On Measuring the Radial Velocity of Stars with Earth-Sized Planets

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ABSTRACT

This project aimed to examine how Earth-sized exoplanets are found around stars using the radial velocity method. Calculations made using the method laid out in Fundamental Photon Noise Limit to Radial Velocity Measurements (Bouchy et al. 2001) attempted to find the quality factor for a synthetic spectrum, but had an error on the magnitude of 10^4, likely due to an error within the code. Using an interpolation function within Python, I recovered a radial velocity from an artificially shifted spectrum, however this also carried an error of 6-9%, depending on the initially chosen velocity. Lastly, I examined other effects that could be seen on stars using the radial velocity method, including a star's rotation and winds on a planet.

Keywords: radial velocity, quality factor, Rossiter-McLaughlin Effect

1. PRELIMINARY RESEARCH

1.1. *Detecting Exoplanets*

To begin this project, I read up on current missions and efforts to find Earth-sized planets, such as MEarth and TESS. The MEarth project uses ground-based telescopes to search nearby M dwarf stars for transiting Earth-sized planets (MEarth 2008), and TESS is an upcoming NASA mission, the Transiting Exoplanet Survey Satellite, that will use a spaced-based satellite to search for transiting planets around the 200,000 brightest stars in the sky (NASA 2015). Both of these efforts use the transit method, looking for regular decreases in the amount of light seen from a star to infer the presence of a planet, and suggest follow up research be conducted, such as transmission or Doppler spectroscopy. Transmission spectroscopy, used by the MEarth team, takes spectra right at the edge of a transit, where the light from the star is passing through the atmosphere of the planet. Doppler spectroscopy is what is more commonly referred to as radial velocity, and measures the change in redshift of a star caused by a planet orbiting it.

A number of Earth-sized planets have been found already due to the MEarth project, as well as others searching for planets. According to data from the Kepler Space Telescope, small planets, which range from 1 to 4 times the size of Earth, are around 26% of Sun-like stars with periods of less than 100 days (Marcy et al. 2014). Such planets are even more common around small stars, like M dwarfs, with a frequency of about 1.4 planets per star, and M dwarfs outnumber Sun-like stars 12:1 (Berta-Thompson et al. 2011) Of these planets, the smallest, those with radii less that 1.5 times the Earth's radius, have densities consistent with rocky planets. Density increases with radius, until about 1.5 times the radius of the Earth, at which point the density begins to decrease. This reveals a low density envelope around the core, such as a large atmosphere, and suggests the planet may be a "mini Neptune (Marcy et al. 2014). Thus for planets up to twice the size of Earth, it is reasonable to assume the presence of a rocky core.

One such planet found my MEarth is GJ 1132b, orbiting a star 12 parsecs away. Doppler mass measurements yield a a mass of 1.2 times the mass of the Earth, and a density consistent with a rocky composition. While GJ 1132b is receiving more stellar radiation than Earth, its substantially cooler that comparable planets' equilibrium temperature suggests it may have maintained an atmosphere. However, uncertainties in the initial size of the planet's hydrogen reservoir, the history star's luminosity, and the evolution of the system make it difficult to determine much of anything about the composition of this atmosphere. Further spectroscopy, particularly transmission spectroscopy could give more information on the atmosphere, and thus potential habitability, of this planet (Berta-Thompson et al. 2011).

1.2. *Habitability Potential*

A number of other factors have the potential to impact the habitability of Earth-sized exoplanets as well, however these can be more difficult to study in tandem. These factors include space weather due to stellar winds and the star's magnetic field, and the possible protection from atmospheric erosion provided by the planet's magnetic field. Planetary magnetic fields can prevent direct stripping away of the atmosphere by stellar wind, but ion escape can still occur at the magnetic poles. This process, the polar wind, is known to occur at Earth and may have contributed to the habitability of Earth's early atmosphere (Garcia-Sage et al. 2017). M dwarf stars, however, like those that MEarth is searching around and like our neighbor Proxima Centauri, have higher ionizing radiation levels than Sun-like stars. Models of Proxima b, an Earth-sized planet around Proxima Centauri, and its magnetic field suggest that the same polar wind process that occurred on Earth would not have completely saved the atmosphere of Proxima b from being stripped. The concept, however, remains, that a planet's magnetic field could protect the atmosphere of the planet from being directly stripped by stellar winds.

The opposite still stands as a factor, the effect of stellar winds and space weather on the planet's atmosphere. Other models of Proxima b show that its proximity to Proxima Centauri leaves it subject to 2000 times the stellar wind pressure that Earth faces from the Sun. During a singular orbit of its star, Proxima b is subject to changes is pressure of 1-3 orders of magnitude. Sudden and periodic changes to the magnetopause combined with the sheer amount of pressure faced by Proxima b make it unlikely to maintain an atmosphere (Garraffo et al. 2016). The MEarth project is searching around stars similar to Proxima Centauri, other M dwarf stars, which would have the same problem. M dwarfs are typically more magnetically active, though it is difficult to determine the history of the star and how its changes in luminosity over time affected the atmosphere of any orbiting planets.

2. MODELING

2.1. *Method*

The majority of this project involved attempting to replicate the work of Fundamental Photon Noise Limit to Radial Velocity Measurements (Bouchy et al. 2001) in order to understand how a radial velocity measurement and the fundamental noise limit are found from a stellar spectrum. This method involves determining a quality factor, Q, to compute the fundamental uncertainty on the radial velocity measurement due to noise. Table 1 shows the parameters of one of the synthetic spectra used in Bouchy and the parameters of

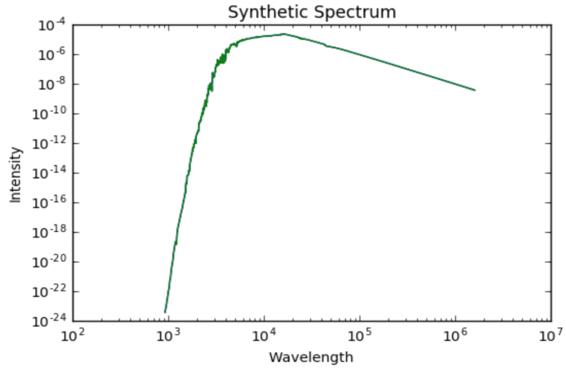

Figure 1. This graph shows the spectrum obtained from a given table of values, shown in wavelength (Å) versus intensity ($ergs/cm^2/s/hz/ster$)

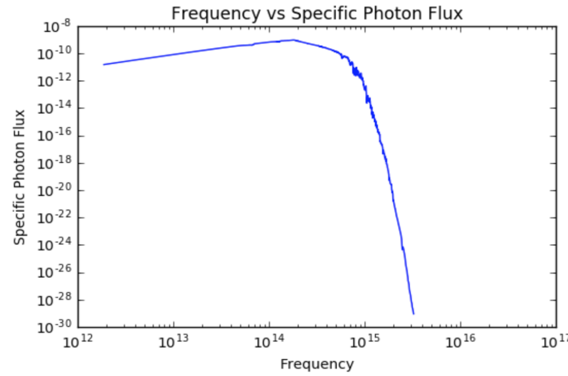

Figure 2. This graph shows the specific photon flux of the spectrum against the frequency bin. It nearly mirrors the synthetic spectrum as frequency is inversely proportional to wavelength

the synthetic spectrum I used to replicate these results. While the spectral type of my synthetic spectrum was not stated, it is clear based on the temperature and surface gravity that it is similar to the K5V spectrum used by Bouchy.

Used By	Spectral Type	Effective Temperature (K)	log(g) (cms^{-2})
Bouchy	K5V	4500	4.5
Casey	unstated	4500	4.5

Table 1. Parameters of synthetic spectra used in Bouchy and here

The synthetic spectrum was given as a table containing wavelength bins, measured in Angstroms, and the intensity for each bin, measured in $ergs/cm^2/s/hz/ster$. To correct an error in the units from the given synthetic spectrum, the intensity was multiplied by a factor of four. Figure 1 shows the spectrum obtained from the initial table. The star represented by this synthetic spectrum was chosen to have a radius of $0.72R_{Sun}$, a calibrated physical parameter as given by Allen's Astrophysical Quantities (Cox 2000), and to be at a distance of 4 parsecs, chosen arbitrarily.

Equation 1 gives the specific photon flux, ϕ_ν.

$$\phi_\nu = \frac{I_\nu \pi (\frac{R_{star}}{D})^2}{h\nu}, \quad (1)$$

Here, ν is the frequency in a given bin. This turns the intensity, or energy flux, from the given spectrum into the specific photon flux per frequency bin. Figure 2 shows the relationship between this specific photon and the frequency. ϕ_ν is necessary, along with the exposure time from the spectrograph and the area of the telescope, to determine A,the number of photons per bin. This is given in Equation 2. For this calculation, I used the area and exposure time of the CORALIE spectrograph, one of the two spectrographs used in the Bouchy paper. The CORALIE spectrograph is mounted on a 1.2m telescope, and a 750s exposure time was used to determine the fundamental uncertainty.

$$A = \Delta\nu\phi_\nu \times t_{exp} \times A_{tel} \quad (2)$$

The factor $\Delta\nu$ gives the width of each frequency bin as $\nu_{i+1} - \nu_i$. This is only valid when i and i+1 are valid indices in the frequency array. For the endpoints, where this is not valid, the adjacent value was simply repeated. As later calculations of the quality factor only look at a range of frequencies in the middle of the array, this is not problematic.

Once I had A, I could run a for loop within Python to determine $\frac{\delta A}{\delta\lambda}$, the observable intensity change at a given pixel for a given wavelength. Here, the endpoints were omitted entirely, as is seen necessary in Equation 3. Again, this will be inconsequential for the final calculation of Q.

$$\frac{\delta A}{\delta\lambda} = \frac{A[i+1] - A[i-1]}{\lambda[i+1] - \lambda[i-1]} \quad (3)$$

One could then find the Doppler shift by Equation 4, where A_0 is the number of photons per bin in a Doppler shifted observation.

$$\frac{\delta V(i)}{c} = \frac{A_0(i) - A(i)}{\lambda(i)(\frac{\delta A}{\delta\lambda})} \quad (4)$$

For this purposes of this model, however, $\frac{\delta A}{\delta\lambda}$ was used to find $W(i)$, the optimum weight, which is proportional to the inverse square of the individual dispersion. Given a small Doppler shift, $W(i)$ can be found with the following equation:

$$W(i) = \frac{\lambda^2(i)[\frac{\delta A}{\delta\lambda}]^2}{A(i) + \sigma_D^2} \quad (5)$$

Finally, Q, the quality factor, is given by Equation 6.

$$Q = \frac{\sqrt{\sum W(i)}}{\sqrt{\sum A(i)}} \quad (6)$$

2.2. *Results and Analysis*

After trimming the array of weights and photons per bin to the wavelength range of the CORALIE spectrograph, 3875-6820 Å, I found a Q of 13.61357, which differs from the expected value obtained by Bouchy of 34,940 by a factor of approximately 2.6e3. Figure 3 shows the graphs of Q for a range of wavelengths in both Bouchy's and my calculations, While there is a difference of a factor of 10^3, the shape of each graph holds some resemblance. My graph declines more steeply, but both have peaks around 4000Å, 5100Å, and 6200Å.

It is probable that the large difference in my found value for Q from the value in Bouchy is due to an error in my code. The precise methodology is not given in the Bouchy paper, and I may be missing a factor that was not explicitly stated in the relationships given in Bouchy. Alternatively, the Bouchy paper states that the calculations are carried out considering both the CORALIE spectrograph, which I used in my considerations, and the HARPS spectrograph, which is on a telescope three times larger than that of CORALIE. I do not suspect this small difference would have such a substantial effect, however, as there is ultimately only a single factor of A, which relies on A_{tel}, in the calculation of Q. Other possible reasons for this difference are not immediately clear to me, though they are likely an error on my part, not on the part of Bouchy and his co-authors.

3. INTERPOLATION

3.1. *Method*

In order to better understand precisely how a radial velocity measurement is obtained from a shifted spectrum, I artificially shifted the same synthetic spectrum, here referred to as A_0 as before, using the following equation:

$$A(\lambda) = A_0(\lambda(1+\frac{v}{c})) \quad (7)$$

Here, the radial velocity, v, is greater than zero. As the synthetic spectrum was given as $A_0(i)$ versus $\lambda(i)$, I first found the shift in wavelength for v=20,000,000 cm/s, and then used an interpolation function within Python to find values for $A(\lambda)$. As figure 4 shows, some wavelengths are shifted a great deal more than others, so the same optimum weight as before must be found per wavelength bin.

Once the weight was found for each wavelength bin, I could calculate $\frac{\delta V(i)}{c}$, which was found with the following equation.

$$\frac{\delta V(i)}{c} = \frac{A(i) - A_0(i)}{\lambda(i)(\frac{\delta A}{\delta \lambda})} \quad (8)$$

V Input (cm/s)	V Output (cm/s)	% Error
9,000,000	9,862,785.73	9.587
20,000,000	21,731,436.97	8.657
50,000,000	53,363,984.86	6.728

Table 2. Inputted and recovered radial velocities and their percent errors

Summing every $\frac{\delta V(i)}{c}$ and applying the optimum weights using the following equation,

$$\frac{\delta V}{c} = \frac{\sum \frac{\delta V(i)}{c} W(i)}{\sum W(i)}, \quad (9)$$

gives the Doppler shift for the spectrum.

3.2. *Results and Analysis*

In every iteration of this method, there was some error. Table 2 shows three iterations of this method, with inputted velocities being those I choose to artificially shift the spectrum and outputted velocities those that were returned by my calculations. It seems that for larger radial velocities, the error is smaller. These are high estimates for radial velocity, however, particularly for a star only 4 parsecs away.

As before, it is likely that this error came from within the code. Should there have been an unstated factor in Bouchy's methodology that caused the error in the determination of Q, it would also likely have an effect here. As the error here is not on the same scale as the error in the determination of Q, I am unlikely to believe this is the root of the problem. There may be an error associated with the interpolation function, but this is also not as likely. While the error in my code is not immediately clear it me, it is likely there somewhere.

4. FURTHER RESEARCH

It is more than just the potential presence of a planet that can be determined from radial velocity measurements of a star. Doppler spectroscopy of a planet's atmosphere can also reveal winds on the planet via wavelength shifts in absorption lines. Spectroscopy of the exoplanet HD 209458b suggested the presence of a strong wind flowing from the irradiated dayside to the non-irradiated nightside of the planet (Snellen et al. 2010).

Doppler spectroscopy can also reveal the rate of rotation of the star being observed. The Rossiter-McLaughlin Effect seen in the amplitude and asymmetry of the radial-velocity distortion, and depends on the stellar rotation rate $vsin(I)$, where I is the line of sight angle to the rotation axis of the star (Cameron et al. 2010). While Collier Cameron succinctly explains the methodology for determining a rotation profile for a star from its Cross Correlation Function (CCF), it is unclear how the CCF is determined from an initial spectrum. It seems as though the CCF is calculated using a program that matches

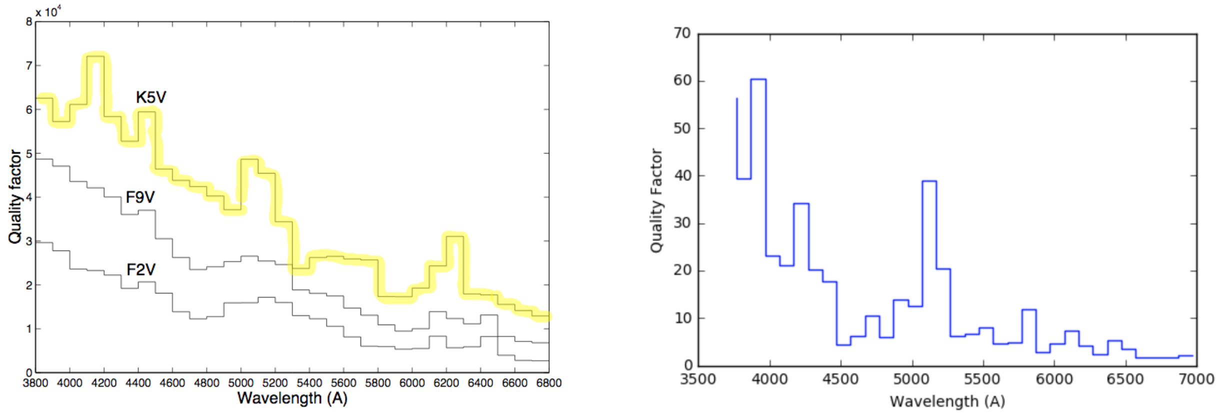

Figure 3. These graphs show the quality factor, Q, for a range of wavelengths, as found in both Bouchy (left, highlighted in yellow) and my work (right).

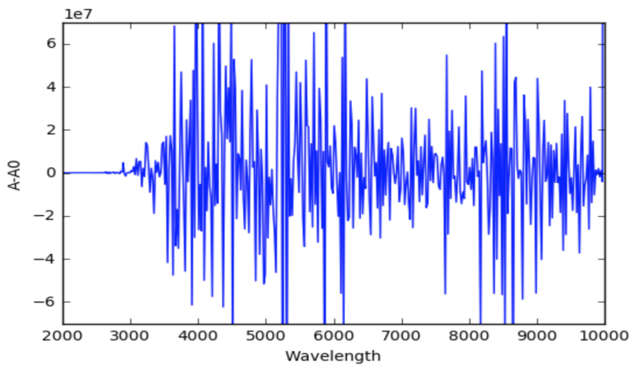

Figure 4. This graph shows the difference in A, the shifted spectrum, and A_0, the original spectrum, for a range of wavelengths.

the stellar spectrum against templates, which are taken as non-rotating to start (Gray 2008). The rotation profile of a star is then the convolution of two functions, Equations 10 and 11, representing the local line profile at any point of a limb-darkened rotating star in the form of a Gaussian, and the limb-darkened rotation profile, respectively.

$$g(x) = \frac{1}{\sqrt{2\pi}s} \exp(\frac{-x^2}{2s^2}) \tag{10}$$

Here, x is the dimensionless velocity $\frac{v}{vsin(I)}$, and s is the Gaussian sigma, $\frac{\sigma}{vsin(I)}$.

$$f(x) = \frac{6[(1-u)\sqrt{1-x^2} - \pi u(x^2-1)/4]}{\pi(3-u)} \tag{11}$$

Here, u is the standard linear limb-darkening coefficient. The convolution of these two yields Equation 12:

$$h(x) = \int_{-1}^{1} f(z)g(x-z)dz. \tag{12}$$

This gives the rotation profile of the star, and analysis of a transiting planet's position throughout the transit can give further depth to details regarding the differentiation of rotation, as a star rarely orbits as a solid body, and I, the inclination to the rotation axis (Snellen et al. 2010).

5. CONCLUSION

Throughout this project, I had hoped to gain a better understanding of the radial velocity method for discovering exoplanets. I have seen that, while discovery is certainly possible, radial velocity measurements often serve as follow up science to transit detections, and can give much more information that simply the mass of the planet. From transmission spectroscopy to determine the composition of an atmosphere or the presence of planetary winds, to Doppler spectroscopy that can give information on the star's rotation, radial velocity measurements are crucial to discovery in the hunt for Earth-sized planets.

I have seen that determining the radial velocity requires precision to a degree that I was unable to achieve with an artificially shifted spectrum, and that a huge number of factors play a role in the quality of a radial velocity measurement's uncertainty. While the physics behind a Doppler shifted spectrum may not seem vastly complicated on the surface, there is far more at work than simply a planet orbiting a star, and all of those details, be they stellar or planetary winds or noise within the detector, have an effect on the quality of a signal and the data that astronomers are able to derive from it. I have learned a great deal on what it means to study Doppler spectroscopy, and what work goes into verifying a possible exoplanet from a mission like TESS or MEarth.

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Software: Numpy, matplotlib

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