WHAT MAKES MATHEMATICS DISCUSSION MEANINGFUL? AN EXAMINATION OF ELEMENTARY TEACHER PRACTICES WHILE ORCHESTRATING MATHEMATICS DISCUSSION

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by

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Abstract

Mathematics reform efforts for the last several decades prioritize teachers facilitating meaningful mathematics discussion in a way that empowers students as thinkers and doers of mathematics (CCSSI, 2021; NCTM, 1991, 2014). While numerous studies have described the benefits of mathematics discussions (Anderson & Boaler, 2008; Kosko et al., 2012; Webb et al., 2021) and the attributes of productive discussions (Boerst et al., 2011; Murata et al., 2017; Webb et al., 2014), teachers still struggle to implement practices that position students as the leaders of discussion. Evidence suggests that specific teacher moves impact the degree to which students share their mathematical thinking and engage with the thinking of others during meaningful mathematics discussion (Chapin et al., 2009; Ellis et al., 2018; Ing et al., 2015; Franke et al., 2009). Teachers at Barron Academy[1], an independent K-12 school in the mid-Atlantic region of the United States, also struggle to facilitate mathematics discussion so that students are leaders of mathematics and engage with their peers' thinking. Through a descriptive case study, I examined three elementary mathematics teachers' instructional practices that support and/or limit meaningful mathematics discussion. To better understand this local problem of practice, I observed mathematics instruction, interviewed teachers, and reviewed the curriculum employed by the school. Findings suggest that teachers 1) recognized meaningful mathematics discussion as student-led but did not facilitate discussion in this way and 2) were heavily reliant on the curriculum to plan for and orchestrate mathematics discussion. These findings informed related recommendations that will actionably support the improvement of mathematics discussion in the school.

Keywords: mathematics instruction, discussion, elementary, qualitative methods

[1] pseudonym

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APPROVAL OF THE CAPSTONE PROJECT

This capstone project, ("What Makes Mathematics Discussion Meaningful? An Examination of Elementary Teacher Practices While Orchestrating Mathematics Discussion", has been approved by the Graduate Faculty of the School of Education and Human Development in partial fulfillment of the requirements for the degree of Doctor of Education.

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Table of Contents

Dedication

To all my students, who taught me that when I talk less, they learn more.

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List of Figures

List of Tables

Chapter 1: Introduction

Background of the Problem

Mathematics discussion, that is, encouraging more students talk in the mathematics classroom, has been a focus of both research and policy encouraging more student talk in the mathematics classroom over the last several decades (NCTM, 1989, 2000, 2014; National Governors' Association [NGA] Center, 2010). Various researchers and policies refer to student talk using a variety of terms, including math talk (Chapin, et al., 2003), talk communities (Hufferd-Ackles et al., 2004), mathematics discourse (NCTM, 2014), and mathematics discussion (NGA Center, 2010), all of which are based on the "common assumption that students learn best when they are given opportunities to speak about mathematics using the language of mathematics" (Cirillo, 2013, p. 1). Encouraging students to communicate their mathematical reasoning to others represents a shift from traditional to reform mathematics instruction. Under the traditional notion of mathematics instruction, teachers transfer knowledge to students through direct instruction and recitation of procedural knowledge. Alternatively, reform mathematics calls for a shift from "recitation to discussion-based lessons" so that students have the potential to become active participants in developing their understanding of mathematics rather than passive recipients of knowledge (Cirillo, 2013, p. 1). At Barron Academy[1], the specific context for this capstone study, teachers and faculty have expressed difficulty in shifting their mathematics classrooms into a space for students to actively participate in discussions with one another. This challenge is not unique to teachers at Barron and research suggests facilitating meaningful mathematics discussion is challenging and complex (Ball 1988b; Bray, 2011). Various national initiatives have been developed to support teachers and students in shifting

[1] pseudonym

from a traditional mathematics classroom to a more reform, discussion-based mathematics classroom.

One such reform initiative includes the introduction of the Standards for Mathematical Practice (SMP), developed as part of the Common Core State Standards for Mathematics (CCSSM; NGA Center, 2010). Rather than focus solely on mastery of specific skills and grade-level standards, the SMP represent the habits students at all grade levels learn as they engage in the process of doing mathematics (NGA Center, 2010). The Common Core State Standards Initiative (CCSI) (NGA Center, 2010) claimed that when taught alongside grade level content standards, the SMP will lead students to develop deep understanding of mathematics concepts and subsequently improve skills as measured on standardized assessments.

The SMP are composed of eight specific standards that mathematically proficient students demonstrate (see Figure 1.1). One of the SMP states that mathematically proficient students "construct viable arguments and critique the reasoning of others" (NGA Center, 2010, para. 4). Mathematically proficient students demonstrate mastery of this SMP by clearly and accurately explaining and communicating their mathematical ideas to their peers. When students construct viable arguments, they use previous knowledge, assumptions, and definitions to explain their mathematical thinking, often involving concrete objects, drawings, and diagrams (NGA Center, 2010). Viable arguments include an explanation and not simply an answer. Not only does this SMP requires students to be able to communicate their own thoughts but also engage with the thinking of their peers. Specifically, the CCSSI stated that "students at all-grades [are expected to] listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments" (NGA Center, 2010, para 4). To achieve mathematical proficiency as outlined by the SMP, student discussion must extend beyond student explanations and position students to engage with the thinking of their peers. Mathematical discussions may provide opportunities for students to engage, develop, and master this specific mathematical practice.

Figure 1.1

Standards for Mathematical Practice (CCSSI, 2021)

Standards for Mathematical Practice (CCSSI)
MP1: Make sense of problems and persevere in solving them
MP2: Reason abstractly and quantitatively
MP3: Construct viable arguments and critique the reasoning of others
MP4: Model with mathematics
MP5: Use appropriate tools strategically
MP6: Attend to precision
MP7: Look for and make use of structure
MP8: Look for and express regularity in repeated reasoning

In response to the SMP, the National Council of Teachers of Mathematics (NCTM; 2014) put forth a set of recommended teaching practices that would assist educators in implementing and teaching the SMP (see Figure 1.2). While the SMP specified what students should be able to do, the NCTM's teaching practices presented guidance for teaching in a way that aids students in mastering the SMP. This list of eight mathematics teaching practices is a research-based framework that includes teaching practices and skills that are "necessary to promote deep learning of mathematics" (NCTM, 2014, p. 9). The NCTM (2014) hoped to bridge the gap between research and practice by providing specific definitions, teacher-actions, student-actions, and narratives describing each of the recommendations. One such teaching practice is for teachers to "facilitate meaningful mathematics discourse" (p. 10). This practice synthesizes major findings from the work of Smith and Stein (2011) and Hufferd-Ackles and colleagues (2004; 2014) to provide teachers with explicit guidance when facilitating meaningful mathematics discourse. The NCTM argued that not all discourse has the potential to be meaningful and defined meaningful mathematics discourse as:

> the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication… [that] gives students opportunities to share ideas and clarify understandings, construct convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives. (p. 29)

Figure 1.2

Mathematics Teaching Practices (NCTM, 2014)

Mathematics Teaching Practices
Establish mathematics goals to focus learning.
Implement Tasks that promote reasoning and problem solving.
Use and connect mathematical representations.
Facilitate meaningful mathematics discourse.
Pose purposeful questions.
Build procedural fluency from conceptual understanding.
Support productive struggle in learning mathematics.
Elicit and use evidence of student thinking.

The NCTM (2014) claimed that the "effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (p. 10). Facilitating a mathematical discussion requires the teacher to step away from a more traditional model of leading and controlling discussion through Initiate-Response-Evaluate (IRE; Sinclair & Coulthard, 1975). In this pattern of discussion, a teacher asks a student to find the solution to 3+4 (initiate), the student responds with 6 (response), and the teacher immediately corrects the student by stating the answer is 7 (evaluate). The teacher controls the narrative by asking a question, calling on a student to respond, and immediately evaluating the response before engaging in direct instruction or calling on an additional student (Sinclair & Coulthard, 1975). On the other hand, when teachers facilitate mathematics discussions, they ask follow-up questions, prompt additional students to engage with the initial response, and yield some control of the discussion to students as they build their own understanding of mathematical ideas (NCTM, 2014). In using the previous example, I present a shift from IRE to a more productive discussion based upon the same initial question and student response. If the teacher asked the student to explain how they found the sum of 6, rather than immediately and negatively evaluating the response, the student is afforded an opportunity to explain their thinking. Additionally, other students are provided an opportunity to listen, potentially argue against the explanation, or compare the student's incorrect answer to their own. As a result, moving away from a traditional IRE pattern of discussion provides an entry for students to engage with mathematical concepts and ideas of their peers.

Facilitating meaningful mathematics discussions encompasses multiple teaching practices recommended by the NCTM (2014). The NCTM recommended discussions as a vehicle for aiding students in achieving a learning goal. Teachers must ask purposeful questions (SMP 5) that elicit student thinking (SMP 8) and urge students to connect various mathematics representations (SMP 3) to the work of their peers, so that discussions may help students to achieve a goal. The connection between the practices, as well as the practices in action, is demonstrated in the following scenario:

In Ms. Dubil's third-grade mathematics class, students have been working on representing multiplication through number lines, modeling equal groups, and repeated addition. Ms. Dubil chose to introduce division through representing multiplication with a missing factor. The goal of the lesson is for students to begin to develop an understanding of how to solve for a missing factor as an introduction to division (SMP 1). Ms. Dubil selects a mathematics task that students could solve using various strategies as they explore the concept of division through a missing factor context (SMP 2). While students work through the problem individually, Ms. Dubil selects students to share during the whole group discussion that solved the problem using different strategies or operations (see Figure 1.3). She intentionally selects two students' work with different strategies so that the discussion may focus on how strategies are related to one another and related to multiplication with a missing factor. A classroom discussion focused on student work follows:

> *Carlos*: I decided to draw pictures of the packs because models make sense for me. I drew a box of 6 and kept drawing boxes of 6 until I got to 42.
>
> *Ms. Dubil:* How did you know when you had drawn 42 tallies? What do those tallies represent?
>
> *Carlos*: Uh… I just counted on starting from 1 up to 42. Each tally is an apple.
>
> *Ms. Dubil*: Does anyone want to ask Carlos anything else about how he solved this problem?

Marci: How did you get your equation -- __ x 6 = 42? I thought it was 6 x 42.

Carlos: Well, multiplication shows equal groups. I knew that we had group of 6 but that there weren't 42 groups. There are 42 apples *total* in the shipment, so I knew that would 42 would be our product.

Marci: But why is it not 42 in each group?

Carlos: The task says there are a total of 42 candy apples. That's our product. Then, we don't know the number of packs or groups but we know there are 6 in each pack. So, we have to solve for the missing number of groups.

Marci: Oh, I see that now. I tried to draw 42 groups with 6 apples in each… but that can't be right. It's way too many… I didn't even finish.

Carlos: Yeah, 42 packs with 6 in each in each doesn't make sense. We usually know the number in the group, but that's what we have to find this time.

Ms. Dubil: Carlos made an interesting point. We had to find the number in each group instead of being given that information in our problem, which is different than other problems we've done before today… Now let's now look at Student 2's work. How does your work relate to Carlos's work?

Nina: Well, I used repeated addition because I really like to add and am good at doubles… and my drawings get messy. But I kept adding 6 until I got to 42.

Ms. Dubil: How does your 6 relate to Carlos's model?

Nina: Hmm… I'm not so sure. I just know repeated addition is a way to practice this.

Ms. Dubil: Carter?

Carter: Well… each addend is one pack of 6 apples. So, two boxes of apples is 6 + 6, then four boxes is 6+6+6+6.

Ms. Dubil: So, Nina… Can you explain to me how your repeated addition relates to Carlos's drawing?

Nina: Every time Student 1 drew a group, I just added 6. We both stopped when we got to 42.

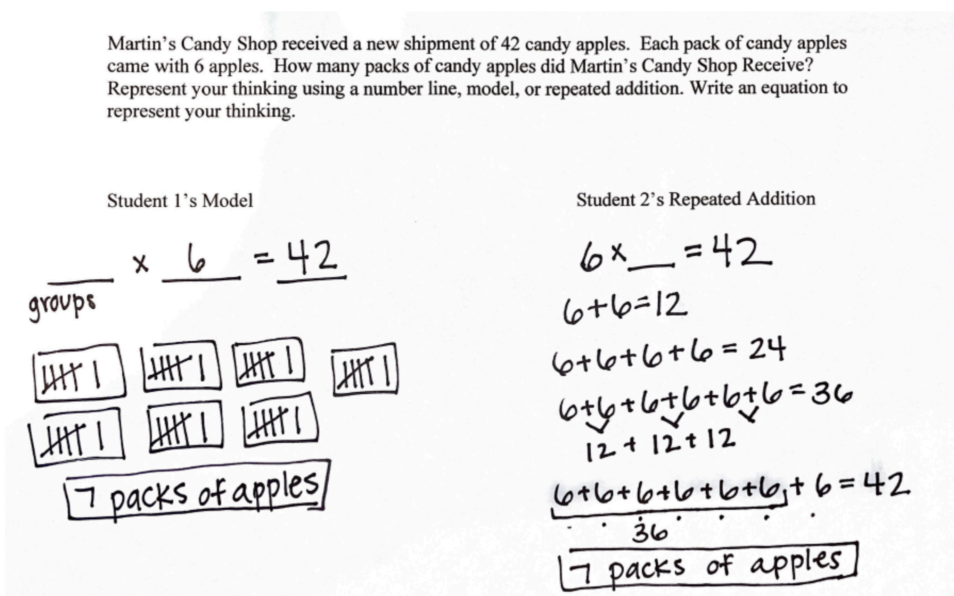
Ms. Dubil: So, how does a pack of apples… or a box of tallies… or a plus 6… relate to our problem? And our equation?

Nina: Well, we are looking for how many packs of apples there are. It's our missing number. But we know how many are in each pack and we know how many apples total.

Ms. Dubil: Trey, would you come label what each term in Nina's multiplication equation represents?

Figure 1.3

Student Work Related to Vignette

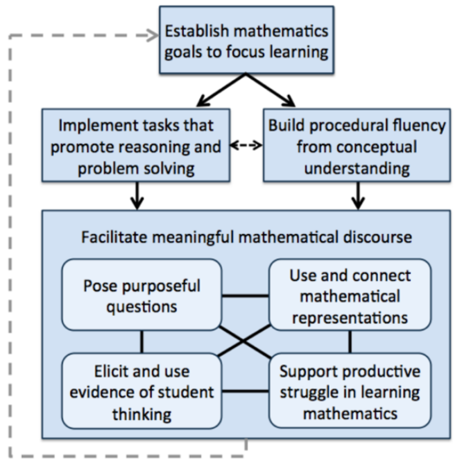


In this vignette of a classroom discussion, the teacher facilitated discussion by posing questions to students which prompted them to show evidence of their thinking and connect their representations to their peers' work. The teacher purposefully selected different strategies so that students could think critically about the work of their peers. She pressed students to understand how strategies were interconnected and how the strategies related back to the initial task. At the end of the vignette, the teacher purposefully questioned students so that they could begin to develop an understanding of multiplication with a missing factor, which was the learning goal. She asked questions that helped students to center their thinking back to her established goal for the lesson, while also giving students the authority to "talk with, respond to, and question one another" (NCTM, 2014, p. 30). Although she did ask most questions, the majority of the thinking was demonstrated by students. Students became responsible for listening to and sharing their thinking with their peers, not just sharing it with the teacher.

Together, these practices set forth a framework for mathematics instruction as developed by Smith et al. (2017) (see Figure 4). According to this framework, mathematics discussions are a substantial component of mathematics teaching because discussions encompass additional mathematical teaching practices, including eliciting student thinking (SMP 8), posing purposeful questions (SMP 5), using and connecting mathematical representations (SMP 3), and supporting students in productively struggling to make sense of mathematics (SMP 1). In the vignette of Ms. Dubil's class discussion, we see how a classroom teacher can purposefully question students in a way that prompts them to make sense of their own ideas and engage with the thinking of their peers.

Figure 1.4

A Framework for Teaching (from Smith et al., 2017, p. 194)



Mathematics Discussion

Mathematics discussion includes a wide array of verbal, visual, and written communication involving mathematics. Throughout the body of literature, various terms including math talk (Chapin et al., 2003), communities of talk (Huford-Ackles et al., 2004), productive discussions (Smith & Stein, 2011), and meaningful mathematics discourse (NCTM, 2014) are used to label the academic conversations that support the construction of knowledge. In this paper, I refer to *mathematics discussions* as the verbal interactions where students share, explain, question, or argue mathematical ideas related to concepts, solutions, and procedures with other students and the teacher. I have chosen to use the word *discussion*, rather than *discourse*, because in research, discourse often involves analysis of the "underlying rules of linguistic or communicative function" (Hodges et al., 2008). Therefore, I will use the term *mathematics discussion* in lieu of the NCTM's use of the term *discourse* in their recommendation. I will limit mathematical discussions to the interactions that exist between teachers and multiple students or between and among multiple students with the purpose of building mathematical understanding for all learners. These discussions can occur in small groups of students, pairs of students, or as a whole class. For the purpose of this capstone study, private conversations that exist exclusively between one teacher and one student were not included in mathematical discussions. I excluded individual student-teacher interactions and instead I focused on the ways students are presented with opportunities to engage in discussion with one another, which is essential to the orchestration of meaningful mathematics discussion.

Pirie and Schwarzenberger (1988) defined *mathematics discussion* as “purposeful talk on a mathematical subject in which there are genuine pupil contributions and interactions” (p. 460). This purposeful talk may support students in developing understanding and achieving mathematical goals established by the teachers. Often, these goals pertain to developing deep mathematical understanding. Pirie and Schwarzenberger (1988) defined mathematical *understanding* as relating to the relationships between mathematical concepts that explain why mathematics works, rather than how to do mathematics. When students understand mathematics, they are able to represent ideas in multiple ways as well as make connections amongst different representations and mathematical concepts (Cramer & Karnowski, 1995). *Meaningful mathematics discussion*, as described by the NCTM (2014), supports all students in developing this deep understanding of mathematical content. Not all use of mathematics discussions leads to meaningful mathematics discussions because not all talk, even when purposeful, supports all students in developing understanding.

There are key features that characterize meaningful mathematics discussion, the first being an equitable distribution of the talking and listening. Since discussion includes genuine contributions and interactions from multiple participants, meaningful mathematics discussions exclude talk that is primarily teacher-led, such as direct lectures with intermittent questions to individual students, and student talk that is entirely show-and-tell where one student talks at another student without interaction. Additionally, meaningful mathematics discussion includes participation from all students, not discussion dominated by a select few students. Mathematics discussions are interactive among participants and do not include instances when one person is "talking at another" (Pirie & Schwarzenberger, 1988, p. 460). Rather, meaningful mathematics discussions engage participants in organic interactions, where they respond to, question, and compare mathematical ideas together. Meaningful mathematics discussions are a collaborative way for students to make sense of mathematics through the talk, questions, ideas, and connections of their peers.

Mathematics discussions, as previously described, have the potential to considerably accelerate student achievement (Hattie et al., 2017; Kosko, 2012; Pirie & Schwarzenberger, 1988). In an ongoing meta-analysis of over 1,600 studies, Hattie and colleagues (2017) sought to measure and rank the various influences that impact student achievement to better understand which influences were the most beneficial to students. Hattie et al. (2017) found that classroom discussion was the 15th most beneficial of the 250 factors. The authors suggested that classroom discussion supports deep mathematical understanding, critical thinking, and reasoning because of the way discussion provides opportunities for students to exchange ideas in detail with one another. When implemented in classrooms, however, mathematics discussions have a large amount of variability in terms of the content discussed, questions asked, information shared by students, levels of student interactions with one another, and general quality of discussion (Kosko, 2012). Therefore, it can be difficult to generalize that all mathematics discussions are meaningful or beneficial to student learning.

Despite the recommendations by the NCTM (2014) for teachers to facilitate mathematical discourse and the potential for positive achievement effects discussion may have on student learning, researchers report that too many classrooms still exhibit teacher-led talk rather than student sustained discussions (Ball, 1988b; Bennett, 2010; Bray, 2011; Hufferd-Ackles et al., 2004; Pirie & Schwarzenberger, 1988; Weaver et al., 2005). Pirie and Schwarzenberger (1988) struggled to find mathematics classrooms that met their expectations when studying the impact of discussion on understanding. While observing elementary mathematics classrooms, researchers found that mathematics discussions ranged in quality and teachers ranged in their ability to facilitate meaningful mathematics discussion (Bray, 2010; Hufferd-Ackles et al., 2004). Bennett (2010) asserted that teachers needed support in "learning how to help students make strong [mathematical] connections" with their peers (p. 87). Both Bennett (2010) and Bray (2011) also concluded that teachers' individual beliefs of students and mathematics may influence how they use discussion for instruction. Even if teachers know the qualities of meaningful mathematics discussions as advised by the NCTM (2014), they may struggle to facilitate discussions because they were not taught nor did they experience such student-centered and conceptually focused methods as students in elementary schools themselves (Ball, 1988a, 1988b).

Local Problem of Practice

Given the potential for mathematics discussion to positively affect student understanding and achievement, it is important that mathematics classrooms demonstrate discussions that support the exchange of ideas between students (NCTM, 2014). Elementary teachers and administrators at Barron Academy reported having difficulty facilitating classroom discussion in a way that engaged students with the ideas of others. In this section, I describe the local problem of practice by describing the local context at Barron Academy, the designated curriculum for mathematics instruction, and evidence of the problem.

Local Context

Barron Academy, the local context of this capstone study, is an independent pre-kindergarten through twelfth-grade school in the mid-Atlantic region of the United States. Barron Academy's elementary school houses 11 classrooms ranging from kindergarten through fifth grade with 197 students. The Barron Academy website advertised that students regularly engage in purposeful activities that encourage students to "question, explore, think, and solve problems". Barron Academy's website also states that the school values a learning environment where students engage with their peers and develop their own understanding through "student-centered programs". Given these values and claims, this capstone project focuses on understanding how teachers currently orchestrate mathematics discussions and how, if at all, discussion encourages students to question and engage with their peers so that the school can improve these aspects of their instructional program. While I am not employed at Barron as an educator, I have long-standing professional relationships with the school's administrators and staff, which led administrators welcoming my capstone study as an opportunity to partner with them on exploring the role of discussions in elementary teachers' classrooms practice. (A more thorough explanation of my positionality will be discussed in Chapter 3).

Math in Focus Curriculum

The Elementary School at Barron Academy has used *Math in Focus: Singapore Math* (Marshall Cavendish Education, 2021b) for its mathematics curriculum in grades K-5 since 2015. Singapore Math is a method for teaching mathematics intended to replicate instructional practices of Singapore's Ministry of Education and follow Singapore's student achievement by replicating the instruction. Singapore consistently ranks at the top of results from international mathematics assessments, such as the Trends in International Mathematics and Science Study (TIMMS) and Programme for International Student Assessment (PISA). *Math in Focus* incorporates real-world problem-solving content that is solved through a concrete-pictorial-abstract (CPA) approach to learning the CCSSM. Through utilizing CPA in classrooms, students first develop understanding through concrete, tangible manipulatives before modeling pictorial representations and finally moving into more symbolic and algorithmic methods of solving problems. The emphasis on problem solving provides teachers with the opportunity to guide students through "acquiring and applying a multitude of concepts and skills in order to solve a wide range of problems in varying non-routine and real-world situations" (Marshall Cavendish, 2020c, p. 10).

The newest edition of the curriculum, which was introduced in the 2021-2022 school year, centered around problem-solving by building students' "conceptual understanding, skills, mathematical processes, attitudes toward math, and self-awareness" (Marshall Cavendish Education, 2020c, p. 11). All elements of the curriculum aim to support students in building their conceptual understanding, skills, mathematical processes, attitudes toward math, and self-awareness. When students engage in mathematical discussions with their peers, they reflect on their own learning (self-awareness) as they seek to communicate and explain their thinking (understanding) about the mathematical processes and to develop mathematical skills. Additionally, the curriculum claims that discussion "appears to help students persist in solving problems and to increase motivation and engagement", which supports building a positive attitude toward math (p. 50). *Math in Focus* emphasizes discussion throughout the curriculum because "talking about mathematics… is an increasingly important way for students to learn and make sense of mathematics" (p. 20) as students are given "access to ideas, relationships among those ideas, strategies, procedures, facts, and more" during discussion with peers (p. 50).

A typical *Math in Focus* lesson contains five main components, which include the *Think, Engage, Learn, Try,* and *Independent Practice,* and additional elements were incorporated throughout the chapter (Marshall Cavendish, 2020a) (see Table 1.1). The *Engage, Learn, and Try* comprise the focus cycle that may be repeated throughout a lesson as students learn new concepts and skills. Each of the components has unique attributes and all include some guidance for how the curriculum intends for teachers to execute instruction, such as suggested grouping. A majority of the components are intended for whole group and include discussion opportunities.

Table 1.1

Math In Focus Lesson Components (Marshall Cavendish, 2020a)

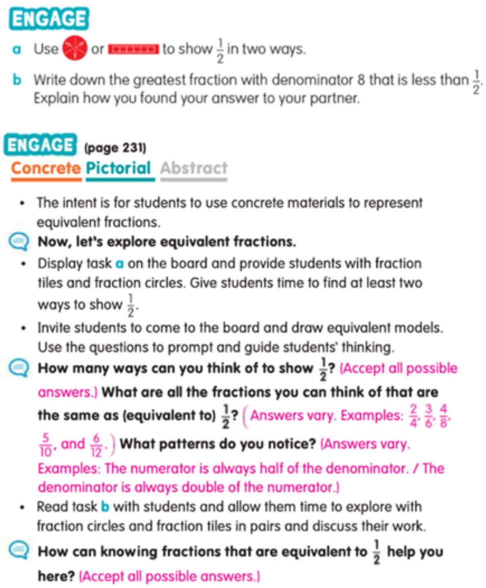
Component	Description	Whole Group
Think	Initial problems that "stimulate critical thinking and require students to use problem-solving methods and communication skills to discover creative solutions" (p. 6). Revisited throughout the chapter.	X
Engage	Mathematics problems or tasks that promote students to make connections between prior knowledge and new learning through inquiry.	X
Learn	Mathematics problems that apply connections made during the engage component. Presence of more direct teaching.	X
Try	Practice problems for students to complete in small groups and review in whole group.	X
Hands-on Activity	Activities for students to reinforce learning and uncover more mathematical concepts with partners or small groups.	
Independent Practice	Practice questions for students to complete independently related to the question.	

Each chapter began with a *Think* activity, which includes a problem-solving task for students to think about solving but not necessarily solve in its entirety. The goal of the *Think* was for students to begin to think critically, with either concrete or pictorial representations, about a new concept and to begin to “discover creative solutions” (p. 8). The *Think* activity included time for students to think, productively struggle, and talk about the problem with a partner as well as a whole group discussion about the activity. Lessons often ended with returning to a discussion of the *Think* problem so that students can compare their initial thinking to their more advanced understanding of the concept (Marshall Cavendish, 2020b).

The focus cycle began with the *Engage* activity, which is a “low-floor, high-ceiling” task, which linked prior knowledge to additional opportunities for exploration (Marshall Cavendish Education, 2021c, p. 85). The *Engage* activity was described in the student edition text as a task that “will have you exploring and discussing math concepts with your classmates” (Marshall Cavendish, 2021b, p. xiii). The Teacher’s Edition (TE) textbook lesson included specific question prompts, which teachers can ask students when discussing solutions to the *Engage* activity as a whole class. A fourth-grade introductory equivalent fraction lesson prompted students to engage with the content by building fractions equivalent to ½ using manipulatives. The corresponding TE included a set of questions teachers should use to elicit and guide student thinking during a whole group discussion of the *engage* problem (see Figure 1.5). The curriculum asserted that these activities combined with “robust questioning… [may] help teachers lead students in meaningful, engaging conversations about the concepts they are learning” (Marshall Cavendish Education, 2020c, p. 51). However, the questions, as included in the TE, did not necessarily direct teachers how to shift from asking individual students questions to promoting students to engage in discussion with one another around these questions.

Figure 1.5

Example of Engage Activity with Teacher Prompts (Marshall Cavendish, 2021, p. 232).



ENGAGE

a Use [fraction circle] or [fraction tiles] to show $\frac{1}{2}$ in two ways.

b Write down the greatest fraction with denominator 8 that is less than $\frac{1}{2}$. Explain how you found your answer to your partner.

ENGAGE (page 231)

Concrete Pictorial Abstract

- The intent is for students to use concrete materials to represent equivalent fractions.

Now, let's explore equivalent fractions.

- Display task a on the board and provide students with fraction tiles and fraction circles. Give students time to find at least two ways to show $\frac{1}{2}$.
- Invite students to come to the board and draw equivalent models. Use the questions to prompt and guide students' thinking.

How many ways can you think of to show $\frac{1}{2}$? (Accept all possible answers.) **What are all the fractions you can think of that are the same as (equivalent to) $\frac{1}{2}$?** (Answers vary. Examples: $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, and $\frac{6}{12}$.) **What patterns do you notice?** (Answers vary. Examples: The numerator is always half of the denominator. / The denominator is always double of the numerator.)

- Read task b with students and allow them time to explore with fraction circles and fraction tiles in pairs and discuss their work.

How can knowing fractions that are equivalent to $\frac{1}{2}$ help you here? (Accept all possible answers.)

The second and third component of the focus cycle were the *Learn* and the *Try*. The *Learn* was a “teacher-facilitated inquiry” of explicit strategies or procedures for students to learn (p. 85). Included in the *Learn* were mathematical questions to solve, followed by a discussion of how-to solve. The *Learn* component most closely aligned with direct teaching through mini lessons. Finally, the *Try* was opportunity for students to apply and practice their learning individually or with peers on practice problems related to the recently learned concept or skill. Integrated within the *Try* were additional instructional activities, such-as hands-on activities, games, or specified prompts for discussion.

Mathematics discussions was one of the key instructional features that supports teachers in “delivering highly effective mathematics instructions” to students (Marshall Cavendish Education, 2020, p. 39). *Math Talk* and *Math Sharing* were two activities embedded throughout the curriculum to support student discussion. *Math Talk* “engages students in questions that encourage reflection and gives students the opportunity to articulate thinking and deepen their understanding of concepts" (Marshall Cavendish, 2022a, p. 27). Throughout each chapter, students were prompted to engage in *Math Talk* with peers around a given prompt (see Figure 1.6). Included in the TE were additional prompts that teachers may ask to support students in clearly communicating their thinking (see Figure 1.7). *Math Talk* was sometimes accompanied by language development tips, such as reviewing definitions for academic vocabulary. *Math Sharing* was another activity supporting a discussion of students' strategies, thoughts, and discoveries. *Math Sharing* prompts intended to support students in using precise mathematical language (MP6) (see Figure 1.8). The suggested questioning in the TE prompted students to share their answers and various ways of writing their answers (Figure 1.9). Neither *Math Talk* nor *Math Sharing* appeared in every lesson. Beyond advice about the group size and language recommendations, however, the teacher and student edition textbooks provided little instruction for how to support students in engaging in math discussion with one another (Marshall Cavendish Education, 2020b).

Figure 1.6

Example of Math Talk (Marshall Cavendish Education, 2021b, p. 232)

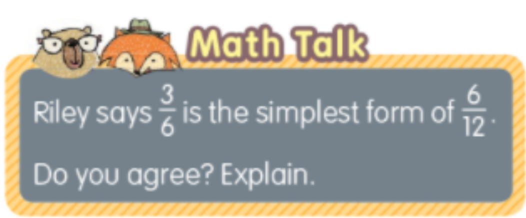


Figure 1.7

Example of Teacher Guidelines for Math Talk (Marshall Cavendish Education, 2021b, p. 233).

Math Talk **(page 232)**

- Display the question and allow students time to discuss it with their partners.
- Provide students with fraction circles to make equivalent fractions of $\frac{6}{12}$, and explore its simplest form visually.
- Now, pose the following questions to students and prompt them for their reasoning.

What does Riley mean by simplest form? (The numerator and the denominator cannot be simplified further.) **How do we know if a fraction is in simplest form?** (The numerator and denominator have no common factors.) **How can we find out?** (Make a list of the factors of both the numerator and denominator. If there are no common factors, apart from 1, the fraction is in simplest form.)

Figure 1.8

Example of Math Sharing (Marshall Cavendish Education, 2021b, p. 74)

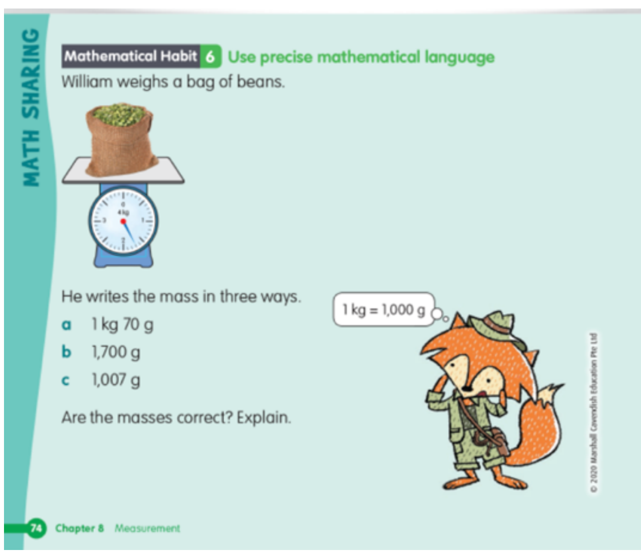


Figure 1.9

Example of Teacher Guidelines for Math Sharing (Marshall Cavendish Education, 2021b, p. 74)

MATH SHARING **(page 74)**

- Display the picture of the bag of beans on the scale. Now, pose the following questions to students and prompt them for their reasoning.

What is the mass of the bag of beans? (1,700 grams or 1 kilogram 700 grams) **What are the different ways we can write the mass?** (See previous answer.)

- Show students the answers that William created.

Are William's answers correct? Why or why not?
a. is not correct. The mass is 1 kg and 70 grams. The scale measures in 100 grams, making it 1 kilogram and 700 grams.
b. is correct.
c. is not correct. The mass is 1 kilogram and 7 grams, rather than 1 kilogram and 700 grams.)

- Encourage students to look at each answer and prove whether it is correct. Select students who used different strategies to share their work with the class.

Barron Academy selected *Math in Focus* as its curriculum for many reasons, including its emphasis on classroom discussion and improving students' ability to communicate reasoning. In recent years, the professional learning (PL) at Barron Academy's Elementary was facilitated by *Math in Focus* representatives. The content of PL focused on how teachers could implement whole group mathematics discussions using the provided curriculum. Specifically, the content of the trainings emphasized teacher questioning and encouraged teachers to increase the frequency of open-ended questions. For example, teachers were encouraged to ask questions such as "How do you know?" and "Why do you think that?" to elicit student thinking. Teachers reported being receptive to the ideas addressed within these training sessions and believed they now ask more open-ended questions because of the PL (personal communication, 9/10/2021). Asking more open-ended questions to individual students, however, does not necessarily constitute a mathematics discussion as it has the potential to lack a collaborative response from multiple students (NCTM, 2014; Pieri & Schwarzenberger, 1988).

Evidence of the Problem

Despite participating in PL on the topic, teachers at Barron Academy have difficulty facilitating classroom discussion in a way that promotes students engaging with the thinking of their peers. Both teachers and administrators admitted having difficulty improving their mathematics instruction and evolving from teacher-led conversations to student-centered discussions. Evidence of this problem arose throughout conversations with administrators and teachers and during observations of mathematics classrooms.

In conversations with administrators, it became clear that the school values the use of mathematics discussions as an opportunity for student learning. In an early conversation with Mr. Samuel Curtis[2], the principal of Barron Academy Elementary School, he shared that mathematics discussions help students "share their answers and strategies and even learn new ones from their peers" (personal communication, 07/12/21). He noted that it is important for students to be able to communicate their thinking to peers and to listen and learn from the ideas of others.

In order to help improve the mathematics discussion practices at Barron Academy, the school's PL included a focus on teacher questioning over the last three years (personal communication, 07/12/21). Administrators viewed teacher questioning as integral to mathematics discussion. In a later conversation with Mr. Curtis, he shared that the school focused on questioning because "teachers can begin to understand the thought process of students through questioning" (personal communication, 9/13/21). He reported having observed an increased number of teacher questions during his classroom walkthroughs and observations since the PL, but he has yet to notice children asking significantly more questions of one another.

[2] pseudonym

When students begin to ask questions of one another, it provides an opportunity for "every child to become a teacher", shifting from a teacher-led discussion to a student-led discussion (personal communication, 9/13/21). An additional administrator, Dr. Tara Klingham[3], also stressed the importance of questions being asked by students to one another. Teachers echoed a similar sentiment and shared that they hoped that asking more open-ended questions would support students in asking questions to one another (personal communication, 9/13/21).

Teachers at Barron worked with the *Math in Focus* representative to count the number of questions during a lesson as a way to monitor discussion. Administrators witnessed an improvement in the quantity of open-ended questions asked by teachers to students but had not considered the quality of the questions. Neither Mr. Curtis nor Dr. Klingham had a system for measuring the quality of discussion or the questions that teachers asked beyond counting the quantity of open-ended questions. An improvement in mathematics discussion might necessitate Barron moving beyond focusing on *quantity* and toward determining the *quality* of the questions and subsequent discussions.

While teachers have perceived growth in their own use of open-ended questions to lead discussions, they admitted to struggling with the move to facilitating discourse in a way that led students to engage in discussion with one another (NCTM, 2014; Personal communication, 9/8/21-9/15/21). In several observations, I noticed teachers asking students "do you agree or disagree with [student 1]?" (Observation, 9/7/21-9/14/21). Students frequently responded with a response that lacked a clear explanation such as "I agree because I also got 14" or "I disagree because I got a different answer" (observation, 9/8/21). Although these students did comment on the answer of the others, they did not provide justification for why they agree or disagree with

[3] pseudonym

their peers. There was no evidence that students considered their peers' thinking, only the answer. Furthermore, when teachers asked students to comment on the work of others or question their peers, students often shared their own strategies and solutions instead of critiquing the work of their peers. For example, I observed a fourth-grade teacher prompt her students by asking "[Student] did a great job explaining his strategy. Can someone compare their strategy to [student's] explanation?" (personal observation, 9/9/21). The teacher then called on a different student who shared their own explanation without a single reference to the work of the initial peer. These examples resemble more of a "show-and-tell" presentation of work rather than students engaging in discussion with one another around student thinking (Pieri & Schwarzenberger, 1988).

Teachers expressed difficulty supporting students in mathematical discussions, particularly when in small groups and partners (personal communication, 9/10/2021). One teacher elaborated by saying that "sometimes students just blindly accept another's answer without stopping to ask why" (personal communication, 9/10/2021). I observed similar interactions during small-group and partner work. Even though student directions included explaining their ideas to their partners, students often had limited conversation and one student talked at another student (personal observation, 9/7/2021-9/12/2021). Considering the curriculum's frequent suggestions for small group work and discussions, it is important to understand how teachers currently support students in communicating their reasoning and sustaining mathematics discussions with one another.

Through both my initial observations of mathematics classrooms and conversations with teachers, it became evident that students did not always communicate their thinking clearly in discussions. One teacher commented that it is "really hard to pull information" out of students when asking students to explain their thinking (personal communication, 9/9/2021). This teacher acknowledged that even with the prompting of open-ended questions, she struggled to prompt more elaborate or thorough responses. She most frequently elicited responses that were brief or vague. A few of her students frequently provided thorough explanations that exceeded her explanations but that she hopes to assist all of her students in providing clear explanations when communicating their thinking.

Through observations and conversations with teachers and administrators at Barron Academy, evidence of this problem of practice became apparent. Although many teachers felt more confident in asking open-ended questions to lead discussions between teacher and student, they were less confident in facilitating discussion between students.

Purpose of the Study

The leadership at Barron Academy endorsed the importance of mathematics discussion in elementary classrooms. As a result, teachers and administrators hoped to improve mathematics discussion at Barron Academy by moving away from teacher-led discussion and toward a student-led discussion, where students eventually sustain the mathematics discussion themselves. As a welcomed but external informant to Barron Academy, I am in a position to provide a rich description of a sample of the current elementary mathematics teachers' enactments of discussion using the Singapore Math curriculum. Through this capstone project, I provide a descriptive analysis of three teachers' practices while orchestrating mathematics discussion using the *Math in Focus* curriculum for the purpose of identifying areas of continued growth and improvement pertaining to mathematics discussion. A better understanding of the current landscape of mathematics instruction and mathematics discussion will assist Barron Academy in shaping strategic instructional decisions, resource allocations, and PL for the improvement of student learning through mathematics discussion. The following research questions informed my study:

1. How do elementary teachers describe the attributes of meaningful mathematics discussions? To what degree are teachers' descriptions aligned to the NCTM's mathematics teaching principles?
2. To what degree do teachers orchestrate meaningful mathematics discussions?
    a. In what ways, if any, do teachers use talk moves to support student-to-student engagement during mathematics discussions?
    b. In what ways, if any, do teachers limit student-to-student engagement during mathematics discussions?
    c. In what ways, if any, does the curriculum support opportunities for meaningful mathematics discussion?

Significance of the Study

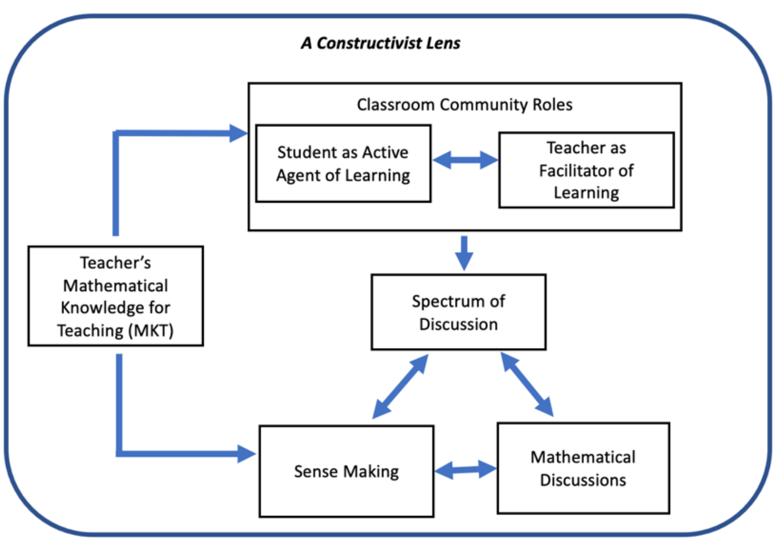
This findings from this study will directly support Barron Academy in addressing the identified problem of practice that teachers do not yet facilitate mathematics discussions in a way that supports students engaging with the ideas of others. A case study approach appropriately provides an in depth understanding of the given problem within the local context through its observations of mathematics instruction and interviews with teachers. Through analyzing observation and interview data collected and conducting a review of available curriculum resources, I now have an in depth understanding of the current landscape of mathematical discussions at Barron Academy, which shaped the recommendations for the administration and teachers.

Conceptual Framework

To further understand mathematics discussion in the context of this study, I developed a conceptual framework built upon a constructivist learning theory and its assumptions for how individuals learn. The assumptions of constructivism influence how the roles of teachers and students in the classroom community contribute to the way in which teachers facilitate discussion across a spectrum impacting both the implementation and utilization of mathematics discussion and the occurrence of sense making in the classroom (see figure 1.10). Additionally, a teacher's mathematical knowledge for teaching (MKT) (Hill et al., 2005) influences the ways in which teacher and student roles take shape, such as how teachers present opportunities for learning, sense making, and discussion (Walkowiak et al., 2017).

Figure 1.10

Conceptual Framework



Constructivism asserts that learners construct their own knowledge and understanding through prior knowledge, experience, and social interactions (Bruner, 1966, 1967; Dewey, 1938;

Vygotsky, 1978). Dewey (1938) asserted that learning is a social activity and that we learn through interactions with others. Vygotsky (1978) extended this idea by recognizing the importance of collaboration in the construction of knowledge. Within a community, individuals can make meaning of the world around them through social interactions. Under constructivist theory, discussions between individuals, viewed as an inherently social interaction, contribute to how those individuals learn and construct both their knowledge and understanding. Viewing this phenomenon through a constructivist lens, I created a framework that identifies how classroom community roles and dialogic discussion are necessary factors in supporting mathematics discussion and sense making in the mathematics classroom.

Teacher and Student Role

In any given classroom community, the individuals within that community have specific roles which influence the teaching and learning within the classroom. Under a traditional role, the teacher provides knowledge directly to students, who passively consume the knowledge, and can be seen as a "dispenser of knowledge" (Stein et al., 2008). Thus, a traditional classroom is a more teacher-centered classroom. Alternatively, in a student-centered classroom, students construct their own knowledge through collaboration and communication with others as social experiences are embedded into the learning process (Honebein, 1996). In accordance with constructivist theory, teachers should "create situations in which students can explore" mathematics (Wen, 2018, p. 236). Teachers are no longer authoritative providers of knowledge, rather they guide students as they develop their own learning (Bruner, 1966). In a constructivist and student-centered classroom, teachers "allow student responses to drive lessons, shift instructional strategies, and alter content" (Kompf, 1996, p. 173). Under this notion, students drive the instruction through discussion and the teacher facilitates the discussion by monitoring it, encouraging students to participate, and deciding when and how to call attention to specific student ideas (White, 2003).

Teacher role and student responsibility within the classroom community are two of the five components of Hufferd-Ackles and colleagues' (2014) framework for understanding the various the levels of classroom discussion (see Figure 1.11). Hufferd-Ackles and colleagues (2004) developed this framework from previous research, which described math-talk learning communities as "classroom communities in which the teacher and students use discourse to support the mathematical learning of all participants" (p. 83). As seen in Figure 1.11, traditional teachers, who stand at the front of the classroom, ask questions, and focus on correctness, do not support meaningful discussion and are the lowest level of a math-talk learning community. Alternatively, when students are positioned as active constructors of knowledge and leaders of discussion, the math-talk learning community is high-level and conducive to meaningful mathematics discussion. This definition of a math-talk learning community expands upon my definition of meaningful mathematics discussion to include the entire classroom community and not simply specific moments of discussion.

Through the framework, teacher and student roles can be distinctly measured using levels (0-3). In a classroom with low levels (level 0 and 1) of discussion, the teacher dominates the conversation and encourages students to share ideas with the class. Students may keep ideas to themselves (level 0) or begin to share ideas with their peers (level 1). Hufferd-Ackles et al. (2014) advance discussions (level 2) when the "teacher facilitates conversation between students and encourages students to ask questions of one another", then the classroom shifts to a more student-centered approach (as cited in NCTM, 2014, p. 32). In the highest level of classroom discussion (level 3), however, the teacher is merely a "guide from the periphery of the conversation" (NCTM, 2014, p. 32). At this point, students believe that they "can shape the thinking of others" and recognize themselves as math leaders in the classroom (Hufferd-Ackles et al., 2014 as cited in NCTM, 2014, p. 32). Hufferd-Ackles et al. (2004) called for students to be active agents of not only their own learning but also the learning of others.

Figure 1.11

A Framework of Levels for a Math-Talk Learning Community (Hufferd-Ackles et al., 2014 as cited in NCTM, 2014, p. 32)

	Teacher role	Questioning	Explaining mathematical thinking	Mathematical representations	Building student responsibility within the community
Level 0	Teacher is at the front of the room and dominates conversation.	Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.	Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.	Representations are missing, or teacher shows them to students.	Culture supports students keeping ideas to themselves or just providing answers when asked.
Level 1	Teacher encourages the sharing of math ideas and directs speaker to talk to the class, not to the teacher only.	Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.	Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.	Students learn to create math drawings to depict their mathematical thinking.	Students believe that their ideas are accepted by the classroom community. They begin to listen to one another supportively and to restate in their own words what another student has said.
Level 2	Teacher facilitates conversation between students, and encourages students to ask questions of one another.	Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.	Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.	Students label their math drawings so that others are able to follow their mathematical thinking.	Students believe that they are math learners and that their ideas and the ideas of their classmates are important. They listen actively so that they can contribute significantly.
Level 3	Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.	Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask "why" and call for justification. Teacher questions may still guide discourse.	Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.	Students follow and help shape the descriptions of others' math thinking through math drawings and may suggest edits in others' math drawings.	Students believe that they are math leaders and can help shape the thinking of others. They help shape others' math thinking in supportive, collegial ways and accept the same support from others.

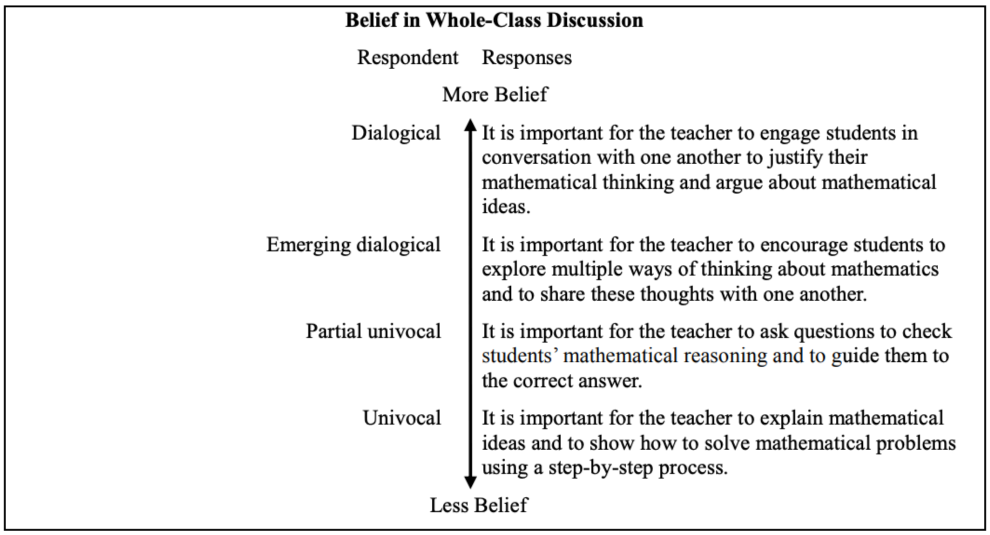
Spectrum of Discussion

Regardless of the role of teachers and students within a classroom community, some talk is present in the mathematics classroom. In a traditional classroom, the talk is primarily from the teacher or univocal. In a reform classroom with meaningful discussion, students participate in dialogic discussion. When peers communicate and actively generate meaning through an equally distributed give-and-take of mathematical thinking, they engage in dialogic discussion. Classroom discussions focused on procedures and solutions typically are more univocal in nature while discussions that focus on student thinking and meaning making are more dialogic (Wertsch, 1991). A dichotomy, however, does not appropriately capture the reality of most classroom discussion. Knuth and Peressini (2001) recognized that individual instances of discussions move along a spectrum of dialogic to univocal and a single classroom can exist across the spectrum of discussion.

Jang (2010) developed a construct for categorizing discussion and teacher beliefs about mathematics discussion. This construct represented teachers across a range of beliefs about mathematical discourse, which included *univocal*, *partial univocal*, *emerging dialogical*, and *dialogical* (see Figure 1.12). While these four categories labeled the beliefs of teachers, they also labeled the types of discussion that may occur in the classroom. The lowest level of discussion is a univocal discussion, which occurs when teachers are the primary voice responsible for explanations. A discussion is partially univocal when teachers question students in a way that guides them toward the right answer. The discussion moves to an emerging dialogical conversation when teachers encourage students to share multiple ways of thinking or strategies with their peers. In this conversation, however, there is still an emphasis on sharing the ideas with their peers and not necessarily explaining and engaging with those ideas. In a dialogical conversation, however, students justify their thinking and engage with the thinking of others as they argue mathematical ideas. While analyzing and interpreting instances of mathematics discussion, I referred to Jang's construct as a way of making sense of the exchanges occurring between teachers and students during discussion. In Chapter 3, I describe my findings using these terms as a way to make sense of the discussion present at Barron Academy.

Figure 1.12

Construct Map of Discussion (Jang, 2010, p. 15)



Sense Making

In a dialogic discussion, students exchange ideas with one another as they build their understanding of mathematics. Sense making, as defined by Battista and colleagues (2017), is the process of "understanding ideas and concepts in order to correctly identify, describe, explain, and apply them" (p. 1). Interaction is not optional, but essential, to the process of sense making (Hiebert et al., 1997). Sense making builds upon constructivist theory as it acknowledges the social construction of understanding through connecting new information to previous knowledge and experiences (Battista et al., 2017). Meaningful mathematics discussion is dependent upon the sense making that occurs during the discussion. Similarly, sense making requires communication and discussion amongst students. My conceptual framework acknowledges the interrelationship between sense making and meaningful mathematics discussion. The two are codependent upon one another; as discussions become more meaningful, more sense making occurs and vice-versa.

Mathematical Knowledge for Teaching (MKT)

The ways in which teachers provide instruction to students is influenced by teachers' MKT (Hill et al., 2005; Walkowiak et al., 2017; Wasserman, 2015). Hill and colleagues (2005) define MKT as the knowledge needed to carry out the work of teaching mathematics. Teachers are required to know more than content knowledge and pedagogical content knowledge to successfully respond to students' statements, use representations accurately, and critically interpret curriculum (Hill et al., 2005). The depth and breadth of the mathematics that teachers should know and understand extends beyond the specific content they teach (Wasserman, 2015, p. 92). Teacher's MKT has been found to not only impact student achievement (Hill et al., 2005) but also impact the ways in which teachers provide students with meaningful opportunities to learn (Walkowiak et al., 2017). Specifically, MKT influences the ways in which teachers use mathematical talk to support students' opportunities to learn (Walkowiak et al., 2017). Although I did not measure teachers' MTK in this capstone study, it is important to include MKT in my conceptual framework because of the way it influences instructional practices and opportunities for learning.

Together, through a constructivist lens, the reform classroom community and dialogic discussions can support teachers and students in engaging in meaningful mathematics discussions and sense making. Meaningful discussion is dialogic, where students build and construct knowledge; however, students must be positioned as active agents of their own learning and the teacher must assume the role of a facilitator of learning within the classroom community. My conceptual framework provided the lens through which I viewed and interpreted mathematics discussion at Barron Academy. Specifically, I examined how the role of teachers and students within the classroom community influenced the types of discussions that occurred during mathematics.

Definitions of Terms

In this section, I define the relevant terms used throughout this capstone. These definitions are defined to provide clarity of meaning throughout the current study:

- Dialogical Discussion: discussion in which students engage in conversation with one another, justify their mathematical thinking, argue mathematical ideas, and make connections among thinking (Jang, 2010).
- Emerging dialogical discussion: discussion where teachers "encourage students to explore multiple ways of thinking and share these thoughts with one another" (Jang, 2010, p. 15)
- Mathematics Discussion: "purposeful talk on a mathematical subject in which there are genuine pupil contributions and interactions" intended to accomplish a specific mathematical goal (Pirie & Schwarzenberger, 1988, p. 460).
- Meaningful mathematics discussion: discussion that is utilized as an "instructional practice [that] can support students' access to mathematical content and discussion practices" and deepen understanding (Herbel-Eisenmann et al., 2013, p. 182; NCTM, 2014).
- Partial Univocal: discussion in which teachers ask questions to check student understanding and guide students to the correct answer or solution (Jang, 2010)
- Partners: refers to the discussions, activities, tasks, etc. that occur between two students.

- Reform Mathematics: an educational shift away from traditional rote learning and toward concept development through problem solving and constructivist beliefs (NCTM, 1989)
- Small Group: refers to the discussions, activities, tasks, etc. that occur in groups between partner and whole group settings.
- Talk Moves: intentional instructional strategies and practices teachers use to support mathematical thinking during discussion (Chapin et al., 2003).
- Understanding: "the ability to represent a mathematical idea in multiple ways to make connections among different representations" (Cramer & Karnowski, 1995, p. 333)
- Univocal discussion: discussion where the teacher is the primary voice explaining mathematic procedures (Jang, 2010).
- Whole Group: refers to the discussions, activities, tasks, etc. that occur in a whole class setting between the teacher and multiple students.

Chapter Summary

In this chapter, I presented how mathematics discussions have become increasingly emphasized in both research and practice in elementary contexts. Various educational organizations, such as the CCSI (2010) and NCTM (2014), suggested that students engage in meaningful mathematics discussions by explaining their reasoning and critiquing the ideas of their peers. In a productive mathematics classroom, the teacher has moved away from leading the discussion and toward facilitating a discussion that is driven by students as they engage with the ideas of others (Hufferd-Ackles et al., 2004; NCTM, 2014). Over the last several years, teachers at Barron Academy have attempted to implement such discussions. However, the teachers and administrators have expressed difficulty in ensuring discussions are meaningful. As a result, this capstone study focuses on the practices enacted by elementary mathematics teachers at Barron

Academy by describing how teachers promote or limit student-to-student discussion through a descriptive case study. In the subsequent chapter, I will provide a review of the relevant literature that informs the direction of my capstone.

Chapter 2: Literature Review

Facilitating mathematics discussion is a complex skill and necessary practice for teachers to both understand and possess so that they may teach high-quality mathematics (NCTM, 2014). At Barron Academy, teachers do not yet facilitate mathematics discussion in a meaningful way driven by student ideas. In Chapter 1, I situated the local problem of practice at Barron Academy through a description of the context and the complexities of facilitating meaningful mathematics discussion. In this chapter, I review literature that is most directly relevant to the context of my study as organized by three main topics – the relationship between discussion and student achievement in mathematics, teacher beliefs about mathematics and the role of a mathematics teacher, and teacher talk moves. I will begin with a discussion of studies that seek to understand the relationship between discussion and achievement in mathematics because it provides evidence for the potential significance of discussion on student understanding. Next, I will discuss how teacher beliefs about mathematics, mathematics discussions, and their role impact what teachers may or may not do when orchestrating mathematics discussion. Finally, I will explore various talk moves that the literature claims support teachers in orchestrating meaningful mathematics discussions.

I chose to focus the literature review on the previous topics even though literature pertaining to mathematical discussion is more expansive. In this capstone, I sought to understand how teachers describe the attributes of meaningful mathematics discussions and to what degree teachers orchestrate meaningful mathematics discussions. Therefore, I bounded the literature to include how beliefs may influence teachers as it relates specifically to discussion and how teacher moves influence students during mathematics discussion. Although teacher content knowledge and pedagogical content knowledge may influence teachers' abilities to facilitate discussion, I will exclude content knowledge and pedagogical content knowledge from this literature review because it is not directly relevant to the scope of this capstone study. The knowledge of teachers at Barron Academy is out of my scope of influence and will not be measured within this study. I also chose to exclude how rich mathematical tasks support discussion. All teachers at Barron Academy utilized the same curriculum, *Math in Focus,* as their core instruction. Barron Academy recently renewed an extended contract with *Math in Focus* and the curriculum itself will not change as a result of the findings of this capstone study. Since *Math in Focus* includes mathematical tasks that claim to support discussion (Marshall Cavendish Education, 2021c), I examined how the literature suggests teachers might further support students in discussion while using any curriculum or mathematical task. Literature that focuses on teacher moves and practices bridges the gap between theory and practice (Boerst et al., 2011, p. 2852). Consequently, I chose to attend to literature that describes teachers' beliefs about mathematics discussion and instructional practices focused on talk moves. For the purpose of this capstone, mathematics discussion is defined as "purposeful talk on a mathematical subject in which they are genuine pupil contributions and interactions" intended to accomplish a specific mathematical goal (Pirie & Shwarzenberger, 1988, p. 460).

This problem of practice, which is the focus of the capstone study, is situated within the specific context of Barron Academy. Although both NCTM (2014) and CCSSI's (2021) recommended practices were intended to support all students in learning high-quality mathematics, much of the literature is focused on public school contexts with different demographics and influences than Barron Academy. A general limitation of the literature reviewed here are the notable differences between those contexts and this study's sample. Barron Academy is a small, independent school serving primarily upper middle class, white, native

English-speaking students. Many of the studies reviewed in this chapter were set in more diverse public-school settings, where instructional practices and research were motivated by achievement gaps within the context. Although achievement is not a primary concern at Barron Academy, the school administration does aim to utilize practices that support student understanding and continued growth. While the contexts of the literature reviewed are different from Barron Academy, they are built upon learning science and practices that are intended to enhance mathematical understanding by supporting all students, despite any socioeconomic, language, or cultural differences (NCTM, 2014).

Relationship between Discussion and Achievement

Reform mathematics efforts, supported by the NCTM (2014) and CCSSI (NGA Center, 2010), claim that discussion supports students' mathematic growth, understanding, and achievement. To better understand the potential impact of discussion on student understanding and achievement, I begin this literature review by examining various studies that seek to identify the relationship between mathematics discussion and achievement. In a metanalysis of factors influencing student achievement, Hattie et al. (2017) found classroom discussion to be the 15th most significant influence on student achievement out of a list of 256 instructional practices. Hattie and colleagues (2017) suggested that it would "be wise [for teachers] to focus their energy on building classroom discourse rather than attempting to teach test-taking" (p. 41). Measuring the specific impact of mathematics discussion on student learning and achievement is difficult, however, due to the variability in discussion and the inability to isolate discussion from other variables within the classroom, such as students' prior knowledge, teacher content knowledge, instructional tasks, or curriculum (Kosko, 2012). Nevertheless, various researchers have sought to identify relationships between the existence of classroom discussion and student achievement through studies measuring frequency of discussion, participation in discussion, and the collaborative nature of discussions. These studies, however, do not always define discussion as purposeful talk with genuine pupil contributions and interactions (Pirie & Schwarzenberger, 1988) or talk that deepens understanding (Herbel-Eisenmann et al., 2013; NCTM, 2014). Rather, they consider discussion to be talk about mathematics.

Frequency of Discussion and Achievement

Some researchers have sought to identify the relationship between the frequency of mathematics discussions and student achievement (Kosko, 2012; Kosko & Miyazaki, 2012; Mercer & Sams, 2006). Using achievement data collected from a longitudinal study by the National Center for Education Statistics (NCES), Kosko (2012) compared student performance on a standardized cognitive domain test to survey results asking students to report how frequently they engaged in discussions during mathematics class. The surveys were administered to students beginning in kindergarten in 1998-1999 through their eighth-grade enrollment in 2006-2007 (n=2,832). The survey asked students to rate how frequently they discussed mathematics from daily, to three or four times per week, once or twice per week, two to three times per month, once per month, to almost never (p. 122-123). Findings indicated that students enrolled in classes with daily mathematics discussion consistently had higher achievement than students in classrooms that hardly ever discussed mathematics (Kosko, 2012). Notably, this study did not examine the quality of discussion nor did the author clearly define discussion for participants; rather, the meaning of discussion was interpreted by the student participants. Additionally, discussion cannot be isolated as the only variable that impacted student achievement. Kosko asserted the implications of these finding did not suggest that more discussion equates to higher quality of discussion (Kazemi & Stipek, 2001). Rather, frequent discussion has the potential to benefit student achievement over time.

Supporting the previous assertion, Kosko and Miyazaki (2012) found that there was no significant relationship between the frequency of discussion and mathematics achievement for fifth-grade students in a single year. Using data from the same longitudinal study as Kosko (2012), Kosko and Miyazaki (2012) used survey data from 3,632 teachers across the country to contrast teachers who claimed students discussed mathematics "at least once a week" to those who reported discussing mathematics "less than once a week" (p. 180). Kosko and Miyazaki (2012) ran multiple analyses measuring the effect of frequency of discussion on student achievement controlling for various covariates such as prior achievement, socio-economic status, and effort. Without controlling for covariates, the results suggested there is a positive effect; however, once controlling for the covariates, they found that there was not a statistically significant difference in achievement for those who engaged in discussion more than once per week and those who did not. These authors attributed this to the idea that discussion varies in each setting. Schools and classrooms that were not positively impacted by the frequency of discussion may not "have structured classroom experiences supportive of effective mathematics discourse" (p. 192). This reiterated the notion that simply talking about math does not yield a meaningful mathematics discussion (Kazemi & Stipek, 2001; Walshaw & Anthony, 2008). Kosko and Miyazaki (2012) recommended that future research should include more school- and teacher- level factors, such as student responses and teacher questions, that qualitatively describe the classroom discussion while also evaluating the "actual gains in mathematics" (p. 192). One such way in which researchers have sought to further investigate the relationship between discussion and achievement beyond the frequency of discussion is through better understanding how student participation in mathematics discussion relates to mathematics achievement.

Participation in Discussion and Achievement

Mathematics reform efforts often claim that discussions have powerful effects on student learning (NGA Center, 2010; NCTM, 1991, 2014) and encourage teachers to utilize discussion in mathematics class (Chapin et al., 2003; Michaels et al., 2016). To support these claims, some research has identified that participation in discussion is positively related, if not predictive, of student achievement (Webb et al., 2014, 2021; see also Hiebert & Wearne, 1993; Hung, 2015; Ing et al., 2015). Hiebert and Wearne (1993) found a positive relationship between student achievement and participation in discussion through a study of six second grade mathematics classrooms. Students in the six study classrooms were assessed on place value, addition, and subtraction three times over the course of a school year using an assessment developed by the research team. Hiebert and Wearne found that students, who spent more time in class asking questions, explaining problems with multiple strategies, and participating in discussions, scored higher on the assessment than those who participated less and used rote procedures more frequently. Hung (2015) further investigated the relationship between student participation in discussion and achievement in mathematics, while also including students' self-image in relation to mathematics. As a teacher-researcher, Hung observed relationships between students' self-image, their engagement in mathematics discussion, and their achievement in his high school mathematics course. Hung concluded that students with higher participation had a higher perception of their own identity within mathematics and higher achievement. While both Hung (2015) and Hiebert and Wearne (1993) identified a relationship between participation and achievement, the relationship cannot be identified as causal and participation in discussion cannot be seen as positively impacting achievement. Rather, the findings do suggest a positive relationship between participation in discussion and achievement (Hiebert & Wearne, 1993; Hung, 2015).

Through an ongoing analysis of various elementary mathematics classrooms in California, Ing et al. (2015) and Webb et al. (2014, 2021) studied the interrelationship between teacher instructional practices, student participation in discussion, and academic achievement. Webb et al. (2014) found that student participation in discussion was positively correlated to student achievement. Moreover, Ing et al. (2015) concluded that student participation significantly predicted student achievement. Webb and colleagues (2021) further examined how student engagement and participation in classroom discussions supported learning through observing classroom discussions in two third-grade mathematics classrooms. Individual students were coded with a score of 0 or 1 for both participating in the discussion and engaging with the ideas of other students. The research team also analyzed the mathematical advances of students over the five months so that they could relate advances to participation. Mathematical advances were defined as "shifts in understanding of mathematical ideas, marked changes in problem-solving strategies, and generation of new mathematical ideas or problem-solving strategies" (p. 6). Webb and colleagues (2021) identified mathematical advances using all available evidence and data including verbal and non-verbal interactions during observations, written work from observations, and written work from days leading up to and following observations. They found that nearly all students made at least one mathematical advancement while explaining their ideas to others and/or engaging with others' ideas during classroom discussion (Webb et al., 2021). They concluded that participation in the classroom discussion and engaging with other students supported students in learning mathematics. This study measured student achievement through noticing advances, or recognizing progress more related to understanding, rather than measuring achievement through standardized assessments alone.

Some research, including Hung (2015), acknowledged that a high correlation between participation and achievement could be explained by more-knowledgeable students being the ones who want to participate (Webb et al., 2021). Webb and colleagues (2021) investigated this correlation through a subsample of students with less than proficient performance on the end-of-year state assessment to determine how participation can support learning for students with lower achievement. In this subsample, 67% of students exhibited at least one mathematical advancement in conjunction with their participation. This finding disproved that the correlation between higher achievement and higher participation is solely due to higher achieving students wanting to participate and lower participating students avoiding participation. Rather, participation has the potential to support students in learning mathematics regardless of their knowledge levels because of the potential to make advances through participation. About half of the observed advances were made despite originally unclear, incomplete, and/or incorrect solutions from students. These misconceptions and mistakes were clarified through further interaction and peer feedback, allowing students to make a mathematical advancement while participating and engaging with peers. Webb et al. (2021) claimed that "through the process of explaining their [student] own thinking and engaging with others' ideas, students not considered to have extensive mathematics knowledge can forge new connections between mathematical ideas and representations and extend their problem-solving strategies" (p. 21). The results indicated that student participation may support all students in making mathematical growth and are strengthened due to the researchers' analyses of a specific subsample, which accounted for a variable of prior knowledge. Although I did not measure student achievement nor did I know students' prior achievement, I attended to how teachers support or inhibit the participation of all students in discussion during data collection.

Collaboration in Discussion and Achievement

In addition to frequency of discussion and participation, some research suggests that discussion supports student achievement because of the collaborative nature of discussions (Boaler, 2002; Boaler & Staples, 2008; Webb et al., 2008, 2009, 2014, 2021). Through extensive research, Webb et al. (2008, 2009, 2014, 2021) found that students' practices of explaining their own ideas and engaging with the ideas of others predicted their mathematics achievement in the areas of algebraic reasoning and understanding of the equal sign. As students engage with the ideas of others in discussion, they collaborate with their peers to develop their mathematical understanding. Webb et al. (2021) claimed that engagement with the ideas of others, in addition to participation, positively relates to students learning outcomes.

Boaler (2002) and Boaler and Staples (2008) also found that high levels of mathematics achievement were related to high participation and engagement in mathematics discussions. Boaler and Staples (2008) followed students from three California high schools to gain a better understanding of equitable and successful teaching practices. Teachers at Railside School, the school with the highest academic gains as measured by content assessments and open-ended mathematics projects over three years, taught mathematics with a reform-oriented curriculum. Mathematics instruction at Railside School frequently included conceptual problems and group work, rather than a more traditional curriculum with direct instruction and practice sets. Students at Railside School spent approximately 72% of their time in group-work collaborating with peers, where they worked together to communicate ideas and make sense of mathematics together. Research argued that more reform-oriented approach provided more opportunities for students at Railside to increase participation and engagement in mathematics through discussions and collaborative group work. Boaler and Staples (2008) did not attribute the greater achievement of Railside School to discussion and collaboration alone, as it was impossible to isolate either as an individual variable. However, they identified discussion and collaboration as two major components of the reform curriculum utilized at Railside School as opposed to the other two schools with less gains and a widening achievement gap. Railside School was a diverse, under-performing public secondary school, which in many ways contrasts with the high-achieving, private elementary program at Barron Academy. Despite the differences in these contexts, the findings from Railside School provided evidence to suggest significant gains that can be achieved through collaborative discussion (Boaler & Staples, 2008).

Although mathematics discussion cannot be directly identified as causing increased mathematics achievement, some research claimed that a positive relationship exists between the mathematics discussion and student achievement (Webb et al., 2014, 2021; see also Boaler & Staples, 2008; Hiebert & Wearne, 1993; Hung, 2015; Ing et al., 2015, Kosko, 2012). However, the research also noted that not all mathematics discussions increase student understanding and achievement. Various factors, such as student participation, collaboration, and classroom experiences, influence how students make sense of mathematics during discussion and build their understanding. To better understand how mathematics discussion can support student understanding and achievement, it is important to consider how the teachers' beliefs about mathematics and discussion may shape how they teach mathematics.

Teacher Beliefs

The ways in which teachers teach mathematics is affected by their individual beliefs as teachers (Boaler, 2016; NCTM, 2014). Rokeach (1968) defined *belief* as "any simple proposition conscious or unconscious inferred from what a person says or does" (p. 113). As constructivist theory assumes, individuals use prior experiences as a foundation for constructing new knowledge and understanding (Bruner, 1966, 1967; Vygotsky, 1978). Teacher beliefs about mathematics, student learning, and instruction are often heavily influenced by their experiences as students and constructed prior to any teacher education programs or professional learning (Ball 1988a, 1998b; Bush, 1986). As such, teachers' experiences and beliefs serve as a foundation underlying what they know about and how they engage with mathematics instruction. To narrow the scope of literature reviewed to those areas most directly related to the focus of my proposed capstone project at Barron Academy, I specifically examined how teachers' beliefs about discussion and their roles as teachers influenced how teachers implemented discussion in instruction.

Teachers' Beliefs about Teaching Mathematics

Reform efforts, supported by the NCTM (1991, 2014), have exemplified the importance of teachers and students thinking productively about mathematics. The NCTM (2014) referred to beliefs associated with reform mathematics instruction as *productive* and beliefs that are associated with more traditional theories of mathematics instruction as *unproductive*. According to the NCTM (2014), beliefs should be considered "unproductive when they hinder the implementation of effective instructional practices or limit important mathematics content and practices" (p. 11). Productive beliefs, conversely, support the implementation of effective practices, mathematics content, and reflect some of the CCSSI's SMP (NGA Center, 2010). A teacher's belief that students can learn mathematics through exploring mathematical problems (NCTM, 2014) parallels encouraging students to make sense of problems and persevere in solving them (MP1) (NGA Center, 2021). The NCTM (2014)'s classification of unproductive and productive beliefs provides a way of understanding both how beliefs influence instructional decisions and which beliefs support meaningful mathematics discussion. The relationship between beliefs and practices is complex; that is, beliefs cannot perfectly predict practices (Bennett, 2010; Conner & Singletary, 2021; Spillane & Zeuli, 1999; Stipek et al., 2001; Yurelki et al., 2020).

Stipek and colleagues (2001) video recorded 21 fourth- through sixth-grade mathematics classrooms and surveyed teachers about their beliefs in mathematics. The survey, using a 6-point Likert Scale, asked teachers to agree or disagree with 57 statements that identified teachers' beliefs about teaching mathematics. They found that "three beliefs – that mathematics is a set of operations and procedures to be learned, that the teacher should be in complete control, and that extrinsic reinforcements are effective strategies to motivate students" were positively associated with classrooms that emphasized performance over understanding (p. 221). In responding this way, these teacher's demonstrated unproductive beliefs according to NCTM (2014). Conversely, they found that the teachers with productive beliefs, which included that "mathematics is a tool for thought, students' goal is to understand mathematics, students should have autonomy, [and] mathematics ability is amenable to change" engaged more in classroom practices that supported conceptual understanding and student thinking (p. 222). These findings suggested a "substantial coherence" among teacher beliefs and their instructional practices as determined by both survey results and classroom observations (p. 1).

In addition to surveying teachers, Stipek et al. (2001) also surveyed students to identify students' self-perceptions of mathematics abilities. The survey included questions, such as "How good are you at math?" and "How much do you like doing math?", on a 6-point Likert Scale (p. 220). Researchers compared student responses to the teacher responses to identify a correlation between students' confidence and teachers' beliefs. The results indicated that students who reported more confidence in mathematics were in classrooms with teachers whose surveys indicated more productive beliefs and vice-versa. The authors suspected that "less confident teachers" are drawn to unproductive beliefs because they can prescribe to simple steps in a textbook, use answer sheets, and make less decisions regarding instruction, which positions students to act as receivers of information and may lead students to feeling less engaged and confident with mathematics (Stipek et al., 2001, p. 223). These speculations, however, are unfounded because the survey did not ask teachers or students to elaborate on their beliefs or choices. While Stipek and colleagues could not definitively claim the reason for the relationship between confidence and practices, they can assert that there exists a relationship between not only teachers' beliefs and teacher practice but also students' beliefs (Stipek et al., 2001). Since meaningful mathematical discussion requires heavy participation from students, it is important to recognize how teacher beliefs may influence student beliefs about and engagement with mathematics.

Additional literature shows that in some instances even those teachers who claimed to possess reform-oriented beliefs, did not consistently demonstrate these beliefs in their practices (Bennett, 2010; Spillane & Zeuli, 1999). In a study of 25 elementary and middle school mathematics teachers, Spillane and Zeuli (1999) sought to identify various trends in instructional practices associated with reform supported by both NCTM (1991) and state legislation. Using data from a larger, five-year study surveying the beliefs of 283 teachers, the researchers randomly sampled teachers who reported practices on the survey that were aligned with reform beliefs. The observations of these teachers, however, suggested that teachers inconsistently adopted reform practices in their classrooms (Spillane & Zeuli, 1999). Many teachers adopted some practices, such as using manipulatives, real-world contextual problems, and multiple representations or strategies for solving problems; however, only four of 25 teachers in this study demonstrated classroom practices that fully reflected reform efforts as evidenced by engaging in classroom discussion where student thinking was shared as the main tool for instruction. One such teacher shared that conceptually centered discussions create “a lot more work because I have to constantly be listening to what they are saying, constantly be working to pull their ideas out, thinking all the time…” (p. 8). This teacher’s description demonstrated how facilitating discussion among students requires more skills than simply believing to be a meaningful practice. Another of the four teachers explained that she “wants [students] to think as often as possible during the day” and demonstrate that thinking to peers through discussion (Spillane & Zeuli, 1999, p. 11). Her beliefs, combined with instructional practices, supported students in *doing* more conceptual mathematics than procedures. The other 21 classroom teachers, despite possessing reform-minded beliefs, primarily used direct instruction focused on procedures and correct answers. Although this study was conducted more than two decades ago, this early study of reform beliefs and classroom practices demonstrates the disparity that can exist between beliefs and practice that is still relevant for the present capstone project. While conducting interviews and observations, I considered misalignments that may exist between beliefs and practice as seen in the work of Spillane and Zeuli (1999).

More recent research has supported Spillane and Zeuli’s (1999) findings that beliefs may misalign from classroom practices (Bennett, 2010; Yurelki et al., 2020). In a case study of two mathematics teachers, Bennett (2010) found that teacher beliefs about their practices varied from the practices that the teachers enacted. One teacher, Steve, claimed that he believed it was very important to engage students in mathematical discussions and that his role was to ask students probing questions to elicit student thinking. However, Bennett rarely observed Steve probing students for justification and evidence of their thinking. Through repeated interviews and observations with teachers, Bennett (2010) argued that teacher "perceptions can be inaccurate because of more immediate demands, decisions, or pressures" in the classroom (p. 88). Although Bennett's case study is bound to the specific context, he suggests the importance of understanding how teachers' beliefs and perceptions may or may not align with the actual classroom practice.

Bennett (2010) claimed that teachers must have the skills to make in-the-moment teaching decisions that mirror productive beliefs. Even if teachers possess these beliefs, they may not possess the necessary skills to demonstrate the practices. Jung and Reifel (2011) observed one kindergarten teacher who, pressured by the constraints of her administration, time, and assessments, utilized more direct instruction than student-centered instruction despite beliefs that student-centered instruction was more beneficial. Jung and Reifel's findings support Bennett's (2010) claim that influences, beyond teachers' beliefs, may impact their instruction. Although I did not specifically study the various influences in my capstone project, I recognized that teachers' perceptions and practices may not align; therefore, I sought to understand how the participating teachers perceive how constraints affect their teaching in order to provide appropriate recommendations to Barron Academy.

Yurelki and colleagues (2020) findings strengthen the argument that disparities may exist between teacher beliefs and practices. Yurelki et al. (2020) assumed that many teachers wished to implement reform-based instructional practices supporting CCSSI but faced various limitations and constraints that prevented them from effectively teaching with reform-based instructional practices. Yurelki and colleagues (2020) developed a survey to identify teachers' beliefs about teaching mathematical connections, whole-class discussions, student input, and mathematical meaning. The survey prompted teachers to rank the importance and frequency of various instructional practices (e.g., drawing models, asking students to solve using two different strategies, facilitating students' connections of ideas). Mismatch scores were calculated to identify the discrepancies that existed between believed importance and the frequency of enacted practices. The survey results indicated that teachers' beliefs generally reflected the mathematics instruction supported by CCSSI. However, they also indicated "that teachers did not always report frequently implementing practices that they believed are important to support students' conceptual understanding of mathematics" (p. 244). The most frequent and significant mismatch revealed was "facilitating students' connection of ideas to arrive at their own explanations of a general mathematical principle" (p. 242).

The NCTM (2014)'s recommendation for teachers to facilitate meaningful mathematics discussion is founded upon the idea that discussion can support students in making connections between representations. Teachers may believe that it is important for them to facilitate or support students in discovering mathematics concepts themselves; however, it can be difficult for teachers to accomplish (Yurelki et al., 2020). While Yurelki and colleagues (2020) measured the relationship between beliefs and practices, in this capstone study I describe the observed practices within the context of their enactment of mathematics instruction at Barron Academy.

To better understand how preservice teachers' attitudes about teaching mathematics through discussion, Casa et al. (2007) developed a survey instrument. The final survey includes 26 questions answered by a 5-point Likert Scale from strongly disagree (1) to strongly agree (5). The survey was piloted to 179 preservice teachers with 27 items including survey and open-ended questions. Questions measured what teachers thought was necessary for productive mathematics discussion, the roles of students and teachers, the importance of conceptual understanding, and the influence of the classroom environment. Items were eliminated that were not found to be reliable through analysis. Casa and colleagues asserted that this instrument is meant to measure attitudes; however, positive attitudes do not ensure that discussion "will be implemented as an effective pedagogical strategy" (Casa et al., 2007, p. 76). Casa et al. suggested that a follow-up study could determine the practices utilized with respect to the pre-service teachers' attitudes. My capstone study sought to determine both the perceptions of discussion and the practices of teachers at Barron Academy. To better understand their perceptions, some of my interview questions were designed to capture select themes included in the survey instrument developed by Casa et al. (2007).

Beliefs about the Roles of Teachers and Students

In addition to general beliefs about mathematics, teachers' beliefs about their roles and responsibilities as a teacher of mathematics may be particularly influential on their ability to facilitate meaningful mathematics discussion (Beswick, 2007; see also Bray, 2004; Conner & Singletary, 2021; Hufferd-Ackles et al., 2004; Stockero et al., 2020). In a study of 25 secondary mathematics teachers, Beswick (2007) sought to identify the beliefs that most significantly underpinned instructional practices for these teachers that were also consistent with constructivist principles. Through surveys, classroom observations, and interviews, Beswick identified nine beliefs that most critically defined the teachers' classrooms which most aligned with reform efforts. Five of the nine beliefs pertained directly to the role and responsibility of the teacher. These beliefs included teachers are responsible for ensuring the classroom is productive and effective, actively facilitating and guiding students' construction of knowledge, supporting students in communicating mathematics clearly, and engaging in continuous learning (Beswick, 2007, p. 115). Beswick asserted that teachers' beliefs about their role as a teacher are possibly more evident and impactful in practice than other beliefs. While Beswick only discussed the beliefs of the teachers that demonstrated these beliefs in their classroom practice, she did not describe these practices in relation to those of the teachers that did not demonstrate constructivist beliefs. Beswick could have provided a description of how the other teachers failed to implement these practices in a way that better supported her arguments for how these beliefs align with constructivist beliefs. Given that mathematical discussions are a constructivist practice and a substantial component of the *Math in Focus* (Marshall Cavendish Education, 2021b, 2021c) curriculum, it was important for me to understand teachers' beliefs about the roles of teachers and students within discussion.

Supporting Beswick's (2007) claim that beliefs about the teacher role specifically relate to classroom practices, Conner and Singletary (2021) found that teachers' beliefs and actions contributed to students' participation in classroom discussions in very different ways. They observed two student teachers in secondary classrooms and their ability to engage students in collaborative argumentation. They found that teachers' beliefs about who was responsible for explanations and justifications in mathematics impacted how teachers prompted students for further explanations or explained mathematical content themselves (Conner & Singletary, 2021). A teacher who viewed giving explanations to students as the responsibility of the teacher, almost never explicitly prompted students to explain their thinking. Given that students' abilities to explain and justify their mathematical thinking and construct a logical argument are two of the SMP (NGA Center, 2010), it is important to understand how we can support teachers in giving students the opportunity to explain their own thinking, rather than the teacher explaining his or her own thinking. Conner and Singletary (2021) observed secondary mathematics in a public-school context, which is remarkably different than a private elementary school; nevertheless, the recommendation for students to explain and justify their thinking is applicable (NGA Center, 2010). My capstone's findings provide Barron Academy with data, specific to their context, about who provides explanations and justifications during discussion.

Stockero et al. (2020) examined how teachers' beliefs were reflected through their use of student thinking as a resource for discussion. Using student thinking as the focus of a whole group discussion provides students with an opportunity to interact with one another, ask for explanations, question, compare, critique, and discuss the ideas of their peers (Stockero et al., 2020, p. 256). Stockero and colleagues (2020) argue that the use of thinking as a resource is critical for enacting quality, student-centered teaching practices and discussions. Through interviews with 13 mathematics teachers across various grade levels, they found that when teachers believed it was their responsibility to explain and demonstrate mathematical thinking, they were less likely to utilize student thinking in discussions (Stockero et al., 2020). On the other hand, teachers, who believed student thinking had the potential to teach everyone including the teacher, were more likely to use student thinking as a resource for learning. These teachers were found to be more likely to ask the class to comment on or ask questions to the student whose thinking had been shared.

Additionally, Stockero et al. (2020) concluded that teachers were heavily influenced by who they believed to be responsible for correcting mistakes. Those teachers, who believed it was their responsibility to immediately correct mistakes, often provided less opportunities for students to engage in discussion with others around thinking. Conversely, teachers who utilized student thinking as a resource believed that "students can identify mistakes and question the shared work of fellow students without teachers intervening to ask questions" (p. 255). When teachers allowed students the opportunity to correct mistakes and misconceptions, they gave more responsibility to the students.

Bray's (2011) multiple case-study of four fourth-grade teachers, also found that whoever teachers believed to be responsible for correcting mistakes influenced classroom discussion. Two teachers, Ms. Larsano and Ms. Rosena, demonstrated these findings. Ms. Larsano, whose initial survey responses showed no reflection of productive beliefs, believed that her role as the classroom teacher was to "provide necessary explanations and directions on how to solve problems" (p. 28). She frequently asked questions that yielded specific responses so that she could funnel student responses to provide direct instruction (Herbel-Eisenmann & Breyfogle, 2005). She believed it was her role to control the classroom narrative, rarely providing time for students to engage in mathematical problem-solving and lead discussions (Bray, 2011). On the other hand, Ms. Rosena believed that teachers should support students in thinking through the mistake by questioning and inviting other students to correct their peers. Bray observed Ms. Rosena asking more open-ended questions that positioned students as responsible for justifying their thinking. Bray's work supported previous findings relating teachers' beliefs about their role and their classroom practices. Bray (2011) and Stockero et al. (2020) provided descriptive evidence of how teachers responded to student mistakes during discussion in different ways, such as a teacher-centered discussion and facilitating a student-centered discussion. These findings informed my capstone study as I specifically observed how teachers facilitated discussions around mistakes as well as discussions involving correct student ideas.

The existing literature shows conflicting evidence of how teachers' beliefs about mathematics may or may not influence their instructional practices while teaching mathematics. While some literature suggested teachers with productive beliefs demonstrate these beliefs in practice (Bray, 2011; Stockero et al. 2020), other studies found that teachers with productive beliefs do not always demonstrate these beliefs while teaching (Beswick, 2010). Regardless of a potential discrepancy between teachers' beliefs about mathematics and the enactment of their practices, teachers' beliefs about mathematics and mathematics discussions are important to better understand the instructional decisions teachers make during mathematics (NCTM, 2014). For my capstone study, I identified and described teachers' beliefs about discussion to gain a more in-depth understanding of the cases.

Talk Moves for Supporting Discussion

Various researchers have developed tools and frameworks that can support teachers in more effectively facilitating productive mathematical discourse (Chapin et al., 2003; Cirillo, 2013; Herbel-Eisenmann et al., 2005, 2013; Kazemi & Stipek, 2001; Mercer & Sams, 2006; Michaels et al., 2016; Smith & Stein, 2018). In this section, I investigate some of verbal instructional practices described in the literature as potentially effective for enhancing mathematical discussions. I refer here to these intentional, verbal strategies and practices teachers use to support mathematical thinking during discussion as *talk moves* (Chapin et al., 2003). For the purpose of this capstone study, I focus on three frequently used categories of teacher talk moves, including repeating students' ideas, questioning to elicit student thinking, and prompting students to engage with the ideas of their peers.

Repeating Student Ideas through Marking and Revoicing

When teachers facilitate meaningful mathematics discussion, the NCTM (2014) asserts that it is the teacher's responsibility to ensure that students make progress toward the mathematical goal. As students share their ideas, strategies, and reasoning with their peers, the teacher is responsible for *marking,* or bringing direct attention to, a notable contribution to the discussion (Michaels et al., 2016). Michaels and colleagues (2016) identified marking as an Accountable Talk practice that teachers can use to engage students in purposeful, coherent, and productive whole group academic discussions. For example, a teacher might mark another student's contribution by saying "Samantha said that…" or "I heard you say….". In marking student ideas, teachers repeat a student's ideas in a clear way that draws the attention from all students in the classroom to a notable contribution that may support students in attaining the mathematical goal.

Other researchers refer to this idea of marking student ideas as revoicing. *Revoicing* student ideas is a talk move utilized when teachers repeat what a student says (Chapin et al., 2003). Chapin and colleagues (2003) suggested that teachers should utilize revoicing to ensure that the teacher understands what the student has said. Concurring with Chapin et al. (2003), Herbel-Eisenmann and colleagues (2013) asserted that revoicing is a talk move that has the potential to support teachers in effectively facilitating discussion amongst students. In a research brief, Cirillo (2013) suggested that revoicing must be purposefully used with other talk moves to support "productive and powerful" discussion (p. 4). Additional researchers suggested that teachers revoice ideas to make their ideas public or mark the ideas for attention by other students (Ellis et al., 2019). For the purpose of this capstone, I refer to both marking and revoicing as *repeating student ideas* because both talk moves include the teacher repeating students' thinking and ideas.

In a study of teacher talk moves, Ellis and colleagues (2019) classified various talk moves by identifying the potential of the talk move for supporting student thinking. Low-potential talk moves led the teacher to have a more prominent role in discussion. High-potential talk moves, on the other hand, positioned students as the prominent role in discussion as they shared their thinking and were doers of mathematics. Ellis and colleagues (2019) found that revoicing has the potential to be both a low-potential and high-potential move depending on how teachers use revoicing. As a low-potential move, teachers repeated what the student said without asking for clarity. Chapin et al. (2003) found revoicing to be impactful because it provided an opportunity to ensure that the teacher clearly understood what the student was saying. Ellis et al. (2019), however, did not observe revoicing being utilized in this way. As a high-potential move, Ellis et al. observed teachers repeating student thinking in a way that sought to "organize, re-frame, or formalize the student's statement or work" (p. 119). This form of revoicing aligns more closely with Michaels et al. (2016)'s idea of marking. Both the low- and high-potential moves, however, involved the teacher repeating what the student shared for the class to hear. Ellis et al. (2019) argued that repeating the student's answer does not necessarily lead to a demonstration of student thinking. Ellis et al. (2019) drew these conclusions from a limited sample of middle school math teachers (n=4). Although this small sample is drawn from a study with older grade levels than those I studied at Barron, Ellis and colleagues' work informed my capstone by challenging the assumption that revoicing student work is always an effective talk move (Chapin et al., 2003; Herbel-Eisenmann et al., 2013; Michaels et al., 2016).

Questioning Student Thinking as a Talk Move

Teachers ask many questions daily and these questions have the potential to support or inhibit meaningful mathematics discussions (NCTM, 2014). Various researchers have developed frameworks for categorizing teacher questions, which can be used to understand the different types of questions that teachers use to facilitate classroom discussions (Boaler & Brodie, 2004; Chapin & O'Connor, 2007; Franke et al., 2009; Ho, 2005; Shaughnessy et al., 2021). By classifying teacher questions, teachers and researchers can begin to think about how different questions are utilized differently throughout instruction. Questions are frequently categorized as open or closed questions, in which open questions have more potential to stimulate complex responses, while closed questions prompt shorter, simpler responses (Ho, 2005). Some researchers, however, criticize such a simplistic dichotomy of questions and argued that questions shouldn not be narrowly defined into two-categories (Ho, 2005).

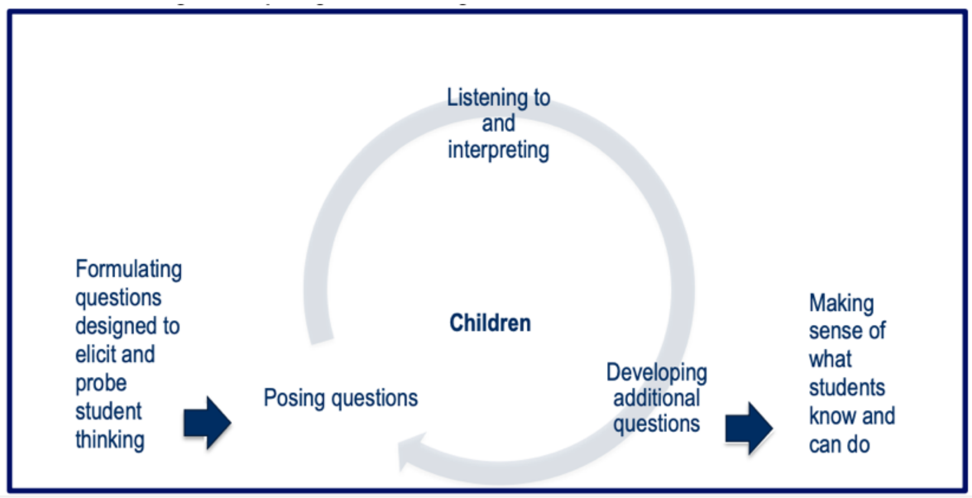
To extend on a dichotomous categorization of questions, Boaler and Brodie (2004) identified nine-types of questions through analyzing observation data from a longitudinal study of seven teachers and over 1,000 students. Boaler and Brodie (2004) developed a framework for categorizing questions that teachers asked during mathematics discussions. After several rounds of data analysis, themes emerged across the question types which led to the development of the nine types of questions. Boaler and Brodie do not suggest that one question type is more effective or significant than the others; rather, there is an appropriate time for all question types. These categories include questions that *gather information* (e.g., How many antelopes are there altogether?), *establish context* of a problem outside of mathematics (e.g., What is an antelope?), *insert terminology* (e.g., What do we call the answer in an addition problem?), *probe student thinking* (e.g., How did you solve this?), *generate discussion* (e.g., Who may think something differently?), *link and apply* (e.g. Where else might we use this?), *extend thinking* (e.g., Does this strategy work with all numbers?), and *orient students to the problem* (e.g., What is important information in the task?). Of these questioning types, only *probing student thinking* asks students to articulate, elaborate, and clarify their ideas and has the potential to elicit student thinking. The NCTM (2014) asserts that for students to engage in meaningful mathematics discussion, teachers must elicit and use evidence of student thinking. While Boaler and Brodie

(2014) provide one general categorization of eliciting student thinking, additional researchers have sought to further unpack how questions that elicit student thinking may vary.

Questions that elicit student thinking have the potential to be especially effective in facilitating meaningful mathematics discussion (Boaler & Brodie, 2004; Chapin & O'Connor, 2007; Ellis et al., 2018; Franke et al., 2009; Ghousseini, 2015; NCTM, 2014). Ghousseini (2015) asserted that when eliciting student thinking, teachers "press[ed] students for explanations and assist[ed] them in articulating their ideas", both of which were necessary components of meaningful discussion (p. 336). Eliciting student thinking allows teachers and other students to gain access to the thoughts, ideas, and methods of their peers about specific academic content (Chapin et al., 2003; Chapin & O'Connor, 2007; TeachingWorks, 2022). When students share their thinking during discussion, teachers and students are exposed to "novel points of view, new ideas, ways of thinking, or alternative conceptions" that contribute to students making connections, making sense of the mathematics, and engaging with the ideas of others (TeachingWorks, 2022, para. 1). Eliciting student thinking requires teachers to carefully listen to student responses so that they may ask additional questions or support students in asking additional questions so that students can develop understanding of the mathematical content (see Figure 2.1). While it is commonly understood that eliciting student thinking is essential for productive discussion, the exact ways in which teachers productively elicit student thinking is less certain (Ellis et al., 2018; Franke et al., 2009; Shaughnessy et al. 2021).

Figure 2.1

Visual Representation of Eliciting and Interpreting Student Thinking (TeachingWorks, 2022)



Similarly to TeachingWorks (2022), Ellis et al. (2019) described eliciting student thinking as just the first step in an "ongoing process of building on and supporting students' mathematical thinking" (p. 118). The subsequent steps of the cyclical process involve teachers responding to student thought specifically by asking for a clarification, prompting error correction by a student or the peer, revoicing student thoughts, drawing comparison to student work, or a variety of other teacher moves. Yet, even in the initial act of attempting to elicit student thinking, the information teachers elicit varies (Ellis et al., 2019). When teachers attempt to elicit students thinking and reasoning, they may in fact just elicit answers, procedures, or ask for clarification, which are low-potential moves because they only provide factual information related to the students' reasoning. Conversely, when teachers use high-potential moves to elicit student thinking, they introduce mathematical ideas, address conceptual connections, or offer alternate strategies. Ellis and colleagues do not claim that high-potential moves yield higher achievement or understanding; rather, they provide for a more student-oriented discussion, “enabling teachers to provide students with a space to engage meaningfully in the process of mathematical reasoning” (p. 127).

In my capstone study, I viewed questions that elicit student thinking through a framework developed by Franke et al. (2009) for analyzing follow-up questions. Franke et al. (2009) studied the ways in which three elementary school mathematics teachers elicited student thinking during classroom discussions. They found that teachers asked students to explain their thinking 98% of the time either during the initial question asking or as prompted after providing a response that did not include evidence of their thinking (Franke et al., 2009). During initial questions, teachers typically prompted students to explain student thinking by asking *why* and *how* questions, such as “How did you solve the problem?” or “What strategy did you use to solve and why?”. Asking for evidence of student thinking did not guarantee students explained their thinking. Follow-up questions, however, increased the likelihood that students responded to teachers with evidence of their thinking. To synthesize their findings, Franke et al. (2009) identified categories of follow-up questions that teachers can ask students after the initial question which may probe for student thinking and clarify student responses (see Table 2.1). These four types of questions all elicit student thinking and can be thought of as a follow-up to an initial question and student response.

Table 2.1

Follow up questions to elicit student thinking (Franke et al., 2009)

Follow-Up Question Type	*Description*	*Example*
Probing Sequence of Questions	Consist of a series of more than two related questions about something specific that a student said	Problem: Are 200 + 1 = 200 + 1 and 200 + 1 = 1 + 200 the same? *Student: It doesn't matter because it still has a partner.* *Teacher: Oh! What partner?* *Student: The numbers.* *Teacher: Could you explain what numbers you are talking about?*
General Question	Do not relate to anything specific the student said and often signal that a student should repeat their initial explanation	*Can you explain more?* *Can you say that one more time?*
Specific Question	Ask students to elaborate or share their thinking around a specific aspect of their initial explanations	Problem: 100 + ___ = 100 + 50 *Student: Well it has to be the same number.* *Teacher: What has to be the same number?*
Leading Questions in Response to Student Explanation	Guide students toward a particular answer, solution, or strategy	Problem: 100 = 50 + ___ *Student: I think it's 50 because 50 + 50 = 100.* *Teacher: If 50 + 50 = 100… 100 is the same as…* *Student: One hundred.*

While all four follow-up question types may lead to students providing additional evidence of their thinking, Franke and colleagues (2009) found that some follow-up questions were more effective at eliciting student thinking than others. Through analyzing classroom discussion, *probing sequences of questions* were most effective and led to a student elaborating on their explanation 100% of the time. The research team argued that asking a sequence of questions has benefits for all students because it enables the teacher to fully understand the student's thinking, provides opportunity for the student to clarify or solidify their own thinking, and allows other students to connect their own thinking to what was being said in class as the thinking is made more visible for all students. Lim et al. (2020) tracked patterns of discussion and teacher questioning by referring to a sequence of follow-up questions as I-R-q-R-q-R-q, extending upon a traditional IRE model of discussion. Lim and colleagues (2020) affirmed Franke and colleagues' (2009) findings by claiming that "frequent follow-up questioning, focused on understanding students' thinking, will support students' engagement in mathematics classroom discussions" (Lim et al., 2020, p. 393).

Other follow-up questions, however, were not found to be as effective as sequences of probing questions (Frank et al., 2009). Individual follow-up questions, whether specific or general, were not always sufficient at leading students to provide evidence of their thinking. As a result, Franke and colleagues (2009) found that individual questions left teachers making assumptions about student thinking, which placed teacher thinking at the center of the discussion rather than student thinking. Assumptions, not drawn directly from student thinking, may lead teachers to making instructional decisions that are not accurate inferences (Franke et al., 2009). Teachers also asked leading questions as a follow-up, where the teacher encouraged students to provide additional detail, but the teacher was primarily responsible for the explanation. Also referred to as funneling, (Wood, 1998; see also Herbel-Eisenman & Breyfogle, 2005), leading questions confine student thinking by suggesting that students respond with a predetermined solution or strategy. Franke et al. (2009)'s framework for labeling follow-up questions provided a way to categorize questions during meaningful mathematics discussions. My capstone at Barron Academy sought to understand how teachers supported or inhibited student engagement through various talk moves, such as follow-up questions that elicit, or hinder, student thinking.

Encouraging Peer Engagement

A third category of talk moves includes questions and prompts that encourage students to engage with the ideas of one another during whole group mathematics discussions. Peer engagement is crucial to ensuring high-level math-talk communities (Hufferd-Ackles et al., 2004) and meaningful mathematical discussion (NCTM, 2014). Some researchers concluded that the level of student engagement with their peers' ideas is positively related to student achievement (Ing et al., 2015; Webb et al., 2014, 2021). Three specific talk moves that teachers may use to encourage and invite students to engage with the ideas of their peers during discussion are asking students to repeat peers' ideas, connect their ideas to those of their peers, and question their peers (Chapin et al., 2003; Franke et al., 2015; Ing et al., 2015; Michaels et al., 2016; Webb et al., 2014).

Students Repeating their Peers' Ideas. Rather than the teacher revoicing student ideas, the teacher may prompt students to repeat what their peers may have shared with the class (Chapin et al., 2003; Michaels et al., 2016). Chapin and colleagues (2003) suggested that "asking one student to repeat or rephrase what another student has said" has several potential benefits (p. 13). Similar to the teacher marking and repeating ideas, asking a student to repeat the ideas of their peers allows the entire class another opportunity to listen and process the initial contribution, which may support students in following the discussion. However, by asking students to repeat the idea, the teacher decreases the amount of teacher talk and increases the amount of student talk, providing another opportunity for participation and student voice. In the Accountable Talk framework, Michaels et al. (2016) named this move "keeping everyone together" because it is an opportunity for the teacher to assess if students understand the contributions of other peers (p. 23). Both Chapin et al. (2003) and Michaels et al. (2016) asserted that when students repeat the ideas of their peers, it is important to confirm with the initial student that their ideas were repeated appropriately.

Asking students to repeat the ideas of their peers has the potential to be an effective tool for engaging students with one another but is not always effective (Franke et al., 2015; Webb et al., 2014). In a mixed-methods study, Webb et al. (2014) collected data from six third- and fourth-grade classrooms to identify a relationship between participation, engagement with peers' ideas, and achievement. The researchers observed teachers prompting students to repeat the ideas of their peers as one potential strategy for supporting student-to-student engagement. Through coding teacher prompts and student explanations, they classified student responses as high-, medium-, or low-levels of engagement (see Table 2.2). Webb et al. (2014) found that teachers prompting students to repeat what other students share is a medium-level practice because students may not provide a contribution related to their own thinking. Additional follow-up may be required to support the students in providing more details about the student's work, thus increasing their level of engagement. Low- and medium-level practices may still provide some opportunity for student engagement, which have the potential to be productive for supporting students in developing their mathematical understanding

Table 2.2

Levels of Engagement (Franke et al., 2015; Webb et al., 2014)

Level of Engagement	Description	Examples
Low	References a peer's idea or strategy in a general way	• Saying "I agree" or "I disagree" without evidence • Saying or gesturing to a strategy that resembles the speakers • "I did __'s way… I drew equal groups…"
Medium	References specific details of a peer's idea or strategy	• Repeating the ideas of another student • "Marcus said that he counted up…"
High	References the details of a peer's idea or strategy while making a new contribution.	• Adds additional details about a student's strategy • Suggests a correction to an incorrect strategy • Suggesting an alternative approach and explicitly referencing how the new approach is similar or different

In an additional study by the same research team, Franke et al. (2015) further investigated the moves teachers employ when engaging students with each other's mathematical ideas by specifically recognizing how teacher follow-up can support students' varied engagement. Franke et al. (2015) observed practices in 12 classrooms at one public elementary school that had been previously identified as having significant opportunities to engage students with one another's work. Unlike Webb et al. (2014), Franke et al. (2015) observed classrooms across all grades pre-K through sixth grade. Franke et al. (2014)'s findings confirmed Webb et al. (2014)'s findings that student engagement may vary dependent upon follow-up questions. Franke et al. (2015) found that teachers prompting students to explain someone else's solution was the third most frequently used talk move and students responded with high-levels of engagement 33% of the time, medium-levels of engagement 54% of the time, and no engagement 13% of the time

(Franke et al., 2015). Students responded by saying that they did not know or understand what the student said (no-level), repeating the details of what the student said (medium-level), or adding additional details that were not included in the initial explanation (high-level). Franke et al. (2015) suggested that teachers should follow-up with probing or scaffolding to increase student engagement after the initial move. A teacher may probe by questioning students further about their response. For example, after repeating a procedure of what a student a did, the teacher may probe by asking why the procedure works or further explain the representations in a model. Alternatively, a teacher may scaffold the student's understanding by providing additional context, information, or clarification for the student. For example, if a student is unable to repeat the explanation of a student, the teacher may suggest looking to a visual representation of the strategy to help in repeating the explanation. Franke et al. (2015)'s findings suggested that a single talk-move, or invitation to engage with peers' work, cannot necessarily be classified as high-level or low-level. Rather, how teachers continue to follow-up with students through scaffolding or additional questioning may support students in higher levels of engagement (Franke et al., 2015), which may better support them in constructing understanding (Webb et al., 2014).

Connecting and Critiquing Ideas. In addition to prompting students to repeat other students' ideas, teachers can also encourage peer engagement by prompting students to connect their ideas to the ideas of the peers. When students engage in discussions sharing their ideas, students are positioned to compare their work with one another to make mathematical connections (Michaels et al., 2016). As advised by the NCTM (2014), "effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematical concepts and procedures" (p. 10). Smith and Stein (2018) believe that helping students to "draw connections between their solutions and other students' solutions as well as to key mathematical ideas" is one of the essential practices required of teachers to orchestrate productive discussions (p. 14). They recommend that discussions should position students to build off the ideas of one another, rather than explain strategies in isolation through show-and-tell (Ball, 2001; Pirie & Schwarzenberger, 1988; Smith & Stein, 2018; Stein et al., 2008). According to Smith and Stein (2018), connecting student solutions requires questions that move beyond clarifying and probing what students did and focus on the mathematical ideas behind the student's thinking. They advised that teachers should prompt students to connect mathematical ideas by asking students to agree, disagree, and share alternate ideas or strategies to solving mathematical solutions (Smith & Stein, 2018). These prompts may support students in making connections to their work, the work of their peers, and the related mathematical ideas.

Prompting students to compare mathematical strategies and solutions has also been found to have the potential to support students in developing their understanding of mathematics (Richland et al., 2017). In a review of the literature, Richland and colleagues (2017) concluded that comparing student work often lies at the heart of rich mathematical discussions. They claimed that in order to optimize the learning outcomes drawn from comparisons, teachers should use visual representations and provide explicit cues prompting students to compare their work (Richland et al., 2017). Different discussion talk moveframeworks may name these cues or prompts differently (Chapin et al., 2003.; Herbel-Eisenmann et al., 2013; Michaels et al., 2016), however, all serve to prompt students to make connections between various mathematical ideas. Examples of teacher talk moves focused on supporting students in connecting their ideas are:

- Do you agree or disagree? (Chapin et al., 2003)

- _____, your strategy was not the same as this one. What did you do differently? (Herbel-Eisenmann et al., 2013)
- What do you notice is missing from ____'s explanation that is in ____'s? (Michaels et al., 2016)
- Who would like to add onto what ____ said? (Smith et al., 2020).
- What about _____'s strategy is similar/different to _____'s strategy?

Research suggests that teachers must ask students to explain why they agree or disagree with the work of their peers to support productive discussion (Franke et al., 2015; Webb et al., 2014; White, 2003). White (2003) described the practices of two elementary mathematics teachers who had participated in Project IMPACT, a teacher enhancement program that supported teachers in adopting reform mathematics practices. White (2003) found that both teachers frequently encouraged student-to-student interactions by asking students to agree or disagree with various student solutions. One teacher claimed that she wanted students to "assume the role of the mathematical judge and jury" (p. 42). White found that students felt free to agree or disagree with their peers and not blindly accept the solutions of another. These teachers often facilitated discussions until students came to a common agreement from building mathematical understanding together. When observing discussion at Barron Academy, I considered how teachers positioned students as "the mathematical judge and jury" (White, 2003, p. 42)

White (2003) also observed teachers frequently asking students to give a symbol or gesture if they agreed or disagreed, without asking students to explain why. Webb et al. (2014) and Franke et al. (2015) found that pressing students to explain why they agree or disagree with another student may generate more meaningful engagement than a symbol without an explanation. Franke et al. (2015) observed teachers prompting students to connect their ideas to the ideas of others more frequently than any other talk movesupporting student engagement. However, their analysis shows that this move solicits no-engagement 46% of the time, low-engagement 32%, medium-engagement 15%, and high-engagement only 7% of the time (Franke et al. 2015). Although various talk moveframeworks (Chapin et al., 2003; Herbel-Eisenman et al., 2013; Michaels et al., 2016) suggest examples of how to invite students to make comparisons, Webb et al. (2014) and Franke et al. (2015) suggest asking a series of follow-up questions to ensure that students attend to specific details in the ideas of their peers. They suggest that it is important to look beyond the initial talk moveof comparing work and look more closely at how teachers prompt students to elaborate on the ideas of their peers as they make comparisons.

In an observation of a fourth-grade mathematics classroom, Langer-Osuna and Avalos (2015) found that prompting students to agree or disagree with each other may lead to more argumentative interactions that may not benefit learners. The classroom observed was a racially and linguistically diverse classroom led by a teacher, who participated in a two-year long research project that trained teachers on best practices in mathematics (NCTM, 2014). Specifically, the teacher had worked to develop meaningful mathematics discussions through using various talk moves and sentence stems such as “I agree with ____ because…” and “I disagree because….” (Langer-Osuna & Avalos, 2015. p. 1316). Langer-Osuna and Avalos found that students in this classroom appropriately employed sentence stems but their utilization was problematic. Students often viewed disagreement as a personal attack and worked to defend their logic, rather than listen to various views. Langer-Osuna and Avalos found that for student argumentation to be productive, students must be willing to revise and revisit their original ideas when presented with more compelling, reasonable ideas. Additionally, the researchers observed students interrupting one another, talking over one another, and uttering insults, such as "you're weird" (p. 1320). Students felt personally attacked when they might have been wrong. Despite the teacher's reinforced use of talk moves and sentence stems (Michaels et al., 2016), some students "drew from traditional notions of school mathematics" focusing on evaluating right and wrong, rather than engaging in productive argumentation (Langer-Osuna & Avalos, 2015, p. 1). Classroom conversation was divisive, and student's felt the need to defend their thinking more than make sense of mathematics. Although just one example, Langer-Osuna and Avalos (2015) demonstrated how teachers may fail to appropriately utilize talk moves, such as connecting student work, to support students in developing shared understanding. Even when utilizing literature supported talk moves, teachers may still struggle to appropriately engage students in discussion.

Peer Questioning. A third strategy for encouraging peer engagement is to prompt students to ask questions directly to their peers. In meaningful mathematics discourse (NCTM, 2014), students are responsible for questioning their peers so that they may better understand their ideas and strategies (Hufferd-Ackles et al., 2004, 2014). Webb et al. (2017) analyzed teachers practices to further identify how teachers can "foster productive collaboration" and discussion in mathematics (p. 179). To support students in engaging with the ideas of their peers, teachers guided students to ask questions of their peers. Rather than teachers elicit an explanation, teachers might engage an additional student by suggesting students ask the questions (*"Jamia, are you sure about Elijah's work? Could you ask him to explain that part of his model?"*). By providing question suggestions or question stems, teachers support students in formulating these questions and engaging with their peers through questioning. While Webb et al. (2019) identified this as a practice that can support engagement, their previous work (Ing et al., 2015; Franke et al., 2015; Webb et al., 2014) did not identify how this specific move may support high-level engagement nor do they relate it to student achievement. Nevertheless, Webb et al. (2019) identified promoting peer-questioning as a practice that supports student-engagement with the ideas of one another, which they claim impacts student achievement in mathematics (Ing et al., 2015).

Research suggests that students, especially elementary students, need explicit teaching to develop their questioning skills (Di Teodoro et al., 2011; Hunter, 2008). Hunter (2008) analyzed the ways in which four elementary mathematics teachers in New Zealand implemented scaffolding to support students in developing questions. Teachers implemented Exploratory Talk[4] (Mercer, 2000 as cited in Hunter, 2008) to promote student inquiry as they co-constructed mathematical reasoning (Hunter, 2008). Over time, Hunter (2008) observed that student questioning became more complex and advanced. Teachers initially began to prompt students to ask questions that might help them to understand their peers such as "What did you do with ____" or "Can you show us what you did with this?" (p. 204). Students used this guidance to develop questions such as "Where did you get the ten from?" (p. 204). Eventually, questions became more complex and students began to challenge the ideas of their peers through questions such as "Would that strategy work with other numbers?" (p. 207). The use of scaffolding supported all teachers in decreased teacher questioning and increased student questioning over time (Hunter, 2008). Teachers explicitly taught student-questioning and modeled this behavior for students to support them in developing the skills necessary to engage with their peers' ideas through questioning (Hunter, 2008). As students began to ask more complex questions, Hunter

[4] Exploratory Talk refers to all students sharing all relevant information pertaining to mathematics, respecting the opinions of others, challenging peers, finding alternative solutions and solution methods, and the group seeking to identify shared understanding and agreements (Mercer & Sams, 2006).

(2008) also observed their explanations and justifications becoming more thorough, which may suggest that students engaged in more sense-making. Two limitations of this study include a small sample size and implementation of a specific program. However, the findings still highlight the ways teachers can support students in peer-questioning that may be relevant to the context of the present study at Barron Academy.

In an action-research project, Di Teodoro and colleagues (2011) also found that with explicit teaching and scaffolds, students can develop and improve their peer-questioning skills. Four second- and third-grade teachers in Ontario explicitly reviewed questions with students through sorting activities where students sorted questions what they thought were surface or deeper level questions. Deeper questions, such as "Why did you do it in that way?" and "Did you draw pictures first and how did they help you?", provided examples for students on the questions they were encouraged to ask of their peers during collaborative discussions (Di Teodoro et al., 2011). Through a discussion of the sorting activity, classrooms developed criteria and descriptions of deeper questions in student-friendly language that was displayed as an anchor chart for students to reference during class. Di Teodoro and colleagues found that between all four classrooms, students increased the number of questions asked (n=97 prior to the lesson; n=163 after the lesson). Questions were also analyzed and coded as either "surface" or "deeper"; originally, only 25% of questions were definitively deeper questions as opposed to 69% of questions after the lesson. The teacher-researchers removed any of the questions that could not be clearly identified and agreed upon from their count, improving the reliability of their findings through intercoder reliability (O'Connor & Joffe, 2020).

Chapter Summary

Although there is not enough evidence to say that engaging in mathematical discussion directly causes increased understanding, significant literature identifies a correlation between student participation in discussion and increased student achievement (Hiebert & Wearne, 1993; Ing et al., 2014; Franke et al., 2015; Kosko, 2012; Webb et al., 2014, 2021). However, the various ways in which teachers facilitate discussion through use of talk moves and follow-up questions varies. Additionally, teachers' perception of their role in discussion and their beliefs about learning mathematics, in addition to other external factors, may influence the ways in which they orchestrate mathematics discussion.

Various frameworks developed by researchers (Chapin et al., 2003; Michaels et al., 2016; Smith & Stein, 2018) suggest that teachers employ specific talk moves that can effectively help teachers facilitate a student-centered discussion. Teachers may revoice student ideas, question students to elicit thinking, or encourage students to engage with the ideas of their peers. In each situation, however, follow-up questioning may be necessary to elicit further elaborations that have the potential to move the discussion forward (Franke et al., 2009; Lim et al., 2020). Within the literature, specific limitations persist including the inability to isolate specific talk moves from other variables that may promote more meaningful discussion. Nevertheless, these talk moves are described as supporting students in participating in discussion and engaging with the ideas of others.

In Chapter 3, I describe the methods that I used to investigate the current landscape of discussion at Barron Academy. Literature informed how I described the practices that teachers employed that promoted and inhibited discussion, as well as provided a basis for how I analyzed and coded specific talk moves (see Appendix A). Additionally, it provided a foundation for how the NCTM (2014) defined meaningful mathematics discussions and the instructional practices that contribute to it.

Chapter 3: Methods

Given that meaningful mathematics discussion is instrumental to quality teaching and learning for all students (NCTM, 2014), my capstone project investigated how teachers supported or limited these types of discussion at Barron Academy. The current curriculum, *Math in Focus (*Marshall Cavendish Education, 2020b), highlighted the importance of student discussion in solidifying understanding of mathematics concepts as students are provided with opportunities to communicate their own reasoning behind mathematics ideas and solutions and critique the reasoning of others. Yet administrators and teachers at Barron Academy noticed that teachers do not always facilitate discussion in a way that supports students in engaging with the ideas of their peers. This capstone study provides a better understanding of the current landscape of mathematical discussions at Barron Academy, so that the school can improve how they support students in learning mathematics, communicating reasoning, and engaging with the ideas of their peers.

I investigated this problem of practice through a descriptive case study addressing the following questions:

1. How do elementary teachers describe the attributes of meaningful mathematics discussions? To what degree are teachers' descriptions aligned to the NCTM's mathematics teaching principles?
2. To what degree do teachers orchestrate meaningful mathematics discussions?
    a. In what ways, if any, do teachers use talk moves to promote and support student-to-student engagement during mathematics discussions?
    b. In what ways, if any, do teachers inhibit student-to-student engagement during mathematics discussions?

c. In what ways, if any, does the curriculum support opportunities for meaningful mathematics discussion?

In Chapter 1, I provided a description of the current problem of practice within the local context of Barron Academy. Additionally, I provided a detailed conceptual framework which informed how I collect and analyze data as part of my study. In Chapter 2, I critically analyzed and reviewed the relevant literature detailing the complex nature of facilitating mathematical discussions through the potential relationships between discussion and achievement, how the literature suggests beliefs may or may not be reflected in practice, and various talk moves teachers utilize in mathematics discussion. In this chapter I provide a detailed description of the research design for this study which includes the setting and context, participants, and data sources collected. Next, I detail procedures I propose to use to collect and analyze data. Then, I describe the ethical considerations taken to preserve trustworthiness as well as the assumptions, limitations, and delimitations of the study.

Study Design

I conducted a qualitative, descriptive case study to investigate the ways in which teachers do or do not facilitate classroom discussion so that students meaningfully engage with the ideas of their peers. This case study was comprised of three elementary mathematics teachers at Barron Academy, the case. Given the context of this capstone study, a case study was appropriate to study the practices pertaining to discussion in their natural context at Barron Academy during mathematics class (Hancock & Algozzine, 2016). A case study approach enabled an in-depth understanding of the specific practices that teachers utilized at Barron Academy when leading and facilitating mathematics discussions. Additionally, a case study acknowledges the contextualized nature of schools by recognizing that "the boundaries between phenomenon and context may not be clearly evident" (Yin, 2016, p. 16). Thus, a study of discussions during mathematics instruction was necessary to appropriately understand the current nature of mathematical discussions. Through my capstone study, I provided "an in-depth description and analysis of a bounded system", which is the mathematics discussion in three K-5 classrooms at Barron Academy (Merriam & Tisdell, 2016, p. 39). In the following sections, I describe the local setting and context of the capstone study as well as the sampling methods and description of the data tools.

Setting and Local Context

Barron Academy is a prekindergarten through twelfth grade independent school in a metropolitan area in the mid-Atlantic region of the United States with 653 total students enrolled. The elementary school at Barron Academy contained 11 kindergarten through fifth grade classrooms, ten of which inclued mathematics. Each of these ten mathematics teachers had at least one year and up to thirty-five years of experience teaching elementary school mathematics. Throughout this project, I consistently communicated with two key stakeholders, Mr. Samuel Curtis[5] and Dr. Tara Klingham[6], from Barron Academy's elementary school. Both stakeholders were administrators at the elementary school and have worked at Barron Academy for more than a decade.

Barron Academy utilized *Math in Focus*, a Singapore Math curriculum, as the basis for mathematics instruction (Marshall Cavendish Education, 2021b). The school first adopted Singapore Math curriculum in 2015, and a new edition of the math curriculum was introduced during the 2021-2022 school year. The *Math in Focus* curriculum referenced the SMP throughout its lessons as mathematical habits, which were designed with CCCSI (NGA Center,

[5] pseudonym
[6] pseudonym

2010) and NCTM (2024) practices in mind (Marshall Cavendish Education, 2020c). Since the curriculum served as both the foundation and guide for Barron's Academy mathematics instruction, it was important to consider how the *Math in Focus* curriculum contributed to mathematics instruction and mathematics discussion.

Sampling

Multiple tiers of purposeful sampling were used to select three teachers from the ten mathematics teachers at Barron Academy (Patton, 2014). I spoke with school administrators to identify the sampling procedures and criteria for the capstone study. Administrators preferred a sample of teachers in various grade levels so to provide for a broader view of teachers' discussion practices across the elementary grades. Maximum variation sampling allowed me to collect data and present findings that represented a range of classrooms from kindergarten through fifth grade at Barron Academy. Administrators then decided that it would be more appropriate to exclude fifth grade from the sampling population because fifth grade teachers were departmentalized by subject. The one fifth grade math teacher did not represent the typical teacher at the school because he was self-contained compared to the other teachers that taught all subjects. As a result, the sampling population was narrowed to nine teachers from across four grade levels. The administration also wished to delimit the sample by excluding teachers with less than three years of experience to eliminate novice teachers. Eliminating novice teachers helped to identify a sample that was more typical since a majority of the teachers at Barron Academy have more than three years of experience.

Given the preferred criteria for participants, I emailed three eligible teachers to invite them to participate in the capstone study (Appendix B). The invitation included a description of the study, an explanation of sampling, and the consent form (Appendix C). I explained that they were nominated by their administrators to participate in the project as they were experienced teachers who were in a position to provide a helpful window into elementary teachers who teach math along with other subjects at Barron. I mentioned that teachers have a choice to participate and that they were not required to participate just because they met the criteria. Two of the original three teachers agreed to consent in the study. One of the previously invited teachers resigned after the eligible teachers were identified. As a result, I invited another teacher to participate in the study. The newly identified teacher taught in the same grade level as the previous teacher but had not previously been contacted to ensure maximum variation in grade levels. The third teacher agreed to participate in the study. I purposefully identified teachers to ensure maximum variation but the sample was also convenient.

Participants

This capstone study included three participants, who were all elementary mathematics teachers at Barron Academy in the 2021-2022 school year. These participants were Mrs. Anna Marzano[7] in kindergarten, Mrs. Nancy Staples[8] in third grade, and Mrs. Wendy Grimes[9] in fourth grade. All three participating teachers identified themselves as White and female. Mrs. Marzano began her teaching career at Barron Academy and was in her 15th year of teaching during this capstone study. Her highest level of education was a Master's in Early Childhood Education. Mrs. Staples was a third-grade teacher during the 2021-2022 school year and had taught for 25 years. She was in her 14th year of teaching at Barron Academy. She received a Bachelor of Arts in History and took additional coursework to meet the requirements of education licensure. Both Anna and Nancy have taught at Barron Academy since it first adopted *Math in Focus*. The third

[7] pseudonym
[8] pseudonym
[9] pseudonym

participants, Mrs. Wendy Grimes, was a fourth-grade teacher and taught for seven years. She was in her fourth year of teaching at Barron Academy. Her highest level of education was a Bachelor of Arts in Elementary Education and Teaching. All three participating teachers have experience teaching multiple grades of elementary school mathematics and multiple years of experience teaching their current level with the *Math in Focus* curriculum.

Data Sources

To answer the research questions, I conducted classroom observations, interviewed teachers, and reviewed documents from the school curriculum. Each of these data sources contributed to a more robust understanding of the current landscape of classroom discussion in relation to opportunities for meaningful mathematics discussion. Table 3.1 illustrates how each data source contributed to answering a research question

Table 3.1

Research Questions and Methods Chart

Research Questions	Classroom Observations	Teacher Interviews	Curriculum Review
How do elementary teachers describe the attributes of meaningful mathematics discussions? To what degree are teachers' descriptions aligned to the NCTM's mathematics teaching principles?	X	X	
To what degree do teachers orchestrate academically productive mathematics discussions?	X	X	X

Observation Protocol

In order to describe meaningful mathematics discussion at Barron Academy, I observed live mathematics instruction in the classroom. Observation was an appropriate source for data collection because I was able to observe the phenomenon, mathematics discussion, in its natural context within the classroom (Merriam & Tisdell, 2016). Additionally, direct observation of the mathematics lessons offered a "better understanding of the contexts" in which the phenomenon occurred (Hatch, 2002, p. 72). Each observation occurred within the given mathematics block, which lasted between 30 to 60 minutes depending upon the day and grade-level. Kindergarten math blocks were typically between 30-45 minutes while third and fourth grade blocks were about one hour.

The observation was guided by a protocol, which provided a framework for what and where I focused my attention to record field notes (Appendix D). The observation protocol included organizational information to identify the specific observation including the name of the teacher, grade level, date of observation, time of observation, number of students present, and corresponding curriculum lesson. The observation template also included a setting diagram to record how the students were seated during mathematics instruction and if they changed locations throughout the lesson. Due to COVID-19 precautions, however, students did not frequently move throughout the room. The remainder of the protocol designated space to record field notes of the mathematics discussion. My running field notes captured direct quotes from teachers and students as they discussed mathematics. Additionally, my protocol included space to record the instructional tools utilized (PowerPoint, textbook, etc.), mathematics activities, and prompts used to guide instruction.

Curriculum Review Protocol

A systematic curriculum review contributed to a better understanding of how teachers used the curriculum to lead or facilitate mathematics discussion. Each *Math in Focus* lesson directed teachers to ask specific questions to "facilitate mathematics discussion" or "guide student thinking" and included various instructional activities that intended to support discussion (Marshall Cavendish, 2020b). Since *Math in Focus* was the designated curriculum at Barron Academy, I conducted a curriculum review for each observed lesson using a designated protocol, which was a simple two-column chart (Appendix E). In the first column, I recorded all the recommendations for discussion or math-talk found in the TE for the given lesson. Recommendations included *Math Talk* prompts, *Math Sharing* activities, suggested question prompts and sequences, and directions related to discussion. The second column provided space to compare the teachers' edition recommendations to the observed lesson during analysis and space to write reflective memos. This protocol helped to develop a better understanding of how the *Math in Focus* curriculum cultivated, if at all, lessons with meaningful mathematics discussions. Additionally, it helped me to understand how the instructional materials related to teachers' instructional choices during discussion.

Interview Protocol

Through interviewing teachers, I gained understanding of how teachers perceived and described meaningful mathematics discussion and why teachers made specific instructional decisions during discussion. Interviews provide unique data relating to the perceptions, beliefs, and experiences of the teachers, which cannot be observed in classroom observations (Hatch, 2002). I used a semi-structured interview protocol as foundation for each interview that I adapted to further question teachers based upon their responses (Appendix F). A semi-structured interview was appropriate so that the interviews provided in-depth insight into teachers' viewpoints and decisions, while also being flexible to alter questions as necessary (Hatch, 2002). Interview questions prompted teachers to describe examples of productive mathematics discussions and unproductive mathematics discussions as well their role in mathematics discussions. Additional questions considered specific choices teachers made during the observed discussion and how teachers planned for discussion using the curriculum, such as how they determined which questions to ask, modify, or omit. Some questions were adapted from a survey tool, developed by Casa et al. (2007), which measured preservice teachers' attitudes about mathematics discussion.

Data Collection

The previously described observation, curriculum review, and interview protocols provided the data necessary to answer the designated research questions. After receiving IRB approval and consent from participating teachers, I met with teachers to schedule observations and identify the corresponding curricular lessons. During this initial meeting, I also introduced myself to students and provided teachers with an informational letter to send to families about my project (Appendix H). Then, I used the following procedures to collect data through curriculum review, observations, and interviews.

Curriculum Review Procedures

Prior to observing the mathematics lesson, I used the *Math in Focus* TE (2021) to conduct a curriculum review of the lessons. Administrators at Barron Academy granted me access to a digital copy of the online curriculum, including TE resources, textbooks, lesson planning documents, and professional learning resources. I first reviewed the sections in the TE textbook associated with the specific observation dates. I referred to the TE and recorded all directions and questioning prompts related to any discussion, including both small-group and whole group. Additionally, I recorded any recommendations for best practice associated with discussion for the observed lessons. These recommendations included notes to teachers about supporting students, specific instructional activities, such as *Math Talk* and *Math Sharing*, as well as any instructions for implementation. Through conducting the curriculum review, I gained an understanding of the lesson objectives and the curriculum's suggested questions for the lessons that I later observed. I reviewed all lessons for the grade prior to conducting the first observation in the grade.

Observation Procedures

For each observation, I recorded raw field notes on my computer using the observation protocol. I recorded, to the best of my ability, all student and teacher interactions that were relevant to mathematics topics during whole group discussions including teacher and student questions, responses, comments, answers, and directions for instructional activities during whole group discussion. I recorded oral talk verbatim as opposed to summarizing or paraphrasing what students and teachers said so that my protocols remained descriptive and not interpretive (Hatch, 2002). I also included any specific instructional strategies and curricular materials that teachers utilized during whole group discussion. When students worked with partners or in small-group discussion, I circulated between groups and tried to follow the teacher so that I could see how the teacher interacted with small groups during discussion. I recognized that it is impossible to record every utterance and every relevant moment of talk within a classroom without an audio recording. When I was unable to record verbatim talk, due to the speed or audibility, I included summaries or paraphrased notes in brackets.

During observations, I took on the role of a "participant as observer" (Cresswell & Báez, 2021, p. 125). As an outsider sitting in an elementary mathematics classroom, it was impossible to remain a complete observer. Both teachers and students engaged with me as a participant in the classroom by talking to me during mathematics lessons. Students addressed me by greeting me, asking questions, or showing me their work. Therefore, positioning myself as a complete observer was impossible as both students and teachers engaged with me during lessons. Since I observed both whole group and small-group mathematics discussions, I moved around the room and interacted with students as necessary (e.g., asking students to repeat what they said in small-group). During whole group discussions, I sat in a corner of the room in an effort to refrain from interfering with instruction as an observer. When students participated in small-group or partner discussions, I walked around the room to capture group talk. I chose to move around the room and observe different groups, as opposed to following a single group or single study so that I could gain an understanding of the entire class and recognize patterns across various small-group discussions. My position as a participant as observer helped me to "gain insider views" as a temporary participant and member of the classroom (Cresswell & Báez, 2021, p. 125).

After completing each observation, I reviewed my field notes and added additional information to provide a more complete description as necessary (Hatch, 2002). When adding information, I utilized bracketing to account for limiting my bias and interpretations. Additionally, I added reflexive notes about what I observed that recorded thoughts, questions, and ideas that came about during the observation or while reviewing the field notes. Questions and ideas that I developed while writing reflexive notes also informed interview questions. Both bracketing and reflexive memos contributed to later analysis because they served as opportunities to record any bias, impressions, interpretations, or questions that came to mind (Hatch, 2002).

Interview Procedures

Following the conclusion of the mathematics instructional block, I interviewed teachers about the observed lessons using the semi-structured protocol. Each interview was approximately 15-20 minutes in a private setting. In kindergarten, interviews began 30 minutes after the math block during Anna's planning. In third grade, interviews began immediately following math during snack time. In fourth grade, interviews started approximately five minutes after the conclusion of math block during recess.

I began the interview by asking for consent to audio record the interview on my phone. Next, I reminded teachers of their ability to refrain from answering any question and stop the interview at any point. During each interview, I took handwritten notes to capture major ideas or phrases that the teacher used in case the recorded file was corrupted. Prior to leaving the campus after the interviews, I wrote a reflexive memo to capture a summary of the interview and my interpretation of the responses. Within 24 hours of the interview, I transcribed the audio-recorded interview and deleted the file from my phone.

After having collected all data and starting analysis, I interviewed teachers one additional time via Zoom. The interview protocol consisted of questions that came about during data analysis (Appendix G). A final interview helped to fill-in gaps that surfaced in the data and confirm that I had understood teachers' views surrounding mathematics discussion. As a result, I had a total of five interviews (n=5) with each of the three teachers (n=3) (for a total of 15 interviews).

Data Analysis

I analyzed data using a systematic process of organizing, coding, and interpreting data from all three data sources. I employed a framework developed by Bazeley (2021) for qualitative data analysis by engaging in a cycle of reading, reflecting, coding, connecting, refining, and reviewing the data to identify relationships and themes among the data. This cycle, however, was not linear and often required "moving back to go forward" (Bazeley, 2021, p. 16). I coded data as I collected it and engaged in additional rounds of coding with a "different set of eyes" to review, describe, compare, and relate the emergent themes in the data (p. 17). While coding data, I referred to previous observations, interviews, curriculum lessons, and the reflexive memos written throughout the data collection process. Eventually, themes and patterns emerged in the data that led me to extracting evidence that led to the development of findings.

Qualitative Coding

Coding is not only a system of naming or describing a unit of data within larger data but also a system for building ideas and questioning the data (Bazeley, 2021). To assist with building these ideas and questioning the data, I used Dedoose, a commercially available computer assisted qualitative data analysis software (CAQDAS). CAQDAS helps researchers to code, organize, and compare data systematically, while also serving as a password protected cloud-based storage system. I chose Dedoose as a CAQDAS for my capstone study because it is both affordable and user-friendly. I easily created codebooks and engaged in coding both deductively with *a priori* codes and inductively by revising the codebook to add emergent codes.

Qualitative coding guided the initial part of my analytic process. I used deductive coding to build upon current ideas previously established through a review of the literature and my own experiences through *a priori* codes. Specifically, I have drawn from the codes of various researchers that have sought to describe and classify various talk moves and questions utilized in discussion (Boaler & Brodie, 2004; Chapin et al., 2003; Ellis et al., 2019; Franke et al., 2015). I created codebooks, using *a priori* codes from the literature, for each of the data sources (Appendices I-K). Although some codes existed across multiple codebooks, I chose to use make codebooks unique to specific data sources so that I would be able to analyze each data source individually and compare across sources on Dedoose. A critical peer, who was familiar with my research and a peer in my doctoral program, reviewed the *a priori* codes.

After multiple rounds of coding the curriculum lessons, observations, and interviews using *a priori* codes, I made multiple revisions to the original codebooks to account for emerging ideas. Through inductively evaluating the data, I concluded the additional codes were necessary to appropriately categorize, describe, and label various segments of data for analysis. As a result, I added emergent codes. Additionally, I eliminated codes that were either not observed at all or did not contribute to analysis. For example, I eliminated the code *repeating* from the curriculum review because it did not occur in any of the lessons reviewed. Additionally, I revised the code *information seeking questions* as a parent-code including the emergent codes *noticing*, *orienting, and focusing*, *answer*, *procedure*, and *vocabulary*. I found it necessary to add emergent codes that further categorized the types of information seeking questions the curriculum suggested and teachers asked so that I could further analyze how various question types supported or inhibited mathematics discussion. In this section, I describe the codebooks and process of coding each data source.

Curriculum Review Codes

Coding the curriculum lessons was a two-pronged process for using the codebook developed for curriculum review (Appendix I). In the first round of coding, I coded the suggestions from the curriculum, such as the question types or recommended group sizes. For example, I coded the questions, included in the curriculum lesson, as *information seeking* or *eliciting student thinking*. After observing lessons, I coded the lesson a second time noting if the lesson suggestions were *utilized*, *adapted*, or *not utilized* by the teacher during the observed lesson. As I coded the curriculum, I wrote analytic memos, where I recorded questions and made notes about common patterns and themes. For example, I noticed that the lesson wrap-up, which included a whole group discussion reflecting on what was learned, was almost always coded as *not utilized*. In an analytic memo, I made a note to ask teachers why they omitted those questions.

As additional codes emerged, I recoded the curriculum suggestions according to the new emergent codes. For example, I developed the emergent code *different strategies/solutions* for any time the curriculum suggested teachers ask students for alternate strategies, solutions, or ideas. In these subsequent rounds of coding with emergent codes, the codes describing utilization did not change.

Observation Codes

Coding the observations followed a similar process as coding the curriculum. Initial rounds of coding utilized *a priori* codes that sought to categorize and describe the various talk moves teachers utilized during mathematics discussion. Coding specific talk moves allowed me to see which talk moves teachers utilized and how these talk moves influenced discussion. Furthermore, coding various talk moves and question types allowed me to compare codes both across and within teachers, which contributed to identifying patterns and themes during analysis. Additionally, initial rounds of coding included codes to describe the size of the discussion –

whole group, *small group*, or *partner*. These codes were useful for data reduction when analyzing whole group discussion specifically.

After multiple rounds of coding, I inductively evaluated the data and determined that revisions to the observation codes were necessary. Several of the *a priori* codes were irrelevant, as I did not observe any discussion that could be described or categorized using these codes. For example, I did not observe teachers *encourage repeating* or *prompting students to question peers* and removed these codes. Additionally, I revised several codes to become parent codes and developed emergent codes under the parent code. For example, *repeating* became a parent code with two sub-codes *repeating with statement* and *repeating with question* to further describe the different ways teachers utilize *repeating* student ideas. The final observation codebook included both *a priori* and emergent codes (Appendix J).

Throughout each round of coding with both *a priori* and emergent codes, I recorded reflexive memos. These reflexive memos included questions that emerged through coding and analysis of the observation. Additionally, I reflected on the patterns and themes that began to emerge through coding.

Interview Codes

Initial rounds of coding interview data used two *a priori* codes that classified teachers' beliefs as *productive* and *unproductive* (NCTM, 2014). These two *a priori* codes served as the foundation of coding as I analyzed how teachers' descriptions of discussion related to the NCTM's definitions of meaningful mathematics discussions to answer research question one. Since I analyzed data while simultaneously collecting additional data, emergent codes developed through the process of inductive coding (Bazeley, 2021). It became clear that I needed to add additional codes that extended beyond *productive beliefs* and *unproductive beliefs.* Because teachers frequently referred to the curriculum during interviews, I added *curriculum reference* as an emergent code. Additionally, I added a code to label whenever teachers mentioned student *participation*. As I revised my codebook, I checked with a critical peer to ensure that the codes were logical and clear (Appendix K). As with coding observations and curricula, I wrote analytic memos to record my interpretations, questions, and recognition of bias while coding interviews.

Thematic Analysis

After engaging in multiple rounds of coding both deductively and inductively, I analyzed the data thematically. Saldaña (2021) suggested that researchers code their data, identify commonalities between codes, and "construct an extended thematic statement" that represents how the researcher interprets the data (p. 259). To begin identifying patterns in the code, I sorted my data by code and ordered the codes by frequency. Although frequency counting is not always recommended in qualitative research (Hays & Singh, 2012), Saldaña (2021) argued that it can help to identify which themes or ideas commonly or rarely occurred. I found that frequency counts were helpful to begin analyzing my data and looking for common patterns across various data sources and teachers. Next, I retrieved data belonging to certain codes and identified patterns that existed between multiple codes.

Through interpreting the data and identifying patterns, I constructed themes to capture the relationships that existed between codes and ideas. I used theme charts (Appendix L) to explain the theme using codes, a description of the patterns, why the patterns were notable, and specific evidence from the data (Bazeley, 2021). During thematic analysis, I triangulated the data by citing evidence from at least two data sources for each theme. Multiple themes contributed toward the development of my eventual findings.

Ethical Considerations

This study took place as part of a capstone study in fulfillment of an Education Doctoral degree. I received research approval from the Institutional Review Board for Social and Behavioral Sciences (IRB-SBS) at the University of Virginia. Since this research study involved human participants, I maintained respect for the teachers as people by fully informing the teachers about the research and obtaining consent. There was little to no risk of harm involved in this capstone study for teachers. Prior to each interview, I sought consent from teachers to participate and record the interview. Teachers received pseudonyms to protect their confidentiality and I reinforced that they were free to withdraw consent at any time during the capstone study. Furthermore, I used a secure, password-protected cloud-based storage system, Dedoose, for securing data.

Research Positionality

The positionality of a researcher has the potential to impact research and findings, particularly in the case of qualitative research (Berger, 2013; Merriam & Tisdell, 2016). As a result, it is necessary that researchers confront and "own one's positionality" rather than ignore it (Merriam & Tisdell, 2016, p. 148). As the researcher in this capstone study, I came to Barron Academy as a current outsider, but I attended Barron Academy as a middle and high school student. I previously worked for seven years as an elementary mathematics teacher and instructional coach in a large, urban public school district. The district's mathematic department's mission included providing students with daily opportunities to participate in mathematics discussions focused on reasoning. As a teacher and coach, I personally worked on improving my own practices pertaining to mathematics discussion and believe that these practices are instrumental in developing all students' mathematical understanding. My experience also included working closely with teachers, other instructional coaches, and district administration to support implementation of mathematics discussions in elementary classrooms across the district. It is important to acknowledge that my experiences as a public-school educator may impact how I view instruction and discussion in the context of this capstone study.

My experiences as an educator impacts my position and beliefs regarding mathematics instruction and mathematics discussion. Informed by the literature and current trends in mathematics education, I believe that meaningful mathematics discussion has the potential to support student learning and understanding. Additionally, I believe that mastering high quality discussion practices improves the ways teachers present opportunities for sensemaking. Understanding my own position and bias towards mathematics instruction and discussion is important as it has the potential to impact my analysis of data.

When I observed mathematics instruction at Barron Academy, I took the role of a participant as observer (Cresswell & Báez, 2021). Although I was an outsider to the classroom, it was difficult to remain a complete nonparticipator. I observed the classroom from a distance so that I was inobtrusive, when applicable; however, there were times when I need to interact with students to properly observe a small group or record interactions that I may not have completely heard. Additionally, teachers and students often spoke to me during instruction. Any observer has the potential to influence activity and participation during an observation; however, direct observations of instruction are necessary to collect in-depth data in a case study.

Trustworthiness

I took several steps to ensure the trustworthiness of my capstone study while collecting and analyzing data. The trustworthiness of a study impacts the value and quality of the study and requires establishing credibility (Lincoln & Guba, 1989, as cited in Bazeley, 2021). To increase the credibility of my findings, I employed triangulation, member-checking, and peer debriefing. Triangulation helped to ensure consistency throughout the data collection process by triangulating data sources over time and using multiple data sources to answer each research question. I triangulated data from interviews, observations, and curriculum review to make appropriate conclusions as evidenced by all data sources. Interviews helped to triangulate data collected during classroom observations. During interviews, teachers mentioned specific instances of classroom discussion or instruction that I recorded but may have interpreted differently during the observation. Additionally, when creating themes charts, I looked for evidence of the themes across data sources. I also engaged in member-checking to ensure that my interpretations and conclusions were accurate. After each observation, I interviewed teachers about the observed lesson and during the interviews, I asked teachers about specific parts of the observation to ensure that my interpretation of the classroom observation was accurate. I also asked teachers clarifying questions to ensure that I understood their responses correctly. For example, I might have said, "So I hear you saying that you…. Is that correct?".

Throughout the analysis process, I was intentional about ensuring that my findings and claims were supported by evidence in the data. I met with three critical peers, who were current and recently graduated students in the same doctoral program, about the codes, patterns, and themes found in the de-identified data. Meeting with critical peers helped to uncover any biases or assumptions as well as verify any emergent themes and findings to ensure that they are reasonable and plausible (Cohen & Crabtree, 2006). During the meetings, I explained my interpretations using data as evidence to ensure that I did not make unjustifiable claims about teachers' beliefs or actions.

Limitations

This capstone study has several limitations that are worth noting. First and foremost, I was not a member of the school community and therefore lack informal relationships with the teachers. This limited the trust that teachers may have had in me as both an individual and researcher. However, my lack of relationship with the teachers at Barron Academy also lessens my biases as I do not have many prior experiences with these teachers. A second limitation is my presence as a researcher, particularly a participant observer, may have impacted instruction. Although I tried to limit my participation, it is likely that the mere presence of an additional adult may have impacted the behaviors and/or actions of both teachers and students. To accommodate for this, I asked teachers during interviews and debriefs if student engagement and behavior was typical. A third limitation of the study is the lack of multiple researchers to support coding and interpretation of data. A fourth limitation is that this case study focused discussion on the oral and verbal talk use to drive mathematics instruction. Discussion is a multi-faceted instructional strategy that is often referred to as discourse and includes more than just verbal talk.

Additionally, the exclusion of accounting for teachers' MTK is limitation of the study. Teachers' MTK influences student learning and teachers' instructional practices. For reasons of scope and access, MTK was not studied in my capstone. The absence of MTK within my findings is a limitation of the study. Additional research would need to be conducted in order to better understand the MTK of teachers at Barron Academy.

Finally, the timing and duration of the study was a limitation. I observed each individual teacher four times (n=4) over the course of two weeks. I observed the end of one unit and beginning of another unit in two classrooms and only observed one unit in the third classroom. Observations occurred in March and April when the major work of the grade had already been taught. Teachers and students were working within the final units of study for the grade. Conducting the study at a different time with different units of study or over a more prolonged period may have yielded different results. Moreover, my findings were gathered from a sample of three teachers and are not directly generalizable to the entire school. While I found commonalities amongst all teachers, differences also existed. For the efficiency of the school, I provide findings and recommendations that assume generalizability across the school; however, I recognize that a study of three individuals is not generalizable.

Assumptions

This capstone proposal made several assumptions regarding mathematics instruction at Barron Academy. First, I assumed that mathematics discussions occur daily; however, I did not assume that all discussions are of a productive or quality nature. I made these assumptions having spent a few weeks at the beginning of the 2021-2022 school year walking through mathematics classrooms at Barron Academy and reviewing the curriculum as part of initial discussions about undertaking the capstone study in this school context. The curriculum includes suggestions for discussion in every lesson to some extent. I also assumed that Barron Academy prioritizes both mathematics and mathematics discussion when considering goals related to instructional improvement. A final assumption was that the findings and recommendations of this research will support Barron Academy in improving instructional practices and students' abilities to communicate reasoning and engage with the ideas of peers

Chapter Summary

In this Methods Chapter, I described both the design and purpose of my capstone study. This qualitative, descriptive case study investigated three elementary mathematics teachers at Barron Academy to better understand the current landscape of mathematics discussion and the ways in which teachers at Barron Academy support or limit students engaging with the ideas of their peers during discussion. I collected data through observations, interviews, and a review of the curriculum, *Math in Focus*. In this chapter, I explained the rationale, protocol, data collection procedures, and analysis for each data source. Finally, I concluded the chapter by disclosing my role as an external researcher, ethical considerations, and limitations of this study. In Chapter 4, I will discuss the findings that I developed through the thematic analysis.

Chapter 4: Findings

This capstone study was designed to describe the current landscape of mathematics discussion at Barron Academy so that teachers can continue to grow in their understanding and readiness to facilitate meaningful mathematics discussion. Mathematics discussion is integral to the effective teaching of mathematics by providing students with opportunities to share ideas, clarify their understanding, construct convincing arguments, and compare mathematical approaches (NCTM, 2014). The research was driven by the following research questions:

1. How do elementary teachers describe the attributes of meaningful mathematics discussions? To what degree are teachers' descriptions aligned to the NCTM's mathematics teaching principles?
2. In what ways do teachers orchestrate meaningful mathematics discussions?
    a. In what ways, if any, do teachers use talk moves to promote and support student-to-student engagement during mathematics discussions?
    b. In what ways, if any, do teachers limit student-to-student engagement during mathematics discussions?
    c. In what ways, if any, does the curriculum support opportunities for meaningful mathematics discussion?

To investigate these research questions, I conducted a case study of three elementary grade teachers across the elementary grades (K-5). The three participating teachers include Mrs. Anna Marzano (kindergarten), Mrs. Nancy Staples (third grade), and Mrs. Wendy Grimes (fourth grade). I collected data through a curriculum review, observations of mathematics instruction, and interviews with the three participating teachers. I observed each of the participating teachers four times and interviewed teachers following each lesson and once more at the conclusion of the study. Additionally, I reviewed the corresponding lessons in the *Math in Focus* curriculum to understand the role the curriculum played in the observed lessons. In this chapter, I use both verbatim quotes from the interviews, classroom observations, and curriculum and narrative descriptions to paint pictures of the classroom contexts and describe the findings. In the following section I present vignettes of the three teachers' classrooms as context for the explanation of findings.

Mrs. Anna Marzano

In kindergarten, the 30-minute mathematics lessons typically began with students sitting on the carpet facing the smartboard. Anna stood at the front of the carpet next to the smartboard, which displayed a photo from the *Math in Focus* textbook (see Figure 4.1). Anna's TE textbook was open to the corresponding lesson page on her desk next to the smartboard. The textbook's close proximity enabled Anna to refer back to the TE textbook throughout the lesson.

Figure 4.1

Addition Sentences Picture (Marshall Cavendish Education, 2020b, p. 99)

Anna initiated the lesson with a discussion on the carpet about what students noticed in the picture. Students began sharing out, "I see shells!" or "I see kids playing with sandcastles", while they waved their hands and pointed from their seats on the carpet toward the objects in the picture. As students shared out, Anna called on individual students to ask follow-up questions and highlight specific students. “Oh, Chase said something interesting! Can you come up and repeat what you said?” Chase walked to the board, circled the objects he saw, turned to face his peers, and repeated “I saw seashells”. Anna, who had stepped to the side of the students on the carpet, asked the class to count the shells. Many students used their fingers to track and count the shells on the board before sharing out, “I see 7 shells!”. Next, Anna asked students to turn-and-talk to their shoulder partner, “Boys and girls, can you make a number sentence about seashells?”. Students turned their bodies to face their partner on the carpet and began sharing ideas. While partners discussed various ways the seashells could be represented in number sentences, Anna glanced at the textbook on her desk and walked around the carpet, listening to the different partners without interjecting in the conversation. As students finished up the partner talk, they returned their attention to the front of the room where Anna asked the group to report what they discussed. “Riley, will you come write the number sentence on the board and explain what it means?” Riley stood up, came forward, walked to the board, wrote an equation on the board, and responded “I saw 3 pink shells and 4 green shells, when I counted them, it was 7. So, I wrote 3 + 4 = 7”. After students explained, other students shared out “Me too! Me too!” or “I know the turnaround fact – 4 and 3 makes 7!”. Anna continued to ask students what else they noticed about the picture and discussion continued along a similar series of questions, turn-and-talks, and student explanations.

After discussing two or three different number sentences from the picture, students returned to their table groups, which consisted of 2-4 students per group. At their desks, students practiced addition workbook pages individually and with partners. The class engaged in other whole group discussions after each completed page and students shared different acceptable answers, strategies, and ideas. Each time, Anna initiated discussion by asking questions about answers and strategies, invited students to the board to model their equation and counting, and encouraged students to share different possible solutions.

Mrs. Nancy Staples

In Nancy's third grade classroom, students sat in groups of three to four students at four table-groups facing the smartboard. The tables were arranged in an arc around the classroom so that each student had a clear line of sight to the board. Nancy most often stood at the front of the classroom next to the smartboard, while students sat at their desks with their *Math in Focus* student workbook opened to the page designated on the board. Throughout the lesson, she referred to her *Math in Focus* TE book, which was open to the lesson on her desk. From the front of the room, Nancy started the mathematics lesson by recalling what students had previously learned about fractions and introducing the new topic. "So far, we have learned a lot about fractions. We have named the parts of the fraction – numerator and denominator. We have practiced drawing fractions and identifying them from pictures. Today, we are going to learn about how we compare fractions." Next, she sought to include student voice and asked, "What does compare mean?", and called on multiple students to share their definition of compare. Nancy reminded students of the goal of the lesson and said "Today, we are going to learn how to compare fractions", which emphasized the procedural nature of the goal with the inclusion of *how to*. Next, she swiped the board to the next slide, which included the initial problem for the day, and prompted students to work at their table groups to compare $1/4$ and $5/8$. She instructed students to use their fraction circles to build the fractions to help compare. As students began building, Nancy walked from table-to-table, stopping to ask students how manipulatives helped them to compare fractions. Although she asked a table of three to four students, only one or two students responded before Nancy moved to the next table. At Table 2, students built fractions with thirds and sixths instead of fourths and eighths. Nancy announced to the class, "I want everyone to check their manipulatives. If you are using fourths, your pieces should be yellow. Eighths should be green. Just double check that you have the right piece!"

After 3-4 minutes of table talk, Nancy returned to the front of the room, next to the board, and redirected students back to the whole group conversation about comparing fractions. Nancy selected Table 3 to share which fraction was greater and one student answered, "five-eighths". She invited students to provide a thumbs-up if they agreed or thumbs-down if they disagreed. Nearly all students provided a thumbs-up and Nancy commented, "Great! We can all see that $^5/_8$ is larger. This time, go ahead and make $^5/_8$ and $^3/_8$ at your seats. Do this on your own." As students finished building, Nancy restarted the conversation by asking "Daniel, which is greater?", to which the student responded "five-eight". Nancy evaluated the student's response, "That's not quite how we say that" and encouraged peer-correction, "Who can help him to name this fraction?". Nancy called on one of the multiple students who quickly raised their hand, and who correctly said "five-eighths". Then, Nancy questioned the class, "How do you know five-eighths is greater?". Nancy selected students as they raised their hands and explanations were brief – "I used fraction circles" or "Three on the top is less than five". After three student explanations, Nancy expanded their response with a more thorough, univocal explanation of how to compare fractions, while drawing on the board.

> So let's think about what Mason said. We can look at the number on top – numerators - and compare the 3 and the 5 because the denominator, or number on the bottom, is the same. Both fractions have an 8 in the denominator. Mason noticed the denominators were

> the same, so we can just look to the numerators to figure out which is more. They have the same size pieces but $^5/_8$ has 5 pieces and $^3/_8$ has 3 pieces. One way we can compare fractions, is by seeing that the denominator is the same and then looking to the numerator.

Next, Nancy presented a new fraction for students to compare, “What if we then compared 3 fractions – $^3/_8$, $^5/_8$, and $^1/_8$. Which would be the least?”. The mathematics lesson continued with students solving new problems individually or in table-groups and sharing their solutions and explanations in whole group. Nancy remained at the front of the classroom, while students remained at their seats. Nancy responded to students by asking the class to signal agreement or disagreement with a thumbs up or down, probed for an explanation, or moved onto the next problem.

Mrs. Wendy Grimes

Similar to Nancy’s classroom, fourth-grade students in Wendy’s mathematics class remained seated in table-groups for the entire duration of the 45-minute mathematics block. Student desks were arranged in four table groups table groups of three students and one table group of two students. Students sat at their tables with their *Math in Focus* student workbooks open to the appropriate page, while Wendy stood at smartboard, which displayed the corresponding pages on a PowerPoint, at the front of the class and held the TE textbook in her arms. The focus of this lesson was finding an unknown side of a rectangle when given the perimeter.

Wendy began the mathematics lesson by asking students to recall what they know about perimeter. A handful of students immediately shot up their hands, ready to answer and share. “It’s the space outside of the shape”, replied one student. Wendy responded to students directly and prompted students to clarify their thinking, “But what space? Everything outside of the shape?”. A few students shouted out, “Just around the outside.” Wendy continued questioning students, “How do you find the perimeter? Student: “You add!”. Again, Wendy pressed further asking, “What do you add?” and selected a student, who had already shared, “You add up the lengths and widths.” Wendy then wrote “P = L + L + W + W” on the board as a reminder of the equation for perimeter of a rectangle. Next, Wendy transitioned to the *Math in Focus* curriculum by drawing attention to an *Engage* task, which was found in the student’s workbooks and displayed on the Smartboard (see Figure 4.2).

Figure 4.2

Perimeter Engage Task (Marshall Cavendish Education, 2020b, p. 112)

ENGAGE

Draw a rectangle with a length of 4 centimeters and a perimeter of 12 centimeters on a square grid. What is the width of the rectangle? What are two different ways to find the width? Share your ideas with your partner.

Engage tasks were designed to help students make new connections between prior knowledge and new learning through inquiry (Marshall Cavendish, 2020b). Wendy read the problem aloud and added “Work with your table to figure out what you need to do to figure this out.” Wendy walked around the room, stopped at tables and asked the table group of students to share their plan with her. At each table, she asked a student “Tell me what you are doing”. When students were uncertain, she prompted them to “Talk about it with your partners!”. After a few minutes of small-group discussions at tables, Wendy drew attention back to the whole group-setting by walking to the board and saying, “Okay, let’s go over it. What do we know about the problem?”. Student turned their heads from the workbooks at their tables to the board. One student responded with information from the problem, “The lengths are 4 and it’s a rectangle.” Wendy again asked for more information, “But how many lengths are there?” and called on a different student, who answered “two”. After students shared what they knew, Wendy asked, “So what would you do?” and called on multiple students to share what they did to solve the problem.

Ty: So, I know that 4 doubled is 8. Then, if I add one to each side… I can do that twice to get 12.

Wendy: Okay, who else did something different?

Kenzie: I drew it out, I put 4 on each length and then saw it was 8. I know that 4 more gives you 12. So it’s 2.

Wendy: Okay… anyone else?

Claire: So, I plugged the numbers in the perimeter equation. I did 12 subtract 8, which is 4 + 4. Then, you get 4 and you have to split 4 in half to get each width is 2.

After Claire shared her response, Wendy wrote Claire’s equation on the smartboard and shared, “We are going to work on this strategy today and solve for missing sides by using an equation.” Then, Wendy taught a mini-lesson covering how-to solve for a missing-length procedurally using the equation, using the *Engage task* as an example. She repeated Claire’s explanation but expanded upon it to include a step-by-step process using the equation (P = L + L + W + W) to solve for the missing side. After the mini-lesson, students worked in table groups to solve problems in the workbook using the prescribed steps, followed up by a whole group review of the answers. During the review, students shared their answers and what they did to solve the problems in whole group. Wendy recorded what students shared on the smartboard to record the correct procedures and responses.

All three teachers' classrooms included discussion of the math content and skills although the ways in which these occurred were different. In the following section I answer the research questions by laying out two key findings about the nuances of the role of math talk in these three teachers' classrooms. Data collected from the interviews primarily informed the first research question and the data collected from the curriculum review and observations primarily informed the second research question; however, patterns emerged across all data sources that contributed to the construction of themes and the eventual findings. To clarify, the findings are not generalizations to teacher practices; rather, the findings are derived from the data collected during this study (n=12 observations and n=15 interviews). Furthermore, the findings do not assume that all teacher practices are the same. Rather, I note commonalities and differences in teacher practice. However, I developed uniform findings for the use of the school. Although teachers described instruction as typical, I conclude that my findings are only directly relevant to the lessons observed and the participating teachers.

- Finding 1: Teachers recognized that meaningful discussions were student-led, yet they did not always facilitate discussions in a student-led way.
- Finding 2: Teachers were heavily reliant on the curriculum's suggested questions to plan for and orchestrate mathematics discussion, which limited meaningful mathematics discussion.

In Chapters 1 and 2, I described meaningful mathematics discussion as a "purposeful exchange of ideas" that supports students in developing understanding (NCTM, 2014, p. 29). Additionally, meaningful mathematics discussions situate students to be active leaders in the discussion as they not only share their ideas but also engage with the thinking of their peers by asking questions, critiquing ideas, and comparing mathematical ideas (NCTM, 2014; Hufferd-Ackles et al., 2014). Meaningful mathematics discussion is dialogical, positioning teachers to support students in engaging with the ideas and thinking of their peers. Mathematics discussion is often conceptualized in a dichotomous way as either univocal or dialogic. However, it often occurs across a spectrum varying between the two extremes or existing somewhere in the middle (Jang, 2010; Knuth & Peressini, 2001). In this chapter I describe instances where the discussion is univocal (teacher provided explanation and instruction), partial univocal (teacher asking questions followed by student answers and explanations), emerging dialogical (teacher encouraging students to share their ideas and strategies with one another), and dialogical (students engaging in discussion with one another to justify and argue their mathematical thinking) (Jang, 2010).

As I describe these findings of my research, I refer to mathematics discussion in terms of the language spoken by teachers and students that was seen and heard during my classroom observations and interviews with each teacher. I refer to discussion as it was described by the teachers and not according to the definition and descriptions provided in Chapter 1 and 2. In future sections I will specifically note where there is alignment or contradiction with the literature on this topic. Generally speaking, these three teachers referred to any component of the lesson that included talk about mathematics between students and teachers as discussion.

Finding 1: Teachers recognized that meaningful discussions were student-led, yet they did not always facilitate discussions in this way.

Across grade-levels, the three teachers in this study described meaningful mathematics discussion in similar ways during their individual interviews. Each consistently described meaningful mathematics discussion as "student-led" with high participation and students "learning from one another". Yet, discrepancies existed between how teachers described student-led discussion and the enactment of these discussions. In this section, I present how descriptions about student-led discussions emerged in both interviews and the curriculum and then describe the ways in which teachers interpreted and executed facilitating discussion. Additionally, I provide specific evidence of how participation and specific teacher talk moves supported and/or limited student-led discussions from occurring. I present this finding through analysis of several themes, which include teachers facilitating discussion, students learning from hearing correct ideas, participation as a tool for engagement, and teachers repeating ideas.

Teachers Facilitating Discussion

The *Math in Focus* curriculum holds student-centered instruction and discussion as core tenants. The curriculum was founded upon the belief that learning occurs when students develop understandings of concepts (Marshall Cavendish, 2020c). This belief centered students at the focus of learning, not the teacher, as they develop those understandings. Throughout the instructional resources, recommendations included teachers "facilitate student learning" and "facilitate discussion"; however, there was no description of what "facilitate" actually meant in concrete instructional terms (Marshall Cavendish Education, 2020a, 2020b). Each lesson of the TE included directions for teachers to "facilitate discussion" using specific question prompts (see Figure 4.3). The TE provided teachers with specific questions and potentially correct answers; however, it did not provide teachers with directions about how *facilitate* differs from *leading* discussion. During interviews, teachers referred to the directions provided in the TE to "facilitate discussion" and operationally defined this as "asking questions".

Figure 4.3

Suggested Questioning for Introducing Mass (Marshall Cavendish, 2020b, p. 68)

Solving real-world problems about mass

Let's recall solving real-world problems involving mass.

- Display and read aloud the word problem, without the numbers. Guide students' thinking by facilitating the following discussions.

What is happening in the problem? (Answers vary. Example: Ms. Davis is using some of the flour in a bag.) **What are we asked to find?** (how much flour is left in the bag) **What operation would we use to solve this?** (subtraction)

- Encourage students to draw a model to show what is happening in the problem after showing them the numbers in the problem. Notice students' use of subtraction.

Teachers in the study described the significance of student-led instruction in helping students to develop mathematical understanding through the idea of facilitating discussion. Anna recalled that the "biggest thing that we [teachers] really took away from professional learning about *Math in Focus* [was] that this is really student-led" (Interview, 4/4/2022). She later emphasized:

> Children learn so much better when it is from another child. If I am giving them the answer, I am not doing them justice because they could be getting it from another student. (Interview, 4/11/2022).

All teachers shared similar sentiments during interviews. Nancy declared that she "wants them to be able to discover something rather than tell them and simply forget three lessons from now" (Interview, 4/7/2022). In both comments, teachers emphasized the importance of the teacher taking a step back and allowing students to learn from one another. Specifically, teachers referenced discussion as a space for student-led instruction to thrive because teachers could act as facilitators to guide students to learn from one another. How these ideas took shape in observations, however, was that teachers more typically led instruction through asking questions.

In the opening vignettes, for example, both Nancy and Wendy used student responses as a launching point for a predetermined mini lesson.

During interviews, I asked teachers to describe for me what facilitating discussion means (see Table 4.1). All teachers commonly described facilitating discussion as asking questions that students answer. These quotes exemplify how these teachers considered the role as a facilitator to be the person asking the questions. Furthermore, both Anna and Nancy claimed that discussion is student-led because the answers or explanations come from the student.

Table 4.1

Teacher Responses to Describing Facilitating Discussion

Teacher	Comment
Mrs. Anna Marzano	"So that's why I like to have them on the carpet when we are talking and discussing. They can come up to the board to show me something. They can turn to a partner and think-pair-share type of thing. So, **I ask the question but want students to provide the answers** versus just me saying the answers to them. It's **student-led conversation** – with me asking questions but the students leading the answer." (Interview, 4/11/22)
Mrs. Nancy Staples	"To **ask those questions**, rather than explain so much. It is hard for me to do that. I want to tell a little more than I ask. But I know when I facilitate, it **should still come from students**." (Interview, 4/1/22)
Mrs. Wendy Grimes	"I think **it just means to ask questions** and then prompt what they are saying when they respond to those questions. So, if I am trying to facilitate discussion, I can ask how are you going to solve this problem… I will first ask students what is going on and then I will ask them to set up a plan." (Interview, 4/20/2022).

Because teachers asked nearly all of the questions, teachers controlled the direction of the conversation. As a result, discussions were not actually student-led (dialogical) but were instead more accurately characterized as partially univocal in that they were led by the teacher whose instructional approach prioritized asking questions. When teacher asked all the questions and students only provide responses, such as what was observed in most of the observations in this study, the discussion could more accurately be characterized as somewhere in-between the dichotomy of student-led (dialogical) and teacher-led (univocal). As seen in the vignette of Wendy's classroom, Wendy asked for students to share their strategies to uncover a specific strategy and solution. Several students shared ideas, which did not garner responses from the teacher. Once a student shared the teacher's desired strategy, Wendy used that student's idea as the launching point for a teacher-led explanation of perimeter. Although Ty and Kenzie presented plausible strategies for solving for an unknown side, the focus of the lesson stemmed only from Claire's response. Wendy did not engage with Ty or Kenzie's ideas, nor did she prompt students to consider their solutions. She only focused attention to Claire's thinking through her mini lesson (univocal) of Claire's strategy. To further emphasize this point, consider an additional example from Nancy's classroom.

Nancy: What does it mean to measure something?

Student 1: You might find a height, or you weigh something.

Nancy: Okay, what do y'all say Table 2?

Student 2: Um… I don't know.

Nancy: What are some things you might measure?

Student 3: You could measure how long the classroom is.

Nancy: Okay, length and width. What else?

Student 4: My backyard.

Nancy: What else… what about at the doctor's office?

Students: Height! Weight! *(Multiple students sharing out)*

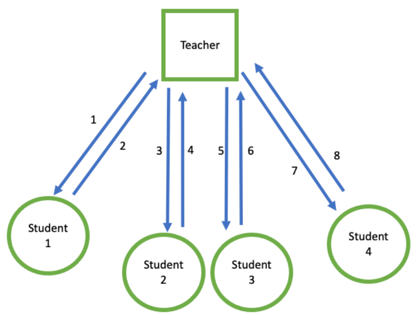
Nancy: Yes, weight. So, let's think about weight… what sort of tools do we use to measure?

(Observation, 4/7/2022)

In Figure 4.4, I mapped how Nancy (the teacher) asked questions to specific students and then asked additional questions until she got the specific answer she was seeking. More than one voice is shared during this conversation; however, the teacher is asking questions to guide student thinking and students are not engaging with their peers. Nancy initiated discussion by asking a question included in the *Math in Focus* textbook that was intended to support teachers in "facilitating a discussion" (1). Then, a student responded (2). Nancy then asked a separate question to Student 2 (3), to which the student responded (4). This individual line of questioning continued with two other students. According to Nancy's definition of facilitating discussion as asking questions to students, Nancy did facilitate a discussion. However, this discussion was a mere exchange of ideas as teachers asked questions that led students to a specific response. Student responses did not drive Nancy's line of questioning; her questions were not dependent or unique to student responses. Student responses and ideas were not the focus of the discussion as intended in a student-led discussion. Rather, Nancy asked questions, students responded, and then the teacher asked additional unrelated questions. Nancy initiated the conversation with a broad question about measurement but continued to ask questions that funneled student responses to the desired topic for the lesson, measuring weight with scales.

Figure 4.4

Map of a Partially Univocal Discussion



Wendy further described facilitating discussion asking questions so that the teacher was a "tool… there to support students in getting [on the right track]" (Interview, 5/11/2022). Wendy emphasized that the facilitator is there to ensure conversation was on the right track and that correct ideas were being shared.

Students Learning by Hearing Correct Answers

During interviews, all teachers referred to mathematics discussions as an opportunity for students to *learn from one another*. Wendy believed that discussion was meaningful when students "gained insight" from one another and commented:

> When discussion is meaningful, students begin to gain insight from each other. You might be seeing that you aren't getting it correctly or you are struggling – but if you hear someone else's answer, it becomes clearer, or you may find one that you want to try it their way. Students are able to learn more from each other when they hear about someone's idea and then they also see it. (Interview, 5/12/2022).

In this description, Wendy highlighted how students learned from one another by seeing or hearing from another student, rather than talking to or doing mathematics with their peers. From this view, one student actively shared a solution and another student passively learned from them or took in the shared idea. Furthermore, this description of students learning from one another in discussion did not include students justifying mathematical thinking or arguing about mathematical ideas (dialogical). Instead, students with correct answered shared their ideas with students that "were struggling" (Interview, 5/12/2022). Although teachers shared that discussion was a space for students to learn from one another, after analyzing the observation data, I concluded that only "struggling students" learned from their peers during discussion.

I observed a similar instance of students learning a correct solution from their peers during a small-group discussion in kindergarten. In the small-group, one student had difficulty creating a subtraction sentence from a picture of 8 ducks.

> **Student 4:** I see 5 brown ducks and 3 green ducks. That makes 8.
>
> **Student 7:** No, no… We're not adding. We start with 8 ducks. We take-away those 5 ducks. We have 3 ducks! *(pointing to the equation on his page)*
>
> **Student 4:** Okay, so 8 – 5 = 3.
>
> (Observation, 4/12/2022).

The "struggling student" understood that the picture of the ducks could represent addition but did not yet demonstrate understanding of the picture representing subtraction. After the observation, Anna mentioned this specific instance as an example of students learning from one another. Anna recalled "When he [Student 8] said it, she [Student 4] started to notice that we needed to start with the total – 8. She quickly got it – she was able to listen to him, get his ideas, and learn from him" (Interview, 4/12/2022). The struggling student, however, did not show evidence of having learned beyond listening and recording the correct response. Students did not engage in discussion with one another about *why* the solution was correct or present the relationship between addition and subtraction; rather, one student voiced the correct answer to his peer (partial univocal). Although Student 7 led the explanation, the explanation was limited to sharing why it was correct and not addressing Student 4's misunderstanding of representing addition instead of subtraction.

Throughout all three classrooms, discussion emphasized the correct answers and correct procedures. Anna asserted that "it makes me happy that they agree because sometimes it's clearly one answer" (Interview, 4/11/2022). Discussions were a space to share what was correct, rather than develop the meaning or understanding of the concept. As a result, conversations exemplified show-and-tell correct answers. The students that "learned" from their peers in discussion were students whose answers were incorrect. This concept was reiterated by a note for best Practice included in the *Math in Focus* TE lessons:

> Students learn by talking and interacting. Use the richness of class discussions to drive the section for the benefit of all your students but especially struggling students who learn from their peers. Students who draw their conclusions will remember the principles they uncover far better than if they are told (Marshall Cavendish Education, 2021b, p. 110).

Similar to what Wendy shared, this "best practice" assumed that struggling students were the greater beneficiaries of discussion. Nancy also stated that discussions were helpful because "someone remembers something but not everyone else did… so it helps to hear from others to create that aha moment" (Interview, 4/7/2022). Here, Nancy explained an "aha moment", which often referred to a climactic moment of learning, coming from just hearing a correct answer or strategy. This explanation assumed that students learned from being told or shown a solution by other students. Both teachers' descriptions and the "best practice" from the text equate learning from peers with a struggling student hearing the correct solution through show-and-tell, which aligned more with univocal discussion than dialogical discussion.

All three teachers mentioned that discussions were more meaningful when students demonstrated mastery of the skill. Specifically, Anna claimed that "more perfect discussions happen towards the end of the unit when they are confident in what they are doing" (Interview, 4/12/2022). Similarly, Wendy commented that some students "don't participate until the end when they know they are right" (Interview, 4/26/2022). Nancy also acknowledged that when students were first learning something new "they hear what their friends say and it's a way off answer. I don't want them to remember that, so I usually reign them back in and explain it to them" (Interview, 4/1/2022). All three teachers stated that discussion was more meaningful when students shared correct answers after having learned the content; on the other hand, when students were learning content, discussions had less participation because students were more likely to mistakes and didn't want to embarrass themselves. Both Wendy and Nancy commented that when students first learned a new skill or concept, they were usually "very quiet" or "shy" and worried about "getting it wrong and being embarrassed" (Interview, 4/1/2022, 4/26/2022). Only in kindergarten, Anna disagreed stating that "students can raise their hands, share an answer that might be wrong and it's okay!" (Interview, 4/12/2022). Nonetheless, she still felt that discussions were more productive at the end of the unit when student learning was "complete" (Interview, 4/12/2022). These comments supported the finding that student discussion was a space to voice correct answers (partial univocal) rather than make sense of concepts together through discussion with their peers (dialogical).

Participation as a Tool for Engagement, not Learning

Teachers in this study commonly defined meaningful discussion in terms of participation from students. Just as teachers mentioned the importance of *hearing* from their peers, all teachers mentioned all students participating in meaningful discussion (see Table 4.2).

Table 4.2

Teacher Description of Meaningful Mathematics Discussion through Participation

Teacher	Comment
Mrs. Anna Marzano	"A perfect discussion would be **everyone being heard**. They want to be heard – kids will have tears because they are upset that they weren't called on." (Interview, 4/12/22)
Mrs. Nancy Staples	"Ideally, I would hear **everyone say something**. I have learned that isn't always going to happen." (Interview, 4/1/22)
Mrs. Wendy Grimes	"I think [today] is what a good math discussion looks like**. Everyone participates. Everyone gets to talk through their ideas. Everyone is heard** and they get to discuss their ideas and **be affirmed by their idea**." (Interview, 4/20/2022).

Both Anna and Wendy described participation as a way for students to be heard, which could be a source of affirmation for students. Both Anna and Wendy shared throughout various interviews that their students were "eager" and "excited" to share what they knew. When students shared the correct ideas during discussion, they were affirmed by positive evaluations or students agreeing with their solutions. Notably, participation was referred to by all teachers as *sharing ideas* rather than participation as a tool for sense-making and learning. Teachers in this study utilized participation as a tool for keeping students engaged with them in the lesson, rather than engaging with the ideas of their peers or the mathematical concepts. Seemingly, student participation was not defined as how students talked with one another, but how they shared their own ideas with the teacher and the class as a whole (partial univocal).

During classroom visits, I observed teachers encouraging participation amongst students by asking numerous direct questions (see Finding 2). In the excerpt below, Wendy solicited participation from half of her class for just one problem. Between each student response, she asked a question that solicited students to respond with more information to develop a plan for how to solve a problem. There was little discussion of the ideas students shared with the class; however, a high quantity of students participated by raising their hands and answering questions.

Wendy: What do we know about this rectangle?

Student 5: The side is 6 feet.

Wendy: What else?

Student 6: So, there is another side that is 6 feet.

Wendy: Who else can tell me something different that we know?

Student 7: The whole perimeter all the way around is 18.

Wendy: Okay, and what do we need to find out?

Student 9: The width of the rectangle.

Teacher: So we know the perimeter and the length. What equation do we use?

Student 3: We can use P = 2L + 2W.

Teacher: Alright, who has a plan for us to use?

Student 7: You could guess numbers. Try out 3 and 3… see if it makes 18.

Teacher: Can you check to see if it works?

Student 4: Uh… it does. Because 12 and 6 is 18.

Teacher: Can anyone think of a way that is mathematical – without using guess and check and using an equation?

Student 11: Uh… well you can write 18 = 2(6) + 2W and then solve for width.

(Observation, 4/25/2022)

In this example, Wendy elicited participation from seven of her 14 students. By asking more questions, she provided more opportunities for individual students to say something during whole group discussion. However, her responses and questions did not respond directly to students' thinking nor did she probe for additional explanations of the responses students offered. Rather, Wendy's questions led students toward getting the correct answer by using a specific equation to solve for the missing side. Students participated by answering the teacher's questions, which led to more individual students having the opportunity to engage. The increased engagement was more accurately described as partially univocal discussion more than organic, student-led learning.

Teachers Repeating Student Voices

Throughout all three grade levels, I witnessed teachers repeating student ideas during mathematics discussions. Although not directly related to their mathematics curriculum, teachers learned to repeat student ideas and comments through their English Language Arts Curriculum. The reading curriculum, as described by Wendy, emphasized building a reading community through talk and carefully listening to one another (Interview, 4/26/2022, 5/11/2022). Wendy said that repeating helps to build a listening community because it showed that you were an active listener. Additionally, repeating allowed for the teacher to "reiterate [ideas] for students that might have missed what others said" (Interview, 4/26/2022). Each teacher acknowledged that repeating was a way to ensure that all students heard a key idea brought out during the mathematics discussion. The ways in which teachers repeated student ideas, however, both limited and supported student-led discussions. Through analysis of the *repeating* code, I found that teachers followed up *repeating* with either a *statement* or a *question*. When teachers repeated followed by a question (n=18), repeating supported more student voice. For example, when a student said that perimeter is the "space outside", Wendy responded by saying "Chris said perimeter is the space outside the shape. Can someone expand on that?" (Observation, 4/21/2022). By following up with a question that prompted further explanation, teachers repeated students and then increased student voice in discussion. On the other hand, when teachers repeated followed by a statement (n=34), repeating limited student-led discussion and student voice. For example, on another day in Wendy's class, she again asked students to define perimeter. This time when a student replied, "the measurement around a shape", Wendy responded with "Yes, so perimeter is the measurement around a shape. We find this measurement by adding up all the sides." (Observation, 4/25/2022). In this example, Wendy repeated a student's idea and then limited student voice by following-up with an additional statement, rather than question. In her explanation, Wendy's voice and elaboration became the center of the explanation for the students to hear.

Repeating as a Limitation

All teachers explained that they repeated students to ensure that students heard the students' idea, answer, or strategy (see Table 4.3). More specifically, teachers noted that when the teacher repeated the idea, it was more likely to bring attention to the idea. Both Wendy and Nancy mentioned that students did not always listen to their peers but were more likely to listen to the teacher. If teachers perceived that students did not listen to the ideas of their peers, than it was likely that they believed that students listened more to the teacher-led discussion, explanation, and lessons. Additionally, teachers emphasized that repeating allowed students to hear a key idea, which reiterated the idea that students could learn from *hearing* the ideas of others.

Table 4.3

Teachers Comments on Repeating Ideas

Teacher	Comment
Mrs. Anna Marzano	"I'll repeat it loudly… So I'll say S12 said she saw '3 and 2 make 5. Did anyone see it a different way?' **I will use the repeating factor to make sure everyone heard what S12 said**. There are lots of mini conversations going on at the same time. So, me repeating **loud enough for everyone to bring their attention back up**." (Interview, 4/4/2022)
Mrs. Nancy Staples	"I repeat what students say for a few reasons. **One, it allows me to say it again for those who inevitably weren't listening or possibly didn't hear it clearly**. They don't always listen to each other. Two, it validates what the student said - telling them it's okay to share…. Sometimes when I repeat what the student says I might use words that are a little more precise or clear." (Interview, 5/22/2022)
Mrs. Wendy Grimes	"I also do it **to reiterate it for students that might have missed what others have said**... So, if I repeat it, **they know that when I am talking, they should listen to me**. So, I can maybe catch 1 or 2 students that might have logged off. Then, I can push others to join in from there. Repeating can help us to listen to others, build off what one another have said, or disagree with others in a kind way. All of that is part of our reading curriculum, too." (Interview, 4/25/2022).

During classroom discussions, I observed teachers repeat student ideas as an introduction into a mini-lesson or univocal explanation from the teacher. Teachers drastically extended student responses to explain mathematical concepts or procedures, which contributed to teacher-led instruction implanted in the middle of a discussion. The following is an excerpt from Nancy's third grade classroom:

Prompt: Which fraction is greater ¼ or ¾? How do you know?

Nancy: What do you notice about these fractions?

Student 7: They both have a 4 on the bottom.

Nancy: Yes, they both have a 4 on the bottom. So, they are divided into the same number of pieces. Even if we didn't have a model, we might be able to figure out

> if they are the same on the bottom, that 3 pieces is greater than 1 piece. But that only works when we have the same kind of pieces and the same number on the bottom of the fraction.
>
> (Observation, 3/30/2022).

This excerpt from Nancy's discussion resembled a form of the IRE pattern of discussion. Nancy began by asking students what they notice about the fractions in the given problem (*initiate)*. One student *responded* to the teacher's question. Then, Nancy immediately *evaluated* the student, positively affirming the response, and then repeated what the student said. Immediately following repeating the student's response, Nancy provided a detailed explanation of how to compare fractions with the same denominator. Rather than ask the student to elaborate further or ask another student to extend upon the response, she limited student voice by shifting the focus of conversation to a teacher provided explanation. This explanation was almost entirely teacher-led (univocal) and did not offer students the opportunity to justify their mathematical thinking with one another (dialogical). Approaching discussion from an IRE stance hindered dialogic discussion by limiting student voices and the opportunity for students to justify their mathematical thinking.

In instances such as the excerpt above, teachers repeated student ideas and provided an additional explanation which led to univocal mini-lessons embedded during a discussion. When analyzing teachers use of repeating, I found examples of teachers repeating student ideas, which led to an mini-lesson or mathematical explanation centered around teachers' ideas rather than student ideas (see Table 4.4).

Table 4.4

Repeating as a Limitation of Student Discussion

Grade Level	Examples
Third Grade	**S3**: Well, it's between 100 and 200…. So 150 **Nancy**: That's correct. It is 150 because it is in the middle. We can think about it that way – the midpoint. There are lines in the middle of the scale and we need to check to see what those lines mean because they may not always mean 10s. (Observation, 4/7/2022)
Fourth Grade	**Wendy**: Are there any rectangles? **S1**: Yes, the basketball court is a rectangle. **Wendy**: Yes, it's a rectangle. We know that because there are two sets of different sides, which means it's a rectangle. (Observation, 4/20/2022)

In both excerpts, teachers repeated student ideas and provided an explanation that included their own thinking rather than allowing space for student thinking. In both segments of dialogue, the teacher missed an opportunity to elicit student thinking by repeating the student's initial response and then launching into their own teacher explanation. As a result, teachers initiated more teacher-led talk (univocal) instead of engaging multiple students in formulating mathematical arguments to justify their ideas (dialogical). In their descriptions of meaningful discussions, teachers shared that student-led discussions, where students provided the explanation, were more meaningful. Yet, in contrast to those stated beliefs, teachers repeated and extended student ideas which led to teacher-led explanation rather than student-led discussion.

Repeating as a Support

Although repeating often surfaced as a limitation of student discussion, teachers supported student-led discussion when they repeated student ideas followed by a question. In these instances, they provided opportunities for students to voice their own ideas, offer explanations of their responses, or engage with the ideas of their peers. The following excerpt is an example of how Anna positioned student thinking at the center of the discussion during a lesson on addition.

> **Anna**: How many of the friends have their hair up?
>
> *students showing various numbers on their fingers*
>
> **Anna**: Student 11, you said that there were 3. Can you point to them?
>
> **Student 11**: 2 are here and 1 is here.
>
> **Anna**: So, 2 and 1 make how many?
>
> **Student 11**: 3
>
> **Anna**: S11 saw 2 and saw 1. Did anyone see it a different way?
>
> **Student 4**: I saw 3 too… but the turnaround fact. I saw 1 first and then 2!
>
> **Anna:** Student 4 mentioned the turnaround fact. Did anyone else see it that way?
>
> (Observation, 4/4/2022)

In this excerpt of discussion, Anna repeated students without adding additional information and instead invited students to come to the board to make their thinking visible to their peers. She positioned student thinking at the center of the discussion by inviting students to speak from the front of the classroom and asking questions that directly responded to student ideas. Additionally, she encouraged multiple ways of thinking about the picture by encouraging other students to share how they saw 3 with turnaround facts (emerging dialogical). The inclusion of sharing multiple ways of thinking shifted toward a more dialogical discussion because it valued the voices and thinking of multiple students.

Although Anna said that she repeated students' so everyone could hear, I also observed Anna asking students to repeat their own ideas. "Could you please repeat that again for everyone to hear?" This teacher move centered student voice, rather than teacher voice, as the source of a key idea. Anna set the precedent that students shared information, not just the teacher. The exchange below demonstrated how Anna prompted a student to repeat themselves, which led to students engaging with the ideas of their peers.

Anna: What do you see in this picture?

Student 9: I see 3 spilled and now there are 5 left.

Anna: Oh S9 sees a number story! Can you repeat that S9?

Student 9: I see 3 spilled and now there are 5 left.

Student 6: Oh, oh! I can write what S9 said as a number sentence!!!

Student 4: And then we can make a turnaround fact.

Anna: Do you want to come write the number sentence, S6?

(Observation, 4/12/2022)

In this exchange, Student 6 responded directly to Student 9 with a new mathematical idea. By asking Student 9 to repeat their idea, Anna focused student thinking at the center of the discussion. Student 6 generated a number sentence to match the number story that Student 9 shared. Additionally, Student 4 continued to think of different ways of representing the story with a turnaround fact. The students did not respond to Anna's thinking or voice but engaged with and responded to their peers, which aligned more closely with dialogical discussion than univocal discussion.

Summary of Finding 1

Although teachers shared that meaningful mathematics discussion was student-led, teachers in this study often led discussion in a teacher-centric way. Teachers frequently maintained control of the narrative by asking questions which funneled student thinking, rather than responding to student thinking. Additionally, teachers described student-led discussions as an opportunity for students to learn from one another; yet, this learning was limited to struggling students hearing the correct solutions and strategies from their more academically capable peers. Finally, teachers often engaged in univocal explanations while repeating student ideas. Anna, however, demonstrated how teachers could repeat student ideas and ask questions which maintained student thinking at the center of the discussion. When considering the spectrum from univocal to dialogical discussion, teachers often asked questions that solicited student responses, which aligned most closely with partial univocal discussion. Furthermore, the quantity of questions increased student participation as a form of engagement but did not increase student participation in a way that produced more dialogical discussion.

Finding 2: Teachers were heavily reliant on the curriculum's suggested questions to plan for and orchestrate mathematics discussion, which limited meaningful mathematics discussion.

Math in Focus served as the foundation of mathematics instruction at Barron Academy. Administrators communicated that the school-wide expectation was for teachers to use the *Math in Focus* curriculum for daily mathematics instruction. Both teachers and administrators explained their expectations about curriculum to me at the start of this capstone study. Teachers could use their discretion to adapt or supplement curriculum to support students, however, teachers should begin planning for mathematics with the curriculum materials. The expectation was that if teachers chose to adapt or omit components of the curriculum from their instruction, they should be able to provide justification for their instructional decision (Personal Communication, 9/10/2022). Through analysis of all data sources, I found that teachers were heavily reliant on the curriculum for planning and orchestrating mathematics discussion. I explain this finding by first describing the curriculum, then how teachers enacted lessons during observations, and finally how teachers described planning for the enacted lessons.

Math in Focus Curriculum Review

Barron Academy adopted *Math in Focus* for kindergarten through fifth grade during the 2015-2016 school year and implemented a new version of the curriculum in the 2021-2022 school year. Teachers accessed the *Math in Focus* curriculum through hard-copies of the student and teacher textbooks, as well as online access. The *Math in Focus* curriculum was comprehensive and included nearly scripted lesson plans, pacing guides, assessments, and lesson materials for an entire year. A *Math in Focus* chapter, or unit of study, was separated into three parts, the Chapter Opener, Learning Sections, and Chapter Wrap Up. The Chapter Opener was a one-day lesson that introduced students to the essential question for the chapter and recalled prior knowledge related to the new content. An example of an essential question for a unit on subtraction was "How do we subtract numbers?" (Marshall Cavendish Education, 2020b). Then, each chapter included about five sections or lessons for learning. Each section included a *Think*, *Engage-Learn-Try*, independent practice, and supplemental additional activities (e.g., game, differentiated instruction, *math sharing*, *math talk*, etc.). A single section was not intended for one day; rather, it took two or three days to teach. Following the learning sections, the chapter concluded with various wrap-up resources such as performance tasks, STEAM projects, cumulative chapter reviews, and assessments.

In the *Research Foundations Paper: Math in Focus,* Marshall Cavendish Education (2020c) identified math talk as one of the key instructional features because it "is an increasingly important way for students to learn and make sense of mathematics" (p. 50). Additionally, it identified NCTM's teaching practices and CCSSI SMP's as initiatives which influenced the development of the curriculum (NCTM, 2014; NGA Center, 2010). As a result, I reviewed the curriculum to identify evidence of how the curriculum supported opportunities for meaningful mathematics discussion as it relates to opportunities for math talk. I reviewed eight learning sections within the curriculum for the capstone study (Table 4.5).

Table 4.5

Lessons from Math in Focus Reviewed for Capstone Study

Grade	Chapter	Section Reviewed
Kindergarten	7: Addition	Section 5: Addition Sentences
	8: Subtraction	Opener
	8: Subtraction	Section 1: Subtraction Stories
3rd	6: Fractions	Section 5: Comparing Fractions
	8: Measurement	Opener
	8: Measurement	Section 1: Mass: Kilograms and Grams
4th	6: Area and Perimeter	Opener
	6: Area and Perimeter	Section 1: Area and Unknown Side

In kindergarten, I reviewed three sections across two chapters. I reviewed the final section in an addition chapter, which was noted to take two days. In this section, students developed and wrote addition stories and number sentences to 10. I also reviewed the opener and initial section of the subtraction chapter (Appendix L), where students were introduced to subtraction for the first time. *Math in Focus* introduced subtraction through subtraction stories where something left (e.g. cats ran away) or was taken away (e.g. someone ate cookies).

I reviewed three sections across two chapters in third grade. I reviewed the final section of the fractions chapter, which centered around students comparing fractions using visual models (Appendix M). Next, I reviewed the following first two sections of the Measurement Chapter. The opening section introduced measurement of mass and liquid volume by recalling what students learned about measurement from third grade and introduced students to thinking about solving mathematical problems involving measurements. The final section I reviewed in third grade focused on measurement of mass using grams and kilograms. In this section, students practiced reading scales and solving multi-step word problems involving mass.

Finally, I reviewed two sections of the measurement unit in fourth grade. This chapter focused on measurement in terms of area and perimeter. In the opening lessons, students were challenged to think about how to find the area of a composite shape using their knowledge of the area of rectangles from third grade. The opening section also included practice problems for review from third grade. The first learning section was designated as 2-3 days in the pacing guide (Appendix N). This learning section taught students how solving for an unknown side of a rectangle when given either area or perimeter.

Through my review of the curriculum, I found that individual learning sections provided many opportunities for students to engage talk with their peers and the teacher about mathematics. The daily lessons presented these opportunities through recommended questions for discussion and the focus mathematical habits.

Opportunities for Talk through Recommended Questioning

Each *Math in Focus* section included multiple opportunities for students and teachers to talk about mathematics in both whole group and small-group settings. Across all grade levels, each component of the lesson included specific opportunities for students to participate in talk or discussion about mathematics. These opportunities were noted in the TE's textbook with a blue speech-bubble. Each opportunity included specific directions for the teacher, bolded teacher questions, and anticipated student responses in pink (see Figure 4.5). Directions to teachers included various phrases, which suggested how teachers should facilitate discussion or talk. These phrases included "facilitate discussion using these questions", "encourage and guide student thinking with these questions", "facilitate productive struggle by asking these questions", "extend student thinking" or "encourage students to discuss with their partners and ask these questions". These phrases supported the idea that students are being provided with an opportunity to talk about math so that mathematics instruction was not entirely univocal from the teacher. Each direction was followed by a series of suggested questions for teachers to ask students.

Figure 4.5

Opportunity for Talk in Kindergarten Lesson (Marshall Cavendish Education, 2020b, p. 123)

For 1, students are required to make a subtraction story about the monkeys. Encourage students to discuss with their partners and ask these questions:

What subtraction story can we make? How many monkeys are there in the tree? (9) **How many monkeys swing away?** (4 monkeys swing away.) **How many monkeys are left in the tree?** (5 monkeys are left in the tree.)

To better understand the nature of questions provided in the curriculum, I coded questions as *information seeking, elicit student thinking,* and *different strategy*. The questions provided in the curriculum were primarily information seeking questions that sought to identify:

- A specific *answer*
- A definition or description of *vocabulary*

- A *procedure* of what students did to solve a problem
- A *noticing* or observation about a picture, problem, or context
- Prior-knowledge or scaffolding that will help *orient and focus* student thinking

Questions were often repeated throughout the learning section. For example, for each of the individual practice problems within the *Try* or independent practice, were at least three similarly structured questions adapted to the context of the individual problem. For example, within the 4 practice problems for subtraction, teachers were suggested to ask nearly identical questions to help students create subtraction stories for each problem (see Figure 4.6). As evidenced by Figure 10, suggested questions often had similar structures but were modified to match the specific context of the problem. Additionally, similar questions were suggested during other parts of the lesson as students were introduced to creating subtraction stories.

Figure 4.6

Repeated Questioning within a Kindergarten Lesson (Marshall Cavendish Education, 2020b, p. 123)

PRACTICE **(pages 123 to 126)**

You may allow students to access the **PRACTICE** questions on **Ed**, which are auto-graded, for students to consolidate learning in the section digitally. For questions that require students to show their work, have them complete the questions in the book.

1 to 4 assess students' ability to make subtraction stories with each given picture.

For 1, students are required to make a subtraction story about the monkeys. Encourage students to discuss with their partners and ask these questions:

What subtraction story can we make? How many monkeys are there in the tree? (9) **How many monkeys swing away?** (4 monkeys swing away.) **How many monkeys are left in the tree?** (5 monkeys are left in the tree.)

Have students go through the subtraction story again in 1. This time, have students cross out the monkeys that swing away. Then, count the remaining monkeys.

For 2, students are required to make a subtraction story about the bees and fill in each blank. Encourage students to discuss with their partners and ask these questions:

What subtraction story can we make? How many bees are there? (8) **How many bees fly away?** (5 bees fly away.) **How many bees are there left?** (There are 3 bees left.)

Mathematical Habits

Every *Math in Focus* lesson included specific mathematical habits for students to practice throughout the lesson. These mathematical habits were derived from the CCSSI (NGA Center, 2010) eight SMP (Marshall Cavendish Education, 2020c). Each lesson, at every grade, included at least one focus mathematical habit. These habits were noted within the chapter planning guide (Figure 4.7) and within each lesson in the TE's textbook (Figure 4.8).

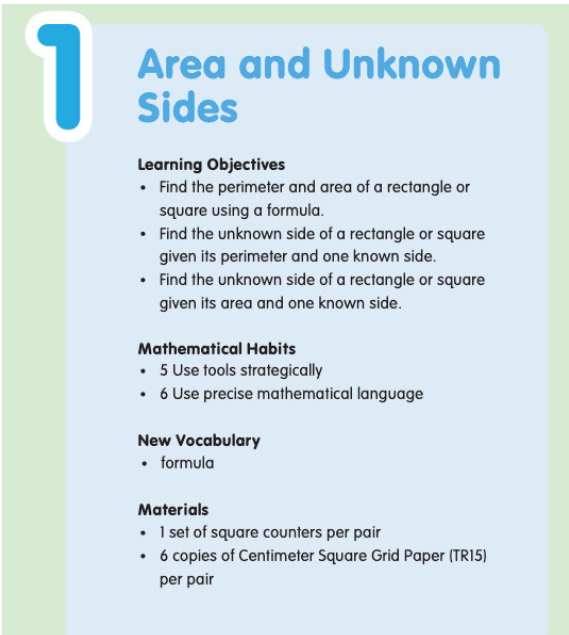
Figure 4.7

Excerpt from the Chapter 6 Planning Guide for Fourth Grade (Marshall Cavendish Education, 202b, p. 101F)

Total pacing: 10 days	Chapter Opener, Recall Prior Knowledge Pages 101 – 106 Pacing: 1 day	1 Area and Unknown Sides Pages 107 – 128 Pacing: 2 days
Pacing	DAY 1 of 10	DAY 2 of 10
Learning Objectives	**How do you find the perimeter and area of a rectangle or square using a formula? How do you find an unknown side of a rectangle or square, given its area or perimeter?** • Review related concepts from previous chapters or grades.	• Find the perimeter and area of a rectangle or square using a formula. • Find the unknown side of a rectangle or square given its perimeter and one known side.
New Vocabulary		formula
Materials	• 1 copy of Centimeter Square Grid Paper (TR15) per student • 1 set of square counters per student, for extra support	• 1 set of square counters per pair • 3 copies of Centimeter Square Grid Paper (TR15) per pair
Instructional Resources	• **Student Edition 4B**, pp. 101 – 106	• **Student Edition 4B**, pp. 107 – 117 • **Extra Practice and Homework 4B**, Activity 1 • **Reteach 4**, Activity 1 • **Enrichment 4**, Activity 1
Mathematical Habits	• 6 Use precise mathematical language • 7 Make use of structure	• 5 Use tools strategically • 6 Use precise mathematical language
Fact Fluency		Fact Strategy Practice, p. 77

Figure 4.8

Excerpt of the Lesson Overview from the Fourth Grade TE Textbook (Marshall Cavendish Education, 2020b, p. 107)


1

Area and Unknown Sides

Learning Objectives
- Find the perimeter and area of a rectangle or square using a formula.
- Find the unknown side of a rectangle or square given its perimeter and one known side.
- Find the unknown side of a rectangle or square given its area and one known side.

Mathematical Habits
- 5 Use tools strategically
- 6 Use precise mathematical language

New Vocabulary
- formula

Materials
- 1 set of square counters per pair
- 6 copies of Centimeter Square Grid Paper (TR15) per pair

Two of the eight habits, “construct viable arguments” (MP3) and “use precise mathematical language” (MP6) specifically support mathematics discussion. These habits were incorporated throughout the curriculum through math talk opportunities in the lesson, which “engage and guide students in productive, collaborative speaking and listening” (Marshall Cavendish Education, 2020c, p. 51). Through the lessons reviewed for this capstone study, I found that there was a greater emphasis on use of precise mathematical language (see Table 4.6).

Table 4.6

Mathematical Habits Included in the Lessons Reviewed

Grade	Learning Section	Mathematical Habit	
		MP3	MP6
Kindergarten	Ch. 7 – Section 5	X	X
	Ch. 8 – Opener		
	Ch. 8 – Section 1	X	
3rd	Ch. 7 – Section 5		X
	Ch. 8 - Opener		X
	Ch. 8 – Section 1		X
4th	Ch. 6 - Opener		X
	Ch. 6 – Section 1		X

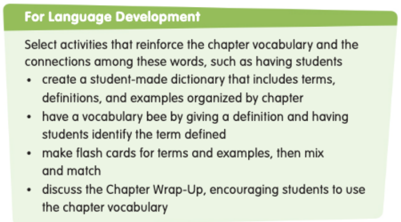
Within each section, the degree to which the habit was referenced varied. In some lessons, the habit was only directly referenced on the Chapter Planning Guide and on the first page of the lesson. There were no further instructions, description, or description for how teachers focused instruction on that mathematical habit, specifically. In other lessons, however, various activities or instructional notes referenced the habit.

Use Precise Mathematical Vocabulary. Six of the eight lessons reviewed highlighted "use precise mathematical vocabulary" (MP6). This habit was referenced throughout the TE's textbook through notes for language development, an emphasis on discussing vocabulary, and specific mathematics problems or activities. The references, however, were inconsistent across grade levels and within the chapters reviewed.

Specific notes for language development that were intended to help teachers reinforce vocabulary and language for students, specifically English Learners or students with language difficulties. Notes for language development were always included in the Chapter Overview following the Chapter Planning Guide (see Figure 4.9).

Figure 4.9

Suggested Note for Language Development in Third Grade Chapter Overview (Marshall Cavendish Education, 2020b, p. 65I)


For Language Development

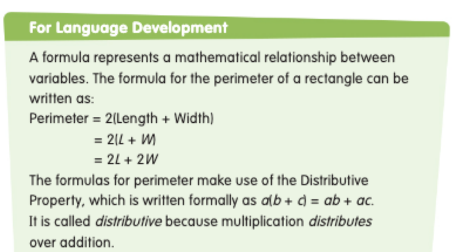
Select activities that reinforce the chapter vocabulary and the connections among these words, such as having students

- create a student-made dictionary that includes terms, definitions, and examples organized by chapter
- have a vocabulary bee by giving a definition and having students identify the term defined
- make flash cards for terms and examples, then mix and match
- discuss the Chapter Wrap-Up, encouraging students to use the chapter vocabulary

The language development notes in the chapter overview included multiple tips to reinforce vocabulary that teachers could use at any point throughout the chapter. The presence of notes for language development was inconsistent throughout the sections within a chapter. Some sections did not include any specific notes for language development, while others did. Of the sections reviewed, none of the third-grade lessons included any specific notes for language development. However, kindergarten and fourth grade did include a specific note for language development in at least one section (see Figure 4.10). The language development note within the section was specific to the content taught during that section.

Figure 4.10

Suggested Note for Language Development in Fourth Grade Section (Marshall Cavendish Education, 2020b, p. 109)


For Language Development

A formula represents a mathematical relationship between variables. The formula for the perimeter of a rectangle can be written as:

Perimeter = 2(Length + Width)

$= 2(L + W)$

$= 2L + 2W$

The formulas for perimeter make use of the Distributive Property, which is written formally as $a(b + c) = ab + ac$. It is called *distributive* because multiplication *distributes* over addition.

In addition to notes for language development, vocabulary was emphasized throughout some of the lessons that included MP6 as a focus through suggested questions that reviewed vocabulary. The *Math in Focus* TE textbook included bolded questions that were intended for teachers to ask students during the lesson. Some of these questions include questions that pertain directly to vocabulary:

- What does it mean to find the volume? (Third Grade, Ch. 8 – Opener)
- What is length? What is width? (Fourth Grade, Ch. 6 – Section 1)

Like the notes for language development, these questions reinforced vocabulary use during the section. In the kindergarten section that highlighted MP6, however, there were no suggested questions that specifically referenced vocabulary or language use, despite it being a section that focused on the use of mathematical language.

Finally, some of the sections that included MP6 as a habit focus included specific activities or mathematics problems that explicitly mentioned their relation to the habit. In third grade, a *math sharing* activity emphasized the use of precise mathematical language by listing the mathematical habit in the activity (see Figure 4.11). As seen in the figure, the *math sharing* activity explicitly suggests students use precise mathematical language. In fourth grade, a hands-on-activity specifically referenced MP6 in one of the questions (see Figure 4.12). Within the TE's textbook, however, there was no description or instruction for how the question specifically supported students in using precise mathematical language. The directions did not include any notes for teachers to tend to language use such as common language mistakes students may make when discussing these questions or notes for language development.

Figure 4.11

Math Sharing Highlighting MP6 in Third Grade (Marshall Cavendish Education, 2020b, p. 74)

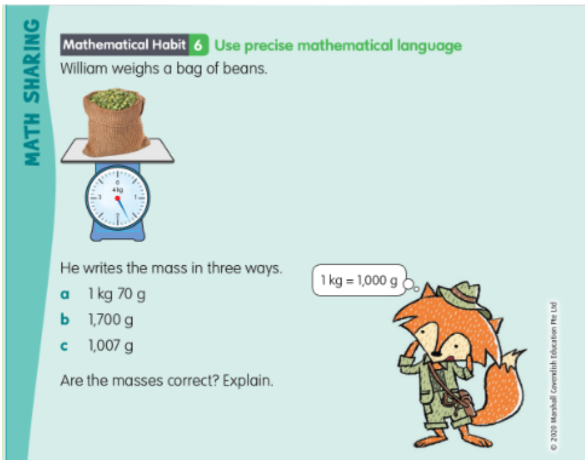
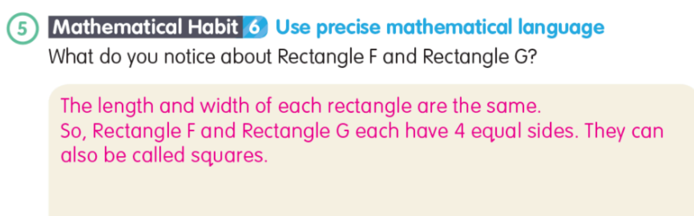


Figure 4.12

Hands-on-activity Highlighting MP6 in Fourth Grade (Marshall Cavendish Education, 2020b, p. 114)



Construct Viable Arguments. Only two of the lessons reviewed listed "construct viable arguments" (MP3) as the mathematical habit focus for the lesson. Unlike "use mathematical language", there was no explicit reference to this habit within the suggested lesson through instructional notes in the TE's edition nor noted on an activity. Within the two kindergarten lessons that highlighted MP3, the written *Math in Focus* lesson lacked any description or direction for how teachers should support students in constructing viable arguments throughout the lessons or where students may demonstrate their ability to do so.

The *Research Foundations Paper* (Marshall Cavendish Education, 2020c) noted that essential questions support this mathematical habit. Questions such as, “Does this argument make sense?” or “Has this been proven?”, aligned with MP3. (p. 46). However, the kindergarten lessons that focused on MP3 did not include suggested questions that resembled the questions noted in the *Research Foundations Paper*. The lack of support related to this habit in the TE textbook is notable because constructing viable arguments is critical for students to engage in dialogical discussions. Dialogical discussions are built upon student arguments and justifications.

Enactment of Mathematics Discussion

Given the previous description about the *Math in Focus* curriculum, I now present insight about how teachers used the curriculum and suggested questions to orchestrate mathematics discussions. As part of my analysis, I compared the curriculum to the observed lessons and analyzed teacher utilization of specific questions. All three teachers asked most of the questions as suggested in the *Math in Focus* text (see Table 4.7). Questions were utilized when teachers asked questions verbatim or adapted the question but retained the focus of the question. As seen through the counts in Table 12, teachers utilized more questions from the curriculum than they omitted. A greater explanation of the ways in which teachers utilized the curriculum as observed during lessons in comparison to the specific sections is provided for each teacher.

Table 4.7

Utilization of Suggested Questions from the Curriculum as Observed

Grade	Learning Section	Questions Utilized	Questions Omitted
Kindergarten	Ch 7 - 5	51	12
	Ch 8 - Opener	14	2
	Ch 8 - 1	36	7
3rd	Ch 7 - 5	24	12
	Ch 8 - Opener	18	3
	Ch 8 – 1	35	14
4th	Ch 6 - Opener	38	6
	Ch 6 – 1	39	9

Observed Use of Curriculum in Anna's Class

In kindergarten, I observed Anna teach three *Math in Focus* sections over four days. For each lesson observed, Anna displayed pages of the Student Workbook or section PowerPoint on the smartboard to guide the lesson. The smartboard always displayed images and mathematics problems from the *Math in Focus* curriculum. Additionally, students used their own workbooks when completing practice problems as instructed by the curriculum.

The following scenario demonstrates how Anna modified the curriculum to orchestrate a discussion during a lesson addition at the beginning of a lesson. The curriculum provided two options for the *Engage* activity at the beginning of the Chapter 7 – Section 5 lesson on addition. The curriculum suggested the following:

> Activity 1: Prepare two groups of objects in different quantities. Place them together and have students point and count them. **I have two teddy bears and three toy soldiers. How many toys do I have in all? Let's point and count to add**. Repeat the activity with different quantities and items. Encourage students to demonstrate how they point and count.

OR

Activity 2: Fill up a ten frame with counters. **What do you notice about the counters?** (There are 3 blue counters and 7 red counters). **How many counters in all?** (10). **What number sentence can we make?** (Answers may vary. Example: 3 + 7 = 10) (Marshall Cavendish Education, 2020c, p. 89)

Rather than choose one activity, Anna modified the *Engage* activities to practice both counting and creating number sentence with students on the carpet. She adapted Activity 1 by drawing shapes on the smartboard to represent addition, rather than use physical objects. Anna used questions from both activities to engage students in counting and creating number sentences. She followed the instructions as provided in the curriculum and rephrased the bolded questions to reflect the problem she drew on the board.

Anna: So, what have we been talking about in math?

S3: We are learning to add!

Anna: What does add mean?

Students: put together!

Anna: So, if I draw 3 triangles and 2 circles, what do I have in all?

Students: "You have 5!" "3 and 2 makes 5"

Anna: What did you do to get 5?

S5: I counted them on the board. (points to shapes on board)

S4: I saw 2, 4, and 1 more. (points to shapes on board)

Anna: Someone said 3 and 2 makes 5. What is the number sentence for that?

S6: 3 + 2 = 5

Anna: Can someone share the turnaround fact?

S10: 2 + 3 = 5

(Observation, 4/4/2022)

Although Anna modified Activity 1, she retained the intended purpose to add to 5. She questioned her students by asking for *answers* (How many in all? What number sentence can we make?) and explain counting *procedures* (What did you do to get to 5?). Both activities included questions with specific answers and procedures. The recommended questions neither elicited student thinking nor prompted students to explain *how* or *why*. In following the TE, Anna did not ask additional questions that prompted students to explain their thinking. The questions provided in both activities were primarily information seeking, which led to Anna asking questions and students responding directly to her. Although Anna did not recite questions verbatim from the text, she did include most of the recommended questions as they applied to her modified activity (see Table 4.8).

Table 4.8

Comparison of Observed Questions to Curriculum's Suggested Questions in Kindergarten

Observed Questions (Observation, 4/4/2022)	Corresponding Suggested Question (Marshall Cavendish, 2020b)
If I draw 3 triangles and 2 circles, what do I have in all?	How many toys do I have in all? How many counters in all?
What does add mean?	
What did you do to get 5?	
What is the number sentence for that?	What number sentence can we make?
Can someone share the turnaround fact?	

When enacting the lesson, Anna omitted one of the recommended questions, which prompted students to share what they noticed. Instead, she initiated discussion by asking students to recall what they had been learning about in mathematics. As noted in the classroom observation excerpt, Anna did ask questions that were not included in the activity. For example, she included questions about the meaning of add and turnaround facts. The additional questions that Anna asked were additional *information seeking questions* that related to the activities. As to be expected, when Anna asked the questions provided in the TE, the conversation around the activity resulted in a series of exchanges between teacher and student. Anna asked questions and individual students responded. The conversation was not dialogical in nature because students were not engaging in conversation with one another about their thinking; rather, they were responding to specific questions from the teacher with desired information.

As seen in the previous example and Table 4.8, Anna asked nearly all the recommended questions provided for discussion in the TE's textbook. When questions were omitted, typically an entire recommended questioning sequence was omitted. For example, the curriculum suggested that teachers ask a series of repeated questions for four *Try* problems (see Figure 4.6). However, Anna only asked the questions for three of the four problems, thus omitting several questions. Anna also omitted specific questions when time required her to end the lesson without having covered every single mathematical prompt, question, or supplemental activity within the learning section.

Anna's Planning for Discussion

When asked about general planning for mathematics instruction, Anna reported, "I use the book a lot! It is helpful to see everything in one place – the workbook pages, the questions I should ask, the materials… its great!" (Interview, 4/11/2022). Throughout interviews, Anna referred to the TE's textbook about planning and describing her instructional choices. Anna described her planning process by saying, "I sit down and read through it… I read through it a couple of times. I talk about it with the other kindergarten teacher, and we discuss it to make sure we both understand what we are supposed to do" (Interview, 4/11/2022). She reiterated that "having a partner to plan with and read [the lesson] through together" is something that she enjoys (Interview, 4/12/2022). Anna emphasized that her planning process is driven by reading the lesson, as written directly from the TE textbook.

Additionally, Anna mentioned multiple times throughout our interviews how helpful and comprehensive she found TE textbook was. Anna stated in an interview:

> With so many questions they want me to ask, I can't get to them all without them losing focus or getting off task! I might not ask all the questions they have, or I might ask different ones that are still similar. Other times, I realize I don't need to ask the same questions again and again. So, I don't ask everything. I mean, I can't have my book attached to me – it's just not feasible and I wouldn't expect any teacher to. But as long as I am going about what they are trying to ask and trying to touch on.
>
> (Interview, 4/4/2022)

As evidenced by Table 11 and this quote, Anna aimed to ask the recommended questions that she felt were appropriate for her class. Although she did not read questions directly from the book, she utilized and modified the questions in the book for her classroom. She modified phrasing but retained the core focus of the suggested questions. Anna omitted specific questions when it may have seemed repetitive or unnecessary for student learning or because of time.

Anna also reported that she felt the recommended questions were "unnecessarily redundant" and that she was "asking the same type of question, just in a different way"

(Interview, 5/11/2022). She said that at times this felt repetitive and unnecessary during class. On the other hand, it was also "redundant in a very good way" because it not only helped her to learn the style of questions but also helped students to anticipate the types of questions she might ask. The *Math in Focus* textbook was very cyclical with lessons asking similar questions and following a similar lesson structure. Kindergarten students in Anna's class were accustomed to the structure of the *Math in Focus* questions because, as was expected of teachers at this school, Anna relied heavily on the *Math in Focus* textbook for her mathematics planning and discussion questions.

Notably, Anna did not mention the mathematical habits in her discussion of planning using the TE textbook. The lessons, which I reviewed, were intended to help students "construct viable arguments" (MP3). As previously mentioned, there was no explicit reference to this habit throughout the suggested questions or lesson activities. When I asked Anna about the curriculum and her planning process, she did not mention how the mathematical habit impacted her planning process or the enactment of the lesson.

Observed Use of Curriculum in Nancy's Class

Similarly, Nancy also used the *Math in Focus* curriculum as the guide for teaching in her third grade mathematics classroom. The *Engage* component of the *Math in Focus*'s lesson on comparing fractions instructed students to compare two sets of fractions. The first question prompted students to "use fraction tiles to show $^{3}/_{8}$ and $^{5}/_{8}$" (Marshall Cavendish Education, 2020b, p. 41). The TE's textbook included the following directions for the first question:

> Invite students to work in pairs and provide each with a set of fraction tiles. Use the questions to prompt and guide student thinking. **What are we asked to do?** (Use

fraction tiles to show $^3/_8$ and $^5/_8$) **How would we use the fraction tiles to compare the two fractions?** (Place a fraction tile completely underneath another to compare).

The TE textbook recommended an additional question, which was intended to "extend students' thinking by asking the following question: "What do we notice about the denominators and numerators in this ENGAGE?" (p. 41). The TE directed teachers to allow students time to discuss their work and encourage students with different reasoning to share their explanation with the class.

I observed Nancy implement this *Engage* activity very similarly to the instructions provided in the textbook. Prior to the following scenario, Nancy provided students with time to work on the engage task with partners. The excerpt of discussion began after students worked in partners and after Nancy has called student attention back to a whole group discussion.

Nancy: What are we being asked to do?

S3: Compare fractions

Nancy: And which fractions?

S7: three-eighths and five-eighths

Nancy: So, how could we compare them?

S9: Look at the numbers.

Nancy: We do need to know the numbers to build them. How does building it with fraction tiles help us compare them?

S7: You can see which one is longer.

Nancy: Right, and what do we notice about the denominators and numerators?

S3: The bottom numbers are the same.

Nancy: Which one is the bottom number – numerator or denominator?

Students: denominator

Nancy: Alright, if you haven't already, go ahead and build the fractions to compare them.

S5: 5/8 is bigger.

Nancy: How do you know?

S5: It's longer (pointing to the tiles)

(Observation, 3/30/2022)

Nancy asked all of the corresponding questions for discussion and delivered additional questions that responded to students. The books questions anticipated responses and students did not always respond as anticipated. For example, Student 7 suggested comparing the fractions by just looking at the numbers, which was not the desired strategy. As result, Nancy employed the suggested question in the text, which asked included fraction tiles as the desired strategy.

As noted in Table 4.9, she asked all of the suggested questions for this problem or asked adapted questions that asked the same idea. Nancy asked additional questions, not included in the text, which reminded students of the desired strategy from the textbook and to use more precise language. In the previous example, she asked, "How do we compare fractions?" and when a student responded with "we look at the numbers", she asked an additional question to focus students toward comparing fractions with fraction tiles, which was the curriculum's desired strategy. She asked additional questions (Which is the bottom number?) that prompted students them to use more complete and precise language, which was the habit for the lesson. The additional questions responded directly to students in her class, which cannot be anticipated by the curriculum. These additional questions, however, still sought specific information related to language by naming the fractions, vocabulary, or providing a more complete response. Any additional questions that Nancy asked still followed the narrow parameters set forth by the curriculum. The discussion was focused on a specific strategy and did not encourage students to share alternate strategies. Dialogical discussions, on the other hand, involve students in making connections among strategies and making sense of peers' explanations of strategies.

Table 4.9

Comparison of Observed Questions to Curriculum's Suggested Questions in Third Grade

Observed Questions (Observation, 4/4/2022)	Corresponding Suggested Question (Marshall Cavendish, 2020b)
What are we being asked to do?	What are we asked to do?
Which fractions [are we comparing]?	
How would we compare them?	
How does building it with fraction tiles help us compare them?	How could we use fraction tiles to compare the two fractions?
What do we notice about the denominators and numerators?	What do we notice about the denominators and numerators?
Which one is the bottom number – the numerator or denominator?	
How do you know?	Encourage students who had different reasoning to share their explanation with the class.

Nancy's Planning for Discussion

Nancy reported that she used the curriculum when planning for lessons and considering discussions during mathematics. Nancy specifically mentioned the questions in the textbook and being mindful of mathematical language, which were the focus habits of the lessons observed. She shared that "I do try to [stick to the question] but sometimes I have to do more" (Interview, 4/1/2022). The suggested questions are “very prescribed”, which did not always align with what students actually said during a lesson (Interview, 4/1/2022). As a result, she shared that she generated extra questions in the moment to respond to students. Alternatively, she sometimes found she “need[ed] to ask a few questions before their questions, to help students get what is going on” so that the discussion can eventually follow the curriculum’s anticipated discussion structure (Interview, 4/7/2022).

Additional questioning often supports Nancy in getting students to elaborate beyond brief or short responses, as seen in the excerpt. She did not report planning specific questions but shared that “I’ll ask students about the words they use or ask them to explain *how* or *why* to try to get them to say more” (Interview, 4/12/2022). Specifically, Nancy mentioned that it was important to ask students about vocabulary to help them “learn and get used to saying words like numerator and denominator” (Interview, 3/30/2022). Seeing the mathematical habit “helps [Nancy] to remind [students] of the vocabulary to use during discussion” (Interview, 3/30/2022). Although Nancy didn’t plan for additional questions regarding the vocabulary, the mathematical habit reminded Nancy to be more mindful of using precise language and vocabulary during the lesson with her students. By using prompting students to use more precise mathematical language and vocabulary, Nancy supported her students in developing their mathematical language; however, this habit alone was not sufficient in creating a meaningful and dialogical discussion between students. Rather, planning of these questions supported a partial univocal discussion where the students respond directly to the teacher.

Nancy also admitted that she planned for all components of the lesson but often had to omit certain components due to time. During my observations of lessons, I noticed that Wendy typically omitted questions during the “wrap-up”, which prompted students to reflect on their learning for the day. Additionally, I observed that she omitted the *Math Sharing* activity (Figure 15) from the classroom lesson. When asked about her choice to omit components of the lesson, she stated "I just ran out of time – sometimes you have limited time and the lessons are designed to take 45 minutes but take much longer" (Interview, 4/7/2022). Many of Nancy's omitted questions were from components skipped due to time, which prioritized instructional time for solving more problems correctly rather than a discussion around reflection.

Observed Use of Curriculum in Wendy's Class

In fourth-grade, Wendy claimed that the questions in the textbook supported "having a discussion to unpack the problem – what the problem is asking, what do we know, what is our plan – [which] can be really helpful" in breaking down complex questions (Interview, 4/25/2022). These suggested questions, as seen in upcoming scenario, involved questions that recall facts, procedures, and information from the problem. Consider the following example from an introductory lesson on perimeter of composite shapes:

> *Engage: Draw a rectangle with a length of 4 centimeters and a perimeter of 12 centimeters on a square grid. What is the width of the rectangle?*
>
> Display the problem and read it aloud with the class (see Figure 11). Give students time to work in pairs. Use the questions to prompt and guide student thinking. **What is the problem asking us to find?** (We need to find the width of a rectangle hat has a length of 4 cm and a perimeter of 12 cm). **If the length of the rectangle is 4 cm, how many sides are 4 cm?** (at least 2) **How many centimeters is that in all?** (8 cm) **What do you know about perimeter of a rectangle?** (Answers may vary: Add up all the side lengths; It is equal to the sum of the two lengths and two widths). Have volunteers share their methods with the class. (Marshall Cavendish Education, 2020b, p. 107)

Wendy followed the directions in the curriculum by reading the problem aloud and providing students two minutes to work with a partner to draw the rectangle on grid-paper. Wendy walked around the room to observe students as they worked; occasionally stopping and prompting students with questions from the text. Then, she returned to the front of the room, stood next to the board, and resumed a whole-class discussion of the problem. Underlined questions represent questions adapted from the suggested questioning in the TE textbook.

Wendy: Okay, so what do we know about this problem?

S11: The lengths are 4 centimeters and it's a rectangle.

Wendy: Okay, so how many lengths are 4 centimeters?

S3: Two!

Wendy: Yes, because it is a rectangle. So, what are we being asked to find?

S9: The unknown side.

Wendy: So, we know two lengths are 4 centimeters. How many centimeters is that?

Students: 8!

Wendy: And what else do we know?

Students: The perimeter is 12 cm.

Wendy: What do we know about the perimeter of a rectangle?

S1: We add up all the sides to get perimeter.

(Observation, 4/25/2022)

The first portion of the discussion of this *Engage* task demonstrated how Wendy used the suggested questions in the curriculum to review the problem in the whole group. She slightly modified the questions but maintained the meaning of each question. Wendy carried the textbook with her throughout the lesson, which allowed her to check to make sure she asked all of the recommended questions. As Wendy described, these questions unpacked the problem through seeking specific information from students. These questions did not solicit student thinking, nor did they prompt students to engage with the thinking of the peers. Next, Wendy prompted students to "share their methods with class" by inviting students to share how they solved the problem.

Wendy: So, what would you do next?

S12: Since 4+4 is 8, if you add 2 more you get 10 and then 2 more is 12.

Wendy: Okay, who did something different?

S1: I just did guess and check… I drew a width of 1 but that wasn't enough. So, then I drew 2.

Wendy: Did anyone use a different method?

S11: I know that 4 x 3 is 12 and I already have two sides of 4. So, if I take the other 4 and split it between the two widths, I get a width of 2.

Wendy: There are so many different ways to solve this!

(Observation, 4/25/2022)

Wendy invited students to share their method for solving with the class. She did not ask any follow-up questions and only inquired about different strategies. Notably, the textbook did not include any specific questions for teachers to ask and help guide discussion for students sharing different strategies. Rather, the textbook prompted students to "share" the method rather than meaningfully discusses and compare methods. As a result, multiple student voices participated by sharing methods through show-and-tell but neither the teacher nor the students engaged in dialogic discussion with one another.

Throughout all fourth-grade lessons, I observed Wendy paying particular attention to student language regarding length and width. Wendy reminded students that *width* referred to the shorter side of the rectangle, while *length* referred to the longer side of the rectangle. She repeatedly asked students, “If we look at the rectangle in the picture, which side is the length and which side is the width?” in reference to specific problems in their workbooks. These questions were not provided directly in the suggested questions; however, the sections observed emphasized “use precise mathematical language” as the mathematical habit.

Wendy’s Planning Process

Wendy echoed Anna and Nancy with a similar description of planning for mathematics instruction. Wendy said that she begins by looking at the pacing for an individual lesson. Then, she stated that she “looks over the lesson” and read through the suggested activities and questions (Interview, 4/20/2022). She did not provide significant details about her planning process beyond reviewing the related materials and anticipating where her discussion may vary from that of the text.

> When I [look at the questions], I will notice that I might need to edit or omit questions because they just aren’t applicable depending on how the lesson goes. I will make sure I get to the same idea but I won’t ask the question exactly [as written]. Like I will skip questions if they just aren’t relevant. I can’t ask students to think about which strategy was better if they only used one strategy. (Interview, 4/20/2022)

As evidenced by the quote, Wendy aimed to follow the curriculum as best as possible within her class. She carried the textbook around the room with her, while orchestrating discussion, to ensure that she asked the recommended questions. Similar to Anna and Nancy, she recognized that she changes the questions but still tried to ask similar questions, supporting the notion that she relied on the curriculum when orchestrating discussion.

Wendy specifically mentioned the importance of mathematical language and vocabulary during our interviews. The presence of the habit reminded Wendy to “reiterate the important language and give [students] time to discuss the problems using precise language” (Interview, 4/26/2022). She planned in advance to incorporate questions that were not included in the TE textbook and reinforced students’ use of precise language. Wendy noted that it was “important to talk about vocabulary and make sure everyone, including me, knows the right language to use” (Interview, 4/26/2022). Additionally, she thought of tips to help students remember vocabulary. For example, “length and long both begin with an L so that is a tip I can share to help them remember that language” (Interview, 4/26/2022). The inclusion of the mathematical habit in the TE’s textbook helped to ensure that Wendy used precise language with her students and expected them to also use precise language during discussion.

Summary of Finding 2

During interviews, teachers shared that they planned for mathematics discussion by reading and reviewing the corresponding section in the *Math in Focus* textbook. Each participating teacher claimed that they tried to follow the curriculum’s suggested lessons and recommended questions during mathematics discussion. Observation data supported this claim because teachers questioning during discussion was very similar to that of the textbook. Additionally, teachers mentioned specific habits, such as using precise mathematical language, when the habits were mentioned in the TE section. Through data analysis and triangulation of all three data sources, I found that each teacher in this study heavily relied on the textbook to plan for and orchestrate discussion.

The questioning, as provided in the curriculum and observed during lessons, were primarily information seeking questions. These questions prompted students to share specific answers, procedures, or recall information from the mathematical problem. The curriculum did not have an abundance of questions that prompted students to justify their thinking and compare their thinking with that of their peers. Consequently, teachers did not plan for additional questions nor facilitate discussion that prompted students to engage in dialogical discussion with one another.

Chapter Summary

Through thematic analysis of data collected from observations, interviews, and curriculum review, I developed two findings to answer the research questions. While teachers knew that meaningful discussion was student-led, they facilitated discussion in a way that positioned teachers as leaders of the discussion by following the curriculum. In following school-wide expectations, teachers followed the curriculum closely when teaching mathematics and were heavily reliant on the curriculum's suggested questioning during discussion. The curriculum provided many suggested questions that sought specific information from students which bounded the discussion to narrow parameters. As a result, mathematics discussion was best described as alternating between partially univocal and emerging dialogical discussion. Teachers questioned and guided students according to the predetermined plan in the curriculum to share specific answers (partially univocal) and strategies with their peers (emerging dialogical). Teachers missed opportunities to engage students in meaningful discussion by not consistently prompting students to justify their thinking and explore their thinking with that of their peers (dialogical) and by centering conversation around their own voices and explanations.

The *Math in Focus* curriculum claimed to support NCTM practices and meaningful mathematics discussion, which stood out to administrators and teachers when they selected a curriculum for Barron's elementary mathematics program. Administrators expected teachers to implement the program with fidelity, in hopes of supporting meaningful discussion. Through examining the practices of three teachers and classrooms, I found that the curriculum as written, however, was not sufficient in supporting meaningful discussion that centers student ideas, elicits thinking through student explanations, and encourages students to engage with the thinking of one another. Mathematics discussion in the three participating classrooms centered around teacher voices and the curriculum's suggested strategies and procedures. Teachers narrowly questioned students, according to the curriculum, to reveal the anticipated insights exactly as provided in the curriculum. Meaningful discussion, however, capitalizes on student ideas and student thinking, which was not always accounted for in a curriculum.

As such, I provide recommendations in Chapter 5 that will support administrators and teachers' in improving opportunities for meaningful mathematics discussion at Barron. The recommendations provided can be used alongside the curriculum to better shape an understanding of meaningful discussion and provide additional resources that can be used when planning for and facilitating discussion with the *Math in Focus* curriculum.

Chapter 5: Recommendations

In identifying the problem of practice, administrators and teachers at Barron Academy recognized the difficulty in facilitating meaningful mathematics discussion. Administrators selected *Math in Focus* as the curriculum for various reasons, including the instructional focus on discussion and opportunities for math talk. Since first implementing *Math in Focus,* teachers perceived an increase in their capacity for facilitating discussion as a result of PL focused on the quantity of teacher questions. Yet, teachers and administrators continued to identify mathematics discussion as an area of growth for the elementary school.

To better understand the landscape of mathematics discussion, I investigated the ways in which teachers described meaningful mathematics discussion and the teacher moves that supported or limited meaningful mathematics discussion. I conducted a case study of three teachers and collected data through observations of mathematics instruction, interviews with participating teachers, and a thorough review of the curriculum. For each teacher, I observed four mathematics blocks and interviewed teachers immediately following the observation. I reviewed the lessons, or sections, within the curriculum that I observed. Additionally, I conducted a final interview with teachers after having coded all data and beginning analysis. Throughout data collection, I ensured the trustworthiness of my research by member checking during interviews, triangulating data, and consulting with critical peers, who were students in the same Education Doctorate program. All data collection occurred between March and May of 2022.

I viewed my study through a conceptual framework built upon constructivists' beliefs that students build their own understanding and knowledge through social interactions with peers (Dewey, 1938; Vygotsky, 1978). The roles of students and teachers in the classroom influences how students construct their knowledge and engage in discussion. These roles specifically influence how discussion falls on the spectrum from dialogical to univocal. Teachers are the leaders of learning in univocal or partially univocal settings; on the other hand, dialogical discussions are led by students actively participating in discussion and engaging with their peers. As students engage in more meaningful, dialogical discussions, they make sense of mathematics with their peers.

In this chapter, I provide actionable recommendations for teachers and administrators at Barron Academy, which are directly aligned to the findings and supported through literature. It is important to acknowledge that recommendations were designed for the entire Barron Academy elementary school, despite being derived from an ungeneralizable sample of three teachers. Given that I am an external researcher, it is most practical for the recommendations to be recommended school wide. If I were an internal researcher, I would provide differentiated recommendations to each teacher. However, my recommendations are more generalized to support the school collectively. First, I contextualize the findings through the literature. Next, I describe the three recommendations, which were developed with both feasibility and impact in mind. The recommendations are actionable with research-based resources and tools to support the implementation of the recommendation.

Contextualizing the Findings

Through analyzing and interpreting the collected data, two key findings emerged. In this section, I situate the findings within the literature. By contextualizing the findings, I provide researched based evidence for my recommendations.

Contextualizing Finding 1

As presented in Chapter 4, I found that the three teachers in this study described meaningful mathematics discussion as student-led, yet they did not orchestrate discussion in this way. Discrepancies existed between the teachers' perception and enactment of meaningful mathematics instruction and the NCTM's definition of meaningful mathematics instruction. First, teachers interpreted "facilitate discussion" as asking questions, to which students responded and provided answers. This, however, is not in alignment with the literature's description of facilitating student-led discussion (Hufferd-Ackles et al., 2004; NCTM, 2014). Under NCTM (2014) guidance, students play an active role leading discussion, asking questions of their peer, providing thorough explanations, and sustaining the dialogue, which extends far beyond replying to teacher-initiated questions. Teachers act as a guide on the side supporting student roles.

Teachers in this capstone study noted that explanations of mathematical ideas should come from students during discussion, which is in agreement with the literature's description of meaningful mathematics discussion (Hufferd-Ackles et al., 2014; NCTM, 2014). During discussions, however, teachers assumed the role of the explainer, providing more elaborate and thorough explanations than those of students. Teacher centered explanations were often initiated by a student idea that teachers repeated and significantly extended. While repeating student ideas is a commonly supported talk move (Chapin et al., 2003; Cirillo, 2013; Ellis et al., 2019; Michaels et al., 2016), the literature advises teachers to repeat student ideas in order to clarify their thinking or highlight a student idea, while keeping student thinking at the center of the repeated statement. Teachers often utilized the move inappropriately without centering student ideas at the forefront of the explanation.

Both the NCTM (2014) and teachers in this capstone study considered participation from all students as a distinguishing attribute of meaningful mathematics discussion. Participation was reported as a crucial component of the teachers' descriptions of meaningful mathematics discussion because of increased engagement rather than increased opportunity for learning. In contrast, the NCTM advocated for all students to participate so that all students can learn from discussion that is both high quality and equitable. NCTM argues that discussion helps students to build understanding, acting as vehicle for learning, and not just a space to share a final, correct solution. During discussions, teachers should make explicit connections between student strategies and approaches to help students connect mathematical ideas (Smith & Stein, 2011). Teacher descriptions of discussion did not emphasize the significance of connecting mathematical ideas during discussion, which is a significant component of meaningful mathematics discussion.

Contextualizing Finding 2

As my second finding illustrates, teachers relied on the curriculum and its suggested questions during discussion, which included primarily information seeking questions and funneled discussion through narrow parameters to predetermined strategies and solutions. As a result, questioning did not make student thinking visible. Literature presents various ways to classify questions such as information seeking, eliciting student thinking, extending thinking, prompting peer engagement, and connecting mathematical relationships (Boaler & Brodie, 2004; Franke et al., 2009). Information seeking questions are more prominent in IRE discussion (Lim et al., 2020). Furthermore, IRE patterns of discussion do not yield meaningful mathematics discussion (Lim et al., 2020; NCTM, 2014). Varied questions, such as questions that elicit student thinking and prompt peer engagement, are needed to support more meaningful mathematics discussion (Ellis et al., 2018; Franke et al., 2009; Lim et al., 2020; Shaughnessy et al., 2020).

The written curriculum did not provide sufficient structures through questioning or directions to teachers which supported students in engaging in meaningful discussion. Directions to “invite students to work in pairs” or “discuss different strategies with the class”, as written in *Math in Focus* (Marshall Cavendish Education, 2020b), did not contribute to discussions with high levels of student-to-student engagement. As a result, discussions were primarily an exchange between the teacher and a series of individual students. Additional questioning, which extends beyond information seeking questions and includes questions that elicit student thinking and promote peer engagement, have the potential to support students in engaging with the ideas of their peers during discussion (Chapin et al., 2013; Franke et al. 2015; Hunter, 2008; Webb et al., 2009, 2014, 2017). Researchers have developed various frameworks that categorize questions to help teachers incorporate various types of questions into their classroom discussions (Boaler & Brodie, 2004; Chapin et al., 2003, 2013; Michaels et al., 2016). Included in these frameworks are research-based questions that specifically teach students to engage with the thinking and ideas of their peers. Notably, these types of questions are missing from the *Math in Focus* curriculum.

Recommendations

To better support teachers and administrators at improving meaningful mathematics discussion across all classrooms, I developed recommendations that directly pertain to the findings of my capstone study. Although my findings were developed from a sample of three teachers and are not generalizable, I present my recommendations as generalizable to the entire school context for the purpose of efficiency and feasibility. When developing recommendations, I considered the feasibility and potential impact of the recommendation, grounding recommendations in research-based practices and tools that were proven to increase teaching and learning opportunities for students. Empirical research contributed to ensuring the recommendations have the potential for high impact, while practitioner focused literature contributed to developing resources that increased the feasibility of implementation. In Table 5.1, I provide an overview of the recommendations as aligned to findings.

Table 5.1

Recommendations Aligned to Findings and Related Literature

Recommendation	Related Finding
Recommendation #1: Identify an individual or team of teacher leaders to serve as Math Leaders for the elementary school.	Finding 1: Teachers recognized that meaningful discussions were student-led, yet they did not always facilitate discussions in this way.
Recommendation #2: Develop professional learning to support the continued growth of meaningful mathematics discussion.	Finding 1: Teachers recognized that meaningful discussions were student-led, yet they did not always facilitate discussions in this way. Finding 2: Teachers were heavily reliant on the curriculum's suggested questions to plan for and orchestrate mathematics discussion, which limited meaningful mathematics discussion
Recommendation #3: Use tools, such as rubrics, checklists, and student surveys, to monitor the quality of mathematics discussion.	Finding 1: Teachers recognized that meaningful discussions were student-led, yet they did not always facilitate discussions in this way.
Recommendation #4: Plan talk moves that support students in justifying their thinking and engaging with the ideas of their peers.	Finding 2: Teachers were heavily reliant on the curriculum's suggested questions to plan for and orchestrate mathematics discussion, which limited meaningful mathematics discussion

Recommendation #1: Identify an individual or team of teacher leaders to serve as math leaders for the elementary school.

Effective school change and improvement require active stakeholders to be champions for change (Beaver & Weinbaum, 2012; Hargreaves & Fullan, 2012). Therefore, I recommend that administrators at Barron Academy identify an individual or team of teacher leaders to act as math leaders for the elementary school. Improvements in instructional practice and teaching are more sustainable and successful when leadership is "located closest to the classroom" (Harris & Muijs, 2003). Teacher leaders, who act as champions of change for mathematics instruction, will be able to work alongside administration at Barron to improve mathematics instruction and implement the additional recommendations.

Teacher leaders are situated alongside their peers, not as superiors, creating a sense of social and professional trust with other teachers (Hargreaves & Fullan, 2012; Harrison Berg et al., 2014). Teacher leaders "slide the doors open" to collaboration and problem solving, while also sharing knowledge, expertise, and experience with their teacher peers (Lumpkin et al., 2016). When teacher leaders advocate for change within a building, they have the potential to directly impact instruction through not only their own classrooms but also the classrooms of their peers through collaboration.

I recommend that the school leaders invest time in identifying teacher leaders, who demonstrate specific knowledge, skills, and dispositions that are critical for teacher leaders to possess (Levin & Schrum, 2017). First, I recommend that they identify teachers who show both an interest in mathematics discussion and in continuing to learn and improve mathematics practice as a leader of the school. Teacher leaders should understand curriculum, mathematics content, and quality instruction because they will work closely with peers to improve teaching using the curriculum (Levin & Schrum, 2017). One way administrators may identify teachers leaders of mathematics discussion is by using the rubric provided in the second recommendation.

Teacher leaders should also know the school community and understand how teachers think, learn, and interact within the school setting so that they can work with the school community to implement change (Hargreaves & Fullan, 2012). A positive relationship, which is built upon trust, with other teachers should assist teacher leaders in implementing change. Collaboration is a key disposition of teacher leaders (Levin & Schrum, 2017; Lumpkin et al., 2016) because they collaboratively learn with their peers to garner instructional improvement (Harris & Muijs, 2003). Together, teachers and teacher leaders learn, inquire, reflect, and plan for improved instruction. Teacher leaders are also in a position to provide feedback and coaching to their peers. Therefore, teacher leaders must be skilled in providing mentorship or coaching to teachers. As teacher leaders collaborate with and coach other teachers, they simultaneously build their rapport, their own teacher leadership skills, and the instructional practices of their peers (Harris & Muijs, 2003; Lumpkin et al., 2016)

Identifying a teacher leader who demonstrates all the aforementioned knowledge, skills, and qualities may require an investment in professional capital (Hargreaves & Fullan, 2012). By investing in professional capital, administrators recognize the talent of individuals, the collaborative power of the entire school, and the wisdom of the teachers to make judgements to improve learning. I recommend that school leaders invest the time and resources in on-going, sustained PL for teacher leaders to increase their capacity to be effective leaders and champions of change. Furthermore, administrators and teacher leaders should engage in professional learning about teacher leadership and quality mathematics instruction together (Appendix O). By learning more about both teacher leadership and mathematical pedagogy, administration and teacher leaders together can gain a better understanding of the foundation for implementing the remaining recommendations.

Recommendation #2: Develop professional learning to support the continued growth of meaningful mathematics discussion.

To continue improving the learning opportunities for students and teacher capacity for orchestrating meaningful discussion, I recommend that the team of math leaders develop ongoing, job-embedded PL that will focus on mathematics discussion. Literature suggests various forms for PL, which can include teacher book clubs or novel studies, collaborative planning, professional development, or a professional learning community (PLC). Regardless of structure, PL at Barron Academy should incorporate research-based practices that will support teachers in improving their capacity for orchestrating meaningful mathematics discussions. Since ongoing, job-embedded professional learning, focused on orchestrating meaningful mathematics discussion, is new to Barron, I developed a high-level guide to support implementation of PL over the course of a year (Appendix P). Within this guide are suggested modules, which may be covered over multiple sessions of PL, and related resources. Two of the three modules were designed to specifically provide PL around the third and fourth recommendations from this capstone study.

Within the plan for PL, I recommend various articles, books, and resources that can be used when planning PL. Specifically, I recommend that two research-based and practitioner friendly books be used in developing PL for Barron. First, *Classroom Discussions in Math* (Chapin et al., 2013) is a teacher's guide for orchestrating discussion using their talk moves. This book includes step-by-step instructions for teachers to develop their capacity for facilitating mathematics discussion. The same authors also wrote a companion book specifically for facilitator's leading professional learning on discussion. An additional book, *The Five Practices in Practice: Successfully Orchestrating Mathematics Discussion in Your Classroom* (Smith et al., 2020), provides specific steps teachers should account for when planning meaningfully discussion. These steps include selecting appropriate mathematics tasks, anticipating student responses, monitoring student work independently and in small groups, select student work, and connect student solutions. PL around this book could center around steps 2-5 since teachers at Barron Academy use the tasks provided in the curriculum. This book is based upon the research of Smith and Stein (2008) and was developed as a resource for practitioners. Both books provide video models, questions for discussion and reflection, and direct opportunities for planning. These books could be used to guide PL as a novel study directly or as supplementary resources during PL. Both books provide planning opportunities, thus, providing job-embedded professional learning intended to change instructional practices and improve student learning (Darling-Hammond et al., 2017).

I also recommend that teacher leaders utilize resources from TeachingWorks (2022), a resource library built by The University of Michigan. Resources on the TeachingWorks website include free, virtual professional learning courses directly related to orchestrating meaningful discussion. I recommend that Barron Academy either integrate the resources and courses directly into PL or use the materials as a resource when planning PL. The TeachingWorks library was built through collaboration between researchers, teacher educators, and practitioners using research-based practices and adult learning pedagogy. The resource library includes information about high leverage practices such as orchestrating discussion, eliciting student thinking, and structuring norms and routines for discussion. Each of these topics are directly relevant to the findings and could support improvement at Barron Academy.

PL opportunities at Barron Academy provide teachers and administrators with opportunities to collaboratively learn, plan, reflect, and improve the quality of mathematics discussion and student learning (Darling-Hammond et al., 2017; Hargreaves & Fullan, 2012). First, effective PL opportunities are ongoing (Desimone, 2009). Developing regularly scheduled PL supports a sustained, ongoing opportunity for development and learning. Effective PL is also an active learning opportunity for teachers (Clarke & Hollingsworth, 2022; Darling-Hammond et al., 2017). Teachers engage in the practices they are learning, apply the practice in their classrooms, and return to the PLC to reflect on the practice (Darling-Hammond et al., 2017). In a PLC, teachers are "active learners shaping their professional growth through reflective participation" (Clarke & Hollingsworth, 2002, p. 948). PLCs provide a space for teachers to engage in reflective conversation about what they learn, their experiences with the practice, and collaborate to improve the learning opportunities for all students. Teachers can collaborate to plan for effective lessons, observe or provide feedback to one another, analyze model videos together, and learn together. When teachers collaborate, they create a collective force for improving instruction and student learning (Darling-Hammond et al., 2017). When implemented well, PLCs are ongoing and provide practical learning that is active, collaborative, and reflective (Darling-Hammond et al., 2017).

Additionally, I recommend that administrators at Barron Academy utilize PL provided by *Math in Focus* to support the improvement of MKT. *Math in Focus*, a Singapore Math curriculum, is designed to replicate the model of teaching and learning utilized in Singapore. Teacher preparation and teacher knowledge, however, is vastly differently in Singapore than the United States (Cheang et al., 2007; Ginsburg et al., 2005). In Singapore, all elementary educators attend the same teacher preparation program, which requires multiple mathematics methods courses covering both mathematics content and mathematics pedagogy (Cheang et al., 2007; Ginsburg et al., 2005). Singaporean teachers demonstrate mathematics skills that are superior to that of U.S. teachers at both the beginning and end of teacher preparation programs as measured by examinations (Ginsburg et al., 2005, p. xiv). Furthermore, requirements for educators in Singapore demand more professional learning to develop skills and knowledge each year. Given the variation in MKT of teachers in Singapore and the United States, it is important to ensure that teachers at Barron Academy have the appropriate MKT to teach *Math in Focus* as designed. Marshall Cavendish provides significant PL related to *Math in Focus*, which could support Barron in implementing mathematics PL.

Recommendation #3: Use practical measures and other tools to monitor the development and quality of mathematics discussion.

Given that teacher descriptions of meaningful mathematics discussion did not align with the NCTM (2014)'s description, I recommend that teachers and administrators use tools as a common system to measure and monitor the development of meaningful discussion at Barron Academy. Administrators at Barron shared that they wanted to improve mathematics discussion; however, they did not have any specified measures for evaluating discussion or goals for determining the current quality of mathematics discussion or monitoring the growth. Practical measures are tools informed by research but specifically designed for practitioners to easily embed into their routines so that they may use the data collected as not only an assessment of improvement but a lever for improvement (Bryk et al., 2015; Jackson et al., 2016; Yeager et al., 2013). By using specific tools, such as rubrics or other practical measures, Barron Academy's teachers, teacher leaders, and administrators can actionably work towards improving mathematics discussion.

A Rubric: Levels of Math-Talk Learning Community

I recommend that teachers and administrators use the framework developed by Hufferd-Ackles et al. (2014) and advocated for by the NCTM (2014) as a tool for monitoring and measuring mathematics discussion at Barron Academy (see Figure 1.11). First, I recommend that administrators and teacher leaders use this framework as a way to assess the quality of mathematics discussion in all classrooms. Using this tool in all classrooms would provide base-line data to inform future decisions about PL and goals for the school, as my findings cannot be generalizable to the entire elementary school. After baselines are established, teacher leaders and administrators at Barron should continue to use this framework as a rubric to monitor the improvement of mathematics discussion.

This framework describes the math-talk learning community across four levels as defined by five different components, teacher role, questioning, explaining mathematical thinking, mathematical representations, and building student responsibility for the community. It was developed following empirical research (Hufferd-Ackles et al., 2004), which sought to describe various math-talk learning communities, to support teachers in advancing classroom discussion (Hufferd-Ackles et al., 2014). The leveled structure provides a description of classrooms under each component so that administrators and teachers can identify both current levels of discussion and identify the qualities of the next level. Specifically, I recommend that teachers and administrators at Barron Academy begin by focusing on three categories, which were directly aligned to the findings of this capstone study and include teacher role, questioning, and mathematical explanation (see Figure 5.1)

Figure 5.1

Focus Areas from Hufferd-Ackles et al. (2014) Framework

	Teacher Role	Questioning	Explain Mathematical Thinking
Level 0	Teacher is at the front of the room and dominates conversation.	Teacher is the only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.	Teacher questions focus on correctness. Students provide short answer-focused responses. Teachers may gave answers.
Level 1	Teacher encourages sharing of math ideas and directs speaker to talk to the class, not to the teacher only.	Teacher questions begin to focus on student thinking and less on answers. Only teacher ask questions.	Teacher probes student thinking somewhat. One or two strategies may be elicited. Teachers may fill in an explanation. Students provide brief descriptions of their thinking in responses to teacher probing.
Level 2	Teacher facilitates conversation between students and encourages students to ask questions of one another.	Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.	Teacher probes more student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.
Level 3	Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.	Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask “why” and call for justification. Teacher questions may still guide discoveries.	Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.

This tool will support both teachers and administrators at Barron Academy in measuring and monitoring meaningful mathematics discussion. Administrators should use this framework as a rubric to monitor the quality of meaningful mathematics discussion throughout all elementary grades. In doing so, administrators may identify teacher leaders, whose classrooms demonstrate meaningful mathematics discussion, and provide specific feedback to teachers to support individual growth. Providing teachers with specific feedback is critical to improving teaching and learning for students (Hargreaves & O'Connor, 2017; Hattie et al., 2017). Using this rubric at Barron Academy can act as an intermittent "temperature read" to determine if desired changes, in relation to mathematics discussion, are occurring (Takahashi et al., 2022).

Additionally, teachers at Barron Academy should use this rubric to reflect on their own instructional practices during discussion by measuring their own orchestration of discussion according to the rubric. Some research asserts that teacher reflection is pivotal for teachers to feel empowered to improve their teaching practices (Rodman, 2010; York-Barr et al., 2001; Zeichner & Liston, 1996). Structured reflection, which uses specific tools or questions to guide reflection, assists teachers in understanding and applying specific instructional practices (Rodman, 2010). Teachers should use the following questions to reflect on their ability to orchestrate meaningful mathematics discussion (Figure 5.2). This reflection tool is intended to be a quick-set of questions, which will support teachers in regularly reflecting on discussions after the conclusion of a lesson. I developed questions from the descriptive levels of the Hufferd-Ackles et al. (2014) framework, which address teacher role, questioning, and explanation of mathematics.

Figure 5.2

Questions to Support Reflection on Discussion

Student Actions	Teacher Actions
❑ Did students share evidence and explanations of their thinking? ❑ Were student responses elaborate or short, few word responses? ❑ Did students show evidence of their thinking using models or manipulatives for the whole class? ❑ Did students ask questions to their peers during discussion? ❑ Did students refer to or comment on their peers work during discussion? ❑ Did students respond to one another or just me? ❑ Did students do most of the talking?	❑ Did I explain most important concepts or skills? ❑ Did I question students to provide their own explanations of important concepts or skills? ❑ Did I probe student to ensure they provided reasoning and thinking in their explanations? ❑ Did I encourage students to refer to a strategy used by a peer? ❑ Did I encourage students to use a specific strategy that I wanted them to use? ❑ Did I help students make connections between strategies? ❑ Did I do most of the talking? ❑ Did I do most of the question-asking?
General Reflection Questions	
❑ Did the discussion use multiple strategies? ❑ Did the discussion support students in learning from one another? ❑ Did the discussion lead to new ideas and thinking? ❑ Do I recall any instances where I missed an opportunity to give students a voice over mine? ❑ Do I recall any instances where I missed an opportunity for students to justify their thinking?	

Student Surveys in the Upper Elementary Grades

In addition to tools designed for teachers and administrators, I recommend that elementary students complete surveys about discussion to provide teachers and administrators with additional data about the quality of mathematics discussion. The role of students during discussion will need to evolve to more active participants and leaders of discussion in order for improvement at Barron Academy. It is the responsibility of the teacher to make changes within the classroom that support students in taking on this role. Measuring student accountability and perceptions through student surveys assists teachers and administrators in practically measuring elements of discussion related to this shift in students' roles (Kochmanski et al., 2015; Nieman et al., 2020; Yilmaz et al., 2022). Attending to student views of their own accountability in discussion can provide data for teachers to better understand their own instruction (Neiman et al., 2020). Additionally, using the surveys over time can provide evidence of change in discussion through the change in student responses. Yilmaz et al. (2022) found that changes in student responses over time can be interpreted as changes in teaching practices.

Specifically, I recommend the use of a survey developed by Kochmanski and colleagues (2015) (see Table 5.2). The survey was designed for practical use and easy implementation which included, limited training needed to administer and read the survey, employ with limited time (1-3 minutes), and a high potential for improving student learning opportunities in discussion (Kochmanski et al., 2015). Specific questions from this survey directly relate to student's accountability during discussion, the extent to which the discussion focused on teacher ideas rather than student ideas, and how students make sense of others' ideas (Yilmaz et al., 2022). Each of these themes relates to the findings of this capstone study and directly align to the framework used by administrators and teachers to monitor discussion (Hufferd-Ackles et al., 2014).

Table 5.2

Student Survey Questions for Whole Group Discussion

Survey Question	Alignment to Hufferd-Ackles et al. (2014)
Were you comfortable sharing your thinking in today's whole-class discussion? o Yes o No	Building responsibility within the community
Who talked the most in today's whole class discussion? o Students o The Teacher	Teacher Role, Explanation of Mathematical Thinking
Did listening to other students in today's whole class discussion help make my thinking better? o Yes o No	Explanation of Mathematical Thinking
Did you have trouble understanding other students' thinking in today's whole class discussion? o Yes o No	Teacher Role, Questioning, Explanation of Mathematical Thinking
What was the purpose of today's whole class discussion? o Share how we solved the problem using steps my teacher showed us. o Learn the way the teacher showed us to solve the problem. o Check to see if our answers are correct o Learn different ways that work to solve a problem from other students. o Share a mathematical idea we came up with on our own.	Teacher Role, Explanation of Mathematical Thinking

While much of the empirical research around the survey was conducted in middle-grades, survey items were written with upper-elementary and middle-grades students in mind (Kochmanski et al., 2015). I developed a modified version of the survey for lower-elementary students (Appendix Q) and used the applicable questions for the upper-elementary student survey (Appendix R). Teacher should administer the survey to students at the end of mathematics lessons at least once a month to monitor progress. Teachers may analyze the survey results individually, collaboratively with peers or teacher leaders, and/or during PL. I developed a tool to support teacher leaders in guiding discussion and reflection with teachers about the survey results (Appendix S). When interpreted appropriately, the survey presents data about how students perceive and participate in discussion, which teachers can use to make informed decisions to move discussion from a space to share correct answers to one for sense-making (Jackson et al., 2015; Kochmanski et al., 2015).

Implementation through PL

I recommend that Barron's PL first focus on helping teachers to understand meaningful mathematics discussion through these tools. During PL, teacher leaders should present school-wide data about mathematics discussion as measured using the rubric (Hufferd-Ackles et al., 2014). This data can provide a baseline for teachers in understanding their own practices and developing school-wide goals for improvement. Additionally, teachers should watch videos of discussion and practice evaluating lessons using the rubric. Recurring use of the rubric during professional learning can support teachers in using the rubric in feeling more comfortable integrating the rubric into their own reflective practices. PL should also address the implementation and analysis of the student surveys. Student surveys should be introduced to teachers during PL and teachers should analyze survey results during PL. Depending on the level of trust between teachers participating in PL, teachers may analyze data individually or collaboratively share survey results during PL. Regardless, teachers and teacher leaders should be able to engage in discussion about the survey results and develop generalizable next steps to improve survey results.

The framework for building math-talk learning communities (Hufferd-Ackles et al., 2014) is recommended by NCTM (2014) as a tool for "moving toward a classroom community centered on discourse" (p. 30). This framework can be viewed as a rubric for measuring classroom discussion and identifying specific goals for continued growth. Administrators can use this tool to provide feedback for teachers, as well as identify teachers that can serve as teacher leaders of math-talk learning communities. I also recommend that teachers use this tool to reflect on their own practices as a teacher and facilitator of mathematics discussion. Student surveys in the upper-elementary grades can also guide teacher reflection and identify areas of improvement. By planning and providing PL around the use of the various tools, teacher leaders and administrators can support teachers in using these tools to improve practices.

Recommendation #4: Plan talk moves that support students in justifying their thinking and engaging with the ideas of their peers.

Finally, I recommend that teachers intentionally plan for talk moves and questions that elicit student thinking and prompt students to engage with the thinking of their peers during mathematics discussion. Literature suggests that meaningful mathematics discussion require careful planning (Smith & Stein, 2011) and explanations of student thinking (Ellis et al., 2019; Franke et al., 2009), while also encouraging students to engage with the thinking of their peers to make mathematical connections (Chapin et al., 2003; Ing et al., 2015; Webb et al., 2014) Specific talk moves and question statements can directly support students in providing evidence of their thinking and engaging with the thinking of their peers.

Teacher Talk Moves

Specific teacher talk moves and questions can support teachers in planning for additional questions that are not included in the TE textbook and support discussion. First, I recommend that teachers at Barron Academy identify a time during lesson and/or unit planning to identify a goal for discussion that is separate from the mathematical goal. Chapin et al. (2003, 2013) identified four goals or steps that can support meaningful mathematics discussion:

1. Helping individual students share, clarify and expand upon their own thoughts

2. Helping students orient to the thinking of others
3. Helping students deepen their own reasoning
4. Helping students engage with the thinking and reasoning of others

Each goal or step contains specific talk moves, which teachers should use when enacting meaningful discussion. Franke and colleagues (2015) studied how teacher questions and prompts support student engagement, leveling them from high to low levels of engagement. I adapted the talk moves suggested by Chapin et al. (2013) to include specific prompts that support medium- and high-level engagement (Franke et al., 2015). Once teachers have identified a goa for discussion, I recommend that they use the following tool to collaboratively plan questions that support the discussion goal (Figure 5.3).

I recommend that teachers use this tool when planning for and orchestrating meaningful mathematics discussion as related to the identified discussion goal. In the *Math in Focus* lesson, teachers are directed to have students share different strategies and discuss as a class. This is one such instance where teachers can plan for specific questions that encourage students to engage with one another during discussion. For example, teachers should plan to use talk moves to promote students comparing and connecting ideas (Goal 4). The curriculum's directions suggest teachers "invite students with different strategies to share", however do not include questions to guide teachers in doing this. In a fourth-grade lesson on perimeter, the curriculum invites students to share their use of one of two equations, $P=2L+2W$ and $P=L+L+W+W$. Teachers could use the productive talk tools framework to develop their own questions that prompt students to identify similarities and differences to recognize the relationship between the two specified equations. This will support discussion in moving from a show-and-tell to meaningfully engaging with the thinking of others and making connections among mathematical ideas (Stein et al., 2008).

Figure 5.3

Productive Talk Tools (adapted from Chapin et al., 2013; Franke et al., 2015)

Goal 1: Help students to share, clarify, and expand upon their thinking
Say more about… ❑ I'm not sure I understand yet. Can you share more about ____________? ❑ What do you mean by ________________? (specifically use student words) ❑ So you decided to do __________. Why did you think to do that? Tell us more. *So, are you saying….?* ❑ Let me see if I've got what you are saying. Are you saying….? (leave space for student to agree, disagree, or say more)
Goal 2: Help students orient to the thinking of others
Who can repeat? ❑ That's an interesting idea. Who can repeat what Jack just shared? ❑ Who can put what Mya said into their own words? ❑ I'd like for you to turn to your should partner and tell them what we just heard from Cory. *Try it out…* ❑ Who can try this out using ______'s strategy? ❑ Paige just showed us how she solved this problem. Who wants to try her strategy?
Goal 3: Help students deepen their own reasoning
Why do you think that? ❑ How can you prove what you just said to us? ❑ Convince the class you are right! ❑ What convinced you that was the answer? ❑ What is your evidence for claiming ________ ? (Specifically use the claim)
Goal 4: Help students engage with the thinking and reasoning of others
Critiquing Student Thinking ❑ Carter, you disagree with Trevor? Tell us why you disagree. ❑ What do you think about Isabela's strategy? ❑ Does anyone want to respond to Ty's idea? ❑ Did something think about it in a different way? If so, what was different? ❑ Does someone have a different solution they want to share? How is it different? *Compare and Connect Ideas* ❑ How is your idea similar to/different from Sam's?

- ❑ What about Jerome's strategy and Kayla's strategy are the same? What is different?
- ❑ Do both strategies always work? Can you prove that?
- ❑ How are these strategies related?

Add On and Explain

- ❑ Rosalee got us started on this problem. Who can pick up where she left off?
- ❑ Can someone add onto Jack's statement?
- ❑ Juan just showed us his work on the board. Who can explain Juan's work and his thinking?
- ❑ When Riley shared _______, what else did you think of?

I recommend that teachers use this tool when planning for and orchestrating meaningful mathematics discussion as related to the identified discussion goal. In the *Math in Focus* lesson, teachers are directed to have students share different strategies and discuss as a class. This is one such instance where teachers can plan for specific questions that encourage students to engage with one another during discussion. For example, teachers should plan to use talk moves to promote students comparing and connecting ideas (Goal 4). The curriculum's directions suggest teachers "invite students with different strategies to share", however do not include questions to guide teachers in doing this. In a fourth-grade lesson on perimeter, the curriculum invites students to share their use of one of two equations, $P=2L+2W$ and $P=L+L+W+W$. Teachers could use the productive talk tools framework to develop their own questions that prompt students to identify similarities and differences to recognize the relationship between the two specified equations. This will support discussion in moving from a show-and-tell to meaningfully engaging with the thinking of others and making connections among mathematical ideas (Stein et al., 2008).

Student Talk Moves

In addition to planning questions for discussion, I recommend that teachers incorporate talk moves that hold students accountable for meaningfully contributing to the class discussion (Michaels et al., 2016; Webb et al., 2017). Given that high quality discussion includes questions from students, teachers should develop structures that encourage students to ask questions during whole group discussion. Discussion requires careful planning (Smith & Stein, 2011; Stein et al., 2008), thus teachers should plan for opportunities where they position students to asks questions instead of the teacher. Planning specific questions that they would like students to ask their peers may also be beneficial. For example, teachers may direct students to ask their peers, "Why did you choose that strategy?" during a partner discussion recommended by the TE textbook. Individual reference sheets or anchor charts can provide students with access to sample questions and statements to use during discussion (see Figure 5.4).

Figure 5.4

Student Facing Questions and Sentence Stems

Mathematicians Ask and Say…

To understand our friend's thinking…

- How did you know to multiply/divide/add/subtract?
- Can you explain what this part of your model means?
- Why did you choose that strategy?
- Can you explain to me how you…
- What do you mean by?
- I think I understand _____'s strategy. Let's try it out.

To agree with a friend's thinking…

- I did something similar, we both….
- I got the same answer as ________ but used ________ strategy. Can we compare our work to see how it is the same?

To disagree with a friend's thinking…

- I got something different. Can you explain to me why you think that?
- I disagree with ______________ because I know…..

Teachers can position students to take up talk moves that are similar to that of the teacher (Hunter, 2008; Webb et al., 2017). For example, a teacher can direct students to ask their peers "How do you know that for sure?" or "Why did you decide to multiply those two numbers?" during a whole group discussion. Positioning students to ask clarifying questions to their peers, rather than just the teacher, supports students in engaging with their peers' thinking at a high level (Webb et al., 2017). As detailed in Chapter 2, teacher support for student questioning can transfer to students developing questioning without the support of the teacher (Hunter, 2018; Webb et al., 2009, 2017). By incorporating opportunities for students to question peers in planning, teachers share the responsibility of asking questions with students, which supports meaningful mathematics discussion.

Implementation through PL

Teacher leaders for mathematics can support the implementation of this recommendation through PL. In order to ensure an appropriate use of the tools, some PL should highlight how teachers can use the tools to support peer engagement during discussion. Additionally, teachers and teacher leaders can collaboratively plan for discussion together during PL using the specified talk move tools. As teachers review the *Math in Focus* suggested discussion questions, they should critically consider if any questions are present that prompt students to share their thinking or engage with the ideas of others. If these questions are not present, teachers should plan for questions that will prompt students to do so. Teachers should also plan for emphasizing specific questions or sentence stems that they want students to practice using during a discussion. Referring to Figure 5.2 and 5.3 while planning will support teachers in intentionally supplementing the current curriculum to include more opportunities for students to engage in meaningful mathematics discussion.

Conclusion

This capstone study sought to describe the current landscape of discussion in elementary mathematics classroom at Barron Academy, an independent school. In this chapter, I provide recommendations that directly relate to mathematics discussion. It must be acknowledged that mathematics discussion is only one component of a complex systems of instructional practices. To improve the quality of mathematics discussion, and thus the learning opportunities presented to students, the school most adopt tools and practical measures to help them assess their current level of discussion and continuously monitor mathematics discussion so that they may measure growth. Without tools, such as rubrics and surveys, there is no practical way to measure improvement. Next, I recommend that teachers intentionally plan for questions that elicit student thinking and prompt students to engage with one another. The *Math in Focus* curriculum provides many opportunities for talk; however, there need to be more opportunities for students to share their thinking so that they may engage with the thinking of others. Planning for additional questions can provide opportunities for students to do so. Finally, I recommend that Barron Academy develop PLCs that will provide space and time for continued collaboration, learning, planning, and reflection surrounding mathematics discussion.

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Appendix A
Literature and Methods Connections

Authors	Related Methods Protocols	Description of Relation	Research Questions
NCTM (2014)	Interview Observation	NCTM (2014) recommendations for high-quality teaching and learning provide a foundation for this capstone study. Teachers' beliefs will be coded as productive and non-productive. Additionally, observations seek to identify how teachers facilitate meaningful mathematics discussion as described by NCTM.	1, 2
Webb et al. (2021)	Observation	During observation and analysis, I will look to see how student participation varies and how teachers support/inhibit participation from all students.	2
Spillane and Zeuli (1999)	Interview	Interviews will seek to understand teachers' beliefs around mathematics discussion.	1
Bennett (2010), Jung (2011), Stipek et al. (2001), Yurelki et al. (2020)	Interview Observation	These studies recognize that the beliefs and practices shared during an interview may be different than those in practice. Misalignment between practice and beliefs may contribute toward findings and recommendations.	1, 2
Casa et al. (2007)	Interview	Survey questions from this study were adapted and influenced questions from the interview protocol.	1
Beswick (2007) and Conner & Singeltary (2021)	Interview Observation	Both interviews and observations will contribute to an understanding of the roles of students and teacher sat Barron Academy, which is essential to the Conceptual Framework of the study.	1, 2
Bray (2011) and Stockero et al. (2020)	Observations	Observations of student mistakes will provide interesting data about how teachers do or do not correct an error or facilitate discussion around the error.	2
Chapin et al. (2003), Ellis et al. (2018), Michaels et al. (2010)	Observations	Several a priori codes were developed from these studies and frameworks, including *repeating, encourage repeating,* and *connecting ideas.*	2
Franke et al. (2009)	Observation	Observed follow-up questions will be coded using Franke et al.'s (2009) classification of *probing sequence of questions, general questions, specific question,* and *leading question*.	2

Franke et al. (2015), Langer-Osuna and Avalos (2015), Richland et al. (2017), Smith and Stein (2018), Webb et al. (2014), White (2003)	Observation	The ways in which teachers do or do not prompt students to engage with the ideas of others through connecting or critiquing their peers ideas or work will be coded as *encourage connecting ideas* or *encourage critique*. These studies provide various examples of how teachers can encourage students to do both of these practices.	2
Di Teodoro (2011), Hufferd-Ackles (2004), Hunter (2008), Ing et al. (2015), and Webb et al. (2019)	Observation	Student-led questioning is a major component of high-level math-talk communities according to Hufferd-Ackles (2004). Observation data will be coded with *encourage questioning* to show how teachers may promote, model, or facilitate student-led questioning.	2
Knuth & Peressini (2001)	Interview and Observation	During analysis of interviews, I will seek to identify how teachers describe discussion and where this description may fall along a dialogic or univocal. Additionally, my analytic memos and analysis of observations will also seek to identify how the conversations in mathematics move across this spectrum.	1, 2

Appendix B

Recruitment Email

Dear [Participant],

I am beginning a study of mathematics discussion at your school for my doctoral program. I am interested in learning more about how teachers lead mathematics discussion and support students in engaging with the ideas of their peers during math discussion. I am inviting you to participate in this study because you have been identified as an experienced teacher who is in an optimal position to provide a helpful window into mathematics classrooms at NCS. You are not, however, required to participate in this study.

Your participation would involve between 3-5 classroom observations and 3-5 interviews following each observation. Interviews would be scheduled at your earliest convenience and last between 15-20 minutes. You would receive a $50 Amazon gift card for participating in this study, paid even if you choose to leave the study. You may choose to leave the study at any point prior to its completion.

Please review the consent form attached to this study. If you choose to consent, please print, and sign a copy of the attached consent form. Respond to this email and I will come to NCS to retrieve the consent form.

Sincerely,

Jamie McLemore
UVA IRB- SBS #5007

Appendix C

Informed Consent Form

Please read this consent agreement carefully before you decide to participate in this study.

Consent Form Key Information:

- Participate in a study about mathematics discussion
- 3-5 classroom observations of mathematics instruction
- 3-5 interviews following the classroom instruction (15-20 minutes each)
- $50 Amazon gift card for full participation

Purpose of the Research Study: The purpose of this study is to examine how elementary mathematics teachers perceive and support productive mathematical discussion between students during mathematics instruction.

What you will do in this study: As a participant in this study, you will be asked to engage in a series of classroom observations and interviews. You will select 3-5 mathematics lessons for the researcher to observe. Following each lesson, you will be briefly interviewed, and each interview will last approximately 15-20 minutes. The interviews will be scheduled at your earliest convenience following the lesson. You can skip any interview questions or stop the interview at any moment. The interview may be audio-recorded, given your consent.

Time required: This study will require between 45 minutes (minimum) and 1 hour and 45 minutes (maximum) of your time for interviews. This time will be spread out in 15–20 minute intervals over the course of 3-weeks. This study will also include 3-5 observations of your scheduled mathematics instruction. The total time required for you will be up to 1 hour and 45 minutes outside of class and up to 5 hours during mathematics class.

Risks: The observations and interview questions should present minimal risk or harm. Your identity will be protected using pseudonyms; thus, there are limited risks of this study.

Benefits: There are no direct benefits to you for participating in this research study. However, your participation may support the findings and recommendations that will help to continue to grow the instructional practices within your school.

Confidentiality: The information that you provide during interviews and obtained during observations will be handled confidentially. You will be assigned a pseudonym and all data collected will be labeled with this pseudonym. Only the researcher and her faculty advisor will have access to the observation and interview data. All recordings of interviews will be deleted once they are transcribed. Data will be stored in a secure online system, UVA Box. Data will not be provided to your employer directly; however, data will be provided to the school through findings and recommendations. While confidentiality will be preserved, I cannot guarantee anonymity due to the nature of this study.

Voluntary participation: Your participation in this study is entirely voluntary. You will not be subject to any penalties or loss for refusing to consent in the study, nor will you receive any benefits for choosing to participate.

Right to withdrawal: You have the right to withdrawal or terminate your participation in this study at any time.

How to withdraw from the study: If you want to withdraw from the study, please tell the researcher immediately. If during an observation, you may tell the researcher to leave the room. If during an interview, you may tell the researcher to stop. There is no penalty for withdrawing. You will still receive prorated payment for your participation in the study.

Payment: You will receive a $50 Amazon gift card for participating in the study.

Using data beyond this study: The data you provide in this study will be retained in a secure manner by the researcher for up to 1 year and then destroyed.

If you have question about the study, contact:

Jamie McLemore (doctoral student researcher)
School of Education and Human Development
University of Virginia
Telephone: (757)-469- 2434
kjm4yw@virginia.edu

Dr. Catherine Brighton (faculty advisor)
School of Education and Human Development
Ridley Hall 102H
University of Virginia, P.O. Box 400873
Charlottesville, 22903
Telephone: (434)-924-1022

To obtain more information about the study, ask questions about the research procedures, express concerns about your participation, or report illness, injury or other problems, please contact:

Tonya R. Moon, Ph.D.
Chair, Institutional Review Board for the Social and Behavioral Sciences
One Morton Dr Suite 500
University of Virginia, P.O. Box 800392
Charlottesville, VA 22908-0392
Telephone: (434) 924-5999
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Website: https://research.virginia.edu/irb-sbs
Website for Research Participants: https://research.virginia.edu/research-participants

UVA IRB-SBS # 5007

Agreement

I agree to participate in the research study described above.

Print Name: ____ Date: ________________

Signature: ____

You will receive a copy of this form for your records.

Appendix D

Observation Protocol

Organizational Information:

Teacher: ______________________________

Grade Level: ______________________________

Date: ______________________________

Tine: ______________________________

Math in Focus Lesson: ______________________________

Number of Students: ______________________________

Setting Diagram: Mark changes as necessary throughout math block.

Field Notes to include:

- Duration of the discussion (start and end time).
- Both whole group and small group discussions.
- Transcription of discussing including questions/comments asked by teachers, questions/comments asked by students in response to either the teacher or student, and other notable interactions between students and teacher regarding mathematical concepts.
- Relevant instructional strategies as observed during discussion (questioning, wait time, sentence stems, directions to students for discussion, etc.).
- Implementation of *Math in Focus* curricular materials (PowerPoints, textbook, workbook, games, etc.).

Field Notes:

Instructional Tools Utilized (Sentence Stems, Visuals, Manipulatives, Curriculum Components, Etc.):

Reflexive Notes/Bracketing:

Appendix E

Curriculum Review Protocol

Teacher: ____________________________________

Dates of Observation: ____________________________________

Grade Level: ____________________________________

Math in Focus Lesson: ____________________________________

Suggestions for Questions, Discussions, and Prompts from the *Math in Focus* Lesson	Notes on Implementation

Appendix F

Interview Protocol #1

Note: This semi-structure interview protocol was used for the interviews immediately following classroom observations.

Interviewee: ________________________

Date and time: ________________________

Consent

- Thank you very much for taking the time to meet with me today and for your willingness to participate in both this interview and my research study. Today, I will be asking you questions about your mathematics instruction in today's lesson so that I can better understand your thoughts and instructional choices. Please feel free to share whatever comes to mind.
- Your answers will not be shared directly with your administrators but will be included in the research project with a pseudonym to protect your identity.
- Before we begin, I wanted to let you know that you can end this interview at any time. You may also skip any questions that you do not wish to answer. Do you consent to participate in this interview?
- I will be audio-recording the interview so that I may ensure accuracy in my write-up of the interview. If you would like me to stop recording at any point or have any concerns about being recorded, please let me know. After I transcribe the interview, I will delete the recording. Are you comfortable if I record our conversation?

General Question:

1. Tell me about how you think today's lesson went.
2. What role do you think the discussion played in the lesson?
3. What goals did you have for the discussion?
4. Tell me about how you think today's discussion went.
5. What role, if any, did the curriculum play in your lesson today?
6. How comfortable were you with the content in today's lesson?

Questions about Discussion (that went well):

7. What in particular do you think really helped make it a productive discussion?
8. What do you think allowed the discussion to come together in the way that you described?
9. Were there any moments that you felt were really impactful for students?
    a. Was there anything you said that you think specifically helped students to achieve the goal?
    b. Was there anything students said that you think specifically helped other students achieve the goal?
10. Let's look at an excerpt of the discussion together… What do you think really went well here?

Questions about Discussion (that did not go well).

11. What do you think contributed to this discussion not going as well as you hoped?
12. What might have been needed to make this discussion more productive?
13. Let's look at a part of the discussion together… (refer to field notes) What might you have wanted to go differently here?

Specific Questions about Lesson (referring to field notes):

14. During the discussion a student said X, why did you choose to respond with Y?
15. What made you choose to discuss X's strategy?
16. I noted that # of students participated in the lesson. What do you think about this number?
    a. What strategies do you use to engage so many/all students?
    b. How might you have engaged more students?

Conclusion: Is there anything else about this lesson or discussion that stood out to you as important that we didn't talk about today? Thank you again for taking the time to speak with me today.

Appendix G

Interview Protocol #2

Note: This interview protocol was designed for the final, concluding interview. I developed this interview protocol after having completed all observations and at least two rounds of coding data.

Introduction: Thank you again for meeting with me following all observations. This interview should take about 20 minutes. I will record this via Zoom so that I am able to go abck and refer to it later. After I transcribe the interview, I will delete the recording. Do you consent to recording this interview? If at any point you wish to terminate the interview, you may. You may also skip any questions that you do not wish to answer. Do you consent to participate in this interview?

General Discussion

1. How, if at all, do you differentiate between just talking about math and a meaningful discussion?
2. Tell me more about your purpose in asking for different strategies and how that contributes to meaningful discussion.
3. Can you describe for me how you distinguish between a productive and an unproductive discussion?
   a. When do you notice productive discussions?
   b. When do you notice more unproductive discussions?

Question Types

4. Tell me a bit more about how you think about different questions to ask studnets.
5. What do you think about when you think of the quality of a question?
   a. What makes a question "good"?
   b. Do you ever reflect on the types of questions you ask?
6. What do you do to support students in having more complete explanations?
7. During observations, you often asked __________. What do you hope to get from that type of question?

Curriculum Related Questions

8. What do you recall about your professional training around *Math in Focus*?
9. Do you remember anything specific about discussions or questioning?
10. The textbook provides several different directions – facilitate discussion, guide discussion, extend thinking with discussion before providing different sentences. Tell me about what these mean to you.
11. Tell me more about how you determine what you may need to supplement in the curriculum.
    a. Why do you think that is important to add?
    b. Do you find that is something that is specific to certain lessons or common?

Conclusions

12. Is there anything else you want to share about how use *Math in Focus?*
13. Is there anything else you want to share about math instruction and discussion?

Thank you so much for participating in this study. I appreciate all of your communication and willingness to have me in your classroom. This will be our last interview. If you have any additional questions for me, feel free to reach out via email.

Appendix H

Parent Notification Letter

This letter will be sent home after receiving consent from the teacher participant and prior to conducting the first observation.

03/25/2022

Dear Parent,

My name is Jamie McLemore and I am conducting a research study in your child's classroom for my doctorate in Education from the University of Virginia's School of Education and Human Development. I am interested in studying how teachers lead discussion in math class.

I will be in your child's math class between three and five times over the next two months for about an hour per session. I will observe mathematics instruction during the normally scheduled time and lessons will continue as normally planned. While I'm in the classroom, I will observe the teacher's instructional methods and take notes. I will not write down your child's name or collect any materials that will identify your child. I will not be collecting or examining any student work. There is no risk to your child and I will be observing the teacher, not your child.

If you have any questions or concerns about the study, please contact me at:

Jamie McLemore
Kjm4yw@virginia.edu

Sincerely,

Jamie McLemore
UVA IRB-SBS #507

Appendix I

Curriculum Review Codebook

Note: This codebook is the final codebook that was used and includes both *a priori* and emergent codes.

Category	Code Name	Description	Example
Group Size	Partner	Lesson activities, questions, or prompts that are suggested to be completed in pairs.	Have students work with a partner to begin solving the problem.
	Small Group	Lessons, activities, questions, or prompts that are suggested to be completed in small groups.	Hands-on-activities and games suggest groups of 3-4 students.
	Whole Group	Lesson activities, questions, or prompts that are suggested to be completed in whole group.	Wrap-up the lesson with a discussion as the whole class. Invite students to share solutions with the class.
	Independent	Lesson activities, questions, or prompts that are suggested to be completed independently.	Independent Practice component of the lesson activity
Utilization	Utilized	When the suggestion from Math in Focus is directly utilized by teacher.	The textbook recommended teachers ask students to discuss in partners how to use place value discs to solve a problem and the teacher does so verbatim.
	Adapted	When the suggestion from Math in Focus is utilized but the teacher adapts the question, group-size, or recommendation in some way.	The textbook recommended teachers ask students to discuss in partners how to use place value discs to solve a problem and the teacher changes to discussing how to solve the problem (without place-value discs) in small-groups.
	Not utilized	When the suggestion from Math in Focus is not utilized or skipped.	Teachers did not wrap-up the lesson that included any of the recommended wrap-up questions.

Suggested Questioning and Directions	Eliciting Student Thinking Question	A question from the text that presses for students to explain and further articulate their ideas	How do you know? Is there a way to prove it?
	Different Strategies	A question from the text to ask for different strategies, solutions, or ideas or direction to discuss various strategies, solutions, or ideas.	Is there another way to find the area? What are the different ways to record mass? Invite students to discuss different strategies.
	Connecting Ideas	Questions that support students in find similarities, differences, or relate two or more solutions, strategies, or mathematical ideas.	How does your thinking compare? Which strategy is more efficient?
	Information Seeking Questions – Answer	An information seeking question that asks students to provide a specific answer to a mathematical problem, idea, or prompt.	What is the sum of 2+6? Which fraction is less?
	Information Seeking Question – Noticing	An information seeking question that asks students to share what they notice about a problem, picture, or idea.	What do you notice about this equation? What do you notice about fourths and sixths?
	Information Seeking Question - Procedure	An information seeking question or prompt that asks students to share a specific procedure or process to account what they *did* to solve a problem.	What did you do to find the area? How can we use side length to find the perimeter? How did you find the measurement?
	Information Seeking Question - Vocabulary	An information seeking question that asks students to define or share the meaning of specific mathematical vocabulary or symbol.	What is area? What is a kilogram? What does subtraction mean?
	Information Seeking Question –	An information seeking question or sequence of questions that helps students to focus on key elements or	What do we know that might help us to solve this problem?

	Orienting and Focusing	aspects of the situation to assist in solving a problem (Boaler & Brodie, 2004).	What information is given? First, let’s count the total number of objects we have. How many do we have?

Appendix J

Observation Codebook

Note: This codebook is the final codebook that was used and includes both *a priori* and emergent codes.

Parent Code	Code Name	Description	Example
	Explaining	When a teacher explains strategies to students without or with little reference to student thinking.	"Before you begin, the green tiles are going to be your sixths and the yellow tiles are going to be your fourths. Use these to help build the fractions and make sure you have the right denominator." "So we call that a compound shape. To find the area of the compound shape we have to…"
	Different Strategy	When the teacher prompts or questions students to share a different strategy, solution, response, or idea.	"What else? Who can tell me something different?" "Who can tell me a different way of finding the area?" "Did anyone make a different addition number sentence?"
	Student Question	When a student asks a question during mathematics discussion.	"I'm a little confused… how did you find the length of that side?"
Repeating	Repeating with Question	When teachers repeat student ideas during discussion followed by a question that seeks additional information or clarification from the student.	"So, I like how you said '30 by 30'. What does that mean?" "Harry wrote the number sentence 2 + 5 = 7. Can you explain that to me?"
	Repeating as Statement	When teachers repeat students ideas without posing a question.	Sally: The length is 12. Teacher: Yes, Sally said the length is 12. We know that the bottom of the compound shape is 12 because the top is made up of two lengths that are both 6, they just aren't a straight line across. But we can use both those lengths by adding up the two sides that are 6.

Information Seeking Question	Answer	An information seeking question that asks students to provide a specific answer to a mathematical problem, idea, or prompt.	"How many students are drinking lemonade all together?" "What is the total mass of all three objects?" "_____ grasshoppers hopped away. Who can tell me the answer?"
	Noticing	An information seeking question that asks students to share what they notice about a problem, picture, or idea.	"Who can share what they notice about this picture with the birds?" "What do we notice about the fractions in this picture?"
	Procedure	An information seeking question or prompt that asks students to share a specific procedure or process to account what they *did* to solve a problem.	"Well, what was the first step that you did to solve this?" "How did you find the perimeter?" "Tell me about what you did to compare ¼ and ½ ." "How do we find the answer in subtraction problems?"
	Vocabulary	An information seeking question that asks students to define or share the meaning of specific mathematical vocabulary or symbol.	"Can someone tell me what this symbol means (=)?" "What is mass?" "Remind me again, what is the length?"
	Orienting and Focusing	An information seeking question, prompt, or sequence of questions that helps students to focus on key elements or aspects of the situation to assist in solving a problem (Boaler & Brodie, 2004).	"Tell me what you know about this problem to help us get started." "What fractions are in this problem?" "What are we being asked to find?"
	Eliciting Student Thinking Question	A question or prompt that presses for students to explain and further articulate their ideas	"How do you know that?" "Why does that work for this problem?" "Tell me more about how you thought about this picture."

Prompting Peer Engagement	Connecting Ideas	When teachers make a statement or pose a question that supports students in comparing two solutions, strategies, or mathematical ideas.	A.J., how is your strategy of using arrays similar to Maria's use of repeated addition?
	Encourage Repeating	When a teacher prompts a student to repeat a previous student's idea.	Shelly, can you describe the strategy that Michael just explained using?
	Critique	When a teacher prompts a student to suggest whether they agree or disagree with a previous students' idea or solution.	Marcus, why do you agree or disagree with Jayden's solution?
Error Correction	Correction by Teacher	When the teacher negatively evaluates and corrects a students' strategy, idea, or solution.	Jessie: You can add up the sides to find the area, that's a strategy. Teacher: I think you are confused. We add to find the perimeter. For area, we multiply the sides.
	Self Correction	When a student makes a self-correction either in explaining their own response unprompted or prompted by the teacher who asked a clarifying question.	Penny: I think that 3+4=8. Teacher: Can you come up to the board and shoe me how you counted that? Penny: Okay! 1, 2, 3… oh wait I get 7 not 8!
	Correction by Peers	When students correct their peers either prompted or unprompted by the teacher.	Cayden: I think I got 18 square units. Students: no, not that. Teacher: It's okay to make mistakes as long as we can figure it out. Who can help us to identify the mistake here?

Appendix K

Interview Codebook

Note: This codebook is the final codebook that was used and includes both *a priori* and emergent codes.

Category	Code Name	Definition	Example
Beliefs	Productive	Beliefs that support the implementation of recommended practices as described by the NCTM (2014)	"Students should ask each other questions, help each other out… not just the teacher" "When students are able to connect their ideas to others during discussion… and can see that these two different strategies are similar… That's a good discussion"
	Unproductive	Beliefs that hinder the implementation of effective instructional practices or limit important mathematics practices as described by the NCTM (2014)	"I'm supposed to fix things… if a student makes a mistake, I need to correct it." "I need to guide the discussion by giving students easy steps to follow."
Curriculum	Benefit of Curriculum	When teachers refer to resources or instructional materials provided by the curriculum that improve teaching, learning, planning, or their experience using the curriculum.	"This program is really good at getting them to hit the mark and learn what they are teach them." "It's really helpful because everything is right here – everything I need to teach."
	Challenge of Curriculum	When teachers refer to resources or instructional materials provided by the curriculum that make teaching, learning, planning, or their experience using the curriculum more challenging or difficult.	"I don't always have the time to go through everything in the curriculum. So, I don't always get to parts of the discussion that are important." "Sometimes the questions are really redundant, I feel its demeaning."
	Purpose of Discussion	When a teacher references the purpose or goal of discussion.	"My goal is for them to be talking about what I have asked them… they listen to their peers share and hear the right answers."

			"Discussion helps for them to hear someone else say it and then they start to remember." "It helps me to see who get its and who might need more help"
	Participation	When a teacher references student participation during discussion.	"I lost a few students but I really did have a lot of participation – which I like. Participation is important!" "A perfect mathematics discussion would be everyone involved and participating."

Appendix L

Themes Chart Example

Note: The chart is adapted from Bazeley (2021).

Theme: Teachers use repeating as a talk move for class discussion in various ways.

The various ways were:

1) Responding to students by repeating their answer and providing additional explanation
2) Responding to students by questioning their answer to prompt correction.
3) Responded to students by repeating, questioning students, and drawing attention to the student's response and explanation.

Teachers also mentioned in their interviews how they viewed repeating as an important move for the teacher to make during discussion.

Related Codes and Definitions:
Repeating – Question: When the teacher repeats what the student said followed by a question.

Repeating – Statement: When the teacher repeats what the student said as a statement.

Explaining: Teacher explains strategies to students without using student work.

Description/Summary of Interpretation:
Teachers recognized that repeating could be a great way to make students heard or identify a correct answer for the class (interviews). In class, teachers did mark ideas in this way (see E3). More often, however, teachers repeated student ideas and drastically up-took student responses. Rather than just repeat to make an idea known or use more accurate vocabulary/mathematical terms, teachers expanded upon student ideas and explained for the child rather than teachers prompting students to explain.

What's missing?

- Students being asked to repeat the ideas of their peers (medium-leverage practice (Franke et al., 2015; Webb et al., 2014)). Repeating can be used to engage students with the ideas of their peers if teachers ask students to repeat what peers said or repeat and add on to the ideas of their peers.
- Teachers repeating an idea and asking students to elaborate further or asking other students to elaborate further.

Notes:

What are my data sources? Why is this notable enough to be a finding?

- Teachers referred to repeating as a strategy to support discussion during interviews from ELA curriculum.

<table>
<tr><td colspan="3">

- Observations showed how teachers repeating occurs during discussion. This repeating did not support more discussion but did support teacher-led explanations.
- This theme helps to support by answering RQ2 by answering what teachers do that supports and/or inhibits productive discussion. As repeating is a common move discussed in literature and practitioner-facing support documents.

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Connections to Literature:

Repeating did not always achieve the same purpose, nor did repeating achieve the purpose as revoicing or marking does according to Chapin, O'Connor, and Anderson (2003). Revoicing when utilized correctly is intended to interact with a student "in a way that will continue to involve that student in clarifying his or her own reasoning (p. 12). However, teachers often revoice and then explain on behalf of the student.

Michaels and colleagues (2010) identify marking as an accountable talk practice where teachers might repeat what a student shares to draw attention to the idea.

Ellis and colleagues (2018) found that it can be utilized as a low-potential move or high-potential move.

</td></tr>
<tr><td>E1 Source</td><td>E1 Excerpt</td><td>E1 Explanation for Choosing</td></tr>
<tr><td>3rd Grade, March 30th Observation</td><td>T: What do you notice about the fractions?

S7: They both have 4 on the bottom.

T: Yes, they both have a 4 on the bottom. So they are divided into the same number of pieces. Even if we didn't have a model, we might be able to figure out if they are the same on the bottom that 3 pieces is greater than 1 piece. But that only works when we have the same kinds of pieces and the same number on the bottom.</td><td>This is a typical example of when the teacher asks the question, the student responds, then the teacher responds by repeating the student and then adding on for additional information.

Note, the teacher did not prompt students to clarify further or ask what that means. Note how the teacher provides a thorough explanation for the student after first repeating the student.</td></tr>
<tr><td>E2 Source</td><td>E2 Excerpt</td><td>E2 Explanation for Choosing</td></tr>
<tr><td>4th Grade, April 20th Observation</td><td>T: How do I use my known sides to figure out the unknown sides?

S12: So, this side is 9 (waving hands to show vertical side), so the other side is also 9.

T: So, S12 thought about if this line (marks on board) was straight down then we would see it was 5 and 4 so it would be 9.</td><td>In this example, the teacher is repeating what the student said to ensure that other students can see the correct response. Additionally, the teacher moves beyond repeating by explaining how the student found at total of 9.

Note, the teacher did not prompt students to clarify further or ask</td></tr>
</table>

		what that means. Note how the teacher provides a thorough explanation for the student after first repeating the student.
E3 Source	*E3 Excerpt*	*E3 Explanation for Choosing*
Kindergarten, April 4 Observation	T: How many of the friends have their hair up? Students: 3 T: S11, you told me that there were 3. Can you point to them? S11: 2 are here and 1 is here. T: So 2 and 1 are how many? S11: 3 T: S11 saw 2 and saw 1…Did anyone see it a different way?	E3 is a common example for this case. The teacher repeats what is said and asks for the students to show her/explain what they did. Then, the teacher repeats what the student said. Note, how the teacher does not provide any additional explanation for students. Only repeats what the students shared themselves for other students to hear. Note, the teacher used a combination of repeating with a question and repeating as a statement.
E4 Source	*E4 Excerpt*	*E4 Explanation for Choosing*
3rd Grade, March 30 Observation	T: The fraction models we have don't show 5/6 and 5/8. But, what do we know about fractions? (Showing a manipulative of 1/6 and 1/8). S3: 8 is bigger than 6. T: 8 is bigger than 6, but is eighth bigger? (showing manipulative) S8: No! Sixth is bigger than eighth because if you have a candy bar and you split it with 8 people instead of 6 people. You will get a bigger piece with sixths. T: That's exactly right, so if we have 5 of our sixths, that will be bigger than 5 of our eighths.	E4 shows how a teacher repeats what a student says but then asks a question that prompts students to correct an error. Note, the teacher repeats and then asks students a question, which allows another student to correct their peer's thinking. Note, the teacher evaluates S8's claim and then applies it to the initial question. This leads to the teacher explaining.
E5 Source	*E5 Excerpt*	*E5 Explanation for Choosing*

Kindergarten, April 11 Interview	Teacher: If an answer is correct, I'll ask students to repeat it is so that everyone can hear it nice and loud. I'll have students repeat something so that there is more attention on it.	In E5, the teacher notes how she does not always repeat what students say herself but will ask students to repeat their choice in order to mark their idea or make the idea the center of everyone's focus.
E6 Source	*E6 Excerpt*	*E6 Explanation for Choosing*
4th Grade, April 25 Interview	Teacher: I think that I do it for two reasons. One, it is something we talk about in reading – confirming what others have said. We have talked about this in our reading curriculum. We kind of did it and continue to do it. It is a good way to show that you are listening to them and that you are an active listener. **But I also do it to reiterate it for students that might have missed what others have said.** Sometimes, I don't know if they are being biased to their peers, but sometimes students switch off when others talk. So, if I repeat it, they know that when I am talking, they should listen to me. So, I can maybe catch 1 or 2 students that might have logged off. **Then, I can push others to join in from there. Repeating can help us to listen to others, build off what one another's have said, disagreeing with others in a kind way**. All of that is part of our reading curriculum, too.	Repeating was most frequently seen in 4th grade. Note, the teacher has several reasons for repeating students. The teacher marks that repeating serves 2 main purposes. She mentioned that repeating is a way for students to listen to an idea of their peer and a way to involve peers with one another.

Additional Memos/Notes on the repeating theme:
As I coded and analyzed codes, I noticed that pieces of data that included repeating were also coded with explaining, prompting self-correction, or prompting correction by peer. I noticed patterns in how the dialogue was coded. Teachers across all grade levels frequently repeated students. The ways in which they leveraged repeating varied. Often, teachers initiated a conversation by asking a question or prompt. After students responded, teachers frequently repeated student responses. Most often teachers repeated followed by an explanation provided by the teacher, not the student. Other times, teachers repeated followed by a question asking students an additional question sought to elicit their thinking. Other times, teachers repeated incorrect student responses with a question that sought to prompt students or peers to correct. Repeating is something that teachers do frequently (see chart below) and reference in their interviews. Yet, the ways in which the repeating is enacted varies.

Repeating Parent Code

	Statement	Question	Total
Kinder	12	5	17
3rd	12	8	20
4th	8	5	13
Total	32	18	50

Appendix L

Excerpt from a Kindergarten *Math in Focus* Lesson
Chapter 8, Lesson 1: Subtraction Stories
(Marshall Cavendish Education, 2020b, 118-126)

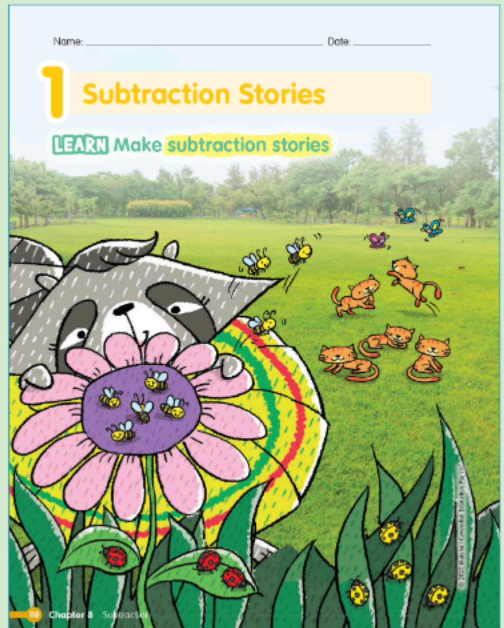
1 Subtraction Stories

Learning Objective
- Make subtraction stories with objects, fingers, pictures, and drawings.

Mathematical Habits
- 3 Construct viable arguments
- 4 Use mathematical models

New Vocabulary
- subtraction story

Materials
- 10 counters per student
- 1 set of craft sticks per pair
- 10 connecting cubes per student
- *Pete the Cat and the Four Groovy Buttons* by Eric Litwin for the teacher



Student Edition Page 118

DAYS 2 – 3 of 16

ENGAGE

Concrete Pictorial Abstract

The intent is for students to explore how to make subtraction stories with objects, fingers, pictures, and drawings.

Activity 1

- Invite six volunteers to the front.
- **Let's pretend we are horses. How many horses do we have?** (6)
- Tap one student.
- **One horse galloped away. How many horses are left?** (5)
- Invite a different group of 5 students up to the front.
- **Now let's pretend we have 5 frogs. 2 frogs jumped away. How many frogs are left?** (3)
- Repeat the activity by varying the number of students.

Activity 2

- Gather students together and read aloud ***Pete the Cat and the Four Groovy Buttons*** by Eric Litwin. As you read each page, encourage students to answer the question: How many buttons are left on Pete's shirt?
- Encourage them to use counters and connecting cubes to model each page.
- You may want to skip the pages that show the subtraction sentences and revisit them in Section 5 later.

Activity 3

- Have students work in groups to dramatize the following subtraction stories:
  - There are 3 fish in a pond. 1 fish swims away. There are 2 fish left in the pond.
  - There 4 ducks in a garden. 2 ducks waddle away. There are 2 ducks left in the garden.
  - There are 5 sheep in a field. 4 of them are grazing on grass. 1 sheep is resting.

LEARN (page 118)

Make subtraction stories

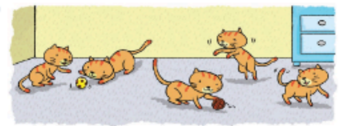
Concrete Pictorial Abstract

- Display the picture on page 118, focusing on the bee.
- **What can we see in the picture?** (bees, cats, butterflies, ladybugs) **What makes a story a subtraction story?** (Accept all possible answers.) **Let's make a subtraction story about the bees. There are eight bees on a flower. Three bees fly away. There are five bees left on the flower. What do you notice about the group of bees on the flower when some bees fly away?** (Answers vary. Example: The group of bees gets smaller and there are fewer bees on the flower now.)

TRY

Use the pictures to make a subtraction story.
Fill in each blank.

1



There are 5 cats in a room.

2 cats run away.

There are 3 cats left.

Student Edition Page 119

continued ...

Let's look at the cats. What stories can we make about the cats? (Answers vary. Example: Finding how many cats are playing.; Finding how many cats are not playing.) **How many cats are there?** (5) **How many cats are playing with the butterflies?** (2 cats are playing with the butterflies.) **How many cats are not playing with the butterflies?** (3 cats are not playing with the butterflies.)

- Encourage students to work in pairs to use the picture and make more subtraction stories. Provide verbal prompts if students are unable to make any subtraction stories. You may want to provide students with counters to model each story.

Best Practice

Encourage students to place counters on the bees as they find out how many bees there are. Then, have students remove the counters as the bees fly away. Repeat the same for the cats.

For Advanced Learners

Encourage students to use the picture to make more subtraction stories. Have them use counters to model the stories. Have some students share their subtraction stories with the class.

TRY **(pages 119 to 122)**

For 1 to 4, provide students with counters.

For 1, encourage and guide students to make subtraction stories about the cats.

What subtraction story can we make? How many cats are in a room? (5) **How many cats run away?** (2 cats run away.) **How many cats are there left in the room?** (There are 3 cats left in the room.)

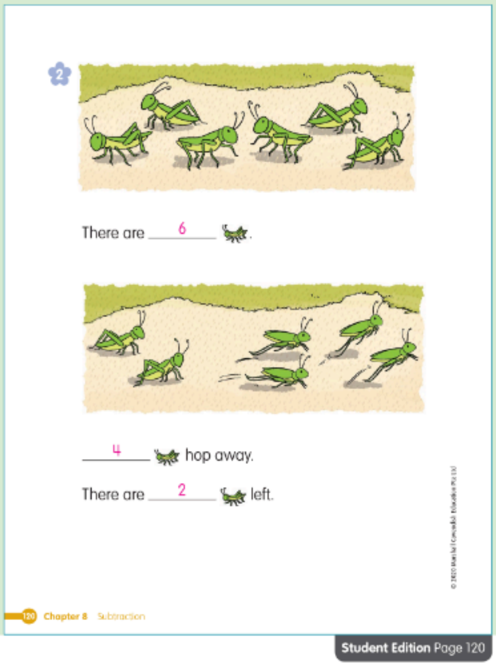
- Have students go through the subtraction story again. This time round, have students cross out the cats that run away. Then, count the remaining cats.

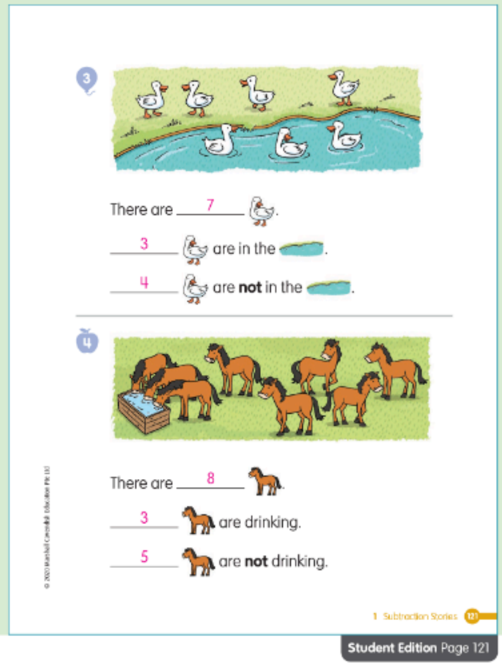
For Advanced Learners

Encourage students to make another subtraction story about the cats in 1. Have them share their stories with the class.

There are 5 cats in a room. 2 cats are playing with balls. How many cats are not playing with balls? (3)

There are 5 cats in a room. 1 cat is jumping. How many cats are not jumping? (4)





For 2, encourage and guide students to make subtraction stories about the grasshoppers.

What subtraction story can we make? How many grasshoppers are there? (6) **How many grasshoppers hop away?** (4 grasshoppers hop away.) **How many grasshoppers are there left?** (There are 2 grasshoppers left.)

Best Practice

Encourage and guide students to place a counter on each grasshopper counted. Then, let the counters "run away" to show the grasshoppers hopping away and count the counters that are left.

For 3, encourage and guide students to make different subtraction stories about the ducks.

What do you see in the picture? (ducks) **How many ducks are there in the picture?** (7) **How many ducks are in the pond?** (3 ducks are in the pond.) **How many ducks are not in the pond?** (4 ducks are not in the pond.) **Let's tell a different story. How many ducks are there in the picture?** (7) **How many ducks are on the grass?** (4 ducks are on the grass.) **How many ducks are not on the grass?** (3 ducks are not on the grass.)

For 4, encourage and guide students to make subtraction stories about the horses.

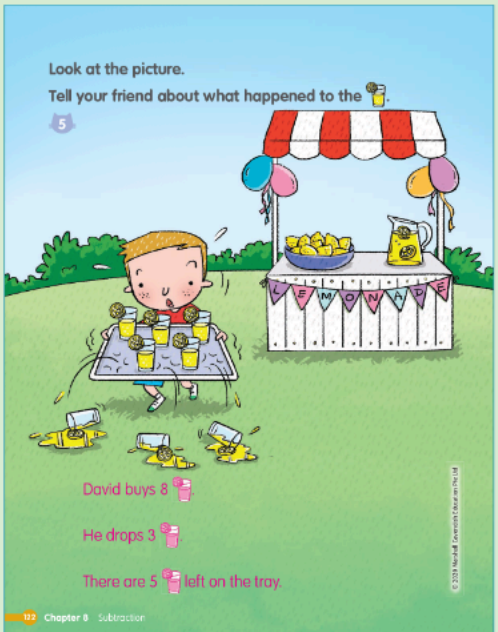
What do you see in the picture? (horses) **How many horses are there?** (8) **How many horses are drinking water?** (3 horses are drinking water.) **How many horses are not drinking water?** (5 horses are not drinking water.)

Best Practice

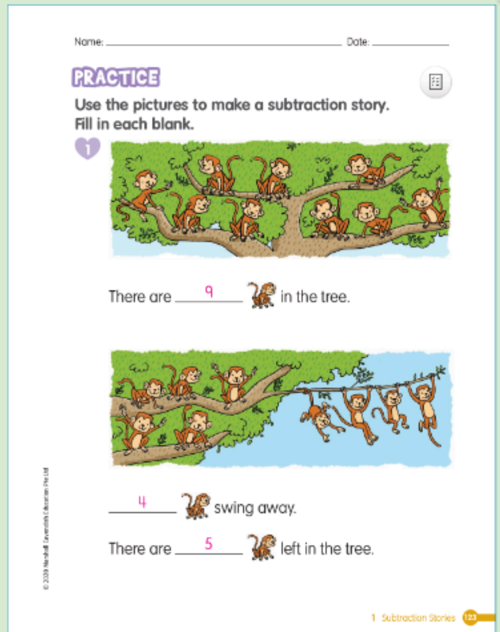
Encourage students to first use their fingers to represent the items shown in 1 to 4. Have students raise their fingers to show the minuend (the greater number). Then, subtract by bending their fingers to show the subtrahend (the number that is less). Finally, the fingers that remain raised shows the answer to the subtraction story. For example, have them raise 5 fingers to show the number of cats in the room at first, then, have them bend 2 fingers to show the number of cats that ran away. Guide students to see that the number of cats that are left is the same as the number of fingers that remain raised.

For Advanced Learners

Encourage students to make their own subtraction stories by drawing their favorite people, food, animals, or objects. Such activities provide opportunities for students to construct viable arguments and model with mathematics through the representations in their pictures. Have students show and tell their subtraction stories.



Student Edition Page 122


Name: ____________ Date: ________

PRACTICE

Use the pictures to make a subtraction story.
Fill in each blank.

1

There are __9__ in the tree.

__4__ swing away.

There are __5__ left in the tree.

1 Subtraction Stories 123

Student Edition Page 123

For 5, encourage and guide students to make their own subtraction story using the visual prompts. Example: David buys 8 glasses of lemonade. He drops 3 glasses of lemonade. There are 5 glasses of lemonade left on the tray.

For Advanced Learners

Encourage students to think of other possible scenarios in 5 and share with their partner.

Days 2 – 3 Wrap Up

- Wrap up by having students reflect on what they have learned.

When might we want to tell a subtraction story? (Answers vary. Example: when we want to show fewer)

- Have students consolidate their learning by working on the **PRACTICE**.
- Use the strategies in **Differentiated Instruction** on page 126A to help students who need additional support or those who could benefit from a challenge.
- Students should be able to make subtraction stories.

Best Practice

Ensure students can solve 1 and 2 before letting them work independently.

PRACTICE (pages 123 to 126)

You may allow students to access the **PRACTICE** questions on **Ed**, which are auto-graded, for students to consolidate learning in the section digitally. For questions that require students to show their work, have them complete the questions in the book.

1 to 4 assess students' ability to make subtraction stories with each given picture.

For 1, students are required to make a subtraction story about the monkeys. Encourage students to discuss with their partners and ask these questions:

What subtraction story can we make? How many monkeys are there in the tree? (9) **How many monkeys swing away?** (4 monkeys swing away.) **How many monkeys are left in the tree?** (5 monkeys are left in the tree.)

Have students go through the subtraction story again in 1. This time, have students cross out the monkeys that swing away. Then, count the remaining monkeys.

Appendix M

Excerpt from a Third Grade *Math in Focus* Lesson
Chapter 7, Lesson 5: Comparing Fractions

(Marshall Cavendish Education, 2020b, p. 41-43)

5 Comparing Fractions

Learning Objective
- Compare fractions using models of the same size.

Mathematical Habits
- 4 Use mathematical models
- 6 Use precise mathematical language

New Vocabulary
- like fractions

Materials
- 1 set of fraction tiles per pair
- 1 set of fraction circles per pair
- 1 copy of Fraction Comparison Cards (TR32) per pair

Name: ______________________ Date: __________

5 Comparing Fractions

Learning Objective:
- Compare fractions using models of the same size.

New Vocabulary
like fractions

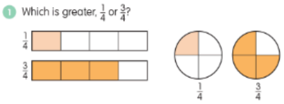
THINK
Fernando writes four fractions in sequence: $\frac{1}{4}$, A, $\frac{5}{8}$, B
What are the possible fractions that A and B can be?

ENGAGE

1. Use to show $\frac{3}{8}$ and $\frac{5}{8}$.
   Which fraction is greater? Which is less?
2. Draw bar models to find the greater fraction between $\frac{5}{6}$ and $\frac{3}{4}$.

LEARN Compare fractions

1. Which is greater, $\frac{1}{4}$ or $\frac{3}{4}$?



$\frac{3}{4}$ is greater than $\frac{1}{4}$.
$\frac{3}{4} > \frac{1}{4}$

$\frac{1}{4}$ and $\frac{3}{4}$ have the same denominator.
They are called like fractions.
The greater fraction is the one with the greater numerator.

Student Edition Page 41

DAYS 9 – 10 of 13

THINK (page 41)

Let's begin by discussing how to write fractions in a sequence.
- Pose the problem and allow students time to work on it in pairs or small groups. Make available whiteboards, fraction tiles, and fraction circles for students to use if desired. Use the question prompts to facilitate productive struggle.

What do we notice about $\frac{1}{4}$ and $\frac{5}{8}$? ($\frac{5}{8}$ is a larger fraction) **How can we state the problem in a different way?** (We are looking for two missing fractions, one that is greater than $\frac{5}{8}$, and another that is greater than $\frac{1}{4}$ but less than $\frac{5}{8}$.) **Since the fractions are written in sequence, what do we know about A and B?** (A is greater than $\frac{1}{4}$ and less than $\frac{5}{8}$. B is greater than $\frac{5}{8}$.) **How can we find possible fractions for A and B?** (Answers vary. Example: We can compare the fractions to $\frac{1}{2}$. $\frac{1}{2}$ lies between $\frac{1}{4}$ and $\frac{5}{8}$. So, A could be $\frac{1}{2}$. B is greater than $\frac{5}{8}$. So, a possible value of B is $\frac{6}{8}$, or $\frac{3}{4}$.)

- After students have discussed their ideas, tell them that they will learn more about comparing fractions before revisiting the problem.
- You may revisit the problem with the students at the start of each day to encourage them to discuss how they might solve the problem differently. A final evaluation of the problem will be found at the end of the section.

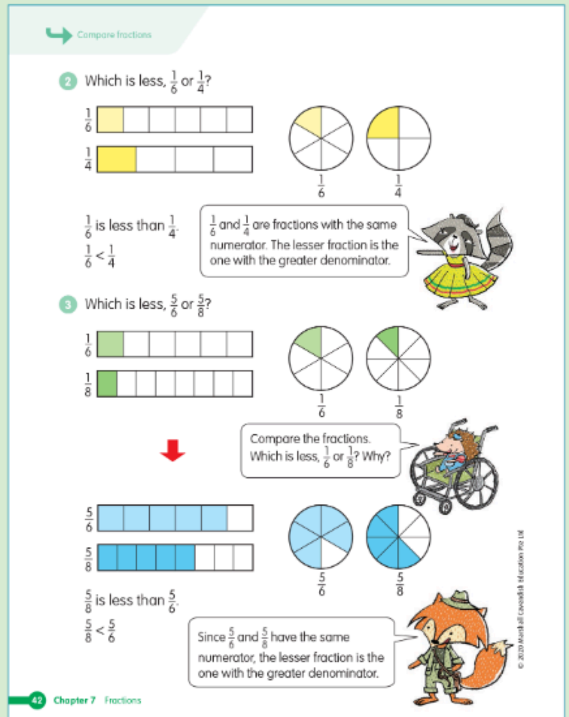
ENGAGE (page 41)
Concrete Pictorial Abstract

- The intent is for students to begin comparing fractions by considering the size of the parts.

Let's explore how we can compare fractions.
- Invite students to work in pairs and provide each pair with a set of fraction tiles. Use the questions to prompt and guide students' thinking.

What are we asked to do? (Use fraction tiles to show $\frac{3}{8}$ and $\frac{5}{8}$.) **How would we use the fraction tiles to compare the two fractions?** (Place a fraction tile completely underneath another to compare their sizes.)

- In 1, ask students to draw the fraction tiles for $\frac{3}{8}$ and $\frac{5}{8}$. Point out to students that drawing the tiles completely underneath another can help them to compare the fractions.
- Encourage students to use different colors to represent the two fractions to help compare them.
- In 2, ask students to draw two bar models of the same size. Support students to represent $\frac{5}{6}$ in one bar model and $\frac{3}{4}$ in the other.



Compare fractions

2 Which is less, $\frac{1}{6}$ or $\frac{1}{4}$?

$\frac{1}{6}$ $\frac{1}{4}$ $\frac{1}{6}$ $\frac{1}{4}$

$\frac{1}{6}$ is less than $\frac{1}{4}$.

$\frac{1}{6} < \frac{1}{4}$

$\frac{1}{6}$ and $\frac{1}{4}$ are fractions with the same numerator. The lesser fraction is the one with the greater denominator.

3 Which is less, $\frac{5}{6}$ or $\frac{5}{8}$?

$\frac{1}{6}$ $\frac{1}{8}$ $\frac{1}{6}$ $\frac{1}{8}$

Compare the fractions. Which is less, $\frac{1}{6}$ or $\frac{1}{8}$? Why?

$\frac{5}{6}$ $\frac{5}{8}$ $\frac{5}{6}$ $\frac{5}{8}$

$\frac{5}{8}$ is less than $\frac{5}{6}$.

$\frac{5}{8} < \frac{5}{6}$

Since $\frac{5}{6}$ and $\frac{5}{8}$ have the same numerator, the lesser fraction is the one with the greater denominator.

Student Edition Page 42

continued ...

What do we notice about fourths and sixths? (There are more equal parts in sixths. The sixths are smaller than the fourths.)

Which fraction is less? How do we know? ($\frac{3}{4}$, because the size of the shaded parts is smaller.)

- Extend student's thinking by asking the following question.

What do we notice about the denominators and numerators in this ENGAGE? (Answers vary. Example: The denominators in 1 are the same. The numerators indicate which fraction is greater or smaller. The denominators and numerators in 2 are different.)

- Allow students time to carry out the task and discuss their work
- Encourage students who had different reasoning to share their explanations with the class.

LEARN (pages 41 and 42)

Compare fractions

Concrete Pictorial Abstract

1

Now, let's compare more fractions.

- You may use the virtual rectangular fraction models to support classroom teaching and learning at appropriate junctures.
- Distribute fraction tiles or circles to students. Ask them to show $\frac{1}{4}$ and $\frac{3}{4}$ using the concrete materials, and then compare them.
- Guide and extend students' thinking by facilitating the discussions.

Which is greater, $\frac{1}{4}$ or $\frac{3}{4}$? How do we know? ($\frac{3}{4}$ is greater, because the size of 3 out of 4 equal parts is larger than the size of 1 out of 4 equal parts.)

- Encourage students to draw models of both fractions to record their thinking.
- Point out that they can compare fractions using the > or < symbol.
- Ask students to consider the denominators of the two fractions, and define fractions with the same denominators as like fractions.

How can we use the numerators to compare like fractions? (The greater fraction is the fraction with the greater numerator.)

2

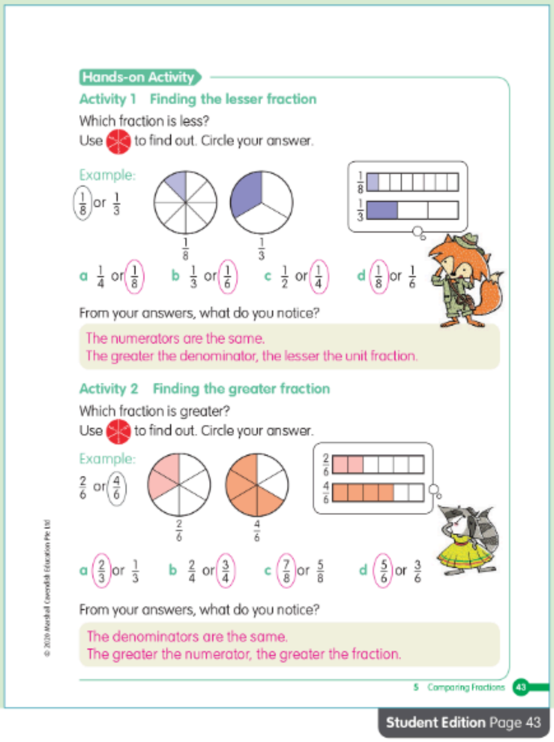
We have compared like fractions in 1. Now, we will compare fractions with different denominators.

- Encourage students to show and compare $\frac{1}{6}$ and $\frac{1}{4}$ using concrete materials and models.
- Guide students' thinking by asking the following questions.

What do we notice about the fractions $\frac{1}{6}$ and $\frac{1}{4}$? (Answers vary. Example: the numerators are the same; the denominators are different) **Which is less, $\frac{1}{6}$ or $\frac{1}{4}$?** ($\frac{1}{6}$ is less, because the size of the shaded part is smaller.)

- Guide students to compare the fractions using the symbols.

What can we notice about comparing fractions with the same numerator? (Answers vary. Example: The greater the denominator, the smaller the size of each equal part. This means the fraction with the greater denominator is less.)



Hands-on Activity

Activity 1 Finding the lesser fraction

Which fraction is less?

Use [fraction circles] to find out. Circle your answer.

Example: ($\frac{1}{8}$) or $\frac{1}{3}$

a $\frac{1}{4}$ or ($\frac{1}{8}$) b $\frac{1}{3}$ or ($\frac{1}{6}$) c $\frac{1}{2}$ or ($\frac{1}{4}$) d ($\frac{1}{8}$) or $\frac{1}{6}$

From your answers, what do you notice?

The numerators are the same.
The greater the denominator, the lesser the unit fraction.

Activity 2 Finding the greater fraction

Which fraction is greater?

Use [fraction circles] to find out. Circle your answer.

Example: $\frac{2}{6}$ or ($\frac{4}{6}$)

a ($\frac{2}{3}$) or $\frac{1}{3}$ b $\frac{2}{4}$ or ($\frac{3}{4}$) c ($\frac{7}{8}$) or $\frac{5}{8}$ d ($\frac{5}{6}$) or $\frac{3}{6}$

From your answers, what do you notice?

The denominators are the same.
The greater the numerator, the greater the fraction.

5 Comparing Fractions 43

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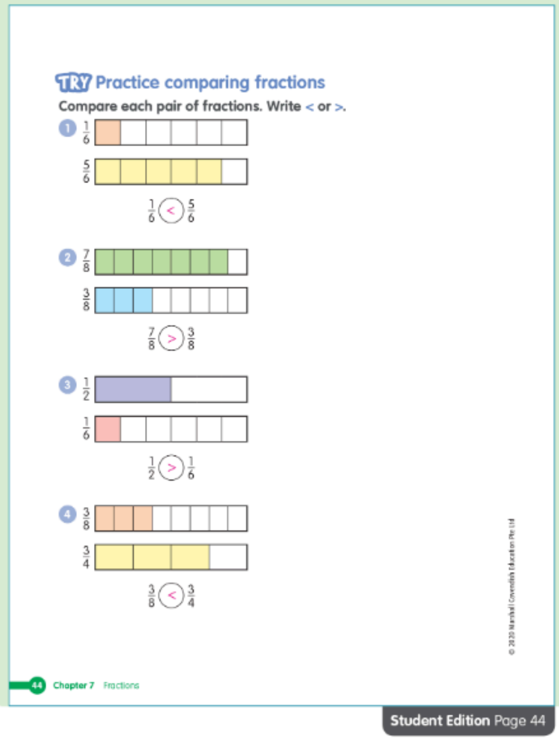
Student Edition Page 43



TRY Practice comparing fractions

Compare each pair of fractions. Write < or >.

1. $\frac{1}{6} < \frac{5}{6}$
2. $\frac{7}{8} > \frac{3}{8}$
3. $\frac{1}{2} > \frac{1}{6}$
4. $\frac{3}{8} < \frac{3}{4}$

44 Chapter 7 Fractions

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Student Edition Page 44

continued ...

3

What can we tell about the fractions $\frac{5}{6}$ and $\frac{5}{8}$? (Answers vary. Example: They have the same numerator.)

- Based on what students concluded from the previous problems, ask them to consider how that applies to this problem.

What do we know about the size of the denominators? (One fraction shows sixths and the other eighths.)

Which equal parts are greater in size? (sixths) **How do we know?** (One whole has been divided into 6 equal parts. Each of these parts is larger than an eighth, where the whole has been divided into 8 equal parts.)

- Ask students to draw models of both fractions to compare $\frac{5}{6}$ and $\frac{5}{8}$.

Which is less, $\frac{5}{6}$ or $\frac{5}{8}$? ($\frac{5}{8}$) **Why is the lesser fraction the one with the greater denominator?** (Answers vary. Example: The whole is cut into smaller equal parts.)

- Ask students to record the comparison using the > or < symbol.

Hands-on Activity (page 43)

Activity 1 Finding the lesser fraction

Let's use fraction circles to compare fractions and find out which fraction is less.

- Organize students into groups and distribute fraction circles to each group of students.
- Encourage students to draw models and shade in the fractions with two different colors to compare them.
- Extend students' thinking with the following questions.

What helped us to compare the fractions? (Accept all possible answers.)

What do we notice about the unit fraction when finding the lesser fraction? (For unit fractions (numerator of 1), the greater the denominator, the lesser the fraction.)

Activity 2 Finding the greater fraction

Now, we will use fraction circles to compare fractions and find out which fraction is greater.

- Extend students' thinking with the following questions.

What helped us compare the fractions? (Accept all possible answers.) **What can we notice about the numerator when finding the greater fraction?** (For fractions with the same denominators, the greater the numerator, the greater the fraction.)

TRY (page 44)

For 1 and 2, encourage and guide students to use the models and the numerators to compare the like fractions.

For 3 and 4, encourage and guide students to use the models and the denominators to compare the fractions with the same numerators.

Appendix N

Excerpt from a Fourth Grade *Math in Focus* Lesson
Chapter 8, Lesson 1: Subtraction Stories
(Marshall Cavendish Education, 2020b, 107-117)

1 Area and Unknown Sides

Learning Objectives
- Find the perimeter and area of a rectangle or square using a formula.
- Find the unknown side of a rectangle or square given its perimeter and one known side.
- Find the unknown side of a rectangle or square given its area and one known side.

Mathematical Habits
- 5 Use tools strategically
- 6 Use precise mathematical language

New Vocabulary
- formula

Materials
- 1 set of square counters per pair
- 6 copies of Centimeter Square Grid Paper (TR15) per pair

Name: ________ Date: ________

1 Area and Unknown Sides

Learning Objectives:
- Find the perimeter and area of a rectangle or square using a formula.
- Find the unknown side of a rectangle or square given its perimeter and one known side.
- Find the unknown side of a rectangle or square given its area and one known side.

New Vocabulary
formula

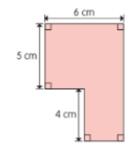
THINK

Daniel has a square piece of paper. The side lengths of the square are whole numbers. He cuts out a smaller square with an area of 16 square centimeters from the paper. What is the least possible area of

a the square piece of paper at first?
b the remaining piece of paper?

ENGAGE

How can you find the perimeter of the figure? How did the sides you know help you to find the sides you do not know? Explain.



Student Edition Page 107

DAY 2 of 10

THINK (page 107)

Let's begin by taking a closer look at the relationship between area and the side lengths of a square.

- Pose the problem and allow students time to work on it in pairs or small groups using the four-step problem-solving model. Use the question prompts to facilitate productive struggle.

What are we asked to do? (Two things: find the size of the paper Daniel started with, and the area of the paper that is left over.) **What do we know that might help us with this problem?** (We know that to find the area of a square, we multiply the side lengths.) **What tools could be used to help solve this problem?** (We could build the figure using square counters or grid paper.) **What squares can you build with your counters?** (1 unit², 4 units², 9 units², 16 units², 25 units², etc.) **Which of these can help you?** (16 units², 25 units²)

- Allow students time to try different ideas. The goal is for students to share their thinking.
- After students have discussed their ideas, tell them that they will learn more about area and perimeter before revisiting the problem.
- You may revisit the problem with the students at the start of each day to encourage them to discuss how they might solve the problem differently. A final evaluation of the problem will be found at the end of the section.

ENGAGE (page 107)

Concrete Pictorial Abstract

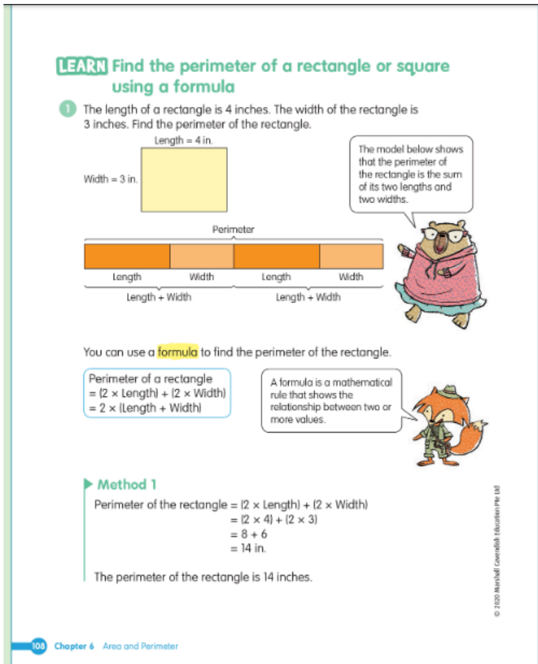
- The intent is for students to investigate how knowing some sides of a figure can help them to find the perimeter of the figure.

Let's begin by looking at a composite figure, and using some side lengths to work out the unknown lengths.

- Display the problem in **ENGAGE** and read it aloud with the class. Invite students to work in pairs. Use the questions to prompt and guide students' thinking.

What is the problem asking us to find? (We are asked to state a method for working out the perimeter of the shape, and to explain how we can determine the unknown side lengths.) **What information is given?** (a diagram showing the lengths of three sides) **What do we already know about rectangles that may help us here?** (Opposite side lengths are equal.) **What is the sum of the sides lengths that are parallel to the side on the right of this shape?** (4 cm + 5 cm = 9 cm) **How can we use parallel sides to determine the length of unknown sides?** (Accept all possible answers.) **How could you find the perimeter of the rectangle?** (Answers vary. Examples: Add up all the side lengths; Add up the known side lengths and multiply by 2.)

- Invite students who used different strategies to share their methods with the class.


LEARN Find the perimeter of a rectangle or square using a formula

1 The length of a rectangle is 4 inches. The width of the rectangle is 3 inches. Find the perimeter of the rectangle.

You can use a formula to find the perimeter of the rectangle.

Perimeter of a rectangle
= (2 × Length) + (2 × Width)
= 2 × (Length + Width)

Method 1

Perimeter of the rectangle = (2 × Length) + (2 × Width)
= (2 × 4) + (2 × 3)
= 8 + 6
= 14 in.

The perimeter of the rectangle is 14 inches.

Student Edition Page 108

LEARN (pages 108 and 109)

Find the perimeter of a rectangle or square using a formula

Concrete Pictorial Abstract

1

Let's learn how to find perimeter using a formula.

- Display only the problem on the board.
- Draw the rectangle on the board. Label the length and width.

What are some ways we could approach this problem? (Answers vary. Examples: Build the rectangle with counters; draw it on grid paper; outline it with a rubber band on the geoboard) **What is perimeter?** (the sum of all the sides) **If we didn't know the measurements for each side, what should we write to show the sum?** (Answers vary. Example: Perimeter = length + width + length + width) **What might a bar model of the perimeter look like?** (Answers vary. Example: The lengths and widths would be the parts, with the total of all parts equal to the perimeter.)

- Display the bar model on page 108. Have students verify their thinking. Make certain they understand before moving on.

How many lengths are there? (2) **How many widths are there?** (2) **Must we always add when there is two of something?** (No, we could also multiply by 2.)

- Allow students time to discuss how they could generate the formula for finding the perimeter of a rectangle.
- Display the remainder of 1 on the board and discuss what a formula is. Define the term 'formula' and have students record it on the word wall and in their math journals.

Now, let's use the formula to find the perimeter of this rectangle.

- Have students prompt you while writing the formula on the board. Substitute values for length and width into the formula and ask students to solve it.

What is the length? (4 inches) **What is the width?** (3 inches)

- Continue through **Method 1**, and move on to **Method 2**.

What are we doing in each method? (In Method 1, we multiply the length by 2 and multiply the width by 2, and then add together. In Method 2, we add the length and width together, and then multiply the total by 2.)

Best Practice

Allow students to model the problems with grid paper, or on a geoboard throughout this section. It will help solidify conceptual understanding and support long-term knowledge retention for this chapter.

Method 2

Perimeter of the rectangle = 2 × (Length + Width)
= 2 × (4 + 3)
= 2 × 7
= 14 in.

The perimeter of the rectangle is 14 inches.

2 The length of a side of a square is 6 centimeters.
Find the perimeter of the square.

6 cm
6 cm

Perimeter of a square = 4 × Length of a side

Perimeter of the square = 4 × 6
= 24 cm

The perimeter of the square is 24 centimeters.

Student Edition Page 109

TRY Practice finding the perimeter of a rectangle or square using a formula

Find the perimeter of the rectangle.

1 7 ft, 4 ft

Method 1

Perimeter of the rectangle = (2 × 7) + (2 × 4)
= 14 + 8
= 22 ft

Method 2

Perimeter of the rectangle = 2 × (7 + 4)
= 2 × 11
= 22 ft

Find the perimeter of the square.

2 5 m

Perimeter of the square = 4 × 5
= 20 m

Student Edition Page 110

2

Now, let's look at finding the perimeter of a square.

- Draw a square on the board and label only one side 6 cm.

What do we know about squares? (Squares have four sides of the same length.) **How might this help us come up with a formula for the perimeter of a square?**
(Perimeter = 4 side length)

- As in 1, allow students to engage in productive struggle and discussion to try to generate the formula on their own.
- Refer students to the square on the board and ask them to find its perimeter using their formula.
- Display 2 and ask them to check their thinking against the solution provided in the Student Edition.

Do you think you generated an effective formula? Which is most efficient, and why? (Accept all possible answers.)

Best Practice

Students learn by talking and interacting. Use the richness of this class discussion to drive the section for the benefit of all your students, but especially struggling students who learn from their peers. Students who draw their own conclusions will remember the principles they uncover far better than if they are told.

TRY (pages 110 and 111)

For 1 and 2, encourage and guide students to find the perimeter of rectangles and squares.

For Language Development

A formula represents a mathematical relationship between variables. The formula for the perimeter of a rectangle can be written as:

Perimeter = 2(Length + Width)
= $2(L + W)$
= $2L + 2W$

The formulas for perimeter make use of the Distributive Property, which is written formally as $a(b + c) = ab + ac$.
It is called *distributive* because multiplication *distributes* over addition.

Appendix O

Professional Learning Resources for Teacher Leaders and Administrators

Resource	Topic	Description and Rationale
Harrison & Killion (2007), *Ten Roles for Teacher Leaders* (article)	Teacher Leadership	This article describes ten roles of teacher leadership in a succinct and practitioner friendly way. Administrators can refer to this article to help identify teachers that may serve as strong teacher leaders.
The Center for Great Teachers and Leaders (GTL) by American Institutes for Research	Teacher Leadership	The GTL is a comprehensive, research-based website and toolkit for teacher leadership. Included on the website are webinars for professional learning, articles about teacher leadership, and strategies to support and sustain teacher leadership.
NCTM (2014). *Principles to Action* (book)	Mathematics Instruction	Teacher leaders and administration should be knowledgeable about high quality mathematics instruction. Reading *Principles to Action* together will help teacher leaders and administrators better understand the qualities of high-quality mathematics instruction set forth by the NCTM (2014).
NCTM Professional Learning – Regional Conferences and Webinars	Mathematics Instruction	*Math in Focus* references NCTM research as a foundation of the curriculum. Sending teachers to regional conferences or paying for NCTM's virtual webinars can support teachers in growing their knowledge and understanding of quality mathematics instruction. Investing in the professional learning of teacher leaders is an investment in the professional capital of the school.
Lipton, L. & Wellman, B. (2011). *Groups at Work*: *Strategies and Structures for Professional Learning*. (book)	Professional Learning	This book provides strategies and structures for leading and guiding PL that supports more engagement. The book includes strategies for activating, goal setting, planning, encouraging dialogue, generating ideas, and reading texts. This book could be useful for supporting the development of active and engaging PL.

Appendix P

Guides for Planning Professional Learning

Below is a high-level overview for year-long professional learning at Barron. Each trimester has a different focus module for professional learning. The quantity of sessions for each module varies and teachers may meet more frequently during some modules than others.

Module 1: Describing and Measuring Mathematics Discussion

The first module consists of teachers understanding and measuring meaningful mathematics discussion. Module 1 sets the foundation for improved action at Barron by providing teachers with descriptions and examples of meaningful mathematics discussion through videos and articles. It also includes an explanation of the tools that the school will use to measure and evaluate discussion, along with practice implementing the tools and interpreting the data together. During each session within the module, teachers should be provided with opportunities to engage collaboratively discuss and reflect on their own practices. A brief example of guidance for sessions is below.

- **Session 1:** Provide a background of meaningful mathematics discussion as described by NCTM (2014) and Hufferd-Ackles et al. (2004). Introduce teachers to the rubric. Provide base-line and school-wide data that was collected by administrators and teacher leaders.
- **Session 2:** Teachers reflect on a 1-3 examples of mathematics discussion in their classroom using the rubric (Figure 5.1) and questions to support reflection (5.2). Collaboratively discuss goals to improve mathematics discussion and advance levels on the rubric. Introduce student surveys as a practical measure and advise teachers to give survey to students.
- **Session 3:** Review student survey data with teachers. Review potential questions that teachers may ask teachers to understand the data revealed in the survey. Make connections between survey data and Hufferd-Ackles et al. (2014) rubric. Collaboratively develop actions related to improving discussion according to the survey.
- **Session 4:** Watch videos and examples of quality mathematics instruction. Practice evaluating lessons using student survey questions and Hufferd-Ackles et al. (2014) rubric. Discuss take-aways from videos that could be implemented in own practice. Advise teachers to issue a second survey to students before Session 5.
- **Session 5**: Review student survey data with teachers. Identify and share areas of growth. Revisit goals set previously in module and discuss supports necessary to achieve the goals.

Recommended Resources:

Articles

- Kalchmann, M. (2022). Revisiting Reinhart for uncertain teaching times. *Mathematics Teacher: Learning & Teaching PK-12, 115*(5), 351-356.
- Nieman, H.J., Kochmanski, N.M., Jackson, K.J, Cobb, P.A., & Henrick, E.C. (2020). Student surveys inform and improve classroom discussion. *Mathematics Teacher: Learning and Teaching PK-12, 113*(12), e91-99.

- Wagganer, E.L. (2015). Creating math talk communities. *Teaching children mathematics, 22*(4), 248-254.

Book Excerpt

- NCTM (2014). *Principles to Action,* pp 29-34.

Additional Resources

- Practical Measure Implementation (www.pmr2.org)
- TeachingWorks (2022) – High Leverage Practices

Module 2: Questions to Support Meaningful Mathematics Discussion

Now that teachers have an understanding of meaningful mathematics discussion, the tools used to measure meaningful mathematics discussion, and have reflected of their own practices using tools, teachers can begin to actionably plan for improved discussion. This module focuses on how teacher questioning supports meaningful mathematics discussion. Teacher videos from Module 1 can be revisited for Module 2. Additional articles and books are recommended.

During this module, I recommend that teacher leaders observe teachers during mathematics discussion and provide feedback. Additionally, teachers may visit other teachers' classrooms and mathematics discussions during their planning periods.

Rather than recommend specific topics for each session, I recommend that Teacher Leaders create collaborative planning structures that support teachers in planning lessons that incorporate questions using the resources provided in Recommendation 4 (Figure 5.3 and Figure 5.4). Chapin and colleagues' goals for discussion should be explicitly taught to teachers during PL. Teachers can continue to use surveys as evidence of improvement and teacher leaders should continue monitoring discussion across the building with the rubric.

Recommended Resources:

Articles

- Blumberg, G. (2022). Teaching students how to have an academic conversation. Edutopia.
- Candela, A.G., Boston, M.D., & Dixon, J. (2020). Discourse actions to prompte student access*. Mathematics Teacher: Learning and Teaching PK-12, 113*(4), 267-277.
- Garcia, N., Shaughnessy, M., & Pynes, D. (2021). Recording student thinking in a mathematics discussion. *Mathematics Teacher: Learning and Teaching PK-12, 114*(12), 927-931.
- Ghousseini, H., Lord, S., & Cardon, A. (2021). Supporting mathematics talk in kindergarten. *Mathematics Teacher: Learning and Teaching PK-12, 114*(5), 363-368.
- Luzniak, C. (2020). Mathematics is personal. Mathematics is debatable. *Mathematics Teacher: Learning and Teaching PK-12, 113*(4).

Books

- Anderson, N.C., Chapin, S.H., O'Connor, C., & (2017). *Talk Moves: A Facilitator's Guide to Support professional Learning of Classroom Discussions in Math*. Math Solutions
- Chapin, S.H., O'Connor, C., & Anderson, N.C. (2013). *Classroom Discussions in Math: A Teacher's Guide*. Math Solutions
- Smith, M.M., Bill, V., & Sherin, M.G. (2020). *The 5 Practices in Practice*. Corwin and NCTM.

Additional Resources

- Teaching Works (2022) – High Leverage Practices

Module 3: Responsive Professional Learning about Discussions

Module 3 is an opportunity for teachers to develop professional learning that is directly responsive to the needs of the school. Math leaders can develop this module using data from:

- Teacher Needs Surveys
- Student Surveys (Practical Measures)
- Evaluation of Discussion School-wide using Hufferd-Ackles et al. (2014) rubric

Depending upon the needs of the school, I recommend that professional learning from March-May be at least once a month.

All modules should include video examples of discussion. Videos of discussion can be found through the following resources:

- Inside Mathematics - University of Texas at Austin's Charles A. Dana Center (2021)
- Teacher Practice Videos - Engage New York on YouTube
- Videos of High Leverage Practices - TeachingWorks (2022)
- Teaching Channel
- Recorded teacher lessons of teachers at Barron

Appendix Q

Student-Facing Survey (K-2)

For each question, select the happy face if you agree and the sad face if you disagree.

1. I was comfortable sharing my ideas in class.

   ☺ ☹

2. Listening to other students helped me understand what I was learning!

   ☺ ☹

Listen carefully to the teacher as they read-aloud the next questions. Select the answer that you agree with the most.

3. Who talked the most in today's whole class discussion?

   ☐ Teacher ☐Students

4. What was the purpose of today's whole class discussion?

   ☐ To check to see if my answer was right!
   ☐ To practice solving problems the way my teacher showed me.
   ☐ Learn different ways to solve math problems from other students.
   ☐ To share an idea about math that I came up with on my own!

Appendix R

Student-Facing Survey (3-5)

For each question, select one response that best describes your experience in today's mathematics class.

1. Were you comfortable sharing your thinking in today's whole-class discussion?
   ☐ Yes ☐ No

2. Who talked the most in today's whole class discussion?
   ☐ Teacher ☐Student

3. Did listening to other students in today's whole class discussion help make my thinking better?
   ☐ Yes ☐ No

4. Did you have trouble understanding the other students' thinking in today's whole class discussion?
   ☐ Yes ☐ No

5. What was the purpose of today's whole class discussion?
   ☐ Share how we solved the problem using steps my teacher showed us.
   ☐ Learn the way the teacher showed us to solve the problem.
   ☐ Check to see if our answers are correct
   ☐ Share a mathematical idea we came up with on our own.
   ☐ Learn different ways that work to solve a problem from other students.

Appendix S

Interpreting Survey Results

The following can support teachers in interpreting survey results as advised from Kochmanski et al. (2015) and *Practical Measures, Routines, and Representations* (2022).

Interpreting Survey Results

Survey Question	Interpretation	Questions and Prompts to Support Reflection and Discussion
Were you comfortable sharing your thinking in today's whole-class discussion? ○ Yes ○ No	Students feeling comfortable and valued is essential for meaningful mathematics discussions.	If yes: - What are some structures and routines you have implemented to help students feel comfortable? - How do students respect one another during discussion in your class? If no: - Let's think for a moment about why students might not feel comfortable. - What can we do to build a better sense of community so that students are comfortable?
Who talked the most in today's whole class discussion? ○ Students ○ The Teacher	Students should talk more during meaningful mathematics discussions.	What does this question suggest about who leads discussion? Does anyone have suggestions for monitoring teacher talk? What are some things we can do to increase student talk?
Did listening to other students in today's whole class discussion help make my thinking better? ○ Yes ○ No	Peer engagement is essential to meaningful mathematics discussion because it can support students in clarifying their own thinking. If students do not listen to their peers to help make their own thinking better, teachers may need to work on supporting peer engagement for sense making.	Why is it important that our students listen to their peers? How does listening to your peers help you during our PLCs? What are some things we can do to help students listen to their peers?

Did you have trouble understanding other students' thinking in today's whole class discussion? o Yes o No (Question eliminated for K-2)	If students have difficulty understanding the ideas of their peers, teachers may need to develop goals that focus on students clearly explaining their thinking and asking students to repeat ideas of their peers.	How can we, as teachers, make sure that student explanations are clear? What are some teachers moves that can help our students to make sure they can listen to understand their peers?
What was the purpose of today's whole class discussion? o Share how we solved the problem using steps my teacher showed us. o Learn the way the teacher showed us to solve the problem. o Check to see if our answers are correct o Learn different ways that work to solve a problem from other students. o Share a mathematical idea we came up with on our own. (Question modified for K-2; combined options 1 and 2)	Options 1-3 suggest a classroom focused on producing the correct answers. Options 3-4 suggest a classroom focused on sense making in mathematics and more meaningful mathematics discussion.	If Options 1-3: - What are the things we do that emphasize correct answers more than sense making? - What are some things we can say in our class to show we want discussion to include sense making and strategies more than just correct answers? If Options 3-4: - How do we show our students that we want them to make sense of math? - What are some things you've done to cultivate a culture of sensemaking?