MONETARY AND FISCAL INTERACTION UNDER DEBT COLLATERALIZATION

Yong Hoon Cho

Seoul, Republic of Korea

B.A. Economics, Indiana University, 2017

B.A. Mathematics, Indiana University, 2017

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University of Virginia

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Professor Eric Leeper, Chair

Professor Eric Young, Chair

Professor Zach Bethune

Professor Michael Gallmeyer

ABSTRACT

This paper studies optimal monetary and fiscal policy rules when government debt and capital serve as collateral in financial markets. When firms face tighter borrowing constraints, bonds and capital carry high premiums leading to distorted investment decisions and lower production. In a New Keynesian model with this financial friction, I examine whether and how monetary policy should respond to collateral premiums. I find that optimal monetary policy should lower interest rates when collateral premiums increase. I also find that fiscal policy complements this by lowering tax rates as debt increases, boosting firms' productivity and labor demand, driving up collateral needs and premiums—and further raising bond prices to finance expanded borrowing. I find that this coordination achieves better welfare outcomes with less aggressive policy interventions than traditional frameworks, but at the cost of higher consumption volatility. I show how financial frictions fundamentally alter optimal policy design, particularly through the interaction between collateral values, inflation, and real economic activity. Moreover, the optimal long-run tax structure features a positive tax on capital and a negative tax (i.e., a subsidy) on labor. These corrective taxes curb firms' demand for capital as collateral and reduce their reliance on collateralized borrowing through a labor subsidy. While this departs from the zero capital tax prescription of standard models, it aligns with findings from frameworks with imperfect financial markets.

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Chapter 1

Introduction

The role of government debt has evolved significantly throughout United States history. Its fundamental roles are to enable the government to finance expenditures by postponing tax collection and to provide a saving device to buyers. Due to their low risk, government bonds are considered safe assets in the economy, commanding a premium in their price. The safeness of government debt stems from what backs their payoff: future primary surpluses of the government. This perception of safety not only awards government debt a premium for reliability but also establishes its function as collateral in financial markets today.

This dual role of government debt generates noteworthy economic effects: private assets—while generally considered less safe than government-issued debt—also serve as collateral and, despite their different risk profiles, substitute for government bonds in financial markets. When government debt can stand in for private assets as collateral, the government can influence financial market conditions and private investment decisions simply by adjusting its debt supply. This dissertation focuses on examining that role and analyzing how shifts in government debt supply affect private investment decisions when these assets are interchangeable as collateral.

In Chapter 2, *Optimal Monetary and Fiscal Policy Rules under Debt Collateralization*, I introduce a model framework where both government debt and private assets, modeled as capital, serve as collateral. This model environment allows agents to endogenously price the collateral value of government debt and private assets. I propose a new type of monetary policy rule where the interest rate responds to the collateral value of government debt in addition to inflation and output gap, and identify the optimal combination of monetary and fiscal policy rules.

In Chapter 3, *Long-run Optimal Capital and Labor Tax Rates under Debt Collateralization*, I employ the same modeling framework to determine the long-run optimal tax rates on capital returns and labor income by solving the Ramsey problem. To facilitate the analysis of capital return taxation, the model is modified to suit this purpose while preserving its fundamental mechanisms.

This work adds to our understanding of how financial frictions transform optimal policy design when assets serve dual roles as investments and collateral. The research reveals the dynamics between monetary and fiscal coordination, highlighting how debt management strategies can enhance welfare by acknowledging collateral premiums. These findings offer new perspectives on how government debt influences macroeconomic outcomes beyond traditional channels, with significant implications for policy frameworks in financially constrained environments.

Chapter 2

Optimal Monetary and Fiscal Policy Rules under Debt Collateralization

2.1 Introduction

Government debt, particularly U.S. Treasury securities, serves as primary collateral in financial markets. Treasury securities back approximately $4 trillion in daily repo transactions, comprising over 60% of all repo collateral (Kolchin, Podziemska, and Mostafa 2021). Moreover, Basel III regulations designate Treasury securities as preferred High-Quality Liquid Assets, requiring financial intermediaries to hold them for liquidity requirements. Private assets, including corporate bonds and equities, also serve as collateral and regulatory assets, though with lower liquidity value. In repo markets, private securities represent approximately 6% of collateral. This coexistence of public and private collateral suggests their substitutability in their roles in the financial market. The substitutability between government debt and private assets suggests that changes in government debt's market value can affect firms' investment decisions. A change in government debt's value changes the effective supply of collateral, which can push firms to alter their investment decision to meet the collateral requirements. This mechanism can create inefficiencies in investment allocation, establishing a link between government debt markets and real economic activity.

This paper's objective is to analyze the inefficiency that government debt collateralization can bring, and to find what policies can do about it. Specifically, the paper answers to the question - should the monetary policy respond to the financial frictions? The model introduced in the paper is specifically designed to answer the question, featuring price rigidity and financial friction through which government debt and capital are used as collateral and carry premiums. When government debt serves as collateral, its market value affects firms' investment decisions. This creates a new transmission channel for both monetary and fiscal policy. Monetary policy affects collateral values through interest rates and inflation, while fiscal policy influences the total supply of collateral through debt management. The interaction between these policies becomes particularly important because government debt and private capital serve as substitute collateral assets. Changes in the value or supply of government debt can force firms to adjust their capital holdings, potentially leading to inefficient investment decisions. Understanding these mechanisms is crucial for designing optimal policy responses to financial market conditions.

This question of how monetary policy should respond to collateral market conditions is particularly relevant today. The size of the U.S. Treasury market has grown significantly, with outstanding debt exceeding $25 trillion, and financial institutions increasingly rely on Treasury securities as collateral. When these securities' market value fluctuates, it affects the financial system's ability to extend credit. The 2008 financial crisis and the March 2020 Treasury market disruption demonstrated how problems in collateral markets can quickly spread to the real economy. Central banks in advanced economies have responded by implementing various tools to address financial market disruptions, from asset purchase programs to emergency lending facilities. Reflecting this policy shift, a growing body of research examines how monetary policy can contribute to financial stability. In this paper, I focus on what conventional monetary policy—the interest rate tool—can achieve in managing financial stability concerns. If collateral values significantly affect investment and production, monetary policy needs to directly consider these financial conditions. This is particularly important because collateral market stress often coincides with economic downturns, potentially amplifying recessions through reduced credit availability. Understanding the optimal policy response to collateral market conditions could help central banks better manage these dynamics and prevent financial market disruptions from severely impacting the real economy.

This paper examines the fundamental trade-offs that monetary policy faces when responding to financial frictions arising from debt and capital collateralization. I analyze how optimal monetary policy should balance traditional price stability objectives against financial stability concerns when both government debt and capital serve as collateral. Using a New Keynesian model where government debt and capital are substitutable collateral assets, I characterize optimal monetary and fiscal policy rules through second-order approximation around the welfare-maximizing steady state.

The analysis yields several key findings. First, a monetary policy rule that responds to collateral premiums, alongside inflation and the output gap, achieves higher welfare than a standard Taylor rule. This premium-sensitive rule works alongside fiscal policy that lowers tax rates as real outstanding debt rises. At the same time, monetary policy adopts a negative response to the collateral premium. This coordination allows the government to finance debt with the premium — by lowering the tax rate when outstanding debt is higher, firms' productivity increases, boosting labor demand. That increase in labor demand raises collateral needs, driving up the collateral premium—and resulting in a higher bond price. Because monetary policy cuts the interest rate when the premium increases, the price of government debt rises further, allowing the debt to finance itself and sustain productivity gains.

Second, the premium-responding monetary rule achieves higher average consumption levels while accepting greater consumption and price volatility compared to the standard Taylor rule, revealing a fundamental trade-off between consumption levels and stability. Furthermore, I demonstrate that divine coincidence fails in this environment not only due to cost-push shocks but also through financial frictions: while inflation creates price distortions, some price flexibility helps adjust the real value of nominal government debt relative to capital used as collateral, thereby affecting financial conditions and real activity.

This work builds on and extends several important contributions in the literature. Like Leeper and Zhou (2021), who show optimal trade-offs between inflation and output stabilization with long-term government debt, I incorporate debt maturity structure but add financial frictions to examine how collateral constraints affect policy interactions. While Curdia and Woodford (2015) similarly find that monetary policy should respond to financial conditions with moderate inflation responses, their results stem from heterogeneous household preferences and consumption patterns. In contrast, I derive similar policy implications through the collateral role of government debt in a homogeneous agent framework. The finding that premium-sensitive monetary policy permits less aggressive fiscal responses represents an important contribution to our understanding of monetary–fiscal coordination under financial frictions.

The model is built on a New Keynesian framework with price adjustment costs. The economy features firms that face price adjustment costs and idiosyncratic productivity shocks, making them susceptible to default. Banks lend to these firms, accepting both government debt and capital as collateral, and seize these assets in the event of default. This creates an explicit channel through which collateral values affect firms' borrowing capacity and their production decisions. The government issues debt that serves as collateral and implements both monetary and fiscal policies. I consider two monetary policy frameworks: a standard Taylor rule (STR) that responds only to inflation and output gap, and a premium-sensitive monetary rule (PSMR) that additionally reacts to collateral premiums. Fiscal policy adjusts tax rates on output to maintain debt sustainability, while providing transfers to households. The economy is subject to aggregate productivity shocks that affect overall production efficiency, government transfer shocks that influence debt dynamics, and a cost-push shock that alters production firms' degree of markup. This environment allows one to analyze how monetary policy should optimally respond to collateral premiums while accounting for price stability concerns and fiscal policy interactions.

The key mechanism in the model operates through firms' needs for government bonds and working capital. Firms must borrow from banks to pay workers' wages in advance of production. These loans require collateral, either in the form of government debt or capital. When collateral requirements increase or collateral values decline, firms face tighter borrowing constraints. These tighter constraints force firms to reduce their labor demand, pushing output below its potential level. The premium-sensitive monetary rule (PSMR) addresses this inefficiency directly. When collateral premiums rise, indicating tighter financial conditions, PSMR responds by lowering interest rates. Lower interest rates increase government bond prices, effectively expanding the collateral value of existing government debt. This expansion in effective collateral relaxes firms' borrowing constraints, allowing them to hire more workers and increase production. Through this channel, monetary policy can help maintain output closer to its potential level by actively managing collateral values.

The implications of collateralizable government bonds or liquid government debt have been discussed in the literature for a long time. Chari and Kehoe (1999) argued that the optimal policy eliminates the liquidity premium on government debt, much like the Friedman rule. However, studies such as Calvo (1978), Woodford (1990), and Sims (2022) demonstrated that when taxes are distortionary, eliminating the liquidity premium is suboptimal, as government debt provision involves a trade-off due to increased debt servicing costs.

The substitution of private assets for government debt has been well explored. Empirical studies, including Gorton, Lewellen, and Metrick (2012) and Krishnamurthy and Vissing-Jorgensen (2012), document private asset substitution for Treasury securities. Gorton and Ordoñez (2014, 2020) provide theoretical frameworks showing how government bonds and private assets function as substitutes in the collateral market. The interaction of fiscal and monetary policy in determining the price level is firmly established in the literature. Foundational contributions from Leeper (1991), Sims (1994), Woodford (1995), and Cochrane (1998) highlight how fiscal and monetary authorities jointly influence inflation dynamics through their policy stances.

The literature on monetary policy's role in financial stability has expanded significantly since the 2008 financial crisis. Brunnermeier and Sannikov (2014) present a framework where financial frictions amplify shocks, generating volatility. Cúrdia and Woodford (2015) show that optimal monetary policy should address credit spreads to improve welfare, while Gertler and Karadi (2011) analyze unconventional monetary interventions, such as central bank credit market operations, to stabilize financial systems during crises.

This paper contributes to this literature by introducing a New Keynesian framework where both government debt and capital serve as substitutable collateral assets with premiums, creating a link between collateral values and firms' investment decisions. By incorporating price rigidity and financial friction, the model uncovers a transmission channel through which monetary and fiscal policies interact through the collateral constraint. The analysis demonstrates that monetary policy rules responding to collateral premiums, in addition to inflation, achieve higher welfare by managing financial conditions more effectively. Moreover, the paper identifies an optimal trade-off between price stability and financial stability, showing that premium-sensitive monetary policy reduces the need for aggressive fiscal adjustments. These findings provide new insights into the design of coordinated monetary and fiscal policies under financial frictions.

2.2 The Model

The model features four types of agents and a government. The representative household consumes goods, supplies labor, and owns both production and banking firms. The household's elasticity of substitution between different varieties of goods varies over time, following an AR(1) process. This variation in elasticity effectively acts as a cost-push shock to the economy, as it directly influences the markup that monopolistically competitive firms can charge. Production firms operate in monopolistic competition with a continuum of firms, each facing Rotemberg-style price adjustment costs. They produce final goods using capital and labor, hold capital and government bonds, and are subject to two types of shocks: macroeconomic and idiosyncratic. The macroeconomic shock, revealed at the start of each period, affects productivity, while the idiosyncratic shock, revealed mid-period, impacts the quality of capital, effectively reducing its usable volume.

Because workers anticipate the idiosyncratic shocks that may affect firms' ability to pay wages, they require wages to be paid upfront. To meet these wage demands, firms borrow from banks, using their capital and government bonds from the previous period as collateral. Banks, modeled as monopolistically competitive entities, provide loans to production firms in exchange for collateral. If a firm experiences a capital quality shock and defaults because the collateral value falls below the loan amount, the bank seizes the collateral. When this happens, a fraction of the collateralized capital is destroyed during this process, adding inefficiency to the system.

The capital-producing firm purchases retiring capital and produces new capital to replace it. This process introduces a friction that establishes a market value for capital, which is essential for determining its worth as collateral.

2.2.1 The Household

The representative household maximizes expected lifetime utility by choosing consumption, labor supply, and bond holdings. Their optimization problem can be expressed as:

$$E_t \left[\sum_{h=0}^{\infty} \beta^h \left(\ln(C_{t+h}) - (1 + N_{t+h}) \right) \right] \tag{2.1}$$

subject to the budget constraint:

$$C_t + \frac{B_t^s}{P_t} = \frac{W_t N_t}{P_t} + (1 + i_{t-1}) \frac{B_{t-1}^s}{P_t} + \Pi_t^f + \Pi_t^b + Z_t \tag{2.2}$$

where consumption C_t is defined as a CES aggregate over differentiated varieties:

$$C_t = \left(\int_0^1 C_t(i)^{\frac{\theta_t - 1}{\theta_t}} di \right)^{\frac{\theta_t}{\theta_t - 1}} \tag{2.3}$$

where $\theta_t > 1$ represents the time-varying elasticity of substitution between different varieties of goods. This elasticity follows an AR(1) process:

$$\frac{1}{\theta_t - 1} = \frac{1}{\bar{\theta} - 1} + \varepsilon_t^\theta \tag{2.4}$$

with $\varepsilon_t^\theta \sim N(0, \sigma_\theta^2)$. A shock to θ_t functions as a cost-push shock in the economy, as it directly affects the degree of monopolistic competition among production firms.

The corresponding price index is given by:

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta_t} di \right)^{\frac{1}{1-\theta_t}} \tag{2.5}$$

and aggregate profits:

$$\Pi_t^f = \int_0^1 \Pi_t^f(i) di, \quad \Pi_t^b = \int_0^1 \Pi_t^b(i) di. \tag{2.6}$$

The household supplies labor N_t at nominal wage W_t, receives profits Π_t^f and Π_t^b from production firms and banks respectively, and government transfers Z_t. They can purchase one-period government bonds B_t^s, which pay a nominal interest rate i_t in the following period. The household takes prices $P_t(i)$, wages W_t, interest rates i_t, profits Π_t^f, Π_t^b, and transfers Z_t as given when solving their optimization problem. The logarithmic utility in consumption and linear disutility in labor follows Hansen (1985), who justifies this functional form through a model where labor is indivisible and agents face employment lotteries. This specification simplifies the labor supply decision while maintaining consistency with observed labor market dynamics.

The optimal allocation of consumption across different varieties implies the standard demand function:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta_t} C_t \tag{2.7}$$

This demand function plays a crucial role in firms' price-setting decisions, as it determines the elasticity of demand they face in the goods market. The parameter θ_t thus influences both household consumption allocation and the degree of market power held by firms.

2.2.2 Production Firms

Production firms in this economy operate under monopolistic competition within a unit continuum. Each firm faces a decision environment characterized by both aggregate and idiosyncratic shocks. The production technology employs capital and labor inputs, with firms making their factor input decisions before the realization of idiosyncratic shocks. This timing structure creates a wedge for financial intermediation and collateral constraints.

Two distinct types of shocks affect production firms' operations and decision-making. The first is a common productivity shock that impacts all firms simultaneously at the start of each period, altering their baseline productivity. The second is an idiosyncratic capital quality shock that affects individual firms differently, manifesting after firms have made their initial production and borrowing decisions but before actual production takes place. The idiosyncratic nature of these shocks, combined with the timing of wage payments, creates the need for the collateral loan.

A key friction in the model arises from the requirement that firms must pay wages to workers before production occurs. Workers, anticipating the possibility of firms facing adverse idiosyncratic shocks and paying lower wages than promised, demand upfront wage payments. To meet this requirement, firms must borrow from banks, offering their capital and maturing government bonds as collateral. The value of this collateral determines firms' borrowing capacity and their ability to hire labor and produce output.

When making decisions, firms must consider the possibility that adverse realizations of the idiosyncratic shock might lead to default. If the realized effective value of a firm's collateral falls below its loan obligation, the firm defaults, and the bank seizes the posted collateral. This default mechanism creates an endogenous borrowing constraint and introduces a meaningful role for both monetary and fiscal policy in affecting firms' production decisions through their impact on collateral values.

Shock processes

The macroeconomic productivity shock follows a standard AR(1) process:

$$A_t = \rho_A A_{t-1} + \varepsilon_t \quad \text{for} \quad \ln(\varepsilon_t) \sim N(0, \sigma_A^2) \tag{2.8}$$

This aggregate shock affects all firms' productivity simultaneously, creating a common movement in output across the economy. The parameter ρ_A is the persistence of productivity shocks, and σ_A^2 determines their volatility.

The idiosyncratic shock to capital quality, $\nu_t(i)$, follows an IID distribution process:

$$\nu_t(i) \sim \phi(1, \sigma_\nu^2) \tag{2.9}$$

where $\phi(\cdot)$ is a probability distribution function with mean 1 and standard deviation σ_ν. This distributional assumption ensures that while individual firms face idiosyncratic risk, the aggregate capital stock remains stable in expectation.

The effective capital holding of firm i, denoted as $\tilde{K}_t(i)$, is defined by:

$$\tilde{K}_t(i) \equiv \nu_t(i) K_{t-1}(i) \tag{2.10}$$

The timing of the idiosyncratic shock's realization - after firms have made their labor and borrowing decisions but before production - creates the potential for default and creates the need for the collateral requirement on loans. By the law of large numbers, the aggregate effective capital in each period equals the aggregate level of physical capital:

$$E_t^-(\tilde{K}_t(i)) = \tilde{K}_t = K_{t-1} \tag{2.11}$$

where E_t^- denotes expectations at the beginning of period t, prior to the realization of the idiosyncratic shock.

Expectation Formation and Information Structure

The timing of firms' decisions and their available information at each point requires careful treatment of expectations in this model. The operator E_t^- denotes expectations formed at the beginning of period t, specifically after the realization of the aggregate productivity shock A_t and the cost-push shock θ_t but before the realization of the idiosyncratic capital quality shock $\nu_t(i)$. This timing structure creates a crucial distinction between information available for different decisions. The superscript minus sign in E_t^- indicates that firms form these expectations with partial information about period-t. Specifically, the information set includes:

- All variables and shocks up through period $t-1$
- The current period's aggregate productivity shock A_t
- The current period's cost-push shock θ_t
- The distribution of the idiosyncratic shock $\nu_t(i)$, but not its realization

This structure implies that when firms make their input and investment decisions, they have no knowledge of their realized capital quality.

The distinction between E_t^- and the standard period-t expectation operator E_t becomes important when analyzing firms' default decisions. While production and borrowing decisions are made under E_t^-, default occurs after the realization of $\nu_t(i)$, creating a role for the collateral constraint in firms' ex-ante decision-making.

The firm's problem

Each firm in this monopolistically competitive environment maximizes expected lifetime profits while facing both nominal and financial frictions. The nominal friction comes from price adjustment costs, while the financial friction arises from the need to collateralize loans with capital and government bonds. The firm's optimization problem can be expressed as:

$$E_t^-[\sum_{h=0}^{\infty} \Delta_{t,t+h} \Pi^f_{t+h}(i)] \tag{2.12}$$

where $\Delta_{t,t+h} \equiv \beta \frac{U'(C_{t+1})}{U'(C_t)}$ represents the stochastic discount factor that firms use to value future profits, and E_t^- denotes expectations formed at the beginning of period-t, before the realization of the idiosyncratic capital quality shock. The period profit

$\Pi_t^f(i)$ consists of:

$$\begin{aligned}\Pi_t^f(i) = (1-\tau_t)\frac{P_t(i)Y_t(i)}{P_t} - \frac{W_t(i)N_t(i)}{P_t} + \\ q_t^o(1-\delta)E_t^-(\tilde{K}_t(i)) - q_t^n K_t(i) - \frac{Q_t B_t(i)}{P_t} + \frac{(1+\zeta Q_t)B_{t-1}(i)}{P_t} + \\ L_t(i) - Min\Big\{R_t L_t(i), \gamma q_t^o(1-\delta)\tilde{K}_t(i) + \frac{(1+\zeta Q_t)B_{t-1}(i)}{P_t}\Big\} \\ - \frac{\phi^f}{2}(\frac{P_t(i)-P_{t-1}(i)}{P_{t-1}})^2 Y_t \end{aligned} \quad (2.13)$$

where the production technology is given by

$$Y_t(i) = A_t \tilde{K}_t(i)^\alpha N_t(i)^{1-\alpha}, \quad (2.14)$$

where the demand function for the firms production is given by

$$Y_t(i) = \Big(\frac{P_t(i)}{P_t}\Big)^{-\theta_t} Y_t. \quad (2.15)$$

The firm's demand for the loan is given by solving the cost minimization problem:

$$L_t(i) = \Big[\int_0^1 L_t(i)(j)^{\frac{\psi-1}{\psi}} dj\Big]^{\frac{\psi}{\psi-1}}, \qquad R_t = \Big[\int_0^1 R_t(j)^{1-\psi} dj\Big]^{\frac{1}{1-\psi}}. \quad (2.16)$$

Additionally, the firm faces following two constraints:

$$\frac{W_t(i)N_t(i)}{P_t} \leq L_t \quad (2.17)$$

$$R_t L_t(i) \leq E_t^-\big(\gamma\chi(1-\delta)q_t^o\tilde{K}_t(i)\big) + \frac{(1+\zeta Q_t)B_{t-1}(i)}{P_t}. \quad (2.18)$$

Take the profit function of the firm (2.13) first. The first term represents revenue after taxes, where τ_t is the tax rate and $P_t(i)/P_t$ captures the firm's relative price. The second term is the real wage bill, where $W_t(i)$ is the nominal wage and $N_t(i)$ is labor input. The third and fourth terms relate to capital transactions: $q_t^o(1-\delta)E_t^-(\tilde{K}_t(i))$ is the expected value of depreciated capital, where q_t^o is the price of the retiring capital, and $\tilde{K}_t(i)$ is the effective capital after the quality shock. q_t^n is the price of the newly purchased capital, making $q_t^n K_t(i)$ the cost of new capital investment. The fifth and sixth terms capture bond-related cash flows: $Q_t B_t(i)/P_t$ is the real cost of purchasing new bonds, while $(1+\zeta Q_t)B_{t-1}(i)/P_t$ represents the return on maturing bonds, where ζ is the coupon decay rate. The seventh term, $L_t(i)$, is the amount borrowed from banks. The eighth term represents loan repayment, which is limited by the smaller of the contracted repayment $R_t L_t(i)$ and the value of posted collateral. The final term captures Rotemberg price adjustment costs.

Production function (2.14) shows that the firm produces with Cobb-Douglas production function where the effective capital volume enters the production, indicating that the firm's productivity relies on the realization of the idiosyncratic capital quality shock. Here, A_t is aggregate productivity and θ_t is the period-specific elasticity of substitution between goods. The firm's demand function (2.15) indicates that when the firms makes decision, it chooses the price instead of quantity, where the production quantity is determined by the firm's choice of price and the demand function.

Constraint (2.17) requires firms to pay wages in advance through bank loans, while constraint (2.18) caps the borrowing by the collateral value. The parameter γ denotes the fraction of capital usable as collateral, and χ represents the recovery rate of capital value in default. Both capital and government bonds function as collateral, with the entire stock of maturing government debt eligible for posting, but only a γ portion of capital. Given that only χ can be recovered after default, the effective capital volume available as collateral equals $\gamma\chi$ times the retiring capital value. This distinction captures the different collateral treatment between private assets and government debt.

Finally, equation (2.18) represents the firm's loan demand $L_t(i)$ and its price R_t. The parameter ψ represents the elasticity of substitution between different bank loans, which characterizes the degree of monopolistic competition in the banking sector. This formulation allows the model to capture the market power of banks in the loan market.

Given that $E_t^-(\tilde{K}_t(i)) = K_{t-1}(i)$ and that the idiosyncratic shock $\nu_t(i)$ follows a known IID process, we can rewrite the firm's expected profit function as:

$$
\begin{aligned}
E_t^-(\Pi_t^f(i)) &= (1-\tau_t)(\frac{P_t(i)}{P_t})^{1-\theta_t}E_t^-(Y_t(i)) - \frac{W_t(i)N_t(i)}{P_t} + q_t^o(1-\delta)K_{t-1}(i) - q_t^n K_t(i) \\
&-\Big(\int_{\bar{v}_t(i)}^{\infty} R_t L_t(i)\phi(v_t(i))dv_t(i) + \int_0^{\bar{v}_t(i)}\Big(\gamma q_t^o(1-\delta)v_t(i)K_{t-1}(i) + \frac{(1+\zeta Q_t)B_{t-1}(i)}{P_t}\Big)\phi(v_t(i))dv_t(i)\Big) \\
&- \frac{Q_t B_t(i)}{P_t} + \frac{(1+\zeta Q_t)B_{t-1}(i)}{P_t} + L_t(i) - \frac{\phi^f}{2}(\frac{P_t(i)-P_{t-1}}{P_{t-1}})^2 E_t^-(Y_t(i)) \quad (2.19)
\end{aligned}
$$

where

$$\bar{v}_t(i) = \frac{R_t W_t(i)N_t(i) - (1+\zeta Q_t)B_{t-1}(i)}{\gamma P_t q_t^o(1-\delta)K_{t-1}(i)}, \quad (2.20)$$

$$L_t(i) = \frac{W_t(i)N_t(i)}{P_t} \quad (2.21)$$

and

$$E_t^-(Y_t(i)) = \Omega A_t K_{t-1}(i)^{\alpha} N_t(i)^{1-\alpha} \quad (2.22)$$

subject to

$$R_t L_t(i) \leq \gamma\chi(1-\delta)q_t^o K_{t-1}(i) + \frac{(1+\zeta Q_t)B_{t-1}(i)}{P_t}. \quad (2.23)$$

This reformulation explicitly accounts for default risk through the integrals over the idiosyncratic shock distribution. In (2.19), the first integral represents expected loan repayment when the firm does not default ($\nu_t(i) \geq \bar{\nu}_t(i)$), while the second integral captures the value of collateral seized by banks in default states ($\nu_t(i) < \bar{\nu}_t(i)$). The default threshold $\bar{\nu}_t(i)$ is determined by (2.20), where the threshold equates the loan repayment obligation with the value of collateral, defining the point at which a firm becomes indifferent between repaying and defaulting. The loan amount is pinned down by the binding wage finance constraint (2.21).

Equation (2.22) is the expected production that incorporates the mean effect of the capital quality shock, where $\Omega \equiv \int_0^\infty \nu_t(i)^\alpha \phi(\nu_t(i)) d\nu_t(i)$ captures the expected impact of the idiosyncratic shock on production efficiency.

This reformulated problem shows how financial frictions, through the collateral constraint and default risk, affect firms' production and investment decisions. The presence of government bonds as collateral creates a direct link between monetary policy, which affects bond prices, and firms' borrowing capacity. This mechanism will prove crucial for understanding the optimal policy responses to financial conditions.

2.2.3 The Capital Firm

The economy incorporates capital adjustment costs through capital production, following the framework of Urban Jermann (1998). These adjustment costs, combined with the presence of a capital-producing firm, establish a market mechanism that assigns value to capital stock based on the supply of "old" capital and the demand for "new" capital. This market valuation mechanism is crucial for determining capital's dual role as both a production input and collateral.

Production firms accumulate capital by purchasing it from the capital-producing firm, which produces new capital using old capital and investment as inputs. The new capital is then used by production firms in the following period for final good production and as collateral for loans. The presence of the capital-producing firm creates a market price for retiring capital that interacts with the collateral constraint faced by production firms.

The capital-producing firm's problem is static and focuses on maximizing profits, which determines the market price of retiring capital. The firm's optimization problem is formalized in equation (2.24)

$$MAX_{K_t,\tilde{K}_t,I_t} \quad q_t^n K_t - q_t^o(1-\vartheta_t)\tilde{K}_t - I_t \tag{2.24}$$

subject to the capital accumulation technology shown in equation (2.25)

$$K_t = (1 - \frac{\vartheta_t}{1-\frac{1}{\xi}})\tilde{K}_t + \frac{\vartheta_t^{\frac{1}{\xi}}}{1-\frac{1}{\xi}}\tilde{K}_t^{\frac{1}{\xi}} I_t^{1-\frac{1}{\xi}} \tag{2.25}$$

where

$$\vartheta_t \equiv \delta + (1-\delta)\gamma(1-\chi)\Lambda_t \tag{2.26}$$

and

$$\Lambda_t \equiv \int_0^{\bar{\nu}_t} \nu\phi(\nu)d\nu. \tag{2.27}$$

In the production of new capital, described by equation (2.25), the parameter $\frac{1}{\xi}$ represents the input share of old capital, while $1-\frac{1}{\xi}$ represents the input share of investment. The term ϑ_t captures the total proportion of capital depreciated or destroyed during the period. It consists of two components: δ, which is the standard depreciation rate, and $(1-\delta)\gamma(1-\chi)\Lambda_t$, which represents the fraction of un-depreciated capital destroyed due to defaults. Here, Λ_t is the weighted average of the idiosyncratic shocks ν for defaulting firms, and $\phi(\cdot)$ is the probability density function of the idiosyncratic shock.

The default threshold $\bar{\nu}_t$ determines which firms default: those with $\nu_t(i) < \bar{\nu}_t$ default, while those with $\nu_t(i) \geq \bar{\nu}_t$ do not. As a result, the production of new capital is directly influenced by the default rate, since higher default rates lead to greater destruction of capital, reducing the effective supply of old capital for production.

2.2.4 The Bank

There is a continuum of banks of unit mass, each operating as a monopolistically competitive firm. Banks set the intratemporal interest rate, which effectively serves as the "price" of loans, and this rate determines the volume of loans based on firms' demand. Recognizing that firms rely on loans to pay workers, banks leverage their market power, which is governed by the elasticity of substitution in loan demand. As a result, banks charge an interest rate on loans that reflects their market power and lending risks.

When a firm defaults, the bank seizes the posted collateral, which consists of retiring capital and maturing government debt. However, a fraction of the seized capital is lost during the process, reflecting the inherent costs of handling defaults. This loss demonstrates the inefficiency introduced by defaults, as it reduces the effective value of the collateral available to banks.

A bank maximizes

$$E_t^{-}\big[\sum_{h=0}^{\infty} \Delta_{t,t+h}\Pi^b_{t+h}(j)\big] \tag{2.28}$$

where

$$\Pi_t^b(j) = \int_0^1 Min\Big\{R_t(j)L_t(i)(j), \chi\gamma q_t^o \tilde{K}_t(i) + \frac{(1+\zeta Q_t)B_{t-1}(i)}{P_t}\Big\}di - L_t(j) \quad (2.29)$$

subject to

$$L_t(j) = (\frac{R_t(j)}{R_t})^{-\psi} L_t. \quad (2.30)$$

Bank j provides a loan $L_t(i)(j)$ to firm i, charging an interest rate $R_t(j)$, which is expected to be repaid by the end of the period. If a borrowing firm defaults, the bank seizes the posted collateral, comprising retiring capital and maturing government bonds along with their payout. The parameter χ represents the fraction of surrendered capital that remains undestroyed from default, while $\gamma\tilde{K}_t$ denotes the collateralized capital. Thus, for each defaulting firm i, the bank loses $(1-\chi)\gamma q_t^o \tilde{K}_t(i)$ in capital.

The bank's profit function can be rewritten using the law of large numbers as:

$$\Pi_t^b(j) = \int_0^{\bar{\nu}_t} \chi\gamma q_t^o \nu_t K_{t-1}\phi(\nu_t) + \frac{(1+\zeta Q_t)B_{t-1}}{P_t} d\nu_t + \int_{\bar{\nu}_t}^{\infty} R_t(j)L_t(j)\phi(\nu_t)d\nu_t - L_t(j). \quad (2.31)$$

This form makes it more straightforward to see that the bank's problem is directly linked to the default threshold $\bar{\nu}_t$, which is determined by the firm's problem in equation (2.20). An increase in the default threshold implies a higher default rate, prompting the bank to charge a higher intratemporal interest rate R_t. Since the default threshold depends on the value of the posted collateral, changes in collateral values directly influence the bank's lending behavior and interest rates.

2.2.5 The Government

The government in this economy collects taxes, issues government bonds, pays interest on these bonds, and distributes transfers to households. Its budget constraint is expressed as:

$$\tau_t Y_t + \frac{B_t^s}{P_t} + \frac{Q_t B_t}{P_t} = \frac{(1+\zeta Q_t)B_{t-1}}{P_t} + (1+i_{t-1})\frac{B_{t-1}^s}{P_t} + Z_t \tag{2.32}$$

Here, τ_t represents the proportional tax on output Y_t, and Z_t denotes the government transfers to households. The transfers is a proportion of output given by:

$$Z_t = z_t Y_t \tag{2.33}$$

where z_t is the transfer-to-output ratio that follows an AR(1) process:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \qquad ln(\varepsilon_t^z) \sim N(0, \sigma_z^2). \tag{2.34}$$

This indicates that government transfers are endogenously linked to the level of output with the stochastic process determining z_t. The proportional tax rate τ_t and the interest rate i_t are determined by the monetary and fiscal policies, as described below.

Monetary Policy

This paper considers two distinct monetary policy regimes: the Standard Taylor Rule (STR) and the Premium-Sensitive Monetary Rule (PSMR).

i) Standard Taylor Rule (STR)

Under STR, the government sets the interest rate on short-term bonds as:

$$i_t - \bar{i} = \varphi^{\pi}(\pi_t - \bar{\pi}) + \varphi^{x} x_t \tag{2.35}$$

The short-term nominal interest rate (i_t) is determined as a deviation from its steady-state value ($\bar{i}$) in response to two factors: deviations of inflation (π_t) from its target ($\bar{\pi}$) and the output gap (x_t). The parameter (φ^{π}) is the monetary policy's inflation response coefficient and (φ^{x}) is the monetary response coefficient to output gap fluctuations. This specification follows the traditional approach to monetary policy, focusing on macroeconomic stabilization through price level and output management.

ii) Premium-sensitive Monetary Rule (PSMR)

The Premium-Sensitive Monetary Rule extends the standard framework by incorporating a response to financial conditions. Under PSMR, the interest rate setting follows equation

$$i_t - \bar{i} = \varphi^{\pi}(\pi_t - \bar{\pi}) + \varphi^{x} x_t + \varphi^{\eta}(\eta_t - \bar{\eta}) \tag{2.36}$$

Equation (2.36) augments the standard rule with an additional term that responds to deviations of the collateral premium (η_t) from its steady-state value ($\bar{\eta}$). The parameter (φ^{η}) determines how strongly monetary policy reacts to changes in financial conditions as measured by the collateral premium. This augmented rule enables the monetary authority to address both traditional macroeconomic objectives and financial stability concerns through a single policy instrument - the nominal interest rate. When $\varphi^{\eta} = 0$, the PSMR collapses to the standard Taylor rule given in equation (2.35), making the STR a nested case of this more general framework.

Fiscal Policy

Fiscal policy determines the supply of short-term bonds and the proportional tax rate according to the following rules:

$$B_t^s = 0 \tag{2.37}$$

$$\tau_t y_t = \bar{\tau}\bar{y} + \phi^B\left(\frac{B_{t-1}}{P_{t-1}} - \frac{\bar{B}}{\bar{P}}\right). \tag{2.38}$$

The government sets the supply of short-term bonds (B_t^s) to zero and adjusts the distortionary tax rate (τ_t) in response to deviations in the past supply of long-term bonds. This suggests that fiscal policy is designed to stabilize the real debt level around its steady-state value.

Policy Interactions

Because the distortionary tax rate τ_t is set by the fiscal policy rule and the interest rate i_t is determined by monetary policy, the only variables that can adjust to clear the government's budget constraint are the supply of long-term debt B_t and the price level P_t, in response to shocks entering the economy through the exogenous lump-sum transfers Z_t.

The price of government debt depends on monetary policy and expectations regarding future bond prices. Following an inflation shock, monetary policy adjusts the interest rate on short-term bonds. Since the government sets the supply of short-term bonds to zero, this adjustment impacts the price of long-term bonds. The resulting changes in the government budget constraint can be resolved in two ways. If the price level (P_t) adjusts, both the interest rate and bond prices change together to clear the constraint. Alternatively, if the supply of long-term debt (B_t) adjusts, it induces changes in future tax rates to restore fiscal balance.

With B_t^s=0, the role of monetary policy is to determine the market value of the nominal interest rate. While fiscal policy fixes the supply of short-term bonds at zero, the household still price these bonds, and production firms make investment decisions on their behalf. As a result, nominal interest rates determine the economy's intertemporal rate of substitution and inflation rate.

2.2.6 Market Clearing Conditions

The market clearing conditions for the economy are given by:

$$C_t + I_t + Y_t \int_0^1 \frac{\phi^f}{2}(\frac{P_t(i) - P_{t-1}(i)}{P_{t-1}(i)})^2 di = Y_t + (1 - \vartheta_t)K_{t-1} \tag{2.39}$$

where C_t is consumption, I_t is investment, and the integral term represents the aggregate price adjustment cost incurred by firms. These conditions ensure that total output (Y_t) and depreciated capital are allocated across consumption, investment, and price adjustment costs.

Additionally, labor and capital are aggregated across all firms as:

$$N_t = \int_0^1 N_t(i)di \qquad and \qquad K_{t-1} = \int_0^1 K_{t-1}(i)di \tag{2.40}$$

where N_t is the total labor supply and K_{t-1} is the total capital stock carried over from the previous period.

Furthermore, the inclusion of price adjustment costs (ϕ^f) reflects frictions in nominal price changes. These costs influence firms' pricing decisions and, in turn, the allocation of resources between consumption, investment, and price adjustment. The market clearing condition highlights the dynamic interactions between these components in maintaining equilibrium.

2.3 Equilibrium Conditions

This section characterizes the equilibrium conditions of the economy, derived from the first-order conditions of the optimization problems faced by households, firms, and banks. These conditions describe how agents make decisions regarding consumption, labor supply, investment, borrowing, and lending while ensuring that the economy's resource constraints and market-clearing conditions are satisfied. By integrating these individual optimization behaviors, the equilibrium conditions provide a complete description of the interactions between households, firms, banks, and the government, as well as the role of fiscal and monetary policies in influencing economic outcomes.

2.3.1 Household Conditions

The household equilibrium condition is given by

$$\omega_t = C_t \tag{2.41}$$

$$\frac{1}{1+i_t} = E_t(\frac{\Delta_{t,t+1}}{\pi_{t+1}}) \tag{2.42}$$

where $\Delta_{t,t+1} \equiv \beta \frac{U'(C_{t+1})}{U'(C_t)}$ is the stochastic discount factor, and $\omega_t \equiv \frac{W_t}{P_t}$ is the real wage. Condition (2.41) states that households allocate their real wage income ω_t entirely to consumption (C_t). Condition (2.42) equates the inverse of the gross nominal interest rate on short-term bonds $1+i_t$ to the household's expected stochastic discount factor, adjusted for expected future inflation π_{t+1}. This implies that the household's intertemporal consumption decisions are influenced by both the expected real return on bonds and expected inflation. By providing labor, the household ensures that the marginal utility of consumption is equal to the real wage, aligning labor supply decisions with the trade-off between consumption and leisure. The intertemporal condition highlights how households balance current and future consumption, taking into account the returns from saving and expected changes in prices.

The fiscal policy decision to set the supply of short-term bonds (B_t^s) to zero plays a significant role in these equilibrium conditions. By fixing $B_t^s = 0$, the government eliminates the household's ability to invest in short-term bonds. Instead, the household engages indirectly in financial markets through pricing these bonds, which influences investment decisions made by production firms on their behalf. This policy ensures that the nominal interest rate (i_t) reflects the intertemporal rate of substitution in the economy, with the price level (P_t) and long-term bonds (B_t) absorbing the adjustments needed to clear the government's budget constraint.

2.3.2 Production firms

The equilibrium conditions of a production firm is given by following conditions:

$$(1-\tau_t)(1-\theta_t) - \phi^f(\pi_t - 1)\pi_t + E_t(\Delta_{t,t+1}\phi^f(\pi_{t+1}-1)\frac{Y_{t+1}}{Y_t}\pi_{t+1}) + \mu_t\theta_t = 0, \quad (2.43)$$

$$\omega_t R_t N_t \left(1 - \Xi_t + \eta_t\right) = \mu_t(1-\alpha)Y_t, \quad (2.44)$$

$$q_t^n = E_t\left[\Delta_{t,t+1}\left(q_{t+1}^o(1-\delta)\left(1-\gamma\Lambda_{t+1}+\gamma\eta_{t+1}\right) + A_{t+1}\alpha\mu_{t+1}\Omega K_t^{\alpha-1}N_{t+1}^{1-\alpha}\right)\right], \quad (2.45)$$

$$Q_t = E_t\left[\Delta_{t,t+1}\frac{(1+\zeta Q_{t+1})}{\pi_{t+1}}\left(1-\Xi_{t+1}+\eta_{t+1}\right)\right]. \quad (2.46)$$

Condition (2.43) is the equilibrium condition for price setting by production firms, where μ_t is the Lagrangean multiplier associated with the household's demand constraint (2.15), representing the inverse of the firm's markup under monopolistic competition. Equation (2.43) mirrors the standard New Keynesian Phillips Curve under Rotemberg pricing, relating current inflation, expected future inflation, output, and the markup. It highlights how firms adjust prices by weighing current and anticipated future economic conditions against the costs of changing prices.

The labor demand condition is given by (2.44), where $\Xi_t = \Phi(\bar{\nu}_t)$ is the default rate derived from the cumulative distribution function Φ of the idiosyncratic shock up to the default threshold $\bar{\nu}_t$, and η_t is the Lagrange multiplier associated with the collateral constraint, representing the collateral premium. The term $(1 - \Xi_t + \eta_t)$ adjusts the effective labor cost for the firm, accounting for default risk and the value of collateral. The condition indicates that the firm's labor demand is influenced not only by the marginal product of labor and the real wage but also by financial variables such as default risk and the collateral premium. Specifically, a higher default rate Ξ_t reduces the effective labor cost (since defaulted loans are not repaid), increasing labor demand, while a higher collateral premium η_t raises the effective labor cost, decreasing labor demand.

The investment demand condition is given by (2.45), where $\Lambda_t \equiv \int_0^{\bar{\nu}_t} \nu \hat{\phi}(\nu) d\nu$ represents the expected loss due to defaults, with $\hat{\phi}(\nu)d\nu$ being the density function of $\nu_t(i)$. This equation equates the cost of investing in new capital today to the expected discounted returns from holding that capital. These returns include both the resale value of capital (adjusted for depreciation and default risk) and the future marginal product of capital in production. Financial frictions, through the terms Λ_{t+1} and η_{t+1}, affect the expected returns and thus the firm's investment decisions.

The bond demand condition is given by (2.46)where Q_t is the price of government bonds and ζ is the coupon decay rate. This condition reflects that the current price of bonds equals the expected discounted payoff from holding the bond, adjusted for default risk and the collateral premium. The terms Ξ_{t+1} and η_{t+1} indicate that financial conditions directly influence the valuation of government bonds in firms' portfolios.

By combining (2.43) and (2.44), we derive a modified New Keynesian Phillips Curve that incorporates financial frictions:

$$(1-\tau_t)(1-\theta_t)-\phi^f(\pi_t-1)\pi_t+E_t\left[\Delta_{t,t+1}\phi^f(\pi_{t+1}-1)\frac{Y_{t+1}}{Y_t}\pi_{t+1}\right]+\theta_t\frac{w_t r_t N_t(1-\Xi_t+\eta_t)}{(1-\alpha)Y_t}=0. \tag{2.47}$$

This equation illustrates how financial frictions—specifically, the default rate Ξ_t and the collateral premium η_t —affect inflation dynamics through their impact on firms' marginal costs. The presence of these financial variables in the Phillips Curve suggests that monetary policy should consider financial conditions alongside traditional macroeconomic variables when aiming to stabilize inflation and output.

The default threshold $\bar{\nu}_t$ is determined by the collateral constraint faced by firms. Combining the definition of $\bar{\nu}_t$ with the collateral constraint (equation (2.18) from earlier), we obtain:

$$\bar{\nu}_t \leq \chi, \tag{2.48}$$

where χ represents the fraction of collateral value recoverable by the bank upon default. When the collateral constraint binds ($\eta_t > 0$), equation (2.48) holds with equality, indicating that firms are operating at the maximum allowable default risk. This relationship provides insight into how default probabilities adjust when the collateral constraint is active, affecting firms' financing and production decisions.

2.3.3 Capital Firm

The capital-producing firm's equilibrium conditions determine the prices of new and old capital, which are crucial for firms' investment decisions and the valuation of capital as collateral. The first equilibrium condition is:

$$q_t^n = \left(\frac{I_t}{\vartheta_t K_{t-1}}\right)^{\frac{1}{\xi}}, \tag{2.49}$$

where q_t^n is the price of new capital, I_t is investment, K_{t-1} is the existing capital stock, ϑ_t represents the effective depreciation rate (accounting for both physical depreciation and capital destroyed due to defaults), and ξ is the elasticity parameter in the capital production function. This equation indicates that the price of new capital increases with higher investment demand relative to the depreciated capital stock, reflecting the convex adjustment costs in capital production.

The second equilibrium condition is:

$$q_t^o(1-\vartheta_t) = q_t^n \left(1 - \frac{\xi\vartheta_t}{\xi-1} + \frac{\vartheta_t^{\frac{1}{\xi}}}{\xi-1}\left(\frac{I_t}{K_{t-1}}\right)^{1-\frac{1}{\xi}}\right), \tag{2.50}$$

where q_t^o is the price of old capital. This condition ensures that the market for capital is in equilibrium, balancing the supply of depreciated capital with the demand for new capital investment. It shows how the price of old capital adjusts based on the depreciation rate, the price of new capital, and the relative levels of investment and existing capital.

An increase in investment demand I_t relative to the depreciated capital stock $\vartheta_t K_{t-1}$ leads to a higher price of new capital q_t^n, as producing additional capital becomes more costly due to adjustment costs. Same happens to q_t^o, reflecting the heightened value of existing capital in response to stronger investment demand. This mechanism captures the capital adjustment costs in the economy, where rapid increases in investment are associated with higher costs. This affects firms' investment decisions and the overall dynamics of capital accumulation.

In the steady state, where investment equals the effective depreciation of capital ($I_t = \vartheta K_{t-1}$) the prices of new and old capital equalize ($q_t^n = q_t^o = 1$), simplifying the analysis and providing a benchmark for dynamic deviations. This implies that in the long run, absent shocks, the cost of producing new capital and the value of existing capital stabilize at unity.

2.3.4 The Banks

In this economy, banks operate under monopolistic competition and determine the intratemporal interest rate $r_t(j)$ for loans extended to firms. The banks' equilibrium condition, derived from profit maximization while considering the probability of firm default, is given by:

$$-(\psi-1)(1-\Xi_t)\left(\frac{R_t(j)}{r_t}\right)^{\psi} L_t + \psi\left(\frac{r_t(j)}{R_t}\right)^{-\psi-1}\frac{L_t}{R_t} = 0, \tag{2.51}$$

where $\psi > 1$ is the elasticity of substitution in loan demand, $1-\Xi_t$ is the probability that a firm does not default on its loan, L_t is the total loan volume in the economy, and r_t is the aggregate intratemporal interest rate. Due to the presence of a continuum of identical banks and firms, and by the law of large numbers, we can assume symmetry in equilibrium. This allows us to set $r_t(j) = r_t$ for all banks j. Substituting this into the equilibrium condition simplifies it to:

$$-(\psi-1)(1-\Xi_t)L_t+\psi\frac{L_t}{r_t}=0. \tag{2.52}$$

Dividing both sides by L_t (assuming $L_t \neq 0$) and rearranging terms, we obtain:

$$R_t=\frac{\psi}{(\psi-1)(1-\Xi_t))} \tag{2.53}$$

Equation (2.53) reveals that the intratemporal interest rate R_t is directly influenced by the default rate Ξ_t. As the probability of default Ξ_t increases, the term $1-\Xi_t$ decreases, leading to a higher interest rate R_t. This reflects the banks' need to compensate for increased expected losses due to a higher likelihood of firm defaults.

This relationship illustrates how financial frictions impact borrowing costs in the economy. A higher default rate not only raises the interest rates charged by banks but can also dampen firms' willingness to borrow, potentially reducing investment and production. Moreover, the linkage between default probabilities and interest rates underscores the crucial role of financial stability in the overall economy. Policies aimed at reducing default risk—such as improved regulatory oversight or measures enhancing firms' collateral values—can lower borrowing costs, stimulate investment, and boost economic output. Understanding how banks adjust interest rates in response to default risk is vital for analyzing the transmission mechanisms of monetary policy, especially in models that incorporate financial frictions and price rigidities.

Additionally, the intratemporal interest rate R_t feeds back into firms' cost structures, affecting their labor and production decisions. As R_t increases, the cost of financing wage payments upfront becomes more expensive, potentially leading firms to reduce labor demand, which can have adverse effects on employment and aggregate demand.

This interconnectedness highlights the importance of coordinating monetary and financial stability policies to mitigate the amplification of shocks through the financial system.

2.3.5 Binding Collateral Constraint Case

We focus primarily on the scenario where the collateral constraint is binding for firms. When the collateral constraint binds, the borrowing constraint (equation (2.18)) holds with equality, which simplifies several expressions in the model. By setting the collateral constraint to equality and combining it with the expression for the default threshold (equation (2.20)), we obtain:

$$\bar{\nu}_t = \chi. \tag{2.54}$$

This equality indicates that the default threshold $\bar{\nu}_t$ equals the fraction χ of collateral value that the bank recovers upon default. As a result, variables such as the default rate Ξ_t and the expected loss due to default Λ_t become constants, since they depend solely on χ, which is a fixed parameter. Furthermore, with Λ_t being constant, the endogenous depreciation rate ϑ_t also becomes a constant. Recall that ϑ_t is defined as

$$\vartheta_t = \delta + (1-\delta)\gamma(1-\chi)\Lambda_t. \tag{2.55}$$

Since $\Lambda_t = \int_0^{\chi} v\phi(v)dv$ is constant due to $\bar{\nu}_t = \chi$, the depreciation rate ϑ_t no longer varies over time.

This constancy implies that when the collateral constraint is binding, minuscule changes in variables associated with the default rate Ξ_t do not influence the dynamics of the model. Around the steady state where the collateral constraint binds, both the default rate and the effective depreciation rate can be treated as fixed parameters. This simplification allows us to analyze the effects of monetary and fiscal policies without the added complexity introduced by variations in default rates and depreciation due to fluctuations in the collateral constraint.

By treating Ξ_t and ϑ_t as constants, we can more precisely examine how the binding collateral constraint affects firms' borrowing, investment, and production decisions, as well as the overall macroeconomic equilibrium. This approach facilitates the analysis of policy interventions aimed at addressing financial frictions arising from collateral constraints, enabling a clearer understanding of how monetary policy can respond to financial conditions in the presence of binding collateral requirements.

2.4 Numerical Analysis

2.4.1 Parameter Calibration

In this section, I numerically solve the model in second-order approximation. The choice of parameter values is crucial, as they significantly influence the model's dynamics and steady-state properties. The following parameter values are selected based on conventional usage in the literature:

$$\alpha = 0.33 \qquad \xi = 0.9. \tag{2.56}$$

The fraction of the defaulted capital that the bank recovers, χ, is set to 0.9, corresponding to a haircut of $1 - \chi = 0.1$ or 10%.[1] The idiosyncratic shock to capital quality, $\nu_t(i)$, is assumed to follow an independent and identically distributed (i.i.d.) uniform distribution with mean 1. This choice reflects symmetric uncertainty faced by firms regarding the effectiveness of their capital. The standard deviation of this uniform distribution is denoted by σ_ν.

The parameters β, δ, and σ_ν are calibrated to achieve the desired deterministic steady-state values, specifically $Y = 1$, $K = 12$, $C = 0.8$, and a default rate $\Xi = 0.01$.

Parameter	Value
β	0.9911
γ	0.0100
δ	0.0167
σ_ν	0.0530
ϕ^f	80

Table 2.1: Key parameter calibration

γ, which represents the fraction of capital accepted as collateral by banks, is set to 0.01 to reflect the small portion of private financial assets utilized as collateral in the financial market[2]. The selection of shock sizes is informed by empirical data. The standard deviation of the total factor productivity (TFP) shock is set to $\sigma_A = 0.005$ under an autoregressive process with persistence parameter $\rho_A = 0.95$. The standard deviation of the productivity was calculated to match the output volatility of 1.5% under the benchmark Taylor rule[3]. The size of the government transfer shock is determined by examining the transfers-to-GDP ratio, resulting in a shock size of

[1] Typical haircuts observed in financial markets vary. Under Basel III regulations, haircuts on private assets can range from as low as 20% to as high as 50%, although in many financial transactions, private assets are subjected to smaller haircuts.

[2] The parameter γ is calibrated so that, in the deterministic steady state, the volume of capital serving as collateral amounts to approximately 20% of the total collateral market. Empirical estimates suggest that private assets constitute between 9% and 35% of collateral or liquid assets used in financial markets.

[3] $\varphi^\pi = 1.5$, $\varphi^x = 0.5$, $\varphi^b = 0.12$. See Section 4.3.1 for details on the benchmark rule.

$\sigma_z = 0.006$ under an autoregressive process with $\rho_z = 0.9$[4]. The size of the cost-push shock was set to be at 10% standard deviation, following Smets and Wouters (2007). I calibrate the Rotemberg price adjustment cost parameter (ϕ^f) to match observed inflation dynamics. Specifically, the parameter is set to generate an inflation volatility of 0.55% (standard deviation) when the model is simulated under the benchmark Taylor rule.

These calibrations ensure that the model's steady state aligns with observed economic indicators and that the simulated dynamics reflect realistic responses to economic shocks.

2.4.2 Deterministic Steady States

Because government debt carries a premium and serves as collateral that enables firms' input transactions, the deterministic steady-state volume of government debt is non-zero. In fact, there exists a unique steady state for each level of government debt that policy rules can target, implying a continuum of steady states depending on the specified value of steady-state bond supply.

By plotting the steady-state welfare against different levels of bond supply (Figure 2.1), we observe an interesting relationship between the volume of government debt and welfare[5]. The plot identifies two distinct regions with different economic dynamics: Region A, where the collateral constraint is binding, and Region B, where it does

[4]The transfers-to-GDP ratio is calculated using data from the Federal Reserve Economic Data (FRED) database. Since the ratio exhibits a long-term growth trend, it is detrended to focus on cyclical fluctuations. The analysis yields the specified shock size.

[5]While the transition between Regions A and B appears discontinuous, there is actually a continuous transition in a narrow range of debt levels. In this region, the collateral constraint becomes slack, causing the default rate to drop rapidly and welfare to increase sharply. Figure A.1 in the Appendix A.1 provides a detailed view of this transition.

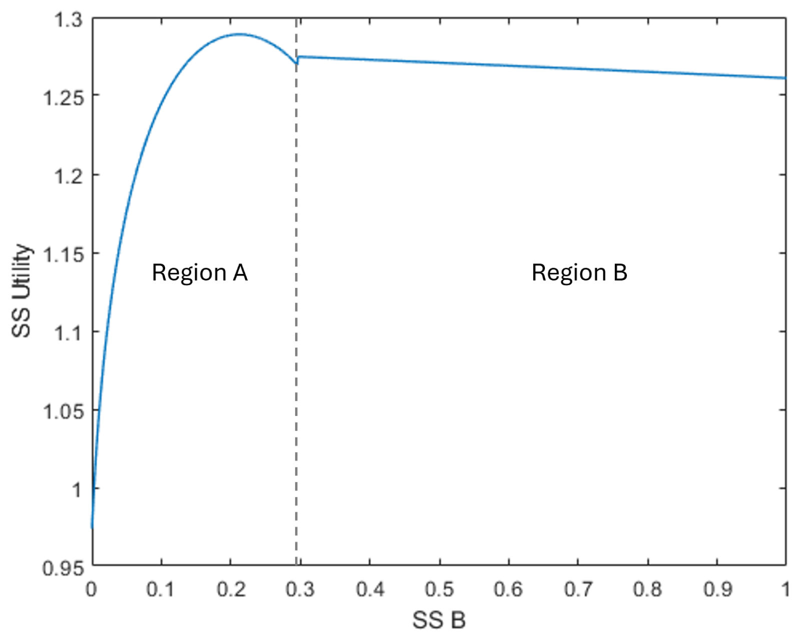

Figure 2.1: Steady-state welfare vs steady-state volume of government debt

not bind.

In Region A, welfare initially increases as government debt supply rises from zero. Figure 2.2 illustrates the underlying mechanism: higher government debt supply reduces collateral premiums (upper left panel) and increases borrowing limits (upper right panel). This improvement occurs because firms can use government debt as collateral instead of capital, allowing them to allocate resources more efficiently and increase production. Throughout Region A, the default rate remains constant because the collateral constraint continues to bind.

However, this welfare improvement eventually reaches its limit as the benefits of additional collateral become overshadowed by the costs of higher distortionary taxes needed to service the growing debt, as shown in the bottom right panel of Figure 2.2.

In contrast, in Region B where the collateral constraint does not bind, the volume

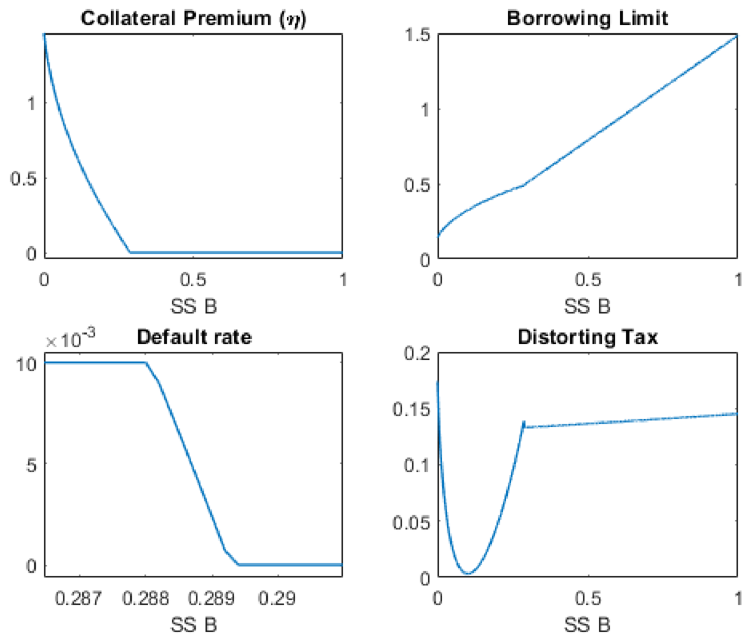

Figure 2.2: Collateral premium, borrowing limit, default rate and distorting tax rate in steady states

of government debt directly influences the default rate. An increase in government debt supply in this region reduces the default rate, as shown in Figure 2.2 (bottom-left panel) because firms have more collateral available, thereby lowering the capital costs associated with defaults. This reduction in default risk positively impacts labor demand, as the expected payout of capital increases with a lower default rate. However, the increase in government debt also necessitates higher distortionary taxes. The peak of Region B represents the point where the welfare gains from reducing the default rate are exactly offset by the welfare losses due to increased tax distortions.

The overall welfare plot resembles an inverted parabola in Region A, peaking at a certain level of government debt supply, followed by a sudden increase around a debt level of 0.3 in Region B before gradually declining. This pattern suggests there is an optimal level of government debt that maximizes steady-state welfare.

When solving the model, the choice of the deterministic steady state around which to approximate the solution is crucial. I select the steady state with the debt level that delivers the highest welfare in this deterministic setting. This welfare-maximizing steady state serves as the reference point for the policy rules and as the approximation point for the second-order solution method. This approach ensures that the dynamic analysis of alternative policy rules is conducted around a steady state that is optimal from a long-run perspective.

2.4.3 Optimal Policy Rules

In this section, I analyze optimal monetary and fiscal policy rules using a second-order approximation around the welfare-maximizing steady state. Due to the model's unique steady state for each level of government bond supply, we select the bond supply that maximizes utility in the deterministic steady state. This approach centers the dynamic analysis on the optimal long-run equilibrium.

Using the second-order solution, I explore two distinct policy regimes:

1. **Standard Taylor Rule (STR)**: Monetary policy responds only to deviations in inflation.

$$i_t = i^{ss} + \varphi^\pi(\pi_t - 1) + \varphi^x x_t \tag{2.57}$$

2. **Premium-Sensitive Monetary Rule (PSMR)**: Monetary policy responds to both inflation deviations and variations in collateral premiums.

$$i_t = i^{ss} + \varphi^\pi(\pi_t - 1) + \varphi^x x_t + \varphi^\eta(\eta_t - \eta^{ss}). \tag{2.58}$$

In both regimes, fiscal policy responds to the debt level:

$$\tau_t Y_t = \tau^{ss} Y^{ss} + \varphi^b (b_t - b^{ss}) \tag{2.59}$$

where $b_t \equiv \frac{B_t}{P_t}$. Here, i^{ss}, τ^{ss}, Y^{ss} and η^{ss} denote the stochastic steady-state values of the nominal interest rate, tax rate, output, and premium, respectively[6].

The optimal policy parameters presented below maximize expected lifetime welfare under different policy rules described[7]. I exclude policy parameter combinations that lead to equilibrium indeterminacy - multiple equilibria would introduce additional sources of volatility that make welfare comparisons unreliable.

Benchmark Taylor Rule (BTR)

To evaluate the welfare implications of optimal policy rules, I establish a benchmark economy with conventional policy parameters. The benchmark monetary policy follows a standard Taylor rule that responds only to inflation, while fiscal policy adjusts taxes in response to debt levels. Specifically, I set:

$$\varphi^\pi = 1.5 \qquad \varphi^x = 0.5 \qquad \varphi^\eta = 0 \qquad \varphi^b = 0.12. \tag{2.60}$$

The inflation response parameter ($\varphi^\pi = 1.5$) and the output gap response parameter (φ^x=0.5) follow Taylor (1993)'s widely adopted specification. The premium response parameter (φ^η) is set to zero, reflecting conventional monetary policy's focus on price

[6] Because different policy parameters lead to different stochastic steady states, for each policy parameter, target policy values are re-calculated in iterative way until they converge to stochastic steady state policy values.

[7] To compute the expected lifetime welfare for each combination of policy parameters without the need for lengthy simulations, I select the ergodic state with the highest probability density among all stable policy parameter combinations. This state serves as the initial condition for the economy, enabling the calculation of expected lifetime welfare under the timeless perspective.

stability. The fiscal policy parameter ($\varphi^b = 0.12$) follows Bohn (2005)'s empirical estimates of U.S. fiscal policy's response to debt levels. Furthermore, I let the monetary policy follow a shock with a standard deviation of 10% of the net nominal interest rate.

Under this benchmark rule, debt stabilization is primarily achieved through fiscal policy adjustments, while monetary policy concentrates on inflation and output gap targeting. To understand the dynamic effects of this policy framework, I examine how the economy responds to an exogenous increase in government debt through an MIT shock analysis. Figure 2.3 displays impulse response functions to a 1% MIT shock to real government debt under the benchmark policy framework. The resulting dynamics reveal the essential mechanism of debt financing under the BTR regime:

Following the initial increase in real debt, there is an immediate expansion in the collateral volume available to firms. This increased collateral availability directly impacts the financial conditions firms face, as shown by the significant drop in the collateral premium. The lower premium indicates that firms face reduced costs for collateralizing their loans, which effectively loosens their borrowing constraints.

This temporary relaxation of financial conditions enables firms to expand their economic activity. The IRFs show that both output and labor increase in response to the improved borrowing conditions. Firms can hire more workers and increase production when collateral is more abundant and less costly to obtain. This illustrates the direct channel through which government debt affects real economic activity through its role as collateral.

However, the fiscal rule quickly responds to the debt deviation by raising the tax rate, which generates additional revenue to bring debt back to its target level. As a result,

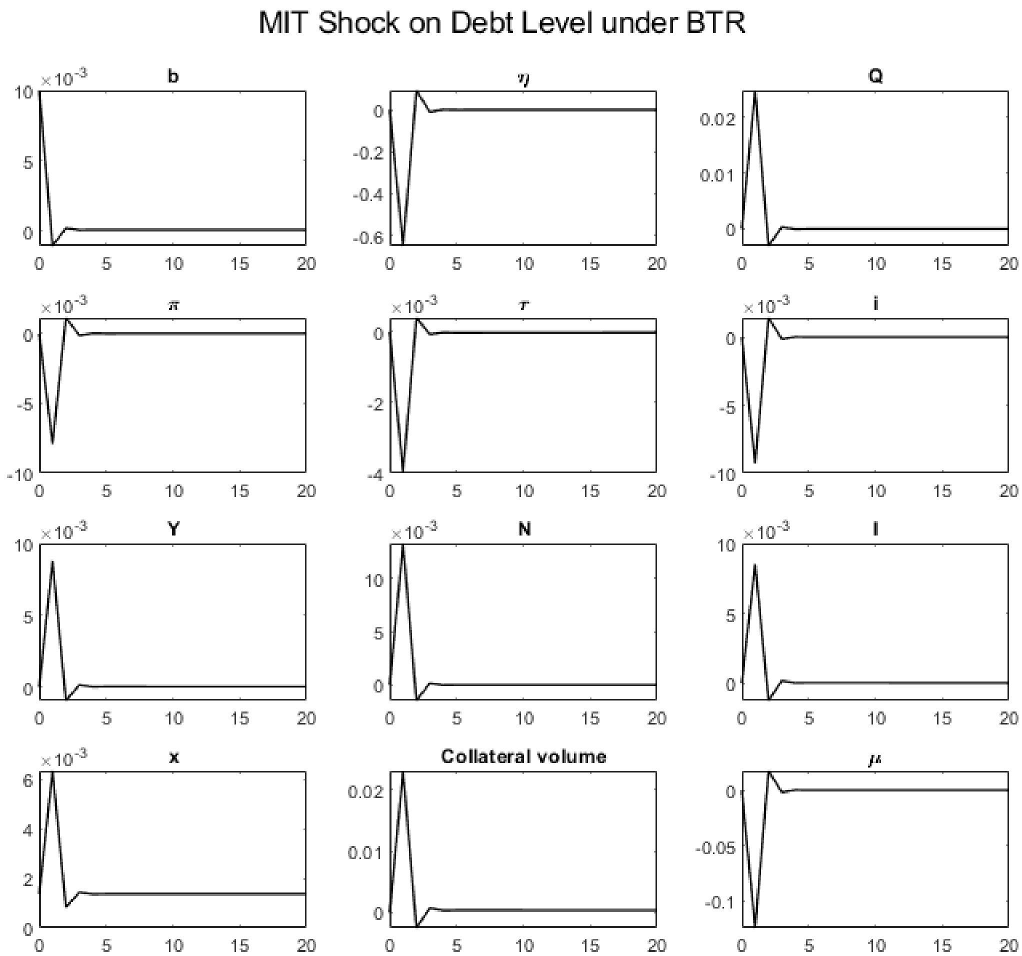

Figure 2.3: IRFs to 1% MIT shock on the real debt under BTR

the economic expansion is short-lived, and all variables - including debt, collateral premium, output, and labor - return to their steady-state values within approximately 5 periods. This rapid stabilization demonstrates how the BTR framework prioritizes quick debt adjustment through fiscal tools rather than allowing persistent changes in debt levels to accommodate financial conditions.

The BTR represents a conventional approach to policy coordination: when government debt rises, fiscal policy actively adjusts through tax increases to restore fiscal sustainability, while monetary policy maintains its focus on inflation and output gap stabilization. Under this approach, the temporary economic benefits from increased collateral availability are sacrificed in favor of rapid debt stabilization.

In subsequent sections, I compare the optimal policy rules against BTR to quantify potential welfare gains and highlight alternative approaches to managing the trade-off between debt stability and financial conditions.

Optimal Standard Taylor Rule (STR)

Under the STR, policy parameters that maximize expected welfare are:

$$\varphi^{\pi} = 10.81, \qquad \varphi^{x} = 7.46 \qquad \varphi^{b} = 2.89.$$

This combination suggests that maintaining price stability requires substantial support from fiscal policy through active debt management - compared to the benchmark. The monetary response to inflation and output gap remains finite, unlike in some standard New Keynesian settings where infinite inflation responses are optimal[8].

[8]Schmitt-Grohé and Uribe (2004) find that under sticky prices and distortionary fiscal policies that adjust to the debt level, an infinite response of monetary policy to inflation is optimal for ensuring price-level determinacy and welfare maximization. However, Leeper and Zhou (2021) show

The finite optimal response arises because divine coincidence fails in this model environment, due not only to cost-push shocks but also to the introduced financial friction. Specifically, the interaction between inflation and the collateral constraint shown in equation (2.61) eliminates divine coincidence:

$$R_t w_t N_t \leq \gamma\chi(1-\delta)q_t^o K_{t-1} + \frac{(1+\zeta Q_t)b_{t-1}}{\pi_t}. \tag{2.61}$$

The right-hand side of equation (2.61) shows the real value of posted collateral. Because government debt is denominated in nominal terms, its real value depends on inflation through the term $\frac{b_{t-1}}{\pi_t}$. Higher inflation depreciates the real value of government debt, causing the collateral constraint to bind more tightly. This tightening increases the collateral premium and reduces labor demand, leading to decreased production. Conversely, deflation relaxes the constraint by appreciating the real value of outstanding debt used as collateral. Therefore, the finite value of φ^π in (2.4.3) reflects an optimal trade-off: while deflation can relax the collateral constraint and boost production, it also incurs price adjustment costs.

Like under the benchmark rule, government debt is primarily financed through fiscal policy. However, the STR regime features a much more aggressive fiscal response to debt deviations ($\varphi^b = 2.89$ compared to 0.12 in BTR), creating distinctive dynamics when the economy experiences a debt shock. Figure 2.4 depicts the impulse response functions to a 1% MIT shock to the debt level.

The most notable feature of these IRFs is the oscillatory pattern visible across multiple

that when the maturity structure of government debt is introduced, there is an optimal trade-off between inflation stabilization and output stabilization. Similarly, this work incorporates a maturity structure of government debt to explore its role in shaping the optimal monetary and fiscal policy trade-offs, emphasizing how longer maturities can influence the allocation of shocks between inflation and other macroeconomic variables.

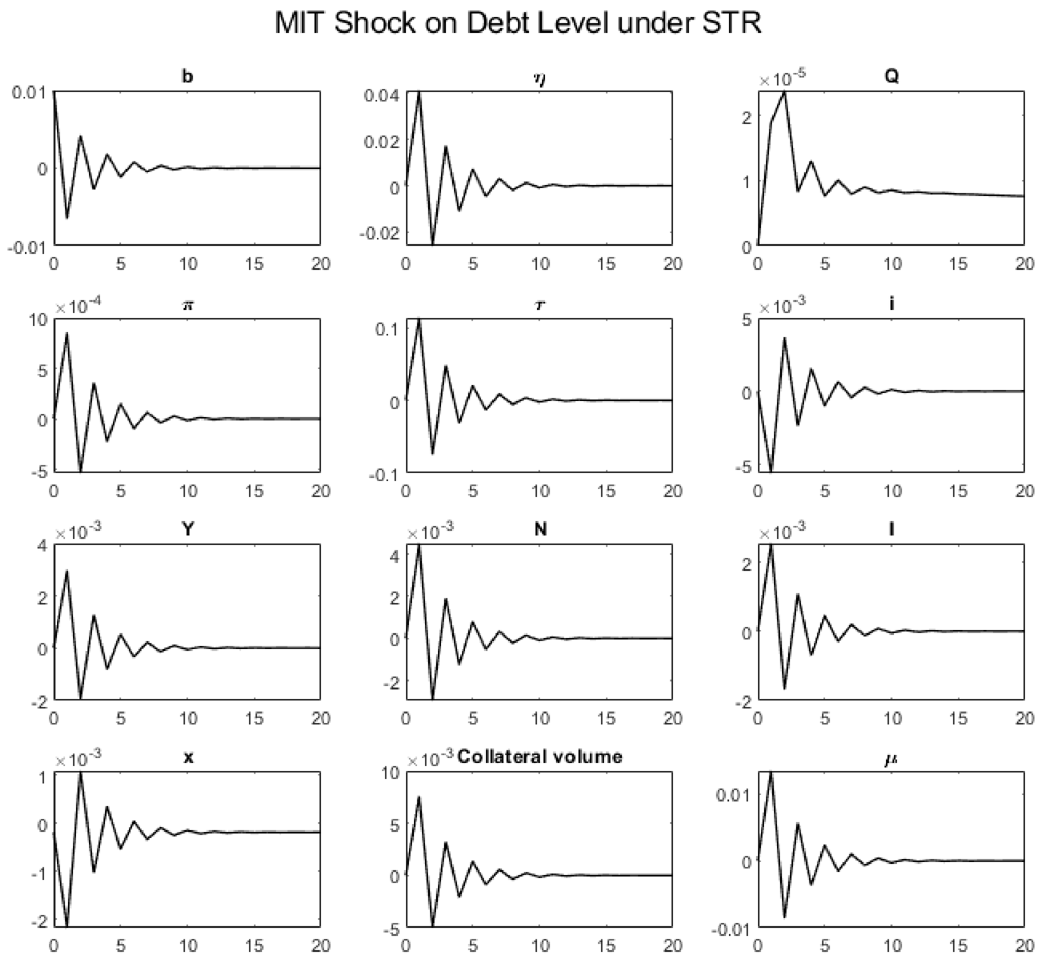

Figure 2.4: IRFs to 1% MIT shock on the real debt under STR

variables. When the debt level initially increases, the aggressive fiscal response quickly drives debt below its steady-state level in the subsequent period, before gradually converging through a series of diminishing oscillations. This pattern emerges because the fiscal policy's response is strong enough to overcorrect the initial debt deviation.

Notably, the collateral premium (η) increases following the debt shock, contrary to what occurred under the BTR regime where it declined. This seemingly counterintuitive result can be explained by examining the relationship between collateral volume, markup, and labor demand. While the debt shock initially increases the total collateral volume available to firms, the figure shows that the markup (μ) rises sharply at the same time. This markup increase drives up firms' labor demand significantly, creating a greater need for collateral to finance wage payments that outpaces the additional collateral provided by the debt increase.

The increase in labor demand is substantial and immediate, as firms anticipate the future effects of the aggressive fiscal rule. Specifically, firms expect negative output growth in the next period due to the anticipated sharp decrease in debt issuance (and consequently collateral availability). This expectation of tightening financial conditions in the future incentivizes firms to expand production immediately, driving up current labor demand and the collateral premium.

The output response mirrors the labor demand pattern, rising sharply in response to the initial shock before exhibiting the same oscillatory convergence. The oscillations in output, labor, and premium all stem from the fiscal policy's strong reaction, which creates a cycle of temporarily relaxed and then tightened collateral constraints as debt levels fluctuate around the steady state.

This analysis reveals a key difference between the BTR and STR regimes: while both finance debt through fiscal adjustments, the STR's more aggressive approach creates pronounced oscillatory dynamics that affect real economic activity through the collateral channel, leading to more volatile transitions back to steady state following a shock.

Premium-sensitive Monetary Rule (PSMR)

The PSMR regime augments the standard Taylor rule by incorporating a response to collateral premiums. The optimal policy parameters that maximize expected lifetime welfare are:

$$\varphi^{\pi} = 3.42, \quad \varphi^{x} = 0.26, \quad \varphi^{\eta} = -0.18, \quad \varphi^{b} = -0.38 \tag{2.62}$$

The optimal monetary policy parameters in (2.62) feature three key characteristics. First, the negative value of φ^{η} implies that monetary policy lowers interest rates when collateral premiums increase. Second, inflation and output responses (φ^{π} and φ^{x}) are lower than in the STR regime, suggesting that the monetary policy responds more moderately to these variables. Third, and most notably, the fiscal response parameter φ^{b} is negative, meaning that the fiscal policy lowers the tax revenue when the carried over debt level is high.

The negative premium response ($\varphi^{\eta} < 0$) has a clear economic intuition. During periods of financial stress (higher η_t), lowering the nominal interest rate raises bond prices Q_t, thereby increasing collateral values. This mechanism operates through the bond pricing equation:

$$Q_t = \frac{1}{1+i_t} + E_t\left(\frac{\Delta_{t,t+1}}{\pi_{t+1}}(\eta_{t+1} - \Xi_{t+1} + \zeta Q_{t+1}(1 - \Xi_{t+1} + \eta_{t+1}))\right) \tag{2.63}$$

As equation (2.63) shows, a lower i_t increases Q_t, which enhances the collateral value in the borrowing constraint (2.61) and relaxes financial conditions.

The negative fiscal response parameter ($\varphi^b = -0.38$) represents a departure from conventional debt management approaches. Figure 2.5 illustrates the dynamic response to a 1% MIT shock to the debt level under PSMR, revealing how this counterintuitive policy stabilizes debt through monetary-fiscal coordination.

When debt increases, several mechanisms operate simultaneously. First, fiscal policy responds by reducing tax rates, which stimulates economic activity. This is visible in the increase in output and labor following the shock. Unlike the oscillatory pattern observed under STR, these variables show a smooth hump-shaped response before gradually returning to steady state.

The debt shock increases the total collateral volume, as shown in the corresponding panel, but the enhanced economic activity driven by tax reduction simultaneously increases firms' demand for collateral to finance wage payments. This increased demand for collateral causes the premium to rise sharply rather than fall. The monetary policy rule responds to the higher premium by lowering the nominal interest rate, which further boosts the bond price.

This coordinated response creates a debt stabilization mechanism: the higher bond price effectively increases the value of existing debt, while the reduced interest rate lowers the cost of new debt issuance. The combination allows the increased debt level to be financed without having to raise the tax rate. Unlike the BTR and STR regimes, the PSMR approach features a relatively persistent debt level following the shock, with a gradual return to steady state. This persistence reflects the policy's accommodation to temporarily higher debt levels when they serve as collateral that supports economic activity. This monetary-fiscal coordination assigns distinct roles across different time horizons. Fiscal policy manages and relaxes collateral needs in the long run through debt level management, while monetary policy addresses immediate collateral needs and ensures sustainable debt financing in the short run. The premium-sensitive approach recognizes that when government debt serves as collateral, debt stabilization should consider not only fiscal sustainability but also the effects on firms' ability to finance production.

The smooth adjustment path visible in Figure 2.5 contrasts with the oscillatory dynamics under STR, suggesting that directly addressing financial frictions through monetary policy can enhance economic stability while reducing the need for aggressive fiscal adjustments. This improved coordination delivers higher welfare by allowing the economy to utilize government debt's dual role as both a fiscal instrument and a source of collateral more efficiently.

Welfare gains

Table 2.2 presents key variables under the BTR, STR, and PSMR. The stochastic steady states reveal important differences across policy regimes. Under PSMR, both consumption (C^{ss}) and capital (K^{ss}) achieve higher levels compared to BTR and STR. The higher capital stock under PSMR suggests that addressing financial frictions directly through monetary policy promotes capital accumulation. This occurs despite a slightly higher steady-state premium (η^{ss}), as the stochastic steady state volume of government debt is higher than in BTR and STR.

Table 2.3 compares expected average welfare at the ergodic mean in consumption units across the three policy regimes. Under both STR and PSMR, the expected welfare

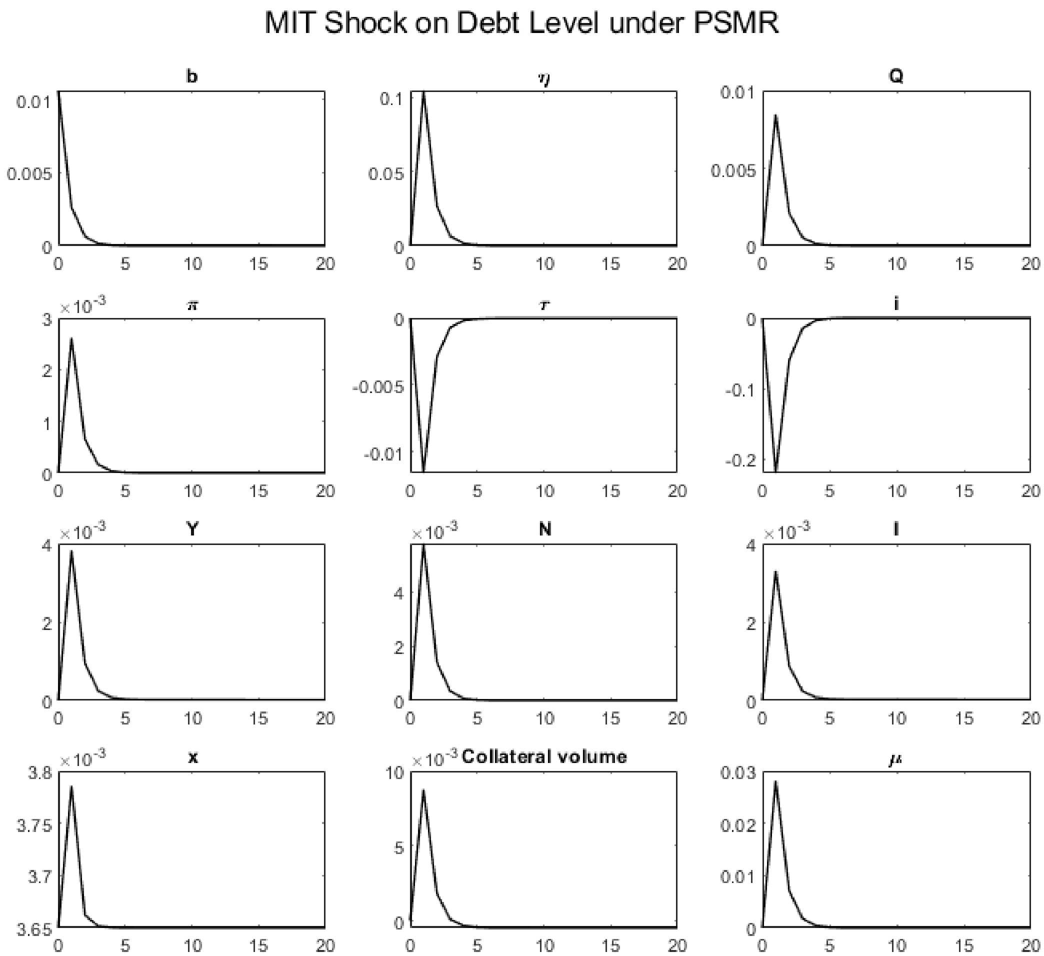

Figure 2.5: IRFs to 1% MIT shock on the real debt under PSMR

Variables	BTR	STR	PSMR
C^{ss}	0.8009	0.8005	0.8065
K^{ss}	12.0439	12.0266	12.3108
i^{ss}	0.0084	0.0091	0.0090
b^{ss}	0.2042	0.2041	0.2067
η^{ss}	0.2670	0.2703	0.2730
τ^{ss}	0.0507	0.0505	0.0490
Y^{ss}	1.0019	1.0009	1.0161

Table 2.2: Key Variables under DSS, STR, and PSMR

improves compared to the benchmark. The adoption of PSMR leads to a welfare gain of approximately 43 basis points over STR. While this gain might appear modest in magnitude, it is achieved with notably less aggressive policy interventions. The required monetary response to inflation falls substantially (φ^{π} = 10.81 to 3.42), and fiscal policy shifts from aggressive debt response (φ^{b} = 2.89) to a more accommodative stance (φ^{b} = -0.38).

	BTR	STR	PSMR
W	1.2898	1.2906	1.2962

Table 2.3: Welfare Comparison under STR and PSMR

Table 2.4 helps in identifying the source of these welfare gains, which reports standard deviations of key variables from simulations of 100,000 periods using identical shock sequences. The STR regime achieves significantly lower consumption volatility (0.0097) compared to both BTR (0.0133) and PSMR (0.0130). It also delivers stable prices, with inflation volatility of just 0.0004 compared to 0.0055 under BTR and 0.0079 under PSMR.

However, these stability gains under STR come at the cost of lower average consumption levels, as shown in Table 2.2. In contrast, PSMR tolerates higher volatility in both consumption and inflation to achieve higher average consumption and capital

Variable	BTR	STR	PSMR
C (σ_c)	0.0133	0.0097	0.0130
η (σ_η)	0.1190	0.0251	0.1424
Q (σ_Q)	0.0371	0.0369	0.0301
π (σ_π)	0.0055	0.0004	0.0079
τ (σ_τ)	0.0006	0.0053	0.0016

Table 2.4: Standard Deviations of Key Variables under STR and PSMR

levels. The higher steady-state output ($Y^{ss} = 1.0161$ under PSMR versus 1.0009 under STR) suggests that this trade-off is welfare-improving. The welfare gains from PSMR thus stem primarily from improved average economic performance rather than reduced volatility.

This trade-off is reflected in the collateral premium volatility (σ_η), which is higher under PSMR (0.1424) than STR (0.0251). Yet PSMR achieves lower bond price volatility ($\sigma_Q = 0.0301$ versus 0.0369), suggesting more stable debt markets despite more variable financial conditions. The lower tax volatility under PSMR ($\sigma_\tau = 0.0016$ versus 0.0053) further indicates that allowing more fluctuation in financial conditions reduces the need for fiscal policy adjustments.

2.5 Conclusion

This paper studies optimal monetary and fiscal policy in an environment where both government debt and capital serve as collateral. The key finding is that monetary policy should actively respond to collateral premiums: when premiums rise, indicating tighter financial conditions, the optimal policy lowers interest rates to boost collateral values. This premium-sensitive monetary rule achieves better welfare outcomes than a standard Taylor rule while requiring less aggressive policy interventions, with both monetary and fiscal responses falling substantially.

The welfare gains under PSMR stem primarily from higher average consumption and capital levels, rather than from reduced economic volatility. Indeed, PSMR tolerates somewhat higher volatility in both consumption and inflation compared to STR, but achieves this with more stable debt markets and less volatile tax rates. Moreover, the optimal inflation and output gap responses are finite under both policy regimes, contrasting with standard New Keynesian results. This finding reflects fundamental trade-offs introduced by financial frictions, particularly through the interaction between inflation, collateral values, and real economic activity.

A particularly intriguing result is the negative fiscal response to debt under PSMR, which becomes optimal through coordination with monetary policy. Lowering tax rates as debt rises boosts firms' productivity and labor demand, driving up collateral needs and premiums — which raises bond prices. Monetary policy then cuts interest rates when those premiums are high, pushing bond prices even higher and allowing debt to finance itself. These results highlight how financial frictions fundamentally alter optimal policy design and emphasize the importance of monetary–fiscal coordination in managing both macroeconomic and financial stability.

Chapter 3

Long-run Optimal Capital and Labor Tax Rates under Debt Collateralization

3.1 Introduction

This chapter finds the optimal long-run capital and labor income tax rates for the economy with financial frictions introduced in the previous chapter. By solving a Ramsey optimal taxation problem and imposing convergence to the steady state, I find that the optimal long-run capital tax rate is significantly positive while the optimal labor income tax is negative, effectively constituting a wage subsidy.

These findings are particularly interesting because the model features incomplete markets through financial frictions that award capital a specific collateral value, creating economic inefficiency. Firms use capital as collateral to secure financing for production inputs, leading to over-accumulation of capital relative to the social optimum. This distortion provides a rationale for corrective taxation on capital returns. Additionally, the negative labor tax reduces firms' dependence on collateralized borrowing by lowering their labor costs, directly addressing the source of inefficiency in the model. This dual tax strategy—positive capital taxation combined with a wage subsidy—represents an approach to correcting distortions arising from financial frictions.

The results stand in contrast to the conventional wisdom in optimal taxation literature, which typically prescribes a zero capital tax in the long run. This classical result, established by Chamley (1986) and Judd (1985), suggests that capital taxation distorts intertemporal choices and discourages investment, ultimately reducing welfare. However, subsequent research has identified various conditions under which positive capital taxation may be optimal. Several studies suggested that under incomplete market, a positive capital tax can become optimal. Aiyagari (1995) demonstrated that positive capital taxes are optimal in economies with incomplete markets and borrowing constraints, as precautionary saving leads to capital over-accumulation relative to the social optimum. This mechanism is in parallel with my findings, while in my model, firms accumulate excessive capital for collateral purposes rather than households building precautionary savings. Lansing (1999) showed that positive capital taxation can be optimal in overlapping generations frameworks or under conditions of market incompleteness. Straub and Werning (2020), without introducing incomplete markets, further specified conditions for optimal positive capital taxation, identifying scenarios where the intertemporal elasticity of substitution is less than one.

The previous chapter of this dissertation introduced a model featuring financial frictions. In this chapter, I modify the original framework to examine long-run optimal tax policies using a Ramsey approach. To properly analyze capital taxation, I restructured the model environment by transferring capital ownership to the household, who then rents it to firms. Firms continue to use this rented capital as collateral for securing financing. This modification creates an explicit rental market for capital with a clear pricing mechanism that the government can influence through taxation policy.

The steady-state results presented in this chapter rely on the convergence of allocations and the Lagrange multipliers associated with the Ramsey problem. As noted by Straub and Werning (2020), the validity of steady-state characterizations depends on such convergence. Therefore, the identified steady state and corresponding optimal tax rates are valid contingent upon this convergence condition being met. This approach aligns with contemporary analyses suggesting fiscal interventions in economies with market incompleteness and financial imperfections, contributing to the growing literature on optimal taxation under various market frictions.

The results highlight an optimal policy framework built on three key components—positive capital taxation, negative labor taxation, and non-negative bond supply. The positive capital tax rate serves as a corrective mechanism to discourage excessive capital accumulation driven by firms' collateral needs. Simultaneously, the negative labor tax rate functions as a substantial wage subsidy that reduces firms' reliance on collateralized borrowing. Complementing these tax instruments, maintaining a non-negative supply of government bonds generates revenue to fund the labor subsidy. Together, these policy components form a comprehensive approach to addressing the inefficiencies arising from financial frictions in the model, demonstrating that optimal fiscal policy must respond to the specific distortions created by market imperfections.

3.2 The Model

The Ramsey problem of this exercise uses the model constraints and optimal conditions derived in the first chapter, with three major modifications to fit this chapter's purpose. First, to determine theoretically optimal tax rates, the government budget constraint is re-written with a new tax scheme that includes taxes on both capital return and labor income. Second, the model specification is modified: households now own and accumulate capital, which firms rent and post as collateral when financing labor costs. Upon default, banks seize this capital and return it to households. Since households lose capital when firms default, their problem incorporates this potential loss, placing a premium on capital rent in the optimal conditions. These modifications preserve the model's fundamental mechanism while allowing it to price capital returns, including both marginal productivity and collateral premium. Finally, because the Ramsey exercise aims to find long-run optimal policy tax rates in steady state, considerations regarding inflation and capital adjustment costs are removed - when steady state conditions are imposed, the related first-order conditions are canceled out. This is achieved by setting parameters $\phi^f = 0$ (Rotemberg price adjustment cost) and $\xi = 1$ (elasticity of input substitution of capital production).

The model modification is described in subsequent subsections. Modifications to the banks' problem are attached in appendix section B.1; a minor change is made to the banks' problem which does not affect the banks' optimal conditions.

3.2.1 The Government

The government's budget constraint becomes

$$\tau^k r_t K_t + \tau_t^w w_t N_t + Q_t b_t = (1 + \zeta Q_t) b_{t-1} + Z_t. \tag{3.1}$$

Note that the government's revenue (left-hand side of the equation) now involves the tax on capital return ($\tau^k r_t K_t$) and tax on labor income ($\tau_t^w w_t N_t$). The right-hand side terms include debt service payments for maturing debt and household transfers.

3.2.2 The Household

With the modification in the model specification, the household's budget constraint can be re-written as follows:

$$
\begin{aligned}
&C_t + K_t = \\
&\qquad (1-\delta)K_{t-1} - \gamma(1-\delta)\int_{lb}^{\bar{\nu}_t} \nu K_{t-1}\phi(\nu)d\nu + \gamma(1-\delta)\chi\int_{lb}^{\bar{\nu}_t} \nu K_{t-1}\phi(\nu)d\nu \\
&\qquad\qquad + (1-\tau_t^k)r_t K_{t-1} + (1-\tau_t^w)w_t N_t + \Pi_t^f + \Pi_t^b + Z_t. \quad (3.2)
\end{aligned}
$$

Here, γ is the collateral ratio of the rented capital and ν is the idiosyncratic shock to the capital quality. lb represents the lower-bound of the distribution of ν, $\phi(\nu)$. $\gamma(1-\delta)\int_{lb}^{\bar{\nu}_t} \nu K_{t-1}\phi(\nu)d\nu$ represents the aggregate quantity of capital the household loses due to defaulting firms, where firms with capital quality below the default threshold value $\bar{\nu}_t$ default. $\gamma(1-\delta)\chi\int_{lb}^{\bar{\nu}_t} \nu K_{t-1}\phi(\nu)d\nu$ is the amount of the capital the bank returns to the household after seizing from the defaulting firms. Putting them together, the household budget constraint can be re-written again as follows:

$$
C_t + K_t = (1-\vartheta_t)K_{t-1} + (1-\tau_t^k)r_t K_{t-1} + (1-\tau_t^w)w_t N_t + \Pi_t^f + \Pi_t^b + Z_t, \quad (3.3)
$$

which relies on the identity of $\vartheta_t \equiv \delta + (1-\delta)\gamma(1-\chi)\Lambda_t$ where $\Lambda_t \equiv \int_{lb}^{ub} \nu\phi(\nu)d\nu$. Note that to focus on the long-term optimal policy, the prices of capital q_t^n and q_t^o do not appear in the household's problem.

3.2.3 Production Firms

Because production firms do not accumulate capital, the profit function of a production firm simplifies to:

$$
\begin{aligned}
E_t^-(\Pi_t^f(i)) = (\frac{P_t(i)}{P_t})^{1-\theta_t} E_t^-(Y_t(i)) - w_t(i)N_t(i) - r_t(i)K_t(i) \\
- \Big(\int_{\bar{v}_t(i)}^{\infty} R_t L_t(i)\phi(v_t(i))dv_t(i) + \int_0^{\bar{v}_t(i)} \frac{(1+\zeta Q_t)B_{t-1}(i)}{P_t}\phi(v_t(i))dv_t(i)\Big) \\
- \frac{Q_t B_t(i)}{P_t} + \frac{(1+\zeta Q_t)B_{t-1}(i)}{P_t} + L_t(i). \quad (3.4)
\end{aligned}
$$

Production firm (i) pays rent on capital, while it does not incur defaulting cost of capital upon default.

3.2.4 Equilibrium Conditions

Under the modified model environment, the household's first-order condition with respect to capital is given as follows:

$$\beta \frac{C_t}{C_{t+1}}((1-\vartheta_{t+1}) + (1-\tau_{t+1}^k)r_{t+1}) = 1. \quad (3.5)$$

Because the household accounts for the lost capital when defaults occur, the rent on capital incorporates this in their capital choice through ϑ.

The aggregated first-order condition of production firms yields the following condition:

$$r_t = \alpha \frac{\theta - 1}{\theta} A_t K_{t-1}^{\alpha} N_t^{1-\alpha} + \eta_t \gamma \chi (1-\delta). \quad (3.6)$$

The rent on capital from the production firms' side equals the sum of the marginal productivity of capital and collateral value of capital.

All other equations, including the resource constraint and first-order conditions are included in section B.2. Government policy rules are left out as the Ramsey planner sets tax rates by solving the Ramsey problem.

Binding collateral constraint case

The aggregated collateral constraint for production firms can be written as

$$R_t w_t N_t \leq \gamma\chi(1-\delta)K_{t-1} + (1+\zeta Q_t)b_{t-1}, \tag{3.7}$$

where the total volume of the posted collateral needs to be larger than the total loan amount. When the collateral constraint (3.7) binds with equality, the default threshold value for capital quality, $\bar{V}_t$, equals χ. Furthermore, by the constraint, $\bar{V}_t$ is bounded above at χ. This means that when the constraint binds and the collateral premium η_t is positive, the default rate Ξ_t equals $\frac{\chi-(1-\sigma\sqrt{3})}{2\sigma\sqrt{3}}$, and the default density Λ_t equals $\frac{\chi^2-(1-\sigma\sqrt{3})^2}{4\sigma\sqrt{3}}$. This also fixes the endogenous default rate ϑ_t at $\vartheta = \delta + (1-\delta)\gamma(1-\chi)\Lambda$.

3.3 The Ramsey problem

Because of the piece-wise nature of the model equations, the Ramsey problem also becomes piecewise. Specifically, when the default threshold $\bar{V}_t$ is between the lower bound and the upper bound, the Ramsey problem would be given as below:

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big(& \ln(C_t) + (1 - N_t) + \lambda_t^{(1)} \left((1 - \vartheta_t) K_{t-1} + Y_t - C_t - K_t \right) \\
& + \lambda_t^{(2)} \left(\tau_t^K r_t K_{t-1} + \tau_t^w w_t N_t + Q_t b_t - (1 + \zeta Q_t) b_{t-1} - Z_t \right) \\
& + \mu_t^{(1)} \left(\delta + (1 - \delta) \gamma (1 - \chi) \Lambda_t - \vartheta_t \right) + \mu_t^{(2)} \left(\frac{\bar{V}_t^2 - (1 - \sigma\sqrt{3})^2}{4\sigma\sqrt{3}} - \Lambda_t \right) \\
& + \mu_t^{(3)} \left(\frac{R_t w_t N_t - (1 + \zeta Q_t) b_{t-1}}{\gamma (1 - \delta) K_{t-1}} - \bar{V}_t \right) + \mu_t^{(4)} \left((1 - \tau_t^w) w_t - C_t \right) \\
& + \mu_t^{(5)} \left(\gamma (1 - \delta) \Lambda_t + \alpha \frac{\theta - 1}{\theta} A_t K_{t-1}^{\alpha - 1} N_t^{1 - \alpha} + \eta_t \gamma \chi (1 - \delta) - r_t \right) \\
& + \mu_t^{(6)} \left(\beta \frac{C_t}{C_{t+1}} \left((1 - \vartheta_{t+1}) + (1 - \tau_{t+1}^K) r_{t+1} \right) - 1 \right) \\
& + \mu_t^{(7)} \left(R_t w_t N_t (1 - \Xi_t + \eta_t) - \frac{\theta - 1}{\theta} (1 - \alpha) Y_t \right) \\
& + \mu_t^{(8)} \left(\frac{\bar{V}_t - (1 - \sigma\sqrt{3})}{2\sigma\sqrt{3}} - \Xi_t \right) \\
& + \mu_t^{(9)} \left(\beta \frac{C_t}{C_{t+1}} (1 + \zeta Q_{t+1}) (1 - \Xi_{t+1} + \eta_{t+1}) - Q_t \right) \\
& + \mu_t^{(10)} \left(\frac{\psi}{(\psi + 1)(1 - \Xi_t)} - R_t \right) + \mu_t^{(11)} \left(A_t K_{t-1}^{\alpha} N_t^{1 - \alpha} - Y_t \right) \\
& + \mu_t^{(12)} \eta_t \left(\gamma \chi (1 - \delta) K_{t-1} + (1 + \zeta Q_t) b_{t-1} - R_t w_t N_t \right) \Big)
\end{aligned}
\tag{3.8}
$$

As discussed in section 3.2.4, the default threshold $\bar{V}$ is bounded above by χ. When $\bar{V}_t < 1 - \sigma\sqrt{3}$ or $\eta > 0$ (meaning that the collateral constraint (3.7) binds and holds with equality), endogenous variables Ξ_t, Λ_t, and ϑ_t are fixed as constants. This makes the Ramsey problem a piecewise function, where some constraints (including the private first-order conditions) do not take effect depending on the values of $\bar{V}$. Specifically, when $\bar{V}_t < 1 - \sigma\sqrt{3}$ or $\bar{V}_t = \chi$, Lagrange multipliers $\mu^{(1)}$, $\mu^{(2)}$, $\mu^{(3)}$, $\mu^{(8)}$, and $\mu^{(10)}$ are fixed at 0, where associated constraints and conditions do not hold.

The first-order conditions and their steady-state conditions are described in the appendix (section B.2.1 and B.2.2)

3.4 Steady State Equilibrium

If the steady state solution to the Ramsey problem exists, the Lagrange multipliers associated with the model equations and optimal conditions should converge in the steady state. Even when the allocation converges, if the Lagrange multipliers do not converge in the steady state, the steady state solution to the Ramsey problem does not exist[1]. If the steady state solution to the Ramsey problem exists, the steady state solves the system of equations described in section B.2.2. Suppose the steady state exists. Then we can solve the system of equations to find the steady state, which is described in table 3.1. The steady state solution occurs where the collateral

Steady-State Values	
Variable	**Value**
Capital tax rate (τ^k)	0.4754
Labor tax rate (τ^w)	-1.7598
Consumption (C)	0.8064
Capital (K)	10.8146
Labor (N)	0.5397
Collateral Premium (η)	2.4789
Government debt (b)	5.6988×10^{-4}
Bond Price (Q)	386.8692
Rent on Capital (r)	0.0489
Wage (w)	0.2922

Table 3.1: Optimal Steady-state values

[1]Straub and Werning (2020) provide examples where Lagrange multipliers do not converge and the Ramsey problem does not have a steady state

constraint binds. This means that constraint (3.7) holds with equality in the steady state, causing $\bar{V}$, Ξ, Λ, ϑ, and R to be fixed as constants. As a result, these variables are no longer choice variables in the optimization problem.

The steady state results presented in Table 1 reveal a few notable characteristics of optimal fiscal policy in this model. First, the government maintains a small positive level of debt (5.6988×10^{-4}) in the steady state, whereas conventional results from standard models typically prescribe negative government bonds (unless bounded below). Second, the optimal policy prescribes a substantial positive tax on capital returns, approximately 47.54%, which diverges from the established optimal taxation literature such as Chamley (1986) and Judd (1985), that prescribes zero or even negative capital taxation to avoid distorting intertemporal choices. Third, this model yields a labor subsidy, with a negative tax rate of -175.98%, contrary to standard results that typically suggest positive labor income taxation as the primary revenue source for government expenditures. From capital subsidies to capital taxation and from labor taxation to labor subsidies, such a departure in the policy prescriptions highlights the impact of financial frictions on optimal fiscal policy design.

These qualitatively different policy implications come from the market inefficiencies generated by the collateral constraints in the model. When firms face binding collateral constraints and tend to over-accumulate capital relative to the socially optimal level, corrective taxation (in the form of positive capital taxes and labor subsidies) becomes desirable to guide the economy toward more efficient outcomes.

The collateral premium ($\eta = 2.4789$) and government bond price ($Q = 386.8692$) in the steady state are notable values in this model. This premium quantifies the additional value assets gain when used as collateral, while the high bond price reflects the scarcity of collateral in the steady state due to low bond supply ($b = 5.6988 \times 10^{-4}$).

With limited government bonds available, capital serves as the primary collateral asset for firms' borrowing needs, which contributes to capital accumulation beyond the level that would be optimal based on productive returns alone.

The high bond valuation creates an important fiscal mechanism within the optimal policy framework. Because government debt carries this premium, the government earns net profit from bond issuance in the steady state. This profit plays a crucial role in balancing the overall fiscal policy. The tax revenue from capital and labor are:

$$\tau^k rK = 0.2512 \qquad \tau^w wN = -0.2775 \tag{3.9}$$

$$Qb - (1 + \zeta Q)b = 0.1563 \tag{3.10}$$

As these equations show, the expenditure from the labor subsidy exceeds the revenue from capital taxation. This fiscal gap and the transfer are financed by bond sales that generate the revenue in the steady state.

Importantly, even at the optimum, the collateral constraint remains binding, as indicated by the positive collateral premium. This reveals a fundamental limitation of fiscal policy in this environment: while taxation can improve welfare by better aligning private and social incentives, it cannot completely eliminate the distortion caused by the underlying financial friction. The constraint remains binding because completely relaxing it would require tax rates that the distortions from taxation would outweigh the benefits of relaxing the constraint.

The positive capital tax serves as a mechanism to discourage excessive capital accumulation resulting from collateral constraints, while the labor subsidy reduces firms' effective labor costs, thereby lessening their need for collateralized borrowing to finance wage payments. This reduction in collateral requirements directly addresses the source of market inefficiency, allowing the labor subsidy to function as a corrective policy instrument that enhances overall economic efficiency.

Chapter 4

Conclusions

Government bonds' dual role as saving device and collateral creates a wedge between private and social returns that standard macro models ignore. Embedding this collateral channel in a New Keynesian framework, the dissertation shows that policy can either correct the resulting misallocation (by realigning marginal incentives) or exploit it (by financing spending with the premium that bonds create as collateral). Recognizing this trade-off awards the conventional policy another tool and clarifies why optimal monetary, tax, and debt-management rules differ from prescriptions of frictionless models.

Optimal Monetary and Fiscal Policy Rules under Debt Collateralization showed that when collateral premiums rise, the welfare-maximizing monetary authority lowers the policy rate. A rule that includes the collateral premium along with inflation and the output gap raises average consumption and capital while requiring smaller tax adjustments than a standard Taylor rule. Fiscal policy complements this strategy with a weak, negative response to debt, which allows the quantity of bonds in circulation to expand when premiums are high. The premium-sensitive rule sacrifices some consumption and price stability for higher mean welfare, yet it also stabilizes bond prices and tax rates.

Long-run Optimal Capital and Labor Tax Rates under Debt Collateralization solved a Ramsey problem to characterize long-run tax policy. Because firms over-accumulate capital to satisfy collateral needs, the optimal steady state features a positive tax on capital returns and a large wage subsidy that is equivalent to a negative labor tax. Even at the optimum, the collateral constraint stays binding, so the premium on collateral persists and government debt generates seigniorage revenue from bond sales that finances the wage subsidy. These results depart from canonical prescriptions of zero capital taxation and positive labor taxation, highlighting the corrective function of taxes when collateral frictions distort investment and hiring decisions.

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Appendices

Appendix A

Optimal Monetary and Fiscal Policy Rules under Debt Collateralization

A.1 Deterministic Steady States

Figure A.1 presented below offers a magnified view of the plot depicted in Figure 2.1. This closer examination reveals that steady state welfare exhibits continuity (though not smoothness) around the boundary between Region A and Region B. When the collateral constraint ceases to bind (approximately at B=0.288), we observe that increasing the debt supply leads to a reduction in the default rate. This reduction in default rate enhances welfare by minimizing capital destruction. However, once the default rate reaches zero (approximately at B=0.29), further increases in debt supply no longer affect the default rate, resulting in diminished welfare gains.

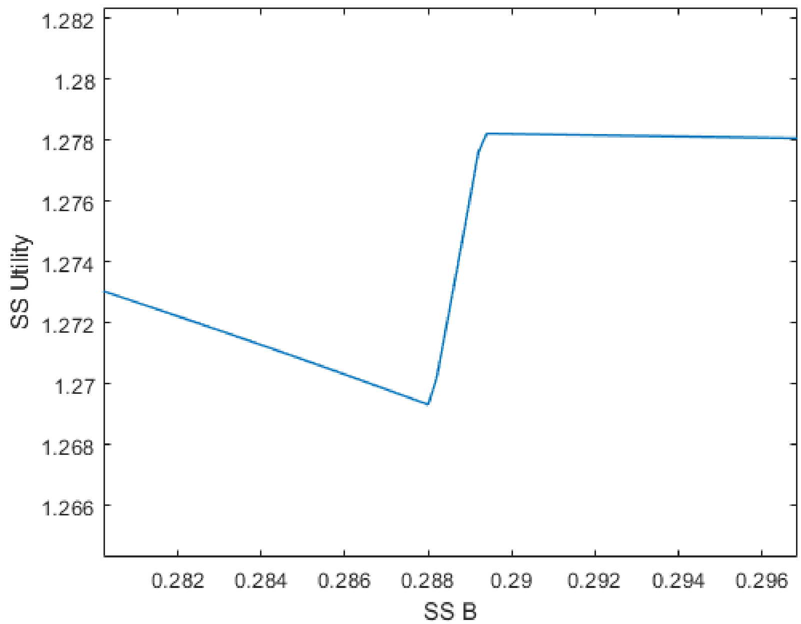

Figure A.1: Zoomed in plot of figure 2.1 around the border of Region A and Region B

A.2 Accuracy

The figure below illustrates the accuracy (in % terms) of the model solution based on an Euler-error test conducted over a simulation spanning 100,000 periods. The results demonstrate that the solution maintains high accuracy throughout the simulation period.

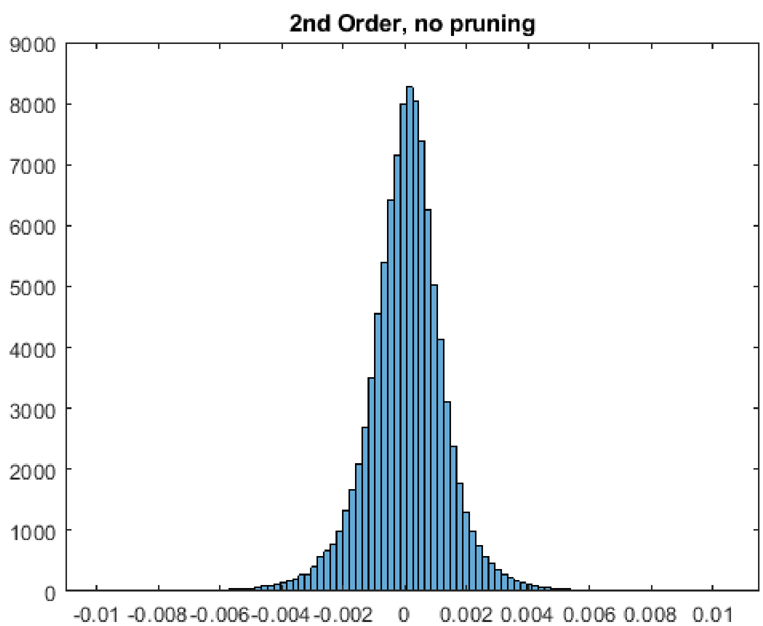

Figure A.2: Steady-state welfare vs steady-state volume of government debt

Appendix B

Long-run Optimal Capital and Labor Tax Rates under Debt Collateralization

B.1 The Model - Banks

A bank's profit function changes due to the change in the model assumption. Bank j's profit function becomes:

$$\Pi_t^b(j) = \frac{(1+\zeta Q_t)B_{t-1}}{P_t} d\nu_t + \int_{\bar{\nu}_t}^{\infty} R_t(j) L_t(j) \phi(\nu_t) d\nu_t - L_t(j). \tag{B.1}$$

The only change in the profit function of the bank is that it does not collect seized capital, as it will be delivered to the household.

B.2 Model Equations

The model has following endogenous variables:

$$C, K, N, Y, R, r, w, Q, b, \vartheta, \Lambda, V, \Xi, \eta$$

The market clearing constraint is:

$$C_t + K_t = (1-\vartheta_t)K_{t-1} + Y_t \tag{B.2}$$

where the production occurs through a Cobb-Douglas technology:

$$Y_t = A_t K_{t-1}^{\alpha} N_t^{1-\alpha}. \tag{B.3}$$

The government's budget constraint is:

$$\tau_t^k r_t K_t + \tau_t^w w_t N_t + \frac{B_t^s}{P_t} + \frac{Q_t B_t}{P_t} = \frac{(1+\zeta Q_t)B_{t-1}}{P_t} + Z_t \tag{B.4}$$

The endogenous depreciation rate is given by:

$$\vartheta_t = \delta + (1-\delta)\gamma(1-\chi)\Lambda_t \tag{B.5}$$

The cumulative density of the defaulting capital (assuming a uniform distribution with standard deviation σ) is:

$$\begin{aligned}
&\Lambda_t = \int_{\tilde{a}}^{\bar{V}_t} V\hat{\phi}(V)dV = \frac{\bar{V}^2 - (1-\sigma\sqrt{(3)})^2}{4\sigma\sqrt{3}} \qquad if \quad \bar{V}_t \in [1-\sigma\sqrt{3}, 1+\sigma\sqrt{3}] \\
&\Lambda_t = 0 \qquad if \quad \bar{V}_t < 1-\sigma\sqrt{3} \\
&\Lambda_t = 1 \qquad if \quad \bar{V}_t > 1+\sigma\sqrt{3}
\end{aligned} \tag{B.6}$$

where $\tilde{a}$ is the lower bound of the distribution: $\tilde{a} = 1-\sigma\sqrt{3}$. $\bar{V}$ is the aggregate default threshold, given by:

$$\bar{V}_t = \frac{R_t w_t N_t - (1+\zeta Q_t)b_{t-1}}{\gamma(1-\delta)K_{t-1}}. \tag{B.7}$$

Here, R_t is the intra-temporal interest rate charged to the firm by the bank. The household's first-order conditions are given by the following two equations:

$$(1-\tau_t^w)w_t = C_t. \tag{B.8}$$

$$\beta\frac{C_t}{C_{t+1}}((1-\vartheta_{t+1}) + (1-\tau_{t+1}^k)r_{t+1}) = 1 \tag{B.9}$$

The production firms' first-order conditions are given by:

$$R_t w_t N_t(1-\Xi_t+\eta_t) = \frac{\theta-1}{\theta}(1-\alpha)Y_t \tag{B.10}$$

where θ represents the degree of firms' monopolistic competition or the elasticity of substitution of households. Ξ is the endogenous default rate, given by the following equations:

$$\begin{aligned}
&\Xi_t = \frac{\bar{V}_t-(1-\sigma\sqrt{3})}{2\sigma\sqrt{3}} \qquad if \quad \bar{V}_t \in [1-\sigma\sqrt{3}, 1+\sigma\sqrt{3}]\\
&\Xi_t = 0 \qquad if \quad \bar{V}_t < 1-\sigma\sqrt{3}\\
&\Xi_t = 1 \qquad if \quad \bar{V}_t > 1+\sigma\sqrt{3}
\end{aligned} \tag{B.11}$$

The first-order condition regarding the firm's capital choice is given by:

$$r_t = \gamma(1-\delta)\Lambda_t + \alpha\frac{\theta-1}{\theta}A_t K_{t-1}^{\alpha} N_t^{1-\alpha} + \eta_t\gamma\chi(1-\delta). \tag{B.12}$$

The first-order condition regarding government bond accumulation is:

$$Q_t = \beta\frac{C_t}{C_{t+1}}(1+\zeta Q_{t+1})(1-\Xi_{t+1}+\eta_{t+1}) \tag{B.13}$$

Firms face the following collateral constraint:

$$R_t w_t N_t \leq \gamma\chi(1-\delta)K_{t-1} + (1+\zeta Q_t)b_{t-1}. \tag{B.14}$$

The banks' first-order condition is given by:

$$R_t = \frac{\psi}{(\psi+1)(1-\Xi_t)} \tag{B.15}$$

where Ψ is the degree of monopolistic competition among banks.

B.2.1 First Order Conditions

Below equations are the first-order conditions of the Ramsey problem in section **??**.

C_{t+1}:

$$\begin{aligned}
&\frac{1}{C_{t+1}} - \lambda^{(1)}_{t+1} - \mu^{(4)}_{t+1} - \mu^{(6)}_t \frac{C_t}{C^2_{t+1}}((1-\vartheta_{t+1}) + (1-\tau^K_{t+1})r_{t+1}) + \beta\mu^{(6)}_{t+1}\frac{1}{C_{t+2}}((1-\vartheta_{t+2}) + (1-\tau^K_{t+2})r_{t+2}) \\
&- \mu^{(9)}_t \frac{C_t}{C^2_{t+1}}(1+\zeta Q_{t+1})(1-\Xi_{t+1}+\eta_{t+1}) + \mu^{(9)}_{t+1}\beta\frac{1}{C_{t+2}}(1+\zeta Q_{t+2})(1-\Xi_{t+2}+\eta_{t+2}) = 0
\end{aligned} \tag{B.16}$$

K_t:

$$\begin{aligned}
&-\lambda^{(1)}_t(1-\beta(1-\vartheta)) + \beta\lambda^{(2)}_{t+1}\tau^K_{t+1}r_{t+1} - \beta\mu^{(3)}_{t+1}\frac{R_{t+1}w_{t+1}N_{t+1} - (1+\zeta Q_{t+1})b_t}{\gamma(1-\delta)K^2_t} \\
&+ \beta\mu^{(5)}_{t+1}\alpha\frac{\theta-1}{\theta}\cdot(\alpha-1)A_{t+1}K^{\alpha-2}_t N^{1-\alpha}_{t+1} + \alpha\beta\mu^{(11)}_{t+1}A_{t+1}K^{\alpha-1}_t N^{1-\alpha}_{t+1} + \mu^{(12)}_{t+1}\eta_{t+1}\cdot\gamma\chi(1-\delta) = 0
\end{aligned} \tag{B.17}$$

N_t:

$$
\begin{aligned}
&-1+\lambda_t^{(2)}\tau_t^w w_t+\mu_t^{(3)}\frac{R_t w_t}{\gamma(1-\delta)K}+\mu_t^{(5)}\alpha\frac{\theta-1}{\theta}A_t K_t^{\alpha-1}N_t^{-\alpha}(1-\alpha)\\
&+\mu_t^{(7)}R_t w_t(1-\Xi_t+\eta_t)+\mu_t^{(11)}(1-\alpha)A_t K_t^{\alpha}N_t^{-\alpha}-\mu_t^{(12)}\eta_t R_t w_t=0
\end{aligned}
\tag{B.18}
$$

Y_t:

$$\lambda_t^{(1)}-\mu_t^{(7)}\frac{\theta-1}{\theta}(1-\alpha)-\mu_t^{(11)}=0 \tag{B.19}$$

r_t:

$$\beta\lambda_t^{(2)}\tau_t^K K_t+\mu_t^{(6)}\beta(1-\tau_t^K)-\beta\mu_t^{(5)}=0 \tag{B.20}$$

w_t:

$$\lambda_t^{(2)}\tau_t^w N_t+\mu_t^{(3)}\frac{R_t N_t}{\gamma(1-\delta)K}+\mu_t^{(4)}(1-\tau_t^w)+\mu_t^{(7)}R_t N_t(1-\Xi_t+\eta_t)-\mu_t^{(12)}\eta_t R_t N_t=0 \tag{B.21}$$

Q_t:

$$\lambda_t^{(2)}(1-\zeta)b_t-\mu_t^{(3)}\frac{\zeta b_t}{\gamma(1-\delta)K_{t-1}}+\mu_t^{(9)}\zeta(1-\Xi_t+\eta_t)-\mu_t^{(9)}+\mu_t^{(12)}\eta_t\zeta b_t=0 \tag{B.22}$$

b_t:

$$\lambda_t^{(2)}Q_t(1-\zeta\beta)-\beta\mu_{t+1}^{(3)}\frac{1}{\gamma(1-\delta)K_t}+\beta\mu_{t+1}^{(12)}\eta_{t+1}(1+\zeta Q_t)=0 \tag{B.23}$$

ϑ_t:

$$-\beta\lambda_t^{(1)}K_t-\beta\mu_t^{(1)}-\beta\mu_t^{(6)}=0 \tag{B.24}$$

Λ_t:

$$\mu_t^{(1)}(1-\delta)\gamma(1-\chi)-\mu_t^{(2)}+\gamma(1-\delta)\mu_t^{(5)}=0 \tag{B.25}$$

$\bar{V}_t$:

$$\mu_t^{(2)} \frac{\bar{V}_t}{2\sigma\sqrt{3}} - \mu_t^{(3)} + \mu_t^{(8)} \frac{1}{2\sigma\sqrt{3}} = 0 \tag{B.26}$$

Ξ_t:

$$-\mu_t^{(7)} R_t w_t N_t + \mu_t^{(8)} - \mu_t^{(9)}(1 + \zeta Q_t) + \mu_t^{(10)} \frac{\psi}{(\psi+1)(1-\Xi_t)^2} = 0 \tag{B.27}$$

r_t:

$$\mu_t^{(3)} \frac{w_t N_t}{\gamma(1-\delta)K} + \mu_t^{(7)} w_t N_t (1 - \Xi_t + \eta_t) - \mu_t^{(10)} - \mu_t^{(12)} \eta_t w_t N_t = 0 \tag{B.28}$$

η_t:

$$\mu_t^{(5)} \gamma\chi(1-\delta) + \mu_t^{(7)} R_t w_t N_t + \mu_t^{(9)}(1+\zeta Q_t) + (\gamma\chi(1-\delta)K_t + (1+\zeta Q_t)b_t - R_t w_t N_t)\mu_t^{(12)} = 0 \tag{B.29}$$

τ_t^K:

$$\lambda_t^{(2)} r_t K_t - \mu_t^{(6)} r_t = 0 \tag{B.30}$$

τ_t^w:

$$\lambda_t^{(2)} w_t N_t - \mu_t^{(4)} w_t = 0 \tag{B.31}$$

B.2.2 Model Equations in Steady State

$$C + \vartheta K = Y \tag{B.32}$$

$$\tau^K RK + \tau^w wN = (1 + \zeta Q - Q)b + Z \tag{B.33}$$

$$\vartheta = \delta + (1-\delta)\gamma(1-\chi)\Lambda \tag{B.34}$$

$$\Lambda = \frac{\bar{V}^2 - (1-\sigma\sqrt{3})^2}{4\sigma\sqrt{3}} \tag{B.35}$$

$$(1-\tau^w)w = C \tag{B.36}$$

$$\bar{V} = \frac{rwN - (1+\zeta Q)b}{\gamma(1-\delta)K} \tag{B.37}$$

$$R = \gamma(1-\delta)\Delta + \alpha\frac{\theta-1}{\theta}AK^{\alpha-1}N^{1-\alpha} + \eta\gamma\chi(1-\delta) \tag{B.38}$$

$$\beta((1-\vartheta) + (1-\tau^K)R) = 1 \tag{B.39}$$

$$rwN(1-\Xi+\eta) = \frac{\theta-1}{\theta}(1-\alpha)Y \tag{B.40}$$

$$\Xi = \frac{\bar{V} - (1-\sigma\sqrt{3})}{2\sigma\sqrt{3}} \tag{B.41}$$

$$Q = \beta(1+\zeta Q)(1-\Xi+\eta) \tag{B.42}$$

$$r = \frac{\psi}{(\psi+1)(1-\Xi)} \tag{B.43}$$

$$rwN \leq \gamma\chi(1-\delta)K + (1+\zeta Q)b \tag{B.44}$$

$$Y = AK^{\alpha}N^{1-\alpha} \tag{B.45}$$

The steady state conditions for the first-order conditions follow:

C:

$$\frac{1}{C} - \lambda^{(1)} - \mu^{(4)} - \mu^{(6)}\frac{(1-\beta)}{C}((1-\vartheta)+(1-\tau^{K})r) - \mu^{(9)}\frac{(1-\beta)}{C}(1+\zeta Q)(1-\Xi+\eta) = 0 \tag{B.46}$$

K:

$$\begin{aligned} &-\lambda^{(1)}(1-\beta(1-\vartheta)) + \beta\lambda^{(2)}\tau^{K}r - \beta\mu^{(3)}\frac{RwN-(1+\zeta Q)b}{\gamma(1-\delta)K^{2}} \\ &+\beta\mu^{(5)}\alpha(\alpha-1)\frac{\theta-1}{\theta}AK^{\alpha-2}N^{1-\alpha} + \alpha\beta\mu^{(11)}AK^{\alpha-1}N^{1-\alpha} + \mu^{(12)}\eta\gamma\chi(1-\delta) = 0 \end{aligned} \tag{B.47}$$

N:

$$\begin{aligned} &-1 + \lambda^{(2)}\tau^{w}w + \mu^{(3)}\frac{Rw}{\gamma(1-\delta)K} + \mu^{(5)}\alpha\frac{\theta-1}{\theta}AK^{\alpha-1}N^{-\alpha}(1-\alpha) \\ &+\mu^{(7)}Rw(1-\Xi+\eta) + \mu^{(11)}(1-\alpha)AK^{\alpha}N^{-\alpha} - \mu^{(12)}\eta Rw = 0 \end{aligned} \tag{B.48}$$

Y:

$$\lambda^{(1)} - \mu^{(7)}\frac{\theta - 1}{\theta}(1 - \alpha) - \mu^{(11)} = 0 \tag{B.49}$$

r:

$$\beta\lambda^{(2)}\tau^K K + \mu^{(6)}\beta(1 - \tau^K) - \beta\mu^{(5)} = 0 \tag{B.50}$$

w:

$$\lambda^{(2)}\tau^w N + \mu^{(3)}\frac{RN}{\gamma(1-\delta)K} + \mu^{(4)}(1 - \tau^w) + \mu^{(7)}RN(1 - \Xi + \eta) - \mu^{(12)}\eta RN = 0 \tag{B.51}$$

Q:

$$\lambda^{(2)}(1 - \zeta)b - \mu^{(3)}\frac{\zeta b}{\gamma(1-\delta)K} + \mu^{(9)}\zeta(1 - \Xi + \eta) - \mu^{(9)} + \mu^{(12)}\eta\zeta b = 0 \tag{B.52}$$

b:

$$\lambda^{(2)}Q(1 - \zeta\beta) - \beta\mu^{(3)}\frac{1}{\gamma(1-\delta)K} + \beta\mu^{(12)}\eta(1 + \zeta Q) = 0 \tag{B.53}$$

ϑ:

$$-\beta\lambda^{(1)}K - \beta\mu^{(1)} - \beta\mu^{(6)} = 0 \tag{B.54}$$

Δ:

$$\mu^{(1)}(1 - \delta)\gamma(1 - \chi) - \mu^{(2)} + \gamma(1 - \delta)\mu^{(5)} = 0 \tag{B.55}$$

$\bar{V}$:

$$\mu^{(2)}\frac{\bar{V}}{2\sigma\sqrt{3}} - \mu^{(3)} + \mu^{(8)}\frac{1}{2\sigma\sqrt{3}} = 0 \tag{B.56}$$

Ξ:

$$-\mu^{(7)}RwN + \mu^{(8)} - \mu^{(9)}(1+\zeta Q) + \mu^{(10)}\frac{\psi}{(\psi+1)(1-\Xi)^2} = 0 \tag{B.57}$$

R:

$$\mu^{(3)}\frac{wN}{\gamma(1-\delta)K} + \mu^{(7)}wN(1-\Xi+\eta) - \mu^{(10)} - \mu^{(12)}\eta wN = 0 \tag{B.58}$$

η:

$$\begin{aligned} &\mu^{(5)}\gamma\chi(1-\delta) + \mu^{(7)}RwN + \mu^{(9)}(1+\zeta Q) \\ &+ (\gamma\chi(1-\delta)K + (1+\zeta Q)b - RwN)\mu^{(12)} = 0 \end{aligned} \tag{B.59}$$

τ^K:

$$\lambda^{(2)}rK - \mu^{(6)}r = 0 \tag{B.60}$$

τ^w:

$$\lambda^{(2)}wN - \mu^{(4)}w = 0 \tag{B.61}$$