Adaptive Control for Distributed Leader-Following Consensus of Multi-Agent Systems

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Abstract

The Distributed leader-following consensus problem for multi-agent systems has drawn increasing attention recently. Consensus is a fundamental approach for distributed coordination. It means that a group of agents are made to reach an agreement on some common states using certain local information. In the leader-following consensus problem, there exists an active leader which specifies the movement of the whole group. A majority of existing research is focused on the leader-following consensus problem assuming that the parameters of follower agents are uncertain, while few papers consider the leader dynamic uncertainty at the same time.

This thesis studies the distributed leader-following consensus problem of multi-agent systems in which the leader and followers both have parametric uncertainties and bounded external disturbances. Follower agents are controlled to follow an active leader with a reference input signal, despite such uncertainties. An Adaptive control method is adopted to solve this problem. This research starts from the basic case that there are one leader and one follower in a multi-agent system. A new adaptive scheme is proposed for dealing with parametric uncertainties. Furthermore, in order to cancel the effect of disturbances, an adaptive disturbance compensator is developed. Then, expanding the size of the multi-agent system under a directed graph, a new distributed control protocol only using local information is adopted, generalizing the previous control scheme. The proposed distributed control protocol has the capability to guarantee that all agents can reach an agreement asymptotically with disturbances acting on the follower agents. Comparing with the classical fixed gain control method, the adaptive control method is capable of effectively handling system ans disturbance uncertainties. Extensive numerical simulation results illustrate the effectiveness of the proposed adaptive control scheme.

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Chapter 1

Introduction

In past two decades, an unprecedented growth in sensing, communications and computation has been witnessed. This growth change the way we collecting and processing the information. The sensor network revolution makes it possible to us for exploring and interacting with the environment. Hence the growing importance of the multi-agent system cooperative systems has also been acknowledge by the research community. Many important scientific results have appeared addressing different aspects of multi-agent cooperative systems. Due to the multi-disciplinary nature of multi-agent cooperative systems, research focusing on this field can address many parallel problems for instance in the area of distributed decision making, in the area of connectivity maintenance and in the area of vehicle formation, etc. [32].

In the meanwhile, adaptive control method is becoming popular in many fields. Since the control community has heavily acknowledged the importance of the adaptive control, lots of useful results on adaptive control appearing to make this modern control method more powerful. Hence, in this thesis, we combine these two popular topics, multi-agent cooperative systems and the adaptive control method, together. We will explore the leader-following consensus problems of multi-agent system by using adaptive control in the coming several chapters.

1.1 Research Motivation

With the development of sensor networks, control of multi-agent systems has been emerging and has drawn lots of attention. Since the main objective in distributed control of multi-agent systems is to enable a group of agents to perform a special task, distributed control is also referred as the cooperative control. There are several typical problems existing in the cooperative control including the consensus problem [9, 37, 45], the formation problem [4, 8, 34] and the flocking problem [36, 39, 41].

Among those three typical cooperative control problems, the consensus problem is the most important one because the consensus algorithm is basic and fundamental [26]. In particular, consensus means to reach an agreement on a certain quantity of interest, namely their position or velocities [37]. As the most fundamental and important control problem in cooperative control, Jadbabaie *et al.* [13] considered such the leader-following consensus problem and proved that if all the agents were jointly connected with their leader, their states would coverage to that of the leader as time goes on. The controllabilty of the leader-following dynamic network was studied in [14, 22, 46]. The leader-following consensus problem can be classified into two different kinds. The first one is the consensus regulation problem, which is also referred as leaderless problem (behavior-based) [20, 27, 31, 35]. The other one is the consensus tracking problem, also known as leader-following consensus(synchronization). For leader-following consensus problems, there should exist an active leader whose states keep changing in the multi-agent system [11, 21, 40, 50]. In another word, leader-following consensus means that all follower agents eventually reach an agreement on the state or output of a preassigned leader, which specifies a desired objective for all other agents to follow and is usually independent of its followers [24].

Formation control requires agents keeping a desired special distance, in another word, formation control requires agents keeping a desired formation configuration which actually is the consensus of relative position and flocking control requires agents to move together with the same velocity and avoid inter-collisions at the same time which obviously is the consensus on velocities [25]. The practical application of co-operative control is broadly used in many areas such as formation control of mobile vehicles and scheduling of automated highway systems.

1.2 Literature Review

In current leader-following consensus relative literature, distributed protocols are designed for multi-agent systems with single-integrator or double-integrator linear dynamics followers with an active leader. The active leader model is also governed by a single-integrator or double-integrator linear dynamics. A plenty of literature focuses on single-integrator and/or double-integrator systems like [9, 10, 33]. For example, [33], it considered a consensus algorithm for double-integrator dynamics with several different cases. In [51], the consensus problem was addressed for a double integrator multi-agent system to track a single-integrator leader with a desired constant velocity, and meanwhile the non-uniform time-varying communication delays were taken into account. Recently, the multi-agent system with general linear dynamics has also been considered. Recent design and analysis tools cover from the output regulation approach in [11] and [40] which based on the output regulation theory [3, 5, 48]. [28] investigated the consensus leader-following problem for a group of identical followers with an autonomous active leader whose parameter matrix is as same as the follower. In [28] only local information can be used by the distributed controller. Being different with [28], [24] solved the consensus problem for a group of followers with different dynamics under an external disturbances with an active leader which model can be also different with the followers. Plant parameters of the followers in [24] are in uncertainty. The distributed control protocol in [24] designed by model reference adaptive control method and also uses the local information of an individual agent. However, although the author said that some part of the leader's parameter can be unknown, this statement is not convinced. Motivations of the aforementioned research are clear but in practical application sometimes conditions of those researches are unsatisfied.

Motivated by the above observations, in this thesis we study the adaptive leader-following consensus problem of a group of linear dynamics agents guided by an active real leader with external disturbances under directed graphs. Different from [28], system parameter matrices in this thesis can be completely different with each other, and also the system parameter matrices of the leader are supposed to be different from all the followers. Comparing to [24], dynamic parameters can be completely unknown including dynamic parameters of all followers and the leader. This assumption is meaningful because in the real engineering environment, sometimes it is difficult to get accurate dynamic parameters. In the meanwhile, like many papers [12, 17–19, 23, 44], this thesis also considers the external disturbances acting on the real application. Hence, we develop a new protocol structure based on the principle of adaptive control method in order to solve the consensus leader-following problem with an external disturbances such that all follower agents can track the active leader with a desired reference signal. Then, the performance of a close-loop system is analyzed to verify the designed control protocol with the corresponding adaptive laws. Finally, several simulation results will be displayed to verify the effectiveness of our proposed theoretical protocol.

1.3 Thesis Outline

The remainder of this thesis is organized as follows. In Chapter 2 we introduce some basic background information to help readers follow the thesis easier including background about control systems and background about multi-agent systems. In Chapter 3, we develop a new adaptive control scheme for leader-following consensus problem with only one leader-one follower with and without disturbances and present simulation results to support the new adaptive scheme. In Chapter 4, we present a new adaptive control scheme for multi-agent systems with directed graph whose structure is modified from the control scheme developed in Chapter 3. Several cases are presented including the disturbance-free case and the disturbance-acting case. The corresponding numerical simulation results illustrate the effectiveness of the adaptive control protocol. Finally, in Chapter 5 we discuss the results in this thesis and the potential future work.

Chapter 2

Background

Before starting to discuss the adaptive control of multi-agent system, some basic background about control system and multi-agent system needs to be presented. In this chapter, topics about control system including control system modeling, system stability, classical control and adaptive control are presented first. Topics following the background of control system are some basic background about topology.

2.1 Control System Models

Usually in order to solve an engineering problem, the first step is to develop an appropriate mathematical model of the system either from physical laws or from experimental data. Dynamic systems are described by differential equations. While most dynamics systems are nonlinear in nature, study of their linearized models and linear systems has played a crucial role in understanding dynamic system behaviors.

In the first coming paragraph, a topic to be discussed is the non-linear system. In the second coming paragraph the linear system is discussed.

Nonlinear Systems A dynamic system can be described by a set of differential equations

$$F_i(y^{(p)}(t), \ldots, y^{(1)}(t), y(t), u^{(p)}(t), \ldots, u^{(1)}(t), u(t), t) = 0, t \geq t_0, \tag{2.1}$$

$i = 1, 2, \ldots, l$. $y^{(i)}(t)$ and $u^{(i)}(t)$ denote the ith time derivatives $\frac{\mathrm{d}^i y(t)}{\mathrm{d}t^i}$ and $\frac{\mathrm{d}^i u(t)}{\mathrm{d}t^i}$ of $y(t)$ and $u(t)$, with a common notation $\dot{y}(t) = y^{(1)}(t), \dot{u}(t) = u^{(1)}(t)$ and $\ddot{y}(t) = y^{(2)}(t), \ddot{u}(t) = u^{(2)}(t)$. A special form of F_i depends on a special system under consideration.

A n-th order dynamic system can be described by a group of n interactive first-order differential equations

$$\dot{x} = f_0(x, u, t),\ y = h_0(x, u, t),\ t \geq t_0, \tag{2.2}$$

for some functions $f_0 \in R^n$ and $h_0 \in R^q$ with $q \geq 1, n \geq 1$, where $x = [x_1, \ldots, x_n]^T \in R^n$ is the system state vector with the state variable x_i, $i = 1, 2, \ldots, n$, physical or artificial, to completely define the system behavior, $u(t) \in R^M$ with $M \geq 1$ is the system input, and $y(t)$ is the system output [42].

The system behavior depends on the control $u(t)$. When a feedback control law $u(t) = \beta(x, t)$ is use, system (2.2) becomes

$$\dot{x} = f(x, u, t),\ y = h(x, u, t),\ t \geq t_0, \tag{2.3}$$

for some functions $f \in R^n$ and $h \in R^q$ with $q \geq 1, n \geq 1$, whose solution is denoted as $x(t) = x(t; t_0, x_0)$, where $x(t_0) = x_0$ is the initial state vector. For a vector function $x(t)$, a measure of its "magnitude" is its norm $\|x\|$ [42].

However, sometimes in order to make system analysis easier, we need to degenerate the non-linear systems to the linear system on (x_0, u_0) by Taylor expansion,[1] which is

$$\delta \dot{x} = A\delta x + B\delta u$$
$$\delta y = A\delta x + B\delta u \tag{2.4}$$

with

$$A = \frac{\partial f}{\partial x}|_{(x_0,u_0)} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}_{|(x_0,u_0)}, \; B = \frac{\partial f}{\partial u}|_{(x_0,u_0)} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_n} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial u_1} & \cdots & \frac{\partial f_n}{\partial u_n} \end{bmatrix}_{|(x_0,u_0)},$$
$$C = \frac{\partial h}{\partial x}|_{(x_0,u_0)} = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \cdots & \frac{\partial h_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial h_n}{\partial x_1} & \cdots & \frac{\partial h_n}{\partial x_n} \end{bmatrix}_{|(x_0,u_0)}, D = \frac{\partial h}{\partial u}|_{(x_0,u_0)} = \begin{bmatrix} \frac{\partial h_1}{\partial u_1} & \cdots & \frac{\partial h_1}{\partial u_n} \\ \vdots & & \vdots \\ \frac{\partial h_n}{\partial u_1} & \cdots & \frac{\partial h_n}{\partial u_n} \end{bmatrix}_{|(x_0,u_0)}.$$

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Since linearization always be made in a small range, in oder to denote conveniently usually people express linearized system (2.4) as

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \tag{2.5}$$

(2.5) is called the "linearized system" and the method used in this linearization process is called 'Lyapunov linearization method". Lyapunov linearization method is concerned with the local stability of a nonlinear system. It is a formalization of the intuition that a nonlinear system method should behave similarly to its linearized

[1] Often, (x_0, u_0) is an equilibrium point which means $f(x_0, u_0) = 0$

approximation for small range motions. Because all physical systems are inherently nonlinear, Lyapunov's linearizion method serves as the fundamental justification of using linear control techniques in practice. More details about Lyapunov linearization method can be found in literature [38].

Linear Systems As we discussed in the previous paragraph, many nonlinear systems are hard to be analyzed or to be controlled. One of the methods to control nonlinear systems is linearization. Thus there is no doubt that researches on linear system are really important.

A linear system time-varying dynamic system is described as

$$\begin{aligned} \dot{x} &= A(t)x + B(t)u \\ y &= C(t)x + D(t)u, \ t \geq t_0, \end{aligned} \tag{2.6}$$

with $x(t_0) = x_0$,where $x \in R^n, u \in R^M, y \in R^q$ are the state, input and output, respectively. $A(t) \in R^{n\times n}, B(t) \in R^{n\times M}, C(t) \in R^{q\times n}$ and $D(t) \in R^{q\times M}$ are continuous functions of t. Systems like (2.6) are linear time varying system. If parameters A, B, C ans D in (2.6) are constant which does not depends on t, the system is called the "linear time invariant (LTI) system". In the time-invariant case, system (2.6) becomes

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du, \ t \geq t_0, \end{aligned} \tag{2.7}$$

where $A \in R^{n\times n}, B \in R^{n\times M}, C \in R^{q\times n}$, and $D \in R^{q\times M}$ are all constant system matrices. In this case, the state solution can be explicitly expressed as

$$x(t) = e^{A(t-t_0)}x_0 + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau \tag{2.8}$$

Gain matrix decomposition Leader-following systems to be discussed in this thesis are LTI systems. Usually, in control input signal, there exists one or more gains making the control input signal work. Introducing an important system characterizations for linear system which is useful to the later control design development.

Assume that all leading principal minors of the gain matrix K_p are nonzero [2]. The LDU decomposition of K_p exists and can be employed for adaptive control of the plant (2.7) [42].

Let Δ_i, $i = 1, 2, \ldots, M$ be the leading principle minors of K_p, The following proposition gives the key result of the LDU decomposition.

Proposition 2.1. *(LDU decomposition) A matrix $K_p \in R^{M\times M}$ with all its leading principle minors being nonzero has a unique decomposition:*

$$K_p = LD^*U \tag{2.9}$$

for some $M \times M$ unit(i.e., with all diagonal elements being 1) lower triangular matrix L and unit upper triangular matrix U, and

$$D^* = diag\left\{d_1^*, d_2^*, \ldots, d_M^*\right\} = diag\left\{\Delta_1, \frac{\Delta_2}{\Delta_1}, \ldots, \frac{\Delta_M}{\Delta_{M-1}}\right\}. \tag{2.10}$$

This is the well-known LDU decomposition of a nonsingular matrix with nonzero

[2]The kth leading principal minor of a matrix $A = a_{ij} \in R^{M\times M}$ is det $\begin{bmatrix} a_{11} & \ldots & a_{1k} \\ \vdots & & \vdots \\ a_{k1} & \ldots & a_{kk} \end{bmatrix}$, $k = 1, 2, \ldots, M$.

leading principal minors.

2.2 Signal Measures

In order to have a unique measurement for vector signals, a new concept called "norm" is defined. Consider a vector signal $x(t) = [x_1(t), \ldots, x_n(t)]^T \in R^n$. $x(t) = [x_1(t), \ldots, x_n(t)]^T$ is a vector at any t, and is a vector function as t changes. Vector norms can measure vectors, and while function norms can measure vector functions.

Definition 2.1. *A real-valued function* $\|\cdot\|$ *on linear space* $\mathcal{S}$ *is a norm if*

(i) $\|x\| \geq 0$ *for all* $x \in \mathcal{S}$ *and* $\|x\| = 0$ *only if* $x = 0$*;*

(ii) $\|\beta x\| = |\beta| \, \|x\|$ *for all* $x \in \mathcal{S}$ *and any scalar* β*; and*

(iii) $\|x + y\| \leq \|x\| + \|y\|$ *for all* $x, y \in \mathcal{S}$*.*

Since in control systems there exist many signal vectors that need to be measured, it is imperative to introduce signal norms before we develop a adaptive control scheme.

The L^1, L^2, and L^∞ norms are defined below as

$$L^1 = \{x(t) \in \mathbb{R}^n : \|x(\cdot)\|_1 < \infty\}, \tag{2.11}$$

$$L^2 = \{x(t) \in \mathbb{R}^n : \|x(\cdot)\|_2 < \infty\}, \tag{2.12}$$

$$L^\infty = \{x(t) \in \mathbb{R}^n : \|x(\cdot)\|_\infty < \infty\}, \tag{2.13}$$

where the vector signal norms are

$$\|x(\cdot)\|_1 = \int_0^\infty (|x_1(t)| + \cdots + |x_n(t)|)dt, \tag{2.14}$$

$$\|x(\cdot)\|_2 = \sqrt{\int_0^\infty x_1^2(t) + \cdots + x_1^n(t)}, \tag{2.15}$$

$$\|x(\cdot)\|_\infty = \sup_{t\geq 0} \max_{1\leq i\leq n} |x_i(t)|. \tag{2.16}$$

Remark 2.1 By definition of L^∞, we can conclude that $x(t) \in L^\infty$ iff $x(t)$ is bounded, i.e., $x(t) \in L^\infty$ and $x(t)$ is bounded are equivalent. □

2.3 System Stability

The concept of stability is crucial to control system design. An unstable control system is useless and dangerous. The methods available to examine the poles depend on the representation of the system model. If the classical approach is taken then the poles of the transfer function can be examined. If the modern approach is used then the eigenvalues, which are the poles, of the system matrix A can be analyzed. Either approach can quickly give information on whether or not the system is inherently stable, marginally stable, or unstable.

For adaptive control systems stability must be defined another way since knowledge of the system parameters are unavailable and possibly changing. The work of Alexander Mikhailovich Lyapunov, who presented definitions and theorems for studying the stability of solutions to a broad class of differential equations, has been used extensively to address this problem [16]. The work of Lyapunov relies on defining an energy function, formally known as a Lyapunov function candidate, that can be used to determine the stability of a system without having to solve for the solutions to the system explicitly. Originally, this Lyapunov function was purely the total mechanical or electrical energy and therefore by nature positive definite.

Lyapunov indirected method can be found in many textbooks about nonlinear system like [42], [38].

Stability definitions Since all systems to be discussed in this thesis are LTI systems, we will introduce the definition of stability for LTI [2]

Definition 2.2. *The response of $\dot{x}(t) = Ax(t)$ is marginally stable or stable in the sense of Lyapunov if every finite initial state x_0 excites a bounded response. It is asymptotically stable if every finite state excites a bounded response which, in addition, approaches 0 as $t \to \infty$.*

Usually, we do not use the definition to check the stability of a LTI system. Theorem 2.1 can help us to check the stability of a LTI system more quickly.

Theorem 2.1. *The equation $\dot{x}(t) = Ax(t)$ is marginally stable if and only if all eigenvalues of A have zero or negative real parts and those with zero real parts are simple roots of the minimal polynomial of A. The equation $\dot{x}(t) = Ax(t)$ is asymptotically stable if and only if all eigenvalues of A have negative real parts.*

Also in the Lyapunov sense, we can check the stability of matrix A by Lyapunov theorem

Theorem 2.2. *The equation $\dot{x} = Ax$, $A \in R^{n\times n}, x \in R^n$ is asymptotically stable if and only if for every positive definite $Q = Q^T \in R^{n\times n}$, the Lyapunov equation $A^TP + PA = -Q$ has a unique and positive definite solution $P = P^T \in R^{n\times n}$.*

Theorem 2.1 and Theorem 2.2 are theorems which we usually used to check the stability of close-looped system by classical control. But for adaptive control, Theorem 2.1 and Theorem 2.2 would not work because in adaptive control there exist uncertainty on the dynamics model, we have no access to get a set of accurate system parameters. So we introduce a new method called Lyapunov direct method to help us check the system stability when applying adaptive control.

Theorem 2.3. *(Lyapunov direct method) If in some Ball $B(h)$ there exists a positive definite function $V(x,t)$ with $\dot{V} \leq 0$, then the equilibrium state $x_e = 0$ of the the system $\dot{x} = Ax$ is stable. If, in addition, $V(x,t)$ is decrescent, then it is uniformly stable.*

Based on definitions and theorems presented before, Barbalart lemma is introduced. This lemma makes the precess to analyze system stability more easier.

Lemma 2.1. ***(Barbalat Lemma)****If a scalar function $\dot{f}(t) \in L^{\infty}$, $f(t) \in L^2$, then* $\lim_{t\to} f(t) = 0$. *[42]*

2.4 Classical Control

The first thing we need to know about the classical control is the feedback is pervasive. Feedback is a very crucial method to stabilize the unstable system stable [1]. Usually, the output $y(t)$ is fed back and compared with the input $u(t)$. The block diagram of the most classical feedback control system is shown in Fig 2.1. System transfer function can be computed by block diagrams like Fig 2.1. Remind that in Fig 2.1, k is a constant through the whole control process. Whether control parameter k can update or not is one of a big differences with adaptive control and the classical control.

With the state space representation it is convenient to feedback the state variables $x(t)$ to the control signal $u(t)$ instead of an input signal $y(t)$. With this configuration each state variable can be adjusted by a gain vector K to give the desired closed loop

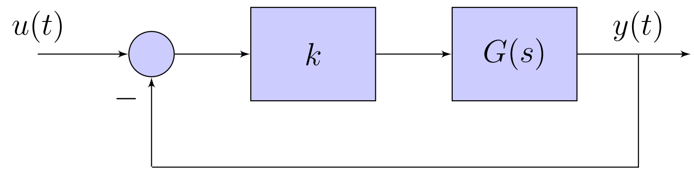

Figure 2.1: Block diagram of a system with output feedback and an adjustable preamplifier gain k.

poles. A typical control system represented with the state space representation utilizing state feedback is displayed in Figure 2.2 [29] where double error lines represent vector signals.

2.5 Adaptive Control

Unlike classical control systems, adaptive control system has capability to payload system uncertainties including unknown system structures, system parameters and other environmental uncertainties. An adaptive controller structure is designed based on a known parameter case, however, the value of the adaptive controller parameter is updated by parameter estimator instantly. Combining an adaptive controller with an parameter estimators, adaptive control can achieve a desired performance by dealing with variety kind of uncertainties.

There are two approaches to adaptive control design. The first one is direct adap-

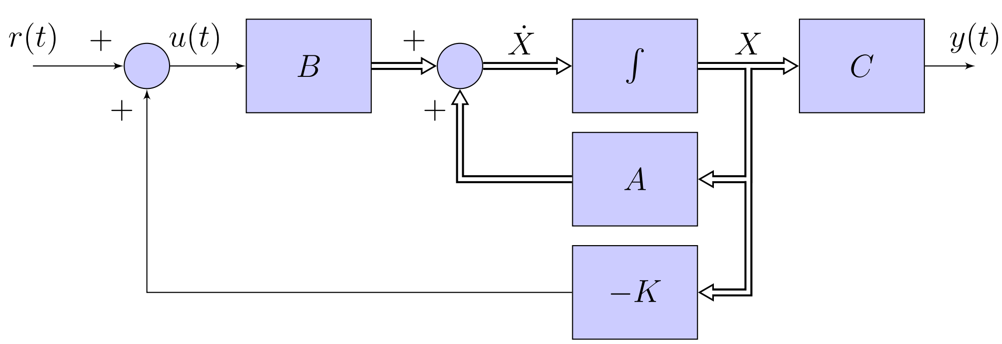

Figure 2.2: State space representation of a plant with state feedback. Reproduced from [29].

tive control. For direct adaptive control, the parameter estimator estimates controller parameters online directly according to the plant information of input/output. Plant parameters are parametrized implicitly in terms of a set of parameters of a nominal controller. The second approach is indirected adaptive control and is characterized by estimating the parameters of the plant first. Then based on some design equation to calculate control parameters to achieve the desired performance. Direct adaptive control will be employed in this thesis.

2.6 Graph Theory and Communication Topology

Information exchange between agents can be represented as a graph. We give some basic terminology and definitions from graph theory [6, 7] which will be used in Chapter 4.

Definition 2.3. *By a graph $\mathcal{G}$ we mean a finite set $\mathcal{V}(\mathcal{G}) = (v_i, \ldots, v_n)$, whose elements are called nodes or vertexes, together with set $\mathcal{E}(\mathcal{G} \subset \mathcal{V} \times \mathcal{V})$, whose elements are called edges. An edge is therefore an ordered pair of distinct vertexes.*

Definition 2.4. *A graph is called direct graph if and only if for all $(v_i, v_j) \in \mathcal{E}(\mathcal{G})$, the edge $(v_j, v_j) \in \mathcal{E}(\mathcal{G})$, then the graph is said to be undirected. Otherwise, it is called a directed graph.*

Definition 2.5. *An edge (v_i, v_j) is said to incoming with respect to v_j and outgoing with respect to v_i) and can be represented as an arrow with vertex v_i as its tail and vertex as its head.*

Definition 2.6. *A path of length r in a directed graph is a sequence $(v_0, \ldots, v_r)$ of $r + 1$ distinct vertexes such that for every $i \in \{0, \ldots, r-1\}$, (v_i, v_{i+1}) is an edge.*

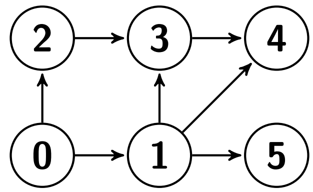

Figure 2.3: An illustrative of a multi-agent system with five followers and one leader

Chapter 3

Adaptive Leader-Following Control

The final objective for this thesis is to design a distributed adaptive control scheme of leader-following consensus problem for multi-agent system. However, before going that far, it is important for us to know how adaptive control works for systems including one follower and one leader. Fully understand this basic problem is important for us to develop a more complex controller for multi-agent systems in Chapter 4.

3.1 Problem Statement

For state tracking control problems, there are two different common cases we are interested in. One is the control problem with a single input, the other one is the control problem with multiple inputs. Adaptive control of single input is easier and more fundamental than multiple input cases and adaptive control of multiple input systems are more significant and practical. In this chapter, both of these two control problems will be discussed.

3.1.1 Adaptive Control of Single Input Systems

Consider a linear time-invariant follower plant in state-space form

$$\dot{x}(t) = Ax(t) + bu(t) + bd(t),\ x(t) \in R^n, u(t) \in R, t \geq 0 \tag{3.1}$$

with $x(0) = x_0$, where $A \in R^{n \times n}$ is an unknown constant parameter matrix, and $b \in R^n$ is an unknown constant parameter vector and $d(t)$ is an external unknown bounded disturbance. The state $x(t) \in R^n$ is measurable. The leader dynamic system is give by

$$\dot{x}_m(t) = A_m x_m(t) + b_m u_m(t) + b_m d_m(t),\ x_m(t) \in R^n, u_m(t) \in R, t \geq 0 \tag{3.2}$$

where A_m is an unknown constant parameter matrix and b_m is an unknown constant parameter vector. $x_m(t)$ and $u_m(t)$ are available for measurement and bounded. $d_m(t)$ is an bounded disturbance acting on the leader dynamic system.

Regarding by the disturbances $d(t)$ and $d_m(t)$, there are three different cases in total.

Case I: Single input systems without disturbances In this case, $d(t) = 0$ and $d_m(t) = 0$. There exist no disturbance in the multi-agent system. The dynamic of the follower and the leader can be re-presented as

$$\dot{x}(t) = Ax(t) + bu(t),\ x(t) \in R^n, u(t) \in R, t \geq 0 \tag{3.3}$$

$$\dot{x}_m(t) = A_m x_m(t) + b_m u_m(t),\ x_m(t) \in R^n, u_m(t) \in R, t \geq 0 \tag{3.4}$$

Case II: Single input systems with a disturbance acting on the followers In this case, $d(t) \neq 0$ and $d_m(t) = 0$. The dynamic plant of the follower is

$$\dot{x}(t) = Ax(t) + bu(t) + bd(t),\ x(t) \in R^n, u(t) \in R, t \geq 0 \tag{3.5}$$

$d(t) \in R$ is an unknown external disturbance acting on the follower system. The disturbance should be bounded and satisfies

$$d(t) = d_0 + \sum_{\beta=1}^{q} d_\beta f_\beta(t) \tag{3.6}$$

where d_0 and d_β are unknown constant and $f_\beta(t)$ is a bunch of **known** continuous bounded functions for $\beta = 1, 2, \ldots, q$ for some $q \geq 0$. The dynamic of the given leader remains as (3.4).

Case III: Single input systems with disturbances acting on both the leader and the followers In this case, $d(t) \neq 0$ and $d_m(t) \neq 0$. When $d(t)$ and $d_m(t)$ are not equal zero, the follower plant is (3.5). The leader dynamic is

$$\dot{x}_m(t) = A_m x_m(t) + b_m u_m(t),\ x_m(t) \in R^n, u_m(t) \in R, t \geq 0 \tag{3.7}$$

where

$$d_m(t) = d_{m0} + \sum_{\beta=1}^{q_m} d_{m\beta} f_{m\beta}(t) \tag{3.8}$$

with d_{m0} and $d_{m\beta}$ are unknown constant and $f_{m\beta}(t)$ are some **known** continuous bounded functions for $\beta = 1, 2, \ldots, p$ for some $p \geq 0$.

The control objective is to design a bounded state feedback control signal $u(t)$ to make the follower system state $x(t)$ bounded and tracking the leader system $x_m(t)$ asymptotically, i.e., $\lim_{t\to\infty}(x(t) - x_m(t)) = 0$.

3.1.2 Adaptive Control of Multiple Inputs Systems

In section 3.1.1, the control input $u(t)$ and the leader reference signal $u_m(t)$ are scalars, and parameters b and b_m are vectors. However, in practical engineering applications, there are many multiple inputs systems with $u \in R^m, m \neq 1$.

Consider a follower plant expressed as (3.9), and the leader is given as (3.10)

$$\dot{x}(t) = Ax(t) + Bu(t) + Bd(t),\ x(t) \in R^n, u(t) \in R^p \tag{3.9}$$

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) + B_m d_m(t),\ x_m(t) \in R^n, u_m(t) \in R^m \tag{3.10}$$

where $A \in R^{n\times n}$ is an unknown parameter matrix and $A_m \in R^{n\times n}$ is an asymptotically stable parameter matrix. However, $B \in R^{n\times p}$ and $B_m \in R^{n\times m}$ become to parameter matrices rather than parameter vectors. Although the dimension of B and B_m are changed, all the parameters matrices here including A, A_m, B and B_m remain unknown in this section. $x_m(t)$ and $u_m(t)$ are available for measurement and bounded and $x(t)$ is measured.

Being similar to single input system, classify multiple inputs systems into three cases regarding by the existence of the disturbances.

Case I: Multiple inputs systems without disturbances $d(t) = 0$ and $d_m(t) = 0$. In this situation, the one leader-one follower system degenerate into

$$\dot{x}(t) = Ax(t) + Bu(t),\ x(t) \in R^n, u(t) \in R^p, t \geq 0 \tag{3.11}$$

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t),\ x_m(t) \in R^n, u_m(t) \in R^m, t \geq 0 \tag{3.12}$$

Case II: Multiple inputs systems with a disturbance acting on the follower systems $d(t) \neq 0$ and $d_m(t) = 0$. The dynamic plant of the follower is

$$\dot{x}(t) = Ax(t) + Bu(t) + Bd(t),\ x(t) \in R^n, u(t) \in R^p, t \geq 0 \tag{3.13}$$

$d(t) \in R^p$ is an unknown external disturbance acting on the follower system. The disturbance should be bounded and satisfies

$$d(t) = d_0 + \sum_{\beta=1}^{q} d_\beta f_\beta(t) \tag{3.14}$$

where $d_0 \in R^p$ and $d_\beta \in R^p$ are unknown constant vectors and $f_\beta(t)$ are a bunch of **known** continuous bounded functions for $\beta = 1, 2, \ldots, q$ for some $q \geq 0$. The given leader remains as

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t),\ x_m(t) \in R^n, u_m(t) \in R^m, t \geq 0 \tag{3.15}$$

Case III: Single input systems with disturbance acting on both leader and follower $d(t) \neq 0$ and $d_m(t) \neq 0$. the follower plant is (3.13). The dynamic of the given leader is

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) + B_m d_m(t),\ x_m(t) \in R^n, u_m(t) \in R^m, t \geq 0 \tag{3.16}$$

where

$$d_m(t) = d_{m0} + \sum_{\beta=1}^{q} d_{m\beta} f_{m\beta}(t) \tag{3.17}$$

with $d_{m0} \in R^m$ and $d_{m\beta} \in R^m$ are unknown constant and $f_{m\beta}(t)$ are some **known** continuous bounded functions for $\beta = 1, 2, \ldots, q$ for some $p \geq 0$.

The control objective is to design a bounded control signal $u(t)$ to make the follower system state $x(t)$ bounded and track the leader system state $x_m(t)$ asymptotically.

3.2 Adaptive Following Control Design for Single Input Systems

In this section, we will solve the single input system leader-following state tracking problem. The single input system without disturbance will be solved first which includes the basic principle of the leader-following state tracking problem. Disturbance rejection is developed in Section 3.2.4.

3.2.1 Design Conditions

As stated in Section 3.1.1, the control objective is to design a bounded state feedback control signal $u(t)$ to make the follower system state $x(t)$ bounded and tracking the leader system $x_m(t)$ asymptotically.

To meet the control objective, we assume

(A3.1) all the eigenvalues of A_m are in the open left-half complex plane;

(A3.2) $u_m(t)$ is bounded and piecewise continuous;

(A3.3) there exist two constant vectors $k_1^* \in R^n$ and $k_3^* \in R^n$ and a non-zero constant scalars $k_2^* \in R$, $k_4^* \in R$ such that the following equations are satisfied:

$$\begin{aligned} A_e &= A + bk_1^{*T}, b_m = bk_2^* \\ A_m &= A + bk_3^{*T}, b_e = bk_4^* \end{aligned} \quad , \tag{3.18}$$

where $A_e \in R^{n\times n}$ is a stable and known matrix and $b_e \in R^n$ is a known vector;

(A3.4) $sign[k_4^*]$, the sign of the parameter k_4^*, is known;

Assumptions (A3.1) and (A3.2) are for a stable, well-defined reference system with a bounded output $y_m(t)$. Assumption (A3.3) is the so-called matching condition such that if the parameters of A_m, b_m, A and b are known and (3.18) is satisfied then the control law

$$u^*(t) = k_1^{*T}(x(t) - x_m(t)) + k_2^* u_m(t) + k_3^{*T} x_m(t) \tag{3.19}$$

achieves the control objectives: the closed-loop system becomes

$$\begin{aligned} \dot{x}(t) &= Ax(t) + b(k_1^{*T}(x(t) - x_m(t)) + k_2^* u_m(t) + k_3^{*T} x_m(t)) \\ &= (A + bk_1^{*T})(x(t) - x_m(t)) + (A + bk_3^{*T})x_m(t) + bk_2^* u_m(t) \\ &= A_e(x(t) - x_m(t)) + A_m x_m(t) + b_m u_m(t) \end{aligned} \tag{3.20}$$

whose state vector $x(t)$ belongs to L^∞, and the tracking error $e(t) = x(t) - x_m(t)$ satisfies:

$$\dot{e}(t) = A_e e(t), e(0) = x(0) - x_m(0) \tag{3.21}$$

which indicates that $\lim_{t\to\infty} e(t) = 0$ exponentially. So that with the nominal controller (3.19) , the desired control objective can be achieved.

3.2.2 Adaptive Control Laws

In our problem, the parameters of A, b, A_m and b_m are unknown, so (3.19) can not be used for control. In this case an adaptive controller which has the same structure of (3.19) is to be used, whose structure is given as

$$u(t) = k_1^T(t)(x(t) - x_m(t)) + k_2(t)u_m(t) + k_3^T(t)x_m(t) \tag{3.22}$$

where $k_1(t), k_2(t)$ and $k_3(t)$ are the estimates of k_1^*, k_2^* and k_3^* respectively. The design task now is to choose adaptive laws update these estimates so that the control objective is still achievable even all the four parameters are unknown.

To be specific, the adaptive laws to update the control parameters are proposed as

$$\dot{\tilde{k}}_1(t) = \dot{k}_1(t) = -sign[k_4^*]\Gamma e(t)e^T(t)Pb_e \tag{3.23}$$

$$\dot{\tilde{k}}_2(t) = \dot{k}_2(t) = -sign[k_4^*]\gamma u_m(t)e^T(t)Pb_e \tag{3.24}$$

$$\dot{\tilde{k}}_3(t) = \dot{k}_3(t) = -sign[k_4^*]\Psi x_m(t)e^T(t)Pb_e \tag{3.25}$$

where $P \in R^{n\times n}$ is a positive definite matrix satisfying $A_e^T P + PA_e = -Q$, for any chosen $Q \in R^{n\times n}$ being constant and $Q = Q^T > 0$. $\Gamma \in R^{n\times n}, \Psi \in R^{n\times n}$ are constant matrices, and $\Gamma = \Gamma^T > 0, \Psi = \Psi^T > 0$, $\gamma > 0$ is a constant scalar. $k_1(0), k_2(0)$ and $k_3(0)$ are arbitrary.

In summary, according to the previous development, we now present the following result.

Theorem 3.1. *The adaptive controller* (3.22)*, with the adaptive laws* (3.23), (3.24) *and* (3.25)*, applied to the plant* (3.3) *guarantees that all closed-loop signals are bounded and the tracking error* $e(t) = x(t) - x_m(t)$ *goes to zero as* t *goes to infinity, i.e.,* $\lim_{t\to\infty} e(t) = 0$*.*

3.2.3 Stability Analysis

To prove Theorem 3.1, we see

$$\begin{aligned}u(t) &= u(t) - u^*(t) + u^*(t)\\&= (k_1(t) - k_1^*)^T(x(t) - x_m(t)) + (k_2(t) - k_2^*)u_m(t)\\&\quad + (k_3(t) - k_3^*)^T x_m(t) + u^*(t),\end{aligned} \tag{3.26}$$

and then we derive the adaptive control based on the tracking error equation

$$\begin{aligned}\dot{e}(t) &= A_e e(t) + b(k_1(t) - k_1^*)^T e(t) + b(k_2(t) - k_2^*)u_m(t) + b(k_3(t) - k_3^*)^T x_m(t)\\&= A_e e(t) + b_e\left(\frac{1}{k_4^*}\tilde{k}_1^T(t)e(t) + \frac{1}{k_4^*}\tilde{k}_2(t)u_m(t) + \frac{1}{k_4^*}\tilde{k}_3^T(t)x_m(t)\right)\end{aligned} \tag{3.27}$$

where

$$\tilde{k}_1(t) = k_1(t) - k_1^*,\ \tilde{k}_2(t) = k_2(t) - k_2^*,\ \tilde{k}_3(t) = k_3(t) - k_3^* \tag{3.28}$$

are parameter errors.

Since the adaptive laws for $k_1(t), k_2(t)$ and $k_3(t)$ are chosen to be dynamics from some adaptive laws, the state vector of the closed-loop error system is

$$e_c(t) = (e^T(t), \tilde{k}_1^T(t), \tilde{k}_2(t), \tilde{k}_3^T(t))^T \in R^{3n+1}. \tag{3.29}$$

We choose a positive definite function as a Lyapunov function candidate

$$V(e_c) = e^T P e + \frac{1}{|k_4^*|}\tilde{k}_1^T\Gamma^{-1}\tilde{k}_1 + \frac{1}{|k_4^*|}\tilde{k}_2^2\gamma^{-1} + \frac{1}{|k_4^*|}\tilde{k}_3^T\Psi^{-1}\tilde{k}_3 \tag{3.30}$$

as a measurement of the system errors. As stated in the adaptive laws (3.23)-(3.25),

$P \in R^{n\times n}$ is positive definite and satisfies:

$$A_e^T P + P A_e = -Q < 0. \tag{3.31}$$

Parameter matrices Γ and Ψ are symmetric positive definite, and parameter γ is a positive scalar as aforementioned in the previous Subsection 3.2.2.

Compute the time derivative of $V(e_c)$

$$\begin{aligned}
\dot{V} &= \frac{\mathrm{d}}{\mathrm{d}t}V(e_c) \\
&= \left(\frac{\partial V(e_c)}{\partial e}\right)^T \dot{e}(t) + \left(\frac{\partial V(e_c)}{\partial \tilde{k}_1}\right)^T \dot{\tilde{k}}_1(t) + \left(\frac{\partial V(e_c)}{\partial \tilde{k}_2}\right) \dot{\tilde{k}}_2(t) + \left(\frac{\partial V(e_c)}{\partial \tilde{k}_3}\right)^T \dot{\tilde{k}}_3(t) \\
&= 2e^T(t)P\dot{e}(t) + \frac{2}{|k_4^*|}\tilde{k}_1^T(t)\Gamma^{-1}\dot{\tilde{k}}_1(t) + + \frac{2}{|k_4^*|}\tilde{k}_2(t)\gamma^{-1}\dot{\tilde{k}}_2(t) + \frac{2}{|k_4^*|}\tilde{k}_3^T(t)\Psi^{-1}\dot{\tilde{k}}_3(t)
\end{aligned} \tag{3.32}$$

Substituting (3.27) and (3.31) in (3.32), we have

$$\begin{aligned}
\dot{V} = &-e^T(t)Qe(t) + e^T(t)Pb_e\frac{2}{k_4^*}\tilde{k}_1^T(t)e(t) + e^T(t)Pb_e\frac{2}{k_4^*}\tilde{k}_2(t)u_m(t) \\
&+ e^T(t)Pb_e\frac{2}{k_4^*}\tilde{k}_3^T(t)x_m(t) + \frac{2}{|k_4^*|}\tilde{k}_1^T\Gamma^{-1}\dot{\tilde{k}}_1(t) + \frac{2}{|k_4^*|}\tilde{k}_2(t)\gamma^{-1}\dot{\tilde{k}}_2(t) \\
&+ \frac{2}{|k_4^*|}\tilde{k}_3^T(t)\Psi^{-1}\dot{\tilde{k}}_3(t)
\end{aligned} \tag{3.33}$$

By the adaptive laws, (3.23)-(3.25), (3.33) becomes

$$\dot{V} = -e^T(t)Qe(t) \leq -q_m \|e(t)\|_2^2 \leq 0 \tag{3.34}$$

where $q_m > 0$ is the minimum eigenvalue of Q. From here on, the desired properties of the proposed adaptive laws are obvious:

(i) $V > 0$ and $\dot{V} \leq 0$ implies that the equilibrium state $e_c = 0$ of the closed-loop system consisting of (3.27), (3.23), (3.24) and (3.25) is uniformly stable and its solution $e_c(t)$ is uniformly bounded, which gives the boundedness of $x(t), k_1(t), k_2(t)$ and $k_3(t)$, and in turn of the boundedness of $\dot{e}(t)$ because of (3.27);

(ii) (3.34) implies $e(t) \in L^2$;

(iii) with $e(t) \in L^2 \cap L^\infty$ and $\dot{e}(t) \in L^\infty$, applying Barbalat lemma, we conclude that $\lim_{t\to\infty} e(t) = 0$. $\bigtriangledown$

From the results demonstrated above, all the properties mentioned in Theorem 3.1 are proved.

3.2.4 Disturbance Rejection

In Section 3.1.1 Case II has a disturbance acting on the follower, and Case III has disturbances acting on the leader and the followers respectively. To reject the effect of unknown disturbances so that the desired system performance can be achieved, certain matching conditions should be satisfied and additional compensation term should be introduced to the controller (3.22).

Design for the disturbance acting on the follower system(Case II) When Assumptions (A3.1)-(A3.4) are satisfied, the ideal control law (3.19) is modified as

$$u(t) = k_1^*(x(t) - x_m(t)) + k_2^* u_m(t) + k_3^* x_m(t) + k_5^*(t), \tag{3.35}$$

where $k_5^* = -d(t) = -d_0 - \sum_{\beta=1}^{q} d_\beta f_\beta(t)$, which leads to the desired closed-loop system

$$\dot{x}(t) = A_e(x(t) - x_m(t)) + A_m x_m(t) + b_m u_m(t) \tag{3.36}$$

an in turn the desired tracking error system $\dot{e}(t) = A_e e(t)$ making $\lim_{t\to\infty} e(t) = 0$, as A_e is stable.

When the parameters A, b, A_m, b_m and the value of $d(t)$ are unknown, the adaptive version of the controller (3.35) is used:

$$u(t) = k_1^T(t)(x(t) - x_m(t)) + k_2(t)u_m(t) + k_3^T(t)x_m(t) + k_5(t), \tag{3.37}$$

where $k_i(t)$ for $i = 1, 2, 3$ are the estimates of k_i^*, respectively, and

$$k_5(t) = k_{50}(t) + \sum_{\beta=1}^{q} k_{5\beta}(t)f_\beta(t) \tag{3.38}$$

with $k_{5\beta}(t)$ being the estimate of $k_{5\beta}^* = -d_\beta,\ \beta = 0, 1, 2, \cdots, q$.

To develop adaptive laws for $k_i(t)$ for $i = 1, 2, 3$ and $k_{5\beta}(t),\ \beta = 0, 1, 2, \ldots, q$, we first derive an system error equation in terms of the tracking error $e(t) = x(t) - x_m(t)$ and the parameter errors

$$\begin{aligned} \tilde{k}_i(t) &= k_i(t) - k_i^*,\ i = 1, 2, 3, \\ \tilde{k}_{5\beta}(t) &= k_{5\beta}(t) - k_{5\beta}^*,\ \beta = 0, 1, 2, \ldots, q. \end{aligned} \tag{3.39}$$

Let $\tilde{k}_5(t) = k_5(t) - k_5^*$. Using (3.37),(3.38) and (3.39), we obtain

$$\begin{aligned} \dot{x}(t) &= Ax(t) + b(k_1^T(t)(x(t) - x_m(t)) + k_2(t)u_m(t) + k_3^T(t)x_m(t) + k_5(t)) \\ &= A_e(x(t) - x_m(t)) + A_m x_m(t) + b_m u_m(t) \\ &\quad + b_e \frac{1}{k_4^*}(\tilde{k}_1^T(t)(x(t) - x_m(t)) + \tilde{k}_2(t)u_m(t) + \tilde{k}_3^T x_m(t) + \tilde{k}_5(t)) \end{aligned} \tag{3.40}$$

Substituting (3.7) into (3.40), we have the tracking error equation

$$\dot{e}(t) = A_e e(t) + b_e \frac{1}{k_4^*}(\tilde{k}_1^T(t)(x(t) - x_m(t)) + \tilde{k}_2(t)u_m(t) + \tilde{k}_3^T x_m(t) + \tilde{k}_5(t)). \quad (3.41)$$

Based on this error equation, we choose the adaptive laws as

$$\dot{k}_1(t) = -sign[k_4^*]\Gamma_1 e(t)e^T(t)Pb_e, \quad (3.42)$$

$$\dot{k}_2(t) = -sign[k_4^*]\gamma_2 u_m(t)e^T(t)Pb_e, \quad (3.43)$$

$$\dot{k}_3(t) = -sign[k_4^*]\Psi_3 x_m(t)e^T(t)Pb_e, \quad (3.44)$$

$$\dot{k}_{50}(t) = -sign[k_4^*]\gamma_{50} e^T(t)Pb_e, \quad (3.45)$$

$$\dot{k}_{5\beta}(t) = -sign[k_4^*]\gamma_{5\beta} f_\beta(t)e^T(t)Pb_e,\ \beta = 1, 2, \cdots, q, \quad (3.46)$$

where $\Gamma_1 = \Gamma_1^T > 0, \gamma_2 > 0, \Psi_3 = \Psi_3^T > 0$ and $\gamma_{5\beta} > 0, \beta = 0, 1, 2, \cdots, q$ are adaptation gains, and $P = P^T > 0$ satisfies (3.31).

To analysis the close-loop system stability, choose a positive definite function as Lyapunov candidate function which is

$$V = e^T Pe + \frac{1}{|k_4^*|}\tilde{k}_1^T \Gamma_1^{-1}\tilde{k}_1 + \frac{1}{|k_4^*|}\tilde{k}_2^2\gamma_2^{-1} + \frac{1}{|k_4^*|}\tilde{k}_3^T \Psi_3^{-1}\tilde{k}_3 + \frac{1}{|k_4^*|}\sum_{\beta=0}^{q}\tilde{k}_{5\beta}^2\gamma_{5\beta}^{-1} \quad (3.47)$$

Taking time derivative and apply the adaptive laws (3.42) - (3.46) into its derivative. The time derivative of (3.47)

$$\begin{aligned}\dot{V} = &-e^T(t)Qe(t) + e^T(t)Pb_e\frac{2}{k_4^*}\tilde{k}_1^T(t)e(t) + e^T(t)Pb_e\frac{2}{k_4^*}\tilde{k}_2(t)u_m(t)\\ &+ e^T(t)Pb_e\frac{2}{k_4^*}\tilde{k}_3^T(t)x_m(t) + e^T(t)Pb_e\frac{2}{k_4^*}\tilde{k}_{5\beta}(t) + \sum_{\beta=1}^{q} e^T(t)Pb_e\frac{2}{k_4^*}\tilde{k}_{5\beta}(t)f_\beta(t)\\ &+ \frac{2}{|k_4^*|}\tilde{k}_1^T\Gamma^{-1}\dot{\tilde{k}}_1(t) + \frac{2}{|k_4^*|}\tilde{k}_2\gamma^{-1}\dot{\tilde{k}}_2(t) + \frac{2}{|k_4^*|}\tilde{k}_3^T\Psi^{-1}\dot{\tilde{k}}_3(t)\\ &+ \frac{2}{|k_4^*|}\tilde{k}_{50}\gamma_{50}^{-1}\dot{\tilde{k}}_{50}(t) + \sum_{\beta=1}^{q}\frac{2}{|k_4^*|}\tilde{k}_{50}\gamma_{5\beta}^{-1}\dot{\tilde{k}}_{5\beta}(t)\end{aligned} \quad (3.48)$$

By the adaptive laws (3.42)-(3.46), (3.48) becomes to

$$\dot{V} = -e^T(t)Qe(t), \; Q = Q^T > 0 \tag{3.49}$$

from which we can obtain the desired system properties, i.e., we can also have the result of Theorem 3.1. $\triangledown$

Remark 3.1 The disturbance $d(t)$ in (3.3) is matched to the control input $u(t)$, that is, both act on the plant dynamics through the same vector b. If they are not matched, for example, for the plant

$$\dot{x}(t) = Ax(t) + bu(t) + b_d(d_0 + d_1 f(t)), \tag{3.50}$$

where b and $b_d \in R^n$ are linear independent, the above adaptive design may not be able to ensure asymptotic state tracking. $\square$

Design for disturbance acting on both leader and follower systems (Case III) For the purpose of designing an adaptive control scheme to address the disturbance acting on both leader and follower systems, we assume all the Assumptions (A3.1)-(A3.4) aforementioned are satisfied. In order to reject the disturbance so that the desired system performance can be achieved, the ideal controller with disturbance compensator is chosen as:

$$u(t) = k_1^{*T}(x(t) - x_m(t)) + k_2^* u_m(t) + k_3^{*T} x_m(t) + k_5^*(t) \tag{3.51}$$

which has the same form with the ideal controller chosen in the last paragraph.

However, when $d(t)$ and $d_m(t)$ present in (3.6) and (3.7) respectively at the same time, we re-define

$$
\begin{aligned}
k_5^* &= k_2^* d_m(t) - d(t) \\
&= (k_2^* d_{m0} - d_0) + \sum_{\beta=1}^{q_m} k_2^* d_{m\beta} f_{m\beta}(t) - \sum_{\beta=1}^{q} d_\beta f_\beta(t) \\
&= k_{50}^* + \sum_{\beta=1}^{q_m} k_{5m\beta}^* f_{m\beta}(t) + \sum_{\beta=1}^{q} k_{5\beta}^* f_\beta(t) \qquad (3.52)
\end{aligned}
$$

with

$$k_{50}^* = k_2^* d_{m0} - d_0,\ k_{5m\beta}^* = k_2^* d_{m\beta},\ k_{5\beta}^* = -d_\beta. \qquad (3.53)$$

The ideal controller (3.51) with the re-defined k_5^* leads (3.5) to a desired system closed-loop system

$$\dot{x}(t) = A_e(x(t) - x_m(t)) + A_m x_m(t) + b_m u_m(t) + b_m d_m(t) \qquad (3.54)$$

In result, we obtain $\lim_{t\to\infty} e(t) = 0$, since $\dot{e}(t) = \dot{x}(t) - \dot{x}_m(t) = A_e e(t)$ and A_e is stable.

When the parameters A, b, A_m, b_m and the value of disturbance $d(t)$ are unknown, we use the adaptive version of controller (3.50) as

$$u(t) = k_1^T(t)(x(t) - x_m(t)) + k_2(t) u_m(t) + k_3^T(t) x_m(t) + k_5(t) \qquad (3.55)$$

where

$$k_5(t) = k_{50}(t) + \sum_{\beta=1}^{q_m} k_{5m\beta}(t) f_{m\beta}(t) + \sum_{\beta=1}^{q} k_{5\beta}(t) f_\beta(t) \qquad (3.56)$$

is the estimate of k_5^*.

To develop adaptive laws for $k_i(t)$ for $i = 1, 2, 3$, $k_{5\beta}(t)$ for $\beta = 0, 1, 2, \ldots, q$ and

$k_{5m\beta}(t)$ for $\beta = 1, 2, \ldots, p$, derive the tracking error $e(t) = x(t) - x_m(t)$ first and the parameter errors

$$\begin{aligned}\tilde{k}_i(t) &= k_i(t) - k_i^*,\ i = 1, 2, 3,\\ \tilde{k}_{5\beta}(t) &= k_{5\beta}(t) - k_{5\beta}^*,\ \beta = 0, 1, 2, \ldots, q,\\ \tilde{k}_{5m\beta}(t) &= k_{5m\beta}(t) - k_{5m\beta}^*,\ \beta = 1, 2, \ldots, q_m.\end{aligned} \tag{3.57}$$

Let $\tilde{k}_5(t) = k_5(t) - k_5^*$. Substituting (3.50), (3.56) and (3.57) into (3.54), we obtain

$$\begin{aligned}\dot{x}(t) = {} & A_e(x(t) - x_m(t)) + b_e\frac{1}{k_4^*}(\tilde{k}_1^T(t)(x(t) - x_m(t)) + \tilde{k}_2(t)u_m(t) + \tilde{k}_3^T x_m(t) + \tilde{k}_5(t))\\ & + A_m x_m(t) + b_m u_m(t) + b_m d_m(t)\end{aligned} \tag{3.58}$$

With $e(t) = x(t) - x_m(t)$, we have the tracking error equation

$$\dot{e}(t) = A_e e(t) + b_e\frac{1}{k_4^*}(\tilde{k}_1^T(t)(x(t) - x_m(t)) + \tilde{k}_2(t)u_m(t) + \tilde{k}_3^T x_m(t) + \tilde{k}_5(t)). \tag{3.59}$$

Based on (3.59), we choose the adaptive laws as

$$\dot{k}_1(t) = -sign[k_4^*]\Gamma_1 e(t)e^T(t)Pb_e, \tag{3.60}$$

$$\dot{k}_2(t) = -sign[k_4^*]\gamma_2 u_m(t)e^T(t)Pb_e, \tag{3.61}$$

$$\dot{k}_3(t) = -sign[k_4^*]\Psi_3 x_m(t)e^T(t)Pb_e, \tag{3.62}$$

$$\dot{k}_{50}(t) = -sign[k_4^*]\gamma_{50} e^T(t)Pb_e, \tag{3.63}$$

$$\dot{k}_{5\beta}(t) = -sign[k_4^*]\gamma_{5j} f_\beta(t)e^T(t)Pb_e,\ \beta = 1, 2, \cdots, q_m, \tag{3.64}$$

$$\dot{k}_{5m\beta}(t) = -sign[k_4^*]\delta_{5m\beta} f_{m\beta}(t)e^T(t)Pb_e,\ \beta = 1, 2, \cdots, p \tag{3.65}$$

Analysis the closed-loop system stability by Lyapunov method. Choose a positive definite function as

$$V = e^T P e + \frac{1}{|k_4^*|}\tilde{k}_1^T \Gamma_1^{-1} \tilde{k}_1 + \frac{1}{|k_4^*|}\tilde{k}_2^2 \gamma_2^{-1} + \frac{1}{|k_4^*|}\tilde{k}_3^T \Psi_3^{-1} \tilde{k}_3 + \frac{1}{|k_4^*|}\sum_{\beta=0}^{q} \tilde{k}_{5\beta}^2 \gamma_{5\beta}^{-1}$$
$$+ \frac{1}{|k_4^*|}\sum_{\beta=1}^{q_m} \tilde{k}_{5m\beta}^2 \delta_{5m\beta}^{-1} \tag{3.66}$$

has the negative semidefinite time derivative after substituting (3.60) -(3.65)

$$\dot{V} = -e^T(t) Q e(t), Q = Q^T > 0 \tag{3.67}$$

from which, we can obtain all the desired system properties, i.e, the control objective is achieved. $\bigtriangledown$

3.3 Adaptive Following Control Design with Multiple Inputs

In this section, we focus on the leader-following consensus problem with multiple inputs. Case I in Section 3.1.2 will be solved first. Case II and Case III in Section 3.1.2 with disturbances are developed in Section 3.3.4. In Section 3.3.5, an adaptive control scheme is designed for multiple inputs systems based on the LDU decomposition.

3.3.1 Design Conditions

Recall that in Section 3.1.2, the control objective for the multiple inputs systems is stated as to design a control input $u(t)$ to make the follower system state $x(t)$ bounded and to track the leader state $x_m(t)$ asymptotically. In order to achieve this control objective, several design conditions are presented as follows.

(A3.5) all the eigenvalues of A_m are in the open left-half complex plane;

(A3.6) $u_m(t)$ is bounded and piecewise continuous;

(A3.7) there exist four parameter matrices $K_1^* \in R^{n\times p}, K_2^* \in R^{p\times m}, K_3^* \in R^{n\times p}$ and $K_4^* \in R^{p\times p}$ such that the following equations are satisfied:

$$A + BK_1^{*T} = A_e,\ BK_2^* = B_m,\ A + BK_3^{*T} = A_m,\ BK_4^* = B_e, \tag{3.68}$$

where $A_e \in R^{n\times n}$ is a stable and known matrix and $B_e \in R^{n\times p}$ is a known matrix;

(A3.8) there is a known matrix $S \in R^{p\times p}$ such that $K_4^* S$ is symmetric and positive definite: $M_s = K_4^* S = (K_4^* S)^T = S^T K_4^{*T} > 0$.

If the parameter of A_m, B_m, A and B are known, (3.68) is satisfied, then the ideal control law

$$u^*(t) = K_1^{*T}(x(t) - x_m(t)) + K_2^* u_m(t) + K_3^{*T} x_m(t) \tag{3.69}$$

results in the closed-loop system

$$\dot{x}(t) = A_e(x(t) - x_m(t)) + A_m x_m(t) + B_m u_m(t) \tag{3.70}$$

whose state vector $x(t)$ belongs to L^∞, i.e., $x(t)$ is bounded. The tracking error $e(t) = x(t) - x_m(t)$ satisfies:

$$\dot{e}(t) = A_e e(t), e(0) = x(0) - x_m(0), \tag{3.71}$$

so that $\lim_{t\to\infty} e(t) = 0$ exponentially since A_e is a stable matrix.

3.3.2 Adaptive Control Scheme

When the parameters of A, b, A_m and b_m are unknown, update (3.69) to the adaptive version,

$$u(t) = K_1^T(t)(x(t) - x_m(t)) + K_2(t)u_m(t) + K_3^T(t)x_m(t) \tag{3.72}$$

where $K_1(t)$, $K_2(t)$ and $K_3(t)$ are the estimates of K_1^*, K_2^* and K_3^* respectively.

Choose the adaptive laws (3.73) - (3.75) to update $K_1(t)$, $K_2(t)$ and $K_3(t)$:

$$\dot{\tilde{K}}_1^T = \dot{K}_1^T(t) = -S^T B_e^T P e(t) e^T(t) \tag{3.73}$$

$$\dot{\tilde{K}}_2 = \dot{K}_2(t) = -S^T B_e^T P e(t) u_m^T(t) \tag{3.74}$$

$$\dot{\tilde{K}}_3^T = \dot{K}_3^T(t) = -S^T B_e^T P e(t) x_m^T(t) \tag{3.75}$$

with S satisfying Assumption (A3.8) and $P = P^T > 0$ satisfying $A_e^T P + P A_e = -Q < 0$ for any chosen symmetric positive definite matrix Q. $K_1(0), K_2(0)$ and $K_3(0)$ are chosen arbitrarily.

Theorem 3.2. *The adaptive controller* (3.72) *with the adaptive laws* (3.73), (3.74) *and* (3.75), *applied to the system* (3.11) *guarantees that all closed-loop signals are bounded and the tracking error* $e(t) = x(t) - x_m(t)$ *goes to zero as* t *goes to* ∞.

3.3.3 Stability Analysis

In order to prove the Theorem 3.2, in another word, to prove that the closed-loop system (3.11) with state feedback adaptive controller (3.72) is stable, we first define the parameter error as

$$\tilde{K}_1(t) = K_1(t) - K_1^*, \tilde{K}_2(t) = K_2(t) - K_2^*, \tilde{K}_3(t) = K_3(t) - K_3^*. \tag{3.76}$$

Use (3.11) and (3.72) to obtain

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t) \\
&= A_e(x(t) - x_m(t)) + A_m x_m(t) + B_m u_m(t) \\
&\quad + B(\tilde{K}_1^T(t)(x(t) - x_m(t)) + \tilde{K}_2(t)u_m(t) + \tilde{K}_3^T(t)x_m(t)) \\
&= A_e(x(t) - x_m(t)) + A_m x_m(t) + B_m u_m(t) \\
&\quad + B_e\left(K_4^{*-1}\tilde{K}_1^T(t)e(t) + K_4^{*-1}\tilde{K}_2(t)u_m(t) + K_4^{*-1}\tilde{K}_3^T(t)x_m(t)\right). \qquad (3.77)
\end{aligned}$$

Substituting (3.12) in $\dot{e}(t) = x(t) - x_m(t)$, we have the tracking error equation

$$\dot{e}(t) = A_e e(t) + B_e\left(K_4^{*-1}\tilde{K}_1^T(t)e(t) + K_4^{*-1}\tilde{K}_2(t)u_m(t) + K_4^{*-1}\tilde{K}_3^T(t)x_m(t)\right). \quad (3.78)$$

Now $K_1(t)$, $K_2(t)$ and $K_3(t)$ are matrices so the state vector of the closed-loop system is

$$e_c(t) = (e^T(t), \tilde{k}_{11}^T(t), \dots, \tilde{k}_{1n}^T(t), \tilde{k}_{21}^T(t), \dots, \tilde{k}_{2m}^T(t), \tilde{k}_{31}^T(t), \dots, \tilde{k}_{3n}^T(t))^T \in R^{(2n+m)p+n} \quad (3.79)$$

where $\tilde{k}_{1i}(t) \in R^p$ is the ith column of $\tilde{K}_1^T(t)$, $i = 1, 2, \dots, n$. $\tilde{k}_{2j}(t) \in R^p$ is the jth column of $\tilde{K}_2(t)$, $j = 1, 2, \dots, m$, and $\tilde{k}_{3q}(t) \in R^p$ is the qth column of $\tilde{K}_3^T(t)$, $q = 1, 2, \dots, n$, that is,

$$\begin{aligned}
\tilde{K}_1^T(t) &= (\tilde{k}_{11}(t), \dots, \tilde{k}_{1n}(t)) \in R^{p\times n}, \\
\tilde{K}_2(t) &= (\tilde{k}_{21}(t), \dots, \tilde{k}_{2m}(t)) \in R^{p\times m}, \\
\tilde{K}_3^T(t) &= (\tilde{k}_{31}(t), \dots, \tilde{k}_{3n}(t)) \in R^{p\times n}. \qquad (3.80)
\end{aligned}$$

We choose the positive definite function

$$V(e_c) = e^T Pe + \sum_{i=1}^{n} \tilde{k}_{1i}^T M_s^{-1} \tilde{k}_{1i} + \sum_{j=1}^{n} \tilde{k}_{2j}^T M_s^{-1} \tilde{k}_{2j} + \sum_{q=1}^{n} \tilde{k}_{3q}^T M_s^{-1} \tilde{k}_{3q} \tag{3.81}$$

as a measurement of these errors, where $P \in R^{n\times n}$ is constant, $P = P^T > 0$ and satisfies (3.31) for some $Q \in R^{n\times n}$ being constant and $Q = Q^T > 0$, and $M_s = M_s^T > 0$ satisfies the Assumption A3.8. With $\mathrm{tr}[M]$ denoting the trace of a square matrix M, we express $V(e_c)$ as

$$V(e_c) = e^T Pe + \mathrm{tr}[\tilde{K}_1 M_s^{-1} \tilde{K}_1^T] + \mathrm{tr}[\tilde{K}_2^T M_s^{-1} \tilde{K}_2] + \mathrm{tr}[\tilde{K}_3 M_s^{-1} \tilde{K}_3^T] \tag{3.82}$$

The time-derivative of $V(e_c)$ is

$$\begin{aligned}
\dot{V} = {} & \left(\frac{\partial V(e_c)}{\partial e}\right)^T \dot{e}(t) + \sum_{i=1}^{n} \left(\frac{\partial V(e_c)}{\partial \tilde{k}_{1i}}\right)^T \dot{\tilde{k}}_{1i}(t) + \sum_{j=1}^{m} \left(\frac{\partial V(e_c)}{\partial \tilde{k}_{2j}}\right)^T \dot{\tilde{k}}_{2j}(t) \\
& + \sum_{q=1}^{n} \left(\frac{\partial V(e_c)}{\partial \tilde{k}_{3q}}\right)^T \dot{\tilde{k}}_{3q}(t) \\
= {} & 2e^T(t)P\dot{e}(t) + 2\sum_{i=1}^{n} \tilde{k}_{1i}^T(t) M_s^{-1} \dot{\tilde{k}}_{1i}(t) + 2\sum_{j=1}^{n} \tilde{k}_{2j}^T(t) M_s^{-1} \dot{\tilde{k}}_{2j}(t) + \\
& 2\sum_{q=1}^{n} \tilde{k}_{3q}^T(t) M_s^{-1} \dot{\tilde{k}}_{3q}(t) \qquad (3.83) \\
= {} & 2e^T(t)P\dot{e}(t) + \mathrm{tr}[\tilde{K}_1(t) M_s^{-1} \dot{\tilde{K}}_1^T(t)] + \mathrm{tr}[\tilde{K}_2^T(t) M_s^{-1} \dot{\tilde{K}}_2(t)] + \mathrm{tr}[\tilde{K}_3(t) M_s^{-1} \dot{\tilde{K}}_3^T(t)]
\end{aligned}$$

Substituting (3.78) and (3.31) in (3.83), we have

$$\begin{aligned}
\dot{V} = {} & -e^T(t)Qe(t) + 2e^T(t)PB_e K_4^{*-1} \tilde{K}_1^T(t)e(t) + 2e^T(t)PB_e K_4^{*-1} \tilde{K}_2(t)u_m(t) \\
& + 2e^T(t)PB_e K_4^{*-1} \tilde{K}_3^T(t)x_m(t) + 2\,\mathrm{tr}[\tilde{K}_1(t) M_s^{-1} \dot{\tilde{K}}_1^T(t)] + 2\,\mathrm{tr}[\tilde{K}_2^T(t) M_s^{-1} \dot{\tilde{K}}_2(t)] \\
& + 2\,\mathrm{tr}[\tilde{K}_3(t) M_s^{-1} \dot{\tilde{K}}_3^T(t)].
\end{aligned} \tag{3.84}$$

Using the definition $M_s = K_4^* S = M_s^T > 0$ and the properties that $\mathrm{tr}[M_1 M_2] = \mathrm{tr}[M_2 M_1], \mathrm{tr}[M_3] = \mathrm{tr}[M_3^T]$ for any matrices M_1, M_2 and M_3 of appropriate dimensions, we obtain

$$\begin{aligned} & e^T(t)PB_eK_4^{*-1}\tilde{K}_1^T(t)e(t) \\ &= \mathrm{tr}[e^T(t)PB_eK_4^{*-1}\tilde{K}_1^T(t)e(t)] = \mathrm{tr}[e^T(t)PB_eSM_s^{-1}\tilde{K}_1^T(t)e(t)] \\ &= \mathrm{tr}[e(t)e^T(t)PB_eSM_s^{-1}\tilde{K}_1^T(t)] = \mathrm{tr}[\tilde{K}_1(t)M_s^{-1}S^TB_e^TPe(t)e^T(t)] \end{aligned} \tag{3.85}$$

$$\begin{aligned} & e^T(t)PB_eK_4^{*-1}\tilde{K}_2(t)u_m(t) \\ &= \mathrm{tr}[e^T(t)PB_eK_4^{*-1}\tilde{K}_2(t)u_m(t)] = \mathrm{tr}[e^T(t)PB_eSM_s^{-1}\tilde{K}_2(t)u_m(t)] \\ &= \mathrm{tr}[u_m(t)e^T(t)PB_eSM_s^{-1}\tilde{K}_2(t)] = \mathrm{tr}[\tilde{K}_2^T(t)M_s^{-1}S^TB_e^TPe(t)u_m^T(t)] \end{aligned} \tag{3.86}$$

$$\begin{aligned} & e^T(t)PB_eK_4^{*-1}\tilde{K}_3^T(t)x_m(t) \\ &= \mathrm{tr}[e^T(t)PB_eK_4^{*-1}\tilde{K}_3^T(t)x_m(t)] = \mathrm{tr}[e^T(t)PB_eSM_s^{-1}\tilde{K}_3^T(t)x_m(t)] \\ &= \mathrm{tr}[x_m(t)e^T(t)PB_eSM_s^{-1}\tilde{K}_3^T(t)] = \mathrm{tr}[\tilde{K}_3(t)M_s^{-1}S^TB_e^TPe(t)x_m^T(t)] \end{aligned} \tag{3.87}$$

Apply adaptive laws (3.73), (3.74) and (3.75) into (3.84), combing with the facts (3.86) and (3.87), we have,

$$\dot{V} = -e^T(t)Qe(t) \le -q_m \left\| e(t) \right\|_2^2 \le 0 \tag{3.88}$$

Hence the equilibrium state $e_c = 0$ of the closed-loop system consisting of (3.73), (3.74), (3.75) and (3.78) is uniformly stable and its solution $e_c(t)$ is uniformly bounded. That is, $y(t)$, $K_1(t)$, $K_2(t)$, $K_3(t)$ and $\dot{e}(t)$ all are bounded. Furthermore (3.88) implies $e(t) \in L^2$ and so $\lim_{t\to 0} e(t) = 0$. Theorem 3.2 is proved. $\triangledown$

3.3.4 Disturbance Rejection

To reject the effect of unknown disturbances so that the desired system performance can be achieved, certain matching conditions should be satisfied and additional compensation is introduced to the controller (3.69).

Case II and Case III in Section 3.1.2 are developed in the later two paragraphs respectively.

Design for disturbance acting on follower systems (Case II) When Assumptions (A3.5)-(A3.8) are satisfied, the ideal control law (3.69) is modified as

$$u(t) = K_1^*(x(t) - x_m(t)) + K_2^* u_m(t) + K_3^* x_m(t) + k_5^*(t), \tag{3.89}$$

where $k_5^* = -d(t) \in R^p$, which leads to the desired closed-loop system

$$\begin{aligned} \dot{x}(t) &= (A + BK_1^{*T})(x(t) - x_m(t)) + (A + BK_3^{*T})x_m(t) + BK_2^* u_m(t) + Bk_5^* + Bd(t) \\ &= A_e(x(t) - x_m(t)) + A_m x_m(t) + B_m u_m(t) \end{aligned} \tag{3.90}$$

an in turn the desired tracking error system $\dot{e}(t) = A_e e(t)$ making $\lim_{t\to\infty} e(t) = 0$, as A_e is stable.

When the parameters of A, B, A_m, B_m and the value of $d(t)$ are unknown, update the controller (3.89) to an adaptive version which is

$$u(t) = K_1(t)^t (x(t) - x_m(t)) + K_2(t) u_m(t) + K_3^T(t) x_m(t) + k_5(t), \tag{3.91}$$

where $K_i(t)$, $i = 1, 2, 3$ are the estimates of K_i^*, respectively, and

$$k_5(t) = k_{50}(t) + \sum_{\beta=1}^{q} k_{5\beta}(t) f_\beta(t) \tag{3.92}$$

with $k_{5\beta}(t)$ being the estimate of $k_{5\beta}^* = -d_\beta$ for $\beta = 0, 1, \ldots, q$.

To develop adaptive laws for $K_i(t)$ for $i = 1, 2, 3$, and $k_{5\beta}(t)$ for $\beta = 0, 1, 2, \cdots, q$, firstly we derive an error equation in terms of the tracking error $e(t) = x(t) - x_m(t)$ and the parameter errors

$$\begin{aligned} \tilde{K}_i(t) &= K_i(t) - K_i^*,\ i = 1, 2, 3, \\ \tilde{k}_{5\beta}(t) &= k_{5\beta}(t) - k_{5\beta}^*,\ \beta = 0, 1, 2, \cdots, q. \end{aligned} \tag{3.93}$$

Substituting (3.89) into (3.13) with the definition of parameter errors, then we obtain

$$\dot{e}(t) = A_e e(t) + B_e K_4^{*-1}(\tilde{K}_1^T(t)(x(t) - x_m(t)) + \tilde{K}_2(t) u_m(t) + \tilde{K}_3^T x_m(t) + \tilde{k}_5(t)) \tag{3.94}$$

where $\tilde{k}_5(t) = k_5(t) - k_5^*$.

Based on this error equation, we choose the adaptive laws as

$$\dot{\tilde{K}}_1^T = \dot{K}_1^T(t) = -S^T B_e^T P e(t) e^T(t) \tag{3.95}$$

$$\dot{\tilde{K}}_2 = \dot{K}_2(t) = -S^T B_e^T P e(t) u_m^T(t) \tag{3.96}$$

$$\dot{\tilde{K}}_3^T = \dot{K}_3^T(t) = -S^T B_e^T P e(t) x_m^T(t) \tag{3.97}$$

$$\dot{\tilde{K}}_{50} = \dot{K}_{50}(t) = -S^T B_e^T P e(t) \tag{3.98}$$

$$\dot{\tilde{K}}_{5\beta} = \dot{K}_{5\beta}(t) = -S^T B_e^T P e(t) f_\beta^T(t) \tag{3.99}$$

where S satisfies Assumption (A3.8) and $P = P^T > 0$ satisfies (3.31).

Applying adaptive control (3.91) with adaptive laws (3.95) - (3.99), (3.13) should have all the desired the properties. In order to show that the controller (3.91) with adaptive laws (3.95) - (3.99) works and can drive $e(t) = x(t) - x_m(t)$ bounded and stable which are the desired properties of trajectories of (3.13). Choose a positive definite function

$$\begin{aligned} V &= e^T P e + \mathrm{tr}[\tilde{K}_1 M_s^{-1} \tilde{K}_1^T] + \mathrm{tr}[\tilde{K}_2^T M_s^{-1} \tilde{K}_2] + \mathrm{tr}[\tilde{K}_3 M_s^{-1} \tilde{K}_3^T] \\ &\quad + \mathrm{tr}[\tilde{k}_{50}^T M_s^{-1} \tilde{k}_{50}] + \sum_{\beta=1}^{q} tr[\tilde{k}_{5j}^T M_s^{-1} \tilde{k}_{5j}]. \end{aligned} \tag{3.100}$$

Then the time derivative of (3.97) is

$$\begin{aligned} \dot{V} &= -e^T(t) Q e(t) + 2e^T(t) P B_e K_4^{*-1} \tilde{K}_1^T(t) e(t) + 2e^T(t) P B_e K_4^{*-1} \tilde{K}_2(t) u_m(t) \\ &\quad + 2e^T(t) P B_e K_4^{*-1} \tilde{K}_3^T(t) x_m(t) + 2e^T(t) P B_e K_4^{*-1} \tilde{k}_{50}^T(t) \\ &\quad + 2e^T(t) P B_e K_4^{*-1} \tilde{k}_{5j} f_{5j} + 2\,\mathrm{tr}[\tilde{K}_1(t) M_s^{-1} \dot{\tilde{K}}_1^T(t)] \\ &\quad + 2\,\mathrm{tr}[\tilde{K}_2^T(t) M_s^{-1} \dot{\tilde{K}}_2(t)] + 2\,\mathrm{tr}[\tilde{K}_3(t) M_s^{-1} \dot{\tilde{K}}_3^T(t)] \\ &\quad + 2\,\mathrm{tr}[\tilde{k}_{50}^T M_s^{-1} \dot{\tilde{k}}_{50}] + 2\,\mathrm{tr}[\tilde{k}_{5j}^T M_s^{-1} \dot{\tilde{k}}_{5j}]. \end{aligned} \tag{3.101}$$

Applying adaptive laws (3.95)-(3.99), the time derivative of (3.101) as

$$\dot{V} = -e^T(t) Q e(t) \leq 0 \tag{3.102}$$

which is negative semidefinite. From (3.102) we can obtain the desired system properties. $\bigtriangledown$

Disturbance acting on both leader and follower systems(Case III) Consider the leader plant (3.13) and follower plant (3.16). In order to reject the disturbance so that the desired system performance can be achieved, we choose the ideal control structure as:

$$u^*(t) = K_1^{*T}(x(t) - x_m(t)) + K_2^* u_m(t) + K_3^{*T} x_m(t) + k_5^*(t). \tag{3.103}$$

Being different with Case II, when $d(t)$ and $d_m(t)$ is present in (3.13) and (3.16) respectively, we re-define

$$\begin{aligned} k_5^* &= K_2^* d_m(t) - d(t) \\ &= (K_2^* d_{m0} - d_0) + \sum_{\beta=1}^{q_m} K_2^* d_{m\beta} f_{m\beta}(t) - \sum_{\beta=1}^{q} d_\beta f_\beta(t) \\ &= k_{50}^* + \sum_{\beta=1}^{q_m} k_{5m\beta}^* f_{m\beta}(t) + \sum_{\beta=1}^{q} k_{5\beta}^* f_\beta(t) \end{aligned} \tag{3.104}$$

with

$$k_{50}^* = K_2^* d_{m0} - d_0,\; k_{5m\beta}^* = K_2^* d_{m\beta},\; k_{5\beta}^* = -d_\beta \tag{3.105}$$

For the ideal controller, the re-defined k_5^* leads to a desired closed-loop system

$$\begin{aligned} \dot{x}(t) &= (A + BK_1^{*T})(x(t) - x_m(t)) + (A + BK_3^{*T})x_m(t) + BK_2^* u_m(t) \\ &\quad + BK_2^* d_{m0} - Bd_0 + B\sum_{\beta=1}^{q_m} K_2^* d_{m\beta} f_{m\beta}(t) - B\sum_{\beta=1}^{q} d_\beta f_\beta(t) + Bd(t) \\ &= A_e(x(t) - x_m(t)) + A_m x_m(t) + B_m u_m(t) + B_m d_m(t) \end{aligned} \tag{3.106}$$

In result, we obtain $\lim_{t\to\infty} e(t) = 0$, since $\dot{e}(t) = \dot{x} - \dot{x}_m = A_e e(t)$ and A_e is stable.

When the parameters A, B, A_m, B_m and the value of disturbance $d(t)$ are unknown, we use the adaptive version of controller (3.103) which is

$$u(t) = K_1^T(x(t) - x_m(t)) + K_2 u_m(t) + K_3^T x_m(t) + k_5(t) \tag{3.107}$$

where

$$k_5(t) = k_{50}(t) + \sum_{\beta=1}^{q_m} k_{5m\beta}(t) f_{5m\beta}(t) + \sum_{\beta=1}^{q} k_{5\beta}(t) f_\beta(t) \tag{3.108}$$

is the estimate of k_5^*.

In adaptive control, it is unaccessible for us to obtain the ideal parameters, so define the parameter errors first.

$$\begin{aligned} \tilde{K}_i(t) &= K_i(t) - K_i^*,\ i = 1, 2, 3. \\ \tilde{k}_{5\beta}(t) &= k_{5\beta}(t) - k_{5\beta}^*,\ \beta = 0, 1, \ldots, q. \\ \tilde{k}_{5m\beta}(t) &= k_{5m\beta}(t) - k_{5m\beta}^*,\ \beta = 1, 2, \ldots, q_m. \end{aligned} \tag{3.109}$$

Let $\tilde{k}_5(t) = k_5(t) - k_5^*(t)$. Substituting (3.103), (3.104) and (3.109) into (3.13), we obtain

$$\begin{aligned} \dot{x}(t) &= A_e(x(t) - x_m(t)) + A_m x_m(t) + B_m u_m(t) + B_m d_m(t) \\ &\quad + B_e K_4^{*-1}\left(\tilde{K}_1^T(t)(x(t) - x_m(t)) + \tilde{K}_2(t) u_m(t) + \tilde{K}_3^T x_m(t) + \tilde{k}_5(t)\right) \end{aligned} \tag{3.110}$$

with $e(t) = x(t) - x_m(t)$, we have the tracking error equation

$$\dot{e}(t) = A_e e(t) + B_e K_4^{*-1}\left(\tilde{K}_1^T(t)(x(t) - x_m(t)) + \tilde{K}_2(t) u_m(t) + \tilde{K}_3^T x_m(t) + \tilde{k}_5(t)\right). \tag{3.111}$$

Based on (3.111), we choose the adaptive laws as

$$\dot{\tilde{K}}_{50} = \dot{K}_{50}(t) = -S^T B_e^T P e(t), \tag{3.112}$$

$$\dot{\tilde{K}}_{5\beta} = \dot{K}_{5\beta}^T(t) = -S^T B_e^T P e(t) f_\beta^T(t), \beta = 1, 2, \ldots, q. \tag{3.113}$$

$$\dot{\tilde{K}}_{5m\beta} = \dot{K}_{5m\beta}(t) = -S^T B_e^T P e(t) f_{m\beta}^T(t), \beta = 1, 2, \cdots, p, \tag{3.114}$$

with $P = P^T > 0$ satisfying $A^T P + PA = -Q, Q = Q^T > 0$ since A_e is stable.

$S \in R^{p \times p}$ satisfies Assumption (A3.8). $K_1(t)$, $K_2(t)$ and $K_3(t)$ are still in (3.95) -(3.97).

Use the Lyapunov direct method to analysis the stability of the closed-loop system (3.13) with adaptive controller (3.103) and adaptive laws (3.95) - (3.97) and (3.112) -(3.114). Choose a positive definite function

$$V = e^T Pe + \mathrm{tr}[\tilde{K}_1 M_s^{-1} \tilde{K}_1^T] + \mathrm{tr}[\tilde{K}_2^T M_s^{-1} \tilde{K}_2] + \mathrm{tr}[\tilde{K}_3 M_s^{-1} \tilde{K}_3^T] + \sum_{\beta=1}^{q} \mathrm{tr}[\tilde{k}_{5\beta}^T M_s^{-1} \tilde{k}_{5\beta}] + \sum_{\beta=1}^{q_m} \mathrm{tr}[\tilde{k}_{5m\beta}^T M_s^{-1} \tilde{k}_{5m\beta}] \quad (3.115)$$

as a measurement of the closed-loop error system. Then get the time derivative of (3.118) which is the negative semidefinite when adaptive laws (3.95) - (3.97) and (3.112) -(3.114) are substituted.

$$\dot{V} = -e^T(t) Q e(t) \leq 0, Q = Q^T > 0 \quad (3.116)$$

from (3.116), we can obtain all the desired system properties. $\bigtriangledown$

Remark 3.2 In fact, there are no essential difference between the disturbance rejection algorithm applied in Case II and Case III. Actually, Case II is a special form of Case III with $d_m(t) = 0$. For example in multiple intputs systems, when $d_m(t) = 0$, it is obviously that $k_5^* = k_2^* d_m(t) - d(t) = -d(t)$. The application of this remark is to be further discussed in Section 4.5. $\square$

3.3.5 Design Based on LDU Parametrization

The adaptive design of previous section for the p-inputs plant (3.11) needs Assumption (A3.8): $M_s = K_4^* S = (K_4^* S)^T = S^T K_4^{*T} > 0$ for some known matrix $S \in R^{p \times p}$. This S matrix is analogous to the sign of k_4^* for the case $p = 1$; however, the knowledge of such an S matrix is more difficult to obtain then that of the sign of k_4^*. Relaxation of such knowledge for multivariable adaptive control is important. In this section, we present an adaptive control scheme for the plant (3.11), using different knowledge for the gain matrix K_4^*. This adaptive design employs a modified controller parametrization based on an LDU decomposition of the gain matrix K_4^{*-1}.

Actually, with LDU decomposition we can address all those six different cases mentioned in Section 3.1.1 and Section 3.1.2. In this section we will solve Case I with multi-input systems as an example. The basic principle of adaptive control scheme and stability analysis for the other two cases are the same.

Gain matrix and design conditions

Proposition 3.1. *(LDU decomposition) A matrix $K_p \in R^{M\times M}$ with all its leading principle minors being nonzero has a unique decomposition:*

$$K_p = LD^*U \tag{3.117}$$

for some $M \times M$ unit(i.e., with all diagonal elements being 1) lower triangular matrix L and unit upper triangular matrix U, and

$$D^* = diag\left\{d_1^*, d_2^*, \ldots, d_M^*\right\} = diag\left\{\Delta_1, \frac{\Delta_2}{\Delta_1}, \ldots, \frac{\Delta_M}{\Delta_{M-1}}\right\}. \tag{3.118}$$

This is the well-known LDU decomposition of a nonsingular matrix with nonzero leading principle minors.

From Proposition 1, we first express the gain matrix $K_4^{*-1} \in R^{p\times p}$ which satisfies Assumption (A3.8) as

$$K_4^{*-1} = LD^*U \tag{3.119}$$

for some $p \times p$ unit (i.e., with all diagonal elements being 1) lower triangular matrix L and unit upper triangular matrix U, and

$$D^* = diag\left\{d_1^*, d_2^*, \ldots, d_p^*\right\} = diag\left\{\Delta_1, \frac{\Delta_2}{\Delta_1}, \ldots, \frac{\Delta_p}{\Delta_{p-1}}\right\} \tag{3.120}$$

with $\Delta_i, i = 1, 2, \ldots, p$, as the leading principle minors of K_4^{*-1} in (3.68). To use this decomposition, we assume

(A3.9) All leading principle minors Δ_i of the matrix K_4^{*-1} are nonzero, and their signs, signs$[d_i^*]$, $i = 1, 2, \ldots, p$, are known;

(A3.10) the matrix L in (3.117) is known.

Controller structure Using (3.68) and (3.117), we express the plant (3.9) as

$$\begin{aligned}
\dot{x}(t) &= A_e(x(t) - x_m(t)) + A_m x_m(t) + B_m u_m(t) \\
&\quad + B(u(t) - K_1^{*T}(x(t) - x_m(t)) - K_2^* u_m(t) - K_3^{*T} x_m(t)) \\
&= A_e(x(t) - x_m(t)) + A_m x_m(t) + B_m u_m(t) \\
&\quad + B_e K_4^{*-1}(u(t) - K_1^{*T}(x(t) - x_m(t)) - K_2^* u_m(t) - K_3^{*T} x_m(t)) \\
&= A_e(x(t) - x_m(t)) + A_m x_m(t) + B_m u_m(t) \\
&\quad + B_e L D^*(U u(t) - U K_1^{*T}(x(t) - x_m(t)) - U K_2^* u_m(t) - U K_3^{*T} x_m(t)) \\
&= A_e(x(t) - x_m(t)) + A_m x_m(t) + B_m u_m(t) \qquad (3.121) \\
&\quad + B_e L D^*(u(t) - (I - U)u(t) - U K_1^{*T}(x(t) - x_m(t)) - U K_2^* u_m(t) - U K_3^{*T} x_m(t))
\end{aligned}$$

where $I - U$ is an upper triangular matrix with zero diagonal elements as U is a unit upper triangular matrix (whose diagonal elements are 1). From (3.10) and (3.121), we have the tracking error equation

$$\dot{e}(t) = A_e(x(t) - x_m(t)) \\ + B_e L D^*(u(t) - (I-U)u(t) - UK_1^{*T}(x(t) - x_m(t)) - UK_2^* u_m(t) - UK_3^{*T} x_m(t)) \tag{3.122}$$

Rewrite the tracking error as,

$$\dot{e}(t) = A_e e(t) + B_e L D^*(u(t) - \Phi_0^* u(t) - \Phi_1^{*T}(x(t) - x_m(t)) - \Phi_2^* u_m(t) - \Phi_3^{*T} x_m(t)) \tag{3.123}$$

where

$$\Phi_0^* = I - U,\ \Phi_1^{*T} = UK_1^{*T},\ \Phi_2^* = UK_2^*,\ \Phi_3^{*T} = UK_3^{*T} \tag{3.124}$$

This error equation motivates the controller structure

$$u(t) = \Phi_0 u(t) + \Phi_1^T(x(t) - x_m(t)) + \Phi_2 u_m(t) + \Phi_3^T x_m(t) \tag{3.125}$$

where $\Phi_0(t)$, $\Phi_1(t)$, $\Phi_2(t)$ and $\Phi_3(t)$ are the estimates of $\Phi_0^*, \Phi_1^*, \Phi_2^*$ and Φ_3^*, respectively; in particular, the parameter matrix Φ_0 has the same special upper triangular form as that of $\Phi_0^* = I - U$, that is,

$$\Phi_0 = \begin{pmatrix} 0 & \phi_{12} & \phi_{13} & \cdots & \phi_{1p} \\ 0 & 0 & \phi_{23} & \cdots & \phi_{2p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \phi_{p-1p} \\ 0 & \cdots & \cdots & 0 & 0 \end{pmatrix} \tag{3.126}$$

This special form ensures that the control signals $u(t)$ is implementable from (3.125) without singularity, that is,

$$
\begin{aligned}
u_p(t) &= [\Phi_1^T(x(t)-x_m(t)) + \Phi_2 u_m(t) + \Phi_3^T x_m(t)]_p, \\
u_{p-1}(t) &= \phi_{p-1p} u_p(t) + [\Phi_1^T(x(t)-x_m(t)) + \Phi_2 u_m(t) + \Phi_3^T x_m(t)]_{p-1}, \\
u_{p-2}(t) &= \phi_{p-2p-1} u_{p-1}(t) + \phi_{p-2p} u_(t) + [\Phi_1^T(x(t)-x_m(t)) + \Phi_2 u_m(t) + \Phi_3^T x_m(t)]_{p-1}, \\
&\vdots \\
u_2(t) &= \sum_{i=3}^{p} \phi_{2i} u_i(t) + [\Phi_0 u(t) + \Phi_1^T(x(t)-x_m(t)) + \Phi_2 u_m(t) + \Phi_3^T x_m(t)]_2, \\
u_1(t) &= \sum_{i=2}^{p} \phi_{1i} u_i(t) + [\Phi_1^T(x(t)-x_m(t)) + \Phi_2 u_m(t) + \Phi_3^T x_m(t)]_1,
\end{aligned} \tag{3.127}
$$

where $[v]_i$ denotes the ith row of the vector v.

Adaptive laws To derive at a compact and exact expression of the control law (3.125) with the special parameter structure (3.126), we let $\Phi_{1i}^T(t)$ be the ith row of $\Phi_1^T(t)$, $\Phi_{2i}^T(t)$ be the ith row of $\Phi_2^T(t)$ and $\Phi_{3i}^T(t)$ be the ith row of $\Phi_3^T(t)$, $i = 1, 2, \ldots, p$, and define

$$
\begin{aligned}
\theta_1(t) &= [\phi_{12}(t), \phi_{13}(t), \ldots, \phi_{1p}(t), \Phi_{11}^T(t), \Phi_{21}^T(t), \Phi_{31}^T(t)]^T \in R^{2n+2p-1} \\
\theta_2(t) &= [\phi_{23}(t), \phi_{24}(t), \ldots, \phi_{2p}(t), \Phi_{12}^T(t), \Phi_{22}^T(t), \Phi_{32}^T(t)]^T \in R^{2n+2p-2} \\
&\vdots \\
\theta_{p-2}(t) &= [\phi_{p-2p-1}(t), \phi_{p-2p}(t), \Phi_{1p-2}^T(t), \Phi_{2p-2}^T(t), \Phi_{3p-2}^T(t)]^T \in R^{2n+p+2} \\
\theta_{p-1}(t) &= [\phi_{p-1p}(t), \Phi_{1p-1}^T(t), \Phi_{2p-1}^T(t), \Phi_{3p-1}^T(t)]^T \in R^{2n+p+1} \\
\theta_p(t) &= [\Phi_{1p}^T(t), \Phi_{2p}^T(t), \Phi_{3p}^T(t)]^T \in R^{2n+p}
\end{aligned} \tag{3.128}
$$

which are the estimates of the corresponding θ_i^* from the rows $\Phi_0^*, \Phi_1^*, \Phi_2^*$ and Φ_3^* in (3.124). It follows from (3.123),(3.125) and (3.128) that

$$\dot{e}(t) = A_e e(t) + B_e L D^* \begin{bmatrix} \tilde{\theta}_1^T(t)\omega_1(t) \\ \tilde{\theta}_2^T(t)\omega_2(t) \\ \vdots \\ \tilde{\theta}_{p-1}^T(t)\omega_{p-1}(t) \\ \tilde{\theta}_p^T(t)\omega_p(t) \end{bmatrix}, \tag{3.129}$$

where $\tilde{\theta}_i(t) = \theta_i(t) - \theta_i^*, i = 1, 2, \ldots, p$, and

$$\begin{aligned} \omega_1(t) &= [u_2(t), u_3(t), \ldots, u_p(t), e^T(t), r^T(t), x_m^T(t)]^T \in R^{2n+2p-1} \\ \omega_2(t) &= [u_3(t), u_4(t), \ldots, u_p(t), e^T(t), r^T(t), x_m^T(t)]^T \in R^{2n+2p-2} \\ &\vdots \\ \omega_{p-2}(t) &= [u_{p-1}(t), u_p(t), e^T(t), r^T(t), x_m^T(t)]^T \in R^{2n+p+2} \\ \omega_{p-1}(t) &= [u_p(t), e^T(t), r^T(t), x_m^T(t)]^T \in R^{2n+p+1} \\ \omega_p(t) &= [e^T(t), r^T(t), x_m^T(t)]^T \in R^{2n+p} \end{aligned} \tag{3.130}$$

we now choose the following adaptive laws for $\theta_i(t)$, $i = 1, 2, \ldots, p$,

$$\dot{\theta}_i(t) = -sign[d_i^*]\Gamma_i \bar{e}_i(t)\omega_i(t), t \geq 0, \tag{3.131}$$

where $\bar{e}_i(t)$ is the ith component of $e^T(t)PB_eL$ with $P = P^T > 0$, satisfying (3.31), $\Gamma_i = \Gamma_i^T > 0$, and $sign[d_i^*]$ is from Assumption A3.9.

Stability Analysis To analyze the stability properties of the adaptive scheme (3.131), we consider the positive definite function

$$V(e,\tilde{\theta}_i, i=1,2,\ldots,p) = e^T Pe + \sum_{i=1}^{p} |d_i^*|\,\tilde{\theta}_i^T \Gamma_i^{-1}(t)\tilde{\theta}_i \tag{3.132}$$

The time derivative of $V(e,\tilde{\theta}_i)$, along the trajectory of (3.131), is

$$\begin{aligned}
\dot{V} &= 2e^T(t)P\dot{e}(t) + 2\sum_{i=1}^{M} |d_i^*|\,\tilde{\theta}_i^T(t)\Gamma_i^{-1}\dot{\theta}_i(t) \\
&= 2e^T(t)P\dot{e}(t) + 2\sum_{i=1}^{M} \bar{e}_i(t)d_i^*\tilde{\theta}_i^T(t)\omega_i(t) + 2\sum_{i=1}^{M} |d_i^*|\,\tilde{\theta}_i^T(t)\Gamma_i^{-1}\dot{\theta}_i(t) \\
&= -e^T(t)Qe(t)
\end{aligned} \tag{3.133}$$

Since $Q = Q^T > 0$, (3.133) implies that the equilibrium state $e_c = 0$, with $e_c = [e^T, \tilde{\theta}_1^T, \ldots, \tilde{\theta}_M^T]^T$, of the closed-loop system consisting of (3.129) and (3.131) is uniformly stable and its solution $e_c(t)$ is uniformly bounded. That is, $\theta_i(t), i = 1,2,\ldots,p$, and $\dot{e}(t)$ all are bounded. From (3.133), we also have $e(t) = x(t) - x_m(t) \in L^2$ and so $\lim_{t\to\infty} e(t) = 0$ as from the Barbalat lemma. $\bigtriangledown$

3.4 Simulation Study

To verify the adaptive laws, simulation studies are displayed below. To simplify the problem and make the result easy to follow, choose $n = 2$ for all systems below. Leader parameters are given for simulation study.

In this section, simulation results for all cases including single input systems and multiple inputs systems are shown. Section 3.4.1 displays simulation results of single input systems. Section 3.4.2 displays simulation results of multiple inputs systems.

3.4.1 Simulation Study for Single Input Systems

This section shows simulation results of single inputs systems. Simulation results of three different cases classifying by the disturbance performance in section 3.1.1 are shown in order. Through the simulation result,we can verify that the adaptive control laws works well.

Case I: Singe input system without disturbances For numerical study, parameter matrices in (3.3) and (3.4) are selected as: $(n = 2)$:

$$A_m = \begin{bmatrix} 1 & 1 \\ -11 & -6 \end{bmatrix}, b_m = \begin{bmatrix} 0 \\ 8 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

and the known A_e and b_e are chosen as

$$A_e = \begin{bmatrix} 1 & 1 \\ -9 & -4 \end{bmatrix} b_e = \begin{bmatrix} 0 \\ 4 \end{bmatrix}.$$

Based on the matching condition (3.8), we can easily obtain the ideal value of k_1^*, k_2^* and k_3^*.

$$k_1^{*T} = [-4,\ -2],\ k_2^* = 4, /, k_3^{*T} = [-5,\ -3].$$

This set of $k_i(i = 1, 2, 3)$ is the values that would make up the nominal controller (3.19). However, this nominal controller maybe unknown in real some experiences. Therefore, the estimates $k_1(t), k_2(t)$ and $k_3(t)$ of k_1^*, k_2^* and k_3^* will be determined from adaptive laws and used in the adaptive controller (3.22).

For the purpose of the simulation, the ideal gains were calculated. They are used to show how a fixed gain controller cannot handle the parameter unknown case. Since $k_1(0), k_2(0)$ and $k_3(0)$ can be chosen arbitrary, here we choose $k_i(0) = 0.5k_i^*, (i =$

$1, 2, 3$). $r = \sin(t) + \cos(t)$. The initial leader and follower state are $x_m = [0, 0]^T$ and $x = [0, 0]^T$, respectively.

The simulation result with a fixed gain controller for the state tracking of x is shown in figure 3.1. Likewise, the simulation result with adaptive controller for x is shown in figure 3.2. Tracking errors in both fixed gain control and adaptive control are shown in figure 3.3. Parameter errors $k_i(t) - k_i^*(t)$ are shown in figure 3.4.

Case II: Single input systems with disturbance acting on follower systems

For numerical study, parameter matrices in (3.5) and (3.4) are selected as: ($n = 2$):

$$A_m = \begin{bmatrix} 1 & 1 \\ -11 & -6 \end{bmatrix}, b_m = \begin{bmatrix} 0 \\ 8 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

The disturbance (3.6) is

$$d(t) = -5 - 4\sin(5t).$$

A_e, b_e are chosen as

$$A_e = \begin{bmatrix} 1 & 1 \\ -9 & -4 \end{bmatrix}, b_e = \begin{bmatrix} 0 \\ 4 \end{bmatrix},$$

Parameter matrices/vectors A, b, A_m, b,A_e and b_e adopted for simulation in Case I and Case II are the same. It is obviously that k_i^*, $i = 1, 2, 3$ are in these two special cases are the same. According to the value of $d(t)$. We can conclude that $k_5^* = 5 + 4\sin(5t)$.

For the purpose of the simulation, the ideal gains were calculated. They are used to show how a fixed gain controller cannot handle the parameter unknown case and in this case, particularly, an unknown external bounded disturbance. Since $k_1(0), k_2(0)$

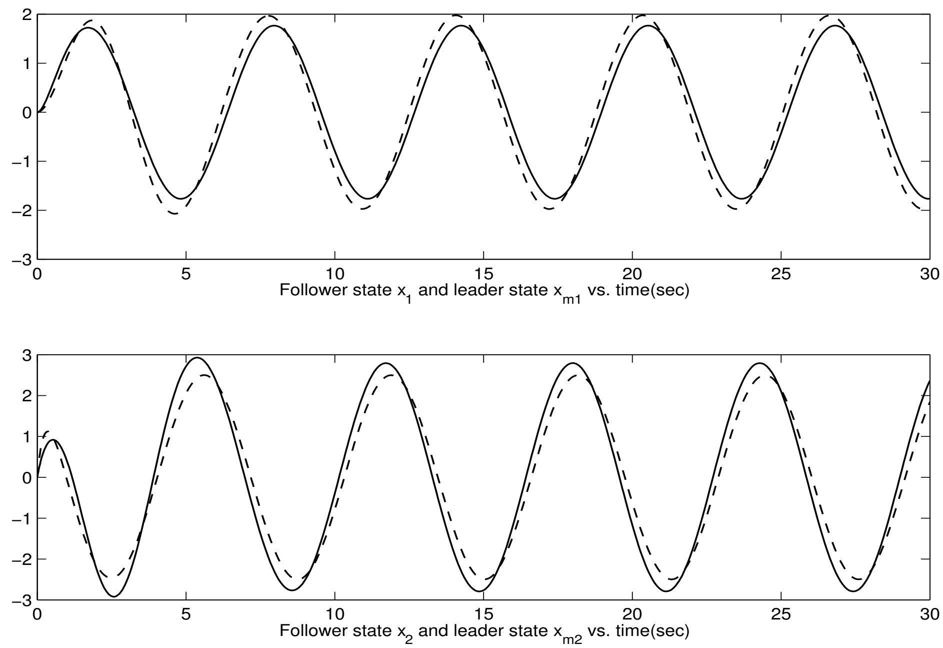

Figure 3.1: Follower state (solid) vs. leader state (dotted) with fixed gain control.

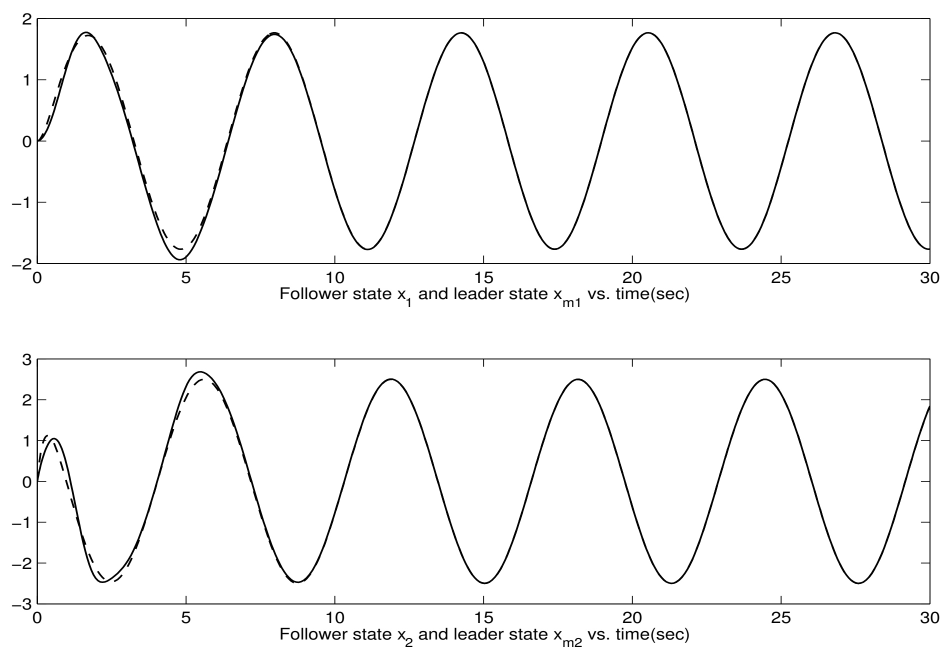

Figure 3.2: Follower state (solid) vs. leader state (dotted) with adaptive gain control.

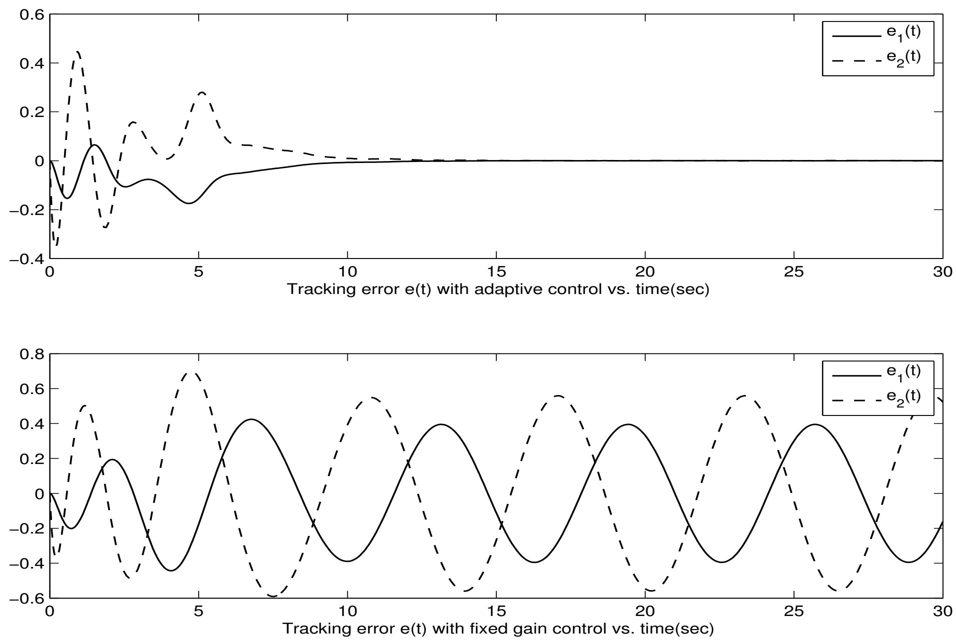

Figure 3.3: Tracking errors with adaptive control vs. fixed gain control.

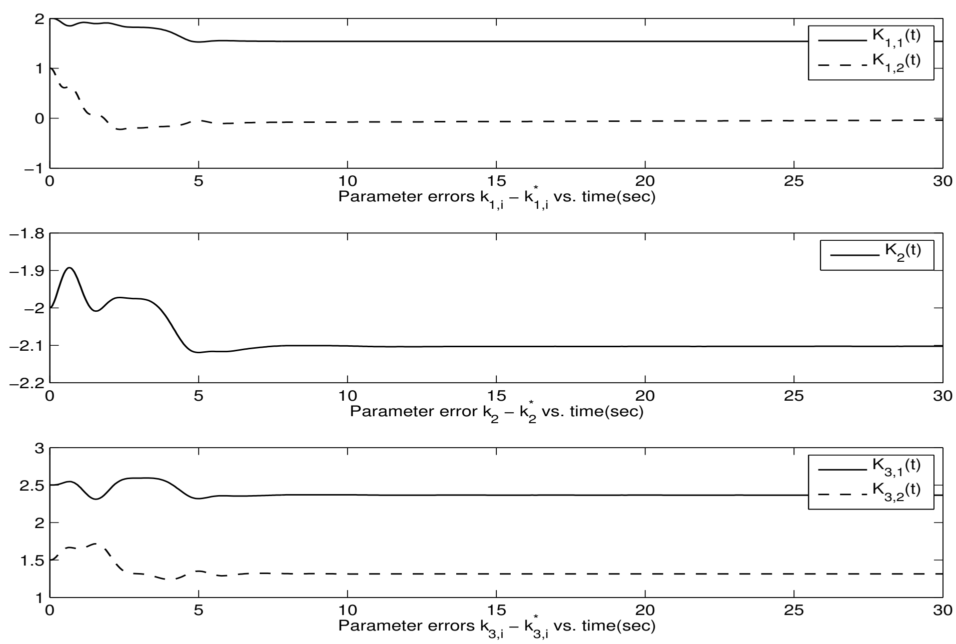

Figure 3.4: Parameter errors $k_i(t) - k_i^*$ with adaptive control $(i = 1, 2, 3)$.

and $k_3(0)$ can be chosen arbitrary, here we choose $k_i(0) = 0.8k_i^*, (i = 1, 2, 3)$. $k_{50}(0) = 5 \times 0.8 = 4, k_{5\beta}(0) = 4 \times 0.8 = 3.6$. $r = \sin(t) + \cos(t)$. The initial leader and follower state are $x_m = [1, 0]^T$ and $x = [-1, 5]^T$, respectively.

The simulation result with a fixed gain controller for the state tracking of x is shown in figure 3.5. Likewise, the simulation result with an adaptive controller for the state tracking of x is shown in figure 3.6. Tracking errors in both fixed gain control and adaptive control are shown in figure 3.7.

Case III: Single input system with disturbance acting on leader and follower systems For numerical study, parameter matrices in (3.5) and (3.7) are selected as: $(n = 2)$:

$$A_m = \begin{bmatrix} 1 & 1 \\ -11 & -6 \end{bmatrix}, b_m = \begin{bmatrix} 0 \\ 8 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 2 \end{bmatrix},$$

Disturbances acting on (3.5) and (3.7) are chosen as follows:

$$d(t) = -5 + (-4)\sin(5t),$$
$$d_m(t) = 4 + 3.6\sin(5t).$$

A_e and b_e are chosen as

$$A_e = \begin{bmatrix} 1 & 1 \\ -9 & -4 \end{bmatrix}, b_e = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Parameter matrices/vectors A, b, A_m and b adopted for simulation in Case I and Case II are the same.So the ideal controller parameters k_i^*, $i = 1, 2, 3$ are the same as the ideal controller parameters in Case I and Case II. According to (3.52), $k_5^* =$

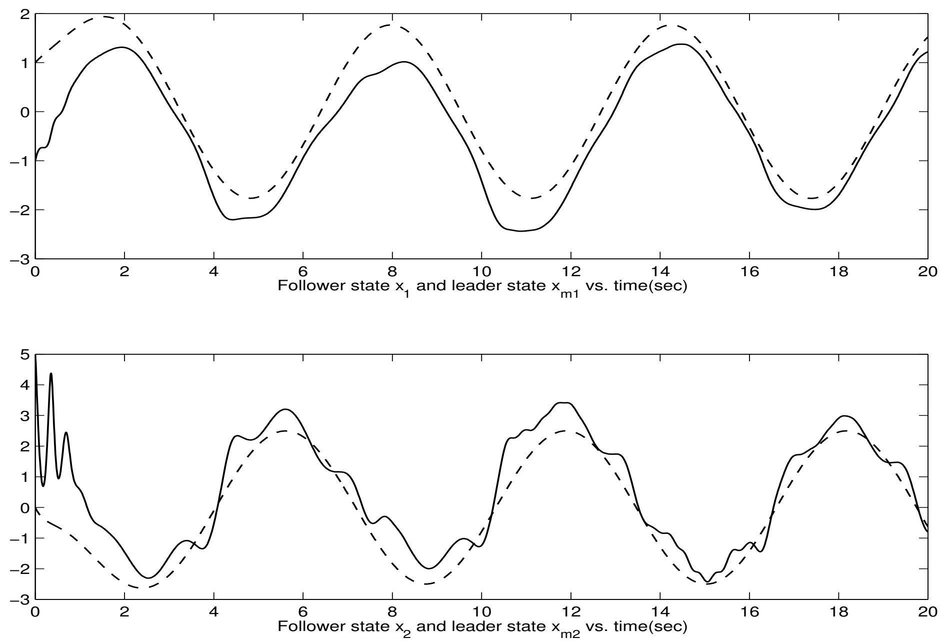

Figure 3.5: Follower state (solid) vs. leader state (dotted) without disturbance rejection.

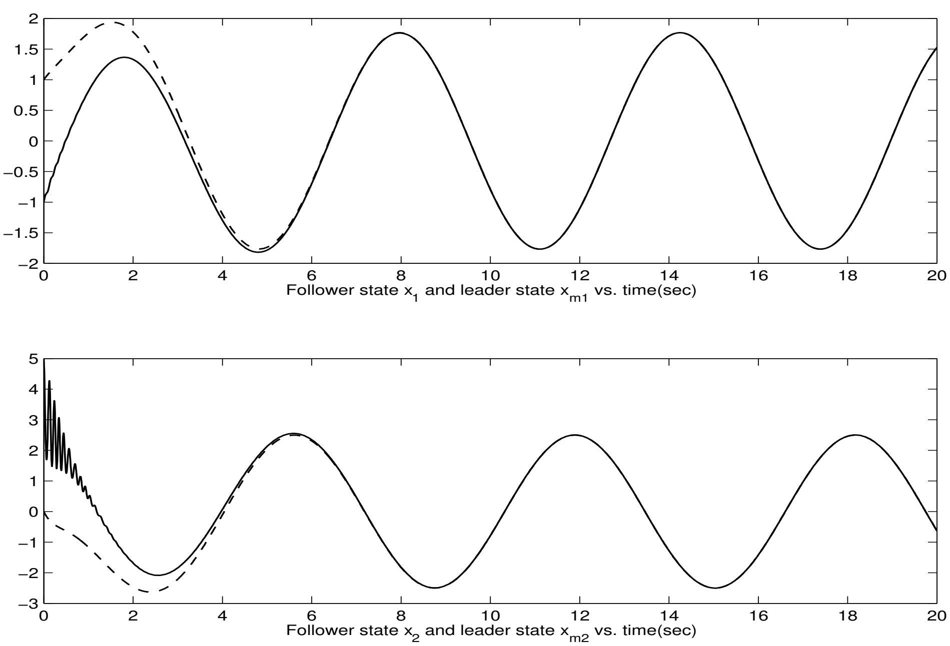

Figure 3.6: Follower state (solid) vs. leader state (dotted) with adaptive disturbance rejection.

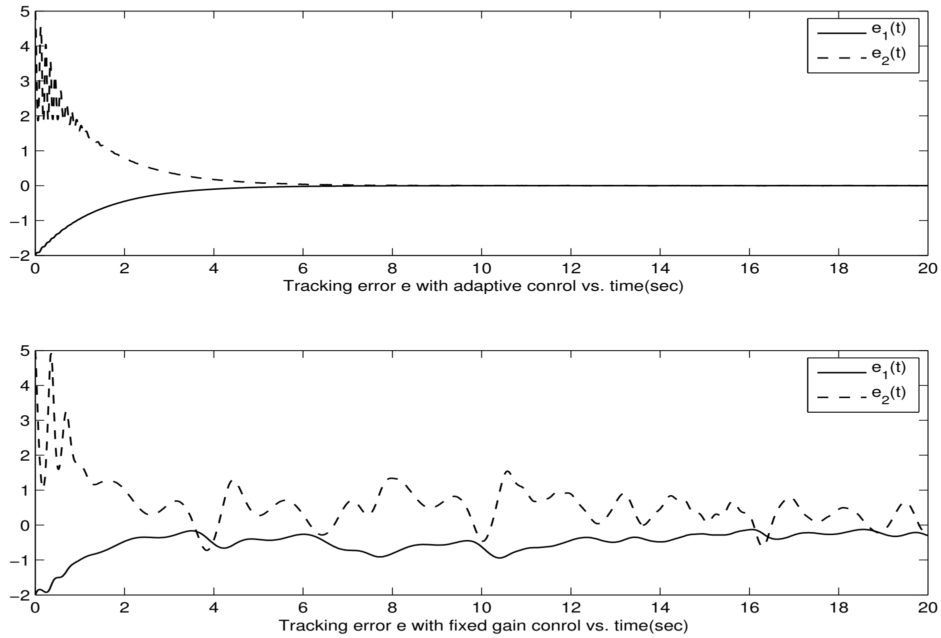

Figure 3.7: Tracking errors with adaptive disturbance rejection vs. without disturbance rejection.

$21 + 16\sin(5t) - 3.6\sin(5t)$.

For the purpose of the simulation, the ideal gains were calculated. They are used to show how a fixed gain controller cannot handle the parameter unknown case. Since $k_1(0), k_2(0)$ and $k_3(0)$ can be chosen arbitrary, here we choose $k_i(0) = 0.8k_i^*, (i = 1, 2, 3)$. $k_{50}(0) = 21 = 16.8$, $k_{5m\beta}(0) = 16 \times 8 = 12.8$, $k_{5\beta}(0) = 3.6 \times 8 = 2.88$. $r = \sin(t)$. The initial leader and follower state are $x_m = [4, 5]^T$ and $x = [3, 1]^T$, respectively.

The simulation result with a fixed gain controller for the state tracking of x is shown in figure 3.8. Likewise, the simulation result with an adaptive controller for the state tracking of x is shown in figure 3.9. Tracking errors in both fixed gain control and adaptive control are shown in figure 3.10.

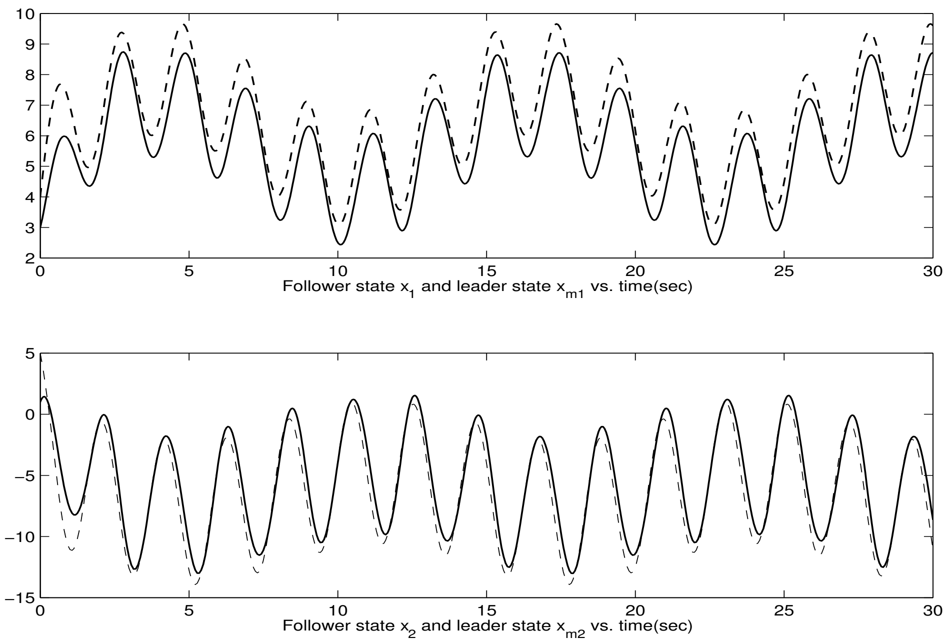

Figure 3.8: Follower state (solid) vs. leader state (dotted) without disturbance rejection

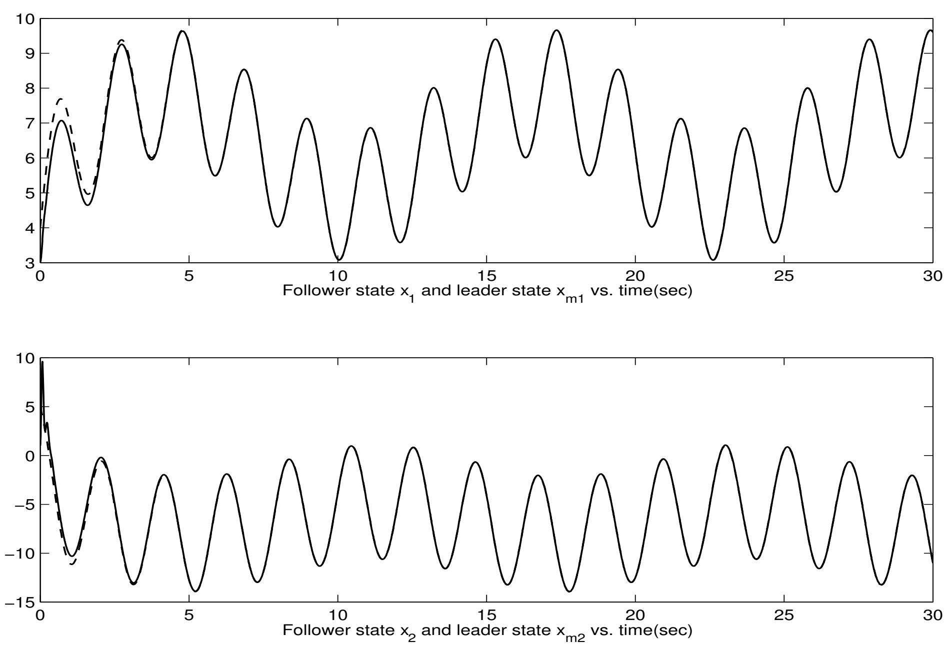

Figure 3.9: Follower state (solid) vs. leader state (dotted) with adaptive disturbance rejection

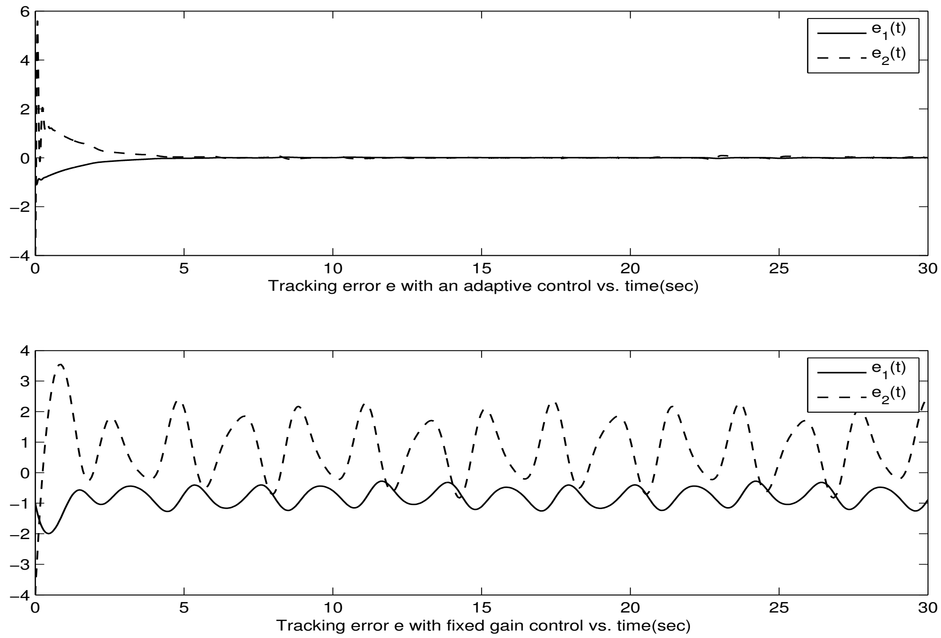

Figure 3.10: Tracking errors with adaptive disturbance rejection vs. without disturbance rejection.

3.4.2 Simulation Study with Multiple Inputs

This section shows simulation results of single inputs systems. Simulation results of three different cases classifying by the disturbance performance in section 3.2.1 are shown in order. Through the simulation result,we can verify that the adaptive control laws works well.

Case I: Multiple inputs systems without disturbance For numerical study, parameter matrices in (3.11) and (3.12) are selected as: ($n = 2$, $p = 2$, $m = 3$):

$$A_m = \begin{bmatrix} 1 & 1 \\ -11 & -6 \end{bmatrix}, B_m = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

The known parameter A_e and B_e are

$$A_e = \begin{bmatrix} 1 & 1 \\ -9 & -4 \end{bmatrix}, B_e = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}.$$

Based on the matching condition (3.68), we can easily obtain the ideal value of K_1^*, K_2^* and K_3^*.

$$K_1^{*T} = \begin{bmatrix} 0 & 0 \\ -4 & -2 \end{bmatrix}, K_2^* = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, K_3^{*T} = \begin{bmatrix} 0 & 0 \\ -5 & -3 \end{bmatrix}.$$

For the purpose of the simulation, the ideal gains were calculated. They are used to show how a fixed gain controller cannot handle the parameter unknown case. Since $K_1(0), K_2(0)$ and $K_3(0)$ can be chosen arbitrary, here we choose $K_i(0) = 0.5K_i^*, (i = 1, 2, 3)$. $r = [\sin(t), \cos(t), \sin(t) + \cos(t)]^T$. The initial leader and follower state are $x_m = [0, 0]^T$ and $x = [0, 0]^T$, respectively.

The simulation result with a fixed gain controller for the state tracking of x is shown in figure 3.11. Likewise, the simulation result with adaptive controller for x is shown in figure 3.12. Tracking errors in both fixed gain control and adaptive control are shown in figure 3.13. Parameter errors $K_i(t) - K_i^*(t)$ are shown in figure 3.14.

Case II: Multiple inputs systems with disturbance acting on follower systems For numerical study, parameter matrices in (3.13) and (3.15) are selected as:

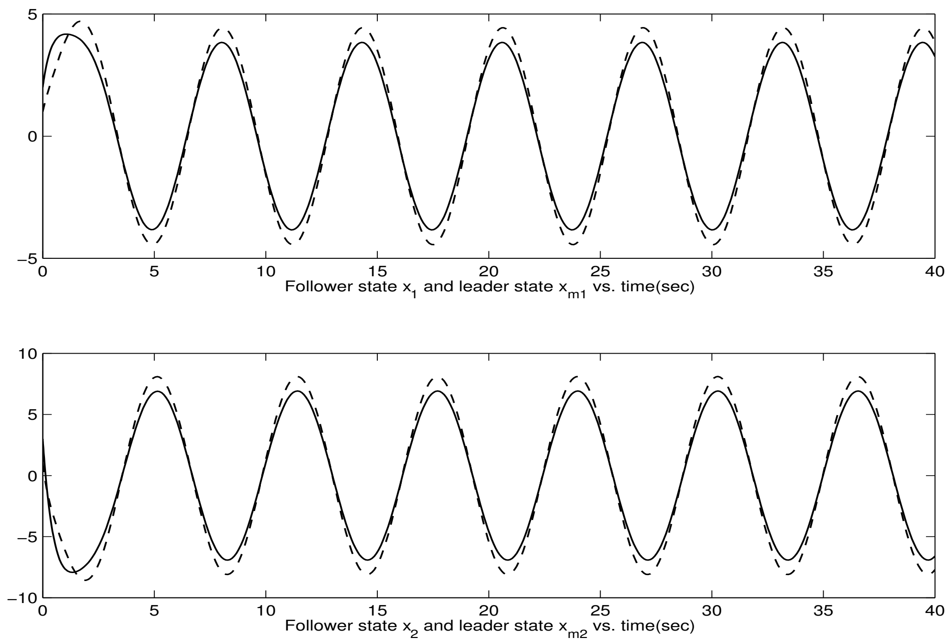

Figure 3.11: Follower state (solid) vs. leader state (dotted) with fixed gain control.

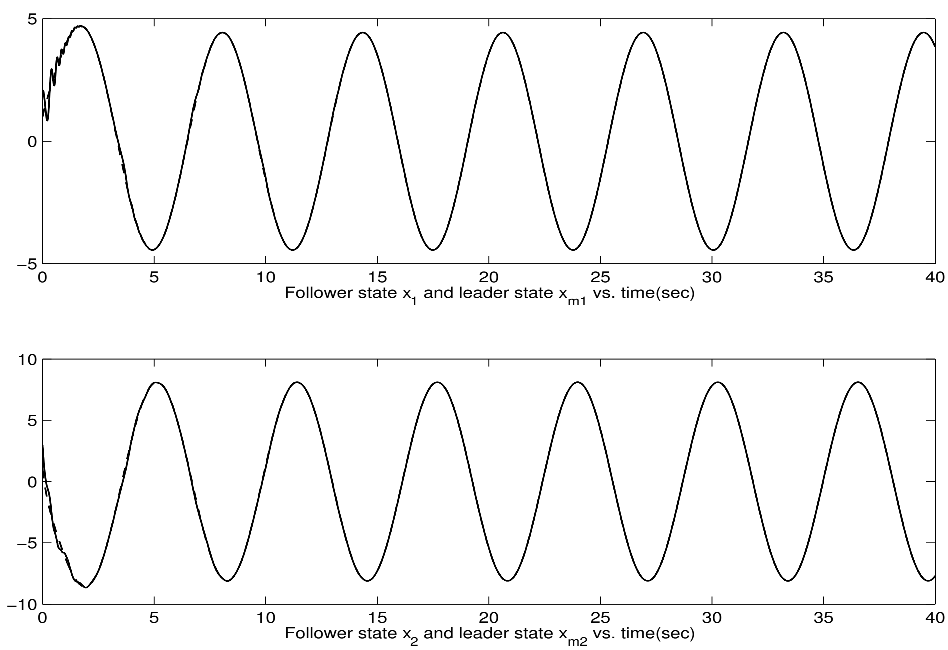

Figure 3.12: Follower state (solid) vs. leader state (dotted) with adaptive gain control.

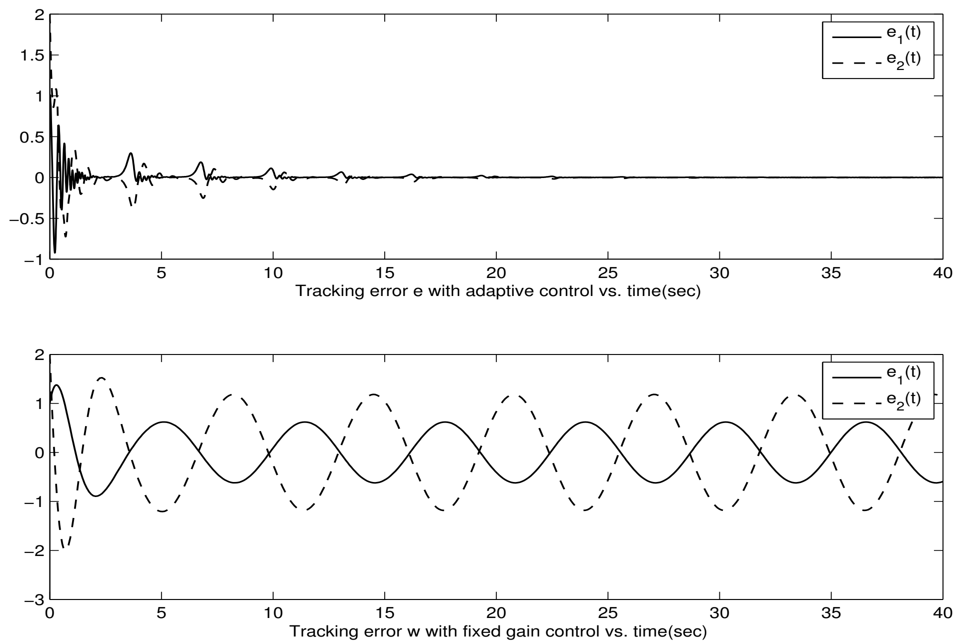

Figure 3.13: Tracking errors with adaptive control vs. fixed gain control.

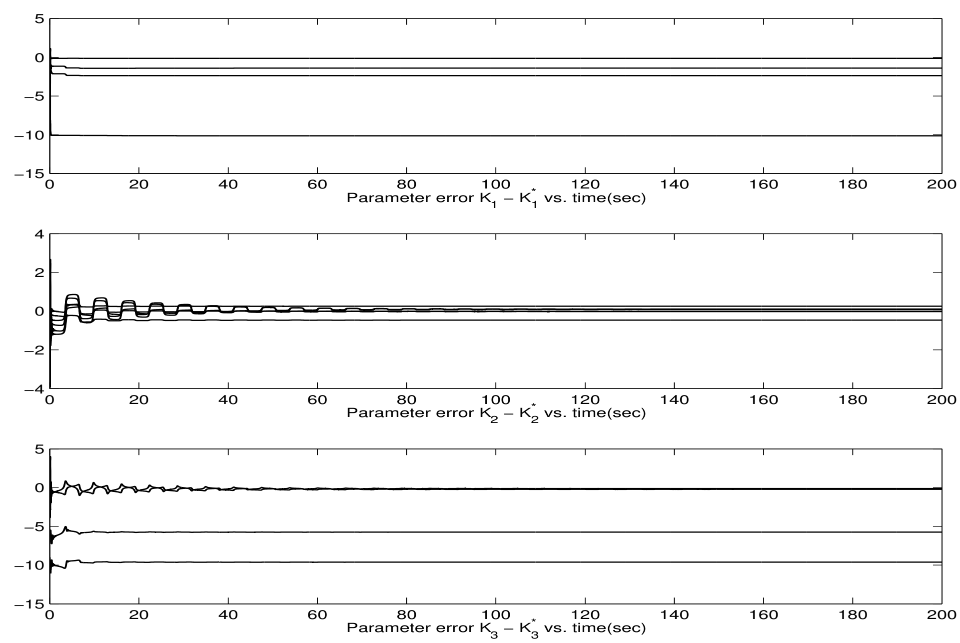

Figure 3.14: Parameter errors $K_i(t) - K_i^*$ with adaptive control $(i = 1, 2, 3)$.

$(n = 2)$:

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix}, B_m = \begin{bmatrix} 1 & -2 \\ 4 & 1 \end{bmatrix}, A = \begin{bmatrix} -2 & 3 \\ 1 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix},$$

The disturbance acting on the follower is

$$d(t) = [-5,\ -5]^T + [1,\ 2]^T \sin(5t)$$

A_e and B_e are

$$A_e = \begin{bmatrix} 1 & 1 \\ -9 & -4 \end{bmatrix}, B_e = \begin{bmatrix} 1 & -2 \\ 4 & 1 \end{bmatrix}.$$

Based on the matching condition (3.68), we can easily obtain the ideal value of K_1^*, K_2^* and K_3^*.

$$K_1^{*T} = \begin{bmatrix} -16 & 2 \\ 3 & -2 \end{bmatrix},\ K_2^* = \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix}, K_3^{*T} = \begin{bmatrix} -6 & 3 \\ 2 & -2 \end{bmatrix},$$

According to the equation $k_{5\beta}^* = -d(t)$, $k_5^* = [5,\ 5]^T + [-1,\ -2]^T \sin(5t)$.

For the purpose of the simulation, the ideal gains were calculated. They are used to show how a fixed gain controller cannot handle the parameter unknown case. Since $K_1(0), K_2(0)$ and $K_3(0)$ can be chosen arbitrary, here we choose $K_i(0) = 0.9k_i^*, (i = 1, 2, 3)$. $k_{50}^* = [-4.5,\ -4.5]^T, k_{5\beta}^* = [-0.8,\ 1.8]^T$. $r = [\sin(t),\ \cos(t)]^T$. The initial leader and follower state are $x_m = [1,\ 1]^T$ and $x = [2,\ 3]^T$, respectively.

The simulation result without adaptive disturbance rejection for the state tracking of x is shown in figure 3.15. Likewise, the simulation result with adaptive disturbance rejection for x is shown in figure 3.16. Tracking errors corresponding with disturbance rejection and non-disturbance rejection are shown in figure 3.17.

Remark 3.3 To show the effectiveness of the adaptive compensator, figure 3.15 is the result making by an adaptive controller without an adaptive disturbance compensator term, i.e., we adopt the adaptive controller (3.72) rather than (3.91). Figure 3.16 is created by the adaptive disturbance compensator (3.91). □

Case III: Multiple inputs systems with disturbance acting on leader and follower systems For numerical study, parameter matrices in (3.13) and (3.16) are selected as: $(n = 2)$:

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix}, B_m = \begin{bmatrix} 1 & -2 \\ 4 & 1 \end{bmatrix}, A = \begin{bmatrix} -2 & 3 \\ 1 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix},$$

The disturbance (3.14) and (3.17) are

$$\begin{aligned} d_m(t) &= [-5,\ -5]^T + [1,\ 2]^T \sin(5t) \\ d_(t) &= [4,\ 4]^T + [2,\ 1]^T \cos(5t) \end{aligned}$$

A_e and B_e are

$$A_e = \begin{bmatrix} 1 & 1 \\ -9 & -4 \end{bmatrix}, B_e = \begin{bmatrix} 1 & -2 \\ 4 & 1 \end{bmatrix}.$$

Based on the matching condition (3.45), we can easily obtain the ideal value of

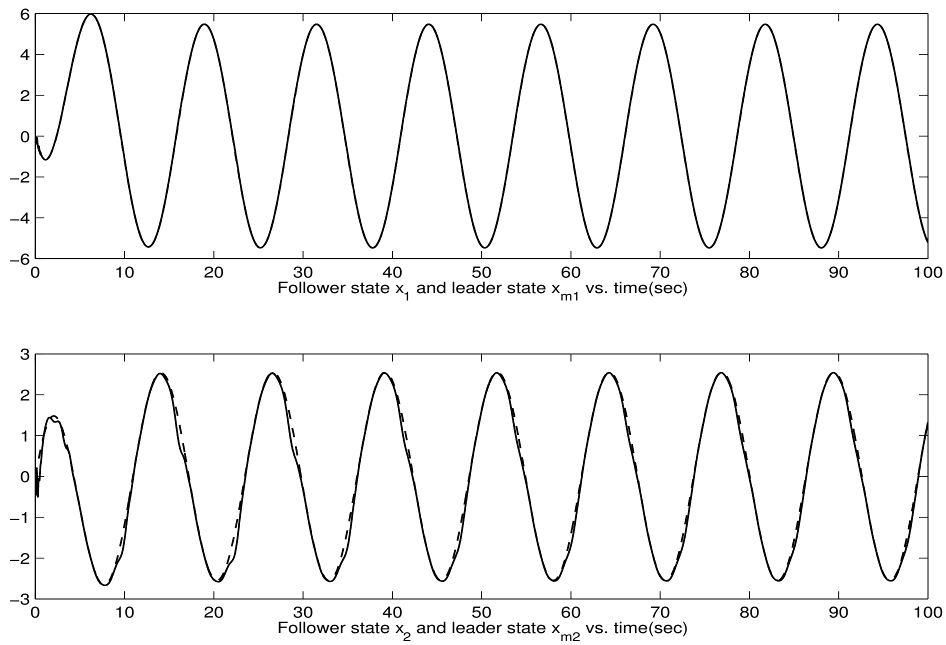

Figure 3.15: Follower state (solid) vs. leader state (dotted) without disturbance rejection.

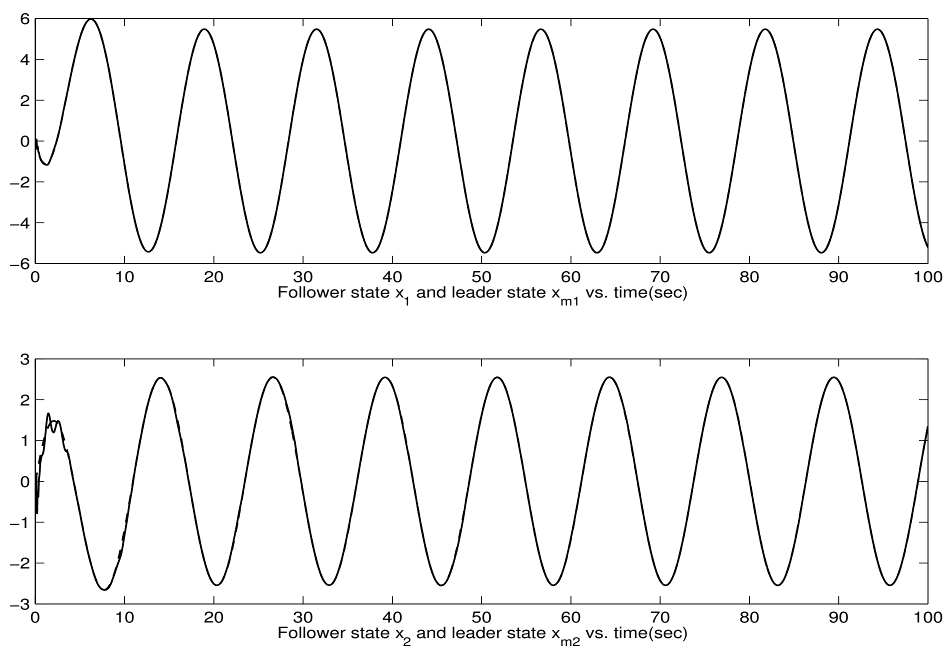

Figure 3.16: Follower state (solid) vs. leader state (dotted) with adaptive disturbance rejection.

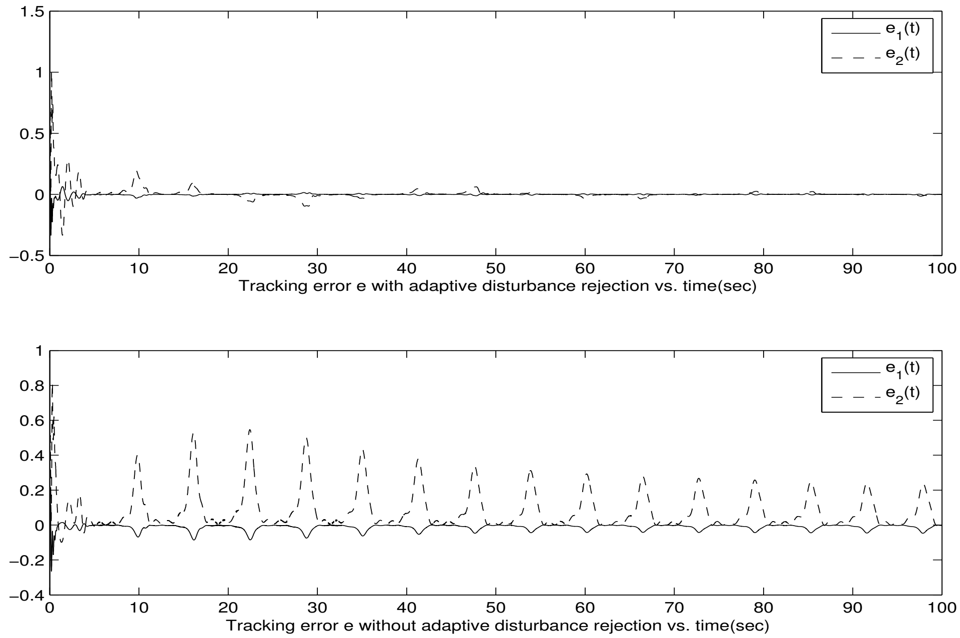

Figure 3.17: Tracking errors with adaptive disturbance rejection vs. without disturbance rejection.

K_1^*, K_2^* and K_3^*.

$$K_1^{*T} = \begin{bmatrix} -16 & 2 \\ 3 & -2 \end{bmatrix},\ K_2^* = \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix},\ K_3^{*T} = \begin{bmatrix} -6 & 3 \\ 2 & -2 \end{bmatrix}.$$

Parameter matrices A, B, A_m and B_m adopted for simulation in Case I and Case II are the same. It is obviously that $k_i^*,\ i = 1, 2, 3$ are in these two special cases are the same. $k_5^* = [23,\ -9]^T + [28,\ -4]^T \sin(5t) - [1,\ 2]^T \cos(5t)$.

For the purpose of the simulation, the ideal gains were calculated. They are used to show how a fixed gain controller cannot handle the parameter unknown case. Since $K_1(0), K_2(0)$ and $K_3(0)$ can be chosen arbitrary, here we choose $K_i(0) = 0.9k_i^*, (i = 1, 2, 3)$. $k_{50}^* = 4, k_{5j}^* = -3.6$. $r = [\sin(t),\ \cos(t)]^T$. The initial leader and follower state are $x_m = [1,\ 1]^T$ and $x = [2,\ 3]^T$, respectively.

The simulation result without adaptive disturbance rejection for the state tracking of x is shown in figure 3.18 (i.e., created by (3.72)). Likewise, the simulation result with adaptive disturbance rejection for x is shown in figure 3.19 (i.e., created by (3.107)). Tracking errors corresponding with disturbance rejection and non-disturbance rejection are shown in figure 3.20.

Simulation study with adaptive control based on LDU decomposition The following simulation results show the tracking performance without disturbances which is exactly the case we discussed in Section 3.3.5.

For numerical study, parameter matrices in (3.6) and (3.7) are selected as: ($n = 2, p = 2$):

$$A_m = \begin{bmatrix} 1 & 1 \\ -11 & 6 \end{bmatrix}, B_m = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},$$

A_e and B_e are

$$A_e = \begin{bmatrix} 1 & 1 \\ -9 & -4 \end{bmatrix}, B_e = \begin{bmatrix} 1 & -2 \\ 4 & 1 \end{bmatrix}.$$

Based on the matching condition (3.68), we can easily obtain the ideal value of $\Phi_0^*, \Phi_1^*, \Phi_2^*$ and Φ_3^*.

$$\Phi_0^* = \begin{bmatrix} 0 & -2 \\ 0 & 0 \end{bmatrix} \Phi_1^{*T} = \begin{bmatrix} -8 & -4 \\ -4 & -2 \end{bmatrix} \Phi_2^* = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \Phi_3^{*T} = \begin{bmatrix} -10 & -6 \\ -5 & -3 \end{bmatrix}$$

For the purpose of the simulation, the ideal gains were calculated. They are used to show how a fixed gain controller cannot handle the parameter unknown case. Since $\theta_1(0), \theta_2(0)$ and $\theta_3(0)$ can be chosen arbitrary, here we choose $\theta_i(0) = 0.8\theta_i^*, (i = 1, 2)$. $r = [\sin(t)\ \cos(t)]^T$. The initial leader and follower state are $x_m = [2.5, 1]^T$ and

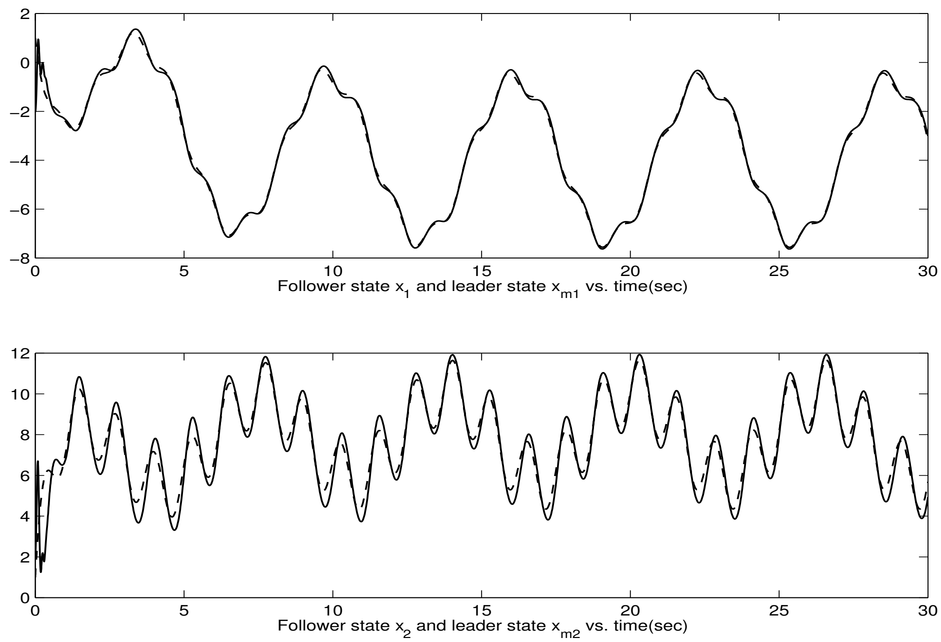

Figure 3.18: Follower state (solid) vs. leader state (dotted) without disturbance rejection.

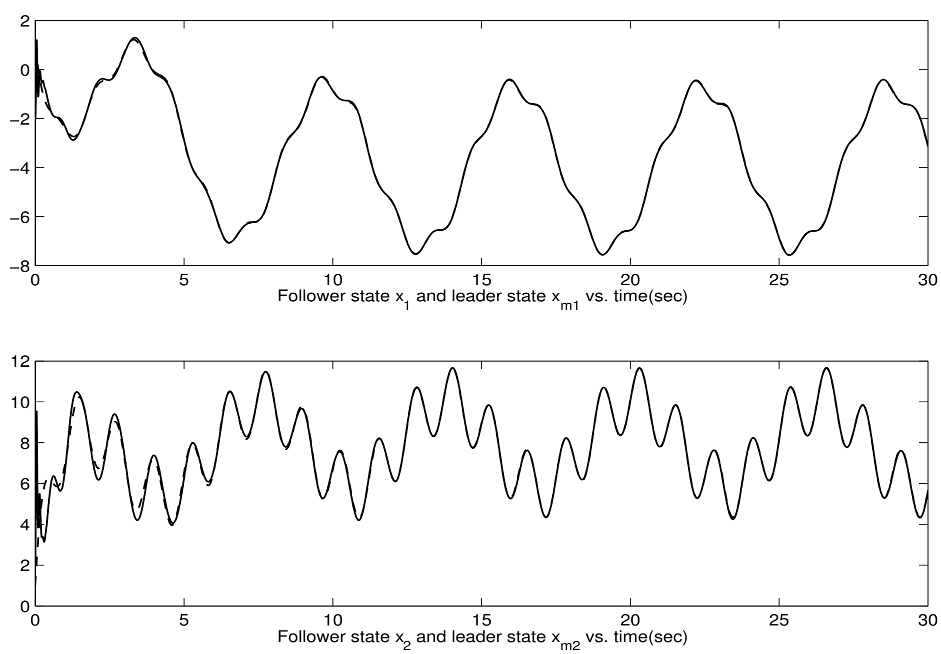

Figure 3.19: Follower state (solid) vs. leader state (dotted) with adaptive disturbance rejection.

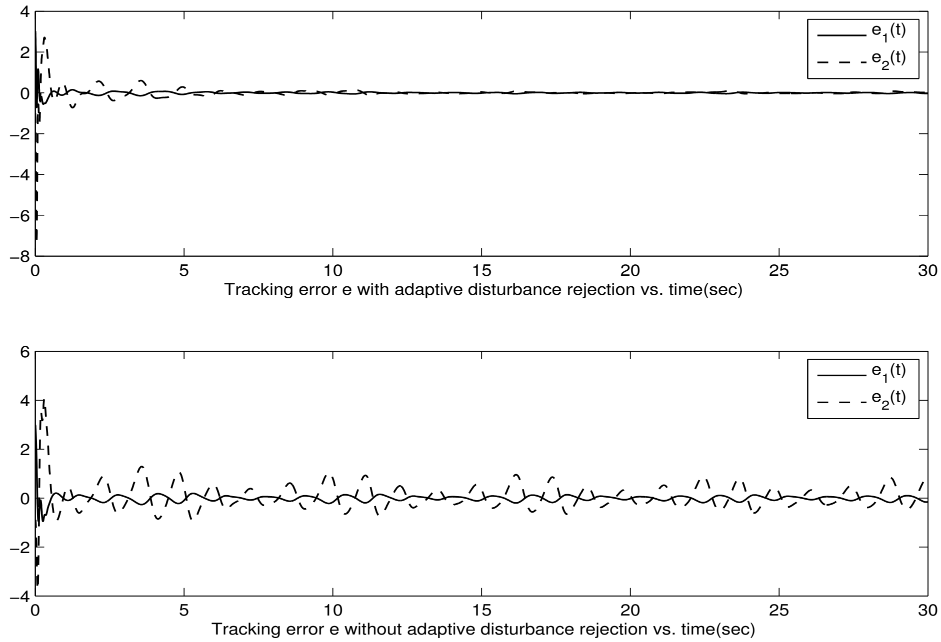

Figure 3.20: Tracking errors with adaptive disturbance rejection vs. without disturbance rejection.

$x = [0.5, 2]^T$, respectively.

The simulation result with a fixed gain controller for the state tracking of x is shown in figure 3.21. Likewise, the simulation result with adaptive controller for x is shown in figure 3.22. Tracking errors in both fixed gain control and adaptive control are shown in figure 3.23.

3.5 Summary

In this chapter basic adaptive control theory relevant to this research is introduced. Since this is a state tracking problem, in multi-agent system we also called this kind of problem as "leader-following consensus" problem. Leader-following consensus problems presented in this chapter are categorized into six cases in total including three single input cases and three multiple inputs cases. Stability analyses are

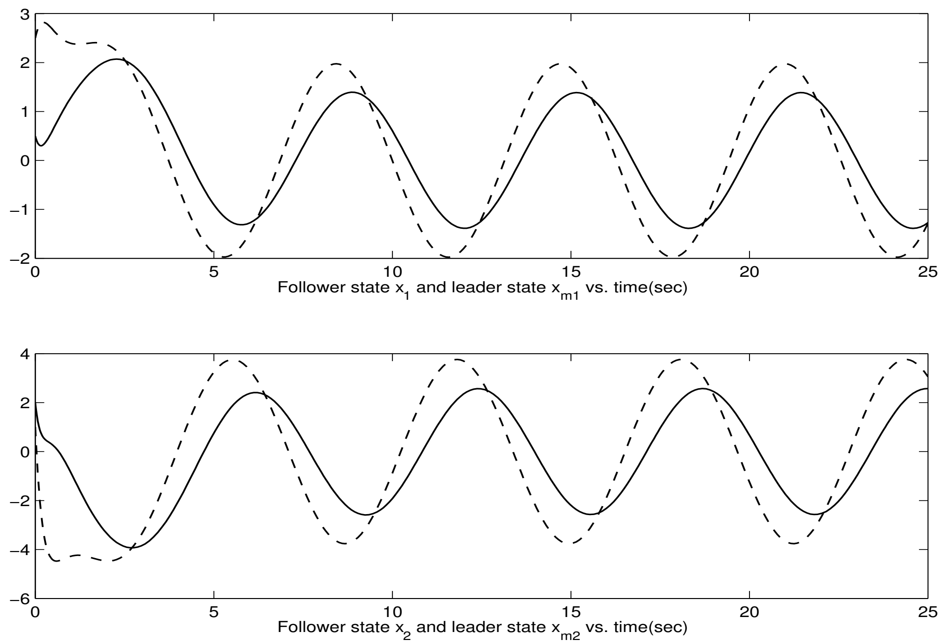

Figure 3.21: Follower state (solid) vs. leader state (dotted) with adaptive gain control (LDU decomposition).

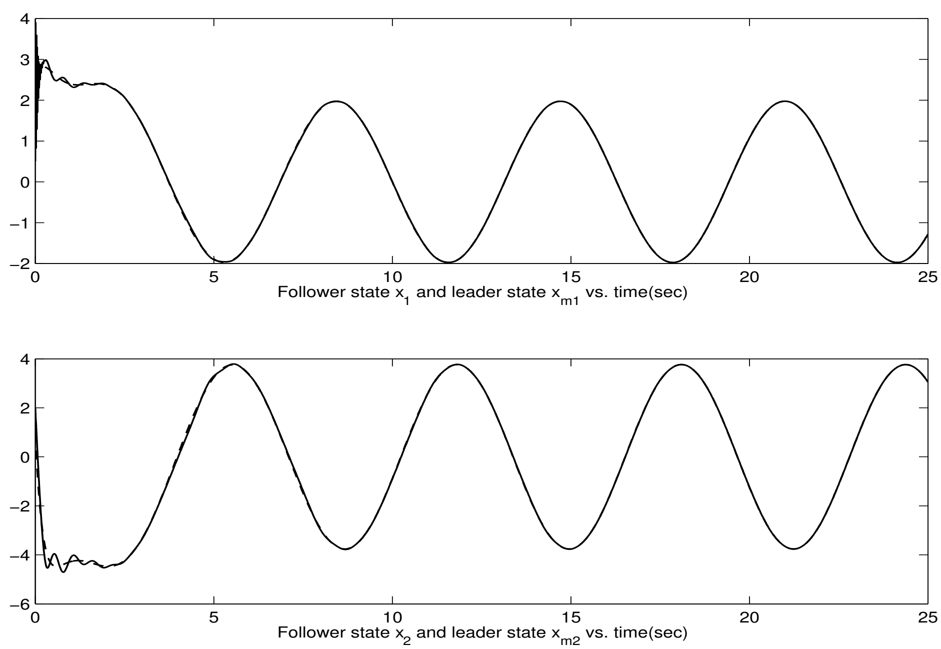

Figure 3.22: Follower state (solid) vs. leader state (dotted) with adaptive gain control (LDU decomposition).

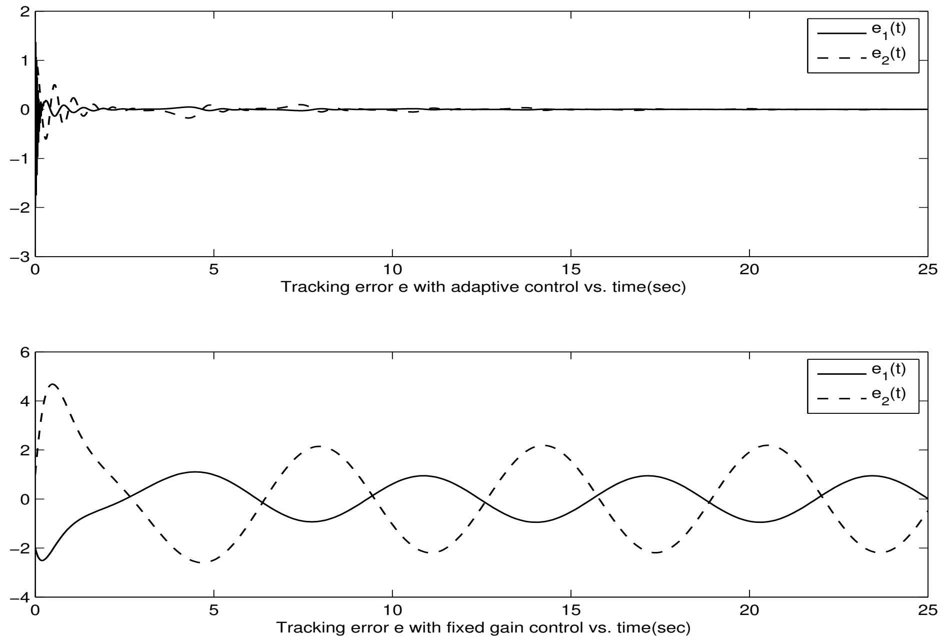

Figure 3.23: Tracking errors with adaptive control vs. fixed gain control (LDU decomposition).

given for each case to show with the adaptive controllers and the corresponding adaptive laws, closed-loop systems in all six cases have obtained the desired properties. Also, in the last subsection in Section 3.3 which focuses on the multiple input systems, LDU decomposition is applied. LDU decomposition is used because in practice some parameters are hard to get, so we need another functional adaptive controller structure with different adaptive laws. Remind that with LDU decomposition, both disturbances-free cases and cases with disturbances can be solved. However, we just show the disturbance-free case to show the basic principle. Many other literatures show more details which are listed in Section 3.3.5. In Section 3.4, simulation results are shown in order to verify that the adaptive controllers with corresponding adaptive laws work well. Simulation results show the difference between adaptive control and fixed gain control directly.

Chapter 4

Adaptive Leader-Following Consensus for Multiple Agents

In Chapter 3, we discussed about one leader-one follower consensus adaptive control strategy, from which we derive a new control strategy can be applied on multi-agent systems with directed graph. In Section 4.1, system dynamic expressions are given and the control objective is discussed. Algebraic graph theory which is used to represent the interactions among multi-agent systems is to be discussed in Section 4.2. In Section 4.3, a distributed adaptive control scheme for multi-agent system describing by direct graphs is developed. Disturbances are introduced in Section 4.4, and adaptive control compensator term will be added and the corresponding adaptive control scheme will be discussed in the same section. In the last section, simulation results are shown to indicate the capability of the distributed adaptive control schemes presented in this chapter.

4.1 Problem Statement

Consider a set of N unknown follower agents. The dynamic system of the ith follow agent can be expressed as

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t),\, i = 1, \ldots, N \tag{4.1}$$

where $x_i(t) \in R^n$ is the state of the ith agent, $u_i(t) \in R^{p_i}$ is the control input(in multi-agent system, we also call it control protocol), All parameter matrices $A_i \in R^{n\times n}, B_i \in R^{n\times p_i}$, $i = 1, \ldots, N$ in (4.1) are unknown. The dynamics of the leader is given by

$$\dot{x}_0 = A_0 x_0 + B_0 u_0(t) \tag{4.2}$$

where $x_0 \in R^n$ is the state of the leader, $u_0(t) \in R^m$ is the bounded reference signal and $A_0 \in R^{n\times n}, B_0 \in R^{n\times m}$ are unknown constant matrices.

The control objective is to design a distributed control protocol using local information for each follower to drive all follower consensus on their states with a given leader, i.e., make all the followers track the give leader on states eventually. Such a control problem is defined as leader-following consensus problem. The control objective can be described mathematically as

$$\lim_{t\to\infty} (x_i(t) - x_0(t)) = 0,\; i = 1, \ldots, N. \tag{4.3}$$

The above equation means if the control objective is achieved, all states errors between follower agents and the leader should go to zero as t goes to infinity.

4.2 Algebraic Graph Theory

The interaction graphs among $N+1$ agents including one leader and N followers are denoted by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with a set of vertices $\mathcal{V}$ and a set of directed edges (ordered pairs of vertices) $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. A vertex represents an agent and the directed edge (v_j, v_i) denotes that agent v_i can obtain the information from agent v_j including both the state information and the input information, but not vice versa. In this case, we say that v_j is one of the neighbors of v_i (although the link between them is directed). Define a neighborhood set $\mathcal{N}_i$ for the follower agent v_i such that $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$ for $i = 1, \ldots, N$. v_0 denotes the leader in the multi-agent system. It also could be a neighbor of the follower agent v_i if v_i can get the information from v_0 (i.e. $(v_0, v_i) \in \mathcal{E}$). A directed path is a sequence of ordered edges of the form $(v_{i1}, v_{i2}), (v_{i2}, v_{i3}), \ldots$ in a directed graph, where $v_{ij} \in \mathcal{V}$. Define an indegree (the number of head endpoints adjacent to a vertex) matrix D such that $D = \mathrm{diag}\{n_1, n_2, \ldots, n_N\} = \mathrm{diag}\{\deg(v_1), \deg(v_2), \ldots, \deg(v_N)\}$ where $\deg(\cdot)$ denotes the indegree of an agent. This indegree matrix D represents the total number of the neighbors of the follower agent v_i, i.e., the total agent number in the neighborhood set $\mathcal{N}_i$ from $i = 1$ to N.

To describe the relationship between the leader and followers, we denote $\mathcal{B} = \mathrm{diag}\{\mu_i\}$ if the agent i can get the information from the leader directly then $\mu_i = 1$ otherwise $\mu_i = 0$. Denote $\mathcal{Q} = \mathcal{L} + \mathcal{B}$. matrix $\mathcal{Q}$ is a matrix which can fully described the $N+1$ order graph including N followers and one leader. It is obviously that elements q_{ii} on diagonal are the total number of agents in the neighbor sets $\mathcal{N}_i$ of ith agent respectively. In order to make the equation more clear, we denote n_i equals q_{ii} to represent the total agent number in neighbor set $\mathcal{N}_i$. More examples show in section 4.5.

In order to make the leader-following consensus realized, we give two assumptions on the interaction graphs.

Assumption 4.1. *For each agent v_i, there should exist at least one directed path $(v_0, v_1), (v_1, v_2), \ldots, (v_{i-1}, v_i)$ which starts from the leader and ends at the agent v_i.*

Assumption 4.2. *Directed graph $\mathcal{G}$ is simple, which means that it has no loops, and no multiple arcs (arcs with same starting and ending nodes).*

Remark 4.1 Many papers focusing on multi-agent leader-following consensus problem demonstrate that the controller they proposed work perfectly for complex multi-agent systems which have many different kinds of edges. But in practical, engineers always consider how to achieve the objective more effectively. From this consideration, we find out the simplest multi-agent system structure which our proposed control protocol can still work with. Being derived from Assumption 4.1 and Assumption 4.2, (4.4) is the corresponding matrix $\mathcal{Q}$ of such the simplest multi-agent systems after elementary transformations.

$$\begin{bmatrix} 1 & & & 0 \\ * & 1 & & \\ & \ddots & \ddots & \\ * & & * & 1 \end{bmatrix}_{N \times N} \tag{4.4}$$

with * representing -1 or 0. (4.4) means for a multi-agent system including N follower agents and one leader, as long as there exist N edges and each follower agent has in-degree one, the distributed adaptive control protocol proposed in this chapter is applicable. □

4.3 Adaptive Control Design

To make adaptive control design work, several design conditions should be satisfied. Design conditions are presented in Section 4.3.1 and a distributed adaptive control protocol with adaptive laws is presented in Section 4.3.2.

4.3.1 Design Conditions

To meet the control objective and solve the leader-following consensus problem, we need several design conditions to be satisfied.

(A4.1) all the eigenvalues of A_0 are in the open left-half complex plane;

(A4.2) $u_0(t)$ is bounded and piecewise continuous;

(A4.3) there exist two matrices $K_{1i}^* \in \mathbb{R}^{n\times p_i}$ and $K_{4i}^* \in \mathbb{R}^{p_i\times p_i}$ for each follower agent v_i, which satisfy the following equations

$$A_e = A_i + B_i K_{1i}^{*T},\ B_e = B_i K_{4i}^*. \tag{4.5}$$

where $A_e \in \mathbb{R}^{n\times n}$ is a stable and known matrix, such that $A_e^T P + P A_e = -Q < 0$, $Q = Q^T > 0$ is satisfied, and $B_e \in \mathbb{R}^{n\times p}$ is also a known matrix.

(A4.4) there exist two matrices $K_{2i0}^* \in \mathbb{R}^{p_i\times m}$, $K_{3i0}^* \in \mathbb{R}^{n\times p_i}$ such that

$$A_0 = A_i + B_i K_{3i0}^{*T},\ B_0 = B_i K_{2i0}^* \tag{4.6}$$

are satisfied, if the leader is one of the neighbors of the follower agent v_i (i.e.,

$(v_0, v_i) \in \mathcal{E}$). Otherwise, if $(v_0, v_i) \notin \mathcal{E}$,

$$A_j = A_i + B_i K_{3ij}^{*T},\ B_j = B_i K_{2ij}^{*} \tag{4.7}$$

should be satisfied for some $K_{3ij}^* \in \mathbb{R}^{n\times p_i}$ and $K_{2ij}^* \in \mathbb{R}^{p_i \times p_j}$ for each pair of $(v_j, v_i) \in \mathcal{E}$ $(j \neq 0)$.

(A4.5) there is a known matrix $S_i \in \mathbb{R}^{p_i\times p_i}$ for each follower i such that $K_{4i}^* S_i$ is symmetric and positive definite: $M_s = K_{4i}^* S_i = (K_{4i}^* S_i)^T = S_i^T K_{4i}^{*T} > 0$.

Assumptions (A4.1) and (A4.2) are for a stable, well-defined reference system (A_0, B_0) with a bounded output $y_0(t)$. Assumptions (A4.3) and (A4.4) are called the matching conditions. Assumption (A4.4) indicates that we classify the N followers into two groups. In the first group, each follower has a direct access to obtain the information of the leader, thus for each pair of (v_0, v_i), there exist two matrices K_{2i0}^* and K_{3i0}^* satisfying (4.6). Followers in the other group have no direct accesses to the leader, thus there exist two matrices K_{2ij}^* and K_{3ij}^* for each pair of (v_j, v_i), $j \neq 0$. In summary, for each directed edges (ordered pair) (v_j, v_i) in the multi-agent system, no matter v_j is the leader or a follower, there exist a set of corresponding K_{2ij}^* and K_{3ij}^* $(0 \leq j \leq N)$.

Suppose all the parameters A_i, B_i for $i = 0, 1, \ldots, N$ are known and Assumptions (4.5) - (4.7) satisfied, an ideal distributed adaptive protocol which can achieve the control objective is designed as

$$u_i^*(t) = \frac{1}{n_i} \sum_{v_j \in \mathcal{N}_i} \left(K_{1i}^{*T}(x_i(t) - x_j(t)) + K_{2ij}^* u_j(t) + K_{3ij}^{*T} x_j(t) \right) \tag{4.8}$$

with n_i being the total agent number in the neighbor set $\mathcal{N}_i$.

We substitute (4.8) into (4.1), the ith closed-loop subsystem (4.1) becomes

$$\begin{aligned}\dot{x}_i(t) &= A_i x_i(t) + \frac{1}{n_i} B_i \sum_{v_j \in \mathcal{N}_i} (K_{1ij}^{*T}(x_i(t) - x_j(t)) + K_{2ij}^{*} u_j(t) + K_{3ij}^{*T} x_j(t)) \qquad (4.9)\\ &= \frac{1}{n_i} \sum_{v_j \in \mathcal{N}_i} (A_e(x_i(t) - x_j(t)) + A_j x_j(t) + B_j u_j(t))\end{aligned}$$

To begin with a control design, define a local tracking error for each agent here in this multi-agent case as follow

$$e_i(t) = x_i(t) - \frac{1}{n_i} \sum_{v_j \in \mathcal{N}_i} x_j(t). \qquad (4.10)$$

for $i = 1, \ldots, N$, as a measurement to show the disagreement between the follower i and the average of its neighbors on the states. The motivation of defining such a local state tracking error is shown in the following lemma.

Lemma 4.1. *Under Assumptions (A4.1) and (A4,2), if* $\lim_{t\to\infty} e_i(t) = 0$ *holds, then* $\lim_{t\to\infty} (x_i(t) - x_0(t)) = 0$ *holds for all* $i = 1, \ldots, N$.

Proof: First we use a one leader and two followers case to verify the Lemma 4.1. For a one leader two followers directed graph, there are four formation possible cases in total.

Case I: In figure 4.1, since we have known that $e_1(t) = x_1(t) - x_0(t) \to 0$ and $e_2(t) = x_2(t) - x_1(t)$ as $t \to \infty$, then obviously we can get $x_1(t) - x_0(t) \to 0$ and $x_2(t) - x_0(t) \to 0$ as $t \to \infty$.

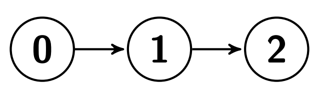

Figure 4.1: Case I: Interaction graph of two followers and one leader

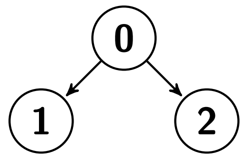

Figure 4.2: Case II: Interaction graph of two followers and one leader

Case II: In figure 4.2, we have known that $e_1(t) = x_1(t) - x_0(t) \to 0$ and $e_2(t) = x_2(t) - x_0(t) \to 0$ as $t \to \infty$, then obviously we can get $x_1(t) \to x_0(t)$ and $x_2(t) \to x_0(t)$ as $t \to \infty$.

Case III: In figure 4.3, since we have known that $e_1(t) = x_1(t) - \frac{1}{2}x_0(t) - \frac{1}{2}x_2(t) \to 0$

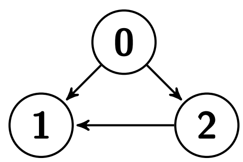

Figure 4.3: Case III: Interaction graph of two followers and one leader

and $e_2(t) = x_2(t) - x_0(t)$ as $t \to \infty$, then obviously we can get $x_1(t) - x_0(t) \to 0$ and $x_2(t) - x_0(t) \to 0$ as $t \to \infty$.

Case IV: Case IV is similar with Case III. We can also verify Lemma 4.1 quickly.

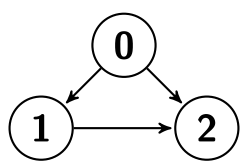

Figure 4.4: Case IV: Interaction graph of two followers and one leader

After the verification, we will start our proof. Under Assumption 4.1, for each follower, there exists at least one directed path from the leader to the follower, which indicates that each directed graph with N followers and one leader in this thesis consists of at least one basic structure which is like figure 4.5. In figure 4.5, $j = 1, \ldots, N$.

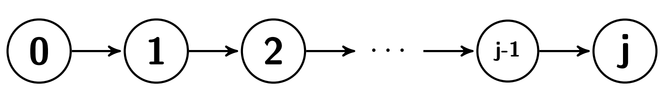

Figure 4.5: A basic structure exists in directed graphs

We assume that for all followers, $\lim_{t\to\infty} e_i(t) = 0$, $i = 1, \ldots, j$, $j < N$ in figure 4.5. Each directed graph may has several basic structures with different orders (i.e., j can be different). Under the condition that $\lim_{t\to\infty} e_i(t) = 0$ holds for $i = 1, \ldots, j$, $j < N$, if we can prove that all followers in figure 4.5 can achieve the global tracking, i.e., $\lim_{t\to\infty}(x_i(t) - x_0(t)) = 0$ no matter how the basic structure changes, we can conclude that the Lemma 4.1 is true.

We have two different methods to make figure 4.5 more complex. The first method is to add neighbors on follower v_1 in figure 4.5 (i.e., the follower who can get the information from the leader directly). The second method is to add neighbors on follower v_2 to v_j (i.e., followers who can not get information from the leader directly).

Before we making changes on figure 4.5, we assume that all followers in figure 4.5 have already achieved the global tracking which means $\lim_{t\to\infty}(x_i(t) - x_0(t)) = 0$, $i = 1, \ldots, j$.

First method: there is only one possibility to add neighbors on follower v_1 under Assumption 4.1 and Assumption 4.2, which is to add a new follower outside into figure 4.5 (maybe a follower from other basic structures in the same directed graph). Figure 4.6 shows the interaction graph when we add a new follower v_{j+1} into figure 4.5. According to the condition of Lemma 4.1, we have $e_{j+1}(t) = x_{j+1}(t) - x_0(t) \to 0$

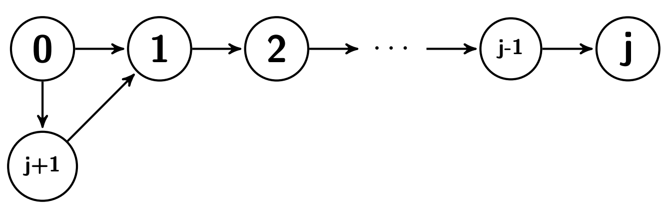

Figure 4.6: Interaction graph after adding one follower on follower v_1

as $t \to \infty$ and $e_1 = x_1 - \frac{1}{2}x_0(t) - \frac{1}{2}x_{j+1}(t)$. So we have $x_1(t) - x_0(t) \to 0$ as $t \to \infty$. Since follower v_2 to v_j do not change, it is obviously that $\lim_{t\to\infty}(x_i(t) - x_0(t)) = 0$. If there are more neighbors adding on follower v_i, the situations will be the same.

Second method: Add followers on the follower who does not have the access to get the information from the leader directedly.

Case I: Add the leader as a neighbor of follower v_k ($k \neq 0$ and $k < N$, see figure 4.7). Here we use mathematic induction to prove that the lemma is true under Case I by the second method.

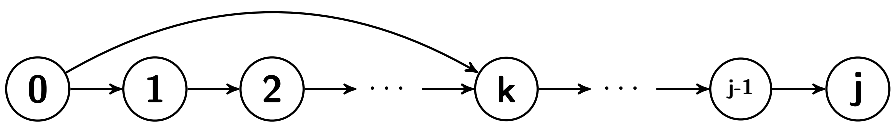

Figure 4.7: Interaction graph after adding one follower on follower v_1

First, when adding the leader as a neighbor of follower v_k, since all $\lim_{t\to\infty} e_i(t) = 0$, we have $\lim_{t\to\infty} e_k(t) = \lim_{t\to\infty}(x_k(t) - \frac{1}{2}x_0(t) - \frac{1}{2}x_{k-1}(t)) = 0$. Since we assume that all the followers have already achieved the global tracking before we start adding neighbors, we have $\lim_{t\to\infty}(x_{k-1}(t) - x_0(t)) = 0$. Thus, obviously we have $\lim_{t\to\infty}(x_k(t) - x_0(t)) = 0$.

Then we assume when we adding n followers on the follower v_k including the leader, we can get

$$\lim_{t\to\infty} e_i(t) = \lim_{t\to\infty}(x_k(t) - \frac{1}{n}\sum_{v_{k-1}\in\mathcal{N}_{k_n}} x_{k-1}(t)) = 0$$

according to $\lim_{t\to\infty}(x_i(t) - x_0(t)) = 0$. $\mathcal{N}_{k_m}$ denotes the neighbors of v_k when it has m neighbors (m¡N).

Under the situation when adding n followers on follower v_k, we add the $(n+1)$th follower v_a on the follower v_k including the leader, we have

$$\lim_{t\to\infty} e_k(t) = \lim_{t\to\infty}\left(x_k(t) - \frac{1}{n+1}\Big(\sum_{v_{k-1}\in\mathcal{N}_{m+1}} x_{k-1}(t) + x_a(t)\Big)\right) = 0$$

Since all the other followers do not change, so $x_a \to x_0$ as $t \to \infty$, $x_{k-1} \to x_0$ as $t \to \infty$, $x_{k-1} \in \mathcal{N}_{k_{n+1}}$ ($\mathcal{N}_{k_{n+1}}$ represents the neighborhood when follower v_k has n neighbors). Thus, when there are $n+1$ followers on followers v_k, according to

$$\lim_{t\to\infty} e_k(t) = \lim_{t\to\infty}(x_k(t) - \frac{1}{n_{k+1}}(\sum_{v_{k-1}\in\mathcal{N}_k} x_{k-1}(t) + x_a(t))) = 0$$

$$\lim_{t\to\infty} x_{k-1}(t) = 0, \qquad \lim_{t\to\infty} x_a(t) = 0$$

we have,

$$\lim_{t\to\infty}(x_k(t) - x_0(t)) = 0$$

Then,

$$\lim_{t\to\infty}(x_i(t) - x_0(t)) = 0,\ i = 1, \ldots, j$$

Case II: Add other followers as neighbors of follower v_k ($k \neq 0$ and $k < N$, see figure 4.8).

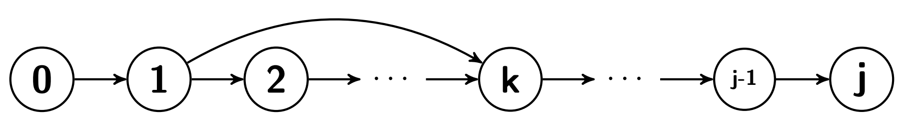

Figure 4.8: Interaction graph after adding one follower for follower v_k

We still use mathematic induction to prove that the lemma is true under Case II by the second method.

First, when adding follower x_a, $(a \neq 0)$, as a neighbor of follower v_k, since all $\lim_{t\to\infty} e_i(t) = 0$, we have $\lim_{t\to\infty} e_k(t) = \lim_{t\to\infty}(x_k(t) - \frac{1}{2}x_a(t) - \frac{1}{2}x_{k-1}(t)) = 0$. Since we assume that all the followers have already achieved the global tracking before we start adding neighbors, we have $\lim_{t\to\infty}(x_{k-1}(t) - x_0(t)) = \lim_{t\to\infty}(x_a(t) - x_0(t)) = 0$. Thus we have $\lim_{t\to\infty}(x_k(t) - x_0(t)) = 0$.

When adding n followers or $n+1$ followers into figure 4.5 under Case II, the situation will be similar with Case I. Then,

$$\lim_{t\to\infty}(x_i(t) - x_0(t)) = 0,\ i = 1, \ldots, j$$

In summary, we have proved that the global tracking can be achieved if we known that all $\lim_{t\to\infty} e_i(t) = 0$ whatever changes we make on a basic structure in directed graph. Also the directed graphs under Assumption (A4.1) and (A4.2) consist of several different basic structures. Thus we can prove that Lemma 4.1 is true. $\triangledown$

From Lemma 4.1, we conclude that as long as all the local tracking errors $e_i(t)$ for $i = 1, \ldots, N$ go to zero as $t \to \infty$, the global tracking will be achieved.

Substituting (4.9) into the derivative of the local tracking error (4.10), we obtain,

$$\begin{aligned}
\dot{e}_i(t) &= \dot{x}_i(t) - \frac{1}{n_i}\sum_{v_j\in\mathcal{N}_i}\dot{x}_j(t) \\
&= \frac{1}{n_i}\sum_{v_j\in\mathcal{N}_i}\Big(A_e(x_i(t) - x_j(t)) + A_j x_j(t) + B_j u_j(t)\Big) - \frac{1}{n_i}\sum_{v_j\in\mathcal{N}_i}\Big(A_j x_j(t) + B_j u_j(t)\Big) \\
&= \frac{1}{n_i}\sum_{v_j\in\mathcal{N}_i}\Big(A_e(x_i(t) - x_j(t))\Big) \\
&= A_e\Big(x_i(t) - \frac{1}{n_i}\sum_{v_j\in\mathcal{N}_i}x_j(t)\Big) = A_e e_i(t)
\end{aligned} \tag{4.11}$$

Recall A_e is a stable matrix, so (4.11) indicates that $\lim_{t\to\infty} e_i(t) = 0$ exponentially.

4.3.2 Adaptive Control Scheme

In our problem, the leader parameter matrices A_0, B_0 and the followers parameter matrices A_i and B_i $(i = 1, \ldots, N)$ are unknown, so the ideal control protocol (4.8) will no longer work. An adaptive version protocol of (4.8) is proposed as follows,

$$u_i(t) = \frac{1}{n_i} \sum_{v_j \in \mathcal{N}_i} K_{1i}^T(t)(x_i(t) - x_j(t)) + K_{2ij}(t)u_j(t) + K_{3ij}^T(t)x_j(t) \tag{4.12}$$

with the following adaptive laws

$$\dot{K}_{1ij}^T(t) = -\frac{1}{n_i} S_i^T B_e^T P e_i(t)(x_i(t) - x_j(t))^T \tag{4.13}$$

$$\dot{K}_{2ij}(t) = -\frac{1}{n_i} S_i^T B_e^T P e_i(t) u_j^T(t) \tag{4.14}$$

$$\dot{K}_{3ij}^T(t) = -\frac{1}{n_i} S_i^T B_e^T P e_i(t) x_j^T(t), (v_j \in \mathcal{N}_i, i = 1, 2, \ldots, N) \tag{4.15}$$

where $K_{1ij}(t), K_{2ij}(t)$ and $K_{3ij}(t)$ are the estimates of K_{1i}^*, K_{2ij}^* and K_{3ij}^* respectively, and S_i satisfies Assumption (A7) for $i = 1, \ldots, N)$ and $P = P^T > 0$ satisfying $A_e^T P + P A_e = -Q < 0$, $Q = Q^T > 0$. $K_{1ij}(0)$, $K_{2ij}(0)$ and $K_{3ij}(0)$ can be chosen arbitrarily.

Remark 4.2 For each follower agent v_i, the ideal controller parameter K_{1i}^* is unique. However, according to (4.12), for each different $v_j \in \mathcal{N}_i$, the estimate K_{1i}^*s are different. In order to distinguish these different estimate values, we denote the estimate K_{1i}^* as $K_{1ij}(t)$. □

We now present the main result in solving the leader-following consensus tracking problem for multi-agent systems as follows.

Theorem 4.1. *The distributed adaptive controller* (4.12)*, with the adaptive laws*

(4.13)-(4.15), *applied to multi-agent systems* (4.1) *guarantees all closed-loop signals are bounded and global tracking is achieved:* $\lim_{t\to\infty}(x_i(t) - x_0(t)) = 0$.

4.4 Stability Analysis

In oder to analyze the stability of the closed-loop system, we first denote

$$\tilde{K}_{1ij}^T(t) = K_{1ij}^T(t) - K_{1i}^{*T},\ \tilde{K}_{2ij}(t) = K_{2ij}(t) - K_{2ij}^*,\ \tilde{K}_{3ij}^T(t) = K_{3ij}^T(t) - K_{3ij}^{*T}, \quad (4.16)$$

Substituting the distributed adaptive protocol (4.12) into the multi-agent subsystem (4.1) leads to

$$\begin{aligned}
\dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) \\
&= A_i x_i(t) + \frac{1}{n_i} B_i \sum_{v_j \in \mathcal{N}_i} \left(K_{1ij}^T(t)(x_i(t) - x_j(t)) + K_{2ij}(t) u_j(t) + K_{3ij}^T(t) x_j(t) \right) \\
&= \frac{1}{n_i} \sum_{v_j \in \mathcal{N}_i} \left(A_e(x_i(t) - x_j(t)) + A_j x_j(t) + B_j u_j(t) \right) \qquad (4.17) \\
&\quad + \frac{1}{n_i} \sum_{v_j \in \mathcal{N}_i} B_e \left(K_{4i}^{*-1} \tilde{K}_{1ij}^T(t)(x_i(t) - x_j(t)) + K_{4i}^{*-1} \tilde{K}_{2ij}(t) u_j(t) + K_{4i}^{*-1} \tilde{K}_{3ij}^T(t) x_j(t) \right).
\end{aligned}$$

Then, tracking error equations evolve to

$$\begin{aligned}
\dot{e}_i(t) &= \dot{x}_i(t) - \frac{1}{n_i} \sum_{v_j \in \mathcal{N}_i} \dot{x}_j(t) \\
&= A_e(x_i(t) - \frac{1}{n_i} \sum_{v_j \in \mathcal{N}_i} x_j(t)) \\
&\quad + \frac{1}{n_i} \sum_{v_j \in \mathcal{N}_i} B_e K_{4i}^{*-1} \left(\tilde{K}_{1i}^T(t)(x_i(t) - x_j(t)) + \tilde{K}_{2ij}(t) u_j(t) + \tilde{K}_{3ij}^T(t) x_j(t) \right) \\
&= A_e e_i(t) \qquad (4.18) \\
&\quad + \frac{1}{n_i} \sum_{v_j \in \mathcal{N}_i} B_e K_{4i}^{*-1} \left(\tilde{K}_{1i}^T(t)(x_i(t) - x_j(t)) + \tilde{K}_{2ij}(t) u_j(t) + \tilde{K}_{3ij}^T(t) x_j(t) \right)
\end{aligned}$$

Choose a positive definite function as a Lyapunov candidate to measure the closed-loop system errors,

$$V = \sum_{i=1}^{N} V_i \tag{4.19}$$

with $V_i = V_{i1} + V_{i2}$ where

$$V_{i1} = e_i^T P e_i \tag{4.20}$$

and

$$V_{2i} = \sum_{v_j \in \mathcal{N}_i} \mathrm{tr}[\tilde{K}_{1i} M_s^{-1} \tilde{K}_{1i}^T] + \sum_{v_j \in \mathcal{N}_i} \mathrm{tr}[\tilde{K}_{2ij}^T M_s^{-1} \tilde{K}_{2ij}] + \sum_{v_j \in \mathcal{N}_i} \mathrm{tr}[\tilde{K}_{3ij} M_s^{-1} \tilde{K}_{3ij}^T] \tag{4.21}$$

where $P = P^T > 0$, such that $A_e^T P + PA_e + = -Q < 0$, $Q = Q^T > 0$, and M_s is a positive symmetric matrix which satisfies $M_s = K_{4i}^* S_i$, $S_i \in R^{p \times p}$ (see Assumption (A4.5)).

Substituting (4.18) into the derivative of (4.20), we have

$$\begin{aligned}
\dot{V}_1 &= 2e_i^T(t) P \dot{e}_i(t) \\
&= e_i^T(t)(PA_e + A_e^T P) e_i(t) \\
&\quad + \frac{2}{n_i} e_i^T(t) P B_e K_{4i}^{*-1} \sum_{v_j \in \mathcal{N}_i} \tilde{K}_{1i}^T(t)(x_i(t) - x_j(t)) \\
&\quad + \frac{2}{n_i} e_i^T(t) P B_e K_{4i}^{*-1} \sum_{v_j \in \mathcal{N}_i} \tilde{K}_{2ij}(t) u_j(t) \\
&\quad + \frac{2}{n_i} e_i^T(t) P B_e K_{4i}^{*-1} \sum_{v_j \in \mathcal{N}_i} \tilde{K}_{3ij}^T(t) x_j(t)
\end{aligned} \tag{4.22}$$

Also, the derivate of V_{2i} becomes to

$$\dot{V}_{2i} = 2\sum_{v_j \in \mathcal{N}_i} \mathrm{tr}[\tilde{K}_{1i} M_s^{-1} \dot{\tilde{K}}_{1i}^T] + 2\sum_{v_j \in \mathcal{N}_i} \mathrm{tr}[\tilde{K}_{2ij}^T M_s^{-1} \dot{\tilde{K}}_{2ij}] + 2\sum_{v_j \in \mathcal{N}_i} \mathrm{tr}[\tilde{K}_{3ij} M_s^{-1} \dot{\tilde{K}}_{3ij}^T] \tag{4.23}$$

Substituting (4.13) into $\sum_{v_j\in\mathcal{N}_i} \mathrm{tr}[\tilde{K}_{1i}M_s^{-1}\dot{\tilde{K}}_{1i}^T]$, it is obtained

$$\begin{aligned}
&\sum_{v_j\in\mathcal{N}_i} \mathrm{tr}[\tilde{K}_{1i}M_s^{-1}\dot{\tilde{K}}_{1i}^T] \\
&= -\frac{1}{n_i}\sum_{v_j\in\mathcal{N}_i} \mathrm{tr}[\tilde{K}_{1i}M_s^{-1}S_i^T B_e^T P e_i(t)(x_i(t)-x_j(t))] \\
&= -\frac{1}{n_i}\sum_{v_j\in\mathcal{N}_i} \mathrm{tr}[\tilde{K}_{1i}K_4^{*-1}B_e^T P e_i(t)(x_i(t)-x_j(t))] \\
&= -\frac{1}{n_i}\sum_{v_j\in\mathcal{N}_i} \mathrm{tr}[e_i^T(t)PB_eK_4^{*-1}\tilde{K}_{1i}^T(x_i(t)-x_j(t))] \\
&= -\frac{1}{n_i}e_i^T(t)PB_eK_4^{*-1}\sum_{v_j\in\mathcal{N}_i}\tilde{K}_{1i}^T(x_i(t)-x_j(t))
\end{aligned} \tag{4.24}$$

Similarly, it is obtained

$$\sum_{v_j\in\mathcal{N}_i} \mathrm{tr}[\tilde{K}_{2ij}^T M_s^{-1}\dot{\tilde{K}}_{2ij}] = -\frac{1}{n_i}e_i^T(t)PB_eK_{4i}^{*-1}\sum_{v_j\in\mathcal{N}_i}\tilde{K}_{2ij}(t)u_j(t) \tag{4.25}$$

$$\sum_{v_j\in\mathcal{N}_i} \mathrm{tr}[\tilde{K}_{3ij}M_s^{-1}\dot{\tilde{K}}_{3ij}^T] = -\frac{1}{n_i}e_i^T(t)PB_eK_{4i}^{*-1}\sum_{v_j\in\mathcal{N}_i}\tilde{K}_{3ij}^T(t)x_j(t) \tag{4.26}$$

Substituting (4.24)-(3.33) into (4.23) leads to

$$\begin{aligned}
\dot{V}_{2i} = &-\frac{2}{n_i}e_i^T(t)PB_eK_{4i}^{*-1}\sum_{v_j\in\mathcal{N}_i}\tilde{K}_{1i}^T(t)(x_i(t)-x_j(t)) \\
&-\frac{2}{n_i}e_i^T(t)PB_eK_{4i}^{*-1}\sum_{v_j\in\mathcal{N}_i}\tilde{K}_{2ij}(t)u_j(t) \\
&-\frac{2}{n_i}e_i^T(t)PB_eK_{4i}^{*-1}\sum_{v_j\in\mathcal{N}_i}\tilde{K}_{3ij}^T(t)x_j(t)
\end{aligned} \tag{4.27}$$

From (4.22) and (4.27), we have

$$\dot{V}_i = \dot{V}_{1i} + \dot{V}_{2i} = e_i^T(t)(PA_e + A_e^T P)e_i(t) = -e_i^T(t)Qe_i(t) \leq 0 \tag{4.28}$$

Finally, the derivative of the positive definite function V is

$$\dot{V} = \sum_{i=1}^{N} \dot{V}_i = \sum_{i=1}^{N} e_i^T(t) Q e_i(t) \leq -q_m \sum_{i=1}^{N} \|e_i(t)\|_2^2 \leq 0 \tag{4.29}$$

where $q_m > 0$ is the minimum eigenvalue of Q.

From (4.29), the desired properties of the proposed adaptive laws are obvious:

- $V > 0$ and $\dot{V} \leq 0$ implies that the equilibrium state $e_{ic} = 0$ of the closed-loop system consisting of (4.13) and (4.15) for $i = 1, \ldots, N$ is uniformly stable and its solution $e_{ic}(t)$ is uniformly bounded, which gives the boundedness of all $x_i(t)$, $K_{1ij}(t)$, K_{2ij} and K_{3ij} in the multi-agent system, and in turn the boundedness of $\dot{e}_i(t)$ for $i = 1, \ldots, N$ because of (4.18);
- (4.29) implies $e_i(t) \in L^2$ for $i = 1, \ldots, N$;
- with $e_i(t) \in L^2 \bigcap L^\infty$ and $\dot{e}_i(t) \in L^\infty$, applying Barbalat lemma, we conclude that $\lim_{t\to\infty} e_i(t) = 0$ for $i = 1, \ldots, N$. Then according to Lemma 1, $\lim_{t\to\infty}(x_i(t) - x_0(t)) = 0$ for $i = 1, \ldots, N$ which means the trajectories of all N follower agents have the desired properties;

4.5 Disturbance Rejection

A distributed adaptive disturbance compensator develops from the basic case in Chapter 3 is presented in this section. It is proofed by a stability analysis that the compensator has the capability to make the followers track the leader on the state and reject the effect of the disturbances as well.

Based on the experience, we have already known that for disturbance rejection, we develop a new term in the adaptive controller comparing with the disturbance-free case and chose an appropriate expression for that new term according to which part does the disturbance act on.

However, in this section, there are $N+1$ followers, for each follower i it could be followed by other agents as well. In order to develop the problem more conveniently, we re-state the followers structure.

Now consider the dynamic system of all the followers becomes as

$$x_i(t) = A_i x_i(t) + B_i u_i(t) + B_i d_i(t) \tag{4.30}$$

where

$$d_i(t) = d_{i0} + \sum_{\beta=1}^{q_i} d_{i\beta} f_{i\beta}(t) \tag{4.31}$$

with d_{i0}, $d_{i\beta} \in R^{p_i}$ being some unknown constants and $f_{i\beta(t)}$ being some known bounded continuous functions, $\beta = 1, 2, \ldots, q_i$ and some $q_i > 0$. The given leader is (4.2).

To cancel the disturbance, the nominal control protocol (4.8) is modified as,

$$u_i^*(t) = \frac{1}{n_i} \sum_{v_j \in \mathcal{N}_i} \left(K_{1i}^{*T}(t)(x_i(t) - x_j(t)) + K_{2ij}^*(t) u_j(t) + K_{3ij}^{*T}(t) x_j(t) + k_{5ij}^*(t) \right), \tag{4.32}$$

where the compensation signal $k_{5ij}^*(t)$ is

$$\begin{aligned} k_{5ij}^* &= K_{2ij}^* d_j(t) - d_i(t) \\ &= (K_{2ij}^* d_{j0} - d_{i0}) + \sum_{\beta=1}^{q_j} K_{2ij}^* d_{j\beta} f_{j\beta}(t) - \sum_{\beta=1}^{q_i} d_{i\beta} f_{i\beta}(t) \\ &= k_{50ij}^* + \sum_{\beta=1}^{q_j} k_{5j\beta}^* f_{j\beta}(t) + \sum_{\beta=1}^{q_i} k_{5i\beta}^* f_{i\beta}(t) \end{aligned} \tag{4.33}$$

for agent $v_j \in \mathcal{N}_i$ where

$$\begin{aligned}
k^*_{50ij} &= K^*_{2ij} d_{j0} - d_{i0} \\
k^*_{5j\beta} &= K^*_{2ij} d_{j\beta} \\
k^*_{5i\beta} &= -d_{i\beta} \qquad (4.34)
\end{aligned}$$

The ideal disturbance compensator (4.32) leads the subsystem (4.1) to a closed-loop result

$$\begin{aligned}
\dot{x}_i(t) = & \frac{1}{n_i} \sum_{v_j \in \mathcal{N}_i} ((A_i + B_i K^{*T}_{1i})(x_i(t) - x_j(t)) + (A_i + B_i K^{*T}_{3ij}) x_m(t) \\
& + BK^*_{2ij} u_j(t) + B_i K^*_{2ij} d_{j0} + B_i \sum_{\beta=1}^{q_j} K^*_2 d_{j\beta} f_{j\beta}(t) - B_i \sum_{\beta=1}^{q_i} d_{i\beta} f_{i\beta}(t) \\
& + B_i d_i(t) - B_i d_{i0}) \\
= & \frac{1}{n_i} \sum_{v_j \in \mathcal{N}_i} (A_e(x_i(t) - x_j(t)) + A_j x_j(t) + B_j u_j(t) + B_j d_j(t)) \qquad (4.35)
\end{aligned}$$

In result, we obtain $\dot{e}_i = A_e e_i(t)$. Since A_e is stable, it is easily to verify that $\lim_{t\to\infty} e_i(t) = 0$ exponentially.

Since the parametric uncertainties, (4.32) does not work for this leader-following consensus problem, update (4.32) to an adaptive version,

$$u_i(t) = \frac{1}{n_i}\left(K^T_{1i}(t)(x_i(t) - x_j(t)) + K_{2ij}(t) u_j(t) + K^T_{3ij}(t) x_j(t) + k_{5ij}(t) \right) \qquad (4.36)$$

where

$$k_{5ij}(t) = k_{50ij}(t) + \sum_{\beta=1}^{q_j} k_{5j\beta}(t) f_{j\beta}(t) + \sum_{\beta=1}^{q_i} k_{5i\beta}(t) f_{i\beta}(t) \qquad (4.37)$$

with $k_{50ij}(t)$ being the estimate of k^*_{50ij} and $k_{5j\beta}(t)$ being the estimate of $k^*_{5j\beta}$. $k_{5i\beta}(t)$ is the estimate of the $k^*_{5i\beta}$. $k_{5ij}(t)$ is the estimate of $k^*_{5ij}(t)$.

The adaptive law for $K_{1i}(t)$, $K_{2ij}(t)$ and $K_{3ij}(t)$ are still given in (4.13), (4.14) and (4.15). The estimate k_{5ij} is developed as

$$\dot{k}_{50ij}(t) = -\frac{1}{n_i} S_i^T B_e^T P e_i(t), \tag{4.38}$$

$$\dot{k}_{5j\beta}(t) = -\frac{1}{n_i} S_i^T B_e^T P e_i(t) f_{j\beta}^T(t),\ \beta = 1, \ldots, q_j, \tag{4.39}$$

$$\dot{k}_{5i\beta}(t) = -\frac{1}{n_i} S_i^T B_e^T P e_i(t) f_{i\beta}^T(t),\ \beta = 1, \ldots, q_i. \tag{4.40}$$

Theorem 4.2. *The distributed adaptive controller* (4.32)*, with the adaptive laws* (4.13)-(4.15) *and* (4.38) - (4.40)*, applied to multi-agent systems* (4.1) *guarantees that all closed-loop signals are bounded and global tracking are achieved:* $\lim_{t\to\infty}(x_i(t) - x_0(t)) = 0$.

Proof: the stability of the closed-loop system, first denote the parameter errors for $k_{5ij}(t)$,

$$\begin{aligned}
\tilde{k}_{50ij}(t) &= k_{50ij}^* - k_{50ij}(t) \\
\tilde{k}_{5j\beta}(t) &= k_{5j\beta}^* - k_{5j\beta}(t) \\
\tilde{k}_{5i\beta}(t) &= k_{5i\beta}^* - k_{5i\beta}(t)
\end{aligned} \tag{4.41}$$

Combining with (4.36), we obtain

$$\begin{aligned}
\dot{e}_i(t) = A_e e_i(t) + \frac{1}{n_i} \sum_{v_j \in \mathcal{N}_i} B_e K_{4i}^{*-1} \Big(& \tilde{K}_{1i}^T(t)(x_i(t) - x_j(t)) + \tilde{K}_{2ij}(t) u_j(t) \\
& + \tilde{K}_{3ij}^T(t) x_j(t) + \tilde{k}_{50ij}(t) + \sum_{\beta=1}^{q_j} \tilde{k}_{5j\beta}(t) f_{j\beta}(t) + \sum_{\beta=1}^{q_i} \tilde{k}_{5i\beta}(t) f_{i\beta}(t) \Big)
\end{aligned} \tag{4.42}$$

Choose a positive definite function as a measurement of closed-loop signals of the multi-agent system

$$\bar{V} = \sum_{i=1}^{N} \bar{V}_i \tag{4.43}$$

with

$$\bar{V}_i = V_i + V_{ai} \tag{4.44}$$

where

$$V_{ai} = \sum_{v_j \in \mathcal{N}_i} \mathrm{tr}[\tilde{k}_{50ij}^T M_s^{-1} \tilde{k}_{50ij}] + \sum_{v_j \in \mathcal{N}_i} \mathrm{tr}[\tilde{k}_{5j\beta}^T M_s^{-1} \tilde{k}_{5j\beta}] + \sum_{v_j \in \mathcal{N}_i} \mathrm{tr}[\tilde{k}_{5i\beta}^T M_s^{-1} \tilde{k}_{5i\beta}] \tag{4.45}$$

and V_i is in (4.19).

Being similar with (4.24), we obtain

$$\sum_{v_j \in \mathcal{N}_i} \mathrm{tr}[\tilde{k}_{50ij}^T M_s^{-1} \dot{\tilde{k}}_{50ij}] = -\frac{1}{n_i} e_i^T(t) P B_e K_{4i}^{*-1} \sum_{v_j \in \mathcal{N}_i} \tilde{k}_{50ij}(t) \tag{4.46}$$

$$\sum_{v_j \in \mathcal{N}_i} \mathrm{tr}[\tilde{k}_{5j\beta} M_s^{-1} \dot{\tilde{k}}_{5j\beta}^T] = -\frac{1}{n_i} e_i^T(t) P B_e K_{4i}^{*-1} \sum_{v_j \in \mathcal{N}_i} \tilde{K}_{5j\beta}^T(t) f_{j\beta}(t) \tag{4.47}$$

$$\sum_{v_j \in \mathcal{N}_i} \mathrm{tr}[\tilde{k}_{5i\beta} M_s^{-1} \dot{\tilde{k}}_{5i\beta}^T] = -\frac{1}{n_i} e_i^T(t) P B_e K_{4i}^{*-1} \sum_{v_j \in \mathcal{N}_i} \tilde{k}_{5i\beta}^T(t) f_{i\beta}(t) \tag{4.48}$$

which helps use to derive the derivative of V as

$$\dot{V} = -\sum_{i=1}^{N} e_i^T(t) Q e_i(t), \; Q = Q^T > 0 \tag{4.49}$$

(the details are similar to the proof of Theorem 1 and are omitted here). From this expression of $\dot{V}$, we can similarly obtain the results of Theorem 2. $\triangledown$

4.6 Simulation Study

Case I: One leader and three followers without disturbance We provide a simulation example to illustrate the leader-following consensus performance of multi-agent systems under the proposed adaptive protocol. In particular, a network consisting of three follower agents and one leader is considered. In systems (4.1) and (4.2),

$$A_1 = \begin{bmatrix} -2 & 3 \\ 1 & -2 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}, B_2 = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -1 & 0 \\ -1 & -2 \end{bmatrix}, B_3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}, A_0 = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix}, B_0 = \begin{bmatrix} 1 & -2 \\ 4 & 1 \end{bmatrix},$$

$$A_e = \begin{bmatrix} 1 & 1 \\ -9 & -4 \end{bmatrix}, B_e = \begin{bmatrix} 1 & -2 \\ 4 & 1 \end{bmatrix}, \tag{4.50}$$

$u_0 = [\sin(t), \cos(t)]^T$. It can be easily verified that Assumption 4.1 is satisfied. Th interaction graph of the three agents and the leader is shown in 4.9, which satisfies the topology precondition Assumption 4.1 and 4.2.

The initial values of the updating matrices and vectors in adaptive laws (4.12)-(4.15) with $K_{1ij}(0), K_{2ij}(0), K_{3ij}(0)$ are 0.8 times of their ideal values, respectively. The performance of the leader-following consensus is presented in Figure 4.10. Tracking errors $x_i(t) - x_0(t), i = 1, 2, 3$ are shown in Figure 4.11. Obviously, the three agents can track the leader with reference signal $u_0(t)$.

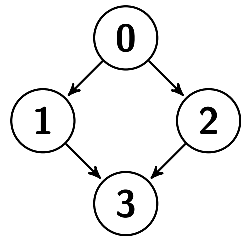

Figure 4.9: Interaction graph of three follower agents and one leader

The corresponding matrix $\mathcal{Q}$ of Fig 4.1 is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix}$.

Case II: One leader and five followers without disturbance Consider a multi-agent systems consisting of five followers and one leader. This case is verified that a follower can achieve the consensus performance no matter the path length from its neighbors to the leader are the same or not.

The interaction relationships between the followers and the leader is shown in Figure 4.4. Leader-following consensus performance is shown in Figure 4.5. Tracking errors are shown in Figure 4.6. In this case, agents 0-3 have the same dynamic models with agents 0-3 in Case I. Also A_e and B_e are the same. And

$$A_4 = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}, B_4 = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}, A_5 = \begin{bmatrix} -1 & -4 \\ 3 & -2 \end{bmatrix}, B_5 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}, \tag{4.51}$$

.

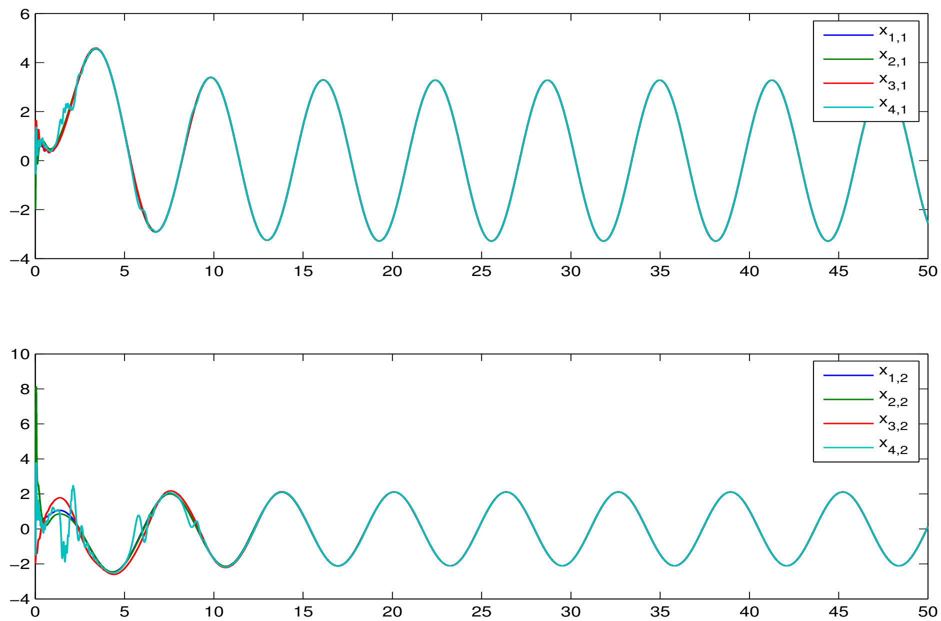

Figure 4.10: State trajectories of three follower states x_i and leader state x_0 vs. time(sec)

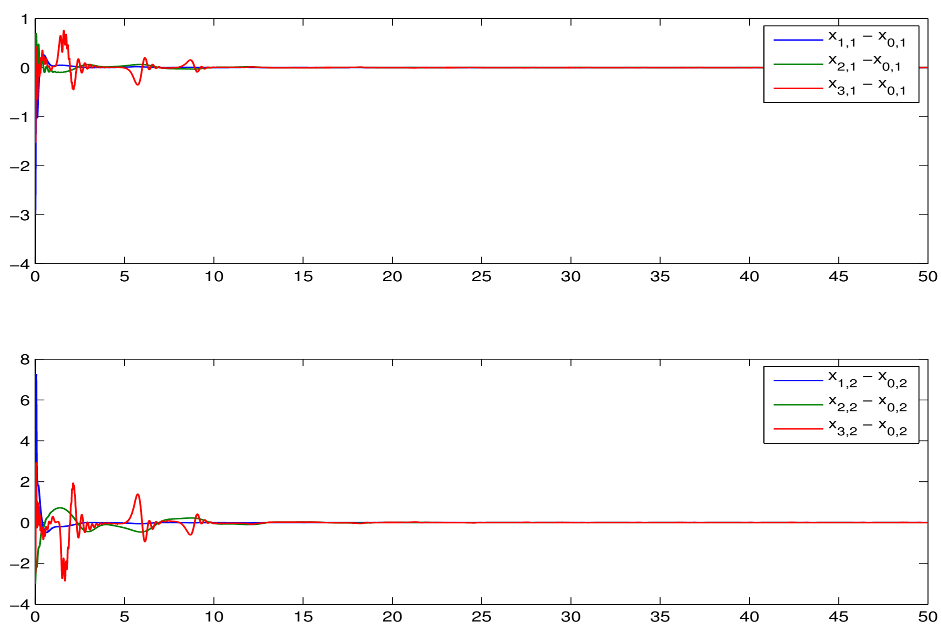

Figure 4.11: Tracking errors between three follower states x_i and leader state x_0 vs. time(sec)

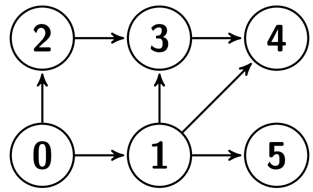

Figure 4.12: Interaction graph of five follower agents and one leader

$$\text{The corresponding } \mathcal{Q} \text{ of Fig 4.4} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ -1 & 0 & -1 & 2 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Case III: One leader and three followers with disturbance Multi-agent system in this case is a system which has disturbances acting on multi-agent system in Case I in Section 4.6. Parameters matrices of each agents including all the followers and the leader are shown in (4.50). Disturbances acting on agent 1-agent 3 are

$$\begin{aligned} d1 &= [-5,\ -5]^T + [1,\ 2]^T \sin(t) \\ d2 &= [-4,\ -4]^T + [2,\ 1]^T \cos(2t) \\ d3 &= [2,\ 6]^T + [6,\ 6]^T \sin(3t) \end{aligned} \tag{4.52}$$

respectively. $u_0 = [\sin(t), \cos(t)]^T$. $K_{1ij}(0), K_{2ij}(0), K_{3ij}(0)$ and $k_{5ij}(0)$ are 0.85 times their ideal value.

Use adaptive disturbance compensator (4.4) we can get the desired state trajectories. Leader-following consensus performance is shown in Figure 4.7. Tracking errors are shown in Figure 4.8.

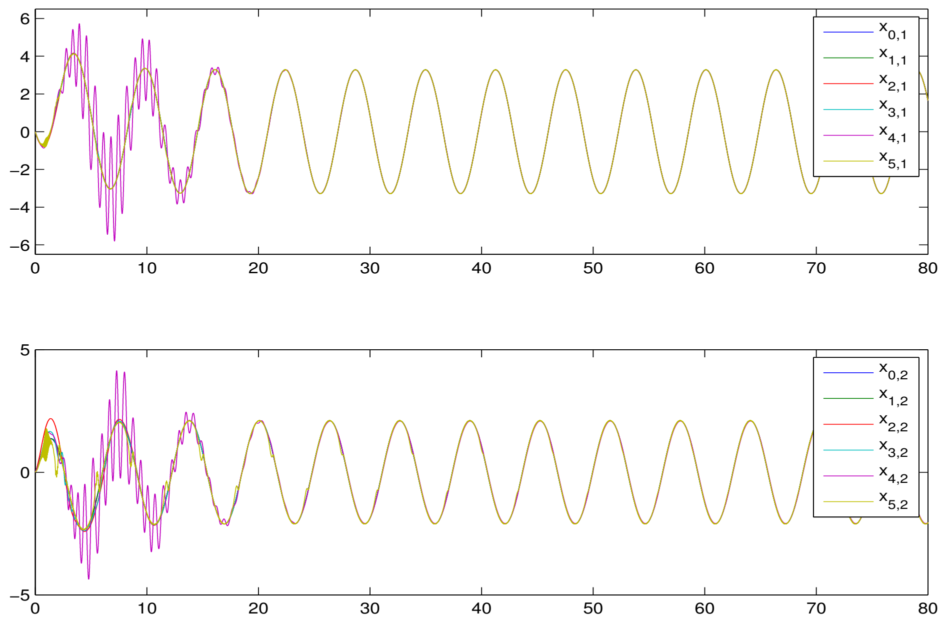

Figure 4.13: State trajectories of five follower states x_i and leader state x_0 vs. time(sec)

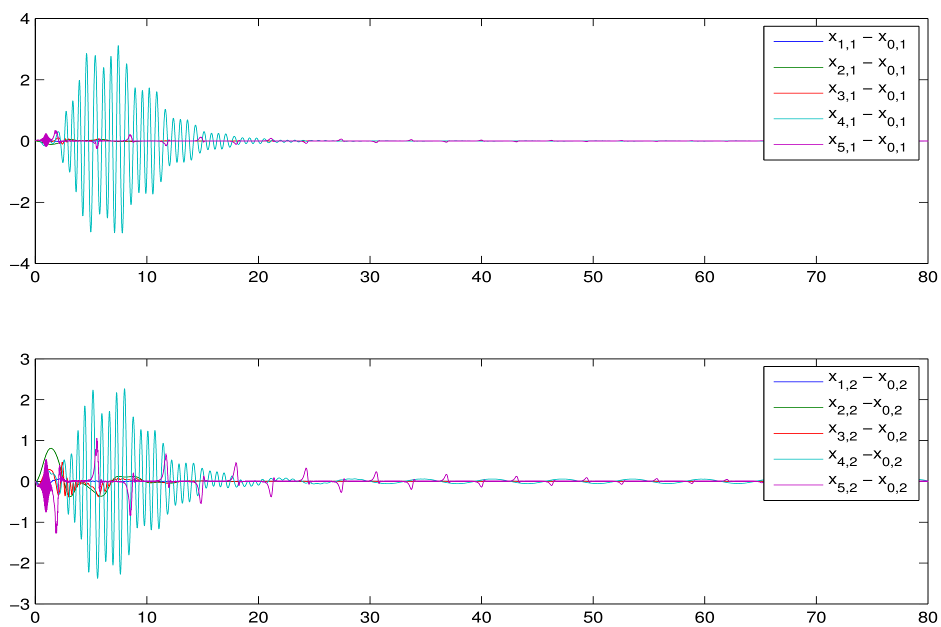

Figure 4.14: Tracking errors between five follower states x_i and leader state x_0 vs. time(sec)

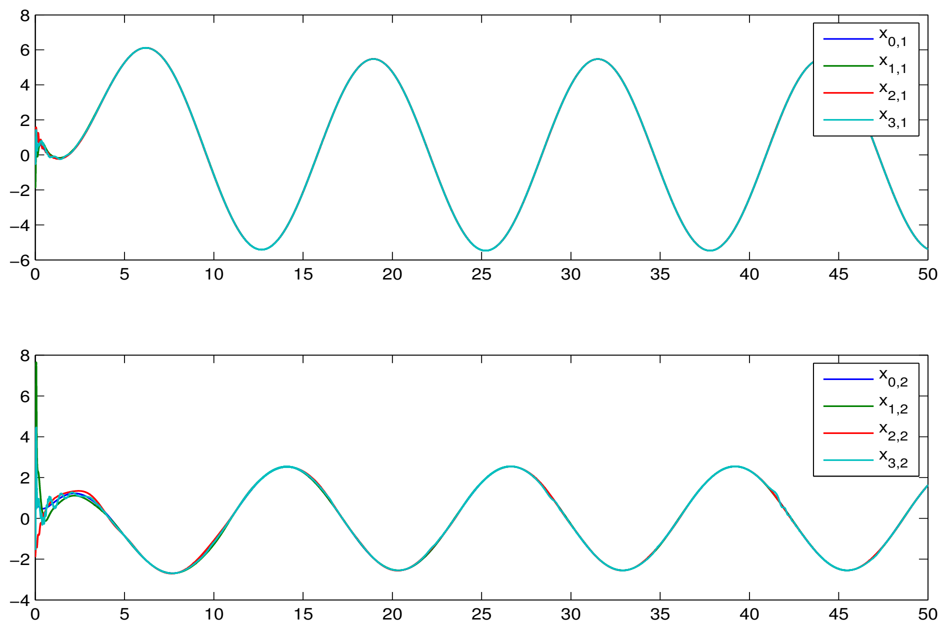

Figure 4.15: State trajectories of three follower states x_i and leader state x_0 with disturbance rejection vs. time(sec)

.

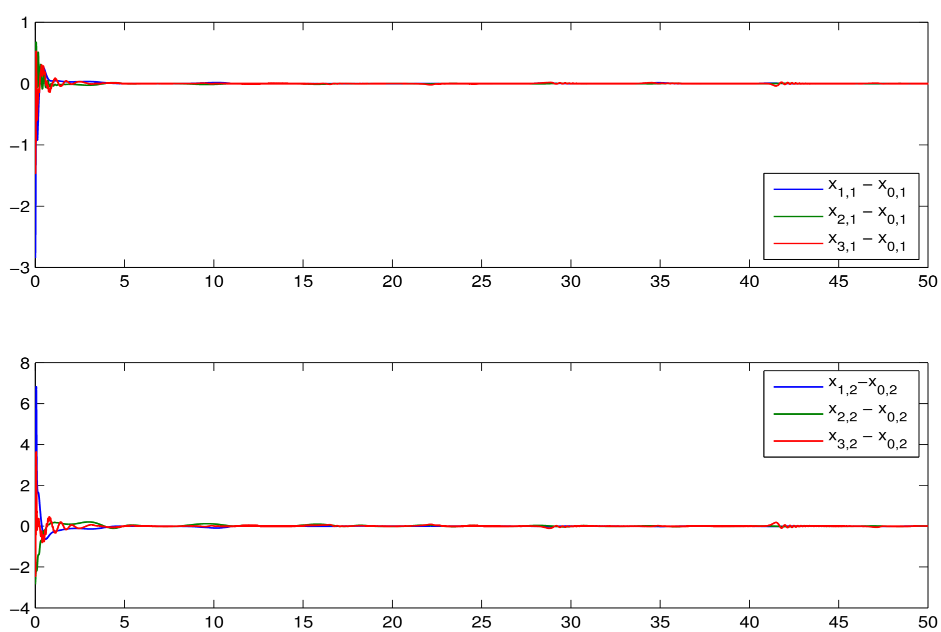

Figure 4.16: Tracking error between follower states x_i and leader state x_0 vs. time(sec)

Chapter 5

Conclusions and Future Work

5.1 Conclusions

In this thesis, in order to solve the distributed leader-following problems for the multi-agent systems, we first solved the one leader-one follower problem with and without the disturbances in Chapter 3. Stability analyses have shown that the proposed controllers in Chapter 3 can achieve the desired properties and the simulation studies also verify the capability of the proposed adaptive control scheme, i.e., all agents can track the prescribed leader eventually. Based on the basic idea used in Chapter 3, we have developed two solutions to the distributed adaptive control and disturbance compensation problems for multi-input multi-agent systems in Chapter 4. Our study has shown that the desired closed-loop system stability and tracking properties can be achieved by control adaptation based on a complete parameterization of the leader and the followers system parameters as well as the disturbance parameters. Simulation results for multi-agent systems have also confirmed the capability of the proposed adaptive control.

5.2 Future Research Topics

Adaptive control for distributed multi-agent coordination with leader and followers parametric uncertainties, as discussed and analyzed in this thesis, is a new research focus. However, there are still many challenges need to be overcame. In this section, we will introduce several potential extensions to our research.

The control framework discussed in Chapter 3 assumes that all of the states of each agent are available for feedback control design and exchange in a communication network. However, in real application, not all of the states can be accessible for measurement and exchange through a communication network. It is both practically and theoretically important to relax the requirement for full state measurement or exchange for multi-agent systems. Therefore,a state observer is needed to achieve the state tracking. Also this useful extension results can be applied to the multi-agent case in Chapter 4. It is worthwhile investigating the estimation of relative state estimation with appropriate estimation accuracy as well as the stability of closed-loop multi-agent systems with estimated information.

At the meanwhile, for some practical multi-agent systems, links between different agents have different weights. It is worth and important to figure out how to make the multi-agent consensus well based on those different weights. Compared to the weighted case, in our thesis, we assume that every edge in the communicate graph have the same weight. This challenge need to be solved for many real practical applications.

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