Conditions for Pulsar Planet Formation

Michele Horner

Department of Astronomy
University of Virginia
May 14, 2021

This thesis is submitted in partial completion of the requirements of the BS Astronomy-Physics Major.

Draft version May 14, 2021
Typeset using LaTeX **twocolumn** style in AASTeX63

Conditions for Pulsar Planet Formation

Michele Horner[1]

[1] *The University of Virginia Department of Astronomy*

1. ABSTRACT

This paper outlines the conditions for planet formation around pulsars. A common model proposes the fallback of mass after an initial supernova explosion, which occurs as a reverse shock propagates the inward accretion of a fraction of bound material back onto the surface of the star. The initial fallback mass spreads and simultaneously decreases in surface density, during which planet formation may take place at sufficiently cool temperatures. The existence of these conditions, and the availability of material to form planets, depends on the time since the initial fallback as well as the fallback radius from the central star. By modeling the profile of a pulsar accretion disk and using solar estimates for dust settling time and planetary isolation mass, we may predict the conditions for the evolution of planetesimals in a pulsar system.

Pulsar	Planetary Object	Semi-major Axis (AU)	Mass
PSR B1620-26	PSR B1620-26 b	23	2.5 M_J
PSR B1257+12	PSR B1257+12 A	.19	0.020 M_E
PSR B1257+12	PSR B1257+12 B	.36	4.3 M_E
PSR B1257+12	PSR B1257+12 C	.46	3.90 M_E
PSR B0943+10	PSR B0943+10 b	1.8	2.8M_J
PSR B0943+10	PSR B0943+10 c	2.9	2.6 M_J
PSR B0329+54	PSR B0329+54 b	10.26 ±0.07	1.97±0.19 M_E
PSR J2322-2650	PSR J2322-2650 b	0.0102	0.7949 M_J

Table 1. List of Known Pulsar Planets.

2. INTRODUCTION

The exact mechanisms of pulsar planet formation are largely unknown, and the existence of the planets themselves is observed to be extremely rare. Estimates from surveys show that only about 5% of pulsars may host planets.(4) There are two primary theories that explain their existence. The first proposes that these planets survive the supernova explosion, and the subsequent formation of a neutron star. It is extremely unlikely that a preexisting planet of a near-Earth mass would survive such an event, however, this case may potentially explain the existence of the exceptionally high-mass planet PSR B16020-26 b, which exists at a ∼20 AU distance from its central star. A second, and more likely theory, is that the fallback material from the initial explosion forms an accretion disk with the materials to form planets. The neutron stars themselves can be classified into one of the four categories: young radio pulsars, millisecond radio pulsars, thermally emitting dim, isolated, neutron stars, or accreting neutron stars. The type of star, and its surrounding environment, has a crucial effect on the formation and composition of the surrounding planets.(1) Three of the first pulsar planets which have been detected orbit around the radio pulsar PSR B1257+12. Two of the three have near-Earth masses and semi-major axes of .3-.4 AU. These planets are believed to have formed from the accretion of matter from a companion star. The location and composition of such planets is first determined by the accretion process, specifically the disk formation and composition following a supernova. Over time, this disk thins, evolves, and cools. In order for dust within a disk to sublimate, the blackbody temperature of material around a neutron star must cool to the sublimation temperature for specific solids, ∼ 2000 K for silicates. This blackbody temperature is determined by distance from the central source and the luminosity of the central source:

$$L = \frac{GM\dot{M}}{r} \tag{1}$$

This value is typically $\sim 10^{38}$ erg s^{-1}, given an accretion rate of $\gtrsim 10^{-8} M_{\odot} y^{-1}$.

3. FALLBACK

After a supernova explosion, although most material is ejected from the system, about 0.001-0.1 $M_\odot$ may remain bound and fall back toward the star.

The radius of the initial fallback disk is described by equation(3):

$$r = (\frac{GMt^2}{\pi^2})^{1/3} \tag{2}$$

where t, the the timescale of this event, is a few hours, and the fallback mass would range from 0.001-0.1 $M_\odot$. This fallback radius is calculated to be 10^8 cm.

From this radius, the inner mass moves inward toward the star and accretes onto the central source, simultaneously decreasing in mass. As the inner mass loses angular momentum, the total angular momentum of the system is conserved. The outer mass gains angular momentum and spreads outward. The angular momentum of the system relates to the mass of the disk by:

$$J = M(t)\sqrt{GMa(t)} \tag{3}$$

where a is the radius of the disk. Planet formation must occur in the outwardly expanding parts of the disk, outside the fallback radius of 10^8 cm. (7) The mass of the disk varies with radius as:

$$M \sim \frac{1}{\sqrt{a}} \tag{4}$$

So, if an initial fallback mass of 0.01 $M_\odot$ is found at a radius of 10^8 cm, the mass at a radius of .5 AU would be around the size of ∼5 Earth masses. Considering planets discovered at this distance are several Earth masses in size, this would imply a near 100% efficiency rate of planet production,(6) which may explain why the observed occurrence of pulsar planets is so unlikely.

4. INITIAL STAGES OF ACCRETION DISK FORMATION

The initial stages of planet production begin with the formation of a protoplanetary disk around a star during its evolution. An initial core within a molecular cloud precedes star formation. The large angular momentum (J∼10^{54}g^2 cm s^{-1}) of that core is consistent with the angular momentum of molecular gas in 10-10^2 AU Keplerian orbit around a Solar mass star. Through these phenomena, large disks may form around these evolving stars. In astronomical observation,the presence of an initial disk is characterized by excesses of infrared and ultra-violet radiation in the Spectral Energy Distribution from that disk, respectively emitted from the hot dust around a star, and the high temperature regions on the star itself where the gas accretion takes place. The formation process of a disk may be either passive or active. (2) In this paper, we focus on passive accretion, during which the disk evolution is driven by the luminosity of the central source. Calculations for source brightness, source temperature, and disk temperature estimate that a thin passive disk absorbs about a quarter of incident flux from its central star. The disk vertical profile consists of a surface layer, which absorbs and re-radiates incident energy from the star, and a cooler inner layer, which intercepts that re-radiated energy, and emits it as thermal energy.(2)

In addition to central source luminosity, the accretion rate and fluid transport within a disk are heavily dependent on the viscosity, or the internal friction of fluid material, within that disk. The time scale of disk evolution decreases with higher viscosity. Commonly, viscosity is characterized by the mean free path of particles within a fluid:

$$\nu \sim c_s\lambda \tag{5}$$

However, the observed formation rates of disks cannot be explained by this molecular viscosity alone, which implies the existence of a much greater viscosity, or turbulent viscosity, as proposed by Shakura Sunyaev (1973). As the hydrodynamic stability of a disk is disrupted by magnetorotational instability, perturbations within the disk occur and increase exponentially. This process creates a high, turbulent viscosity which sufficiently increases the rate of angular momentum transport. The equation for this turbulent viscosity is:

$$\nu = \alpha c_s h \tag{6}$$

where c_s is the speed of sound, and α is a constant $\gtrsim$ 0.01-0.1. This equation can be subsequently be used to determine viscosity as a function of radius and surface density. A profile of the initial make up of accretion disks can be determined using viscosity, as well as the different opacity values for different ranges of radii within the disk(6). The opacities of a circumpulsar disk differ from those of a protosolar disk through the characteristic nucleation process of the disk grains, as well as the low ratio of gas to dust initially present in the disk. In a pulsar system, the materials for planet production come from stellar remnants, and the layers of heavy elements within an original star, rather than the molecular clouds that precede solar systems. The size of these initial grains may be about 10 times larger than those in protosolar disks. Additionally, the concentration of heavy elements, such as C, O, or Si may much be higher. The opacities throughout a pulsar disk can be differentiated using separate laws, boundaries, and

constants for different regions. These various regions include electron scattering, free-free/bound-free absorption, areas of intermediate temperatures, and lower temperature areas composed of grains and ice grains. These opacity laws can be applied to solve for temperature, viscosity, scale height, and surface density over a disk. The fourth opacity law specifically accounts for temperatures within the disk bounded from above by T=3000 K and below by T=$4.6\times10^3\rho^{1/15}$, at which solid materials may sublimate.

5. SURFACE DENSITY

The surface density of a given part of a disk is dependent on both radius and time. With both time and radius, the surface density of a disk decreases. The parameters for the calculation of surface density at different radii of the disk depend on the different opacity laws associated with the varying temperature of the disk. Surface density is found through:

$$\Sigma = \Sigma_0 x^{-3}\tau^{-2\lambda}\xi^c(1-k\xi^b)^a \tag{7}$$

The value for surface density, and the values for the constants in the equation can be determined using the opacity laws in the form $\kappa = \kappa_0\rho^m T^n$, which can be substituted in equations for viscosity, temperature, and scale height:

$$\nu_0 = \frac{2}{3}\Omega^{\frac{m+2n-4}{6-2n+m}}\alpha^{\frac{8-2n+m}{6-2n+m}}\left(\frac{9k_0}{32\sigma_b}\right)^{\frac{2}{6-2n+m}}\left(\frac{k_b}{\mu m_p}\right)^{\frac{8-2n}{6-2n+m}}$$

$$\mathrm{T}_0 = \left(\frac{9\alpha k_0}{32\sigma_b}\right)^{\frac{2}{6-2n+m}}\left(\frac{k_b}{\mu m_p}\right)^{\frac{2-m}{6-2n+m}}\Sigma^{\frac{2m+4}{6-2n+m}}\Omega^{\frac{2m+2}{6-2n+m}}$$

$$h = \left(\frac{k_b}{\mu m_p}\right)^{1/2}\frac{T_0^{1/2}}{\Omega} \tag{8}$$

where $\Omega = (GM_s/r^3)^{1/2}$. From these, p and q in the equation for the viscosity prescription $\nu = \nu_0\Sigma^q r^p$ can be determined, and the the surface density can be fully solved using formulas(6):

$$q = \frac{2m+4}{6-2n+m}$$
$$p = (-3/2)\frac{m+2n-4}{6-2n+m}$$
$$a = 1/q$$
$$c = \frac{3q+3-2p}{1+q}$$
$$b = 1+c$$
$$\lambda = 1/(5q+4-2p)$$
$$k = q\lambda/(4q+4-2p)$$
$$x = (\tfrac{r}{r_0})^{1/2}$$
$$\tau = \tfrac{3}{4}\nu_0 r_0^{p-2}\Sigma_0^q t$$
$$\xi = x(\tau^{-1\lambda}$$

The calculated surface densities for each of the given opacities at a time of 10^5 years is shown in Figure 1, over a scale of $x = (r/r_0)^{1/2}$, where r_0=10^{11}.

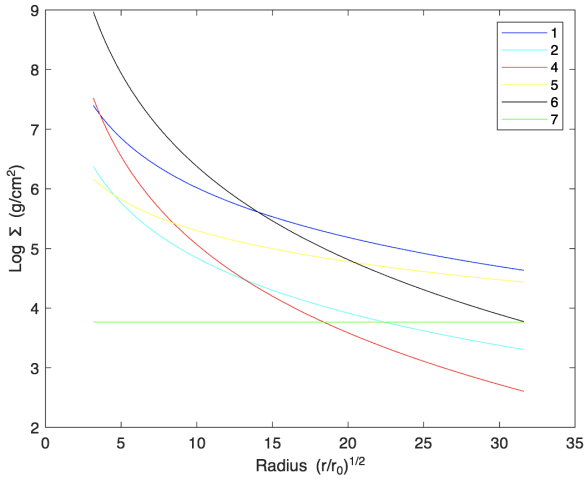

Figure 1. Surface density over scale of $x = (r/r_0)^{1/2}$ for opacity Laws 1-7 at t=10^5 years

.

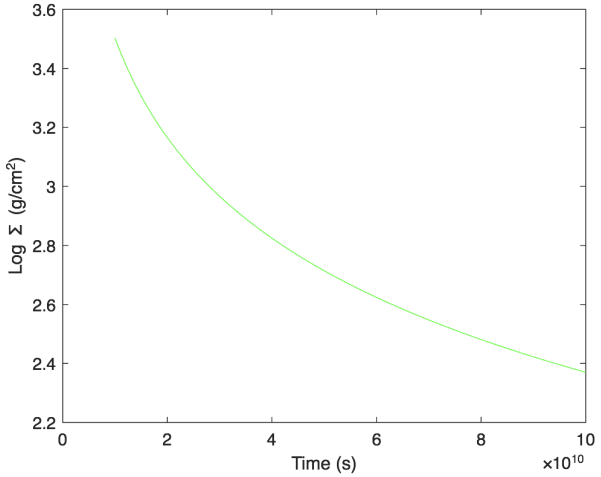

Figure 2. Variation of surface density over time at r=2 AU

.

The variation of surface density with time can be depicted as well. Given the fourth case opacity condition, where temperatures are within the range of planet formation, at a radius of 2 AU, the surface density of a disk over a time scale of 10^3-10^4 years is shown in Figure 2. As shown in the figure, with increasing time, the surface density of the disk declines due to viscous spreading.

The different surface density functions given by different opacity laws can be plotted for a given time, and matched to one another at specific boundary radii. By assuming the mass rate remains constant across the boundary between two similarity solutions, the bound-

ary conditions between surface density solutions can be determined. This mass rate is described by equation:

$$\dot{M} = 6\pi\nu_0 r^{1/2}\frac{\delta}{\delta r}[r^{p+1/2}\Sigma^{1+q}] \tag{9}$$

Using two neighboring opacity conditions such as electron scattering and free-free/bound free absorption, and solving for their respective mass rates a time t=10^5 years, the intersection of those mass rates produces boundary conditions necessary for the matching of surface densities for different opacities, which allows for a complete surface density profile of a disk. One such matching, between neighboring opacities 1 and 2, is shown in Figure 3 over a scale of $x = (r/r_0)^{1/2}$, where r_0=10^{11}. The intersection of the two surface density functions is found at the boundary of about 1 AU.

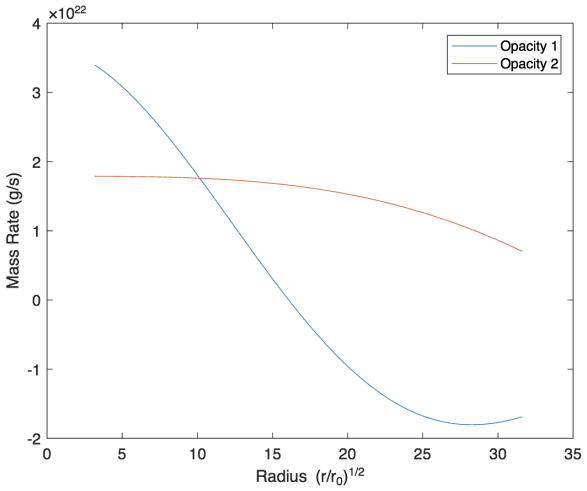

Figure 3. Intersection of mass rates for opacity laws 1 and 2 over scale of $x = (r/r_0)^{1/2}$ at t=10^5 years

.

6. PARTICLE GROWTH

The planetary growth process consists of five stages. Initial small particles within a disk first agglomerate, forming dust as they physically collide. The dust proceeds to form meter-sized rocks through an unknown mechanism. The next stage proceeds with a population of planetesimals, which is considered the initial condition for larger planet formation. These planetesimals may grow through aerodynamic interactions with small bodies, or "pebble accretion". They may then grow into Earth mass planets, which, through increased gravitational force, become coupled to the accretion disk. These Earth mass planets may then evolve into planetary cores, with masses around 10 times that of Earth.

The first stage occurs when a disk is sufficiently cooled (below ∼2000 K), and solid grains nucleate. Unlike in the solar system, in which dust nuclei grow by accumulating atoms to their surfaces, the grain formation process around a pulsar may only occur below a sublimation temperature, once gas has cooled enough for grains to nucleate. Densities of nucleation sites and rates of grain growth vary throughout the disk profile. The different grain size distributions and opacities around a pulsar suggest that the nucleated grains would be 10^-3 times less numerous, but 10 times larger than the grains in a solar disk. Additionally a pulsar accretion disk may have higher compositions of heavy elements due to the difference in initial debris and metallicity, resulting in higher opacities. (6).

Dust growth occurs as small particles settle toward the midplane of the disk, and with their movement, accrete other particles. The height of the plane, h, varies directly with temperature, and inversely with time. Once the disk cools to a sufficient temperature, grains may nucleate within the disk. However over the period of cooling time, less material remains throughout the vertical distribution of the disk. The time scale of particle growth in an accretion disk around a neutron star can be determined by relating dust grain radius to the time-dependent surface density function of the disk. The grain size relates to surface density by

$$a = \frac{f\Sigma}{8\rho_d} \tag{10}$$

where f is the dust to gas ratio 10^-2, and ρ_d is the material density 3 g cm^3.

As depicted in Figure 4, as surface density decreases over time, and the disk spreads, the size of the particle that may form decreases.

The settling time for a dust particle is characterized by the equation:

$$t_{settle} = \frac{2}{\pi}\frac{\Sigma}{\Omega_K\rho_d a}e^{-z^2/2h^2} \tag{11}$$

In the midplane, when z=0, this equation becomes

$$t_{settle} = \frac{2}{\pi}\frac{\Sigma}{\Omega_K\rho_d a} \tag{12}$$

This equation for settling time may be combined with the time-dependent equation for Σ (equation 6) to determine a solution. Using the fourth opacity law that corresponds to the bounds T=$4.6\times10^3\rho^{1/15}$ to 3000 K, this time is equivalent to roughly 1.15×10^{10} seconds, or $\sim 10^3$ years, at 2 AU. This settling period corresponds with a final dust particle radius of 1.12 cm and a temperature of 1600 K, which is meets the requirements for maximum sublimation temperature.

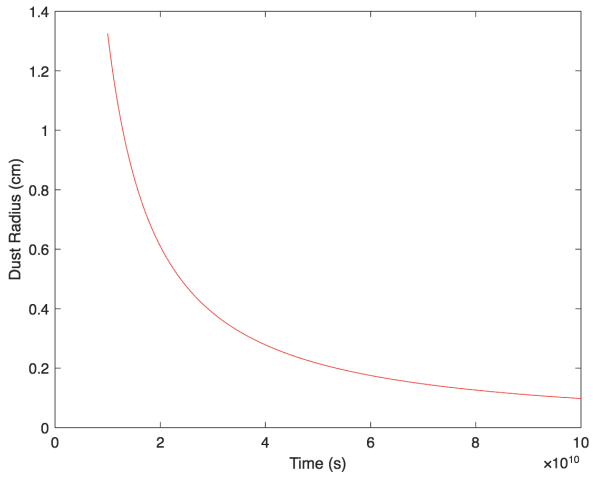

Figure 4. Variation of dust radius over time in a pulsar disk

.

The isolation mass during planet growth describes the mass at which planetesimals no longer collide with a growing planet, and mass accumulation slows down.

$$M_{iso} = \frac{8}{\sqrt{3}}\pi^{3/2}C^{3/2}M_*^{-1/2}\Sigma_p^{3/2}a^3 \qquad (13)$$

Figure 6 shows that as available mass in a disk decreases with time, the isolation mass does as well. This mass, in a pulsar planetary system at a time of 10^5 years, with $\Sigma = 100$, $a = 1AU$, and $M = .01M_{solar}$ is equal to ∼10 Earth masses, which represents the expected size of planets at that radius in a pulsar accretion disk.

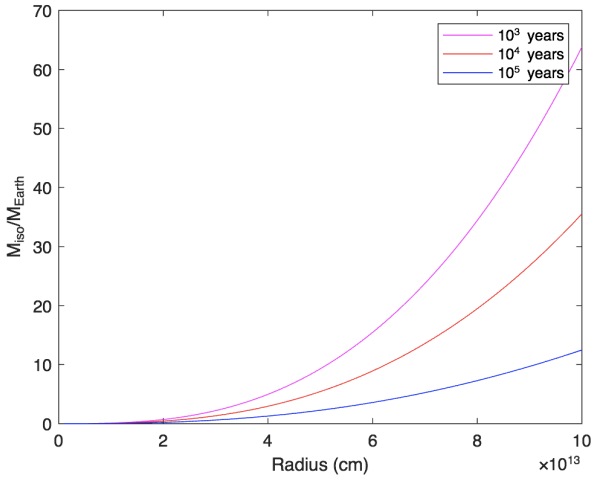

Figure 5. Isolation mass over radius for 10^3, 10^4, and 10^5 years

.

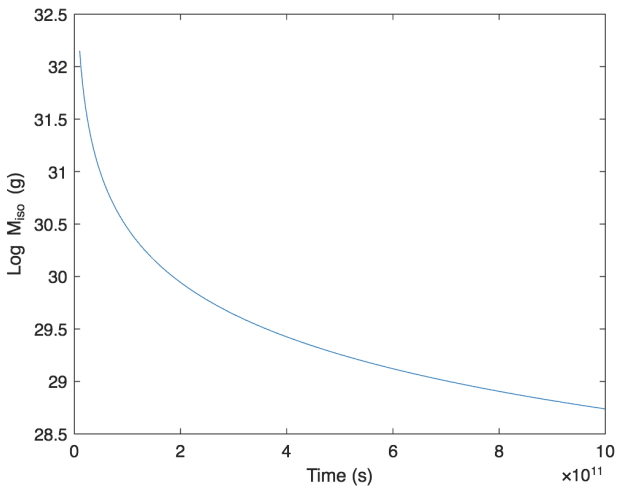

Figure 6. Isolation mass over timescale of 10^3-10^4 years at 2 AU radius

.

7. RECENT FINDINGS

Recent studies have found that the planetary system around PSR B1257+12 may be the product of the merge of white dwarf-neutron star binary system, rather than a fallback disk. This theory accounts for the discrepancy between the expected and the observed abundance of pulsar planets. This scenario, and the high metallicity of WD debris, would also account for the composition of disks and the existence of large solid materials. The merger begins as a 0.6 M⊙ WD, composed of half carbon and half oxygen, is tidally disrupted by a 1.4 M⊙ neutron star, and subsequently forms an accretion torus, which later cools to form a thin disk structure and undergoes the range of opacity regimes. Similar to the previous model, planetary bodies may form throughout the process of viscous spreading of mass on radii of AU scales. The WD-NS merger scenario case implies an initial angular momentum a factor of 10^2 greater than that in the fallback scenario, accounting for a greater available mass at planet formation radii, and allowing for a much lower efficiency rate than previously calculated. Unlike in the fallback scenario, the disk mass available at the planet formation radius in the PSR B1257+12 system greatly exceeds the combined mass of the observed planets. This model would additionally account for the observed diamond composition of pulsar planet PSR J17191438 b. Due to the C/O composition of the WD star, the outer portions of the accretion disk are dominated by carbon and oxygen as it evolves. As the temperature of the disk decreases, the formation of carbon solids may exceed the formation of

CO. The carbonaceous dust may then condense, sink to the midplane of the disk, and grow via collisions.(5)

8. CONCLUSION

The variation in observed characteristics of discovered pulsar planets leaves room for several theories regarding the specifics of the planet production process. Using a model for the formation of the planets in own our solar system may serve as a basic and incomplete template. However, with the addition of fallback models, or more recent models such as that proposed by Margalit et al. (2016) (5), more comprehensive and complete conditions for pulsar planet production, size, location, and composition may be estimated, and used as predictions for future pulsar planet findings.

REFERENCES

[1]Neutron star planets: Atmospheric processes and irradiation.

[2]Philip Armitage. Lecture notes on the formation and early evolution of planetary systems, February 2017.

[3]Robert A. Chevalier. Neutron star accretion in a supernova. *The Astrophysical Journal*, 346:847–859, November 1989.

[4]M. Kerr, S. Johnston, G. Hobbs, and R. M. Shannon. Limits On Planet Formation Around Young Pulsars And Implications For Supernova Fallback Disks. *The Astrophysical Journal*, 809(1):L11, aug 2015.

[5]Ben Margalit and Brian D. Metzger. Merger of a white dwarf–neutron star binary to 1029 carat diamonds: origin of the pulsar planets. *Monthly Notices of the Royal Astronomical Society*, 465(3):2790–2803, Oct 2016.

[6]E.S. Phinney and B. M. S. Hansen. The pulsar planet production process. *Planets around Pulsars; Proceedings of the Conference*, pages 371–390, February 1993.

[7]P. Podsiadlowski. The formation of planet pulsars. 72:411, January 1995.