahl, Steven Backer, and Julius O. Smith III, "If I Had a Hammer: Design and Theory of an Electromagnetically Prepared Piano," *Proceedings of the 2005 International Computer Music Conference.* https://ccrma.stanford.edu/~eberdahl/Papers/ICMC2005.pdf (accessed August 5, 2019).

Certainly, works for electro-magnetic actuation and piano are not without precedent. Similar projects by other researchers and composers attest to the significant interest in actuated piano, as well as the ubiquity and apparent consensus regarding the continued relevance of prepared piano in contemporary compositional practice. Looking to previous work, Maggi Payne's *Holding Patterns* for piano and three Ebows (2001) and Stephen Scott's *Resonant Resources* (1984) both employ similar modes of solenoid-based, electro-magnetic actuation.[41] Likewise, more recent applications involving actuated piano strings include the Overtone Harp, an electro-magnetically actuated instrument developed by sound artist and luthier Andy Cavatorta.[42] Of particular relevance to our discussion, Cavatorta exhorts the just tuning capabilities for his instrument, stating that the Overtone Harp "creates layers of sound like a pipe organ and gives access to additional consonant intervals not found in twelve-tone equal temperament, such as the Perfect third and Perfect [septimal minor] seventh."[43] Earlier and contemporary works aside, insights gained from the MRP and EMPP arguably provide the most concise framework of techniques and practical considerations for subsequent developments in electro-magnetically actuated instrumentation.

As with other solenoid-based topologies, derivative expressions based upon Coulombs law reveal properties applicable to actuated string instrument design. Specifically, McPherson identifies three significant properties. Firstly, force (F) acting upon a body is linearly proportional to the current (I) and number of turns (N) applied to a coil. Secondly, the square of the distance between an actuator and magnetized object (r) is inversely proportional to the applied force. As

[41] Dan Overholt, Edgar Berdahl, and Robert Hamilton, "Advancements in Actuated Instruments," *Organised Sound.* Vol. 16, Issue 2 (2011): 154-165. https://www.cambridge.org/core/journals/organised-sound/article/advancements-in-actuated-musical-instruments/AFBD83D9E53F8C0270492F06CD0F2380 (accessed August 5, 2019).

[42] Xavier Aaronson, "How to Build a Magnetized Harp (Sound Builders—Andy Cavatorta)," *Vice Motherboard*, 2014. https://www.vice.com/amp/en_us/article/mgb9x8/%7B%7Bcontributor.public_url%7D%7D (accessed September 14, 2019)

[43] Andy Cavatorta, "The Overtone Harp," Andy Cavatorta, 2018. http://andycavatorta.com/overtoneharp.html (accessed September 14, 2019).

noted by McPherson, maintaining a minimal distance between the solenoid and the affected string improves the efficacy of this actuation model. However, this property retains certain physical limitations. For example, the maximum vertical displacement of the actuated string must not exceed the value of r, lest physical contact between the string and solenoid disrupt the intended vibrational mode. As observed, such disruptions may introduce substantial non-linearities to the resultant spectra. Concerns over linearity aside, we shall address the aesthetic affordances for these disruptions in later chapters. Finally, given a fixed current (I), coils occupying a larger surface area (A) and shorter length (ℓ) produce greater force.[44] Thus, force exerted upon a ferrous object by electro-magnetic actuation can be expressed in accordance with the following equation:

$$F = \frac{\mu N^2 I^2 A}{4r^2} \tag{2 - 4}$$

2.4.1 Mitigating Spectral Non-Linearities

As observed by McPherson and others, ferromagnetic actuation may also introduce non-linearities in frequency response for the actuated signal. While ferrous metal objects alone lack the degree of magnetic permeability exhibited by a fixed magnet, magnetic polarity can be induced by external fields in close proximity. These external fields include those generated by a solenoid actuator or a nearby permanent magnet.[45] However, in the absence of a permanent magnetic field, actuating an alternating bi-polar current will induce a "frequency-doubling effect" upon the

[44] Andrew McPherson, Edgar Berdahl, Jeff Snyder, and Cameron Britt, Actuated Instruments Workshop (slides), *Presented May 20, 2012 at the Conference on New Interfaces for Musical Expression (NIME 2012),* Ann Arbor, Michigan.

[45] Edgar Berdahl, Steven Backer, and Julius O. Smith III, "If I Had a Hammer: Design and Theory of an Electromagnetically Prepared Piano," *Proceedings of the 2005 International Computer Music Conference.* https://ccrma.stanford.edu/~eberdahl/Papers/ICMC2005.pdf (accessed August 5, 2019).

sounding spectrum of the string.[46] In this scenario, a bi-polar audio signal consisting of a 110 Hertz sinusoid will induce a 220 Hertz vibration in the string. This phenomenon may persist, even if the fundamental frequency of the string is tuned to 110 Hertz. The reason for this phenomenon is rather simple. Upon actuation, the force exerted upon a non- or weakly-polarized ferromagnetic object, such as a steel string, induces magnetic attraction. However, the field does not repel the object in equal measure. In acoustic terms, the resulting force induces a periodic state of rarefaction; without compression. Since the non-magnetized string responds attractively to both positive and negative peaks in the magnetic field, the duration of each wave-period is halved. As such, both the negative and positive amplitude components of our 110 Hertz sine wave attract the steel string periodically, at a rate twice the frequency of the actuated audio signal.

To address and potentially mitigate spectral artifacts generated by this phenomenon, researchers from both teams (EMPP and MRP) have proposed two respective solutions: addition of a permanent magnetic field or implementation of a unipolar input signal. For researchers at CCRMA and the Instrumentation Lab at Miami University this mitigation process began with analysis of the relationship between physical variables involved in electro-magnetic actuation of strings. In line with Coulombs law, Berdahl, Backer, and Smith observed a non-linear relation between current applied to the electromagnet and resultant force applied to a ferrous metal string. As expected, a decrease in distance between an actuator and magnetized object (r) corresponds to a similarly exponential increase in force applied to the string. However, when a magnetic field reaches such a high level of intensity so as to magnetically "saturate" the ferrous metal string, the relationship between current and force retains a more or less linear trajectory. As stated, an implicit goal of the team's research is to produce a linear response from the instrument "capable of receiving an arbitrary waveform from any common computer soundcard or electronic signal source, and in turn relaying this information to a piano string."[47] To this end, magnetic saturation

[46] Andrew McPherson, "Techniques and Circuits for Electromagnetic Instrument Actuation," *Proceedings of the 2012 Conference on New Interfaces for Musical Expression (NIME 2012)*. http://www.nime.org/proceedings/2012/nime2012_117.pdf (accessed March 3, 2017).

[47] Edgar Berdahl, Steven Backer, and Julius O. Smith III, "If I Had a Hammer: Design and Theory of an Electromagnetically Prepared Piano," *Proceedings of the 2005 International Computer Music Conference.* https://ccrma.stanford.edu/~eberdahl/Papers/ICMC2005.pdf (accessed August 5, 2019).

provides one means for achieving this goal—thus affecting a more-or-less predictable and linear relationship between source signal and actuated audio output. Optionally, the application of current (*I*) in excess of a given threshold can generate a sufficiently strong electro-magnetic field so as to magnetize the string. However, in developing the first iterations of the EMPP, Berdahl, Backer, and Smith instead opted for the inclusion of a permanent field. Produced independently to the amount of current (*I*) already applied to the electromagnet, the permanent field provides what the researchers equate to a form of "biasing" or polarity offset.[48] To achieve these results, two permanent magnets of opposite polarity are positioned at either side of each electro-magnet, thus allowing for near linearity in current-force response. Together, the two permanent magnets induce sufficient polar response from the steel strings so as to respond to both attractive and repulsive forces generated by the solenoid actuator. In this sense, elements of the EMPP can be considered analogous to techniques involving "permanent magnet actuation," such as those employed in Britt's *EM-Vibe*.[49]

Alternately, McPherson forgoes the use of permanent magnets in the Magnetic Resonator Piano. Instead, he addresses the issue of frequency-doubling by maintaining a unipolar current at the amplification stage. Here, the addition of an offset current, provided by a separate power supply, lifts the overall amplitude of the input signal. This form of DC-offset ensures that nearly all portions of the actuated waveform maintain positive polarity. Furthermore, in place of a fixed voltage offset, the circuit utilizes peak-detection to ensure that "the troughs of the waveform will

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[48] Edgar Berdahl, Steven Backer, and Julius O. Smith III, "If I Had a Hammer: Design and Theory of an Electromagnetically Prepared Piano," *Proceedings of the 2005 International Computer Music Conference.* https://ccrma.stanford.edu/~eberdahl/Papers/ICMC2005.pdf (accessed August 5, 2019).

[49] Cameron N. Britt, Jeff Snyder, and Andrew McPherson, "The EM-Vibe: An Electromagnetically Actuated Vibraphone," *Proceedings of the 2012 Conference on New Interfaces for Musical Expression (NIME 2012).* http://vhosts.eecs.umich.edu/nime2012//Proceedings/papers/101_Final_Manuscript.pdf (accessed February 5, 2020).

always stay above ground."[50] Even in the absence of a permanent magnetic field, these techniques allow only positive components of a periodic signal to apply attractive force upon the steel strings of the MRP—thereby eliminating frequency-doubling artifacts introduced by actuating a bi-polar audio signal. Thusly, frequency content from an original input signal can be accurately reproduced via ferromagnetic actuation.

In addition to addressing issues surrounding magnetic polarity, McPherson also proposes novel solutions for mitigating other non-linearities associated with ferromagnetic actuation. As noted by McPherson and others, most commercial audio amplifiers function as voltage amplifiers, wherein output voltage exhibits a proportional gain structure in relationship with input voltage. When driving loudspeakers, linearity is maintained. However, when driving solenoid actuators, flux density—and thus, the resultant forces acting upon strings or other ferrous-metal objects—is instead proportional to an amplifier's output current.[51] As such, voltage-gain applied to an input signal will not always drive electromagnetically-actuated strings with a linearly proportional force. Here, an actuator's inductance (*L*) limits changes in current, or "slew rate".[52] When driven by a standard voltage amplifier, an electromagnetic actuator behaves like a low-pass filter, attenuating higher frequencies from the actuated signal. To affect a more linear frequency response and improve overall performance, output current—not voltage—must be proportional to input voltage for an actuated signal. Consequently, McPherson employs an array of 88 "transconductance" amplifiers for each actuator and corresponding course strings within the MRP. When the input

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[50] Andrew McPherson, "Techniques and Circuits for Electromagnetic Instrument Actuation," *Proceedings of the 2012 Conference on New Interfaces for Musical Expression (NIME 2012)*. http://www.nime.org/proceedings/2012/nime2012_117.pdf (accessed March 3, 2017).

[51] Andrew McPherson and Youngmoo E. Kim, "Augmenting the Acoustic Piano with Electromagnetic String Actuation and Continuous Key Position Sensing," *Proceedings of the 2010 Conference on New Interfaces for Musical Expression (NIME 2010)*. http://www.educ.dab.uts.edu.au/nime/PROCEEDINGS/papers/Paper%20K1-K5/P217_McPherson.pdf (accessed August 5, 2019).

[52] Per Bloland, "The Electromagnetically-Prepared Piano and its Compositional Implications," *Proceedings of the 2007 International Computer Music Conference.* http://www.perbloland.com/userfiles/file/EMPP-Comp-Implications.pdf (accessed August 5, 2019).

signal’s frequency increases, impedance rises and an increased voltage swing becomes necessary to produce the same current output. As McPherson describes, for a transconductance amplifier “*voltage swing*, and not total output power determines the maximum change in current (*slew rate*).”[53] In performance, McPherson’s transconductance amplifier drives actuators more efficiently at higher frequencies than standard voltage amplifiers. At the behest of composers working with the Magnetic Resonator Piano, later iterations of the instrument increased both the slew rate and voltage swing, thus affording the effective actuation of high-order harmonics.

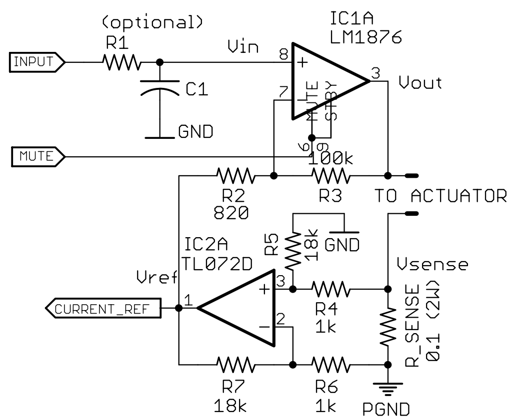

Figure 2. 1—Transconductance Amplification Circuit Proposed by Andrew McPherson[54]

2.5 Application of Ferromagnetic Actuation Techniques in ‘Rosebud I’

Designed and constructed by the author, Rosebud I is an actuated string instrument consisting of six, electro-magnetically actuated strings and two electronic pickups. An integral element in the composition, performance, and recording of *Artemsia*, Rosebud I embodies the same principles and techniques of ferromagnetic actuation pioneered by Nicolas Collins, Edgar Berdhal, Per Bloland, Andrew McPherson, and other researchers at CCRMA and the

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[53] Andrew McPherson, “Techniques and Circuits for Electromagnetic Instrument Actuation,” *Proceedings of the 2012 Conference on New Interfaces for Musical Expression (NIME 2012)*. http://www.nime.org/proceedings/2012/nime2012_117.pdf (accessed March 3, 2017).

[54] Ibid.

Instrumentation Lab at Miami University. Likewise, our design reflects a tradition of creative reuse in actuated instrumentation. We constructed the body of the instrument using both prefabricated and re-purposed materials. To reduce overall weight, we chose an open-frame design comprised of two 36” lengths of 1 ½” × 1 ½” × $^1/_8$” 6061-T6 extruded aluminum angle. Per the manufacturer specifications, this material is generally utilized in “aerospace, marine, electronic, ornamental, machinery, and structural applications” where strength, low-weight, and “resistance to corrosion” is most valued.[55] Speaking to the unique tonal properties of 6061-T6 aluminum, this material has also been utilized extensively in the construction of Travis Bean guitars.[56] Notably, various models of Travis Bean guitars have appeared in iconic recordings by Jerry Garcia (The Grateful Dead), Colin Newman (Wire), Steve Albini (Big Black, Shellac), Lee Ranaldo (Sonic Youth), Duane Denison (The Jesus Lizard), Stephen O'Malley (Sunn O))), and Buzz Osborne (The Melvins).[57]

Two 5” × 7 ¼” × ¾” poplar boards connect the aluminum frame at each end, while two $^1/_8$” sheets of acrylic house the humbucker pickups and actuators. The author utilized Computer Numerical Control (CNC) and laser-cutting techniques to fashion cavities of precise depth and dimension to house hardware components—including machine-tuners, pickups, adjustable bridges, pickups, actuators, and associated electronics.[58] [59] In addition to pre-fabricated components, such as a hard-tail electric guitar bridge, we also re-purposed various hardware to accommodate less conventional design features.[60] For example, to match the fixed spacing of the

[55] OnlineMetals.com, “Aluminum Angle 6061 Extruded with Equal Legs,” https://www.onlinemetals.com/en/buy/aluminum-angle-equal-6061-extruded-t6 (accessed September 16, 2016).

[56] Clifford Travis Bean, Stringed Instrument with Aluminum Made Integral Unit, US Patent 5,516,157.

[57] Art Thompson, “Artifacts: Travis Bean Guitars,” *Guitar Player*, vol. 39, no. 12, 1 Dec. 2005, 140 - 141.

[58] Saliency Chrome Guitar Machine Head Tuners (3L 3R).

[59] BQLZR, Humbucker Double-Coil Electric Guitar Pickups

[60] Kmise A0052 6 Saddle Hardtail Bridge, Top Load (78 mm).

instrument’s six solenoid-actuators, strings must be placed at a non-orthagonal orientation. While spacing between each string occupies a distance of approximately $^3/_8$” at the hard-tail bridge nearest the pickups, this distance widens to nearly ¾” as the strings pass over each of the six actuators. To accommodate this unusual spacing, we fashioned a custom bridge from a ¼” stainless steel bar. Spanning the width of the instrument (7 ¼”), the bar is suspended between two metal rope clamps, procured from a local hardware store in Charlottesville, Virginia.[61] Shallow grooves carved into the metal bar secure each string in place.

Figure 2. 2—‘Rosebud I’—Electro-Magnetically Actuated String Instrument

Even in the absence of external amplification, the solenoid-actuators generate enough force to induce strong vibrations from the instrument’s strings. Alone, these vibrations produce an audible acoustic output. To supplement this acoustic response, a dual-coil humbucker and piezo transducer provide additional gain and the ability to process the instrument’s output through guitar-style effects pedals and other analog or digital signal processing devices. Output levels for each pickup can be adjusted by the performer using two potentiometers, with signal from each source wired to one of two ¼” audio jacks. This modular arrangement allows audio from each pickup to be routed to separate channels on a mixing console or signal processor, enabling independent

[61] Martin Hardware, https://www.martinhardwareinc.com/

control over a variety of coloristic effects and spatialization parameters. Similar to later iterations of Collins' Backwards Guitar, as well as the MRP and EMPP, an array of external amplifiers drives each solenoid-actuator using a separate audio stream. In the current configuration, positive and negative leads from three, 20-Watt Class-D stereo amplifiers (Lepai LP-2020A) connect to each actuator via twelve 'banana plug'-style patch-points.[62] Importantly, pairing one solenoid to each string affords the ability to actuate audio frequencies exclusive to the harmonic spectrum of a given string. By changing the frequency content of the incoming audio stream, the performer may activate specific partials at will. Therein, this process capitalizes upon the phenomenon of sympathetic resonance.

Audio input for each actuator can originate from a variety of sources. In most performance scenarios, we route individual line-level outputs from the digital to analog convertors (DAC) of a Motu Ultralite or comparable multi-channel audio interface. However, audio signals for actuation could feasibly originate from any line-level source, including auxiliary outputs from an analog mixing console, preamplifier, synthesizer, or another electronically-transduced instrument. In turn, each audio channel feeds directly into either the left or right input for one of three stereo, 20-Watt Class-D amplifiers. Standard 18-gauge (AWG) speaker cable carries positive and negative leads for each amplified signal to twelve patch-points, located behind the hard-tail bridge. Finally, amplified current from these patch-point leads to the solenoid-actuators, positioned under each string. On the opposite end of the instrument, a humbucker pickup and piezo-electric disc (attached to the bottom surface of the hard-tail bridge) transduce resulting vibrations from the six strings.

[62] Lepai LP-2020A 2 × 20-Watt Stereo Power Amplifier. https://www.parts-express.com/lepai-lp-2020a-2x20w-hi-fi-audio-stereo-power-amplifier-with-3a-power-supply--310-294 (accessed February 10, 2017).

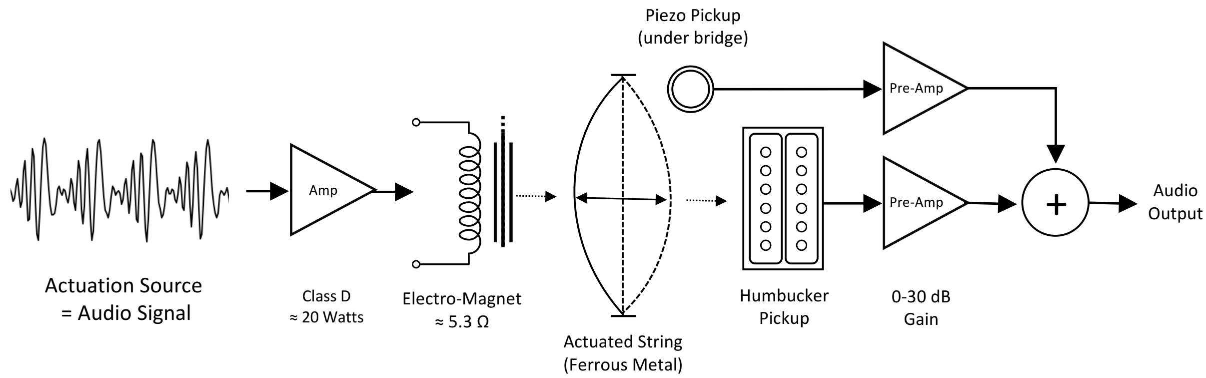

Figure 2. 3—Ferromagnetic Actuation Topology for 'Rosebud I'

2.5.1 Actuator Design and Implementation

Ferromagnetic actuation techniques for Rosebud I mirror those established by McPherson, as well as teams at CCRMA and the Instrumentation Lab at Miami University. In fact, we utilized some of the same physical components found within the MRP and EMPP in construction of our instrument. Consequently, Rosebud I retains many of the same traits and limitations presented by other actuated instruments. In regards to this design, McPherson, Berdahl, and others note the following limiting factors. First, given a fixed current, increasing the number of turns (N) within a solenoid's coil yields a stronger magnetic field. In musical contexts, reproducing a suitably wide range of frequencies requires the ability to modulate current at audio rates (≈ 20 – 20,000 Hertz). However, McPherson observes that changes in current, as well as physical properties of the solenoid, are necessarily limited by an actuator's inductance (L).[63] [64]

$$L = \mu \frac{N^2 A}{\ell} \tag{2 - 5}$$

[63] Andrew McPherson, Edgar Berdahl, Jeff Snyder, and Cameron Britt, Actuated Instruments Workshop (slides), *Presented May 20, 2012 at the Conference on New Interfaces for Musical Expression (NIME 2012),* Ann Arbor, Michigan.

[64] Edgar Berdahl, Steven Backer, and Julius O. Smith III, "If I Had a Hammer: Design and Theory of an Electromagnetically Prepared Piano," *Proceedings of the 2005 International Computer Music Conference.* https://ccrma.stanford.edu/~eberdahl/Papers/ICMC2005.pdf (accessed August 5, 2019).

In consideration for the contributing factors of turns (*N*), surface area of the solenoid (*A*), magnetic permeability (μ), coil length (ℓ), and total inductance (*L*), the earliest iterations of the MRP employed numerous "hand-wound" actuators consisting of 600 turns of 30 AWG copper wire. Including the solenoid's ferromagnetic core—a $^{3}/_{8}$" steel threaded rod—this actuator measures approximately 0.6" in length and 0.8" in diameter. Altogether, this first iteration yielded a total inductance (*L*) of 20mH and 9 Ohms of resistance.[65] Notably, the actuator's impedance falls within a range comparable to many consumer loudspeakers: 4-16 Ohms. Consequently, actuators matching these specifications may be driven by standard audio amplifiers—an important feature in considering amplification options for Rosebud I. For ease and reproducibility, later iterations of the MRP, EMPP, and Rosebud I replaced hand-wound coils with pre-fabricated electromagnets measuring 0.82" in diameter and 0.77" in length. Composed of 28 AWG wire, these E-77-82 actuators yield a total inductance of 19 mH and approximate resistance of 5.3 Ohms. In developing the second iteration of the EMPP, Bloland, Berdahl, and Backer suggest deploying electromagnets with similar properties, including an "infinite [100%] duty cycle."[66] Following the same design specifications, we purchased and installed a similar array of six E-77-82 electromagnets in Rosebud I. [67]

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[65] Andrew McPherson, "Techniques and Circuits for Electromagnetic Instrument Actuation," *Proceedings of the 2012 Conference on New Interfaces for Musical Expression (NIME 2012)*. http://www.nime.org/proceedings/2012/nime2012_117.pdf (accessed March 3, 2017).

[66] Per Bloland, "The Electromagnetically-Prepared Piano and its Compositional Implications," *Proceedings of the 2007 International Computer Music Conference.* http://www.perbloland.com/userfiles/file/EMPP-Comp-Implications.pdf (accessed August 5, 2019).

[67] Magnetic Sensor Systems. "E-77-82 Tubular Electro-Magnet." http://www.magneticsensorsystems.com/electromagnet/tubular/E-77-82.asp (accessed November 11, 2016).

Figure 2. 4—Array of Six (E-77-82) Electro-Magnetic Actuators Installed in 'Rosebud I'

As discussed, amplification of actuated signals for Rosebud I is accomplished using an array of six 20-Watt, class-D amplifiers. These components are readily-available, relatively inexpensive, and closely match impedance values for the model of electromagnet (E-77-82) suggested by McPherson, Bloland, Berdahl. As stated before, the development of these two instruments provided a tangible model for ferromagnetic actuation techniques applied in the construction of Rosebud I. Similarly, our mode of amplification exhibits many of the pitfalls described by McPherson and others. Like other commercial audio amplifiers, the Lepai LP-2020A does not provide any form of DC-offset. Thus, the amplified signal retains bi-polar properties, resulting in the phenomenon of "frequency-doubling."[68] Similarly, our chosen amplifier does not exhibit the efficiency, nor the spectral linearity of McPherson's transconductance amplifier. Consequently, high-frequency partials must be driven with increased gain, so as to maintain comparable amplitude with the fundamental frequency or other low-order harmonics. Instead, our

[68] Andrew McPherson, Edgar Berdahl, Jeff Snyder, and Cameron Britt, Actuated Instruments Workshop (slides), *Presented May 20, 2012 at the Conference on New Interfaces for Musical Expression (NIME 2012),* Ann Arbor, Michigan.

design assumes some variant of permanent magnetic field. This approach enables both attractive and repellent forces to act upon each ferrous metal string. As a precedent for this design choice, the first two iterations of the EMPP placed neodymium magnets of opposite polarity adjacent to each actuator.[69] Here, implementation of a permanent field appears as an additive feature, whereas no other substantial source of magnetism exists within the body of a grand piano or near the instrument's strings.

In contrast to the MRP and EMPP, the humbucker pickup installed in Rosebud I provides a permanent magnetic field. Designed to reduce hum and other electronic interference endemic to single-coil pickups, the dual-coil humbucker includes two permanent magnets of opposite polarity.[70] Placed in close proximity, each positive and negative field induces some magnetic force upon each ferrous metal string. As follows, these fields behave in part like the neodymium magnets placed near the actuated strings of the EMPP. Our initial impressions suggest that the addition of the humbucker's permanent field bears some influence upon each string's response to a bi-polar actuated signal, thus mitigating at least some of the frequency-doubling effects. Supporting this assertion, we observed substantial linearity in frequency response between actuated source signals and spectra produced by strings actuated by the same signal. For example, actuating a string tuned to a fundamental frequency of 50 Hertz with an audio signal containing a similar spectrum induced vibrations from the string at the same frequencies. Importantly, spectral analysis of the actuated string retained a prominent 50 Hertz fundamental frequency. Admittedly, our limited observations on this effect are anecdotal and warrant further investigation. That said, preliminary results suggest at least some mitigation of frequency artifacts otherwise produced in the absence of any permanent magnetic field.

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[69] Per Bloland, "The Electromagnetically-Prepared Piano and its Compositional Implications," *Proceedings of the 2007 International Computer Music Conference.* http://www.perbloland.com/userfiles/file/EMPP-Comp-Implications.pdf (accessed August 5, 2019).

[70] Seth E. Lover, 1955, Magnetic Pickup for Stringed Instrument, US Patent 2,896,491A.

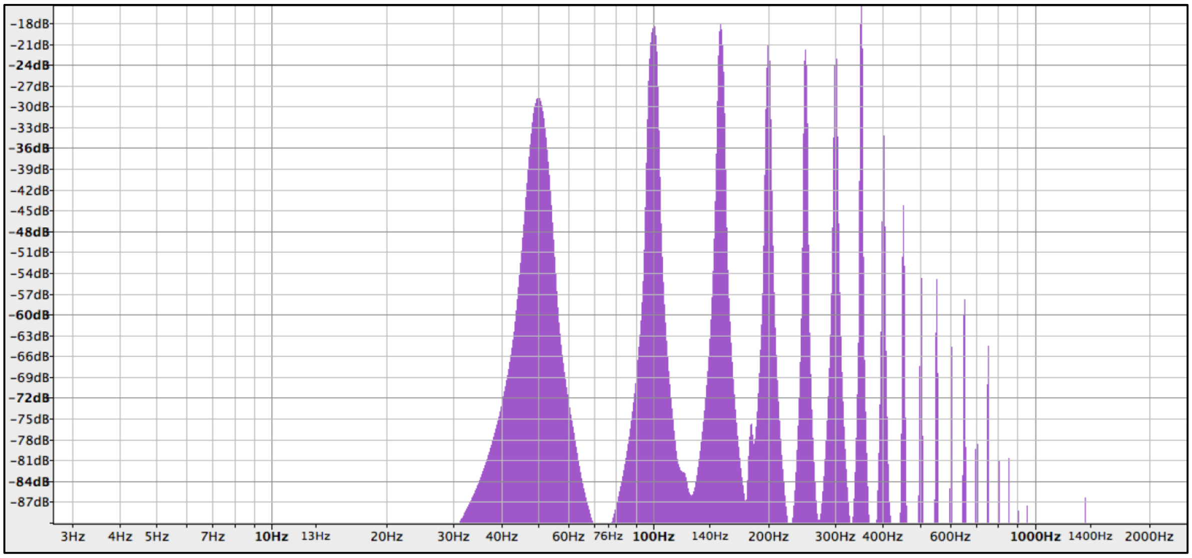

Figure 2. 5—Spectral Analysis of Actuated String—Rosebud I[71]
Fundamental Frequency for String and Actuated Audio Signal = 50 Hertz
Frequency Peaks (> -60 dB) at 50, 100, 150, 200, 250, 300, 350, 400, 450, 500, 550, and 600 Hertz

Speaking to the issue of linearity, researchers have generally favored predictive models in performance. In a broad sense, Berdahl, Backer, and Smith's stated aim to achieve a "roughly linear and time-invariant" translation between "arbitrary" source materials and actuated sonorities appears consistent with this dominant tendency in actuated instrument design; one which privileges transparency over affect.[72] Here, transparency and physical artifact appear as oppositional forces. Certainly, the ability to understand, quantify, and perhaps mitigate unintentional artifacts produced by non-linear systems warrants consideration for those endeavoring to manifest particular compositional results. These contributions cannot be overstated. However, in designing actuated instruments which exhibit overtly non-linear behaviors, one might also seek to upend the primacy of linear reproduction and propose a practice which embraces artifact as a potent aesthetic force.

[71] Spectral analysis performed using an FFT size of 16384 (Hanning window).

[72] Edgar Berdahl, Steven Backer, and Julius O. Smith III, "If I Had a Hammer: Design and Theory of an Electromagnetically Prepared Piano," *Proceedings of the 2005 International Computer Music Conference.* https://ccrma.stanford.edu/~eberdahl/Papers/ICMC2005.pdf (accessed August 5, 2019).

2.6 Representative Applications of Electro-Mechanical Actuation

As demonstrated, electro-magnetism can provide an effective means for inducing sympathetic vibration in strings. However, other non-simulative actuation techniques have also been deployed in the design and construction of historical and contemporary instrumentation. Henceforth, we examine electronically-mediated methods exclusive to those stringed instruments which employ electro-mechanical actuation. As demonstrated in the compositional work of David Coll, Matthew Goodheart, and others, surface transducers and other kinetic means of actuation have been applied to a variety of instruments and resonant objects.[73] While these contributions remain relevant, the scope of this essay is purposefully limited to those instruments which employ strings as their primary, sounding body. Here we define electro-mechanical actuation by the transference of acoustic energy via direct, physical contact between a string or set of strings and an electronic transducer. Alternately, transmission of sympathetic vibrations may also occur through a secondary acoustic body or medium, such as a resonant chamber or bridge(s). In either case, physical contact between the actuator and sounding body help distinguish electro-mechanical actuation from electro-magnetic methods. Of all electronically-mediated techniques, electro-mechanical actuation represents the most kinetically-embodied methodology.

Instrumental applications of electro-mechanical actuation for strings date to at least the early 1950's with the introduction of the *Palme Diffuseur*. Designed for use with the Ondes Martenot, the device was intended to provide additional resonance for chromatic pitches produced by the instrument. To achieve pitch-specific actuation, the Palme's transducer transmits sympathetic vibrations to twelve metal strings positioned over a wooden chamber. The instrument's inventor, Maurice Martenot, developed this device as of one four modified loudspeakers: the *Diffuseur Principal*, *Métallique*, and *Resonance*.[74] Similar to the Palme, the *Diffuseur Resonance* utilizes stretched coils to imbue comparable effect. Unfortunately, with

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[73] Reembodied Sound, "Artists and Researchers," http://evolvingdoormusic.net/mw/index.php?title=People, (Accessed September 8, 2019).

[74] David Madden, "Advocating for Sonic Restoration." *Wi: Journal of Mobile Culture*. Vol. 7 (2013). http://wi.mobilities.ca/advocating-sonic-restoration-les-ondes-martenot-in-practice/ (accessed October 7, 2019).

exception of the Sustain Man "Electro-acoustic" sustainer by Sustainiac™, few other electro-mechanically actuated devices have been developed commercially for use with stringed instruments.[75] Unlike the ubiquitous Ebow™, electro-mechanical actuation tends to retain more specialized features, appearing as singular prototypes or limited-production instruments for research-based applications. Consequently, we shall begin by examining a few representative examples of instruments exhibiting similar properties.

2.6.1 Referencing Existing Instruments and Design Morphologies

As with other actuated string instruments, electro-mechanically actuated devices tend to retain formal elements consistent with existing instrumentation. Similar morphologies and functional components, including resonant chambers and other dimensional characteristics, are often either duplicated or referenced. Like electro-magnetically actuated instruments, such as the MRP or EMPP, electro-mechanical actuation may be employed as a means for augmenting or preparing an existing instrument. In line with this paradigm, Dan Overholt's Overtone Fiddle retains morphological features of a violin. In performance, this instrument employs electro-mechanical actuation via a tactile transducer, affixed to the body of the instrument. Actuation occurs through both the main body and a second acoustically-resonant chamber, positioned below the instrument. This secondary resonator consists of a thin wall constructed from carbon-fiber and balsa wood. Thus, Overholt's design achieves relative independence between harmonically-sympathetic vibrations emanating from the strings and those primary vibrations occurring near the point of contact between the tactile transducer and resonant chamber.[76] Like many contemporary actuated instruments, a small Class-T amplifier drives the embedded transducer.

While each resonant structure operates as an efficient medium for acoustic transmission, the designer employs a magnetic pickup to detect vibrations from the strings for subsequent

[75] Alan Hoover, 2005, Electroacoustic Sustainer for Musical Instruments, US Patent 2,005,008,170,3A1.

[76] Dan Overholt, "The Overtone Fiddle: An Actuated Acoustic Instrument," *Proceedings of the 2011 Conference on New Interfaces for Musical Expression (NIME 2011)*. https://www.nime.org/proceedings/2011/nime2011_004.pdf (accessed February 24, 2021).

amplification and processing. An important structural feature in the application of DSP-mediated feedback networks, Overholt describes this separation between actuator and strings as a "forced-sensing approach."[77] In this context, the electro-magnetic pickup functions as a *sensor*, rather than a means of acoustic amplification. The distinguishing language and stated functionality between *pickups* and *sensors* mirrors broader tendencies within the field—one which maintains independence between actuated topologies and those involved in supplemental sound-reinforcement. In addition to acoustic interactions afforded by DSP-driven feedback networks, Overholt also embeds an array of sensing technology within a modified bow. A CUI32 microprocessor, wireless 802.11g radio module, and an absolute orientation sensor facilitate the mapping of real-timer performance data to control various actuation parameters.[78]

Referencing similar performative and morphological features to a cello, the *Halldorophone* represents an analogous approach to instrument design; one which embodies both imitative and extensible modalities of existing stringed instruments. The instrument's inventor and namesake, Halldór Úlfarsson, describes the relationship between existing and emergent properties in design and performance practices in calculated terms:

> "Referencing the classical strings is a tactical choice because of their association in the collective consciousness to virtuosic players and luthiers and the propensity of those instruments to occupy centre stage, all of which seemed like an asset to the identity of new instrument. Being cello-like also makes the instrument more relevant to numerous well-trained performers of the cello as it allows for the recycling of their playing skills."[79]

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[77] Dan Overholt, Edgar Berdahl, and Robert Hamilton, "Advancements in Actuated Instruments," *Organised Sound,* Vol. 16, Issue 2 (2011): 154-165. https://www.cambridge.org/core/journals/organised-sound/article/advancements-in-actuated-musical-instruments/AFBD83D9E53F8C0270492F06CD0F2380 (accessed August 5, 2019).

[78] Dan Overholt, "The Overtone Fiddle: An Actuated Acoustic Instrument," *Proceedings of the 2011 Conference on New Interfaces for Musical Expression (NIME 2011).* https://www.nime.org/proceedings/2011/nime2011_004.pdf (accessed February 24, 2021).

[79] Halldór Úlfarsson, "The Halldorophone: The Ongoing Innovation of a Cello-like Drone Instrument," *Proceedings of the 2018 Conference on New Interfaces for Musical Expression (NIME 2018).* http://www.nime.org/proceedings/2018/nime2018_paper0058.pdf (accessed August 14, 2018).

As suggested by Úlfarsson, these referential design features and the instrument's familiar physical proportions and layout are strategic. Combined, these properties afford the performer a sense of immediacy in translating conventional techniques, while introducing newly assessable performance modalities and sonic textures. Like the Overtone Fiddle, the Halldorophone utilizes electro-mechanical actuation to affect feedback-based resonances. A familiar arrangement of four bowed strings pairs with a second set of four sympathetically-tuned strings, positioned below the fingerboard. Dedicated electro-magnetic pickups sense vibrations from any combination of the eight strings, wherein embedded electronics boost each signal to line-level. Here, the process of parsing signals from separate strings appears reminiscent of later iterations of the Backwards Guitar. However, instead of piezo-electric pickups, Úlfarsson utilizes separate electro-magnetic pickups installed under the Halldorophone's eight strings.[80] Applying one's right-hand, the performer can adjust individual volume levels for the four bowed strings using a set of faders, placed on the left-facing side of the instrument. To supplement this control structure, Úlfarsson suggests controlling the volume of the four sympathetic strings using two stereo volume pedals. The resulting mix is then fed through a 50-watt amplifier and routed to a mid-range speaker cone "embedded in the back of the instrument."[81] Acting as actuator, mechanical vibrations from the speaker induce sympathetic vibrations from the eight strings to complete a positive-feedback structure. Additionally, the resonant properties of the instrument's hollow body behave like an acoustic filter, thereby inducing variable states of self-sustained oscillation for different frequencies.

As with Piñeyro's Electric Slide Organistrum and Trail's STARI, tuning capabilities for certain iterations of the Halldorophone (2008-18) suggest a framework conducive to extended just intonation. In addition to a fretless neck, earlier versions also included a means for adjusting the tension for individual strings during performance. This added feature enables continuous changes

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[80] Nicolas Collins, *Handmade Electronic Music: The Art of Hardware Hacking*, Second Edition, New York, NY: Routledge, 2009.

[81] Halldór Úlfarsson, "The Halldorophone: The Ongoing Innovation of a Cello-like Drone Instrument," *Proceedings of the 2018 Conference on New Interfaces for Musical Expression (NIME 2018)*. http://www.nime.org/proceedings/2018/nime2018_paper0058.pdf (accessed August 14, 2018).

in pitch. In part, alternate tuning structures have defined the instrument's compositional history. For example, in scoring *Composition for Viola and halldorophone* (2012), Johan Svensson assessed a variety of tuning structures before settling upon the following open-string arrangement: IV (32.7 Hertz), III (49 Hertz), II (73.42 Hertz), I (110 Hertz).[82] Certainly, this manner of scordatura is not without precedent. Likewise, Svensson's open tuning does not necessarily reflect just proportions. Instead, as Úlfarsson recounts in his correspondence with the composer, this tuning structure arose as an exploration of "feedback behavior."[83] Here, the prominence of certain sonorities generated through recursive, electro-mechanical actuation informed how the instrument was tuned for the piece. The composer recalls an intense process of deliberation in settling upon a single, fixed system. With eight available strings—four bowed and four sympathetic—one could feasibly envision a variety of tuning structures which capitalize upon this phenomenon. As with other actuated string instruments, sympathetic approaches to intonation certainly warrant further investigation.

Related to Overholt and Úlfarsson's respective work with extant instrumentation, Finnish researcher and composer Otso Lähdeoja's developments in "active acoustics" and "structure-borne sound" engage electro-mechanical actuation as means for re-embodying electronic sound within the physical form of acoustic stringed instruments. As defined by Lähdeoja, the term "active acoustics" signifies "the use of structure-borne sound drivers to drive electronic sounds into the physical structures of an acoustic instrument, inducing air-borne vibration, analogously to a diaphragm loudspeaker."[84] Moreover, the principle of "structure-borne sound" re-envisions the primacy of loudspeaker-based paradigms in electro-acoustic performance. Here, the acoustic

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[82] Johan Svensson, "Works." *Johan Svensson: Composer*, http://johansvensson.nu/works/ (accessed February 26, 2021).

[83] Halldór Úlfarsson, "The Halldorophone: The Ongoing Innovation of a Cello-like Drone Instrument," *Proceedings of the 2018 Conference on New Interfaces for Musical Expression (NIME 2018).* http://www.nime.org/proceedings/2018/nime2018_paper0058.pdf (accessed August 14, 2018).

[84] Otso Lähdeoja, "Active Acoustic Instruments for Electronic Chamber Music," *Proceedings of the 2016 Conference on New Interfaces for Musical Expression (NIME 2016)*, https://www.nime.org/proceedings/2016/nime2016_paper0027.pdf (accessed August 5, 2019).

instrument retains two roles: a performable object capable of generating sound through physical interventions by the player; and a vessel for diffusing electronically generated or processed audio. In the latter regard, Lähdeoja extolls the unique spatial and coloristic properties of structure-borne sound, stating:

> "The aural image given by the electro-acoustic instrument [electro-mechanically actuated guitar] is radically different from a loudspeaker: the guitar's directionality points towards a cardioid pattern over the whole spectrum, while loudspeakers radiate in a closed angle at high frequencies. Also, the audio spectrum modified by the tonewood results in a characteristic aural imprint for both acoustic and electronic sounds."[85]

Much of his research in active acoustics centers around development of the "augmented guitar." Beginning with a Breedlove C20 acoustic steel string guitar, Lähdeoja's design extends the performative capabilities of the instrument using a combination of two commercially-available drivers, a 30-Watt Class-D stereo amplifier, Ubertar hexaphonic (electro-magnetic) pickups, Fishman preamplifiers, and an RME audio interface. To generate and process actuated signals, Lähdeoja developed custom signal-processing using Max/MSP and Pure Data ('Extended') programming environments. While each iteration of the instrument employed two Hiwave 32C30-4B surface transducers, placement of these electro-mechanical drivers on the body of the instrument shifted according to measured and perceived frequency-frequency response. Balancing considerations for signal strength and applied weight, later versions placed the drivers on the back and side panels of the guitar.[86]

The instrument's actuation topology tracks six discrete audio signals, representing vibrations captured from each string. Here, the hexaphonic pickup allows each signal to be routed to a separate channel on the audio interface and processed independently using custom software.

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[85] Otso Lähdeoja, "An Augmented Guitar with Active Acoustics," *Proceedings of the 12th International Conference on Sound and Music Computing (SMC 2015, Maynooth, Ireland),* 98-102. http://www.maynoothuniversity.ie/smc15/files/FinalProceedings.pdf (Accessed August, 2015).

[86] Otso Lähdeoja, "Active Acoustic Instruments for Electronic Chamber Music," *Proceedings of the 2016 Conference on New Interfaces for Musical Expression (NIME 2016)*, https://www.nime.org/proceedings/2016/nime2016_paper0027.pdf (accessed August 5, 2019).

Following a recursive signal-path, the processed audio passes through another stage of amplification before reaching the two drivers. Concurrent vibrations couple with the resonant properties of the instrument's body and strings to generate an audible output. This configuration affords a variety of feedback-induced effects.[87] While mechanical vibrations transmitted through the resonant body of the guitar likely account for the majority of the instrument's acoustic output, secondary vibrations also induce sympathetic resonance via the six strings. In this regard, Lähdeoja's approach to active acoustics retains features consistent with our definition of electro-mechanical actuation, wherein electronic mediation confers the transference of acoustic energy to a set of strings through direct, physical contact.

2.7 Rosebud ('Louise') II

Rosebud ('Louise') II is an electro-mechanically actuated string instrument, designed and constructed by the author.[88] This instrument retains many formal characteristics of Rosebud I, including a 36-inch open-frame construction comprised of 6061-T6 extruded aluminum angle and humbucker pickups. However, frame dimensions between the two instruments vary, with a pair of 2" × 1 ½" × $^1/_8$" angles contributing to a slightly elevated profile for Rosebud II. Other material differences include CNC-routed oak boards connecting the frame, bridges, actuator, and tuning hardware. An additional 8" × 7 ½" × $^3/_{32}$" 6061-T6 aluminum panel secures a moveable third-bridge, two humbucker pickups, and associated electronics.[89] Whereas the low-frequency range for Rosebud I necessitates the use of baritone-gauge strings (0.074"), we strung Rosebud II with twelve, 0.008" (plain-steel) guitar strings.[90] These relatively narrow-gauge strings span a total

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[87] Etienne Thuillier, Otso Lähdeoja, and Vesa Välimäki, "Feedback Control in an Actuated Acoustic Guitar Using Frequency Shifting," *Journal of the Audio Engineering Society*, Vol. 67, No. 6, June 2019, http://www.aes.org/e-lib/browse.cfm?elib=20480 (accessed August 5, 2019).

[88] The author applied the 'Louise' moniker for Rosebud II in tribute to his mother, Laura Louise Luca (b. 1951 – d. 2011).

[89] "0.09 Aluminum Sheet 6061-T6," https://www.onlinemetals.com/en/buy/aluminum/0-09-aluminum-sheet-6061-t6/pid/1244 (accessed September 1, 2019).

[90] "Plain-Steel 0.008 Single Guitar Strings." *JustStrings.com.* https://www.juststrings.com/jsb-008p-1.html (accessed September 22, 2019).

scale-length of approximately 28 inches. To accommodate additional strings, we utilize two Les Paul (LP)-style stop-tail bridges.[91] As these pre-fabricated bridges are designed for use with twelve-string electric guitars, we arrange each set into six, two-string courses. The increased string-count necessitates other unique design features, including dual-headstocks. This unconventional arrangement conserves space by distributing six of the twelve tuning machines at opposite ends of the instrument. Luthier Yuri Landman employs a similar configuration for his multi-string instruments, including the Electric Harmonic Canon and Brooklyn Bridge.[92] [93]

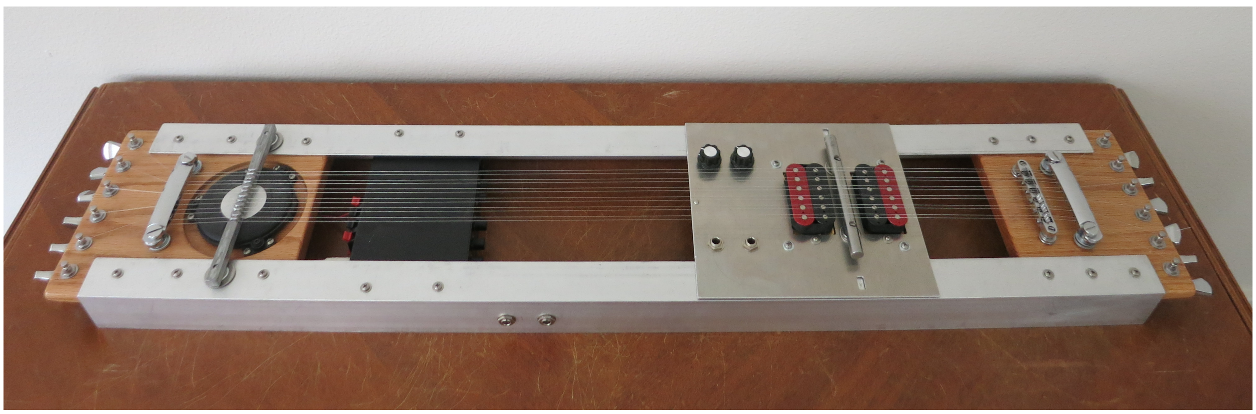

Figure 2. 6—Rosebud ('Louise') II—Electro-Mechanically Actuated String Instrument

Like other electro-mechanically actuated instruments, our design re-purposes commercially-available transducers. Retaining many functional features of standard loudspeakers, these so-called "bass shaker" devices consist of a speaker coil and permanent magnet, housed within a plastic puck-shaped enclosure. Unlike the mid-range loudspeaker embedded into the body of the Halldorophone, these specialized transducers lack the cone typically found in most speakers.

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[91] M Y Fly Young Bridge and Stop Tail Bar for 12-String Electric Guitar (LP)

[92] Yuri Landman, "Hypercustom Brochure April 2012," *Hypercustom.nl*. https://issuu.com/yurilandman/docs/brochureapril2012 (accessed March 4, 2020).

[93] Yuri Landman, "DIY: Yuri Landman's Flying Double Dutchman Crunch Project," *Premier Guitar,* March 2013. https://www.premierguitar.com/articles/Yuri_Landman_Makeover_Flying_Double_Dutchman_Crunch (accessed September 11, 2019).

Utilized in home theatre applications, these devices can be attached to walls and other surfaces—essentially converting any object into a subwoofer, capable of reproducing low-frequency audio. Similarly, by placing the transducer in direct contact with an instrument's bridge(s), one can induce sympathetic vibrations in one or more strings. As noted by Jiffer Harriman, efficient transference of kinetic energy between the transducer element, bridge, and strings requires significant calibration and material considerations. For example, in prototyping earlier iterations of the Feedback Lap Steel, Harriman initially attached a Dayton Audio TT25-8 ('Puck') tactile transducer to underside his instrument. According to Harriman's observations, this configuration induced a negligible amount of acoustic energy upon the instrument's strings, resulting in a relatively inefficient mode of actuation. In later versions of the Feedback Lap Steel, Harriman couples the transducer to a customized acrylic bridge. Laser-cut to match the physical contours of TT25-8, the increased contact between the transducer and bridge appears to facilitate a more efficient transfer of acoustic vibrations to the instrument's strings.[94]

With the goal of implementing a comparable means for electro-mechanical actuation in Rosebud II, we followed many of the same design conventions and material properties established by Harriman. In fact, our instrument employs the same model of tactile transducer and a similar method for coupling the transducer, bridge, and strings. Varying our approach slightly, we substitute an acrylic bridge for a metal bridge fashioned from a 6 ¾" × $^3/_8$" × $^3/_8$" stainless steel bar. Hand-cut grooves secure each of the twelve strings in place, while two metal posts position the bridge in a fixed, perpendicular orientation to the instrument's aluminum frame. While the lateral position of the bridge remains static, our design affords some degree of vertical displacement. As such, overhead pressure from the twelve strings helps maintain maximal contact between the bottom the bridge and top surface of the transducer. Like Harriman, our aim with this configuration is to increase the surface area for transmitting vibrations, thusly increasing efficiency

[94] "Dayton Audio TT25-8 PUCK Tactile Transducer Mini Bass-Shaker—8 Ohm," *Parts-Express.com*, https://www.parts-express.com/Dayton-Audio-TT25-8-PUCK-Tactile-Transducer-Mini-Bass-Shake-300-386?gclid=Cj0KCQiA4feBBhC9ARIsABp_nbXo3n2vON2vBlEf3jEFWBzkTeL8uo6Ee5Bb9QMxZejTIwt8R0BhLGAaAqPzEALw_wcB (accessed September 1, 2019).

in actuation. While qualitative observations in performance support this hypothesis, further study is needed to corroborate these initial results.

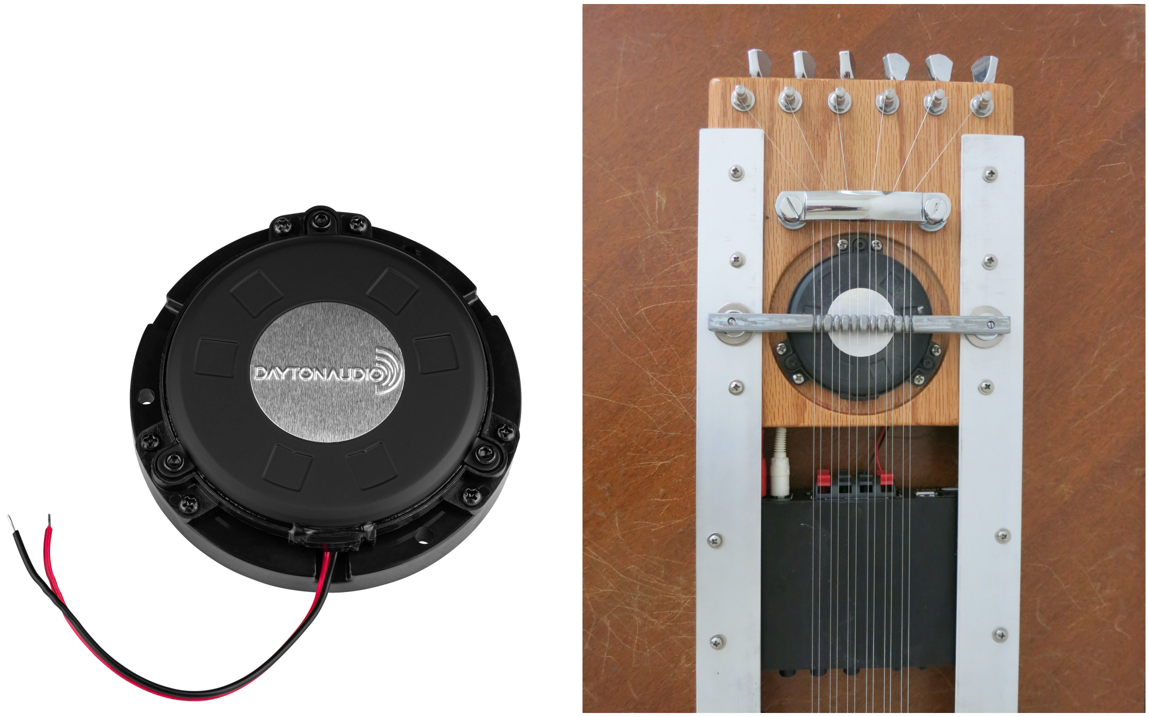

Figure 2. 7—Dayton Audio TT25-8 ('Puck') Tactile Transducer Installed Under the Bridge of Rosebud II

As with other modes of actuation, a preference for inexpensive and readily-available components guided our choice in amplification. Here, we employ a 40-Watt Class-T amplifier to drive a single, tactile transducer.[95] A pair of ¼" inputs connect to the stereo power amplifier, allowing a performer to actuate the instrument's strings using any external audio signal. Unlike the modular routing configuration for Rosebud I, amplification remains embedded within the body of the instrument, with all audio and other signal connections normalized. In the present

[95] "Lepai LP-168HA 2 × 40-Watt Stereo Power Amplifier," *Parts-Express.com,* https://www.parts-express.com/Lepai-LP-168HA-2.1-2x40W-Amplifier-1x68W-Sub-Output-310-308?gclid=Cj0KCQiA4feBBhC9ARIsABp_nbVB8ZPtmawkcZFIZ8IaxyWkD5XdOMuTlsf5WK_LejJq05hroQ5ATUYaAnnbEALw_wcB (accessed September 1, 2019).

configuration, Rosebud II engages only a single mode of electro-mechanical actuation. However, in anticipating future modifications, we chose to install a second ¼" input for connecting external audio sources to the built-in stereo amplifier. This extensible framework provides a means for independent control over a secondary mode of actuation. For example, we are currently planning for the installation of three solenoid actuators. When implemented, Rosebud II will embody a *hybrid* modality, exhibiting the unique properties and affordances of both electro-mechanical and electro-magnetic actuation.

III. RELEVANT PRACTICES IN JUST INTONATION

Few can doubt Harry Partch's significance as a seminal figure in the theory, compositional practice, and codification of just intonation in the twentieth century. Speaking to the latter point, the language and conventions presented in his major treatise, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments* have laid the foundation for structural models and approaches taken *and* extended by composers and theorists to follow. In fact, three of the composers and theorists presented in this chapter, Ben Johnston, James Tenney, and Erv Wilson, have either studied or collaborated with Partch at some point during their respective careers. One could also argue that work in the field of just intonation which preceded the bulk of Partch's contributions may also be described, analyzed, and understood through the lens of the composer's specific language and theoretical conventions. Such is the case with the research conducted during the first decades of the twentieth century by Kathleen Schlesinger (b. 1862 – d. 1953). For our purposes, Partch's conception of a Monophonic Fabric suggests a concise structural and functional framework for apprehending the construction, performance, and compositional affordances of actuated string instruments. Thus, we must begin our analysis of relevant practices in just intonation with a general explication of Partch's theories, focusing on his proprietary and often, invented language to describe specific organizational features and phenomenon.

3.1 The Overtone and Undertone Series' and Conceptions of Major and Minor Tonality

The body of Partch's theories reside within the structure of what the composer refers to as a "Monophonic Fabric" or simply, "Monophony."[96] Related, though not be confused with the concept of a single voice, Partch's definition of Monophony can be expressed in three forms. First, the Monophonic fabric can be reduced to the organization of musical intervals (e.g. dyads) in relation to a central tone, or *unity*, represented by the ratio '1/1'. In standard practice, we refer to this most basic interval as a *unison* or—within specific contexts—as a *tonic*. All subsequent ratios in just intonation exist as proportional derivatives of the unity ratio, 1/1. For example, if our unity ratio is equal to the frequency 55 Hertz, then the ratio 2/1 shall represent a frequency equivalent to 110 Hertz—or twice the frequency of unity ($55\ Hz \cdot 2/1 = 110\ Hz$). Within the same tuning

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[96] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

structure, the just ratio 4/3 represents the frequency 73.333 Hertz ($55\ Hz \cdot 4/3 = 73.333\ Hz$), while 5/4 is equal to 68.75 Hertz ($55\ Hz \cdot 5/4 = 68.75\ Hz$), and so on.

Secondly, Partch correlates the proportional structure of Monophony to harmonic or arithmetic divisions along the string of a monochord. As an outgrowth of experiments conducted by Pythagoras of Samos during the sixth century B.C.E., Partch assigns the frequency of an undivided string as the unity ratio, 1/1.[97] By successive divisions of a string's length into whole-number proportions, we perceive an array of tones whose frequencies are equivalent to corresponding integral ratios of the undivided string's fundamental frequency (F). When sounded, each integral division in string length generates a frequency equivalent to a harmonic overtone of the undivided string's fundamental frequency. In Figure 3.1, we see division of a single string into 1/2, 1/3rd 1/4th, and 1/5th the length of the undivided string (L). Notice how each corresponding frequency ratio appears in direct proportion to the reciprocal value of each divided length. For example, dividing a string into two equal parts yields the pitch ratio 2/1—one octave, or twice the frequency of the fundamental. Dividing the string into three equal parts produces the ratio 3/1, equivalent to one octave and a *Pythagorean* perfect fifth. According to Partch, the resulting structure of small, whole-number ratios represents salient musical intervals and, therein may be defined as 'Just Intonation'. Alternately, the term Monophony describes the specific procedure for generating this system.

[97] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

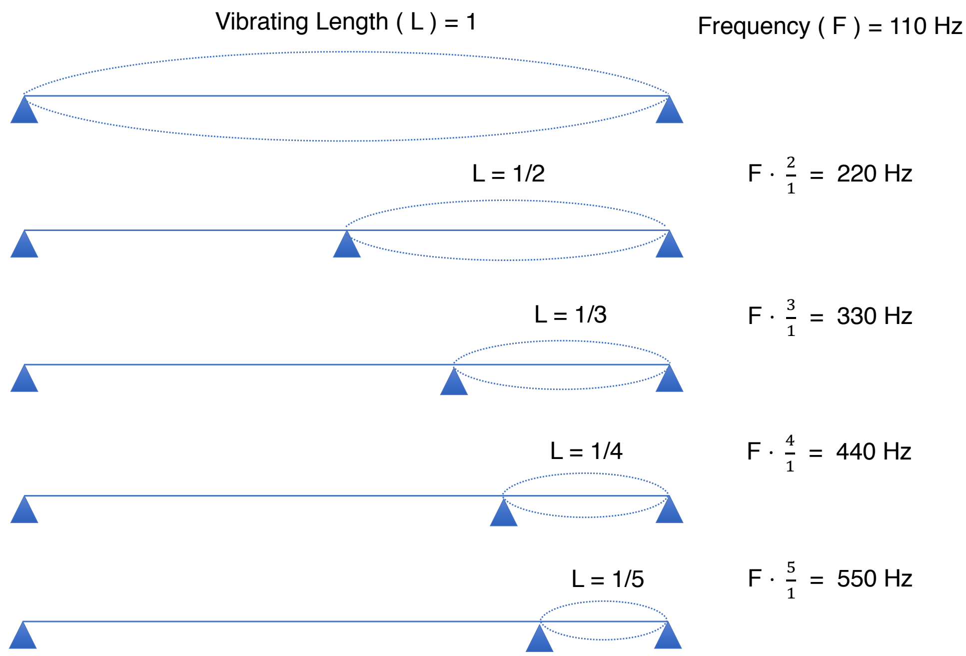

Figure 3. 1—Harmonic Divisions for a Single, Vibrating String (Monochord)

In a similar manner, *Arithmetic* procedures also employ equal length divisions of a vibrating string to derive just proportions.[98] In Figure 3.2, we see a single string divided into twelve equal lengths, with each segment representing 1/12th the length of the undivided string (L). However, instead of sounding each single division of our string, arithmetic procedures compel us to activate the *remaining* string length. For example, by shortening the string by 1/12th of the undivided length ($L/12$), the remaining 11/12th of the original length vibrates at 12/11th the frequency of the undivided string's fundamental frequency. As follows, if the fundamental frequency of our undivided string is 110 Hertz, this divided length ($11/12 \cdot L$) would vibrate at 120 Hertz ($110\,Hz \cdot 12/11 = 120\,Hz$). Again, notice that the divided string length and frequency ratio represent reciprocal values. Moving forward, if we shorten the same length of string by another 1/12th of the total length, the vibrating section is reduced to 10/12th the length of the undivided length—or 5/6, in reduced-ratio form. Sounding this section of string yields a

[98] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

frequency 6/5th the frequency of the undivided string's fundamental frequency ($110\ Hz \cdot 6/5 = 132\ Hz$).

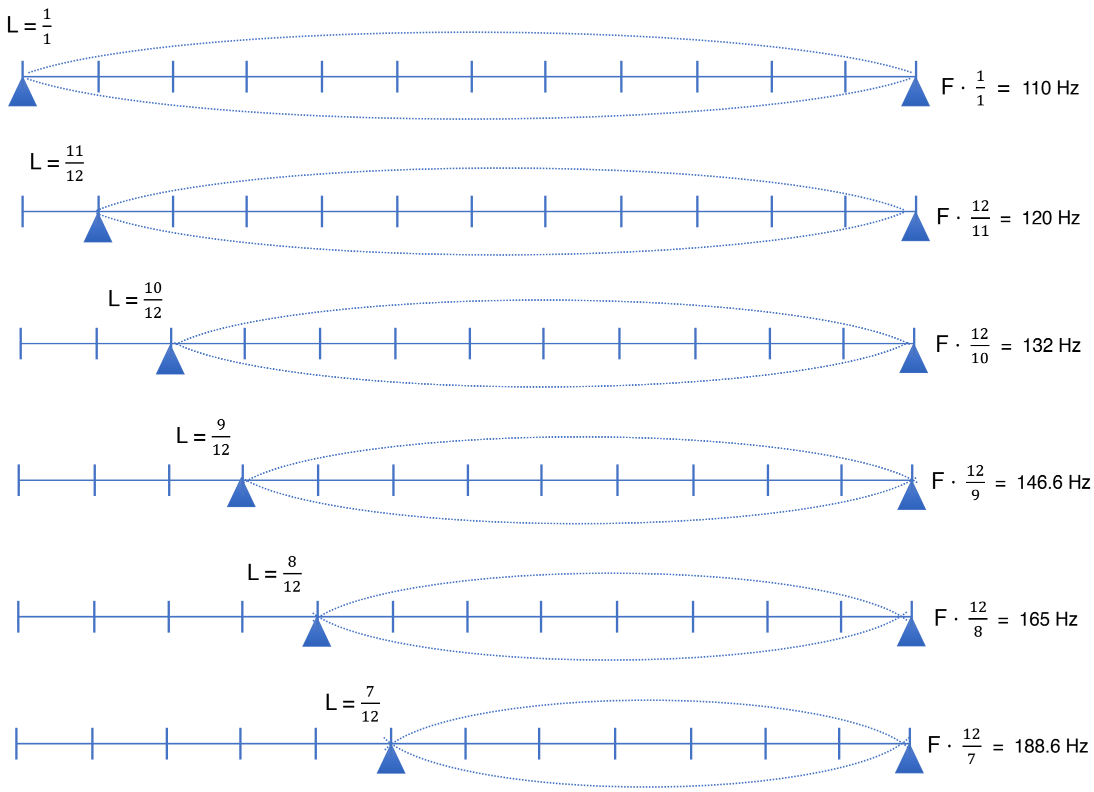

Figure 3. 2—Arithmetic Divisions of a Single, Vibrating String (Monochord)

Notably, instead of producing harmonic frequencies proportional to an ascending overtone series (e.g. integral multiples of a fundamental frequency), arithmetic divisions yield frequency ratios proportional to a descending 'undertone' series. Here each resultant sub-harmonic represents a frequency equivalent to an integral division of the undivided string's fundamental frequency.[99] Within the undertone series, the second sub-harmonic sounds at half the frequency of the fundamental, while the 3rd, 4th, and 5th subharmonics sound at 1/3, 1/4, and 1/5th the frequency of the fundamental, respectively. Returning to our previous example, these six frequencies

[99] Kathleen Schlesinger, "The Origin of the Major and Minor Modes," *The Musical Times*, Vol. 58, No. 893 (July 1917): 297-301. http://www.jstor.org/stable/908417 (accessed Dec. 28, 2017).

generated through arithmetic division represent the 12th, 11th, 10th, 9th, 8th, and 7th sub-harmonics of an undertone series whose fundamental frequency is twelve times the frequency of our undivided string ($F \cdot 12 = 1320\ Hz$).

(3 - 1)

$$\mathit{1^{st}\ Subharmonic} = F \cdot \frac{12}{1} = 1320\ Hz$$

$$[\ \dots\ ]$$

$$\mathit{7^{th}\ Subharmonic} = F \cdot \frac{12}{7} = 188.6\ Hz$$

$$\mathit{8^{th}\ Subharmonic} = F \cdot \frac{12}{8}\ (or\ \frac{3}{2}) = 165\ Hz$$

$$\mathit{9^{th}\ Subharmonic} = F \cdot \frac{12}{9}\ (or\ \frac{4}{3}) = 146.666\ Hz$$

$$\mathit{10^{th}\ Subharmonic} = F \cdot \frac{12}{10}\ (or\ \frac{6}{5}) = 132\ Hz$$

$$\mathit{11^{th}\ Subharmonic} = F \cdot \frac{12}{11} = 120\ Hz$$

$$\mathit{12^{th}\ Subharmonic} = F \cdot \frac{12}{12}\ (or\ \frac{1}{1}) = 110\ Hz$$

Finally, as an extension of spectral phenomenon first observed by Marin Mersenne during the seventeenth century, Partch frames the Monophonic Fabric as a subset of relationships between harmonic partials in the overtone series.[100] From this perspective, a collection of consecutive overtones constitutes a series of just ratios, whereby the proportional relationships between their frequencies is analogous to a specific class of intervals. In this example, frequency intervals between the fourth, fifth, and sixth partials [4:5:6] represent the just ratios 5/4, 6/5, and 3/2 (or 6/4). Together, this subset of consecutive harmonics forms a just major triad. By similar procedures, we can derive a collection of 23 just intervals from interceding frequency ratios found between the first fifteen harmonics of the overtone series.[101]

[100] Albion Gruber, "Mersenne and Evolving Tonal Theory," *Journal of Music Theory* 14, no. 1 (1970): 36-67. http://www.jstor.org/stable/843036 (accessed January 22, 2019).

[101] This collection of 23 just intervals excludes octave-equivalent ratios.

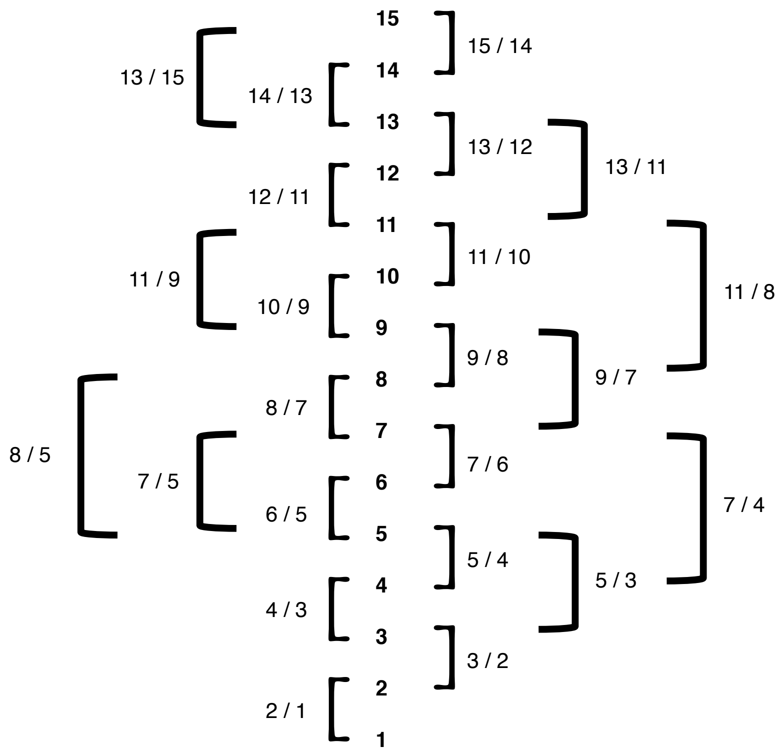

Figure 3. 3—First Fifteen Harmonics of the Overtone Series and 23 Interceding Just Ratios

3.2 Odentities, Udentities, and the Numerary Nexus

Specific conventions for the Monophonic Fabric are perhaps best described in taxonomical terms. Thereby, Partch classifies structural behaviors and perceptual affordances shared by different just ratios according to common, numerical features. Despite the bewildering quantity and potential variety of intervallic relationships presented in just intonation, the properties of Monophony maintain a cohesive tonal quality. That is to say that certain combinations of just ratios generate a perceptual pull towards a tonal center, or *unity*. Taken in spectral terms, the unity value represents the fundamental frequency of a given overtone or undertone series. Distinguishing ratios which share a mutual tonality is as simple as identifying a common numerator or denominator. Partch refers to this value as the ratio's *Numerary Nexus*.[102] For example, take the following set of just ratios:

[102] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

1/1, 7/6, 5/4, 7/5, 3/2, 7/4

The ratios 1/1, 5/4, 3/2, and 7/4 all retain a common denominator reducible to a factor of '/2'. According to Partch's terminology, this tonality may be identified by a Numerary Nexus of '2'. Likewise, when arranged in ascending order, numerators within the same tonality form a particular subset of integers: 1:3:5:7. These values correspond to the ratios 1/1, 3/2, 5/4, and 7/4, respectively. Interceding ratios within this subset present intervals proportional to relationships found between the first, third, fifth, and seventh harmonics of an ascending overtone series. Here, the ratio 1/1 is proportionally equivalent to the first harmonic, or fundamental frequency. Thus, 1/1 defines our unity, or tonal center. To further codify this intervallic arrangement, Partch refers to any set of ratios sharing a common denominator as an *Otonality*. Likewise, within any Otonal structure, each numerator value signifies a separate *Odentity*. In relation to unity, each ratio bears a tonal identity equivalent to a specific, odd-numbered partial within the harmonic series.[103]

As noted previously, the simultaneous sounding of ratios 1/1, 5/4, and 3/2—a proportional facsimile of intervals present between the fourth, fifth, and sixth harmonics—produces a just major triad. By transposing the same 4:5:6 triad to the subdominant (IV) and dominant (V) position, we yield a seven-note scale known as the "Ptolemaic Sequence"—sometimes called the "Intense Diatonic." Originating in Alexandria during the second century, this sequence of intervals establishes a procedural framework for modern tonality and scale construction.[104] In just intonation, transposition of a given ratio is achieved via multiplication. As the just counterparts of the perfect fourth (IV) and fifth (VI) embody the ratios 4/3 and 3/2, the products of these values and our original, tonic position major triad (1/1—5/4—3/2) generate a seven-note sequence which exhibits intervallic properties akin to the major scale. Thereby, the implicit features of the overtone series provide a basis for the conception of major tonality.

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103 Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

104 Douglas Keislar, "Six American Composers of Non-standard Tunings," *Perspectives of New Music*, Vol. 29, No. 1 (Winter, 1991): 176-211. http://www.jstor.org/stable/833076 (accessed Dec. 17, 2017).

(3 - 2)

Harmonic Partial		=	4	:	5	:	6		
I	$\times$ 1/1	=	1/1		5/4		3/2		
IV	$\times$ 4/3	=	4/3		5/3		2/1		
V	$\times$ 3/2	=	3/2		15/8		9/4 (reduced to 9/8)		

Scale Position	=	I	II	III	IV	V	VI	VII
Just Ratio	=	1/1	9/8	5/4	4/3	3/2	5/3	15/8
Cents Value	≈	0	204	386	498	702	884	1088

Returning to our original set of six ratios (1/1, 7/6, 5/4, 7/5, 3/2, 7/4), one may also observe the presence of a Numerary Nexus equal to '7', shared by the ratios 7/6, 7/5, and 7/4. In this instance, the Numerary Nexus takes the form of a common numerator. Similarly, if we factor the ratio 1/1 so as to retain a numerator value of '7' ($\frac{1}{1} \cdot \frac{7}{7} = \frac{7}{7}$), the process yields a complete subset consisting of four ratios: 7/7 (or 1/1), 7/6, 7/5, 7/4. Partch classifies this or any arrangement of ratios sharing a common numerator as an *Utonality*. Notice how the denominator values from this subset form a descending series of consecutive integers: /7, /6, /5, /4. Taken in reciprocal form, these values represent four distinct *Udentities*.[105] While this tonality exhibits the same intervallic proportions found between the seventh, sixth, fifth, and fourth harmonics of the overtone series, their inverse order reflects a mirrored image of the overtone series. This arrangement appears analogous to a series of sub-harmonics (or undertones) descending from the same fundamental frequency.[106] Like the unity ratio 1/1 in our previous Otonality, the ratio 7/4 fulfills a complimentary role as fundamental frequency in this descending, sub-harmonic series. Thus, 7/4 defines a *second* unity value in our original set of six just ratios. Importantly, this example demonstrates how a single ratio—in this case, 7/4—can fulfill multiple roles, or *identities*, within a scale.

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[105] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

[106] Yuri Landman, "Comparison of Utonal Scales with 12-Tone Equal Temperament," *Hypercustom.nl*. http://www.hypercustom.nl/utonaldiagram.jpg (accessed March 4, 2020).

Embedded within the three ratios (7/6—7/5—7/4) of our Utonality, one can also observe an inverse arrangement of the 4:5:6 major (or *Otonal*) triad. Just as multiplication of two ratios enables transposition, dividing one ratio by another reveals the interval interceding the two tones. For example, dividing 7/5 by 7/6 yields the interval 6/5—a just minor third. Likewise, the quotient of 7/4 and 7/5 is equal to 5/4—a just major third. Together, these intervals form the just equivalent of a minor triad.

(3 - 3)

Utonal Ratios	=	7/6		7/5		7/4
Interceding Interval	=		6/5		5/4	
Equivalent Dyad	≈		minor 3rd		major 3rd	
Cents Value	≈		316		386	

Composers and theorists have long debated the role, or even the *existence* of the undertone series as an acoustical basis for minor tonality. Unlike Partch, James Tenney omits Utonalities entirely from his harmonic language. The basis for this determination is rooted in disagreement over the origin of the minor triad in the undertone series. According to Partch's conception of tonal identity, the overtone (*Otonal*) and undertone (*Utonal*) series are reciprocal manifestations of the same proportional structure. Therein, they occupy both an inverse *and* equal status in musical usage. Speaking to the perceptual saliency of Utonality and integral ratios at large, Partch unequivocally defends this key tenet of Monophony, stating, "Under-number Tonality, or Utonality […] is the immutable faculty of ratios, which in turn represent an immutable faculty of the human ear."[107] None the less, Tenney sees no acoustical basis for this equivalency. Taking a definitively spectral tack, Tenney contends that minor tonality, as a function of Utonal identity, obscures the position of the root note (1/1) in a chord. Instead, Tenney offers three alternate propositions: harmonic proportions do not occupy a reciprocal hierarchy; harmonic directionality

[107] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

matters; and, for the unison (1/1) to be unambiguously identifiable as the root of a chord, it must also retain a pitch equivalent to the fundamental for a given spectrum.[108]

Composer and Theorist, Ben Johnston expresses similar reticence. Though Johnston *does* incorporate Utonality within his own compositions, like Tenney, he acknowledges that just intonation implies a common tonal reference. As this system is derived from the overtone series, the psychoacoustic phenomenon of apprehending a "missing fundamental" or "root" from chords or tone clusters appears all the more salient in just intonation. Supporting this assertion, Johnston states in his 1983 essay, *Scalar Order as a Compositional Resource*:

> "The most significant difference between proportional and linear organization is that the former makes possible the relation of all musical sounds to a common reference point. A group of pitches may be very complexly related to each other, but often all of them can be simply relate to another pitch, which need not even be present. Thus, the missing pitch is strongly implied by the complex group. The root of a chord, the tonic of a tonality, the principal tonality of a modulating movement are examples of this principle." [109]

Certainly, Tenney favors the overtone series and, by proxy, Otonality as the basis for chord construction. Even in *Critical Band* (1988)—a piece which, by design, pushes the limits of perceptual ambiguity and pitch "tolerance"—Tenney limits his palette to Otonal ratios; albeit those of the highest harmonic order.[110] By way of exclusively Otonal harmonies, Tenney's implicit engagement with a variant of spectralism is also evident in *Spectral CANON for CONLON Nancarrow* (1974), *Saxony* (1978), and *Voices* (1983-84).[111] In essence, Partch and Tenney embody opposite views regarding the causal relationship between harmonic perception and

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[108] James Tenney, "John Cage and the Theory of Harmony," (1983), http://www.plainsound.org/pdfs/JC&ToH.pdf (accessed March 2, 2019).

[109] Ben Johnston, "Scalar Order as a Compositional Resource." *Perspectives of New Music* 2, no. 2 (1964): 56-76. https://www.jstororg.proxy01.its.virginia.edu/stable/pdf/832482 (accessed March 19, 2019).

[110] James Tenney, *Critical Band*, Lebanon, New Hampshire, Frog Peak Music, 1988.

[111] Bob Gilmour, "Changing the Metaphor: Ratio Models of Musical Pitch in the Work of Harry Partch, Ben Johnston, and James Tenney," *Perspectives of New Music*, Vol. 33, No. 1/2 (Winter-Summer, 1995): 458-503. http://www.jstor.org/stable/833715 (accessed Jan. 3, 2018).

structure. Whereas Partch sees our favorable response to small, integral ratios as emerging from an innate recognition of the physical structure of the overtone series, Tenney views perceptual recognition of harmonic proportions as a coincidental feature of the human auditory system. As stated in Tenney's primary treatise, *John Cage and the Theory of Harmony*, "And it is not—as Rameau postulated—the 'son fondamental' which 'generates' the triad, but the other way around: when there is a sense that a particular pitch is the root of a chord it is surely the chord itself which creates that sense."[112]

3.3 Prime Limits: Extending Harry Partch's Theories and Practice

Some of the most fertile ground for extension of Partch's theories arise through analysis of numerical properties implicit to integral ratios. These properties are effectively addressed through the classification of various thresholds of divisibility. In Partch's system, as well as other systems to follow, the foundational metric for classifying the structural and aesthetic affordances of a given ratio lie in the divisibility of its numerator and denominator values by a particular prime-number factor; or *prime-limit*.[113] According to this convention, the interval 7/5 is considered a 7-limit ratio. Between the numerator and denominator, the highest prime-factor is '7'. However, a degree of ambiguity exists within Partch's system in how to regard ratios whose numerator or denominator retain a high odd-numbered value, yet exhibit a low prime limit. Partch deems these ambiguous intervals, "multiple-number ratios."[114] For example, according to their prime-limit values, the ratios 9/8 and 15/8 may be classified as 3-limit and 5-limit, respectively. None the less, Partch assigns each "multiple-number" ratio an identity according to the highest *odd* number present: '9'

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112 James Tenney, "John Cage and the Theory of Harmony," (1983), http://www.plainsound.org/pdfs/JC&ToH.pdf (accessed March 2, 2019).

113 John Chalmers, "Combination Product Sets and Other Harmonic and Melodic Structures," *Proceedings of the 7th International Computer Music Conference*, North Texas State University, Denton, TX (1981): 348–362. https://quod.lib.umich.edu/i/icmc/bbp2372.1981.045/1/--combination-product-sets-and-other-harmonic-and-melodic?page=root;size=150;view=pdf (accessed March 27, 2019)

114 Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

and '15'. Thus, the composer obfuscates the structural primacy of the prime-limit as a categorical metric.

Expressed as a finite threshold, a ratio's prime-limit reveals its structural complexity and therein, can be attributed to the relative consonance or dissonance of harmonic intervals. Accordingly, small whole-numbered ratios embody greater consonance, while ratios divisible by larger, prime numbers exhibit increasingly high degrees of dissonance.[115] As well as an attempt to codify structural features in just intonation, prime-limit thresholds also afford particular aesthetic properties. Here, a more qualitative approach to analysis informs compositional decisions. Not surprisingly, composers have treated prime-limit thresholds as a malleable parameter for affecting specific musical outcomes within their respective practices. Most notably, Ben Johnston attributes certain prime-limits to different aesthetic and expressive characteristics; or what he terms "rasas." By Johnston's estimation, a ratio's prime-limit may imbue intervals with certain indelible qualities. Here, the characteristic *rasa* ascribed to each prime-limit arises from its relative location in the overtone series. For example, Johnston describes ratios derived from the third partial, including 4/3 (perfect fourth) and 3/2 (perfect fifth), as contributing "stability and strength," while 5-limit thirds and sixths (5/4, 6/5, 8/5, 5/3) convey a sense of "…warmth of emotion; ordinary human warmth."[116] One may recognize an air of familiarity that pervades the language Johnston uses in describing the evocative qualities afforded by 3- and 5-limit ratios. According to Johnston, the "warmth" of a 5-limit major third (5/4) is "ordinary," while the "strength" of the 3-limit perfect fifth (3/2) pairs with a sense of "stability." As previously explicated, the basis for this sense of familiarity is clear: in common practice, both major and minor tonality find analogous origins in the Ptolemaic Sequence, a structure delimited by transposition of intervals relating to the second, third, and fifth harmonic partials.

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[115] Bob Gilmour, "Changing the Metaphor: Ratio Models of Musical Pitch in the Work of Harry Partch, Ben Johnston, and James Tenney," *Perspectives of New Music*, Vol. 33, No. 1/2 (Winter-Summer, 1995): 458-503. http://www.jstor.org/stable/833715 (accessed Jan. 3, 2018).

[116] Douglas Keislar, "Six American Composers of Non-standard Tunings," *Perspectives of New Music*, Vol. 29, No. 1 (Winter, 1991): 176-211. http://www.jstor.org/stable/833076 (accessed Dec. 17, 2017).

Moving into less familiar territory, Johnston attributes 7-limit, or *septimal* ratios (7/4, 8/7, 7/6, 12/7, 7/5, 10/7) with creating "…a sensuality, for example in vernacular music like the blues."[117] Speaking to the latter point, scholars have long sought to quantify the exact intervallic characteristics of the "blue note" associated with music of African American and West African origins. Notably, instrument designer and microtonalist Bart Hopkin proposes the ratio 7/6 (≈ 267 cents) as one possible derivative for this historically-significant interval, spanning a distance between a whole-tone and minor third.[118]

Similarly, Johnston charges 11-limit ratios (11/8, 12/11, 11/10, 11/9, 16/11, 11/6, 20/11, 18/11) with a sense of "ambiguity" or "neutrality." These intervals occupy a space *in-between* major and minor tonality; yet fully embody neither.[119] Historically, intervals derived through proportional relationships with the eleventh partial have been approximated—albeit crudely—through the use of equal-tempered quarter-tones. However, such coarse approximations do not account for both subtle and significant variation between 11-limit ratios. For example, transcriptions of Arabic modal systems, or *Maqam*, often conflate the ratios 12/11 (≈ 151 cents) and 11/10 (≈ 165 cents), classifying both intervals as a "three-quarter-tone" (≈ 150 cents).[120] The problem becomes all the more acute when attempting to translate the subtle inflections of certain maqam, such as *Bayati Husseini*, using standard or quarter-tone notation.

117 Douglas Keislar, "Six American Composers of Non-standard Tunings," *Perspectives of New Music*, Vol. 29, No. 1 (Winter, 1991): 176-211. http://www.jstor.org/stable/833076 (accessed Dec. 17, 2017).

118 Bart Hopkin, *Musical Instrument Design: Practical Information for Instrument Making*. Tuscon, AZ: See Sharp Press, 1996.

119 Douglas Keislar, "Six American Composers of Non-standard Tunings," *Perspectives of New Music*, Vol. 29, No. 1 (Winter, 1991): 176-211. http://www.jstor.org/stable/833076 (accessed Dec. 17, 2017).

120 Sami Abu Shumays, "Maqam Analysis: A Primer." *Music Theory Spectrum* 35, no. 2 (2013), https://www.jstor.org/stable/10.1525/mts.2013.35.2.235 (accessed April 6, 2019).

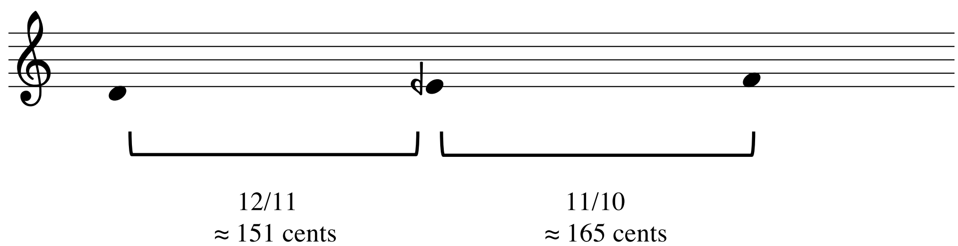

Figure 3. 4—First three scale degrees of Maqam Bayati Husseini (Just Intervals Approximated using Quarter-Tone Notation)

Moving higher up the series, Johnston's most evocative designation of rasa resides with 13-limit ratios (13/8, 13/12, 13/10, 13/7, 13/11, 16/13, 24/13, 20/13, 14/13, 22/13). These intervals appear to belie any definitive reference to standard practice or comparable musical forms. In an interview with Douglas Keislar, the composer extends his description of these ratios to imply symbolic values or meanings associated with a "melancholy, dark quality." Citing these associative properties in composition, Johnston recalls that "nearly every time I've used it [13-limit ratio], it has something to do with death, which would square with the meaning of thirteen in numerology."[121] While the figurative qualities Johnston ascribes to 13-limit ratios may spring from anecdotal origins, the depth of expression and importance to the composer's creative practice elevate the role of qualitative analysis in assessing the parametric affordances of prime-limits in just intonation.

Suggestive of Johnston's qualitative assessment for rasas and prime-limit, La Monte Young reveals an aesthetic preference for 7-limit ratios in both *The Well-Tuned Piano* (1964-) and *The Two Systems of Eleven Categories* (1966-).[122] In tandem with his predilection for septimal intervals, Young intentionally omits 5-limit thirds and sixths from these pieces. The motivation

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[121] Douglas Keislar, "Six American Composers of Non-standard Tunings," *Perspectives of New Music*, Vol. 29, No. 1 (Winter, 1991): 176-211. http://www.jstor.org/stable/833076 (accessed Dec. 17, 2017).

[122] La Monte Young and Dia Art Foundation, *The Well-Tuned Piano: 81 X 25: 6:17:50-11:18:59 Pm*, New York: Gramavision, 1987.

for this omission appears referential, if not somewhat reactionary. Twelve-tone equal temperament represents grossly mistuned approximations of both the 5-limit major and minor thirds: 5/4 (≈ 386 cents) 6/5 (≈ 316 cents), respectively. Likewise, tempered estimations of their respective inversions, 8/5 (≈ 814 cents) and 5/3 (≈ 884 cents), appear equally out-of-tune.[123] Deeply conscious of the false equivalencies implied by equal temperament, Young forgoes 5-limit intervals entirely in earlier works, *Trio for Strings* (1958) and *for Brass* (1957). Instead, the composer fills comparable intervallic spans with septimal thirds, 9/7 (≈ 435 cents) and 7/6 (≈ 267 cents), and their respective inversions: 14/9 (≈ 765 cents) and 12/7 (≈ 933 cents). As Young states in a 1990 interview:

> "I don't know to what degree I thought other music was out of tune before I really completely became interested in just intonation and aware of the possibility of analyzing it with the integers, but I do know that from the very beginnings in *Trio for Strings*, and even to some degree in *for Brass* (but not as strictly), I began to exclude thirds and sixths. And that has been a characteristic of my music until this time. The kinds of thirds and sixths that appear in *The Well-Tuned Piano* are not 5/4ths, but 9/7ths and 14/9ths. What I established when I worked on *The Two Systems of Eleven Categories* is that I was interested in excluding those intervals which were factorable by five or its multiples, because that's how we generate the major third as we know it in Western music and much Eastern music, too." [124]

Through structural and qualitative analysis of numerical properties intrinsic to just intonation, both Johnston and Young have extended and refined the language and procedures of Partch's Monophonic Fabric to suit their respective objectives and aesthetic preferences. As discussed, these properties are most effectively illustrated through the classification of various thresholds of divisibility, or prime-limits. Johnston ascribes expressive qualities according to these limits, thus aestheticizing the harmonic series. Meanwhile, Young's predilection for intervallic structures exceeding 5-limit threshold defines a clear and intentional break with the common practice. In later sections, we will explore how Johnston and others implement different visual

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[123] Kyle Gann, "La Monte Young's the Well-Tuned Piano," *Perspectives of New Music*, Vol. 31, No. 1 (Winter, 1993), 134-162, http://www.jstor.org/stable/833045 (accessed December 7, 2017).

[124] William Duckworth, *Talking Music: Conversations with John Cage, Philip Glass, Laurie Anderson, and Five Generations of American Experimental Composers*, New York, NY: Da Capo Press, 1999.

projections to convey these procedures. Moreover, we shall examine how these projections inform instrument design and configuration.

3.4 Kathleen Schlesinger: Connecting Greek Modes, Utonality, and 13-Limit Ratios

Unlike the work of other theorists described in this essay, Kathleen Schlesinger's research does not emerge from her *own* compositional practice. Instead, her research arises from time served as curator of musical instruments for the British Museum, where she specialized in musical archeology during the first decades of the twentieth century. Professional discipline aside, the chronological scope and importance of her work precedes many of Partch's theories by decades and in turn, deserves recognition. For Partch, Kathleen Schlesinger's primary contributions to the field of just intonation appear two-fold. In resolving to map a logical framework to the music of the Ancient Greeks—both in terms of tuning practice and instrumentation—Schlesinger posits an arithmetically-derived explanation of modality parallel to Partch's conception of Utonality. Like Johnston, she describes systems of just intonation which embraces 13-limit ratios. To the latter point, Schlesinger's mapping of Ancient Greek modes exceeds the 11-limit presented in Partch's Monophonic fabric. Both the significance and perceptual saliency of intervals within *and* exceeding his 11-limit are not lost to Partch, who writes:

> "…can the human ear perceive so many degrees in 2/1 [the octave]? The unequivocal answer is: it can, and frequently a good many more, depending largely on the range of pitch in which a test is made […] The smallest interval in the Monophonic fabric measures 14.4 cents […] and this potential in the average devotee of music has been amply demonstrated; the persons I have encountered who are unable to hear this particular interval of 14.4 cents can be numbered on one hand."[125]

Despite steadfast assurance in one's ability to perceive, if not appreciate intervals exceeding the 11-limit, Partch stops short of including 13-limit ratios as part of his Monophonic fabric. Here, his reasoning reverts to a more qualitative evaluation and alludes to the underlying corporeality of Partch's own musical practice:

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[125] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

> "The reasons why Monophony proceeds to the limit of eleven are basic and quite specific, as will be seen, but the reason for resting at the limit of eleven is a purely personal and arbitrary one. When a hungry man has a large table of aromatic and unusual viands spread before him he is unlikely to go tramping along the seashore and in the woods for still other exotic fare. And however skeptical his of the many warnings regarding the unwholesomeness of his fare—like the 'poison' of the 'love-apple' tomato of a comparatively few generations ago—he has no desire to provoke further alarums." [126]

Partch's own temperance aside, his appreciation for Schlesinger's theories remains nested in similarities with his own. From a musicological standpoint, Schlesinger's inclusion of 13-limit intervals resides in her study of the physical proportions of the *aulos*, a wind instrument dating to at least the six-century B.C.E.[127] Extending even further into antiquity, analogous reed-flutes made from stalks of the riparian species *Arundo domax*, may in fact date to the third millennium B.C.E.[128] Drawing from a substantial body of archeological evidence, Schlesinger's contends that the equidistant placement of finger-holes found on extant examples of the aulos suggest that Ancient Greek modes were derived from whole-number proportions—a procedure akin to modern conceptions of just intonation. Accordingly, the number of equal divisions between finger-holes along the length of the reed-pipe determines the specific modal identity of the generated scale. For example, twelve equal divisions yield a just scale equivalent to the Phrygian mode, while fourteen equal divisions produce a just Mixolydian mode.[129] In the Mixolydian mode, seven finger-holes spaced at $\frac{1}{14}$ the distance of the entire length of reed-pipe make up the seven notes of the octave.

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[126] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

[127] Kathleen Schlesinger, "Further Notes on Aristoxenus and Musical Intervals," *The Classical Quarterly*, Vol. 27, No. 2 (April 1933): 88-96. http://www.jstor.org/stable/6363812 (accessed Dec. 28, 2017).

[128] Kathleen Schlesinger, *The Greek Aulos: A Study of its Mechanism and of its Relation to the Modal System of Ancient Greek Music – A Survey of the Greek Harmoniai in Survival or Rebirth in Folk Music*, London: Methuen & Co, 1939.

[129] Kathleen Schlesinger, "The Origin of the Major and Minor Modes," *The Musical Times*, Vol. 58, No. 893 (July 1917): 297-301. http://www.jstor.org/stable/908417 (accessed Dec. 28, 2017).

Schlesinger refers to this generating ratio as the *Modal Determinant*.[130] Aligned to Partch's convention, the Modal Determinate is equivalent to the value of a sounding ratio's Numerary Nexus, as it pertains to the common numerator or denominator value between tones in a given Utonality or Otonality.[131]

It should be noted that, while the body of the instrument is physically divided into equidistant parts, the resultant intervals between each step of the scale still retain just ratios, and by definition represent unequal divisions of the octave. To better illustrate this point, it is helpful to refer back to arithmetic divisions on a monochord. For example, by dividing a vibrating string into two unequal parts at $\frac{1}{14}$ of its entire length, we can generate two distinct frequencies. This division, or stop, is analogous to the position of a given finger-hole on the aulos. Plucking the string between the nut and our stop ($\frac{1}{14}$ of the string's entire length) will produce a pitch fourteen times the frequency of the undivided string's fundamental: a ratio value equivalent the 14th harmonic partial. The remaining length of string between our stop and the bridge constitutes $\frac{13}{14}$ of our entire string. Plucking the string here produces a frequency $\frac{14}{13}$ times that of our fundamental. In either case, our sounding frequency ratio is always equal to the reciprocal of the length of the plucked string. If we repeat this procedure in $\frac{1}{14}$ length increments—treating this value as our Modal Determinate—we generate seven equal divisions, spanning the first half of our string. Measuring from the bridge, each stop occurs at $\frac{14}{14}, \frac{13}{14}, \frac{12}{14}, \frac{11}{14}, \frac{10}{14}, \frac{9}{14}$, and $\frac{8}{14}$ the length of our string. By taking the reciprocal of each length and reducing each fraction, we produce a just Mixolydian scale whose notes sound at $\frac{1}{1}, \frac{14}{13}, \frac{7}{6}, \frac{14}{11}, \frac{7}{5}, \frac{14}{9}$, and $\frac{7}{4}$ times the fundamental frequency.

Importantly, every ratio shares a common numerator (or Numerary Nexus) equivalent to a value of '7'. Thus, our scale constitutes a single Utonality, consisting of ratios equivalent to the

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[130] Kathleen Schlesinger, "Further Notes on Aristoxenus and Musical Intervals," *The Classical Quarterly*, Vol. 27, No. 2 (April 1933): 88-96. http://www.jstor.org/stable/6363812 (accessed Dec. 28, 2017).

[131] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

8th, 9th, 10th, 11th, 12th, 13th, and 14th sub-harmonics of the ratio 7/4. As this ratio retains a binary denominator, Schlesinger treats the ratio 7/4 as the unison identity of the undertone series embedded with this mode. In turn, each of Schlesinger's seven reconstructed Greek *Harmoniai* retain a single tonal identity—or *Mese*. However, depending upon the size of the Modal Determinant, the Mese may occupy a different scalar position within each mode.[132] Likewise, not all equidistant divisions on the monochord or aulos define a discrete scale step. In certain Greek Harmoniai, physical stops are sometimes skipped, while others are retained.

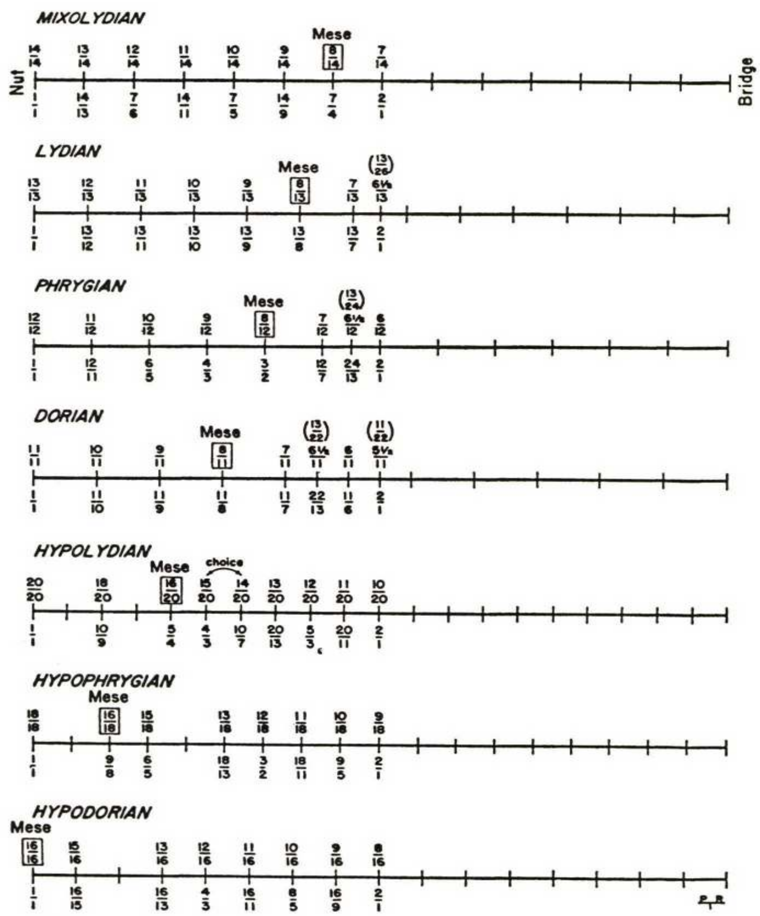

Figure 3. 5—Harry Partch's Adaptation of Kathleen Schlesinger's Seven Harmoniai for Monochord[133]

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132 Kathleen Schlesinger, *The Greek Aulos: A Study of its Mechanism and of its Relation to the Modal System of Ancient Greek Music – A Survey of the Greek Harmoniai in Survival or Rebirth in Folk Music*, London: Methuen & Co, 1939.

133 Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

Strangely, Schlesinger concludes that neither a desire for proportionality in pitch, nor an innate understanding of harmonic spectra guided the Greeks in constructing the aulos or defining any particular scalar structure for the instrument. Instead, she views intonation practices in antiquity following visual form. Accordingly, ratio-based proportionality in pitch originates from a visual aesthetic which favors symmetry and repeating patterns. In this case, a preference for equal distribution of visual markers (e.g. finger-holes) happens to coincide with particular harmonic properties. Schlesinger comments on this implied causality in her 1917 essay:

> "We may assume that the inborn feeling of the eye for harmonious proportions and symmetry possessed by primitive and untutored man [sic] guided him in placing the holes at equal distances along his reed-pipe, just as it did when he decorated his pottery with rows of dots, circles, or other symbolical figures equally spaced and symmetrically grouped […] This may be considered as the first cause, quite unrelated in the mind to its effect as sound; thus a system of scales came into being quite naturally, without preconceived musical notions or arbitrary interference, by purely mechanical means, and as consequence of embodying natural laws." [134]

While Schlesinger's assumptions regarding an innate and causal relationship between visual preferences and acoustic proportions may appear conflated, her analysis of resultant intonation remains solvent. By connecting archeological records and measurements from surviving artifacts with known physical and acoustical properties, Schlesinger presents a salient model for tuning practice rooted in reciprocal, sub-harmonic proportions. Analogous to Partch's theories on arithmetic division and Utonality, her theories expand the field of just intonation to include ratios derived from the 13th partial and its inverse, undertone equivalent. Here, Schlesinger presents high-order prime-limits not as arbitrarily complex or ornamental figures; but as an integral component in the history and construction of modal scales.

3.5 Tonality Diamonds and other Visual Representations of Just Tuning Systems

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[134] Kathleen Schlesinger, "The Origin of the Major and Minor Modes," *The Musical Times*, Vol. 58, No. 893 (July 1917): 297-301. http://www.jstor.org/stable/908417 (accessed Dec. 28, 2017).

Composers and theorists have long deployed various graphical and notational models to communicate specific structural and acoustic properties of just tuning systems. According to Bob Gilmour, the efficacy of any "pitch model"—graphical or otherwise—resides in its ability to "manifest processes" using cognitive representations of materials which embody a given pitch space. In turn, these models affect the perception of a system's most salient features, as well as the compositional process itself.[135] In particular, visual projections can provide an effective means for elucidating important pitch relationships within a scalar or harmonic structures, represent acoustic phenomenon, and generate spatial prototypes for instrument design.

Focusing on pitch models as generative, as opposed to fixed structures; the development of Partch's 'Tonality Diamond' can be considered a prototype for other visual projections of just intonation. Here, proprietary features, such as the number of tones-per-octave, exist as secondary elements to the organizational principles underlying the system. A Tonality Diamond, for example, may be arranged to accommodate any prime-limit threshold. Consequently, the number of scalar divisions within an octave develops as a resultant property of the prime-limit. Therein, a ratio's prime limit represents a clearly delineated marker within the visual projection. According to these procedures, one may derive pitch ratios within a Tonality Diamond by dividing two sets of factors: one set consists of integers arranged on a horizontal plane, while a second set of reciprocal values occupies a vertical plane.[136] In composition, we constrain the range of integer factors and their respective reciprocal values according to a specified prime (or odd)-limit. For example, by multiplying two axes consisting of the factors {2, 3, 5, 7, 9, 11} and {/2, /3, /5, /7, /9, /11}, we generate a 29-tone, 11-limit scale: 1/1, 12/11, 11/10, 10/9, 9/8, 8/7, 7/6, 6/5, 11/9, 5/4, 14/11, 9/7, 4/3, 11/8, 7/5, 10/7, 16/11, 3/2, 14/9, 11/7, 8/5, 18/11, 5/3, 12/7, 7/4, 16/9, 9/5, 20/11, 11/6. Note that all ratios with a value greater than '2' or less than '1' have been reduced to confine our scale to a single octave (displayed in parentheses within Figure 3.6). Herein, we refer to these ratios as *octave-reduced* or retaining *octave equivalency*.

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135 Bob Gilmour, "Changing the Metaphor: Ratio Models of Musical Pitch in the Work of Harry Partch, Ben Johnston, and James Tenney," *Perspectives of New Music*, Vol. 33, No. 1/2 (Winter-Summer, 1995): 458-503. http://www.jstor.org/stable/833715 (accessed Jan. 3, 2018).

136 Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

Additional properties of Partch's Monophony become evident through the spatial features of this projection. Otonalities and Utonalities occur, respectively, along horizontal and vertical planes. Where these planes intersect, a row of Otonal ratios may share a common tone with a column of Utonal ratios. For example, the Otonality (4/3—1/1—5/3—7/6—3/2—11/6) shares the ratio 4/3 with the Utonality (1/1—4/3—8/5—8/7—16/9—16/11). However, while both sonorities retain a common ratio, the same note embodies multiple identities. Whereas the ratio's (4/3) position in the first column (2) of the second (/3) row signifies a '2' Odentity, its position in the second row (/3) of the first column (2) indicates a '3' Udentity. Thus, by deploying a visual projection, Partch illustrates another structural property of this tuning model.

Otonalities →

Utonalities ↓

→ Odentity ↓ Udentity	**2**	**3**	**5**	**7**	**9**	**11**
2	1/1	3/2	5/2 (5/4)	7/2 (7/4)	9/2 (9/8)	11/2 (11/8)
3	2/3 (4/3)	1/1	5/3	7/3 (7/6)	3/1 (3/2)	11/3 (11/6)
5	2/5 (8/5)	3/5 (6/5)	1/1	7/5	9/5	11/5 (11/10)
7	2/7 (8/7)	3/7 (12/7)	5/7 (10/7)	1/1	9/7	11/7
9	2/9 (16/9)	1/3 (4/3)	5/9 (10/9)	7/9 (14/9)	1/1	11/9
11	2/11 (16/11)	3/11 (12/11)	5/11 (20/11)	7/11 (14/11)	9/11 (18/11)	1/1

Figure 3. 6—11-Limit Tonality Diamond

In certain applications, Partch shifts the orientation of the Tonality Diamond by 45 degrees. Though this modification retains the same tonal properties, the composer also re-orders rows and columns to achieve ascending and descending patterns. These same organizational principles often influenced the layout of his instruments, particularly in the regards to the design of idiophones. For example, following the arrangement of his Diamond Marimba, six 11-limit Utonalities run at right angles to six Otonalities. This configuration affords rapid ascent across each of the six Otonalties via upward, left-to-right gesture, while Utonal descent can be achieved using bottom-right to top-left motions. Both gestures occur perpendicular to the diamond's axis.[137]

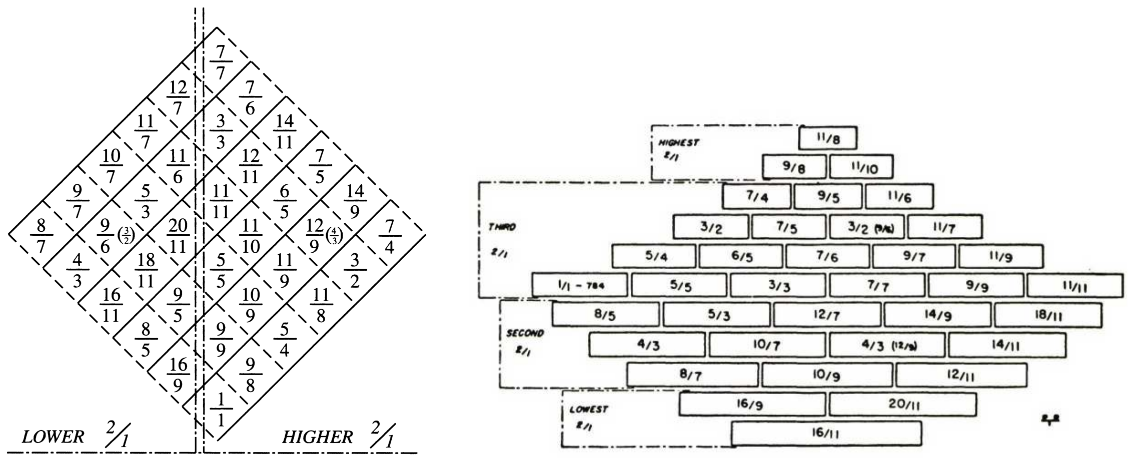

Figure 3. 7—Partch's 11-Limit Tonality Diamond and Block Diagram for the Diamond Marimba [138]

Similar to Partch's Tonality Diamond, Erv Wilson's conception of "cross-sets" provide another means of combining two scalar or chordal structures via the multiplication of concurrent axes'. The composer first came into contact with Partch during the mid-1960's, assisting with design and construction of the Quandrangularis Reversum.[139] Notably, Wilson also created diagrams for the second edition of *Genesis of a Music*. However, while Partch models his diamonds on divisional procedures, Wilson sometimes employs a multiplicative or matrix-based

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[137] Bob Gilmour, "Changing the Metaphor: Ratio Models of Musical Pitch in the Work of Harry Partch, Ben Johnston, and James Tenney," *Perspectives of New Music*, Vol. 33, No. 1/2 (Winter-Summer, 1995): 458-503. http://www.jstor.org/stable/833715 (accessed Jan. 3, 2018).

[138] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

[139] Terumi Narushima, *Microtonality and the Tuning Systems of Erv Wilson*, New York, NY: Routledge, 2018.

approach. For example, the cross-set {1, 3, 5} · {1, 3, 5} produces the harmonic ratios: 1/1, 3/1, 5/1, 9/1, 15/1, 25/1 (1st, 3rd, 5th, 9th, 15th, and 25th harmonic partials) or 1/1, 9/8, 5/4, 3/2, 25/16, 15/8 in octave-reduced form. According to Partch's convention, this cross-set generates exclusively Otonal ratios. A reciprocal cross-set, such as {1, 3, 5} · {/1, /3, /5}, brings us close to Partch's divisional model, represented in the Tonality Diamond. According to Wilson's model, whole-number generators (top row of the cross-set) represent components within the harmonic series, while reciprocal inversions of these values (located in the far-left column) embody a complimentary, sub-harmonic series. Wilson classifies this particular arrangement of the cross-set as a *Lambdoma*.[140] Here, each cross-set yields identical results to Partch's 5-limit Tonality Diamond. Likewise, both Wilson and Partch's 5-limit models generate corresponding Otonalities and Utonalities, consisting of three sets of just major and minor triads, respectively:

(3 - 4)

Major Triads (Otonalities) =	1/1—5/4—3/2
	4/3—5/3—3/3 (1/1)
	8/5—5/5 (1/1)—6/5
Minor Triads (Utonalities) =	4/3—8/5—1/1
	3/3 (1/1)—6/5—3/2
	5/3, 5/5 (1/1), 5/4

Tuning scholar, Terumi Narushima frames Wilson's cross-sets as a means of transposing one set of harmonic ratios by another set, thereby "imbuing" or hybridizing resultant intervals with mutual qualities. This concept bears similarity to Ben Johnston classification of *rasa* for a convening ratio. Following this principle, the resultant intervallic quality can be perceived as a derivative of each ratio's prime-limit. According to Johnston's interpretation, 6/5—a cross-set derivative of '3' and '/5'—would retain a sense of "stability and strength" indicative of the third harmonic, while simultaneously expressing a "…warmth of emotion" resulting from a 5-limit relationship."[141] From this perspective, multifarious states of evocation may emerge from strict,

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[140] Terumi Narushima, *Microtonality and the Tuning Systems of Erv Wilson*, New York, NY: Routledge, 2018.

[141] Douglas Keislar, "Six American Composers of Non-standard Tunings," *Perspectives of New Music*, Vol. 29, No. 1 (Winter, 1991): 176-211. http://www.jstor.org/stable/833076 (accessed Dec. 17, 2017).

numerical proportions. Notably, Johnston's assessment of a ratio's rasa does not address the position of the contributing prime-limited value within the ratio—be it numerator or denominator. Likewise, he does not appear to base any distinction in quality upon whether a value occupies the role of Numerary Nexus or other functional identity, such as Otonality or Utonality. Notably, this view appears at odds with Tenney's distinction between the respective functionality of Otonal and Utonal structures.

3.6 Commas, Chromas, Diesis', and other Enharmonic Intervals

As discussed, visual or notational conventions for just intonation emerge from both idiomatic usage and individual preference. In contrast to Wilson, and perhaps more than any other composer discussed in this essay, Ben Johnston's compositions embody distinct features of the Western Classical tradition. Consequently, it should come as no surprise that Johnston chose to project his model (what he terms "extended just intonation") through the medium of modified, standard notation. On one hand, we may view his approach to just intonation as a means of reconciling Partch's theories with those held within the conservatory. However, one would be mistaken in assuming that Johnston holds established conventions in greater regard. To the contrary, Johnston views Partch's theories as a remedy for an ailing tradition, asserting that "serious music" needed Partch "…more than he ever needed it. He [Harry Partch] addressed problems it is ignoring with far more than tentative success, and he diagnosed many of its more serious ills with uncanny accuracy." In the same essay, Johnston clarifies the position of his own "extension" of monophony and his relationship with European art music, stating:

> "The most significant aspect of my own work as a composer is a very extensive development of microtonal just intonation. I have developed a theory in support of this which greatly extends Partch's. Since I am dealing with traditions of performing and with instruments and players which are in the European tradition, I have steeped myself in that music and have studied the techniques and aesthetic attitudes of all its phases of development up through the present. But my purpose has not been to Europeanize Partch's ideas. Rather it has been to alter that tradition so as to render it pervious to his way of thinking." [142]

[142] Ben Johnston, "Beyond Harry Partch." *Perspectives of New Music* 22, no. 1/2 (1983), 223-32, https://www.jstor.org.proxy01.its.virginia.edu/stable/832943?seq=1#metadata_info_tab_contents (accessed March 12, 2019).

Johnston's relationship with Partch's theories originate from time spent with the composer himself. Between 1950-51, Johnston lived, worked, and studied with Harry Partch. Like James Tenney, Johnston would not compose his first pieces in just intonation, *Knocking Piece*, *Sonata for Microtonal Piano* (1962) and *String Quartet No. 2* (1964), until more than a decade after his apprenticeship with Partch.[143] The implied philosophy of Johnston's early work emphasizes the process of framing modernistic complexity within concise, logical systems. In this regard, Johnston certainly draws from Helmholtz's writings of the mid nineteenth-century.[144] In essence these systems exist as a kind of *Natural Law* whose virtues are qualified in relation to small, whole-numbered ratios. Taking a categorically Aristotelian tack, the composer seems to draw parallels between the clarity of whole-number ratios and an ordered universe, contending that "…the extreme complexity of contemporary life [can] be reconciled with the simplifying and clarifying influences of systems of order based upon ratio scales."[145] For Johnston, consonance is a function of a ratio's simplicity.

Accordingly, the composer's style reveals an intensely hierarchical approach to proportion and form, indicative of the Western Classical tradition. Like Tenney and Partch, Johnston also argues that, when assessed through proportional relationships, human auditory perception is indeed capable of making acute distinctions in pitch beyond those described in common practice. Consequently, equal temperament, as a structural feature embedded within the common practice, omits intervallic relationships that are perceptually distinct. Fundamentally, Johnston's interest

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[143] Bob Gilmour, "Changing the Metaphor: Ratio Models of Musical Pitch in the Work of Harry Partch, Ben Johnston, and James Tenney," *Perspectives of New Music*, Vol. 33, No. 1/2 (Winter-Summer, 1995): 458-503. http://www.jstor.org/stable/833715 (accessed Jan. 3, 2018).

[144] Hermann von Helmholtz, *On the Sensations of Tone as a Physiological Basis for the Theory of Music*. 2d English ed. New York: Dover Publications, 1954.

[145] Ben Johnston, "Extended Just Intonation: A Position Paper." *Perspectives of New Music* 25, no. 1/2 (1987), 517-19. http://www.jstor.org.proxy01.its.virginia.edu/stable/833124 (accessed March 2, 2019).

in just intonation resides in reconciling these distinctions and composing music that is truly "in-tune."[146]

In formulating a generalized system for notating just intonation, Johnston sets off where Partch left off in the 1930's. In those following decades, Partch had already abandoned a single system of notation; instead devising proprietary and instrument-specific visual models.[147] It follows that, as Partch became more invested in instrument design and construction, the desire to create a generalized system of notation applicable to conventional instrumentation and performers would wane. Much of Partch's later notation takes the form of tablature, further divorcing ratio proportions from the fixed spatial proportions of the staff. Unlike contemporaries, including Lou Harrison, Johnston does not utilize tablature-based notation.

At its core, "extended just intonation" is both cumulative and multiplicative, affording an "expandable pitch space" unhindered by the fixity of a closed scale structure, gamut, or pitch set.[148] As such, notating music in extended just intonation necessitates the inclusion of numerous accidentals, traditional and otherwise. Here, the composer notates increasingly complex ratios using a combination of accidentals to represent various sharps, flats, commas, and chromas. In many cases, Johnston applies multiple accidentals to the same note-head. The 53-tone scale used in *String Quartet No. 2* (1964) is a prime example of this cumulative approach to intonation.[149] However, within the expanding field of tonality afforded through Johnston's extended just

[146] William Duckworth, *Talking Music: Conversations with John Cage, Philip Glass, Laurie Anderson, and Five Generations of American Experimental Composers*, New York, NY: Da Capo Press, 1999.

[147] Bob Gilmour, "Changing the Metaphor: Ratio Models of Musical Pitch in the Work of Harry Partch, Ben Johnston, and James Tenney," *Perspectives of New Music*, Vol. 33, No. 1/2 (Winter-Summer, 1995): 458-503. http://www.jstor.org/stable/833715 (accessed Jan. 3, 2018).

[148] Ben Johnston, "Extended Just Intonation: A Position Paper," *Perspectives of New Music* 25, no. 1/2 (1987), 517-19. http://www.jstor.org.proxy01.its.virginia.edu/stable/833124 (accessed March 2, 2019).

[149] Ben Johnston, "Scalar Order as a Compositional Resource." *Perspectives of New Music* 2, no. 2 (1964): 56-76. https://www.jstororg.proxy01.its.virginia.edu/stable/pdf/832482 (accessed March 19, 2019).

intonation, the number of notes generated within a given octave becomes a secondary consideration to the generative processes by which chords, hexads, and other intervallic structures are defined. Highlighting the arbitrary nature of note-count, an excerpt from the program notes for *String Quartet No. 5* (1979) reads, "I have no idea as to how many different pitches it used per octave." [150]

Despite the proportional complexity, increasing number of notes-per-octave, and extensive microtonal variation generated through extended just intonation, Johnston models his notation around the comparatively simple structure of the Ptolemaic Sequence.[151] As discussed, this seven-note series of 5-limit ratios forms the basis for major tonality. Lacking accidentals, the key of 'C' provides a proverbial 'clean slate' to build upon. Thus, Johnston establishes the C-major scale as a structural foundation for his notation. This scale consists of the following just ratios and respective note values:

(3 - 5)

C		D		E		F		G		A		B
1/1		9/8		5/4		4/3		3/2		5/3		15/8
	9/8		10/9		16/15		9/8		10/9		9/8	

Johnston's application of the Ptolemaic Sequence in 'C' generates three just triads consisting of C, E, G (1/1, 5/4, 15/8); F, A, C (4/3, 5/3, 2/1); and G, B, D (3/2, 15/8, 9/8). By appending standard accidentals (e.g. sharps and flats) to the third, sixth, and seventh scale degrees (5/4, 5/3, 15/8); one may transpose each pitch by a ratio value of 25/24 (≈ 70.67 cents). In most contexts, we refer to this ratio as a single *chroma*. Thus, through conventional notation, Johnston achieves a salient representation of both major and minor tonality within a fixed scale.

[150] Heidi Von Gunden, *The Music of Ben Johnston* (Metuchen, N.J.: Scarecrow Press, 1986).

[151] Ben Johnston, "Scalar Order as a Compositional Resource." *Perspectives of New Music* 2, no. 2 (1964): 56-76. https://www.jstororg.proxy01.its.virginia.edu/stable/pdf/832482 (accessed March 19, 2019).

(3 - 6)

C	D	Eb	F	G	Ab	Bb
1/1	9/8	6/5	4/3	3/2	8/5	9/5

9/8	16/15	10/9	9/8	16/15	9/8

The predominance of conventional notation and governing role of a familiar—albeit, extremely precise—intervallic structure cement Johnston's notational model as a bridge between the Western Classical tradition and Partch's more radical ideas. Here, triadic structure reigns supreme! Using standard accidentals, consonant triads can be rooted in all chromatic notes, with the exception of the following combinations: F's and A's tuned to D (4:5:6 major triad = 9/8, 45/32, 27/16; 10:12:15 'minor' triad = 9/8, 27/20, 27/16), Bb or D tuned to F (3:4:5 = 4/3, 16/9, 10/9). These outlying triads demonstrate a perennial issue for any fixed system of just intonation: transposition. In fixed just intonation, transposition by certain scalar degrees introduces high-order ratios, thus compromising the consonance of certain intervals—particularly fifths. For example, the dyad between 9/8 and 5/3 spans the rather high-order ratio 40/27, an interval approximately 22 cents flat of the consonant Pythagorean perfect fifth (3/2). This difference in frequency corresponds with the ratio 81/80, an interval historically referred to as the *Syntonic Comma*, *Diatonic Comma*, or *Comma of Didymus*.[152]

D	A
9/8	5/3

$$\frac{5}{3} \div \frac{9}{8} = \frac{40}{27}$$

Figure 3. 8—Dissonant, High-Order ('Wolf') Fifth

[152] Ben Johnston, "Scalar Order as a Compositional Resource." *Perspectives of New Music* 2, no. 2 (1964): 56-76. https://www.jstororg.proxy01.its.virginia.edu/stable/pdf/832482 (accessed March 19, 2019).

So as to retain consonant fifths, Johnston employs enharmonic variations of certain ratios. To this end, he introduces two new accidentals (+ and -) to signify microtonal transposition by a single syntonic comma, or 81/80 (≈ 22 cents). Both *Sea Dirge* (1962) and *Knocking Piece* (1962) institute similar notational models and concurrent, 5-limit structures.[153] For example, to construct a consonant 4:5:6 triad when transposing to the second scale degree of 9/8 (D), we must raise 5/3 (A) to 27/16 (A+), or $\frac{5}{3} \cdot \frac{81}{80} = \frac{27}{16}$.

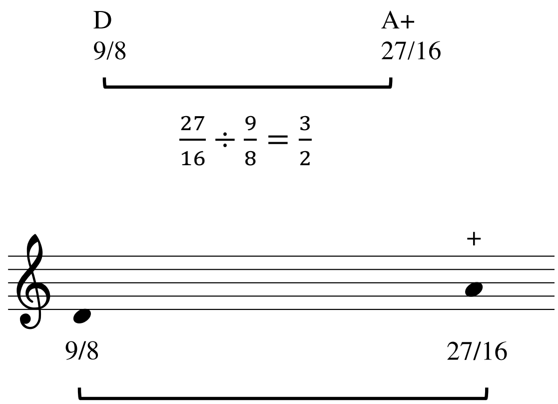

Figure 3. 9—Consonant "Pythagorean" Perfect Fifth

In more recent work, Johnston has extended his notation system to address just ratios derived from prime numbers in excess of seven. As such, this notation requires an array of new accidentals to signify higher-order harmonic relations. Johnston terms microtonal commas derived through harmonic ratios exceeding the 5-limit as *chromas*. For example, by appending an upside-down '7' to Bb (9/5), the resulting note sounds as the ratio 7/4. In regards to spectra, this ratio signifies the octave-reduced seventh harmonic. This septimal chroma is equivalent to the ratio 36/35 (≈ 49 cents), or $\frac{9}{5} \div \frac{7}{4} = \frac{36}{35}$. As to convey ratios of increasing complexity, Johnston deploys proprietary accidentals projecting commas and chromas through at least the 19-limit.[154]

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[153] Heidi Von Gunden, *The Music of Ben Johnston*, Netuchen, New Jersey: Scarecrow Press, Inc, 1986.

[154] Ibid.

raise	lower	ratio	cents	amount by which. . . .	exceeds
♯	♭	25/24	71	5/4	6/5
+	−	81/80	22	9/8	10/9
∠	7	36/35	49	9/5	7/4
↑	↓	33/32	53	11/8	4/3
13	ƐI	65/64	27	13/8	8/5
17	ㄥI	51/50	34	17/16	25/24
6I	19	96/95	18	6/5	19/16

Figure 3. 10—Ben Johnston's Notational Conventions for Commas and Chromas (19-Limit)[155]

Beyond those signified by Johnston's 19-limit system of accidentals, an array of other intervals can be derived through similar, divisional procedures. In practice, theorists often classify commas, chromas, diesis', and other enharmonic intervals according to both procedural and historical terms. Here, naming conventions often reference significant figures and formal constructs from Western Antiquity. Likewise, certain ratios draw correlation with their closest equal-tempered equivalents, taking the form of so-called third-, quarter-, fifth-, or sixth-tones. In either case, divisional procedures define these precise intervallic proportions, while revealing features implicit to the tuning structure from which they derive. For example, taking the quotients for sets of ratios generated by the 11-limit Tonality Diamond yields fourteen commas, chromas, diesis', and other enharmonic intervals spanning one 5-limit Chroma or less ($\leq 25/24$ or $\approx$ 71 cents).[156]

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[155] Bob Gilmour, "Changing the Metaphor: Ratio Models of Musical Pitch in the Work of Harry Partch, Ben Johnston, and James Tenney," *Perspectives of New Music*, Vol. 33, No. 1/2 (Winter-Summer, 1995): 458-503. http://www.jstor.org/stable/833715 (accessed Jan. 3, 2018).

[156] In whole, an 11-Limit Tonality Diamond generates fifteen interceding ratios, spanning intervals less than or equal to one 5-Limit Chroma ($\leq 25/24$ or $\approx$ 71 cents). While the ratio $80/77$ – $\left(\frac{8}{7} \div \frac{11}{10}\right)$ *or* $\left(\frac{20}{11} \div \frac{7}{4}\right)$ – spans approximately 66 cents, the numerator and denominator values do not exhibit epimoric properties indicative of other commas. Henceforth, for the purposes of our discussion, we shall omit this ratio.

$$[\ 5\ Limit\ Chroma\ ]$$
$$\left(\frac{5}{4} \div \frac{6}{5}\right)\ or\ \left(\frac{5}{3} \div \frac{8}{5}\right)$$
$$= \frac{25}{24} \approx 71\ cents$$

$$[\ Septimal\ ¼\ Tone\ ]$$
$$\left(\frac{6}{5} \div \frac{7}{6}\right)\ or\ \left(\frac{12}{7} \div \frac{5}{3}\right)\ or\ \left(\frac{9}{7} \div \frac{5}{4}\right)\ or\ \left(\frac{8}{5} \div \frac{14}{9}\right)$$
$$= \frac{36}{35} \approx 49\ cents$$

$$[\ Septimal\ Comma\ ]$$
$$\left(\frac{8}{7} \div \frac{9}{8}\right)\ or\ \left(\frac{16}{9} \div \frac{7}{4}\right)$$
$$= \frac{64}{63} \approx 27\ cents$$

$$[\ Ptolemy's\ Comma\ ]$$
$$\left(\frac{10}{9} \div \frac{11}{10}\right)\ or\ \left(\frac{20}{11} \div \frac{9}{5}\right)$$
$$= \frac{100}{99} \approx 17\ cents$$

$$[\ Unidecimal\ \tfrac{1}{5}\ Tone\ ]$$
$$\left(\frac{9}{8} \div \frac{11}{10}\right)\ or\ \left(\frac{20}{11} \div \frac{16}{9}\right)\ or\ \left(\frac{5}{4} \div \frac{11}{9}\right)\ or\ \left(\frac{18}{11} \div \frac{8}{5}\right)$$
$$= \frac{45}{44} \approx 39\ cents$$

$$[\ Unidecimal\ Comma\ ]$$
$$\left(\frac{9}{8} \div \frac{12}{11}\right)\ or\ \left(\frac{11}{6} \div \frac{16}{9}\right) or\ \left(\frac{11}{8} \div \frac{4}{3}\right)\ or \left(\frac{3}{2} \div \frac{16}{11}\right)$$
$$= \frac{33}{32} \approx 53\ cents$$

$$[\ Mothwellsma\ Comma\ ]$$
$$\left(\frac{9}{7} \div \frac{14}{11}\right)\ or\ \left(\frac{11}{7} \div \frac{14}{9}\right)$$
$$= \frac{99}{98} \approx 18\ cents$$

$$[\ Syntonic\ Comma\ ]$$
$$\left(\frac{9}{8} \div \frac{10}{9}\right)\ or\ \left(\frac{9}{5} \div \frac{16}{9}\right)$$
$$= \frac{81}{80} \approx 22\ cents$$

$$[\ Large\ Septimal\ Deisis\ ]$$
$$\left(\frac{7}{6} \div \frac{8}{7}\right)\ or\ \left(\frac{7}{4} \div \frac{12}{7}\right)$$
$$= \frac{49}{48} \approx 36\ cents$$

$$[\ Septimal\ \tfrac{1}{3}\ Tone\ ]$$
$$\left(\frac{7}{6} \div \frac{9}{8}\right)\ or\ \left(\frac{16}{9} \div \frac{12}{7}\right)\ or\ \left(\frac{4}{3} \div \frac{9}{7}\right)\ or\ \left(\frac{14}{9} \div \frac{3}{2}\right)$$
$$= \frac{28}{27} \approx 63\ cents$$

$$[\ Biyatisma\ Comma\ ]$$
$$\left(\frac{12}{11} \div \frac{11}{10}\right)\ or\ \left(\frac{11}{6} \div \frac{20}{11}\right)$$
$$= \frac{121}{120} \approx 14\ cents$$

$$[\ Unidecimal\ Diasecundal\ Comma\ ]$$
$$\left(\frac{10}{9} \div \frac{12}{11}\right)\ or\ \left(\frac{11}{6} \div \frac{9}{5}\right)\ or\ \left(\frac{11}{9} \div \frac{6}{5}\right)\ or\ \left(\frac{5}{3} \div \frac{18}{11}\right)$$
$$= \frac{55}{54} \approx 32\ cents$$

$$[\ Ptolemy's\ Enharmonic\ ]$$
$$\left(\frac{14}{11} \div \frac{5}{4}\right)\ or\ \left(\frac{8}{5} \div \frac{11}{7}\right)\ or\ \left(\frac{7}{5} \div \frac{11}{8}\right)\ or\ \left(\frac{16}{11} \div \frac{10}{7}\right)$$
$$= \frac{56}{55} \approx 31\ cents$$

$$[\ Tritonic\ Diesis\ ]$$
$$\left(\frac{10}{7} \div \frac{7}{5}\right)$$
$$= \frac{50}{49} \approx 35\ cents$$

3.7 Tonal Flux

Procedural implications for commas and other enharmonics extend beyond the stated thresholds of intervallic size. As noted by Partch and others, the same divisional forces which derive these high-order ratios from our Tonality Diamond also imply a structural framework for harmonic progression within the Monophonic Fabric. Therein, transitional states between two consonant (e.g. low-order) tonalities—Otonal *and* Utonal—entail forms of linear or contrapuntal motion spanning relatively narrow, microtonal intervals. In context to just intonation, these intervals retain proportional equivalency with one or more commas, chromas, diesis', or other enharmonic structures. In the eleventh chapter of his primary treatise, Partch refers to these interceding states between chords and other consonant sonorities as moments of "tonal[ity] flux."[157] As an implicit feature of just harmonic progressions, he deploys this technique with some regularity in his compositions. Some of the earliest applications include *On the City Street* and *The Intruder*—both composed in Santa Rosa during 1931.[158] In analysis of these works, Bob Gilmour speaks to the integral relationship between harmonic progression and tonal flux, stating that "both [pieces] use two contrasting tonalities (i.e., chords) whose constituent degrees are narrow, microtonal distances apart, and the tonal progression through each setting is a structural analog of this type of resolution. In both cases the two tonalities are used to symbolize different things-opposing tensions-in the verse."[159] This structural feature is not exclusive to Partch's compositions. In fact, Kyle Gann deploys a similar, chordal approach for *Hyperchromatica* (2015-2017). Here, Gann distributes chordal structures from a 13-limit system between three, justly-tuned disklaviers (or other, computer-driven pianos). Throughout the piece, chordal harmony shifts between familiar triadic structures, consisting of 1—3—5 tonal identities, and less-familiar 7—9—11 (or 13) identities. In describing the work, Gann highlights both distinctions between

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[157] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

[158] Harry Partch, John Cage, and Inc. *Recorded Anthology of American Music*. New York, N.Y: New World Records, 1978.

[159] Bob Gilmore, "On Harry Partch's "Seventeen Lyrics by Li Po"," *Perspectives of New Music* 30, no. 2 (1992): 22-58. doi:10.2307/3090619. (accessed January 21, 2019).

these low and high-order harmonic identities, and their equal relevance in regards to his formal intent, stating:

> "Let me put it more simply: I'm trying to make microtonality attractive and seductive, not scary as it is to most people and in most microtonal music. A lot of people, mostly composers, want to hear the most weird-ass and transgressive shit I can throw at them [sic], and I try to gratify that in some movements. But more, I want to suggest (and prove) that we can keep conventional tonality and augment it with higher-overtone relationships." [160]

Citing Partch's conception of "tonality flux", Gann attributes the transitional states between major (Otonal) and minor (Utonal) triadic structures, as well as high-order identities in *Hyperchromatica* as exhibiting only "the slightest changes of pitch."[161] Thusly, we may apprehend a rather extensive array of 13-Limit commas bridging each harmonic intersection—be they familiar or "transgressive."

As with Gann's 13-limit system, navigating between respective Otonalities and Utonalities within the 11-limit Tonality Diamond also makes explicit this integral relationship between harmonic progression, commas, and tonal flux. For example, one may derive two consonant triads from the second, third, and fifth Odentities and Udentities for ratios retaining a Numerary Nexus of '7'. Given their position within the Tonality Diamond, just major and minor thirds (5/4 and 6/5, respectively) intercede each ratio within the two chords (see Figure 3.11). Thus, each dyad reinforced the triad's perceived consonance. However, linear progression from one triad to the other yields a new set of high-order ratios. Dividing the second, third, and fifth Udentities (7/6—7/5—7/4) by their corresponding Odentities (8/7—10/7—12/7) produces three, nearly identical commas: two *Large Septimal Diesis'* (49/48 ≈ 36 cents) and one *Tritonic Diesis* (50/49 ≈ 35 cents). Here, divisional procedures within our Tonality Diamond determine intervallic properties exhibited during states of tonal flux.

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[160] Kyle Gann, "Hyperchromatica." www.kylegann.com/Hyperchromatica.html (Accessed January 28, 2021).

[161] Robert Carl, David DeBoor Canfield, Colin Clarke and Marc Medwin, "'To Get the Ball Rolling Again': Kyle Gann Speaks About His Hyperchromatica," Fanfare: The Magazine for Serious Record Collectors, vol. 42, no. 1, 1 Sep. 2018, pp. 75 - 84.

Otonalities →

Utonalities ↓

→ Odentity ↓ Udentity	2	3	5	7	9	11
2				**7/4**		
3				**7/6**		
5				**7/5**		
7	**8/7**	**12/7**	**10/7**	1/1	9/7	11/7
9				14/9		
11				14/11		

Figure 3. 11—11-Limit Tonality Diamond—Consonant Otonal and Utonal Triads (Numerary Nexus = 7)

(3 - 8)

$$\text{Otonal Triad} \;=\; \underbrace{8/7 \qquad 10/7}_{5/4}\;\underbrace{\qquad 12/7}_{6/5}$$

$$\text{Utonal Triad} \;=\; \underbrace{7/6 \qquad 7/5}_{6/5}\;\underbrace{\qquad 7/4}_{5/4}$$

$$\frac{2\!-\!Udentity}{2\!-\!Odentity} = \;7/6 \;\div\; 8/7 = \;49/48 \;\approx 36\,cents \quad [\,Large\ Septimal\ Deisis\,]$$

$$\frac{3\!-\!Udentity}{3\!-\!Odentity} = \;7/4 \;\div\; 12/7 = \;49/48 \;\approx 36\,cents \;\; [\,Large\ Septimal\ Deisis\,]$$

$$\frac{5\!-\!Odentity}{5\!-\!Udentity} = \;10/7 \;\div\; 7/5 = \;50/49 \;\approx 35\,cents \;\; [\,Tritonic\ Deisis\,]$$

3.8 Epimoric (or 'Super-Particular') Ratios

Concurrent with Partch and Johnston's conceptions of commas, chromas, and tonal flux, Erv Wilson goes further by connecting intervallic derivatives of his Lambdomas to the so-called 'Farey series'. This emergent structure presents a complete sequence of reduced fractions between '0' and '1', whose denominators are always equal or less than a specified value (*n*).[162] Intervals spanning the distance between fractions in the Farey series always represent ratios whose numerator and denominator values are separated by a difference of '1' (see Figure 3.12). Theorists refer to these intervals as 'super-particular' or 'epimoric' ratios, often favoring them in scale construction. Notably, such ratios appear with great frequency in Lou Harrison compositions for gamelan and strings.[163] As described by Narushima, Canright, and others, preference for super-particular ratios may result from a variety of factors. Most conspicuously, these ratios correspond to intervals formed between consecutive partials in the harmonic series. As such, difference tones between two frequencies separated by a super-particular ratio always correspond to the fundamental frequency of a common overtone series. Of special relevance to just scale construction, super-particular ratios also represent the "simplest" ratio form for a given intervallic range or scalar step.[164] For example, a minor third interval can be described by a number of harmonic proportions, including: 32/27 (≈ 294 cents; Pythagorean minor third), 19/16 (≈ 298 cents; octave equivalent of the 19th harmonic), and 13/11 (≈ 289 cents). However, the 5-limit super-particular ratio 6/5 (≈ 316 cents) retains the simplest intervallic form.[165]

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162 "So-Called Farey Series, Extended 0/1 to 1/0 (Full Set of Gear Ratios), and Lambdoma," *The Wilson Archives* (1996), http://anaphoria.com/lamb.pdf (accessed March 27, 2019).

163 Douglas Keislar, "Six American Composers of Non-standard Tunings," *Perspectives of New Music*, Vol. 29, No. 1 (Winter, 1991): 176-211. http://www.jstor.org/stable/833076 (accessed Dec. 17, 2017).

164 David Canright, "Superparticular Pentatonics," *1/1: Journal of the Just Intonation Network*, vol. 9, no. 1 (1995): 10–13, https://sites.google.com/site/davidrcanright/music-articles/superparticular-pentatonics (accessed March 27, 2019).

165 G. D. Halsey and Edwin Hewitt, "More on the Superparticular Ratios in Music," *The American Mathematical Monthly* 79, no. 10 (1972): 1096-100. www.jstor.org/stable/2317424 (accessed March 27, 2019).

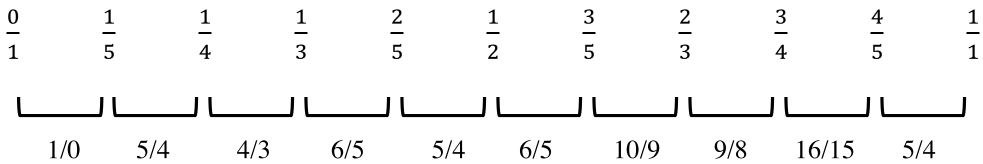

Figure 3. 12—Farey Series (whereas *n* = 5) and Interceding Super-Particular Ratios

James Tenney further codifies this notion of intervallic simplicity, its perceptual affordances, and how these factors pertain to super-particular ratios through the principle of "*Harmonic Distance*." These theories arise primarily through examination of harmonic aggregates produced by the concurrent sounding of two harmonic spectra. Importantly, Tenney's determinations regarding harmonic distance and other derivatives for harmonic aggregates assume the concurrence of two contiguous spectra, consisting of *all* even and odd-numbered harmonics for a given frequency-range. By the composer's own admission, aggregates formed by *only* odd, even, or other continuous collections of partials generally produce different harmonic distance values.[166] It is also worth noting that perceptual models for tuning these and other inharmonic spectra have been proposed by other researchers, including roughness-based analysis by Pantelis Vassilakis and William Sethares.[167] [168]

In musical terms, an aggregate appears analogous to a single dyad. Accordingly, the proportional relationship between the aggregate's two fundamental frequencies determines the number of harmonic partials whose frequencies coincide within a given range. As expected, certain integral ratios generate fewer common (or *coincidental*) harmonic frequencies than others. Tenney describes such ratios as retaining higher degrees of "*Harmonic Disjunction*." In contrast,

[166] James Tenney, Lauren Pratt, Rob Wannamaker, Michael Winter and Larry Polansky. From Scratch: Writings in Music Theory. Edited by Lauren Pratt, Rob Wannamaker, Michael 1980 Winter and Larry 1954 Polansky. Urbana: University of Illinois Press, 2015.

[167] Pantelis Vassilakis, "Auditory roughness as means of musical expression," *Selected Reports in Ethnomusicology* 12 (Perspectives in Systematic Musicology): 119-144. http://www.acousticslab.org/papers/SRE12.htm (accessed August 5, 2018)

[168] William A. Sethares, *Tuning, Timbre, Spectrum, Scale*. London: Springer-Verlag, 1998.

integral (or just) ratios containing relatively small numerator and denominator values produce more common harmonic frequencies and thus, exhibit lesser degrees of harmonic disjunction. Tenney defines this value as a fraction of harmonic frequencies within an aggregate which do *not* coincide with another. He terms this fractional value the "*Disjunction Ratio*."[169]

(3 - 9)

$$Disjunction\ Ratio\ (a/b) = \left(1 - \frac{1}{a}\right) \cdot \left(1 - \frac{1}{b}\right)$$

$$whereas:$$

$$a = numerator\ value\ for\ a\ just\ ratio\ separating\ two\ fundamental\ frequencies\ of\ an\ aggregate$$

$$b = denominator\ value\ for\ a\ just\ ratio\ separating\ two\ fundamental\ frequencies\ of\ an\ aggregate$$

Perhaps most significant to our discussion, Tenney correlates values derived through this function with "traditional estimates of dissonance," thus establishing one codified metric for evaluating perceptual affordances of just ratios. In defining a second function for Harmonic Distance (*HD*), Tenney refines his model further by integrating objective evaluations of other spectral properties. These values remain similar to those derived from harmonic disjunction, while fulfilling the mathematical criteria for a distance function: symmetry ($HD_{(a,b)} = HD_{(b,a)}$), non-negativity ($HD_{(a,b)} \geq 0$), and non-degeneracy ($HD_{(a,b)} = 0$ if and only if $a = b$).[170]

(3 - 10)

$$Harmonic\ Distance\ (a/b) = log_2\ (a \cdot b)$$

$$whereas:$$

$$a = numerator\ value\ for\ a\ just\ ratio\ separating\ two\ fundamental\ frequencies\ of\ an\ aggregate$$

$$b = denominator\ value\ for\ a\ just\ ratio\ separating\ two\ fundamental\ frequencies\ of\ an\ aggregate$$

While expressed as two distinct functions, harmonic disjunction and harmonic distance values assume relative equivalence when mapped to the same array of just ratios. As stated, their

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[169] James Tenney, Lauren Pratt, Rob Wannamaker, Michael Winter and Larry Polansky. From Scratch: Writings in Music Theory. Edited by Lauren Pratt, Rob Wannamaker, Michael 1980 Winter and Larry 1954 Polansky. Urbana: University of Illinois Press, 2015.

[170] Ibid.

efficacy in evaluating the consonance or dissonance of intervals as a function of proportional simplicity between numerator and denominator values appears consistent with assertions by Partch and others. Preceding this work, Partch's graphical projection, the 'One-Footed Bride', provides a more qualitative evaluation of perceptual properties reflected in the codified metrics of harmonic disjunction and harmonic distance. For example, notice how peaks in "power" and "emotion" correspond to the same low-harmonic distance (and disjunction) ratios (1/1, 8/7, 7/6, 6/5, 5/4, 4/3, 3/2, 8/5, 5/3, 12/7, 7/4, 2/1) defined by Tenney.[171]

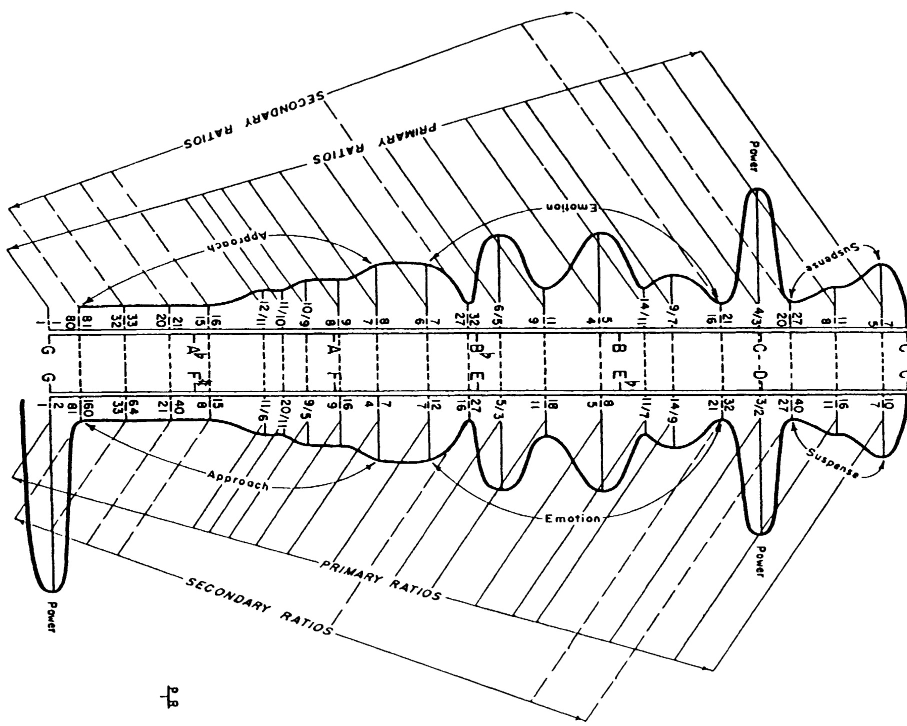

Figure 3. 13—Harry Partch's 'One-Footed Bride' [172]

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[171] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

[172] Ibid.

Likewise, contemporary data from roughness-based models by Vassilakis and Sethares also support the saliency of Tenney's findings, so far as they pertain to harmonic dyads.[173] Derived from perceptual models introduced by Terhardt, Sethares, and others, Vassilakis' model calculates the cumulative roughness value for pairs of sinusoids within a harmonic aggregate, consisting of at least two partials—or sinusoid pairs.[174] Here, cumulative *Spectral Roughness Analysis* (SRA) of an aggregate derives the sum of roughness values for each sinusoid pair within the aggregate spectrum. This iterative approach takes into account the mutual influence of each partial's amplitude and frequency. However, unlike harmonic distance or disjunction functions, the SRA model does not assume octave equivalency for a given interval; nor does it imply strict harmonicity for the aggregate spectrum. Instead, roughness-based models take into account both the frequency distribution, range, and respective amplitudes of constituent partials. Perhaps the most notable distinction between SRA and other perceptual models can be illustrated by comparing the perceived consonance or dissonance of a minor third played on the lowest range of a piano keyboard (A0—C1 or ≈ 55—65.4 Hertz), versus the same interval performed in a higher range (A4—C5 or ≈ 440—523.2 Hertz). While calculating the harmonic distance for both dyads yields the same value, Vassilakis' SRA model produces significantly elevated roughness values for the minor third dyad performed in a lower frequency range. In this regard, SRA appears to more effectively model how we perceive consonance or dissonance as a byproduct of both interval *and* frequency range. To illustrate the cumulative interaction between these factors, Vassilakis assigns separate coefficient values to amplitude (*X* and *Y*) and frequency components (*Z*) in his equation. Thus, his model demonstrates that roughness is mutually dependent upon both the differences in amplitude and frequency between two sinusoids. For example, partials exhibiting greater differences in amplitude tend to present lower roughness values than partials of near equal amplitude. Likewise, the rate of amplitude fluctuation (e.g. 'beating') between two frequencies

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[173] Pantelis Vassilakis, "SRA: A Web-based Research Tool for Spectral and Roughness Analysis of Sound Signals," *Proceedings of the Fourth Sound and Music Computing Conference (SMC 2007), Lefkada, Greece*. http://www.acousticslab.org/papers/Vassilakis2007SMC.pdf (accessed April 22, 2020).

[174] Ernst Terhardt, "On the Perception of Periodic Sound Fluctuations (Roughness)," *Acoustica* 30 (4): 201-213 (1974).

influences our perception of roughness and, by proxy, tension or dissonance of an aggregate spectra.

(3 - 11)

$$Roughness\ for\ a\ Pair\ of\ Sinusoids\ (R) = X^{0.1} \cdot 0.5 \cdot (Y^{3.11}) \cdot Z$$

$$Whereas:$$

$$X = A_{min} \cdot A_{max}$$

$$Y = \frac{2 \cdot A_{min}}{(A_{min} + A_{max})}$$

$$Z = e^{-b1s(f_{max} - f_{min})} - e^{-b2s(f_{max} - f_{min})}$$

$$A_{min} = minimum\ peak\ amplitude\ value\ for\ pair\ of\ sinusoids$$

$$A_{max} = maximum\ peak\ amplitude\ value\ for\ pair\ of\ sinusoids$$

$$F_{min} = minimum\ Frequency\ (Hertz)\ for\ pair\ of\ sinusoids$$

$$F_{max} = maximum\ Frequency\ (Hertz)\ for\ pair\ of\ sinusoids$$

$$b_1 = 3.5 \qquad b_2 = 5.75 \qquad s = \frac{0.24}{(s_1 \cdot f_{min} + s_2)}$$

$$s_1 = 0.0207 \qquad s_2 = 18.96$$

In examining a combined, visual representation for each respective model, we notice an immediate correlation between localized minima for harmonic distance, harmonic disjunction, and spectral roughness values. Figure 3.14 illustrates this point by displaying harmonic distance and harmonic disjunction values derived from the 29 tones of our 11-Limit Tonality Diamond, as well as SRA values for the same intervallic range. As roughness derives from both frequency and amplitude coefficients, we chose to analyze a linear spectrum consisting of twelve contiguous harmonics ($n = 1 - 12$), whose fundamental frequency ranges from A4 (1/1 = 440 Hertz) to A5 (2/1 = 880 Hertz). Amplitude for each harmonic decrease linearly, wherein the amplitude of the n^{th} harmonic is equal to $\frac{1}{n}$. In general form, this arrangement emulates the spectral profile for the first twelve harmonics of a saw-tooth waveform. While admittedly less than exhaustive, the harmonic richness and range afforded by this timbre provides a general approximation of many stringed instruments.

Relative to other just intervals displayed within a given range, those ratios retaining the smallest numerator and denominator values consistently yield the lowest derivatives across each of the three perceptual models. While output for each model reflects a different metric range, normalizing data to match a standard, maximum height reveals additional insights. For example, substantial decreases in both harmonic distance and harmonic disjunction values occur at the following ratios: 1/1, 6/5, 5/4, 4/3, 3/2, 8/5, 5/3, 7/4, 9/5, 2/1. Mirroring these results, localized minima—or *valleys*—in spectral roughness coincide with the same small-number ratios. Moreover, each of these ratios exhibit epimoric qualities. By definition, a difference of '1' separates the numerator and denominator of ratios 6/5, 5/4, 4/3, 3/2, and 2/1. Octave-transposed equivalents of the remaining ratios (1/2, 4/5, 5/6, 7/8, and 9/10, respectively) retain similar epimoric properties. As discussed, these proportions are indicative of super-particular ratios valued in just scale construction.

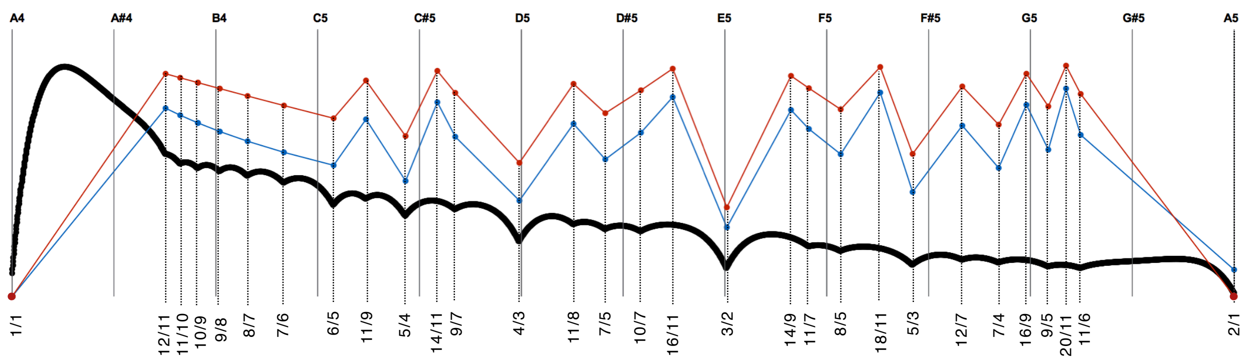


Figure 3. 14—Harmonic Distance (Blue), Harmonic Disjunction (Red), and Spectral Roughness Analysis (Black) for Ratios Derived from an 11-Limit Tonality Diamond (A4 = 440 Hertz)

3.8.1 Applying Epimoric Tuning Structures to Actuated Instrumentation

In determining appropriate tuning structures for actuated instrumentation, super-particular ratios afford a similar perceptual framework. Thus, we derive the tuning of individual strings for our actuated instruments, Rosebud I and Rosebud ('Louise') II, according to epimoric proportions. Consistent with all manner of actuation discussed in this essay, frequencies shared by actuated signals and the spectra of vibrating strings contribute to the sounding of discrete tones. For our purposes, this sympathetic relationship between input source and actuated body therein defines all subsequent performance practices—particularly, tuning. As discussed, the concurrence of super-particular ratios with points of minimal roughness, harmonic distance, and harmonic disjunction reinforces the perceptual salience of these intervals. In regards to actuation of strings, the spectral properties by which these three values derive lay manifest in the innate harmonicity of vibrating strings. Moreover, from the intrinsic proportionality of epimoric ratios emerge other compositionally-relevant just tuning structures and procedures.

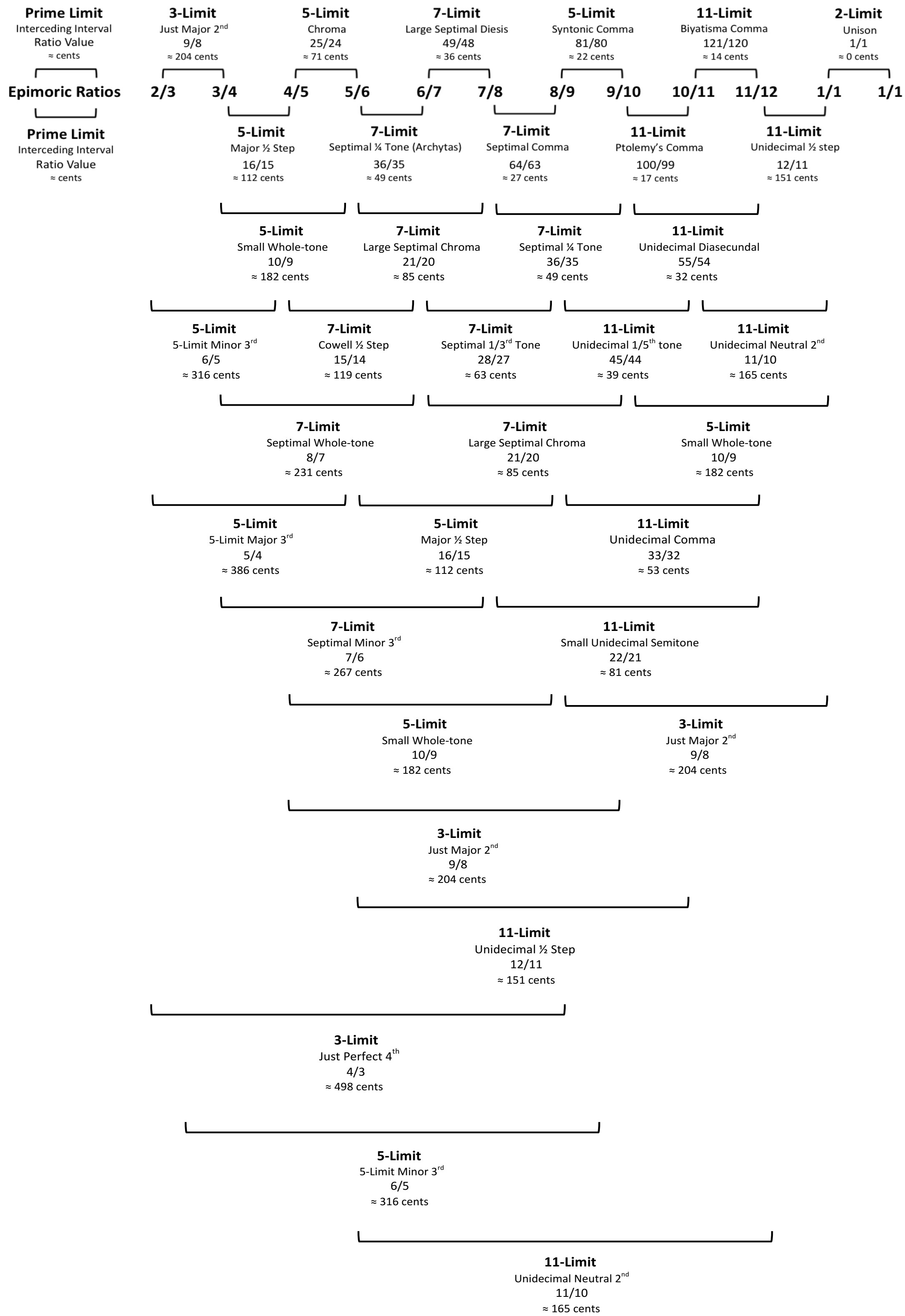
These emergent properties appear most evident in the tuning structure for Rosebud II (see Figure 3.15). Here, an 11-limit series of super-particular ratios defines tuning for the instrument's twelve strings (XII-I). Expressed in octave-equivalent form, this ascending series represents all eleven epimoric intervals found within Partch's 11-limit Tonality Diamond: 1/2 (I/II), 2/3 (XII), 3/4 (XI), 4/5 (X), 5/6 (IX), 6/7 (VIII), 7/8 (VII), 8/9 (VI), 9/10 (V), 10/11 (IV), and 11/12 (III). Tuned as a single course, strings I and II both retain the unity ratio 1/1—a frequency of 330 Hertz[175]. Furthermore, interceding intervals between these strings form a secondary set of twenty-three 11-limit epimoric ratios, consisting of: 5/4, 6/5, 7/6, 8/7, 9/8, 10/9, 11/10, 12/11, 15/14, 16/15, 21/20, 22/21, 25/24, 28/27, 33/32, 36/35, 45/44, 49/48, 55/54, 64/63, 81/80, 100/99, 121/120. These interceding ratios include eleven commas, chromas, diesis', or other enharmonic intervals spanning less than a single 5-limit chroma (25/24 ≈ 71 cents): *Septimal 1/3rd Tone* (28/27 ≈ 63 cents), *Unidecimal Comma* (33/32 ≈ 53 cents), *Septimal ¼ Tone* (36/35 ≈ 49 cents), *Unidecimal 1/5th Tone* (45/44 ≈ 39 cents), *Large Septimal Diesis* (49/48 ≈ 36 cents), *Unidecimal Diasecundal Comma* (55/54 ≈ 32 cents), *Septimal Comma* (64/63 ≈ 27 cents), *Diatonic Comma* (81/80 ≈ 22 cents), *Ptolemy's Comma* (100/99 ≈ 17 cents), and *Biyatisma Comma* (121/120 ≈ 14 cents).

Moving forward, we shall examine how these emergent features of prime limit, Odentities and Udentities, commas, tonal flux, and epimoric proportions inform the compositional structure and form of *Artemisia* and other significant works for actuated string instruments. Codified by

[175] In subsequent descriptions of tuning for actuated instruments performed on *Artemisia*, we assign a value of 55 Hertz to the unity ratio (1/1). However, as frequencies actuated by Rosebud II embody specific spectral functions in the piece, a transposed value of 1/1 = 330 Hertz (55 Hertz · $\frac{6}{1}$) accompanies certain diagrams.

Partch and others, this Monophonic Fabric provides the language, conventions, and theoretical underpinning for describing this work. Together, these features define a unified practice in which tuning, spectra, and perceptual affordance intersect instrument design, composition, and performance practice.

Figure 3. 15—Epimoric Tuning Structure for Rosebud II



IV. COMPOSITIONAL STUCTURE AND CONVENTIONS OF *ARTEMISIA*

As a recorded composition, *Artemisia* demonstrates a variety of electro-magnetic and electro-mechanical actuation techniques. We witness these techniques applied in composition, performance, and post-production. In addressing the intersections between systematic approaches to tuning and actuated instrument design and performance practices, the structure of the composition embodies specific, foundational principles of just intonation. These principles and practices include: Tonal Flux, commas, chromas, and the application of epimoric (or super-particular) ratios.[176] By actuating harmonic frequencies derived through these systems and procedures, each instrument acts as both a medium for re-embodiment of sound *and* a physical representation of the spectral proportions by which all just ratios are defined. Therein, physically-tangible properties of sound directly inform the principles of design, performance, and intonation.

4.1 Overtone Structure for Actuated Strings

To further cement the foundational relationship between actuated instrumentation and just intonation, nearly all harmonic content for the piece derives from the first seven overtones of six actuated strings. Here, the physical design and tuning of the electro-magnetically actuated instrument Rosebud I (RBI) provide a tangible vessel for spectral features which govern structure for the entire piece. Originating from a unity frequency ($F_{1/1}$) of 55 Hertz (A1), we tune the instrument's six strings (VI-I) to a set of corresponding, just ratios: VI = 8/9, V = 9/10, IV = 4/3, III = 20/11, II = 11/6, I = 2/1. In octave-reduced form, this tuning represents an ascending series of four 11-limit, super-particular ratios—8/9, 9/10, 10/11 (or 20/11), 11/12 (or 11/6)—and two harmonic derivatives of the ratio 8/9: 4/3 and 2/1. Referencing Pythagorean proportions, these latter ratios form through an ascending series of perfect fifths (3/2) above the ratio 8/9. Intervals spanning octave-reduced equivalents of the original series of epimoric ratios (8/9, 9/10, 10/11, 11/12) form a set of six commas: *Syntonic Comma* ($9/10 \div 8/9 = 81/80$), *Ptolemy's Comma* ($10/11 \div 9/10 = 100/99$), *Biyatisma Comma* ($11/12 \div 10/11 = 121/120$), *Unidecimal*

[176] "So-Called Farey Series, Extended 0/1 to 1/0 (Full Set of Gear Ratios), and Lambdoma," *The Wilson Archives* (1996), http://anaphoria.com/lamb.pdf (accessed March 27, 2019).

Comma ($11/12 \div 8/9 = 33/32$), *Unidecimal 1/5th Tone* ($10/11 \div 8/9 = 45/44$), and *Unidecimal Diasecundal Comma* ($11/12 \div 9/10 = 55/54$).[177]

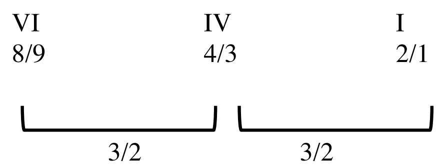


Figure 4. 1—Two Pythagorean Derivatives for the Epimoric Ratio '8/9'

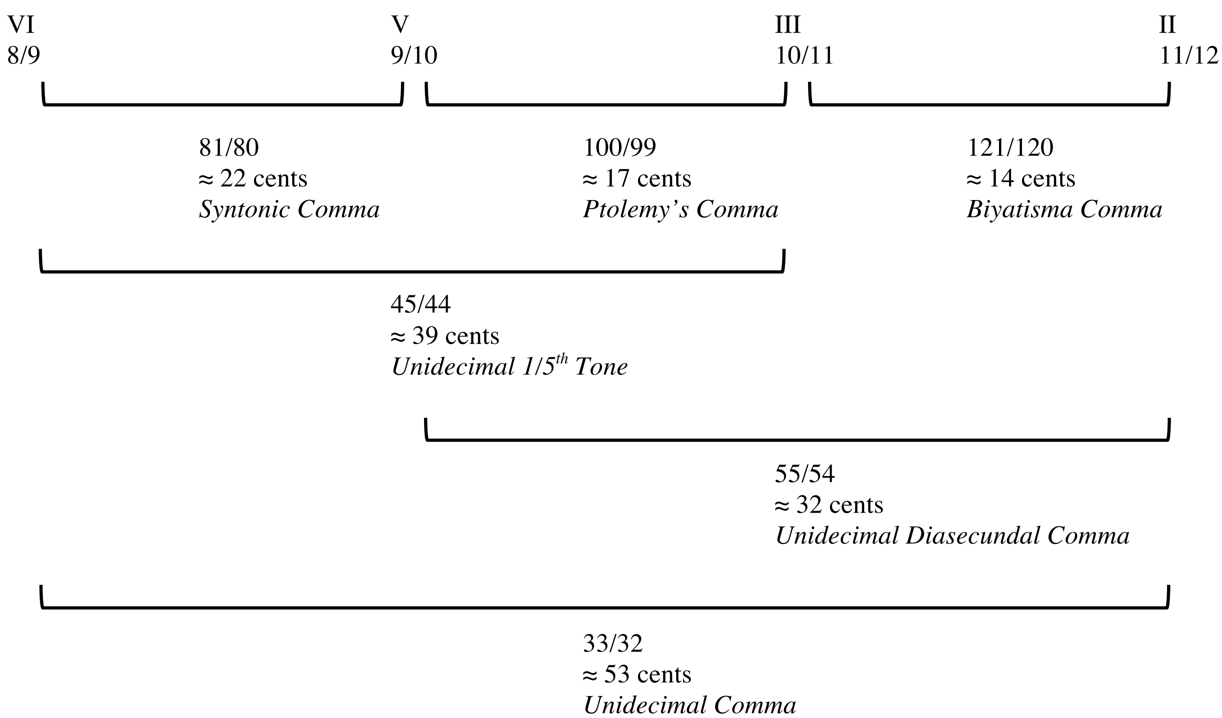


Figure 4. 2—Interceding Commas for 11-Limit Epimoric (Super-Particular) Ratios – 8/9, 9/10, 10/11, 11/12

In whole, actuated partials for all six strings generate a reservoir of 42 harmonic tones, utilized exclusively throughout the piece. Each tone arises as the product of one or more fundamental ratios ($F_{RBI(1-6)} = \frac{8}{9}, \frac{9}{10}, \frac{4}{3}, \frac{20}{11}, \frac{11}{6}, \frac{2}{1}$) and integral, harmonic multiples ($n =$

[177] Kyle Gann, "Anatomy of an Octave," *Just Intonation: General Theory and Reference*, https://www.kylegann.com/Octave.html, (accessed August 15, 2017).

$1, 2, 3, 4, 5, 6, 7$) for each string. Viewed as 6×7 matrix (see Figure 4.3), multiplication of the first seven harmonics in the left column (n) by the fundamental ratios from the bottom row ($F_{RBI(1-6)}$) yields an ascending series of overtones for each actuated string (VI-I). For example, actuating the third harmonic ($n = 3$) for string VI ($F_{RBI(6)} = \frac{8}{9} \cdot F_{1/1}$) generates a harmonic tone with a frequency ratio of $\frac{8}{3} \cdot F_{1/1}$ (or 146.66 Hertz). Referencing equal-temperament, the same pitch can be approximated by note value D3 -2 cents. Notice that the same ratio occurs at the second harmonic ($n = 2$) of string IV ($F_{RBI(4)} = \frac{4}{3} \cdot F_{1/1}$). In this case, the same frequency can be actuated across multiple strings. Moving higher in the overtone series, intervallic complexity across strings affords increasingly high-order Otonalities and Utonalities between the respective y- and x-axis' of our harmonic matrix.

Harmonic (n)	Just Ratio ≈ Equal-Tempered Note Value (+/- Cents)					
7	56/9 F4 -35	63/10 F4 -14	28/3 C5 -33	140/11 F5 -4	77/6 F5 +18	14/1 G4 -31
6	16/3 D4 -2	27/5 D4 +20	8/1 A4 +0	120/11 D5 +37	11/1 D#5 -49	12/1 E5 +2
5	40/9 B3 -18	9/2 B3 +4	20/3 F#4 -16	100/11 B4 +21	55/6 B4 +36	10/1 C#5 -14
4	32/9 G3 -4	18/5 G3 +18	16/3 D4 -2	80/11 G4 +35	22/3 G4 +49	8/1 A4 +0
3	8/3 D3 -2	27/10 D3 +20	4/1 A3 +0	60/11 D4 +37	11/2 D#4 -49	6/1 E4 +2
2	16/9 G2 -4	9/5 G2 +18	8/3 D3 -2	40/11 G3 +35	11/3 G3 +49	4/1 A3 +0
1	8/9 G1 -4	9/10 G1 +18	4/3 D2 -2	20/11 G2 +35	11/6 G2 +49	2/1 A2 +0
$F_{RBI(1-6)}$	$8/9 \cdot F_{1/1}$	$9/10 \cdot F_{1/1}$	$4/3 \cdot F_{1/1}$	$20/11 \cdot F_{1/1}$	$11/6 \cdot F_{1/1}$	$2/1 \cdot F_{1/1}$
String #	VI	V	IV	III	II	I

Figure 4. 3—Reservoir of 42 Harmonic Tones for Actuation

4.2 Nearest Actuated String Harmonics (NASH) for Inharmonic Timbres

Integrating past research in spectral composition, our work endeavors to bridge certain practices and procedures of just intonation with the production of inharmonic sonorities and other procedural non-linearities. In general, we experience these phenomena when the distribution of partials stray from simple, harmonic proportions. That is to say, inharmonicity occurs when one or more frequencies within a spectrum no longer retain an integral relationship with the fundamental frequency.[178] Such non-linearities appear incongruous with the implied harmonicity of vibrating strings. As listeners, we most commonly associate inharmonicity with bells, gongs, and other metallic percussion or idiophones. However, as many piano technicians can attest, strings of substantial mass or rigidity may exhibit audible inharmonicity.[179] [180] While calculating the exact frequency of partials for a given string involves a detailed assessment of length, diameter, tension, mass, and other material properties, simplified models have been developed which distill these complex properties into a few, basic coefficients.[181] [182] From a compositional perspective, this economized approach presents a means for assessing the most salient features of complex

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[178] William A. Sethares, *Tuning, Timbre, Spectrum, Scale*, London: Springer-Verlag, 1998.

[179] Harvey Fletcher, E. Donnell Blackman, and Richard Stratton, "Quality of Piano Tones." *The Journal of the Acoustical Society of America,* Volume 34, No. 6 (June 1962), http://www.physics.byu.edu/download/publication (accessed March 19, 2020).

[180] "Wire-Strung Harp: The Complete Resource for the Wire-Strung Harp," https://www.wirestrungharp.com/material/strings/table_3_wound_strings/ (accessed March 27, 2020).

[181] Robert W. Young, "Inharmonicity of Plain Wire Piano Strings," *The Journal of the Acoustical Society of America,* Volume 24, No. 3 (May 1952). http://www.afn.org/~afn49304/youngnew.htm (accessed March 19, 2020).

[182] Robert D. Polak, Adam R.V. Davenport, Andrew Fischer, and Jared Rafferty, "Determining Young's modulus by measuring guitar string frequency," *The Physics teacher* 56, no. 2 (February 01, 2018). https://aapt.scitation.org/doi/pdf/10.1119/1.5021447 (accessed March 27, 2020).

sonorities. Notably, French spectralist composers, including Tristan Murail and Gerard Grisey, have applied comparable techniques for the synthesis and orchestration of inharmonic timbres.[183] According to Joshua Fineberg and others, one can apprehend the intervallic displacement of partials away from strictly harmonic proportions as a product of spectral "stretching." By adapting an equation from Finberg's essay, "A Guide to Basic Concepts and Techniques of Spectral Music," one can generate an array of stretched spectra using a single *Stretch Coefficient* value (x).[184] Here, coefficient values greater than one ($x > 1.0$) correspond with stretched spectra. Progressively raising the absolute value for this coefficient results in increasingly inharmonic timbres.

$$Stretched\ Harmonic\ Frequency = F \cdot n^{x} \tag{4 - 1}$$

$$\begin{gathered} whereas: \\ F = Fundamental\ Frequency \\ n = Harmonic\ Number\ (1, 2, 3, 4, 5, 6, 7 \ldots \infty) \\ x = Stretch\ Coefficient \end{gathered}$$

In composing the spectral structure for *Artemisia*, we deploy a variant of the single-coefficient equation cited by Fineberg. As byproducts of an exponential function, the intervallic proportions between partials generated in this manner is innately non-linear—if not, harmonically arbitrary. In stark contrast, the integral quality of just intervals ensures that all harmonic proportions exhibit absolute linearity reflected in the overtone (or undertone) series. To reconcile the intrinsic non-linearities of stretched spectra with the strictly just proportions of the overtone series, we propose a method for mapping the frequencies of stretched partials to our reservoir of 42 harmonic tones for actuation. Here, a simple scalar function (programmed in Max/MSP) quantizes stretched harmonic frequencies (f_{st}) generated by the aforementioned equation ($F \cdot n^{x}$) to match the nearest frequency produced by one (or more) of the first seven harmonics for the six actuated strings of Rosebud I (VI-I). Therein, we refer to the quantized equivalent of a stretched partial as the *Nearest Actuated String Harmonic* (NASH).

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183 Francois Rose, "Introduction to the Pitch Organization of French Spectral Music," *Perspectives of New Music,* Vol. 34, No. 2 (1996): 6-39.

184 Joshua Fineberg, "A Guide to Basic Concepts and Techniques of Spectral Music," *Contemporary Music Review*. Vol. 19, Part 2 (2000): 81-113.

(4 - 2)

$$Frequency\ of\ Stretched\ Harmonic\ (f_{st})\ =\ F_{RBI(1-6)} \cdot n^{x}$$

$$n\ = Integral\ Value\ of\ Harmonic\ Partial$$

$$x\ = Stretch\ Coefficient$$

$$Unity\ or\ \text{"Tonal Center"}\ \left(F_{1/1}\right) = 55\ Hz$$

$$Fundamental\ Frequencies\ for\ Strings\ of\ Rosebud\ \ I\ (F_{RBI(1-6)})$$

$$F_{RBI\,(6)} = \frac{8}{9} \cdot F_{1/1}$$

$$F_{RBI\,(5)} = \frac{9}{10} \cdot F_{1/1}$$

$$F_{RBI\,(4)} = \frac{4}{3} \cdot F_{1/1}$$

$$F_{RBI\,(3)} = \frac{20}{11} \cdot F_{1/1}$$

$$F_{RBI\,(2)} = \frac{11}{6} \cdot F_{1/1}$$

$$F_{RBI\,(1)} = \frac{2}{1} \cdot F_{1/1}$$

$$Nearest\ Actuated\ String\ Harmonic\ (NASH) = \frac{F_{RBI(1-6)} \cdot n}{F_{1/1}}$$

As harmonic derivatives for the stretch coefficient (x) are exponential, each octave within the series is subject to variable intervallic displacement. In the case of spectral stretching, lower octaves may exhibit displacement by intervals equivalent to one of the six commas interceding the five epimoric ratios which define fundamental frequencies ($F_{RBI(1-6)}$) for strings VI, V, III, II, and I. For example, applying a stretch coefficient value of $x = 1.015$ to the fourth harmonic ($n = 4$) of string V ($\frac{9}{10} \cdot F_{1/1}$) generates an inharmonic partial (f_{st}) with a frequency of approximately $3.676 \cdot F_{1/1}$ (≈ 202.16 Hertz or G#3 -46 cents). The Nearest Actuated String Harmonic (NASH) for this frequency resides at the second harmonic ($n = 2$) of string II ($\frac{11}{6} \cdot F_{1/1}$); a frequency of $\frac{11}{3} \cdot F_{1/1}$ (≈ 201.66 Hertz or G3 +49 cents). Here, the interval between the stretched harmonic and its NASH value spans one *Unidecimal Diasecundal Comma* (55/54 or ≈ 32 cents).

(4 - 3)

$$Frequency\ of\ Stretched\ Harmonic\ (f_{st})\ =\ \left(\frac{9}{10}\cdot F_{1/1}\right)\cdot 4^{\,1.015} \approx 3.676\ \cdot\ F_{1/1}$$

$$Nearest\ Actuated\ String\ Harmonic\ (NASH) = \frac{\left(\frac{11}{6}\right)\cdot 2}{F_{1/1}} \approx 3.666\ \cdot\ F_{1/1}$$

$$\frac{f_{st}}{NASH} = \left[\left(\frac{9}{10}\cdot F_{1/1}\right)\cdot 4^{\,1.015}\right] \div \left[\frac{\left(\frac{11}{6}\right)\cdot 2}{F_{1/1}}\right] \approx\ 55/54$$

As noted in previous studies, the cumulative effect for this form of inharmonic distortion suggests the perception of an altered or ambiguous fundamental frequency—thus eliciting a sense of movement in the lower voices analogous to features Partch and others attribute to Tonal Flux.[185] [186] In composing this piece, we intend to capitalize upon this phenomenon. Moreover, linking these systemic approaches to intonation with the spectral properties of actuated strings demonstrates an intersecting framework for composing with actuated string instruments.

4.3 Visual Representation of Formal Structure Within the Score

Using the methods described, the structure of the piece follows 120 cyclical states of tonal flux. Therein, five sonorities of maximal spectral similarity are followed by a sixth sonority whose spectrum shares the least number of common partials with the preceding state. Each transition between states signifies a new fundamental frequency ($F_{RBI(1-6)}$) and stretch coefficient value (x). As discussed, we intentionally limit these values to match fundamental frequencies of the sixth, fifth, third, second, and first strings (XI, V, III, II, I) of the electro-magnetically actuated instrument, Rosebud I. These frequencies correspond to the epimoric ratios 8/9, 9/10, 20/11, 11/6, and 2/1, respectively.

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[185] Joshua Fineberg, "A Guide to Basic Concepts and Techniques of Spectral Music," *Contemporary Music Review*. Vol. 19, Part 2 (2000): 81-113.

[186] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

By constraining stretch coefficient values to a specified range (1.0 ≤ 1.5) and including only those values which yield at least one new partial from the quantized spectra (NASH), we generate an array of up to 78 distinct timbres from a single string. To explore the shared properties of emergent spectra, a programmable matrix stores stretch coefficient (x), fundamental frequency ($F_{RBI(1-6)}$), and NASH values within a multi-dimensional array of numbered cells.[187] Therein, we divide these cells into six separate rows (0-5) according to the fundamental frequency of the corresponding string. By querying the matrix, one can determine which stretch coefficient value generates the greatest or least number of shared partials for a given pair of strings. Thus, one may ranks two spectra as being 'most similar' or 'most different', respectively. The capacity to determine similarity and difference between two spectra affords the composer the ability to either seamlessly interpolate between sonorities or create discrete sections in time.

We organize each of the 120 states according to 24 non-repeating 'sets' of five string ratios ($120\ states\ \div 5 = 24\ sets$). Each set always begins with the ratio for string VI ($\frac{8}{9} \cdot F_{1/1}$), while internal order within a set is determined via two, nested permutations. However, as the sonority for every sixth state contrasts with the spectra of the previous sonority, our perception of stasis and change alternates across twenty sections ($120\ states\ \div 6 = 20\ sections$). Consequently, the order of fundamental frequencies for each state varies by section, giving each a unique spectral identity within the score.

For the dual purposes of analysis and reproduction of state-specific spectral form, we deploy a specific set of visual and notational conventions. As seen below (Figure 4.4), these graphical projections convey formal makers, including state number, as well as other parameters contributing to harmonic structure and subsequent actuation. In the column labeled $F_{RBI(1-6)}$, we find the fundamental frequency ratio for one of six strings. As mentioned, this value indicates one of three coefficients utilized in calculating the frequencies of stretched, harmonic partials. Directly to the right, the second coefficient (x) determines the stretch coefficient value for our calculation, while up to seven harmonic multiples (n) provide the base for the exponential function. Corresponding decimal values ($f_{st}\ /\ F_{1/1}$) report the resulting stretched frequency ratios in relation

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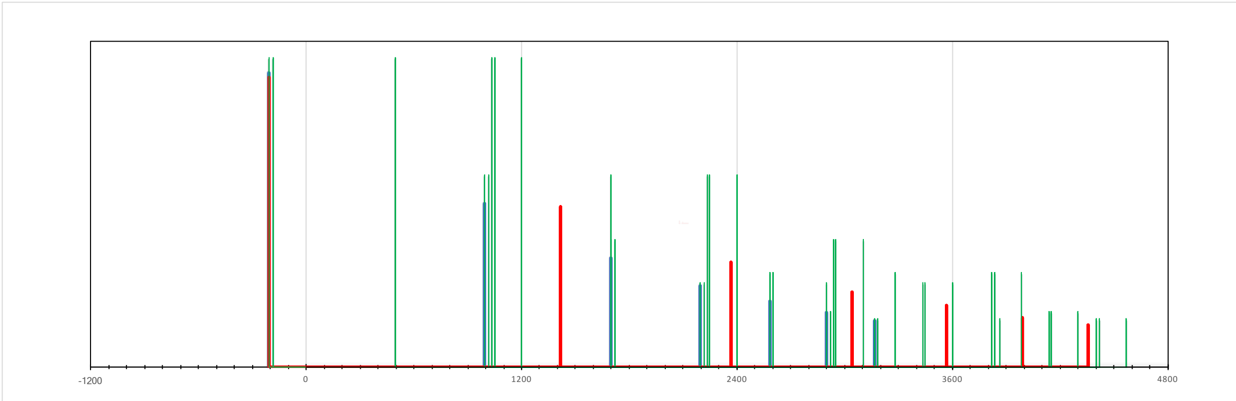
[187] The composer would like to acknowledge the generous assistance of our colleague, Michele Zaccagnini, in implementing this matrix using Jitter.

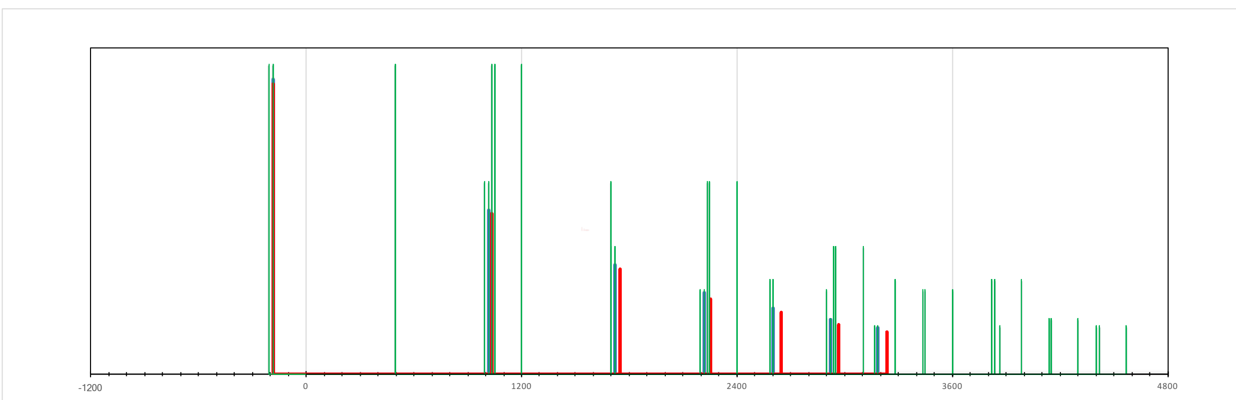
to the unity frequency ($F_{1/1}$ = 55 Hertz) prior to quantization. Finally, NASH values appear to the right of the un-quantized, (in)harmonic equivalents for a given state.

Above each table, a visual projection illustrates the position of stretched partials (shown as red vertical lines) in relation to the first seven harmonics of an overtone series ($F_{RBI(1-6)} \cdot n$) and reservoir of 42 harmonic tones for actuation (*NASH*). These latter values appear in blue and green, respectively. Each chart retains a five-octave range, spanning -1200 cents to 4800 cents above unity ($F_{1/1}$). Here, the '0' cents mark is equivalent to $F_{1/1}$ (55 Hertz). In this context, a logarithmic representation of pitch supports visual clarity. The column labeled *'Time'* references the track number (*Artemisia_track_1.wav, Artemisia_track_2.wav, Artemisia_track_3.wav*) and starting time (minutes: seconds) for the audio file associated with each state.

In addition to conveying spectral features, other annotations denote recurrence of sections. As discussed, new sections generally occur after six succeeding states of maximal spectral similarity. Given a new fundamental string ratio ($F_{RBI(1-6)}$), each state within a section retains a stretch coefficient (x) which generates the greatest number of common harmonic frequencies (NASH) in relation to the preceding state. However, with the onset of a new section, we introduce a sonority exhibiting the fewest common partials for a given fundamental string ratio. According to the chosen convention, we denote this occurrence with the abbreviation **diff.* appearing below the current state number. Henceforth, we shall reference the aforementioned visual conventions in analysis of spectra and intonation throughout the score (see APPENDIX A), as well as how these properties mutually inform actuation in design, composition, performance, and production of *Artemisia*.



State #	Time	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
41	Track 2 2:24	$\frac{8}{9} \cdot F_{1/1}$	1.353	1	0.889	$8/9\ \cdot 1$
				2	2.271	$2/1\ \cdot 1$
				3	3.930	$4/3\ \cdot 3$
				4	5.800	$2/1\ \cdot 3$
				5	7.844	$4/3\ \cdot 6$
				6	10.039	$2/1\ \cdot 5$
				7	12.367	$20/11\ \cdot 7$



State #	Time	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
42 *diff.	Track 2 2:46	$\frac{9}{10} \cdot F_{1/1}$	1.015	1	0.900	$9/10\ \cdot 1$
				2	1.819	$20/11\ \cdot 1$
				3	2.745	$9/10\ \cdot 3$
				4	3.676	$11/6\ \cdot 2$
				5	4.610	$9/10\ \cdot 5$
				6	5.547	$11/6\ \cdot 3$
				7	6.487	$2/1\ \cdot 3$

$$f_{st}\ /\ F_{1/1} \quad =$$

$$F_{RBI(1-6)} \cdot n \quad =$$

$$NASH \quad =$$

Figure 4. 4—Transition Between States of Maximal Similarity (#36-41) to Maximal Difference (#42)

V. CATEGORIES OF ACTUATED STRING INSTRUMENT PERFORMANCE PRACTICE

Expanding upon Overholt, Berdahl, and Hamilton's three, source-based modalities, we define a total of four categories for actuated instrument performance practice: "computer-mediated" electronic signals, "self-sustaining oscillation" (or recursive actuation), "third-party" audio streams from other instruments, and disruptive preparation of an actuated body.

5.1 "Computer (Electronically)-Mediated" Signals

Berhdahl and others suggest that computer-mediated sources for actuation offer the performer an enhanced sense of agency over parametric control and acoustic outcomes. Thusly, computer-mediated actuation augments both the structure of extant instrumentation and experience of instrumentalist alike, "granting them access to properties and functionalities normally associated with computer systems."[188] Essentially, actuated instruments afford the performer access to sonorities and control structures more typically associated with electro-acoustic or digital instrumentation. In the case of certain actuated string instruments, such as the Feedback Resonance Guitar, researchers describe the interaction between performable attributes intrinsic to the physical properties of the extant instrument and sources of actuation as a performable parameter. For example, while electronically-generated signals may induce physical vibration within the strings of a guitar at a given frequency, the performer—by altering the length of the physical string (e.g. changing fret position)—determines whether or not the string may be sympathetically activated by said frequency.[189]

Similarly, Nicolas Collin's first piece for the electro-magnetically actuated Backwards Guitar, *Killed in a Bar When He Was Only Three* (1982), employs pre-recorded actuation sources in the form of arbitrarily selected radio signals and the percussive sounds produced by six toy

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[188] Dan Overholt, Edgar Berdahl, and Robert Hamilton, "Advancements in Actuated Instruments," *Organised Sound.* Vol. 16, Issue 2 (2011): 154-165. https://www.cambridge.org/core/journals/organised-sound/article/advancements-in-actuated-musical-instruments/AFBD83D9E53F8C0270492F06CD0F2380 (accessed August 5, 2019).

[189] Ibid.

"panda bears."[190] While the incidental performances captured via AM/FM transmission—as well as the "performance" of the six ursine automatons—might also suggest elements of "third-party" audio streams, the guitarist's mode of selecting and modifying these input sources places the piece within the broad category of "computer-" or "electronically-mediated" performance practice. Here, the performer institutes both continuous and matrix-based agencies over actuated, electronic signals. By design, Collins enables these methods of control by installing binary switching mechanisms on the pickguard, as well as access to the tuning dial for a short-wave radio. The composer assesses similar matrix-based source selection in piece, *A Letter from My Uncle* (1984) for actuated bass guitar—this time actuating signals from "radios and prepared tape machines," as well as microphones.[191] [192] Later pieces by Collins, such as *Lightning Strikes Not Once but Twice* (1993), also employ electronically-generated source signals, including analog oscillators performing continuous glissandi.[193]

Comparable methods of computer-mediated actuation employing physical models represent a merging of virtual and physically-tangible components. Notably, long string instruments developed by John Bowers and Alex Sanders retain a similar "dynamic coupling" between virtual models of plucked strings and their corporeal "counterpart(s)"—in this case, a monochord driven by various electro-magnetic and electro-mechanical actuators. Simon Waters and other researchers working at the University of East Anglia describe this mode of electronically-mediation within a broader class of "Virtual/Physical Feedback Instruments (VPFI's)," in which "a physical instrument excites its virtual counterpart which in turn drives the physical instrument,

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[190] Nicolas Collins and Ron Kuivila, *Going Out with Slow Smoke*, Lovely Music LP, 1982.

[191] Nicolas Collins, "A Brief History of the 'Backwards Electric Guitar' (2009)," https://www.nicolascollins.com/texts/BackwardsElectricGuitar.pdf (Accessed August 9, 2019).

[192] Nicolas Collins, *Let the State Make the Selection*, Lovely Music LP, 1984.

[193] Nicolas Collins, *Sound Without Picture*, Periplum CD, 1999.

and so on."[194] This common thread of mutual excitation between virtual and physical models appears consistently throughout the design and performance practices of actuated instrumentation.

Mutual excitation—or "coupling"—of virtual and physical elements also plays an important role in design and performance modalities of Troy Rogers' Monochord-Aerophone Robotic Instrument Ensemble (MARIE). Performing as a single ensemble, Rogers' device pairs one or more electronically-mediated aerophone instruments (Cylindrical Aerophone Robotic Instruments, or CARI's) with a single or multiple Automatic Monochord Instruments (AMI's). Each element may perform independently or as a "tunable, acoustic filter" for the transduced output of its counterpart. Rogers, Kemper, and Barton describe the latter scenario within the virtual framework of "physical modeling technologies." As with purely virtual models, Roger's physically-embodied design affords the creation of similar hybrid timbres, including "plucked air column(s)" and "blown string" sounds.[195] In the manner in which this device couples two ostensibly independent "instrumental" components (AMI and CARI), one could argue that MARIE retains performative modalities indicative of third-party actuation. However, Rogers, Kemper, and Barton make explicit the surrogate role that the AMI and CARI play as corporeal stand-ins for virtual wave-guides, most commonly employed in digital physical-modeling techniques. The researchers further illustrate the affordances of this virtual-corporeal duality, stating that "each AMI-CARI pair can also operate as an interconnected hybrid instrument, allowing for effects that have heretofore been the domain of physical modeling technologies."[196]

While Rogers' model retains many definitional features of actuated instrumentation, electro-mechanical components of MARIE also demonstrate simulative properties indicative of

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[194] Simon Waters, "Performance Ecosystems: Ecological Approaches to Musical Interaction," *Proceedings of the Electroacoustic Music Studies Network*, Leicester: De Montfort: 1-20. http://www.ems-network.org/IMG/pdf_WatersEMS07.pdf. (accessed January 1, 2020).

[195] Troy Rogers, Steven Kemper, and Scott Barton. "MARIE: Monochord-Aerophone Robotic Instrument Ensemble," *Proceedings of the 2015 Conference on New Interfaces for Musical Expression (NIME 2015)*, https://www.researchgate.net/publication/280742870_MARIE_Monochord-Aerophone_Robotic_Instrument_Ensemble (accessed March 25, 2019).

[196] Ibid.

musical robotics. For example, the aerophone component utilizes changes in air pressure analogous to human breath to generate acoustic response. Likewise, in a manner analogous to human fingers, both the AMI and CRI employ solenoid-driven stops and dampeners to selectively mute and change the fret or key position of a vibrating string or valve, respectively. However, activation of the monochord's string involves other non-simulative methods, including electro-magnetic actuation. Thus, MARIE demonstrates a truly novel means of inducing acoustic response, indicative of actuated instrumentation.

Speaking in general terms, Rogers, Kemper, and Barton describe both the AMI and CARI as containing "a resonant acoustic element that can function as a filter and a control system with automated electro-mechanical actuators that excite, tune and dampen this acoustic element."[197] Coupling the transduced output of one resonant element or sounding body with the actuation source for the other determines the modes of excitation for each acoustic element. Accordingly, both the AMI and CARI retain three primary elements: some form of actuator, an acoustically resonant body—a vibrating string and air-column, respectively, and a means of transducing signal output from the resonant body. Each AMI employs an electro-magnetic actuator, electro-mechanical "picking mechanism," and a piezo-electric transducer to pick up and amplify resulting vibrations from the string. Modeled on a clarinet, the CARI's resonant body consists of a cylindrical air column with nineteen equally spaced key-holes. Here, a compression driver functions as actuator, changing the air pressure within the column according to fluctuations in amplitude from a selected audio source. An array of microphones transduces the resulting audio output. As with other examples of actuated instruments, prominent frequency components common to both the actuation source and the harmonic spectrum of the actuated body tend to produce strong resonances. A programmable control structure within Max/MSP allows the performer to selectively route signals to and from the AMI and CARI modules, thereby producing

[197] Troy Rogers, Steven Kemper, and Scott Barton. "MARIE: Monochord-Aerophone Robotic Instrument Ensemble," *Proceedings of the 2015 Conference on New Interfaces for Musical Expression (NIME 2015)*, https://www.researchgate.net/publication/280742870_MARIE_Monochord-Aerophone_Robotic_Instrument_Ensemble (accessed March 25, 2019).

complex hybrid timbres. Thus, MARIE embodies virtual qualities of "computer (electronically)-mediated" actuation in both sound-generation and control structures.

5.1.1 Computer-Mediated Actuation in *Artemisia*

Computer-mediated modes of actuation underlie the structure and performance of *Artemisia*. Even when other performance modalities, such as recursive or third-party audio streams, appear most prominent in the score; computer-mediated signals either initiate actuation or provide a secondary harmonic texture. Following the lead of Bowers and Sanders, these actuation methods embody a similar "dynamic coupling" between virtual models of plucked strings and their actuated equivalents.[198 199] As discussed, similar iterations of this virtual-physical coupling occur in Rogers, Barton, and Kemper's design for MARIE. However, whereas Rogers, Kemper, and Barton's pairing of aerophone (CARI) and monochord (AMI) components manifest a variety of hybrid timbres, embodying traits of each physical model, our implementation derives exclusively from physical models of strings.

Here, we employ a novel extension of Karplus-Strong synthesis as our physical model *and* actuation source. Initially conceived in 1978 by Alex Strong, this algorithm provides a computationally-efficient means for synthesizing plucked string timbres. The realism for this physical model resides in the ability to recreate variable decay rates for different harmonics—a trait indicative of the natural envelope for plucked strings.[200] In turn, its efficiency derives from the ability to emulate these spectral properties using minimal components, primarily: a high-order filter (e.g. sample delay), signal modifier (usually another filter or attenuator), and an initial

[198] Simon Waters, "Performance Ecosystems: Ecological Approaches to Musical Interaction," *Proceedings of the Electroacoustic Music Studies Network*, Leicester: De Montfort: 1-20. http://www.ems-network.org/IMG/pdf_WatersEMS07.pdf. (accessed January 1, 2020).

[199] John Bowers and Sten Olof Hellström, "Simple Interfaces to Complex Sound in Improvised Music," *Extended Abstracts on Human Factors in Computing Systems*. April, 2000: 125-126. https://dl.acm.org/doi/10.1145/633292.633364 (accessed December 4, 2020).

[200] Kevin Karplus and Alex Strong, "Digital Synthesis of Plucked-String and Drum Timbres," *Computer Music Journal* 7, no. 2 (1983): 43-55. http://www.jstor.org/stable/3680062 (accessed Apr. 9, 2017).

impulse. The latter may consist of a short, aperiodic signal (e.g. "noise burst") or another transient source. Arranged in a recursive configuration, the total "round-trip" time (in samples) determines the fundamental frequency of the sounding spectrum, while phase-delay properties of the filter influence the frequency and rate of attenuation for individual partials. [201]

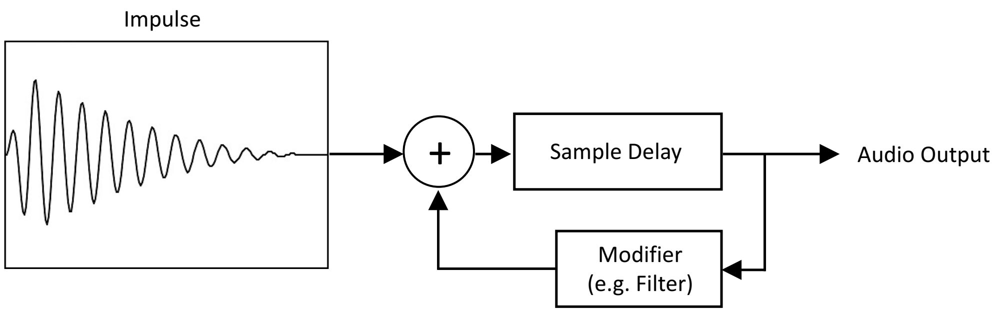

Figure 5. 1—Basic Karplus-Strong Algorithm

Extending this model involves modifying two of the three components for the basic algorithm: the modifier and impulse source. Our design retains features of physical models employed in other extant virtual-physical instruments. In this regard, we engage a very similar synthesis model to that of the Harmonic Wand—a gestural instrument developed previously by the author.[202] Here, modifier type and implementation inform various properties of pitch and spectra for synthesized sonorities. In determining pitch, the fundamental frequency of the synthesized spectra (F_R) remains a function of the time it takes for audio to move through the recursive structure. This value can be controlled by setting the length of the sample-delay. Importantly, if the delay-line is restricted to an integer number of samples, then control of the pitch is limited to quantized derivatives of these integer values. Without addressing this discrepancy,

[201] David A. Jaffe and Julius O. Smith, "Extensions of the Karplus-Strong Plucked-String Algorithm," *Computer Music Journal* 7, no. 2 (1983): 56-69.

[202] Ben Luca Robertson and Luke Dahl, "Harmonic Wand: An Instrument for Microtonal Control and Gestural Excitation," *Proceedings of the 2018 Conference on New Interfaces for Musical Expression (NIME 2018)*. http://www.nime.org/proceedings/2018/nime2018_paper0017.pdf (accessed August 6, 2018).

sample quantization results in inaccurate tuning of the fundamental frequency. As to mitigate this issue and enable more precise control of F_R, we employ the delread4~ object in Pure Data (or equivalent tapout~ object in Max/MSP). This object implements fractional delay using a four-point FIR interpolation.[203]

(5 - 1)

$$Delay\ Time = F_s/F_R$$
$$where\ F_s = Sampling\ Rate$$

In regards to overall amplitude characteristics and decay times, modifiers within the algorithm bear significant influence. Together with the fundamental frequency value, the inclusion of a gain modifier (e.g. amplifier or attenuator) affords precise control over the harmonic decay of our physically-modelled string. Given the algorithm's recursive nature, we refer to amount of attenuation applied by our modifier as 'Feedback Gain'. Speaking to the temporal component of our envelope, the 'Harmonic Decay Time' (HDT) value—expressed here in milliseconds—defines the time in which energy from an initial impulse is attenuated by 60 dB (≈ 0.001 of the original amplitude). For any fundamental frequency (F_R), the feedback gain needed to achieve a specific HDT can be expressed according to the following function:

(5 - 2)

$$Feedback\ Gain = 0.001^{(1000/(HDT \cdot F_R))}$$

$$where$$
$$F_R = Fundamental\ Frequency$$
$$HDT = Harmonic\ Decay\ Time$$

Achieving maximal pitch accuracy for our fundamental frequency and constituent harmonics presents a unique set of challenges and inherent compromises. As discussed, the efficacy of Karplus-Strong synthesis resides in the ability to recreate variable decay rates for different harmonics. Moreover, allowing the performer to dynamically control the perceived *darkness* or *brightness* for our virtual string denotes a key facet of expressivity within this actuated

203 Charles R. Sullivan, "Extending the Karplus-Strong Algorithm to Synthesize Electric Guitar Timbres with Distortion and Feedback," *Computer Music Journal* 14, no. 3 (1990): 26-37. http://www.jstor.org/stable/3679957 (accessed Apr. 9, 2017).

performance modality. To this end, we include a low-pass filter as a second modifier within the recursive algorithm, thereby reducing the energy of high-order harmonic frequencies each time audio circulates through the loop.[204] While necessary for modeling the rapid decay of upper partials, placement of a low-pass filter introduces additional latency, or *phase delay*. Consequently, the overall delay-time is lengthened, resulting in de-tuning of both fundamental and harmonic frequencies.

As with previous instruments, including the Harmonic Wand, a primary objective for this research involved implementing a low-pass filter whose properties allow for the mitigation of artifacts resulting from cumulative phase delay. Concurrently, the model must also afford expressive control over spectral parameters. Minimizing these artifacts necessitates the accurate calculation of phase delay across a broad range of frequency and filter coefficient values, as well as the ability to compensate for these variances within the algorithm itself. In choosing a low-pass filter, we posited a variety of factors, including variance in phase delay across frequencies and amplitude response. However, desirable properties, such as consistent phase delay and expressive control over spectral content, do not necessarily function in concert.

With pitch accuracy in mind, a Finite Impulse Response (FIR) filter—similar to the four-point interpolation used in the delread4~ object—appears the most fitting solution. As this type of filter exhibits a linear phase response, compensation for phase delay across all frequencies remains consistent. In this scenario, a one-sample reduction in the length of each delay line is sufficient in accurately reproducing any given target frequency below Nyquist. Use of this filter-type in signal processing applications is quite ubiquitous. For example, Charles Sullivan proposes a variation on this filter for creating physical models of electric guitars.[205] His design consists of a 3-tap FIR with a symmetrical arrangement of coefficient values, wherein the first and third

[204] Lindroos, Niklas, Henri Penttinen, and Vesa Välimäki, "Parametric Electric Guitar Synthesis," *Computer Music Journal* 35, no. 3 (2011): 18-27. http://www.jstor.org/stable/41241762 (accessed Apr. 9, 2017).

[205] Sullivan, Charles R. "Extending the Karplus-Strong Algorithm to Synthesize Electric Guitar Timbres with Distortion and Feedback," *Computer Music Journal* 14, no. 3 (1990): 26-37. http://www.jstor.org/stable/3679957 (accessed Apr. 9, 2017).

coefficient values of the difference equation remain equal. Sullivan's filter exhibits a linear phase response and a group delay of one-sample across all frequencies, allowing us to reduce the delay length by one sample and maintain accurate intonation across all frequencies.

(5 - 3)

3-tap FIR Filter (Difference Equation)

$$y_n = a_0 x_{[n]} + a_1 x_{[n-1]} + a_0 x_{[n-2]}$$

$$Where\ a_1 \geq 2a_0 \geq 0$$

Transfer Function

$$H[z] = a_0 + a_1 Z^{-1} + a_0 Z^{-2}$$

Unfortunately, while the linear phase properties of this FIR filter afford accurate intonation, attenuation (roll-off) of upper partials is rather subtle—even at low coefficient values. Thus, the overall range and variation in spectra remains relatively static, privileging bright timbres and precluding the kind of expressivity desired for this mode of actuated performance. These limitations become particularly apparent at lower fundamental frequencies (< 200 Hz). It is certainly feasible that exploration of other configurations of multi-tap FIR filters may yield a more pronounced roll-off of upper partials and, thus, greater expressivity. That said, Sullivan's implementation is not necessarily appropriate for our purposes. In order to achieve additional dynamic control over high frequency partials, we chose to employ a DC-normalized, one-pole IIR filter with a roll-off of -6 dB per octave. With some modification to the algorithm, this filter can be controlled quite intuitively using a -3 dB cutoff. To achieve a desired cutoff frequency (f_c), the feedback coefficient (a_1) can be calculated from the amplitude response [$G(f_c)$] of the filter.[206]

(5 - 4)

$$G(f_c) = \frac{1 - |a_1|}{\sqrt{1 + \ a_1{}^2 + 2 \cdot a_1 \cdot \ \cos\left(2\pi\, f_c / F_s\right)}}$$

[206] Julius O. Smith, "Introduction to Digital Filters," CCRMA: Stanford University, https://ccrma.stanford.edu/~jos/fp/One_Pole.html (accessed May. 2, 2017).

Unlike the symmetric FIR filter proposed by Sullivan, the phase response of the one-pole filter is not linear, and thus the phase delay varies across frequency. To program this filter, we utilized the rpole~ object (Pure Data), a one-pole low pass IIR filter which accepts an audio-rate signal to control the coefficient value [a_1].[207] Consistent with other IIR filters, rpole~ exhibits a distinctly non-linear phase response. When embedded within the Karplus-Strong algorithm, this variability in phase delay produces spectra whose partials exhibit varying degrees of inharmonicity. This inharmonicity becomes particularly evident with increased coefficient values. Drawing further correlations within our physical model, it is worth noting that strings bearing significant mass and rigidity tend to exhibit natural inharmonicity.[208] This behavior has been extensively researched, and parametric control of inharmonicity has been successfully integrated into physical models of electric guitars developed by Niklas Lindroos, Henri Penttinen, Vesa Välimäki, and others.[209] [210]

So as to maintain precise control of the fundamental frequency (F_R), we must compensate for varying amounts of phase delay introduced by a one-pole IIR filter. We can account for this difference by calculating the phase delay for the given frequency and then subtracting this amount

[207] When implementing the program in Max/MSP, we utilize the onepole~ object. In nearly all regards, this object appears functionally identical to rpole~. However, onepole~ allows the user to control filter coefficients via a variable cutoff frequency value, expressed in Hertz.

[208] Jonathan A. Kemp, "The Physics of Unwound and Wound Strings on the Electric Guitar Applied to the Pitch Intervals Produced by Tremolo / Vibrato Arm Systems," PLOS ONE, vol. 12, no. 9, 2017, pp. 1 - 25. https://search-proquest-com.proxy01.its.virginia.edu/docview/1941348198/fulltextPDF/1A6196E3AD5F47D4PQ/1?accountid=14678 (accessed March 27, 2020).

[209] Niklas Lindroos, Henri Penttinen, and Vesa Välimäki, "Parametric Electric Guitar Synthesis," *Computer Music Journal* 35, no. 3 (2011): 18-27. http://www.jstor.org/stable/41241762 (accessed Apr. 9, 2017).

[210] Matti Karjalainen, Vesa Välimäki, and Tero Tolonen. "Plucked-String Models: From the Karplus-Strong Algorithm to Digital Waveguides and beyond." *Computer Music Journal* 22, no. 3 (1998).

from the overall delay length.[211] As both Pure Data and Max/MSP process delay times in milliseconds, additional calculations derive compensation values using three variable parameters: filter coefficient value (a_1), frequency (F_R), and sampling rate (F_s). This value is then subtracted from the overall delay-time (F_s/F_R) to yield an accurate, sounding pitch.

(5 - 5)

$$Phase\ Delay\ at\ F_R = \frac{1}{\omega} \tan^{-1} \left[\frac{a_1 \sin(\omega)}{1 - a_1 \cos(\omega)} \right]$$

$$where\ \omega = 2\pi \left[\frac{F_R}{F_s} \right]$$

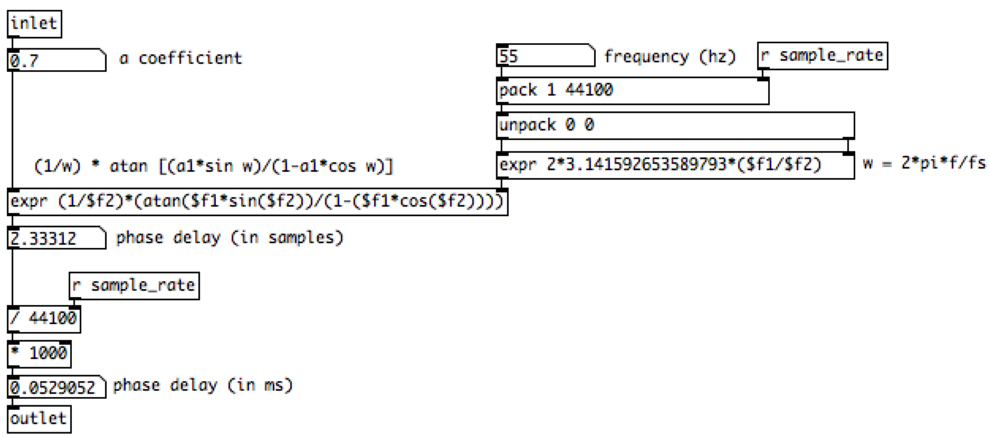

Figure 5. 2—Phase-delay Compensation in Pure Data

5.1.2 Actuated Impulse Topologies

Having addressed the functionality and procedural extensions of different filters and other modifiers, we shift our discussion to the role of impulse signals as another extensible component within our physical model. Beginning with the earliest iterations, Kevin Karplus and Alex Strong propose a short impulse, composed of a wavetable filled with random signal values. As noted by the researchers, the aperiodic—or *noisy*—quality of this initial transient fulfills two functions which contribute to the realism of the physical model. First, the impulse produces numerous high-

[211] Julius O. Smith, "Introduction to Digital Filters," CCRMA: Stanford University, https://ccrma.stanford.edu/~jos/fp/One_Pole.html (accessed May. 2, 2017).

order partials, indicative of the initial spectral envelope for plucked strings. Secondly, the randomized nature of the wavetable ensures that each instantiation will generate a unique balance of harmonics, varying slightly from the last.[212] In the same manner, no two notes performed with a stringed instrument are identical.

In consideration of the ease in implementation and realism afforded by aperiodic wavetables, it comes as no surprise that this type of impulse has come to represent a generalized topology for Karplus-Strong (KS) synthesis—particularly when synthesizing plucked string timbres. However, subsequent observations by Karplus and Strong provide basis for further extensions. Notably, previous research indicates that decay rates vary in approximate proportion to the round-trip delay-time in samples (F_s/F_R). That is to say, with all other factors equal, shorter delay times used to synthesize higher pitches yield proportionally shortened decay times. When implementing a basic KS algorithm, harmonic decay time (HDT in samples) for the fundamental frequency (F_R) is roughly equivalent to $(F_s/F_R)^3$, with each harmonic (n) decaying at a rate of $(F_s/F_R)^3/n^2$.[213] Accordingly, a fundamental frequency of 880 Hertz (F_R) will retain a harmonic decay time (HDT) of approximately 125,854 samples, or 2.853 seconds at a sampling rate (F_s) of 44,100 Hertz.

(5 - 6)

$$HDT_{(F_R \cdot n)} = (F_s/F_R)^3/n^2$$

$$whereas:$$
$$F_s = 44{,}100\ Hz$$
$$F_r = 880\ Hz$$
$$n = 1$$

$$HDT = (44{,}100/880)^3/(1)^2 = 125{,}854.211\ samples\ \ or\ 2.853\ seconds$$

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[212] Kevin Karplus and Alex Strong, "Digital Synthesis of Plucked-String and Drum Timbres," *Computer Music Journal* 7, no. 2 (1983): 43-55. http://www.jstor.org/stable/3680062 (accessed Apr. 9, 2017).

[213] The basic KS algorithm referenced for these calculations utilizes a simple averaging filter proposed by Kevin Karplus and Alex Strong. Actual decay times vary according to filter type and implementation.

To compensate for these differences in decay time, Karplus and Strong propose a method they term the "harmonic trick."[214] By both lengthening the total, round-trip delay time (F_s/F_R) and populating this recursive structure with several identical waveforms—each with a wavelength equal to $(F_s/F_R)/n$, one may increase the harmonic decay time (HDT) for the frequency ($F_R \cdot n$). To increase round-trip delay time, we simply transpose F_R by the sub-harmonic factor: $(1/n)$. Working from our previous example, we can apply similar procedures to lengthen the HDT for the frequency 880 Hertz. First, we transpose F_R down one octave ($n = 2$), yielding a new F_R value of 440 Hertz and lengthening our round-trip delay-time. Next, we populate the recursive structure with two identical waveforms, each with a wavelength of $(F_s/F_R)/(2)$. Treating 880 Hertz as our second harmonic ($F_R \cdot 2$), we derive a larger HDT value of approximately 251,708 samples—or 5.707 seconds.

(5 - 7)

$$HDT_{(F_R \cdot n)} = (F_s/F_R)^3/n^2$$

$$whereas:$$
$$F_s = 44{,}100\ Hz$$
$$F_r = 440\ Hz$$
$$n = 2$$

$$HDT = (44{,}100/440)^3/(2)^2 = 251{,}708.422\ samples\ \ or\ 5.707\ seconds$$

Besides extending decay time, Karplus and Strong's "harmonic trick" implies an extensible framework for further modifications to our physical model. As discussed, populating the delay-line with identical copies of a given waveform allows the user to extend the duration for individual harmonic components. Building upon this model, we populate the delay-line using a repeating sequence of 20-100 millisecond impulses—each with its own attack and decay envelope. Whereas previous KS models generally employ randomized impulse signals, ours consist of discrete sinusoids whose frequencies correspond to integral multiples (e.g. harmonics) of the virtual string's fundamental (F_r). Here, KS synthesis functions as a digital waveguide, resonating and

[214] Kevin Karplus and Alex Strong, "Digital Synthesis of Plucked-String and Drum Timbres," *Computer Music Journal* 7, no. 2 (1983): 43-55. http://www.jstor.org/stable/3680062 (accessed Apr. 9, 2017).

sustaining specific harmonic frequencies. By tuning F_r to the fundamental frequency of one or more actuated strings, we—as Bowers and Sanders suggest, "dynamically couple" the spectra of our virtual string with its corporeal counterpart.[215] As the primary technique for computer-mediated performance in *Artemisia*, this method affords the performer the ability to select different harmonics for actuation within the same overtone series or across multiple strings. According to this topology, synthesized output from our virtual string induces sympathetic vibration(s) from one or more strings of the electro-magnetically actuated instrument, Rosebud I.

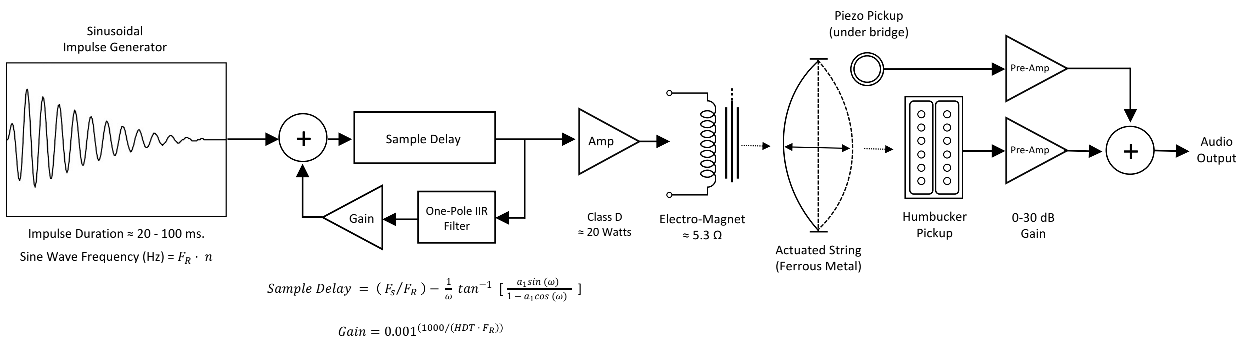

Figure 5. 3—Computer-Mediated Actuation Topology for Rosebud I: Modified Karplus-Strong Synthesis

5.2 "Self-sustaining Oscillation" (or Recursive Actuation)

In addition to the sense of "tangibility" achieved by re-embodying virtual qualities within the sonic structure of vibrating strings and other resonant objects, Overholt, Berdahl, and Hamilton suggest that electronically-mediated actuation techniques afford certain performative possibilities, distinct from other modes of actuated performance. Such affordances are of particular relevance to stringed instruments, which—in the absence of electronic augmentation or processing—require intensive physical effort and attention to generate sustained sonorities. Freed from the necessity of continually activating a string into vibration, "self-sustaining oscillation" can free the performer

[215] Simon Waters, "Performance Ecosystems: Ecological Approaches to Musical Interaction," *Proceedings of the Electroacoustic Music Studies Network*. Leicester: De Montfort: 1-20, http://www.ems-network.org/IMG/pdf_WatersEMS07.pdf. (accessed January 1, 2020).

to explore various extended techniques and otherwise impractical modes of interaction.[216] For our purposes, self-sustaining oscillation is most commonly achieved using recursive topologies.

Generally speaking, recursive actuation involves some variant of feedback loop. Applied to stringed instruments, the fundamental components of this topology include: some means of actuation (either electro-magnetic or electro-mechanical), a resonant body (e.g. one or more strings), and output transducer(s) capable of converting acoustic vibrations from the vibrating string into an electrical signal. Depending upon the instrument's structure and the needs of the performance, the output transducer(s) may take the form of an electro-magnetic pickup (e.g. humbucker or single-coil guitar pickup), piezo-electric transducer (e.g. contact microphone.), air microphone (either dynamic or condenser), or any combination thereof. More often than not, recursive actuation also employs some form of amplification or attenuation circuit between the output transducer and actuator, and—in some cases—additional signal modifiers, such as filters or delay-lines. Invariably, the arrangement of these components follows a similar topology, with signal from the output transducer(s) coupled to the input of the actuator.

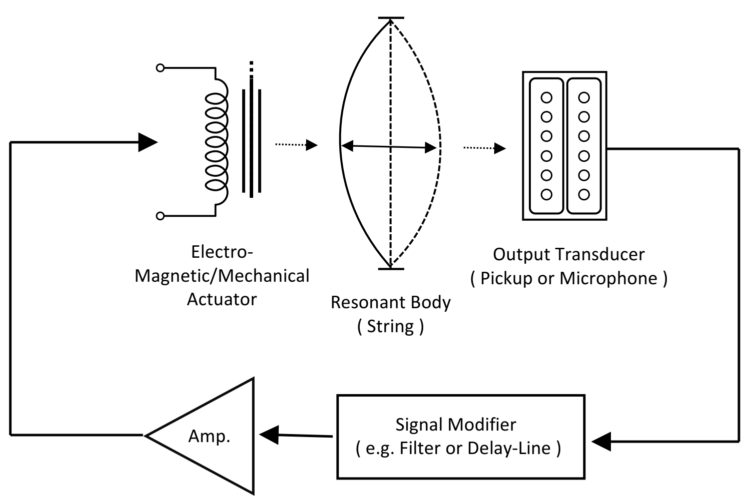

Figure 5. 4—Recursive Actuation Topology for Strings

[216] Dan Overholt, Edgar Berdahl, and Robert Hamilton, "Advancements in Actuated Instruments," *Organised Sound,* Vol. 16, Issue 2 (2011): 154-165. https://www.cambridge.org/core/journals/organised-sound/article/advancements-in-actuated-musical-instruments/AFBD83D9E53F8C0270492F06CD0F2380 (accessed August 5, 2019).

Owing to its relative simplicity in implementation and integral coupling of source and output materials, recursive actuation presents the performer, composer, and luthier alike with an attractive (and efficient) means of generating compelling effects—including near-infinite sustain. Consequently, its application has become ubiquitous in actuated instrument design. Moving forward, we shall examine a few examples from two sub-classes of recursive actuation typologies. We term these sub-classes, *Linear Recursive Actuation* (LRA) and *Non-Linear Recursive Actuation* (NLRA). Here, we reserve the term Linear Recursive Actuation for those instruments and devices whose actuated input remains synchronous with its transduced output. Therein, no perceivable time-delay occurs within the signal feedback loop or network. In contrast, NLRA demonstrates asynchronous capture and playback of actuated signals within a feedback network. Here, asynchronous capture and playback can be achieved through delay-lines or sample buffers. In either NLRA scenario, all actuated source materials derive from the instrument's transduced output—albeit displaced in time. Finally, we will examine performance typologies which employ both *Linear* and *Non-Linear Recursive Actuation* techniques.

5.2.1 Linear Recursive Actuation (LRA)

Early examples of Linear Recursive Actuation include David Behrman's composition *Wave Train* (1966). Here the performer resonates multiple piano strings using a feedback array between guitar pickups, positioned loosely underneath the strings, and loudspeakers underneath the piano.[217] [218] Contemporary to, if not preceding Behrman's work during the 1960s, intentional manipulation of acoustic feedback remains a mainstay in the performance practice of rock music. Resulting from extreme levels of amplification and proximity of strings to the amplification source, this mode of recursive actuation affords increased, if not continuous sustain. However, to achieve self-sustaining oscillations, factors of acoustic volume and proximity must remain coupled. As Overholt, Berdahl, and Hamilton note, in contrast to such "idiomatic" feedback

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[217] Nicolas Collins, "A Brief History of the 'Backwards Electric Guitar' (2009)," https://www.nicolascollins.com/texts/BackwardsElectricGuitar.pdf (Accessed August 9, 2019).

[218] David Behrman, *Wavetrain*, Alga Marghen CD, 1998.

effects which rely upon interactions between high-volume amplification, strings, and electro-magnetic pickups; other emergent systems of recursive actuation assume no correlation between output volume and sustain. The musician may thus treat performance volume and note duration as independent forces.[219] Over the last four decades multiple electro-magnetically actuated devices have been produced and marketed for use with electric guitars. By employing specialized recursive topologies, these devices often decouple factors of volume and proximity, affording infinite sustain—even at very low volumes. Such products include: EBow—Direct String Synthesis™ , Fernandes Sustainer™ , Moog Guitar™ , Sustainiac (Stealth)™ , and TC Electronic Aeon™ .[220] [221] [222]

Beyond the issue of de-coupling forces of volume and proximity, other aspects of design, composition, performance, and notational conventions can present definitional challenges in the development of integrated practices for actuated instrumentation. In performing self-sustained oscillation, the ability to prolong a sonority independently from instrument's natural decay has necessitated proprietary measures, both in regards to performance instructions and notation. For example, in scoring for the Magnetic Resonator Piano, composers found themselves tasked with the challenge of developing a notational convention distinguishing electromagnetically-actuated sustain from the natural mode of sustain produced by depressing the *sustain* pedal—thereby lifting dampers from the strings. In the end, both composers and researchers came to adopt the terms "organ sustain" and "piano sustain," respectively.[223]

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[219] Dan Overholt, Edgar Berdahl, and Robert Hamilton, "Advancements in Actuated Instruments," *Organised Sound,* Vol. 16, Issue 2 (2011): 154-165. https://www.cambridge.org/core/journals/organised-sound/article/advancements-in-actuated-musical-instruments/AFBD83D9E53F8C0270492F06CD0F2380 (accessed August 5, 2019).

[220] Gregory Heet,1978, String Instrument Vibration Initiator and Sustainer, US Patent 4,075,921.

[221] Floyd D. Rose, Steven M. Moore, and Richard W. Knotts, 1992, Musical Instrument Sustainers and Transducers, US Patent 5,123,324.

[222] Alan Hoover, 2000, Controls for Musical Instrument Sustainers, US Patent 6,034,316.

[223] Andrew McPherson and Youngmoo E. Kim, "The Problem of the Second Performer: Building a Community Around an Augmented Piano." *Computer Music Journal* 36, no. 4 (2012): 10-27. http://www.jstor.org/stable/41819545. (accessed July 16, 2019).

Similarly, self-sustaining oscillation may also be achieved using electro-mechanical actuation techniques. However, with the exception of a handful of products, very few electro-mechanical devices for stringed instruments have appeared on the market. Even fewer of these models employ explicitly recursive techniques; the most notable exception being the 'Sustain Man' "Electro-acoustic" sustainer by Sustainiac™ .[224] According to the manufacturer's copy:

> "The Sustainiac *Sustain Man* sustainer is an electroacoustic [sic] type sustainer. It makes your string vibrations sustain by making feedback. It is like getting natural amp feedback from a very large, loud amp. Only it is much, much more intense and predictable. […] The *Sustain Man* consists of two separate parts: The string-driver transducer [actuator] and the control box *Sustainiamp* controller/amplifier. The *Sustainiamp* takes your guitar signal, amplifies it, and then sends this amplified signal to the transducer. There is a "to amp" jack on the *Sustainiamp* that splits off the raw guitar signal and sends it directly to your regular guitar amp or effects chain. This signal is hard-wired, so you hear only your raw, unprocessed guitar signal. […] It [the transducer] mounts onto the instrument body, preferably the headstock. The transducer produces sound vibrations in that part of the instrument body. You can feel the instrument vibrate as you play. Some of this vibration energy coming from the transducer gets transferred to the strings, where they sustain their vibration for as long as you want them to. […] The pickup senses the string vibration, and sends this signal to the "Sustainiamp" amplifier. The Sustainiamp amplifies the pickup signal, and the sends this to the transducer."[225]

Here, the manufacturer describes components roughly equivalent to our basic, recursive actuation topology. The electro-mechanical actuator (or "string-driver transducer") affixes to the headstock, while the "Sustainiamp" accepts signal input from the guitar's pickup. In addition, the "Sustainiamp" controller houses amplification (1/2 watt to 1-Watt, in "TURBO mode") and signal modifier circuitry indicative of our established recursive topology. As described below, the guitar's frets act as the primary point of contact between the actuator and string(s). Similar to other applications utilizing acoustic feedback, the physical proximity of fretted notes influences both efficiency in inducing physical vibration *and* activation of particular harmonic modes. To

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[224] Alan Hoover, 2005, Electroacoustic Sustainer for Musical Instruments, US Patent 2,005,008,170,3A1.

[225] "Sustain Man Electro-Acoustic Sustainer," *Sustainiac*, https://www.sustainiac.com/s-man.htm#overview (accessed December 7, 2020).

address the resultant latency and associated phase-delay, additional signal-modifiers within the circuitry allows the performer to bypass or activate phase-correction (e.g. "AUTOMATIC mode") processing. As described by the manufacturer:

> "Sound travels from the transducer to the fret. The distance from the transducer to a fret varies from about six inches to about two feet, depending on what fret is being used for that note. This takes about 0.1 to maybe two milliseconds, depending on what fret is being used. During this time period, the phase of the note will change from about 10 degrees to a full 360 degrees, depending on the note frequency and also upon the distance between transducer and fret. Furthermore, additional phase shift occurs due to the way pickups work. If the transducer energy that reaches the fret happens to be in phase with the string vibration, the sustained note vibration will be a fundamental. But most likely, the energy will be out of phase with the fundamental mode string vibration. How much out of phase depends on all the variables just explained. The energy will usually reach the string more or less in phase with SOME harmonic mode (but not necessarily). That is the mode that will end up being the final vibration mode of the string at the particular fret being used. The harmonic mode that is the most in phase with the energy reaching the string has the most gain, and therefore will "win out". But you cannot say you will get "second harmonics" or "third harmonics" as some people want to generalize, as you can understand from the above explanation. In some cases, the energy will reach the fret almost precisely 180 degrees out of phase with the string vibration, which quickly stops the note. If you are in AUTOMATIC mode, the sustainer will automatically change phase in this case, which then causes the vibration to be in phase with the note."[226]

Implementation of signal modifiers within recursive topologies remains a consistent feature of many actuated string instruments, particularly when computer-mediated control structures are involved. For instance, Jiffer Harriman's Feedback Lap-Steel employs an electro-mechanical transducer and recursive topology similar to the Sustain Man. However, the point of contact between the tactile actuator and string(s) remains fixed at the instrument's stationary bridge. Modeled after existing lap-steel or slide guitars, the performer determines the vibrating length of each string and the resulting pitch. While presumably subject to similar variations in harmonic intensity, issues of latency and subsequent phase-delay primarily reflect the additional sample-delay introduced to the feedback loop via digital signal processing. When implementing a recursive performance topology, the performer routes the output of the instrument into either a laptop or a Raspberry Pi running the 'Satellite' CCRMA sound synthesis and processing

[226] "Sustain Man Electro-Acoustic Sustainer," *Sustainiac*, https://www.sustainiac.com/s-man.htm#overview (accessed December 7, 2020).

platform.[227] The processed output is then fed back into the electro-mechanical actuator after passing through one or more volume pedals and a 40-Watt Class-T amplifier. These digital signal modifiers serve two purposes. Like the Sustainiamp controller, programmable filters afford precise control over harmonic content and, as Harriman notes, attenuate "high order [...] harmonics" otherwise present in the unprocessed output.[228]

In addition to mitigating less-desirable artifacts, processing the actuated signal with filters, delay-lines, and pitch-shifting algorithms allows for a variety of novel effects. As Harriman observes, "Shifting down an octave down, the upper harmonics remain stable. By doing microtonal pitch shifts slightly off the original output from the instrument beating effects are achieved that create a dense yet controllable texture not available with a traditional instrument."[229] While the implementation of delay-lines, filters, and other means of either introducing or mitigating the effects of phase-delay certainly introduce latency within these recursive systems, the majority of the signal modifier discussed generally operate at relatively short delay times (< 50 milliseconds). Thus, each instrument's transduced output and actuated source signal remain perceptually synchronous, as representative examples of Linear Recursive Actuation (LRA).

5.2.2 Non-Linear Recursive Actuation (NLRA)

In contrast to Linear Recursive Actuation, Non-Linear Recursive Actuation (NLRA) methods demonstrate asynchronous capture and playback of actuated signals within a feedback network. Here, a discernable time-delay (> 50 milliseconds) occurs between the onset of actuated input and output signals. This latency usually results from the implementation of delay-lines,

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[227] Edgar Berdahl and W. Ju, "Satellite CCRMA: A Musical Interaction and Sound Synthesis Platform," *Proceedings of the International Conference on New Interfaces for Musical Expression*, 2011. https://www.semanticscholar.org/paper/Satellite-CCRMA%3A-A-Musical-Interaction-and-Sound-Berdahl-Ju/35f234146369d3ff179870e999bff51fd801d36d (Accessed December 7, 2020).

[228] Jiffer Harriman, "Feedback Lap Steel: Exploring Tactile Transducers as String Actuators," *Proceedings of the 2015 Conference on New Interfaces for Musical Expression (NIME 2015)*, 178-179. https://nime2015.lsu.edu/proceedings/152/0152-paper.pdf (Accessed June 6, 2019).

[229] Ibid.

sample buffers, or other time-based signal modifiers. Regardless of the means for temporal displacement, by definition, all actuated signals emanate exclusively from the resonant body or string. All recursions are essentially self-contained and wholly integral. This feature distinguishes NLRA methods from both computer-mediated and third-party audio streams, which employ external input sources from other instruments or electronic media. Of course, NLRA and LRA topologies may be used in consort. Going forward, we shall examine examples of instruments and creative works which combine multiple recursive topologies.

Often applications of NLRA arise as practical extensions of linear methods. For example, later iterations of Daniel Fishkin's actuated instrument, the Lady's Harp, and performances of his continuously-evolving piece, *The Tinnitus Suites (a, b, c, and d)*, demonstrate a progression from linear to non-linear recursion methods in design and performance. A long-string instrument in excess of twenty feet, Fishkin's Lady's Harp employs electro-mechanical actuators positioned under wooden bridges and placed near the terminating end of one or more lengths of piano wire. Moveable electro-magnetic pickups (either single coil or humbucker) capture vibrations along various harmonic nodes. Early iterations of *The Tinntus Suites* (2008-2015) embody explicitly linear actuation techniques, with output from each pickup fed through various gain stages, attenuation, and equalization before returning to the actuator.[230] In adapting the instrument for remote installations, Fishkin introduced computer-mediated control, similar in some respects to Harriman's implementation of signal-modifiers for the Feedback Lap Steel. However, as recalled by the composer, these automated processes lacked the expressivity afforded by hands-on control and the innate "sensitivity" of a performance-centered system.

> "I had been developing software to play the Lady's Harp automatically/remotely as an installation since 2013—using Max [Max/MSP] to control digital filters/gain cells within the feedback loop, to toggle through different memory states of set feedback chords. But the problem here was that I had to completely re-imagine the interface, where the analog mixer I use was perfectly sensitive, a marvelous analog to a piano keyboard."[231]

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[230] Daniel Fishkin, "Composing the Tinnitus Suites: 2015," http://dfiction.com/ctts-2015/ (accessed December 3, 2020).

[231] Ibid.

In reconciling the necessity of autonomous control with performative nuance, non-linear approaches to actuation—particularly in the application of recursive topologies—present a viable solution. For example, recording the transduced signal output from a recursively-actuated (LRA) string provides an actuation source that can be recalled at a later time or location—what we term, a *re-embodied performance*. So long as the tuning and gain structure of the system remain consistent, similar results may be achieved regardless of time and location. Here, the composer recalls a similar application of NLRA techniques:

> "I found a different solution in my friend, the legendary engineer Bob Bielecki, who told to me one evening a powerful observation. I retell it now. Bob said, take a resonant situation—any room or transduced object—and produce feedback in/through it, as our ancestors of electronic music have always done. Now, if you simultaneously record that feedback "direct to tape" from your mixer, computer, or recorder, you have recorded a powerful spell. This recording won't sound as interesting as the sound of feedback coming to life in the room. However, it can be used thusly: play the spell through the same resonant situation and you will stir a mirror image of the recorded feedback—the object/room is resonated at the same frequencies once more. […] This would not be a mere recording, but a re-performance, for the strings would actually move in resonance to the source recording." [232]

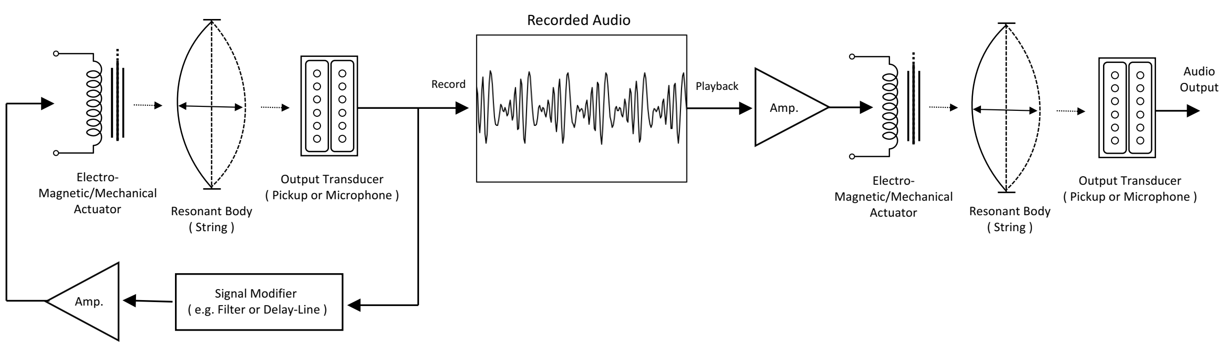

Figure 5. 5—'Re-Embodied Performance' using Non-Linear Recursive Actuation (NLRA)

[232] Daniel Fishkin, "Composing the Tinnitus Suites: 2015," http://dfiction.com/ctts-2015/ (accessed December 3, 2020).

5.2.3 Concurrent Instances of Linear and Non-Linear Recursion in *Artemisia*

As in the *Tinnitus Suites*, both linear and non-linear approaches to inducing self-sustaining oscillation play an essential role in the composition, performance, and recording of *Artemisia*. Notably, in states 18-23, the emergence of a unified set of sustained partials defines the structure and sonic character for this section of the piece. We exercise similar, recursive topologies to achieve sustained harmonic textures with our instruments. In design and performance, both Fishkin's Lady's Harp and Rosebud II employ electro-mechanical means for actuation. However, whereas Fishkin and Bielecki's conception of recorded "spell(s)" derive exclusively from the instrument's own output, in orchestrating *Artemisia*, we deploy external sources to initiate actuation. Here, pre-recorded tones pass through various gain stages, amplifiers, and a single electro-mechanical actuator to induce sympathetic resonance in one or more strings. During succeeding recursions, vibrations captured by the instrument's humbucker pickup replace the pre-recorded audio buffer. The length of the audio buffer spans the entire duration of states 18-23—approximately 1'52". In a decisively non-linear fashion, playback of the actuated signal ensues at a later time, eliciting additional resonance from the strings. Concurrently, direct output from the Rosebud II blends with these previously-recorded signals to form a single source for subsequent actuation. The merging of past and present events within a single feedback network is indicative of an emergent, hybrid topology—one which embodies both linear *and* non-linear recursion.

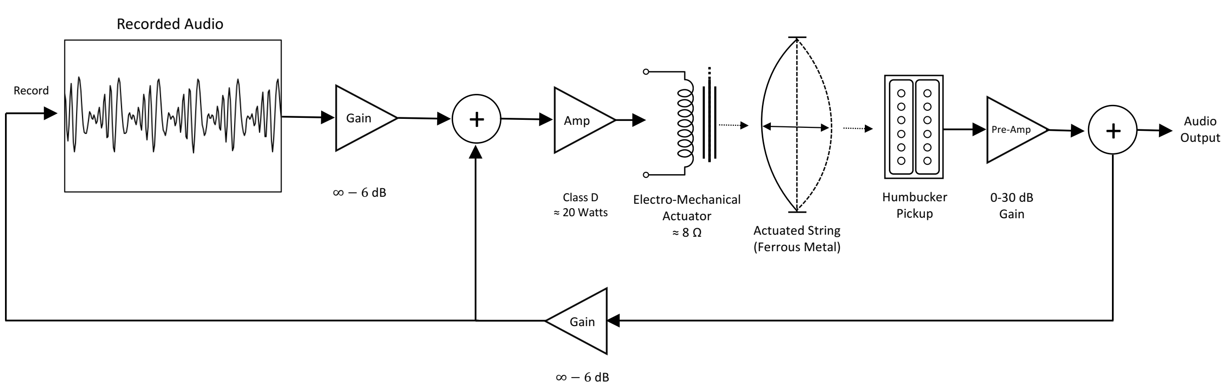

Figure 5. 6—Linear and Non-Linear Recursive Actuation Topologies (LRA/NLRA) for Rosebud II

Returning to the compositional function of self-sustaining oscillations, we witness the steady passage of a single set of integrally-related partials across states 18-23. Taken as components within a near-continuous sequence of harmonics, we may perceive the section as a singular sustained spectrum. Significantly, sustained partials for the section constitute the fourth, fifth, sixth, and eleventh harmonics of the unity ratio, $F_{1/1}$ (55 Hertz). Here, self-oscillation enables continuous sounding of common partials across multiple states. For example, beginning in state eighteen, actuation of the fundamental of string III $\left(\frac{11}{2} \cdot F_{1/1}\right)$ of Rosebud II extends through state number twenty (18-20). Notice the frequency equivalence between this string's fundamental and the third harmonic of string II $\left(\frac{11}{6} \cdot 3\right)$ for Rosebud I. Concurrent actuation in such instances indicates a transpositional relationship between tunings for the two instruments, representing an interval of one octave and a perfect fifth $\left(\frac{6}{1}\right)$. Starting in the same state, the second harmonic of string XII $\left(\frac{4}{1} \cdot 2\right)$ sustains throughout the entire the section (18-23), followed by the second partials of strings IX $\left(\frac{5}{1} \cdot 2\right)$ and I $\left(\frac{6}{1} \cdot 1\right)$, respectively. Again, recursive actuation functions to highlight the voicing of common harmonics between states, emphasizing similar features within continuously evolving spectra.

(5 - 8)

State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*Linear & Non-Linear Recursive Actuation (LRA & NLRA)*
18 *diff.	Track 1 4:17	$\frac{9}{10} \cdot F_{1/1}$	1.1425	1	0.900	9/10 · 1	
				2	1.987	2/1 · 1	
				3	3.158	8/9 · 4	
				4	4.386	8/9 · 5	
				5	5.660	11/6 · 3	RBII → RBII (III) (11/2 · 1)
				6	6.971	20/11 · 4	
				7	8.313	4/3 · 6	RBII → RBII (XII) (4/1 · 2)

State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*Linear & Non-Linear Recursive Actuation (LRA & NLRA)*
19	Track 1 4:33	$\frac{20}{11} \cdot F_{1/1}$	1.0385	1	1.818	20/11 · 1	
				2	3.735	11/6 · 2	
				3	5.690	11/6 · 3	RBII → RBII (III) (11/2 · 1)
				4	7.671	4/3 · 6	RBII → RBII (XII) (4/1 · 2)
				5	9.672	2/1 · 5	RBII → RBII (IX) (5/1 · 2)
				6	11.688	2/1 · 6	RBII → RBII (I) (6/1 · 2)
				7	13.717	2/1 · 7	

State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*Linear & Non-Linear Recursive Actuation (LRA & NLRA)*
20	Track 1 4:50	$\frac{11}{6} \cdot F_{1/1}$	1.033	1	1.833	11/6 · 1	
				2	3.752	11/6 · 2	
				3	5.703	11/6 · 3	RBII → RBII (III) (11/2 · 1)
				4	7.677	4/3 · 3	RBII → RBII (XII) (4/1 · 1)
				5	9.667	2/1 · 5	RBII → RBII (IX) (5/1 · 2)
				6	11.670	2/1 · 6	RBII → RBII (I) (6/1 · 2)
				7	13.684	2/1 · 7	

21	Track 1 5:06	$\frac{8}{9} \cdot F_{1/1}$	1.339	1	0.889	$8/9 \cdot 1$	
				2	2.249	$2/1 \cdot 1$	
				3	3.870	$4/3 \cdot 3$	RBII → RBII (XII) ($4/1 \cdot 1$)
				4	5.689	$11/6 \cdot 3$	RBII → RBII (III) ($11/2 \cdot 1$)
				5	7.670	$4/3 \cdot 6$	RBII → RBII (XII) ($4/1 \cdot 2$)
				6	9.790	$2/1 \cdot 5$	RBII → RBII (IX) ($5/1 \cdot 2$)
				7	12.035	$2/1 \cdot 6$	RBII → RBII (I) ($6/1 \cdot 2$)

22	Track 1 5:22	$\frac{9}{10} \cdot F_{1/1}$	1.3315	1	0.900	$9/10 \cdot 1$	
				2	2.265	$2/1 \cdot 1$	
				3	3.886	$4/3 \cdot 3$	RBII → RBII (XII) ($4/1 \cdot 1$)
				4	5.700	$11/6 \cdot 3$	RBII → RBII (III) ($11/2 \cdot 1$)
				5	7.672	$4/3 \cdot 6$	RBII → RBII (XII) ($4/1 \cdot 2$)
				6	9.780	$2/1 \cdot 5$	RBII → RBII (IX) ($5/1 \cdot 2$)
				7	12.009	$2/1 \cdot 6$	RBII → RBII (I) ($6/1 \cdot 2$)

23	Track 1 5:38	$\frac{20}{11} \cdot F_{1/1}$	1.0765	1	1.818	$20/11 \cdot 1$	
				2	3.834	$4/3 \cdot 3$	RBII → RBII (XII) ($4/1 \cdot 1$)
				3	5.933	$2/1 \cdot 3$	RBII → RBII (I) ($6/1 \cdot 1$)
				4	8.086	$4/3 \cdot 6$	RBII → RBII (XII) ($4/1 \cdot 2$)
				5	10.282	$2/1 \cdot 5$	RBII → RBII (IX) ($5/1 \cdot 2$)
				6	12.512	$20/11 \cdot 7$	
				7	14.770	$2/1 \cdot 7$	

5.3 "Third-Party Audio Streams" from Other Instruments

Overholt, Berdahl, and Hamilton term "third-party audio streams" to denote the routing of audio signals from one or more instrumental performers—acoustic or electronic—to actuate another acoustic or electronically-amplified instrument. In performance, these researchers describe certain instruments capable of actuating third-party audio sources as contributing to an "augmented virtuality paradigm." That is to say, by actuating one instrument using the output of another, source signals embody the roles of both "elements in the virtual environment," as well as "avatars representing other real objects."[233] In essence, third-party audio streams signify an act of performative ventriloquism, with one physical voice speaking through the resonant, physical body of another.

Drawing upon similar principles, instances of instrumental sonorities re-embodied through the sounding bodies of other instruments occur in both historical and contemporary compositional

[233] Dan Overholt, Edgar Berdahl, and Robert Hamilton, "Advancements in Actuated Instruments," *Organised Sound.* Vol. 16, Issue 2 (2011): 154-165, https://www.cambridge.org/core/journals/organised-sound/article/advancements-in-actuated-musical-instruments/AFBD83D9E53F8C0270492F06CD0F2380 (accessed August 5, 2019).

contexts. Even in the absence of electronic transduction, the acoustical phenomenon of sympathetic resonance provides a direct means by which harmonic properties exhibited by one instrument may imbue similar sonorities upon another instrument within close spatial proximity and tuning. For example, Per Bloland cites sympathetic resonance in Luciano Berio's *Sequenza X* (1984) as an important precedent for his development of and composition for the Electromagnetically Prepared Piano.[234] In this earlier work, Berio directs the trumpet player to produce periodic bursts of notes into a piano whilst the pianist depresses certain keys. Here, the combination of extreme amplitude and dense harmonic spectra from the trumpet induce sympathetic vibrations from comparably-tuned partials of the piano's un-dampened strings.[235] Bloland's piece *Thingvellir* (2001) employs similar techniques and instrumentation. Extending this principle using electronics, the composer involves a microphone and loudspeaker placed within the piano to further amplify sympathetic partials from the trumpet.[236]

One could feasibly extend the definition of third-party actuation to include the re-embodiment of both audio signals *and* real-time performance data from one or more instruments into another. In the latter scenario, capture, analysis, and re-synthesis of salient musical parameters presents one possible mode for assessing what Overholt, Berdahl, and Hamilton term "augmented virtuality." For example, Andrew McPherson's composition, *d'Amour* for Magnetic Resonator Piano and viola, embodies elements for this form of third-party performance. Here, the composer employs pitch-tracking to selectively capture, re-synthesize, and actuate notes performed by a solo

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[234] Per Bloland, "The Electromagnetically-Prepared Piano and its Compositional Implications," *Proceedings of the 2007 International Computer Music Conference,* http://www.perbloland.com/userfiles/file/EMPP-Comp-Implications.pdf (accessed August 5, 2019).

[235] Darrett Adkins, Tony Arnold, Luciano Berio, Boris Berman, Steven Dann, Guy Few, et al. Sequenzas I-XIV (Complete), Hong Kong: Naxos Digital Services Ltd, 2006. (Accessed November 26, 2020).

[236] Per Bloland, "The Electromagnetically-Prepared Piano and its Compositional Implications," *Proceedings of the 2007 International Computer Music Conference,* http://www.perbloland.com/userfiles/file/EMPP-Comp-Implications.pdf (accessed August 5, 2019).

violist.[237] While the original, unprocessed audio-stream from the viola does not serve as the actuated signal, one can argue that the most salient elements of the performance are still retained through the process of actuated re-embodiment. In accordance with Berdahl, Overholt, and Hamilton's conception of "augmented virtuality," pitch data from the viola retains the role of "avatar(s) representing other real objects"—in this case, the viola.[238] Through actuation, these virtualities become re-embodied within one or more tangible acoustic objects, represented by the vibrating strings of the piano.

While specifically citing the use of "live" instruments in their description of "third-party" performance modalities, one could feasibly extend this practice to include actuation via previously recorded signals from other instruments. Here, the distinction between "live" and recorded sources appears merely temporal. Henceforth, we contend that recorded performances—though separated in time from the instance of actuation—continue to embody both sonic and gestural qualities imbued by the pairing of instrument(s) and performer(s). Moreover, in calling into question the notion of a temporal criteria for third-party actuation, we begin to blur the line between what distinguishes an *instrument* from a *signal processor*, as well as what constitutes an act of *performance* versus *post-production*.

For example, in describing the performance of third-party audio streams in Robert Hamilton's piece *six in one hand,* for Feedback Resonance Guitar and six telematically-networked performers, the composer retains language usually reserved for conveying aspects of post-production—most notably, the procedures of signal routing and processing. Here, Hamilton conveys a process in which "live audio signals are routed into the Feedback Resonance Guitar

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[237] Andrew McPherson and Youngmoo E. Kim, "Augmenting the Acoustic Piano with Electromagnetic String Actuation and Continuous Key Position Sensing," *Proceedings of the 2010 Conference on New Interfaces for Musical Expression (NIME 2010).* http://www.educ.dab.uts.edu.au/nime/PROCEEDINGS/papers/Paper%20K1-K5/P217_McPherson.pdf (accessed August 5, 2019).

[238] Dan Overholt, Edgar Berdahl, and Robert Hamilton, "Advancements in Actuated Instruments," *Organised Sound.* Vol. 16, Issue 2 (2011): 154-165, https://www.cambridge.org/core/journals/organised-sound/article/advancements-in-actuated-musical-instruments/AFBD83D9E53F8C0270492F06CD0F2380 (accessed August 5, 2019).

from six guitarists around the world."[239] While the performance he portrays is ostensibly *live*, elements of proximity and temporality certainly belie aspects of this definition. By virtue of the physical distance and unavoidable latency inherent to telematic performance, the actuated instrument and six performers occupy neither the same space, nor the same time.

In addressing these temporal thresholds which arbitrarily distinguish modes of production from actuated performance practice, one is reminded of the first of four "transgressive principles of interaction design" by John Bowers and Sten Olof Hellström:

> "Algorithmically-mediated interaction separates out a layer of algorithmic mediation which is distinct from direct manipulation—often by capturing or storing input data for use out-of-time—so that different peripheral devices, transformation algorithms, and sound models can be freely exchanged." [240]

While distinguishing algorithmically-mediated modes of interaction from "direct manipulation," Bowers and Hellström posit a free exchange of musical materials accomplished through a process of recording and recalling previous "data." Here, "data" may take on the form of audio streams or other, quantized representations of sonic materials. Furthermore, this exchange may occur as a temporally independent event or series of events, appearing "out-of-time" from their initial instantiation.[241] Such displacement in time can result from physical distance or network latency—as in the case of telematic performance, triggered audio playback from a previous performance, recall of real-time performance data, or time-based effects. Thus, the processing of previously-performed content by strings or other actuated bodies suggests a form of third-party performance practice.

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[239] Dan Overholt, Edgar Berdahl, and Robert Hamilton, "Advancements in Actuated Instruments," *Organised Sound.* Vol. 16, Issue 2 (2011): 154-165, https://www.cambridge.org/core/journals/organised-sound/article/advancements-in-actuated-musical-instruments/AFBD83D9E53F8C0270492F06CD0F2380 (accessed August 5, 2019).

[240] John Bowers and Sten Olof Hellström, "Simple Interfaces to Complex Sound in Improvised Music," *Extended Abstracts on Human Factors in Computing Systems*, April, 2000: 125-126. https://dl.acm.org/doi/10.1145/633292.633364 (accessed December 4, 2020).

[241] Simon Waters, "Performance Ecosystems: Ecological Approaches to Musical Interaction," *Proceedings of the Electroacoustic Music Studies Network*, Leicester: De Montfort: 1-20, http://www.ems-network.org/IMG/pdf_WatersEMS07.pdf. (accessed January 1, 2020).

Occupying a similarly liminal space between analog signal-processing and actuated performance practice, Nicolas Collins describes an early iteration of his Backwards Guitar as combining "characteristics of a spring reverb (in the way that it sustains sounds) and a six-band resonant filter or vocoder […] Some sounds remain quite identifiable through the string processing, while others were rendered completely unintelligible."[242] Collins not only likens the sonic qualities of his instrument to three notable modes of signal processing—the spring reverb, resonant filter, and vocoder—but, in fact coins the method of actuation, "string processing." Furthermore, his description of the continuum of intelligibility between the actuated output and the input source alludes to similar parametric procedures employed on many audio-processing devices. Specifically, we refer to adjustments made between a 'dry' (unprocessed) signal and a 'wet' (fully processed) mix. Speaking to his compositional intent, Collins makes further, unequivocal comparison between performance and production modalities, stating, "My goal, however, was not to elicit 'pure' string tones, but to inject outside sounds into the strings in pursuit of unusual, performable analog signal processing."[243] Live implementation of Collin's actuated "signal processing" also occur in works for solo performer, mixed ensemble, and iterative variations on his Backwards Guitar. Notably, in *Sound for Picture* (1992), a solo performer generates multiple third-party audio streams using their voice, bird calls, drum machine, and trumpet.[244]

5.3.1 "Third-Party Audio Streams" in *Artemisia*

Implementation of third-party audio streams in the composition, performance, and recording of *Artemisia* retains procedural traits analogous to Collins' approach to the Backwards Guitar. Similarities in morphology and modes of actuation include the appropriation of guitar strings, hardware, electro-magnetic transducers, and frames composed of aluminum. Likewise, in

[242] Nicolas Collins, "A Brief History of the 'Backwards Electric Guitar' (2009)," https://www.nicolascollins.com/texts/BackwardsElectricGuitar.pdf (Accessed August 9, 2019).

[243] Ibid.

[244] Nicolas Collins, *Sound Without Picture*, Periplum CD, 1999.

performance, both instruments (Rosebud I, II) and the Backwards Guitar fulfill the dual roles of analog signal-processor *and* instrument. This functional duality is most evident during states 35-53 of *Artemisia.* In a manner akin to what Collins terms "string processing," actuated strings of Rosebud I and II resonate and sustain spectral components from other instruments. Here, sources for third-party actuation include the four voices of the Viola da Gamba consort, *Science Ficta*. Moreover, the role of the consort exemplifies other definitional features of third-party audio streams. Citing Overholt, Berdahl, and Hamilton's conception of the "augmented virtuality paradigm," actuated signals from the ensemble embody the roles of both "elements in the virtual environment" and "avatars representing other real objects."[245] These "real objects" include the single treble and three bass viols of the consort.

In the mixed recording, we hear the unprocessed sound of each viol and subsequent "string processing" performed in parallel. However, the simultaneous nature of these sources is, in fact illusionary. The original recording with Science Ficta occurred on March 1st, 2020 in Charlottesville, Virginia. Whereas, actuation of these recorded signals occurred months later, during subsequent recording sessions in Boise, Idaho. Once again, we extend Overholt, Berdahl, and Hamilton's definition of third-party actuation to include signals which, as Bowers and Hellström suggest, may occur "out-of-time" and physically-displaced from their initial instantiation.[246] As described with recursive topologies (LRA and NLRA), temporal and proximal linearity remain fluid forces.

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[245] Dan Overholt, Edgar Berdahl, and Robert Hamilton, "Advancements in Actuated Instruments," *Organised Sound.* Vol. 16, Issue 2 (2011): 154-165. https://www.cambridge.org/core/journals/organised-sound/article/advancements-in-actuated-musical-instruments/AFBD83D9E53F8C0270492F06CD0F2380 (accessed August 5, 2019).

[246] John Bowers and Sten Olof Hellström, "Simple Interfaces to Complex Sound in Improvised Music," *Extended Abstracts on Human Factors in Computing Systems*, April, 2000: 125-126. https://dl.acm.org/doi/10.1145/633292.633364 (accessed December 4, 2020).

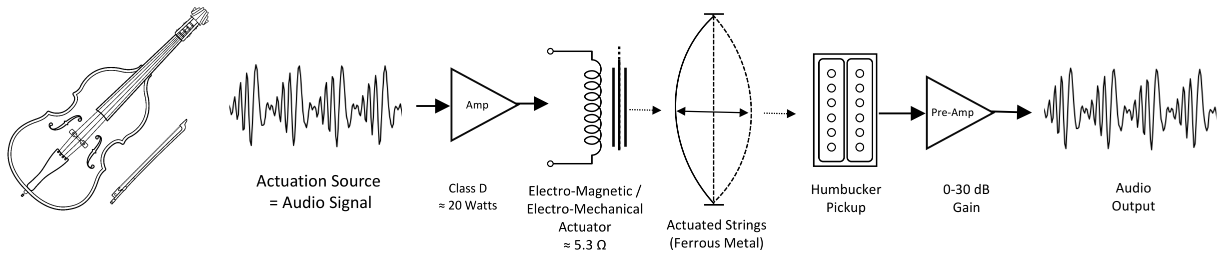

Figure 5. 7—“Third-Party” Actuation Topology in *Artemisia* (States 35-53)

Consistent with other modes of actuation, audibility of spectral features originating from third-party audio streams emerge as properties of sympathetic resonance. Therein, frequencies common to both pitches performed by the consort and low-order harmonics of the actuated strings appear most prominent. Consequently, tuning structure for both third-party instrumental source(s) and actuated string(s) inform the orchestration of spectra within the score. Fortunately, the moveable frets of the Viola da Gamba accommodate a wide range of possible tuning structures. Generally, spacing for frets follows conventional intervallic distribution, with the second, fourth, fifth, and seventh frets positioned at 1/9th, 1/5th, 1/4th, and 1/3rd the length of the fingerboard. These proportions correspond to the 5-limit just ratios: 9/8 (Major 2nd), 5/4 (Major 3rd), 4/3 (Perfect 4th), and 3/2 (Perfect 5th). Analogous to the first five notes of a diatonic scale, these ratios deviate from equal temperament by approximately +4, -14, -2, and +2 cents, respectively.[247] However, more substantial alteration occurs at the third fret. Here, the performers re-position the fret to a distance of 1/7th the length of the fingerboard. Fretting at this positions produces a note 7/6th the frequency of the open string, or approximately 31 cents flat of an equal-tempered minor third. Moreover, the inclusion of 7-limit (septimal) ratios facilitates the sympathetic actuation of high-order partials from the strings of Rosebud I and II. While the score calls for some alteration in fret position, tuning for each string in the viol consort follows simple, Pythagorean proportions. With little deviation from equal-temperament, strings for the treble and bass viols are tuned in ascending sets of perfect fourths (4/3).[248]

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[247] Hermann von Helmholtz, *On the Sensations of Tone as a Physiological Basis for the Theory of Music*, 2nd English ed., New York: Dover Publications, 1954.

[248] Whereas, 1/1 = 55 Hertz (A1)

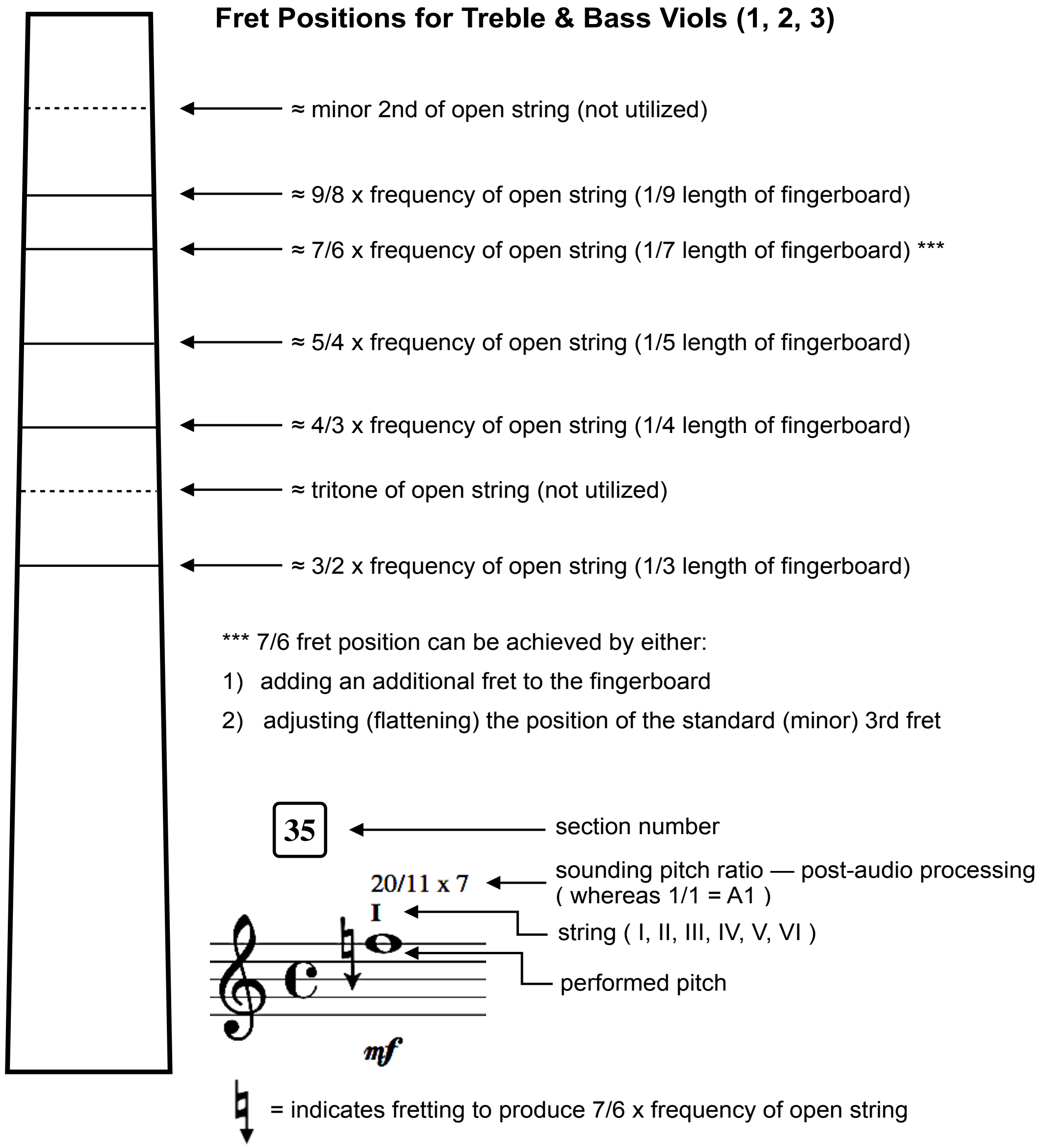

Figure 5. 8—Tuning Key for Bass and Treble Viols

(5 - 9)

Tuning: Treble Viol

VI	V	IV	III	II	I
D3 (-2 cents)	G3 (-4 cents)	C4	E4 (+2 cents)	A4	D4 (-2 cents)
8/3	32/9	n/a	6/1	8/1	32/3
146.66 Hz	195.55 Hz		330 Hz	440 Hz	586.66 Hz

4/3 4/3 4/3

(5 - 10)

Tuning: Bass Viols (1, 2, 3)

VI	V	IV	III	II	I
D2 (-2 cents)	G2 (-4 cents)	C3	E3 (+2 cents)	A3	D4 (-2 cents)
4/3	16/9	n/a	3/1	4/1	16/3
73.33 Hz	97.77 Hz		165 Hz	220 Hz	293.33 Hz

4/3 4/3 4/3

Commensurate to tuning and fret position, performers also utilize custom software to continuously alter the pitch of two electric (amplified) instruments within the consort: Treble Viol and Bass Viol #1. Depressing a MIDI footswitch enables a member of the ensemble to toggle between several state-specific, transposition values for each instrument. Therein, the score directs performers to apply microtonal shifts to performed pitches. These shifts correspond to one of three structurally-significant commas: 81/80 (*Syntonic Comma* or ≈ 22 cents), 45/44 (*Unidecimal 1/5th Tone* or ≈ 39 cents), or 33/32 (*Unidecimal Comma* or ≈ 53 cents).[249] [250] Importantly, these transposition values afford the sounding of pitches concurrent with harmonic frequencies for Rosebud I and II's strings. Applied to the open strings VI, V, III, II, I of each viol, resultant frequency ratios retain octave equivalence with fundamental frequencies for all strings (VI-I) of Rosebud I and strings XII, VI, II, and I of Rosebud II. Moreover, fretted notes also maintain octave equivalence with the first, second, and third harmonics for strings XII, XI, X, IX, VI, V, IV, III, II, and I of Rosebud II (see Table 5 – 11). In sharing common frequency components—albeit, through digital transposition—sympathetic resonance provides the acoustical means for effective translation of spectral materials from consort to actuated instrumentation.

[249] "So-Called Farey Series, Extended 0/1 to 1/0 (Full Set of Gear Ratios), and Lambdoma," *The Wilson Archives* (1996), http://anaphoria.com/lamb.pdf (accessed March 27, 2019)

[250] Kyle Gann, "Anatomy of an Octave," *Just Intonation: General Theory and Reference*, https://www.kylegann.com/Octave.html, (accessed August 15, 2017).

(5 - 11)

Octave Equivalent Ratios:	Viol	Rosebud I	Rosebud II
VI/I – open	4/3	IV $\left(\frac{4}{3} \cdot 1,2,4\right)$	VI $\left(\frac{16}{3} \cdot 1,2\right)$
VI/I – open $(\cdot 81/80)$	27/20	V $\left(\frac{9}{10} \cdot 3,6\right)$	V $\left(\frac{27}{5} \cdot 1,2\right)$
VI/I – open $(\cdot 45/44)$	15/11	III $\left(\frac{20}{11} \cdot 3,6\right)$	IV $\left(\frac{60}{11} \cdot 1,2\right)$
VI/I – open $(\cdot 33/32)$	11/8	II $\left(\frac{11}{6} \cdot 3,6\right)$	III $\left(\frac{11}{2} \cdot 1,2\right)$
VI/I – fret 2 $(\cdot 9/8)$	3/2	I $\left(\frac{2}{1} \cdot 3,6\right)$	I/II $\left(\frac{6}{1} \cdot 1,2\right)$
VI/I – fret 2 $(\cdot 9/8 \cdot 81/80)$	243/160	n/a	n/a
VI/I – fret 2 $(\cdot 9/8 \cdot 45/44)$	135/88	n/a	n/a
VI/I – open $(\cdot 9/8 \cdot 33/32)$	99/64	n/a	n/a
VI/I – fret 3 $(\cdot 7/6)$	14/9	VI $\left(\frac{8}{9} \cdot 7\right)$	n/a
VI/I – fret 3 $(\cdot 7/6 \cdot 81/80)$	63/40	V $\left(\frac{9}{10} \cdot 7\right)$	n/a
VI/I – fret 3 $(\cdot 7/6 \cdot 45/44)$	35/22	n/a	n/a
VI/I – fret 3 $(\cdot 7/6 \cdot 33/32)$	77/48	II $\left(\frac{11}{6} \cdot 7\right)$	n/a
VI/I – fret 4 $(\cdot 5/4)$	5/3	IV $\left(\frac{2}{1} \cdot 5\right)$	n/a
VI/I – fret 4 $(\cdot 5/4 \cdot 81/80)$	27/16	n/a	XI $\left(\frac{9}{2} \cdot 3\right)$
VI/I – fret 4 $(\cdot 5/4 \cdot 45/44)$	75/44	n/a	n/a
VI/I – fret 4 $(\cdot 5/4 \cdot 33/32)$	55/32	n/a	n/a
VI/I – fret 5 $(\cdot 4/3)$	16/9	VI $\left(\frac{8}{9} \cdot 1,2,4\right)$	n/a
VI/I – fret 5 $(\cdot 4/3 \cdot 81/80)$	9/5	V $\left(\frac{9}{10} \cdot 1,2,4\right)$	X $\left(\frac{24}{5} \cdot 3\right)$
VI/I – fret 5 $(\cdot 4/3 \cdot 45/44)$	20/11	III $\left(\frac{20}{11} \cdot 1,2,4\right)$	n/a
VI/I – fret 5 $(\cdot 4/3 \cdot 33/32)$	11/6	II $\left(\frac{11}{6} \cdot 1,2,4\right)$	n/a
VI/I – fret 7 $(\cdot 3/2)$	2/1	I $\left(\frac{2}{1} \cdot 1,2,4\right)$	XII $\left(\frac{4}{1} \cdot 1,2\right)$
VI/I – fret 5 $(\cdot 3/2 \cdot 81/80)$	81/80	n/a	V $\left(\frac{27}{5} \cdot 3\right)$
VI/I – fret 5 $(\cdot 3/2 \cdot 45/44)$	45/44	n/a	IV $\left(\frac{60}{11} \cdot 3\right)$
VI/I – fret 5 $(\cdot 3/2 \cdot 33/32)$	33/32	n/a	III $\left(\frac{11}{2} \cdot 3\right)$
V – open	16/9	VI $\left(\frac{8}{9} \cdot 1,2,4\right)$	n/a
V – open $(\cdot 81/80)$	9/5	V $\left(\frac{9}{10} \cdot 1,2,4\right)$	X $\left(\frac{24}{5} \cdot 3\right)$
V – open $(\cdot 45/44)$	20/11	III $\left(\frac{20}{11} \cdot 1,2,4\right)$	n/a
V – open $(\cdot 33/32)$	11/6	II $\left(\frac{11}{6} \cdot 1,2,4\right)$	n/a
V – fret 2 $(\cdot 9/8)$	2/1	I $\left(\frac{2}{1} \cdot 1,2,4\right)$	XII $\left(\frac{4}{1} \cdot 1,2\right)$
V – fret 2 $(\cdot 9/8 \cdot 81/80)$	81/80	n/a	V $\left(\frac{27}{5} \cdot 3\right)$
V – fret 2 $(\cdot 9/8 \cdot 45/44)$	45/44	n/a	IV $\left(\frac{60}{11} \cdot 3\right)$
V – fret 2 $(\cdot 9/8 \cdot 33/32)$	33/32	n/a	III $\left(\frac{11}{2} \cdot 3\right)$
V – fret 3 $(\cdot 7/6)$	28/27	n/a	n/a
V – fret 3 $(\cdot 7/6 \cdot 81/80)$	21/20	n/a	n/a
V – fret 3 $(\cdot 7/6 \cdot 45/44)$	35/33	n/a	n/a
V – fret 3 $(\cdot 7/6 \cdot 33/32)$	77/72	n/a	n/a
V – fret 4 $(\cdot 5/4)$	10/9	VI $\left(\frac{8}{9} \cdot 5\right)$	n/a
V – fret 4 $(\cdot 5/4 \cdot 81/80)$	9/8	V $\left(\frac{9}{10} \cdot 5\right)$	XI $\left(\frac{9}{2} \cdot 1,2\right)$
V – fret 4 $(\cdot 5/4 \cdot 45/44)$	25/22	III $\left(\frac{20}{11} \cdot 5\right)$	n/a
V – fret 4 $(\cdot 5/4 \cdot 33/32)$	55/48	I $\left(\frac{11}{6} \cdot 5\right)$	n/a
V – fret 5 $(\cdot 4/3)$	32/27	n/a	n/a
V – fret 5 $(\cdot 4/3 \cdot 81/80)$	6/5	n/a	X $\left(\frac{24}{5} \cdot 1,2\right)$
V – fret 5 $(\cdot 4/3 \cdot 45/44)$	40/33	n/a	n/a
V – fret 5 $(\cdot 4/3 \cdot 33/32)$	11/9	n/a	n/a
V – fret 7 $(\cdot 3/2)$	4/3	IV $\left(\frac{4}{3} \cdot 1,2,4\right)$	VI $\left(\frac{16}{3} \cdot 1,2\right)$
V – fret 7 $(\cdot 3/2 \cdot 81/80)$	27/20	V $\left(\frac{9}{10} \cdot 3,6\right)$	V $\left(\frac{27}{5} \cdot 1,2\right)$
V – fret 7 $(\cdot 3/2 \cdot 45/44)$	15/11	III $\left(\frac{20}{11} \cdot 3,6\right)$	IV $\left(\frac{60}{11} \cdot 1,2\right)$
V – fret 7 $(\cdot 3/2 \cdot 33/32)$	11/8	II $\left(\frac{11}{6} \cdot 3,6\right)$	III $\left(\frac{11}{2} \cdot 1,2\right)$

Octave Equivalent Ratios:	Viol	Rosebud I	Rosebud II
III – open	3/2	I $\left(\frac{2}{1} \cdot 3,6\right)$	I/II $\left(\frac{6}{1} \cdot 1,2\right)$
III – open $(\cdot\, 81/80)$	243/160	n/a	n/a
III – open $(\cdot\, 45/44)$	135/88	n/a	n/a
III – open $(\cdot\, 33/32)$	99/64	n/a	n/a
III – fret 2 $(\cdot\, 9/8)$	27/16	n/a	XI $\left(\frac{9}{2} \cdot 3\right)$
III – fret 2 $(\cdot\, 9/8 \cdot 81/80)$	2187/1280	n/a	n/a
III – fret 2 $(\cdot\, 9/8 \cdot 45/44)$	1215/704	n/a	n/a
III – fret 2 $(\cdot\, 9/8 \cdot 33/32)$	891/512	n/a	n/a
III – fret 3 $(\cdot\, 7/6)$	7/4	I $\left(\frac{2}{1} \cdot 7\right)$	n/a
III – fret 3 $(\cdot\, 7/6 \cdot 81/80)$	567/320	n/a	n/a
III – fret 3 $(\cdot\, 7/6 \cdot 45/44)$	315/176	n/a	n/a
III – fret 3 $(\cdot\, 7/6 \cdot 33/32)$	231/128	n/a	n/a
III – fret 4 $(\cdot\, 5/4)$	15/8	n/a	IX $\left(\frac{5}{1} \cdot 3\right)$
III – fret 4 $(\cdot\, 5/4 \cdot 81/80)$	243/128	n/a	n/a
III – fret 4 $(\cdot\, 5/4 \cdot 45/44)$	675/352	n/a	n/a
III – fret 4 $(\cdot\, 5/4 \cdot 33/32)$	495/256	n/a	n/a
III – fret 5 $(\cdot\, 4/3)$	2/1	I $\left(\frac{2}{1} \cdot 1,2,4\right)$	XII $\left(\frac{4}{1} \cdot 1,2\right)$
III – fret 5 $(\cdot\, 4/3 \cdot 81/80)$	81/80	n/a	V $\left(\frac{27}{5} \cdot 3\right)$
III – fret 5 $(\cdot\, 4/3 \cdot 45/44)$	45/44	n/a	IV $\left(\frac{60}{11} \cdot 3\right)$
III – fret 5 $(\cdot\, 4/3 \cdot 33/32)$	33/32	n/a	III $\left(\frac{11}{2} \cdot 3\right)$
III – fret 7 $(\cdot\, 3/2)$	9/8	V $\left(\frac{9}{10} \cdot 5\right)$	XI $\left(\frac{9}{2} \cdot 1,2\right)$
III – fret 7 $(\cdot\, 3/2 \cdot 81/80)$	729/640	n/a	n/a
III – fret 7 $(\cdot\, 3/2 \cdot 45/44)$	405/352	n/a	n/a
III – fret 7 $(\cdot\, 3/2 \cdot 33/32)$	297/256	n/a	n/a
II – open	1/1	I $\left(\frac{2}{1} \cdot 1,2,4\right)$	XII $\left(\frac{4}{1} \cdot 1,2\right)$
II – open $(\cdot\, 81/80)$	81/80	n/a	V $\left(\frac{27}{5} \cdot 3\right)$
II – open $(\cdot\, 45/44)$	45/44	n/a	IV $\left(\frac{60}{11} \cdot 3\right)$
II – open $(\cdot\, 33/32)$	33/32	n/a	III $\left(\frac{11}{2} \cdot 3\right)$
II – fret 2 $(\cdot\, 9/8)$	9/8	V $\left(\frac{9}{10} \cdot 5\right)$	XI $\left(\frac{9}{2} \cdot 1,2\right)$
II – fret 2 $(\cdot\, 9/8 \cdot 81/80)$	729/640	n/a	n/a
II – fret 2 $(\cdot\, 9/8 \cdot 45/44)$	405/352	n/a	n/a
II – fret 2 $(\cdot\, 9/8 \cdot 33/32)$	297/256	n/a	n/a
II – fret 3 $(\cdot\, 7/6)$	7/6	IV $\left(\frac{4}{3} \cdot 7\right)$	n/a
II – fret 3 $(\cdot\, 7/6 \cdot 81/80)$	189/160	n/a	n/a
II – fret 3 $(\cdot\, 7/6 \cdot 45/44)$	105/88	n/a	n/a
II – fret 3 $(\cdot\, 7/6 \cdot 33/32)$	77/64	n/a	n/a
II – fret 4 $(\cdot\, 5/4)$	5/4	I $\left(\frac{2}{1} \cdot 5\right)$	IX $\left(\frac{5}{1} \cdot 1,2\right)$
II – fret 4 $(\cdot\, 5/4 \cdot 81/80)$	81/64	n/a	n/a
II – fret 4 $(\cdot\, 5/4 \cdot 45/44)$	225/176	n/a	n/a
II – fret 4 $(\cdot\, 5/4 \cdot 33/32)$	165/128	n/a	n/a
II – fret 5 $(\cdot\, 4/3)$	4/3	IV $\left(\frac{4}{3} \cdot 1\right)$	VI $\left(\frac{16}{3} \cdot 1,2\right)$
II – fret 5 $(\cdot\, 4/3 \cdot 81/80)$	27/20	V $\left(\frac{9}{10} \cdot 3,6\right)$	V $\left(\frac{27}{5} \cdot 1,2\right)$
II – fret 5 $(\cdot\, 4/3 \cdot 45/44)$	15/11	III $\left(\frac{20}{11} \cdot 3,6\right)$	IV $\left(\frac{60}{11} \cdot 1,2\right)$
II – fret 5 $(\cdot\, 4/3 \cdot 33/32)$	11/8	II $\left(\frac{11}{6} \cdot 3,6\right)$	III $\left(\frac{11}{2} \cdot 1,2\right)$
II – fret 7 $(\cdot\, 3/2)$	3/2	I $\left(\frac{2}{1} \cdot 3,6\right)$	I/II $\left(\frac{6}{1} \cdot 1,2\right)$
II – fret 7 $(\cdot\, 3/2 \cdot 81/80)$	243/160	n/a	n/a
II – fret 7 $(\cdot\, 3/2 \cdot 45/44)$	135/88	n/a	n/a
II – fret 7 $(\cdot\, 3/2 \cdot 33/32)$	99/64	n/a	n/a

Through the culmination of tuning, fret position, and digital signal processing, performers produce pitches sympathetically-resonant with the overtone structure of each actuated string. Beginning in the first system (states 35-38), third-party audio streams from the consort actuate concurrent fundamental frequencies and common partials from Rosebud (RB) I and II. From the start, digital transposition of signal output from the two electric instruments (Bass Viol #1 and Treble Viol) transforms *performed* notes into *sounding* frequencies common to the spectra for one or more actuated strings. For example, in state 35, transposing a note (G2) sounded by the open string V $\left(\frac{16}{9} \cdot F_{1/1}\right)$ of Bass Viol #1 produces a frequency equivalent to the fundamental frequency for string III $\left(\frac{20}{11} \cdot F_{1/1}\right)$ of Rosebud I.[251] Following in state 36, we hear the contributing effects of tuning, fret position, and digital signal processing. To actuate the seventh harmonic of string V $\left(\frac{9}{10} \cdot 7\right)$ for Rosebud I, the performer stops string VI $\left(\frac{4}{3} \cdot F_{1/1}\right)$ of Bass Viol #1 at the third fret. This action produces a pitch 7/6th the frequency of the open string VI $\left(\frac{4}{3} \cdot \frac{7}{6} = \frac{14}{9}\right)$. Executed in the same instance, digital transposition of the performed pitch by one *Syntonic Comma* $\left(\frac{14}{9} \cdot \frac{81}{80} = \frac{63}{40}\right)$ yields an actuated frequency two octaves below the seventh harmonic of string V $\left(\frac{9}{10} \cdot 7\right)$ for Rosebud I.[252]

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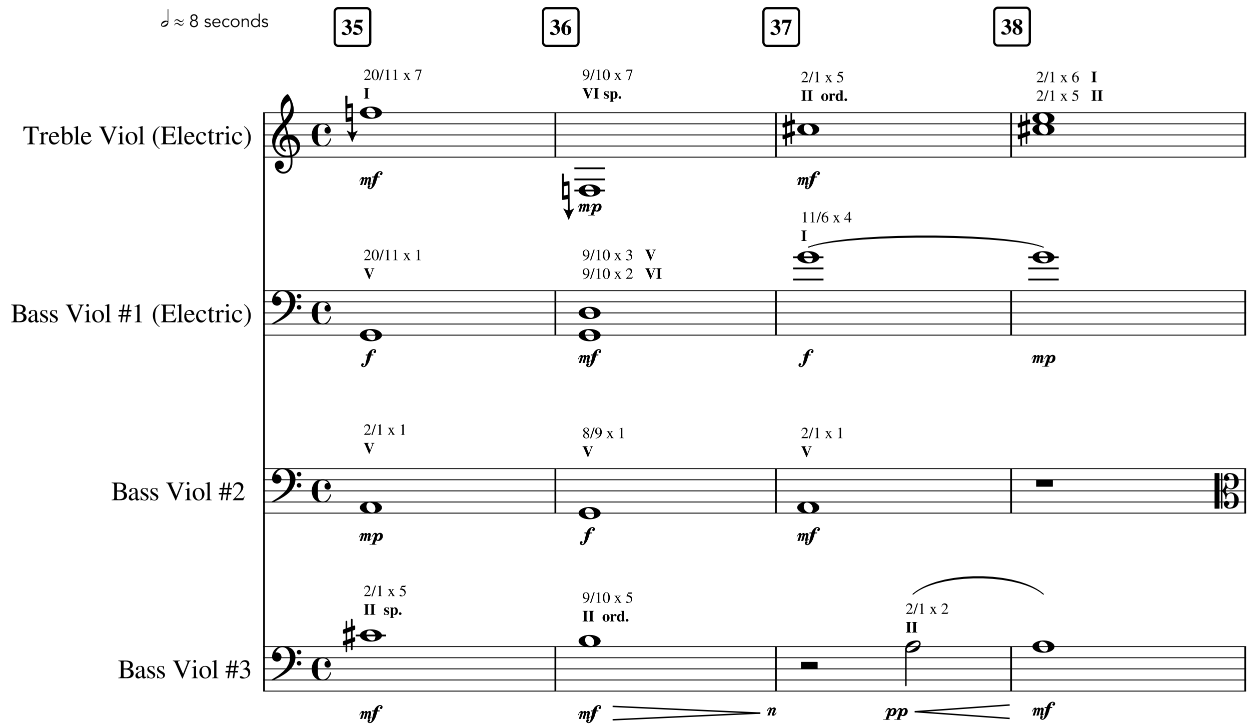
[251] $V\left(\frac{16}{9} \cdot F_{1/1}\right) \cdot 45/44\ \left(Unidecimal \frac{1}{5}\ Tone\right) = III\left(\frac{20}{11} \cdot F_{1/1}\right)$

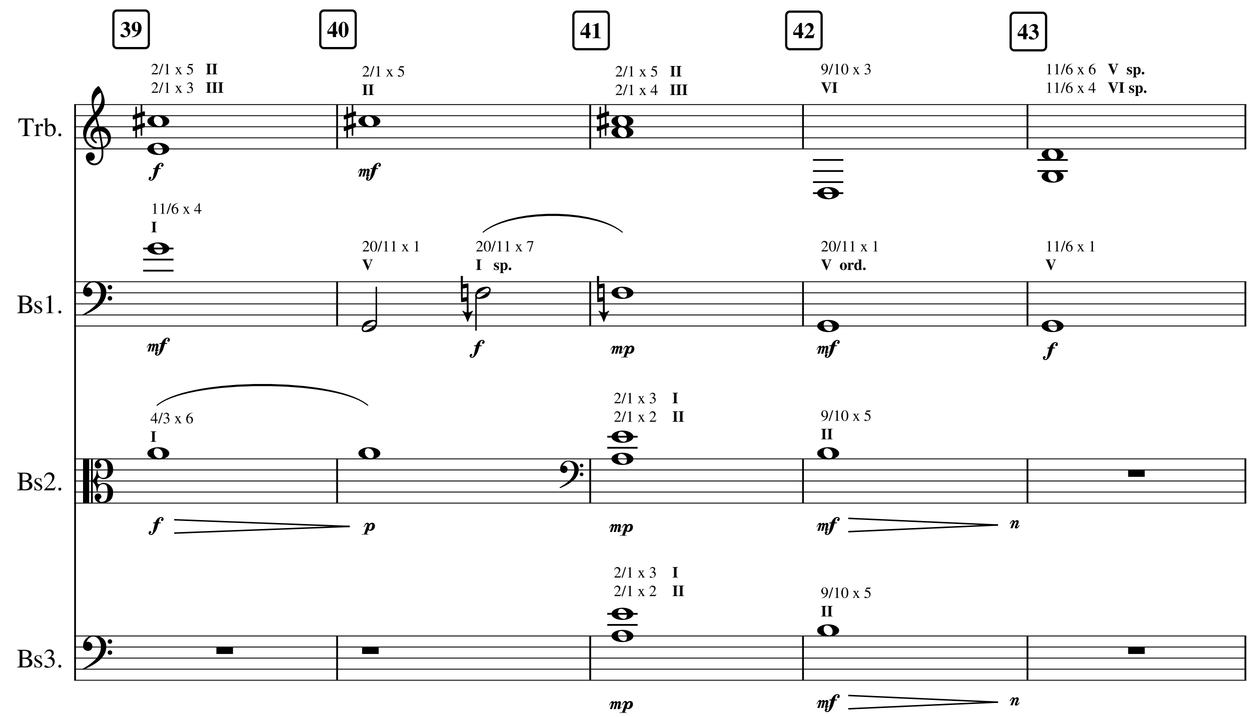
[252] While the just ratio 63/40 occurs two octaves below our actuated frequency $\left(\frac{9}{10} \cdot 7\right)$, application of *Sul Ponticello (sp.)* techniques by the performer emphasizes higher, octave-equivalent frequencies for the same performed pitch.



State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*"Third-Party" Audio Stream(s)*
35	Track 2 0:16	$\frac{20}{11} \cdot F_{1/1}$	1.0765	1	1.818	$20/11 \cdot 1$	Bass Viol #1 → RBI (III)
				2	3.834	$4/3 \cdot 3$	
				3	5.933	$2/1 \cdot 3$	
				4	8.086	$4/3 \cdot 6$	
				5	10.282	$2/1 \cdot 5$	
				6	12.512	$20/11 \cdot 7$	
				7	14.770	$2/1 \cdot 7$	
36 *diff.	Track 2 0:38	$\frac{8}{9} \cdot F_{1/1}$	1.0193	1	0.889	$8/9 \cdot 1$	
				2	1.802	$9/10 \cdot 2$	Bass Viol #1 → RBI (V)
				3	2.724	$9/10 \cdot 3$	Bass Viol #1 → RBI (V)
				4	3.652	$11/6 \cdot 2$	
				5	4.585	$9/10 \cdot 5$	Bass Viol #3 → RBII (XI)
				6	5.521	$11/6 \cdot 3$	
				7	6.460	$9/10 \cdot 7$	Treble Viol → RBI (V)
37	Track 2 0:58	$\frac{2}{1} \cdot F_{1/1}$	1.183	1	2.000	$2/1 \cdot 1$	Bass Viol #2 → RBII (XII)
				2	4.541	$9/10 \cdot 5$	
				3	7.336	$11/6 \cdot 4$	
				4	10.310	$2/1 \cdot 5$	Treble Viol → RBI (I)
				5	13.425	$2/1 \cdot 7$	
				6	16.656	n/a	
				7	19.989	n/a	

38	Track 2 1:19	$\frac{9}{10} \cdot F_{1/1}$	1.325	1	0.900	$9/10 \cdot 1$	
				2	2.255	$2/1 \cdot 1$	
				3	3.859	$4/3 \cdot 3$	
				4	5.649	$11/6 \cdot 3$	
				5	7.592	$11/6 \cdot 4$	
				6	9.667	$2/1 \cdot 5$	Treble Viol → RBI (I)
				7	11.858	$2/1 \cdot 6$	Treble Viol → RBI (I)

Moving forward, we witness instances of concurrent actuation of common frequencies across multiple instruments. Here, three distinct third-party topologies unfold: 1) one third-party audio stream actuating the same frequency across multiple instruments; 2) multiple third-party audio streams actuating the same frequency from a single instrument; and 3) multiple third-party audio streams actuating the same frequency from multiple instruments. Beginning in state 39, we witness the first scenario. Sustained audio from the second harmonic of the Treble Viol's open string III $\left(\frac{6}{1} \cdot 2\right)$ actuates the sixth harmonic of string I $\left(\frac{2}{1} \cdot 6\right)$ for Rosebud I, as well as the second harmonic of strings I and II $\left(\frac{6}{1} \cdot 2\right)$ for Rosebud II. In each case, the actuated partials sound at a common frequency of 660 Hertz $\left(\frac{12}{1} \cdot F_{1/1}\right)$. A similar topology ensues in state 42, wherein the pitch-shifted, second harmonic from the open string VI $\left(\frac{8}{3} \cdot 2 \cdot \frac{81}{80}\right)$ of the Treble Viol actuates the same frequency via the sixth harmonic and fundamental frequency of string V $\left(\frac{9}{10} \cdot 6\right)$; $\left(\frac{27}{5} \cdot F_{1/1}\right)$ for Rosebud I and II, respectively. Within the same state, we hear a frequency common to multiple, third-party audio streams actuating a single instrument. In this case, stopping string II $\left(\frac{4}{1} \cdot \frac{9}{8}\right)$ of Bass Viols #2 and 3 at the second fret sounds a unison pitch equivalent the fundamental frequency of string XI $\left(\frac{9}{2} \cdot F_{1/1}\right)$ for Rosebud II. Thusly, the performer initiates mutual third-party actuation. Finally, an example of our third topology transpires at state 41. Here, a common frequency sounded by the Treble Viol's open string II $\left(\frac{8}{1} \cdot F_{1/1}\right)$ and the second harmonic of string II $\left(\frac{4}{1} \cdot 2\right)$ for Bass Viol #3 concurrently actuate the sixth harmonic of string IV $\left(\frac{4}{3} \cdot 6\right)$ for Rosebud I and the second harmonic of string XII $\left(\frac{4}{1} \cdot 2\right)$ for Rosebud II.



State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*"Third-Party" Audio Stream(s)*
39	Track 2 1:40	$\frac{11}{6} \cdot F_{1/1}$	1.0645	1	1.833	$11/6 \cdot 1$	
				2	3.834	$4/3 \cdot 3$	
				3	5.904	$2/1 \cdot 3$	
				4	8.019	$4/3 \cdot 6$	Bass Viol #2 → RBII (XII)
				5	10.169	$2/1 \cdot 5$	Treble Viol → RBI (I)
				6	12.348	$2/1 \cdot 6$	Treble Viol → RBI (I), Treble Viol → RBII (I/II)
				7	14.550	$2/1 \cdot 7$	

40	Track 2 2:02	$\frac{20}{11} \cdot F_{1/1}$	1.0765	1	1.818	$20/11 \cdot 1$	Bass Viol #1 → RBI (III)
				2	3.834	$4/3 \cdot 3$	
				3	5.933	$2/1 \cdot 3$	Treble Viol → RBI (I)
				4	8.086	$4/3 \cdot 6$	
				5	10.282	$2/1 \cdot 5$	Treble Viol → RBI (I)
				6	12.512	$20/11 \cdot 7$	
				7	14.770	$2/1 \cdot 7$	

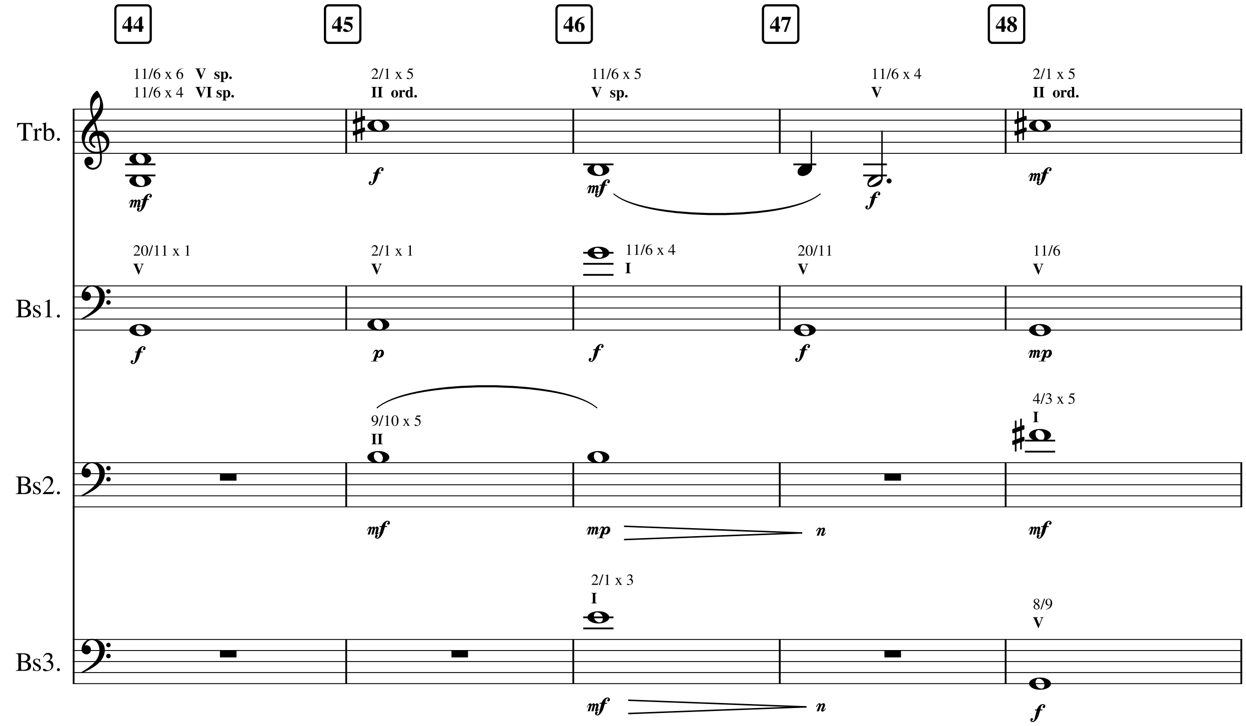
41	Track 2 2:24	$\frac{8}{9} \cdot F_{1/1}$	1.353	1	0.889	$8/9 \cdot 1$	
				2	2.271	$2/1 \cdot 1$	
				3	3.930	$4/3 \cdot 3$	
				4	5.800	$2/1 \cdot 3$	
				5	7.844	$4/3 \cdot 6$	Treble Viol → RBI (IV), Bass Viol #3 → RBII (XII)
				6	10.039	$2/1 \cdot 5$	
				7	12.367	$20/11 \cdot 7$	

42 *diff.	Track 2 2:46	$\frac{9}{10} \cdot F_{1/1}$	1.015	1	0.900	$9/10 \cdot 1$	
				2	1.819	$20/11 \cdot 1$	Bass Viol #1 → RBI (III)
				3	2.745	$9/10 \cdot 3$	Treble Viol → RBI (V), Treble Viol → RBII (V)
				4	3.676	$11/6 \cdot 2$	
				5	4.610	$9/10 \cdot 5$	Bass Viol #2 → RBII, Bass Viol #3 → RBII (XI)
				6	5.547	$11/6 \cdot 3$	
				7	6.487	$2/1 \cdot 3$	

43	Track 2 3:10	$\frac{11}{6} \cdot F_{1/1}$	1.006	1	1.833	$11/6 \cdot 1$	Bass Viol #1 → RBI (II)
				2	3.682	$11/6 \cdot 2$	
				3	5.536	$11/6 \cdot 3$	
				4	7.395	$11/6 \cdot 4$	Treble Viol → RBI (II)
				5	9.256	$4/3 \cdot 7$	
				6	11.119	$11/6 \cdot 6$	Treble Viol → RBII (III)
				7	12.984	$11/6 \cdot 7$	

Within the following system, we witness another example of a single third-party audio stream actuating the same frequency from two instruments. In state 45, stopping the second string of the Treble Viol at the fourth fret II $\left(\frac{4}{1} \cdot \frac{5}{4}\right)$ sounds a frequency of 550 Hertz. This sustained frequency concurrently actuates the fifth harmonic of string I $\left(\frac{2}{1} \cdot 5\right)$ for Rosebud I and the second harmonic of string IX $\left(\frac{5}{1} \cdot 2\right)$ of Rosebud II. At the same moment, third-party streams from Bass Viol #1 and #2 actuate the fundamental frequencies of string I $\left(\frac{2}{1} \cdot F_{1/1}\right)$ for Rosebud I and string XI $\left(\frac{9}{2} \cdot F_{1/1}\right)$ for Rosebud II.

(5 - 14)



State #	Time	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	*NASH*	*"Third-Party" Audio Stream(s)*
44	Track 2 3:33	$\frac{20}{11}\cdot F_{1/1}$	1.011	1	1.818	$20/11\ \cdot 1$	Bass Viol #1 → RBI (III)
				2	3.664	$11/6\ \cdot 2$	
				3	5.521	$11/6\ \cdot 3$	
				4	7.384	$11/6\ \cdot 4$	Treble Viol → RBI (II)
				5	9.253	$4/3\ \cdot 7$	
				6	11.126	$11/6\ \cdot 6$	Treble Viol → RBII (III)
				7	13.003	$11/6\ \cdot 7$	

45	Track 2 3:54	$\frac{2}{1}\cdot F_{1/1}$	1.179	1	2.000	$2/1\ \cdot 1$	Bass Viol #1 – RBI (I)
				2	4.528	$9/10\ \cdot 5$	Bass Viol #2 → RBII (XI)
				3	7.304	$11/6\ \cdot 4$	
				4	10.253	$2/1\ \cdot 5$	Treble Viol → RBI (I), Treble Viol → RBII (IX)
				5	13.339	$11/6\ \cdot 7$	
				6	16.538	$2/1\ \cdot 7$	
				7	19.834	n/a	

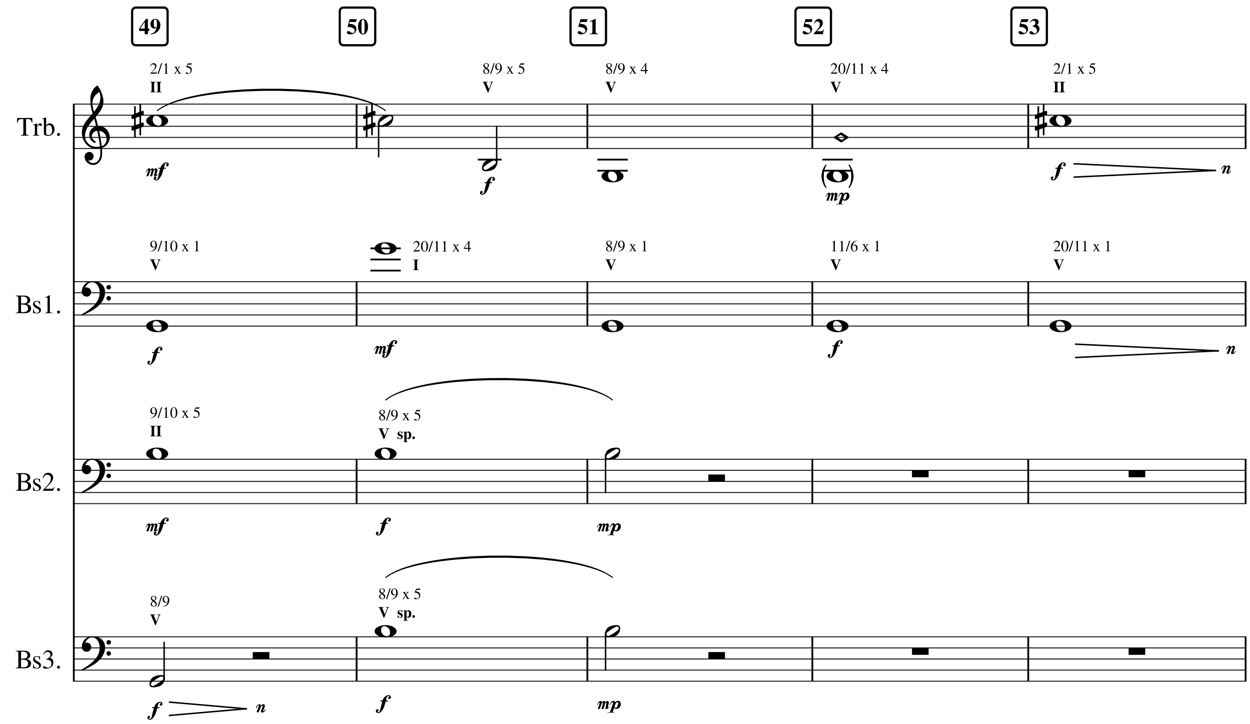
46	Track 2 4:14	$\frac{8}{9}\cdot F_{1/1}$	1.197	1	0.889	$8/9\ \cdot 1$	
				2	2.038	$2/1\ \cdot 1$	
				3	3.311	$8/9\ \cdot 4$	
				4	4.672	$9/10\ \cdot 5$	
				5	6.103	$2/1\ \cdot 3$	Bass Viol #3 → RBII (I/II)
				6	7.591	$11/6\ \cdot 4$	
				7	9.129	$11/6\ \cdot 5$	Treble Viol → RBI (II)

47	Track 2 4:36	$\frac{20}{11} \cdot F_{1/1}$	1.003	1	1.818	$20/11 \cdot 1$	Bass Viol #1 → RBI (III)
				2	3.644	$20/11 \cdot 2$	
				3	5.473	$20/11 \cdot 3$	
				4	7.303	$11/6 \cdot 4$	
				5	9.135	$11/6 \cdot 5$	Treble Viol → RBI (II)
				6	10.968	$11/6 \cdot 6$	
				7	12.802	$11/6 \cdot 7$	

48 *diff.	Track 2 4:59	$\frac{11}{6} \cdot F_{1/1}$	1.204	1	1.833	$11/6 \cdot 1$	Bass Viol #1 → RBI (II)
				2	4.224	$8/9 \cdot 5$	
				3	6.882	$4/3 \cdot 5$	
				4	9.730	$2/1 \cdot 5$	Treble Viol → RBI (I)
				5	12.729	$20/11 \cdot 7$	
				6	15.854	$2/1 \cdot 7$	
				7	19.087	n/a	

Throughout the following system (states 49-53), third-party actuation functions to highlight differential features between interceding states. Here, conventions established as part of Partch's "Monophonic Fabric" aide us in analyzing state-specific, intervallic traits.[253] Notably, between states 49-51, we witness an alternating pattern of Otonalities and Utonalities formed between actuated, third-party audio streams. Moreover, harmonic intervals represented within each succeeding state maintain a prime-limit distinct from surrounding states. For example, in state 49, actuation of the fundamental and fifth harmonic of string V$\left(\frac{9}{10} \cdot 1, 5\right)$ form the first and fifth odentities of a single dyad, unified by Numerary Nexus of '5'. State 50 follows with the same Numerary Nexus. However, the actuated ratios 40/9 $\left(\frac{8}{9} \cdot 5\right)$, 40/11 $\left(\frac{20}{11} \cdot 4\right)$, and 10/1 $\left(\frac{2}{1} \cdot 5\right)$ denote a contrasting shift towards an 11-limit Utonality. Intervallic complexity contracts noticeably in state 51, with the actuation of two partials of string I $\left(\frac{8}{9} \cdot 1, 4\right)$ separated by two octaves. Moving in parallel to the next state Treble and Bass Viol #1 actuate a detuned double-octave, spanning the ratios 11/6 $\left(\frac{11}{6} \cdot F_{1/1}\right)$ and 80/11$\left(\frac{20}{11} \cdot 4\right)$ across strings II and III of Rosebud I and II, respectively. The section terminates at state 53 with another 11-limit Utonality—this time formed between the fundamental and fifth harmonics of strings III $\left(\frac{20}{11} \cdot F_{1/1}\right)$ and I $\left(\frac{2}{1} \cdot 5\right)$ for Rosebud I.

[253] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.



State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*"Third-Party" Audio Stream(s)*
49	Track 2 5:20	$\frac{9}{10} \cdot F_{1/1}$	1.4755	1	0.900	$9/10 \cdot 1$	Bass Viol #1 → RBI (V)
				2	2.503	$8/9 \cdot 3$	
				3	4.552	$9/10 \cdot 5$	Bass Viol #2 → RBII (XI)
				4	6.960	$4/3 \cdot 5$	
				5	9.673	$2/1 \cdot 5$	
				6	12.659	$20/11 \cdot 7$	
				7	15.892	$2/1 \cdot 7$	

50	Track 2 5:41	$\frac{2}{1} \cdot F_{1/1}$	1.137	1	2.000	$2/1 \cdot 1$	
				2	4.398	$8/9 \cdot 5$	Treble Viol → RBI (VI)
				3	6.975	$20/11 \cdot 4$	Bass Viol #1 → RBI (III)
				4	9.673	$2/1 \cdot 5$	Treble → RBII (IX)
				5	12.467	$20/11 \cdot 7$	
				6	15.339	$2/1 \cdot 7$	
				7	18.277	n/a	

51	Track 2 6:03	$\frac{8}{9} \cdot F_{1/1}$	1.1635	1	0.889	$8/9 \cdot 1$	Bass Viol #1 → RBI (VI)
				2	1.991	$2/1 \cdot 1$	
				3	3.191	$8/9 \cdot 4$	Treble Viol → RBI (VI)
				4	4.460	$8/9 \cdot 5$	
				5	5.782	$2/1 \cdot 3$	
				6	7.149	$20/11 \cdot 4$	
				7	8.553	$20/11 \cdot 5$	

52	Track 2 6:24	$\frac{11}{6} \cdot F_{1/1}$	1.237	1	1.833	$11/6 \cdot 1$	Bass Viol #1 → RBI (II)
				2	4.321	$8/9 \cdot 5$	
				3	7.136	$20/11 \cdot 4$	Treble Viol → RBI (III)
				4	10.186	$2/1 \cdot 5$	
				5	13.424	$2/1 \cdot 7$	
				6	16.820	n/a	
				7	20.353	n/a	

53	Track 2 6:44	$\frac{20}{11} \cdot F_{1/1}$	1.2235	1	1.818	$20/11 \cdot 1$	Bass Viol #1 → RBI (III)
				2	4.246	$8/9 \cdot 5$	
				3	6.973	$20/11 \cdot 4$	
				4	9.914	$2/1 \cdot 5$	Treble Viol → RBI (I)
				5	13.026	$11/6 \cdot 7$	
				6	16.282	$2/1 \cdot 7$	
				7	19.661	n/a	

5.4 Disruptive Preparation of an Actuated Body

While definitional features for the three previous categories of actuated string performance practice reference the source(s) for actuation, disruptive preparation entails the physical or procedural treatment of an actuated body already in motion. In the case of vibrating strings, acoustic outcomes induced by these treatments tend to manifest spectrally as a range of subtle to increasingly radical transformations in timbre. With strings, some of the earliest widely acknowledged precedents for disruptive preparation originate during the first half of the twentieth century with the prepared piano. Contemporary accounts generally position John Cage's *Bacchanale* (1938-1940) as the essential prototype for prepared piano performance and associated practices to follow. Furthermore, conventional wisdom frames the origins of preparation as a pragmatic alternative to Cage's full percussion ensemble. Here, preparation affords the performance of a variety of timbres from a single, ubiquitous instrument: the piano. Famously, Cage first devised the prepared piano for a performance of *Bacchanale* at Syvilla Fort's dance recital on April 28, 1940. As the stage at Seattle's Repertory Playhouse was too small to accommodate his percussion ensemble, *preparing* the piano provided a very practical solution.[254] By placing different objects across the strings of the piano, thus disrupting the natural harmonic modes of vibration, Cage creates what he terms "a percussion orchestra of the original sound and

[254] Leta E. Miller, "Henry Cowell and John Cage: Intersections and Influences, 1933-1941," *Journal of the American Musicological Society*, Vol. 59, No. 1 (Spring 2006), 47-112. https://www.jstor.org/stable/10.1525/jams.2006.59.1.47 (accessed December 8, 2020).

the decibel range of a harpsichord directly under the control of a pianist's fingertips."[255]

However, similar disruptive preparations by Cage and others precede the 1940 performance of *Bacchanale*. Both *First Construction (in Metal)* (1939) and *Second Construction* (1940) employ a so-called "string piano"—a term coined by Henry Cowell. This term refers to performance techniques involving physical engagement with or modification of the strings of a grand piano. Cage draws upon comparable techniques in composing these earlier works. For example, in one section of *Second Construction*, Cage asks the performer to "mute" strings within the piano's middle range using a piece of cardboard. Furthermore, the composer elicits a "siren-like sound" by allowing a metal cylinder to roll across the strings, while the pianist performs a rapid trill. Similar disruptive preparations of the "string piano" occur within Cowell's scores more than a decade prior. As noted by musicologist Leta E. Miller, in the third movement of *A Composition for String Piano with Ensemble* (1925), Cowell instructs the performer to place a "flat metal object" upon the strings.[256] In either case, we witness intensive modifications to timbre resulting from intentional, object-centered disruptions to the vibrational modes of the instrument's strings. Perhaps as an extension of this tradition, noteworthy examples of disruptive preparation also appear in works for actuated piano strings. In a commission for pianist Lois Svard, *Music for Piano with Magnetic Strings* (1995), Alvin Lucier instructs the performer to position (and reposition) five Ebows across the strings of a grand piano. This technique induces self-sustained oscillations of varying pitch and harmonicity. In addition to the overtly spectral features of the composition, Lucier also draws our attention to incidental structures resulting from physically-disruptive events, including "occasional rhythms produced as one or more magnets vibrates against

[255] John Cage, "A Composer's Confessions." Address at Vassar College, Poughkeepsie, New York, 28 February 1948. Published in *Musicworks* 52 (Spring 1992): 6–15; reprinted in *John Cage, Writer: Previously Uncollected Pieces*, edited by Richard Kostelanetz. New York: Limelight Editions, 1993, 27–44. https://www.editions-allia.com/files/pdf_595_file.pdf (accessed December 14, 2020).

[256] Leta E. Miller, "Henry Cowell and John Cage: Intersections and Influences, 1933-1941," *Journal of the American Musicological Society*, Vol. 59, No. 1 (Spring 2006), 47-112. https://www.jstor.org/stable/10.1525/jams.2006.59.1.47 (accessed December 8, 2020).

adjacent strings."[257]

While modern examples of disruptive preparation of strings arguably begin with the piano, similar practices extend to other extant instrumentation. Most notably, musicians and composers have applied similar techniques to the guitar—both acoustic and electric. In their book, *Nice Noise: Modifications and Preparations for Guitar*, experimental luthiers Bart Hopkin and Yuri Landman define "prepared guitar" or "guitar preparations" as "temporary modifications to the guitar to alter the tone, often involving small objects attached to or inserted between the strings."[258] The relevance of disruptive preparations for guitar extend, perhaps, more deeply into actuated instrument design and performance practice than any other extant instrumental tradition. The ubiquity of the guitar in popular music(s), its varied morphology, availability of external signal-processing devices (e.g. pedals), and commercial development of purpose-built actuated device, such as the EBow™ , cement the instrument's role in our discussion of disruptive preparation and actuated strings.[259] Furthermore, the varied form and functionality of guitars have provided models for the design and augmentation of actuated string instruments, including the Backwards Guitar, Feedback Lap Steel, and Feedback Resonance Guitar. Certainly, many actuated string instruments cannibalize hardware guitar hardware. Notably, both Rosebud I and II employ proprietary electric guitar components in their construction, including: machine-tuners, humbucker pickups, adjustable 'hard-tail' or 'tune-o-matic' bridges, and standard-gauge or baritone guitar strings. [260]

[257] Alvin Lucier, "Music for Piano with Magnetic Strings," Theme (Liner Notes). http://www.lovely.com/albumnotes/notes5011.html (accessed August 5, 2019).

[258] Bart Hopkin and Yuri Landman, Nice Noise: Preparations and Modifications for Guitar, Point Reyes Station, CA: Experimental Musical Instruments, 2012.

[259] Gregory Heet, 1978, String Instrument Vibration Initiator and Sustainer, US Patent 4,075,921.

[260] Saliency Chrome Guitar Machine Head Tuners (3L 3R).

[261] [262] [263] [264] [265] In regards to form, construction, history, and practice, prepared guitar and actuated instrumentation retain many mutual properties.

Amongst countless others, a few notable practitioners of prepared guitar include: Bradford Reed, Hans Reichel, Fred Frith, Glenn Branca, Thurston Moore, Lee Ranaldo, Bill Horist, Yuri Landman, Erhard Hirt, Frank Rühl, Annette Krebs, Hainer Wörmann, Hans Tammen, Jean-Marc Montera, Sharif Sehnaoui, Roger Kleier, Nick Didkovasky, Chris Forsyth, Kazuhisa Uchihashi, John Russell, Neil Feather, and Mic Levi.[266] Unsurprisingly, specific preparation techniques vary drastically between guitarists, often reflecting the proprietary goals and aesthetic inclinations of the individual performer. For the purpose of this essay, we shall examine approaches most suited to or previously demonstrated with actuated string instruments. According to Hopkin and Landman, these techniques can be classified according to the following physical criteria: "rattles on the strings," "buzzing bridges," "weighted Strings," "middle [third] bridges," and "retuning."[267] Of course, beyond scordatura and dropped tunings addressed by Hopkin and Landman, we shall continue to address intonation throughout this essay. More relevant to the scope of our current discussion, we instead focus upon the application of "rattles on the strings," "buzzing bridges," and "weighted strings" in actuated string performance.

Moving beyond prepared piano or guitar, we begin by examining more explicit examples

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[261] BQLZR, Humbucker Double-Coil Electric Guitar Pickups.

[262] Kmise A0052 6-Saddle Hardtail Bridge, Top Load (78 mm).

[263] M Y Fly Young Bridge and Stop Tail Bar for 12-String Electric Guitar (LP).

[264] Just Strings, Plain Steel Electric Guitar Strings—single (0.009, 0.010").

[265] Ernie Ball Nickel-wound Baritone Guitar String Set (0.013, 0.018, 0.030, 0.044, 0.056, 0.072").

[266] Yuri Landman, "DIY: Thurston Moore's Drone Guitar Project," *Premier Guitar*. April 2016, https://www.premierguitar.com/articles/23988-diy-thurston-moores-drone-guitar-project (accessed September 11, 2019).

[267] Bart Hopkin and Yuri Landman, Nice Noise: Preparations and Modifications for Guitar, Point Reyes Station, CA: Experimental Musical Instruments, 2012.

of disruptive preparation involving actuated string instruments. While not entirely common, accounts of both disruptive string preparation and subsequent signal processing of actuated instrumentation appear in multiple performances with Nicolas Collins' *Backwards Guitar*, as well as earlier compositions for other actuated string instruments. For example, as means for emulating specific spectral qualities, Collins describes attaching "alligator clips" to the instrument's strings, thereby producing "gamelan-style sounds."[268] Presumably, these modifications in timbre result from inharmonic partials introduced through the additional weight and arbitrary disruption of harmonic nodes. Hopkin and Landman describe similar "weighted string" techniques (and resulting harmonic displacement) involving fishing sinkers, screws, nuts, washers, glue, solder, and wire windings.[269] Like Cage in decades prior, we find Collins recreating and embodying the proprietary timbres of a percussion ensemble within the sounding bodies of a *single* stringed instrument, under the control of a *single* performer. Assessing further expansions in timbre, Collins recounts routing the audio output from the Backwards Guitar through various guitar pedals, including harmonic distortion. Here, we witness substantial modifications to the resultant spectra. In the latter example, distortion functions as a means of reinforcing certain harmonic overtones. Preceding this performative work, Collins' first audio installation, *Under the Sun—A Pythagorean Experiment* (1976), employs similar disruptive preparation to an electro-mechanically actuated string. In this piece, the artist attaches a solenoid to induce periodic vibrations along one section of long steel wire. As these vibrations occur, a Teflon ring travels up and down the length of the wire, it's position occasionally coinciding with various harmonic nodes.[270] Again, the act of disruptive preparation affords specific spectral outcomes.

As well as the objects or methods employed in disruptive preparation, the extent to which the performer actively intervenes with the actuated body determines both aural and relational

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[268] Nicolas Collins, "A Brief History of the 'Backwards Electric Guitar' (2009)," https://www.nicolascollins.com/texts/BackwardsElectricGuitar.pdf (Accessed August 9, 2019).

[269] Bart Hopkin and Yuri Landman, Nice Noise: Preparations and Modifications for Guitar, Point Reyes Station, CA: Experimental Musical Instruments, 2012.

[270] Nicolas Collins, "A Brief History of the 'Backwards Electric Guitar' (2009)," https://www.nicolascollins.com/texts/BackwardsElectricGuitar.pdf (Accessed August 9, 2019).

outcomes. The question arises: does the act of disruptive preparation suggest an active interplay between the responses of a sounding body, disrupting object (or procedure), and the performer(s)? Alternately, does the act of disruption constitute a preparatory procedure that—once set into motion—unfolds with few or no further interventions by the performer? Like Collins, Fishkin explores each scenario in separate performances of *The Tinnitus Suites* for Lady's Harp. In earlier iterations (2008-2012), disruptive preparation appears as an active procedure in which performers continually modify the pitch and resultant harmonics by means of "metal slide mallets" (e.g. stops or bridges) moving along the length of piano wire. Fishkin likens this mode of active preparation to "playing" a guitar, with the "metal slide mallets" acting as surrogates for "big frets." As noted by the composer, the interventional nature of this activity appears incongruous with his stated goals of "hearing what the system is doing, rather than trying to coax sounds out of it."[271]

> "I was never satisfied with the metal slide mallets we used to play the [Lady's] harp back then—it felt too guitaristic, and the music that came from these "big frets" didn't feel like the Tinnitus Suites. Oliver [Jones] had brought in a cache of old wood that he pulled from the rotting windowsills of his house, and hung them on the wall as a component of his visual work. While we were talking, one of us lifted a hunk of wood and rested it on the strings. It wobbled back and forth, making beautiful rhythms and patterns. Was this our eureka moment? Fred Frith mentioned once that he turned the guitar horizontal as a method of treating the guitar less as an instrument and more of a sound source. I'm not sure I'm in this territory. But I do know that by placing objects on the strings, we were able to walk away from them and listen to their subtle shifts amidst the feedback. It felt more like hearing what the system is doing, rather than trying to coax sounds out of it—less like playing, more like placing. Balancing the object is like setting a capo on a guitar, except that the object will dance along with the vibrating string."[272]

From this account, we understand disruptive preparation as an explicit act of performance. In recalling these conceptual inconsistencies, Fishkin and his collaborators came upon modes of disruptive actuation initiated through the *placement,* rather than the *performance* of physical objects. Moving forward, we shall examine these and other modes of disruption as distinct compositional forces.

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[271] Daniel Fishkin, "Composing the Tinnitus Suites: 2015," http://dfiction.com/ctts-2015/ (accessed December 3, 2020).

[272] Ibid.

5.4.1 Disruptive Preparation as a Structuring Feature in *Artemisia*

In the performance and recording of *Artemisia,* disruptive preparation takes the form of two, distinct topologies: external signal processing and physical preparation of actuated strings. In both topologies, three transducer types—including an electro-magnetic pickup (dual-coil humbucker), a bridge-mounted piezo-electric pickup, and a small diaphragm condenser microphone (Neumann KM184)—capture the signal output from one or more of Rosebud I's actuated strings. As follows, external analog or digital signal processors modify the transduced signal by either transforming the resultant spectra or applying time-based functions, such as reverberation. To these ends, we employ only three external signal-processing devices: an Electro-Harmonix 'Big Muff' (π) distortion/sustainer [sic], a Lexicon 'Alex' digital effects processor, and the 'ReaPitch' Fourier-based pitch-shifting plugin (native to the digital audio workstation, *Reaper*).[273] Each method of processing disrupts or displaces the linearity of some aspect of the string's spectral envelope or temporal placement.

As discussed, physical preparations hold significant bearing over spectral structure. During specified sections in the piece, we position one of two objects so as to make contact with the instrument's vibrating string(s). These objects include a 3/32"× 7" plastic straw, arranged in a perpendicular orientation to the length of each string, and a thin 3"× 5" foil wrapper covering the lateral width all six strings. To maximize the range of vertical motion during actuation, both preparations are placed loosely on top of the strings and above the instrument's six electro-magnetic actuators—located three to four inches from the left bridge. According to Hopkin and Landman's criteria, the majority of preparatory techniques employed on *Artemisia* fall within the category of "rattles on the strings," a method achieved "by attaching loose objects to the strings." Therein, nearly all preparations in the piece involve objects of nominal weight, making intermittent contact with the string during actuation. These limiting features—weight and contact—distinguish this category of preparation from "weighted strings." As described by Hopkin and Landman:

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[273] When utilized, the Lexicon 'Alex' is set to the "Large Hall" reverberation setting, with the decay time set to the device's maximum value. Similarly, I consistently position the 'volume', 'sustain', and 'tone' controls of the Electro-Harmonix 'Big Muff (π)' pedal to twelve, three, and four o'clock, respectively.

> "Adding small weights to a string distorts the frequency relationship [between harmonic partials], causing the overtones to become inharmonic […] In the case of weighted strings, the effect is often strangely gong-like; not at all like what we normally think of as a string tone […] Adding buzzes and rattles to the string affects the situation a little differently. These additions may vibrate along with the string in ways that bring out certain harmonic overtones very conspicuously." [274]

Similarly, whereas Collin's attachment of alligator clips to the actuated strings of the Backwards Guitar introduced substantial and sustained weight—thereby disrupting the strings' natural overtone series and producing inharmonic "gamelan-style" timbres—the intermittent contact and relatively low weight of the Teflon ring preparation for *Under the Sun—A Pythagorean Experiment* (1976) retains a greater degree of harmonicity.[275] While intrinsic to the physical structure of the instrument, we also include secondary vibrations and other mechanical artifacts. By increasing the gain of signals sent to the actuator(s), strings may be driven to the extent that sympathetic vibrations occur between the string(s), bridge(s), and aluminum body of the instrument. Thus, the act of controlling volume produces sonorities similar to Hopkin and Landman's account of "buzzing bridge" techniques. In an earlier publication, Hopkin describes two additional examples of extant instrumentation employing sympathetic or secondary mechanical vibrations between strings, bridge, and body. Notably, the vina, tamboura, and sitar all employ some form of "buzzing bridge" consisting of a curved surface—or *jawari*—positioned within an inch of where the strings terminate. As the vibrating strings make intermittent contact with the jawari, these secondary vibrations produce additional, high-order harmonics.[276]

While Hopkin's first example describes sympathetic vibrations between the string and bridge, his account of another instrument, the Trumpet Marine, appears more analogous to the mechanical artifacts and resultant sonorities produced by Rosebud I. Here, the rich timbre results

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[274] Bart Hopkin and Yuri Landman, Nice Noise: Preparations and Modifications for Guitar, Point Reyes Station, CA: Experimental Musical Instruments, 2012.

[275] Nicolas Collins, "A Brief History of the 'Backwards Electric Guitar' (2009)," https://www.nicolascollins.com/texts/BackwardsElectricGuitar.pdf (Accessed August 9, 2019).

[276] Bart Hopkin, *Musical Instrument Design: Practical Information for Instrument Making*, Tuscon, AZ: See Sharp Press, 1996.

from mechanical interactions between the string, bridge, *and* body of the instrument. By design, the Trumpet Marine's single string passes over a U-shaped bridge, with one "foot" firmly attached to the instrument's sound board and the other barely making contact with the same surface. However, as the performer bows the string, periodic variations in pressure cause the shorter foot to make intermittent contact with the sound board. The resultant buzzing between the two surfaces produces a bright "trumpet-like" timbre indicative of the instrument's namesake.[277] Similarly, strong vibrations from Rosebud I's strings sometimes generate intermittent "buzzes" and other mechanical artifacts—presumably, originating from the small gaps between the bridge and body of the instrument. However, further investigation may yet yield the exact source.

In terms of compositional structure, external signal processing and physical preparations often function in concert, with each topology fulfilling a similar objective. For example, during the first nine states of *Artemisia*, we witness a progressive thickening in spectral density resulting from both forms of disruptive preparation. Beginning in state one, a swell of harmonic distortion imbues each of the seven actuated harmonics of string VI $(\frac{8}{9} \cdot F_{1/1})$ with their own ascending overtone series'. This exponential increase in harmonic content distributes energy into higher frequency registers, thus expanding both the range, density, and intervallic complexity between actuated partials *and* frequencies introduced via harmonic distortion.

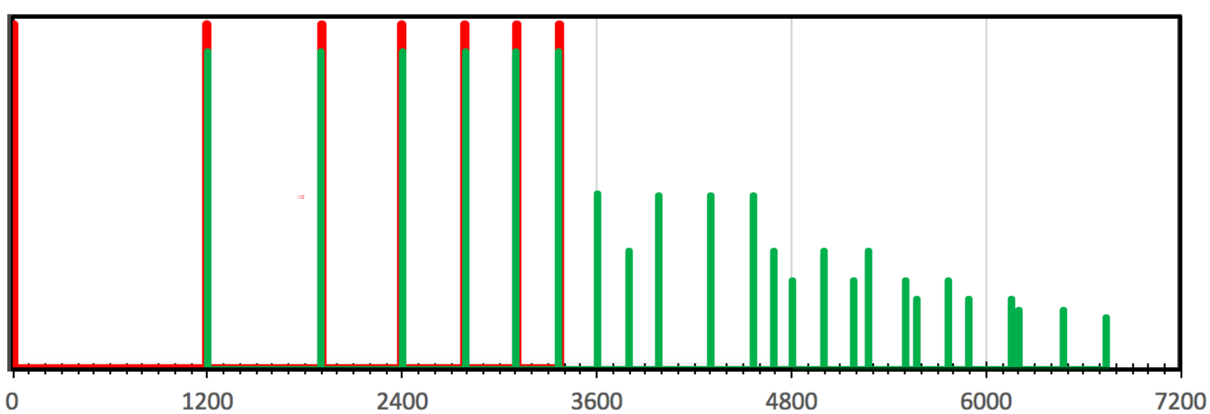

Figure 5. 9—Actuated Harmonics (Red) and Additional Partials Generated Via Harmonic Distortion (Green)

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[277] Bart Hopkin, *Musical Instrument Design: Practical Information for Instrument Making*, Tuscon, AZ: See Sharp Press, 1996.

Moving into the second and third states, intervallic complexity continues to increase as the performer actuates and distorts harmonics from adjacent strings: V ($\frac{9}{10} \cdot F_{1/1}$), IV ($\frac{4}{3} \cdot 5, 6$), III ($\frac{20}{1} \cdot F_{1/1}$), and I ($\frac{2}{1} \cdot 3, 5, 7$). Transitioning between the third and fourth states, distorted harmonics merge with a synthesized ostinato pattern, mirroring the fundamental frequency and ascending overtones of string II ($\frac{11}{6} \cdot F_{1/1}$). Here, as in other sections to follow, both harmonic distortion and physical preparations introduce ambiguity between actuated and synthesized partials. Moreover, the synthesized ostinato acts as a bridge between physical and procedurally disruptive topologies.

(5 - 16)

State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*Disruptive Preparation(s)*
1	Track 1 0:00	$\frac{8}{9} \cdot F_{1/1}$	1.0	1	0.889	$8/9 \cdot 1$	Electro-Harmonix (π) Distortion/Sustainer (VI)
				2	1.778	$8/9 \cdot 2$	Electro-Harmonix (π) Distortion/Sustainer (VI)
				3	2.667	$8/9 \cdot 3$	Electro-Harmonix (π) Distortion/Sustainer (VI)
				4	3.556	$8/9 \cdot 4$	Electro-Harmonix (π) Distortion/Sustainer (VI)
				5	4.444	$8/9 \cdot 5$	Electro-Harmonix (π) Distortion/Sustainer (VI)
				6	5.333	$8/9 \cdot 6$	Electro-Harmonix (π) Distortion/Sustainer (VI)
				7	6.222	$8/9 \cdot 7$	Electro-Harmonix (π) Distortion/Sustainer (VI)
2	Track 1 0:16	$\frac{9}{10} \cdot F_{1/1}$	1.227	1	0.900	$9/10 \cdot 1$	Electro-Harmonix (π) Distortion/Sustainer (V)
				2	2.107	$2/1 \cdot 1$	Electro-Harmonix (π) Distortion/Sustainer (I)
				3	3.465	$8/9 \cdot 4$	Electro-Harmonix (π) Distortion/Sustainer (VI)
				4	4.931	$8/9 \cdot 6$	Electro-Harmonix (π) Distortion/Sustainer (VI)
				5	6.485	$4/3 \cdot 5$	Electro-Harmonix (π) Distortion/Sustainer (IV)
				6	8.110	$4/3 \cdot 6$	Electro-Harmonix (π) Distortion/Sustainer (IV)
				7	9.799	$2/1 \cdot 5$	Electro-Harmonix (π) Distortion/Sustainer (I)
3	Track 1 0:32	$\frac{20}{11} \cdot F_{1/1}$	1.0765	1	1.818	$20/11 \cdot 1$	Electro-Harmonix (π) Distortion/Sustainer (III)
				2	3.834	$4/3 \cdot 3$	Electro-Harmonix (π) Distortion/Sustainer (IV)
				3	5.933	$2/1 \cdot 3$	Electro-Harmonix (π) Distortion/Sustainer (I)
				4	8.086	$4/3 \cdot 6$	Electro-Harmonix (π) Distortion/Sustainer (IV)
				5	10.282	$2/1 \cdot 5$	Electro-Harmonix (π) Distortion/Sustainer (I)
				6	12.512	$20/11 \cdot 7$	Electro-Harmonix (π) Distortion/Sustainer (III)
				7	14.770	$2/1 \cdot 7$	Electro-Harmonix (π) Distortion/Sustainer (I)
4	Track 1 0:40	$\frac{11}{6} \cdot F_{1/1}$	1.0655	1	1.833	$11/6 \cdot 1$	3/32"× 7" Plastic Straw – Piezo / KM184 (II)
				2	3.837	$2/1 \cdot 2$	3/32"× 7" Plastic Straw – Piezo / KM184 (I)
				3	5.910	$2/1 \cdot 3$	3/32"× 7" Plastic Straw – Piezo / KM184 (I)
				4	8.030	$2/1 \cdot 4$	3/32"× 7" Plastic Straw – Piezo / KM184 (I)
				5	10.186	$2/1 \cdot 5$	3/32"× 7" Plastic Straw – Piezo / KM184 (I)
				6	12.370	$20/11 \cdot 7$	
				7	14.578	$2/1 \cdot 7$	3/32"× 7" Plastic Straw – Piezo / KM184 (I)
5	Track 1 0:46	$\frac{2}{1} \cdot F_{1/1}$	1.0	1	2	$2/1 \cdot 1$	3/32"× 7" Plastic Straw – Piezo / KM184 (I)
				2	4	$2/1 \cdot 2$	3/32"× 7" Plastic Straw – Piezo / KM184 (I)
				3	6	$2/1 \cdot 3$	3/32"× 7" Plastic Straw – Piezo / KM184 (I)
				4	8	$2/1 \cdot 4$	3/32"× 7" Plastic Straw – Piezo / KM184 (I)
				5	10	$2/1 \cdot 5$	3/32"× 7" Plastic Straw – Piezo / KM184 (I)
				6	12	$2/1 \cdot 6$	3/32"× 7" Plastic Straw – Piezo / KM 184 (I)
				7	14	$2/1 \cdot 7$	3/32"× 7" Plastic Straw – Piezo / KM 184 (I)

With the physical preparation of strings II $(\frac{11}{6} \cdot F_{1/1})$ and I $(\frac{2}{1} \cdot F_{1/1})$ in state number four, actuated impulses cause the light-weight plastic straw to dance across the adjacent strings, resulting in a rapid flurry of lateral collisions between the straw and strings. In motion and timbre, this mode of preparation resembles the effect produced by a mallet striking the strings of a hammered dulcimer, Santoor, or other stringed, percussion instrument. As the straw collides with each string, we hear not only the actuated partial; but also, the fundamental frequency and overtones generated by each percussive impact. Distinct from the procedural effects of harmonic distortion, we perceive not only the addition of overtones; but also, the root or fundamental frequency from which the actuated partials originate. In certain instances, spectral density results from ascending *and* descending artifacts—thus signifying intervallic features representative of both the overtone *and* undertone series. For example, as a result of physical preparations in state number four, actuation of the second, third, fourth, fifth, and seventh harmonics of string I $(\frac{2}{1} \cdot F_{1/1})$ also generates the missing fundamental frequency $(2/1 \cdot 1)$, as well as sixth overtone $(2/1 \cdot 6)$ of the same string. Taken as sub-harmonics of a descending series, string I's fundamental frequency $(\frac{2}{1} \cdot F_{1/1})$ functions dually, as the second, third, fourth, fifth, and seventh undertone for actuated harmonics $(2/1 \cdot 2)$, $(2/1 \cdot 3)$, $(2/1 \cdot 4)$, $(2/1 \cdot 5)$, and $(2/1 \cdot 7)$. Certainly, the psychoacoustic phenomenon behind the perception of missing fundamental frequencies is not without precedent. Here, audible tones whose frequencies represent a consecutive sequence of integral multiples of a common frequency may elicit the perception of a lower tone—even if this frequency component is absent from the spectrum.[278] [279] In this sense, the form of disruptive preparation described in *Artemisia* provides a tangible analog for addressing the perceptual qualities of similarly, incomplete spectra. While actuation places sole emphasis upon certain

[278] Ernst Terhardt, "The Concept of Musical Consonance: A Link between Music and Psychoacoustics," *Music Perception: An Interdisciplinary Journal 1*, no. 3 (1984): 276-95. https://www.jstor.org/stable/pdf/40285261.pdf?ab_segments=0%2Fbasic_SYC-5187_SYC-5188%2F5188&refreqid=fastly-default%3A1f8efd92f12af9e46e9705e9e30066ab (accessed December 22, 2020).

[279] Stephen McAdams and Daniel Pressnitzer, "Acoustics, Psychoacoustics, and Spectral Music," *Contemporary Music Review*, Vol. 19, Part 2 (2000): 33-59.

partials, disruptive preparation *fills-in* the gaps between these actuated harmonics or missing fundamentals. Thus, we perceive a more cohesive spectrum.

As in state number one, where the additive properties of harmonic distortion generate an ascending overtone series for each of the seven actuated partials of string VI $\left(\frac{8}{9} \cdot F_{1/1}\right)$, physical preparation of string I $\left(\frac{2}{1} \cdot F_{1/1}\right)$ produces similar spectral effects in state number five. In each case, the performer actuates a linear sequence of harmonic partials beginning with the fundamental frequency of a single string: VI or I. Unlike other states, no spectral stretching, transposition, or other inharmonic procedures affect the intervals between actuated partials. At the point of actuation, all proportions remain integral, or *just*. Instead, spectral distortion derives from subsequent, physical preparations. Here, the transition to state five unfolds as a backdrop to the sustained actuation and similar preparation of string II $\left(\frac{11}{6} \cdot F_{1/1}\right)$, with frequency content from the previous state remaining most prominent.

Methods of physical preparation also remain consistent throughout the next three states (6-8), with the same $3/32" \times 7"$ plastic straw placed in a perpendicular orientation across the actuated strings. However, the introduction of new actuated partials and a subtle change in tone color belie these similarities. In line with the established structure for *Artemisia,* each of the 120 states represents a distinct sonority. Therein, five to six states of maximal spectral similarity precede a sonority whose spectrum shares the least number of common partials with the previous state. As such, the introduction of state number six represents a demarcation between maximal and minimal spectral similarity, thus heralding a new section in the piece (state numbers 6-11). While state number six shares no common partials with state number five, each of following five states share at least one common partial with the previous, actuated sonority. For example, in state number six we no longer hear actuated harmonics from string I $\left(\frac{2}{1} \cdot F_{1/1}\right)$. Instead, actuated partials from strings VI $\left(\frac{8}{9} \cdot 1\right)$, V $\left(\frac{9}{10} \cdot 3\right)$, IV $\left(\frac{4}{3} \cdot 5\right)$, III $\left(\frac{20}{11} \cdot 1\right)$, and II $\left(\frac{11}{6} \cdot 2, 3\right)$ gain prominence. However, while the fundamental frequency and other actuated partials shift between states, the consistent presence of actuated partials from string II $\left(\frac{11}{6} \cdot F_{1/1}\right)$ contribute to a sense of continuity within the section. Furthermore, physical preparations of the same string ensure that both actuated and interceding partials, as well the fundamental frequency $\left(\frac{11}{6} \cdot 1\right)$, remain audible throughout the section.

(5 - 17)

State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*Disruptive Preparation(s)*
6 *diff.	Track 1 0:52	$\frac{8}{9} \cdot F_{1/1}$	1.0252	1	0.889	$8/9 \cdot 1$	3/32"× 7" Plastic Straw – Piezo / Humbucker (VI)
				2	1.809	$20/11 \cdot 1$	3/32"× 7" Plastic Straw – Piezo / Humbucker (III)
				3	2.742	$9/10 \cdot 3$	3/32"× 7" Plastic Straw – Piezo / Humbucker (V)
				4	3.682	$11/6 \cdot 2$	3/32"× 7" Plastic Straw – Piezo / Humbucker (II)
				5	4.628	$9/10 \cdot 5$	3/32"× 7" Plastic Straw – Piezo / Humbucker (V)
				6	5.580	$4/3 \cdot 5$	3/32"× 7" Plastic Straw – Piezo / Humbucker (IV)
				7	6.535	$11/6 \cdot 3$	3/32"× 7" Plastic Straw – Piezo / Humbucker (II)
7	Track 1 1:08	$\frac{20}{11} \cdot F_{1/1}$	1.011	1	1.818	$20/11 \cdot 1$	3/32"× 7" Plastic Straw – KM184 / Humbucker (III)
				2	3.664	$11/6 \cdot 2$	3/32"× 7" Plastic Straw – KM184 / Humbucker (II)
				3	5.521	$11/6 \cdot 3$	3/32"× 7" Plastic Straw – KM184 / Humbucker (II)
				4	7.384	$11/6 \cdot 4$	3/32"× 7" Plastic Straw – KM184 / Humbucker (II)
				5	9.253	$4/3 \cdot 7$	3/32"× 7" Plastic Straw – KM184 / Humbucker (IV)
				6	11.126	$11/6 \cdot 6$	3/32"× 7" Plastic Straw – KM184 / Humbucker (II)
				7	13.003	$11/6 \cdot 7$	3/32"× 7" Plastic Straw – KM184 / Humbucker (II)
8	Track 1 1:24	$\frac{9}{10} \cdot F_{1/1}$	1.303	1	0.900	$9/10 \cdot 1$	3/32"× 7" Plastic Straw – KM184 (V)
				2	2.221	$2/1 \cdot 1$	3/32"× 7" Plastic Straw – KM184 (I)
				3	3.766	$11/6 \cdot 2$	3/32"× 7" Plastic Straw – KM184 (II)
				4	5.479	$11/6 \cdot 3$	3/32"× 7" Plastic Straw – KM184 (II)
				5	7.328	$11/6 \cdot 4$	3/32"× 7" Plastic Straw – KM184 (II)
				6	9.293	$4/3 \cdot 7$	3/32"× 7" Plastic Straw – KM184 (IV)
				7	11.361	$11/6 \cdot 6$	3/32"× 7" Plastic Straw – KM184 (II)
9	Track 1 1:40	$\frac{11}{6} \cdot F_{1/1}$	1.006	1	1.833	$11/6 \cdot 1$	3"× 5" Foil Wrapper – KM184 (II)
				2	3.682	$11/6 \cdot 2$	
				3	5.536	$11/6 \cdot 3$	
				4	7.395	$11/6 \cdot 4$	
				5	9.256	$4/3 \cdot 7$	
				6	11.119	$11/6 \cdot 6$	
				7	12.984	$11/6 \cdot 7$	3"× 5" Foil Wrapper – KM184 (II)

Though the method of physical preparation remains fixed between states five and six, techniques for transducing the instrument's audio output shift as we transition between sections. Whereas the performer captures and amplifies audio output in states four and five using a Neumann KM184 microphone and piezo-electric pickup, state number six introduces signal from the humbucker pickup.[280] As state numbers 6-8 unfold, we hear a subtle transformation in tone color as the performer blends audio outputs from the piezo-electric, humbucker, and KM184 signal paths.

[280] While testing a number of microphone configurations, we ultimately settled upon positioning the Neumann KM184 approximately one-inch above string IV and three inches away from the terminating bridge, near the humbucker pickup.

Throughout the piece, similar gestures recur as a result of the same physical preparations. For example, actuating the fundamental frequency of string V $\left(\frac{9}{10} \cdot 1\right)$ during state 33 induces a rapid series of percussive impulses, as the straw ricochets off the single string. In contrast, a separate audio stream actuates the remaining partials—a nearly linear reproduction of the first six harmonics of string I $\left(\frac{2}{1} \cdot F_{1/1}\right)$, sans preparation. Here, physical modes of disruptive preparation signify a coda. However, the same methods and distinctive timbres also introduce later sections. Again, in state 54, comparable ricochet effects serve to emphasize the instantiation of a new section. Recorded in multiple takes, a combination of piezo-electric and humbucker pickups transduces vibrations from strings V $\left(\frac{9}{10} \cdot F_{1/1}\right)$ and I $\left(\frac{2}{1} \cdot F_{1/1}\right)$, while the KM184 captures audio from string II $\left(\frac{11}{6} \cdot F_{1/1}\right)$. Equivalent modes of preparations occur in states 74-75, as well.

(5 - 18)

State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*Disruptive Preparation(s)*
33	Track 1 8:02	$\frac{9}{10} \cdot F_{1/1}$	1.338	1	0.900	$9/10 \cdot 1$	3/32"× 7" Plastic Straw – KM184 (V)
				2	2.275	$2/1 \cdot 1$	
				3	3.914	$4/3 \cdot 3$	
				4	5.752	$2/1 \cdot 3$	
				5	7.753	$4/3 \cdot 6$	
				6	9.895	$2/1 \cdot 5$	
				7	12.161	$2/1 \cdot 6$	

State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*Disruptive Preparation(s)*
54 *diff.	Track 3 0:00	$\frac{9}{10} \cdot F_{1/1}$	1.3095	1	0.900	$9/10 \cdot 1$	3/32"× 7" Plastic Straw – Piezo / Humbucker (V)
				2	2.231	$2/1 \cdot 1$	3/32"× 7" Plastic Straw – Piezo / Humbucker (I)
				3	3.793	$11/6 \cdot 2$	3/32"× 7" Plastic Straw – KM184 (II)
				4	5.529	$11/6 \cdot 3$	3/32"× 7" Plastic Straw –KM184 (II)
				5	7.405	$11/6 \cdot 4$	3/32"× 7" Plastic Straw –KM184 (II)
				6	9.402	$4/3 \cdot 7$	3/32"× 7" Plastic Straw – Piezo / Humbucker (IV)
				7	11.505	$2/1 \cdot 6$	3/32"× 7" Plastic Straw – Piezo / Humbucker (I)

74	Track 3 6:26	$\frac{2}{1} \cdot F_{1/1}$	1.071	1	2.000	$2/1 \cdot 1$	3/32"× 7" Plastic Straw – Humbucker (I)
				2	4.202	$2/1 \cdot 2$	3/32"× 7" Plastic Straw – Humbucker (I)
				3	6.487	$4/3 \cdot 5$	
				4	8.827	$20/11 \cdot 5$	
				5	11.211	$11/6 \cdot 6$	
				6	13.628	$2/1 \cdot 7$	
				7	16.074	n/a	

75	Track 3 6:41	$\frac{9}{10} \cdot F_{1/1}$	1.3985	1	0.900	$9/10 \cdot 1$	3/32"× 7" Plastic Straw – KM184 (V)
				2	2.373	$8/9 \cdot 3$	3/32"× 7" Plastic Straw – KM184 (VI)
				3	4.183	$4/3 \cdot 3$	
				4	6.255	$20/11 \cdot 5$	
				5	8.546	$8/9 \cdot 7$	
				6	11.028	$11/6 \cdot 6$	
				7	13.681	$2/1 \cdot 7$	

In state number nine tonal quality shifts more radically, with the introduction of a second prepared object: a 3"× 5" foil wrapper. Like the plastic straw, the performer places the foil across the six strings and directly above the electro-magnetic actuators. As with the majority of preparatory techniques in the piece, the method and materials employed fall within Hopkin and Landman's broad category of "rattles on the strings." Remaining un-affixed to the instrument and applying negligible weight to the actuated strings, the thin foil introduces very few inharmonic artifacts to the resultant spectra. Instead, interactions between the vibrating strings and loose foil create additional, high-order harmonic partials. Similar to other procedural forms of harmonic distortion, this mode of disruptive preparation tends to concentrate energy in higher frequency registers. In fact, the aural effect is, at times, difficult to distinguish from harmonic distortion introduced by the Electro-Harmonix pedal or the "nasal" buzz generated by saw-tooth and pulse-wave oscillators from the JUNO-6 synthesizer. Parsing these sources becomes particularly difficult when the respective preparations and synthesis techniques occur simultaneously. None the less, material properties of the straw and foil retain distinct features. Though rapid in succession, we perceive the intermittent contact between the straw and string in states 4-8 as discrete, percussive attacks. Holding the straw loosely between the thumb and middle finger, the performer may also vary the volume and attack-rate by applying manual pressure upon the straw and strings. With practice, such performative interventions can produce percussive gestures akin to a "multiple-bounce" or "buzz roll."[281] In contrast, vibrations induced between the actuated strings and irregular surfaces of the foil produce a steady, buzzing timbre. Structural features aside, the sustained effect closely resembles sounds produced by interactions between the strings and *jawari* of Hopkin's "buzzing bridge."[282]

Continuing through states 12-14, disruptive preparations maintain certain formal objectives. Here, harmonic artifacts highlight specific, intervallic structures formed between actuated partials. With the introduction of state number twelve, an ascending series of overtones

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[281] Percussive Arts Society, "The Forty Percussive Arts Society International Drum Rudiments (1984)," https://www.pas.org/resources/rudiments (accessed December 23, 2020).

[282] Bart Hopkin, *Musical Instrument Design: Practical Information for Instrument Making*, Tuscon, AZ: See Sharp Press, 1996.

sound from each actuated partial for five of the instrument's six strings: V $\left(\frac{9}{10} \cdot 7\right)$, IV $\left(\frac{4}{3} \cdot 3\right)$, III $\left(\frac{20}{11} \cdot 5\right)$, II $\left(\frac{11}{6} \cdot 6\right)$, and I $\left(\frac{2}{1} \cdot 7\right)$. Assessing a broad array of partials for five strings generates substantial harmonic complexity, while revealing differential features which define the onset of a new section. In contrast, using foil to prepare only two of the instruments strings and associated harmonics—V $\left(\frac{9}{10} \cdot 1\right)$ and IV $\left(\frac{4}{3} \cdot 3\right)$—focuses our attention on fewer intervallic structures. The combined effect of actuating the fundamental frequency of string V and the ascending series of high-order overtones afforded through physical preparation emphasize the just ratio, $9/10\left(\frac{9}{10} \cdot 55\,Hz = 49.5\,Hz\right)$. Coincidentally, the cumulative effect of actuating the third partial of string IV $\left(\frac{4}{3} \cdot 3\ or\ \frac{4}{1}\right)$ and subsequent physical preparation emphasizes both the harmonic and fundamental frequency of the string. Thus, we perceive a ratio derivative for the actuated partial, $4/1$ $\left(\frac{4}{1} \cdot 55\,Hz = 220\,Hz\right)$ and string IV's fundamental frequency, $4/3$ $\left(\frac{4}{3} \cdot 55\,Hz = 73.333\,Hz\right)$. Divisional procedures between the ratios yield three intervallic relations whose unique characteristics define the tonal structure of state number thirteen:

(5 - 19)

$$4/1\ (220\,Hz) \div 4/3\ (73.333\,Hz) = 3/1\ \ (or\ 1\ Octave + Just\ Perfect\ 5th)$$

$$4/1\ (220\,Hz) \div 9/10\ (49.5\,Hz) = 40/9\ \ (or\ 2\ Octave + 5\ limit\ Just\ Whole\ Tone)$$

$$4/3\ (73.333\,Hz) \div 9/10\ (49.5\,Hz) = 40/27\ \ (5\ limit\ \text{"Wolf"}\ 5th)$$ [283]

Moving onto state number fourteen, similar preparations bear less influence upon the intervallic characteristics of subsequent states within the section. Instead, we perceive a shift in timbre affecting the fundamental and fifth harmonic of string III $\left(\frac{20}{11} \cdot 1, 5\right)$, as well as other harmonic artifacts, introduced via physical preparation of the same string.

[283] Kyle Gann, "Anatomy of an Octave," *Just Intonation: General Theory and Reference*, https://www.kylegann.com/Octave.html, (accessed August 15, 2017).

(5 - 20)

State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*Disruptive Preparation(s)*
12 *diff.	Track 1 2:36	$\frac{11}{6} \cdot F_{1/1}$	1.118	1	1.833	$11/6 \cdot 1$	3"× 5" Foil Wrapper – KM184 (II)
				2	3.979	$4/3 \cdot 3$	3"× 5" Foil Wrapper – KM184 (IV)
				3	6.261	$9/10 \cdot 7$	3"× 5" Foil Wrapper – KM184 (V)
				4	8.637	$20/11 \cdot 5$	3"× 5" Foil Wrapper – KM184 (III)
				5	11.084	$11/6 \cdot 6$	3"× 5" Foil Wrapper – KM184 (II)
				6	13.590	$2/1 \cdot 7$	3"× 5" Foil Wrapper – KM184 (I)
				7	16.146	n/a	

13	Track 1 2:53	$\frac{9}{10} \cdot F_{1/1}$	1.3995	1	0.900	$9/10 \cdot 1$	3"× 5" Foil Wrapper – KM184 (V)
				2	2.374	$8/9 \cdot 3$	
				3	4.188	$4/3 \cdot 3$	3"× 5" Foil Wrapper – KM184 (IV)
				4	6.264	$9/10 \cdot 7$	
				5	8.560	$20/11 \cdot 5$	
				6	11.048	$11/6 \cdot 6$	
				7	13.707	$2/1 \cdot 7$	

14	Track 1 3:09	$\frac{20}{11} \cdot F_{1/1}$	1.126	1	1.818	$20/11 \cdot 1$	3"× 5" Foil Wrapper – KM184 (III)
				2	3.968	$4/3 \cdot 3$	
				3	6.264	$9/10 \cdot 7$	
				4	8.661	$20/11 \cdot 5$	3"× 5" Foil Wrapper – KM184 (III)
				5	11.135	$11/6 \cdot 6$	
				6	13.672	$2/1 \cdot 7$	
				7	16.264	n/a	

Again, in states 19-22 disruptive preparations serve to focus our attention upon discrete, intervallic properties—specifically those intrinsic to the principles and practice of just intonation. Within this section, harmonic distortion functions additively by introducing an ascending overtone series to each of four, closely-tuned fundamental frequencies: $\left(\frac{10}{11} \cdot F_{1/1} \ or \ \frac{20}{11} \cdot 1/2\right)$, $\left(\frac{11}{12} \cdot F_{1/1} \ or \ \frac{20}{11} \cdot 1/2\right)$, $\left(\frac{8}{9} \cdot F_{1/1}\right)$, and $\left(\frac{9}{10} \cdot F_{1/1}\right)$. In the case of states nineteen and twenty, these ratios represent sub-octave transpositions of strings III $\left(\frac{20}{11} \cdot F_{1/1}\right)$ and II $\left(\frac{11}{6} \cdot F_{1/1}\right)$, respectively. To accommodate octave transposition within the section (states 18-23), the performer tunes strings VI and V to the fundamental ratios $\left(\frac{20}{11} \cdot 1/2\right)$ and $\left(\frac{11}{12} \cdot 1/2\right)$—notated as VI+ and V+.

(5 - 21)

State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*Disruptive Preparation(s)*
19	Track 1 4:33	$\frac{20}{11} \cdot F_{1/1}$	1.0385	1	1.818	$20/11 \cdot 1/2$	JUNO→RBI—EH (π) Distortion/Sustainer (VI+)
				2	3.735	$11/6 \cdot 2$	
				3	5.690	$11/6 \cdot 3$	
				4	7.671	$4/3 \cdot 6$	
				5	9.672	$2/1 \cdot 5$	
				6	11.688	$2/1 \cdot 6$	
				7	13.717	$2/1 \cdot 7$	

20	Track 1 4:50	$\frac{11}{6} \cdot F_{1/1}$	1.033	1	1.833	$11/6 \cdot 1/2$	JUNO→RBI—EH (π) Distortion/Sustainer (V+)
				2	3.752	$11/6 \cdot 2$	
				3	5.703	$11/6 \cdot 3$	
				4	7.677	$4/3 \cdot 3$	
				5	9.667	$2/1 \cdot 5$	
				6	11.670	$2/1 \cdot 6$	
				7	13.684	$2/1 \cdot 7$	

21	Track 1 5:06	$\frac{8}{9} \cdot F_{1/1}$	1.339	1	0.889	$8/9 \cdot 1$	JUNO→RBI—EH (π) Distortion/Sustainer (VI)
				2	2.249	$2/1 \cdot 1$	
				3	3.870	$4/3 \cdot 3$	
				4	5.689	$11/6 \cdot 3$	
				5	7.670	$4/3 \cdot 6$	
				6	9.790	$2/1 \cdot 5$	
				7	12.035	$2/1 \cdot 6$	

22	Track 1 5:22	$\frac{9}{10} \cdot F_{1/1}$	1.3315	1	0.900	$9/10 \cdot 1$	JUNO→RBI—EH (π) Distortion/Sustainer (V)
				2	2.265	$2/1 \cdot 1$	
				3	3.886	$4/3 \cdot 3$	
				4	5.700	$11/6 \cdot 3$	
				5	7.672	$4/3 \cdot 6$	
				6	9.780	$2/1 \cdot 5$	
				7	12.009	$2/1 \cdot 6$	

As described, this set of four fundamental frequencies represent a consecutive sequence of epimoric (super-particular) ratios: $8/9, 9/10, 10/11,$ and $11/12$. Progressing through states 19-22, the interceding intervals between the epimoric ratios form three significant commas: $121/120$ or ≈ 14 cents (*Biyatisma Comma*), $33/32$ or ≈ 53 cents (*Unidecimal Comma*), and $81/80$ or ≈ 22 cents (*Syntonic Comma*). Superimposed over other actuated partials, distorted harmonics for each fundamental ratio fuse to form secondary spectra. In performance, transition between states 19-22 unfolds in a fluid fashion, allowing significant overlap between consecutive spectra. Consequently, interactions between these interceding sonorities generate complex interference patterns. We perceive the results of this phenomenon as periodic changes in amplitude, or *beating*. During transitory these states, resultant commas retain beat frequencies equivalent to the arithmetic difference between the fundamental frequencies of consecutively actuated strings: VI+, V+, VI, and V. Of course, differential beating occurs between other actuated partials, as well as secondary frequencies introduced via harmonic distortion. Furthermore, these secondary frequencies double those produced by third-party audio streams from the JUNO-6 synthesizer. Within this section, audio from the synthesizer can be heard as both an actuation source *and* accompanying instrumentation. In fact, as previously demonstrated in state one, similar spectral qualities of the two instruments obscure their individual identities during playback.

However, the most salient (and lowest frequency) interference patterns originate from interactions between fundamental frequencies. Spanning states 19-22, we perceive a sequence of ascending and descending beat frequencies of approximately: 0.416 Hertz $\{(\frac{11}{12} \cdot F_{1/1} \approx 50.416\,Hz) - (\frac{11}{12} \cdot F_{1/1} \approx 50\,Hz)\}$, 1.527 Hertz $\{(\frac{11}{12} \cdot F_{1/1} \approx 50.416\,Hz) - (\frac{8}{9} \cdot F_{1/1} \approx 48.888\,Hz)\}$, and 0.611 Hertz $\{(\frac{9}{10} \cdot F_{1/1} \approx 49.5\,Hz) - (\frac{8}{9} \cdot F_{1/1} \approx 48.888\,Hz)\}$. These resulting pulsations represent the combined rhythmic manifestation of physical actuation, disruptive preparation, and just tuning structures—namely those practices which employ epimoric ratios and their derivative commas.

(5 - 22)

# 19 10/11 (VI+) ≈ 50 Hz	→	# 20 11/12 (V+) ≈ 50.416 Hz	→	# 21 8/9 (VI) ≈ 48.888 Hz	→	# 22 9/10 (V) ≈ 49.5 Hz
	121/120 ≈ 14 cents *Biyatisma Comma* Beat Freq. ≈ 0.416 Hz		33/32 ≈ 53 cents *Unidecimal Comma* Beat Freq. ≈ 1.527 Hz		81/80 ≈ 22 cents *Syntonic Comma* Beat Freq. ≈ 0.611 Hz	

Moving to states 34 and 35, the performer employs procedural preparations to transpose and sustain actuated spectra. Structurally, these two states function as a preamble to the instantiation of a new section, beginning at state 36. After recording the actuated output of Rosebud I, signal from each track is then routed (via an auxillary bus) through Reaper's native pitch-shifting algorithm (*ReaPitch*), before reaching the electro-mechanical actuator for Rosebud II's twelve strings. Here, we witness another instance of third-party audio streams—this time, involving signals from two actuated instruments. Moreover, integral features of procedural preparation, just tuning structures, and third-party actuation perform intersect within each state. For example, in both states 34 and 35, transposing actuated partials by a Pythagorean perfect fifth—the just ratio of 3/2—affords the actuation of sympathetically tuned partials within certain pairings of Rosebud II's strings. In state 34, all actuated partials from Rosebud I represent a linear sequence of harmonics from string I $(\frac{2}{1} \cdot 1, 2, 3, 4, 5, 6, 7)$. Transposed by a Pythagorean perfect fifth ($\cdot$ 3/2), many of the effected partials now share common harmonics (2^{nd} – 6^{th}) with strings I

$\left(\frac{6}{1} \cdot 1, 2, 3\right)$, II $\left(\frac{6}{1} \cdot 1, 2, 3\right)$, IX $\left(\frac{5}{1} \cdot 3\right)$, XI $\left(\frac{9}{2} \cdot 2, 4\right)$, and XII $\left(\frac{4}{1} \cdot 3\right)$ of Rosebud II. Third-party actuation of the same set of transposed partials recur through state 35, as well.

(5 - 23)

State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*Disruptive Preparation(s)*
34	Track 2 0:00	$\frac{2}{1} \cdot F_{1/1}$	1.0	1	2.000	$2/1 \cdot 1$	RBI (I) – Pitch-Shift (· 3/2) → RBII – Lexicon
				2	4.000	$2/1 \cdot 2$	RBI (I/IV) – Pitch-Shift (· 3/2) → RBII – Lexicon
				3	6.000	$2/1 \cdot 3$	RBI (I) – Pitch-Shift (· 3/2) → RBII – Lexicon
				4	8.000	$2/1 \cdot 4$	RBI (I/IV) – Pitch-Shift (· 3/2) → RBII – Lexicon
				5	10.000	$2/1 \cdot 5$	RBI (I) – Pitch-Shift (· 3/2) → RBII – Lexicon
				6	12.000	$2/1 \cdot 6$	RBI (I) – Pitch-Shift (· 3/2) → RBII – Lexicon
				7	14.000	$2/1 \cdot 7$	RBI (I) – Pitch-Shift (· 3/2) → RBII – Lexicon

35	Track 2 0:16	$\frac{20}{11} \cdot F_{1/1}$	1.0765	1	1.818	$20/11 \cdot 1$	RBI (III) – Pitch-Shift (· 3/2) → RBII – Lexicon
				2	3.834	$4/3 \cdot 3$	RBI (I/IV) – Pitch-Shift (· 3/2) → RBII – Lexicon
				3	5.933	$2/1 \cdot 3$	RBI (I) – Pitch-Shift (· 3/2) → RBII – Lexicon
				4	8.086	$4/3 \cdot 6$	RBI (I/IV) – Pitch-Shift (· 3/2) → RBII – Lexicon
				5	10.282	$2/1 \cdot 5$	RBI (I) – Pitch-Shift (· 3/2) → RBII – Lexicon
				6	12.512	$20/11 \cdot 7$	RBI (III) – Pitch-Shift (· 3/2) → RBII – Lexicon
				7	14.770	$2/1 \cdot 7$	RBI (I) – Pitch-Shift (· 3/2) → RBII – Lexicon

(5 - 24)

Rosebud I (Transposed)	*Electro-Mechanical Actuation*	*Rosebud II*
I $\left(\frac{2}{1} \cdot 2, 4, 6\right) \cdot \frac{3}{2}$	→	I, II $\left(\frac{6}{1} \cdot 1, 2, 3\right)$
I $\left(\frac{2}{1} \cdot 5\right) \cdot \frac{3}{2}$	→	IX $\left(\frac{5}{1} \cdot 3\right)$
I $\left(\frac{2}{1} \cdot 3, 6\right) \cdot \frac{3}{2}$	→	XI $\left(\frac{9}{2} \cdot 2, 4\right)$
I $\left(\frac{2}{1} \cdot 6\right) \cdot \frac{3}{2}$	→	XII $\left(\frac{4}{1} \cdot 3\right)$

Transpositional forms of preparation also occur in states 73 and 77, with Pythagorean proportions informing the pitch structure for each state. Here, successive transposition of the ratio 3/2 determines a ratio of 9/4 $(3/2)^2$. Partch attributes similar "cyclical" procedures to crude conceptions of the "circle of fifths."[284] Accordingly, the ratio 9/4 yields an interval roughly

[284] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

equivalent to the addition of one octave and a major second. Applying the same pitch-shifting algorithm to actuated partials of strings I $\left(\frac{2}{1} \cdot 1, 2, 6\right)$, IV $\left(\frac{4}{3} \cdot 3, 4\right)$, and III $\left(\frac{20}{11} \cdot 1\right)$ derives the just ratios 9/2, 9/1, 27/1, 12/1, and 45/11.

(5 - 25)

State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*Disruptive Preparation(s)*
73	Track 3 6:10	$\frac{20}{11} \cdot F_{1/1}$	1.1575	1	1.818	$20/11 \cdot 1$	RBI (III) – Pitch-Shift ($\cdot$ 9/4) – Lexicon
				2	4.056	$4/3 \cdot 4$	RBI (IV) – Pitch-Shift ($\cdot$ 9/4) – Lexicon
				3	6.485	$4/3 \cdot 5$	
				4	9.047	$20/11 \cdot 5$	
				5	11.714	$2/1 \cdot 6$	RBI (I) – Pitch-Shift ($\cdot$ 9/4) – Lexicon
				6	14.466	$2/1 \cdot 7$	
				7	17.292	n/a	

77	Track 3 7:15	$\frac{2}{1} \cdot F_{1/1}$	1.071	1	2.000	$2/1 \cdot 1$	RBI (I) – Pitch-Shift ($\cdot$ 9/4) – Lexicon
				2	4.202	$2/1 \cdot 2$	RBI (I/IV) – Pitch-Shift ($\cdot$ 9/4) – Lexicon
				3	6.487	$4/3 \cdot 5$	
				4	8.827	$20/11 \cdot 5$	
				5	11.211	$11/6 \cdot 6$	
				6	13.628	$2/1 \cdot 7$	
				7	16.074	n/a	

This combination of disruptive preparations affords the realization of complex textures, unattainable through singular methodologies. Following the iteration of a new section, states 61-63 employ both procedural (harmonic distortion and reverberation) and physical preparations, including mechanical artifacts. In recording, the performer parses modes of preparation according to the spectral structure for a given state. For example, to emphasize and expand the range of harmonicity around the fundamental frequency for each state ($F_{RBI\,(1-6)}$), we apply harmonic distortion to strings VI $\left(\frac{8}{9} \cdot F_{1/1}\right)$, V $\left(\frac{9}{10} \cdot F_{1/1}\right)$, an V+ $\left(\frac{11}{6} \cdot 1/2 \cdot F_{1/1}\right)$. Akin to procedures employed in states 19-22, these interceding sonorities highlight a sequence of commas. Between states 61 and 62, the progression of fundamental frequencies VI $\left(\frac{8}{9} \cdot F_{1/1}\right)$ and V $\left(\frac{9}{10} \cdot F_{1/1}\right)$ yield a *Syntonic comma* (81/80 or ≈ 22 cents), while the following passage from V $\left(\frac{9}{10} \cdot F_{1/1}\right)$ to V+ $\left(\frac{11}{6} \cdot 1/2 \cdot F_{1/1}\right)$ produces the wider, *Unidecimal Diasecundal Comma* (55/54 or ≈ 32 cents).[285] The

[285] "So-Called Farey Series, Extended 0/1 to 1/0 (Full Set of Gear Ratios), and Lambdoma," *The Wilson Archives* (1996), http://anaphoria.com/lamb.pdf (accessed March 27, 2019)

resulting harmonic spectra unfold concurrently with other physical preparations. In state 62, both forms of physical preparation affect the harmonics for separate strings. Here, preparing string VI $\left(\frac{8}{9} \cdot F_{1/1}\right)$ with a 3"× 5" foil wrapper introduces additional, high-order partials; thus, focusing our attention to an overtone series originating in the third partial of the actuated string VI $\left(\frac{8}{9} \cdot 3\right)$. Recorded as a separate take, placement of a 3/32"× 7" plastic straw across the remaining actuated strings (I, II, III, IV, and VI) generates rapid, percussive gestures and harmonic features reminiscent of states 4-8, 33, and 54. Closing the passage, a secondary layer of reverberation and mechanical artifacts—presumably, the product of loose hardware and overdriven actuators—augments the remaining actuated partials: VI $\left(\frac{8}{9} \cdot 7\right)$, IV $\left(\frac{4}{3} \cdot 4\right)$, III $\left(\frac{20}{11} \cdot 5\right)$, II $\left(\frac{11}{6} \cdot 6\right)$, and I $\left(\frac{2}{1} \cdot 7\right)$.

(5 - 26)

State #	Time	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	*NASH*	*Disruptive Preparation(s)*
61	Track 3 2:15	$\frac{8}{9} \cdot F_{1/1}$	1.4065	1	0.889	$8/9 \cdot 1$	JUNO→RBI—EH (π) Distortion/Sustainer (VI)
				2	2.356	$8/9 \cdot 3$	JUNO→RBI—EH (π) Distortion/Sustainer (VI)
				3	4.168	$4/3 \cdot 3$	
				4	6.247	$8/9 \cdot 7$	JUNO→RBI—EH (π) Distortion/Sustainer (VI)
				5	8.550	$20/11 \cdot 5$	
				6	11.049	$11/6 \cdot 6$	
				7	13.724	$2/1 \cdot 7$	

62	Track 3 2:30	$\frac{9}{10} \cdot F_{1/1}$	1.3985	1	0.900	$9/10 \cdot 1$	JUNO→RBI—EH (π) Distortion/Sustainer (V)
				2	2.373	$8/9 \cdot 3$	3"× 5" Foil Wrapper – KM184 (VI)
				3	4.183	$4/3 \cdot 3$	3/32"× 7" Plastic Straw – Piezo / Humbucker (IV)
				4	6.255	$8/9 \cdot 7$	3/32"× 7" Plastic Straw – Piezo / Humbucker (VI)
				5	8.546	$20/11 \cdot 5$	3/32"× 7" Plastic Straw – Piezo / Humbucker (III)
				6	11.028	$11/6 \cdot 6$	3/32"× 7" Plastic Straw – Piezo / Humbucker (II)
				7	13.681	$2/1 \cdot 7$	3/32"× 7" Plastic Straw – Piezo / Humbucker (I)

63	Track 3 2:49	$\frac{11}{6} \cdot F_{1/1}$	1.111	1	1.833	$11/6 \cdot 1/2$	JUNO→RBI—EH (π) Distortion/Sustainer (V+)
				2	3.960	$4/3 \cdot 4$	Mechanical Artifacts – Piezo (IV) – Lexicon
				3	6.213	$8/9 \cdot 7$	Mechanical Artifacts – Piezo (VI) – Lexicon
				4	8.553	$20/11 \cdot 5$	Mechanical Artifacts – Piezo (III) – Lexicon
				5	10.960	$11/6 \cdot 6$	Mechanical Artifacts – Piezo (II) – Lexicon
				6	13.420	$2/1 \cdot 7$	Mechanical Artifacts – Piezo (I) – Lexicon
				7	15.927	n/a	

Whether the product of strategic intervention or unintentional artifact, disruptive preparations influence both the spectral structure and associated tuning practices for actuated strings. While largely absent from previous discussions of performance practice for actuated instrumentation, implementation of physical and procedural forms of disruptive preparation in the compositional, performative, and production stages of *Artemisia* provide a clear precedent for further research and creative application. Speaking to the overarching themes of this essay, disruptive preparation as an emergent performance category supports an intersectional framework for composing with actuated string instruments and just intonation.

VI. CONCLUSION

In examining historical and contemporary applications of actuated string instrument design and just tuning practices, we have presented an integrated framework for composing and performing using these resources. As discussed, a broad body of research has been conducted in the distinct areas of actuated instrumentation and just intonation. However, prior to our work, few scholarly or creative works sought to bridge the fields of actuated string instrument design, performance, and just tuning practices. In addition to offering a tangible demonstration of actuated string techniques, established performance modalities, and relevant practices in just intonation, the accompanying score and recording of *Artemisia* documents the intersection of these fields and the emergence of an original creative praxis. Therein, our work addresses how features of actuated string instrument design and tuning dually inform one another.

Building upon past approaches, we established a contextual definition for actuated instrumentation exclusive to those instruments, devices, and algorithms which employ a secondary or indirect method of inducing acoustic response from one or more strings. Bearing in mind the substantial body of research already conducted with musical robots, we intentionally refined the scope of our analysis to exclude methods of actuation which simulate human faculties. Instead, our contributions lie in the practical application of non-simulative and electronically-mediated methods. Here, we defined two broad categories of actuated instruments employing electro-magnetic and electro-mechanical transducers, respectively.

By providing a survey of new and existing instruments, designers, and practitioners, we sought to elucidate a variety of methods for actuation. Explanations and examples of ferromagnetic actuation, Lorentz-Force actuation, and tactile transducers established context, as well as a discussion of creative implications and affordances for each distinct mode. Notably, ferromagnetic actuation principles modeled by Andrew McPherson and other researchers working at CCRMA and the Instrumentation Lab at Miami University bore significant influence over our design and construction of two new instruments: Rosebud I and Rosebud (‘Louise’) II. While some of McPherson and others’ most substantial contributions lie in the development of efficient and highly linear systems for actuation, we alternately chose to explore non-linear behaviors and proposed practices which embrace artifact as a potent aesthetic force.

Citing theoretical conventions instituted by Harry Partch, Kathleen Schlesinger, Ben Johnston, and others, we applied similar principles in just intonation to both the composition of *Artemisia* and tuning structures for actuated instruments deployed in recording the piece. Here, we focused primarily upon Otonality, Utonality, Commas, Tonal Flux, Epimoric ratios and other structural features relevant to composing and performing with actuated strings. Moreover, we connected these principles to perceptual models of Harmonic Distance and Spectral Roughness Analysis (SRA) developed by James Tenney, William Sethares, and Pantelis Vassilakis.

In addition to assessing methods for actuation and tuning, we described nascent performance practices associated with actuated instrumentation. Prior to our work, Dan Overholt, Edgar Berdahl, and Robert Hamilton defined three categories of actuated instrument performance practice: "computer-mediated" electronic signals, "self-sustaining oscillation," and "third-party" audio streams.[286] While all three modes of performance reference an initial source for actuation, we proposed a fourth category rooted in earlier experiments by John Cage and Henry Cowell. Termed 'disruptive preparation', this practice extends Overholt, Berdahl, and Hamilton's source-based classification to include acoustic artifacts and other nonlinearities generated through interactions between actuated strings and foreign objects, external processing, or other performative interventions. Through spectral and notational analysis of *Artemisia*, we demonstrated how each category manifests in practicum.

Moving forward, we aim to develop more extensible frameworks for actuation and tuning. As discussed, current iterations of both Rosebud I and II each engage a single mode of actuation—electro-magnetic and electromechanical, respectively. However, with the addition of a secondary input and corresponding channel for amplification, we anticipate the installation of three solenoid actuators adjacent to the strings of Rosebud II. Once implemented, the instrument will retain the unique properties and affordances of both electro-mechanical *and* electro-magnetic actuation. Our

[286] Overholt, Dan, Edgar Berdahl, and Robert Hamilton. "Advancements in Actuated Instruments." *Organised Sound.* Vol. 16, Issue 2 (2011): 154-165. https://www.cambridge.org/core/journals/organised-sound/article/advancements-in-actuated-musical-instruments/AFBD83D9E53F8C0270492F06CD0F2380 (accessed August 5, 2019).

plans reflect a general tendency towards hybrid actuation modalities and increasing modularity in design. As well as modifying the means for actuation, hybrid modalities enable multiple and concurrent actuation topologies during a single performance. For example, a performer could drive a tactile transducer using “third party” audio streams from a string quartet, while simultaneously initiating recursive actuation via the instrument’s solenoid actuators. Similarly, the possibility for different combinatory arrangements of inputs, actuators, and pickup types afford a modular approach to actuation, rife with creative potential.

Speaking to the issue of tuning, comparably extensible features reside in the intrinsic properties of vibrating strings. Commensurate with the ability to vary pitch through tension or string gauge, application of harmonic or arithmetically divisional procedures present another promising avenue for extended intonation. While not explicitly assessed in the composition or recording of *Artemisia*, the addition of a moveable third bridge within Rosebud II enables proportional divisions in string length and sounding pitch. Pickups mounted at either side of the instrument’s moveable bridge allow the performer to independently actuate and amplify two, vibrating lengths of string for each course. Dividing the string by integral proportions generates two acoustic vibrations, separated in frequency by a just interval. When divided according to harmonic proportions (e.g. 1:1, 2:1, 3:1, 4:1, 5:1), frequencies activated on one length of string may induce sympathetic vibrations upon the length of string opposite the third bridge. Notably, this phenomenon has been applied by instrument makers Hans Reichel, Yuri Landman, and others.[287] [288] However, in the field of actuated instrument design and performance, comparable modes of harmonic activation remain relatively unexplored. With a moveable third-bridge already in place, subsequent compositions for Rosebud II will almost certainly integrate similar practices. Looking to the future, we aim to continue developing extensible modalities for the design, tuning, and performance practices of actuated string instruments.

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[287] Joe Gore, “Crossing the Bridge by Hans Reichel,” *Guitar Player*. January 1989. https://issuu.com/yurilandman/docs/hans_reichel_guitar_player_magazine_1989 (accessed March 4, 2020).

[288] Yuri Landman, “Third Bridge Diagram,” http://www.hypercustom.nl/3rdbridge.jpg (accessed October 21, 2019).

APPENDIX A—*Artemisia* (Spectral Score)

$$Frequency\ of\ Stretched\ Harmonic\ (f_{st})\ =\ F_{RBI(1-6)} \cdot n^{x}$$

$$n\ = Integral\ Value\ of\ Harmonic\ Partial$$

$$x\ = Stretch\ Coefficient$$

$$Unity\ or\ \text{"Tonal Center"}\ \left(F_{1/1}\right) = 55\ Hz$$

$$Fundamental\ Frequencies\ for\ Strings\ (VI - I) - Rosebud\ \ I\ (F_{RBI(1-6)})$$

$$F_{RBI\ (1)} = \frac{8}{9} \cdot F_{1/1}$$

$$F_{RBI\ (2)} = \frac{9}{10} \cdot F_{1/1}$$

$$F_{RBI\ (4)} = \frac{4}{3} \cdot F_{1/1}$$

$$F_{RBI\ (5)} = \frac{20}{11} \cdot F_{1/1}$$

$$F_{RBI\ (6)} = \frac{11}{6} \cdot F_{1/1}$$

$$F_{RBI\ (6)} = \frac{2}{1} \cdot F_{1/1}$$

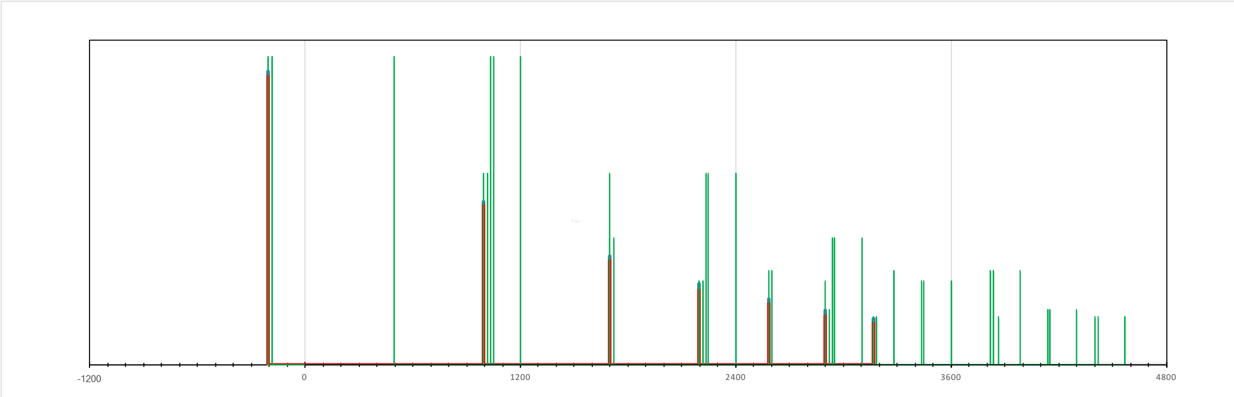
$$Nearest\ Actuated\ String\ Harmonic\ (NASH) = \frac{F_{RBI(1-6)} \cdot n}{F_{1/1}}$$

$$Visual\ Reference\ for\ Spectral\ Score:$$

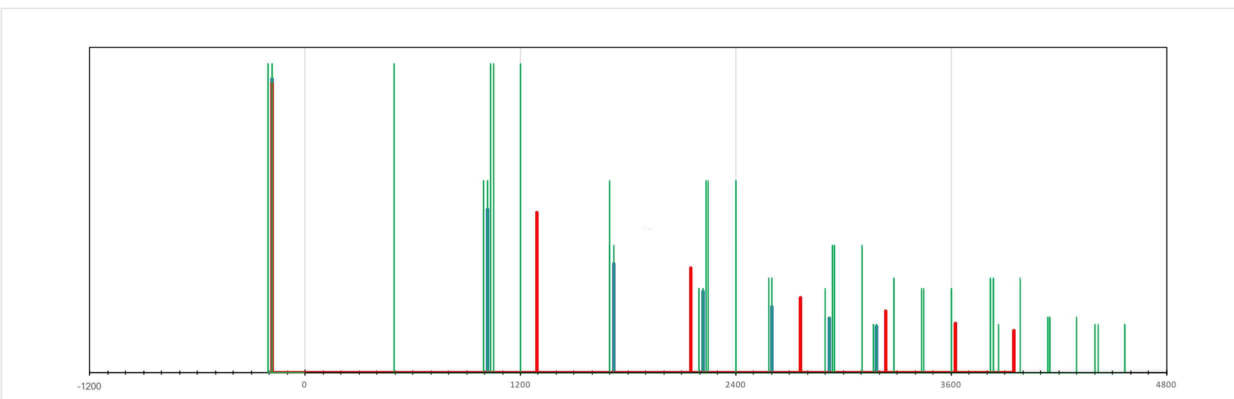
$f_{st}\ /\ F_{1/1}$ =

$F_{RBI(1-6)} \cdot n$ =

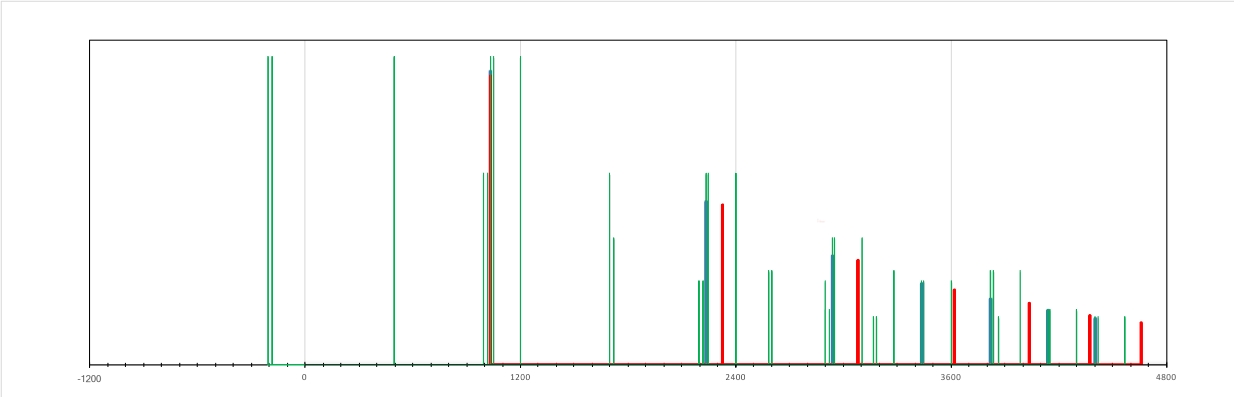
$NASH$ =



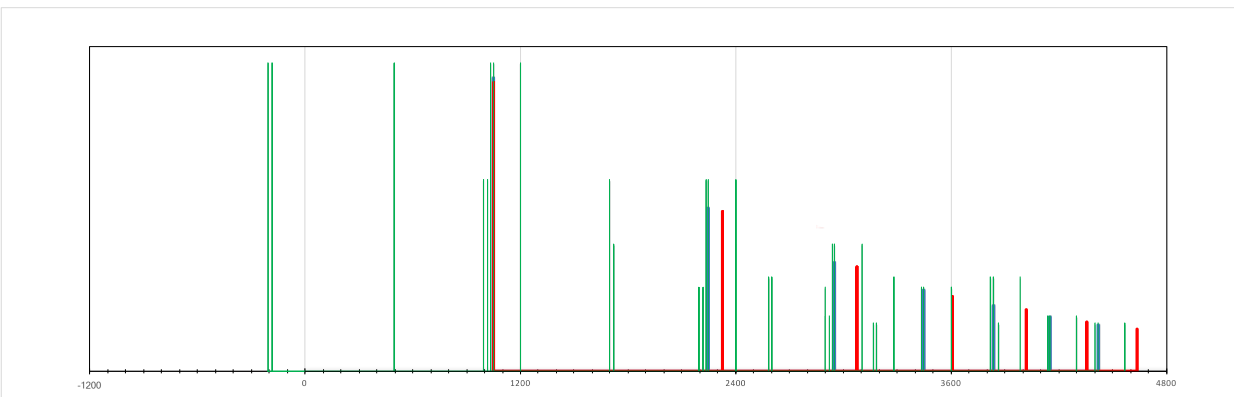
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
1	$\frac{8}{9} \cdot F_{1/1}$	1.0	1	0.889	$8/9 \cdot 1$
			2	1.778	$8/9 \cdot 2$
			3	2.667	$8/9 \cdot 3$
			4	3.556	$8/9 \cdot 4$
			5	4.444	$8/9 \cdot 5$
			6	5.333	$8/9 \cdot 6$
			7	6.222	$8/9 \cdot 7$



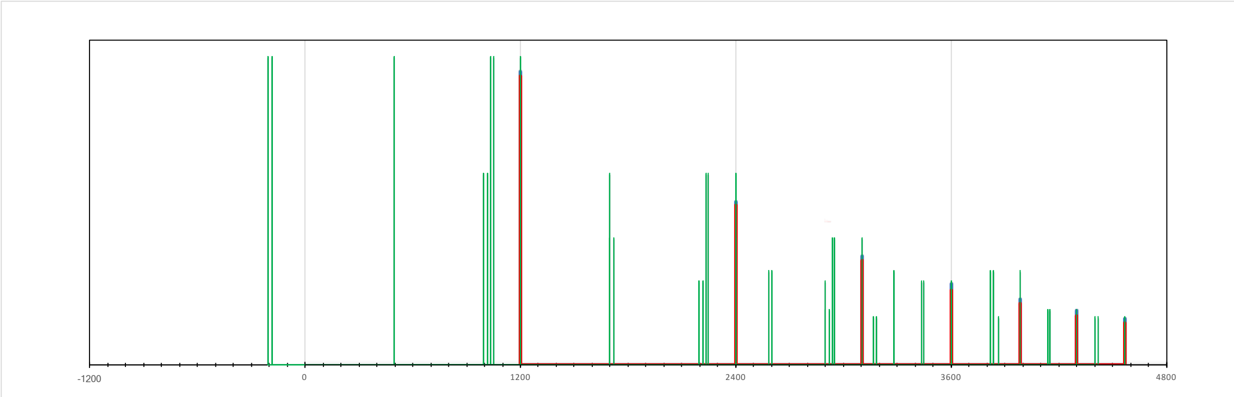
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
2	$\frac{9}{10} \cdot F_{1/1}$	1.227	1	0.900	$9/10 \cdot 1$
			2	2.107	$2/1 \cdot 1$
			3	3.465	$8/9 \cdot 4$
			4	4.931	$8/9 \cdot 6$
			5	6.485	$4/3 \cdot 5$
			6	8.110	$4/3 \cdot 6$
			7	9.799	$2/1 \cdot 5$



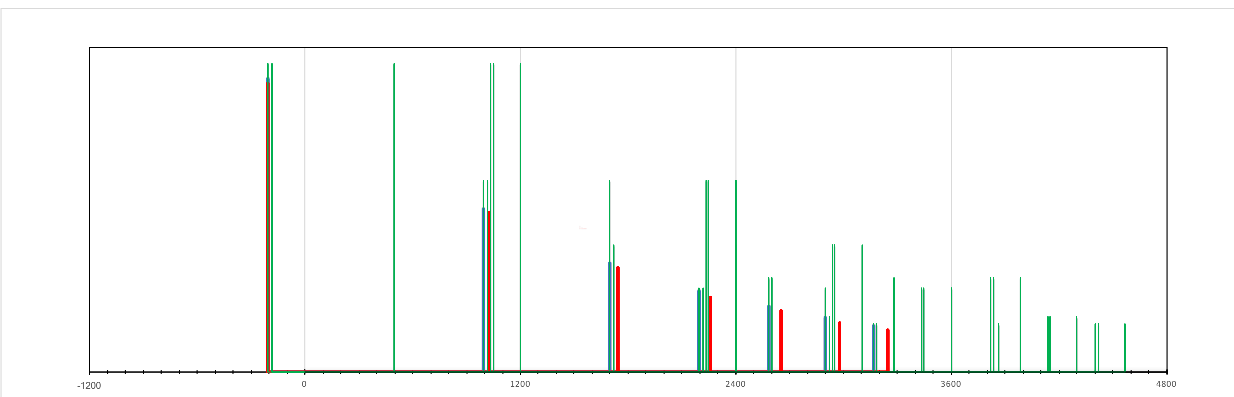
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
3	$\frac{20}{11} \cdot F_{1/1}$	1.0765	1	1.818	$20/11\ \cdot 1$
			2	3.834	$4/3\ \cdot\ 3$
			3	5.933	$2/1 \cdot 3$
			4	8.086	$4/3\ \cdot 6$
			5	10.282	$2/1\ \cdot 5$
			6	12.512	$20/11\ \cdot 7$
			7	14.770	$2/1\ \cdot 7$



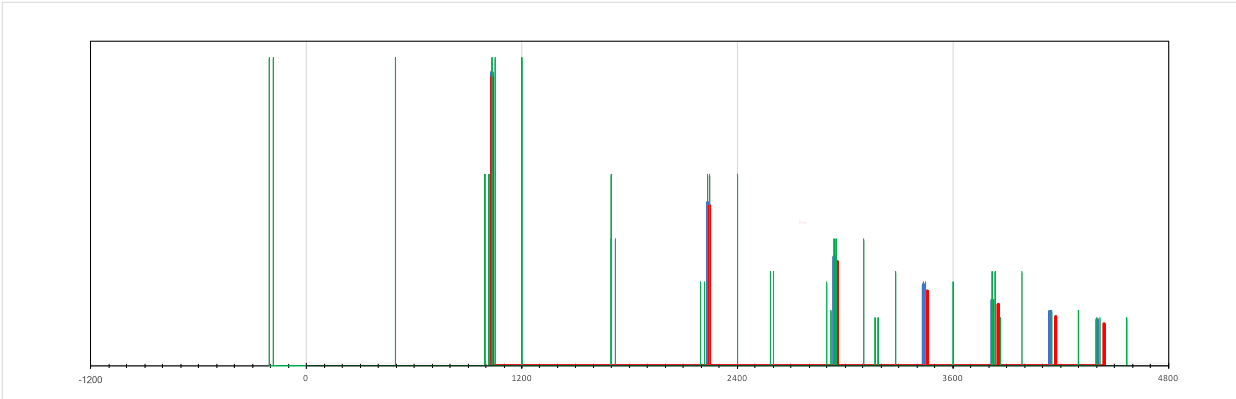
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
4	$\frac{11}{6} \cdot F_{1/1}$	1.0655	1	1.833	$11/6\ \cdot 1$
			2	3.837	$4/3\ \cdot 3$
			3	5.910	$4/3\ \cdot 6$
			4	8.030	$2/1\ \cdot 4$
			5	10.186	$2/1\ \cdot 5$
			6	12.370	$20/11\ \cdot 7$
			7	14.578	$2/1\ \cdot 7$



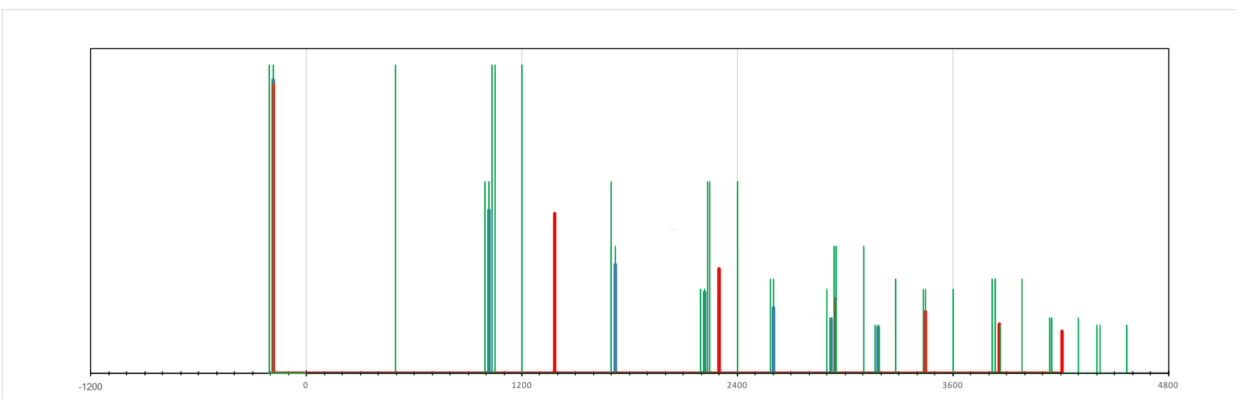
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
5	$\frac{2}{1}\ \cdot\ F_{1/1}$	1.0	1	2	$2/1\ \cdot 1$
			2	4	$2/1\ \cdot 2$
			3	6	$2/1\ \cdot 3$
			4	8	$2/1\ \cdot 4$
			5	10	$2/1\ \cdot 5$
			6	12	$2/1\ \cdot 6$
			7	14	$2/1\ \cdot 7$



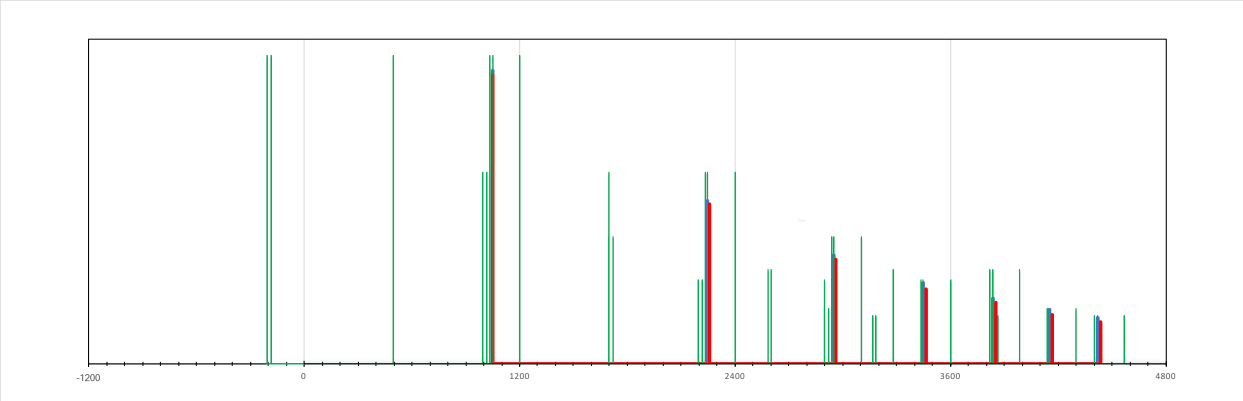
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
6 *diff.	$\frac{8}{9}\ \cdot\ F_{1/1}$	1.0252	1	0.889	$8/9\ \cdot 1$
			2	1.809	$20/11\ \cdot 1$
			3	2.742	$9/10\ \cdot 3$
			4	3.682	$11/6\ \cdot 2$
			5	4.628	$9/10\ \cdot 5$
			6	5.580	$4/3\ \cdot 5$
			7	6.535	$11/6\ \cdot 3$



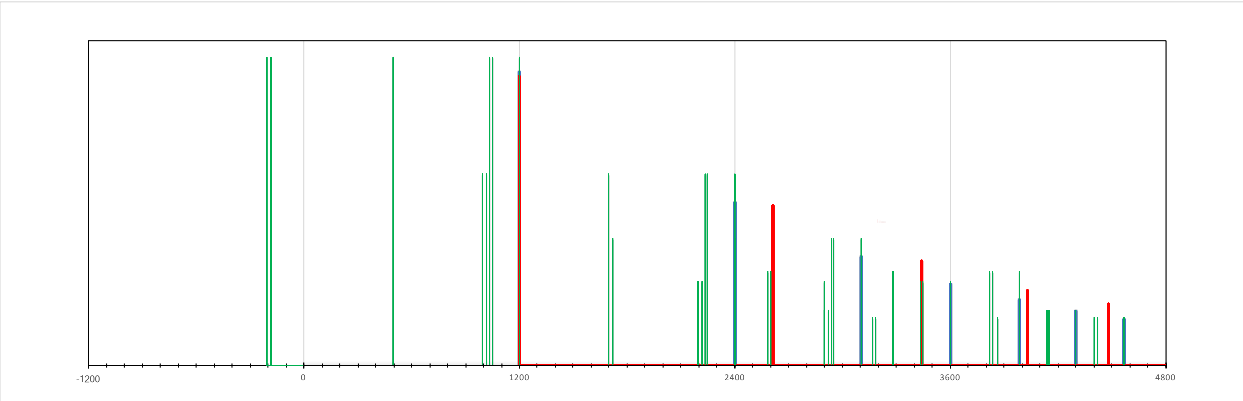
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
7	$\frac{20}{11}\cdot F_{1/1}$	1.011	1	1.818	$20/11\ \cdot 1$
			2	3.664	$11/6\ \cdot 2$
			3	5.521	$11/6\ \cdot 3$
			4	7.384	$11/6\ \cdot 4$
			5	9.253	$4/3\ \cdot 7$
			6	11.126	$11/6\ \cdot 6$
			7	13.003	$11/6\ \cdot 7$



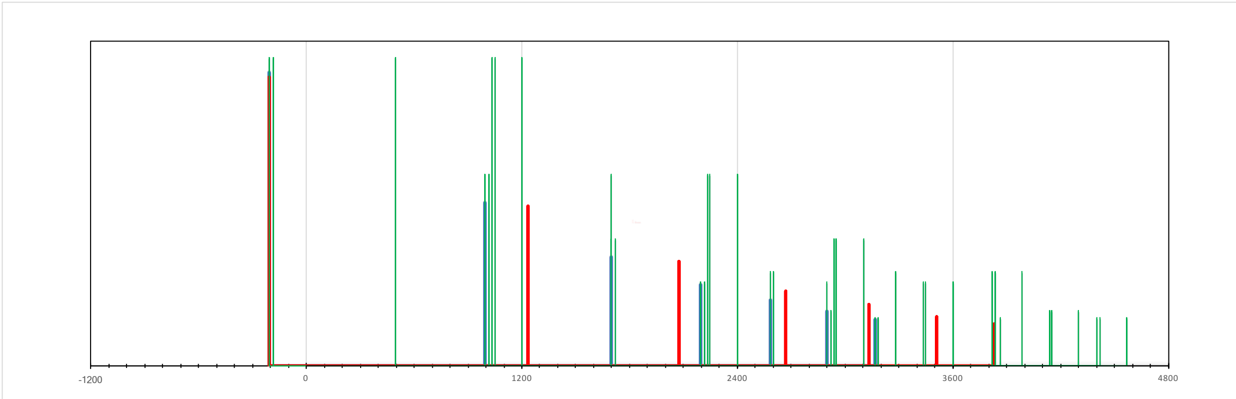
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
8	$\frac{9}{10}\cdot F_{1/1}$	1.303	1	0.900	$9/10\ \cdot 1$
			2	2.221	$2/1\ \cdot 1$
			3	3.766	$11/6\ \cdot 2$
			4	5.479	$11/6\ \cdot 3$
			5	7.328	$11/6\ \cdot 4$
			6	9.293	$4/3\ \cdot 7$
			7	11.361	$11/6\ \cdot 6$



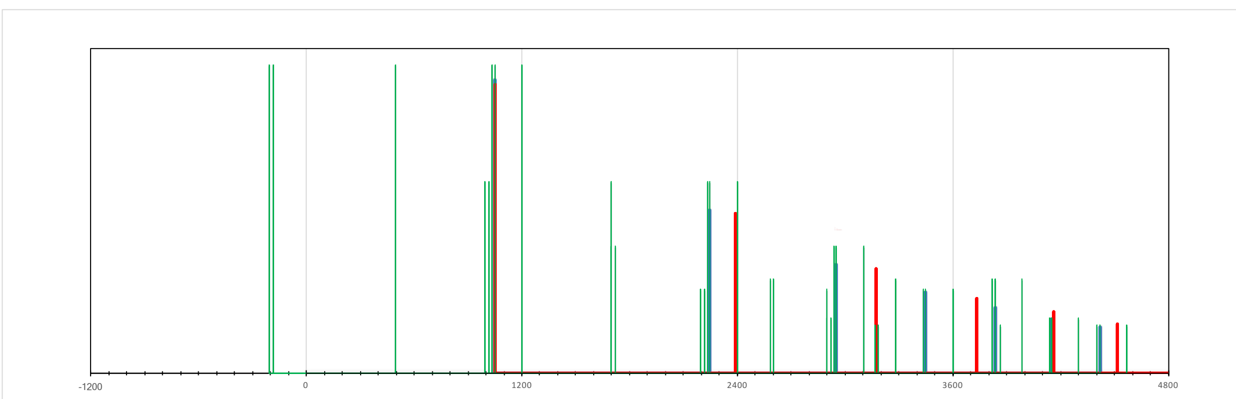
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
9	$\frac{11}{6} \cdot F_{1/1}$	1.006	1	1.833	$11/6\ \cdot 1$
			2	3.682	$11/6\ \cdot 2$
			3	5.536	$11/6\ \cdot 3$
			4	7.395	$11/6\ \cdot 4$
			5	9.256	$4/3\ \cdot 7$
			6	11.119	$11/6\ \cdot 6$
			7	12.984	$11/6\ \cdot 7$



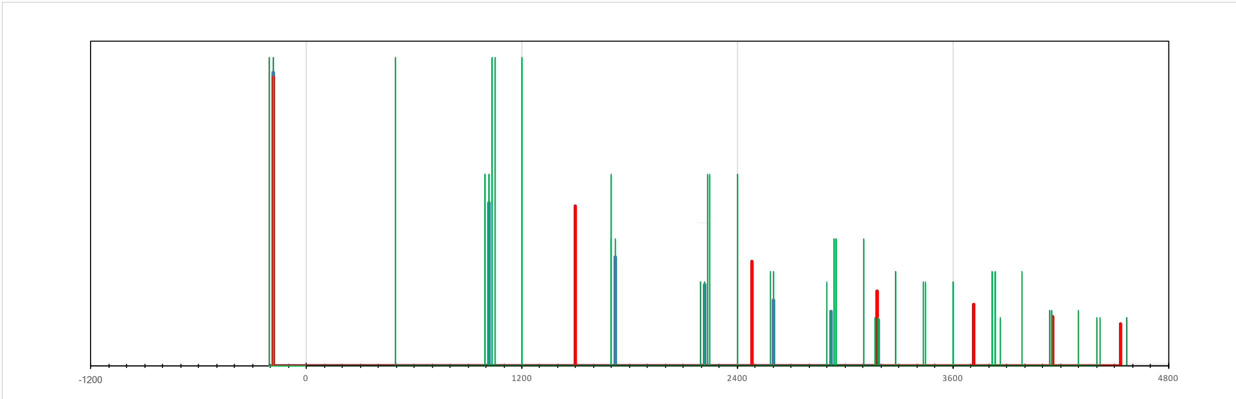
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
10	$\frac{2}{1} \cdot F_{1/1}$	1.179	1	2.000	$2/1\ \cdot 1$
			2	4.528	$9/10\ \cdot 5$
			3	7.304	$11/6\ \cdot 4$
			4	10.253	$2/1\ \cdot 5$
			5	13.339	$11/6\ \cdot 7$
			6	16.538	$2/1\ \cdot 7$
			7	19.834	n/a



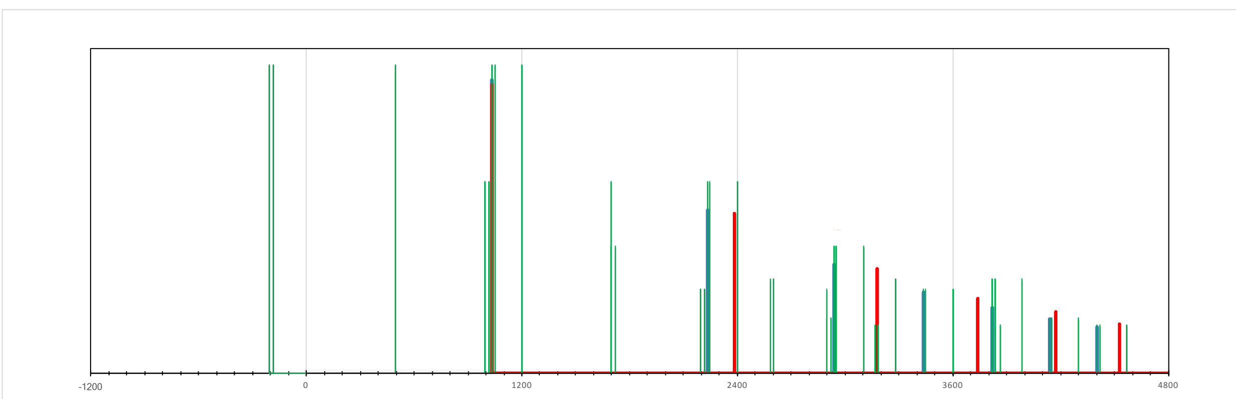
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
11	$\frac{8}{9}\cdot F_{1/1}$		1	0.889	$8/9\ \cdot 1$
			2	2.038	$2/1\ \cdot 1$
			3	3.311	$8/9\ \cdot 4$
			4	4.672	$9/10\ \cdot 5$
			5	6.103	$2/1\ \cdot 3$
			6	7.591	$11/6\ \cdot 4$
			7	9.129	$11/6\ \cdot 5$



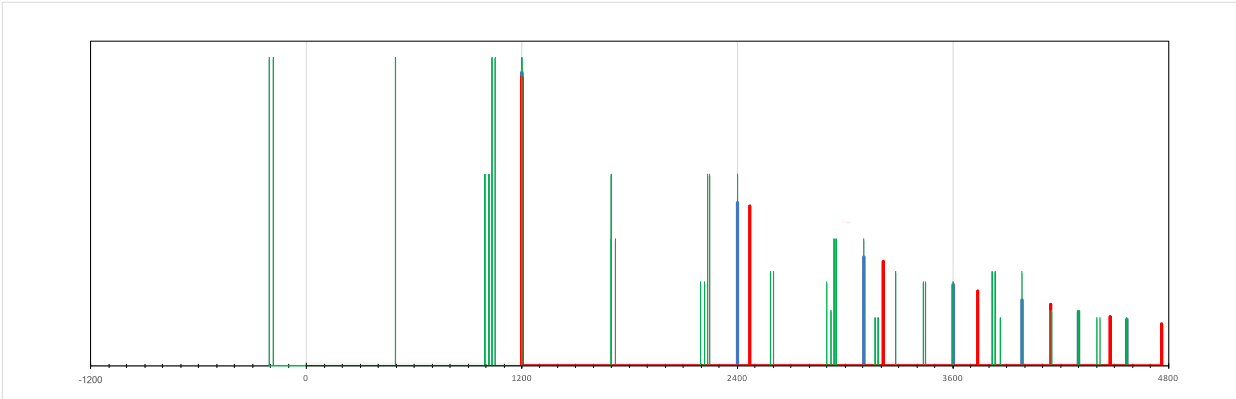
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
12 *diff.	$\frac{11}{6}\cdot F_{1/1}$	1.118	1	1.833	$11/6\ \cdot 1$
			2	3.979	$4/3\ \cdot 3$
			3	6.261	$9/10\ \cdot 7$
			4	8.637	$20/11\ \cdot 5$
			5	11.084	$11/6\ \cdot 6$
			6	13.590	$2/1\ \cdot 7$
			7	16.146	n/a



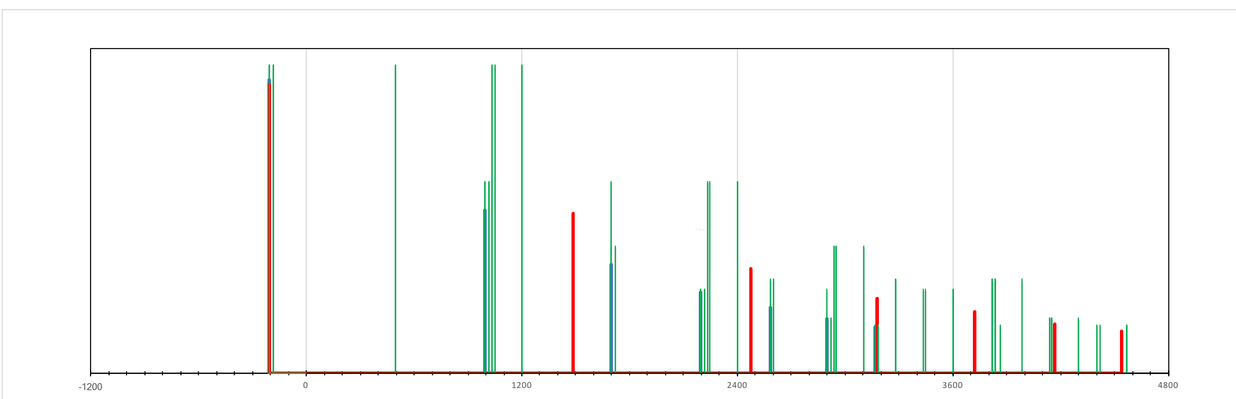
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
13	$\frac{9}{10} \cdot F_{1/1}$	1.3995	1	0.900	$9/10 \cdot 1$
			2	2.374	$8/9 \cdot 3$
			3	4.188	$4/3 \cdot 3$
			4	6.264	$9/10 \cdot 7$
			5	8.560	$20/11 \cdot 5$
			6	11.048	$11/6 \cdot 6$
			7	13.707	$2/1 \cdot 7$



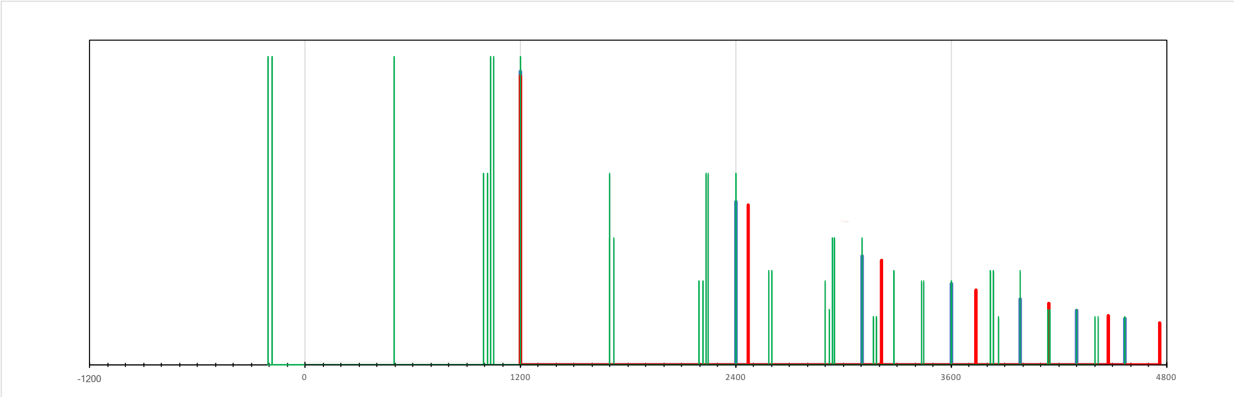
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
14	$\frac{20}{11} \cdot F_{1/1}$	1.126	1	1.818	$20/11 \cdot 1$
			2	3.968	$4/3 \cdot 3$
			3	6.264	$9/10 \cdot 7$
			4	8.661	$20/11 \cdot 5$
			5	11.135	$11/6 \cdot 6$
			6	13.672	$2/1 \cdot 7$
			7	16.264	n/a



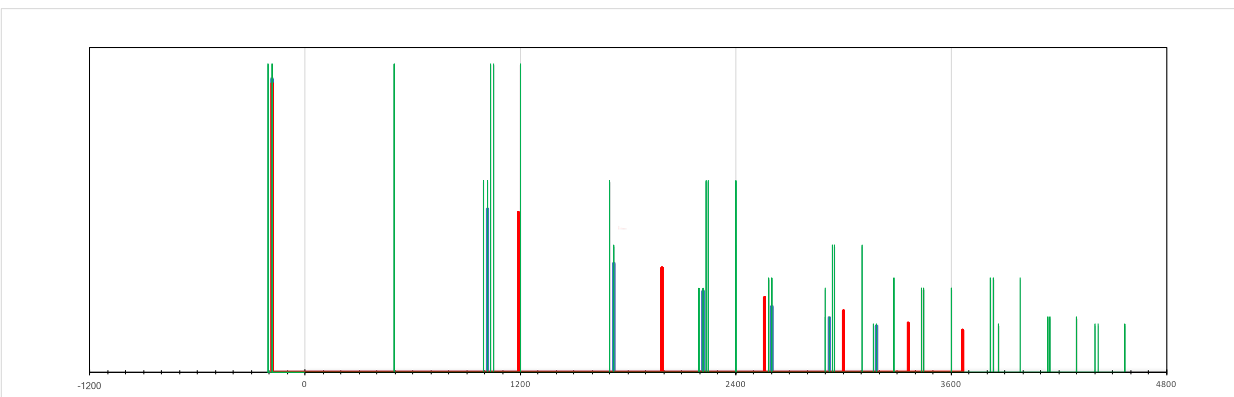
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
15	$\frac{2}{1}\cdot F_{1/1}$	1.057	1	2.000	$2/1\ \cdot 1$
			2	4.161	$4/3\ \cdot 1$
			3	6.388	$9/10\ \cdot 7$
			4	8.658	$20/11\ \cdot 5$
			5	10.961	$11/6\ \cdot 6$
			6	13.290	$11/6\ \cdot 7$
			7	15.642	$2/1\ \cdot 7$



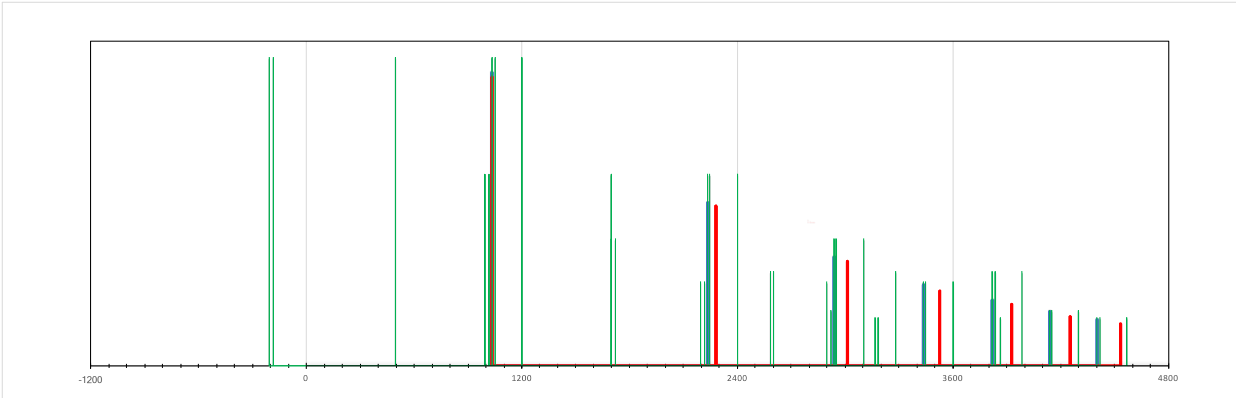
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
16	$\frac{8}{9}\cdot F_{1/1}$	1.4085	1	0.889	$8/9\ \cdot 1$
			2	2.360	$8/9\ \cdot 3$
			3	4.177	$4/3\ \cdot 3$
			4	6.264	$9/10\ \cdot 7$
			5	8.577	$20/11\ \cdot 5$
			6	11.089	$11/6\ \cdot 6$
			7	13.777	$2/1\ \cdot 7$



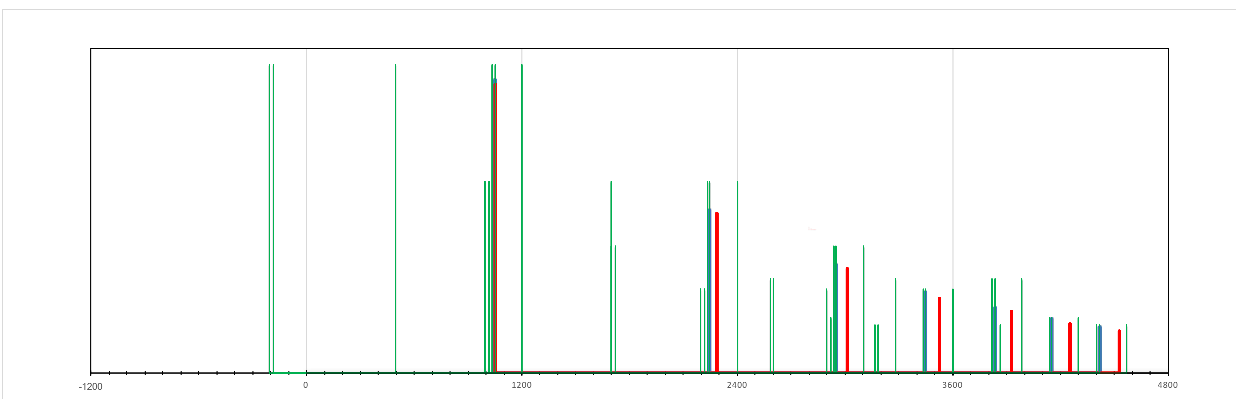
State #	$F_{RBI\ (1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
17	$\frac{2}{1} \cdot F_{1/1}$	1.057	1	2.000	$2/1 \cdot 1$
			2	4.161	$4/3 \cdot 3$
			3	6.388	$9/10 \cdot 7$
			4	8.658	$20/11 \cdot 5$
			5	10.961	$11/6 \cdot 6$
			6	13.290	$11/6 \cdot 7$
			7	15.642	$2/1 \cdot 7$



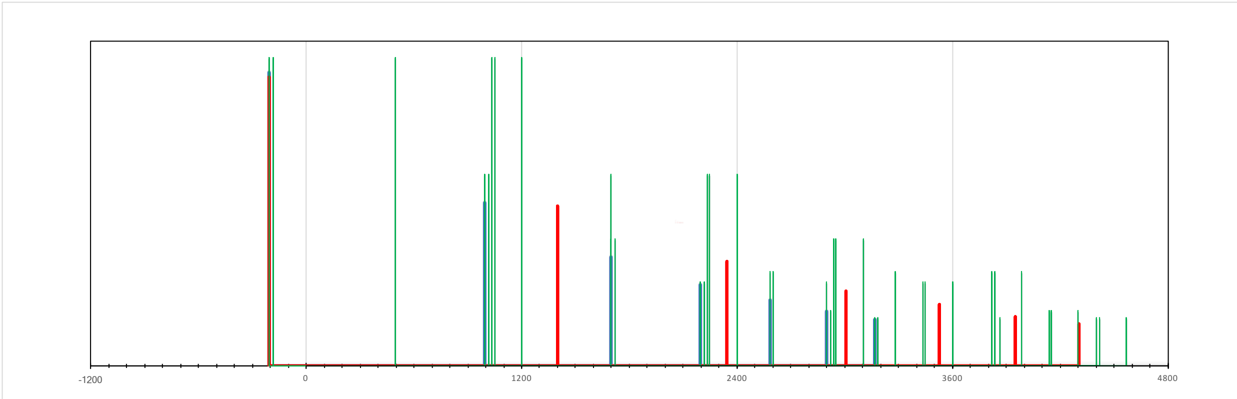
State #	$F_{RBI\ (1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
18 *diff.	$\frac{9}{10} \cdot F_{1/1}$	1.1425	1	0.900	$9/10 \cdot 1$
			2	1.987	$2/1 \cdot 1$
			3	3.158	$8/9 \cdot 4$
			4	4.386	$8/9 \cdot 5$
			5	5.660	$11/6 \cdot 3$
			6	6.971	$20/11 \cdot 4$
			7	8.313	$4/3 \cdot 6$



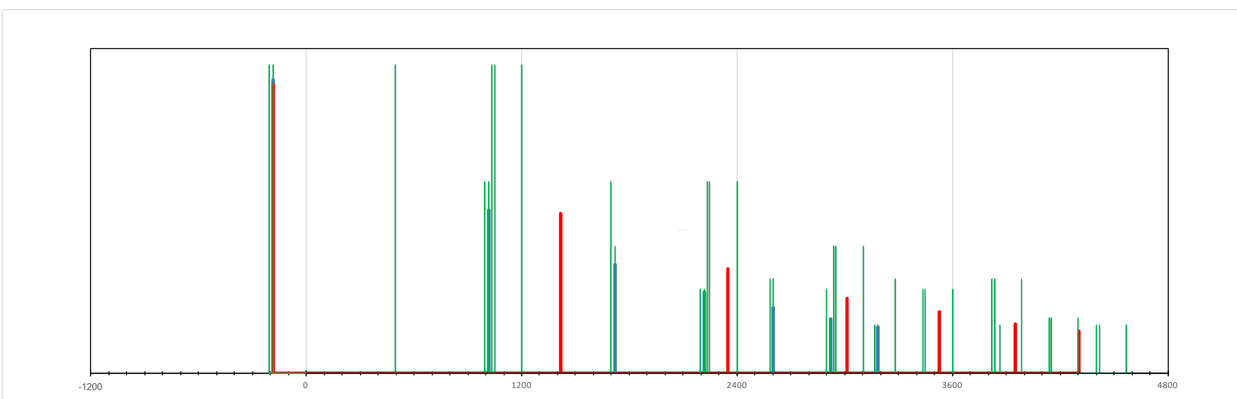
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
19	$\frac{20}{11}\cdot F_{1/1}$	1.0385	1	1.818	$20/11\cdot 1$
			2	3.735	$11/6\cdot 2$
			3	5.690	$11/6\cdot 3$
			4	7.671	$4/3\cdot 6$
			5	9.672	$2/1\cdot 5$
			6	11.688	$2/1\cdot 6$
			7	13.717	$2/1\cdot 7$



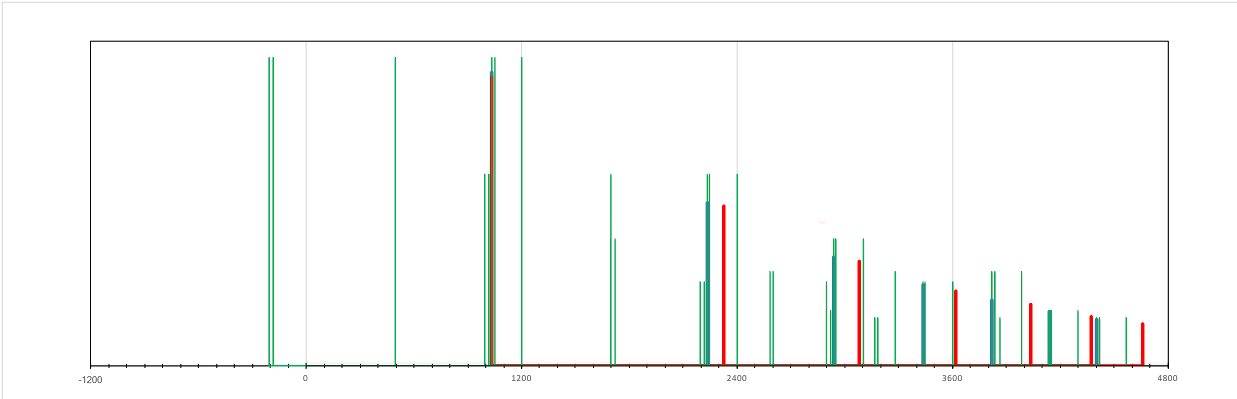
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
20	$\frac{11}{6}\cdot F_{1/1}$	1.033	1	1.833	$11/6\cdot 1$
			2	3.752	$11/6\cdot 2$
			3	5.703	$11/6\cdot 3$
			4	7.677	$4/3\cdot 3$
			5	9.667	$2/1\cdot 5$
			6	11.670	$2/1\cdot 6$
			7	13.684	$2/1\cdot 7$



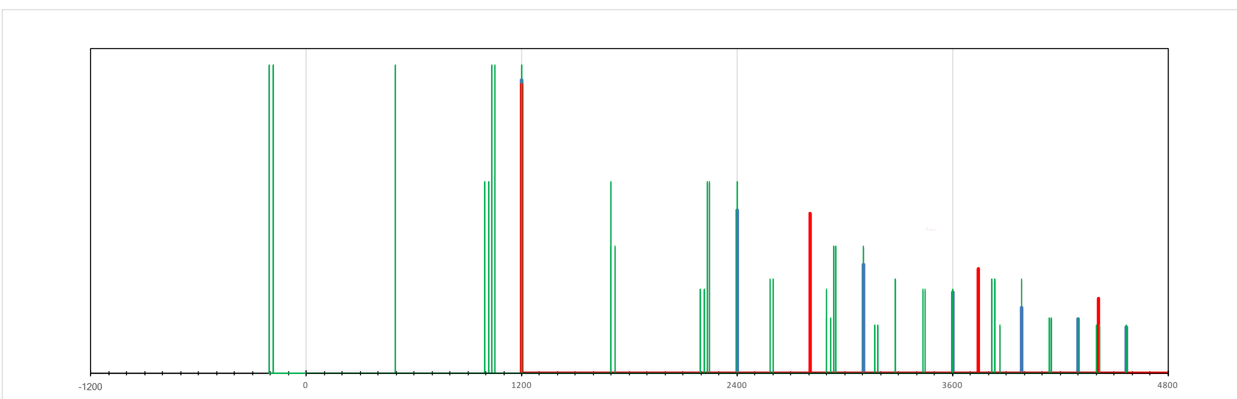
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
21	$\frac{8}{9} \cdot F_{1/1}$	1.339	1	0.889	$8/9 \cdot 1$
			2	2.249	$2/1 \cdot 1$
			3	3.870	$4/3 \cdot 3$
			4	5.689	$11/6 \cdot 3$
			5	7.670	$4/3 \cdot 6$
			6	9.790	$2/1 \cdot 5$
			7	12.035	$2/1 \cdot 6$



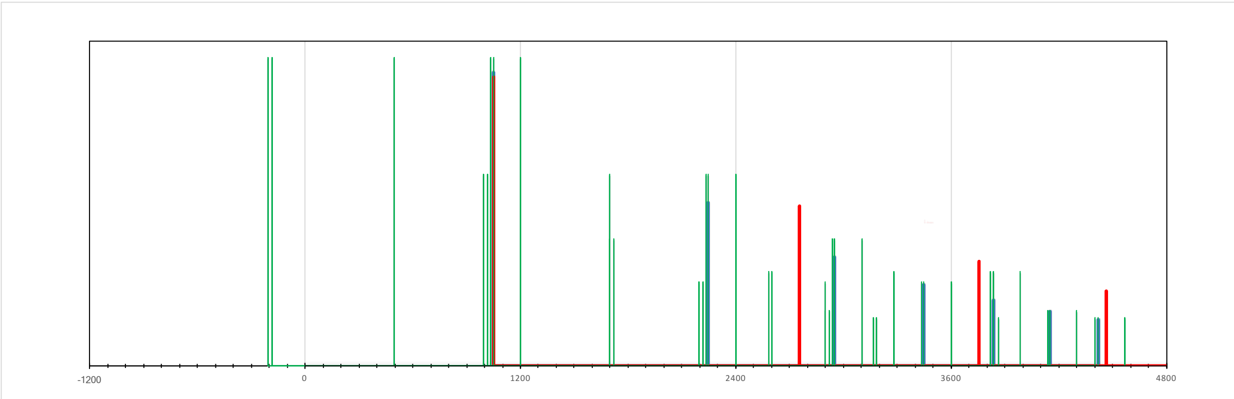
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
22	$\frac{9}{10} \cdot F_{1/1}$	1.3315	1	0.900	$9/10 \cdot 1$
			2	2.265	$2/1 \cdot 1$
			3	3.886	$4/3 \cdot 3$
			4	5.700	$11/6 \cdot 3$
			5	7.672	$4/3 \cdot 6$
			6	9.780	$2/1 \cdot 5$
			7	12.009	$2/1 \cdot 6$



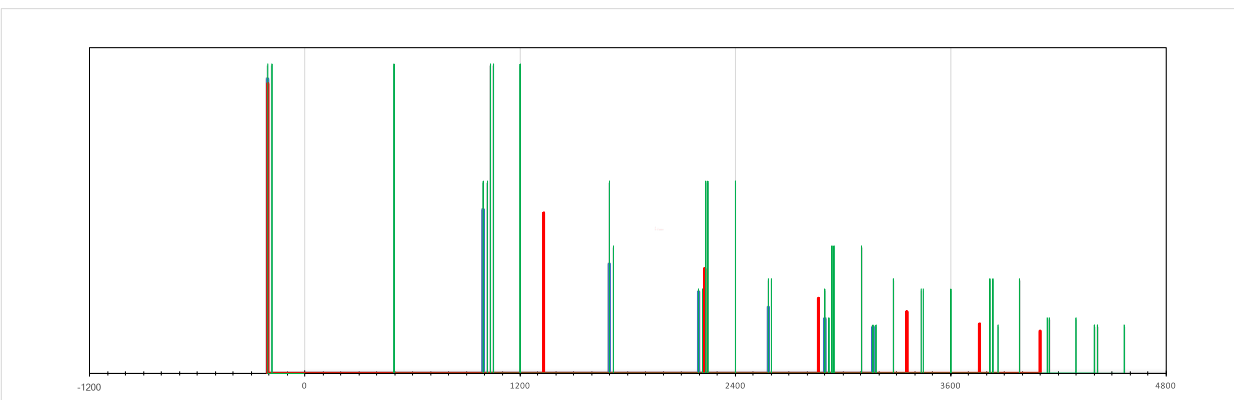
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
23	$\frac{20}{11} \cdot F_{1/1}$	1.0765	1	1.818	$20/11 \cdot 1$
			2	3.834	$4/3 \cdot 3$
			3	5.933	$2/1 \cdot 3$
			4	8.086	$4/3 \cdot 6$
			5	10.282	$2/1 \cdot 5$
			6	12.512	$20/11 \cdot 7$
			7	14.770	$2/1 \cdot 7$



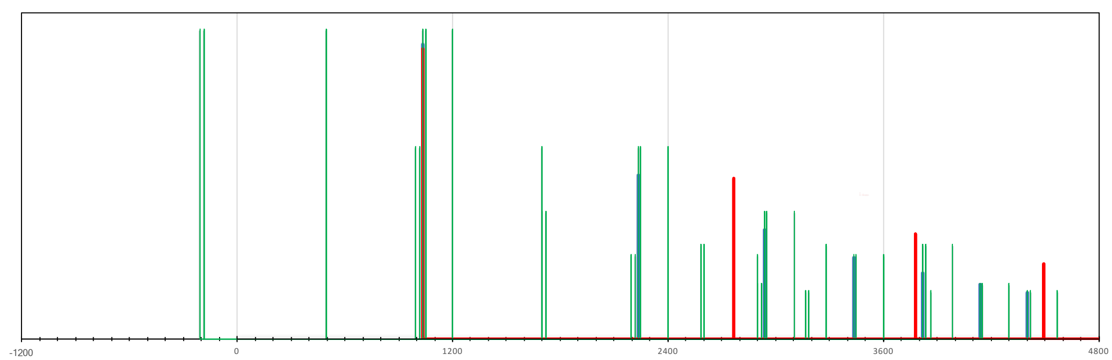
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
24 *diff.	$\frac{2}{1} \cdot F_{1/1}$	1.338	1	2.000	$2/1 \cdot 1$
			2	5.056	$8/9 \cdot 6$
			3	8.698	$20/11 \cdot 5$
			4	12.782	$2/1 \cdot 7$
			5	17.229	n/a
			6	21.989	n/a
			7	27.025	n/a



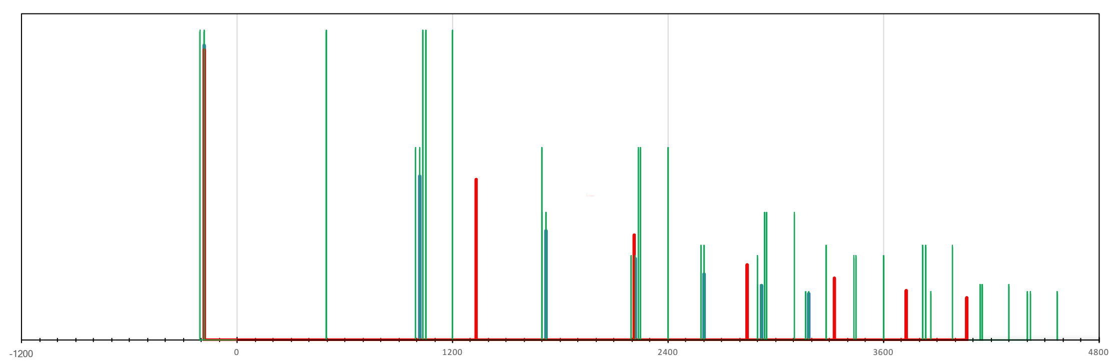
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
25	$\frac{11}{6} \cdot F_{1/1}$	1.4235	1	1.833	$11/6\ \cdot 1$
			2	4.918	$8/9\ \cdot 6$
			3	8.758	$20/11\ \cdot 5$
			4	13.191	$11/6\ \cdot 7$
			5	18.123	n/a
			6	23.493	n/a
			7	29.258	n/a



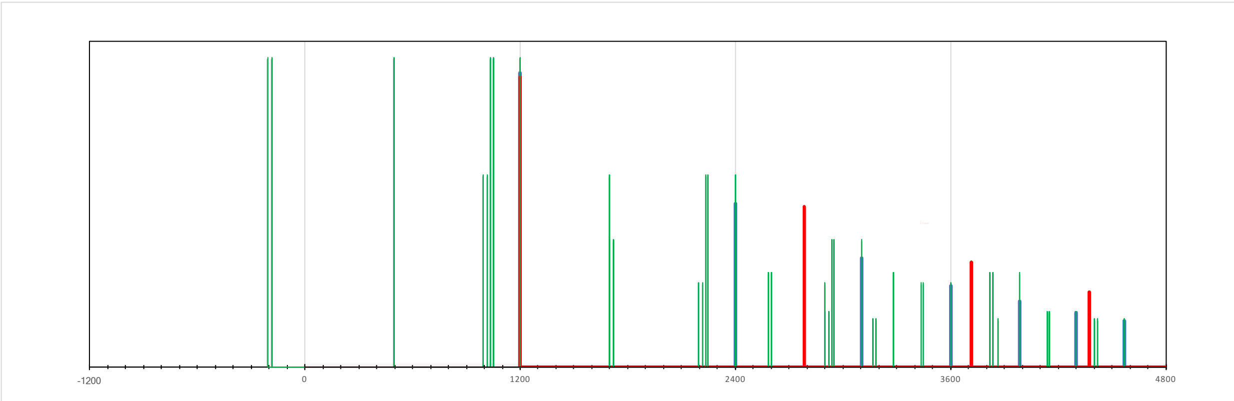
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
26	$\frac{8}{9} \cdot F_{1/1}$	1.278	1	0.889	$8/9\ \cdot 1$
			2	2.156	$2/1\ \cdot 1$
			3	3.619	$20/11\ \cdot 2$
			4	5.227	$8/9\ \cdot 6$
			5	6.952	$4/3\ \cdot 5$
			6	8.777	$20/11\ \cdot 5$
			7	10.688	$20/11\ \cdot 6$



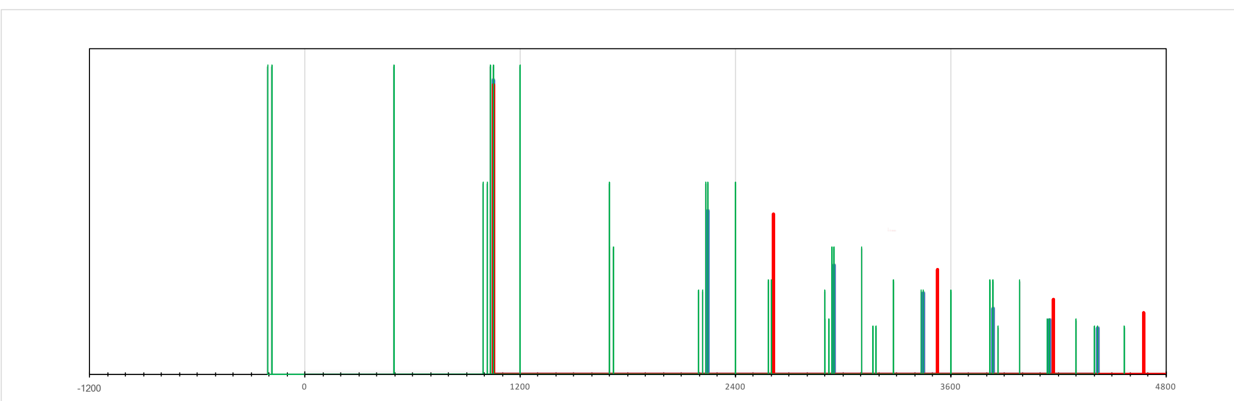
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
27	$\frac{20}{11} \cdot F_{1/1}$	1.442	1	1.818	$20/11 \cdot 1$
			2	4.940	$8/9 \cdot 6$
			3	8.864	$20/11 \cdot 5$
			4	13.422	$2/1 \cdot 7$
			5	18.516	n/a
			6	24.084	n/a
			7	30.079	n/a



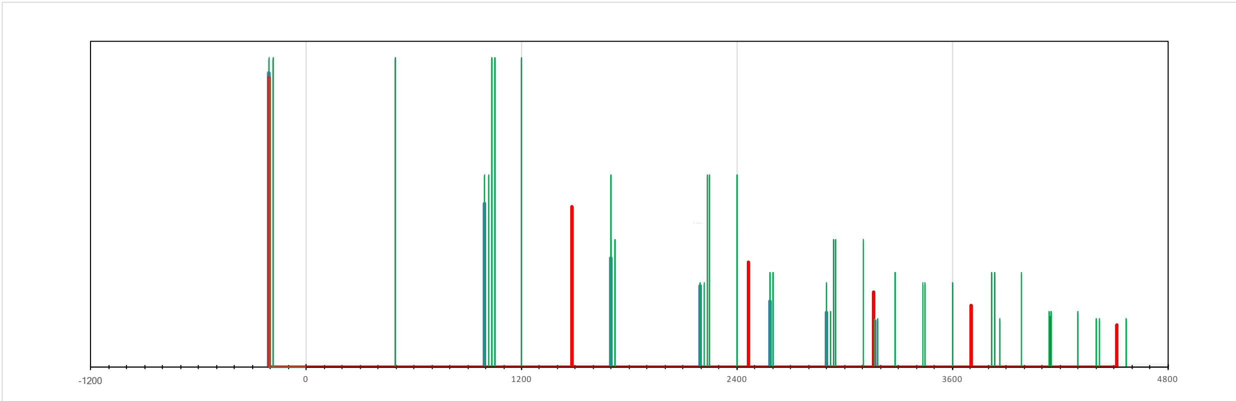
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
28	$\frac{9}{10} \cdot F_{1/1}$	1.2605	1	0.900	$9/10 \cdot 1$
			2	2.156	$2/1 \cdot 1$
			3	3.595	$9/10 \cdot 4$
			4	5.166	$4/3 \cdot 4$
			5	6.844	$4/3 \cdot 5$
			6	8.612	$20/11 \cdot 5$
			7	10.459	$20/11 \cdot 6$



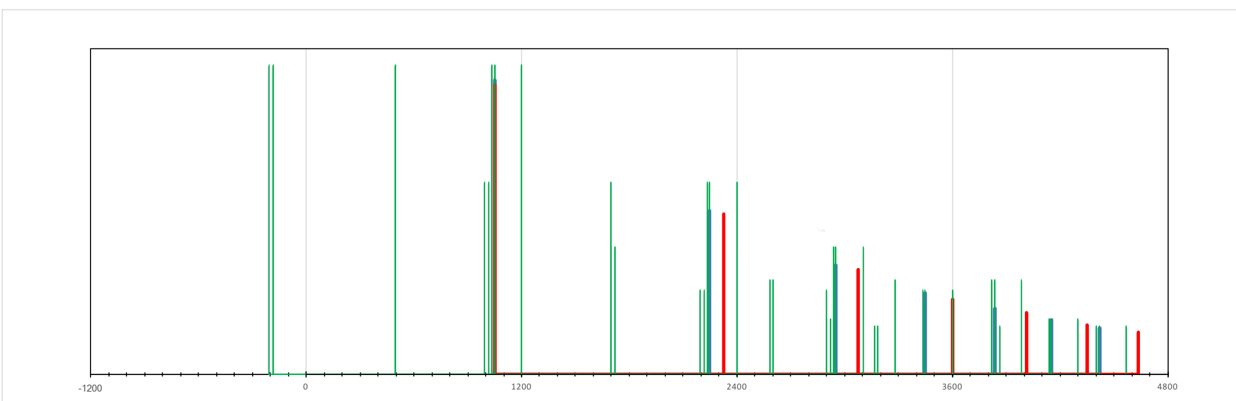
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\,/\,F_{1/1}$	$NASH$
29	$\frac{2}{1} \cdot F_{1/1}$	1.322	1	2.000	$2/1 \cdot 1$
			2	5.000	$8/9 \cdot 6$
			3	8.546	$20/11 \cdot 5$
			4	12.501	$20/11 \cdot 7$
			5	16.791	$2/1 \cdot 7$
			6	21.367	n/a
			7	26.197	n/a



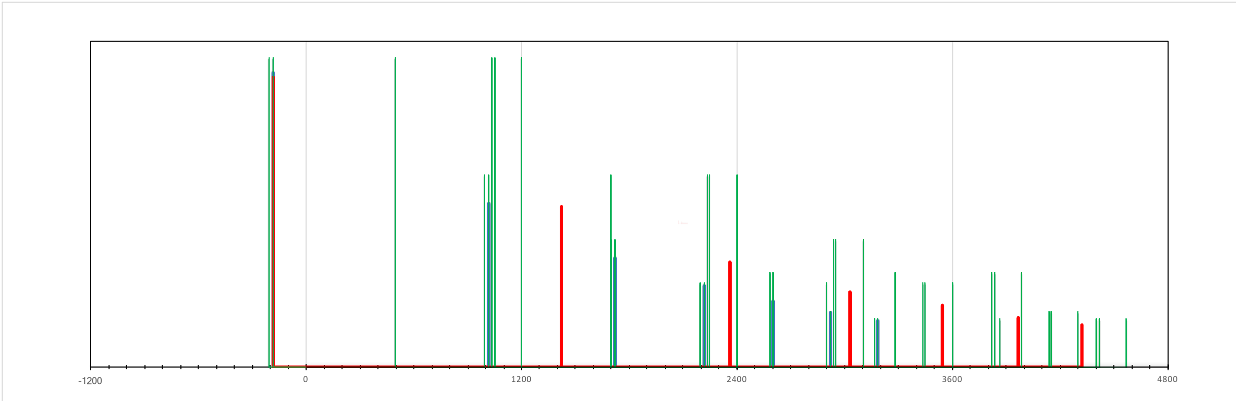
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\,/\,F_{1/1}$	$NASH$
30 *diff.	$\frac{11}{6} \cdot F_{1/1}$	1.3025	1	1.833	$11/6 \cdot 1$
			2	4.522	$9/10 \cdot 5$
			3	7.668	$4/3 \cdot 6$
			4	11.154	$11/6 \cdot 6$
			5	14.916	$2/1 \cdot 7$
			6	18.914	n/a
			7	23.120	n/a



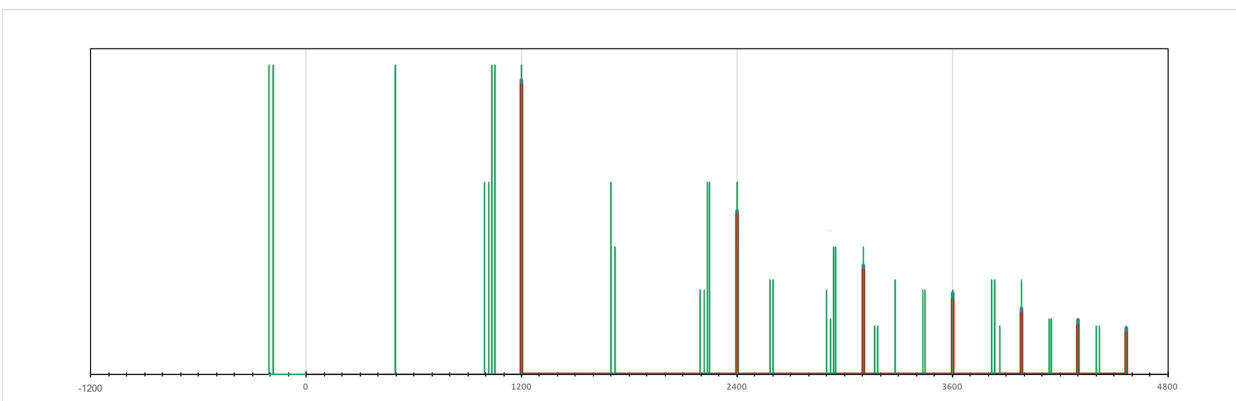
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
31	$\frac{8}{9} \cdot F_{1/1}$	1.402	1	0.889	$8/9\ \cdot 1$
			2	2.349	$8/9\ \cdot 3$
			3	4.147	$4/3\ \cdot 3$
			4	6.208	$8/9\ \cdot 7$
			5	8.488	$4/3\ \cdot 6$
			6	10.960	$11/6\ \cdot 6$
			7	13.604	$2/1\ \cdot 7$



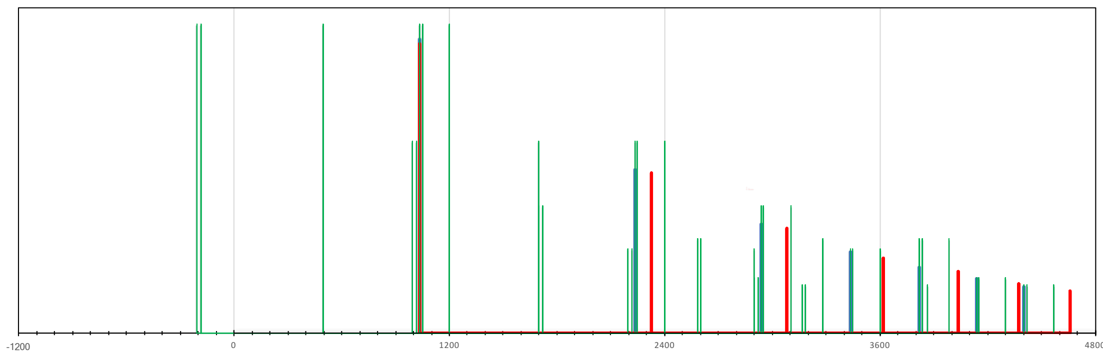
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
32	$\frac{11}{6} \cdot F_{1/1}$	1.0645	1	1.833	$11/6\ \cdot 1$
			2	3.834	$4/3\ \cdot 3$
			3	5.904	$2/1\ \cdot 3$
			4	8.019	$4/3\ \cdot 6$
			5	10.169	$2/1\ \cdot 5$
			6	12.348	$2/1\ \cdot 6$
			7	14.550	$2/1\ \cdot 7$



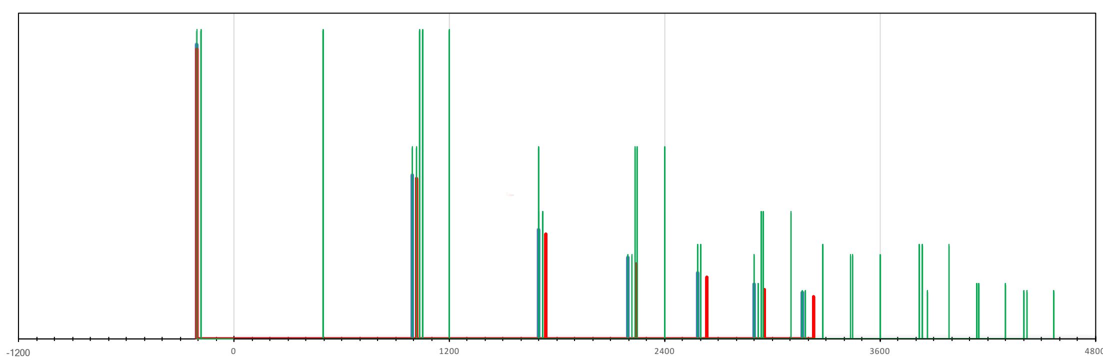
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
33	$\frac{9}{10} \cdot F_{1/1}$	1.338	1	0.900	$9/10 \cdot 1$
			2	2.275	$2/1 \cdot 1$
			3	3.914	$4/3 \cdot 3$
			4	5.752	$2/1 \cdot 3$
			5	7.753	$4/3 \cdot 6$
			6	9.895	$2/1 \cdot 5$
			7	12.161	$2/1 \cdot 6$



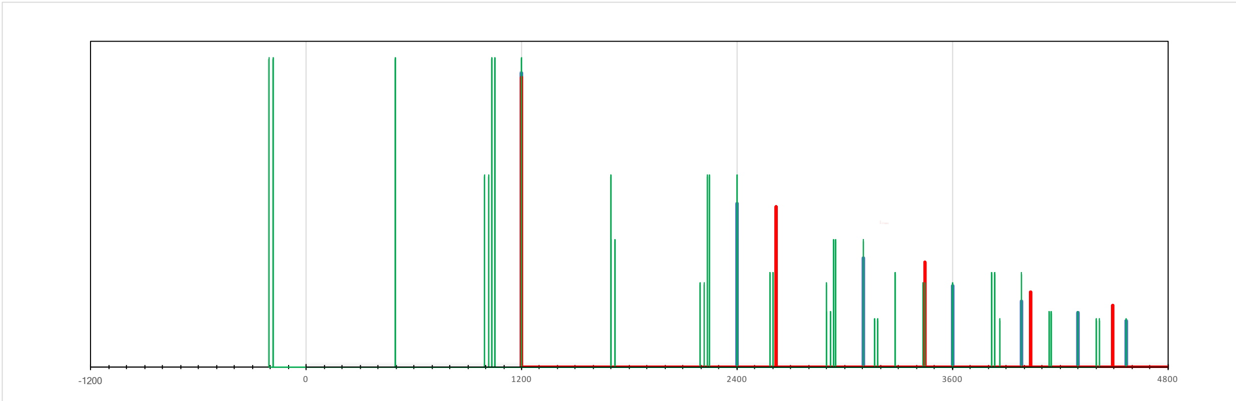
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
34	$\frac{2}{1} \cdot F_{1/1}$	1.0	1	2.000	$2/1 \cdot 1$
			2	4.000	$2/1 \cdot 2$
			3	6.000	$2/1 \cdot 3$
			4	8.000	$2/1 \cdot 4$
			5	10.000	$2/1 \cdot 5$
			6	12.000	$2/1 \cdot 6$
			7	14.000	$2/1 \cdot 7$



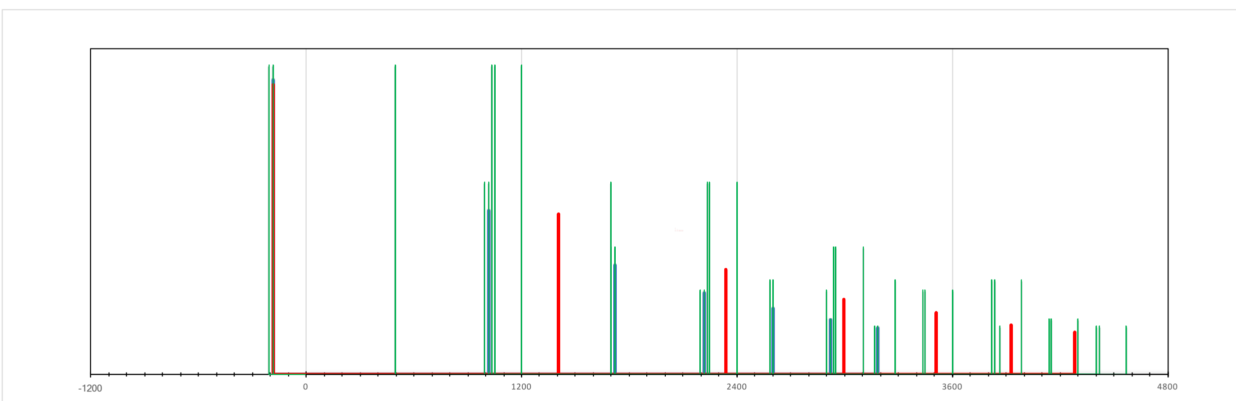
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
35	$\frac{20}{11} \cdot F_{1/1}$	1.0765	1	1.818	$20/11\ \cdot 1$
			2	3.834	$4/3\ \cdot 3$
			3	5.933	$2/1\ \cdot 3$
			4	8.086	$4/3\ \cdot 6$
			5	10.282	$2/1\ \cdot 5$
			6	12.512	$20/11\ \cdot 7$
			7	14.770	$2/1\ \cdot 7$



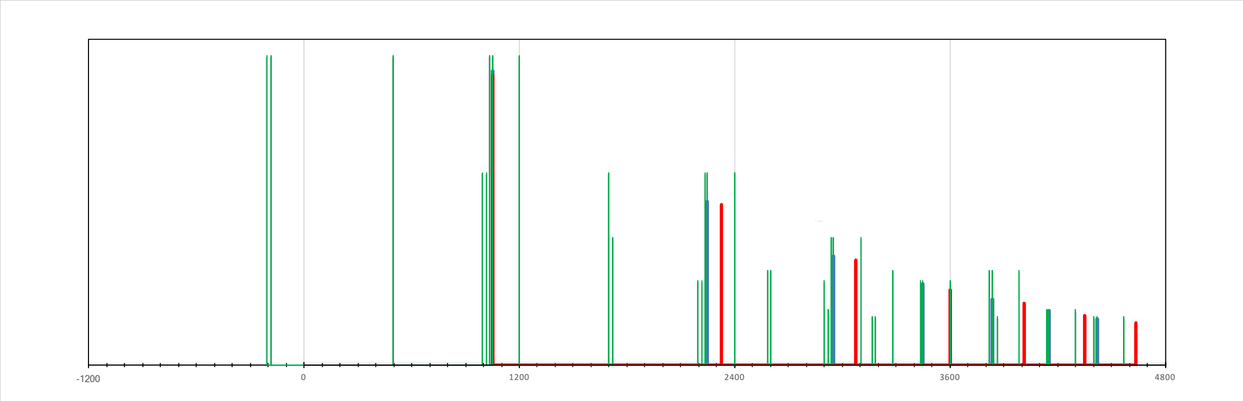
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
36 *diff.	$\frac{8}{9} \cdot F_{1/1}$	1.0193	1	0.889	$8/9\ \cdot 1$
			2	1.802	$9/10\ \cdot 2$
			3	2.724	$9/10\ \cdot 3$
			4	3.652	$11/6\ \cdot 2$
			5	4.585	$9/10\ \cdot 5$
			6	5.521	$11/6\ \cdot 3$
			7	6.460	$9/10\ \cdot 7$



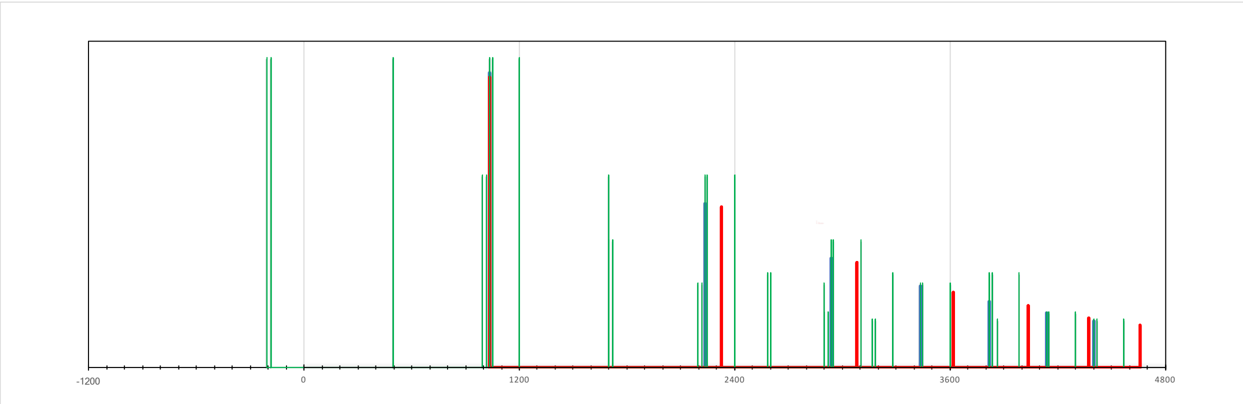
State #	$F_{RBI\ (1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
37	$\frac{2}{1} \cdot F_{1/1}$	1.183	1	2.000	$2/1 \cdot 1$
			2	4.541	$9/10 \cdot 5$
			3	7.336	$11/6 \cdot 4$
			4	10.310	$2/1 \cdot 5$
			5	13.425	$2/1 \cdot 7$
			6	16.656	n/a
			7	19.989	n/a



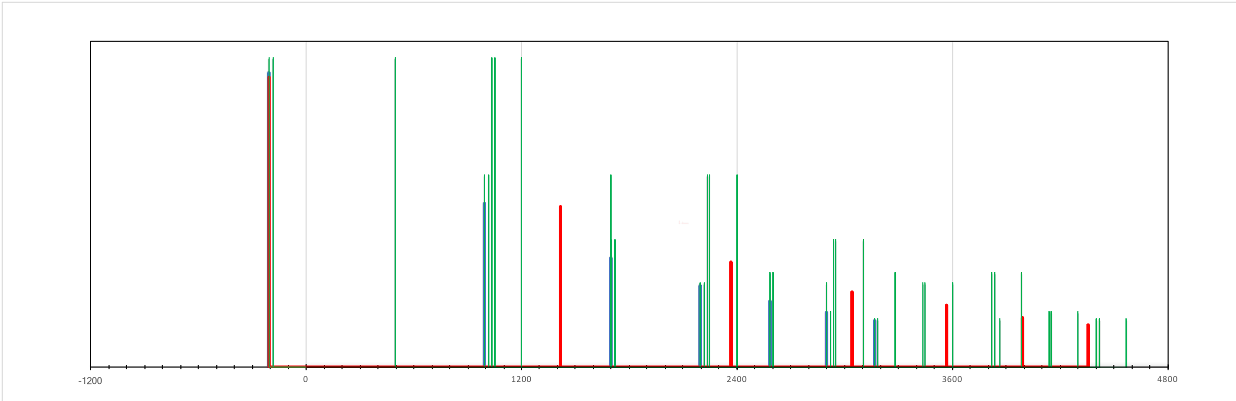
State #	$F_{RBI\ (1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
38	$\frac{9}{10} \cdot F_{1/1}$	1.325	1	0.900	$9/10 \cdot 1$
			2	2.255	$2/1 \cdot 1$
			3	3.859	$4/3 \cdot 3$
			4	5.649	$11/6 \cdot 3$
			5	7.592	$11/6 \cdot 4$
			6	9.667	$2/1 \cdot 5$
			7	11.858	$2/1 \cdot 6$



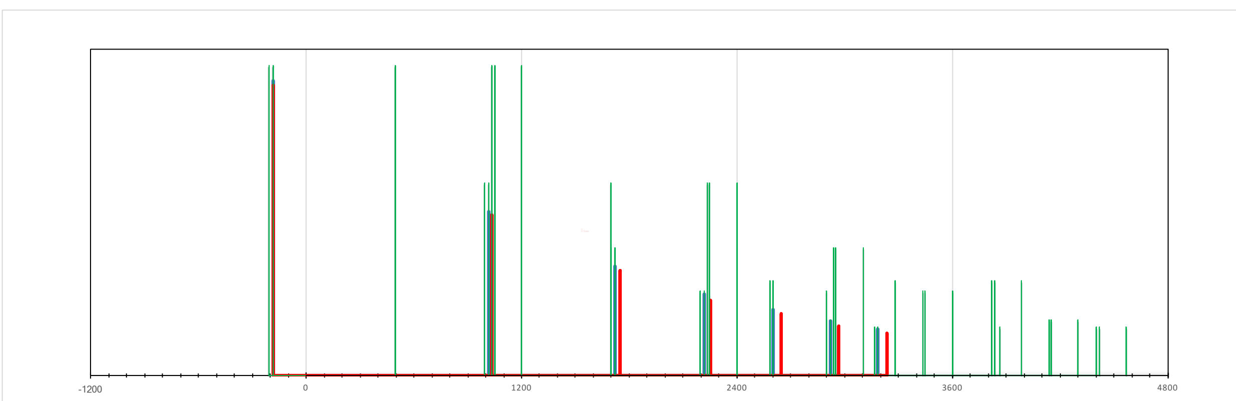
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
39	$\frac{11}{6} \cdot F_{1/1}$	1.0645	1	1.833	$11/6 \cdot 1$
			2	3.834	$4/3 \cdot 3$
			3	5.904	$2/1 \cdot 3$
			4	8.019	$4/3 \cdot 6$
			5	10.169	$2/1 \cdot 5$
			6	12.348	$2/1 \cdot 6$
			7	14.550	$2/1 \cdot 7$



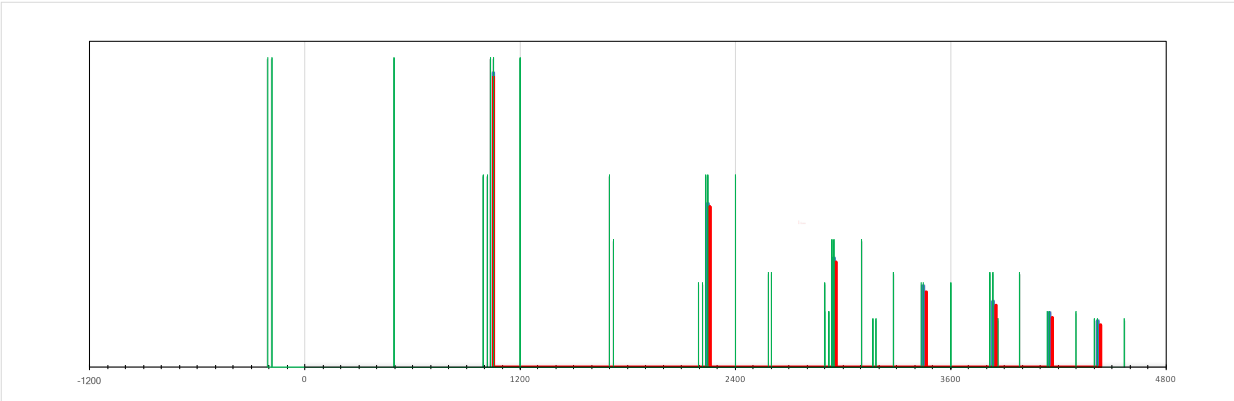
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
40	$\frac{20}{11} \cdot F_{1/1}$	1.0765	1	1.818	$20/11 \cdot 1$
			2	3.834	$4/3 \cdot 3$
			3	5.933	$2/1 \cdot 3$
			4	8.086	$4/3 \cdot 6$
			5	10.282	$2/1 \cdot 5$
			6	12.512	$20/11 \cdot 7$
			7	14.770	$2/1 \cdot 7$



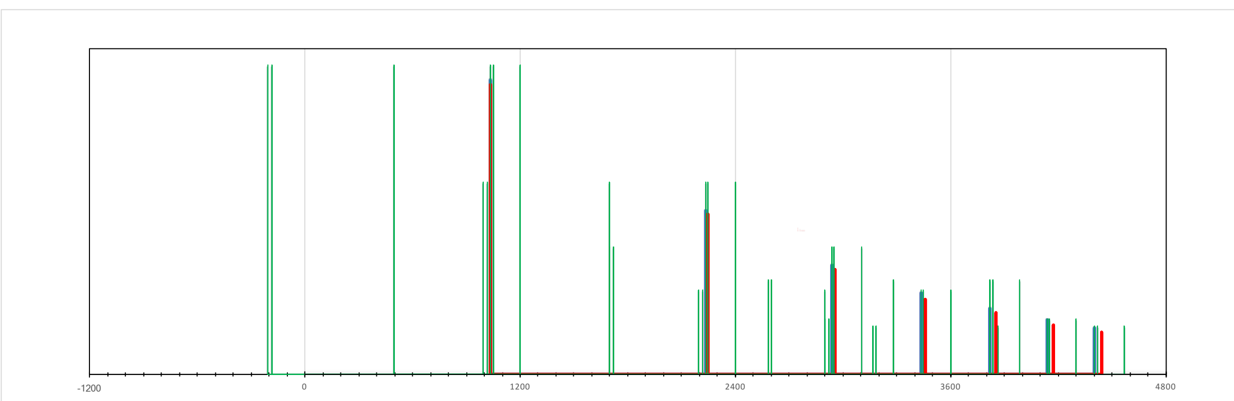
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
41	$\frac{8}{9} \cdot F_{1/1}$	1.353	1	0.889	$8/9 \cdot 1$
			2	2.271	$2/1 \cdot 1$
			3	3.930	$4/3 \cdot 3$
			4	5.800	$2/1 \cdot 3$
			5	7.844	$4/3 \cdot 6$
			6	10.039	$2/1 \cdot 5$
			7	12.367	$20/11 \cdot 7$



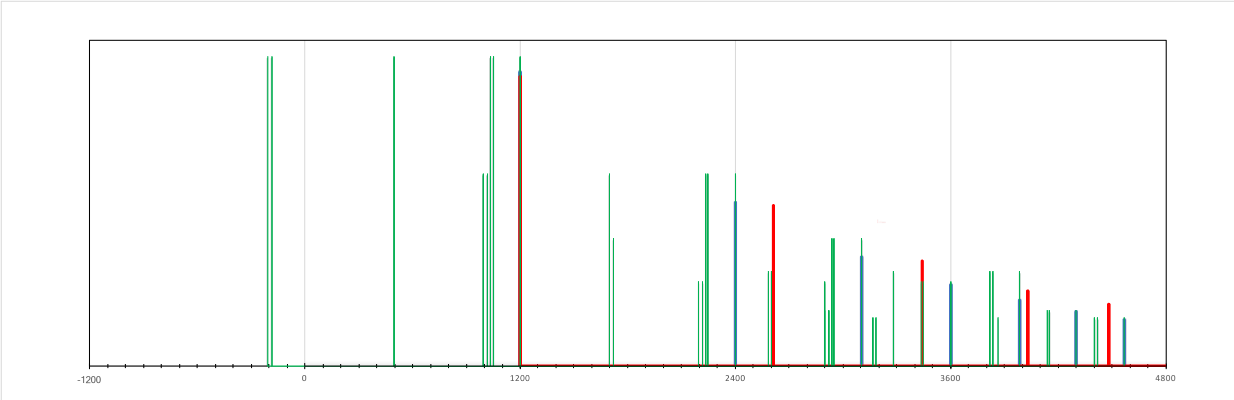
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
42 *diff.	$\frac{9}{10} \cdot F_{1/1}$	1.015	1	0.900	$9/10 \cdot 1$
			2	1.819	$20/11 \cdot 1$
			3	2.745	$9/10 \cdot 3$
			4	3.676	$11/6 \cdot 2$
			5	4.610	$9/10 \cdot 5$
			6	5.547	$11/6 \cdot 3$
			7	6.487	$2/1 \cdot 3$



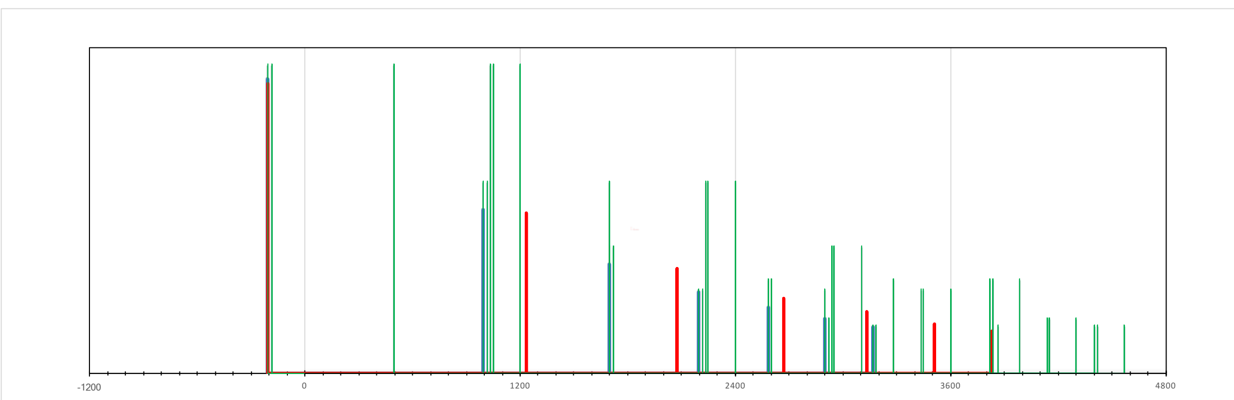
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
43	$\frac{11}{6} \cdot F_{1/1}$	1.006	1	1.833	$11/6 \cdot 1$
			2	3.682	$11/6 \cdot 2$
			3	5.536	$11/6 \cdot 3$
			4	7.395	$11/6 \cdot 4$
			5	9.256	$4/3 \cdot 7$
			6	11.119	$11/6 \cdot 6$
			7	12.984	$11/6 \cdot 7$



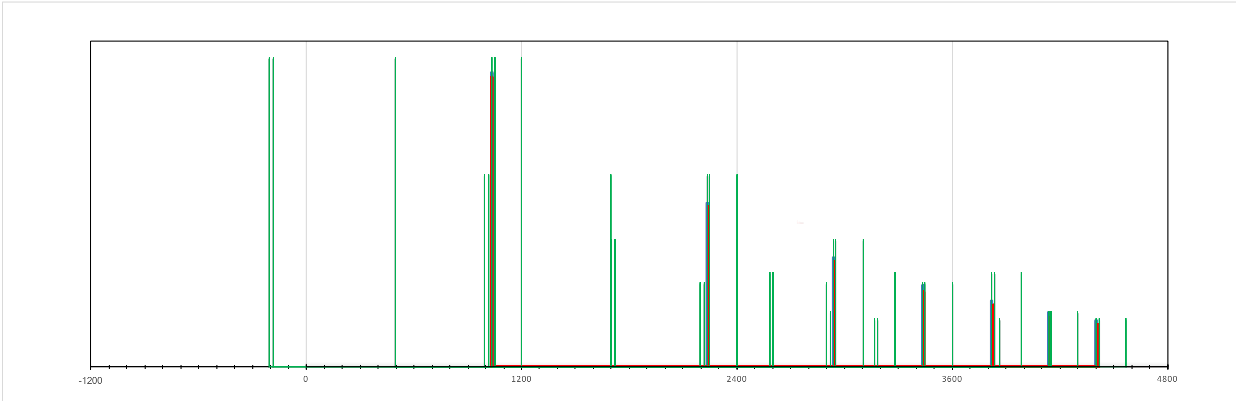
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
44	$\frac{20}{11} \cdot F_{1/1}$	1.011	1	1.818	$20/11 \cdot 1$
			2	3.664	$11/6 \cdot 2$
			3	5.521	$11/6 \cdot 3$
			4	7.384	$11/6 \cdot 4$
			5	9.253	$4/3 \cdot 7$
			6	11.126	$11/6 \cdot 6$
			7	13.003	$11/6 \cdot 7$



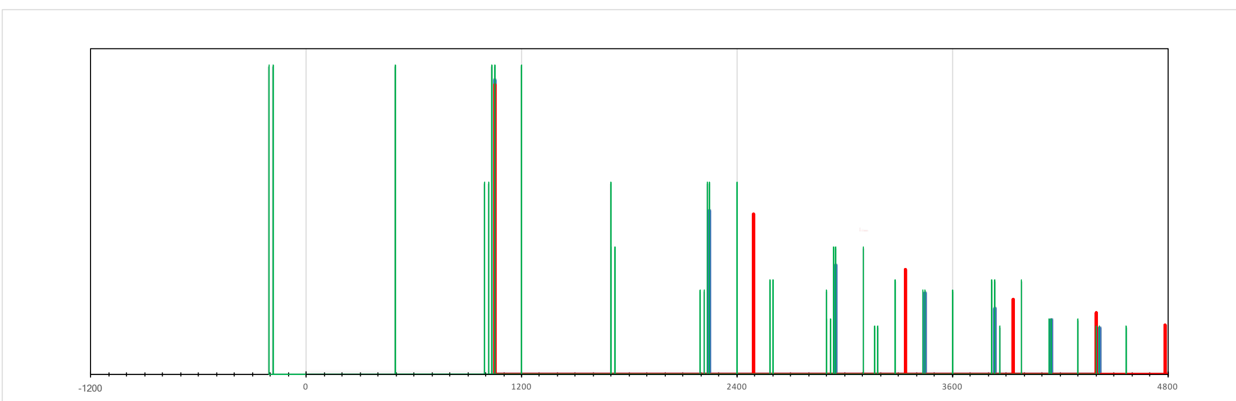
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
45	$\frac{2}{1} \cdot F_{1/1}$	1.179	1	2.000	$2/1\ \cdot 1$
			2	4.528	$9/10\ \cdot 5$
			3	7.304	$11/6\ \cdot 4$
			4	10.253	$2/1\ \cdot 5$
			5	13.339	$11/6\ \cdot 7$
			6	16.538	$2/1\ \cdot 7$
			7	19.834	$n/a\ \cdot$



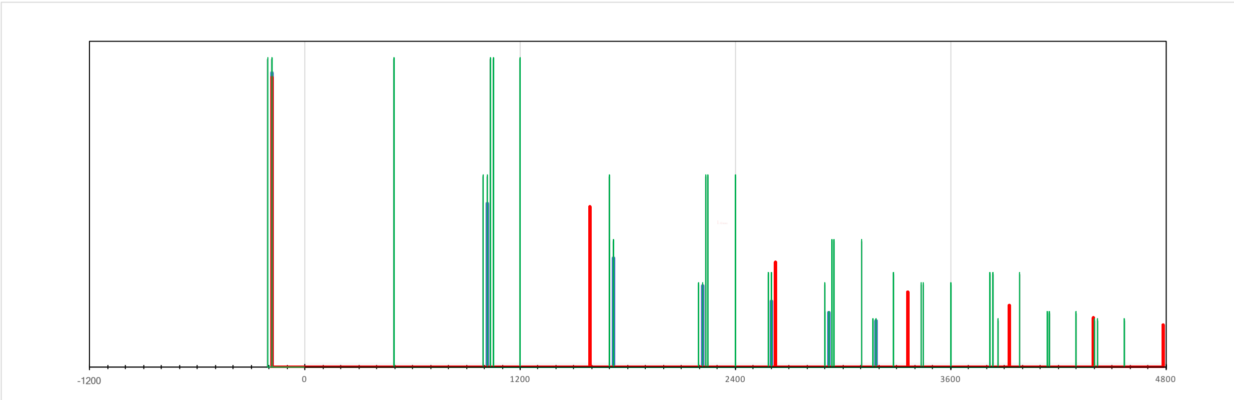
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
46	$\frac{8}{9} \cdot F_{1/1}$	1.197	1	0.889	$8/9\ \cdot 1$
			2	2.038	$2/1\ \cdot 1$
			3	3.311	$8/9\ \cdot 4$
			4	4.672	$9/10\ \cdot 5$
			5	6.103	$2/1\ \cdot 3$
			6	7.591	$11/6\ \cdot 4$
			7	9.129	$11/6\ \cdot 5$



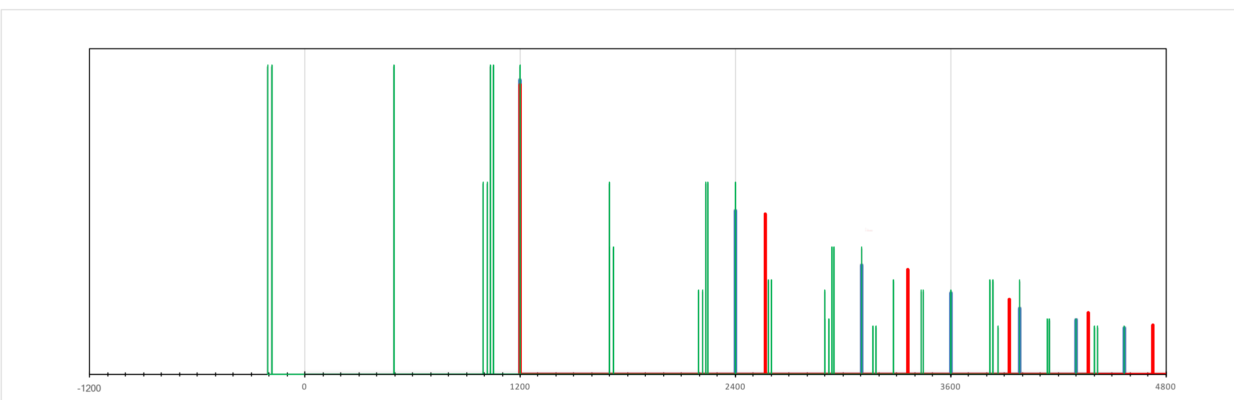
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
47	$\frac{20}{11} \cdot F_{1/1}$	1.003	1	1.818	$20/11 \cdot 1$
			2	3.644	$20/11 \cdot 2$
			3	5.473	$20/11 \cdot 3$
			4	7.303	$11/6 \cdot 4$
			5	9.135	$11/6 \cdot 5$
			6	10.968	$11/6 \cdot 6$
			7	12.802	$11/6 \cdot 7$



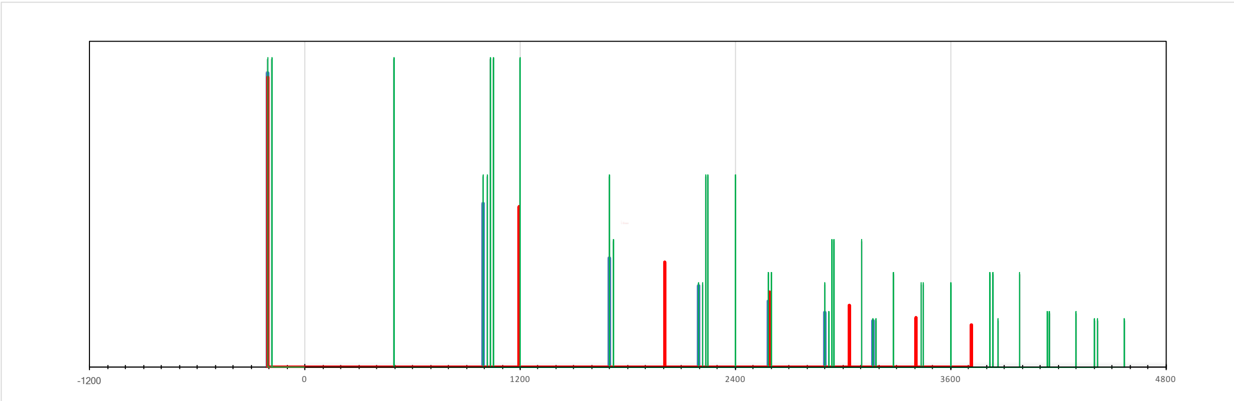
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
48 *diff.	$\frac{11}{6} \cdot F_{1/1}$	1.204	1	1.833	$11/6 \cdot 1$
			2	4.224	$8/9 \cdot 5$
			3	6.882	$4/3 \cdot 5$
			4	9.730	$2/1 \cdot 5$
			5	12.729	$20/11 \cdot 7$
			6	15.854	$2/1 \cdot 7$
			7	19.087	n/a



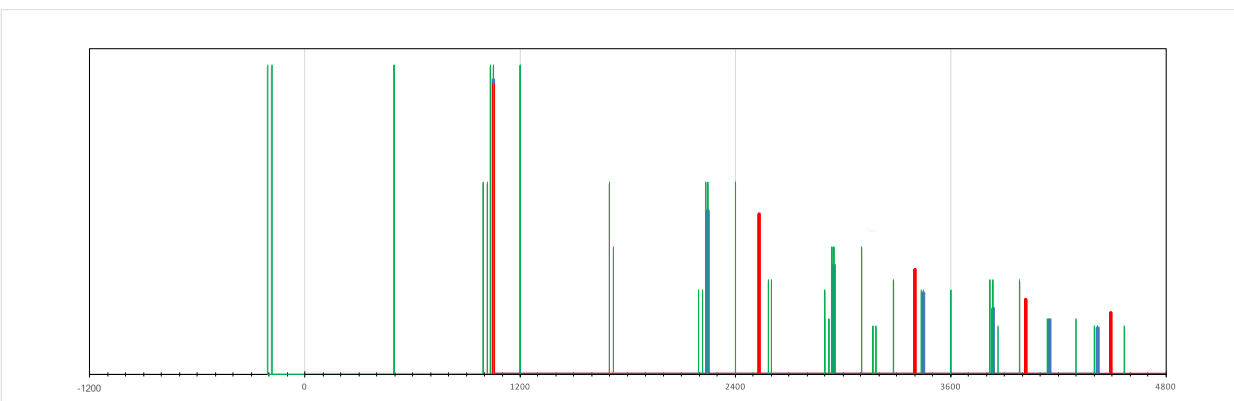
State #	$F_{RBI\ (1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
49	$\frac{9}{10} \cdot F_{1/1}$	1.4755	1	0.900	$9/10\ \cdot 1$
			2	2.503	$8/9\ \cdot 3$
			3	4.552	$9/10\ \cdot 5$
			4	6.960	$4/3\ \cdot 5$
			5	9.673	$2/1\ \cdot 5$
			6	12.659	$20/11\ \cdot 7$
			7	15.892	$2/1\ \cdot 7$



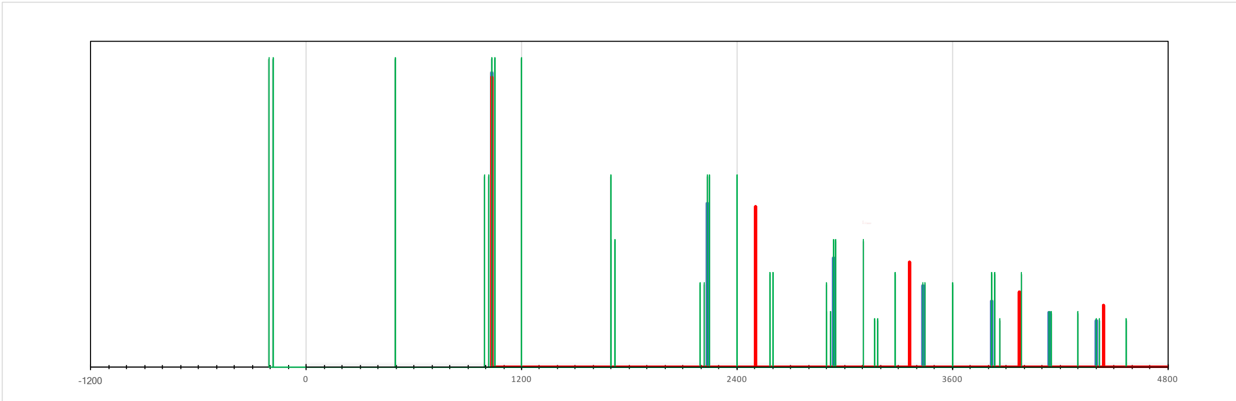
State #	$F_{RBI\ (1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
50	$\frac{2}{1} \cdot F_{1/1}$	1.137	1	2.000	$2/1\ \cdot 1$
			2	4.398	$8/9\ \cdot 5$
			3	6.975	$20/11\ \cdot 4$
			4	9.673	$2/1\ \cdot 5$
			5	12.467	$20/11\ \cdot 7$
			6	15.339	$2/1\ \cdot 7$
			7	18.277	n/a



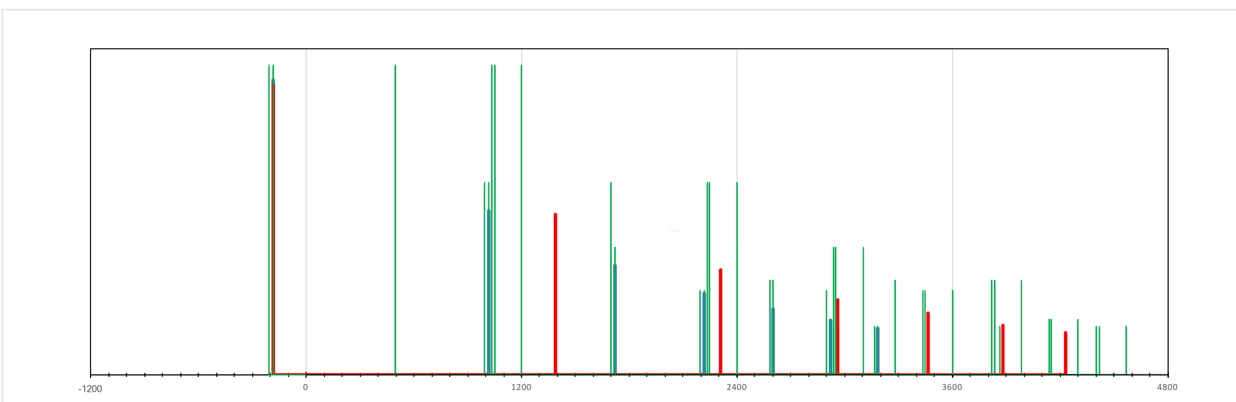
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
51	$\frac{8}{9}\cdot F_{1/1}$	1.1635	1	0.889	$8/9\ \cdot 1$
			2	1.991	$2/1\ \cdot 1$
			3	3.191	$8/9\ \cdot 4$
			4	4.460	$8/9\ \cdot 5$
			5	5.782	$2/1\ \cdot 3$
			6	7.149	$20/11\ \cdot 4$
			7	8.553	$20/11\ \cdot 5$



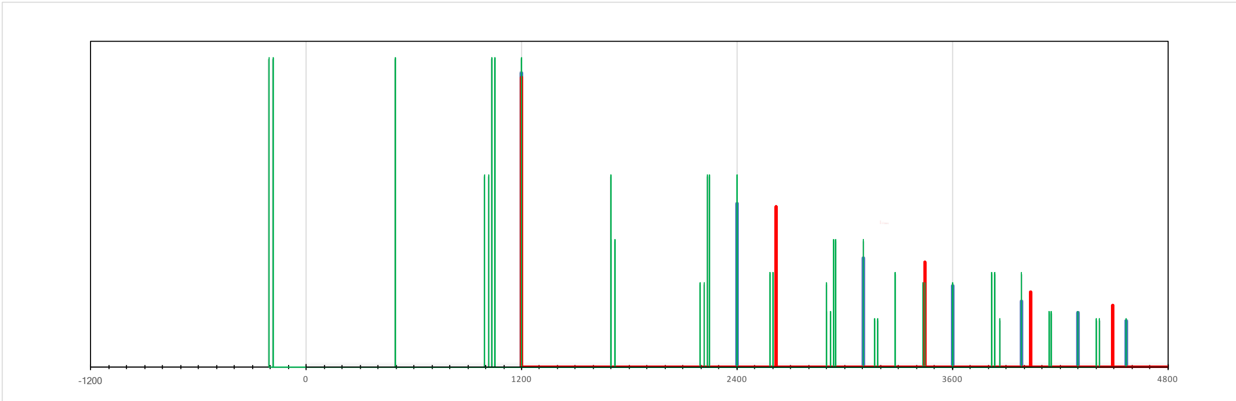
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
52	$\frac{11}{6}\cdot F_{1/1}$	1.237	1	1.833	$11/6\ \cdot 1$
			2	4.321	$8/9\ \cdot 5$
			3	7.136	$20/11\ \cdot 4$
			4	10.186	$2/1\ \cdot 5$
			5	13.424	$2/1\ \cdot 7$
			6	16.820	n/a
			7	20.353	n/a



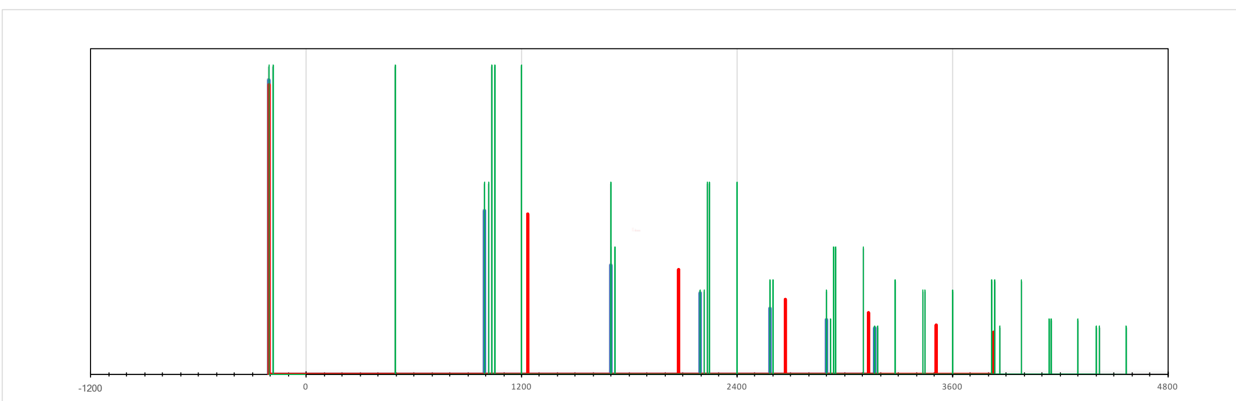
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
53	$\frac{20}{11} \cdot F_{1/1}$	1.2235	1	1.818	$20/11 \cdot 1$
			2	4.246	$8/9 \cdot 5$
			3	6.973	$20/11 \cdot 4$
			4	9.914	$2/1 \cdot 5$
			5	13.026	$11/6 \cdot 7$
			6	16.282	$2/1 \cdot 7$
			7	19.661	n/a



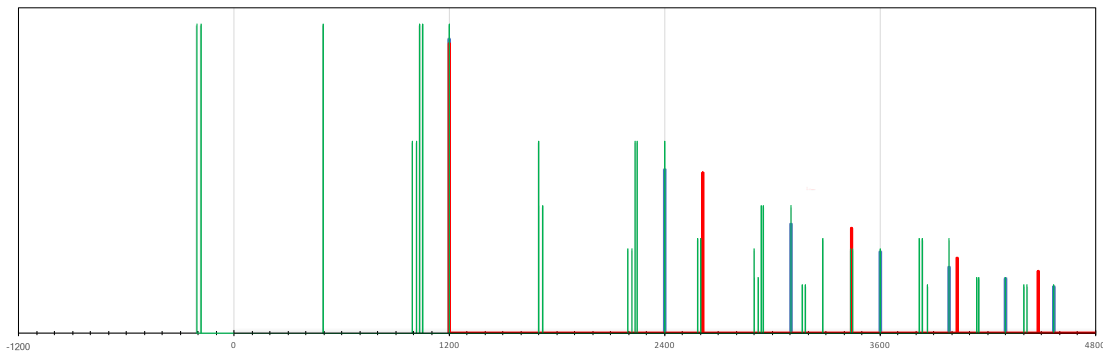
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
54 *diff.	$\frac{9}{10} \cdot F_{1/1}$	1.3095	1	0.900	$9/10 \cdot 1$
			2	2.231	$2/1 \cdot 1$
			3	3.793	$11/6 \cdot 2$
			4	5.529	$11/6 \cdot 3$
			5	7.405	$11/6 \cdot 4$
			6	9.402	$4/3 \cdot 7$
			7	11.505	$2/1 \cdot 6$



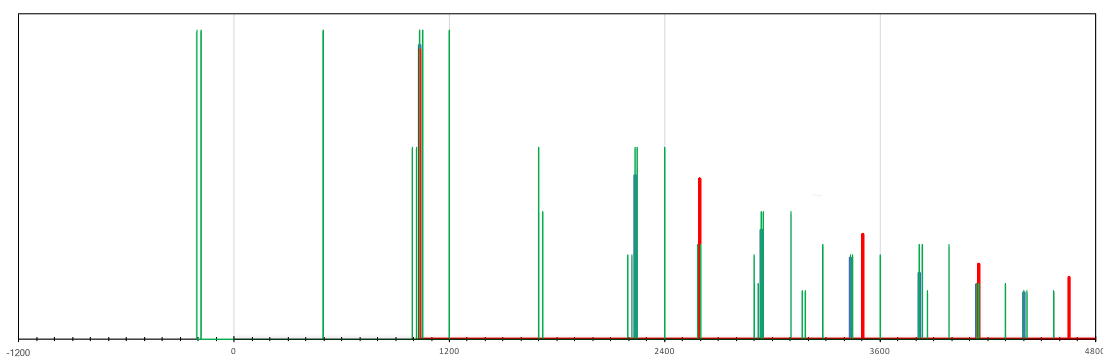
State #	$F_{RBI\ (1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
55	$\frac{2}{1} \cdot F_{1/1}$	1.183	1	2.000	$2/1 \cdot 1$
			2	4.541	$9/10 \cdot 5$
			3	7.336	$11/6 \cdot 4$
			4	10.310	$2/1 \cdot 5$
			5	13.425	$2/1 \cdot 7$
			6	16.656	n/a
			7	19.989	n/a



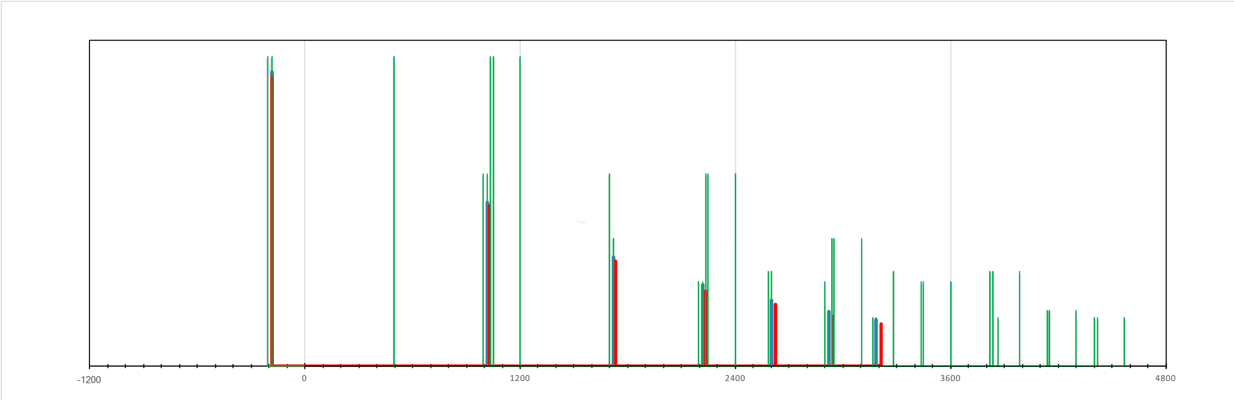
State #	$F_{RBI\ (1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
56	$\frac{8}{9} \cdot F_{1/1}$	1.197	1	0.889	$8/9 \cdot 1$
			2	2.038	$2/1 \cdot 1$
			3	3.311	$8/9 \cdot 4$
			4	4.672	$9/10 \cdot 5$
			5	6.103	$2/1 \cdot 3$
			6	7.591	$11/6 \cdot 4$
			7	9.129	$11/6 \cdot 5$



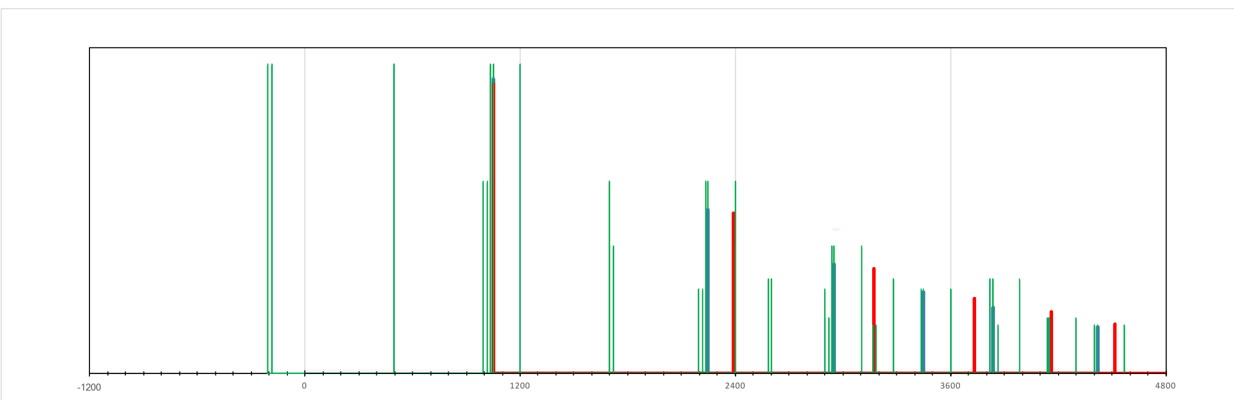
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\,/\,F_{1/1}$	$NASH$
57	$\frac{2}{1}\cdot F_{1/1}$	1.179	1	2.000	$2/1\ \cdot 1$
			2	4.528	$9/10\ \cdot 5$
			3	7.304	$11/6\ \cdot 4$
			4	10.253	$2/1\ \cdot 5$
			5	13.339	$11/6\ \cdot 7$
			6	16.538	$2/1\ \cdot 7$
			7	19.834	n/a



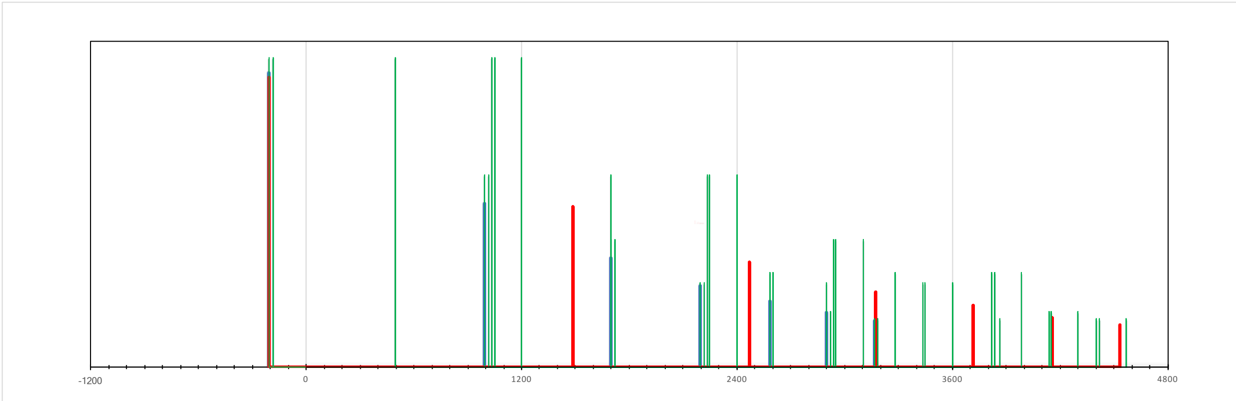
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\,/\,F_{1/1}$	$NASH$
58	$\frac{20}{11}\cdot F_{1/1}$	1.2985	1	1.818	$20/11\ \cdot 1$
			2	4.472	$9/10\ \cdot 5$
			3	7.571	$11/6\ \cdot 4$
			4	11.000	$11/6\ \cdot 6$
			5	14.698	$2/1\ \cdot 7$
			6	18.624	n/a
			7	22.751	n/a



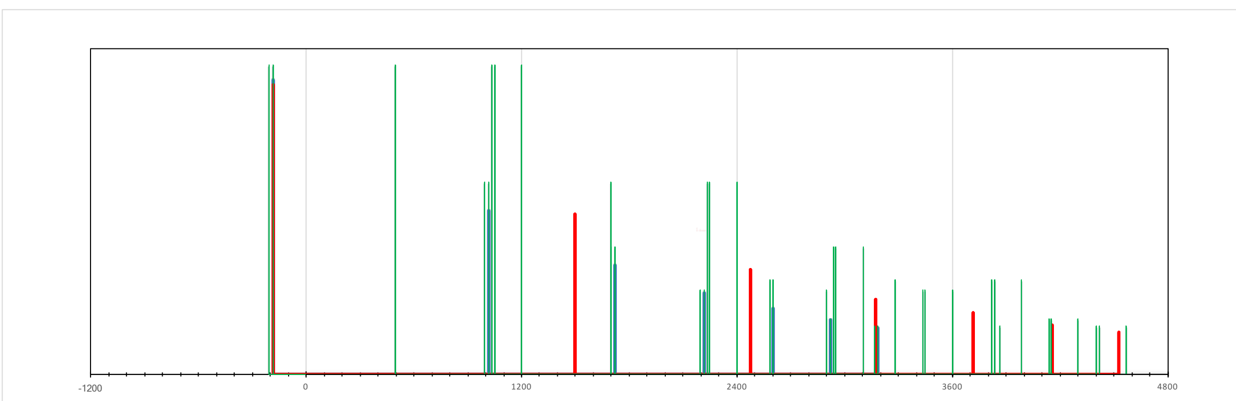
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
59	$\frac{9}{10} \cdot F_{1/1}$	1.0075	1	0.900	$9/10\ \cdot 1$
			2	1.809	$20/11\ \cdot 1$
			3	2.722	$9/10\ \cdot 3$
			4	3.638	$20/11\ \cdot 2$
			5	4.555	$9/10\ \cdot 5$
			6	5.473	$20/11\ \cdot 3$
			7	6.393	$9/10\ \cdot 7$



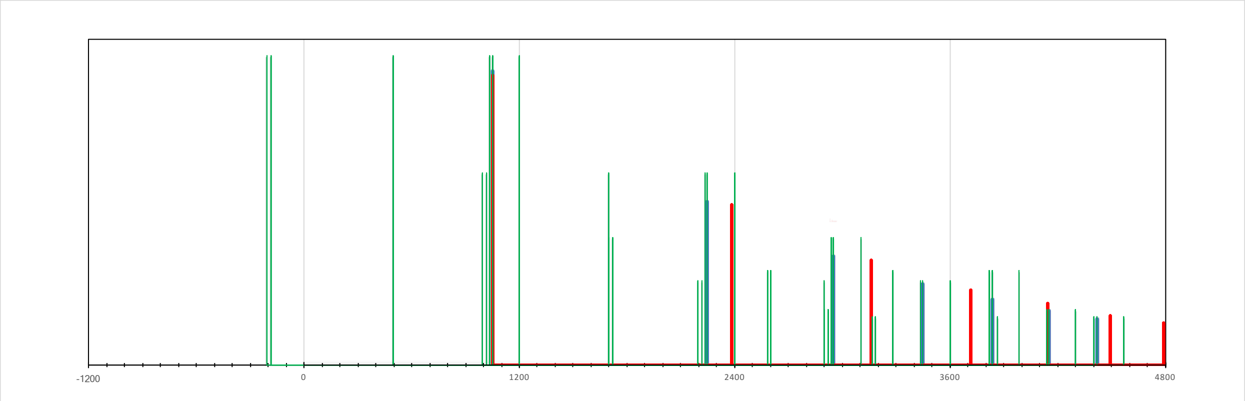
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
60 *diff.	$\frac{11}{6} \cdot F_{1/1}$	1.118	1	1.833	$11/6\ \cdot 1$
			2	3.979	$4/3\ \cdot 4$
			3	6.261	$8/9\ \cdot 7$
			4	8.637	$20/11\ \cdot 5$
			5	11.084	$11/6\ \cdot 6$
			6	13.590	$2/1\ \cdot 7$
			7	16.146	n/a



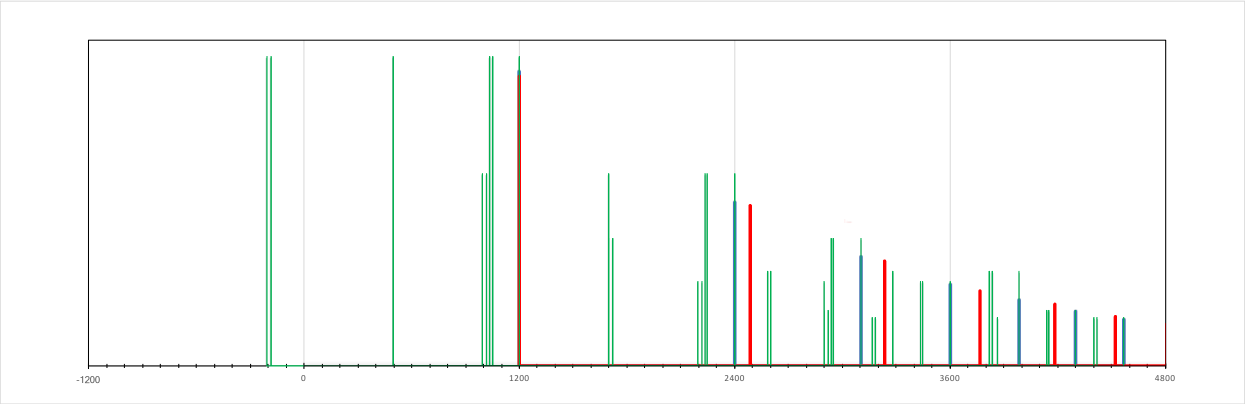
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
61	$\frac{8}{9} \cdot F_{1/1}$	1.4065	1	0.889	$8/9\ \cdot 1$
			2	2.356	$8/9\ \cdot 3$
			3	4.168	$4/3\ \cdot 3$
			4	6.247	$8/9\ \cdot 7$
			5	8.550	$20/11\ \cdot 5$
			6	11.049	$11/6\ \cdot 6$
			7	13.724	$2/1\ \cdot 7$



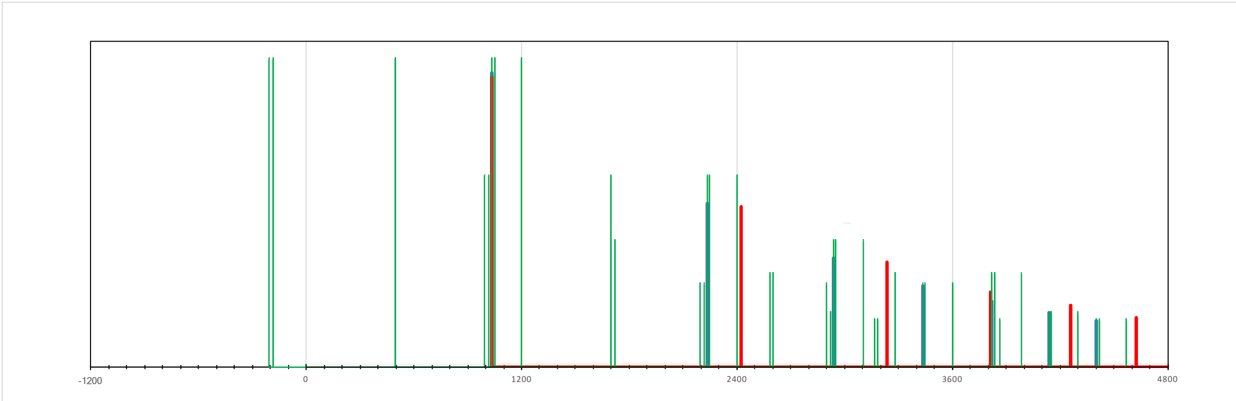
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
62	$\frac{9}{10} \cdot F_{1/1}$	1.3985	1	0.900	$9/10\ \cdot 1$
			2	2.373	$8/9\ \cdot 3$
			3	4.183	$4/3\ \cdot 3$
			4	6.255	$8/9\ \cdot 7$
			5	8.546	$20/11\ \cdot 5$
			6	11.028	$11/6\ \cdot 6$
			7	13.681	$2/1\ \cdot 7$



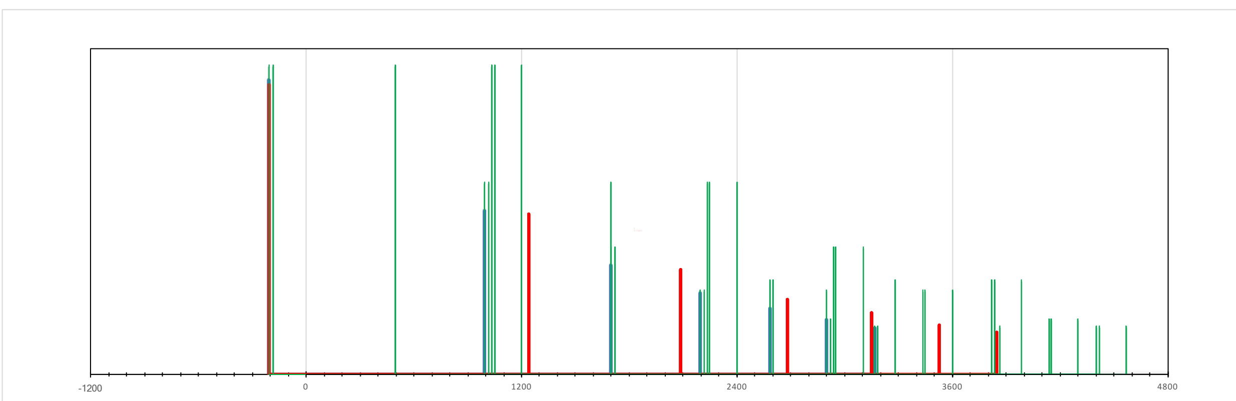
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
63	$\frac{11}{6} \cdot F_{1/1}$	1.111	1	1.833	$11/6\ \cdot 1$
			2	3.960	$4/3\ \cdot 4$
			3	6.213	$8/9\ \cdot 7$
			4	8.553	$20/11\ \cdot 5$
			5	10.960	$11/6\ \cdot 6$
			6	13.420	$2/1\ \cdot 7$
			7	15.927	n/a



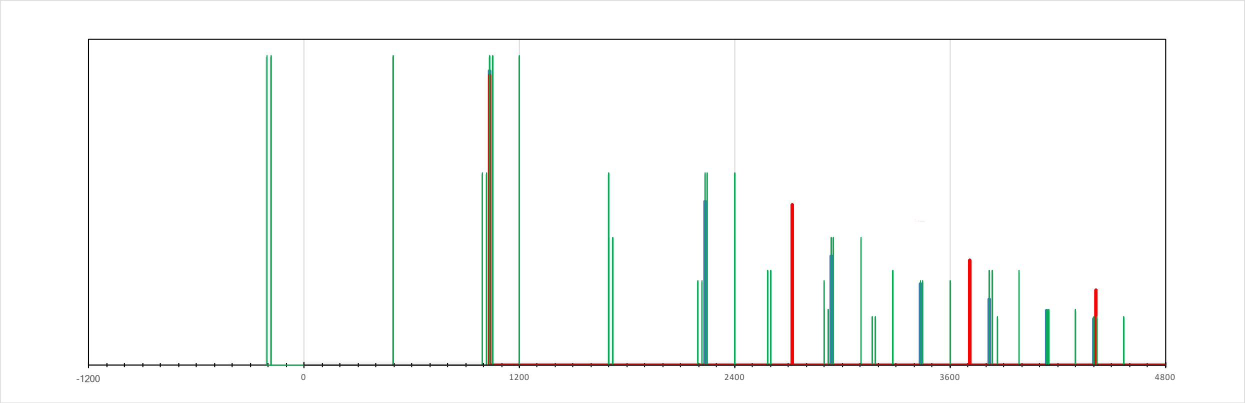
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
64	$\frac{2}{1} \cdot F_{1/1}$	1.071	1	2.000	$2/1\ \cdot 1$
			2	4.202	$4/3\ \cdot 3$
			3	6.487	$4/3\ \cdot 5$
			4	8.827	$20/11\ \cdot 5$
			5	11.211	$11/6\ \cdot 6$
			6	13.628	$2/1\ \cdot 7$
			7	16.074	n/a



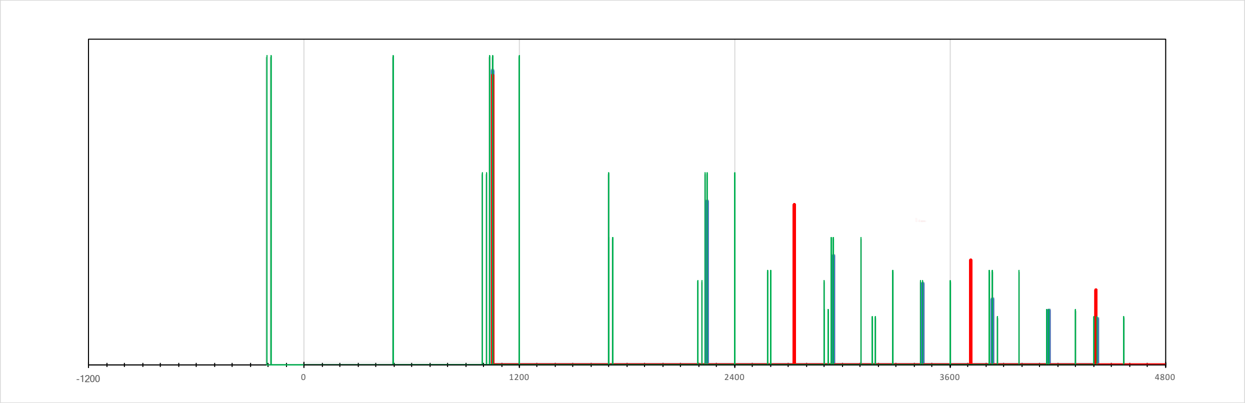
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
65	$\frac{20}{11} \cdot F_{1/1}$	1.1575	1	1.818	$20/11\ \cdot 1$
			2	4.056	$4/3\ \cdot 3$
			3	6.485	$4/3\ \cdot 5$
			4	9.047	$20/11\ \cdot 5$
			5	11.714	$2/1\ \cdot 6$
			6	14.466	$2/1\ \cdot 7$
			7	17.292	n/a



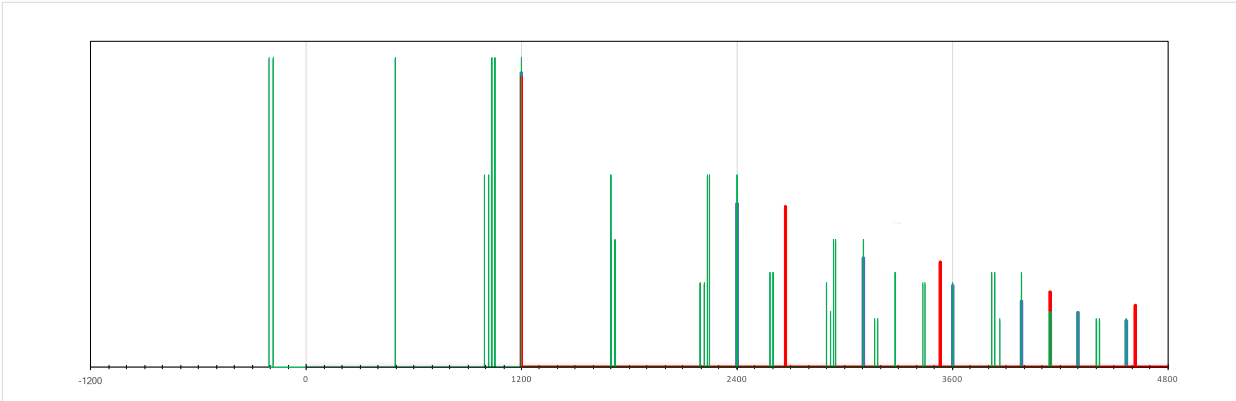
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
66 *diff.	$\frac{8}{9} \cdot F_{1/1}$	1.203	1	0.889	$8/9\ \cdot 1$
			2	2.046	$2/1\ \cdot 1$
			3	3.333	$8/9\ \cdot 4$
			4	4.711	$9/10\ \cdot 5$
			5	6.162	$8/9\ \cdot 7$
			6	7.673	$4/3\ \cdot 6$
			7	9.236	$11/6\ \cdot 5$



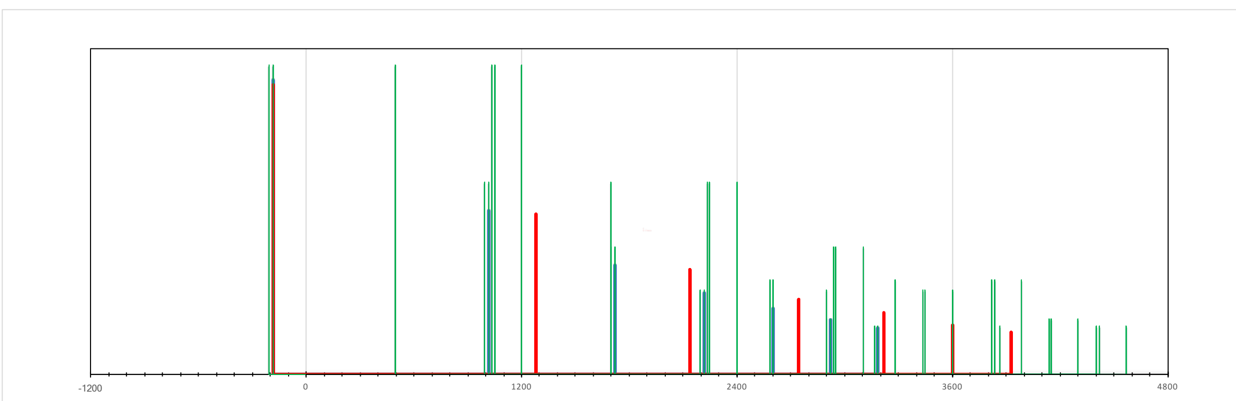
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
67	$\frac{20}{11} \cdot F_{1/1}$	1.407	1	1.818	$20/11\ \cdot 1$
			2	4.822	$9/10\ \cdot 5$
			3	8.530	$4/3\ \cdot 6$
			4	12.786	$11/6\ \cdot 7$
			5	17.502	$2/1\ \cdot 7$
			6	22.620	n/a
			7	28.099	n/a



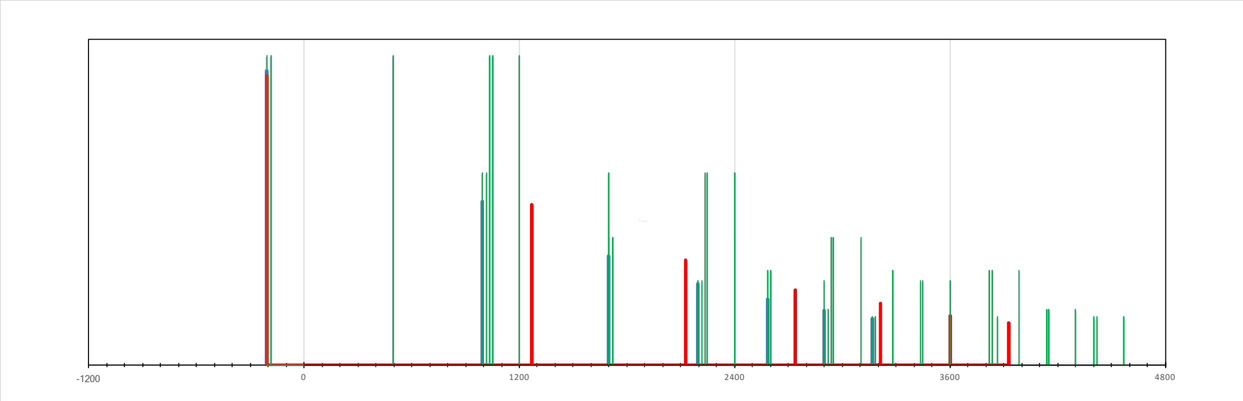
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
68	$\frac{11}{6} \cdot F_{1/1}$	1.401	1	1.833	$11/6\ \cdot 1$
			2	4.842	$9/10\ \cdot 5$
			3	8.545	$4/3\ \cdot 6$
			4	12.786	$11/6\ \cdot 7$
			5	17.478	$2/1\ \cdot 7$
			6	22.565	n/a
			7	28.004	n/a



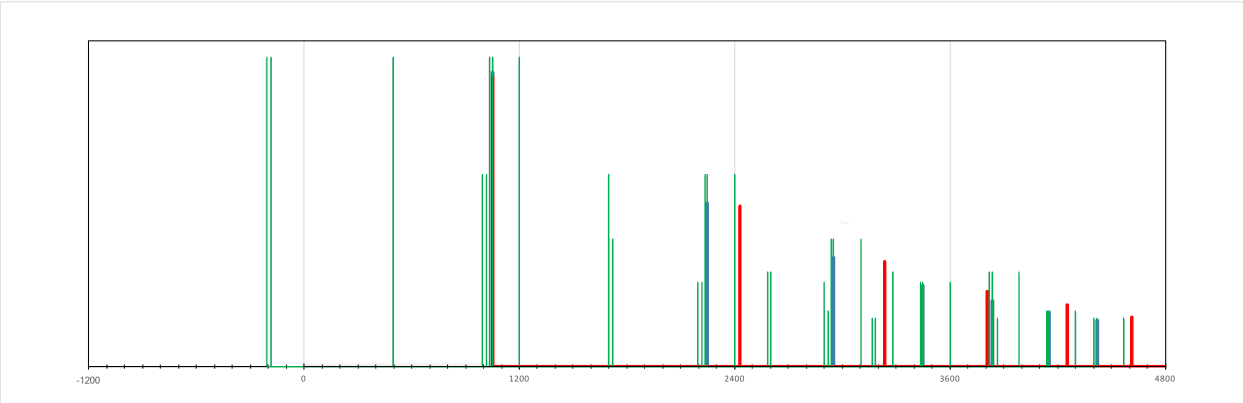
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
69	$\frac{2}{1}\ \cdot\ F_{1/1}$	1.227	1	2.000	$2/1\ \cdot 1$
			2	4.682	$9/10\ \cdot 5$
			3	7.699	$4/3\ \cdot 6$
			4	10.959	$11/6\ \cdot 6$
			5	14.410	$2/1\ \cdot 7$
			6	18.023	n/a
			7	21.775	n/a



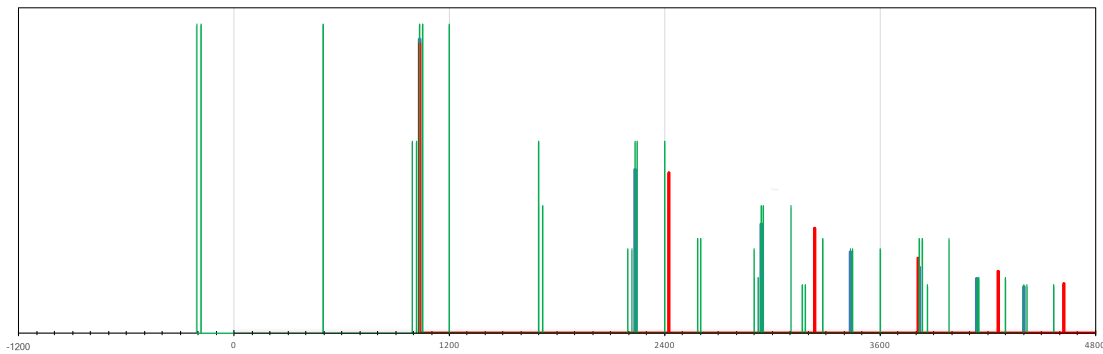
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
70	$\frac{9}{10}\ \cdot\ F_{1/1}$	1.2205	1	0.900	$9/10\ \cdot 1$
			2	2.097	$2/1\ \cdot 1$
			3	3.440	$8/9\ \cdot 4$
			4	4.887	$9/10\ \cdot 5$
			5	6.417	$9/10\ \cdot 7$
			6	8.016	$4/3\ \cdot 6$
			7	9.676	$2/1\ \cdot 5$



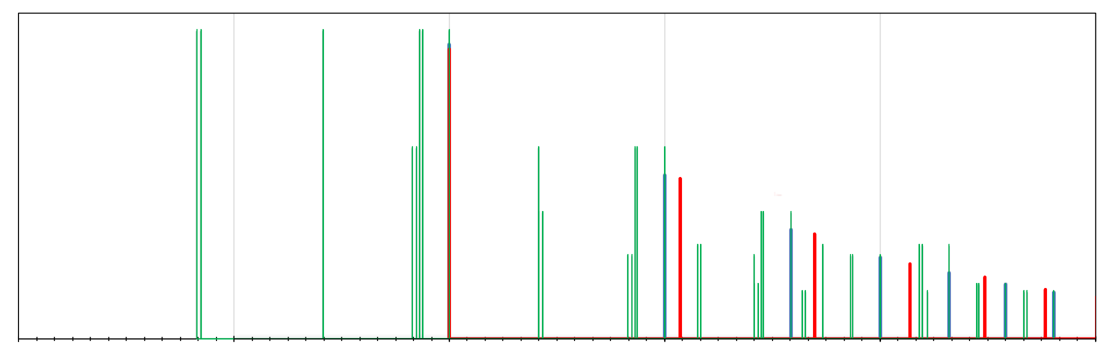
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
71	$\frac{8}{9}\cdot F_{1/1}$	1.2265	1	0.889	$8/9\ \cdot 1$
			2	2.080	$2/1\ \cdot 1$
			3	3.420	$8/9\ \cdot 4$
			4	4.867	$9/10\ \cdot 5$
			5	6.399	$9/10\ \cdot 7$
			6	8.003	$4/3\ \cdot 6$
			7	9.669	$2/1\ \cdot 5$



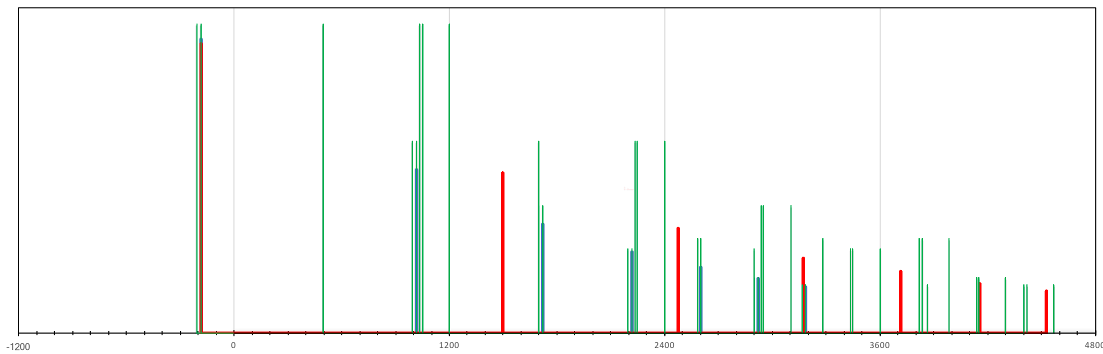
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
72 *diff.	$\frac{11}{6}\cdot F_{1/1}$	1.15	1	1.833	$11/6\ \cdot 1$
			2	4.068	$4/3\ \cdot 3$
			3	6.485	$4/3\ \cdot 5$
			4	9.028	$20/11\ \cdot 5$
			5	11.670	$2/1\ \cdot 6$
			6	14.392	$2/1\ \cdot 7$
			7	17.183	n/a



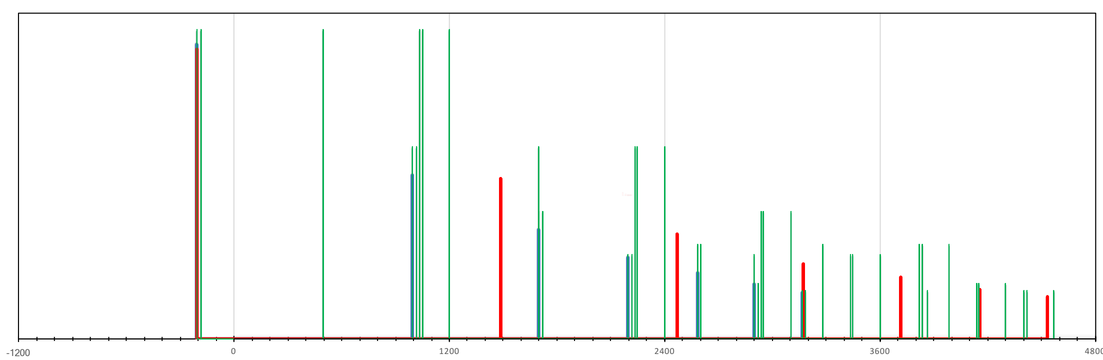
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
73	$\frac{20}{11} \cdot F_{1/1}$	1.1575	1	1.818	$20/11 \cdot 1$
			2	4.056	$4/3 \cdot 4$
			3	6.485	$4/3 \cdot 5$
			4	9.047	$20/11 \cdot 5$
			5	11.714	$2/1 \cdot 6$
			6	14.466	$2/1 \cdot 7$
			7	17.292	n/a



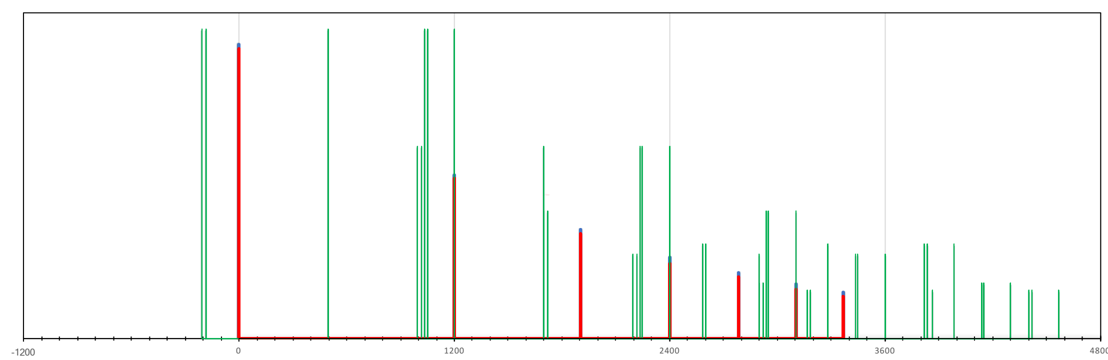
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
74	$\frac{2}{1} \cdot F_{1/1}$	1.071	1	2.000	$2/1 \cdot 1$
			2	4.202	$4/3 \cdot 3$
			3	6.487	$4/3 \cdot 5$
			4	8.827	$20/11 \cdot 5$
			5	11.211	$11/6 \cdot 6$
			6	13.628	$2/1 \cdot 7$
			7	16.074	n/a



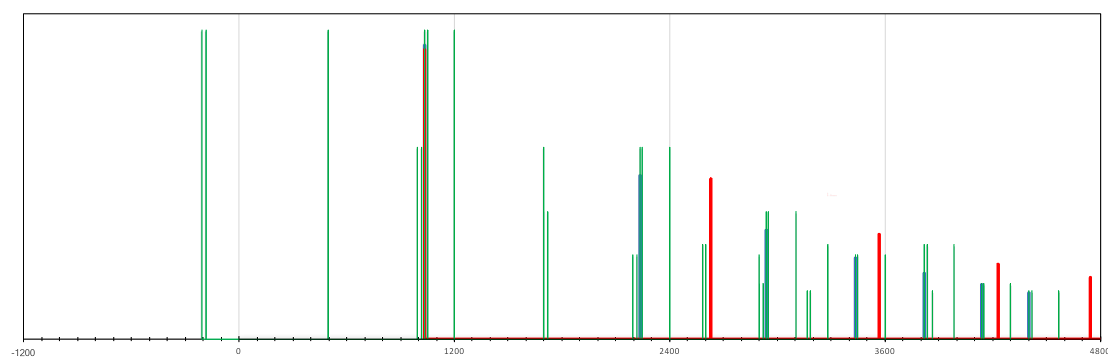
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\,/\,F_{1/1}$	$NASH$
75	$\frac{9}{10}\cdot F_{1/1}$	1.3985	1	0.900	$9/10\cdot 1$
			2	2.373	$8/9\cdot 3$
			3	4.183	$4/3\cdot 3$
			4	6.255	$20/11\cdot 5$
			5	8.546	$8/9\cdot 7$
			6	11.028	$11/6\cdot 6$
			7	13.681	$2/1\cdot 7$



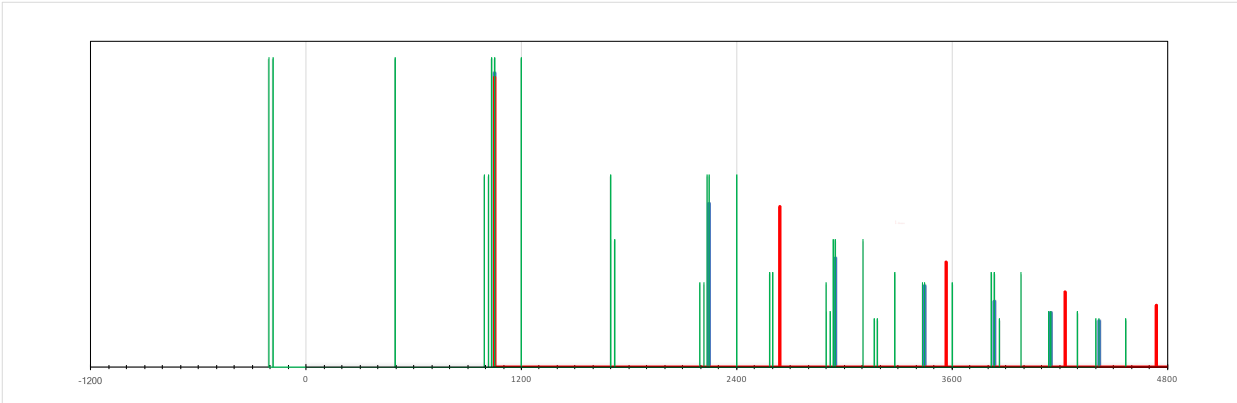
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\,/\,F_{1/1}$	$NASH$
76	$\frac{8}{9}\cdot F_{1/1}$	1.4065	1	0.889	$8/9\cdot 1$
			2	2.356	$8/9\cdot 3$
			3	4.168	$4/3\cdot 3$
			4	6.247	$8/9\cdot 7$
			5	8.550	$20/11\cdot 5$
			6	11.049	$11/6\cdot 6$
			7	13.724	$2/1\cdot 7$



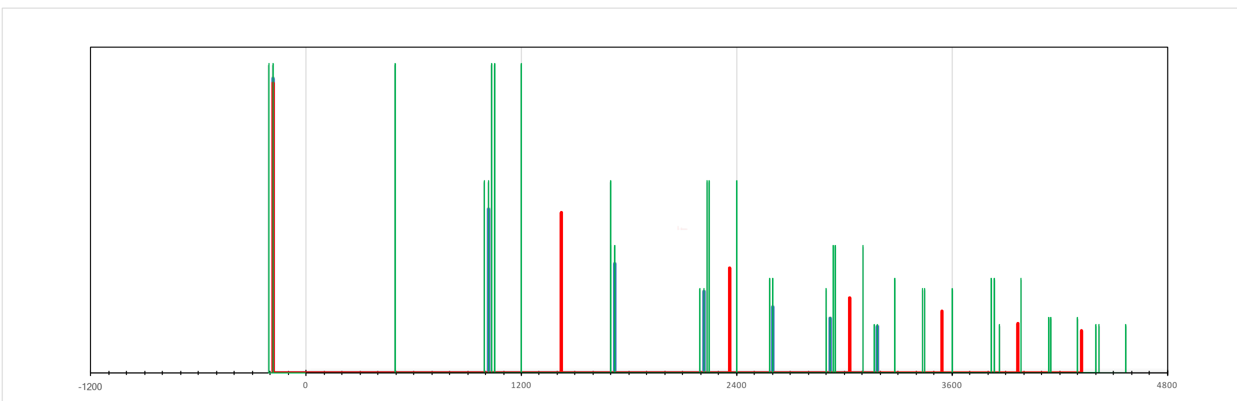
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
77	$\frac{2}{1} \cdot F_{1/1}$	1.071	1	2.000	$2/1 \cdot 1$
			2	4.202	$4/3 \cdot 3$
			3	6.487	$4/3 \cdot 5$
			4	8.827	$20/11 \cdot 5$
			5	11.211	$11/6 \cdot 6$
			6	13.628	$2/1 \cdot 7$
			7	16.074	n/a



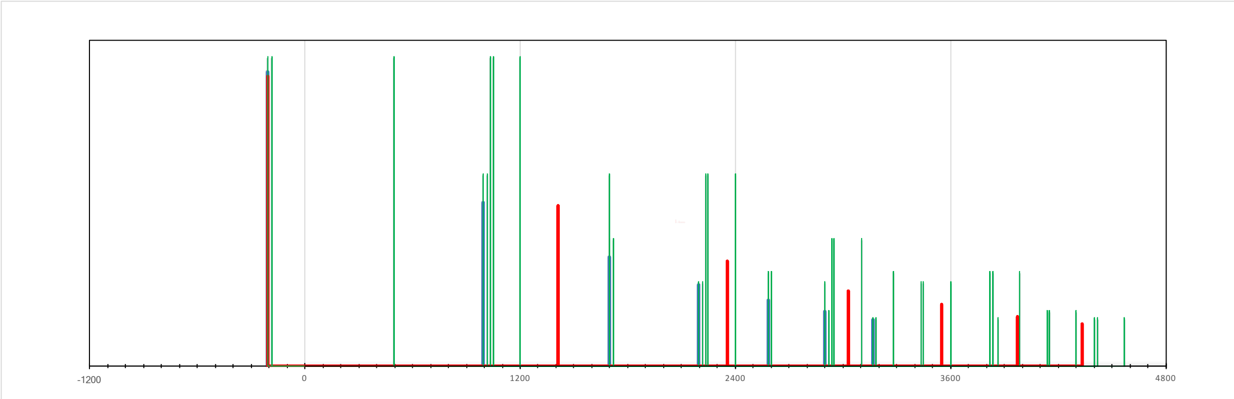
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
78 *diff.	$\frac{20}{11} \cdot F_{1/1}$	1.331	1	1.818	$20/11 \cdot 1$
			2	4.574	$9/10 \cdot 5$
			3	7.847	$4/3 \cdot 6$
			4	11.507	$2/1 \cdot 6$
			5	15.487	$2/1 \cdot 7$
			6	19.740	n/a
			7	24.236	n/a



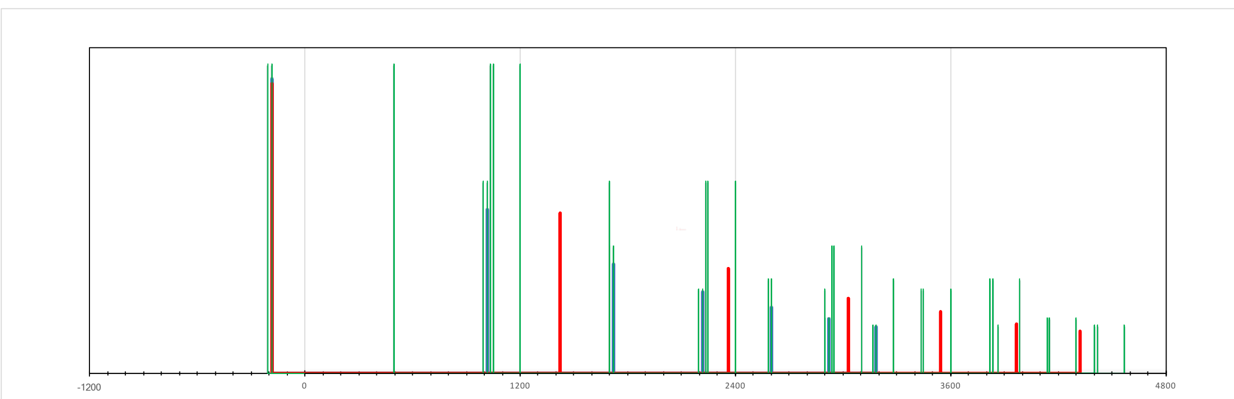
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
79	$\frac{11}{6} \cdot F_{1/1}$	1.325	1	1.833	$11/6\ \cdot 1$
			2	4.593	$9/10\ \cdot 5$
			3	7.860	$4/3\ \cdot 6$
			4	11.507	$2/1\ \cdot 6$
			5	15.466	$2/1\ \cdot 7$
			6	19.692	n/a
			7	24.154	n/a



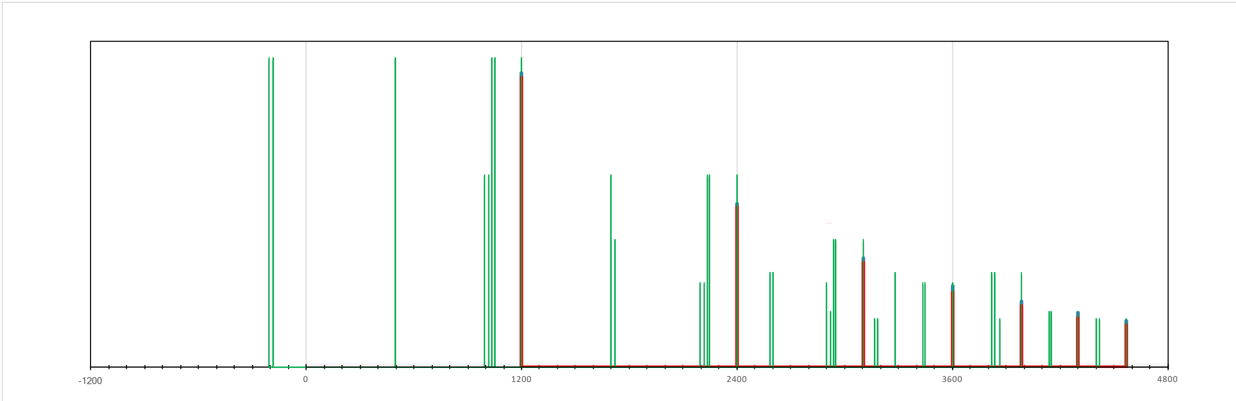
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH$
80	$\frac{9}{10} \cdot F_{1/1}$	1.338	1	0.900	$9/10\ \cdot 1$
			2	2.275	$2/1\ \cdot 2$
			3	3.914	$4/3\ \cdot 3$
			4	5.752	$2/1\ \cdot 3$
			5	7.753	$4/3\ \cdot 6$
			6	9.895	$2/1\ \cdot 5$
			7	12.161	$2/1\ \cdot 6$



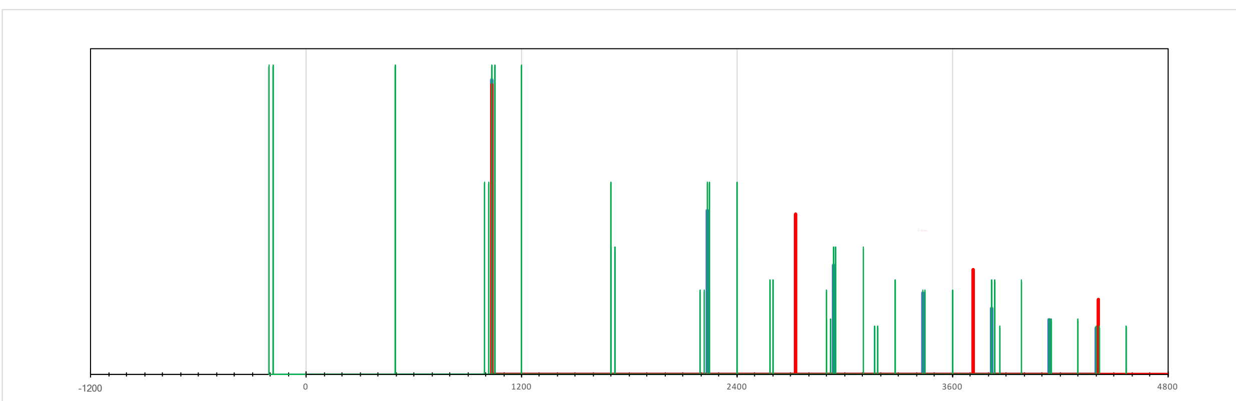
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
81	$\frac{8}{9} \cdot F_{1/1}$	1.347	1	0.889	$8/9 \cdot 1$
			2	2.261	$2/1 \cdot 1$
			3	3.904	$4/3 \cdot 3$
			4	5.752	$2/1 \cdot 3$
			5	7.769	$4/3 \cdot 6$
			6	9.932	$2/1 \cdot 5$
			7	12.223	$2/1 \cdot 6$



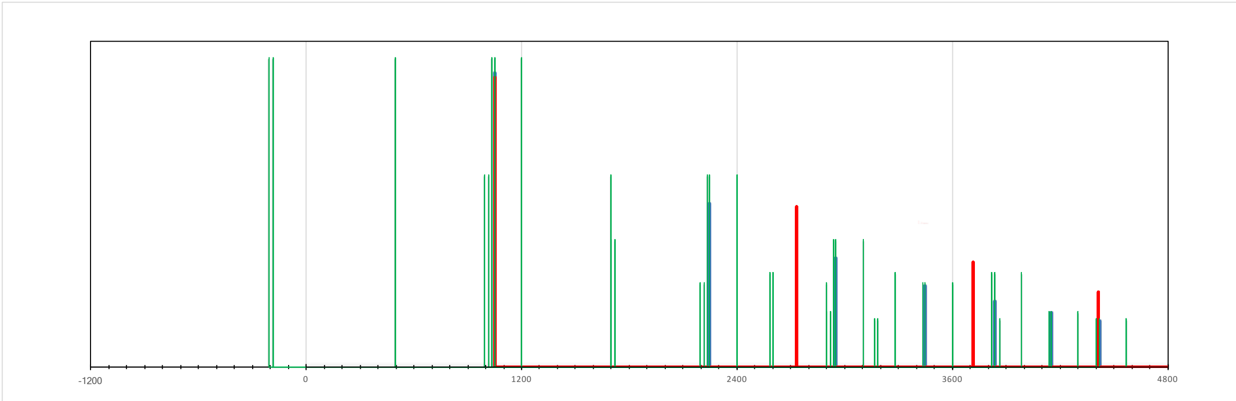
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
82	$\frac{9}{10} \cdot F_{1/1}$	1.338	1	0.900	$9/10 \cdot 1$
			2	2.275	$2/1 \cdot 1$
			3	3.914	$4/3 \cdot 3$
			4	5.752	$2/1 \cdot 3$
			5	7.753	$4/3 \cdot 6$
			6	9.895	$2/1 \cdot 5$
			7	12.161	$2/1 \cdot 6$



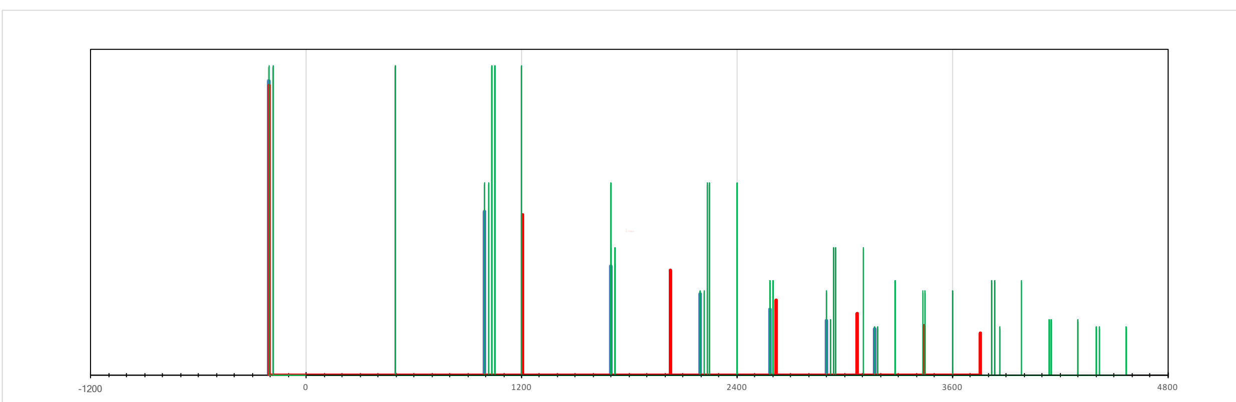
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH$
83	$\frac{2}{1} \cdot F_{1/1}$	1.0	1	2.000	$2/1 \cdot 1$
			2	4.000	$2/1 \cdot 2$
			3	6.000	$2/1 \cdot 3$
			4	8.000	$2/1 \cdot 4$
			5	10.000	$2/1 \cdot 5$
			6	12.000	$2/1 \cdot 6$
			7	14.000	$2/1 \cdot 7$



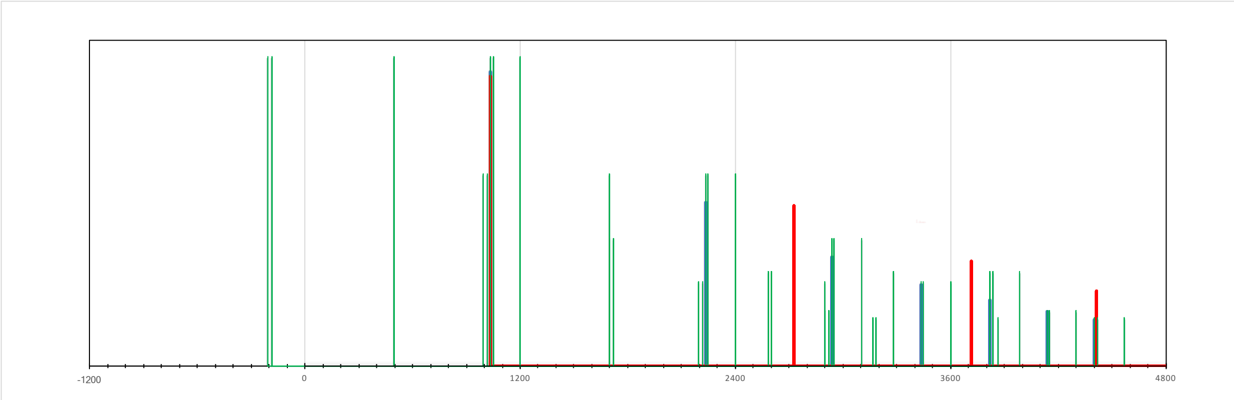
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
84 *diff.	$\frac{20}{11} \cdot F_{1/1}$	1.409	1	1.818	$20/11 \cdot 1$	
			2	4.828	$9/10 \cdot 5$	$(4/5 \cdot 6) \cdot 1$
			3	8.549	$20/11 \cdot 5$	$(3/4 \cdot 6) \cdot 2$
			4	12.822	$11/6 \cdot 7$	
			5	17.558	$2/1 \cdot 7$	$(1/1 \cdot 6) \cdot 3$
			6	22.701	n/a	
			7	28.209	n/a	



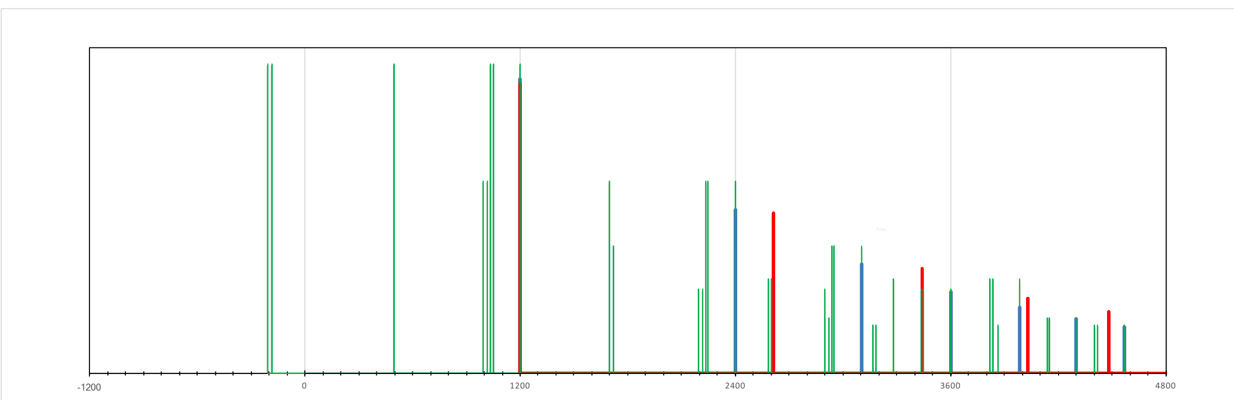
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
85	$\frac{11}{6} \cdot F_{1/1}$	1.4015	1	1.833	$11/6\ \cdot 1$	
			2	4.843	$9/10\ \cdot 5$	$(4/5 \cdot 6) \cdot 1$
			3	8.549	$20/11\ \cdot 5$	$(3/4 \cdot 6) \cdot 2$
			4	12.795	$11/6\ \cdot 7$	
			5	17.492	$2/1\ \cdot 7$	$(1/1 \cdot 6) \cdot 3$
			6	22.585	n/a	
			7	28.031	n/a	



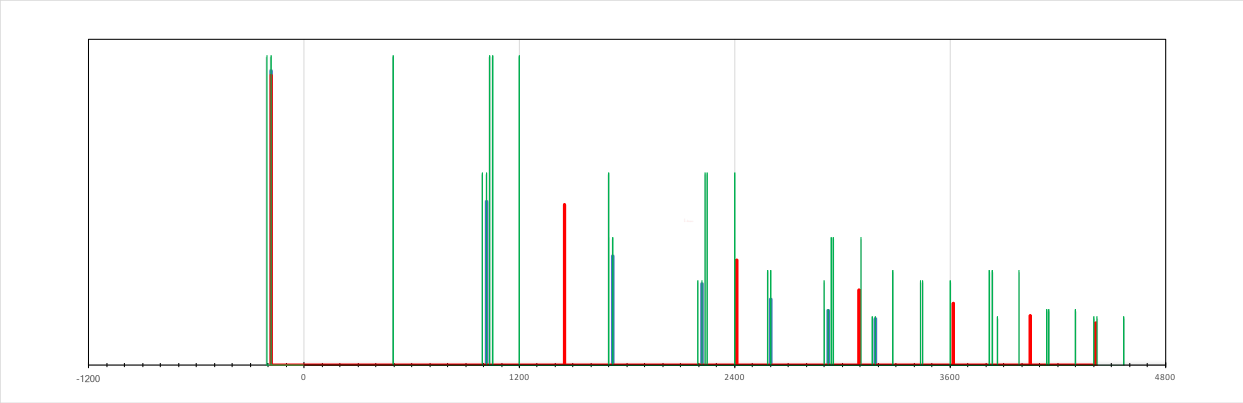
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
86	$\frac{8}{9} \cdot F_{1/1}$	1.1755	1	0.889	$8/9\ \cdot 1$	
			2	2.008	$2/1\ \cdot 1$	
			3	3.234	$8/9\ \cdot 4$	
			4	4.535	$9/10\ \cdot 5$	$(3/4 \cdot 6) \cdot 1$
			5	5.895	$2/1\ \cdot 3$	$(1/1 \cdot 6) \cdot 1$
			6	7.304	$11/6\ \cdot 4$	
			7	8.755	$20/11\ \cdot 5$	$(3/4 \cdot 6) \cdot 2$



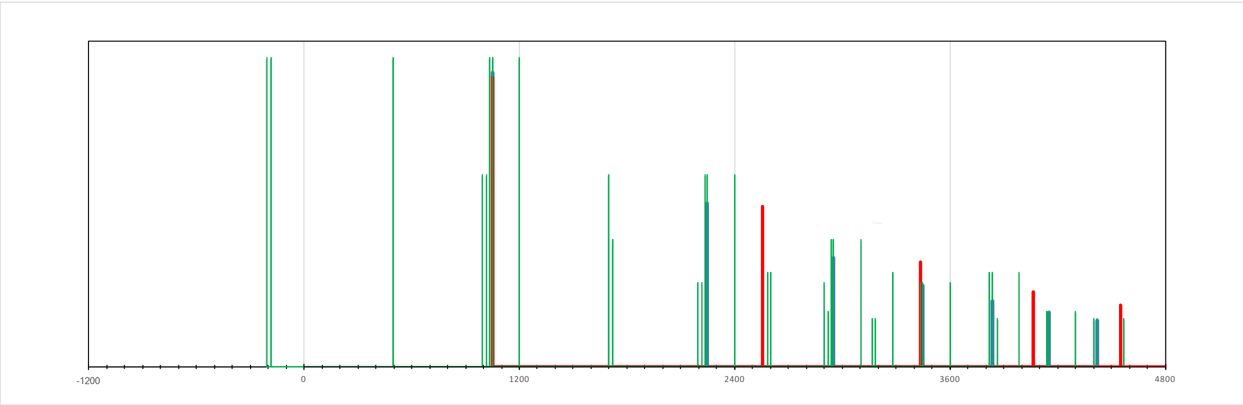
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
87	$\frac{20}{11} \cdot F_{1/1}$	1.409	1	1.818	$20/11 \cdot 1$	
			2	4.828	$9/10 \cdot 5$	$(4/5 \cdot 6) \cdot 1$
			3	8.549	$20/11 \cdot 5$	$(3/4 \cdot 6) \cdot 2$
			4	12.822	$11/6 \cdot 7$	
			5	17.558	$2/1 \cdot 7$	$(1/1 \cdot 6) \cdot 3$
			6	22.701	n/a	
			7	28.209	n/a	



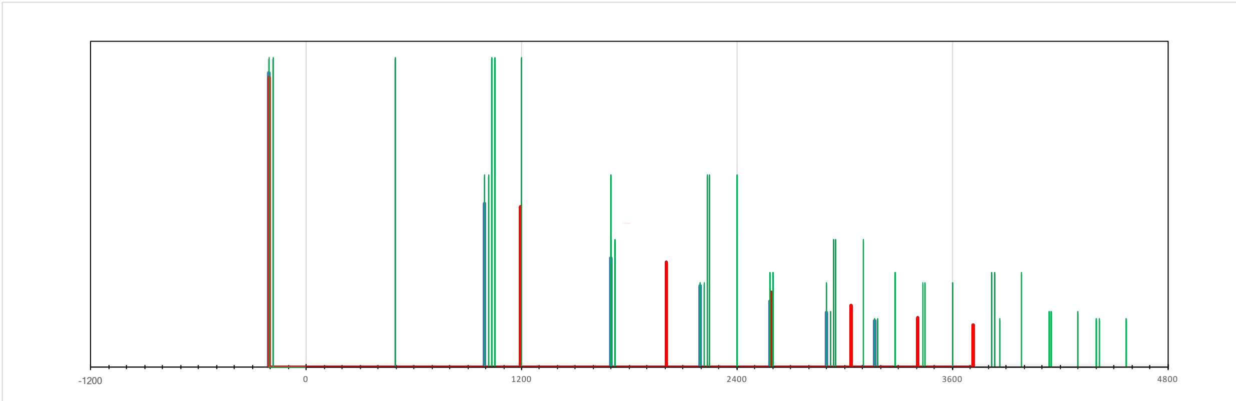
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
88	$\frac{2}{1} \cdot F_{1/1}$	1.179	1	2.000	$2/1 \cdot 1$	
			2	4.528	$9/10 \cdot 5$	$(3/4 \cdot 6) \cdot 1$
			3	7.304	$11/6 \cdot 4$	
			4	10.253	$2/1 \cdot 5$	$(6/7 \cdot 6) \cdot 2$
			5	13.339	$11/6 \cdot 7$	$(3/4 \cdot 6) \cdot 3$
			6	16.538	$2/1 \cdot 7$	$(11/12 \cdot 6) \cdot 3$
			7	19.834	n/a	$(1/1 \cdot 6) \cdot 3$



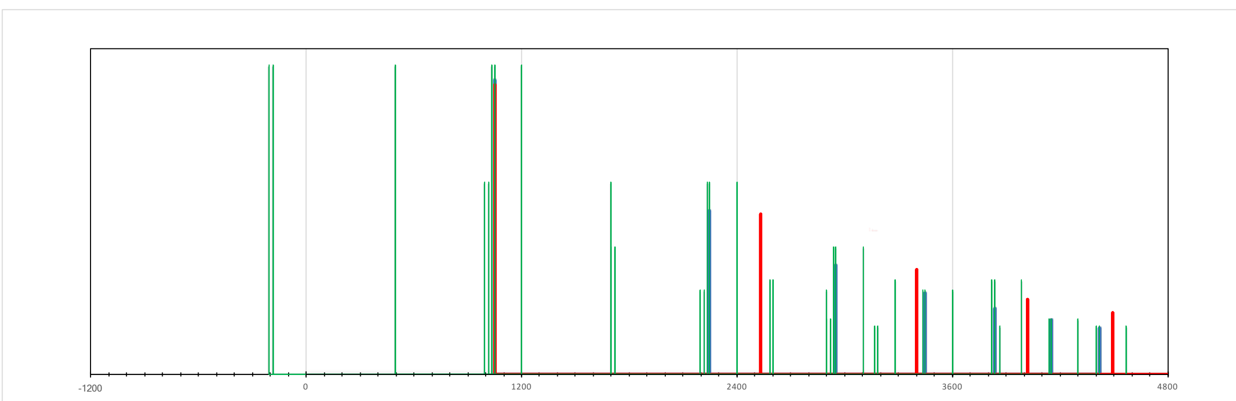
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
89	$\frac{9}{10} \cdot F_{1/1}$	1.364	1	0.900	$9/10 \cdot 1$	
			2	2.317	$2/1 \cdot 1$	
			3	4.028	$4/3 \cdot 3$	$(2/3 \cdot 6) \cdot 1$
			4	5.963	$2/1 \cdot 3$	$(1/1 \cdot 6) \cdot 1$
			5	8.084	$4/3 \cdot 6$	$(2/3 \cdot 6) \cdot 2$
			6	10.367	$2/1 \cdot 5$	$(6/7 \cdot 6) \cdot 2$
			7	12.793	$11/6 \cdot 7$	



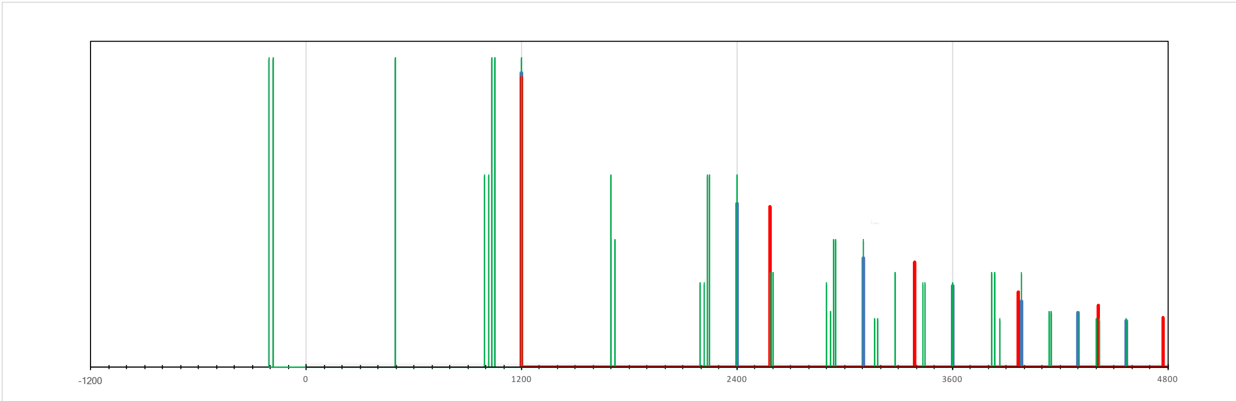
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
90 *diff.	$\frac{11}{6} \cdot F_{1/1}$	1.256	1	1.833	$11/6 \cdot 1$	
			2	4.379	$8/9 \cdot 5$	
			3	7.286	$20/11 \cdot 4$	
			4	10.458	$20/11 \cdot 6$	$(7/8 \cdot 6) \cdot 2$
			5	13.840	$2/1 \cdot 7$	
			6	17.402	n/a	$(1/1 \cdot 6) \cdot 3$
			7	21.120	n/a	



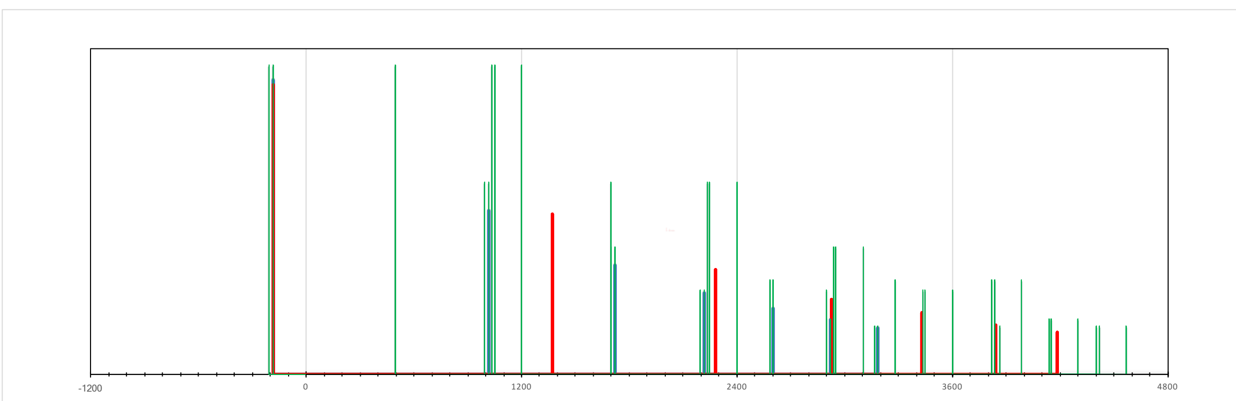
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
91	$\frac{8}{9}\cdot F_{1/1}$	1.1635	1	0.889	$8/9\ \cdot 1$	
			2	1.991	$2/1\ \cdot 1$	
			3	3.191	$8/9\ \cdot 4$	
			4	4.460	$8/9\ \cdot 5$	
			5	5.782	$2/1\ \cdot 3$	$(1/1 \cdot 6) \cdot 1$
			6	7.149	$20/11\ \cdot 4$	
			7	8.553	$20/11\ \cdot 5$	$(3/4 \cdot 6) \cdot 2$



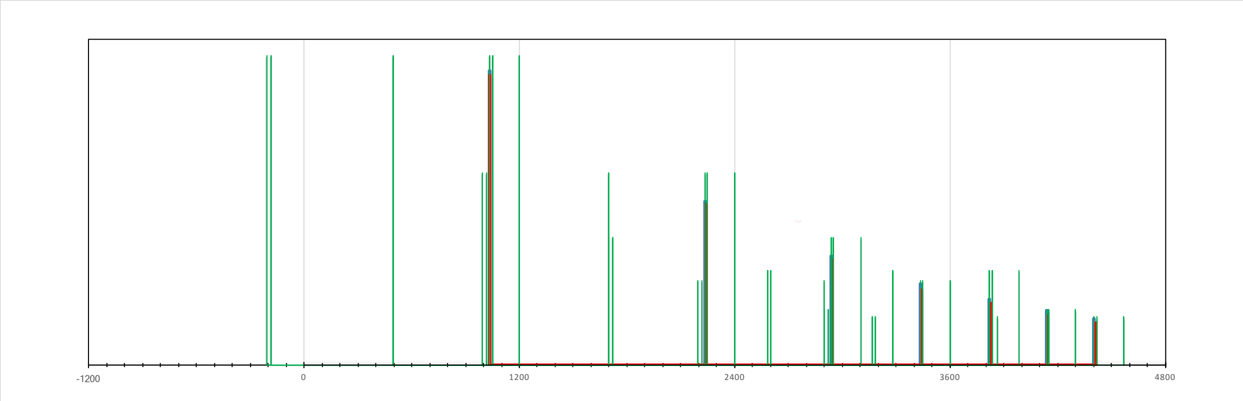
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
92	$\frac{11}{6}\cdot F_{1/1}$	1.237	1	1.833	$11/6\ \cdot 1$	
			2	4.321	$8/9\ \cdot 5$	
			3	7.136	$20/11\ \cdot 4$	
			4	10.186	$2/1\ \cdot 5$	$(6/7 \cdot 6) \cdot 2$
			5	13.424	$2/1\ \cdot 7$	$(3/4 \cdot 6) \cdot 3$
			6	16.820	n/a	$(11/12 \cdot 6) \cdot 3$
			7	20.353	n/a	$(1/1 \cdot 6) \cdot 3$



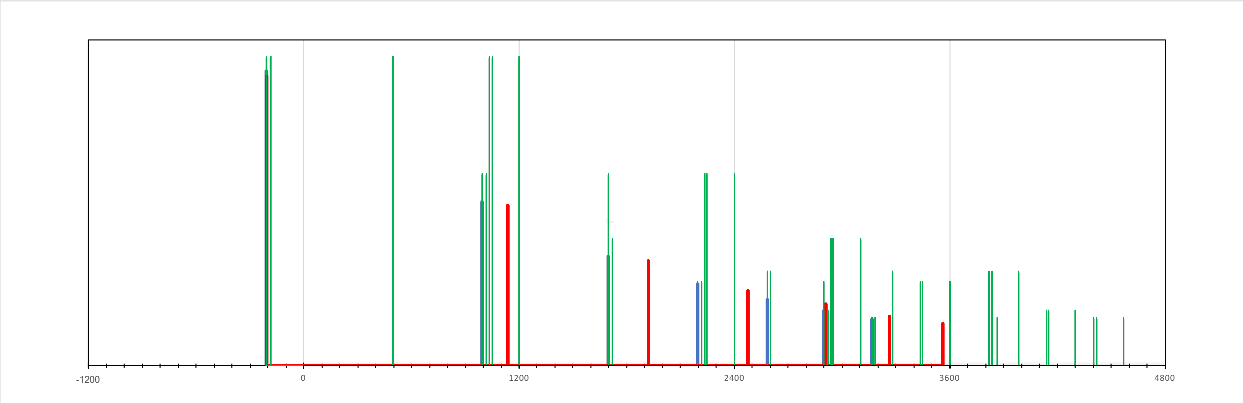
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
93	$\frac{2}{1}\cdot F_{1/1}$	1.1525	1	2.000	$2/1\ \cdot 1$	
			2	4.446	$8/9\ \cdot 5$	
			3	7.094	$20/11\ \cdot 4$	
			4	9.883	$2/1\ \cdot 5$	$(5/6\cdot 6)\cdot 2$
			5	12.782	$11/6\ \cdot 7$	
			6	15.771	$2/1\ \cdot 7$	$(7/8\cdot 6)\cdot 3$
			7	18.837	n/a	$(1/1\cdot 6)\cdot 3$



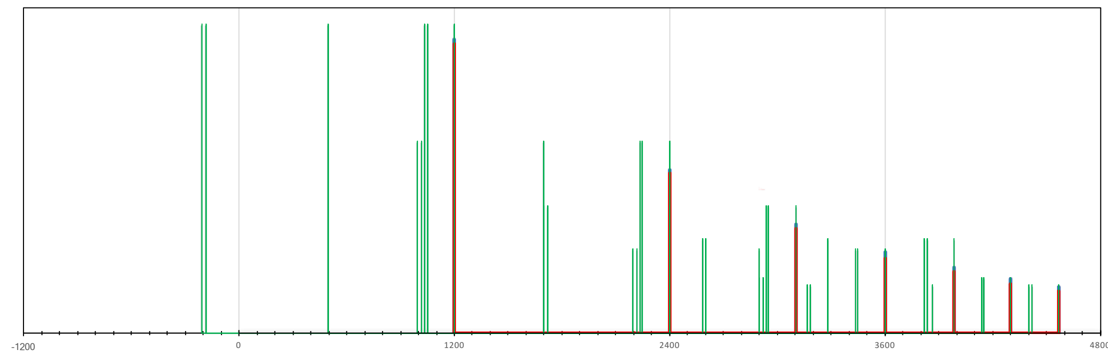
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
94	$\frac{9}{10}\cdot F_{1/1}$	1.2965	1	0.900	$9/10\ \cdot 1$	
			2	2.211	$2/1\ \cdot 1$	
			3	3.740	$11/6\ \cdot 2$	
			4	5.430	$20/11\ \cdot 3$	$(10/11\cdot 6)\cdot 1$
			5	7.252	$20/11\ \cdot 4$	
			6	9.186	$11/6\ \cdot 5$	
			7	11.218	$11/6\ \cdot 6$	$(11/12\cdot 6)\cdot 2$



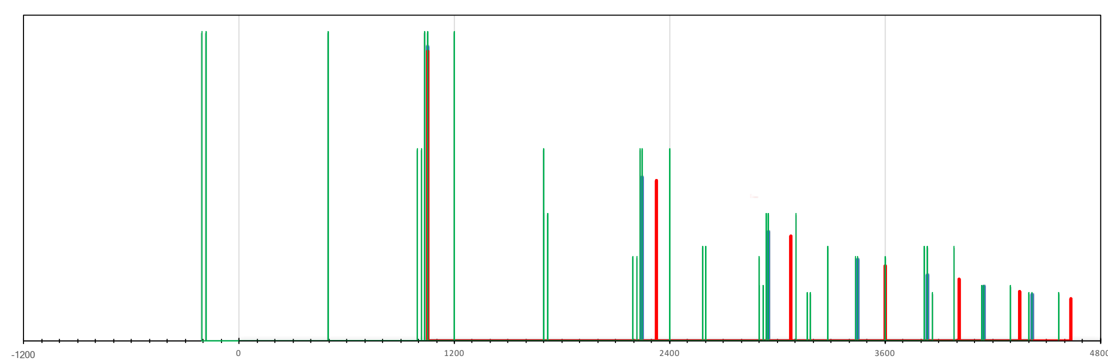
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\,/\,F_{1/1}$	$NASH-RBI$	$NASH-RBII$
95	$\frac{20}{11} \cdot F_{1/1}$	1.003	1	1.818	$20/11 \cdot 1$	
			2	3.644	$20/11 \cdot 2$	
			3	5.473	$20/11 \cdot 3$	$(10/11 \cdot 6) \cdot 1$
			4	7.303	$11/6 \cdot 4$	
			5	9.135	$11/6 \cdot 5$	
			6	10.968	$11/6 \cdot 6$	$(11/12 \cdot 6) \cdot 2$
			7	12.802	$11/6 \cdot 7$	



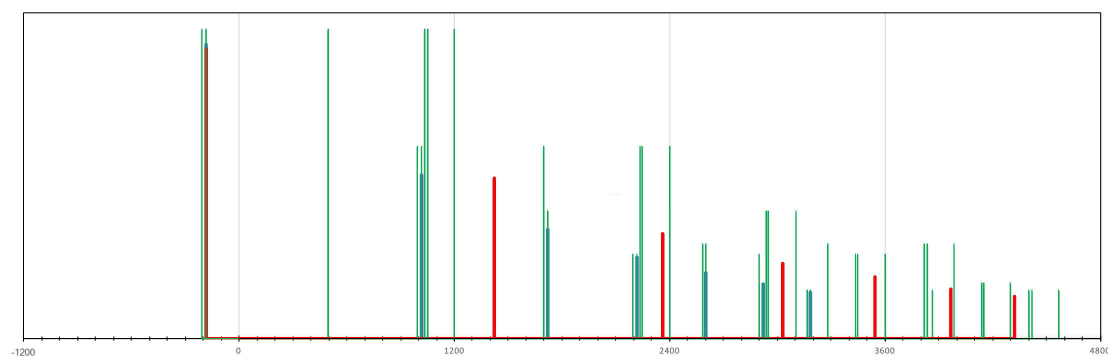
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\,/\,F_{1/1}$	$NASH-RBI$	$NASH-RBII$
96 *diff.	$\frac{8}{9} \cdot F_{1/1}$	1.1175	1	0.889	$8/9 \cdot 1$	
			2	1.929	$2/1 \cdot 1$	
			3	3.034	$9/10 \cdot 3$	
			4	4.185	$4/3 \cdot 3$	$(2/3 \cdot 6) \cdot 1$
			5	5.370	$9/10 \cdot 6$	$(9/10 \cdot 6) \cdot 1$
			6	6.583	$4/3 \cdot 5$	
			7	7.821	$4/3 \cdot 6$	$(2/3 \cdot 6) \cdot 2$



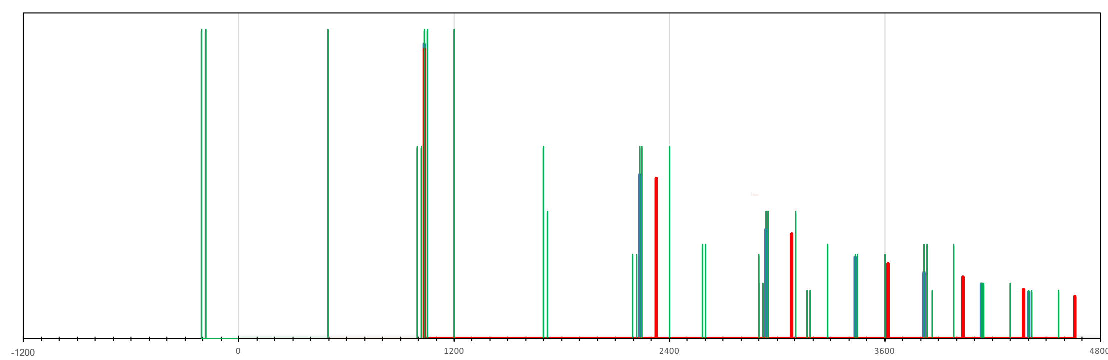
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
97	$\frac{2}{1}\ \cdot\ F_{1/1}$	1.0	1	2.000	$2/1\ \cdot 1$	
			2	4.000	$2/1\ \cdot 2$	$(2/3 \cdot 6) \cdot 1$
			3	6.000	$2/1\ \cdot 3$	$(1/1 \cdot 6) \cdot 1$
			4	8.000	$2/1\ \cdot 4$	$(2/3 \cdot 6) \cdot 2$
			5	10.000	$2/1\ \cdot 5$	$(5/6 \cdot 6) \cdot 2$
			6	12.000	$2/1\ \cdot 6$	$(2/3 \cdot 6) \cdot 3$
			7	14.000	$2/1\ \cdot 7$	



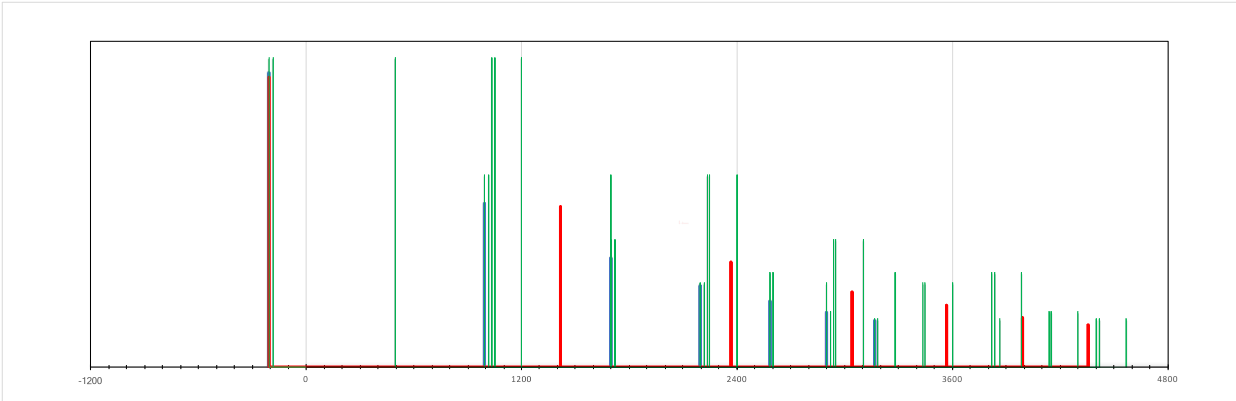
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
98	$\frac{11}{6}\ \cdot\ F_{1/1}$	1.0645	1	1.833	$11/6\ \cdot 1$	
			2	3.834	$4/3\ \cdot 3$	$(2/3 \cdot 6) \cdot 1$
			3	5.904	$2/1\ \cdot 3$	$(1/1 \cdot 6) \cdot 1$
			4	8.019	$4/3\ \cdot 6$	$(2/3 \cdot 6) \cdot 2$
			5	10.169	$2/1\ \cdot 5$	$(6/7 \cdot 6) \cdot 2$
			6	12.348	$2/1\ \cdot 6$	$(2/3 \cdot 6) \cdot 3$
			7	14.550	$2/1\ \cdot 7$	$(4/5 \cdot 6) \cdot 3$



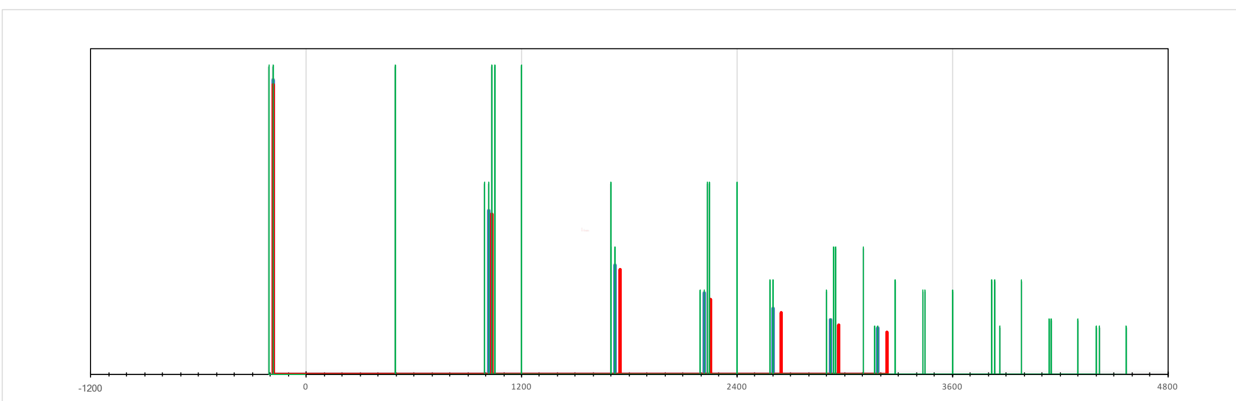
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
99	$\frac{9}{10} \cdot F_{1/1}$	1.338	1	0.900	$9/10\ \cdot 1$	
			2	2.275	$2/1\ \cdot 1$	
			3	3.914	$4/3\ \cdot 3$	$(2/3 \cdot 6) \cdot 1$
			4	5.752	$2/1\ \cdot 3$	$(1/1 \cdot 6) \cdot 1$
			5	7.753	$4/3\ \cdot 6$	$(2/3 \cdot 6) \cdot 2$
			6	9.895	$2/1\ \cdot 5$	$(5/6 \cdot 6) \cdot 2$
			7	12.161	$2/1\ \cdot 6$	$(2/3 \cdot 6) \cdot 3$



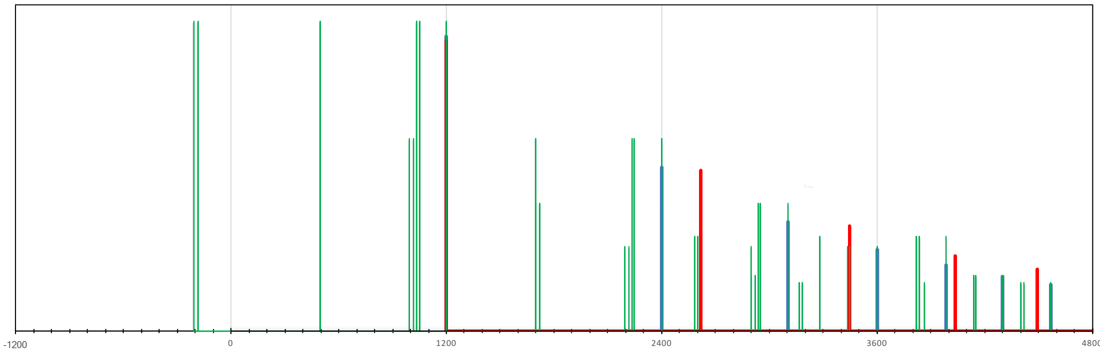
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
100	$\frac{20}{11} \cdot F_{1/1}$	1.0765	1	1.818	$20/11\ \cdot 1$	
			2	3.834	$4/3\ \cdot 3$	$(2/3 \cdot 6) \cdot 1$
			3	5.933	$2/1\ \cdot 3$	$(1/1 \cdot 6) \cdot 1$
			4	8.086	$4/3\ \cdot 6$	$(2/3 \cdot 6) \cdot 2$
			5	10.282	$2/1\ \cdot 5$	$(6/7 \cdot 6) \cdot 2$
			6	12.512	$20/11\ \cdot 7$	
			7	14.770	$2/1\ \cdot 7$	$(5/6 \cdot 6) \cdot 3$



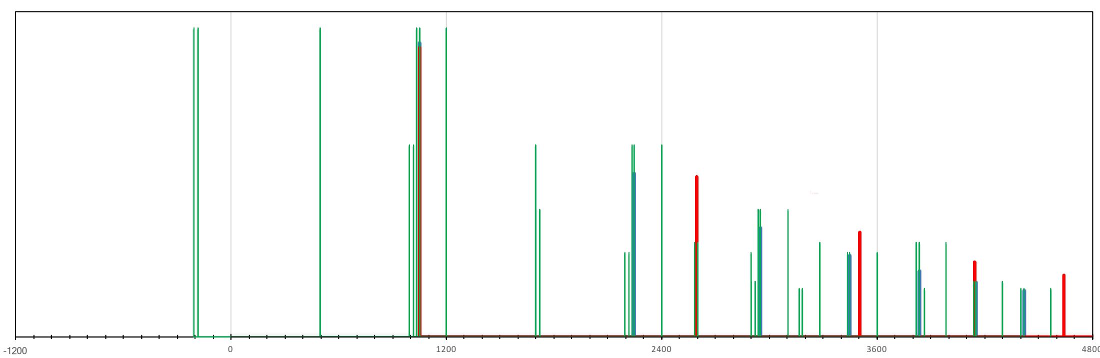
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
101	$\frac{8}{9}\cdot F_{1/1}$	1.353	1	0.889	$8/9\ \cdot 1$	
			2	2.271	$2/1\ \cdot 1$	
			3	3.930	$4/3\ \cdot 3$	$(2/3\cdot 6)\cdot 1$
			4	5.800	$2/1\ \cdot 3$	$(1/1\cdot 6)\cdot 1$
			5	7.844	$4/3\ \cdot 6$	$(2/3\cdot 6)\cdot 2$
			6	10.039	$2/1\ \cdot 5$	$(5/6\cdot 6)\cdot 2$
			7	12.367	$20/11\ \cdot 7$	



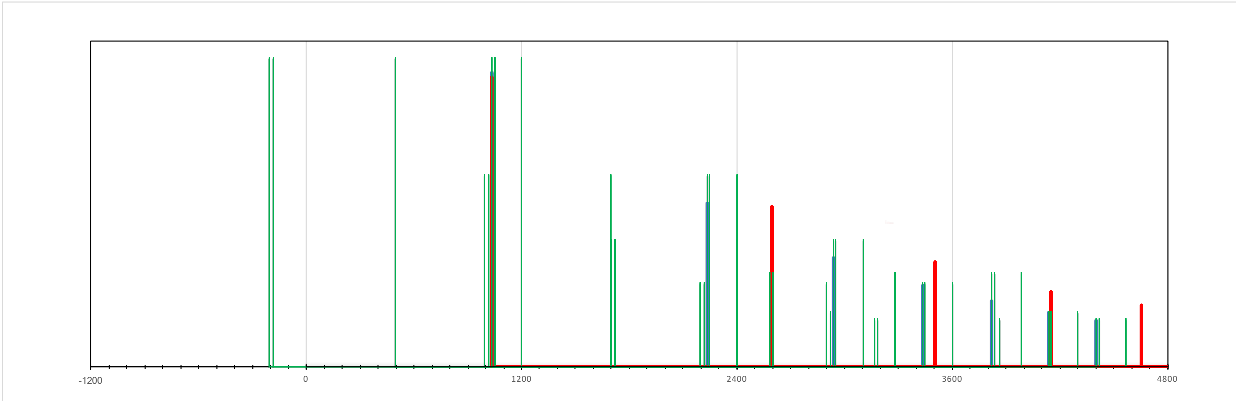
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
102 *diff.	$\frac{9}{10}\cdot F_{1/1}$	1.015	1	0.900	$9/10\ \cdot 1$	
			2	1.819	$20/11\ \cdot 1$	
			3	2.745	$9/10\ \cdot 3$	
			4	3.676	$11/6\ \cdot 2$	
			5	4.610	$9/10\ \cdot 5$	$(3/4\cdot 6)\cdot 1$
			6	5.547	$11/6\ \cdot 3$	$(11/12\cdot 6)\cdot 1$
			7	6.487	$4/3\ \cdot 5$	



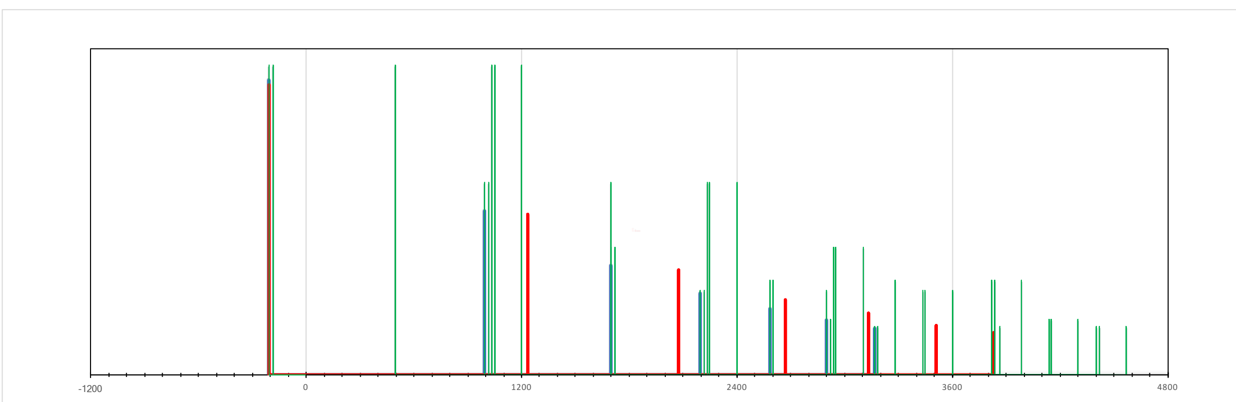
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
103	$\frac{2}{1}\cdot F_{1/1}$	1.183	1	2.000	$2/1\ \cdot 1$	
			2	4.541	$9/10\ \cdot 5$	$(3/4\cdot 6)\cdot 1$
			3	7.336	$11/6\ \cdot 4$	
			4	10.310	$2/1\ \cdot 5$	$(6/7\cdot 6)\cdot 2$
			5	13.425	$2/1\ \cdot 7$	$(3/4\cdot 6)\cdot 3$
			6	16.656	n/a	$(11/12\cdot 6)\cdot 3$
			7	19.989	n/a	$(1/1\cdot 6)\cdot 3$



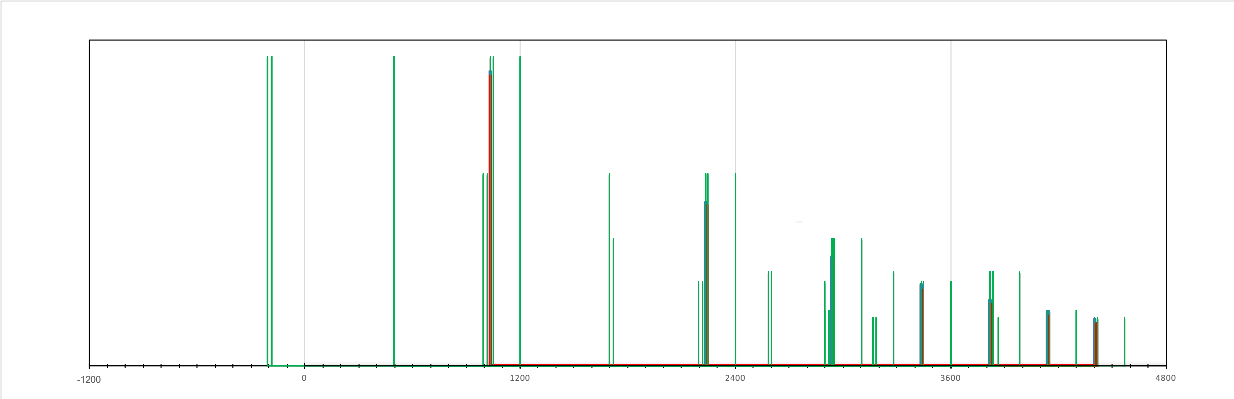
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
104	$\frac{11}{6}\cdot F_{1/1}$	1.2895	1	1.833	$11/6\ \cdot 1$	
			2	4.481	$9/10\ \cdot 5$	$(3/4\cdot 6)\cdot 1$
			3	7.559	$11/6\ \cdot 4$	
			4	10.955	$11/6\ \cdot 6$	$(11/12\cdot 6)\cdot 2$
			5	14.607	$2/1\ \cdot 7$	$(4/5\cdot 6)\cdot 3$
			6	18.479	n/a	$(1/1\cdot 6)\cdot 3$
			7	22.542	n/a	



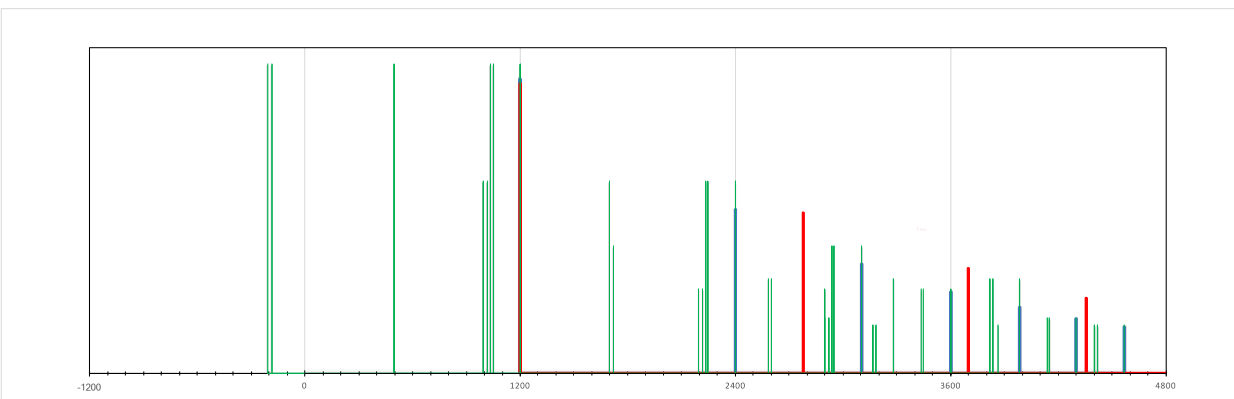
State #	$F_{RBI\ (1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
105	$\frac{20}{11} \cdot F_{1/1}$	1.2985	1	1.818	$20/11\ \cdot 1$	
			2	4.472	$9/10\ \cdot 5$	$(3/4 \cdot 6) \cdot 1$
			3	7.571	$11/6\ \cdot 4$	
			4	11.000	$11/6\ \cdot 6$	$(11/12 \cdot 6) \cdot 2$
			5	14.698	$2/1\ \cdot 7$	$(4/5 \cdot 6) \cdot 3$
			6	18.624	n/a	$(1/1 \cdot 6) \cdot 3$
			7	22.751	n/a	



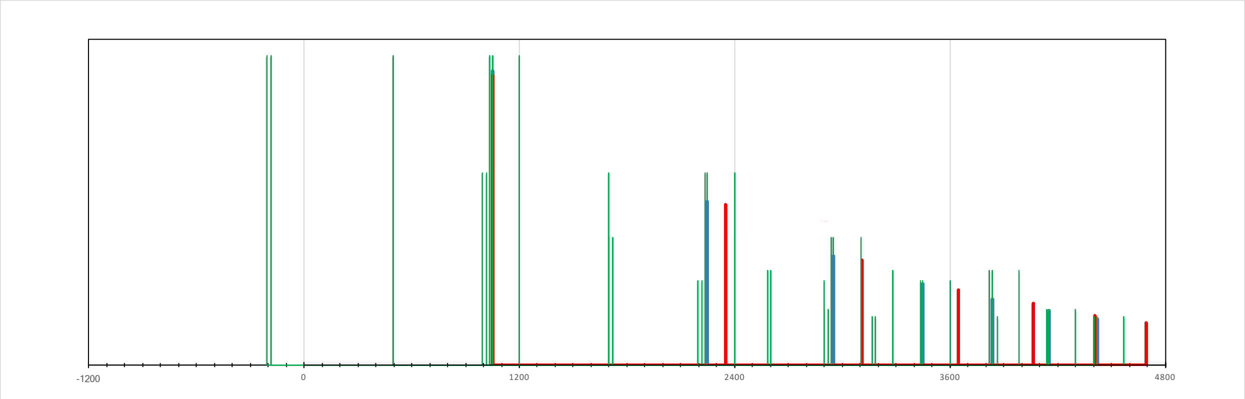
State #	$F_{RBI\ (1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
106	$\frac{8}{9} \cdot F_{1/1}$	1.197	1	0.889	$8/9\ \cdot 1$	
			2	2.038	$2/1\ \cdot 1$	
			3	3.311	$8/9\ \cdot 4$	
			4	4.672	$9/10\ \cdot 5$	$(4/5 \cdot 6) \cdot 1$
			5	6.103	$2/1\ \cdot 3$	$(1/1 \cdot 6) \cdot 1$
			6	7.591	$11/6\ \cdot 4$	
			7	9.129	$11/6\ \cdot 5$	$(8/9 \cdot 6) \cdot 2$



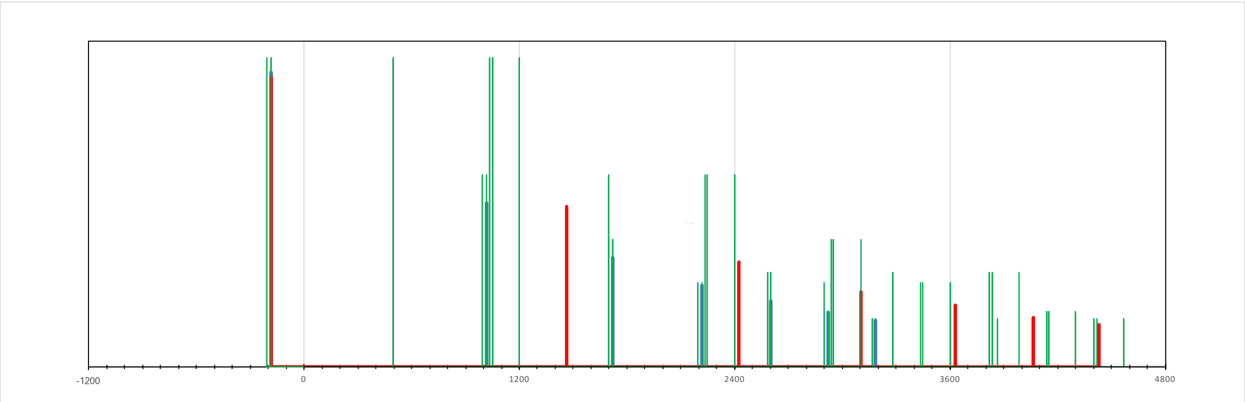
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
107	$\frac{20}{11}\cdot F_{1/1}$	1.003	1	1.818	$20/11\ \cdot 1$	
			2	3.644	$20/11\ \cdot 2$	
			3	5.473	$20/11\ \cdot 3$	$(10/11\cdot 6)\cdot 1$
			4	7.303	$11/6\ \cdot 4$	
			5	9.135	$11/6\ \cdot 5$	
			6	10.968	$11/6\ \cdot 6$	$(11/12\cdot 6)\cdot 2$
			7	12.802	$11/6\ \cdot 7$	



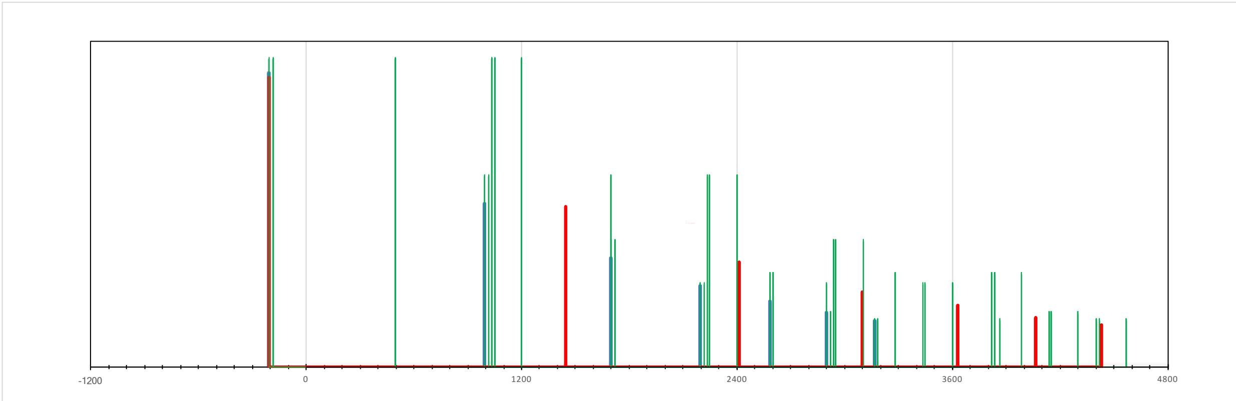
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
108 *diff.	$\frac{2}{1}\cdot F_{1/1}$	1.3154	1	2.000	$2/1\ \cdot 1$	
			2	4.977	$8/9\ \cdot 6$	$(5/6\cdot 6)\cdot 1$
			3	8.485	$4/3\ \cdot 6$	$(2/3\cdot 6)\cdot 2$
			4	12.387	$20/11\ \cdot 7$	
			5	16.613	$2/1\ \cdot 7$	$(11/12\cdot 6)\cdot 3$
			6	21.116	n/a	$(1/1\cdot 6)\cdot 3$
			7	25.863	n/a	



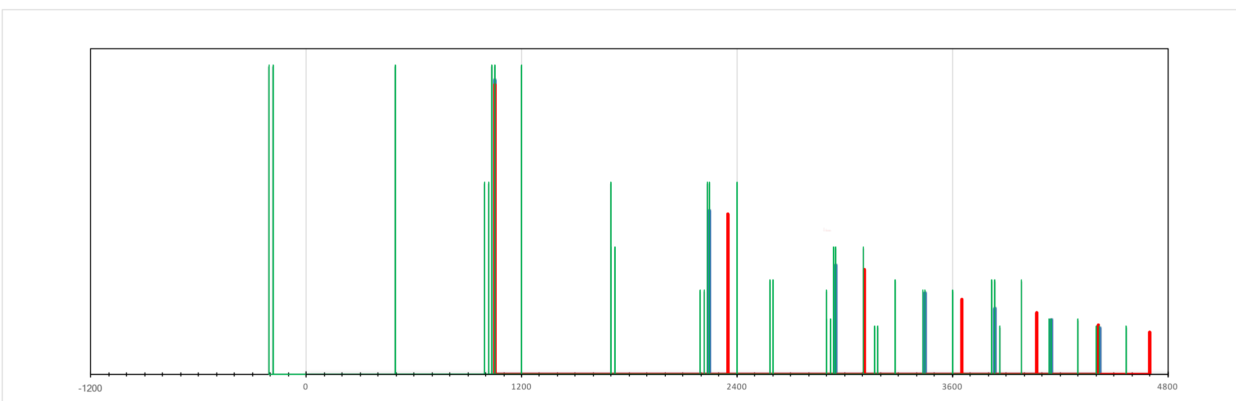
State #	$F_{RBI\ (1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
109	$\frac{11}{6} \cdot F_{1/1}$	1.082	1	1.833	$11/6\ \cdot 1$	
			2	3.881	$4/3\ \cdot 3$	$(2/3 \cdot 6) \cdot 1$
			3	6.018	$2/1\ \cdot 3$	$(1/1 \cdot 6) \cdot 1$
			4	8.216	$4/3\ \cdot 6$	$(2/3 \cdot 6) \cdot 2$
			5	10.460	$20/11\ \cdot 6$	$(7/8 \cdot 6) \cdot 2$
			6	12.741	$20/11\ \cdot 7$	
			7	15.054	$2/1\ \cdot 7$	$(5/6 \cdot 6) \cdot 3$



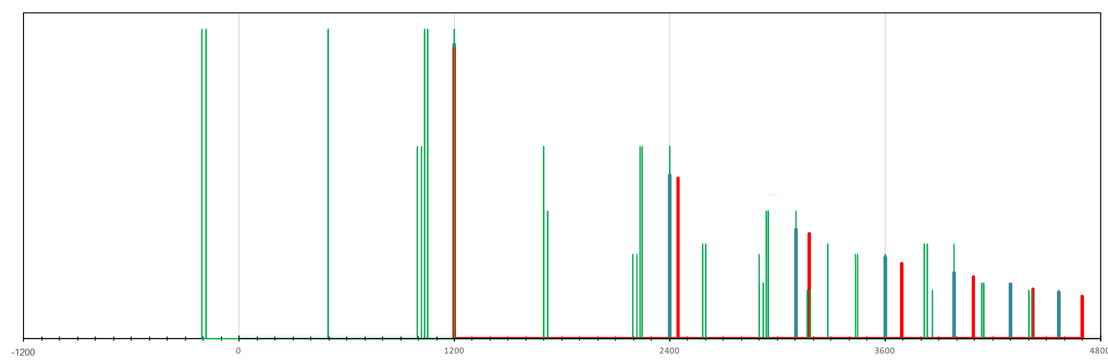
State #	$F_{RBI\ (1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
110	$\frac{9}{10} \cdot F_{1/1}$	1.369	1	0.900	$9/10\ \cdot 1$	
			2	2.325	$2/1\ \cdot 1$	
			3	4.050	$4/3\ \cdot 3$	$(2/3 \cdot 6) \cdot 1$
			4	6.004	$2/1\ \cdot 3$	$(1/1 \cdot 6) \cdot 1$
			5	8.150	$4/3\ \cdot 6$	$(2/3 \cdot 6) \cdot 2$
			6	10.460	$20/11\ \cdot 6$	$(7/8 \cdot 6) \cdot 2$
			7	12.918	$11/6\ \cdot 7$	



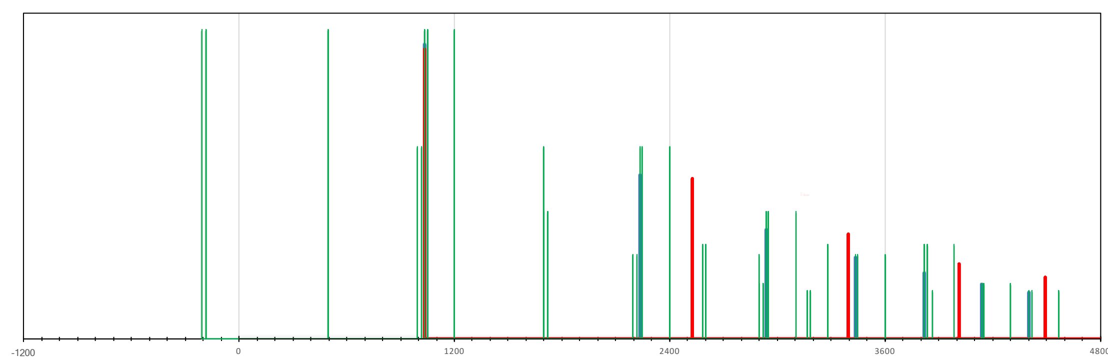
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
111	$\frac{8}{9} \cdot F_{1/1}$	1.376	1	0.889	$8/9\ \cdot 1$	
			2	2.307	$2/1\ \cdot 1$	
			3	4.031	$4/3\ \cdot 3$	$(2/3 \cdot 6) \cdot 1$
			4	5.988	$2/1\ \cdot 3$	$(1/1 \cdot 6) \cdot 1$
			5	8.140	$4/3\ \cdot 6$	$(2/3 \cdot 6) \cdot 2$
			6	10.461	$20/11\ \cdot 6$	$(7/8 \cdot 6) \cdot 2$
			7	12.933	$11/6\ \cdot 7$	



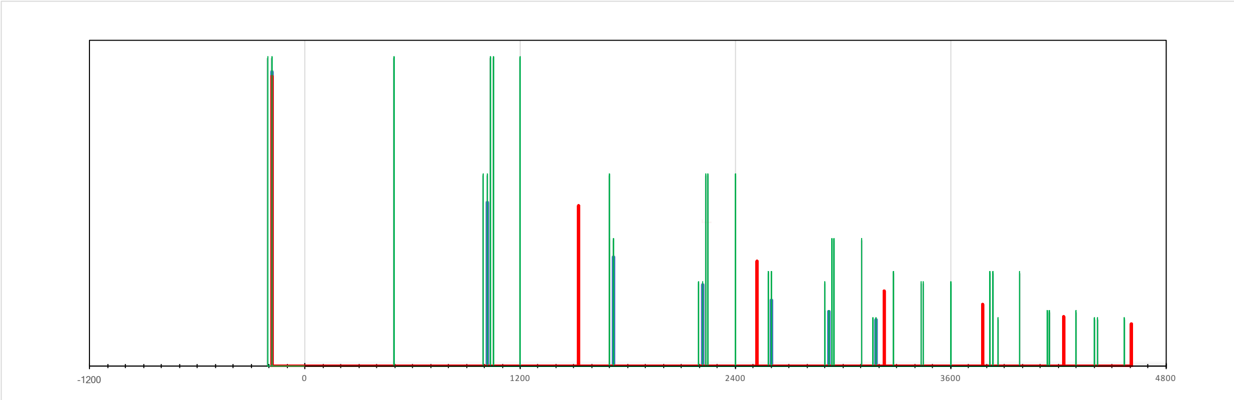
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
112	$\frac{11}{6} \cdot F_{1/1}$	1.084	1	1.833	$11/6\ \cdot 1$	
			2	3.886	$4/3\ \cdot 3$	$(2/3 \cdot 6) \cdot 1$
			3	6.032	$2/1\ \cdot 3$	$(1/1 \cdot 6) \cdot 1$
			4	8.239	$4/3\ \cdot 6$	$(2/3 \cdot 6) \cdot 2$
			5	10.494	$20/11\ \cdot 6$	$(7/8 \cdot 6) \cdot 2$
			6	12.787	$11/6\ \cdot 7$	
			7	15.112	$2/1\ \cdot 7$	$(5/6 \cdot 6) \cdot 3$



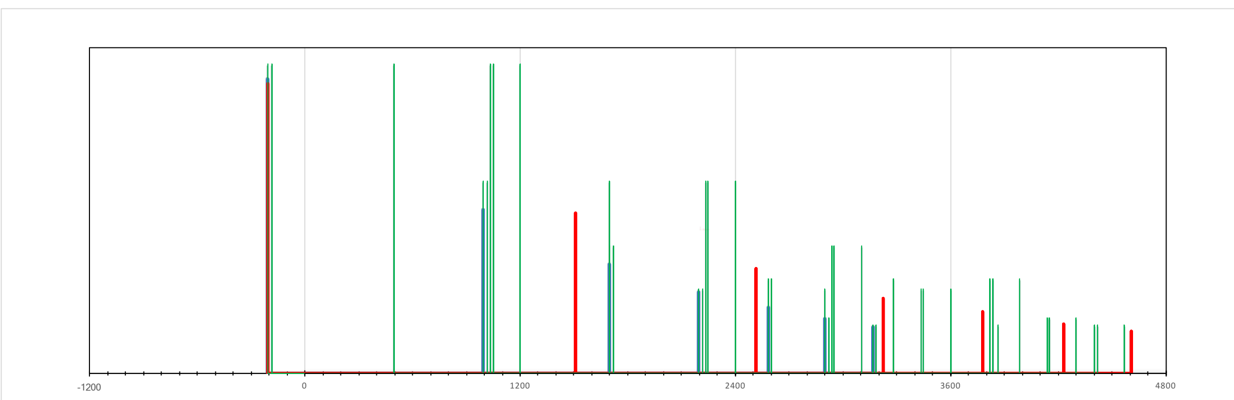
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
113	$\frac{2}{1}\cdot F_{1/1}$	1.039	1	2.000	$2/1\ \cdot 1$	
			2	4.110	$4/3\ \cdot 3$	$(2/3\cdot 6)\cdot 1$
			3	6.263	$9/10\ \cdot 7$	
			4	8.444	$4/3\ \cdot 6$	$(2/3\cdot 6)\cdot 2$
			5	10.648	$20/11\ \cdot 6$	$(8/9\cdot 6)\cdot 2$
			6	12.869	$11/6\ \cdot 7$	
			7	15.104	$2/1\ \cdot 7$	$(5/6\cdot 6)\cdot 3$



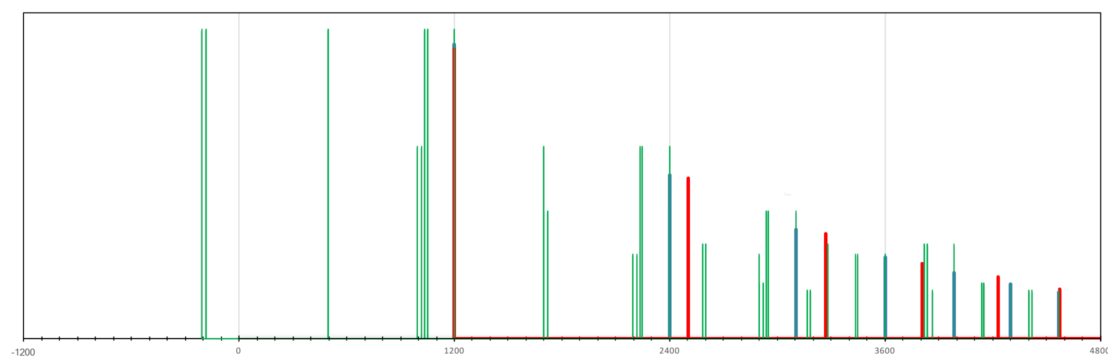
State #	$F_{RBI\ (1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
114 *diff.	$\frac{20}{11}\cdot F_{1/1}$	1.242	1	1.818	$20/11\ \cdot 1$	
			2	4.300	$8/9\ \cdot 5$	
			3	7.116	$20/11\ \cdot 4$	
			4	10.172	$2/1\ \cdot 5$	$(6/7\cdot 6)\cdot 2$
			5	13.420	$2/1\ \cdot 7$	$(3/4\cdot 6)\cdot 3$
			6	16.831	n/a	$(11/12\cdot 6)\cdot 3$
			7	20.382	n/a	$(1/1\cdot 6)\cdot 3$



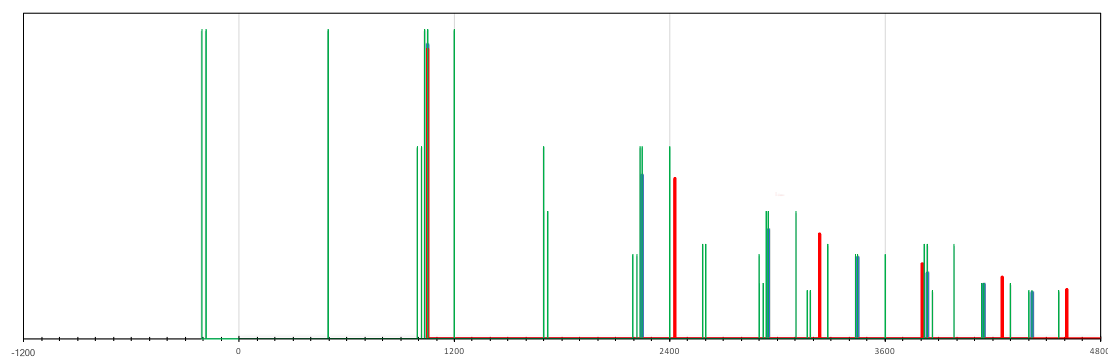
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
115	$\frac{9}{10} \cdot F_{1/1}$	1.422	1	0.900	$9/10 \cdot 1$	
			2	2.412	$8/9 \cdot 3$	
			3	4.292	$8/9 \cdot 5$	
			4	6.462	$9/10 \cdot 7$	
			5	8.875	$20/11 \cdot 5$	$(3/4 \cdot 6) \cdot 2$
			6	11.502	$2/1 \cdot 6$	$(2/3 \cdot 6) \cdot 3$
			7	14.321	$2/1 \cdot 7$	$(4/5 \cdot 6) \cdot 3$



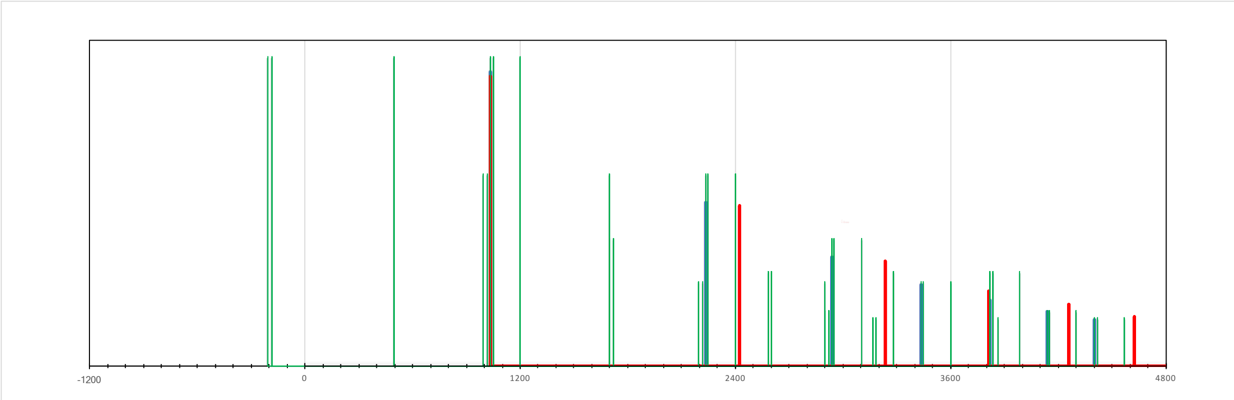
State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
116	$\frac{8}{9} \cdot F_{1/1}$	1.429	1	0.889	$8/9 \cdot 1$	
			2	2.393	$8/9 \cdot 3$	
			3	4.272	$8/9 \cdot 5$	
			4	6.445	$9/10 \cdot 7$	
			5	8.865	$20/11 \cdot 5$	$(3/4 \cdot 6) \cdot 2$
			6	11.503	$2/1 \cdot 6$	$(2/3 \cdot 6) \cdot 3$
			7	14.338	$2/1 \cdot 7$	$(4/5 \cdot 6) \cdot 3$



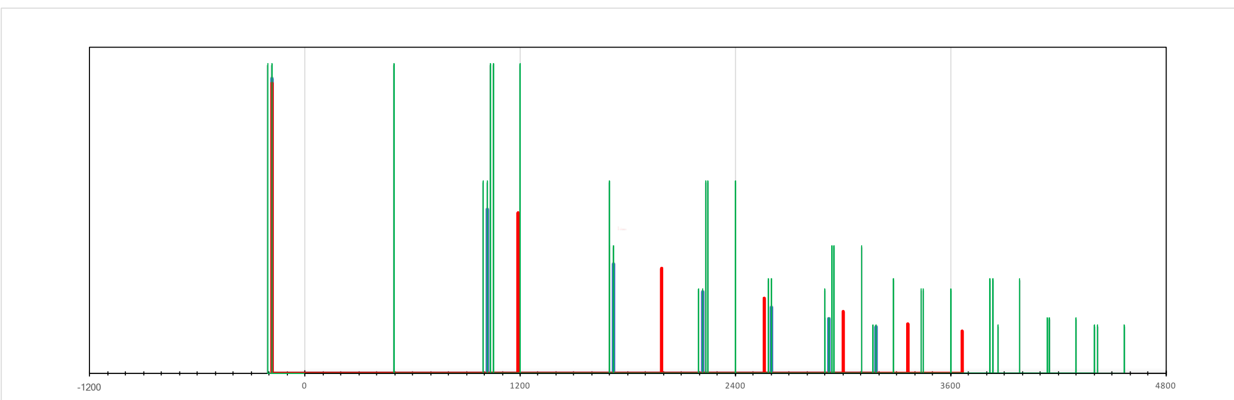
State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
117	$\frac{2}{1} \cdot F_{1/1}$	1.087	1	2.000	$2/1\ \cdot 1$	
			2	4.249	$8/9\ \cdot 5$	
			3	6.602	$4/3\ \cdot 5$	
			4	9.025	$20/11\ \cdot 5$	$(3/4 \cdot 6) \cdot 2$
			5	11.503	$2/1\ \cdot 6$	$(2/3 \cdot 6) \cdot 3$
			6	14.024	$2/1\ \cdot 7$	
			7	16.583	n/a	$(11/12 \cdot 6) \cdot 3$



State #	$F_{RBI\,(1-6)}$	x	n	$f_{st}\ /\ F_{1/1}$	$NASH-RBI$	$NASH-RBII$
118	$\frac{11}{6} \cdot F_{1/1}$	1.15	1	1.833	$11/6\ \cdot 1$	
			2	4.068	$4/3\ \cdot 3$	$(2/3 \cdot 6) \cdot 1$
			3	6.485	$4/3\ \cdot 5$	
			4	9.028	$20/11\ \cdot 5$	$(3/4 \cdot 6) \cdot 2$
			5	11.670	$2/1\ \cdot 6$	$(2/3 \cdot 6) \cdot 3$
			6	14.392	$2/1\ \cdot 7$	$(4/5 \cdot 6) \cdot 3$
			7	17.183	n/a	$(11/12 \cdot 6) \cdot 3$



State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
119	$\frac{20}{11} \cdot F_{1/1}$	1.1575	1	1.818	$20/11 \cdot 1$	
			2	4.056	$4/3 \cdot 3$	$(2/3 \cdot 6) \cdot 1$
			3	6.485	$4/3 \cdot 5$	
			4	9.047	$20/11 \cdot 5$	
			5	11.714	$2/1 \cdot 6$	$(2/3 \cdot 6) \cdot 3$
			6	14.466	$2/1 \cdot 7$	$(4/5 \cdot 6) \cdot 3$
			7	17.292	n/a	$(1/1 \cdot 6) \cdot 3$



State #	$F_{RBI\,(1-6)}$	x	n	f_{st} / $F_{1/1}$	$NASH-RBI$	$NASH-RBII$
120 *diff.	$\frac{9}{10} \cdot F_{1/1}$	1.1425	1	0.900	$9/10 \cdot 1$	
			2	1.987	$2/1 \cdot 1$	
			3	3.158	$8/9 \cdot 4$	
			4	4.386	$8/9 \cdot 5$	
			5	5.660	$11/6 \cdot 3$	$(11/12 \cdot 6) \cdot 1$
			6	6.971	$20/11 \cdot 4$	
			7	8.313	$2/1 \cdot 4$	$(2/3 \cdot 6) \cdot 2$

APPENDIX B—*Artemisia* (Score for Viol Consort)

Artemisia (35-53, 64-67, 70-71, 80-82)

For Viol Consort & Electronics

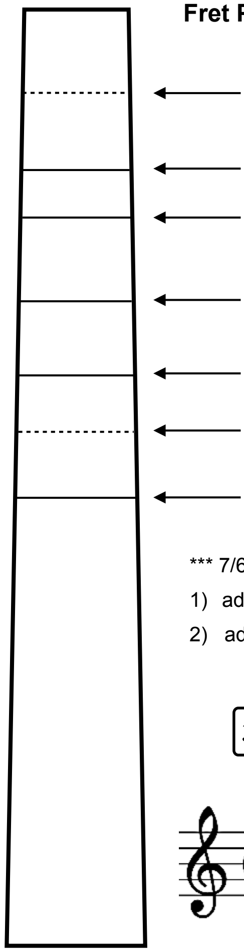
Ben Luca Robertson

Tuning

Treble Viol	
I	= D5
II	= A4
III	= E4
IV	= C4
V	= G3
VI	= D3

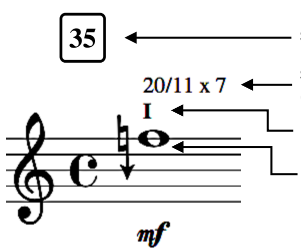
Bass Viols (1, 2, 3)	
I	= D4
II	= A3
III	= E3
IV	= C3
V	= G2
VI	= D2

Fret Positions for Treble & Bass Viols (1, 2, 3)



*** 7/6 fret position can be achieved by either:

1) adding an additional fret to the fingerboard
2) adjusting (flattening) the position of the standard (minor) 3rd fret



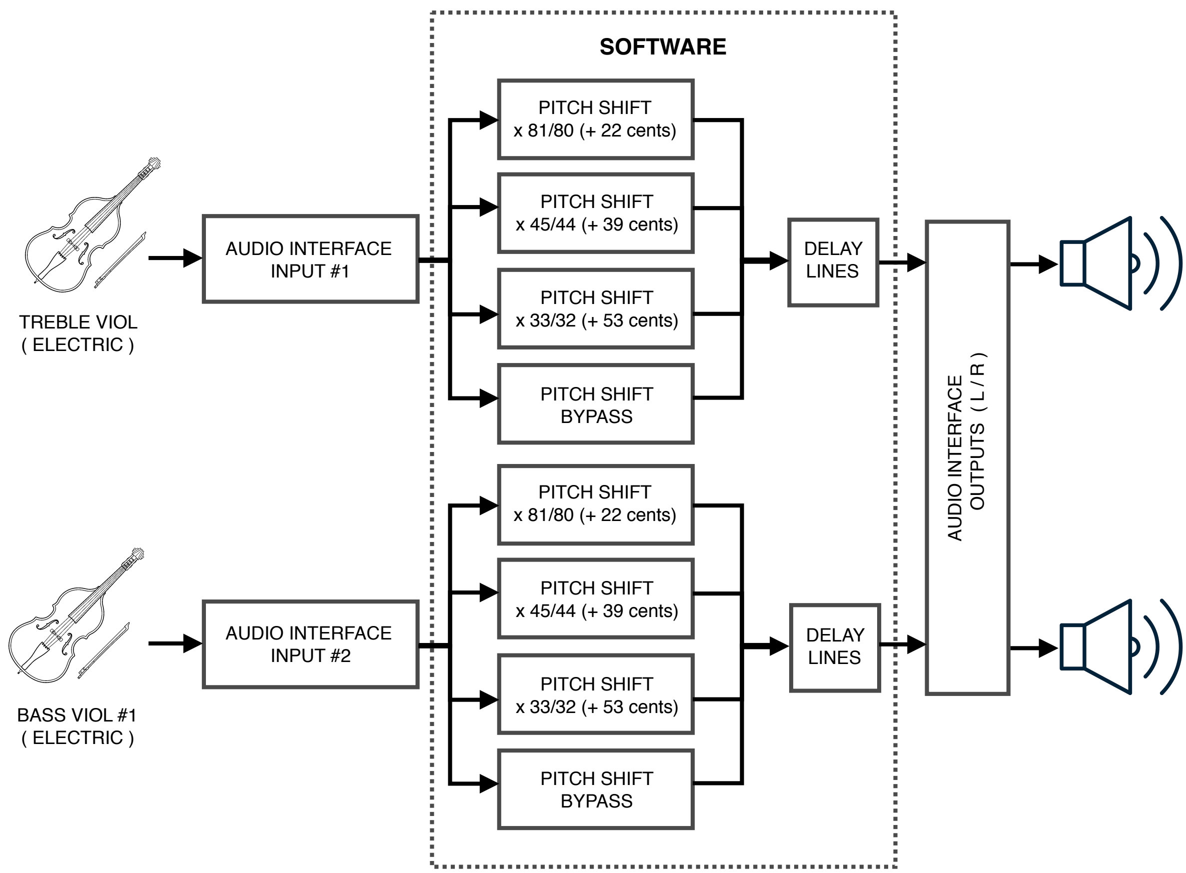
 = indicates fretting to produce 7/6 x frequency of open string

Electronics (Audio Processing)

The piece employs an extended system of 11-Limit just intonation to model the phenomenon of “stretched” octaves and other spectral non-linearities associated with inharmonic timbres. For example, de-tuned octaves exhibit intervallic displacement equivalent to at least three forms of just commas (e.g. 81/80, 45/44, 33/32). The cumulative effect for this mode of inharmonic distortion suggests the perception of an altered or otherwise, ambiguous fundamental frequency—thus eliciting a sense of movement analogous to ‘Tonal Flux’.

So as to afford the precise transformations in pitch and spectra necessary in generating these phenomena, performers use the attached software (artemisia_1.maxpat) to process the signals from two electric (solid body) instruments in the consort—namely, the Treble Viol & Bass Viol #1. Within certain sections of the piece, this software applies real-time pitch-shifting to change the “performed” (e.g. notated) pitch into a different “sounding” pitch (indicated in ratio form above the staff).

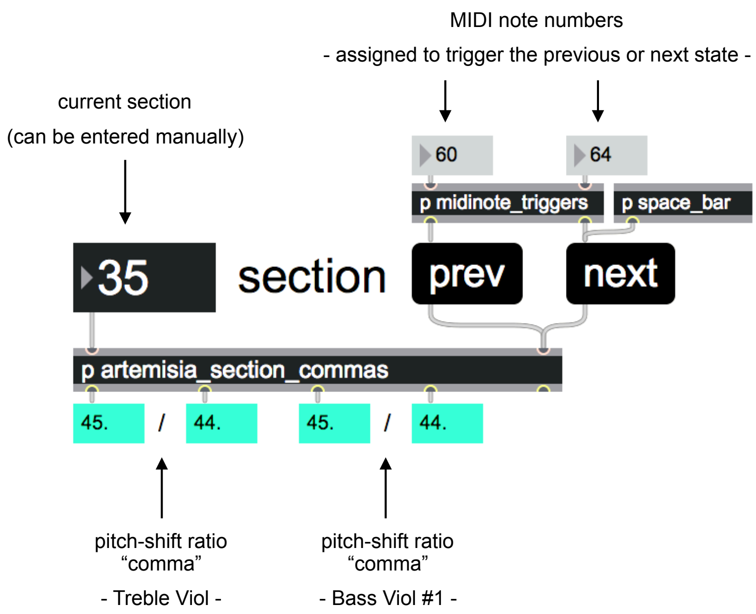
For the program to function correctly, output from the Treble Viol’s pickup must be routed to the first input (ADC~1) of the audio interface (connected to a laptop running the software). Similarly, output from Bass Viol #1 should be connected to the second input (ADC~2). Concurrently, stereo output from the audio interface shall be amplified using a pair of powered loudspeakers or similar sound-system. To balance levels between instruments in a concert setting, it may be necessary to mic and amplify bass Viols 2 & 3, as well.



Electronics (Triggering Different States)

Structure for the piece follows 28 measured states: 35-53, 64-67, 70-71, 80-82. As states appearing within the score constitute a series of sections taken from an extended suite, state numbers appear discontiguous. Regardless, each state should be performed in order, as written. Here, every state corresponds to a single measure (≈ 16 seconds) and each measure constitutes a unique array of audio-processing parameters (pitch-shifting, delay, & reverberation values) to be triggered in real-time.

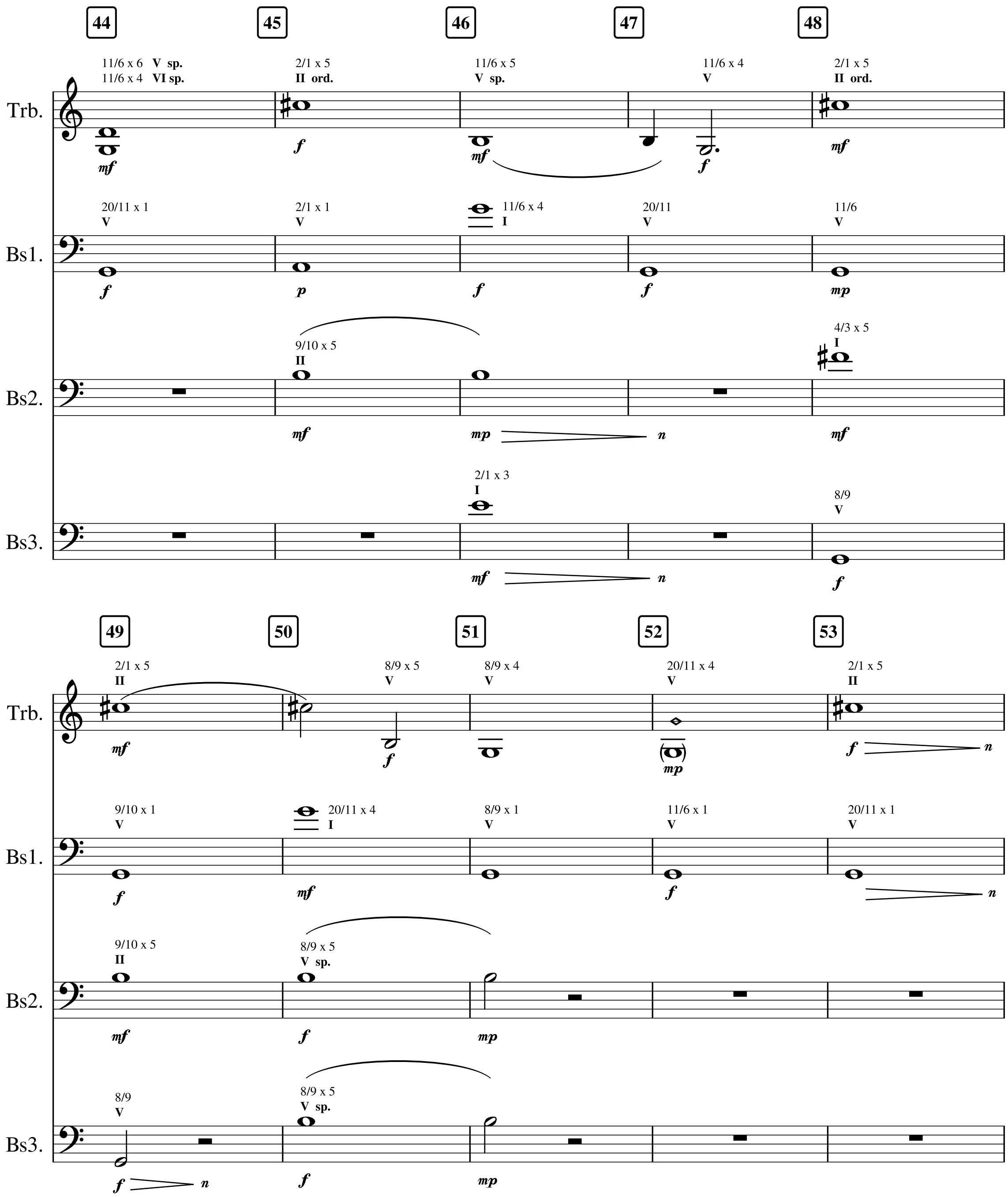
To ensure accurate processing for each section during performance, one member of the consort (or a conductor) shall be tasked with triggering the software. The program will respond to either note messages from a MIDI pedal or the 'space bar' on a computer keyboard. By default, activating MIDI note-number 64 (or pressing the 'space bar') will trigger the next state in sequence. Alternately, one may scroll back to the previous section by activating MIDI note-number 60 or enter the state number manually. If needed, these values may be re-assigned in the software to accommodate different MIDI device settings.



Artemisia

Ben Luca Robertson

2019


44
45
46
47
48
11/6 x 6 V sp.
11/6 x 4 VI sp.
2/1 x 5
II ord.
11/6 x 5
V sp.
11/6 x 4
V
2/1 x 5
II ord.
Trb.
20/11 x 1
V
2/1 x 1
V
11/6 x 4
I
20/11
V
11/6
V
Bs1.
9/10 x 5
II
4/3 x 5
I
Bs2.
2/1 x 3
I
8/9
V
Bs3.
49
50
51
52
53
2/1 x 5
II
8/9 x 5
V
8/9 x 4
V
20/11 x 4
V
2/1 x 5
II
Trb.
9/10 x 1
V
20/11 x 4
I
8/9 x 1
V
11/6 x 1
V
20/11 x 1
V
Bs1.
9/10 x 5
II
8/9 x 5
V sp.
Bs2.
8/9
V
8/9 x 5
V sp.
Bs3.


64
65
66
67
Trb.
20/11 x 5
V sp.
2/1 x 4
II ord.
9/10 x 5
V
Bs1.
2/1 x 1
V
20/11 x 1
V
8/9 x 7
I
20/11 x 1
V
Bs2.
4/3 x 5
I
8/9 x 4
V
Bs3.
4/3 x 5
I
8/9 x 1
VI
70
71
80
81
82
9/10 x 5
V
9/10 x 3
VI
4/3 x 6
II
9/10 x 7
I
9/10 x 1
V
8/9 x 1
V
9/10 x 5
II
8/9 x 1
V

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