Speed Harmonization Using Connected and Automated Vehicles

A Thesis

Presented to

the faculty of the School of Engineering and Applied Science

of the University of Virginia

In partial fulfillment of

the requirements for the degree of

Master of Science

By

Stacy Learn

December

2016

APPROVAL SHEET

The thesis

Is submitted in partial fulfillment of the requirements

for the degree of

MASTER OF SCIENCE

Stacy Learn, Author

This Thesis has been read and approved by the following committee:

Dr. Brian Smith, Advisor

Dr. Donna Chen

Dr. Brian Park

Accepted for the School of Engineering and Applied Science:

Dean, School of Engineering and Applied Science

December

2016

ACKNOWLEDGEMENTS

I would like to thank a number of people for their support and contribution to my work:

Dr. Brian Smith, my advisor, for providing me the opportunity to join the graduate research team. Your support and praise has contributed immensely to my development as an engineer.

Dr. Jiaqi Ma, whose guidance has been tremendously vital in this research. Your strong work ethic and passion for your work constantly inspires me to always work even harder.

Dr. Hyungjun Park, for his time and commitment to the project and being readily available to address any questions or concerns I had regarding my research.

Dr. Mike Fontaine, who kindly offered assistance in my statistical analysis and taught me everything I know about transportation data analysis.

My parents, Connie and Chun Learn, and brother, Scott Learn, for their endless love, support, and commitment to my success. Everything I have accomplished is because of you all, I couldn't have done it without you.

My best friends, My-Anh, Alice, and Becca, for their positivity, encouragement, and love that never fails to uplift me.

ABSTRACT

Traffic oscillations, known more commonly as "stop-and-go" driving conditions in heavily congested traffic, result in numerous negative impacts. Not only do they reduce effective capacity, but they also increase safety risks, travel delay, and fuel consumption and emissions. Researchers have hypothesized that strategies intended to harmonize the speed of vehicles will reduce traffic oscillations and reduce the adverse impacts. A speed harmonization system that utilizes the capabilities of connected and automated vehicles (CAV) has been developed that dynamically and automatically adjusts the speeds of vehicles based on current traffic conditions. While researchers have evaluated speed harmonization in simulation studies, there have been no large-scale field evaluations of speed harmonization systems. This thesis presents the results of the continuation of the first large-scale field test of a speed harmonization system using CAVs. The objectives of this research effort are threefold: to demonstrate the feasibility of the implementation of speed harmonization in a real-world environment, to determine the effectiveness of speed harmonization in reducing traffic oscillations, and to investigate the potential of speed harmonization in improving traffic performance.

In a field test of a speed harmonization system prototype, a fleet consisting of three CAVs and five probe vehicles was deployed on a roadway segment that experiences daily recurring traffic congestion. The fleet was led by one probe vehicle, followed by the three CAVs driving nearly parallel to one another, followed by the four remaining probe vehicles. The objective of the CAVs was to regulate traffic upstream of the bottleneck so that vehicles move with uniform speed, thereby creating a steadier flow of traffic.

To demonstrate the feasibility of the implementation of speed harmonization in a real-world environment, recommended and actual speeds of the CAVs were compared. Results of these comparisons showed that the speeds of the CAVs compared favorably to recommended speeds, thereby implying that the implementation of speed harmonization using CAVs is feasible. To evaluate the effectiveness of speed harmonization on smoothing the flow of traffic by reducing traffic oscillations, the oscillatory behavior of the leading probe vehicle, which represents the "general" flow of traffic unaffected by the CAVs, and the following probe vehicles that followed the CAVs were analyzed and compared. Power spectral density (PSD) analysis was used to analyze traffic oscillations. Results showed that there was a statistically significant difference in each PSD comparison between the lead probe vehicle (P0) and the following probe vehicles. The PSDs of P1, P2, P3, and P4 were reduced by 29%, 36%, 90%, and 84%, respectively, when compared to the PSD of P0. Therefore, it can be concluded that speed harmonization can reduce speed oscillations.

Finally, to determine whether speed harmonization had additional benefits, specifically with regards to mobility and the environment, travel time and fuel consumption were measured. Results showed that the average travel time of each probe vehicle was very similar, thus implying there is no statistically significant difference in travel time. On the other hand, it was found that there were statistically significant differences in the fuel consumption of the probe vehicles. However, these results showed increases in the fuel consumption of the following probe vehicles in comparison to that of the leading probe vehicle. This was not unexpected, as the speed harmonization algorithm primarily focused on smoothing the flow of traffic by reducing traffic oscillations, not explicitly considering other performance measures. The results

presented in this thesis will provide more knowledge and a better understanding of the potential benefits of speed harmonization using CAVs.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS i
ABSTRACTii
LIST OF FIGURES
LIST OF TABLES
Chapter 1. INTRODUCTION 1
Chapter 2. LITERATURE REVIEW
2.1 Connected and Automated Vehicles
2.2 Simulation Tests
2.3 Field Test7
2.4 Power Spectral Density Analysis
2.4.1 Algorithm for Estimating the Power Spectral Density
Chapter 3. METHODOLOGY 11
3.1 System Architecture
3.1.1 Research Platform
3.1.2 Algorithm for Speed Recommendations
3.2 Experimental Setup 14
Chapter 4. RESULTS 18
4.1 Feasibility of Speed Harmonization Implementation in a Real-World Environment

4.2 Effectiveness of Speed Harmonization in Reducing Traffic Oscillations	21
4.3 Potential of Speed Harmonization in Improving Traffic Performance	27
Chapter 5. CONCLUSIONS	34
Chapter 6. FUTURE WORK	37
REFERENCES	39

LIST OF FIGURES

Figure 1 Plot of the PSD estimate using MATLAB [®] 's <i>periodogram</i> function	10
Figure 2 Experimental vehicles and devices	13
Figure 3 Diagram of the speed harmonization field test experiment	16
Figure 4 Field experiment map of system integration.	17
Figure 5 Field test site and trailer locations.	17
Figure 6 Recommended and actual speeds of controlled vehicle 1 (C1)	20
Figure 7 Spatial speed trajectories of all experimental vehicles	21
Figure 8 Detrended speed trajectories of the prob vehicles	22
Figure 9 Power spectral density of detrended speed trajectories of the probe vehicles	24

LIST OF TABLES

Table 1 85 th Percentile Power Spectral Density	
Table 2 Paired T-Test Results – 85 th Percentile PSD	
Table 3 Kruskall-Wallis Test – PSD Ranks	
Table 4 Kruskall-Wallis Test – PSD Test Statistics	
Table 5 Travel Times from T4 to T5 (seconds)	
Table 6 Paired T-Test Results – Travel Times	29
Table 7 Kruskall-Wallis Test – Travel Time Ranks	29
Table 8 Kruskall-Wallis Test – Travel Time Test Statistics	

Table 9 Fuel Consumption (L)	31
Table 10 Paired T-Test Results – Fuel Consumption	32
Table 11 Kruskall-Wallis Test – Fuel Consumption Ranks	32
Table 12 Kruskall-Wallis Test – Fuel Consumption Test Statistics	32

CHAPTER 1. INTRODUCTION

Traffic oscillations are defined as the fluctuating patterns in congested traffic where vehicles engage in repeated deceleration-acceleration cycles, unable to maintain a constant, steady speed (Tian et al., 2016). Such speed variations impose safety risks, increase traffic congestion, and often increase in amplitude as they propagate (Zheng et al., 2011). In an effort to minimize speed variations, active traffic management strategies such as variable speed limits (VSL), adaptive ramp metering, and coordinated traffic signal timing are implemented to dynamically manage congestion based on prevailing traffic conditions (FHWA, 2014). The study presented in this thesis focuses on a specific active management strategy, namely speed harmonization, and its potential in reducing traffic oscillations. Speed harmonization aims to minimize speed variations primarily by modifying the suggested speed according to prevailing traffic conditions, reducing the number of traffic incidents, maximizing traffic throughput, and ultimately maintaining a steady flow of traffic.

As new technologies emerge and evolve, there are alternative approaches to improve traffic performance. Connected and automated vehicles (CAV) are among the new technologies that can be used to harmonize the speeds of vehicles in a traffic stream by creating a uniform flow of traffic and reducing traffic oscillations. CAVs are still in the exploration stage, as there have been few research studies conducted on their performance in mitigating traffic congestion. Most studies in the connected vehicle (CV) and automated vehicle (AV) realm focus on individual vehicle performance rather than aggregate traffic performance. Although simulation studies have

been conducted, there have been a limited number of field tests implementing CAVs. Hale et al. (2016) performed simulation tests, which set the foundation for an experimental field test that was later conducted. In the simulation tests, system performance was studied on a macroscopic level. The results of the tests revealed no significant improvements in average speed, throughput, or travel time. However, the study did not analyze the microscopic system performance, more specifically the smoothing potential of individual vehicle trajectories. The approach to speed harmonization presented in this study uses CAVs to control the flow of traffic and analyzes the system performance on a microscopic level.

This thesis presents the results of the continuation of the first large-scale field test of a speed harmonization system using CAVs. The field tests were conducted on a roadway segment of I-66 eastbound, near Washington, D.C, a site of daily recurring, high traffic congestion. The CAVs are equipped with an adaptive cruise control (ACC) system which enables automated longitudinal control of the vehicle. This capability aims to regulate the speed of the vehicle to maintain safe and adequate spacing between vehicles. The use of vehicle-to-infrastructure (V2I) communication systems allows for the CAVs to drive at a recommended speed determined by the speed recommendation strategies for upstream traffic based on current downstream traffic conditions (Iteris, 2015).

The objectives of this research effort are:

- 1. To demonstrate the feasibility of speed harmonization implementation in a real-world environment;
- 2. To investigate the effectiveness of speed harmonization in reducing traffic oscillations;

3. To study the potential of speed harmonization in improving traffic performance.

Dailey et al. (2015) previously documented analysis results from a single run of the field experiment. The field experiments conducted for the research presented in this thesis built upon this analysis to develop an improved vehicle control platform, and a total of 12 runs were conducted to develop more insight into the effectiveness of speed harmonization using various performance measures.

The remainder of this thesis is organized as follows. Chapter 2 includes a literature review that covers connected and automated vehicles, previously conducted speed harmonization simulation and field tests, and power spectral density analysis used to analyze the traffic oscillations. Chapter 3 describes the system architecture and the methodology of the field experiment. Chapter 4 presents the data analysis using performance measures to analyze the field results. Chapter 5 follows with the conclusions of the results and the objectives achieved in this study. Finally, chapter 6 discusses future research on improvements of this work.

CHAPTER 2. LITERATURE REVIEW

2.1 Connected and Automated Vehicles

Connected vehicles are relatively new intelligent transportation system (ITS) technologies that allow vehicles to communicate with other vehicles (vehicle-to-vehicle, V2V) and with the infrastructure (vehicle-to-infrastructure, V2I) (Guériau et al., 2016). These vehicles are equipped with bidirectional communication and sensors that allow them to collect and report data on vehicles' surrounding traffic conditions and environment (Guériau et al., 2016).

V2V communication provides detailed information in regards to the movement of the vehicle in addition to the drivers' operational decisions (e.g., speed, acceleration, and location). V2I communication provides information on traffic breakdown downstream (e.g., due to crashes or lane closures), changes in speed limit, work zone condition, weather condition, roadway condition, geometry, and more (Talebpour & Mahmassani, 2016). These communication capabilities improve drivers' knowledge and perception of their driving environment, thereby enhancing the reliability of driving-related decisions. Thus, such communication and exchange of information are expected to improve drivers' efficiency, response, and comfort while simultaneously enhancing safety and mobility (Talebpour & Mahmassani, 2016). Connected and automated vehicles integrate both CV and AV technologies to go one step further, allowing the "automated" control of a vehicle according to information that is provided in real-time (Ma et al., 2016).

4

Connected and automated vehicles have the potential to effectively achieve speed harmonization, if properly deployed. For example, with individual vehicle trajectory data, traffic oscillations can be identified in a timely and accurate manner and thus can be dampened at an early stage with proper speed harmonization control (Talebpour, Mahmassani, & Hamdar, 2013). Talebpour et al. (2016) conducted simulation tests that confirmed that automation is likely to be more effective than connectivity alone in preventing traffic oscillation formation and propagation. In addition to reducing traffic oscillations, CAVs are intended to enhance mobility, safey, comfort, and fuel consumption, while reducing emissions (Talebpour et al., 2016). The CAVs used in this research are equipped with adaptive cruise control (ACC) systems, which controls the vehicle in the longitudinal direction. Longitudinal control aims to regulate a vehicle's speed to maintain adequate spacing between vehicles (Ma et al., 2016). Research conducted by Bose and Ioannou (2013) showed that in mixed traffic consisting of ACC and regular vehicles, the ACC vehicles could improve the stability of traffic flow, reduce emissions, and improve fuel efficiency.

Early versions of autonomous vehicles, regardles of automation level, relied only on on-board sensors to collect information, which have certain range and accuracy limitations (Talebpour et al., 2016). For example, maximum speed of an autonomous vehicle is limited by its radar range that has a specific detection range. CV technology can overcome the limitations of these sensors to provide smoother, safer, and more reliable driving experiences with autonomous vehicles (Talebpour et al., 2016), as CVs alone still require humans to process the information received via V2V or V2I communication and make decisions accordingly. CAV technology provides a collaborative plaform that can optimally utilize the information received to improve traffic operations.

2.2 Simulation Tests

Due to the limited deployment of CV and AV technologies, most research and modeling platforms that implement these technologies are evaluated using computer simulation. It has been found that many of these studies revealed improvements in traffic performance. The simulation results presented by Lu et al. (2015) showed that even with a 10 percent driver compliance rate, the overall system performance of their algorithm – taking into consideration total travel time, total distance traveled, average speed variation, and flow at bottleneck locations – improved significantly. Similarly, Talebpour et al. (2013) found that their CV-based speed harmonization algorithm can effectively delay or eliminate traffic breakdowns and also improves traffic safety – even at a low penetration rate of 10%. The research conducted by Wang et al. (2015) showed that their CAV-based speed harmonization system improves traffic efficiency and sustainability.

Hale et al. (2016) tested two speed harmonization algorithms: one that was speed-based (tested using Aimsum[®]) and another that was density-based (tested using INTEGRATION[©] and VISSIM[®]). The algorithms had the following objectives:

- Speed-based algorithm: determines advisory speeds for corridors upstream and downstream of the location of a known bottleneck based on measured speeds within the vicinity of the bottleneck area.
- 2. Density-based algorithm: to prevent upstream density from exceeding a critical density.

Thus, this research evaluated the traffic system performance using simulation on a macroscopic level, which did not reveal significant improvements in travel time, average speed, and throughput. However, the potential benefits of speed harmonization on a microscopic level (i.e., smoothing individual vehicle speed trajectories) were not analyzed. Although the simulation experiments produced mixed results, they were positive enough to warrant follow-up field experiments (Hale et al., 2016). The research conducted by Hale et al. (2016) set the foundation for the field test discussed in the following subsection, whereby the effectiveness of speed harmonization on the system performance of traffic was analyzed on a microscopic level.

2.3 Field Test

Because CV and AV based speed harmonization concepts are relatively new, there are no realworld deployments of such systems. Dailey et al. (2015) conducted a field test experiment on a segment of the I-66 freeway inside the I-495 beltway near Washington, D.C. The experiment used three controlled vehicles equipped with I2V technology, a lead probe vehicle that drove approximately 100 meters ahead of the controlled vehicles, and two probe vehicles that followed approximately 50 meters behind the controlled vehicles. The downstream probe vehicles detect congestion level at the bottleneck while the upstream probe vehicles record speed profiles for evaluating the effectiveness of the control algorithm on reducing traffic oscillations (Ma et al., 2016). This experiment used a simple algorithm on a longitudinally automated vehicle to smooth the flow of traffic on a congested roadway (Ma et al., 2016). To determine the effects of the speed harmonization on smoothing the flow of traffic by reducing traffic oscillations, the oscillatory behavior of the speeds of the probe vehicles were analyzed using the power spectral densities of the measurements. This analysis is described in the following subsection. Spectral analyses showed that speed oscillation magnitudes of upstream traffic (i.e., traffic beyond the active control) were significantly reduced by the speed harmonization approach. Further, the field experiments demonstrated that from a mechanical standpoint, the CAVs equipped with the manufacturer-supplied adaptive cruise control (ACC) could successfully implement V2I-based speed harmonization. However, there were constraints to the field experiment from an operational standpoint, due to the limited availability of only three CAVs (Hale et al., 2016). Similar to the results of the speed harmonization simulation tests, the CAVs showed they significantly reduced speed oscillations in their vicinity, but did not have significant impacts on aggregate average speeds or travel times. The research presented in this thesis replicated the field test conducted by Dailey et al. (2015) for the same segment of I-66, however with the addition of two probe vehicles to measure the effects of speed harmonization further upstream of the active control.

2.4 Power Spectral Density Analysis

Power spectral density (PSD) is used to analyze the oscillatory behavior of traffic. PSD describes the amount of signal energy found at each frequency in the signal (Ma et al., 2016). A signal can be converted between the time and frequency domains with a pair of mathematical operators called a transform; an example is the Fourier transform, which decomposes a function into the sum of a number of sine wave frequency components (MathWorks, 2016). The PSD analysis method aims to extract oscillation attributes using frequency-domain signal processing techniques. The process has three steps: (i) detrending the data to remove low frequency demand effects; (ii) identifying stationary time intervals for analysis; and (iii) diagnosing the oscillations of interest in these intervals (Li, Peng, & Ouyang, 2010). The detrended data contain mid-range oscillation and high-frequency noise components that can be easily distinguished in the frequency spectrum (Li, Peng, & Ouyang, 2010). This method of analysis measures trajectory oscillations, where significant oscillatory behavior is indicated by a large PSD.

PSD was estimated using MATLAB[®]'s *periodogram* method. The remainder of this section is excerpted from the documentation of the *periodogram* method as explained by MathWorks (2016). The *periodogram* estimates the PSD by finding the discrete-time Fourier transform of the samples of the process, and appropriately scales the magnitude squared of the result. The periodogram estimate of the PSD of a signal $x_L(n)$ of length *L* is

$$P_{xx}(f) = \frac{1}{LF_s} \left| \sum_{n=0}^{L-1} x_L(n) e^{-j2\pi f n/F_s} \right|^2$$
(1)

Where F_s is the sampling frequency.

MATLAB[®]'s *periodogram* function computes the signal's fast Fourier transform (FFT), which is used to find the frequency components of a signal buried in a noisy time domain signal, and normalizes the output to obtain a PSD – a power spectrum from which power can be measured. The PSD, which has units of watts/Hz, describes how the power of a time signal is distributed with frequency.

2.4.1 Algorithm for Estimating the Power Spectral Density

Periodogram computes and scales the output of the FFT to produce the power vs. frequency plot as follows.

- If the input signal is real-valued, the magnitude of the resulting FFT is symmetric with respect to zero frequency (DC). For an even-length FFT, only the first (1 + nfft/2) points are unique. Determine the number of unique values and keep only those unique points.
- 2. Take the squared magnitudes of the unique FFT values. Scale the squared magnitudes (except for DC) by $2/(F_sN)$ where N is the length of signal prior to any zero padding. Scale the DC value by $1/(F_sN)$.
- Create a frequency vector from the number of unique points, the nfft and the sampling frequency.
- 4. Plot the resulting magnitude squared FFT against the frequency.

Figure 1 presents an example of a plot of a power spectral density estimate using MATLAB[®]'s *periodogram* function.



Figure 1 Plot of the PSD estimate using MATLAB[®]'s *periodogram* function.

CHAPTER 3. METHODOLOGY

Using CAVs, speed harmonization was implemented on a segment of I-66 eastbound, a site of daily recurring traffic congestion, near Washington, D.C. A total of 12 field runs were conducted in September 2015 and October 2015. The following sections describing the methodology of the field experiment was adapted and slightly modified from the technical report written by Hale et al (2016).

3.1 System Architecture

3.1.1 Research Platform

To execute the speed harmonization experiment on a microscopic scale but in active traffic on a congested freeway, the project team built a fleet of three CAVs. These vehicles were modified such that longitudinal control (e.g., vehicle set speed and gap) was accomplished from a central control center using V2I communication over a cellular digital network. Commands were sent over a V2I connection into an onboard computer. Commands generated by the onboard computer were communicated over a controller area network (CAN) bus to the original equipment manufacturer (OEM)-supplied ACC system. This experimental equipment allowed remote control over the longitudinal speed of individual vehicles. The modified vehicles were operated by drivers in a mode where safety was enhanced, as the OEM-supplied ACC maintained a minimum time gap (1.1 seconds). Vehicle operators had complete control over the lateral control (i.e., steering) and could override V2I recommendations using the brake and accelerator pedals.

The intent for the vehicle fleet was that speed recommendations based on measurements from a variety of locations on the roadway could be used with algorithms selected for laboratory testing.

The connected and automated vehicles (CAV) used in the field experiment are part of the Federal Highway Administration's (FHWA) fleet of research vehicles. All vehicles were designed to be a complete research platform. Each research platform is outfitted with the components listed below, as shown in Figure 2(a). Figure 2(b) shows the automated vehicle fleet used in this experiment.

- A proprietary longitudinal controller: a set of custom electronic control units (ECUs) that enable fully automatic control of vehicle acceleration and braking by integrating directly with the existing vehicle adaptive cruise control (ACC) system
- A dSPACE MicroAutoBox II controller (dSPACE, 2016): a specialized real-time computing platform that provides commands to the longitudinal controller, accessed via dSPACE ControlDesk through a MATLAB/Simulink library (MathWorks, 2016)
- An Arada LocoMate On-Board Unit (ARADA Systems, 2016): a dedicated short range communications (DSRC) onboard unit (OBU) that enables the transmission and reception of Basic Safety Messages (BSMs)
- An in-vehicle computer that integrates with MicroAutoBox, gathers vehicle measures, operates the algorithms and communicates with the Human Machine Interface (HMI)
- A tablet computer serving as an HMI used to select vehicle role and display the transmission and receipt of algorithm-specific, DSRC-based messages during this project



(a) Vehicle control systems



(b) Automated vehicle fleet

(c) Trailers with RTMS sensors

Figure 2 Experimental vehicles and devices.

Speed recommendation inputs to the CAVs were based on traffic data collected from connected mobile traffic sensing (CMTS) trailers located along the roadway segment. Three trailers are deployed at upstream and downstream of the experimental segments. Figure 2(c) shows the trailers used in the study. The trailers included remote traffic microwave sensors (RTMS) that measured 15-second averaged speeds, volumes, and occupancies (Dailey et al., 2015). These

data were transmitted to a central computer in the Saxton Transportation Operations Laboratory at the Turner Fairbank Highway Research Center (TFHRC) in McLean, Virginia. These measurements were then used with algorithms by the central computer to calculate speed recommendations to be injected into the CAVs.

3.1.2 Algorithm for Speed Recommendations

A central computer in the laboratory at TFHRC controlled the speeds of the controlled vehicles by injecting speed recommendations into the V2I systems of the vehicles. The vehicles received these speed recommendations every two seconds (Ma et al., 2016), directing the ACC system to adjust the speed of the vehicle accordingly. The algorithm executed by the central computer to generate the speed recommendations, s(x,t), was a linear function of space (x), which was dependent upon temporal (t) speed measurements (Dailey et al., 2015) obtained from the roadside CMTS trailers. The algorithm is as follows.

$$s(x,t) = \left(\frac{s_n(t) - s_m(t)}{\Delta x_{nm}}\right) x + s_n(t)$$
⁽²⁾

Where $s_n(t)$ represents the speed measurement at trailer *n* at time *t*, Δx_{nm} is the distance between trailers *n* and *m* and x_{T_n} is the location of trailer *n*. The lowest possible set speed (Ma et al., 2015) of the ACC system was 25 mi/h (40.2 km/h).

3.2 Experimental Setup

In the field test runs, a fleet of three controlled vehicles (i.e., the CAVs) and five probe vehicles was deployed. The fleet was led by one probe vehicle, P0, followed by the three controlled

vehicles driving nearly parallel to one another. Following behind the controlled vehicles were the remaining four probe vehicles: P1, P2, P3, and P4, as seen in Figure 3. The leading probe vehicle was positioned to be in the center lane (of the three lanes), approximately 100 meters in front of the controlled vehicles, while the other probe vehicles were placed approximately 100 meters behind the controlled vehicles.

The probe vehicles were used to observe the speed trajectories of the traffic stream. The purpose of the leading probe vehicle, P0, was to act as a baseline by representing the surrounding natural flow of traffic downstream of the controlled vehicles. The four other probe vehicles were placed behind the controlled vehicles to observe the impacts of the controlled vehicles in controlling the flow of traffic. While P1 and P2 were used to observe the more immediate impacts of the controlled vehicles, being that they were within a close distance, P3 and P4 were used to observe the impacts further along the traffic stream. The objective of the controlled vehicles was to regulate the speed of following vehicles, thereby creating a steadier flow of traffic. The four probe vehicles trailing behind the controlled vehicles were used to observe the effects of the CAVs on reducing speed oscillation amplitudes and smoothing the flow of traffic.



Figure 3 Diagram of the speed harmonization field test experiment.

Input to the CAVs was based on corridor traffic speed measurement processing, as shown in Figure 4. Three CMTS trailers were deployed along the roadway segment to collect these data. Trailer 4, located at mile marker 67.49, demarcated the start of the control section, and trailer 5, located at mile marker 68.785, demarcated the end of the section, as seen in Figure 5. During the experiment, all drivers were to remain in their respective lanes. Each vehicle was equipped with a Bluetooth Global Positioning System (GPS) receiver that recorded its location and speed. This information was communicated at a 10-Hz frequency in real-time to servers in the laboratory at TFHRC (Dailey et al., 2015). Real-time traffic data recorded and obtained from the RTMSs in the trailers were used to calculate speed recommendations for the controlled vehicles.



©2016 Google®

Figure 4 Field experiment map of system integration.



©2016 Google®



CHAPTER 4. RESULTS

Using CAVs in the experimental design discussed previously, speed harmonization was implemented on the selected segment of I-66 eastbound, a site of daily recurring traffic congestion, near Washington, D.C. A total of 12 field runs were conducted in September and October 2015. In the data analysis, two field runs were removed as these runs were conducted on days with extreme congestion during which traffic was complete stop-and-go. Therefore, ten days of data are used in the analysis.

The results of the analysis are presented in three parts, with each part corresponding to one of three objectives presented in Chapter 1:

- 1. To demonstrate the feasibility of speed harmonization implementation in a real-world environment;
- 2. To investigate the effectiveness of speed harmonization in reducing traffic oscillations;
- 3. To study the potential of speed harmonization in improving traffic performance.

4.1 Feasibility of Speed Harmonization Implementation in a Real-World Environment

The first objective of this research was to demonstrate that speed harmonization can be implemented in a real-world setting using CAVs. In this section, four days of data are presented to visually examine whether the recommended speeds were successfully applied and automatically implemented to the CAVs. Note that the four selected days – September 29, 2015, October 7, 2015, October 14, 2015, and October 20, 2015 – are considered as relatively good

runs (i.e., traffic slows but does not completely stop) with the exception of October 14, 2015, when traffic on the experimental roadway segment was rather congested.

Figure 6 presents examples of the CAV control performance in this experiment. In the figure, CAV trajectory (speed vs. space) is plotted as a solid red line and the speed recommendations are plotted as black dots. In most of the cases, as seen in Figure 6(a), 3(b), and 3(d), the speeds of the CAVs compared favorable to recommended speeds. The significant discrepancy between the CAV speed trajectory and the recommendations in Figure 6(c) is due to the CAVs being blocked by existing traffic, most likely due to traffic congestion. In summary, the field experiment results show that when there are no vehicles blocking the CAVs, the recommended speeds were successfully and automatically applied to the CAVs.



(a) September 29, 2015

(b) October 7, 2015



Figure 6 Recommended and actual speeds of controlled vehicle 1 (C1).

Figure 7 demonstrates the speed profiles of all CAVs and probe vehicles. As seen in the figures, the trajectories of all following probe vehicles are impacted by CAVs. It can therefore be concluded that the speed trajectories of probe vehicles P1 – P4 follow the smoother trends of controlled vehicles C1 and C2, indicating the smoothing effect of the CAVs on the "following" probe vehicles' behavior. Theoretically, smoother trajectories usually mean less fuel consumption and improved safety, and even shorter travel times in some cases (Ma et al., 2016). Therefore, it is necessary to calculate different quantitative performance measures (i.e., traffic oscillations, fuel consumption, and travel time) to determine whether these effects exist, which will be discussed in the following sections.



Figure 7 Spatial speed trajectories of all experimental vehicles.

4.2 Effectiveness of Speed Harmonization in Reducing Traffic Oscillations

Next, we examine the effects of speed harmonization in terms of smoothing the flow of traffic and reducing speed oscillations. For this, power spectral density was estimated and is presented in this section. As seen in Figure 7 in the previous section, there are clear trends in the original speeds of the vehicles at different locations of the roadway segment. Thus, to examine the stationary statistics of the trajectories, the overall trend was removed from the original speed trajectories, thus resulting in the detrended trajectories, which is approximately second order stationary (i.e., constant temporal mean and variance for each probe vehicle). The detrended speed trajectories for all probe vehicles (P0-P4) are presented in Figure 8.



Figure 8 Detrended speed trajectories of the probe vehicles.

The detrended speed trajectories are Fourier transformed and the power spectral density (PSD) is estimated to analyze the oscillatory behavior of the probe vehicles (Bendat & Piersol, 1980). Figure 9 shows the PSD plots for the four selected days. As shown, the PSD of P0 is significantly larger than those of the other probe vehicles (P1-P4), with the exception of the field run presented in Figure 9(c), during which severe traffic congestion blocked the CAVs at most parts of the roadway segment. This limited the mobility of the test vehicles; thus, the smoothing effect was not possible.





Figure 9 Power spectral density of detrended speed trajectories of the probe vehicles.

The 85th percentile PSD values of all ten runs are tabulated in Table 1. As can be seen, the PSDs of P1, P2, P3 and P4 were reduced by 29%, 36%, 90%, and 84%, respectively, with the PSD of P0 as the baseline.

Table 1 85th Percentile Power Spectral Density

Date	PO	P1	P2	P3	P4
Average	17.59	12.44	11.20	1.76	2.73
Standard Deviation	17.07	13.71	11.64	0.67	3.53
% Change Compared to P0	-	-29%	-36%	-90%	-84%
25-Sep-15	1.38	1.26	1.75	1.75	1.67
29-Sep-15	8.86	12.22	9.62	2.27	0.47
30-Sep-15	35.47	32.06	9.17	1.55	5.99
1-Oct-15	15.43	7.39	11.74	1.88	11.59
6-Oct-15	16.44	8.64	14.44	0.27	0.77
7-Oct-15	1.08	1.63	0.47	1.06	0.86
14-Oct-15	16.66	18.07	27.52	2.36	0.72
20-Oct-15	7.14	2.04	1.64	2.07	3.05
21-Oct-15	16.23	1.11	1.19	1.97	0.76
22-Oct-15	57.24	39.97	34.43	2.44	1.44

The average 85th percentile PSD values in Table 1 show that the PSD of the leading probe vehicle (P0) is considerably larger than those of all following probe vehicles (P1-P4), indicating the significant smoothing effect of speed harmonization. However, the PSDs of the first following probe vehicle group (P1 and P2) were found to be larger than those of the second following probe group (P3 and P4), which raises interesting questions. Theoretically, the smoothing effect should decrease as the distance from the CAVs increases, thereby having a greater impact on the vehicles closest to the CAVs. The results, however, indicate an increase in the smoothing effects on vehicles further upstream of the active control. One possible reason could be attributed to different driving behavior. Drivers of P1 and P2 were within close proximity of the CAVs, thus they can see clearly that the CAVs were slowing the speeds of following vehicles. As a result, P1 and P2 may follow very closely to the vehicles directly preceding as they anticipate traffic to move faster due to the significant space created in front of the CAVs. The drivers of P3 and P4, on the other hand, are following traffic that is more smoothed, as they are not expecting the vehicles in front of them to accelerate due to the moderate spacing between each vehicle.

To determine whether there was a statistically significant difference between the 85th percentile PSD values of the probe vehicles, a paired t-test was used, where the null hypothesis, H₀, assumes there is no difference in the PSDs of the pre- and post-speed harmonization effects. P0 represents the pre-speed harmonization condition and the following probe vehicles represent the post-speed harmonization effect. The alternative hypothesis, H_A, assumes there is a difference in the PSDs of the pre- and post-speed harmonization effects. Additionally, paired t-tests for P1-P3 and P2-P4 were used to determine whether there was a statistically significant difference

between the immediate following probe group (P1 and P2) and the next following probe group (P3 and P4) which were further upstream from the CAVs. As shown in Table 2, based on the p-values presented, the null hypothesis is rejected for all observations. For pairs P0-P1, P0-P3, P0-P4, P1-P3, and P2-P4, there is evidence to suggest that there are statistically significant differences in the PSDs of the pairs at a 95% confidence level, and a statistically significant difference in the PSDs of P0 and P2 at a 90% confidence level.

Table 2 Paired T-Test Results – 85th Percentile PSD

Pair	Significance (1-tailed)
P0 - P1	0.022*
P0 - P2	0.057**
P0 - P3	0.008*
P0 - P4	0.011*
P1 - P3	0.017*
P2 - P4	0.029*

* 95% confidence level, ** 90% confidence level

The paired t-test was done under the assumption that the data is normally distributed. However, because of the limited number of samples, the assumption of normality may be violated. Therefore, to validate the results of the paired t-test, a second, nonparametric test was done: the Kruskal-Wallis test. The null hypothesis, H₀, states that all populations have the same locations (Washington, Karlaftis, & Mannering, 2011); in other words, that the mean ranks of the sample populations are expected to be the same. The alternative hypothesis, H_A, assumes that the samples come from different populations. Table 3 and Table 4 present the results from the Kruskall-Wallis test from IBM SPSS Statistics, a software package used for statistical analysis.

	Group	N	Mean Rank
PSD	0	10	34.90
	1	10	30.10
	2	10	28.75
	3	10	18.85
	4	10	14.90
	Total	50	

Table 3 Kruskall-Wallis Test – PSD Ranks

Table 4 Kruskall-Wallis Test – PSD Test Statistics

	PSD
Chi- Square	13.020
df	4
Asymp. Sig.	.011

As seen in the tables, $\chi^2(2) = 13.02$, p = 0.011. Therefore, the Kruskall-Wallis test showed that there is a statistically significant difference between the 85th percentile PSD values of the probe vehicles.

4.3 Potential of Speed Harmonization in Improving Traffic Performance

The last objective of this research was to determine whether speed harmonization has the potential to improve traffic performance in regards to mobility and the environment. For this, two measures were selected: travel time for mobility and fuel consumption for the environment.

First, using individual vehicle data collected, travel times were calculated and are presented in Table 5. To determine whether there was a statistically significant difference between the average travel times of the probe vehicles, a paired t-test was used, where the null hypothesis (H₀) assumes there is no difference in the PSDs of the pre- and post-speed harmonization effects, where P0 represents the pre-speed harmonization condition and the following probe vehicles represent the post-speed harmonization effect. The results of the paired t-test are shown in Table 6, which shows that there is no statistically significant difference between any of the probe vehicle comparisons. This is not unexpected, since the algorithm in this research only attempts to smooth the traffic and does not explicitly consider travel time.

Date	PO	P1	P2	P3	P4
Average	141	147	143	148	146
Standard Deviation	58	57	53	56	53
% Change Compared to P0	-	4%	1%	5%	4%
25-Sep-15	76	84	84	85	87
29-Sep-15	172	194	185	193	185
30-Sep-15	157	189	154	191	182
1-Oct-15	251	237	236	230	231
6-Oct-15	205	209	203	208	204
7-Oct-15	88	97	103	100	104
14-Oct-15	132	130	128	135	133
20-Oct-15	94	89	89	89	91
21-Oct-15	156	157	156	162	159
22-Oct-15	82	87	88	86	87

Table 5 Travel Times from T4 to T5 (seconds)

Pair	Significance (2-tailed)
P0 - P1	0.183
P0 - P2	0.664
P0 - P3	0.187
P0 - P4	0.231
P1 - P3	0.604
P2 - P4	0.224

Table 6 Paired T-Test Results – Travel Times

Similar to the analysis of the PSDs in the previous section, the paired t-test for analyzing travel time was done under the assumption that the data is normally distributed. However, because of the limited number of samples, the assumption of normality may be violated. Therefore, the Kruskal-Wallis test was done to validate the results of the paired t-test. The results of the test are shown in Table 7 and Table 8.

Table 7 Kruskall-Wallis Test – Travel Time Ranks

	Group	N	Mean Rank
Travel Time	0	10	23.55
	1	10	26.40
	2	10	24.50
	3	10	26.60
	4	10	26.45
	Total	50	

Table 8 Kruskall-Wallis Test – Travel Time Test Statistics

	Travel Time
Chi- Square	.364
df	4
Asymp. Sig.	.985

As presented in the tables, $\chi^2(2) = 0.364$, p = 0.985. Therefore, the Kruskall-Wallis test showed that there is no statistically significant difference between the travel times of the probe vehicles, as the paired t-test showed previously.

To measure vehicle fuel consumption, the VT-Micro model was used, incorporating the speed profiles of the probe vehicles, as shown in Equation 3, where coefficient $K_{ij}(\ddot{p}_n(t))$ depends on the sign of $\ddot{p}_n(t)$, the type of vehicle, and the measure-of-effectiveness (MOE) (e.g., fuel consumption) (Ma et al., 2016).

$$e(\dot{p}_{n}(t), \ddot{p}_{n}(t)) = \exp\left\{\sum_{i=0}^{3}\sum_{j=0}^{3}K_{ij}(\ddot{p}_{n}(t))\left(\lfloor\dot{p}_{n}(t)\rfloor_{0}^{120} \text{ km/h}\right)^{i}\left(\lfloor\ddot{p}_{n}(t)\rfloor_{-5}^{13} \text{ km/h/sec}\right)^{j}\right\}$$
(3)

The results obtained from 10 field test runs are presented in Table 9. In terms of fuel consumption, it was found that P0 generally consumes less than the other probe vehicles. Similar to travel time, this reiterates that simply achieving the smoothing effect of traffic does not guarantee reduced fuel consumption. Still, it was found that P3 and P4 perform better than P1

and P2, as indicated by the less average fuel consumption by P3 and P4 (1.53L and 1.49L,

respectively) compared to that of P1 and P2 (2.06L and 1.74L, respectively).

Date	PO	P1	P2	P3	P4
Average	1.37	2.06	1.74	1.53	1.49
Standard Deviation	0.13	0.50	0.33	0.22	0.25
% Change Compared to P0	-	50.29%	27.41%	11.48%	8.99%
25-Sep-15	1.22	2.05	2.24	1.55	1.56
29-Sep-15	1.49	2.57	1.40	1.14	1.40
30-Sep-15	1.52	2.11	1.95	1.59	1.44
1-Oct-15	1.35	3.07	1.99	1.36	1.56
6-Oct-15	1.55	2.04	1.65	1.82	1.91
7-Oct-15	1.49	1.51	1.88	1.61	1.38
14-Oct-15	1.27	2.17	1.52	1.69	1.87
20-Oct-15	1.25	1.32	1.49	1.65	1.44
21-Oct-15	1.34	1.77	2.09	1.62	1.18
22-Oct-15	1.20	1.95	1.22	1.22	1.17

Table 9 Fuel Consumption (L)

To determine whether there was a statistically significant difference between the average fuel consumption of the probe vehicles, a paired t-test was used. The paired t-test results presented in Table 10 show that there was a statistically significant difference in each observed pair with the exception of P0-P4, where there was no significant difference in fuel consumption between P0 and P4. For all other pairs, however, there was a statistically significant difference in fuel consumption at the 90% or 95% confidence level.

Pair	Significance (2-tailed)
P0 - P1	0.002*
P0 - P2	0.008*
P0 - P3	0.063**
P0 - P4	0.162
P1 - P3	0.027*
P2 - P4	0.087**

Table 10 Paired T-Test Results – Fuel Consumption

* 95% confidence level, ** 90% confidence level

The results of the Kruskal-Wallis test, which was done to validate the results of the paired t-test,

are shown in Table 11 and Table 12.

Table 11 Kruskall-Wallis Test – Fuel Consumption Ranks

	Group	Ν	Mean Rank
Fuel Consumption	0	10	13.90
	1	10	38.65
	2	10	31.10
	3	10	23.80
	4	10	20.05
	Total	50	

Table 12 Kruskall-Wallis Test – Fuel Consumption Test Statistics

	Fuel Consumption
Chi- Square	17.492
df	4
Asymp. Sig.	.002

As presented in the tables, $\chi^2(2) = 17.492$, p = 0.002. Therefore, the Kruskall-Wallis test showed that there is a statistically significant difference between the fuel consumption of the probe vehicles, as the paired t-test showed previously. The results show that the fuel consumption of the following probe vehicles increased in comparison to that of the leading probe vehicle. As mentioned in the analysis of the PSD results in Chapter 4.2, in theory, as the smoothing effect should decrease as the distance from the CAVs increases, so should fuel consumption. One possible reason could be attributed to different driving behavior, as discussed previously.

In summary, based on the data collected from 10 field runs, it was found out that speed harmonization has positive benefits in reducing traffic oscillations. However, the effects of speed harmonization on travel time and fuel consumption showed that in some cases, there was a decline in the performance measure. Several possible causes include:

- The timing of the speed harmonization implementation might have been off. If the leading probe (P0) or the controlled vehicle (C1 or C2) does not hit the exact timing of a bottleneck formation, speed harmonization may not be able to "improve" traffic operations.
- The results were drawn from only a limited number of test runs.
- This may be also related to the algorithm itself. It is highly likely that the simple, heuristic algorithm only smoothed the traffic, but not in a way to optimize travel time and fuel consumption performance.

CHAPTER 5. CONCLUSIONS

Traffic oscillations, known more commonly as "stop-and-go" driving conditions in heavily congested traffic, result in numerous negative impacts: they reduce effective capacity and increase safety risks, travel delay, and fuel consumption and emissions. Researchers have hypothesized that strategies intended to harmonize the speed of vehicles will reduce traffic oscillations and reduce the adverse impacts. While there have been speed harmonization simulation studies, there have been no large-scale field evaluations of speed harmonization systems. This thesis presents the results of the continuation of the first large-scale field test of a speed harmonization system using connected and automated vehicles (CAV).

Connected and automated vehicles are equipped with communication technologies that allow them to exchange information (e.g., changes in speed limit, weather condition, etc.) with other vehicles (vehicle-to-vehicle, V2V) and infrastructure (vehicle-to-infrastructure, V2I), and also allow the "automated" control of a vehicle on the basis of this information provided in real-time (Ma et al., 2016). The performance and effects of CAVs in reducing traffic oscillations was observed in a speed harmonization project.

Speed harmonization was implemented using three CAVs on a segment of the I-66 freeway near Washington, D.C., where the speeds of the CAVs were dynamically and automatically adjusted based on current traffic conditions. The objective of the CAVs was to regulate traffic upstream of the bottleneck so that vehicles move with uniform speed, thereby creating a steadier flow of traffic. The results of the field experiment demonstrate the potential CAVs have on improving the flow of traffic on a microscopic level (i.e., traffic oscillations). Based on the PSD analysis of the speed trajectories of the probe vehicles, the PSDs of P1, P2, P3, and P4 were reduced by 29%, 36%, 90%, and 84%, respectively. Therefore, it can be concluded that there is potential for the CAVs to reduce the oscillatory behavior of traffic. While no significant difference in travel time between the probe vehicles was observed, the experiment showed that there was a statistically significant difference in fuel consumption between the leading probe vehicle and the following probe vehicles.

Although this difference was not necessarily an improvement, as the results showed that the fuel consumption of the following probe vehicles was greater than that of the leading probe vehicle, it is important to note the parallels observed between fuel consumption and traffic oscillations of the following probe vehicles. In other words, the results of the PSD analysis showed that P1 and P2, the two following probe vehicles closest to the active control, had greater values (12.44 and 11.20, respectively) than the PSDs of the last two probe vehicles, P3 and P4 (1.76 and 2.73, respectively). Similarly, the fuel consumption values of P1 and P2 (2.06 and 1.74 liters, respectively), were greater than those of P3 and P4 (1.53 and 1.49 liters, respectively). This shows that despite the interesting observation that the smoothing effect of speed harmonization decreased as distance from the active control increased, so did the fuel consumption of the following vehicles. Thus, it can be concluded that there is a relationship between the two performance measures, whereas traffic oscillations decrease, so does fuel consumption.

In summary, this research has demonstrated that speed harmonization 1) is implementable in real-world situations, 2) can stabilize traffic streams by reducing traffic oscillations, and 3) has

the potential to improve traffic performance. The promising results of this speed harmonization experiment call for future research to validate the potential benefits as well as make advancements in the integration of CAVs in future speed harmonization efforts to improve traffic operations.

CHAPTER 6. FUTURE WORK

A list of recommendations for future research is summarized below.

- In the analysis, the impacts of the controlled vehicles in regulating the speed of upstream traffic were assessed by comparing the performance measures of the following probe vehicles with those of the leading probe vehicle. This was executed under the assumption that the leading probe vehicle would represent the characteristics of the "general" traffic stream, acting as a surrogate for traffic downstream (Dailey et al., 2015) of the controlled vehicles, or the active control. Ideally, there would be many more field runs, where one scenario would include the controlled vehicles and one scenario would not. However, due to the limited number of field runs, only the scenario with the controlled vehicles was performed. Thus, the experiment was set up as such to accommodate for the limitations.
- More test runs needed: the results were drawn from only a limited number (10) of test runs. For example, in the field test conducted on October 14, 2015, the CAVs were blocked, most likely due to traffic congestion. This affects how the rest of the field test is executed and may ultimately have an effect on the overall analysis. More field tests would allow us to draw better and more significant conclusions.
- Additionally, future field experiments should include a larger number of CAVs.
 More sophisticated methodology for field test run is needed: the timing of the speed harmonization implementation might have not been executed at the exact anticipated time.
 For example, if the leading probe (P0) or the controlled vehicle (C1 or C2) does not hit the exact timing of a bottleneck formation, speed harmonization may be limited in its smoothing effects and not be able to "improve" traffic operations.

• The speed harmonization algorithm needs to be enhanced to accommodate multiple objectives and performance measures (mobility, environment, etc.) in addition to smoothing traffic. The simple, heuristic algorithm implemented in this thesis did not explicitly consider or aim to optimize travel time and fuel consumption performance.

REFERENCES

ARADA Systems. LocoMate classic on board unit OBU-200, 2016.

- Bendat, J.S., and A.G. Piersol. Engineering applications of correlation and spectral analysis. New York: Wiley, 1980.
- Bose, A., Ioannou, P.A., 2003. Analysis of traffic flow with mixed manual and semiautomated vehicles. IEEE Trans. Intell. Transport. Syst. 4 (4), 173–188
- Dailey, D. J., Jagannathan, R., Jawad, I.S., Ma, J., Phillips, T.H., and Bared J.G. A freeway speed harmonization experiment using I2V communication with connected automated vehicles. IEEE International Intelligent Vehicle Symposium, 2015.
- dSPACE. MicroAutoBox II, compact and robust protoyping system for in-vehicle applications. dSPACE GmbH, 2016.
- Federal Highway Administration. Active traffic management: The next step in congestion management. Office of International Programs. U.S. DOT, 2014.
- Guériau, M., Billot, R., El Faouzi, N., Monteil, J., Armetta, F., and S. Hassas. How to assess the benefits of connected vehicles? A simulation framework for the design of cooperative traffic management strategies. Transportation Research Part C, 2016.
- Hale, D., Phillips, T.H., Raboy, K., Ma, J., Su, P., Lu, X.Y., Rakha, H., and D.J. Dailey.
 Introduction of cooperative vehicle-to-infrastructure systems to improve speed harmonization. Federal Highway Administration, 2016.

- Iteris. Speed harmonization. Connected Vehicle Reference Implementation Architecture, 2015.
- L. X.-Y., S. S. E., I. Jawad, R. Jagannathan and P. T. H., "A Novel Speed-Measurement Based Variable Speed Limit / Advisory Algorithm for a Freeway Corridor with Multiple Bottlenecks," *94th Transportation Research Board Annual Conference*, 2015.
- Li, X., Peng, F., and Y. Ouyang. Measurement and estimation of traffic oscillation properties. Transportation Research Part B, 2010.
- Ma, J., Li, X., Shladover, S., Rakha, H., Xiao-Yun, L. U., Jagannathan, R., & Dailey, D. (2016).Freeway Speed Harmonization. IEEE Transactions on Intelligent Vehicles, 1(1): 1 13.
- Ma, J., Li, X., Zhou, F., Hu, J., and B. Park. Parsimonious shooting heuristic for trajectory design of connected autonomous traffic part II: Computational issues and optimization,

Transportation Research Part B, 2016.

MathWorks. Simulation and model-based design. The MathWorks, Inc., 2016.

- Talebpour, A., and H. Mahmassani. Influence of connected and autonomous vehicles on traffic flow stability and throughput. Transportation Research Part C, 2016.
- Talebpour A., Mahmassani H. S. and Hamdar S. H., "Speed Harmonization: Evaluation of Effectiveness Under Congested Conditions[J].," *Transportation Research Record Journal* of the Transportation Research Board, vol. 65(2391), pp. 69-79, 2013.

Tian, J., Jiang, R., Jia, B., Gao, Z., & S. Ma. Empirical analysis and simulation of the concave

growth pattern of traffic oscillations. Transportation Research Part B, 2016.

- Washington, S., Karlaftis, M., & F. Mannering. Statistical and econometric methods for transportation data analysis. CRC Press, 2011.
- Zheng, Z.,Ahn, S., Chen, D., and J. Laval. Freeway traffic oscillations: Microscopic analysis of formations and propagations using wavelet transform. Procedia – Social and Behavioral Sciences, 2011.