The Migration of Hot Jupiters and our Outer Planets

Author: Jeremy Lawson
Instructor: Paul Seidelmann
Department of Astronomy
University of Virginia

This thesis is submitted in completion of the requirements of the BS Astronomy-Physics Major.

Date: May 7, 2019

Abstract

Our understanding of planetary migration has greatly increased since the discovery of exoplanets. A few decades ago, we had little to no knowledge of exoplanets, and our understanding of planetary migration came from the eight planets that are in our solar system. In this paper I will discuss various methods planets migrate and how they settle into stable orbits. I will also discuss some interesting examples of planetary migration such as the migration of the gas giants in our solar system and the unusual formation of Hot Jupiters.

1 Introduction

Planetary migration is a very complex problem. Since the discovery of over three thousand exoplanets our understanding of planetary migration has increased exponentially. We have classified multiple different types of planetary migration, such as type I, type II, planet-planet scattering and planetesimal scattering. There are other types of migration, but these are the main four that seem to be the most influential. All four of these types will be discussed in greater detail in Section 2. The migration of planets is a very slow process, relative to our life span. The time it takes a planet to migrate can be tens of millions of years, or as much as 1.1 Gy in simulations by Gomes et al., (2005). Knowing this it should be noted that a significant amount of the information we have of planetary migration comes from computer simulations, not from direct observations of these migrations occurring from start to finish. Type I and Type II migration are important when determining the final position of a planet. While Type I migration occurs for smaller planets and Type II occurs for larger planets, both are important due to the fact that large planets were once small, when they were still just protoplanets. These two methods are the main methods that planets migrate toward their stable orbits (Lubow & Ida, 2010). Planet-planet scattering can be used to explain more unusual positions of planets, such as Hot Jupiters, and is relevant to the migration mechanics of typical planets as well. Planet-planet scattering occurs when two planets come close enough to one another to have a significant gravitational influence over one another. This influence will lead one be launched further away from the star, or possibly even out of the system, and the other launched closer to its star (Ford & Raio, 2008). Planet migration due to planetesimal scattering is very similar to planet-planet scattering, but the affect is less significant and more random. The randomness is due to the fact that these planetesimals can come from all directions, causing many small "nudges", instead of a single large one due to another planet. These many random nudges cause a small random walk for the planets being affected. Most of these migration types are only possible early in the evolution of a solar system, due to the plentiful gas, dust, and planetesimals. The only one that seems possible outside of this period is planet-planet scattering, which is unlikely to happen in developed solar systems where the planets have achieved resonance and are in stable orbits.

From the resources I have read for this report, it seems that planetary migration stops, or at least "pauses" when the planet reaches a resonance with other planets. The mechanism that seems to cause this is a combination of two factors. The first is when substances that are causing the instabilities (i.e. gas, dust, and numerous planetesimals) begin to deplete and, therefore, no longer affect the planet. The second is when the planet is in resonance with the other planets, the torques that affect the planet counter one another and the planet is now balanced among the other planets.

Planetary migration is the only method that can explain the position of some planets, such as Hot Jupiters. A Hot Jupiter is a planet that has a mass comparable to the mass of Jupiter's, but it orbits much closer to its parent star. Hot Jupiters simply could not have formed as close to their parent star as they are. This is due to the fact that the ability for a planet to collect gas during its formation is limited by its temperature (Cossou et al., 2014). This means that by the time the protoplanet would have cooled down enough to capture a significant amount of gas, the majority of the gas in the inner solar system would have been blown away or captured by the star. With this information it is reasonable to say that planetary migration is the only method that can form Hot Jupiters. However, the type of migration that is most likely to create these Hot Jupiters is still up for debate.

Our own solar system seems to have gone through significant migration in its early days. Uranus and Neptune posed a challenge to the theories of planetary formation due to the fact that they exist in a region of the solar system where long dynamical timescales and a lack of material to accrete would have made the formation of planets that are ~ 15 and $17 M_{\oplus}$, respectively, very difficult (Thommes et al., 2002). The migration of the planets in our solar system seem to be understood far better than that of other systems, due to the fact that we have far more reliable data of our own system than others. The simulations used to show some scenarios of the migration of the giant planets can be used to explain phenomena that we see in our solar system today. For example, the "Nice Model" (Levison 2007) can be used to explain the positions of the giants now, as well as the Late Heavy Bombardment of the terrestrial planets. (Levison 2007) This will be explored further in Section 4.

2 Planetary Migration

Planetary migration is the process where a planet's orbital radius changes over time. The main cause for gas giant migration is the gravitational interaction of the protoplanet with the gaseous disk from which it was formed (Lubow & Ida, 2010). This type of migration has two main types, which are simply called Type I and Type II.

2.1 Type I Migration

For Type I migration, the gaseous disk that is near the planet cause torques to arise. This torque can cause either inward or outward migration. For outward migration the gas must be somewhat inside the orbit of the planet $r < a$ (but outside of the coorbital region). Torques that are caused by gas outside the orbit of the planet cause inward migration. When modeling Type I migration a cylindrical coordinate system centered on the star was used by Lubow & Iba (2010). The perturbing potential of a circular orbit planet was expanded in a Fourier series as

$$\Phi(r, \theta, t) = \sum_m \Phi(r, a) cos[m(\theta - \Omega_p t)] \qquad (1)$$

Here, m is a nonnegative integer, and Ω_p is the orbital frequency of the planet. There is a torque associated with each m-value due to the effects of the density perturbation for each m. The sum of these torques is the net torque on the planet. Results by Goldreich & Tremaine, (1979, 1980) show that the gas responds by the effects of two types of resonances, Linblad resonances and corotational resonances.

At certain radii, r_m, the gas respsonse to a particular Fourier potential component, m in equation (1), is strong. This is where the Lindblad resonance is satisfied. The Lindblad resonance occurs for gas that periodically passes by the planet. These passes at specific radii are referred to as mean motion resonances. For each value of m, there are two Lindblad resonances, shown in Equation 2.

$$\Omega(r_m) = \frac{m\Omega_p}{m \mp 1} \qquad (2)$$

Here $\Omega(r)$ refers to Equation 3.

$$\Omega(r) = \sqrt{\frac{GM_s}{r^3}} \qquad (3)$$

where M_s is the mass of the parent star. The two resonances refer to an outer Lindblad resonance, which occurs outside the orbital radius of the planet, and an inner Lindblad resonance. At both of these Lindblad resonances, spiral waves are launched into the disk and propagate away from the source. These waves can modify the underlying disk density distribution, $\Sigma(r)$, and open gaps as they are damped.

The co-rotation resonance in the disk is simpler than the Lindblad resonance. The condition is described simply in Equation 4.

$$\Omega(r_m) = \Omega_p \qquad (4)$$

This means that the co-rotation resonance occurs at the orbital radius of the planet. This resonance does create disturbances within the disk, but they do not propagate and instead remain trapped within a radial region whose size is of the order of the disk thickness. (Lubow & Ida, 2010)

2.2 Type II Migration

In Type II migration a planet's tidal torques cause material interior to the orbit of the planet to lose angular momentum and material exterior to the orbit of the planet to gain angular momentum. The material losing angular momentum accretes onto the planet and the material that gains angular momentum migrates away from the planet. This process opens gaps through the orbit of the planet. This type of migration occurs for larger planets due to the fact that they need to clear out their orbit early in the development of the system. To open a gap in the disk, the torque causing the gap must be greater than the torque closing the gap, $T_o \gtrsim T_c$. Using this the condition on the planet's mass required to open a gap can be found.

$$\frac{M_p}{M_s} \gtrsim C_g \left(\frac{\nu}{a^2\Omega_p}\right)^{1/2} \left(\frac{H}{a}\right)^{3/2} \qquad (5)$$

Here, C_g is a dimensionless number of order unity, which is estimated to be $2\sqrt{10}$ by Lin & Papaloizou (1986). H is the thickness of the disk and ν is the viscosity of it. The migration of a planet within a gap is quite different from the Type I migration discussed earlier. A planet that opens a gap in a disk, where the disk has a much greater mass than the planet, would be expected to move inward, pushed along with the disk accretion inflow, but the planet communicates the viscous torques across the gap by means of tidal torques that balance them. This means that Type II migration timescale is of the order of the disk viscous timescale, which is shown in Equation 6.

$$t_{vis} \sim \frac{a^2}{\nu} \sim \frac{a^2}{\alpha c H} \sim \left(\frac{a}{H}\right)^2 \frac{1}{\alpha\Omega_p} \qquad (6)$$

This shows that the migration timescale can be much longer than the Type I migration timescale for higher mass planets that open gaps (Lubow & Ida, 2010).

2.3 Planet-Planet Scattering

As mentioned previously, planet-planet scattering is the change of orbital radii of planets due to close gravitational interaction with one another. Unlike the previous two migrations methods described, Type I and II, this type of migration can happen at any point of time in the lifetime of a solar system. Even after the system stabilizes there is always a possibility of a rouge planet coming through the system, or a planet that was moved to an extremely long and eccentric orbit can make a pass through the system and possibly scatter planets. However, the scientific consensus seems to show that planet-planet scattering is not a common occurrence later in the system but is possible (D'Angelo et al. 2006). According to Ford & Raio (2008), in planetary systems with two or more giant planets that are in dynamical instability, planets can collide with one another or be scattered through strong planet-planet scattering. These scattered planets can explain the unexpected existence of Hot Jupiters. A combination of planet-planet scattering, and tidal circularization can help explain them. Orbital migration due to planet scattering can also play an important role in explaining the distribution of orbital periods found by radial velocity surveys. For these reasons it is important to understand planet-planet scattering and under what conditions they can occur. Through simulations conducted by Ford & Raio (2008), it has been observed that when two planets are not near equal mass, they tend to collide, while planets of near equal mass are more likely to scatter. This simulation reproduced observed eccentricities with a plausible distribution of planet mass ratios.

2.4 Planetesimal Scattering

Planets that are embedded in a planetesimal disk will migrate as a result of angular momentum and energy conservation, like that of planet-planet scattering, but with smaller effects. Planetesimal scattering is a significant form of migration that becomes most significant when the gas disk in a forming system has dissipated. When the gas dissipates and there are still a significant number of planetesimals in the system, then planets can migrate as a result of gravitational encounters with these objects. For instance, if a planet is surrounded by small bodies it will recoil every time it gravitationally scatters one of these objects. However, since the objects can come from any direction, the planets that are affected by this undergo small random walks in the semimajor axis. Although, since the objects tend to be on one side or another of the planet, it would cause a net flux of material being scattered inward or outward. The planet, of course, would move in the opposite direction to conserve energy and angular momentum. If there are no other strong gravitational perturbations from other planets, the semimajor axis of the planet will smoothly change with time. This is called simple migration by Levison in *Planet Migration in Planetesimal Disks* (2007). Planet migration like this is heavily dependent on specific features of the system, such as: number of planets, their separations, their masses and mass ratios, the disk's mass and radial extent, its radial surface density profile etc.

2.5 Resonance Capture

To stop the migration of a planet it must be captured in a resonance or a chain of resonances. A chain of resonances is when multiple planets are in resonance. An example of this is the resonance chain of Jupiter's moons Ganymede, Europa and Io, which have resonance 1:2:4. These resonances can be temporary, if the planetary system is unstable due to many planetesimals that have not been ejected or if it still has gas that has not been accreted or blown away by solar winds If these factors are still present then the planets can still migrate. Once the planetesimals have been scattered or been formed into planets, and the gas has successively accreted or been blown away, the planets can come into resonance with one another. (Cossou et al., 2014)

3 Hot Jupiters

Hot Jupiters are an unexpected discovery when exoplanets were being discovered. These planets were not expected because our understanding of planetary formation does not allow the formation of large gas giants to form close to its parent star. From these observations we had to form theories of how these unusual planets could have formed. The scientific consensus is that Hot Jupiters formed far from the star, like all other gas giants, and then migrate inward to find a stable orbit near the star.

3.1 Formation of Giant Planets

To understand why Hot Jupiters cannot form near the star it would be useful to have an overview of how giant planets are formed. In the core accretion model, gas giants form in two steps. The solid core, which is $5 - 10M_\oplus$ is formed. The core then gravitationally captures gas from the surrounding gaseous disk. This capture is slow at first and is limited by the core's ability to cool and contract. The gas begins to form an envelope around the core and once the envelope mass becomes comparable to the core mass, it can enter a runaway phase where the protoplanet begins to quickly accrete gas and it can grow to a Jupiter-size planet within $\sim 10^5$ years. (Cossou et al., 2014)

3.2 Formation of Hot Jupiters

Runaway migration seems to be a good candidate to account for the characteristics of Hot Jupiters. Planets that undergo runaway migration are known to cluster at short periods. However, planets of greater than two Jovian masses rarely have short periods. This indicates different types of migration process operated for the two classes of objects. Furthermore, it is shown that in the runaway regime, migration can be directed outward. This makes the regime potentially rich in a variety of important effects in shaping a planetary system, during the last stages of its formation. Hot Jupiters are likely to have started to form further in the disk and migrate inward to the positions they are found in. When the planet's mass is still small enough for the Hill radius to be much smaller than the disk thickness, the migration rate can be evaluated. Using linear analysis, the proportionality of the planet mass and disk surface density is shown to be inversely proportional to the square of the disk aspect ratio. Type I migration is too fast to produce Hot Jupiters. This shown by the fact that the timescale of Type I migration is shorter than the time it takes for giant protocores to form. (Masset & Papaloizou, 2003)

Next, the formation of Hot Jupiters is looked into using Type II migration. As mentioned before, Type II migration of a protoplanet occurs when it has a mass sufficient to open a gap in the disk. This opening of the disk splits it into an inner disk and an outer disk. The protoplanet then finds itself locked into the disk. The protoplanet can then undergo Type II migration toward the star. While it is migrating toward the star it can accrete the surrounding gas and dust from the disk. The time it takes Type II migration to occur can be shown by $\tau_{migII} \sim r^3/3\nu$, where τ_{migII} is the timescale of Type II migration and ν is the viscosity of the disk. Using this timescale and the time it takes for the giant protocore to form, it has been shown that Type II migration is a process that can form Hot Jupiters. (Masset & Papaloizou, 2003)

Another possible explanation of Hot Jupiters is migration through planet-planet scattering. In this model of planetary migration, with tidal circularization, Hot Jupiters were shown to have formed by simulations by D'Angelo et al., 2006. A planetary system with multiple giant planets can become unstable. This can lead to a collision or the ejection of one of the planets from the system. The ejection of one planet from the system can lead to the other one migrating further inward and reaching a stable orbit with a short period. However, simulations for two planets of unequal masses showed a reduced frequency of collisions, but planets of equal mass were scattered more frequently. These simulations reproduced the observed eccentricities of Hot Jupiters. (Masset & Papaloizou, 2003)

4 Migration of the Outer Planets

The existence of Uranus and Neptune, at their current positions, is a challenge to our current understanding of planetary formation. The problem is that these planets exist in a place of the solar system where there was little gas available to accrete. This would make the formation, of bodies with ~ 15 and $17 M_{\oplus}$ very difficult (Thommes et al., 2002). Instead of changing our theories of planetary formation it was thought that the planets had migrated instead. Thommes et al., (2002) shows that Uranus and Neptune can be scattered from the Jupiter-Saturn region by Jupiter and Saturn, and this scattering had the ice giants ending up on the orbits we see today. There is a model, called the Nice Model, that can explain many phenomena in our solar system, which seems to be more in depth and accurate than the simulations in Thommes et al., 2002. Some of these phenomena are: the capture of Jupiter's Trojans (Morbidelli et al., 2005), the capture of the irregular of the giant planets (Nesvorný et al., 2007), the absence of regular moons beyond Oberon's orbit at Uranus (Deienno et al., 2001), and others. However, the original version of the Nice Model has a major problem. The initial conditions had the planets migrate from a specific resonant configuration. This is a problem because there are other resonant configurations that are possible. Some of these configurations even explain more phenomena than the original Nice Model. (Deienno et al., 2017)

4.1 Nice Model

The "Nice Model" is a model that is used to explain the present conditions of the giant planets. (Levison et al., 2007) This model has initial conditions that are intended to represent the outer solar system at the time when the gaseous disk had dissipated. The giant planets are assumed to be on nearly circular orbits as well as coplanar orbits, which is what is expected from the theory of formation of giant planets. A configuration of the planets that have orbits much smaller than the orbits seen today was assumed for this model. To be more precise, the giants in the planetary system are assumed to be in the range of ~ 5.5 to 14au. Jupiter and Saturn are placed closer to the Sun than the Uranus and Neptune, Saturn is assumed to be closer to Jupiter than their mutual 1:2 Mean Motion Resonance (MMR). This is a condition required to avoid a substantial amount of Type II migration during the life time of the gaseous disk. This compact system is consistent with the constraints on the formation timescales for Uranus and Neptune (Levison et al., 2007). A planetesimal disk is assumed to be beyond the orbit of the giant planets, so the planets can scatter them inward and have the giant planets migrate outward. It is also assumed that the particles inhabit regions where the dynamical lifetime of the individual objects is of the order of the gas disk lifetime, which is about a million years, or possibly longer. Using the lifetime of the gas disk and the lifetime of the planetesimal disk, the inner edge of the planetesimal disk was found to be 1.5au beyond the location of the outermost planet. The outer edge was assumed to be $\sim 34au$. To reproduce characteristics we observe in our solar system a mass of $\sim 35M_{\oplus}$ was assumed for the disk. (Levison et al., 2007)

With the configuration described above, the planetesimals in the inner disk began to go through planet-scattering orbits on a timescale of a few million years. Due to the small masses of the planetesimals compared to the giant planets, the migration of the giant planets is very slow, because the migration is governed by the rate of the planetesimal escape rate, which is slow. After a period of time from 60My to 1.1Gy, from simulations by Gomes et al. (2005), Jupiter and Saturn cross their mutual 1:2 MMR. The timing of this migration was found to be consistent with the timing of the Late Heavy Bombardment (LHB), which is estimated to be 650My after the formation of the planets. It should be noted that the resonance crossing excites their eccentricities to values that are slightly larger than the values currently observed. This increase in eccentricities is a problem because they also increase the eccentricities of Uranus and Neptune to where they begin to approach each other. Both of the ice giants scattered outward, onto large eccentric orbits ($e \sim 0.25 - 0.4$). This migration destabilizes the rest of the planetesimal disk and scatters them throughout the solar system. This causes the eccentricities of Uranus and Neptune to dampen, as well as the eccentricities of Jupiter and Saturn, to a lesser extent. This happens on a timescale of a few My. (Levison et al., 2007)

During and after the eccentricity damping phase, the giant planets continue to migrate radially and eventually reach their final orbits. By this time the majority of the disk has been scattered. The final outcome of the simulations of the Nice Model reproduce quantitatively the current aspects of our giant planets, such as the semi-major axes, eccentricities and the inclinations. (Levison et al. 2007)

As mentioned before, this model also explains the LHB. The sudden destabilization of the planetesimal disk, caused by the migration of Uranus and Neptune into it, scattered many objects from the outer solar system into the inner system. The rapid migration of Jupiter and Saturn, from their 1:2 MMR, destabilized approximately 90% of the asteroids in the asteroid belt. With both of these destabilizations, the LHB was caused and was shown to last $\lesssim$ 100My. The magnitude of the bombardment is also consistent with the constraints placed on it by the Lunar crater rate. (Levison et al., 2007)

A fraction of the planetesimal disk was captured into Trojan orbits for Jupiter and Saturn as they migrated away from their 1:2 MMR. The total mass of these Trojan asteroids is very similar to the total observed mass. Neptune's Trojans are captured as the giants migrated as well. These Trojans are at many different inclinations, but they are consistent with the observed inclinations. The final phenomena that was explained by the Nice Model is that during the planet-planet interactions with Saturn, Uranus, and Neptune, some irregular satellites were captured, which are similar to ones observed. (Levison et al., 2007)

There is a major problem with the Nice Model. The initial conditions of the planets were chosen in an ad hoc manner. It was discovered that when the giant planets were still embedded in a disk of gas, the planets should have acquired a fully resonant configuration. This resonant configuration should be: Jupiter and Saturn preferentially locked in their mutual 3:2 MMR with Jupiter at about 5.4au, Saturn and Uranus in 3:2 MMR, and Saturn and Neptune in a 4:3 MMR. Although, other configurations are possible. (Deienno et al., 2017)

The Nice Model was improved by adding a fifth giant planet that encounters Jupiter while it is migrating through the planetesimal scattering period. The fifth planet was ejected onto a hyperbolic orbit. Nesvorný (2011) and Batygin et al., (2012) suggested that the solar system originally had a fifth giant planet. The planet is thought to be an ice-giant that had mass comparable to Uranus or Neptune's mass. This scenario increases the probability that when Jupiter migrates, due to planetesimal scattering, its evolution ends with four planets near their current orbits.

Nesvorný & Morbidelli (2012) investigated which resonant configurations would be more likely to reproduce the current orbital configuration. They considered four criteria of success and they found multiple initial conditions that met all of the non-negligible fraction of the simulations. All four of these configurations require the existence of a fifth giant, with a mass comparable to Uranus or Neptune, which was ejected from the system.

5 Conclusion

In conclusion, planetary migration was researched, and four main methods were described. Type I migration was shown to last for a shorter period than Type II, it occurs for lower mass planets than Type II does, and it can occur for smaller planets. Although, once the planet accretes enough mass it can clear a gap in the gas disk and the migration can transition from Type I to Type II. The next two types of migrations that were mentioned were planet-planet scattering and planetesimal scattering. Both of these types of scattering operate in similar ways. When one object comes close to another, they gravitationally interact with one another and they are "pushed" to different positions. The main difference between the two is that planet-planet scattering causes a massive effect and significant migration. Planetesimal scattering has a much smaller effect than planet-planet scattering, but due to the massive amount of planetesimals in the disk, it can still have a significant effect. The processes of stopping these migrations was also looked into, and from the resources available it seems that migration stops when the gaseous disk or planetesimal disk are dissipated and the planets gain a balance with one another.

Next, the formation of Hot Jupiters was researched and it was discovered that they could not have formed where they are found. The planets would have to migrate inward. With the types of migration that were discussed it is apparent that the most likely scenario for gas giants to migrate inward is Type II migration or planet-planet scattering.

Lastly, the migration of the outer planets were discussed. The fact that Uranus and Neptune were not likely to form were they presently are was looked into. Using a model of the solar system called the Nice Model and an improvement of it, it was shown that there are multiple initial conditions that can reproduce what is observed in our solar system today.

References:

Batygin, K. & Brown, M. E. & Betts, H. 2012. ApJL, 744, L3

Cossou, C. & Raymond, S. N. & Hersant, F. & Pierens, A. 2014. *Hot super-Earths and giant planet cores from different migration histories.* arXiv:1407.6011

D'Angelo, G. & Lubow, S. H. & Bate, M. R. 2006. *Evolution of Giant Planets in Eccentric Disks* Astrophys. J., 652, 1698-1714.

Deienno, R. & Yokoyama, T. & Nogueira, E. C. & Callegari, N. Jr. & Santos, M. T. 2011. A&A, 536, A57

Deienno, R. & Morbidelli, A. & Gomes, R. S. & Nesvorný D. 2017. *Constraining the giant planets' initial configuration from their evolution: implications for the planetary instability* arXiv:1702.02094

Ford, E. B. & Raio, F. A. 2008. *Origins of Eccentric Extrasolar Planets: Testing the Planet-Planet Scattering Model.* https://doi.org/10.1086/590926

Goldreich, P. & Tremaine, S. 1979. *The Excitation of Density Waves At the Linblad and Corotation Resonances By an External Potential.* Astrophys. J., 233, 857-871.

Goldreich, P. & Tremaine, S. 1980. *The Dynamics of Planetary Rings.* Ann. Rev. Astron. Astrophys., 20, 249-283.

Gomes, R. & Levison, H. F. & Tsiganis, K. & Morbidelli, A. 2005. *Origin of the cataclysmic Late Heavy Bombardment period of the terrestrial planets.* Nature 435, 466-469.

Levison, H. F. & Morbidelli, A. & Gomes, R. & Backman, D. 2007. *Planet Migration in Plan-*

etesimal Disks. Protostars and Planets V, ed. B. Reipurth, D. Jewitt, & K. Keil (Tucson: Univ. Arizona Press), 669

Levison, H. F. & Morbidelli, A. & Laerhoven, C. V. & Gomes, R. & Tsiganis, K. 2007. *Origin of the Structure of the Kuiper Belt during a Dynamical Instability in the Orbits of Uranus and Neptune.* arXiv:0712.0553.

Lin, D. N. C. & Papaloizou, J. C. B. 1986. *On the tidal interaction between protoplanets and the protoplanetary disk. III - Orbital migration of protoplanets.* Astrophys. J., 309, 846-857.

Lubow, S. H. & Ida, S. 2010. *Planet Migration* arXiv:1004.4137

Masset, F. S. & Papaloizou, J. C. B. 2003. *Runaway Migration and the Formation of Hot Jupiters* Astrophys. J., 588, 494-508.

Morbidelli, A. & Levison, H. F. & Tsiganis, K. & Gomes, R. 2005. Nature, 435, 462

Nesvorný, D. & Vokrouhlický, D. & Morbidelli, A. 2007. AJ, 133, 1962

Nesvorný, D. 2011, ApJ, 742, 22

Nesvorný, N. & Morbidelli, A. 2012. AJ, 144, 117

Thommes, E. W. & Duncan, M. J. & Levison, H. F. 2002. *The Formation of Uranus and Neptune among Jupiter and Saturn.* The Astronomical Journal 123, 2862-2883.