Possible Effects of a Course Enhancement on Elementary Pre-Service Teachers

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Table of Contents

Abstract

There is a growing need for qualified teachers at all levels of education in the United States, but it is becoming hard to find such candidates. Traditional teacher preparation programs have recently experienced an increase in criticism regarding their lack of ability to produce qualified teachers. One reason for such criticism is based on a lack of consensus regarding the best practices for teaching pre-service teachers. Many colleges and universities face similar difficulties related to teacher preparation programs, yet it seems hard to find a remedy for these teacher preparation shortcomings. This research focuses on the effects that a course enhancement has on pre-service elementary school teachers' mathematical content knowledge, general teacher readiness, as well as their self-efficacy in teaching. The course enhancement involves careful consideration of components that adhere to the Virginia Standards of Learning, teaching licensure requirements for the state of Virginia, as well as topics seen on the Praxis core mathematics exam. There is a very little to no research focusing on the effects of a course enhancement, in which case the results from this study provide insight into the most beneficial components related to teacher preparation programs.

Keywords: teacher preparation program, elementary pre-service teacher, mathematical content knowledge, mathematic pedagogical content knowledge, self-efficacy, course enhancement

Introduction

There are many things that influence an individual on a daily basis. These influences include a person's background, financial status, family, as well as educational experiences. With such a heavy reliance on education to shape the lives of young individuals, it is important to ensure that education is as effective as possible. To this end, the importance for pre-service teacher preparation programs to produce quality teachers is exposed. This investigation examines the effects that a course enhancement has on elementary pre-service teachers. The study takes place at Waverly College (WC), which is a small, private, liberal arts college located in southeastern United States.

In addition to traditional university-based teacher preparation programs, there are a number of alternate route programs for an individual to obtain his or her teaching licensure. Regardless of the type of program, evidence suggests that preparation programs are not adequately preparing teaching candidates for a career in teaching and therefore too many candidates do not demonstrate a suitable level of content knowledge required to be an effective teacher (Olson, Tippett, Milford, Ohana, & Clough, 2015; Waddell & Vartuli, 2015; Darling-Hammond, Chung, & Frelow, 2002). Criticisms surrounding university-based teacher preparation programs have been evident since the 1980's, but with the recent questions surrounding the effectiveness of preparation programs, criticism has once again surfaced (Darling-Hammond, 2010).

Teaching is an ever changing profession, not only from year to year through the implementation of new standards, but also on a daily basis in the classroom. Teachers must adapt to constantly changing situations and monitor and adjust instruction based on the conditions they encounter. While it may be easy for someone to criticize teachers, as well as preparation programs, many critics are unaware of the complexities involved with teaching and teacher preparation programs. Needless to say, teaching is difficult even when done correctly. In an effort to ease critics and to gain an understanding of some of the difficulties associated within Teacher Preparation Programs (TPP), I present my findings of an investigation regarding the enhancement of a mathematical course sequence for pre-service elementary school teachers.

Problem Statement

The purpose of this study is to investigate pre-service elementary school teachers' mathematical content knowledge (MCK) at WC. Teaching candidates are required to complete 12 hours in mathematics as part of their degree program. Half of these credits come from a two-semester math course sequence titled Math 117 and Math 118, Introduction to School Mathematics I and II. The major topics within these courses include: problem solving, whole numbers and computation, rational numbers, geometry, as well as probability and statistics. The other six credits include a 3-credit general education mathematics course and a 3-credit elementary math / science methods course. The general education course options include basic liberal arts mathematics which overviews a number of different mathematical topics, problem solving mathematics, or statistics. The elementary science methods course integrates science principles with a laboratory opportunity to expose teaching candidates to practical teaching methodology.

The main focus and rationale for the Math 117 / 118 course sequence, which exists specifically for pre-service teachers, is not only to prepare candidates for a career in teaching mathematics, but also to ready them for the Praxis core mathematics exam. A passing score on this exam is needed for teaching candidates to be admitted into the TPP at WC, as well as to obtain their teaching licenses. This study investigates how a newly designed mathematical course sequence affects pre-service teachers' mathematical content knowledge, general readiness for teaching, and self-efficacy in teaching mathematics. The course sequence was structured in a way that adheres to the requirements set forth within the Virginia Standards of Learning (SOL's), state licensure requirements, as well as topics seen on the Praxis exam. In addition to addressing these requirements, the course offers teaching candidates practical teaching experiences through video lecture cases, the use of mathematical manipulatives, and analysis of student errors via whole class discussion. Furthermore, pre-service teachers are exposed to the proper scaffolding techniques used within a specific mathematical topic, learn how to construct unit lesson plans, and finally gain insight regarding all components involved with different forms of assessment.

Recently Waverly College experienced a low pass rate on the Praxis core mathematics examination, presenting major concerns. For pre-service teachers, similar to many other university-based programs, a passing score on the Praxis is needed in order to formally gain admittance into a TPP and obtain a teaching license. From Waverly College's standpoint, the importance of pass rates on the Praxis exam is necessary for the accreditation of preparation programs. For example, in order for Waverly College to maintain their teacher education accreditation, there must be an 80% pass rate on the exam, something that was not being regularly achieved. During the past academic year (2015-2016), teaching candidates completed the necessary mathematical course sequence and earned very acceptable grades, yet when the results of the Praxis core mathematics exam were released, there was only a 40% pass rate. With this in mind, it was evident that the course sequence was not adequately preparing teaching candidates for the Praxis exam. Waverly College is not alone in experiencing difficulties with TPP as many other colleges and universities across the United States face similar issues.

To make matters even more difficult and complex, it is becoming harder and harder for schools to find qualified teachers and teaching candidates. Recent research points out that we have witnessed a decline in enrollment numbers within teacher preparation programs (Sawchuk, 2014; Darling-Hammond, Holtzman, Gatlin, & Heilig, 2005). However, there is good reason for such a decline. For an individual to become a teacher, he or she must take on a large financial burden in order to overcome a myriad of requirements, where remuneration is far less than other career choices. Perhaps the main reason why we have such difficulties developing qualified teachers is based on the fact that the most qualified teaching candidates are obtaining jobs outside of education because of the more favorable financial compensation found in private sector jobs.

The significance of this research can also be demonstrated by looking at global comparisons. Even with all of the requirements that are part of teacher preparation programs, data by Schmidt et al. (2011) revel how the United States is lagging behind many other countries with regard to teacher preparedness and consequently student achievement on mathematical content assessments. Through the listed requirements needed for a TPP to be accredited, it is apparent that the demands of these programs put teaching candidates through a sufficient amount of rigor, yet we are still not up to par with other countries in the field of mathematics. For one reason or another, even though teaching candidates are taking the necessary courses to complete a program, they still seem to struggle with the material. There is a large body of research, spearheaded by Shulman's (1986) work, which highlights the importance of content knowledge (CK) and pedagogical content knowledge (PCK) and the differences between them. As defined by Shulman (1987) PCK is "a blending of content and pedagogy related to how particular topics, problems, or issues are organized, represented, adapted, and presented for instruction" (p. 8).

More specifically, mathematics pedagogical content knowledge (MPCK) was another area of focus for this research. There is much debate regarding where more emphasis should be placed within TPP because finding the right balance can be difficult. Generalist teachers, such as elementary education teachers who must teach a number of different subjects, may not have a strong content knowledge background in specific subjects, while some subject specialist teachers lack pedagogy (Olson et al., 2015). When comparing the United States to other countries, pre-service teachers took two fewer mathematics classes, and one more pedagogical class, on average, than A+ performing countries (Schmidt et al., 2011). This investigation looks into the possible effects that a course enhancement has regarding the mathematical content knowledge of elementary pre-service teachers.

Exposure to methods on how to teach mathematics is tough to comprehend. The balance of MCK and MPCK within an elementary pre-service teacher's course preparation program is especially important because these teachers are faced with the responsibility of teaching a number of different subjects. While it is imperative for teachers to understand the content, at times the content they will be teaching is rather basic. Teaching the basic foundational material often proves to be the most difficult because many elementary students have little prior knowledge to build from or reference. When a student does not understand a very fundamental topic, it is left up to teachers to be able to teach that topic in a number of different ways in order to clear up any misconceptions. Whether it is through different modes of representation, using a different entry point, using student errors in a meaningful way, or task selection, an elementary math teacher faces a tough challenge in teaching beginning students. Grossman (2010) states how novice teachers need structured opportunities in order to gain experience in actual teaching practices. Even though this can occur in a field experience course, the course enhancement that is described in this research offers pre-service teachers opportunities to become exposed to actual teaching experiences in a classroom setting.

This investigation serves another purpose by looking into general teacher readiness prior to entering the work force. Many studies and coursework often emphasize specific subject area material. On the other hand, in addition to the subject material, general day to day teaching procedures were emphasized. For example, as defined by Guardino and Fullerton (2014), "*transitions* are open blocks of time when students are not engaged in traditional learning, but are moving from one activity to another" (p. 12). Within a mathematical block, smooth transitions can make for an improved learning environment for students. Fernández and Figueiras (2014) highlight a teacher's impact on the continuity of a mathematics education. Not only must a teacher be sequential with their lessons from day to day, but they must also possess an understanding on how to transition from topic to topic, and conduct sound vertical articulation of subject matter from one grade level to the next. Even though it may be hard to directly measure how well a teacher is able to accomplish continuity within the classroom, such competencies should be addressed within a mathematical course sequence for pre-service teachers. Without this exposure, teaching candidates may be unaware of the role that transitions play in the classroom.

The final purpose of this investigation was to look into the self-efficacy of teaching candidates. The mathematical courses under investigation often produce the most anxiety for some pre-service teachers for various reasons. Research describes how confidence is one of three themes which must be used to classify a teacher as "excellent" (McCullouch, 2016). In many facets of life, the more confident you are in something, the better product you will be able to produce or vice versa. This can be seen in many TPP as pre-service teachers experience additional responsibilities in a clinical experience as they gain confidence and expertise (Hollins, 2011). Darling-Hammond et al. (2002) observe how teachers felt they could reach more students if they were better prepared and had more confidence in their teaching ability. That being said, through a course enhancement, it only makes sense to start building a sense of confidence earlier through a mix of MCK and MPCK. This approach can enable pre-service teachers to be further along when they enter their clinical experiences and subsequent classrooms.

Results from this study are beneficial for instructors to gain an understanding of pre-service teacher struggles within mathematics courses associated with elementary teacher preparation programs. The results obtained will not only be able to aid in the efforts to improve the pre-service teacher programs at Waverly College, but also to aid other post-secondary schools and their understanding of quality pre-service teacher preparation programs. The analysis attempts to provide a rationale for a university-based teacher preparation mathematics course. In other words, I am aiming to provide evidence toward the possible benefits that structuring a mathematics course around certain requirements will have for pre-service teachers. These insights would help to improve university-based teacher preparation programs by answering the following research questions:

1. How is teaching candidates' exposure to various topics and instructional strategies in an elementary mathematics methods course sequence associated with their subsequent mathematical content knowledge and self-efficacy?
2. How are changes in teaching candidates' knowledge associated with changes in their self-efficacy?

Literature Review

In this section, I review research that has been conducted in the area of pre-service teacher preparation programs. I provide view of similar research conducted in the field regarding course enhancements, factors associated with elementary teaching candidates' preparations, as well as areas that are influenced by quality teacher preparation programs including self-efficacy of teachers. Although many importances exist surrounding such topics, Quigley (2011) stresses how important teacher preparation programs and certifications are for education reform efforts. Additional research conducted by Young, Range, Hvidston, and Mette (2015) highlights the importance of the influences that teachers have on student learning. If we hope to be able to effectively implement reforms to education, we must start by ensuring teachers are properly informed and trained.

While many studies have examined teaching candidates' performance dealing with specific mathematical topics and methodology, there is considerably less research regarding the effectiveness of pre-service teacher training based on specific university-based coursework. Hill (2010) notes the importance of elementary teachers' content knowledge and the need for further descriptive information regarding mathematical knowledge for teaching. With the recent subpar performance on the Praxis core mathematics exam by Waverly College students, additional research situated in this topic will aid in the efforts to understand difficulties that teaching candidates are experiencing regarding content knowledge. It is understood that the Praxis I core exam has been a long standing assessment required for teacher licensure and program accreditation (Mahoney, 2015). This exam offers the ability to measure candidates' basic skills, specifically in mathematics, prior to being admitted to a teacher preparation program, a prerequisite that many colleges require as part of their program (Quatroche, Watkins, & Boliner, 2004). Research by Clotfelter, Ladd, and Vigdor (2011) demonstrates a significant correlation between scores earned on the Praxis I core exam and teacher effectiveness. Teachers are the forefront of education and through the results obtained in this research I hope to add to the understanding of improving teacher preparation programs.

Studies of Efforts to Enhance Teacher Education Courses

Coinciding with the notion of enhancing a course through addressing particular requirements set forth by the SOL's and licensure standards, there are numerous studies which focus their efforts on "ambitious instruction" through an instructional activities approach (Kazemi, Franke, & Lampert, 2009; Kucan et al., 2011; Lampert el al., 2013; Steele, 2005). As described by Kazemi et al. (2009), ambitious instruction "requires that teachers teach in response to what students do as they engage in problem solving performances, all while holding students accountable to learning goals" (p. 11). Furthermore, an instructional activity can include any hands-on task conducted in a classroom setting, instructional modules, rehearsals, or other conceptual tasks that present an opportunity to learn.

Teaching candidates are likely to structure their own instruction based on the instruction which guided them through all levels of education. Incorporating the positive effects that instructional activities have had on student development, there is supporting evidence for the implementation of instructional activities and a variety of field experiences integrated into coursework associated with preparation programs (Darling-Hammond, 2007). Results from Steele (2005) indicate significant gains in teaching candidates' MPCK and MCK through the use of video cases in their courses. Video cases provide teaching candidates the opportunity to experience actual situations that they may encounter during instruction. Candidates were then given the opportunity to reflect on how a particular situation was handled and offer insight regarding alternative approaches that could have been used through the means of a whole class discussion. Even though this may be considered a “non-traditional” teaching task, candidates experienced valuable gains to their knowledge for teaching mathematics and were better prepared for ambitious instruction (Steele, 2005).

Ambitious instruction is not limited to particular subjects, such as mathematics. The use of modules for literature education used by Kucan et al. (2011) introduced teaching candidates to many different features of actual teaching. These components included various approaches to understanding content, the importance of selecting appropriate texts, assessment development, as well as enacting a text-based discussion. To this end, not only would teaching candidates learn the content surrounding a particular subject, but they would also be able to experience and familiarize themselves with how they might teach the subject once in that role. These results are supported by similar research conducted by Lampert et al. (2013) who introduce the idea of using “rehearsals” in coursework as a means to convey knowledge. Teaching candidates would first benefit from the knowledge gained regarding a mathematical topic as delivered by the instructor of the course. They would then plan and deliver a shortened enactment of a particular lesson where they were asked purposeful questions by the instructor, similar to what they might expect to encounter in their own classrooms. Finally, they would be provided with feedback from three sources; the teacher evaluator, their peers, and the students they taught. The mathematical course enhancement presented in this research paper includes similar methods as in the aforementioned examples.

Using international comparison to high performing countries, research shows shortcomings in the preparation of teaching candidates in the United States. High performing countries will be referred to as those whose students scored highest on international assessments such as the Third International Mathematics and Science Study (TIMSS). When looking at mathematical content knowledge, teaching candidates in the United States take, on average, two fewer mathematics courses in their preparation programs compared to high performing countries. Lannin et al. (2015) revels that teacher preparation programs in high performing countries offer more experiences that are designed to develop not only mathematical content knowledge, but emphasize the importance of mathematics pedagogical content knowledge.

There is a substantial evidence base which supports certain enhancement approaches. This evidence includes a number of positive effects on teaching candidates' knowledge when they are able to experience actual teaching situations and requirements associated with the profession. These types of enhancement approaches require appropriate task selection which involves a considerable amount of addition preparation time and expose instructors, as well as teaching candidates, to vulnerabilities due to the unknowns that arise (Kazemi et al., 2009). While these types of gains will not come easy and further research supporting similar enhancement efforts is needed, my research, which incorporates some of the same enhancement ideas, will hopefully aid in those efforts.

Factors Associated with Elementary Teaching Candidates' Knowledge

The number of factors which are comprised within the teaching profession present a high demand for additional research of elementary teachers to determine where to focus efforts regarding teacher preparation programs (Hill, 2010). With all of the salient points involved within the teaching profession, there is an ongoing debate regarding which ones deserve highest priority. Many university-based teacher preparation programs have their own vision as to what they believe a highly qualified math teacher should know, leaving little consensus about the best practices for preparing pre-service teachers (Kunker & Murry Orr, 2015; Hollins, 2011; Schmidt et al., 2011). On the other hand, until a body of research, which outlines the qualifications deemed necessary to be considered "highly qualified", is developed, we will continue down this path of disorganization within our preparation programs.

Content knowledge plays a large role in teacher preparation programs and there is a substantial amount of evidence which suggests that teacher preparation programs are not adequately preparing elementary pre-service teachers (Olson et al., 2015; Waddell & Vartuli, 2015; Ball, Thames, & Phelps, 2008; Steele, 2005; Darling-Hammond et al., 2002). This is concerning because in most university-based preparation programs, candidates are required to complete a number of mathematics courses specific to pre-service teachers which focus on content knowledge. In research conducted by Steele (2005), the author indicates significant gaps in knowledge of mathematics and knowledge for teaching mathematics for American teachers. When looking at factors associated with the top performing preparation programs, Darling-Hammond (2010) finds that these programs have certain commonalities among them, including extensive coursework in reading and mathematics content.

Even with positive supporting evidence that content knowledge has on teacher preparation programs, debates over the most important factors continue. Ball et al. (2008) expresses how "just knowing a subject well may not be sufficient for teaching" (p. 404). Pedagogical content knowledge is believed to be an important component of teacher knowledge (Hill, Ball, & Schilling, 2008). As defined by Shulman (1987), PCK is "a blending of content and pedagogy related to how particular topics, problems, or issues are organized, represented, adapted, and presented for instruction" (p. 8).

Through the use of PCK, teachers are able to use a variety of different approaches based on what may be the best for students with different learning styles. In which case, even though it may be hard to find the right combination between the amount of content knowledge and pedagogical content knowledge within teacher preparation programs, there is a large amount of research indicating the importance of PCK required to be an effective teacher (Lannin et al., 2015; Turnuklu & Yesildere, 2007). In fact, the quality of teaching and learning may be influenced by distinguishing among three types of knowledge; subject matter, pedagogical content knowledge, and curricular knowledge (Shulman, 1986).

When looking at ways to increase teaching candidates' PCK and CK, learning opportunities within teaching preparation programs become an important discussion point. Similar to what was discussed in the previous section related to enhancement examples, an example of how a learning opportunity is able to enhance teachers' PCK can be seen through the implementation of video cases within the course setting (Steele, 2005). Learning opportunities also have the ability to impact teaching candidates' content knowledge (Qian & Youngs, 2015; Schmidt et al., 2011; Schmidt et al., 2007). Evidence can be seen in work by Schmidt et al. (2007), where the authors demonstrate how teachers from high performing countries have been trained with extensive education opportunities and in the practical aspects of teaching mathematics. The authors compare preparation programs among six countries and investigate how these preparations influence teaching candidates' preparations and student outcomes as presented by TIMSS data. The practical aspects of the study include pedagogical components of what is involved in the actual teaching of mathematics, such as completing lesson plans, task selection, assessment development, and an understanding of child development. Schmidt et al. (2007) find, when compared to other countries, the United States offers few opportunities related to practical aspects. Higher performing countries, whose preparation programs demonstrate a higher level of teaching candidates' success, offer more opportunities to learn through practical aspects of actual teaching experiences.

Darling-Hammond and Bransford (2007) demonstrate evidence of successful teacher preparation programs through a variety of field experiences integrated with coursework. In their investigation of seven successful teacher preparation programs, the authors are able to identify characteristics that rank these programs above others. Even though subject matter is stressed, pedagogy and the ability to promote theory-to-practice are also emphasized. Successful programs offer teaching candidates opportunities to practice and reflect on their practice. In addition to a strong content background, Darling-Hammond and Bransford (2007) suggest high quality teaching preparation programs must include similar pedagogical approaches.

A final example of how learning opportunities benefit teaching candidates' preparations comes from research conducted by Qian and Youngs (2015), who find that exposure to topics, such as developing lesson plans, improves CK and PCK. It was found that some preparation components, such as what actually happens in coursework and student teaching, were more important than other components including the number of content courses and methods courses that teaching candidates completed. Similar to Schmidt et al. (2007), Qian and Youngs (2015) discuss the association of teaching candidates' practical experiences with their MCK and MPCK. Even though the findings across the five countries studied were not unanimous, the importance of opportunities to learn within teacher preparation programs was further established.

With substantial evidence towards the benefits of added learning opportunities and other course attributes, my research provides additional insight related to this topic within a mathematical course sequence. Implementation of these opportunities to learn integrated into preparation programs will hopefully be able to provide additional details regarding their benefits.

There is no reason why preparation programs should delay field experiences when it is possible to introduce and expose teaching candidates to these opportunities to learn within specific content area courses.

Factors Associated with Elementary Teaching Candidates' Self-Efficacy

The training of pre-service mathematics teachers varies from traditional university-based programs to alternate route programs. According to Darling-Hammond (2010), the differences associated with the training of pre-service teachers have "enormous implications for the nature of professional work and of the teaching career" (p. 5). Research demonstrates the importance that teacher quality has on student achievement, through investigating teacher preparation, earned degrees, as well as any other certifications (Darling-Hammond, 2000; Goe, 2007). While there exists variability between the different types of programs, there is also variability within the programs. Variability among different programs has a direct impact on teacher effectiveness. Some institutions are producing teaching candidates who perform equally to that of the highest performing countries, while other institutions yield candidates whose skills compare to those of the lowest performing countries, a characteristic unique only to the United States (Hiebert et al., 2005; Schmidt et al., 2011).

Varied instruction has influences on self-efficacy, which is another crucial factor of teaching performance and student achievement. Bandura (1977) states how self-efficacy represents a person's belief in his or her ability to perform a difficult task and identifies four main factors which contribute to an individual's self-efficacy. Of these four factors, "mastery experiences" and "vicarious experiences" directly relate to the course enhancement components that I am presenting through this research. Mastery experiences can be thought of as experiences which take place in a real class setting and vicarious experiences can be thought of as observational opportunities for teaching candidates (Lee, Walkowiak, & Nietfeld, 2017).

There are a number of studies which demonstrate a positive correlation between teaching candidates' feelings of preparedness and their self-efficacy in teaching (Darling-Hammond et al., 2002; Bleicher, 2007; Palmer, 2006; Lee et al., 2017). Another body of research exhibits the relationship that teacher's beliefs have on their classroom behaviors and practices through the use of different measurement instruments and results (McGee & Wang, 2014; Tschannen-Moran & Hoy, 2001). While it is apparent that studies involving self-efficacy are not easy, a number of instruments have been developed, and reformed, in attempt to adequately measure this construct. In a study by Tschannen-Moran and Hoy (2001), the authors reviewed many of the major self-efficacy measures and found problems within each of the instruments. Concerns of these measures include reliability, lack of evidence regarding successful implementation, as well as the inclusion of confounding variables found outside of academics. Tschannen-Moran and Hoy (2001) were able to offer validity and reliability data in reference to an earlier self-efficacy measure known as an Integrated Model which was developed by Tschannen-Moran, Hoy, and Hoy (1998). This measure examines the development and modification of self-efficacy. When investigating teaching candidates, Tschannen-Moran et al. (1998) demonstrated the need for teacher preparation programs to include more opportunities for instructing and managing children. With the implementation of these experiences, the self-efficacy of teaching candidates increased. Along the same lines, McGee and Wang (2014) investigated and confirmed the reliability and validity of another measure, the Self-Efficacy for Teaching Math Instrument for measuring self-efficacy on teaching mathematical content. The authors express the importance of teaching candidates' content knowledge on their self-efficacy, as it is an integral part to be able to teach mathematics successfully (McGee & Wang, 2014). Certain components from each of these models may be beneficial toward the understanding within the analysis of my research.

Bleicher (2007) investigated the effects that a conceptual, hands-on science activities based course has on self-efficacy of teaching candidates. These types of learning opportunities not only enhanced teaching candidates' content knowledge, but also an increase in self-efficacy was observed (Bleicher, 2007). Results from this study have been implemented into teacher preparation programs whose aim is to increase conceptual understanding and teacher confidence. Palmer (2006) provides additional evidence of how enactive mastery experiences in a methods course are able to increase science content knowledge leading to an increased level of confidence.

In general, literature related to teaching candidates' self-efficacy is best summarized by Darling-Hammond et al. (2002) who state that, "a sense of preparation is by far the strongest predictor of teaching efficacy" (p. 9). Simply put, the more prepared a teaching candidate feels through their preparatory work, the more confidence they will have headed into the classroom, and consequently the better their students will perform. Higher levels of self-efficacy lead to higher levels of teaching practices and behaviors, which ultimately leads to increased learning opportunities for students (Lee et al., 2017). With adequate preparation and higher confidence, teachers can offer more to their students. On the other hand, there is no set standard for teacher preparation programs and not all teachers are exposed to such experiences.

With a wide range of teacher quality, we should expect to see a vast difference in student achievement. Teachers with more exposure to the practical aspects of teaching can offer their students more learning opportunities when compared to inadequately trained teachers. Differences in preparation programs then lead to differences with student achievement in the classroom. Clotfelter et al. (2009) demonstrate the effects that teacher credentials have on student achievement by providing evidence of how subject-specific certification positively influences student achievement. Their results reveal that the variation in teacher credentials account for at least 20% of the distribution in teacher quality (Clotfelter et al., 2009).

In reviewing literature, it is apparent that a few gaps are present. With the variability of teacher preparation programs, it is hard to find a standard set of topics and pedagogy required within university-based programs, causing concerns. The factors associated with effective teaching are debated, although research demonstrates that considerable knowledge in mathematics and pedagogy is required for a teacher to be effective in the classroom (Steele, 2005). While there has been extensive research regarding the topics of CK and PCK, there appears to be considerably less in other areas. This includes a lack of information regarding how certain course experiences seem to affect some of the major factors associated with effective teaching including math content knowledge and self-efficacy. Finally, even though there exist many research documents outlining teacher preparation programs, there seems to be a lack of research related to specific course features. For this reason, it may be hard to construct accurate conclusions that stem from specific teacher preparation program course changes. The research that I propose has a very specific focus that aligns with the topics seen on the Virginia SOL's, state licensure requirements, and Praxis core mathematics exam. With this in mind, I hope to offer further insight into the benefits, as well as possible drawbacks, of specific components within a mathematical course sequence for pre-service teachers. Therefore in my study, I observed how exposing pre-service teachers to the conditions presented by the SOL's and state licensure requirements, through a course enhancement, has on their development. This will enable me to test the following hypotheses:

Hypothesis 1: An enhancement to a mathematical course sequence will offer significant gains to pre-service teachers' mathematical content knowledge and self-efficacy. This will be demonstrated by their performance on formal pre-post assessment measures in two content areas, whole number computation, geometry and measurement, as well as survey materials related to self-efficacy.

Hypothesis 2: Pre-service teachers' self-efficacy will improve through increased mathematical content knowledge, introduction to relevant mathematical representations, and exposure to the requirements of everyday teaching practices.

Methods

In order to gather the most relevant and informative data sources, this investigation uses an explanatory sequential mixed methods research design broken down into three phases. While the mathematical abilities vary among teaching candidates, a mixed methods design provides a more in-depth understanding regarding elementary pre-service teachers' knowledge. As stated by Creswell (2014), the type of design proposed offers the ability to first analyze quantitative data and then build on the results through more descriptive qualitative research components. This type of design aids in the efforts to attempt to understand how an enhancement to a course sequence affects pre-service teacher's performance. All necessary forms and approval for the investigation has been granted through the Institutional Review Board (IRB) at Waverly College.

Participants

The participants were invited to be part of this study at the beginning of the 2016 academic year. Participants all aged 18 or over, with 80% being Caucasian females, consisted of roughly 25 undergraduate teaching candidates per course. Of these participants, some were enrolled in one or both courses associated with a two-semester mathematical course sequence for elementary pre-service teachers. While teaching candidates may take the courses in any order, most tend to take Math 117, Introduction to School Mathematics I, in the fall semester, followed by Math 118, Introduction to School Mathematics II, in the subsequent spring semester. These courses are a large component of the core set of mathematics courses within the major, typically taken in the second year of study. Adhering to IRB protocol, all privacy rights and consent were acknowledged for each participant.

Phase I: Quantitative – Whole Number / Computation

The first phase mainly focused on quantitative measures related to teaching candidates' content knowledge in mathematics. This phase was carried out over the fall semester and the sample consisted of 21 teaching candidates who were enrolled in first of two mathematical courses within the sequence. Mathematical content knowledge was measured using an instrument known as the Diagnostic Teacher Assessments in Mathematics and Science (DTAMS). The DTAMS assessments were developed by Bill Bush and the Center for Research in Mathematics and Science Teacher Development (CRiMSTeD) located at the University of Louisville. Even though more specific details regarding these assessments are described within the measures section of this paper, examples of the assessments are found in Appendix G. This assessment focused on teaching candidates' content knowledge regarding whole numbers and computation, which also included problem solving strategies. The assessment was administered as pre-test during the beginning weeks of the fall 2016 semester and then again as a post-test during the final week of the fall 2016 semester. The topics under investigation were part of the normal topics covered during the fall semester of Math 117, which is the first of two courses in the sequence for pre-service teachers.

In addition to the DTAMS measures, survey data was collected in order to attempt to understand teaching candidates' self-efficacy related to teaching mathematics during the beginning of the semester. Additional details regarding the chosen survey are provided within the measures section and the survey itself can be found in Appendix J. Powerful survey software, known as Qualtrics, hosted the survey and manage all responses. Each participant was provided with a unique link through their e-mail and were required to complete the survey in the first week of the semester.

The last form of data collection in this phase included teaching candidates' course lesson plans, which involved some, but not all, components of a full lesson plan. While many more specific details regarding the course lesson plans are outlined in the measures section and Appendix K, the work on these assignments was able to provide further supporting evidence. Specifically, I was able to use this data to support possible findings related to the implementation of MCK and MPCK by elementary teaching candidates within the practical aspects of teaching.

Phase II: Quantitative- Geometry / Measurement

Similar to the first phase, quantitative measures of pre-service teachers' content knowledge was collected using a pre-, post-test design using the DTAMS assessment tool. This took place during the spring 2017 semester within the second course of the sequence, titled Math 118. While the major topics covered in this course include rational numbers, geometry, measurement, probability and statistics, the chosen DTAMS assessment focused on geometry and measurement. The sample included 25 pre-service teachers, which was slightly larger than the first course based on several factors. First, some education majors require teaching candidates to complete both courses in the sequence, while other majors simply require fewer credits in mathematical courses, regardless of the course title. Based on course offerings, teaching candidates typically elect to take the second course in the sequence because their spring schedules are a bit more flexible. Next, some teaching candidates may need to re-take the course in order to achieve an adequate grade for the major. Finally, some teaching candidates may have taken the first course in a previous academic year and now are completing the second course.

The specific topics under investigation during this phase included geometry and measurement. Similar to the fall semester, the DTAMS assessment was administered twice. During the first class meeting of the spring semester, teaching candidates completed the DTAMS pre-test, and during the latter half of the semester (after the completion of lectures on these topics), teaching candidates completed the post test. Similar to the first phase, survey data dealing with pre-service teachers' self-efficacy and teaching beliefs was administered for a second time towards the end of the academic year. For teaching candidates who did not take the first course, Math 117, their initial survey responses were gathered at the beginning of the semester. Finally, data regarding one additional course lesson plan was obtained during the spring semester.

Phase III: Qualitative

The third and final phase occurred in the beginning stages of the following academic year, 2017-2018. This phase consisted of teaching candidate interviews and the analysis of course lesson plan assignments. After the results of the semester assessments and DTAMS measures were provided, I was be able to purposefully select a sample of five teaching candidates in which to interview on an individual basis. Subjects were chosen based on their performances in the course sequence and DTAMS measures. Based on their grades in Math 117 and Math 118, DTAMS assessments, as well as their performance on key assessments, lower-, middle-, and high-performing teaching candidates were selected for interviews. A lower-performing teaching candidate was an individual who ranked in the lowest quartile of course grades and DTAMS measures among all other subjects. A middle-performing teaching candidate ranked in the middle two quartiles, and a high-performing teaching candidate ranked in the highest quartile in the respective categories.

This breakdown of lower-, middle-, high-performing teaching candidates permitted a wide range of mathematical abilities to be further investigated. With an in-depth understanding across all ability levels, I was able to use a cross-sectional analysis to draw further conclusions related to performance and thought processes.

The interview protocol, see Appendix F, includes general mathematical background questions, possible influences that the course sequence had on teacher preparation, as well as two task-based questions. All interviews were recorded, transcribed, and coded. Responses regarding the course sequence and how well it prepared pre-service teachers for a career in teaching, as well as preparedness to pass the Praxis core mathematics exam, served as additional backing to the quantitative data that was collected during the previous academic year. Task based questions were purposefully chosen in the areas of whole numbers and computation, as well as geometry and measurement. The purpose of these task based questions was to understand how changes associated with teaching candidates' CK might affect self-efficacy, PCK, and overall preparations for teaching.

In addition to interview data, I also analyzed course lesson plans for the same group of teaching candidates who were interviewed. This analysis offered another opportunity to gain a true understanding of teaching candidates. Specifically, I was able to obtain further insight into the CK and PCK of these teaching candidates. The quality of work, methodology, and pedagogical components were all closely investigated in an effort to provide an additional layer and depth of understanding to support any quantitative findings.

Data Collection

Below is an overview of when and how this data was gathered. In order to keep all responses confidential and protect the identity of the teaching candidates, each participant was assigned a unique identification number.

In the first phase, DTAMS assessment and survey data was collected during the beginning weeks of the fall semester. DTAMS measures consist of 20 questions, 10 multiple choice and 10 open-ended, which were completed using paper and pencil. These assessments were then collected, made into an electronic version by scanning the images, and finally sent off to be graded by the developers of the assessment. Survey data included responses to a 21-item survey through the means of an online survey tool, Qualtrics. The 21-items were grouped into seven categories each with a number of sub-items. All responses were stored in an account on the software's webpage. The last major component of data collection came in the form of post-test DTAMS data, which was collected during the final week of the semester. Other data, including traditional semester assessments and lesson plans, were collected throughout the semester and scanned in order to save an electronic copy of all work permitting later reference.

The second phase followed a similar schedule for data being collected during the spring semester, with the only difference coming from the survey data. Instead of a pre-survey questionnaire being administered at the beginning of the semester, post-survey data was collected on the last day of classes through an electronic software site.

For the third phase, the interview phase, data was recorded and then transcribed for each interviewee. All data was stored and backed up monthly on a password protected laptop.

Measures

DTAMS. The DTAMS assessments are evaluation tools which are used to measure teacher knowledge, as well as their strengths and weaknesses, across four different domains: whole number / computation, rational number / computation, geometry / measurement, and probability / statistics / algebra. Each domain has six different versions and each version consists of 10 multiple choice questions and 10 open-response questions. In order to establish validity, each topic was reviewed by at least 36 different reviewers across the country, which included teams of mathematicians, mathematics educators, and teachers. Reviewers were asked to provide an analysis of the proposed tasks to be used on the assessments. If a task was not found to be suitable, it was altered or replaced based on the comments from the reviewers.

Once all tasks were approved by the reviewers, the DTAMS measures were made available for use. In order to use these assessments, I simply put in a request to use the assessments by providing a short description of my project. The requested assessments were sent to me over e-mail, printed out, and then administered to my classes.

For a nominal fee, in order to ensure consistent grading and that the results from these assessments were reliable, the staff at CRiMSTeD offers the option to grade the assessments. The grade report includes a detailed score report with a number of different options to analyze the data. For example, the report offers an itemized breakdown for each question, as well as cumulative scores in particular categories. Specifically, each question is grouped into one of four categories based on the type of knowledge it is attempting to assess: (1) Type I – memorized / factual knowledge, (2) Type II – conceptual understanding, (3) Type III – reasoning / problem solving, (4) Type IV – pedagogical content knowledge.

Self-efficacy survey. Items used on the self-efficacy surveys, see Appendix J, were selected from a similar survey used in research conducted by Pogodzinski, Youngs, and Frank (2013) who investigated novice teachers' intentions to remain teaching. Within that research, the authors demonstrated how the survey items expressed a high level of internal consistency, with Cronbach alpha levels of most constructs at or above 0.80. This indicates that there is a high correlation between items. With this in mind, I selected a subset of the questions, or question groups, which were most applicable to my study and developed a shorter survey with a specific focus. The final survey consisted of seven question groups, each with 4 to 8 sub-items sharing a common theme related to self-efficacy. For example, the first question group themed *self-efficacy related to teaching* included the following survey items: (1) I like answering questions during mathematics lessons, (2) I get anxious when I have to teach some mathematics topics, (3) Even if I work hard, I will not teach math as well as I will most students, (4) The mathematics achievement of some students cannot generally be attributed to their teachers, (5) I will continually find better ways to teach mathematics. Furthermore, each of these sub-items had four possible responses to choose from: (1) strongly disagree, (2) disagree, (3) agree, and (4) strongly agree.

Interview data. Other measures to be used in this investigation include interview data from interviews conducted after the course was complete. The purpose of incorporating this form of data was to hopefully generate rich, descriptive data leading to further insight into teaching candidates' mathematical thinking (Rossman & Rallis, 2012). This approach would hopefully aid in the overall understanding of the quantitative results. The individual interviews were conducted in a face-to-face setting during the subsequent academic year. Interviews typically lasted between 20-35 minutes and the protocol was grounded in topics concerning CK and PCK. This protocol was developed through the collaborative efforts of the Mathematics and Education departments at Waverly College, as well as my capstone advisor, in conjunction with the objectives and research questions presented within this investigation. Questions centered on topics related to how well teaching candidates felt the CK prepared them for the Praxis core mathematics exam, if the structure of the curriculum aligned with the specific course goals, as well as how the PCK course components prepared teaching candidates for teaching mathematics. Using experts in each of the subject areas ensured that all necessary components regarding pre-service teacher performances were included in the interview through member checking in each department. The initial protocol was piloted during the summer months before the academic year of 2017-2018 on two teaching candidates and further altered after presenting the questions to my doctorial committee during my capstone proposal defense. The revised protocol questions were shared with my committee for a final review in order to refine the questions further. Upon those revisions, the final interview protocol was established in order for this protocol to provide the best available data related to the proposed research questions.

Teaching candidates were purposefully selected based on their performance in the course sequence so that interview data would span all mathematical ability levels. The interviews were recorded and then transcribed by typing out the interviewee’s responses under the respective question. Prior to coding these transcriptions, I developed an initial coding sequence based off of the research questions and what I hoped to show through this investigation. The initial and final coding sequences can be found in Appendix L.

Throughout the coding process, it became apparent that the initial coding sequence would not be sufficient. Within the interview transcriptions there was meaningful data that simply did not fit into the initial categories. In which case, reoccurring data themes transformed the initial coding sequence in order to better organize the teaching candidate interview data that was available. While certain categories were added, I feel as if the category titled “Memorable take-a-ways” held the highest significance over the others. This category was able to capture the true personal and academic gains that teaching candidates experienced through the course sequence.

Even in properly coding all of the teaching candidate interview data, I was left in a position where I did not know what to do next. There certainly were emerging themes from the data, however if I were to just list the themes, I felt as if this approach would not capture the true interpretation of the interview data. One of my doctoral committee members suggested that I highlight the most pertinent interview data by providing a short summary of each interviewee. This would permit for a better situational understanding of each teaching candidate, their experiences, and a detailed insight into their overall impressions of the course sequence. With this in mind, a summary for each teaching candidate’s interview is provided within the results section of the investigation.

Course lesson plans. In addition to interview data, I reviewed lesson plans for the same group of teaching candidates that were selected to be interviewed. Based on the fact that most of the pre-service teachers complete these courses during their second year, I did not expect them to have a full understanding of how to create a lesson plan. Instead, the lesson plans that were completed during the semester as part of the teaching candidates’ workload were somewhat informal and offered pre-service teachers a chance to become exposed to lesson plan development.

The lesson plan requirements were developed using strict guidelines related to course material, see Appendix K. This ensured that the information included within the lesson plans aligned to topics covered in the course, to the teacher licensure requirements, and the Virginia

SOL's. No formal validity tests were conducted on the teaching candidate interview protocol or the lesson plan requirements. On the other hand, to ensure the effectiveness of the lesson plan, I had two of my colleagues review the assignment requirements.

Reliability and validity. Many of the possible issues related to reliability and validity stem from the fact that this investigation was composed of a small sample size. When investigating the quantitative analysis, internal validity of the findings is jeopardized with high kurtosis and skewness values from the DTAMS score data, see Table 1 and 3, respectively. In which case, even though the procedural analysis is accurate, the data may not support all criteria or assumptions for the given statistical test. Therefore, the true variability may not be captured through the analysis. In not knowing the actual population distribution, I had to investigate the data using a non-parametric approach. In order to support rationale and provide a sound argument, I included several different analytic strategies. By collecting and analyzing multiple forms of data, the intention was that each data type would support findings related to other forms of data. In most cases, this approach provided me with robust results across several different forms of analysis.

Researcher as an instrument. As an employee at the college being studied, specifically the course instructor, there are a few potential issues that may exist in the study. First, it could be thought that I had power of authority over the elementary pre-service teachers, affecting results. Also, certain bias may exist in knowing the importance for teaching candidates to pass the course, as well as the Praxis core mathematics exam to acquire their teaching license. As much as someone would hope to stick to planned course material, it could have been possible to go against this course description in order to "teach to the test" or get through a certain amount of material. In regards to the qualitative information collected in the third phase, selection of participants and question selection could have been influenced by outside factors, such as the background knowledge of teaching candidates from previous semesters. To this end, it is possible for the results of the investigation to be biased.

In order to address and limit any potential issues related to the researcher as an instrument, I implemented certain measures. First, data was collected from a variety of different sources including DTAMS measures, course lesson plans, survey data, and interview data. This triangulation of both quantitative data and qualitative data hopefully add to the soundness of the research regardless of the researcher. Next, I utilized my colleagues for peer review and debriefing associated with specific components related to the course, specifically the overall design and mathematical content aspects. This process was carried out through discussion and review of material related to certain course components. Last, I used member checking in an effort to validate the interview questions associated with the qualitative components. My capstone committee reviewed my proposed interview protocol and offered feedback as to how I could refine certain items to ensure they were aligned to my research questions. While these efforts may not eliminate all bias, they hopefully help to add to the credibility.

Results

Through analysis and cross-reference, it was observed that teaching candidates experienced significant gains in reasoning / problem solving for both mathematical topics, whole number / computation and geometry / measurement. The factual / memorized knowledge of elementary pre-service teachers was also significant for whole number / computation topics. While there were discrepancies among some of the quantitative results provided by the DTAMS measures, the qualitative analysis provided extensive evidence related to the effects of the course sequence on the MPCK and self-efficacy of teaching candidates. Themes related to MPCK and conceptual understanding, while not significant through quantitative analysis, were consistently revealed as having high impacts on teaching candidates through their qualitative measures.

All results were supported by data obtained from four sources. The quantitative components included pre-test and post-test DTAMS measures in two content areas, along with a pre-survey and post-survey related to teaching candidates' self-efficacy in teaching. Qualitative components included interview data from five pre-service teachers and written work acquired from these same teaching candidates' course lesson plans. In using an explanatory sequential mixed methods research design, I first investigated the results obtained from the DTAMS measures and then analyze the qualitative components to see how the qualitative data corresponds to the quantitative data.

DTAMS Measures

The score report from these assessments listed an overall compiled score for each teaching candidate, as well as a more specific score breakdown into four knowledge types for each content area, whole number / computation and geometry / measurement. The specific knowledge type scores were broken down into the following sub-categories:

1. Type I – memorized / factual knowledge
2. Type II – conceptual understanding
3. Type III – reasoning / problem solving
4. Type IV – pedagogical content

With all of the different possible options to analyze the data, I decided to use a few different types of statistical tests, specifically for the sub-categories for each different type of knowledge. These types of tests included a paired *t*-test, independent *t*-test, regression analysis, and a repeated measures ANOVA test, which not only boosts internal validity, but also indicates potential instances of Lord's paradox (Lord, 1967), signifying inconsistency according to different statistical tests in pre- and post-test design. Before investigating specific sub-categories, in order to determine if the course enhancement influenced teaching candidates' combined scores, results from a two-sample independent *t*-test indicate significant differences on the pre-, post-test DTAMS measure examining whole number / computation, $t(35) = 2.91$, $p = 0.006$. However, the results on the DTAMS measure examining geometry / measurement were not significant, $t(47) = 0.89$, $p = 0.377$. While these scores may offer a general idea of how teaching candidates were affected by the course, investigating the four sub-categories for each knowledge type offered a more specific focus as to where possible significant findings occurred.

Whole number / computation. The descriptive statistics for the DTAMS measure on whole number / computation are listed in Table 1 and the actual data are found in Appendix A.

Table 1

Descriptive Statistics for DTAMS Pre-, Post-Assessments on Whole Number / Computation

Assessment / Knowledge	*n*	Mean	Std. Dev	Skewness	Kurtosis
Pre / Type I	20	3.95	1.791	1.060*	1.072*
Pre / Type II	20	4.45	1.605	-0.578	0.709
Pre / Type III	20	1.20	1.735	1.475*	1.618*
Pre / Type IV	20	2.45	2.038	-0.065	-1.769*
Post / Type I	18	5.56	1.464	0.257	0.870
Post / Type II	18	5.61	2.004	0.063	-0.574
Post / Type III	18	2.39	2.004	1.367*	2.322**
Post / Type IV	18	4.00	1.940	0.598	0.528

Note: Measures denoted with a * indicate that the data would not be considered normal. ** indicate measures that are well beyond acceptable, meaning that the distribution is non-normal.

For the most part, the skewness and kurtosis values range between -2 and 2, indicating that the distribution of data for each type of knowledge is fairly normal with respect to the small sample size. Even though it may be most meaningful to explore this data using a paired sample *t*-test based on design of the study, analysis from other types of statistical tests could provide further evidence related to significance. For example, in most cases the $\boldsymbol{R^2}$ value from the regression analysis is relatively low. Usually this would imply that the model does not support the data well; however, I am still able to use this form of analysis to help to interpret and support my findings. The summary from all of the different types of statistical tests used is listed in Table 2 and the calculation tables for the paired samples *t*-test, independent samples *t*-test, regression analysis, and repeated measures ANOVA test, are all found in Appendix B, respectively.

Table 2

Summary of Statistical Findings for Whole Number / Computation DTAMS Assessments

WNC Knowledge Type	Paired/Dependent T-Test	Independent T	Regression	RM ANOVA
Type I: Memorized/factual	*t*(16) = 3.625, p = 0.002**	*t*(36)= 3.000, p = 0.004**	$R^2 = 0.312$, F(1,15)= 6.805, p = 0.020*	F(1,16) = 13.143, p = 0.020*
Type II: Conceptual understanding	*t*(16) = 2.167, p = 0.046*	*t*(36)= 1.980, p = 0.055	$R^2 = 0.192$, F(1,15)= 3.563, p = 0.079	F(1,16) = 4.696, p = 0.046*
Type III: Reasoning/problem solving	*t*(16) = 3.163, p = 0.006**	*t*(36)=1.960, p = 0.058	$R^2 = 0.355$, F(1,15)= 8.261, p = 0.012*	F(1,16) = 10.005, p = 0.006**
Type IV: PCK	*t*(16) = 1.776, p = 0.095	*t*(36)= 2.364, p = 0.022*	$R^2 = 0.020$, F(1,15)= 0.303, p = 0.590	F(1,16) = 3.135, p = 0.095

Note: Statistical significance indicates that there is a difference between pre-, post-test scores of each respective knowledge type using the statistical test indicated in the column header.
*p < 0.05. **p < 0.01.

Given the results of all of the different statistical tests, there is confirming evidence that significant differences are observed within knowledge type I, memorized / factual knowledge, with $p < 0.05$ for all tests. For knowledge type II, conceptual understanding, results are inconclusive among the different types of tests. When using a paired sample *t*-test, significant results are observed, $t(16) = 2.167$, $p = 0.046$. Yet the results from regression analysis using the same data are non-significant, $F(1,15) = 3.563$, $p = 0.079$. The findings from knowledge type III, reasoning / problem solving, are considered significant from the results obtained in both the paired samples *t*-test, $t(16) = 3.163$, $p = 0.006$, and regression analysis, $F(1,15) = 8.261$, $p = 0.012$. Even though the independent samples *t*-test concluded a p-value = 0.058, there is substantial evidence that a significant change has occurred from pre-test to post-test among the different statistical tests. When investigating the results obtained for knowledge type IV, the paired samples *t*-test, $t(16) = 1.776$, $p = 0.095$, and regression analysis, $F(1,15) = 0.303$, $p = 0.590$, both indicate confirming evidence that there are no significant changes between the pre-, post-test measures. Even though the results from the independent samples *t*-test demonstrated significant gains, $t(38) = 2.364$, $p = 0.022$, the samples were different sizes and the influences of including high-performing teaching candidates and excluding low-performing teaching candidates in certain measures could have influenced these results.

Geometry / measurement. The descriptive statistics for the DTAMS measure on geometry / measurement are listed in Table 3 and the actual data are found in Appendix C. The results for the second DTAMS measure, which are summarized in Table 4, proved to be a bit more complex. Knowledge type III was the only sub-category to have confirming evidence across all statistical tests, $p < 0.05$. On the other hand, Lord's paradox exists when investigating the results to knowledge type I, knowledge type II, and knowledge type IV. The significance calculated by a paired samples *t*-test for these three knowledge types were found to be $p = 0.086$, $p = 0.519$, and $p = 0.225$, respectively. However, when running a regression analysis to support

Table 3

Descriptive Statistics for DTAMS Pre-, Post-Assessments on Geometry / Measurement

Assessment / Knowledge	*n*	Mean	Std. Dev	Skewness	Kurtosis
Pre / Type I	25	3.56	1.083	1.650*	3.927**
Pre / Type II	25	2.96	1.241	-0.061	0.382
Pre / Type III	25	0.32	0.748	2.624**	6.895**
Pre / Type IV	25	1.64	1.524	1.054*	1.125*
Post / Type I	24	3.08	1.100	-0.177	-0.486
Post / Type II	24	3.17	1.880	0.294	-0.378
Post / Type III	24	1.04	1.301	1.724*	3.202**
Post / Type IV	24	2.25	2.251	1.432*	2.388**

Note: Measures denoted with a * indicate that the data would not be considered normal. ** indicate measures that are well beyond acceptable, meaning that the distribution is non-normal. Although this was to be expected with smaller samples sizes.

Table 4

Summary of Statistical Findings for Whole Number / Computation DTAMS Assessments

GM Knowledge Type	Paired/Dependent T-Test	Independent T	Regression	RM ANOVA
Type I: Memorized/factual	*t*(23) = -1.796, p = 0.086	*t*(47) = -1.528, p = 0.133	$R^2 = 0.126$, F(1,22) = 6.805, p = 0.020*	F(1,23) = 3.225, p = 0.086
Type II: Conceptual understanding	*t*(23) = 0.654, p = 0.519	*t*(47) = 0.456, p = 0.651	$R^2 = 0.323$, F(1,22)= 10.503, p = 0.004**	F(1,23) = 0.428, p = 0.519
Type III: Reasoning/problem solving	*t*(23) = 3.093, p = 0.005**	*t*(47) = 2.392, p = 0.021*	$R^2 = 0.262$, F(1,22) = 7.812, p = 0.011*	F(1,23) = 9.564, p = 0.005**
Type IV: PCK	*t*(23) = 1.248, p = 0.225	*t*(47) = 1.115, p = 0.271	$R^2 = 0.174$, F(1,22) = 4.629, p = 0.043*	F(1,23) = 1.558, p = 0.225

Note: Statistical significance indicates that there is a difference between pre-, post-test scores of each respective knowledge type using the statistical test indicated in the column header.
*p < 0.05. **p < 0.01.

the rationale of results found in the paired samples *t*-test, all three knowledge types demonstrated significant results, p = 0.020, p = 0.004, and p = 0.043, respectively.

Even though the results from the different tests are inconclusive, my final conclusions use the results obtained from the paired samples *t*-test. In which case, only knowledge type III yielded significant changes between the pre-test and post-test measures. While these results will be discussed in greater detail within the discussion, based on the nature of the presented materials in the course, there is a reason why only knowledge type III was found to be significant. The course sequence focused heavily on 2-column proofs for topics in geometry and asked teaching candidates to provide reasoning to their answers for topics in measurement.

Self-Efficacy Survey Data

In order to evaluate self-efficacy, teaching candidates were asked to complete a pre-survey and post-survey, see Appendix J. The self-efficacy survey was administered at the beginning and end of the course sequence. Even though the survey consisted of seven question groups, each with sub-items, I focused my efforts on four of the seven question groups. The selected question groups, along with their themes were (1) Question group 1: Self-efficacy related to teaching, (2) Question group 2: Conceptual understanding, (3) Question group 3: Beliefs / math identity, and (4) Question group 5: Self-view regarding teaching abilities. I selected these specific question groups based on the fact that the underlying theme of each related to the proposed research questions and purpose of my study.

I used confirmatory factor analysis (CFA) to examine the sub-items within each of these question groups in order to see how well the sub-items captured the underlying constructs. CFA provides the ability to determine the contribution that each latent variable has on describing a given construct. In other words, we now have a way to understand how well each of the sub-items “fits” with the others and determine how much of an influence each sub-item has on describing the overall theme of a specific question group. In order to determine if a factor loading falls within an acceptable range of measuring a construct in a meaningful way, I use the criterion of $|factor\ loading| > 0.3$, consistent to measurement methods used by McDonald (1985).

Nevertheless, due to a small sample size issue, CFA was only successful in modeling two of the four question groups using all sub-items, question groups 1 and 3.

Self-efficacy related to teaching. The results from question group 1 of the pre-, post-survey indicate that pre-service teachers now realize the impact they potentially have on student achievement. These significant results are expressed through sub-item Q1_d where the factor loading on the pre-survey indicates that teaching candidates agreed with the statement, “The mathematics achievement of some students cannot generally be attributed to their teachers.” However, on the post-survey the factor loading indicated that teaching candidates disagreed with this statement. These results are supported by the pre-survey parameter estimates that are listed in Table 5 and the corresponding CFA model diagram that is presented in Figure 1. The model fit

Table 5

Parameter Estimates for Pre-Survey Model Related to Question Group 1: Self-Efficacy

Sub-item	Estimate	S.E.	p-value
Q1_a (fixed)	1.000*		
Q1_b	-1.594*	1.088	0.143
Q1_c	-0.898*	0.485	0.064
Q1_d	0.130	0.440	0.768
Q1_e	0.380*	0.370	0.304

Note: The contribution of parameter estimates is not scaled to other latent variables. In order to determine their significance instead of referencing the p-value, parameter estimates are considered significant, denoted by “*”, if $|estimate| > 0.3$. With Q1_a being fixed, positive loadings indicate teaching candidates agree with the sub-item.

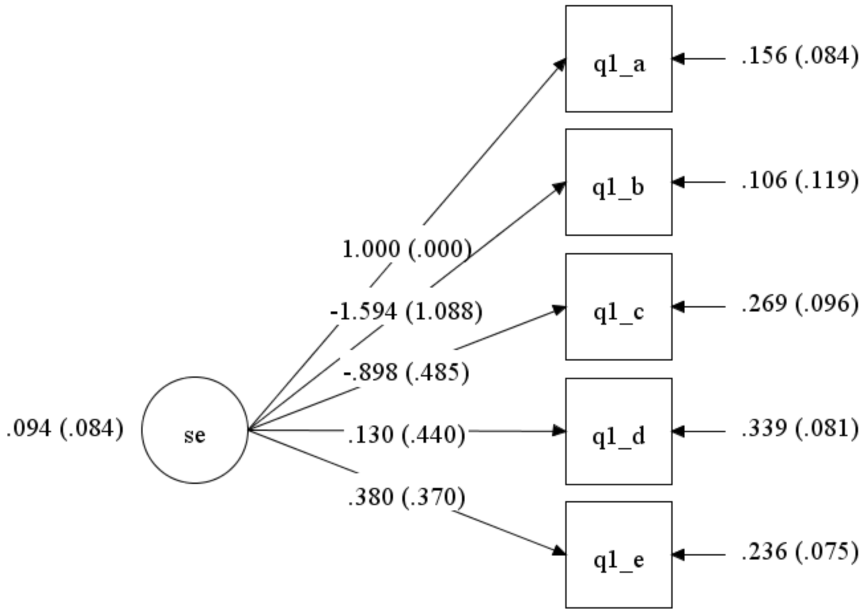

Figure 1: CFA Model Diagram for Question Group 1 of the Self-Efficacy Pre-Survey Data.

indices for the pre-survey of question group 1 included RMSEA = 0.000 (strong), CFI = 1.000 (strong), X^2 = 5.151 → p-value = 0.8808 (fail to reject), SRMR = 0.093 (weak). Overall, this model fit is acceptable and with a p-value of 0.8808 the hypothesized model is supported by the data.

Parameter estimates for post-survey data related to question group 1 are listed in Table 6, along with the corresponding CFA model diagram in Figure 2. The model fit indices for the post-survey data were calculated to be RMSEA = 0.177 (weak), CFI = 0.601 (weak), X^2 = 35.484 → p-value = 0.0021 (reject null), SRMR = 0.106 (weak). Even though this model converged in CFA, the overall fit is below the typical acceptance standards.

When attempting to modify the model by dropping one of the latent variables, there were no instances where the model fit improved significantly. In fact, when attempting to modify the model by dropping one latent variable, improvement was only observed in two of the individual model fit indices of RMSEA, CFI, or SRMR, for different modification attempts, compared to the full model.

Table 6

Parameter Estimates for Post-Survey Model Related to Question Group 1: Self-Efficacy

Sub-item	Estimate	S.E.	p-value
Q1_a (fixed)	1.000*		
Q1_b	-1.147*	0.407	0.005
Q1_c	-1.117*	0.728	0.125
Q1_d	-0.388*	0.482	0.421
Q1_e	0.526*	0.532	0.323

Note: The contribution of parameter estimates is not scaled to other latent variables. In order to determine their significance instead of referencing the p-value, parameter estimates are considered significant, denoted by "*", if $|estimate| > 0.3$. With Q1_a being fixed, positive loadings indicate teaching candidates agree with the sub-item.

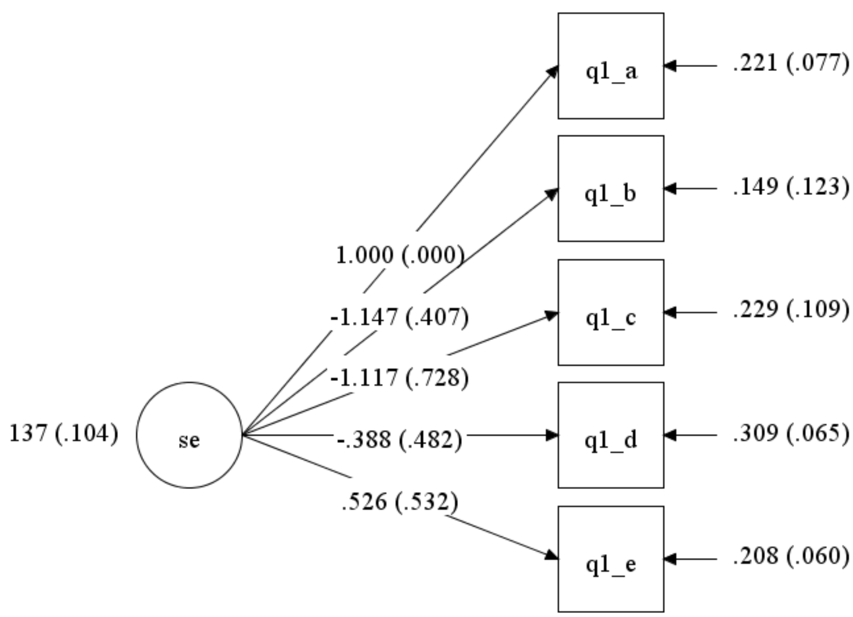

Figure 2: CFA Model Diagram for Question Group 1 of the Self-Efficacy Post-Survey Data.

Beliefs / math identity. The second question group that CFA was successful in modeling was question group 3, which describes teaching candidates' beliefs / math identity. The sub-items associated with this question group explore teaching candidates' attitudes towards mathematics. For example, sub-items include prompts such as, “I enjoy thinking about different ways to solve a mathematics problem”, “No matter how much effort I put forth, I can only do so well in mathematics”, and “My effort is the key to my success in mathematics.” The results from the CFA related to this question group do not show any significant changes from pre-survey to post-survey, however. The model fit indices for the pre-survey were RMSEA = 0.300 (weak), CFI = 0.622 (weak), $\mathbf{X}^2$ = 70.087 → p-value = 0.0000 (reject the null), and SRMR = 0.111 (weak). Similar to the post-survey model fit from question group 1, the model fit is below the typical acceptance standards. It is most common in small sample studies to modify these models. However, when I tried to modify the model by dropping one of the sub-items, I did not observe any significant gains in terms of the fit indices. By considering the small sample issue as a study limitation, I plan to use the parameter estimates that are listed in Table 7 and the corresponding CFA model diagram in Figure 3.

Table 7

Parameter Estimates for Pre-Survey Model Related to Question Group 3: Belief / Identity

Sub-item	Estimate	S.E.	p-value
Q3_a (fixed)	1.000*		
Q3_b	0.693*	0.596	0.245
Q3_c	-3.494*	3.770	0.354
Q3_d	-4.507*	4.895	0.357
Q3_e	-3.051*	2.883	0.290
Q3_f	1.841*	1.495	0.218
Q3_g	2.641*	2.255	0.242

Note: The contribution of parameter estimates is not scaled to other latent variables. In order to determine their significance instead of referencing the p-value, parameter estimates will be considered significant, denoted by “*”, if $|estimate| > 0.3$. With Q3_a being fixed, positive loadings indicate teaching candidates agree with the sub-item.

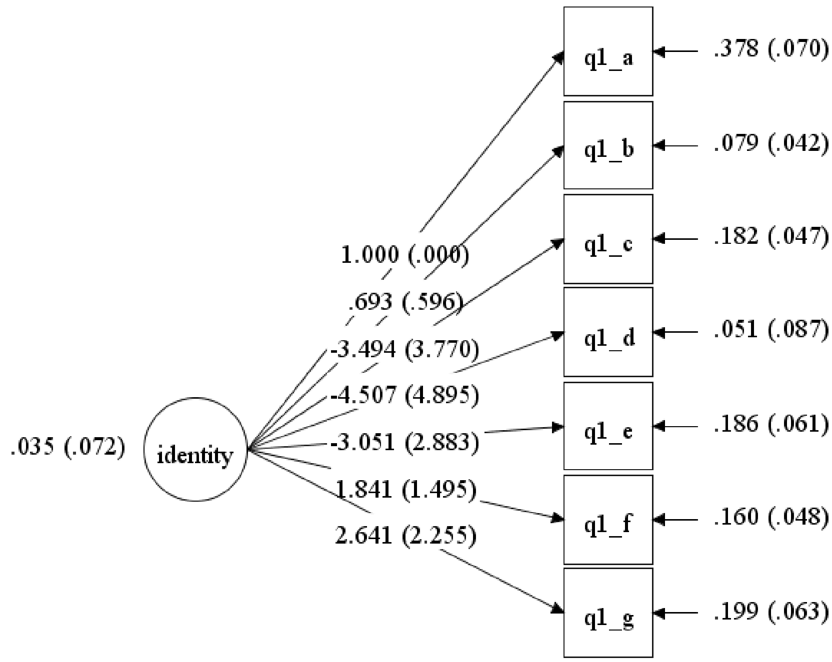

Figure 3: CFA Model Diagram for Question Group 3 of the Self-Efficacy Pre-Survey Data.

The model fit indices for the post-survey for question group 3 were RMSEA = 0.247 (weak), CFI = 0.657 (weak), X^2 = 124.94 → p-value = 0.0000 (reject null), SRMR = 0.134 (weak). The parameter estimates are listed in Table 8 and CFA model diagram in Figure 4.

Table 8

Parameter Estimates for Post-Survey Model Related to Question Group 3: Belief / Identity

Sub-item	Estimate	S.E.	p-value
Q3_a (fixed)	1.000*		
Q3_b	1.062*	0.513	0.038
Q3_c	-2.276*	1.098	0.038
Q3_d	-3.503*	1.791	0.050
Q3_e	-3.109*	1.531	0.042
Q3_f	1.333*	0.559	0.017
Q3_g	1.199*	0.749	0.109

Note: The contribution of parameter estimates is not scaled to other latent variables. In order to determine their significance instead of referencing the p-value, parameter estimates will be considered significant, denoted by "*", if $|estimate| > 0.3$. With Q3_a being fixed, positive loadings indicate teaching candidates agree with the sub-item.

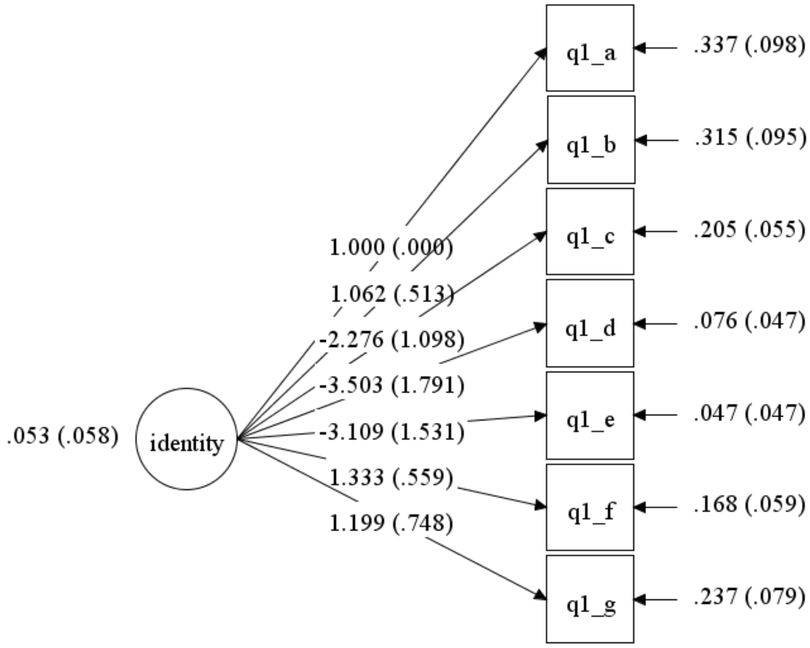

Figure 4: CFA Model Diagram for Question Group 3 of the Self-Efficacy Post-Survey Data.

Conceptual understanding. When conducting a CFA for question group 2, convergence of variables was unsuccessful when all latent variables were included in the model. With this being the case, I attempted to modify the model. First, I tested for any correlation between each possible pair of sub-item latent variables, which did not exist. This was true for both the pre-survey and post-survey data.

The next model modification I attempted to drop one of the variables from the model to see if CFA would be able to model the remaining pre-survey data. In two instances using the pre-survey data, by dropping Q2_b and Q2_d the model did converge and the model fit appeared to be more acceptable when dropping Q2_b. RMSEA = 0.069 (weak), CFI = 0.808 (weak), X^2 = 14.472 → p-value = 0.1525 (fail to reject null), SRMR = 0.144 (weak).

Using the same approach with the post-survey data, CFA was able to model the remaining data when Q2_a was dropped. In doing so, the resulting model fit is summarized as RMSEA = 0.000 (strong), CFI = 1.000 (strong), X^2 = 23.950 → p-value = 0.0077 (reject null), SRMR = 0.111 (weak). Due to the fact that CFA was able to model the pre-, post-survey data only when omitting different sub-items, the CFA for this question group was not included. Instead, I simply calculated the mean response of each sub-item within the pre-, post-survey with the results displayed in Table 9. In quantifying a four point Likert scale categorized as: (1) strongly disagree, (2) disagree, (3) agree, (4) strongly agree, an average of 2.5 would be neutral. Although certain sub-items demonstrate minimal change, sub-item Q2_c reflects a significant change from pre-survey to post-survey means. Pre-survey responses yield an average of 2.62 (agree) compared to the post-survey response average of 1.96 (disagree). This difference indicates that teaching candidates possess more confidence in the use of manipulatives.

Table 9

Mean of Survey Responses for Sub-Items related to Question Group 2: Conceptual Understanding

Sub-item	Pre-Survey	Post-Survey	Difference
Q2_a	2.38	2.44	0.06*
Q2_b	2.48	3.08	0.60*
Q2_c	2.62	1.96	-0.66*
Q2_d	2.95	3.00	0.05*

Note: The sample size for the pre-survey was n = 21 and for the post-survey was n = 25.The mean was calculated by quantifying the Likert scale responses on the survey which ranged from strongly disagree (1) to strongly agree (4). The value of the difference of means should be interpreted correctly as a (-) value often demonstrates an increased change based on the structure of the sub-item. For example, looking at item Q2_c, a -0.66 change is interpreted as an increase to teaching candidates' knowledge of mathematical manipulatives. Differences that indicate a beneficial change are denoted by "*".

Self-view regarding teaching ability. Question group 5 which investigated teaching candidates' self-view regarding their teaching abilities. Sub-items in this question group asked teaching candidates how important it was for them to be portrayed as a strong mathematics teacher. Examples of these prompts included, "It is important for me to teach mathematics better than other teachers" and "I don't want to look like an incompetent teacher of mathematics to my fellow teachers."

Similar to model fit attempts associated with question group 2, CFA was unable to fit a model when all latent variables were included in question group 5. Also, no pair of latent variables were correlated. This was true for both the pre-survey and post-survey. The next model modification attempted to drop one of the sub-item latent variables from the model. This approach proved to be successful when omitting either Q5_d or Q5_e in both the pre-survey and post-survey. Even though the model fit indices of the modified model are not great, they appeared to be slightly better when omitting Q5_e. For the pre-survey, RMSEA = 0.171 (weak), CFI = 0.566 (weak), X^2 = 34.299 → p-value = 0.0031 (reject null), and SRMR = 0.177 (weak). The parameter estimates, found in Table 10, indicate changes for all sub-items related to teaching candidates' self-view regarding their teaching ability. The factor loading for each sub-item on the pre-survey indicate that teaching candidates disagreed with items related toward their self-view in teaching mathematics, whereas all of the factor loadings on the post-survey indicated the exact opposite response from the teaching candidates.

Table 10

Parameter Estimates for Pre-Survey Model Related to Question Group 5: Self-View

Sub-item	Estimate	S.E.	p-value
Q5_a (fixed)	1.000*		
Q5_b	-0.236	0.422	0.576
Q5_c	-0.129	0.202	0.523
Q5_d	-0.050	0.084	0.555
Q5_e (omitted)			

Note: The contribution of parameter estimates is not scaled to other latent variables. In order to determine their significance instead of referencing the p-value, parameter estimates will be considered significant, denoted by "*", if $|estimate| > 0.3$.

For the post-survey RMSEA = 0.318 (weak), CFI = 0.290 (weak), X^2 = 55.744 → p-value = 0.0000 (reject null), and SRMR = 0.221 (weak). The parameter estimates for this model are found in Table 11.

Table 11

Parameter Estimates for Post-Survey Model Related to Question Group 5: Self-View

Sub-item	Estimate	S.E.	p-value
Q5_a (fixed)	1.000*		
Q5_b	1.517*	0.819	0.064
Q5_c	0.759*	0.434	0.080
Q5_d	0.600*	0.384	0.118
Q5_e (omitted)			

Note: The contribution of parameter estimates is not scaled to other latent variables. In order to determine their significance instead of referencing the p-value, parameter estimates will be considered significant, denoted by "*", if $|estimate| > 0.3$.

Qualitative Analysis

The qualitative data obtained through teaching candidate interviews provided me with additional insight into elementary teaching candidates' knowledge and outcomes from the course sequence. Interviewees were purposefully selected in order to exemplify lower-, middle-, high-performing teaching candidates. More specific details regarding this breakdown, along with specific performance indicators, are found in Table 12.

Through the transcription and coding of the teaching candidate's interview data, it was apparent that certain themes were present among the interviewee responses. In order to emphasize these themes I summarized the interview data from each teaching candidate, highlighting specific examples that support the emerging themes which are presented and discussed following the summaries.

Table 12

Interview Participant Performance Overview

		DTAMS WNC		DTAMS GM		Semester Grades		Lesson Plans	
Candidate	Rank	Pre	Post	Pre	Post	Fall	Spring	Fall	Spring
Ashley	High	25	34	20	21	A+	A+	29	30
Laura	Low	4	11	7	-	C-	D	25	15
Canton	Mid	16	23	9	10	B	B	30	23
Anne	High	12	27	11	18	A	B	30	22
Rich	Mid	-	-	7	7	-	C+	-	19

Note: The range of DTAMS assessment scores is 0-40 points for the pre-, post-tests in Whole Number / Computation (WNC) and Geometry / Measurement (GM). Semester grades used a typical 10 point per letter grade scale and lesson plans were scored 0-30 based on the rubric found in Appendix K.

Case study: Ashley. Coming from an advanced mathematical track in high school, this teaching candidate proved herself to be among the top performing teaching candidates within the course sequence. During high school she completed courses up through calculus and was invited to be part of the Westover Honors program at Waverly College. Even though her mathematical background established a significant amount of mathematical content knowledge, it was apparent that this course sequence had drastic effects on her *preparations to be a teacher*. She stated, “the content wasn’t hard or that useful, but seeing how to use multiple methods to solve a problem helped a lot.”

Even though this teaching candidate performed academically well throughout the course sequence, as well as in her high school courses, on several occasions throughout the interview she brought up the fact that in the past she had only ever been taught using a procedural approach. While she was able to obtain the correct answer, she explained how it bothered her not knowing why she was doing certain mathematical steps. Specifically, when investigating the task concerning the division of two fractions, she stated, “I had always been taught straight to the point and I was told to multiply by the reciprocal and that’s it.” *Understanding foundational* *mathematical* principles, along with being exposed to different teaching methods, such as the use of manipulatives, were other takeaways that she mentioned repeatedly throughout her interview.

In the end, Ashley expressed how it was necessary to put forth more effort in mathematics, compared to other subjects, to ensure success on the larger assessments. On the other hand, she disclosed how she put forth minimal effort in regards to the homework and most of the other content items in these courses. Regardless, with this new foundational knowledge her *confidence in teaching* had increased, along with her content knowledge in certain areas. She was able to apply the knowledge she had towards teaching situations making her a much stronger teaching candidate. I think she expressed it best saying, “I wish we had courses like this in other subjects to get us ready to teach those subjects and not only focus solely on learning material.”

Case study: Laura. In high school, this teaching candidate completed courses in Algebra 1, Algebra 2, and Geometry. She ranked in the lowest 10% of all teaching candidates in the course sequence, however I feel as if these grades did not reflect her true academic ability. Even with having gone through a standard high school course sequence, she had no hesitation expressing some of her previous subpar mathematical learning experiences. “When I don’t get something (in any math class), my confidence would go down…because I would do it in elementary school and I’m used to not getting it.” These types of experiences extended into her high school mathematical courses, where she expressed how she “never got a deep understanding of topics, so I never really think that I learned the topics.” While other teaching candidates might have been able to succeed in this type of learning environment, Laura had to work much harder and put forth a noticeable amount of additional effort to succeed, if at all possible.

On a more positive note, through completing the course sequence for pre-service teachers, Laura expressed how some topics helped to *boost her confidence* because she now had a true understanding regarding those topics. “I’ve had a lot of trouble in math growing up, so being able to understand it has been really helpful.” Before these courses she stated how she may have only known one way to solve a task, but now she has *learned different methods* and alternate approaches to teach topics in different ways. For example, she stated how much she liked the use of manipulatives, which during the task based question of dividing two fractions she referenced as a possible method to solve the problem. Finally, this teaching candidate pointed out the benefits of engaging in *practical teaching opportunities*. She cited an example where she was required to create her own assessment, which “put me in the role of the teacher, which I never really got to do before.”

Case study: Canton. This teaching candidate offers a great example of how dedication can result in educational gains. In high school Canton was part of a lower mathematical track which consisted of Algebra 1, Algebra 2, Geometry, and Statistics. He felt more confident in algebra topics over Geometry and Statistics, which was consistent with his thoughts regarding the same topics in this course sequence. He also explained the importance of hard work, which for him meant that he had to try much harder in math compared to other subjects. Unlike history, which comes more naturally to him, he described that “when you get stuck in math, it makes it harder to keep focus and stay on task.” He then went on to compare mathematical process to writing a history paper, expressing that when writing a paper you can just re-word something in order to keep your thoughts going.

When discussing specific components related to the course sequence, there were a few items that stood above others. While, Canton considers a true understanding as an important educational component and he stated, “These courses exposed me to a number of different teaching components and teaching ideas.” He cited how *practical teaching experiences* such as creating his own word problems, developing lesson plans, exposure to the Virginia SOL's for the first time, and just knowing how to talk to students, were all acquired knowledge.

Furthermore, Canton went on to explain how he feels a lot *more confident* in his teaching abilities after completing this course sequence. "I feel like I can teach counting methods and other topics with more knowledge than when I started." During the course we emphasized three essential counting principles, which he used as an example during the interview to demonstrate the importance of knowing multiple ways to explain a mathematical task. As he said, "This course really helped me understand how knowing a variety of ways to teach will make me a more effective teacher."

Case study: Anne. While Anne was one of the highest achieving teaching candidates in the first course, her ranking was drastically decreased during the second course of the sequence. With her mathematical background she tested out of Algebra and Geometry in high school, meaning that she took Pre-Calculus as her only high school mathematics course. She is certified in ESL and has plans on attending graduate school.

Throughout the course sequence it was apparent that Anne possessed a good deal of mathematical content knowledge, although she claimed, "I'm not math oriented, it's something that I have to work at really hard." Additionally, when discussing different mathematical topics she claimed to always have been better in Algebra compared to Geometry. While she couldn't give me a specific reason, she felt that algebra simply came more naturally to her. Even though this could have accounted for some of the drop in performance from the first semester to the second (based on the fact that more Geometry is covered in the second course), Anne noted that the second course was more of a struggle. Reasons for this included the fact that she was taking 16 credits and was heavily involved in obligations with her sorority.

This teaching candidate indicated that she was a procedural learner at the beginning of this course sequence. “I don’t feel as if other classes prepared me for teaching whereas this class did, specifically how to *apply different teaching and pedagogical* methods.” Even though certain methods worked better for her than others, with her prior content knowledge she was able to grasp the true understanding and connection of different mathematical methods. Most of the education classes she took focused on content knowledge and not how to not apply material to practical teaching experiences. As an example, “I liked how we were put into the role of a teacher through the different tactics used on homework and lesson plans, it was as if we were an actual teacher.” Even though this course sequence focused on content knowledge, teaching candidates were then required to use this knowledge within other course assignments related to *practical teaching experiences*.

Case study: Rich. Taking only the second course in the sequence during his senior year at Waverly College, Rich had not taken a math course in a few years. His background included Algebra 1 in 8^{th} grade, and then an Algebra 2 preparation course, Algebra 2, Geometry, and an advanced math course, similar to applied pre-calculus, in high school. Of these courses, he noted how Geometry was the most difficult for him to understand, but in general he stated that, “I need to work more in math than other subjects.” On the other hand, if he was able to relate mathematical topics to a real world experience or visually, he usually noticed greater academic gains.

“Without this class, I wouldn’t feel as confident with material, I can now teach topics in different ways.” Compared to high school, Rich explained how he now had an *understanding behind the rationale* for doing certain steps in order to solve a mathematical task. “The procedures demonstrated in this course really helped out in my understanding of ideas to use in my future classroom." Without being exposed to different modes of representation which included varied procedural approaches, visuals, and manipulatives, this teaching candidate said how he would have most likely just used a traditional approach in his own classroom. "Counters were very useful, I never saw that process before taking this course." When discussing the task based interview prompt dealing with division of two fractions, Rich struggled at first trying to recall procedural steps. However, he made much more significant progress when thinking about the task using the methods learned in the course sequence using counters.

His past preparations as a student involved studying a formula, learn to apply it, and then get to know the meaning behind the topic. "At times it was hard for me to follow this traditional approach, however different methods keep students more engaged and fresh." By taking this course, he now has a better understanding of methods that he can use in his own teaching.

Emerging themes from coding of interview data. The themes that emerged from the interview data include (1) commonalities among an increased effort in mathematics compared to other subjects, regardless of academic background, (2) content knowledge was mainly based on procedural approaches in their high school backgrounds, (3) the benefits of exposure to additional pedagogical techniques, and (4) teaching preparation gains through being exposed to practical teaching experiences. These themes serve as the foundation of support to the quantitative findings.

Increased effort in mathematics. When investigating the mathematical background of the elementary teaching candidates, even if they were part of an advanced mathematics track in high school, they still found themselves having to work harder in these courses compared to other subject areas. This was evident in all 5 teaching candidate interviews that I conducted. Even with the content in these courses being rather simple, the higher performing teaching candidates, who had mathematical backgrounds as advanced as pre-calculus and calculus, expressed the need to have an increased effort in the course sequence. It could have been that these elementary pre-service teachers finally had the opportunity to explore mathematical properties and develop a true understanding of certain mathematical topics that were simply assumed to be true in past courses. For example, when discussing topics such as dividing fractions, one of the higher performing teaching candidates elaborated on the fact that, “I never even realized why I did that (multiply by the reciprocal) and it bothered me that I didn’t know why.” Even though the procedural components of dividing fractions was not that difficult, observed by her performance on the quantitative assessment measures, perhaps increased attention and effort was needed in order to gain a true understanding of such topics.

Content knowledge based on procedural approach. A further look into the high school background of the interviewees revealed the fact that they were mostly taught using procedural techniques, leaving them without a true understanding of certain mathematical topics. This was true in 3 of the 5 cases within teaching candidate interview data. Without a deep, conceptual understanding in the foundational mathematics courses, teaching candidates expressed how they were often confused and searching for a reason as to why they were doing certain procedural steps in previous mathematics courses. While empirical data from the quantitative results support the importance of content knowledge through significant gains in memorized / factual knowledge, the majority of the teaching candidates considered these courses as a “refresher”, or a review of topics they already knew. Even though there were significant gains, it required the course sequence to re-familiarize teaching candidates to the necessary procedural processes in order for them to be able to answer rudimentary mathematical tasks. On the other hand, non-significant results on the DTAMS measures regarding an increase to teaching candidates’ conceptual understanding were found. Similar difficulties regarding the lack of conceptual understanding were also apparent through the mathematical tasks presented to them during the interview process, as well as some of their coursework. Teaching candidates were able to obtain an answer to certain mathematical tasks, yet they were not able to fully explain the reasoning behind their work.

Exposure to pedagogical techniques. Noticeable in all 5 interviews, understanding the existence of different pedagogical techniques served as the third theme among interview data. Teaching candidates felt as if they were more prepared to become a teacher through the acknowledgement of knowing multiple forms of pedagogical techniques. Exposure to different pedagogical styles offered teaching candidates the ability to complete a mathematical task in more than one way. They began to understand the benefits this could have on student learning. As explained by Rich during his interview, "Multiple modes of representation and different approaches will be able to keep students engaged and fresh." It seemed for many of the teaching candidates, that this was the first time they had been exposed to different forms of pedagogy. While the idea of pedagogy is part of their core methods courses within the major, the teaching candidates mentioned how this was the first time that they were able to apply it to an actual teaching experience. Teaching candidates communicated how they now realized the importance of knowing multiple methods to present materials, which increased their confidence in teaching. Instead of knowing one way to present a topic, teaching candidates now understood the importance of preparing an alternate instructional approach in order to reach more students. Different students may not be able to understand a particular topic using a typical approach, in which case it may be necessary to offer these students a different method of instruction in order to grasp the material. Laura stated that she was aware of different forms of pedagogy, but this course “made the benefits of teaching in different ways a little more clear when we were put into the role of a mathematics teacher.” Other teaching candidates discussed pedagogy surrounding the potential educational impacts that manipulatives could have on student learning. In any case, interview data made it very clear that for many of the teaching candidates, this was the first time they had ever used mathematical manipulatives to supplement instruction, which is something that could not have been known through quantitative data alone.

Practical teaching experiences. Last, an overwhelming theme observed in each and every interview came through teaching candidates’ explanations surrounding exposure to practical teaching situations. Through this course sequence, elementary pre-service teachers stated how this was the first course in which they felt as if they were put in the role of an actual teacher and asked to complete assignments related to situational cases they might encounter in an everyday teaching environment.

It was appreciated that this course did not focus solely on content, but rather how to actually teach the content to future students and apply what they are learning to their own classroom. Ashley explained how knowing mathematical content has limitations on teaching by stating, “Doing well in a calculus class wouldn’t have helped my confidence as a teacher because those topics don’t translate at all to teaching.” Even though the content covered in calculus would expose her to a high level of MCK, this knowledge would not have helped to prepare her for actual teaching and classroom experiences. Another example included Canton’s statement regarding the SOL’s where, “This was the first time being exposed to specific SOL’s and having to think about how to incorporate them into my lessons and assessments.” Other practical teaching items which were mentioned by teaching candidates included: assessment creation, investigating student errors, how to talk to students, creating a gradebook, and discussing how to handle specific teaching situations that were presented in videos.

Summary

Results observed through quantitative analysis show significant gains in certain types of knowledge, such as memorized / factual knowledge and reasoning / problem solving. On the other hand, qualitative analysis was able to provide further insight regarding the influences that the enhancement to a mathematical course sequence had on elementary pre-service teachers' MCK, MPCK, as well as self-efficacy. Even though some of these constructs may not have been significant through the quantitative analysis, different forms of qualitative data were able to express specific gains in other knowledge type sub-categories, which will be focused on in the discussion section.

Discussion

Results from the analysis of DTAMS measures, teaching candidates' self-efficacy survey responses, interview data, and informal semester lesson plan entries suggest several conclusions. There is no hiding the fact that the sample size used in my research was small, with $n = 21$ teaching candidates in the fall course and $n = 25$ teaching candidates in the spring course. When using a small sample size, variability exists and validity of my results is jeopardized. For this reason, I chose to use an explanatory sequential mixed methods research design in an effort to minimize variability. These efforts included triangulation of data, as well as the use of different statistical tests on the same data set. Even with a small sample size, if different forms of analysis support particular findings, results would be considered more robust and trustworthy. I discuss how one form of data supports another, the implications of these findings, as well as possible limitations of my study.

General Findings

Given the pre-, post-test design of this investigation, which involves dependent variable data sets, a paired samples *t*-test is the most appropriate type of analysis to use. This analysis revealed significant gains for memorized / factual knowledge, conceptual understanding, and reasoning / problem solving on the whole number / computation assessment. With regard to the geometry / measurement assessment, only reasoning / problem solving was found to have significant gains when using a paired sample *t*-test. Carefully planned course material and methodology certainly contributed to these findings. Rather than simply teach mathematical topics procedurally, a deep conceptual understanding was stressed. In past mathematical courses the majority of teaching candidates were taught using through procedural approaches. However, in transitioning to become future teachers, teaching candidates were exposed to a variety of ways to teach certain topics in order to demonstrate a true understanding and be considered a highly qualified teacher. Mathematical proofs, inquiry based tasks, as well as attempts to generalize answers all contributed to a deeper conceptual understanding. Even though some tasks did not directly correspond to specific mathematical topics, the tasks provided opportunities for mathematical exploration and promoted a collaborative work environment. Through these course elements, I was expecting to observe significant gains across all types of knowledge, but this was not the case. While there is a lack of research examining effects of a course enhancement on specific mathematical topics, from the non-significant results observed in my research it appears that teaching candidates struggled more with geometry / measurement compared to whole number / computation. Typically the topics covered in geometry / measurement are more abstract and teaching candidates did not have as large of a recent background in these topics compared to whole number / computation. These conclusions were supported by the background data obtained through interviews.

Unlike some of the MCK findings, when focusing my efforts on mathematics pedagogical content knowledge, significant gains were not evident in either mathematical topic. These results are consistent with existing research which expresses concerns related to teacher preparation programs not adequately preparing teaching candidates (Olson et al., 2015). On the other hand, the quantitative results obtained through the DTAMS pre-, post-tests may not have been able to provide a comprehensive understanding of all MPCK gains within the course sequence. Upon reviewing the DTAMS questions that assessed MPCK, each of those questions asked teaching candidates to offer a possible instructional activity which would help to correct a student’s misconception within the given prompt. Even though investigating student errors were part of the course enhancement, very little lecture time was dedicated towards correcting student misconceptions directly. Instead there was more emphasis towards developing course materials which would guide the instruction of introductory geometry / measurement topics. For these reasons, through different forms of statistical analysis it was apparent that Lord’s paradox of the MPCK conclusions existed.

While there may be some inconsistency among the results from individual knowledge types, perhaps there is an association between the level of difficulty of the different knowledge types and the significant findings. Using Bloom’s taxonomy as a guide, the lower levels include the categories of remember, understand, and apply, while the higher levels include analyze, evaluate, and create (Krathwohl, Anderson, & Bloom, 2001). The four knowledge types which make up this study can be categorized into the distinct levels. Memorized / factual knowledge and reasoning / problem solving fit into the lower level categories, while conceptual understanding and PCK fit into the higher level. Between the two mathematical topics under investigation, the lower level objectives demonstrated significance in 3 of 4 possible situations, while the higher level objectives only observed significance in 1 of 4 situations, which was conceptual understanding on the whole number / computation assessment. While it is possible that an association between the levels of difficulty of different knowledge types exists, addition evidence may be needed to support such claims.

In an attempt to further understand the inconsistencies within the quantitative results regarding MPCK, qualitative results offer an in-depth perspective of how elementary pre-service teachers felt the course sequence impacted their learning. Similar to research conducted by Lannin et al. (2015), teaching candidates' interview data expressed how they now understood the importance of using different pedagogies in the classroom.

Even though the quantitative analysis may not have shown significant gains, interview and course lesson plan data completed by teaching candidates inferred that there was an *increased understanding of MPCK*. These ideas are supported by interview comments such as, "It's better to teach in more than one way," and "Before I only knew one way and understood it, but now I learned different methods and different steps on how to teach it."

When comparing the initial course lesson plans to later lesson plans, there were obvious improvements. In later course lesson plans, teaching candidates were able to effectively implement multiple modes of representation, appropriate guiding questions, correct use of mathematical language, clear organization, and proper formatting of the course lesson materials which they constructed from scratch. Even though improvements to course lesson materials were observed in each of the five teaching candidate's submission, the overall scores on these assignments were directly correlated to DTAMS and semester performances of each candidate.

The highest-performing teaching candidate's baseline DTAMS scores in geometry / measurement were higher in each of the four knowledge type categories compared to that of the lowest-performing teaching candidate. Out of 10 points, the highest-performing teaching candidate outscored the lowest-performing teaching candidate by 3 points in memorized / factual knowledge, 1 point in conceptual understanding, 3 points in reasoning / problem solving, and 6 points in PCK. In general, it is apparent that higher performances on DTAMS assessments translate to a higher, more professional level of course lesson plan as this was not an isolated case. Furthermore, these findings reveal gains to the MPCK of pre-service teachers which may not have been fully captured by other measures.

Practical teaching experiences were also found to have drastic effects on teaching candidates' MPCK and self-efficacy. These results were consistent to research conducted by Darling-Hammond and Bransford (2007) who expressed that successful teacher preparation programs often include a variety of field experience integrated with coursework. In each of the five interviews, teaching candidates noted the effects that similar methodology used in this course sequence had on their educational gains by permitting them to be in the role of a teacher and not a student. Mentioned examples included being able to develop assessments, understanding how to use mathematic manipulatives, investigate student errors and provide appropriate feedback, how to talk to elementary students, as well as the first time being having to pay attention to SOL's related to coursework. Research conducted by Bandura (1977) identifies "mastery experiences", or those which take place in a real class setting, as one of four main factors which contribute to an individual's self-efficacy. Therefore, by offering such experiences to elementary pre-service teachers through coursework, an increase to their self-efficacy is evident. As one teaching candidate explains, "Doing well in a Calculus class wouldn't have helped my confidence as a teacher because those topics don't translate at all." Another teaching candidate states, "I can now teach topics in different ways, which boost my confidence and without this class I wouldn't feel as confident with the material." Additional evidence provided by conducting a confirmatory factor analysis on the self-efficacy pre-, post-survey suggests two main findings which are consistent with the research findings of Tschannen-Moran et al. (1998). First, teaching candidates realize the importance that a conceptual understanding has on instructional approaches and student learning. Second, post-survey data suggests that teaching candidates now associate the influences that teachers can have on student achievement. Given the overwhelming evidence supporting the impacts of practical teaching experiences, perhaps more emphasis should be considered within the course structure of teacher preparation programs.

Specific Findings

Whole number / computation. Results of the DTAMS whole number / computation assessment indicate that there were significant changes to teaching candidates' memorized / factual knowledge and reasoning / problem solving across all types of statistical tests, see Table 2. This is due to the fact that much of the course content was focused on mathematical content knowledge related to topics that teaching candidates may have seen such before in prior courses, as well as new topics. Not only were teaching candidates required to learn the material as if they were a student, but also a considerable amount of time was spent on the mathematical reasoning behind specific answers. Furthermore, there were course goals specific to problem solving, which was a large focus of the lectures. On the other hand, knowledge type IV, pedagogical content knowledge, was found to be non-significant. This was the first time that many of these teaching candidates had been exposed to this style of instruction which blended MCK and MPCK. Unlike other courses that simply focused on content, I do not feel as if elementary pre-service teachers fully understood how to adapt their learning into actual teaching situations. Often, this type of knowledge takes time to develop and implement into actual classroom experiences. These ideas are supported by interview data where teaching candidates struggled to explain how to teach a topic or describe students' mathematical difficulties related to topics covered in the course. Finally, results dealing with knowledge type II, conceptual understanding, were inconclusive. Some statistical tests, such as the paired samples *t*-tests, indicated that there were significant findings, while regression analysis results were non-significant. Even though the course may have had certain influences on teaching candidates' MCK and conceptual understanding in certain topics, I did not stress a full conceptual understanding for all topics. This could have been a reason for inconclusive findings on the DTAMS assessment measures. Similar conclusions were evident through interview data, where 3 of the 5 teaching candidates communicated academic gains related to a conceptual understanding. These elementary pre-service teachers explained how they had a much deeper understanding of particular topics than before taking these courses, but still struggled with other topics. Even though the course focused on learning the material and exposing teaching candidates to different pedagogical methods, time constraints limited how much time I could spend in certain areas. With this in mind, development of a deep conceptual understanding could have taken place in certain topics, but not others.

Geometry / measurement. The only significant finding within the DTAMS measure on geometry / measurement was within knowledge type III, reasoning / problem solving. When teaching the sections related to geometry / measurement, many of the lectures involved 2-column proofs, as well as a discussion involving a deep understanding on how to link certain geometry facts together. To this end, teaching candidates were forced to reason their way through a given task and provide justification for their work, rather than simply provide an answer.

On the other hand, non-significant results were noted in memorized / factual knowledge, conceptual understanding, and pedagogical content knowledge. Unlike content that was taught in the first course which may have been familiar to more teaching candidates and easier to relate to, geometry / measurement topics appear to be more difficult for elementary pre-service teachers. Even though memorized / factual knowledge was stressed during the beginning phases of the geometry / measurement unit, the course focused more in certain areas over others. For example, the course sequence spent a significant amount of time on the properties of polygons, specifically quadrilaterals. While these topics were included on the DTAMS assessments, they may have only accounted for a small percentage of the problems which evaluated the memorized / factual knowledge type. In which case, even though teaching candidates may have experienced significant gains in specific geometry topics, the DTAMS assessments may not have been successful in capturing these gains because they included many different geometry topics.

Interview data supports the fact that teaching candidates were able to relate to topics seen in the first course more easily. Topics in that course were more arithmetic based, compared to topics in geometry. Perhaps it was more difficult for teaching candidates to rely on their previous knowledge in geometry to help aid with their understandings of similar topics presented in this course. It may take longer to develop a deep understanding of topics in this course, compared to how quickly a teaching candidate might be able to re-develop a previous understanding within whole number computations. For these reasons, it is understandable why non-significant results were observed in memorized / factual knowledge and conceptual understanding. Along the same lines, with these topics being more difficult to grasp for elementary pre-service teachers, it becomes harder to make use of different pedagogical methods when teaching candidates lack the content knowledge needed to understand topics. Even though I was able to present different approaches on how teaching candidates might go about teaching these topics to their future classes, I felt as if I was not able to teach as many different styles since I had to spend more time explaining mathematical content.

Concerns with DTAMS measures. Looking at individual teaching candidates' responses to DTAMS measures, two concerns with the assessment were noticed. The first concern dealt with the structure of open ended questions. When reviewing each of the DTAMS pre-, post-test assessments, teaching candidates answered all of the multiple choice questions on each assessment. However, a closer look at the open-ended questions reveled a concerning trend, where a number of open ended questions were left entirely blank. For example, 28% of all open-ended tasks on all four assessments were left completely blank. This included a non-response rate of 42% on the open-ended questions presented on the whole number / computation pre-test. On specific assessment tasks, such as the first open ended question on the whole number / computation pre-test, I observed that 64% of teaching candidates did not provide any response.

While it could have been the case that the teaching candidates simply did not know how to respond to the task, when comparing the number of blanks on the post-test for the corresponding task, only 17% did not respond. However, even with a 47% increase in attempts at an answer for that task, the cumulative score, as a class, did not improve by a single point on this task. Therefore it could be the case that the presentation of the task was not clear. More specific details of these concerns are presented in Appendix H.

The second major concern with the DTAMS assessment involved the grading of the open ended questions. As an overview, multiple choice questions were graded as 0 for an incorrect answer and 1 for a correct answer. Questions 11-20 were open ended and were each graded out of 3 points. According to the documentation provided by the DTAMS on how they interpreted scores for the open-ended questions, “These 3 points are distributed such that 0 points or 1 point is assigned depending on whether or not teachers demonstrated memorized / factual knowledge or conceptual understanding. In addition, 0 points, 1 point, or 2 points were awarded depending on the degree to which teacher’s demonstrated appropriate reasoning or problem solving strategies in items 11-15 or pedagogical content knowledge in items 16-20.” Upon further review of teaching candidates’ responses to certain opened ended questions, I noticed inconsistencies to the grades assigned based on the mathematical understanding that was provided by different teaching candidates, see Appendix I. For example, even though a teaching candidate may have included an appropriate amount of mathematical reasoning in their answer, they did not earn the same grade as another teaching candidate who had a similar answer.

With the role that grades associated with the DTAMS assessments have on the outcomes of this investigation, I reached out to the developers of the assessment for further clarification regarding the grading of the open-ended questions. J. H. Jones (personal communication, November 3, 2017) provided clarification stating, “The open response mathematics items are scored using a rubric… answers need to be very specific and contain enough information to determine that the participant had a correct understanding of the concepts involved in the task.” In knowing this information, even though I feel as if there are inconsistencies within the grading of certain items which may have impacted the outcomes, reliability measures through the grading rubric of the DTAMS assessments were implemented.

Self-efficacy survey. CFA guided the analysis of the self-efficacy survey for two of the four question groups, question group 1 and 3, using all sub-items. For question group 5, I was able to use a model modification by dropping a sub-item in order to get an acceptable model fit. However, unsuccessful in modeling question group 2, I analyzed the results by comparing the difference of means of each sub-item from the pre-test to the post test. Below is the listing of constructs measured by each question group:

1. Question group 1: Self-efficacy related to teaching
2. Question group 2: Conceptual understanding
3. Question group 3: Beliefs / math identity
4. Question group 5: Self-view regarding teaching abilities

Even though CFA does not scale latent variables with other variables, it is able to express the contribution that latent variables have on a given construct. It should also be noted that the model fit indices of most models used in my study were not necessarily acceptable. However, I plan to use the models as if the fit indices were categorized as “strong”. The use of survey data findings, even with weak model fit indices, can be endorsed through evidence in the research where the survey was adopted. Pogodzinski, Youngs, and Frank (2013) were able to provide evidence of sound psychometrics related to the items on the survey using a “moderate” sample size of $n = 184$, close to nine times as large as the sample in this investigation. It is very likely that small sample sizes are to blame for the model fit issues within my study since the sample characteristics were fairly similar. The sample used in research conducted by Pogodzinski, Youngs, and Frank (2013) consisted of 90% Caucasians and 83% females, very similar to my study which consisted of 80% Caucasians, 91% females. The only main difference was that my participants consisted of teaching candidates who were in their second year undergraduate study, while the authors in the other investigation studied a sample of beginning teachers. These concerns are addressed in the limitations section.

Self-efficacy related to teaching. CFA suggested that there were no significant changes, from pre-survey to post-survey, in the contribution of four of the five latent variable loadings on the construct self-efficacy. However, when looking at the fourth sub-item which asks, "The mathematics achievement of some students cannot generally be attributed to their teachers," it appears that the responses to sub-item four are initially positive contributions to a teaching candidate's self-efficacy. However, on the post-survey this loading is found to have negative contributions. With the loading of the first item fixed, negative contributions are associated with the "disagree" categories. Therefore, this suggests that by completing this course sequence, teaching candidates' view's regarding the influences that they have on students changed. Consistent to findings in research conducted by Young et al. (2015) pre-service teachers now realized the impact they would have on their students' mathematical achievements.

Beliefs / math identity. CFA on this question group did not show any differences in the latent variable loadings on the construct of beliefs / math identity from pre-survey to post-survey. While it is possible to look at the portion of the construct described by individual latent variables, it appears that the course sequence did not have significant effects on teaching candidates' beliefs / math identity.

Assuming the poor model fit might be to blame for not showing any significant differences, upon reviewing the mean differences in responses from the pre-survey to post-survey, the results were mixed. For some sub-items, Q3_a, Q3_b, Q3_f, Q3_g, it appeared that the course had beneficial results towards a teaching candidate's beliefs/math identity. This could be based on the fact that the course sequence exposed teaching candidates to different mathematical methods and the benefits of looking at certain tasks in more than one way. However, for other sub-items, Q3_c, Q3_d, and Q3_e, the course actually had a negative influence on a pre-service teacher's beliefs / math identity. These sub-items included prompts such as, "No matter how much effort I put forth, I can only do so well in mathematics" and "I'm not the type to do well in math." In which case, even in completing the course sequence, teaching candidates still were troubled by mathematics and how a negative view is very hard to change. Perhaps, the course sequence made teaching candidates realize complexities in mathematics which were not evident before.

Conceptual understanding. CFA was not able to successfully model the sub-item responses in question group 2. I attempted to modify the model by dropping certain items, but I was unable to fit a corresponding model for the pre-survey and post-survey data. In attempts to look for correlations between pairs of sub-items, no two items were found to correlate. In which case, looking at the difference in the mean response score between the pre-survey compared to the post-survey was utilized, summarized on Table 9.

For each sub-item it was found that the course had beneficial impacts on the conceptual understanding of teaching candidates. Therefore, it appears that teaching candidates realized the importance that a conceptual understanding had on instructional approaches and student learning. Additionally, a negative mean difference was noted on sub-item Q2_c indicating that teaching candidates disagreed with the statement that, "I find it difficult to use manipulatives to explain why mathematics works" on the post-survey. This is beneficial evidence as it expresses the confidence teaching candidates gained through the course sequence and being exposed to mathematical manipulates. Many teaching candidates may not have ever been exposed to manipulatives before this course sequence, yet now they would be able to use manipulatives to explain a mathematical topic in a different way.

Self-view regarding teaching ability. While CFA was unable to fit a model using all latent variables, when I dropped the last sub-item, Q5_e, I was able to fit a model for both the pre-survey and post-survey. The CFA results reported that the contribution of all sub-items within this question group went from a negative loading to a positive loading on a teaching candidates self-view. Pre-service teachers now had a view that they wanted to teach mathematics better than others. They did not want their teaching abilities in mathematics to be inferior to others. And finally, they did not want to look like an incompetent teacher of mathematics to their fellow teachers.

From completing the course sequence and becoming exposed to different pedagogical methods which they may have never seen before, teaching candidates had a new appreciation for mathematics. Perhaps in knowing multiple ways to teach a mathematical topic, or having a stronger conceptual understanding as expressed through the results from Question group 2, teaching candidates felt the need to present themselves as a knowledgeable mathematics teacher in front of their colleagues. Nonetheless, it was apparent that elementary pre-service teachers expressed more confidence and wanted to make this apparent by characterizing themselves as knowledgeable in mathematical instruction to others.

Implications

The results obtained through my investigation offer additional information for other institutions of higher education that might experience similar issues within mathematical courses that are part of teacher preparation programs. By conducting this study, I now have a better understanding, which extends beyond assessment data, of the influences that an enhancement to a course sequence had on elementary pre-service teachers. It is also obvious that teaching candidates were able to relate more to topics in whole number / computation, compared to geometry / measurement. These findings are able to offer insight to issues raised by Hill (2010) who states, “Teacher education programs must be focused where they will be most useful, and knowing which topics and tasks teachers find to be challenging provides one source of guidance.” Consequently, the results obtained in this study are able to narrow this focus to topics concerning geometry / measurement.

Findings also suggest several possible directions for future research. First, replication of the presented study would permit a larger sample size and subsequently more sound results. Second, in order to understand the results of this study further, I could attempt to collect additional post-test data at a later date. In a subsequent semester or academic year, additional post-test data would permit a repeated measure which might help to explain and understand certain conclusions. Third, with many of the teaching candidates at Waverly College finding employment locally, it may be interesting to collect longitudinal data related to teaching effectiveness as the teaching candidates, who were part of this investigation, enter the work force. To this end, I would be able to link conclusions found within this investigation to teaching candidates’ beginning teaching experiences.

Other avenues for future research include comparisons of TPP at different colleges and universities, use other mathematical topics or explore entirely different subject areas, as well as explore further course revisions. By investigating other mathematical topics, such as the remaining two DTAMS assessment measures focused on rational numbers or probability / statistics / algebra, teacher preparation programs would have additional insight towards specific areas of concern. The possibility of investigating other subject areas could provide a more general understanding of similar effects in other subject areas. There is an obvious demand for such research and information as one teaching candidate stated, “I wish we had courses like this in other subjects, to get us ready to *teach* those subjects and not focus solely on learning material."

Limitations

The major limitation regarding my study is the small sample size, which ranged between 20-25 teaching candidates per semester. Small sample sizes affect model fit and internal validity. Even though I may have used sound procedural analyses and proven data measures, it is hard for small samples to satisfy all criteria and assumptions needed for statistical analyses. Therefore, there is no way to be sure that my results captured the variability within the study and it would be hard to generalize my findings to a larger population, even if I were to assume appropriate model fit based on successful evidence in other research.

Another limitation was present through the inconsistent grading of the DTAMS measures. Even though the developers of the assessment were able to provide me with a permissible rationale for the grading of the open-ended questions, I feel as if inconsistences were present. With a larger sample size, these minor inconsistences might have been absorbed into the overall score reports, however with a small sample size these minor inconsistences could have had major influence on the final results.

Conclusions

While there may not be a simple solution to the criticisms of teacher preparation programs not adequately preparing teaching candidates for a career in teaching, this investigation offers different forms of beneficial evidence regarding the enhancement to a mathematics course sequence. When looking at the different forms of data that were collected, there were clear contrasts between high-, middle-, and low-performing teaching candidates. High performing teaching candidates' scores on the DTAMS assessments were consistently 5 points higher on average than middle-performing teaching candidates and 9 points higher than the lower-performing teaching candidates within the course. These differences were within a score range of 0-40, where the average total score across all assessments was a 10. In addition to higher quantitative DTAMS scores, high-performing teaching candidates provided a higher quality of work demonstrated on course lesson plans. Finally, notable differences were evident through interview data where higher-performing teaching candidates were able to retain more MCK and used appropriate, clear mathematical language to explain concepts.

For most of the teaching candidates, it was apparent that this course sequence was the first time they had been exposed to a number of pedagogical techniques involved with teaching mathematics. As a whole, there were significant gains in factual / memorized knowledge and reasoning / problem solving. Even though PCK measures on the DTAMS assessments were found to be non-significant, the interview data provided a much different conclusion. Elementary pre-service teachers were more cognizant of different pedagogical methods and had a higher level of self-efficacy in teaching mathematics. Additionally, these results were further supported by CFA on the self-efficacy survey where results showed that teaching candidates had an increased attention to their self-view in mathematics. These findings were consistent with my hypothesis which stated that the course sequence would offer significant gains to pre-services teachers' MCK and self-efficacy through course materials.

By looking at the DTAMS results, teaching candidates experienced positive gains in their overall knowledge scores for whole number / computation. Even though overall gains in geometry / measurement were not found to be significant, specific analysis was able to show significance on the knowledge sub-category of reasoning / problem solving within the same assessment. In any case, it was evident that teaching candidates experienced positive gains in particular knowledge sub-categories on both assessments. Content knowledge gains, coupled with interview and self-efficacy data revealed how teaching candidates now had a higher level of self-efficacy in teaching mathematics. They were made aware of different ways to present material, were exposed to more thorough descriptions of particular mathematical topics, and understood practical aspects of the profession, all of which may not have been aware of before taking this course. In conclusion, from the results of this study there is a direct influence of teaching candidates' knowledge toward their self-efficacy. While there are debates that these courses should focus on either MCK or MPCK, my results show the benefits for using a blending of the two types of knowledge. Elementary pre-service teachers were appreciative of a class that pushed them mathematically, while at the same time offered meaningful insight into the pedagogical and practical teaching aspects that they will see in their future careers.

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APPENDIX A

Table 13

DTAMS Data for Whole Number / Computation Pre-, Post-Assessments

	Pre-Test				Post-Test			
ID	Type I	Type II	Type III	Type IV	Type I	Type II	Type III	Type IV
0108	4	3	0	0	6	5	2	5
0109	-	-	-	-	-	-	-	-
0120	3	1	0	0	5	4	1	1
0123	3	6	0	4	5	7	0	5
0212	3	4	1	4	7	9	3	8
0406	4	4	1	5	6	5	2	2
0415	5	5	1	5	7	6	4	4
0504	8	5	3	0	6	7	2	5
0506	1	5	4	1				
0515	3	1	0	0	5	6	1	3
0526	3	5	0	1	6	4	5	3
0530	-	-	-	-	-	-	-	-
0601	5	4	0	2	5	5	1	3
0602	5	4	3	0				
0613	6	6	3	4	5	5	0	3
0624	-	-	-	-	7	7	4	5
0722	-	-	-	-	-	-	-	-
0803	-	-	-	-	-	-	-	-
0811	-	-	-	-	-	-	-	-
0828	2	6	0	3	3	6	2	4
0929	8	7	6	4	9	9	8	8
0930	3	7	2	4	6	8	4	5
1024	4	4	0	2	4	2	2	3
1031	3	4	0	0	-	-	-	-
1220	3	4	0	5	3	3	1	1
1225	3	4	0	5	5	3	1	4

Note: Pre-, post-test scores were broken down into 4 knowledge types; Type 1 – Memorized / Factual Knowledge, Type 2 – Conceptual Understanding, Type 3 – Reasoning / Problem Solving, Type 4 – Pedagogical Content Knowledge. Scores for teaching candidates who did not complete the assessment are indicated with a dash.

APPENDIX B

Table 14

Paired Sample T-Test: Whole Number / Computation

Pre/Post Pair	Mean	Std. Dev	Std. Error	95% CI	*t*	df	Sig. (2-tailed)
Type I	1.353	1.539	0.373	[0.56, 2.14]	3.625	16	0.002**
Type II	1.059	2.015	0.489	[0.02, 2.10]	2.167	16	0.046*
Type III	1.294	1.687	0.409	[0.43, 2.16]	3.163	16	0.006**
Type IV	1.118	2.595	0.629	[-0.22,2.45]	1.776	16	0.095

Note: Type 1 – Memorized / Factual Knowledge, Type 2 – Conceptual Understanding, Type 3 – Reasoning / Problem Solving, Type 4 – Pedagogical Content Knowledge. $*p < 0.05$. $**p < 0.01$.

Table 15

Independent Samples T-Test: Whole Number / Computation

	Pre-Test		Post-Test		
	Mean	Std. Dev	Mean	Std. Dev	Sig. (2-tailed)
Type I	3.95	3.21	5.56	2.14	0.005**
Type II	4.45	2.58	5.61	4.02	0.055
Type III	1.20	3.01	2.39	4.02	0.058
Type IV	2.45	4.16	4.00	3.76	0.021*

Note: Type 1 – Memorized / Factual Knowledge, Type 2 – Conceptual Understanding, Type 3 – Reasoning / Problem Solving, Type 4 – Pedagogical Content Knowledge. Pre-Test data consisted of n = 18 observations and the post-test consisted of n = 20 observations. *p < 0.05. **p < 0.01.

Table 16

Regression Analysis: Whole Number / Computation

	Pre-Test		Post- Test					
Model	Mean	Std. Dev	Mean	Std. Dev	R^2	Std. Error of Estimate	F Change	Sig. F Change
Type I	4.12	1.76	5.47	1.46	0.312	1.253	6.805	0.020*
Type II	4.47	1.74	5.53	2.04	0.192	1.889	3.563	0.079
Type III	1.00	1.66	2.29	2.02	0.355	1.678	8.261	0.012*
Type IV	2.82	1.98	3.94	1.98	0.020	2.028	0.303	0.590

Note: The post-test measurement was the dependent variable for all models and the sample size was $n = 17$.
*p < 0.05. **p < 0.01.

Table 17

Repeated Measures ANOVA: Whole Number / Computation

Knowledge	Sum of Squares	Error	df	Sig.
Type I	15.559	18.941	(1,16)	0.002**
Type II	9.529	32.471	(1,16)	0.046*
Type III	14.235	22.765	(1.16)	0.006**
Type IV	10.618	53.882	(1,16)	0.095

Note: Conditions for ANOVA are assumed to be satisfied. *p < 0.05. **p < 0.01.

APPENDIX C

Table 18

DTAMS Data for Geometry / Measurement Pre-, Post-Assessments

	Pre-Test				Post-Test			
ID	Type I	Type II	Type III	Type IV	Type I	Type II	Type III	Type IV
0108	3	3	0	1	3	4	1	1
0109	3	0	0	0	1	0	0	2
0120	4	3	0	0	-	-	-	-
0123	4	3	1	0	4	4	2	1
0212	4	3	1	3	3	6	0	9
0406	4	3	0	3	4	4	0	6
0415	3	5	2	1	5	6	1	3
0504	-	-	-	-	-	-	-	-
0506	4	2	0	1	4	1	0	0
0515	3	3	0	2	2	2	1	0
0526	4	2	0	2	4	3	2	2
0530	4	3	0	0	1	2	1	3
0601	4	1	0	0	3	3	0	1
0602	6	5	1	3	3	6	1	3
0613	3	2	0	1	3	3	0	1
0624	3	2	0	3	4	3	4	2
0722	2	2	0	1	2	0	0	0
0803	3	5	0	1	4	4	1	6
0811	2	5	0	3	3	2	1	2
0828	3	3	0	0	2	1	0	0
0929	7	4	3	6	5	7	5	4
0930	3	2	0	4	4	2	2	2
1024	3	3	0	1	3	2	0	0
1031	4	3	0	3	3	3	1	3
1220	3	3	0	1	2	5	2	0
1225	3	4	0	1	2	3	0	3

Note: Pre-, post-test scores were broken down into 4 knowledge types; Type 1 – Memorized / Factual Knowledge, Type 2 – Conceptual Understanding, Type 3 – Reasoning / Problem Solving, Type 4 – Pedagogical Content Knowledge. Scores for teaching candidates who did not complete the assessment are indicated with a dash.

APPENDIX D

Table 19

Paired Sample T-Test of the DTAMS Geometry / Measurement Pre-, Post- Assessment

Pre/Post Pair	Mean	Std. Dev	Std. Error	95% CI	*t*	df	Sig. (2-tailed)
Type I	-0.458	1.250	0.255	[-0.99, 0.07]	-1.796	23	0.086
Type II	0.208	1.560	0.318	[-0.45, 0.87]	0.654	23	0.519
Type III	0.708	1.122	0.229	[0.24, 1.18]	3.093	23	0.005**
Type IV	0.542	2.126	0.434	[-0.36, 1.44]	1.248	23	0.225

Note: Type 1 – Memorized / Factual Knowledge, Type 2 – Conceptual Understanding, Type 3 – Reasoning / Problem Solving, Type 4 – Pedagogical Content Knowledge. *p < 0.05. **p < 0.01.

Table 20

Independent Samples T-Test: Geometry / Measurement

	Pre-Test		Post-Test		
	Mean	Std. Dev	Mean	Std. Dev	Sig. (2-tailed)
Type I	3.56	1.17	3.08	1.21	0.133
Type II	2.96	1.54	3.17	3.54	0.651
Type III	0.32	0.56	1.04	1.69	0.021*
Type IV	1.64	2.32	2.25	5.07	0.271

Note: Type 1 – Memorized / Factual Knowledge, Type 2 – Conceptual Understanding, Type 3 – Reasoning / Problem Solving, Type 4 – Pedagogical Content Knowledge. Pre-Test data consisted of $n = 24$ observations and the post-test consisted of $n = 25$ observations. *p < 0.05. **p < 0.01.

Table 21

Regression Analysis: Geometry / Measurement

Model	Pre-Test Mean	Pre-Test Std. Dev	Post- Test Mean	Post- Test Std. Dev	R^2	Std. Error of Estimate	F Change	Sig. F Change
Type I	3.54	1.10	3.08	1.10	0.126	1.051	3.182	0.088
Type II	2.96	1.27	3.17	1.88	0.323	1.582	10.503	0.004**
Type III	0.33	0.76	1.04	1.30	0.512	1.143	7.812	0.011*
Type IV	1.71	1.52	2.25	2.25	0.174	2.092	4.629	0.043*

Note: The post-test measurement was the dependent variable for all models and the sample size was $n = 24$.
*p < 0.05. **p < 0.01.

Table 22

Repeated Measures ANOVA: Geometry / Measurement

Knowledge	Sum of Squares	Error	df	Sig.
Type I	2.521	17.979	(1,23)	0.086
Type II	0.521	27.979	(1,23)	0.519
Type III	6.021	14.479	(1,23)	0.005**
Type IV	3.521	51.979	(1,23)	0.225

Note: Sphericity condition for ANOVA testing is satisfied. *p < 0.05. **p < 0.01.

APPENDIX E

Informed Consent Agreement

Please read this consent agreement or listen carefully as it is read to you before you decide to participate in the research study. You are being given a copy of what you read or what is read to you – keep your copy.

Project Title: Possible Effects of a Course Enhancement on Elementary Pre-Service Teachers

Purpose: The purpose of this study is to investigate the effects of the enhancement of a two course mathematical sequence for pre-service teachers. Mathematical content knowledge, self-efficacy, and Praxis core mathematics exam pass rates will be the main focal points.

Participation: You are being asked to participate in this study because you are a student enrolled in the pre-service teacher education program, specifically the course sequence of Math 117 and Math 118. This study will take place Waverly College. You will be asked to complete two online surveys, as well as two pre-tests and post-tests on the topics of whole numbers and operations, as well as geometry. 5 to 8 select students will be invited to participate in an optional task-based interview at the end of the academic year.

Time Required: Your participation is expected to take about 3 hours. Roughly 10 minutes for each online survey and around 45 minutes for each pre-test and post-test.

Risks & Benefits: There are no anticipated risks in this study.

Compensation: There is no compensation for your participation.

Voluntary Participation: Please understand that participation is completely voluntary. You have the right to refuse to participate and/or answer any question(s) for any reason, without penalty. You also have the right to withdraw from the research study at any time without penalty. If you want to withdraw from the study please tell the researcher or a member of the research team who is present during your participation. For any student who may not complete both courses, or those which do not provide all required data, the researcher has the right to end that students participation in the study.

Confidentiality: Your individual privacy will be maintained throughout this study by Professor Thomasey. In order to preserve the confidentiality of your responses, all data will be kept in my office or on my password protected laptop.

Whom to Contact with Questions: If you have any questions or would like additional information about this research, please contact Professor Thomasey at thomasey@waverly.edu. The Waverly College Institutional Review Board (IRB) for Human Subjects Research has approved this project. This IRB currently does not stamp approval on the informed consent/assent documents; however, an approval number is assigned to approved studies – the approval number for this study is ______________________________. You may contact the IRB Director, Dr. Tom Bowman, through the Office of the Associate Dean for Academic Affairs at

Waverly College at 434.544.8327 or irb-hs@waverly.edu with any questions or concerns related to this research study.

Agreement: I understand the above information and have had all of my questions about participation in this research study answered. By signing below I voluntarily agree to participate in the research study described above and verify that I am 18 years of age or older.

Signature of Participant ______________________________ Date ______________________

Printed Name of Participant ______________________

Signature of Researcher ______________________________ Date ______________________

Printed Name of Researcher ______________________

APPENDIX F

Interview Protocol Questions

1. Please provide a short description of your background in mathematics, specifically the mathematical courses you have taken in high school and college and any teaching experience you may have.
2. Describe the amount of effort you needed to put forth during these mathematical courses compared to other courses required for your major in order to develop an understanding.
3. How might these courses have helped to improve your knowledge in mathematics, if at all?
4. How have these courses helped in your preparations to be teacher? Please cite specific examples.
5. Can you provide some general ideas of how you would help an elementary student learn mathematics?
6. Do any of the mathematical teaching methods, taught in these courses, stand above others which you may use in your own teaching some day? Explain.
7. Using the area of a triangle, $A = \frac{1}{2}bh$:
    a. Can you explain how to derive this equation?
    b. How might you go about teaching this formula to your students?
8. When dividing fractions such as, $\frac{1}{2} \div \frac{1}{8}$:
    a. How would you teach your students to divide these fractions?
    b. Do you recall how this process is possible? In other words, what properties of mathematics permit us to alter the expression to an equivalent form? Explain.
9. Do you have any other information to add regarding how this course sequence has helped to improve your mathematical content knowledge or confidence in teaching?

APPENDIX G

1. Whole Number & Computation Pre-Assessment (Version 2.3)
2. Whole Number & Computation Post-Assessment (Version 4.3)
3. Geometry & Measurement Pre-Assessment (Version 2.3)
4. Geometry & Measurement Post-Assessment (Version 4.3)

Date ___________

Finish Time ________

Start Time ________

Elementary Mathematics

Please provide the following information about yourself: Gender: M D F D **Last 4 digits of Soc. Sec. # ___________**

Years of teaching experience: (0 if preservice teacher) _______________

Grade level(s) currently teaching: (Check all that apply)

K	1	2	3	4	5	6	7	8	9	10	11	12
D	D	D	D	D	D	D	D	D	D	D	D	D

Number of ***college*** math courses:

0-3	4-6	7-9	10-12	>12
D	D	D	D	D

Teaching certificate grade levels: (Check all that apply) D D D D D D D D D D D D D

The ***YEAR*** you received your most recent *teaching* degree or Rank:

Teaching certificate content area(s): (Check all that apply)

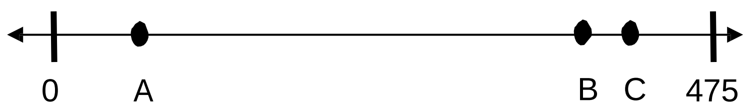
Elem.	M.S.	H.S.	Spec. Ed.	Admin.	Other
D	D	D	D	D	D

Directions for completing items:
Please record date and starting and finishing times in the spaces in the upper right-hand corner of this page. It is *very important* to fill out the demographic information above, ***especially*** the last 4 digits of your SSN, as test results will be reported using that as your ID.

Please answer all questions as completely as possible. Show all work in responding to items and briefly explain your thinking on all items. Let the test facilitator know when you are finished. Thank you very much for your time.

#	Item	Answer
1	Which of the following is expanded notation for the number 720,340 a. 720,000 + 340 b. 720 × 1,000 + 340 × 1 c. 72 × 10,000 + 3 × 100 + 40 × 1 d. 7×100,000 + 2×10,000 + 0×1,000 + 3×100 + 4×10 + 0×1	
2	Which of the following numbers, when rounded to the nearest thousand, becomes 50,000? a. 50,487 b. 51,089 c. 51,490 d. 50,581	

3	Using whole numbers, for which two operations does the associative property hold? a. addition and subtraction b. multiplication and division c. addition and multiplication d. subtraction and division	
4	Solve: 36 ÷ (⁻9 ÷ ⁻3) = [ ] a. 12 b. -12 c. -6 d. 6	

5	Which one of the following statements is true about all prime numbers? a. They are odd numbers b. They are multiples of 1 c. They have more than 2 factors d. They are greater than 2	
6	Which means the same as 50 thousands, 200 hundreds, and 1,000 tens? a. 80,000 b. 50,300 c. 53,000 d. 52,100	
7	What numbers do A, B, and C probably represent on the number line below?  a. A = 100, B = 385, C = 415 b. A = 10, B = 400, C = 410 c. A = 100, B = 250, C = 400 d. A = 120, B = 160, C = 415	
8	Which of the following shows the meaning of 4 × 5? a. 4 × 5 = b. O O O O O O O O c. OOOO × OOOOO d. O O O O O O O O O O O O O O O O O O O O	
9	If a number N has exactly two divisors, then N can only be a. an even number b. a square number c. an odd number d. a prime number	
10	What is the sum of the prime factors of 294? a. 12 b. 18 c. 19 d. 20	

11	Francis James, a math teacher, teaches in a special school. The school board pays Francis 1 dollar for each <u>minute</u> of teaching. Francis teaches (with no breaks) from 8 am until 4 pm each day, Monday through Friday. Francis teaches 180 days each year. a. When will Francis earn the one-millionth dollar? Give your answer to the nearest day, such as 2 years and 43 days. b. Explainy our reasoning	
12	Each of the letters in the following addition problem stands for a unique number 0-9. Find the value of each letter and justify your answers. $$\begin{array}{r} \mathbf{D}\ \mathbf{I} \\ +\ \mathbf{I}\ \mathbf{S} \\ \hline \mathbf{I}\ \mathbf{L}\ \mathbf{L} \end{array}$$	

13	I started thinking about the number of sandwiches I could make with individual ingredients. A loaf of bread contains twenty slices, although I always feed the two end slices to the birds. Each 16-ounce jar of peanut butter will make 12 sandwiches, and each 48-ounce jar of jelly will make 60 sandwiches. If I were to start with full loaves of bread and new jars of peanut butter and jelly, how many PB&J sandwiches would I have to make before emptying a bread bag, a jar of jelly, and a jar of peanut butter at the same time?	
14	When three numbers are multiplied their product is -735. When they are added their sum is -1. What are the three numbers?	

15	Justify that the difference of an odd number and an even number is an odd number.	
16	A student uses the counting strategy named ‘counting up’ to solve 3 +______= 11. She explains, “I start at 3 and count up to 11. That’s 3, 4, 5, 6, 7, 8, 9, 10, 11. (Each time she says a number she raises another finger.) That’s 9 fingers so the answer is 9.” a. How would you help the student understand her misconception? b. How would you help the student understand the correct procedure? Use a drawing or diagram in your explanation.	

17	One student estimates $24\overline{)6543}$ by first rounding 24 to 20 and 6543 to 6000. Explain how you would help the student understand <u>two</u> *other* methods of estimation.	

18	A student during your mathematics lesson used the following steps to solve the multiplication problem 28 × 32 = [ ]. Mathematically speaking, why did her method work? Explain.
	
	    3 2
	    6 4
	1 2 8
	2 5 6
	5 1 2
	
	First, I double 32 until I have enough. Then I add the ones I need. So 28 × 32 is:
	512 + 256 + 128 = 896	

19	Consider the expression 4 × -3. Explain how you would help students understand this expression by using two real world applications of negative integers.	
20	A student is looking for **<u>all</u>** of the factors of 90. She says to you, "My mom said that after I try 1, 2, 3, 4, 5, 6, 7, 8 and 9, I don't need to test any numbers larger than 9." Is her mom correct? Explain why or why not.	

Date ___________

Start Time ________ Finish Time ________

Elementary Mathematics

Please provide the following information about yourself: Gender: M D F D **Last 4 digits of Soc. Sec. # ___________**

Years of teaching experience: (0 if preservice teacher) _______________

Grade level(s) currently teaching: (Check all that apply) K D 1 D 2 D 3 D 4 D 5 D 6 D 7 D 8 D 9 D 10 D 11 D 12 D

Number of ***college*** math courses: 0-3 D 4-6 D 7-9 D 10-12 D >12 D

Teaching certificate grade levels: (Check all that apply) D D D D D D D D D D D D D

The ***YEAR*** you received your most recent *teaching* degree or Rank:

Teaching certificate content area(s): (Check all that apply) Elem. D M.S. D H.S. D Spec. Ed. D Admin. D Other D

Directions for completing items:
Please record date and starting and finishing times in the spaces in the upper right-hand corner of this page. It is *very important* to fill out the demographic information above, ***especially*** the last 4 digits of your SSN, as test results will be reported using that as your ID.

Please answer all questions as completely as possible. Show all work in responding to items and briefly explain your thinking on all items. Let the test facilitator know when you are finished. Thank you very much for your time.

#	Item	Answer
1	Which of the following is expanded notation for the number 207,035? a. 207,000 + 35 b. 207 × 1,000 + 35 × 1 c. 20 × 10,000 + 70 × 100 + 35 × 1 d. 2 × 100,000 + 0 × 10,000 + 7 × 1,000 + 0 × 100 + 3 × 10 + 5 × 1	
2	Which of the following numbers, when rounded to the nearest thousand, becomes 21,000? a. 21,523 b. 21,379 c. 20,089 d. 20,492	
3	The distributive property holds for a. addition over multiplication b. multiplication over addition c. addition over subtraction d. multiplication over subtraction	
4	Solve: $^{-}36 \div (^{-}9 + 3) = [\ ]$ a. 3 b. –3 c. –6 d. 6	

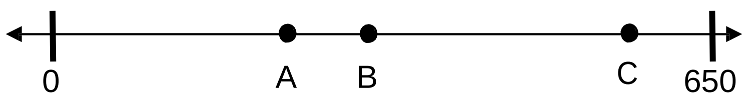
5 Which one of the following statements is true about all factors of 12?

a. They are odd numbers
b. They are divisible by 12
c. They are divisors of 12
d. They are composite numbers

6 Which means the same as 4 ten thousands, 40 hundreds, 400 tens, and 4,000 ones?

a. 44,400 b. 48,000 c. 44,444 d. 52,000

7 What numbers do A, B, and C probably represent on the number line below?



a. A = 105, B = 175, C = 625
b. A = 40, B = 300, C = 640
c. A = 230, B = 310, C = 600
d. A = 220, B = 300, C = 350

8 Which of the following shows the meaning of 2×3?

a. OO × OOO

c. 2 × 3 =

b.
O O
O
O

d.
O O O
O O O

9 If a number N has an odd number of factors, then N can only be

a. a prime number
b. an odd number
c. an even number
d. a square number

10	What is the sum of the prime factors of 315? a. 18 b. 15 c. 21 d. 20	
11	Francis James, a math teacher, teaches in a special school. The school board pays Francis 1 dollar for each <u>minute</u> of teaching. Francis teaches (with no breaks) from 9 am until 4 pm each day, Monday through Friday. Francis teaches 180 days each year. a. When will Francis earn the one-millionth dollar? Give your answer to the nearest week, such as 4 years and 12 weeks. (Consider a week to be 5 days of teaching.) b. Explain your reasoning	
12	Each of the letters in the following addition problem stands for a unique number 0-9. Find the value of each letter and justify your answers. $$\begin{array}{r} \mathbf{S}\ \mathbf{O} \\ +\ \mathbf{S}\ \mathbf{O} \\ \hline \mathbf{T}\ \mathbf{O}\ \mathbf{O} \end{array}$$	
13	I started thinking about the number of sandwiches I could make with individual ingredients. A loaf of bread contains twenty slices. Each 16-ounce jar of peanut butter will make 12 sandwiches, and each 48-ounce jar of jelly will make 60 sandwiches. If I were to start with full loaves of bread and new jars of peanut butter and jelly, how many PB&J sandwiches would I have to make before emptying a bread bag, a jar of jelly, and a jar of peanut butter at the same time?	

14	When three numbers are multiplied their product is -308. When they are added their sum is 23. What are the three numbers?	
15	Justify that the difference of an even number and an even number is an even number.	
16	A student uses the counting strategy 'counting down' to solve 16 - _____ = 10. She explains, "I start at 16 and count down to 10. That's 16, 15, 14, 13, 12, 11, 10." (Each time she says a number she raises another finger.) "That's 7 fingers, so the answer is 7." a. How would you help the student understand her misconception? b. How would you help the student understand the correct procedure? Use a drawing or diagram in your explanation.	

17	One student estimates $27\overline{)3456}$ by first rounding 27 to 30 and 3456 to 3300. Explain how you would help the student understand <u>two</u> *other* methods of estimation.	
18	A student during your mathematics lesson used the following steps to solve the multiplication problem 21 × 45 = [ ]. Mathematically speaking, why did her method work? Explain. 45 90 180 360 720 First, I double 45 until I have enough. Then I add the ones I need. So 21 × 45 is: 720 + 180 + 45 = 945	

19	Consider the expression 3 × -4. Explain how you would help students understand this expression by using two real world applications of negative integers.	
20	A student is looking for **<u>all</u>** of the factors of 70. She says to you, "My mom said that after I try 1, 2, 3, 4, 5, 6, 7 and 8, I don't need to test any numbers larger than 8." Is her mom correct? Explain why or why not.	

Diagnostic Teacher Assessments in Mathematics and Science
Elementary Mathematics

Date _______________
Start Time __________ Finish Time _________

Please provide the following information about yourself: Gender: M D F D **Last 4 digits of Soc. Sec. #** _______________

Years of teaching experience: (0 if preservice teacher) _______________

Grade level(s) currently teaching: (Check all that apply)

K	1	2	3	4	5	6	7	8	9	10	11	12
D	D	D	D	D	D	D	D	D	D	D	D	D

Number of ***college*** math courses:

0-3	4-6	7-9	10-12	>12
D	D	D	D	D

Teaching certificate grade levels: (Check all that apply) D D D D D D D D D D D D D

The ***YEAR*** you received your most recent *teaching* degree or Rank:

Teaching certificate content area(s): (Check all that apply)

Elem.	M.S.	H.S.	Spec. Ed.	Admin.	Other
D	D	D	D	D	D

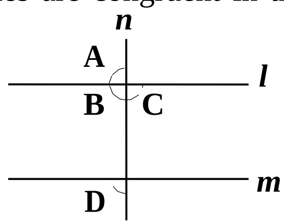
Directions for completing items:
Please record date and starting and finishing times in the spaces in the upper right-hand corner of this page. It is *very important* to fill out the demographic information above, ***especially*** the last 4 digits of your SSN, as test results will be reported using that as your ID.

Please answer all questions as completely as possible. Show all work in responding to items and briefly explain your thinking on all items.
Let the test facilitator know when you are finished. Thank you very much for your time.

#	Item	Answer
1	Which of the following is a quadrilateral with four right angles and four sides of equal length? a. rectangle b. rhombus c. square d. parallelogram	
2	A graduated beaker is used to measure… a. temperature b. volume c. weight d. surface area	
3	Select an appropriate metric unit of measurement for the capacity of a coffee cup. a. kilogram b. centimeter c. milliliter d. liter	
4	Which of these is the formula for finding the area of a square? a. s^2 b. 4s c. s^s d. s^4	

5 Lines *l* and *m* are parallel. Line *n* is not perpendicular to line *l* and it is not perpendicular to line *m*. Which angles are congruent in the drawing?

a. all angles are congruent
b. ∠A ≅ ∠B and ∠C ≅ ∠D
c. ∠A ≅ ∠D and ∠B ≅ ∠C
d. ∠A ≅ ∠C and ∠B ≅ ∠D



6 Folding Figure E along the line segments at the base of each triangle can create which three-dimensional shape?

a. prism
b. pyramid
c. tetrahedron
d. octagon

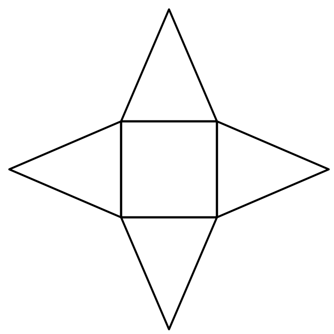


Figure E

7 Four children measured the *length* of a diving board by walking it off heel-toe. The chart shows their measurements. Who had the longest foot?

a. Lisa
b. Arlene
c. Eric
d. Shelton

Name	# footsteps
Lisa	12
Arlene	15
Eric	11
Shelton	13

8 Which pair of properties is common to a rectangle and rhombus?

a. The diagonals are parallel and perpendicular
b. The opposite angles are right angles
c. The diagonals bisect each other
d. The diagonals bisect the opposite angles

9

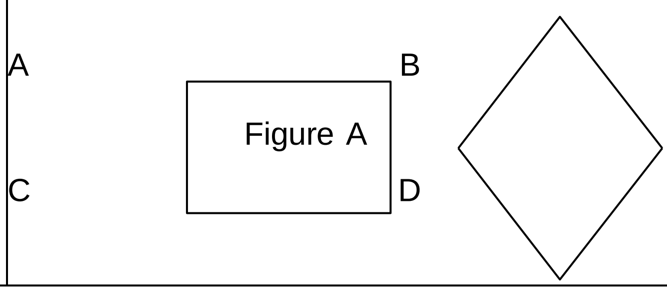
If BF is perpendicular to EA, which statement below is true about the diagram?

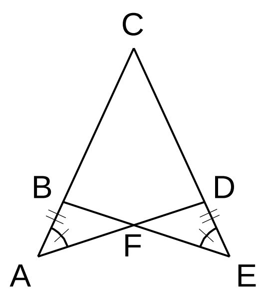
a. Angles EFD and CFA are complementary angles
b. Angles EFC and CFE are supplementary angles
c. Angles DFB and CFB are adjacent angles
d. Angles DFA and DFC are both acute angles

10 How many faces, edges and vertices does a pentagonal prism have?

a. 5 faces, 5 edges, 5 vertices
b. 5 faces, 10 edges, 7 vertices
c. 7 faces, 15 edges, 5 vertices
d. 7 faces, 15 edges, 10 vertices

11 How would you subdivide the rectangle ABCD to reassemble the pieces to create Figure A?



12	After 2 hours and 58 minutes at 360º a 7 pound 8 ounce turkey will be fully cooked. Assume a linear relationship between weight and cooking time. a. How long would it take to cook an 11 pound 3 ounce turkey at 360º? b. What weight turkey would take 4 hours and 20 minutes to cook at 360º?	
13	Explain why ΔCBE ≅ ΔCDA. 	

14	Given the dimensions of a rectangular solid $3 \times 5 \times 7$. Double two of the dimensions and examine the new solid. a. What happened to the volume? b. Will this be true for any rectangular solid? Justify your reasoning.	

15 The weight of the smaller rock below is 1 lb.

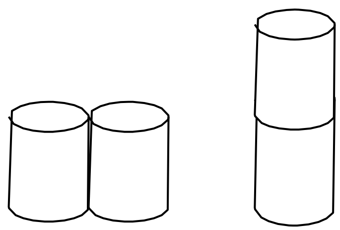
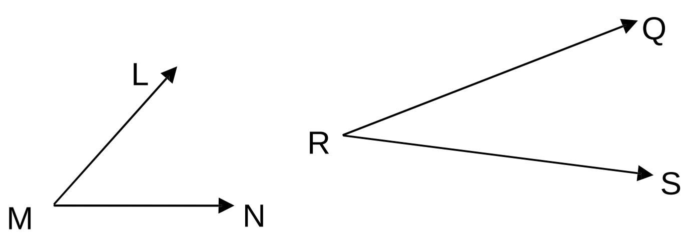
a. Describe how you can use the information about the weight of the smaller rock to estimate the weight of the larger rock.
b. Estimate how many lbs. the larger rock weighs and explain your reasoning.

http://www.promotega.org/ksu 00006/igneous.htm

16	Student A says that any 2 triangles with three congruent angles are congruent. Student B disagrees and says that any two triangles with three congruent angles are not congruent. Describe an instructional activity that you could use to address any misconceptions.	

17	Students arrived at the following conjectures after studying their textbook: • Kites have two pairs of congruent sides, so they are parallel. • Opposite sides of parallelograms are congruent so they are rectangles. a. Identify which conclusion(s) is/are incorrect. Explain. b. Describe an instructional activity to address any misconceptions.	

18	Rex and Mary were putting cork tiles on their walls. Rex's wall measured 8 × 6 and Mary's measured 12 × 4. Rex concluded that they both need the same amount of tile to cover them since Mary's wall is longer, but his is wider. Mary argued that she would need more tile because her wall had a greater perimeter. a. Explain how you would address the students' conclusions. b. Describe an instructional activity that you would use to address the errors or misconceptions.	

19	Students were asked to compare the surface area of the two shapes below. Shape 1 is composed of two cylindrical cans side by side. Shape 2 is composed by the same two cylindrical cans stacked on top of each other. Student A determined that the surface area would be identical to each other because both were created from the same two cans. Student B determined that Shape 1 would have less surface area since Shape 2 was taller. a. Describe what you can conclude about the two students' understanding about surface area. b. Describe an instructional activity that you would use to address the misconceptions.  Shape 1 Shape 2	
20	A student concluded that ∠LMN and ∠QRS were not congruent. The student said that the ∠LMN was 50º and ∠QRS was 30º but the student felt that the measurements were incorrect. The measure of ∠QRS should be larger due to its line segments. a. Explain what is wrong with the student's thinking about the measurement of angles, and b. How you would address this misconception?  	

Date _______________

Start Time __________ Finish Time _________

Elementary Mathematics

Please provide the following information about yourself: Gender: M D F D **Last 4 digits of Soc. Sec. #** ________________

Years of teaching experience:
(0 if preservice teacher) _______________

Grade level(s) currently teaching: (Check all that apply)

K	1	2	3	4	5	6	7	8	9	10	11	12
D	D	D	D	D	D	D	D	D	D	D	D	D

Number of *college* math courses: 0-3 D 4-6 D 7-9 D 10-12 D >12 D

Teaching certificate grade levels: (Check all that apply)
D D D D D D D D D D D D D

The ***YEAR*** you received your most recent *teaching* degree or Rank: _______________

Teaching certificate content area(s): (Check all that apply)

Elem.	M.S.	H.S.	Spec. Ed.	Admin.	Other
D	D	D	D	D	D

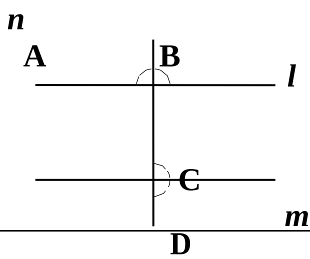
Directions for completing items:

Please record date and starting and finishing times in the spaces in the upper right-hand corner of this page. It is *very important* to fill out the demographic information above, ***especially*** the last 4 digits of your SSN, as test results will be reported using that as your ID.

Please answer all questions as completely as possible. Show all work in responding to items and briefly explain your thinking on all items. Let the test facilitator know when you are finished. Thank you very much for your time.

#	Item	Answer
1	Which of the following is a quadrilateral with one pair of opposite sides that are parallel and one pair of opposite sides that are not parallel? a. rectangle b. trapezoid c. square d. parallelogram	
2	A spring scale is used to measure… a. magnification b. cost c. weight d. volume	
3	Select an appropriate metric unit of measurement for the weight of a textbook. a. kilogram b. centimeter c. millimeter d. liter	
4	Which of these is the formula for finding the circumference of a circle? a. πr^2 b. πd c. πr d. $2\pi r^2$	

5 Lines *l* and *m* are parallel. Line *n* is not perpendicular to line *l* and it is not perpendicular to line *m*. Which angles are congruent in the drawing?



a. All angles are congruent

b. ∠A ≅ ∠B and ∠C ≅ ∠D
c. ∠A ≅ ∠D and ∠B ≅ ∠C
d. ∠A ≅ ∠C and ∠B ≅ ∠D

6 Folding Figure E along the line segments formed (as shown in the diagram) can create which three-dimensional shape?

a. Rectangular prism
b. Pentagonal pyramid
c. Pentagonal prism
d. Square pyramid

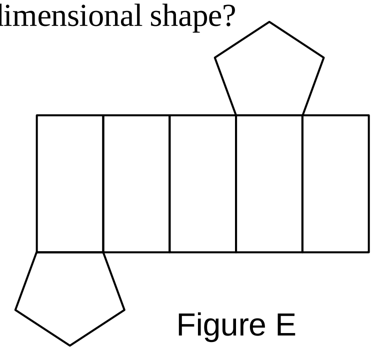


Figure E

7 Four children measured the *length* of a chalkboard by counting how many hand-spans it took them to reach end to end. The chart shows their measurements. Who had the largest hand-span?

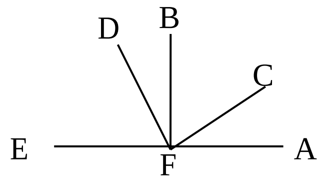
a. Lisa
b. Arlene
c. Eric
d. Shelton

Name	# hand-spans
Lisa	40
Arlene	59
Eric	61
Shelton	58

8 Which pair of properties is common to a parallelogram and an isosceles trapezoid?

a. The diagonals are perpendicular
b. The opposite angles are right angles
c. The diagonals are congruent
d. The diagonals bisect each other

9



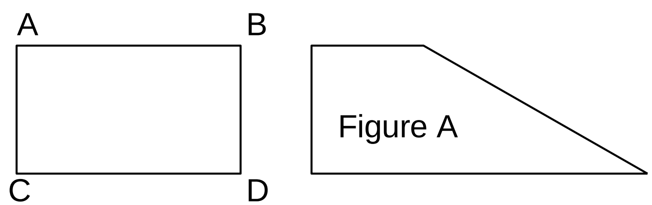
If BF is perpendicular to EA, which statement below is true about the diagram?

a. Angles EFB and BFA are complementary angles
b. Angles EFC and CFE are supplementary angles
c. Angles EFC and DFA are both obtuse angles
d. Angles BFA and BFE are vertical angles

10 How many faces, edges and vertices does an octagonal pyramid have?

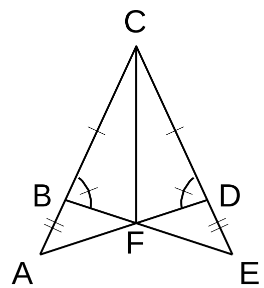
a. 9 faces, 16 vertices, 16 edges
b. 8 faces, 16 vertices, 16 edges
c. 8 faces, 16 vertices, 9 edges
d. 9 faces, 9 vertices, 16 edges

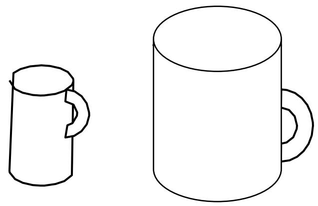
11 How would you subdivide the rectangle ABCD to reassemble the pieces to create a figure A?



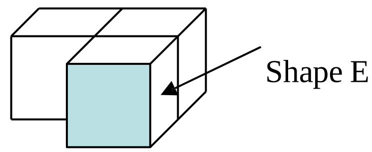
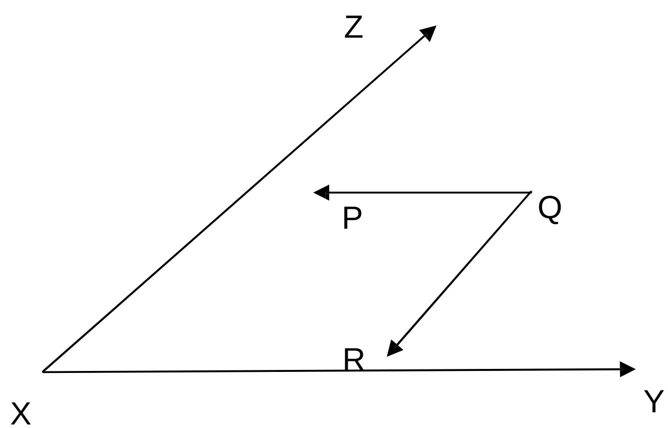
12 It takes one hour and twenty minutes to cook a 4 pound 6 ounce turkey at 350º. Assume a linear relationship between weight and cooking time.

a. How long would it take to cook a 9 pound 10 ounce turkey at 350º?
b. What weight turkey would take 3 hours and 33 minutes to cook at 350º?

13	Explain why $\Delta BFC \cong \Delta DFC$.  	
14	Given the dimensions of a rectangular solid $2 \times 3 \times 4$. Double one of the dimensions and examine the new solid. a. What happened to the volume? b. Will this be true for any rectangular solid? Justify your reasoning.	

15 The smaller mug below can fit 5 pieces of ice inside of it. a. Describe how you can use the information about the capacity of the smaller mug to estimate how much ice will fit into the larger mug. b. Estimate how many ice cubes the larger mug will hold and explain your reasoning. 	
16 Student A says any 2 quadrilaterals with the same perimeter are congruent. Student B disagrees and says any two quadrilaterals with the same perimeter are not congruent. Describe an instructional activity that you could use to address any misconceptions.	

17	Students arrived at the following conjectures after studying their textbook: • Each pair of opposite sides of a parallelogram are congruent so they are all rhombuses. • A trapezoid can be a rectangle if it has two pairs of parallel sides. a. Identify which conclusion(s) is/are incorrect. Explain. b. Describe an instructional activity to address any misconceptions.	
18	Zach and Rita were each getting paper to cover their murals. Zach's mural measured 9×9 and Rita's measured 27×3. Zach concluded that they both need the same amount of paper to cover them since Rita's mural is longer but his is wider. Rita argued that she would need more paper because her mural had a greater perimeter. a. Explain how you would address the students' conclusions. b. Describe an instructional activity that you would use to address the errors or misconceptions.	

19	Students were asked to add a cube to Shape E at the shaded face and determine the increase in the surface area of the new shape. (Assume each face is one square unit.) Student A determined that the surface area increased by six square units, because you added 6 faces. Student B disagreed and reasoned that it was 5 square units because one face abuts to the shaded face. a. Describe what you can conclude about the two students' understanding about surface area. b. Describe an instructional activity that you would use to address the misconceptions.  	
20	A student measured $\angle ZXY$ and $\angle PQR$ and found the measure of $\angle ZXY$ to be 44º and $\angle POR$ to be 50º. The student however felt the $\angle ZXY$ measurement was incorrect because $\angle PQR$ fit inside $\angle ZXY$ so it had to be smaller. (a) Explain what is wrong with the student's thinking about the measurement of angles, and (b) how you would address this misconception.  	

APPENDIX H

Concerns with DTAMS Open-Ended Questions Being Left Blank

When investigating the differences between the response data between the geometry / measurement pre-test and post-test I noticed a substantial number of additional teaching candidates attempted questions 11 and 16 on the post-test compared to the pre-test, see Figures 5 and 6.

Problem # ->	11a	11b	12a	12b	13a	13b	14a	14b	15a	15b	16a	16b	17a	17b	18a	18b	19a	19b	20a	20b
Total Students	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
Blanks	16	16	9	9	8	8	6	6	4	4	11	11	8	8	2	2	9	9	5	5
Attempts	9	9	16	16	17	17	19	19	21	21	14	14	17	17	23	23	16	16	20	20
Awarded no points	8	7	14	14	17	17	17	16	21	21	13	12	17	16	21	10	16	15	19	4
Earned Credit	1	2	2	2	0	0	2	3	0	0	1	2	0	1	2	13	0	1	1	16
Not attempted KT1			9				6		4		11									
Not attempted KT2	16				8								8		2		9		5	
Not attempted KT3		16		9		8		6		4										
Not attempted KT4												11		8		2		9		5

Figure 5: Geometry / Measurement Assessment Response Pre-Test Data.

Problem # ->	11a	11b	12a	12b	13a	13b	14a	14b	15a	15b	16a	16b	17a	17b	18a	18b	19a	19b	20a	20b
Total Students	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24
Blanks	4	4	4	4	0	0	4	4	0	0	1	1	3	3	4	4	9	9	4	4
Attempts	20	20	20	20	24	24	20	20	24	24	23	23	21	21	20	20	15	15	20	20
Awarded no points	19	18	16	12	24	24	15	11	24	24	23	10	19	16	16	10	13	10	20	9
Earned Credit	1	2	4	8	0	0	5	9	0	0	0	13	2	5	4	10	2	5	0	11
Not attempted KT1			4				4		0		1									
Not attempted KT2	4				0								3		4		9		4	
Not attempted KT3		4		4		0		4		0										
Not attempted KT4												1		3		4		9		4

Figure 6: Geometry / Measurement Assessment Response Post-Test Data.

For example, when looking at question 11, see Figure 7, there were 16 teaching candidates who did not respond on the pre-test, compared to 4 who did not respond on the post-test. When investigating the cumulative scores earned on this task as a class, there was no change in the total number of points earned on the post-test compared to the pre-test. In other words, only 3 total points were earned on this task for the pre-test and even though 12 additional teaching candidates attempted the problem on the post-test the cumulative class score on this task remained at 3 points. This leads me to believe that there was a structural problem with this question, being that teaching candidates simply did not know how to respond to the question.

11 How would you subdivide the rectangle ABCD to reassemble the pieces to create Figure A?

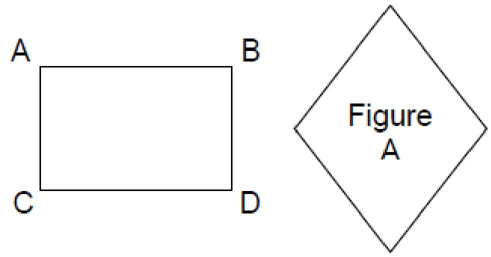


Figure 7: Geometry / Measurement Assessment Differences Between Pre-Test and Post-Test.

In contrast to question 11, question 16 on the geometry / measurement assessment, see Figure 8, demonstrated substantial gains on the post-test despite the fact that there was a high number of blank responses for the pre-test. There were 11 blanks on the pre-test compared to 1 on the post-test. Unlike possible structural issues, as presented for question 11, differences for this task are legitimized by the fact that educational gains from the semester sequence influenced responses by observing an increased in the cumulative class score.

16 Student A says that any 2 triangles with three congruent angles are congruent. Student B disagrees and says that any two triangles with three congruent angles are not congruent.

Describe an instructional activity that you could use to address any misconceptions.

Figure 8: Geometry and Measurement Assessment Differences Between Pre-Test and Post-Test.

Question 16, assessed knowledge type IV – Pedagogical Content Knowledge. It could have been the case that teaching candidates did not possess the necessary PCK to respond to the task before the semester. On the other hand, based on the instructional gains acquired through the course sequence, they had a much better understanding of different pedagogical techniques in which to apply to this task and could better respond to the task within the assessment. This is validated by looking at cumulative class scores, where only 3 total points were earned on this task for the pre-test, compared to 16 points earned on the post-test. In other words, I observed considerable gains on this task for the pre-test compared to the post-test related to the 10 additional teaching candidates who attempted this task on the post-test.

A final example of a question which may have been confusing for teaching candidates was open-ended question 12, see Figure 9.

12 After 2 hours and 58 minutes at 360° a 7 pound 8 ounce turkey will be fully cooked. Assume a linear relationship between weight and cooking time.

a. How long would it take to cook an 11 pound 3 ounce turkey at 360°?
b. What weight turkey would take 4 hours and 20 minutes to cook at 360°?

Figure 9: Question 12 from the Geometry / Measurement Assessment which Teaching Candidates Misinterpreted as a Multiple Choice Task.

A number of teaching candidates responded to this question as if it were a multiple choice question. Even though the directions are fairly straight forward, the layout of the question appears as if it were a multiple choice task. It is understandable why teaching candidates may have thought this, based on the fact that the first 10 questions of the assessment had the exact same layout, with similar bulleted letter options as answers, compared to what was presented in this task.

APPENDIX I

Concerns with Grading of DTAMS Assessments

In order to expose the concerns related to the grading of the open-ended questions, I will use teaching candidates' responses to question 11 on the pre-test and post-test, seen in Figures 10, 11, and 12.

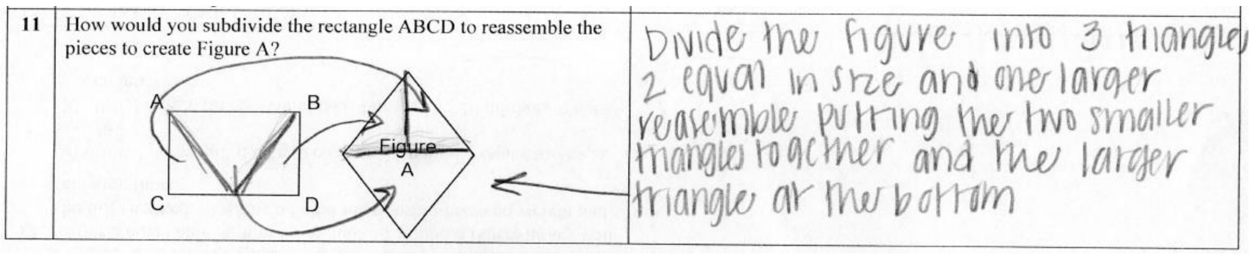


Figure 10 : Work of Teaching Candidate 0415 on Question 11 of the Geometry / Measurement Pre-Test.

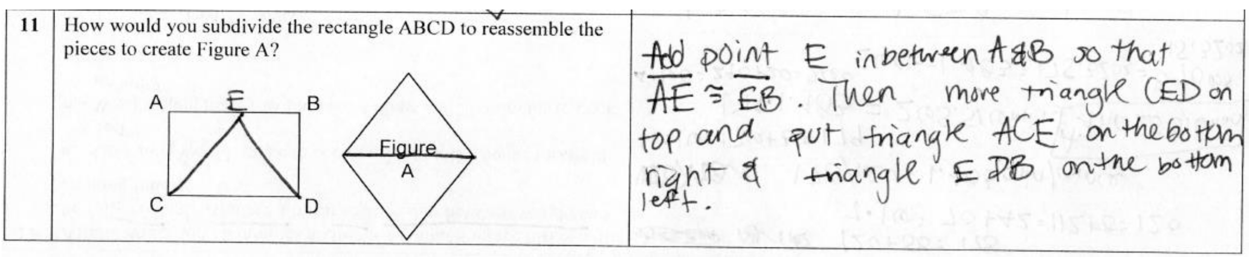


Figure 11 : Work of Teaching Candidate 0929 on Question 11 of the Geometry / Measurement Pre-Test.

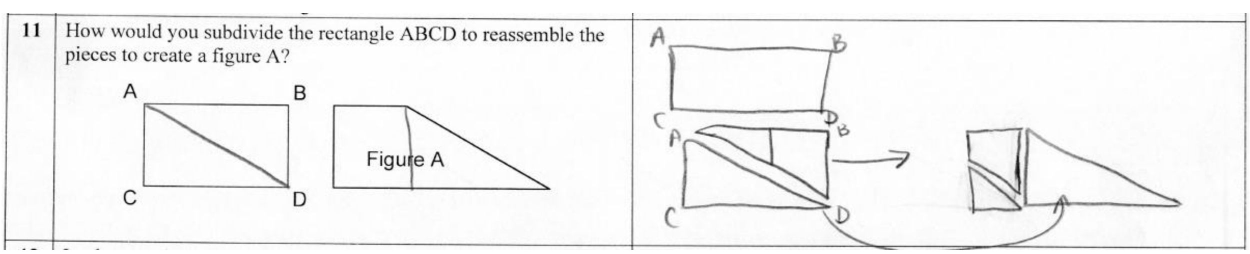
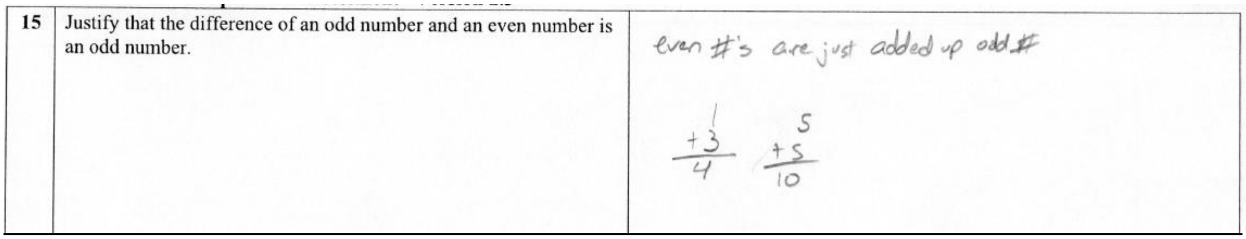


Figure 12: Work of Teaching Candidate 0929 on Question 11 of the Geometry / Measurement Post-Test.

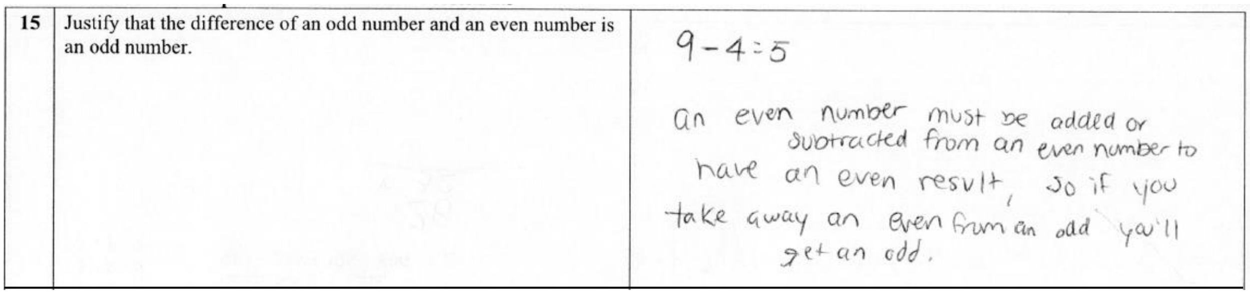
The grades awarded to these example responses were 3, 1, and 3, respectively. However, when investigating the grades associated with the first two examples, Figures 10 and 11, I feel as if the grading is inconsistent. The work of teaching candidate 0929 in Figure 11, used more appropriate mathematical notation and language, and certainly demonstrated a fair amount of description to convey their understanding, but was awarded zero points for "appropriate reasoning or problem solving strategies." Furthermore, when it came to the grade awarded to an equivalent task on the post-test, this same teaching candidate, who responded to the task using basically the same approach (with less formal mathematical notation), was awarded the full credit of 3 points. To this end, not only does this question pose a problem associated with the grading of the assessments, but it could have been that teaching candidates simply did not know what a type of response was suitable for the assessment. This is especially true based on the fact that 11 more teaching candidates attempted this question on the post-test compared to the pre-test, yet the total number of points earned remained the exact same from the pre-test class total. Hence, even though more teaching candidates attempted the problem, they may have been unaware of what the question was asking. In which case, perhaps the directions for this question should be clarified.

In contrast to the inconsistent grades assigned to tasks on the DTAMS measure dealing with geometry / measurement, the grading on the whole number / computation assessment were more sound. When investigating a typical problem on the whole number / computation pre-test, the assigned grades match the understanding that was displayed by the teaching candidate's response. Figure 13, Figure 14, and Figure 15 were given grades of 0, 2, and 3, respectively. These grades are accurate and acceptable in quantifying the understanding of the task displayed by the teaching candidate.



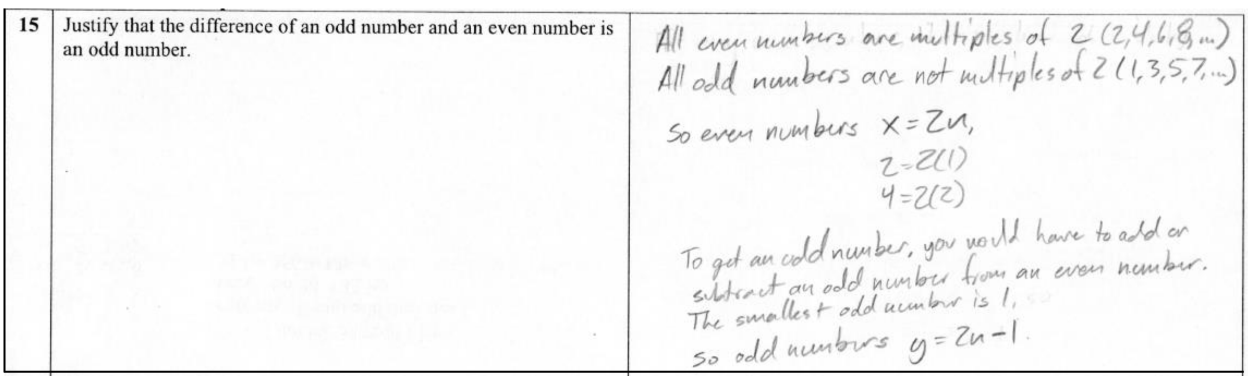
15	Justify that the difference of an odd number and an even number is an odd number.	even #'s are just added up odd # 1 + 3 = 4 5 + 5 = 10

Figure 13: Example Work for Question 15 of the Whole Number / Computation Pre-Test, Earning 0 Points.



15	Justify that the difference of an odd number and an even number is an odd number.	9 − 4 = 5 an even number must be added or subtracted from an even number to have an even result, so if you take away an even from an odd you'll get an odd.

Figure 14: Example Work for Question 15 of the Whole Number / Computation Pre-Test, Earning 2 Points.



15	Justify that the difference of an odd number and an even number is an odd number.	All even numbers are multiples of 2 (2,4,6,8,...) All odd numbers are not multiples of 2 (1,3,5,7,...) So even numbers x = 2n, 2 = 2(1) 4 = 2(2) To get an odd number, you would have to add or subtract an odd number from an even number. The smallest odd number is 1, so odd numbers y = 2n − 1.

Figure 15: Example Work for Question 15 of the Whole Number / Computation Pre-Test, Earning 3 Points.

APPENDIX J

Survey on Self-Efficacy and Teaching Beliefs

(starting on next page)

Math 117 - Perceptions regarding the teaching of mathematics

ID#:__________Date:__________

The purpose of this survey is to help aid in the improvements to the Math 117/118 course sequence. Please answer each prompt to the best of your ability and your own personal views regarding mathematics and the teaching profession. There are no right or wrong answers. Honest answers will help in the improvements over answers that you may think someone would want to hear. The survey is not graded.

Q1: To what extent do you agree with each of the following?

	Strongly disagree (1)	Disagree (2)	Agree (3)	Strongly agree (4)
I like answering questions during mathematics lessons.	❍	❍	❍	❍
I get anxious when I have to teach some mathematics topics.	❍	❍	❍	❍
Even if I work hard, I will not teach math as well as I will most students.	❍	❍	❍	❍
The mathematics achievement of some students cannot generally be attributed to their teachers.	❍	❍	❍	❍
I will continually find better ways to teach mathematics.	❍	❍	❍	❍

Q2: To what extent do you agree with each of the following?

	Strongly disagree (1)	Disagree (2)	Agree (3)	Strongly agree (4)
If students are underachieving in mathematics, it is most likely due to ineffective mathematics instruction.	❍	❍	❍	❍
I understand mathematics concepts well enough to be effective in teaching mathematics.	❍	❍	❍	❍
I find it difficult to use manipulatives to explain why mathematics works.	❍	❍	❍	❍
When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach.	❍	❍	❍	❍

Q3: To what extent do you agree with each of the following?

	Strongly disagree (1)	Disagree (2)	Agree (3)	Strongly agree (4)
I enjoy thinking about different ways to solve a mathematics problem.	❍	❍	❍	❍
If I work hard, I am confident in my ability to learn new mathematics.	❍	❍	❍	❍
No matter how much effort I put forth, I can only do so well in mathematics.	❍	❍	❍	❍
For some reason, even though I study, math seems unusually hard for me.	❍	❍	❍	❍
I'm not the type to do well in math.	❍	❍	❍	❍
My effort is the key to my success in mathematics.	❍	❍	❍	❍
I can change my mathematics intelligence.	❍	❍	❍	❍

Q4: To what extent do you agree with each of the following?

	Strongly disagree (1)	Disagree (2)	Agree (3)	Strongly agree (4)
It is important for me to continue to learn more about teaching mathematics.	❍	❍	❍	❍
One of my goals for teaching mathematics is to develop more effective teaching methods.	❍	❍	❍	❍
The development of my teaching abilities in mathematics is important to me.	❍	❍	❍	❍
It is important for me to be praised for having higher teaching abilities in mathematics than other teachers.	❍	❍	❍	❍
It is important that my students think that I am a better teacher of mathematics than other teachers in the school.	❍	❍	❍	❍

Q5: To what extent do you agree with each of the following?

	Strongly disagree (1)	Disagree (2)	Agree (3)	Strongly agree (4)
One of my goals for teaching is to be recognized as one of the best teachers of mathematics in the school.	❍	❍	❍	❍
It is important for me to teach mathematics better than other teachers.	❍	❍	❍	❍
It is important that my teaching abilities in mathematics are not inferior to that of most of my colleagues.	❍	❍	❍	❍
I don't want to look like an incompetent teacher of mathematics to my fellow teachers.	❍	❍	❍	❍
I don't want to show poor teaching skills in mathematics when the principal or parents observe one of my lessons.	❍	❍	❍	❍

Q6: To what extent do you agree with each of the following?

	Strongly disagree (1)	Disagree (2)	Agree (3)	Strongly agree (4)
If I really try hard, I can get through to even the most difficult or unmotivated students.	❍	❍	❍	❍
If a student did not remember information I gave in a previous lesson, I would know how to increase his/her retention in the next lesson.	❍	❍	❍	❍
If some of my students couldn't do a class assignment, I would be able to accurately assess whether the assignment was at the correct level of difficulty.	❍	❍	❍	❍
If a student in my class becomes disruptive and noisy, I feel assured that I know techniques to redirect him/her quickly.	❍	❍	❍	❍

Q7: What percentage of teachers in general or at your school (if you are employed) share the following beliefs?

	None (1)	1% to 25% (2)	26% to 50% (3)	51% to 75% (4)	76% to 100% (5)
Students just aren't motivated to learn.	❍	❍	❍	❍	❍
The opportunities in the community (-ies) where our students live help ensure that they will learn.	❍	❍	❍	❍	❍
Every child can learn.	❍	❍	❍	❍	❍
Teachers are able to motivate their students.	❍	❍	❍	❍	❍
Teachers are able to get through to difficult students.	❍	❍	❍	❍	❍
If a child doesn't want to learn, teachers give up on him/her.	❍	❍	❍	❍	❍
Students come to school ready to learn.	❍	❍	❍	❍	❍
Home life provides so many advantages that students are bound to learn.	❍	❍	❍	❍	❍

APPENDIX K

Lesson Plan Description and Grading Rubric

(starting on next page)

Math 117 – Lesson Plan Development

Description
Choose one mathematical topic covered up to this point during the course. While you are not required to submit a formal lesson plan, the lesson must include all necessary notes, descriptions, worksheets, activities, etc. There should be enough material to cover a 45-minute mathematical block and written in such a way that another teacher could look at your lesson and use it in their classroom.

Requirements
1. Develop an outline which will guide your overall lesson
    a. Use your own words/style as if you were teaching the material
    b. Provide an idea of how much time will be spent on each section
    c. Give short descriptions of what you would need to do throughout the lesson.
        i. Purposefully chosen tasks, fat/skinny questions, conceptual meanings, etc.
2. Include: SOL's or the Common Core objective(s) that you plan to cover.
3. Lessons must include at least one supporting element.
    a. Worksheet, manipulative, activity, stations, real world example, etc.
        i. Internet worksheets are not acceptable, but you could reconstruct them.
4. Include one extension activity related to the chosen topic.

Method of Submission
1. Your work must be submitted as a single PDF document and uploaded to Moodle.
2. Save the file as: Lastname_Journal#.pdf
    a. For example: "Thomasey_Journal1.pdf"

Due Dates:
- All projects are due by midnight on Saturday 9/24.

Grading
- Please see the grading rubric for details regarding the grading of this assignment.

Scoring Rubric for Math 117 Lesson Plan

Dimension	Exemplary	Proficient	Needs Work
General Outline (Clarity and organization)	Clearly organized through the use of proper headings and subheadings. The structure follows a logical order appropriate for the topic.	At times the structure is difficult to follow and/or the arrangement of topics could be improved.	Very hard to follow the structure of the lesson and/or there is no logical order for the chosen topic.
Outline Content	Includes all necessary material needed for a successful lesson, including appropriately chosen examples. Information is clear and there are very few grammatical mistakes.	Most essential material is provided within the outline; however the lesson could be improved with additional insight, details and/or examples. Grammatical mistakes are evident, but not overwhelming.	Major gaps within the necessary material needed to teach a particular topic. Lack of necessary examples and/or other required materials. Major grammar mistakes. Topic does not align with those covered in class.
Amount of material	Lesson included an appropriate amount of material that would effectively use the allotted time.	While the lesson included all necessary components, the proposed allotted time is incorrect. Certain objectives will fall short or exceed the available time.	Lesson lacks major components and/or would fall well short of the allotted time. Or the amount of material provided is well beyond what could be covered in the time slot.
SOL or Common Core listing	Items are included and are in alignment with the topics covered in the lesson.	Items are included, but are missing certain components and/or do not align with the topics being covered.	Not present and/or do not align with the lesson topics.
Supporting element/activity	Lesson includes an original, well thought out, appropriate activity and/or lesson element to support the topic being taught.	Supporting element is evident, however it lacks in creativity and/or necessary supporting components to mathematical topic. Procedural steps need further clarification.	Supporting element is not present and/or does not pertain to the given topic being taught.
Extension activity	Included and relevant to topic and more advanced learners.	Activity is included, but lacks in details and/or connectivity towards the topic.	Not included and/or does not pertain to any aspect of the topic being taught.
Submission requirements	Project was submitted on time using all necessary requirements.	Project lacked some submission requirements (eg. File name, document type, etc.)	Project did not follow proper submission requirements and/or was submitted late.

Comments:

Grade:

APPENDIX L

Codes for Interview Transcriptions

Initial Coding Sequence

1. Mathematical background
2. Content knowledge
3. Pedagogy (method and practice of teaching)
4. Preparations to be a teacher
5. Confidence

Final Coding Sequence

1. Content knowledge
2. Teaching preparations
3. Mathematical Language
4. Confidence
5. Memorable take-a-ways
6. Preparations as a student / Personal mathematical practices
7. Past Math Experiences
8. Background and experience