Essays in Household Heterogeneity and Monetary Policy

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Abstract

My dissertation delves into household heterogeneity and monetary policy. The first essay studies optimal monetary policy in a multi-sector model with heterogeneous consumption baskets and different price indices across households. Based on micro-founded welfare, the first-best outcome is not achievable even in the absence of nominal rigidities: Optimal monetary policy targets non-zero output gaps and benefits borrowing-constrained households. Heterogeneity opens up new redistributive channels for monetary policy that operate through sectoral inflation and relative prices, and leads the central bank to target inflation rates that are weighted toward the goods consumed more intensively by the constrained households and not merely the goods with less flexible prices. Income inequality across households strengthens the results. A policy neglecting heterogeneous baskets benefits the richer households more than optimal at the cost of the poorer.

The second essay revisits classic questions in monetary economics. We show that the extent of risk-sharing among workers is a determinant of the degree of monetary non-neutrality in a multi-sector sticky-price model. Workers are employed in different sectors of the economy and, as a consequence, earn different wages. The inability of workers to insure fully against their labor income risks generates strategic complementarity in price-setting decisions of firms with respect to aggregate shocks and strategic substitutability with respect to idiosyncratic shocks. Such pricing interactions lead to slow price adjustments to monetary and other aggregate shocks, thereby producing large fluctuations of the output gap, without dampening price responses to idiosyncratic shocks. This in turn allows for large responses of sectoral and aggregate outputs to idiosyncratic productivity shocks. We illustrate our results under three stylized asset market setups: complete markets, non-contingent bond-only markets, and financial autarky.

To YJ, SJ and SH...

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Right after the second year of my PhD program in the summer of 2017, I was abruptly thrown to confront a life-threatening situation. But I clearly remember the moment I left the hospital with the joy of overcoming lymphoma. More clearly the moment I decided not to surrender but to return to continue my PhD program. This decision was not made only for my passion but also for the patients struggling against blood cancer. So my accomplishment is indebted to them. I hope my dissertation serves as a ray of light of hope to those who are in dire need of it.

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Chapter 1

Optimal Monetary Policy under Heterogeneous Consumption Baskets

1.1 Introduction

Consumption baskets are heterogeneous across households of different income levels. Cravino et al. (2020) and Vieyra (2018) find that the prices of luxuries, which are consumed more intensively by higher-income households, are stickier and less volatile than those of necessities. Argente and Lee (2020) and Cavallo (2020) document that lower-income groups experienced higher inflation rates during the Great Recession and the recent pandemic, respectively. Since heterogeneous consumption baskets translate into different price indices across households, shocks that have differential effects on sectoral inflation alter relative prices to generate distributional effects through households' budget sets. Monetary policy also has redistributive effects, because it can respond to and influence sectoral inflation differently, affecting relative prices. This phenomenon calls for better understanding of how monetary policy affects different groups in the economy differently and how policy should address the distributional issues that arise from heterogeneous consumption baskets.

We extend the optimal monetary policy work of Aoki (2001), Benigno (2004), and Bilbiie (2008) to analyze consumption basket heterogeneity and its distributional implications for the policy. How do heterogeneous consumption baskets affect equilibrium dynamics? Does heterogeneity generate new inefficiencies and policy trade-offs? How do the new redistributive channels of monetary policy operate? How does optimal monetary policy change? What are the consequences if the central bank neglects heterogeneity? What are the implications of income inequality in this environment? Answers to these questions will fill a gap in the literature.

This paper contributes to the literature in three respects: study the new redistributive channels of monetary policy that are absent under homogeneous consumption baskets; derive micro-founded welfare loss functions and conduct normative analyses by comparing heterogeneous and homogeneous consumption baskets; and draw implications for designing an inflation rate a central bank targets that accounts for the distributional consequences of heterogeneity.

We show that to maximize social welfare, the central bank can and should deal with distributional issues at the cost of overall price instability. Two main conclusions emerge: (1) optimal monetary policy targets non-zero output gaps; (2) optimal policy benefits borrowing-constrained households at the expense of the unconstrained households by targeting inflation rates weighted toward the goods that are consumed more intensively by the constrained households. The existing literature, such as Aoki (2001), Benigno (2004), Mankiw and Reis (2003a), and Eusepi et al. (2011), find that a central bank should stabilize a price index that is weighted heavily toward sectors with less flexible prices. In contrast, this paper finds that optimal policy does not necessarily seek to stabilize less flexible prices, and identifies a new rationale for stabilizing inflation in sectors with more flexible prices.

We employ a two-agent—financially constrained and unconstrained—New Keynesian (TANK) framework to model the fact that 25-40% of households live hand-to-mouth based on either net worth or liquid wealth, with limited access to financial markets. They are at a kink in their budget set and insensitive to small changes in interest rates (Kaplan et al., 2014, 2018; Aguiar et al., 2020; Bilbiie, 2008; Debortoli and Galí, 2018). We extend the TANK model to two sectors, which are subject to aggregate and sector-specific productivity shocks. To be consistent with the empirical evidence that consumption baskets are heterogeneous across different income levels and that hand-to-mouth households are relatively poor, we assume that the two types of households consume different shares of goods. They have different CES preferences over the goods, consume different baskets, and face different price indices. This causes households to face different real wages, even in an economy-wide labor market with perfect labor mobility and substitutability, and thus they face idiosyncratic real wage risk. Households also face idiosyncratic non-labor income risk due to the asymmetric distribution of dividend and transfers.

In this economy, monetary policy has redistributive channels through sectoral inflation and relative prices that are absent under homogeneous consumption baskets. Although monetary policy cannot fully stabilize sectoral inflation in both sectors simultaneously under asymmetric disturbances, it can still choose which sectoral inflation to stabilize more, effectively redistributing *across sectors*. When consumption baskets are homogeneous across households, monetary policy has few distributional consequences across households through sectoral inflation, because households face the same price indices and real wages, and hence sectoral inflation and relative prices affect them symmetrically. Thus, optimal policy under homogeneous consumption baskets focuses mostly on price rigidities as demonstrated in existing work. As we introduce heterogeneous baskets, however, we find that monetary policy has significant distributional implications for the welfare of households, because stabilizing inflation in a specific sector more is more beneficial to households that consume goods more intensively from the corresponding sector, and translates into effectively redistributing *across households*. The more stable are a household's consumption-relevant inflation rates and real wages, the lower its consumption volatility (*Real Wage Stabilization Channel*), the less its consumption loss from price dispersion (*Consumption Support Channel*), and the higher its expected welfare. Consequently, optimal policy considers the redistributive effects as well as the distortions from price rigidities.

Under heterogeneous consumption baskets, imperfect risk-sharing gives monetary policy a new role to deal with the distributional inefficiencies. First, the impossibility of achieving the first-best outcome and new trade-offs lead optimal policy to *target non-zero output gaps*. Suppose asymmetric productivity across sectors under flexible prices. The more a household consumes from the higher productivity sector, the lower its price index and the higher its real wage become. Thus, the labor hours of households diverge. They would trade financial instruments to insure against idiosyncratic real wage risk in the frictionless economy, but due to the borrowing constraints, households cannot equalize the marginal disutility of labor and fail to achieve the first-best outcome even in the absence of nominal frictions. Consequently, monetary policy confronts a trade-off whereby sectoral output gaps and labor hour gaps cannot be closed simultaneously. This is the distributional inefficiency from imperfect sharing of idiosyncratic real wage risk. In order to balance the marginal utilities of consumption and marginal disutilities of labor across households, optimal policy targets non-zero output gaps, as we show in the micro-founded welfare-theoretic loss function.

Second, due to the asymmetric responsiveness of consumption across households, optimal inflation targeting policy *benefits the constrained households more and targets an inflation rate weighted toward them.* The constrained households have higher wage elasticity of consumption than the unconstrained households due to the countercyclicality of markups under demand shocks and imperfect risk-sharing. Hence the marginal utility of consumption diverges inefficiently between households. This is the distributional inefficiency from imperfect sharing of idiosyncratic non-labor income risk. Optimal policy benefits the hand-to-mouth households more in order to redistribute toward reducing differences between households' marginal utility. By stabilizing the constrained households' consumption-relevant inflation rates to a greater degree, the variations of their real wage and consumption are subdued (*Real Wage Stabilization Channel*) and consumption loss from price dispersion is also reduced (*Consumption Support Channel*). As such, the central bank effectively redistributes resources from households with lower marginal utility to those with higher marginal utility, which maximizes social welfare. In the end, heterogeneous consumption baskets lead the central bank to target inflation rates that are weighted toward the goods that are consumed more intensively by the constrained households—and not merely the goods with less flexible prices as existing work finds.

Under homogeneous baskets, however, this is not the case. First, in the absence of nominal rigidities, households face no idiosyncratic real wage risk, thus the borrowing constraints are not binding. There is no trade-off between distributional variables and optimal policy targets zero output gaps. Second, despite imperfect sharing of non-labor income risk and the asymmetric responsiveness across households, the central bank cannot redistribute marginal utility across households through sectoral inflation, because the redistributive channels of monetary policy that operate through different price indices degenerate. The inefficient variations of distributional variables are rather at the aggregate level and cannot be addressed by redistribution across sectors.

This study finds that income inequality across households significantly strengthens the main results: As we introduce larger degrees of income inequality, optimal policy assigns even more weight to the stabilization of inflation in the sector of goods that the constrained or the poorer households consume more intensively.[1] Since the hand-to-mouth or the poorer households have higher marginal utility and higher responsiveness of consumption, the utilitarian central bank cares disproportionately more about them and redistributes marginal utilities in their favor to maximize the social welfare.

Through numerical experiments, we also find that if the central bank neglects heterogeneous consumption baskets across different income levels, the policy would worsen inequality. The consequences would then be more beneficial to the richer or unconstrained households than optimal, at the cost of the poorer or constrained households.

Related literature This work contributes to various strands of the literature. First, this study relates to the literature on heterogeneous consumption baskets. Vieyra (2018), Clayton et al. (2019), and Cravino et al. (2020) find the evidence on heterogeneity in consumption baskets across households of different income and education levels and investigate its implication for dynamics in quantitative models. Specifically, Cravino et al. (2020) and Vieyra (2018) find that the prices of luxuries are stickier and less volatile than those of necessities, and Clayton et al. (2019) establish that prices are more rigid in sectors that sell to college-educated households. Argente and Lee (2020) construct income-specific price indices from 2004 to 2010 and investigate the mechanism behind the differences between them. Cavallo (2020) finds a significant difference in inflation rates across income groups after the outbreak of COVID-19. However, these studies do not address the normative questions of optimal monetary policy. We construct a model that allows comparison of heterogeneous and homogeneous consumption baskets, derive a micro-founded welfare-theoretic loss

[1]We check that the results are robust to the degrees of heterogeneity in consumption baskets, relative degrees of price stickiness, distortions from monopolistic competition, and whom to tax to finance subsidies.

function for each, and conduct normative analysis to draw implications of heterogeneity for the redistributive channels of monetary policy and optimal policy.

This study is also related to the literature that examines heterogeneous agents, particularly in a two-agent framework. Bilbiie (2008) sets up a TANK model and studies the implications of limited asset market participation for dynamics and optimal monetary policy. Debortoli and Galí (2018) also build on a TANK model and study the implications for aggregate dynamics, comparing it with dynamics from RANK and HANK models. These studies employ a single-sector framework in which households consume homogeneous baskets. Our multi-sector TANK model nests both heterogeneous and homogeneous consumption baskets, which allows us to extend the existing analyses to heterogeneous consumption baskets in a two-agent two-sector framework. Moreover, we extend our numerical analyses to cases with nonlinear production functions that allow income inequality across households.

This study is also related to the extensive literature on optimal monetary policy. Most research on optimal policy, such as Aoki (2001), Benigno (2004), Woodford (2003), and Bhattarai et al. (2015) has been conducted under a framework in which consumption baskets are homogeneous. There are some studies that consider home bias in the open economy framework. De Paoli (2009) and Faia and Monacelli (2008) study optimal monetary policy in a small open economy characterized by home bias. Auray and Eyquem (2013) examine optimal monetary policy in a monetary union with home bias. However, these works do not fit the study of an economy with hand-to-mouth households and labor mobility. To the best of our knowledge, this paper is the first to derive a micro-founded welfare-analytic loss function and to study the normative implications of heterogeneous consumption baskets for optimal monetary policy in an economy that features heterogeneous-agent with differential access to financial markets and multi-sector with perfect labor mobility.

Lastly, this paper contributes to the literature that studies which price indices central banks should target. Aoki (2001), Benigno (2004), Mankiw and Reis (2003a), and Eusepi et al. (2011) find that a central bank should stabilize a price index that is weighted heavily toward sectors with less flexible prices. This implies that a central bank should target core inflation rather than headline inflation. In contrast, we identify a new rationale for stabilizing inflation in sectors with more flexible price and for targeting headline inflation.

The rest of the paper is organized as follows. Section 2 presents the structure of the model and examines equilibrium dynamics for both heterogeneous and homogeneous consumption baskets. Section 3 considers the redistributive channels of monetary policy and the asymmetric responsiveness across households. Section 4 derives the welfare loss functions and optimal monetary policy. Section 5 discusses the consequences of neglecting heterogeneity and studies optimal inflation targeting policy. Section 6 outlines some possible extensions.

1.2 Model

We build on a two-agent framework to model that some 25-40 percent of households live hand-to-mouth (HtM) based on either net worth or low liquid wealth, facing limited access to financial markets. HtM households are at a kink in their budget set and are insensitive to small changes in interest rates; they have a high marginal propensity to consume out of transitory income changes, which can account for the high correlation between consumption and the transitory component of income growth. (Kaplan et al., 2014, 2018; Aguiar et al., 2020; Bilbiie, 2008; Debortoli and Galí, 2018). We extend a TANK model to a two-sector framework that nests heterogeneous and homogeneous consumption baskets. To be consistent with the empirical evidence that consumption baskets are heterogeneous across different income levels and that hand-to-mouth households are relatively poor, we assume that the two types of households consume different shares of goods.

1.2.1 Households

Households are either one of the two types, *Constrained* or *Unconstrained*, indexed by $h \in \{C, U\}$. They are populated by measures λ and $1-\lambda$, respectively, so the total population is normalized to 1. Type U households have access to financial markets, while type C households do not.

Both types of households get utility from consumption and disutility from labor supply,

$$\begin{aligned}\mathcal{U}(C_{h,t}, N_{h,t}) &\equiv U(C_{h,t}) - V(N_{h,t}) \\ &\equiv \frac{{C_{h,t}}^{1-\sigma}}{1-\sigma} - \frac{{N_{h,t}}^{1+\varphi}}{1+\varphi}\end{aligned}$$

but their preferences on sectoral good 1 and 2 are different, generating "heterogeneous consumption baskets."[2] Each type of household consumes heterogeneous baskets or different final goods, $C_{U,t}$ and $C_{C,t}$, according to their CES preference parameters, ω_U and ω_C,

$$C_{U,t} \equiv \left[\omega_U^{\frac{1}{\eta}} {C_{U,1,t}}^{\frac{\eta-1}{\eta}} + (1-\omega_U)^{\frac{1}{\eta}} {C_{U,2,t}}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{1.1}$$

$$C_{C,t} \equiv \left[\omega_C^{\frac{1}{\eta}} {C_{C,1,t}}^{\frac{\eta-1}{\eta}} + (1-\omega_C)^{\frac{1}{\eta}} {C_{C,2,t}}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{1.2}$$

where $C_{h,j,t} \equiv \left(\int_{\mathcal{I}_j} (\frac{1}{z_j})^{\frac{1}{\theta}} C_{h,j,t}(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}},\ j \in \{1, 2\}$ are indices of household h's consumption of sectoral good j that are CES aggregates of a continuum of differentiated goods, $C_{h,j,t}(i)$, produced in sector 1 if $i \in \mathcal{I}_1 = [0, z_1]$, and in sector 2 if $i \in \mathcal{I}_2 = (z_1, 1]$. The parameters z_1 and $z_2 (= 1 - z_1)$ measure the economic size of each sector. σ^{-1} is the elasticity of intertemporal substitution and φ^{-1} is the Frisch elasticity of labor supply, while η and θ denote the elasticity of substitution between sectoral good 1 and 2, and that across differentiated goods produced within each sector, respectively. We assume that (sectoral) good 1 is the numeraire.

Since consumption baskets are different, each type of households face "heteroge-

[2]There are various ways to generate heterogeneous consumption baskets. One of them is to assume non-homothetic preferences where consumption baskets are endogenously different across households of different income levels. Another way is to assume homothetic preference but with exogenously different weight on each good. In this paper, we adopt the latter assumption.

neous consumer price indices (CPIs)" of their own final consumption good

$$P_{U,t} = \left[\omega_U P_{1,t}^{1-\eta} + (1-\omega_U) P_{2,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{1.3}$$

$$P_{C,t} = \left[\omega_C P_{1,t}^{1-\eta} + (1-\omega_C) P_{2,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{1.4}$$

where $P_{j,t} = \left(\int_{\mathcal{I}_j} \frac{1}{z_j} P_{j,t}(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$, $j \in \{1,2\}$ are price indices of sectoral goods, $C_{h,j,t}$, determined by the supply side as Eq.(1.20) in Section 1.2.2. The (consumption-relevant) real wages for each type of households are derived as $W_{h,t} = \frac{P_{1,t} W_t}{P_{h,t}}$, and we define the relative price as $Q_t \equiv \frac{P_{2,t}}{P_{1,t}}$.

Labor market is economy-wide with perfect labor mobility across sectors and labor supplies are perfect substitutes.[3] Despite a single equilibrium nominal wage that applies identically to all the households and firms, each household faces "heterogeneous real wages" due to heterogeneous consumer price indices. Thus households face *idiosyncratic real wage risk* under heterogeneous consumption baskets. In addition, they face *idiosyncratic non-labor income risk*, because two types of households have different sources of non-labor income such as dividend, transfer and tax.

The Financially Unconstrained

Type U households, populated with mass $1-\lambda$, have access to the bond market and the stock market, thus earn dividend from the firm's profit as well as labor income. They maximize present value of expected lifetime utility Eq.(1.5) subject to the budget constraint Eq.(1.6),

[3] We do not make any assumptions on differences in labor productivity nor restrictions on labor mobility to focus on heterogeneous consumption baskets and resulting heterogeneous price indices.

$$\max_{\{C_{U,t},N_{U,t}\}_{t=0}^{\infty}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{U,t}^{1-\sigma}}{1-\sigma} - \frac{N_{U,t}^{1+\varphi}}{1+\varphi} \right] \tag{1.5}$$

$$\begin{aligned} s.t.\quad & P_{U,t}C_{U,t} + B_{U,t} + P_{1,t}V_tS_{U,t} \\ & = B_{U,t-1}(1+i_{t-1}) + P_{1,t}W_tN_{U,t} + P_{1,t}(D_t+V_t)S_{U,t-1} + P_{1,t}T_{U,t} \end{aligned} \tag{1.6}$$

where $B_{U,t}$ and $S_{U,t}$ denote holdings of one-period nominally riskless bond, and of the share in a fund that owns all the firms where the total supply of stock is normalized to 1. In each period t, bonds that mature in period $t+1$ are traded at the nominal interest rate i_t, while shares, a claim to dividend D_t, are traded at price V_t. The dividend D_t is defined as

$$D_t = \sum_{j=1,2} \int_{\mathcal{I}_j} \left(\frac{P_{j,t}(i)}{P_{1,t}} - \frac{(1-\tau)W_t}{A_tA_{j,t}} \right) Y_{j,t}(i)di$$

where τ is subsidy rate on labor cost that will be covered in Section 1.2.2. $N_{U,t}$ and W_t are labor supply of type U and the wage, and $T_{U,t}$ is the net lump-sum transfers from the government. W_t, D_t, and $T_{U,t}$ are measured in units of the numeraire (good 1). $0<\beta<1$ is the intertemporal discount factor.

The first order conditions with respect to $C_{U,t}$, $N_{U,t}$ and $B_{U,t}$ from Eq.(1.5) and Eq.(1.6) give the Euler equation and optimal condition for labor supply

$$\frac{1}{1+i_t} = E_t\left[\beta \frac{C_{U,t+1}^{-\sigma}}{C_{U,t}^{-\sigma}} \frac{P_{U,t}}{P_{U,t+1}} \right] = E_t\left[\Lambda_{t,t+1}\right] \tag{1.7}$$

$$\frac{N_{U,t}^{\varphi}}{C_{U,t}^{-\sigma}} = \frac{P_{1,t}W_t}{P_{U,t}} \tag{1.8}$$

where $\Lambda_{t,t+1} \equiv \beta\frac{C_{U,t+1}^{-\sigma}}{C_{U,t}^{-\sigma}}\frac{P_{U,t}}{P_{U,t+1}}$ is the stochastic discount factor. Given decisions on $C_{U,t}$, households optimally allocate the expenditure on $C_{U,1,t}$ and $C_{U,2,t}$ by minimizing the total expenditure $P_{U,t}C_{U,t}$ under the constraint given by Eq.(1.1)

$$C_{U,1,t} = \omega_U \left(\frac{P_{1,t}}{P_{U,t}}\right)^{-\eta} C_{U,t} \tag{1.9}$$

$$C_{U,2,t} = (1-\omega_U)\left(\frac{P_{2,t}}{P_{U,t}}\right)^{-\eta} C_{U,t} \tag{1.10}$$

Now given decisions on $C_{U,1,t}$ and $C_{U,2,t}$, households optimally allocate the expenditure on $C_{U,1,t}(i)$ and $C_{U,2,t}(i)$ by minimizing the total expenditure $P_{1,t}C_{U,1,t}$ and $P_{2,t}C_{U,2,t}$ under the constraint given by the definitions of CES aggregates $C_{U,1,t}$ and $C_{U,2,t}$

$$C_{U,1,t}(i) = \frac{1}{z_1}\left(\frac{P_{1,t}(i)}{P_{1,t}}\right)^{-\theta} C_{U,1,t} \tag{1.11}$$

$$C_{U,2,t}(i) = \frac{1}{z_2}\left(\frac{P_{2,t}(i)}{P_{2,t}}\right)^{-\theta} C_{U,2,t} \tag{1.12}$$

The Financially Constrained

Type C households, populated with mass λ, live hand-to-mouth, have no access to the bond market and the stock market, and face borrowing and savings constraints. Wage income is the only source of their income except transfers. They maximize utility Eq.(1.13) each period subject to the budget constraint Eq.(1.14),

$$\max_{\{C_{C,t},N_{C,t}\}} \left[\frac{C_{C,t}^{1-\sigma}}{1-\sigma} - \frac{N_{C,t}^{1+\varphi}}{1+\varphi}\right] \tag{1.13}$$

$$s.t. \;\; P_{C,t}C_{C,t} = P_{1,t}W_tN_{C,t} + P_{1,t}T_{C,t} \tag{1.14}$$

where $N_{C,t}$ is labor supply and $T_{C,t}$ is the net lump-sum transfer from the government measured in units of the numeraire (good 1).

The first order conditions with respect to $C_{C,t}$ and $N_{C,t}$ from Eq.(1.13) and Eq.(1.14) give the optimal conditions for labor supply

$$\frac{N_{C,t}^{\varphi}}{C_{C,t}^{-\sigma}} = \frac{P_{1,t}W_t}{P_{C,t}} \tag{1.15}$$

Given decisions on $C_{C,t}$, households optimally allocate the expenditure on $C_{C,1,t}$ and $C_{C,2,t}$ by minimizing the total expenditure $P_{C,t}C_{C,t}$ under the constraint given by Eq.(1.2)

$$C_{C,1,t} = \omega_C \left(\frac{P_{1,t}}{P_{C,t}}\right)^{-\eta} C_{C,t} \tag{1.16}$$

$$C_{C,2,t} = (1-\omega_C)\left(\frac{P_{2,t}}{P_{C,t}}\right)^{-\eta} C_{C,t} \tag{1.17}$$

Now given decisions on $C_{C,1,t}$ and $C_{C,2,t}$, households optimally allocate the expenditure on $C_{C,1,t}(i)$ and $C_{C,2,t}(i)$ by minimizing the total expenditure $P_{1,t}C_{C,1,t}$ and $P_{2,t}C_{C,2,t}$ under the constraint given by the definitions of CES aggregates $C_{C,1,t}$ and $C_{C,2,t}$

$$C_{C,1,t}(i) = \frac{1}{z_1}\left(\frac{P_{1,t}(i)}{P_{1,t}}\right)^{-\theta} C_{C,1,t} \tag{1.18}$$

$$C_{C,2,t}(i) = \frac{1}{z_2}\left(\frac{P_{2,t}(i)}{P_{2,t}}\right)^{-\theta} C_{C,2,t} \tag{1.19}$$

Two Special Cases

For the analytical study, we focus on the comparison of the following two cases for simplicity and tractability:[4]

(1) HetCB completely *hetero*geneous consumption baskets ($\omega_U = 0,\ \omega_C = 1$)

Households specialize their consumption: $\omega_U = 0$ denotes that type U households consume only good 2, thus $C_{U,t} = C_{U,2,t}$, $P_{U,t} = P_{2,t}$ and $W_{U,t} = \frac{W_t}{Q_t}$, while $\omega_C = 1$ denotes that type C households consume only good 1, $C_{C,t} = C_{C,1,t}$, $P_{C,t} = P_{1,t}$ and $W_{C,t} = W_t$. Heterogeneous consumption baskets result in heterogeneous price indices between two household types, which in turn leads to heterogeneous real wages despite one nominal wage under economy-wide labor market.

(2) HomCB completely *homo*geneous consumption baskets ($\omega_U = \omega_C = \frac{1}{2}$)

[4] We extend our study to the general cases of heterogeneous consumption baskets in Section 1.5, and find that the main results are robust.

If $\omega_U = \omega_C = \omega$ holds, both types of households consume the same baskets of goods or final good. Thus, they face identical price indices, $P_{U,t} = P_{C,t}$, and real wages.

1.2.2 Firms

Firm $i \in [0,1]$ in each sector $j \in \{1,2\}$ is a monopolistically competitive producer that produces differentiated good $Y_{j,t}(i)$ through a constant returns to scale production function

$$Y_{j,t}(i) = A_t A_{j,t} N_{j,t}(i)$$

where $Y_{j,t}(i)$ and $N_{j,t}(i)$ are output and labor employed by firm i.[5] [6] A_t and $A_{j,t}$ are economy-wide and sector-specific productivity, respectively, that follow AR(1) process in log.[7] Each firm faces its own demand function from both types of households' optimization

$$\begin{aligned} Y_{1,t}(i) &= (1-\lambda)\ \frac{\omega_U}{z_1}\left(\frac{P_{1,t}(i)}{P_{1,t}}\right)^{-\theta}\left(\frac{P_{1,t}}{P_{U,t}}\right)^{-\eta} C_{U,t} + \lambda\ \frac{\omega_C}{z_1}\left(\frac{P_{1,t}(i)}{P_{1,t}}\right)^{-\theta}\left(\frac{P_{1,t}}{P_{C,t}}\right)^{-\eta} C_{C,t} \\ Y_{2,t}(i) &= (1-\lambda)\frac{1-\omega_U}{z_2}\left(\frac{P_{2,t}(i)}{P_{2,t}}\right)^{-\theta}\left(\frac{P_{2,t}}{P_{U,t}}\right)^{-\eta} C_{U,t} + \lambda\frac{1-\omega_C}{z_2}\left(\frac{P_{2,t}(i)}{P_{2,t}}\right)^{-\theta}\left(\frac{P_{2,t}}{P_{C,t}}\right)^{-\eta} C_{C,t} \end{aligned}$$

Given the outputs and labor employments of a continuum of firms, we define the sectoral output as a CES aggregate of differentiated goods, $Y_{j,t} \equiv \left(\int_{\mathcal{I}_j} (\frac{1}{z_j})^{\frac{1}{\theta}} Y_{j,t}(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$, and sectoral labor employment as the sum of labor employment in each sector j, $N_{j,t} \equiv \int_{\mathcal{I}_j} N_{j,t}(i)$.[8]

We model nominal friction as in Calvo (1983) and Yun (1996). Firms in each sector re-adjust their prices with probability $1-\alpha_j$ each period. A firm that resets its

[5] We extend the model to introduce a decreasing returns to scale production function for numerical study in Section 1.5, and find that the main results are further strengthened as inequality gets larger.

[6] Firm i is in sector 1 if $i \in \mathcal{I}_1 = [0, z_1]$, and in sector 2 if $i \in \mathcal{I}_2 = (z_1, 1]$.

[7] $a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a,\ \ \varepsilon^a \sim N(0,1)$ where $a_t \equiv logA_t$

$a_{j,t} = \rho_{a_j} a_{j,t-1} + \sigma_{a_j} \varepsilon_t^{a_j},\ \ \varepsilon^{a_j} \sim N(0,1)$ where $a_{j,t} \equiv logA_{j,t}$

[8] In equilibrium, sectoral output equals sectoral consumption which is the weighted sum of demand from both types of households: $Y_{j,t} = (1-\lambda)C_{U,j,t} + \lambda C_{C,j,t}$. Thus we have that $Y_{j,t}(i) = \frac{1}{z_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} Y_{j,t}$.

price $P^*_{j,t}(i)$ at period t maximizes its expected sum of discounted profit

$$\max_{P^*_{j,t}(i)} \; E_t \sum_{s=0}^{\infty} \alpha_j^s \Lambda_{t,t+s} \left[P^*_{j,t}(i) - \frac{(1-\tau) P_{1,t+s} W_{t+s}}{A_{t+s} A_{j,t+s}} \right] \left(\frac{P^*_{j,t}(i)}{P_{j,t+s}} \right)^{-\theta} Y_{j,t+s}$$

where $\Lambda_{t,t+s} = \beta^s \frac{C_{t+s}^{U^{-\sigma}}}{C_t^{U^{-\sigma}}} \frac{P_t^U}{P_{t+s}^U}$ is stochastic discount factor between period t and $t+s$. Since type C households are financially constrained and type U households own all the firms in the economy, the shareholders use their own discount factor in discounting expected future profits of each firm. We eliminate the inefficiency that originates from imperfect competition at the steady state by introducing a proportional subsidy on labor cost at rate τ.[9]

The first-order condition of a price-setting firm's problem is:

$$E_t \sum_{s=0}^{\infty} \alpha_j^s \Lambda_{t,t+s} \left(\frac{P^*_{j,t}(i)}{P_{j,t+s}} \right)^{-\theta} Y_{j,t+s} \left[P^*_{j,t}(i) - \frac{\theta}{\theta-1} \frac{(1-\tau) P_{1,t+s} W_{t+s}}{A_{t+s} A_{j,t+s}} \right] = 0$$

All the price-setting firms at a certain period within each sector choose the same optimal price in equilibrium, $P^*_{j,t}(i) = P^*_{j,t}$. Considering all the firms that adjust prices and do not, the sectoral price level in sector j is determined by:

$$P_{j,t} = \left[(1-\alpha_j) {P^*_{j,t}}^{1-\theta} + \alpha_j {P_{j,t-1}}^{1-\theta} \right]^{\frac{1}{1-\theta}} \qquad (1.20)$$

Given sectoral price levels, each type of households' price index, $P_{h,t}$, is differently determined by Eq.(1.3) and Eq.(1.4) according to the corresponding consumption baskets.

[9] A positive markup, $\frac{\theta}{\theta-1}$, arising from monopolistic competition, lowers output below its efficient level. Since it is irrelevant to this study, we eliminate this inefficiency by assuming subsidy on a firm's labor employment cost at the rate τ that satisfies $1-\tau = \frac{\theta-1}{\theta}$.

1.2.3 Fiscal and Monetary Policy

The government budget constraint is given by:

$$B_{G,t-1} \;=\; \frac{B_{G,t}}{1+i_t} + P_{1,t}G_t + (1-\lambda)P_{1,t}T_{U,t} + \lambda P_{1,t}T_{C,t} + \tau P_{1,t}W_t(N_{1,t}+N_{2,t})$$

The government buys goods, G_t, transfers lump-sum (net of tax) to each type of households, $T_{U,t}$ and $T_{C,t}$, and subsidizes firms proportionally for their labor cost at subsidy rate $\tau = \frac{1}{\theta}$ to remove monopolistic distortion in steady state. The government participates in the bond market to borrow ($B_{G,t} < 0$) or lend ($B_{G,t} > 0$), or to implement open market operations.

Fiscal policy is characterized as follows:[10] there is no government expenditure ($G_t = 0$); the government does not transfer lump-sum to and from type C households ($T_{C,t} = 0$), and does not issue nor buy bonds ($B_{G,t} = 0$). The government needs to finance employment subsidy by tax or issuing bonds, and its decision on whom to tax has nontrivial effects on dynamics and income inequality as we discuss in the following section. We assume that the government tax only the unconstrained households.[11] This assumption results in a symmetric steady state with no income inequality between households; since the source of the firms' profit is monopolistic competition under linear production function, subsidy induces no profit and no non-labor income for the unconstrained households at the steady state.[12] But we find in

[10]Unlike in models with a representative agent, the aggregate and distributional consequences of monetary policy are nontrivially affected by the details of fiscal rules in models with heterogeneous agents, because Ricardian equivalence generically fails to hold. As explained in Kaplan et al. (2018), monetary policy has an indirect effect that operates through fiscal policy; for example, an exogenous shock on interest rate affects the government budget constraint, which in turn affects each households' budget constraints and their decisions through fiscal rules.

[11]Then, bond holdings and transfer (net of tax) terms in the type U households' budget constraint cancel out by the government budget constraint and bond market clearing condition, leaving subsidy term only; this is exactly the same as in models with representative agent. As a result, $B_{G,t}$ plays little role in the bond market mechanism of monetary policy implementation, shutting down the indirect channel of monetary policy through fiscal sides. Hence we can simply assume $B_{G,t} = 0$. Consequently, type U households ends up financing the subsidy, which is ultimately rebated back to them in the form of dividend.

[12]Considering that this study focuses primarily on qualitative aspects rather than on quantitative aspects, we suppose the assumptions are innocuous. Moreover, those assumptions put aside the indirect channel of monetary policy enabling us to shed light more on the implications of heterogeneous consumption baskets itself, and make welfare analysis simpler facilitating comparisons of this

Section 1.5 that our main results are robust to whom to tax to finance subsidy, and are further strengthened as we introduce income inequality by relaxing assumptions on tax rules and introducing a decreasing returns to scale production function.

Lastly, monetary policy characterized by a Taylor rule closes the model.

$$1+i_t = \frac{1}{\beta}\left(\frac{\Pi_{1,t}}{\overline{\Pi}_1}\right)^{\phi_{\pi_1}}\left(\frac{\Pi_{2,t}}{\overline{\Pi}_2}\right)^{\phi_{\pi_2}}\left(\frac{Y_{1,t}}{Y_{1,t}^E}\right)^{\phi_{y_1}}\left(\frac{Y_{2,t}}{Y_{2,t}^E}\right)^{\phi_{y_2}}\exp(\nu_t)$$

where $Y_{j,t}^E$ is the efficient level of sectoral output j and ν_t is monetary policy shock that follows AR(1) process. We assume zero inflation steady state ($\overline{\Pi}_1 = \overline{\Pi}_2 = 1$).

1.2.4 Market Clearing

All the markets clear in equilibrium: clearing conditions for the goods markets (sectoral good j and a continuum of differentiated good i), economy-wide labor market, bond market, and stock market are given by

$$\begin{aligned}
Y_{j,t} &= (1-\lambda)C_{U,j,t} + \lambda C_{C,j,t} \\
Y_{j,t}(i) &= (1-\lambda)C_{U,j,t}(i) + \lambda C_{C,j,t}(i) \\
N_{1,t} + N_{2,t} &= (1-\lambda)N_{U,t} + \lambda N_{C,t} \\
0 &= (1-\lambda)B_{U,t} + B_{G,t} \\
1 &= (1-\lambda)S_{U,t}
\end{aligned}$$

1.2.5 Equilibrium under HetCB

Now we characterize the equilibrium under completely heterogeneous consumption baskets (**HetCB**, $\omega_U = 0,\ \omega_C = 1$). We establish the efficient (first-best) allocation, and characterize the model equilibrium in terms of percentage deviation from the efficient allocation.[13] Imperfect risk-sharing in real wage leads to the impossibility of achieving efficiency and a new trade-off, generating distributional inefficiencies from

study to the findings in the literature such as Benigno (2004).

[13] We set the parameters z_1 and $z_2 (= 1 - z_1)$ as λ and $1-\lambda$ to measure the economic size of each sector.

idiosyncratic real wage risk.

Efficient Allocation

We derive the economy's efficient allocation by solving a social planner's problem that maximizes the weighted sum of utility of both types of households, subject to the resource and technology constraints

$$\max_{\{C_{h,t},N_{h,t},Y_{j,t}(i)\}} \left\{\varpi_U(1-\lambda)\left[\frac{C_{U,t}^{1-\sigma}}{1-\sigma}-\frac{N_{U,t}^{1+\varphi}}{1+\varphi}\right]+\varpi_C\lambda\left[\frac{C_{C,t}^{1-\sigma}}{1-\sigma}-\frac{N_{C,t}^{1+\varphi}}{1+\varphi}\right]\right\}$$

$$\begin{aligned} s.t. \qquad \lambda C_{C,t} &= \left(\int_{\mathcal{I}_1}\left(\frac{1}{z_1}\right)^{\frac{1}{\theta}} Y_{1,t}(i)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}} \\ (1-\lambda)C_{U,t} &= \left(\int_{\mathcal{I}_2}\left(\frac{1}{z_2}\right)^{\frac{1}{\theta}} Y_{2,t}(i)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}} \\ (1-\lambda)N_{U,t}+\lambda N_{C,t} &= \int_{\mathcal{I}_1}\frac{Y_{1,t}(i)}{A_tA_{1,t}}di+\int_{\mathcal{I}_2}\frac{Y_{2,t}(i)}{A_tA_{2,t}}di \end{aligned}$$

where $\{\varpi_h\}$ denotes Pareto weights. First order conditions with respect to $C_{h,t}$, $N_{h,t}$, and $Y_{j,t}(i)$ are given by

$$\begin{aligned} \varpi_C C_{C,t}^{-\sigma} &= \mu_1 \\ \varpi_U C_{U,t}^{-\sigma} &= \mu_2 \\ \varpi_C N_{C,t}^{\varphi} &= \mu_3 \\ \varpi_U N_{U,t}^{\varphi} &= \mu_3 \\ \mu_1 Y_{1,t}^{\frac{1}{\theta}}\ z_1^{-\frac{1}{\theta}} Y_{1,t}(i)^{-\frac{1}{\theta}} &= \mu_3\frac{1}{A_tA_{1,t}} \\ \mu_2 Y_{2,t}^{\frac{1}{\theta}}\ z_2^{-\frac{1}{\theta}} Y_{2,t}(i)^{-\frac{1}{\theta}} &= \mu_3\frac{1}{A_tA_{2,t}} \end{aligned}$$

where μ_1, μ_2 and μ_3 are Lagrange multipliers. According to the last two conditions, $Y_{j,t}(i)$ should have a common value, $Y_{j,t}(i)=\frac{Y_{j,t}}{z_j}$, implying no output dispersion within sector in the efficient allocation. By simplifying the first order conditions and the constraints, the efficient allocation is characterized by

$$
\begin{aligned}
{N_{C,t}^{E}}^{\varphi} &= {C_{C,t}^{E}}^{-\sigma} A_t A_{1,t} \\
{N_{U,t}^{E}}^{\varphi} &= {C_{U,t}^{E}}^{-\sigma} A_t A_{2,t} \\
\frac{N_{C,t}^{E}}{N_{U,t}^{E}} &= \left(\frac{\varpi_C}{\varpi_U}\right)^{-\varphi} \\
\lambda C_{C,t}^{E} &= Y_{1,t}^{E} \\
(1-\lambda) C_{U,t}^{E} &= Y_{2,t}^{E} \\
(1-\lambda) N_{U,t}^{E} + \lambda N_{C,t}^{E} &= \frac{Y_{1,t}^{E}}{A_t A_{1,t}} + \frac{Y_{2,t}^{E}}{A_t A_{2,t}}
\end{aligned}
$$

where E stands for "*Efficient.*" The intuition for the first two efficiency conditions is straightforward: marginal utility earned from the goods marginally produced should equal marginal disutility when a household supplies one more unit of labor to the sector of its consumption.

The efficient allocation is affected by relative Pareto weights, $\frac{\varpi_C}{\varpi_U}$; how much a social planner values each household determines its corresponding efficient allocation. In this study, we assume that a social planner is utilitarian ($\varpi_U = \varpi_C$), so that the market outcome without nominal and financial constraints coincides with the efficient allocation, and the steady state of the market outcome regardless of frictions coincides with that of the efficient allocation.

The dynamics of log-linearized variables expressed in terms of exogenous processes are:

$$n_t^E = n_{C,t}^E = n_{U,t}^E = \underbrace{\frac{1-\sigma}{\sigma+\varphi}}_{+/-}\left(a_t + n_1 a_{1,t} + n_2 a_{2,t}\right)$$

$$y_{1,t}^E = c_{C,t}^E = \underbrace{\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)a_t}_{+/+} \underbrace{+\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1\right)a_{1,t}}_{+/+} \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2}_{-/+} a_{2,t}$$

$$y_{2,t}^E = c_{U,t}^E = \underbrace{\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)}_{+/+}a_t \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1}_{-/+} a_{1,t} \underbrace{+\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2\right)}_{+/+} a_{2,t}$$

$$n_{1,t}^E = \underbrace{\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)}_{+/-}a_t \underbrace{+\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1\right)}_{+/-} a_{1,t} \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2}_{-/+} a_{2,t}$$

$$n_{2,t}^E = \underbrace{\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)}_{+/-}a_t \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1}_{-/+} a_{1,t} + \underbrace{\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2\right)}_{+/-} a_{2,t}$$

where the signs are when $\sigma < 1$ and $\sigma > 1$, respectively. The lower-case letters denote percentage deviations from the steady state, and the sectoral output equals the consumption of the corresponding type of households. The implied wage and relative price in the efficient allocation are derived as $w_t^E = a_t + a_{1,t}$ and $q_t^E = a_{1,t} - a_{2,t}$, so we identify heterogeneous real wages, $w_{C,t}^E (= w_t^E) = a_t + a_{1,t}$ and $w_{U,t}^E (= w_t^E - q_t^E) = a_t + a_{2,t}$.

In the efficient allocation, labor hours are equalized between households: perfect substitutability of labor hours with identical productivity and the convexity of disutility of labor lead the social planner to equalize marginal disutility of labor to minimize the social disutility cost in production of any sets of outputs. However, consumption would not equalize generically due to heterogeneity: because marginal utility gain is higher in the sector with higher productivity given one additional unit of labor hour, the social planner finds it efficient to produce more goods in that sector. Thus it is efficient that households who consume goods from the sector with higher productivity more intensively consume more.

Note that the value of σ matters for the scale and direction of each sectoral and distributional variable in their dynamics, because σ measures the relative size of the income effect compared to the substitution effect in labor supply decision and the extent to which households care about the variation of consumption.[14] Throughout the paper, we make a baseline assumption that $\sigma < 1$, because it is more intuitive that labor supply schedule on wage is upward sloping, and it is shown by some studies on labor supply that the income effect is not big enough to dominate the substitution effect. However, the main results of this paper do not change qualitatively with the assumptions on σ.

Let us check how the efficient allocation can be achieved by the frictionless market outcome under a positive shock on sector-specific productivity $a_{1,t}$.[15] Higher productivity in sector 1 affects the real wages differently: it increases the real wage and consumption of type C households who consume good 1 intensively, and if $\sigma < 1$, a higher wage leads to an increase in labor supply of type C households, which is reconciled with a large increase in demand for good 1 following the shock. However, there is no direct effect on the real wage of type U households who consume good 2 intensively. As the labor hours of both types of household diverge, *an incentive to trade financial instruments to insure against idiosyncratic real wage risk* is created: due to the convexity of disutility of labor, both types benefit from it and achieve Pareto improvement by equalizing marginal disutility of labor; the real wage risk is perfectly shared, achieving efficiency conditions. As a result, labor supply of type U households increases while that of type C households decreases. Since more labor supply translates to a higher disutility for type U households, their consumption decreases.[16] [17] We will discuss more in Section 1.2.5, that if we introduce borrowing and

[14]If $\sigma < 1$, the substitution effect dominates the income effect, and an increase in wage leads to more labor hours. In addition, the elasticity of intertemporal substitution is higher, because households care less about consumption smoothing. If $\sigma > 1$, the opposites hold true.

[15]The symmetric mechanism applies for the other sector-specific shock, $a_{2,t}$.

[16]As the elasticity of intertemporal substitution is high ($\sigma < 1$), households care less about consumption smoothing and the responses of their consumption to shocks are large. Thus, the labor employment in sector 1 increases despite a positive sector-specific productivity shock due to a larger increase in demand for good 1, while labor employment in sector 2 decreases as the demand falls by higher disutility of labor supply of type U households.

[17]If $\sigma > 1$, however, a higher wage lowers labor supply of type C households, and this is reconciled with a small increase in demand for good 1 following the shock. As the real wage risk is perfectly shared, labor hours of type C households decreases, from which they would have lower disutility leading to an increase in their consumption. $\sigma > 1$ implies that households care more about consumption smoothing, and their responses are relatively smaller. Thus, labor employment in sector

savings constraints into the frictionless economy, the market outcome cannot obtain the first-best allocation.

Approximate Allocation

We approximate the decentralized model by log-linearizing the equilibrium conditions around the deterministic efficient zero-inflation steady state. The market outcomes with no frictions coincide with the first-best allocation. However, as we introduce nominal friction and financial constraints, the market outcome would deviate from the first-best; we find that the first-best outcome is not implementable even in the absence of nominal rigidities.[18] We provide the system of equations expressed in welfare-relevant gaps: variables with *tilde* denote percentage deviations from the efficient allocation.[19] Note that output is aggregated at the sector-level and we have $\widetilde{c}_{C,t}=\widetilde{y}_{1,t}$ and $\widetilde{c}_{U,t}=\widetilde{y}_{2,t}$, because each type of households consume goods of different sectors under **HetCB**.

The first set of equations are from the household side:

$$\widetilde{y}_{2,t} - E_t[\widetilde{y}_{2,t+1}] = -\frac{1}{\sigma}\left(\widetilde{i}_t - E_t[\pi_{2,t+1}] - r_t^E\right) \tag{1.21}$$

$$\varphi\widetilde{n}_{U,t} + \sigma\widetilde{y}_{2,t} = \widetilde{w}_t - \widetilde{q}_t \tag{1.22}$$

$$\varphi\widetilde{n}_{C,t} + \sigma\widetilde{y}_{1,t} = \widetilde{w}_t \tag{1.23}$$

$$\widetilde{w}_t + \widetilde{n}_{C,t} = \widetilde{y}_{1,t} + \frac{1-\sigma}{\sigma} z_2 q_t^E \tag{1.24}$$

where the real interest rate in the efficient allocation is $r_t^E \equiv \sigma(E_t[y_{2,t+1}^E] - y_{2,t}^E)$.

Eq.(1.21) is the Euler equation of type U households: the output gap in sector 2 is a function of the sum of the current and the expected future real interest rate gaps. Since type U households consume good 2 intensively, the Euler equation is expressed in variables from sector 2. There is no Euler equation for type C households who make purely static decisions due to the financial constraints. Eq.(1.22) and Eq.(1.23)

1 rather decreases due to a higher sector-specific productivity, while labor employment in sector 2 increases as the demand rises by lower disutility of labor supply of type U households.

[18]This is discussed in Section 1.2.5

[19]We provide the full system of equations and their derivations in the Appendix Section A.2.

are the labor supply schedules of each type of households who face different real wages and idiosyncratic real wage risk: $w_{C,t}(\equiv w_t) \neq w_{U,t}(\equiv w_t - q_t)$.

Financial constraints are shown in Eq.(1.24), which is the budget constraint of the constrained households.[20] Note the adjustment term in q_t^E, the relative productivity; this term is created due to the impossibility of achieving efficiency under asymmetric disturbances, and implies the amount of bond that type C households would desire to trade to share real wage risk if efficiency were to achieve. We discuss the impossibility of achieving efficiency in Section 1.2.5, and identify a novel trade-off between output gaps and labor supply gaps in Section 1.2.5, which further leads to shifts in target output gaps in Section 1.4.2.

The second set of equations are from the firm side, the sectoral Phillips curves:

$$\pi_{1,t} = \beta E_t[\pi_{1,t+1}] + \kappa_1 \widetilde{w}_t \tag{1.25}$$

$$\pi_{2,t} = \beta E_t[\pi_{2,t+1}] + \kappa_2 \big(\widetilde{w}_t - \widetilde{q}_t\big) \tag{1.26}$$

where $\kappa_j \equiv \frac{(1-\alpha_j\beta)(1-\alpha_j)}{\alpha_j}$. In the presence of nominal friction in each sector ($\alpha_j \neq 0$), sectoral inflation is the weighted sum of the current and the expected future real marginal costs. In the absence of nominal friction, $\alpha_j = 0$, the real marginal cost is constant and the sectoral Phillips curve in the corresponding sector would degenerate, with inflation causing no inefficiency as standard.

Since the wage is applied economy-wide and measured in units of numeraire (good 1), the real marginal cost in sector 1, $w_t - a_t - a_{1,t}$, equals the real wage gap of the constrained households, and that in sector 2, $w_t - q_t - a_t - a_{2,t}$, equals the real wage gap of the unconstrained households. Thus both real marginal cost terms can be expressed in terms of output gaps using the equilibrium conditions from the demand side. Each sectoral output gap and adjustment terms have asymmetric effects on

[20] Type U households' budget constraint, $y_{2,t} = w_t - q_t + n_{U,t} + \frac{1}{z_2\theta}(d_t - t_{U,t})$ is excluded from the system of equations here to focus more on the implications of the financially constrained households, but it plays a nontrivial role in the analysis of optimal monetary policy. For later use, note that $d_t - t_{U,t} = -\theta\{z_1(w_t - a_t - a_{1,t}) + z_2(w_t - q_t - a_t - a_{2,t})\}$

sectoral inflation as we discuss in Section 1.2.5.

$$\widetilde{w}_t \;=\; \frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + z_2\frac{\varphi}{1+\varphi}\frac{1-\sigma}{\sigma}q_t^E$$
$$\widetilde{w}_t - \widetilde{q}_t = \frac{z_1}{z_2}\varphi\frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + (\sigma+\varphi)\widetilde{y}_{2,t} - z_1\frac{\varphi}{1+\varphi}\frac{1-\sigma}{\sigma}q_t^E$$

Lastly, the (economy-wide) labor market clearing condition is given by

$$z_1\widetilde{y}_{1,t} + z_2\widetilde{y}_{2,t} = z_1\widetilde{n}_{C,t} + z_2\widetilde{n}_{U,t} \tag{1.27}$$

Eq.(1.27) shows that the weighted sum of output gaps equals that of labor supply gaps.

Impossibility of achieving efficiency

Heterogeneous consumption baskets make market outcomes impossible to achieve the efficient allocation under asymmetric disturbances. For clarity, we check this in a flexible-price variant of the model in which the wage and the relative price trace the efficient levels. N stands for *natural* or flexible-price economy.[21]

Recall that idiosyncratic real wage risk is perfectly shared through bond market in the frictionless economy as seen in Section 1.2.5: a positive shock on sector-specific productivity $a_{1,t}$ affects the real wages differently: it raises the real wage of type C, $w_{C,t}^E = w_t^E = a_t + a_{1,t}$, but has no effect on that of type U, $w_{U,t}^E = w_t^E - q_t^E = a_t + a_{2,t}$. To insure against the idiosyncratic real wage risk and equalize marginal disutility of labor, type C households borrow with their consumption increasing, and type U households save with their consumption decreasing.

Now we introduce financial constraint – no risk-sharing between two types – into the frictionless economy. Then, with binding borrowing and savings constraints, a positive shock on $a_{1,t}$ only affects type C household with their consumption and labor supply increasing, while type U households are unaffected. Due to imperfect risk-sharing, households cannot equalize marginal disutility of labor. This results in failure to achieve efficient distribution of labor hours across households, and hence in

[21] We provide the full system of equations of flexible-price allocation in the Appendix Section A.2.

Table 1.1: The effects of $a_{1,t}$ shock with and without risk-sharing

x_t	$w_{C,t}$	$w_{U,t}$	$n_{C,t}$	$y_{1,t}$	$n_{U,t}$	$y_{2,t}$	Risk-sharing
x_t^N	$\uparrow$	$-$	$\Uparrow$	$\uparrow$	$-$	$-$	No
x_t^E	$\uparrow$	$-$	$\uparrow$	$\Uparrow$	$\uparrow$	$\downarrow$	Perfect
$\widetilde{x}_t^N$	$-$	$-$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	No

failure to achieve the first-best outcome even in the absence of nominal frictions.

The following proposition summarizes the above analysis.

Proposition 1.1 (Impossibility of achieving efficiency)**.** *Under heterogeneous consumption baskets and financial constraints that prevent perfect sharing of idiosyncratic real wage risk, market outcomes cannot obtain the first-best outcome unless* $\sigma = 1$*, even in the absence of nominal frictions.*[22]

Proof. Please refer to the Appendix Section A.1. □

The impossibility is attributable to both heterogeneous consumption basket and the existence of HtM households together. On the one hand, if the consumption basket is homogeneous, both types of households face the same CPI and real wage; even under asymmetric disturbances, they make the same decisions with no idiosyncratic real wage risk. Thus, with the flexible prices, financial constraints are not binding anymore in achieving efficiency, and market outcomes can support the first-best outcome. On the other hand, if there is no borrowing and savings constraint, households can trade bonds to share risk. The bond holdings terms fix the constrained households' budget constraint so market outcomes can support the efficient allocations.[23]

An immediate result of the impossibility of achieving efficiency is the adjustment term that shows up in fitting the efficient allocation into the constrained households' budget constraint, which cannot support the first-best outcome. We need to add

[22]If $\sigma = 1$, labor supply schedule degenerates to a constant term because the income effect and the substitution effect exactly cancel out. Thus labor hour is always the same, making borrowing and savings constraint not binding in the absence of nominal rigidity.

[23]If we remove borrowing and savings constraint, we can derive type C households' budget constraint as $w_t + n_{C,t} = y_{1,t} + \lambda b_{C,t} + \frac{\lambda}{\beta} b_{C,t-1}$, where $b_{C,t}$ is defined as $b_{C,t} \equiv \frac{B_{C,t}}{P_1 Y_1}$. Since $b_{C,t-1}$ is predetermined, $b_{C,t}$ would trace its corresponding efficient level to support the first-best outcome.

an adjustment term as in Eq.(1.29) to take into account *the amount of bond type C households would sell if they were under perfect risk-sharing.* By definition, Eq.(1.24) is derived by subtracting Eq.(1.29) from Eq.(1.28):

$$w_t + n_{C_t} = y_{1,t} \tag{1.28}$$

$$w_t^E + n_{C_t}^E = y_{1,t}^E - \frac{1-\sigma}{\sigma} z_2 q_t^E \tag{1.29}$$

$$\widetilde{w}_t + \widetilde{n}_{C_t} = \widetilde{y}_{1,t} + \frac{1-\sigma}{\sigma} z_2 q_t^E \tag{1.24}$$

An intuitive interpretation is that: under perfect risk-sharing, households would borrow to equate marginal disutility of labor achieving efficiency (Eq.(1.29)). Due to the financial constraints, however, they cannot borrow anymore (Eq.(1.28)), and cannot consume goods or leisure as much by the amount $\frac{1-\sigma}{\sigma} z_2 q_t^E$, failing to achieve efficiency (Eq.(1.24)). Thus under market outcomes in the absence of risk-sharing, consumption is smaller than wage income by $\frac{1-\sigma}{\sigma} z_2 q_t^E$ than under perfect risk-sharing.

A Trade-off between Output Gaps and Labor Supply gaps

In this section, we discuss the distribution of labor demand – how labor hours from each household are determined in equilibrium – and identify a novel trade-off between output gaps and labor supply gaps that is generated by the impossibility.

Assuming no transfers to them, the constrained households' decisions on labor hours and consumption are affected only by their wage, because they are hand-to-mouth depending entirely on their labor income: given wage, their consumption and labor are optimally chosen by $C_{C,t} = W_t^{\frac{1+\varphi}{\sigma+\varphi}}$, and $N_{C,t} = W_t^{\frac{1-\sigma}{\sigma+\varphi}}$. Defining $LE(X)$ as labor-equivalent of variable X to denote the amount of (market) labor to produce X under technology constraint, we have

$$LE(C_{C,t}) - LE(N_{C,t}) = \frac{\lambda C_{C,t}}{A_t A_{1,t}} - \lambda N_{C,t} = \lambda W_t^{\frac{1-\sigma}{\sigma+\varphi}} \left(\frac{W_t}{A_t A_{1,t}} - 1 \right) \begin{cases} > 0 & \text{if } W_t > A_t A_{1,t} \\ < 0 & \text{if } W_t < A_t A_{1,t}. \end{cases}$$

This implies that their labor hours are smaller (larger) than the labor-equivalent of their consumption when their real wage gap is positive (negative), with the rest of the labor demand is, in effect, filled by the unconstrained households' labor hours through the labor market clearing condition. For instance, if an expansionary monetary policy shock raises real wages through sticky prices, hand-to-mouth households consume more labor-equivalent than their labor supply, and type U households backs this up implying that the latter consume less labor-equivalent than their labor supply in equilibrium. This is reconciled with the countercyclicality of non-labor income for the unconstrained, which is the difference in the income sources between households abstracting from heterogeneous CPIs. As standard in New Keynesian models, markups and dividend are countercyclical in response to demand shocks. Due to this negative income effect of non-labor income, type U decides to work more hours.[24] In this way, labor demand is redistributed from type C to type U by the amount $\frac{\sigma+\varphi}{1+\varphi}y_{1,t}$.[25] The relations between labor gaps and output gaps are summarized by:

$$\widetilde{n}_{C,t} = \frac{1-\sigma}{1+\varphi}\widetilde{y}_{1,t} + \frac{1-\sigma}{\sigma}\frac{1}{1+\varphi}z_2 q_t^E \tag{1.30}$$

$$\widetilde{n}_{U,t} = \widetilde{y}_{2,t} + \frac{z_1}{z_2}\frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} - \frac{1-\sigma}{\sigma}\frac{1}{1+\varphi}z_1 q_t^E \tag{1.31}$$

Note the adjustment terms: since the budget constraint of HtM households which cannot support the first-best outcome is crucial in deriving them, the relations cannot support efficiency either; hence the adjustment terms should be added to the relations to reflect the lack of risk-sharing.[26]

Imperfect sharing in real wage risk and the impossibility lead to a novel *trade-*

[24]The representative agent in the basic New-Keynesian model is in the same situation, but it receives negative dividend that induces negative income effect. Thus the labor-equivalent of consumption and labor supply are equalized as $\frac{C_t}{A_t} = \frac{Y_t}{A_t} = N_t$.

[25]This has a nontrivial implication for dynamics of sectoral inflation which is discussed in Section 1.2.5.

[26]By definition, Eq.(1.30) is derived as the gap between two equations: $\begin{cases} n_{C,t} = \frac{1-\sigma}{1+\varphi}y_{1,t} \\ n_{C,t}^E = \frac{1-\sigma}{1+\varphi}y_{1,t}^E - \frac{1-\sigma}{\sigma}\frac{1}{1+\varphi}z_2 q_t^E. \end{cases}$ Recalling that $n_{C,t}^N(\Uparrow) > n_{C,t}^E(\uparrow)$, and $y_{1,t}^N(\uparrow) < y_{1,t}^E(\Uparrow)$ under a positive shock on $a_{1,t}$, we can find an adjustment term that captures type C households' borrowing under perfect risk-sharing for this labor supply–output relation to support the efficient outcome. Eq.(1.31) is analogous to this. Note that the adjustment terms are in the opposite directions to each other and of the size by the lack of risk-sharing, so that the population-weighted sum of adjustment terms in each relation is zero.

off between output gaps and labor supply gaps under asymmetric disturbances that generates *distributional inefficiency from idiosyncratic real wage risk*: we cannot close output gaps and labor supply gaps simultaneously, $\widetilde{y}_{1,t} = \widetilde{y}_{2,t} = \widetilde{n}_{C,t} = \widetilde{n}_{U,t} = 0$. Even though we can close both output gaps, labor gaps cannot be closed due to the lack of risk-sharing, and vice versa. What is more, we cannot even close both output gaps simultaneously, regardless of nominal frictions.

Proposition 1.2 (Trade-off between output gaps and labor supply gaps)**.** *In a model with heterogeneous consumption baskets and borrowing and savings constraints under asymmetric disturbances,*
1) It is impossible to close all the sectoral output gaps and labor supply gaps simultaneously.
2) It is impossible to close both sectoral output gaps simultaneously.

Proof. Please refer to the Appendix Section A.1. □

The trade-off gives monetary policy a new role to deal with the distributional inefficiency in addition to traditional objectives. We will discuss more in detail in Section 1.4.2, where we find that the trade-off leads the central bank to target non-zero output gaps.

Asymmetric redistribution of inflationary pressure across sectors

The effects of sectoral output gaps and adjustment terms on dynamics of sectoral inflation are asymmetric as shown in the Phillips curves rewritten in terms of sectoral output gaps:[27] (1) inflation in sector 1 is affected only by output gap 1, while (2) inflation in sector 2 is affected by both output gaps; (3) a relative productivity shock q_t^E has the opposite consequences in each sector. (1) and (2) imply the redistribution of inflationary pressure across sectors as the labor demand is redistributed across households, and (3) is due to the lack of risk-sharing. We discuss more in detail in

[27]In case of **HomCB**, sectoral output gap has symmetric effects on both sectoral inflations aside from asymmetric price stickiness, as shown in Section 1.2.6

the Appendix Section A.2.4.

$$\pi_{1,t} = \beta E_t[\pi_{1,t+1}] + \kappa_1 \left(\frac{\sigma+\varphi}{1+\varphi} \widetilde{y}_{1,t} + z_2 \frac{\varphi}{1+\varphi} \frac{1-\sigma}{\sigma} q_t^E \right)$$
$$\pi_{2,t} = \beta E_t[\pi_{2,t+1}] + \kappa_2 \left(\frac{z_1}{z_2} \varphi \frac{\sigma+\varphi}{1+\varphi} \widetilde{y}_{1,t} + (\sigma+\varphi) \widetilde{y}_{2,t} - z_1 \frac{\varphi}{1+\varphi} \frac{1-\sigma}{\sigma} q_t^E \right)$$

Note the inefficient distribution of inflation, which is represented by the adjustment terms in the Phillips curves: they are similar to cost-push shocks in that they add stochasticity to inflation dynamics even under zero output gaps, but different in that the former always disappears as we aggregate sectoral inflation with the economic size of each sector. Suppose a positive shock on sector-specific productivity $a_{1,t}$. Due to financial constraints, type C households work more, and type U households work less than under efficient allocation. Since marginal disutility of labor supply gap is higher (lower) for type C (type U) households, their real wage gap that equals to real marginal cost, $\widetilde{w}_t$ ($\widetilde{w}_t - \widetilde{q}_t$), and inflation in the sector of goods they consume more intensively, $\pi_{1,t}$ ($\pi_{2,t}$), are higher (lower) in equilibrium due to the lack of risk-sharing, implying that inefficient distribution of labor supply translates to inefficient distribution of inflationary pressure across sectors. As a result, inflation dynamics in both sectors are amplified if $\sigma < 1$, or subdued otherwise, considering that the shock leads to a negative output gap in sector 1 and a positive output gap in sector 2 due to nominal rigidities.[28]

1.2.6 Equilibrium under HomCB

Now we characterize the equilibrium under completely homogeneous consumption baskets (**HomCB**, $\omega_U = \omega_C = \frac{1}{2}$). The main purpose of studying the case of **HomCB** is to better understand the implications of heterogeneous consumption baskets by comparing **HetCB** and **HomCB**. We first establish the efficient allocation, and then characterize the model equilibrium in percentage deviation from the efficient allocation.[29] Unlike **HetCB**, households face the same CPI and real wages, so there is no

[28] Please refer to the Appendix Section A.2.4 for more detail.

[29] We set the parameters z_1 and $z_2 (= 1 - z_1)$ as ω and $1 - \omega$ to measure the economic size of each sector.

distributional inefficiency from idiosyncratic real wage risk with no trade-off between distributional variables.

Efficient Allocation

We derive the economy's efficient allocation by solving a social planner's problem that maximizes the weighted sum of utility of both types of households, subject to the resource and technology constraints

$$\max_{\{C_{h,t},N_{h,t},Y_{j,t}(i)\}} \left\{ \varpi_U(1-\lambda)\left[\frac{C_{U,t}^{1-\sigma}}{1-\sigma}-\frac{N_{U,t}^{1+\varphi}}{1+\varphi}\right]+\varpi_C\lambda\left[\frac{C_{C,t}^{1-\sigma}}{1-\sigma}-\frac{N_{C,t}^{1+\varphi}}{1+\varphi}\right]\right\}$$

$$\begin{aligned} s.t. \qquad (1-\lambda)C_{U,1,t}+\lambda C_{C,1,t} &= \left(\int_{\mathcal{I}_1}\left(\frac{1}{z_1}\right)^{\frac{1}{\theta}} Y_{1,t}(i)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}} \\ (1-\lambda)C_{U,2,t}+\lambda C_{C,2,t} &= \left(\int_{\mathcal{I}_2}\left(\frac{1}{z_2}\right)^{\frac{1}{\theta}} Y_{2,t}(i)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}} \\ (1-\lambda)N_{U,t}+\lambda N_{C,t} &= \int_{\mathcal{I}_1}\frac{Y_{1,t}(i)}{A_tA_{1,t}}di+\int_{\mathcal{I}_2}\frac{Y_{2,t}(i)}{A_tA_{2,t}}di \end{aligned}$$

where $\{\varpi_h\}$ denotes Pareto weights, and $C_{h,t}$ are defined as Eq.(1.1) and Eq.(1.2). As we did for the case of **hetCB** in Section 1.2.5, we assume a utilitarian social planner ($\varpi_U = \varpi_C$).

Since both types of households are identical with the same preference consuming homogeneous consumption baskets, both consumption and labor hours are equalized across all the households in the first-best allocation, as if there is a representative household:[30]

$$
\begin{aligned}
y_t^E = c_t^E \equiv c_{C,t}^E = c_{U,t}^E &= \frac{1+\varphi}{\sigma+\varphi} a_t + \frac{1+\varphi}{\sigma+\varphi} z_1 a_{1,t} + \frac{1+\varphi}{\sigma+\varphi} z_2 a_{2,t} \\
y_{1,t}^E = c_{1,t}^E \equiv c_{C,1,t}^E = c_{U,1,t}^E &= \frac{1+\varphi}{\sigma+\varphi} a_t + \left(\frac{1+\varphi}{\sigma+\varphi} z_1 + z_2\eta\right) a_{1,t} + \left(\frac{1+\varphi}{\sigma+\varphi} z_2 - z_2\eta\right) a_{2,t} \\
y_{2,t}^E = c_{2,t}^E \equiv c_{C,2,t}^E = c_{U,2,t}^E &= \frac{1+\varphi}{\sigma+\varphi} a_t + \left(\frac{1+\varphi}{\sigma+\varphi} z_1 - z_1\eta\right) a_{1,t} + \left(\frac{1+\varphi}{\sigma+\varphi} z_2 + z_1\eta\right) a_{2,t} \\
n_t^E \equiv n_{C,t}^E = n_{U,t}^E &= \frac{1-\sigma}{\sigma+\varphi} a_t + \frac{1-\sigma}{\sigma+\varphi} z_1 a_{1,t} + \frac{1-\sigma}{\sigma+\varphi} z_2 a_{2,t}
\end{aligned}
$$

Note that sectoral outputs in the first-best outcomes are different between **HomCB** and **HetCB**, depending on the relative size of the elasticity of substitution between sectors, η, and the elasticity of intertemporal substitution, $\frac{1}{\sigma}$. Suppose a positive shock on sector-specific productivity $a_{1,t}$: output in sector 1 would directly increase in both cases, but under **HomCB**, the increase is larger as households substitute goods from the higher-productivity sector for goods from the lower-productivity sector; however, the intertemporal substitution effect on good 1 would be weaker under **HomCB**, because the positive income effect of the shock is distributed to both sectors. If we assume that the elasticity of substitution between sectors dominates the elasticity of intertemporal substitution, $\eta > \frac{1}{\sigma}$, the former effect outweighs the latter, so output in sector 1 would be larger while output in sector 2 would be smaller under **HomCB** than under **HetCB**.

$$
\frac{\partial}{\partial a_{1,t}}\left[y_{1,t}^{E,\textbf{HomCB}} - y_{1,t}^{E,\textbf{HetCB}}\right] = \left(\eta - \frac{1}{\sigma}\right) z_2; \quad \frac{\partial}{\partial a_{2,t}}\left[y_{1,t}^{E,\textbf{HomCB}} - y_{1,t}^{E,\textbf{HetCB}}\right] = -\left(\eta - \frac{1}{\sigma}\right) z_2
$$
$$
\frac{\partial}{\partial a_{1,t}}\left[y_{2,t}^{E,\textbf{HomCB}} - y_{2,t}^{E,\textbf{HetCB}}\right] = -\left(\eta - \frac{1}{\sigma}\right) z_1; \quad \frac{\partial}{\partial a_{2,t}}\left[y_{2,t}^{E,\textbf{HomCB}} - y_{2,t}^{E,\textbf{HetCB}}\right] = \left(\eta - \frac{1}{\sigma}\right) z_1
$$

Approximate Allocation

We approximate the decentralized model by log-linearizing the equilibrium conditions around the deterministic efficient zero-inflation steady state. We focus on the

[30] We provide more details including the log-linearized system of equations in the Appendix Section A.3.

different features of **HomCB** from **HetCB**.[31]

The first set of equations are from the household side:

$$\widetilde{c}_{U,t} - E_t[\widetilde{c}_{U,t+1}] = -\frac{1}{\sigma}\left(\widetilde{i_t} - (\omega E_t[\pi_{1,t+1}] + (1-\omega)E_t[\pi_{2,t+1}]) - r_t^E\right) \tag{1.32}$$

$$\varphi\widetilde{n}_{U,t} + \sigma\widetilde{c}_{U,t} = \widetilde{w}_t - (1-\omega)\widetilde{q}_t \tag{1.33}$$

$$\varphi\widetilde{n}_{C,t} + \sigma\widetilde{y}_{1,t} = \widetilde{w}_t - (1-\omega)\widetilde{q}_t \tag{1.34}$$

$$\widetilde{w}_t - (1-\omega)\widetilde{q}_t + \widetilde{n}_{C,t} = \widetilde{c}_{C,t} \tag{1.35}$$

where the real interest rate in the efficient allocation is $r_t^E \equiv \sigma(E_t[c_{U,t+1}^E] - c_{U,t}^E)$.

Homogeneous consumption baskets make non-trivial differences: first, both households face the same real wage, $w_{C,t} = w_{U,t} = w_t - (1-\omega)q_t$ even under asymmetric disturbances.[32] Households do not have idiosyncratic real wage risk to insure against anymore, making borrowing and savings constraint not binding in achieving the first-best outcome in the absence of nominal rigidity. Hence market outcomes can support the efficient allocation, creating no adjustment term in Eq.(1.35), the budget constraint of HtM households, and no trade-off shown in Section 1.2.5.[33] And there is no distributional inefficiency from idiosyncratic real wage risk. We define the aggregate output gap as $\widetilde{y}_t \equiv \omega\widetilde{y}_{1,t} + (1-\omega)\widetilde{y}_{2,t}$. Then the distributional variables are perfectly correlated (in log) with the aggregate output gap, implying that the inefficient variations of distributional variables are rather at an aggregate level under **HomCB**.

$$\widetilde{c}_{C,t} = (1+\varphi)\widetilde{y}_t; \quad \widetilde{c}_{U,t} = \frac{1-\lambda(1+\varphi)}{1-\lambda}\widetilde{y}_t; \quad \widetilde{n}_{C,t} = (1-\sigma)\widetilde{y}_t; \quad \widetilde{n}_{U,t} = \frac{1-\lambda(1-\sigma)}{1-\lambda}\widetilde{y}_t$$

[31] We provide the full system of equations and some derivations in the Appendix Section A.3.

[32] We can simplify the expression for real wage by defining wage to be expressed in units of the final good, $\ddot{w}_t \equiv w_t - (1-\omega)q_t$. But for consistency with the **HetCB** case, we maintain the previous definition.

[33] Unlike the **HetCB** case, the flexible-price allocation under **HomCB** achieve efficiency closing both output gaps and labor supply gaps simultaneously despite constraints on risk-sharing.

The second set of equations are from the firm side:

$$\pi_{1,t} = \beta E_t[\pi_{1,t+1}] + \frac{(1-\alpha_1\beta)(1-\alpha_1)}{\alpha_1}\widetilde{w}_t$$
$$\pi_{2,t} = \beta E_t[\pi_{2,t+1}] + \frac{(1-\alpha_2\beta)(1-\alpha_2)}{\alpha_2}\big(\widetilde{w}_t - \widetilde{q}_t\big)$$

where the real marginal cost terms in the sectoral Phillips curves are different from those under **HetCB** and given by

$$\begin{aligned} \widetilde{w}_t &= (\sigma+\varphi)\widetilde{y}_t + (1-\omega)\widetilde{q}_t \\ \widetilde{w}_t - \widetilde{q}_t &= (\sigma+\varphi)\widetilde{y}_t - \omega\widetilde{q}_t \end{aligned}$$

The dynamics of sectoral inflation are affected by the current and expected future aggregate output gap and the relative price gap. Hence unlike the **HetCB** case, each sectoral output gap has symmetric effects on both sectoral inflations aside from asymmetric price stickiness.[34]

Lastly, the economy-wide labor market clearing condition is given by

$$\omega\widetilde{y}_{1,t} + (1-\omega)\widetilde{y}_{2,t} = \lambda\widetilde{n}_{C,t} + (1-\lambda)\widetilde{n}_{U,t}$$

1.3 Model dynamics

This section studies monetary policy transmission mechanism and the redistributive effects that operates through sectoral inflation and relative prices under heterogeneity. Then we examine the features that induce asymmetric responsiveness across households.

[34] We define the aggregate inflation as $\pi_t \equiv \omega\pi_{1,t} + (1-\omega)\pi_{2,t}$. If the price stickiness in both sectors are the same, $\alpha_1 = \alpha_2$, the aggregate Phillips curve that explains the dynamics of the aggregate inflation can easily be established as a weighted sum of sectoral Phillips curves.

1.3.1 Monetary Policy Transmission Mechanism

Table 1.2 shows the baseline parameter values assumed in the numerical analysis.[35] We assume $\sigma = 0.67$, because it is more intuitive that labor supply schedule on wage is upward sloping, and it is shown by some studies that the income effect on labor supply is not big enough to dominate the substitution effect. However, the main results of this paper do not depend on the assumptions on σ. The mass of HtM households is 40% to be consistent with empirical evidence ($\lambda = z_1 = 0.4$).[36] The inverse of the Frisch elasticity of labor supply is assumed to be unity as standard in the literature.[37]

Table 1.2: Baseline parameter values in the numerical analysis

β	0.99	λ	0.4	AD	4	ρ_a	0.9	σ_a	0.01	ϕ_{π_1}	0.75
φ	1	$1-\lambda$	0.6	RD	0.5	ρ_{a_1}	0.9	σ_{a_1}	0.01	ϕ_{π_2}	0.75
θ	6	z_1	0.4	α_1	0.65	ρ_{a_2}	0.9	σ_{a_1}	0.01	$\phi_{\widetilde{y}_1}$	0
σ	0.67	z_2	0.6	α_2	0.82	ρ_v	0.0	σ_v	0.01	$\phi_{\widetilde{y}_2}$	0

We introduce the concepts of average duration – $AD \equiv (1-\alpha_1)^{-z_1}(1-\alpha_2)^{-z_2}$ – and relative duration – $RD \equiv (1-\alpha_2)(1-\alpha_1)^{-1}$ – as in Benigno (2004), where the duration of price contract in each sector is $(1-\alpha_j)^{-1}$.[38] For the study of transmission mechanism, we follow the empirical evidence (Vieyra, 2018; Cravino et al., 2020; Argente and Lee, 2020; Clayton et al., 2019) that the prices in luxury good sector adjust more frequently than those in necessity good sector, and assume $AD = 4$ and $RD = 0.5$, or $\alpha_1 = 0.65$ and $\alpha_2 = 0.82$, which implies average duration of both sectors is 4 quarters while duration in sector 2 is double that in sector 1. We do not confine this study to this parameterization, but consider a variety of combinations of Calvo

[35] We conduct robustness check for a variety of combinations of parameterizations.

[36] To facilitate the comparison of our numerical results under **HetCB** and **HomCB** to those of Benigno (2004), we assume $\lambda = z_1 = 0.5$ in the numerical analysis of optimal monetary policy.

[37] A large share of financially constrained households can lead to "Inverted Aggregate Demand Logic" as shown by Bilbiie (2008) by which an increase in real interest rate is rather expansionary. In this case, we need inverted Taylor principle for determinacy: only passive policy is consistent with a unique rational expectations equilibrium. The IADL occur when the share of non-asset holders is high enough (high λ) and/or the Frisch elasticity of labor supply is low enough (high φ). But we do not face this under baseline specification.

[38] From the definitions, we derive that $\alpha_1 \equiv 1 - AD^{-1}RD^{-z_2}$ and $\alpha_2 \equiv 1 - AD^{-1}RD^{z_1}$

parameters, α_1 and α_2, in the normative analysis. Monetary policy is characterized by a simple Taylor rule responding only to sectoral inflation ($\phi_{\pi_1} = \phi_{\pi_1} = 0.75$).

Now we examine the monetary policy transmission mechanism of an expansionary shock. In the model, type U households have Euler equation and respond to changes in interest rates, while type C households do not. Thus monetary policy shock is injected in sector 2 whose goods are consumed intensively by type U, then propagated to sector 1 through the labor market: an interest rate cut is followed by an increase in demand for good 2, leading to higher labor demand and wage; a higher marginal cost induces inflation, but the price in sector 1 rises faster than those in sector 2, because the price in sector 1 is stickier, leading to a decrease in relative price.[39] Consequently, the real wage of type U, $w_t - q_t$, is higher than that of type C, w_t, having different effects on consumption and labor supply across households. This is the *redistributive channel* of monetary policy that operates through heterogeneous real wages.

Consumption of type C increases more than that of type U in equilibrium, despite a higher real wage of type U. This is due to the counter-cyclicality of dividend under demand shocks and to the assumption on fiscal policy that finances subsidy on employment cost by lump-sum tax on type U.[40] As the real wage increases, type C raises their labor hours but not enough to cover all their consumption. The rest is backed up by type U in equilibrium who are under negative income effects of dividend. Above illustration is shown in Figure 1.1.

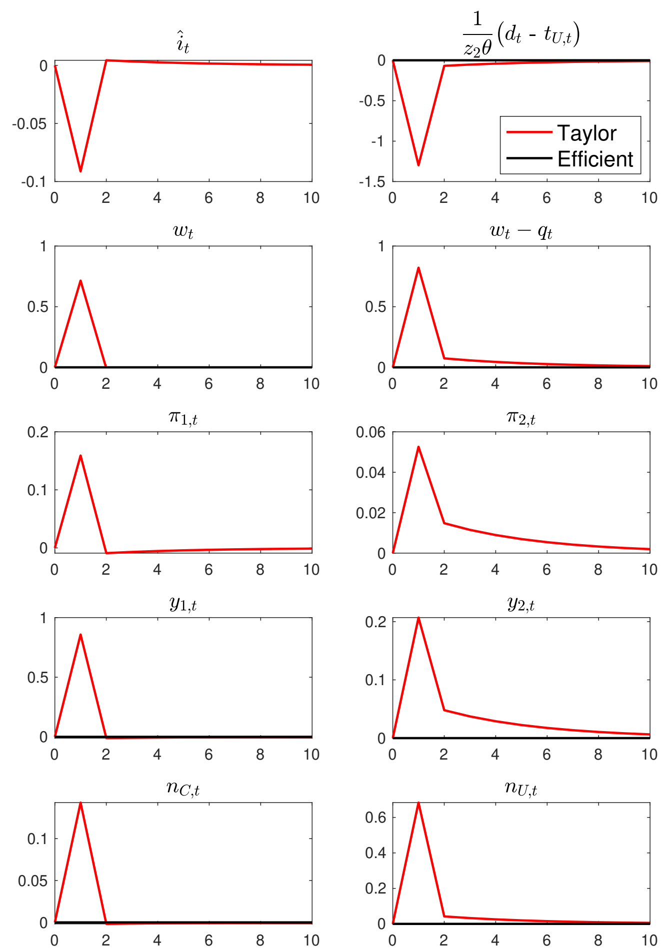
1.3.2 Redistributive Effects of Monetary Policy

Heterogeneity creates nontrivial redistributive channels of monetary policy, which operate through relative prices and sectoral inflation. Monetary policy can have different effects on the real wages across households through relative prices, affecting their consumption and labor hours differently. In addition, although it cannot stabilize sectoral inflation in both sectors simultaneously under asymmetric disturbances, monetary policy can choose which one to stabilize more than the other, which af-

[39] Real marginal cost is higher in sector 2 considering that real wage is higher for type U, although inflation is higher in sector 1. This is explained by the asymmetry in nominal rigidity.

[40] The sum of dividend and lump-sum transfer (net of tax) terms in type U households' budget constraint are linearized as $\frac{1}{z_2\theta}(d_t - t_{U,t}) = -\frac{1}{z_2}\{z_1(w_t - a_t - a_{1,t}) + z_2(w_t - q_t - a_t - a_{2,t})\}$.

Figure 1.1: Transmission mechanism of an expansionary monetary policy shock



fects the variations of consumption-relevant inflation rates and real wages differently. This has important distributional implications for the welfare of households: The more stable a households' consumption-relevant inflation rates, the more stable its real wages, the lower volatility of its consumption and labor hours, with its welfare increasing. This is the *Real Wage Stabilization Channel.*[41] Moreover, as inflation in its consumption sector stabilizes more, a household benefits more by lower price

[41]We show this in Section 1.5.

dispersion and smaller output loss. This is the *Consumption Support Channel*, that operates in second-order. Under homogeneous baskets, however, those channels do not work, because relative prices and sectoral inflation have only symmetric effects on households through the same real wages;[42] relative price affects the distribution of demands across sectors, but has no distributional consequences across households, because they consume the same composition of baskets.

Heterogeneity also confronts monetary policy with a nontrivial distributional issue on balancing welfare-relevant output gaps. Relative productivity shocks directly affect relative price, but it shows a sluggish adjustment due to nominal rigidity, leading to a negative output gap in the sector with higher productivity and a positive output gap in the sector with lower productivity. Which output gap to close more does not have distributional implications across households under **HomCB**, because its effects are symmetric, but does have under **HetCB**. Monetary policy faces a trade-off regarding whom to care about more: A more expansionary policy would benefit households who consume goods from the sector with higher productivity intensively by reducing the variation of its output gap, while having the opposite effects on households who consume goods from the sector with lower productivity intensively by raising variation of its output gap.

In a similar context, heterogeneity causes monetary policy to balance different efficient rates of interest across households. Suppose a positive shock on the sector-specific productivity $a_{1,t}$. Under **HomCB**, the efficient levels of consumption for both types increase with the efficient rate of real interest ($r_t^E = -\sigma E_t[c_t^E - c_{t+1}^E]$) decreasing; nominal rigidity would lead to a negative aggregate output gap. Although HtM households' Euler equation does not work, the central bank would largely trace the unique efficient rate and implement expansionary policy to benefit both types. Under **HetCB**, however, the efficient rates of real interest diverge: It decreases for type C ($r_{C,t}^E = -\sigma E_t[c_{C,t}^E - c_{C,t+1}^E]$) but increases for type U ($r_{U,t}^E = -\sigma E_t[c_{U,t}^E - c_{U,t+1}^E]$). The population-weighted average of the efficient rates of real interest coincides with that under homogeneous baskets, but since HtM households' Euler equation does

[42] Monetary policy still has a distributional effect under **HomCB** through dividend, that is inversely correlated with price dispersion in second-order. Monetary policy can benefit the unconstrained (constrained) households more by assigning more weight to overall inflation stabilization (output stabilization).

not work, monetary policy needs another real interest rate to target. This would have distributional consequences for welfare-relevant output gaps; the central bank's objective function will characterize the policy.

Monetary policy has direct effects through intertemporal substitution and indirect effects through labor demand and real wages in general equilibrium. The unconstrained households have the Euler equation (Eq.(1.21), Eq.(1.32)) and respond to changes in interest rates, while HtM households do not. Thus monetary policy affects the former through both direct and indirect channels, but the latter is affected only by indirect effects, with the policy having disproportionate effects on the unconstrained households.

Moreover, monetary policy can have an indirect distributional effect through dividend that is inversely correlated with price dispersion, which transforms into inflation terms in the welfare loss function.

1.3.3 Asymmetric Responsiveness across Households

In the model, HtM households show larger responsiveness to shocks, as is standard in TANK models. This is attributable to the imperfect sharing of idiosyncratic non-labor income risk and idiosyncratic real wage risk, which leads the marginal utility of consumption and marginal disutility of labor to diverge inefficiently across households. Three main factors determine asymmetric responsiveness of consumption across households: differences in (1) wage elasticities of consumption, (2) real wages, and (3) responses to interest rate changes.

Wage Elasticity of Consumption

To understand the responses of consumption to changes in wages, we use a simple example of a household that makes a static decision on consumption and labor supply given the wage with utility function and budget constraint below:

$$\max U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$$
$$s.t. \;\; WN+M=C$$

where M indicates sources of income other than wage income. From this analysis, we find that the responsiveness or the wage elasticity of consumption of a household depends nontrivially on the dynamics of other sources of income M,

$$\varepsilon_{C,W} \equiv \frac{\partial C}{\partial W}\frac{W}{C} = \frac{1+\varphi(1+\frac{\frac{\partial M}{\partial W}W}{WN})}{\sigma+\varphi(1+\frac{M}{WN})} = \frac{1+\varphi(1+\frac{\varepsilon_{M,W}M}{WN})}{\sigma+\varphi(1+\frac{M}{WN})}$$

where $\varepsilon_{M,W} \equiv \frac{\partial M}{\partial W}\frac{W}{M}$ is the wage elasticity of non-labor income.

Let us consider the cases of both types of households in the model. Since HtM households depend entirely on wage income ($M=0,\ \varepsilon_{M,W}=0$), their wage elasticity of consumption would be $\frac{1+\varphi}{\sigma+\varphi}$. However, the unconstrained households have other sources of income, dividend, which is countercyclical ($M>0,\ \varepsilon_{M,W}<0$) in response to demand shocks, as in standard New Keynesian models.[43] Thus their wage elasticity of consumption is smaller than that of HtM ($\varepsilon_{C,W,\text{type C}} = \frac{1+\varphi}{\sigma+\varphi} > \varepsilon_{C,W,\text{type U}}$), with the marginal utility of consumption diverging.[44] This is the *distributional inefficiency from idiosyncratic non-labor income risk* that arises from the financial constraints.

We derive similar results for the wage elasticity of labor hours. Refer to Section A.2.5 for more detail.

Real Wage

Due to heterogeneity in consumption baskets, households face different price indicess and real wages. Thus shocks that affect sectoral inflation differently alter relative prices and have differential effects on households' real wages and their varia-

[43]As is standard in New Keynesian models, markup and dividends are countercyclical, leading to a stabilized consumption and labor hours for the unconstrained households. Cyclicality of markups is still controversial, but a recent study such as Hong (2019) shows that markups are countercyclical with an average elasticity of -1.1 with respect to real GDP. In reality, the richer or unconstrained households would be able to smooth their consumption making use of financial instruments, while the poorer or HtM households cannot. In the TANK model, there is effectively no instrument for savings. So we can consider countercyclicality of markups as an important model feature that generates a smoother consumption for the richer or unconstrained households than the poorer or HtM households, even in a simple model with no features such as assets and wage rigidities.

[44]Fiscal rules are important because they affect the dynamics and cyclicality of macroeconomic variables; if we introduce transfers (or tax), they also play nontrivial roles along with other sources of income in the determination of the responsiveness of consumption. We conduct various robustness check for the main results in Section 1.5.

tions, and thereby on their marginal utilities of consumption and marginal disutilities of labor. It is through this mechanism that monetary policy can have redistributive effects; it can respond to and influence sectoral inflation differently affecting relative prices. Thus the policy can redistribute marginal utilities between households to maximize social welfare, as we discuss in the following sections.

Responses to Interest Rate Changes

Unlike type U, HtM households are insensitive to changes in interest rates. Thus monetary policy has a stronger effect on the unconstrained households through the direct channels.

1.4 Optimal Monetary Policy

We study optimal monetary policy under commitment by using a linear-quadratic approach following Woodford (2003). First, we take a second-order approximation to the equally-weighted sum of present valued utilities of both types of households around the deterministic efficient zero-inflation steady state, to derive a quadratic welfare-theoretic loss function of the utilitarian central bank. Then, we analyze optimal monetary policy by solving a Ramsey problem of the central bank that minimizes the welfare loss under the constraints that consist of first-order approximations to the equilibrium conditions.

We draw implications of the existence of HtM households and heterogeneous consumption baskets separately. Under **HomCB**, the distributional inefficiencies from idiosyncratic non-labor income risk are rather at the aggregate level, creating no additional trade-off. We find that financial constraint itself makes little difference to the results provided by Benigno (2004). Under **HetCB**, however, optimal policy changes significantly from Benigno (2004). The distributional inefficiencies are non-trivial from both idiosyncratic real wage risk and idiosyncratic non-labor income risk. Since monetary policy has redistributive effects, it should deal with the distributional inefficiencies at the cost of some price instability.

1.4.1 OMP under HomCB

Welfare-theoretic Loss Function

The welfare-theoretic loss function of the utilitarian central bank is derived as follows:

Proposition 1.3. *Under homogeneous consumption baskets, a second-order approximation to the equally-weighted present valued sum of both types of households' utilities is given by*

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \beta^t \Big[\lambda \big\{ U(C_{C,t}) - V(N_{C,t}) \big\} + (1-\lambda) \big\{ U(C_{U,t}) - V(N_{U,t}) \big\} \Big] \\ &= -\frac{U_c \overline{Y}}{2} E_0 \sum_{t=0}^{\infty} \beta^t \mathbb{L}_t + t.i.p. + o(||\xi||^3) \end{aligned}$$

where t.i.p. denotes "the terms independent of monetary policy" and $o(||\xi||^3)$ *includes all the terms of third order or above. The loss function is defined as*[45]

$$\mathbb{L}_t = \Phi_{\pi_1} \pi_{1,t}^2 + \Phi_{\pi_2} \pi_{2,t}^2 + \Phi_y \widetilde{y}_t^2 + \Phi_q \widetilde{q}_t^2$$

$$\Phi_{\pi_1} \equiv \omega \frac{\theta}{\kappa_1}; \quad \Phi_{\pi_2} \equiv (1-\omega) \frac{\theta}{\kappa_2}; \quad \Phi_y \equiv (\sigma+\varphi) \frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}; \quad \Phi_q \equiv \eta\omega(1-\omega)$$

Proof. Please refer to the Appendix Section A.1. □

Nominal rigidity is a source of inefficiencies: it causes price dispersion within each sector that leads to output losses in second-order and transforms to sectoral inflation, $\pi_{1,t}$ and $\pi_{2,t}$; it induces inefficient variations in demand for goods, shown by the aggregate output gap, $\widetilde{y}_t$; also, it creates cross-sectoral distortion, $(\widetilde{y}_{1,t} - \widetilde{y}_{2,t})^2$, affected by inefficient variations of relative price, $\widetilde{q}_t$.

[45] For a comparison to the loss function under **HetCB**, we can rewrite the loss function in terms of sectoral output gaps:

$$\mathbb{L}_t^{(\text{HomCB})} = \Phi_{\pi_1} \pi_{1,t}^2 + \Phi_{\pi_2} \pi_{2,t}^2 + \Phi_{y_{11}} \widetilde{y}_{1,t}^2 + \Phi_{y_{12}} \widetilde{y}_{1,t} \widetilde{y}_{2,t} + \Phi_{y_{22}} \widetilde{y}_{2,t}^2$$

$$\text{where} \quad \Phi_{y_{11}} \equiv \Phi_y \omega^2 + \frac{\Phi_q}{\eta^2}; \quad \Phi_{y_{22}} \equiv \Phi_y (1-\omega)^2 + \frac{\Phi_q}{\eta^2}; \quad \Phi_{y_{12}} \equiv 2\bigg(\Phi_y - \frac{1}{\eta} \bigg) \omega(1-\omega);$$

Financial constraints generate distributional inefficiencies from idiosyncratic non-labor income risk and are reflected in the coefficient of the aggregate output gap, Φ_y. Since the distributional variables are perfectly correlated with the aggregate output gap, distributional inefficiencies such as differences in consumption or labor hours between two types can be explained by the aggregate output gap. Thus distributional inefficiency is rather at an aggregate level, and the central bank's problem of balancing welfare loss from sectoral inflation, aggregate output gap and relative price gap is essentially unaffected.[46] Note that Φ_y is increasing in λ: as the share of HtM households increases, output stabilization becomes relatively more important than price stabilization. This is because the dividend is inversely correlated with price dispersion in second-order:

$$\int D_t(i)di = \sum_{j=1,2} Y_{j,t}\left[\frac{P_{j,t}}{P_{1,t}} - \frac{W_t}{A_t A_{j,t}} \int_{\mathcal{I}_j} \frac{1}{z_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} di\right]$$

where $d_{j,t} \equiv \log \frac{1}{z_j} \int_{\mathcal{I}_j} \left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} = \frac{\theta^2}{2} Var_i^j\{p_{j,t}(i)\} + o(||\xi||^3)$, which is associated with sectoral inflation.[47] As λ increases, the share of households receiving dividend decreases, and the central bank care relatively less about price dispersion, putting a relatively higher weight on output gap stabilization. This finding is in line with Bilbiie (2008), which studies in a single-sector framework with cost-push shocks. In our multi-sector model, we have the policy trade-off even in the absence of the inefficient cost-push shock due to the asymmetric disturbances.

Optimal Monetary Policy under Commitment

Now we investigate optimal monetary policy of the central bank under commitment that chooses target variables and nominal interest rate to maximize the objective function under equilibrium constraints.[48] Under **HomCB**, the distributional ineffi-

[46] We will see in Section 1.4.2 that distributional inefficiencies lead to a shift in target output gaps under **HetCB**.

[47] Refer to the proof of Proposition 1.3 provided in Appendix Section A.1 for the derivation.

[48] Studying optimal monetary policy is deriving one more condition, a "targeting rule", to minimize the welfare loss among all the possible candidate rules including simple Taylor rules that can close the model.

ciencies from idiosyncratic non-labor income risk are at the aggregate level, in that inefficient variations of distributional variables are perfectly correlated with aggregate output gap, creating no additional trade-off. Thus, monetary policy focus on dealing with nominal distortions. We find that financial constraint itself makes little difference to the results provided by Benigno (2004) where the market is complete, only raising the relative importance of aggregate output gap in the loss function. We briefly discuss them in the following propositions.

We study in three different cases of price stickiness: (i) flexible price in one sector and sticky price in the other sector ($\alpha_1=0$ or $\alpha_2=0$); (ii) sticky price in both sectors to the same degree ($0<\alpha_1=\alpha_2$); and (iii) sticky price in each sector but to different degrees ($0<\alpha_1<\alpha_2$).

(i) Flexible Price in One Sector

Proposition 1.4. *If the price of either one of the two sectors is fully flexible, it is optimal to fully stabilize inflation of the sticky sector. Under the optimal monetary policy, the market outcome can achieve efficiency.*

Proof. Please refer to the Appendix Section A.1. □

In this case, the only distortion is from nominal rigidity in the sticky sector. Since the central bank has one instrument and effectively one distortion, it can perfectly fix the distortion achieving efficiency; inflation in the flexible sector is innocuous because there is no price or output dispersion; the price in the sector with no nominal friction adjusts flexibly so that relative price traces its efficient level; if inflation in the sticky sector is fully stabilized, there would be no inefficiency from nominal friction. Since real marginal costs in both sectors are closed to zero inducing no non-labor income source for type U households with dividend and tax summing up to zero, financial constraints are not binding, and the first-best is obtained.

(ii) Equal Degrees of Nominal Rigidity across Sectors

Proposition 1.5. *If the prices of both sectors are sticky to the same degree, it is optimal to fully stabilize the aggregate inflation weighted by sector size. However, the optimal monetary policy cannot achieve efficiency.*

Proof. Please refer to the Appendix Section A.1. □

In this case, the central bank deals with two distortions – nominal rigidity in each sector – with one instrument. Moreover, monetary policy loses control over relative price which is affected only by the exogenous asymmetric shocks, and it cannot fix the inefficiencies induced by sluggish adjustment of relative price.[49] Although distributional variables, $c_{h,t}$ and $n_{h,t}$, aggregate output and real wage, $w_t-(1-\omega)q_t$, are on their efficient paths, relative price, wage, and sectoral output fail to achieve efficiency.

(iii) General Case

Proposition 1.6. *If the prices of both sectors are sticky to different degrees, efficiency cannot be obtained. A targeting rule is derived as follows:*

$$\frac{1}{\kappa_2-\kappa_1}\left[\begin{array}{c}\kappa_2\{\theta\pi_t+\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t\}+A(L)\{\theta\pi_t+\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t\}\\ -\beta A(L)\{\theta E_t[\pi_{t+1}]+\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)E_t[\widetilde{y}_{t+1}]\}\end{array}\right]$$
$$=(1-\omega)\theta\pi_{2,t}-\eta\omega(1-\omega)A(L)\widetilde{q}_t+(1-\omega)\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t$$

where $A(L)\equiv 1-L$. If the central bank commits to the class of "inflation targeting policy", it is optimal to give higher weight to the sector with higher degrees of nominal rigidity.

Proof. Please refer to the Appendix Section A.1 for proof, and Section 1.5 for numerical results. □

With one instrument and two distortions to deal with, monetary policy fails to achieve efficiency. Since the targeting rule that we derive is complicated to get an intuition from, we draw implications from the perspective of "optimal inflation targeting policy": what is optimal weight δ that minimize welfare loss among the class of policy rules that fully stabilizes a weighted average inflation? Through numerical experiments in Section 1.5, we find that optimal inflation targeting policy give higher

[49] Note that if $\alpha_1=\alpha_2$, the dynamics of relative price is derived only by sectoral Phillips curves and the definition of relative price.

weight to the sector whose price is stickier, which is consistent with the findings of Benigno (2004).

$$\begin{cases} \delta^{hom} > z_1, & \text{if } \alpha_1 > \alpha_2 \\ \delta^{hom} < z_1, & \text{if } \alpha_1 < \alpha_2 \end{cases}$$

1.4.2 OMP under HetCB

Welfare-theoretic Loss Function

We find that the trade-off generated by the impossibility under **HetCB** leads the central bank to target non-zero output gaps, as shown in the welfare-theoretic loss function.

Proposition 1.7. *Under heterogeneous consumption baskets, a second-order approximation to the equally-weighted present valued sum of both types of households' utilities is given by*

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \beta^t \Big[\lambda \big\{ U(C_{C,t}) - V(N_{C,t}) \big\} + (1-\lambda) \big\{ U(C_{U,t}) - V(N_{U,t}) \big\} \Big] \\ & = -\frac{U_c \overline{Y}}{2} E_0 \sum_{t=0}^{\infty} \beta^t \mathbb{L}_t + t.i.p. + o(||\xi||^3) \end{aligned}$$

where t.i.p. denotes "the terms independent of monetary policy", $o(||\xi||^3)$ *includes all* *the terms of third order or above, and the loss function is defined as*[50]

$$\begin{aligned}\mathbb{L}_t &= \frac{z_1\theta}{\kappa_1}\pi_{1,t}^2 + \frac{z_2\theta}{\kappa_2}\pi_{2,t}^2 + z_1\sigma\widetilde{y}_{1,t}^2 + z_2\sigma\widetilde{y}_{2,t}^2 + z_1\varphi\widetilde{n}_{C,t}^2 + z_2\varphi\widetilde{n}_{U,t}^2 \\ &= \Gamma_{\pi_1}\pi_{1,t}^2 + \Gamma_{\pi_2}\pi_{2,t}^2 + \Gamma_{y_{11}}(\widetilde{y}_{1,t} - x_{1,t}^*)^2 + \Gamma_{y_{12}}(\widetilde{y}_{1,t} - x_{1,t}^*)(\widetilde{y}_{2,t} - x_{2,t}^*) + \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x_{2,t}^*)^2\end{aligned}$$

$$x_{1,t}^* \equiv \frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{(\sigma-z_2)z_2}{\sigma\varphi+z_2}q_t^E; \quad x_{2,t}^* \equiv \frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{z_1 z_2}{\sigma\varphi+z_2}q_t^E;$$

$$\Gamma_{\pi_1} \equiv \frac{z_1\theta}{\kappa_1}; \quad \Gamma_{\pi_2} \equiv \frac{z_2\theta}{\kappa_2};$$

$$\Gamma_{y_{11}} \equiv z_1\left[\sigma + \left(\frac{1-\sigma}{1+\varphi}\right)^2\varphi + \frac{z_1}{z_2}\left(\frac{\sigma+\varphi}{1+\varphi}\right)^2\varphi\right]; \quad \Gamma_{y_{12}} \equiv 2z_1\varphi\frac{\sigma+\varphi}{1+\varphi}; \quad \Gamma_{y_{22}} \equiv z_2(\sigma+\varphi)$$

Proof. Please refer to the Appendix Section A.1. □

Nominal rigidity is a source of inefficiencies: it causes price dispersion within each sector that leads to output losses in second-order and transforms to sectoral inflation, $\pi_{1,t}$ and $\pi_{2,t}$; it induces inefficient variations of households' real wages and hence of their demand for goods, sectoral outputs and labor hours, shown by output gaps and labor hour gaps, $\widetilde{y}_{1,t}, \widetilde{y}_{2,t}, \widetilde{n}_{C,t}$ and $\widetilde{n}_{U,t}$.[51] Taking into account the distributional inefficiencies from idiosyncratic real wage risk shown by the relations between labor supply gaps and output gaps (Eqs.(1.30)-(1.31)), we find that the output gaps that the central bank should target, $x_{1,t}^*$ and $x_{2,t}^*$, move away from zero following asymmetric disturbances. This is the consequences of the central bank's optimal balancing of marginal utility of consumption and marginal disutility of labor between households under imperfect risk-sharing.[52]

Suppose a positive shock on sector-specific productivity $a_{1,t}$ when $\sigma < 1$.[53] If we

[50] We express the loss function in terms of distributional variables, because **HetCB** creates new trade-offs between distributional variables and cannot be explained by the aggregate variables.

[51] $\widetilde{q}_t$ does not appear in the loss function for two reasons. First, since it captures differences in real wages, it is reflected in the distributional variables. Second, since we are assuming completely heterogeneous consumption baskets with no substitution between sectoral goods, the cross-sectoral distortion, $(\widetilde{y}_{1,t} - \widetilde{y}_{2,t})^2$, is not penalized, nor is correlated with inefficient variations of relative price, $\widetilde{q}_t$.

[52] Note that distributional inefficiency from idiosyncratic real wage risk is reflected in the target output gap terms, while distributional inefficiency from idiosyncratic non-labor income risk is reflected in the weight of output gap terms and the covariance term.

[53] If we set $\sigma > 1$, both target output gaps unambiguously decreases below zero. If we set $\sigma = 1$,

suppose the central bank can close output gaps, marginal disutility of labor of type C is larger and that of type U is smaller than efficient levels. Hence the central bank would try to lower marginal disutility of type C at the cost of their consumption (negative output gap 1), and raise that of type U by boosting consumption of both types (positive output gaps in both sectors). Thus, target output gap 2 should obviously be raised above zero, $x^*_{2,t} > 0$, but the direction of target output gap 1 depends on the value of σ that measures the extent households care about consumption smoothing, the relative size of income effect in labor supply, and the size of redistribution of labor demand.[54]

If σ is small enough ($\sigma < z_2$), the target output gap 1 is lowered below zero, $x^*_{1,t} < 0$. On the one hand, households care less about consumption smoothing and their responses of consumption to shocks are stronger; the shock affects labor hour gaps by a larger amount generating larger inefficiency; the benefit from balancing also increases, because households care relatively more about variations in labor hour gap. On the other hand, since the redistribution of labor demand is smaller when the income effect is smaller, output gap 1 is more effective in adjusting labor hour gap C than labor hour gap U. Consequently, optimal balancing is to lower target output gap 1. If σ is not small enough ($z_2 < \sigma < 1$), the opposite holds, and output gap 1 should be targeted above zero. We summarize the direction of shifts in target output gaps under an increase in relative productivity $q^E_t (\equiv a_{1,t} - a_{2,t})$ in Table 1.3 with varying values of σ.

Table 1.3: Directions of shifts in target output gaps under an increase in q^E_t

	$\sigma < z_2$	$\sigma = z_2$	$z_2 < \sigma < 1$	$\sigma = 1$	$\sigma > 1$
$x^*_{1,t}$	↓	–	↑	–	↓
$x^*_{2,t}$	↑	↑	↑	–	↓

The covariance term shows up in the loss function as a result of the redistribution of labor demand in equilibrium from type C to type U households, whose labor hour gap is positively correlated with both output gaps; the weight $\Gamma_{y_{12}} \equiv 2z_1\varphi\frac{\sigma+\varphi}{1+\varphi}$ reflects

labor hours degenerate to a constant, so labor hours are equalized always.

[54]Note that target output gap in sector 2 shifts by a larger amount than in sector 1 under $\sigma < 1$.

the amount of the redistribution, $\frac{\sigma+\varphi}{1+\varphi}y_{1,t}$. Redistribution is also shown in the weight of output gap 1, $\Gamma_{y_{11}} \equiv z_1[\sigma+(\frac{1-\sigma}{1+\varphi})^2\varphi+\frac{z_1}{z_2}(\frac{\sigma+\varphi}{1+\varphi})^2\varphi]$: the second and the third term indicate labor demanded by sector 1 that is distributed to type C and type U, respectively.

Optimal Monetary Policy under Commitment

Heterogeneous consumption baskets make significant differences to the results under **HomCB** or provided by Benigno (2004) where the market is complete. This is because distributional inefficiencies are non-trivial from both idiosyncratic real wage risk, and idiosyncratic non-labor income risk: the impossibility creates trade-offs at the distributional level, leading the central bank to target non-zero output gaps in order to balance marginal utilities and marginal disutilities between households; optimal policy benefits more HtM households, whose wage elasticity of consumption is higher, to redistribute towards reducing differences between households' marginal utility. Since monetary policy has redistributive channels in operation, it should deal with the distributional inefficiencies as well as nominal rigidity, but at the cost of some price instability.

We study in four different cases of price stickiness: (i) flexible price in sector 1 and sticky price in sector 2 ($\alpha_1 = 0 < \alpha_2$); (ii) flexible price in sector 2 and sticky price in sector 1 ($\alpha_2 = 0 < \alpha_1$); (iii) sticky price in both sectors to the same degree ($0 < \alpha_1 = \alpha_2$); and (iv) sticky price in each sector but to different degrees ($0 < \alpha_1 < \alpha_2$).[55]

(i) Flexible Prices of Goods Consumed Intensively by the Constrained

Proposition 1.8. *If the price of the goods consumed more intensively by the constrained households is fully flexible, it is optimal to stabilize inflation of the sticky sector. Under the optimal monetary policy, the market outcome fails to obtain efficiency, but achieves flexible-price allocation.*

Proof. Please refer to the Appendix Section A.1. □

In this case, the central bank with one instrument should deal with two distortions – nominal rigidity in sector 2 and distributional inefficiencies. Moreover, due

[55]Note that case (i) and (ii) are effectively the same under homogeneous consumption baskets.

to its flexible price and the insensitivity of its consumers to interest rate, sector 1 is insulated from monetary policy and monetary policy cannot deal with the distributional inefficiency. Thus, it is optimal to eliminate distortion from nominal rigidity in sector 2, achieving flexible-price allocation, which is generically not efficient due to imperfect risk-sharing.[56]

(ii) Flexible Prices of Goods Consumed Intensively by the Unconstrained

Proposition 1.9. *If the price of the goods consumed more intensively by the unconstrained households is fully flexible, flexible price allocation is feasible by fully stabilizing inflation of the sector with nominal friction, but sub-optimal. Under optimal policy, the deviations of output gaps from their target levels are optimally distributed as functions of the current and the past shocks:*

$$\widetilde{y}_{1,t}^{OMP} = \underbrace{z_2 \frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{\sigma-z_2}{\sigma\varphi+z_2} q_t^E}_{=x_{1,t}^*} - z_2 \frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{\sigma\varphi+\sigma}{\sigma\varphi+z_2}\frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho}\sum_{k=0}^{\infty}\lambda_2^k q_{t-k}^E$$

$$\widetilde{y}_{2,t}^{OMP} = \underbrace{z_1 \frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{z_2}{\sigma\varphi+z_2} q_t^E}_{=x_{2,t}^*} + z_1 \frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{\sigma\varphi}{\sigma\varphi+z_2}\frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho}\sum_{k=0}^{\infty}\lambda_2^k q_{t-k}^E$$

Under optimal policy, a weighted average of the deviation of output gaps from the target level is fully stabilized, giving higher weight to the sector with flexible price:

$$\varphi z_1(\widetilde{y}_{1,t} - x_{1,t}^*) + (1+\varphi) z_2(\widetilde{y}_{2,t} - x_{2,t}^*) = 0$$

Proof. Please refer to the Appendix Section A.1. □

We find a policy trade-off in which the central bank has an incentive to deal with the distributional inefficiency from financial constraints at the cost of some price instability: it tolerates inflation or deflation to some degrees. As in case (i), the central bank should deal with two distortions – nominal rigidity in sector 1 and

[56] Other policy rules may be able to affect sector 2 and type U households, but they are sub-optimal because any effects on them are at the cost of inflation as in the representative-agent New-Keynesian model.

distributional inefficiencies – and can perfectly eliminate nominal distortion by fully stabilizing inflation of the sticky sector. Unlike that however, monetary policy can and should deal with the distributional inefficiency as well as distortions from nominal rigidities.[57] Note from the case (i) and (ii) that monetary policy should deal with the distributional inefficiencies when it has redistributive effects. Optimal policy balances between two distortions, although it fails to achieve efficiency.

In the following cases (iii) and (iv), optimal policy balances between three distortions – nominal rigidity in each sector and distributional inefficiencies.

(iii) Equal Degrees of Nominal Rigidity across Sectors

Proposition 1.10. *If the prices of both sectors are sticky to the same degree, it is no longer optimal to stabilize the aggregate inflation weighted by sector size. A targeting rule is derived as a function of the current and past variables under commitment:*

$$z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t}) = 0$$

Proof. Please refer to the Appendix Section A.1. □

Note from the aggregate Phillips curve, $\pi_t \equiv \beta E_t[\pi_{t+1}] + \kappa(\sigma+\varphi)(z_1\widetilde{y}_{1,t} + z_2\widetilde{y}_{2,t})$, and the targeting rule that full stabilization of the aggregate inflation is no longer optimal: optimal plan is a mix of price stabilization and output stabilization. In this case, the central bank loses control over relative price which is affected only by the exogenous asymmetric shocks. Thus it cannot fix the inefficiencies induced by sluggish adjustment of relative price failing to achieve efficiency.

(iv) General Case

[57] Since monetary policy that is injected into the flexible sector propagates to the sticky sector through the labor market, sector 1 and type C households are under the effects of monetary policy. Suppose an interest rate cut that changes demand for goods and labor, leading to changes in wage. Real marginal cost gap in sector 2 or real wage gap of type U households, $\widetilde{w}_t - \widetilde{q}_t$ is closed due to flexible price, but adjustments of real marginal cost in sector 1 or real wage of type C households, $\widetilde{w}_t$ are sluggish due to sticky prices in sector 1.

Proposition 1.11. *If the prices of both sectors are sticky to different degrees, a targeting rule is derived as a function of the current and past variables under commitment:*

$$\frac{\kappa_2}{\kappa_2-\kappa_1}\left[z_1\theta\pi_{1,t}+z_2\theta\pi_{2,t}+\left(z_1+\frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t}-x^*_{1,t})+z_2A(L)(\widetilde{y}_{2,t}-x^*_{2,t})\right]$$
$$+\frac{1}{\kappa_2-\kappa_1}A(L)\left[z_1\theta\pi_{1,t}+z_2\theta\pi_{2,t}+\left(z_1+\frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t}-x^*_{1,t})+z_2A(L)(\widetilde{y}_{2,t}-x^*_{2,t})\right]$$
$$-\frac{\beta}{\kappa_2-\kappa_1}A(L)E_t\left[z_1\theta\pi_{1,t+1}+z_2\theta\pi_{2,t+1}+\left(z_1+\frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t+1}-x^*_{1,t+1})+z_2A(L)(\widetilde{y}_{2,t+1}-x^*_{2,t+1})\right]$$
$$=z_2\theta\pi_{2,t}+\frac{z_1\varphi}{1+\varphi}A(L)(\widetilde{y}_{1,t}-x^*_{1,t})+z_2A(L)(\widetilde{y}_{2,t}-x^*_{2,t})$$

where $A(L)\equiv 1-L$. If the central bank commits to the class of "inflation targeting policy", heterogeneous consumption baskets give higher weight to the sector consumed by the constrained households than the optimal weight implied by homogeneous consumption baskets.

Proof. Please refer to the Appendix Section A.1 for proof, and Section 1.5 for numerical results. □

To overcome the complexity of the targeting rules in case (iii) and (iv), we get intuition from the "optimal inflation targeting policy": heterogeneous consumption baskets put higher weight to the sector of goods consumed intensively by the constrained households than under homogeneous baskets, $\delta^{het}>\delta^{hom}$, regardless of how nominal rigidities are distributed between sectors. We discuss further in Section 1.5.

1.5 Some Numerical Analysis

This section conducts numerical experiments on the consequences of neglecting heterogeneity and the implications of heterogeneity for the optimal inflation targeting policy. We also rationalize the redistributive effects by welfare analysis and discuss the robustness.

1.5.1 Consequences of Neglecting Heterogeneity

What would the consequences be if the central bank neglects heterogeneous consumption baskets? We posit a scenario in which the central bank minimize welfare loss under **HomCB** instead of the true one under **HetCB**.

The experiment shows significant implications. Neglect of heterogeneity would lead to: (1) *under*stabilization of consumption-relevant inflation and real wages of the constrained households and of the output gap in the sector of goods type C consumes more intensively; (2) *over*stabilization of inflation and real wages of the unconstrained households and of the output gap in the sector of goods type U consumes more intensively.[58]

Let us discuss why under a positive shock on sector-specific productivity $a_{1,t}$.

- Distributional inefficiencies from idiosyncratic real wage risk are not considered, neglecting shifts in target output gap above zero by the shock; monetary policy would be less expansionary than optimal.

- Distributional inefficiencies from idiosyncratic non-labor income risk under **HetCB** require stabilizing more the real wage of HtM households who are more responsive, to reduce the difference in marginal utility of consumption between households. Since the real wage gap of the constrained decreases due to nominal rigidity, neglect would lead to less expansionary policy than optimal.

- The loss function under **HomCB** penalizes the relative price gap, $\widetilde{q}_t$, for cross-sectoral distortion, which doesn't need to be cared for under heterogeneity because the substitution between sectors is absent or weak. Since the shock leads to a decrease in $\widetilde{q}_t$, under plausible assumptions on nominal rigidities ($RD < 1$), the loss function under **HomCB** would falsely require a contractionary policy to reduce the gap.

- The loss function under **HomCB** penalizes the covariance more strongly, because the inter-sector connection is tighter than under **HetCB**.[59] Since the

[58] This result is qualitatively robust under plausible values of σ.

[59] Under **HetCB**, two sectors are connected as long as labor demand from HtM households' consumption sector is distributed from them to the unconstrained households; the connection through

shock leads to negative output gap in sector 1 and positive output gap in sector 2, a larger size of the product, $|\widetilde{y}_{1,t}\widetilde{y}_{2,t}|$, would reduce welfare loss more. Thus the central bank would let $\widetilde{y}_{1,t}$, which is more volatile, to deviate more by a contractionary policy than optimal.[60]

- A misperception may arise due to the difference between each efficient allocation, $\frac{\partial}{\partial a_{1,t}}\big[y_{1,t}^{E,\mathbf{HomCB}} - y_{1,t}^{E,\mathbf{HetCB}}\big] = \big(\eta - \frac{1}{\sigma}\big)z_2$, $\frac{\partial}{\partial a_{1,t}}\big[y_{2,t}^{E,\mathbf{HomCB}} - y_{2,t}^{E,\mathbf{HetCB}}\big] = -\big(\eta - \frac{1}{\sigma}\big)z_1$. The loss function under **HomCB** can misperceive with an upward bias on $y_{1,t}$ and a downward bias on $y_{2,t}$, leading to a less expansionary policy.

The results are compatible with those under optimal inflation targeting policy in Section 1.5.2 that puts more weight on the sector of goods consumed more intensively by HtM households, allowing more variation of inflation in the other sector.

1.5.2 Optimal Inflation Targeting Policy

We derive the implications of heterogeneous consumption baskets under the inflation targeting policy by solving for the optimal weight δ^* that minimizes welfare loss in the class of policy rules that fully stabilize a weighted average inflation $\pi_t^\delta \equiv \delta\pi_{1,t} + (1-\delta)\pi_{2,t}$.[61]

Distributional Consequences and the Expected Welfare

First, we shed light on the distributional consequences of inflation targeting policy. Figure 1.4 shows how the expected welfare, defined as the sum of present-valued utilities, changes with the weight, δ, given to sector 1 on the horizontal axis. We find clear redistributive effects under **HetCB** in Figure 1.4a, whereby the expected welfare of each type of household, $\mathbb{W}_C$ and $\mathbb{W}_U$, is monotonically increasing in the weight the

consumption is absent or weak.

[60] $\widetilde{y}_{1,t}$ is more volatile than $\widetilde{y}_{2,t}$, because HtM households, who consume goods from sector 1 more intensively, are more responsive to shocks. Moreover, they do not respond to the interest rates and affected by monetary policy only through the indirect channels.

[61] For the numerical study, we solve the model using a second-order approximation method to the policy functions. For easier comparison with the literature, we set the sector sizes equal, $z_1 = z_2 = 0.5$. We vary them as needed for the robustness checks. The results do not change qualitatively and are robust.

inflation targeting policy assigns to each type's consumption sector: $\frac{\Delta \mathbb{W}_C}{\Delta \delta} > 0$ and $\frac{\Delta \mathbb{W}_U}{\Delta (1-\delta)} > 0$.

The intuition is that the more consumption-relevant inflation is stabilized, the more real wages stabilize, the more consumption and labor hours stabilize. Households dislike volatility due to the concavity of utility from consumption and the convexity of disutility from labor, as shown in the loss functions. This is *the real wage stabilization channel*. Moreover, households benefit from the stabilization of inflation in the sector of goods they consume more intensively, because output loss or consumption loss from price dispersion are also reduced in second-order. This is *the consumption support channel*. Through these channels, the expected welfare of a household increases as its price indices are more stabilized.[62] Thus under **HetCB**, monetary policy can effectively redistribute welfare and marginal utilities across households by changing the weight δ, and deals with the distributional inefficiencies as well as distortions from nominal rigidities.[63]

Let us discuss the implication of a policy change that gives higher weight δ to sector 1. In the welfare loss function below, the red terms are related with type C households, and the blue terms are related with type U households. Note that a household's consumption and labor hours are functions of its real wages. As the price in sector 1 stabilizes more, type C households' consumption and labor hours stabilize more, and they experience less consumption loss in second-order. Thus their expected welfare increases. However, as the price in sector 2 is less stabilized, type U households' expected welfare decreases. Table 1.4 summarizes this.

$$\mathbb{L}_t^{(\text{HetCB})} = \frac{z_1 \theta}{\kappa_1} \pi_{1,t}^2 + \frac{z_2 \theta}{\kappa_2} \pi_{2,t}^2 + z_1 \sigma \widetilde{y}_{1,t}^2 + z_2 \sigma \widetilde{y}_{2,t}^2 + z_1 \varphi \widetilde{n}_{C,t}^2 + z_2 \varphi \widetilde{n}_{U,t}^2$$

Under **HomCB**, however, we confirm that monetary policy has little redistribu-

[62]This result is robust to the general cases of heterogeneous consumption baskets in which households consume some common share of goods. Please refer to Figure A.1 in the Appendix.

[63]We also find that the curvature of the welfare curves is affected by nominal rigidity in each sector. In Figure 1.4a, the curvature of the welfare curve is larger for HtM households. This is because the sector of goods they consume intensively has a lower degree of nominal rigidity and hence the benefit of reducing the variation of their real wages and output loss by stabilizing their CPIs more gets smaller as δ increases.

Channel	Type C	Type U
Real Wage Stabilization	$\underbrace{\widetilde{y}_{1,t}, \quad \widetilde{n}_{C,t}}_{\text{more stabilized}}$	$\underbrace{\widetilde{y}_{2,t}, \quad \widetilde{n}_{U,t}}_{\text{less stabilized}}$
Consumption Support	$\underbrace{\pi_{1,t}}_{\text{less consumption loss}}$	$\underbrace{\pi_{2,t}}_{\text{more consumption loss}}$
Expected Welfare	$\Uparrow$	$\Downarrow$

Table 1.4: The distributional effect of higher δ on welfare

tive effect, because sectoral inflation, relative prices, and sectoral output gaps have only symmetric effects on both types with distributional variables being correlated only with the aggregate output gap. Thus the central bank cannot deal with the distributional inefficiencies, but only addresses distortions from nominal rigidity. We discuss more in detail in the next subsection.

Optimal Weight

Table 1.5 compares optimal δ under **HomCB** and **HetCB** with 4-quarter average duration and varying relative duration; for example, $RD = 0.5$ is equivalent to $(\alpha_1, \ \alpha_2) = (0.65, \ 0.82)$. Note that we assumed symmetric sectoral size for both **HomCB** and **HetCB**, $z_1 = z_2 = 0.5$.

Under **HomCB**, financial constraint itself induces no significantly different implications from those of Benigno (2004), as the analytical results did: More weight is assigned to the sector with higher nominal rigidity.[64] This result is in line with the previous finding that under **HomCB**, distributional inefficiencies are at the aggregate level; with no redistributive effects through sectoral inflation and relative prices, an inflation targeting policy deals only with distortions from nominal rigidities.

However, **HetCB** makes significant differences, and gives consistently higher weight to the goods consumed more intensively by HtM households than under **HomCB** regardless of relative degrees of nominal rigidities. This is because heterogeneity gives monetary policy a new role to deal with distributional inefficiencies

[64] The values of optimal δ^{hom} are very close to those of Benigno (2004).

from imperfect sharing of idiosyncratic real wage risk and idiosyncratic non-labor income risk. The policy redistributes in favor of HtM households who has higher responsiveness of consumption. In order to benefit them more through the redistributive channels, the central bank targets inflation rates that are weighted toward the goods that are consumed more intensively by the constrained households and not merely the goods with less flexible prices. We find that income inequality further strengthens this result in the next Section 1.5.2.

Table 1.5: Optimal inflation targeting policy under **HomCB** and **HetCB**

AD	$RD \equiv \frac{1-\alpha_2}{1-\alpha_1}$	δ^{hom}	δ^{het}	$\delta^{het} - \delta^{hom}$
4 quarters	2	0.77	0.82	+0.05
	1.5	0.67	0.73	+0.06
	1.2	0.58	0.65	+0.07
	1	0.50	0.58	+0.08
	0.83	0.42	0.50	+0.08
	0.67	0.33	0.40	+0.07
	0.5	0.23	0.30	+0.07

Given RD, the additional weight put on sector 1 by **HetCB** decreases as σ increases. A smaller elasticity of intertemporal substitution and the larger income effect of labor supply lead to a more stabilized variation of consumption for HtM households, and the policy has less incentive to stabilize inflation in their consumption baskets. Table 1.6 shows the optimal weight under $RD < 1$ that is compatible with empirical findings.

Table 1.6: Optimal inflation targeting policy with varying σ

$AD = 4$	δ^{hom}	$\delta^{het}\|_{\sigma}$	$\sigma = \frac{1}{3}$	$\frac{2}{3}$	1	2	3
$RD = 1$	0.50		0.61	0.58	0.55	0.51	0.49
0.83	0.42		0.53	0.50	0.48	0.44	0.43
0.67	0.33		0.43	0.40	0.39	0.36	0.35
0.50	0.23		0.32	0.30	0.29	0.27	0.26

Policy under Inequality

We find that income inequality between households significantly strengthens the main results. To introduce income inequality, we extend the model: (1) A nonlinear production function, $Y_{j,t}(i) = A_t A_{j,t} N_{j,t}(i)^{\alpha}$, that induces an additional source of profits through a convex cost function aside from monopolistic competition; and (2) fiscal rules that finance the share $\bar{s}$ of the subsidy by taxing HtM households: Tax only type U if $\bar{s}=0$, tax both types equally if $\bar{s}=\lambda$, and tax only type C if $\bar{s}=1$. As α decreases from unity and $\bar{s}$ increases from zero, inequality would get wider.

To examine the implications of income inequality for the redistributive effect of monetary policy and optimal inflation targeting policy, we vary α from unity to $\frac{2}{3}$. In this case, a moderate degree of income inequality is generated where the richer households income is about 50% higher than the poorer households, where the size of sector 1 is 0.38. When the sector size is controlled for under **HomCB**, optimal weight δ is 0.15. With no redistributive effects, the policy deals only with the distortions from nominal rigidities, giving much higher weight to the goods with less flexible prices compared to the sector size. Under **HetCB**, however, optimal weight δ is 0.34, which is much higher than under **HomCB**. This is because the policy deals with the distributional inefficiencies as well as the distortions from nominal rigidities.

Comparing the Figure 1.5 to Figure 1.4, we find that wider income inequality strengthens the result even more. The utilitarian central bank benefits more the households with higher marginal utility and higher responsiveness by stabilizing inflation in the sector of goods consumed more intensively by the poorer or the constrained households. If the central bank mistakenly sets it to be 0.15, the policy would benefit the richer households more than optimal, at the cost of the poorer households' welfare.

Now we conduct experiments on a few more specifications of income inequality. We assume that $(\omega_U,\ \omega_C) = (0.1,\ 0.9)$ for heterogeneous baskets. Since sector sizes are different $(z_j \neq 0.5)$ due to inequality, we compare each case with its homogeneous-basket counterpart $(\omega_U = \omega_C = z_1)$ with the same sector size z_1.

A strong policy implication of income inequality is drawn in every case of $\bar{s}$: As we introduce a nonlinear production function, the size of sector 1 (z_1) decreases due

Table 1.7: Optimal inflation targeting policy under inequality

		$\bar{s}=0$	λ	1
$(\alpha=1)$	z_1	0.50	0.46	0.42
	δ^{het}	0.28	0.27	0.26
	δ^{hom}	0.23	0.20	0.18
$(\alpha=\frac{2}{3})$	z_1	0.41	0.38	0.35
	δ^{het}	0.31	0.29	0.28
	δ^{hom}	0.17	0.15	0.14

to the inequality; despite this, δ^{het} increases, whereas δ^{hom} decreases, leading to even wider differences between them. The intuition is that since the hand-to-mouth or the poorer households have higher marginal utility with a higher volatility and are more responsive to real wages, the utilitarian central bank cares disproportionately more about them and redistributes marginal utilities in their favor to maximize the social welfare.

We also find that the dynamics and distribution of non-labor income, such as tax and dividend, are nontrivial. Let us compare $(\alpha,\ \bar{s}) = (1,\ 1)$ and $(\frac{2}{3},\ 0)$: Both have similar degrees of inequality at the steady state, sector size, and hence δ^{hom}. However, δ^{het} is smaller for the former, although they are both higher than δ^{hom}; this is attributable to the lump-sum tax on HtM households, which can be regarded as countercyclical non-labor income for them that stabilizes their consumption and labor hours to some degree.

Robustness

We conduct the robustness checks, and the results are robust to the following features: the degrees of heterogeneity in consumption baskets; income inequality; the specifications of whom to tax to finance subsidies; whether monopolistic distortion is eliminated or not at the steady state; and relative degrees of nominal rigidities across sectors. The results are significantly strengthened as income inequality deepens.

1.6 Conclusion

We analyze optimal monetary policy in a model with households that differ along two dimensions: They consume different baskets of consumption goods and have differential access to financial markets. Households face idiosyncratic real wage risk and non-labor income risk. Imperfect risk-sharing gives monetary policy a new role to address the distributional inefficiencies at the cost of some price instability. Based on a micro-founded welfare criterion, the first-best outcome is not achievable even in the absence of nominal rigidities: Optimal monetary policy targets non-zero output gaps due to new trade-offs, and benefits borrowing-constrained or poorer households more by targeting inflation rates that are weighted toward the goods that are consumed more intensively by the constrained or poorer households and not merely the goods with less flexible prices. This is because the utilitarian central bank benefits more the households with higher marginal utility and higher responsiveness to changes in real wages. If the central bank neglect heterogeneous consumption baskets, the policy would be more beneficial to the richer households than optimal at the cost of the poorer households.

This study focuses on the qualitative aspects of the mechanisms that are newly generated by HetCB, and the new redistributive channel that operates through different price indices across different income levels. But it would be of interest to extend this paper to several dimensions. First, since we abstract from unemployment, it would be an important extension to study the normative implications of the asymmetry in unemployment risk observed in the real world under the heterogeneous consumption baskets framework. Second, we simplified the role of the fiscal sides, but heterogeneous consumption baskets may also have important implications for fiscal policy as we examined shortly in the main text. Monetary and fiscal policy interaction under heterogeneous consumption baskets merits further study. Third, in this study we focused on the differences in the sectors of goods that households consume. Not only that, the differences in the sector households work would also have important implication for monetary policy, because the weight given to each sector by inflation targeting policy would benefit households who work in some sectors at the cost of households who work in other sectors.

Figure 1.2: Consequences of neglecting heterogeneity under $a_{1,t}$ shock

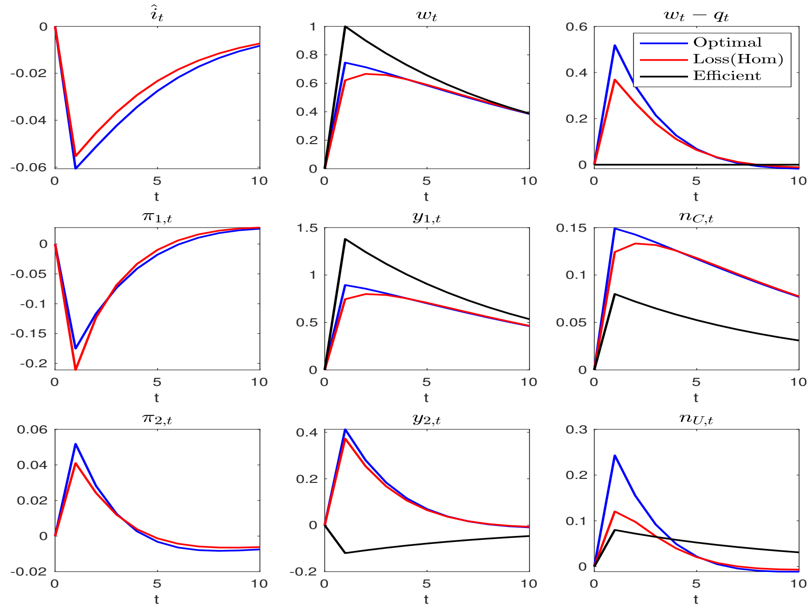
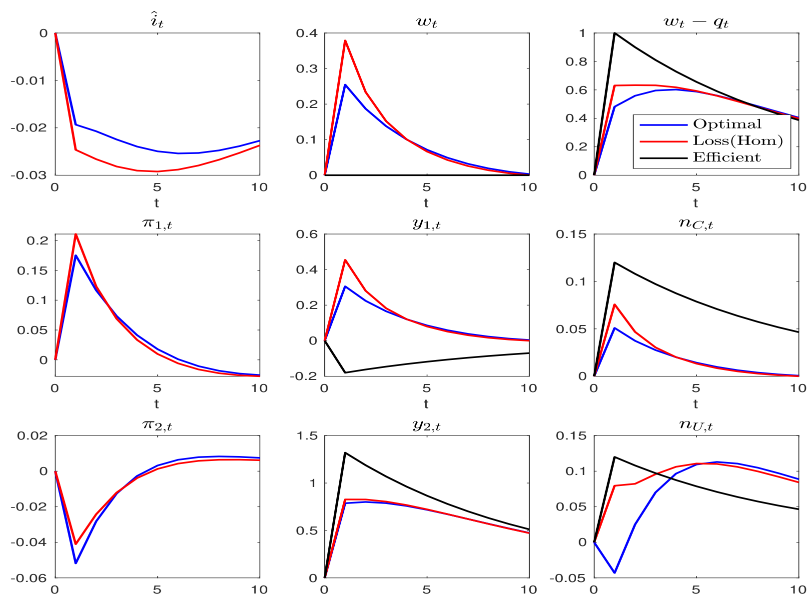
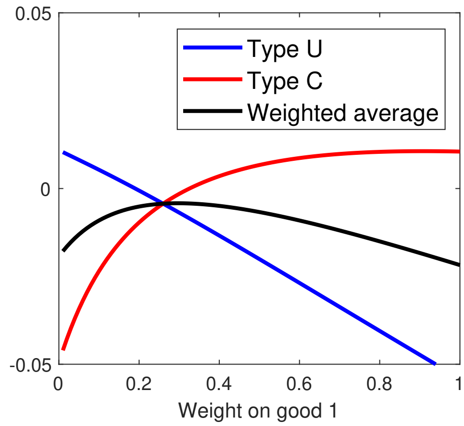
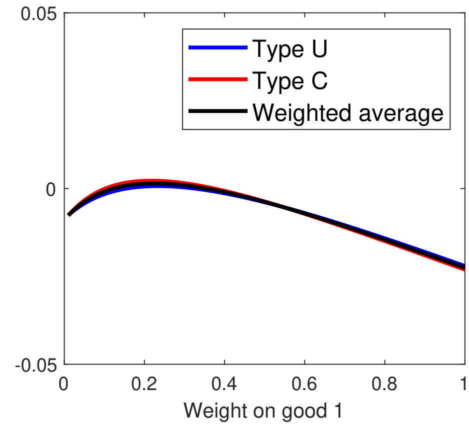


Figure 1.3: Consequences of neglecting heterogeneity under $a_{2,t}$ shock



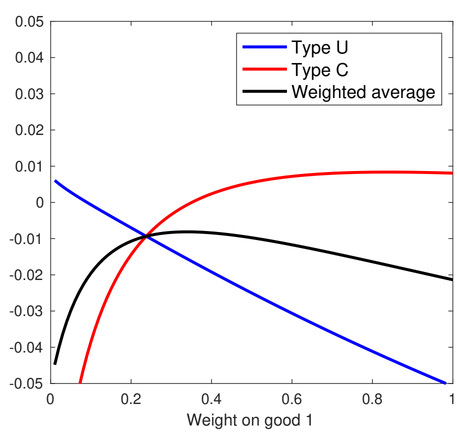


(a) **HetCB**

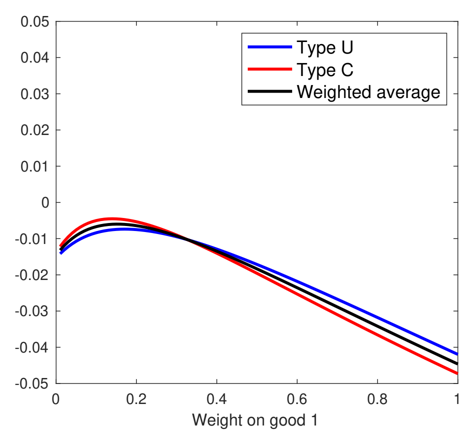


(b) **HomCB**

Figure 1.4: Expected welfare and redistributive effects of an inflation targeting policy



(a) **HetCB** ($\delta^{het} = 0.34$)



(b) **HomCB** ($\delta^{hom} = 0.15$)

Figure 1.5: Redistributive effects of an inflation targeting policy under income inequality ($\alpha = \frac{2}{3}$)

Chapter 2

Monetary Non-Neutrality in a Multisector Economy: The Role of Risk-Sharing[1]

2.1 Introduction

This paper revisits classic questions in monetary economics. What are the mechanisms that cause aggregate output to deviate from its natural level? How do aggregate demand shocks, such as an exogenous change in monetary policy, affect output significantly in the short run? We address these questions in a multisector sticky-price model, focusing on the role of risk-sharing among workers who work in different sectors of the economy and, as a consequence, earn different wages.

While a variety of frictions could potentially contribute to significant short-run variations in the output gap that are apparent in the data, nominal price rigidity remains as the major source for the "non-neutrality" in a large part of the literature. Empirical studies based on identified vector autoregressions (VARs) point to sluggish adjustment of the price level to various macroeconomic shocks (e.g., Christiano et al. (1999)). The evidence, in turn, motivates quantitative macroeconomic analyses using structural models with a high degree of nominal rigidities (e.g., Smets and Wouters (2007)).

The empirical literature based on disaggregated or micro-level price data, however, paints a seemingly conflicting picture of nominal rigidities.[2] For example, Bils and

[1]This chapter describes joint work with Jae Won Lee, an assistant professor at UVA.

[2]See Klenow and Malin (2010) and Nakamura and Steinsson (2013), for a survey of the literature.

Klenow (2004), Klenow and Kryvtsov (2008), and Nakamura and Steinsson (2008) all point out that individual prices are highly volatile and adjust more frequently than suggested by macroeconomic data. The microevidence implies that nominal price rigidity *by itself* is unable to generate the persistent non-neutrality observed in the aggregate data.

In an effort to narrow the gap between the evidence of relatively flexible individual prices and sluggish aggregate price adjustment, monetary models typically combine nominal rigidities with other features, often referred to as "real rigidities," that generate strategic complementarities among firms whose pricing decisions are asynchronous. Such real rigidities render a firm's optimal price dependent upon the prices of other firms in the economy. Given the dependence, when a shock hits the economy, the firms that have an opportunity to change their prices in a particular period adjust only partially because of prices that have not yet adjusted. Strategic complementarities thus, when coupled with nominal rigidities, can produce a significant amplification and propagation mechanism, thereby bridging the gap between the macro and the micro rigidity of prices.

There are, however, credible critiques against models with strategic complementarities in price setting (Bils et al. (2012); Klenow and Willis (2016)). The essence of the critics is that price adjustments, under certain types of real rigidities, are small in response to *all* shocks, not only to aggregate shocks. The model property is at odd with the microevidence that individual prices are quite volatile, implying prices must be relatively flexible in response to *some* shocks.[3] In a closely related study, Boivin et al. (2009) show that highly disaggregated price indices (or sectoral prices) respond much faster to idiosyncratic (sector-specific) shocks than to aggregate shocks, and much of the variations in prices reflects the former shocks.

The micro- and macro-evidence together disciplines the type of real rigidities a modeler may include in his or her model. A *right* type would generate strategic complementarities conditional only on aggregate shocks, thereby allowing prices to respond relatively swiftly to idiosyncratic shocks.

We argue that asset market imperfections can produce such real rigidities in a specific way. We consider an environment in which each worker has a labor skill specific

[3]Otherwise, some shocks would have to be implausibly large.

to a particular sector and cannot learn new skills and migrate to other sectors for the time horizon of our interest. That is, labor markets are sector specific, and workers of different sectors earn different wages. Wages vary across sectors not only because of sector-specific labor productivity. Aggregate shocks also induce divergent sectoral wages as sectors differ in nominal rigidities. Workers may or may not be able to insure fully against their labor income risks depending on the assumption of asset markets. We show that, in this environment, imperfect risk-sharing between workers of different sectors generates strategic complementarities in price setting conditional on aggregate shocks and strategic substitutabilities conditional on sector-specific shocks. These *two-way* pricing interactions slow down price responses to the former type of shocks and speed up price responses to the latter type of shocks.

Therefore, such across-sector imperfect risk-sharing – a natural extension to standard multisector sticky-price models, in our view – provides a new amplification and propagation mechanism of aggregate shocks without producing the implausible implication at the micro level. This result applies to all aggregate shocks: *Any* aggregate shocks would have greater effects on the output gap with the new mechanism. When illustrating and quantifying the result, however, we will focus on the effect of monetary shocks, which has been the source of significant debate in the literature.

Section 2.3 provides detailed intuition for the mechanism. We first discuss the nature of pricing interactions by looking at individual firms, and subsequently show how the interactions alter the form of the aggregate Phillips curve. Notice that, regardless of risk-sharing, nominal marginal costs of a firm in a given sector depend *negatively* on the sectoral relative price – the sectoral price level relative to the aggregate price level – due to an expenditure-switching effect in a model with sector-specific labor markets. A decrease in the relative price leads to an increase in the sector's production and hours, which in turn raises the sector's wage rate and marginal costs.

Given the expenditure-switching effect and the resulting negative dependence of a firm's marginal costs upon the sectoral relative price, we compare two types of shocks. First, consider an aggregate shock that increases nominal marginal costs of *all* sectors (e.g., an expansionary monetary shock). The presence of fixed prices in many sectors of the economy prevents the aggregate price level from increasing by a "*full*" amount, which *ceteris paribus* causes sectoral relative prices to increase by more and a firm's marginal costs to increase by less – relative to the case where all firms adjust their prices. Consequently, re-pricing firms would increase their prices by *less* to the aggregate shock. Now consider a sector-specific shock that increases nominal marginal costs of a particular sector (e.g., an adverse sector-specific technology/productivity shock). When each sector is small compared to the economy, the aggregate price level is exogenous to this shock, which allows us to focus on the sector's price level. The presence of fixed prices in that sector, for the same reason as before, causes the sector's price level to increase by less than a *full* amount. In contrast to the case of aggregate shocks, however, the smaller increase of the sector's price level results in a less (not greater) increase in the sectoral relative price. Consequently, the sector's marginal costs increase by more, and a re-pricing firm in that sector raises its price by *more* to the idiosyncratic shock.

The strength of the two-way pricing interaction, however, depends on how well workers can insure against their labor income risks. Imperfect risk-sharing amplifies the aforementioned mechanism because a sector's marginal costs depend more heavily on its sectoral relative price in this case. As mentioned above, due to the expenditure switching effect, a decrease in the sectoral relative price leads to an increase in the sector's hours and wage rates. However, with imperfect risk sharing, the resulting increase in labor income causes consumption of workers in that sector to rise, which in turn shifts in the sector's labor supply curve through a wealth effect. Consequently, the sector's wage rate, and thus marginal costs, rise further. The wealth effect therefore produces a greater dependence of a sector's marginal costs upon the sectoral relative price, which in turn renders prices even less responsive to aggregate shocks and even more responsive to idiosyncratic shocks.

While the finding, qualitatively, is not specific to *why* or *how* workers fail to insure fully against income risks, in section 2.4 we illustrate and quantify the effects of monetary shocks under three stylized asset market setups that are exogenously given: i) complete asset markets, ii) non-contingent bond-only market, and iii) financial autarky. The first case is equivalent to the model with a representative agent and serves as our reference point. Clearly, none of the three asset market setups resembles the real world which has a multiplicity of risk-sharing institutions as well as of sources of financial frictions. Nevertheless, our exercise suggests an importance of the mechanism for monetary non-neutrality.

Section 2.5 supplements the main results by considering two modified versions of the model. In these models, labor markets are segmented not only across sectors (as in our main model) but also across firms within each sector, and each agent works for a particular firm. Given the labor market setting, we consider two cases for risk-sharing. First, risk-sharing is perfect within each sector and imperfect across sectors. Second, risk-sharing is imperfect within and across sectors. We contrast the nature of pricing interactions arise in these new cases with that in our main model to further understand the mechanism.

Finally, although the focus of this paper is on the propagation of aggregate – especially monetary – shocks as mentioned above, we in section 2.6 present some quantitative results on the role of sector-specific technology shocks in aggregate fluctuations. Since imperfect risk-sharing causes prices to respond by more to these shocks, sectoral outputs also respond by more, which in turn generates a greater aggregate output response. We thus show that pricing interactions created by across-sector imperfect risk-sharing enhance the ability of the (supply-type) sectoral shocks, as a whole, to drive aggregate fluctuations.

Related literature Our work is greatly indebted to earlier papers that develop multisector sticky-price models to address a variety of questions. These include normative analyses such as Aoki (2001), Mankiw and Reis (2003b), Benigno (2004), and Eusepi et al. (2011). Closely related with our study, Carvalho (2006) and Nakamura and Steinsson (2008) first highlight the role of sectoral heterogeneity of nominal price rigidities in amplifying the real effects of monetary shocks in the Calvo and menu cost models respectively. Recently, Pasten et al. (2019) confirms the importance of heterogeneity in price stickiness in a model with heterogeneous input-output linkages. Although most studies in the literature rely on calibrated models owing to the large dimension typical multisector models entail, some relatively recent papers have estimated the models to study the propagations of various shocks (e.g., Bouakez et al. (2014); Carvalho et al. (2020); Carvalho et al. (2021); Smets et al. (2018)).

What these studies have in common is that they use models with a representative agent or equivalently assume perfect risk-sharing among agents, thereby precluding any role of worker or household heterogeneity. This paper gives new theoretical insights into the transmission of aggregate and idiosyncratic shocks when household heterogeneity interacts with price stickiness heterogeneity in a multisector environment.

Our paper also builds on the original work on real rigidities that generate pricing interactions. Ball and Romer (1990) divide real rigidities into the "micro" and "macro" types. The micro type includes the kinked demand of Kimball (1995), diminishing returns to scale in production (Galí (2008), chapter 2), and firm-specific factors of production (Woodford (2003), chapter 3; Woodford (2005); Sveen and Weinke (2005); Altig et al. (2011)). These features are popular in sticky-price business cycle models, yet are subject to the aforementioned criticism as they slow down price responses to both aggregate and idiosyncratic shocks. A prominent example of the macro type is input-output production structures as in Basu (1995). The macro type generates pricing interaction in a way such that prices respond slowly only to aggregate shocks, and thus are immune to the criticism. A third type, which does not fit into a single-sector framework, has been proposed by Carvalho et al. (2021) in a multisector environment. They show that labor market segmentation at the sectoral level causes prices to respond slowly to aggregate shocks, similar to the previous two types. What separates the third type is that it also generates faster price responses to idiosyncratic shocks.[4]

We point out that asset market imperfections that prevent perfect risk-sharing between households can significantly enhance the role of the third type of real rigidities that lead to such two-way pricing interactions. This gives rise to slow aggregate price adjustments and large and persistent output deviations from the natural level, without sacrificing volatile individual prices. Our work thus contributes to the vast literature on real rigidities and monetary non-neutrality.

Finally, our analysis produces a tangential contribution to the literature on production networks.[5] Studies in this growing literature focus on the importance of

[4] Carvalho and Nechio (2016) provide a comparison of three types of factor markets: firm-specific, sector-specific, and economy-wide markets.

[5] The literature builds on early contributions such as Long and Plosser (1983), Horvath (1998), Horvath (2000) and Dupor (1999). More recent studies include Foerster et al. (2011), Acemoglu et al. (2012), Coibion and Gorodnichenko (2011), and Miranda-Pinto and Young (2019). See Carvalho

idiosyncratic shocks in generating aggregate fluctuations – usually in flexible-price multisector models.[6] Our quantitative analysis here concentrates on aggregate – more specifically, monetary – shocks. However, the discussion in section 2.3 and the numerical result in section 2.6 both suggest the possibility of pricing interactions induced by imperfect risk-sharing as an amplification mechanism of idiosyncratic productivity shocks. Whether the mechanism is relevant in a more elaborate quantitative model is an open question that we do not address in the current paper.

2.2 The model

Our model is an extension to the standard multisector sticky-price model of Carvalho et al. (2021). Firms, indexed by $i \in [0, 1]$, produce differentiated goods that are aggregated into final consumption goods. Firms are divided into a finite number of sectors indexed by $k \in \{1, 2, \cdots, K\}$. We use $\mathcal{I}_k$ to denote the set that contains firms in sector k. The sectors are characterized by different degrees of nominal rigidities $\{\alpha_k\}_{k=1}^K$ and different sizes $\{n_k\}_{k=1}^K$. Labor markets are sector-specific. Households are heterogeneous in labor skills: "Type-k household" possesses labor skills specialized for goods produced in sector k.

The main departure from standard multisector models is that households working in different sectors can fail to insure perfectly against their labor income risks. We consider three stylized asset market setups that are exogenously given: i) complete asset markets, ii) non-contingent bond-only market, and iii) financial autarky. The first case is equivalent to the model with a representative household and will serve as our reference point

2.2.1 Households

Members of each household are simultaneously consumers and workers. Households who work in sector k (or type-k households) maximize a discounted sum of

and Tahbaz-Salehi (2019) for a survey of the literature.

[6]See Pasten et al. (2020) for a model with sticky prices.

utilities of the form:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\log C_{k,t}-\omega_k\frac{H_{k,t}^{1+\varphi}}{1+\varphi}\right)\right],$$

where $C_{k,t}$ and $H_{k,t}$ denote respectively household consumption and labor hours supplied in sector k. The parameter $\beta \in (0,1)$ is the discount factor, $\varphi \geq 0$ is the inverse of the Frisch elasticity of labor supply, and $\omega_k > 0$ is the relative disutility of supplying labor.

This paper considers three stylized asset market environment. In the bond-only economy, the flow budget constraint is given by:

$$P_tC_{k,t}+B_{k,t}+\Omega\left(B_{k,t}\right)=R_{t-1}B_{k,t-1}+W_{k,t}H_{k,t}+P_tT_t+\Pi_t, \tag{2.1}$$

where $W_{k,t}$, P_t, R_t, T_t and Π_t denote respectively the nominal wage rate in sector k, the aggregate price level, the gross nominal interest, net transfers from the government, and dividend. Different types of households earn different labor incomes, $W_{k,t}H_{k,t}$, and share income risks by trading nominal bonds. We use $B_{k,t}$ to denote type-k household's bond holdings at time t. The convex term, $\Omega\left(B_{k,t}\right)$, is introduced mostly for convenience; it is useful to pin down a well-defined steady state (Schmitt-Grohe and Uribe (2003)).[7] It also captures the costs for the households, as a reduced form, of undertaking positions in the bond market – as in Heaton and Lucas (1996).[8]

In the case of financial autarky, there are no markets that allow households to insure against their labor income risks. The budget constraint (2.1) simplifies to

$$P_tC_{k,t}=W_{k,t}H_{k,t}+P_tT_t+\Pi_t.$$

One can numerically approximate financial autarky by assuming the cost term in (2.1) is sufficiently large. Finally, the model with complete markets is standard in the literature and is omitted here.

[7]The households are, in the absence of shocks, identical and endowed with zero initial debt $B_{k,-1}=0$.

[8]It creates a wedge between the lending rate and the borrowing rate.

Type-k household's first order conditions are:

$$1+\Omega^{'}\left(B_{k,t}\right) = \beta R_t \mathbb{E}_t\left[\left(\frac{C_{k,t}}{C_{k,t+1}}\right)\left(\frac{P_t}{P_{t+1}}\right)\right],$$
$$\omega_k H_{k,t}^{\varphi} C_{k,t} = \frac{W_{k,t}}{P_t},$$

where $\Omega^{'}\left(B_{k,t}\right)$ is the first derivative.

2.2.2 Firms

The final consumption good, Y_t, is produced by perfectly competitive firms using sectoral goods, $\{Y_{k,t}\}_{k=1}^{K}$, with a CES production technology:

$$Y_t = \left(\sum_{k=1}^{K} n_k^{1/\eta} Y_{k,t}{}^{(\eta-1)/\eta}\right)^{\eta/(\eta-1)},$$

where η is the elasticity of substitution between sectoral goods. The appropriate price index for the final consumption good is:

$$P_t = \left(\sum_{k=1}^{K} n_k P_{k,t}^{1-\eta}\right)^{1/(1-\eta)}, \tag{2.2}$$

where $P_{k,t}$ is the sectoral price index associated with $Y_{k,t}$. Given Y_t, $P_{k,t}$ and P_t, the optimal demand for sector-k good minimizes total expenditure, $P_t Y_t$, and is given by

$$Y_{k,t} = n_k \left(\frac{P_{k,t}}{P_t}\right)^{-\eta} Y_t \quad \forall k. \tag{2.3}$$

Each sectoral good is a composite of $\{Y_{k,t}(i)\}_{i\in\mathcal{I}_k}$ that are produced by firms in sector k:

$$Y_{k,t} = \left(\left(\frac{1}{n_k}\right)^{1/\theta} \int_{\mathcal{I}_k} Y_{k,t}(i)^{(\theta-1)/\theta} di\right)^{\theta/(\theta-1)} \quad \forall k,$$

where $\theta > 1$ is the elasticity of substitution between different types of goods. The corresponding price index for a sectoral good is given by:

$$P_{k,t} = \left(\frac{1}{n_k} \int_{\mathcal{I}_k} P_{k,t}(i)^{1-\theta} di \right)^{1/(1-\theta)} \quad \forall k.$$

The optimal demand for good i is given by:

$$Y_{k,t}(i) = \frac{1}{n_k} \left(\frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\theta} Y_{k,t}.$$

Firm i in sector k uses a linear production function to produce $Y_{k,t}(i)$:

$$Y_{k,t}(i) = A_{k,t} H_{k,t}(i), \tag{2.4}$$

where $H_{k,t}(i)$ denotes hours employed by firm i, and $A_{k,t}$ is exogenous sector-specific productivity.

Prices are sticky as in Calvo (1983) and Yun (1996). Firms in sector k adjust their prices with probability $1 - \alpha_k$ each period, which results in the sector's price level, $P_{k,t}$, evolving as:

$$P_{k,t} = \left[\frac{1}{n_k} \int_{\mathcal{I}_k^*} P_{k,t}^*(i)^{1-\theta} di + \alpha_k P_{k,t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{2.5}$$

where $P_{k,t}^*(i)$ is an optimal price chosen by firm i when $i \in \mathcal{I}_k^*$. The set $\mathcal{I}_k^* \subset \mathcal{I}_k$, with measure $n_k (1 - \alpha_k)$, is a randomly chosen subset in which firms adjust their prices.

A firm that adjusts its price at time t choose $P_{k,t}^*(i)$ that maximizes its expected discounted profits:

$$\max_{P_{k,t}^*(i)} \mathbb{E}_t \sum_{j=0}^{\infty} \alpha_k^j q_{k,t,t+j} \frac{\Pi_{k,t+j}(i)}{P_{t+j}},$$

where $q_{k,t,t+j}$ is the real stochastic discount factor between time t and $t + j$, and $\Pi_{k,t+j}(i)$ is the firm's nominal profit at time $t + j$ given that the price chosen at time t is still being charged:

$$\Pi_{k,t+j}(i) = P_{k,t}(i) Y_{k,t+j}(i) - W_{k,t+j} H_{k,t+j}(i).$$

When asset markets are incomplete and a firm is owned by multiple households, a uniquely obvious way to discount future profits does not exist. We assume that firms use the real interest rate, in which case, $q_{k,t,t+j} = \prod_{z=0}^{j} R_{t+z}^{-1} \frac{P_{t+z+1}}{P_{t+z}}$.[9]

The first order condition is given by:

$$0 = \mathbb{E}_t \sum_{j=0}^{\infty} \alpha_j^k q_{k,t,t+j} Y_{t+j} \left(\frac{P_{k,t}^*(i)}{P_{k,t+j}} \right)^{-\theta} \left(\frac{P_{k,t+j}}{P_{t+j}} \right)^{-\eta} \left\{ \left(\frac{P_{k,t}^*(i)}{P_{t+j}} \right) - \left(\frac{\theta}{\theta-1} \right) MC_{k,t+j} \right\}, \tag{2.6}$$

where $MC_{k,t+j} = \frac{W_{k,t+j}}{A_{k,t+j} P_{t+j}}$ denotes sector-k real marginal costs at $t+j$. The optimal prices chosen at time t, $\left\{ P_{k,t}^*(i) \right\}_{i \in \mathcal{I}_k^*}$ that satisfy the first order condition (2.6) determine the equilibrium dynamics of the sectoral price level $P_{k,t}$ through (2.5). The dynamics of the aggregate price level are then determined by aggregating these sectoral prices through (2.2).

2.2.3 Government

The government budget constraint is:

$$\frac{B_t - R_{t-1} B_{t-1}}{P_t} + \sum_{k=1}^{K} n_k \Omega\left(B_{k,t}\right) = T_t + G_t, \tag{2.7}$$

where B_t is the supply of government bonds, and G_t is government purchases. For simplicity, we assume $B_t = G_t = 0$, thereby abstracting from any meaningful influence of fiscal policy on equilibrium. The government simply collects the bond market participation costs and returns them to the households as a transfer. This assumption has no important consequences for the results.

Monetary policy is characterized by a Taylor-type rule :

$$R_t = \beta^{-1} \left(\frac{P_t}{P_{t-1}} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} \exp(\mu_t), \tag{2.8}$$

[9] Alternatively, one could assume that a firm maximizes the weighted average of its shareholders' objectives. In this case, the discount factor of firm i would be $\sum_{k=1}^{K} n_k \beta^j \frac{C_{k,t}}{C_{k,t+j}}$. This alternative discount factor generates the same dynamics in the first order approximation. Pescatori (2007) has made the same argument.

where μ_t represents exogenous variations in monetary policy, and Y is the steady-state value of output.

2.2.4 Equilibrium and additional notations

The definition of equilibrium is standard, given the maximization problems of the private sector and the monetary and fiscal policy described above. Goods, asset and labor markets clear in equilibrium:

$$Y_t = \sum_{k=1}^{K} n_k C_{k,t}$$
$$0 = \sum_{k=1}^{K} n_k B_{k,t}$$
$$n_k H_{k,t} = \int_{\mathcal{I}_k} H_{k,t}(i) di$$

We solve the model by log-linearizing the equilibrium conditions around the deterministic zero-inflation steady state. The appendix provides a detailed derivation of the steady-state equilibrium as well as the full set of log-linearized equations. In what follows, lowercase letters denote log-deviation from their steady state counterparts. One exception is nominal bond holdings: $b_{k,t} \equiv \frac{B_{k,t}-B}{PY}$ denotes the deviation from the steady-state level $B = 0$, relative to steady-state nominal income.

2.3 The mechanism

Imperfect risk-sharing produces two types of pricing interdependence: strategic complementarity in price setting in response to aggregate shocks and strategic substitutability in response to sector-specific shocks. Such *two-way* pricing interactions lead to slow aggregate price adjustments and large output gap responses, while allowing for relatively fast price adjustments to idiosyncratic shocks.

We first discuss the nature of pricing interactions by looking at individual firms' pricing behaviors. We then show how the interactions influence aggregate dynamics.

2.3.1 The nature of pricing interactions

As mentioned, the nature of pricing interactions differs depending on whether a shock affects all sectors or a specific sector. To understand this, it is instructive to consider how a flexible-price firm, that optimizes frictionlessly every period without the Calvo constraint, would set its price. The firm's price equals its nominal marginal costs in a log-linear approximation:

$$\begin{aligned} p^{**}_{k,t}(i) &= \text{sector-}k \text{ nominal marginal costs} \\ &= p_t + mc_{k,t} \\ &= p_t + (1+\varphi)\, y_t - (1+\varphi)\, a_{k,t} \\ &+ \quad \underbrace{\varphi\,(y_{k,t} - y_t)}_{\text{Segmented labor markets}} \quad + \quad \underbrace{(c_{k,t} - y_t)}_{\text{Imperfect risk-sharing}} \quad , \end{aligned} \tag{2.9}$$

where $p^{**}_{k,t}(i)$ is firm i's *frictionless optimal price.*

Let us focus on the two terms in the second line of equation (2.9) – sectoral output and consumption relative to aggregate output. A sector's marginal costs depend positively on the relative output, $y_{k,t} - y_t$. For a given amount of aggregate output, a high level of production in a sector requires more labor hours. This shifts out the sector's labor demand curve, which in turn raises the wage rate, and thus marginal costs, in that sector. The relative production would not appear in a model with an economy-wide labor market because a change in labor hours in a *small* sector would not affect the economy-wide wage rate.

When asset markets are incomplete, a sector's marginal costs depend also positively on the sector's household consumption relative to the economy's average consumption, $c_{k,t} - y_t$. A high level of consumption, ceteris paribus, raises the wage rate through wealth effects on labor supply, which leads to a rise in marginal costs. Under complete markets, a household's consumption equals aggregate income, which drops the last term from equation (2.9).

The presence of the two terms in a firm's marginal costs causes firms' pricing decisions to be strategic complements with respect to aggregate shocks and strategic substitutes with respect to idiosyncratic shocks. We start with the role of the relative output. Notice that it can be replaced by the relative price according to the demand function (2.3):

$$\varphi\left(y_{k,t}-y_t\right)=-\varphi\eta p_{k,t}+\varphi\eta p_t.$$

As the coefficients on the aggregate price level (p_t) and on the sectoral price ($p_{k,t}$) have the opposite signs, pricing interactions are different, depending on whether firms are in the same sector or in different sectors. To fix ideas, first consider an aggregate shock that increases marginal costs of *all* sectors (e.g., an expansionary monetary shock). If prices were fully flexible, all firms would increase their prices, and the price level (p_t) would adjust by a "*full*" amount. However, when some prices in many sectors do not move, the price level increases by less than the full amount. As a consequence, a re-optimizing firm increases its price also by less as its marginal costs depend positively on the price level (as shown in $\varphi\eta p_t$). Now, consider a sector-specific shock that increases marginal costs of only one sector (e.g., an adverse sector-specific productivity shock). The price level is (almost) exogenous with respect to this shock because each sector is small, which allows us to focus on $-\varphi\eta p_{k,t}$. The negative coefficient ($-\varphi\eta$) indicates that the sector's price level ($p_{k,t}$) – that is *lower* than the potential "*full*" level due to the existence of non-adjusting firms in the same sector – induces adjusting firms to increase their prices by more than they would if $p_{k,t}$ increased fully. The presence of the relative output (or the relative price) in marginal costs, therefore, causes firms to react differently depending on types of shocks.[10]

We now turn to the role of the relative consumption, $c_{k,t}-y_t$. Notice that this term has qualitatively the same effect on a firm's pricing decision as the relative output $y_{k,t}-y_t$, to the extent that $c_{k,t}$ and $y_{k,t}$ are positively correlated, or equivalently when $c_{k,t}$ and $p_{k,t}$ are negatively correlated. This is what happens under incomplete asset markets. Intuitively, a decrease in the sectoral price ($p_{k,t}$) leads to an increase in $y_{k,t}$, $h_{k,t}$ and $w_{k,t}$ through the expenditure switching effect, as explained above. As labor incomes ($h_{k,t}+w_{k,t}$) in that sector increase, the sector's household consumption ($c_{k,t}$) rises too.

In summary, the magnitude of the negative relationship between a sector's marginal costs and the sectoral relative price depends on how well households can insure against

[10] See Carvalho and Nechio (2016) and Carvalho et al. (2021) for additional discussions.

their labor income risks. The harder risk-sharing is, the stronger the relationship is – because of wealth effects and a resulting smaller wage elasticity of household labor supply. The stronger relationship in turn causes prices to respond more differently depending on whether a given shock is economy-wide or sector-specific.

In what follows (until section 2.5), we focus on the propagation of monetary shocks, for which the aforementioned strategic substitutability plays no role. However, it will continue to operate in the background, and individual prices would respond relatively fast to idiosyncratic shocks – as documented by empirical studies (e.g., Boivin et al. (2009)). Imperfect risk sharing between households, thus, provides a new amplification and propagation mechanism of aggregate shocks without sacrificing volatile individual prices. We then revisit the role of strategic substitutability in section 2.6 and confirm that disaggregated prices are indeed more volatile under imperfect risk-sharing.

2.3.2 The Phillips curve and aggregate implications

We now discuss the implications for aggregate dynamics. Aggregating prices in sector k leads to the sectoral Phillips curve of the form:

$$\pi_{k,t} = \beta \mathbb{E}_t \pi_{k,t+1} + \lambda_k \left[(1+\varphi)\, y_t + \left(\varphi + \frac{1}{\eta} \right) y^R_{k,t} + c^R_{k,t} \right], \tag{2.10}$$

where $\lambda_k \equiv \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}$ is convexly decreasing in α_k.[11] The expression between square brackets is sector-k marginal costs deflated by sector-k price level: $mc_{k,t} + p_t - p_{k,t}$. The superscript R is used to define a variable relative to its mean; that is, $y^R_{k,t} \equiv y_{k,t} - y_t$ and $c^R_{k,t} \equiv c_{k,t} - y_t$ are respectively sectoral output and consumption relative to aggregate output. The two variables affect sectoral marginal costs for the reasons outlined in the previous subsection.

The aggregate Phillips curve is then obtained by taking a weighted sum of the sectoral Phillips curves:

$$\pi_t = \beta \mathbb{E}_t \left[\pi_{t+1} \right] + \kappa y_t + \Theta_{y,t} + \Theta_{c,t}, \tag{2.11}$$

[11] A detailed derivation is given in the appendix. The idiosyncratic shock ($a_{k,t}$) is omitted here.

where $\kappa \equiv (1+\varphi)\sum_{k=1}^{K} n_k\lambda_k$, and the shift terms are given as

$$\Theta_{y,t} \equiv \left(\varphi + \eta^{-1}\right)\sum_{k=1}^{K} n_k\lambda_k y_{k,t}^R,$$
$$\Theta_{c,t} \equiv \sum_{k=1}^{K} n_k\lambda_k c_{k,t}^R.$$

Under complete asset markets, the second shift term, $\Theta_{c,t}$, is zero.

Both shift terms, $\Theta_{y,t}$ and $\Theta_{c,t}$, move in the opposite direction as the rate of aggregate inflation π_t, in response to aggregate shocks. This leads to slower aggregate price adjustments and greater output deviations from the natural level. The finding results from the strategic complementarity discussed above and the property that a majority of price-adjusting firms are in high-frequency sectors.

To understand the dynamics of the shift terms, let us consider an example in which a contractionary monetary shock hits the economy. This shock decreases marginal costs of all sectors, and firms will reduce their prices when an opportunity to do so comes.

Such opportunities, however, arrive more frequently in *high-frequency* sectors – those with a low value of Calvo parameter, α_k. Consequently, a high-frequency sector's price level decreases more than a low-frequency sector's price level; that is, the relative price of a high-frequency to a low-frequency sector decreases. This in turn leads to: a high relative demand for a high-frequency sector's good (say sector k); a high relative demand for labor hours in the sector; a high relative wage in the sector; and thus an increase in sector-k marginal costs:

$$shock \longrightarrow mc_{k,t}\downarrow\longrightarrow p_{k,t}^R\downarrow\longrightarrow y_{k,t}^R\uparrow\longrightarrow h_{k,t}^R\uparrow\longrightarrow w_{k,t}^R\uparrow\longrightarrow mc_{k,t}\uparrow.$$

As the latter rise in $mc_{k,t}$ partially offsets the initial fall in $mc_{k,t}$, firms in high-frequency sectors do not decrease their prices as much. Consequently, sectoral inflation $\pi_{k,t}$ in (2.10) moves more sluggishly in those sectors.

Across-sector imperfect risk-sharing increases marginal costs of high-frequency sectors further, thereby generating even smaller adjustments of prices in such sectors.

A rise in labor hours and the wage rate in a sector leads to a rise in consumption of workers in the same sector. This generates wealth effects, producing a further increase in the wage rate:

$$shock \longrightarrow mc_{k,t} \downarrow \longrightarrow p^R_{k,t} \downarrow \longrightarrow y^R_{k,t} \uparrow \longrightarrow \underbrace{h^R_{k,t} \uparrow \longrightarrow w^R_{k,t} \uparrow} \longrightarrow mc_{k,t} \uparrow$$

$$\Downarrow$$

$$\underbrace{c^R_{k,t} \uparrow \longrightarrow w^R_{k,t} \uparrow}_{\text{Wealth effects}} \longrightarrow mc_{k,t} \uparrow$$

Consequently, firms in high-frequency sectors decrease their prices by even less, and sectoral inflation $\pi_{k,t}$ in (2.10) moves even more sluggishly.

On the flip side, the exactly opposite process occurs in low-frequency sectors. Since only a small number of firms adjust their prices here, these sectors' price levels remain relatively high after the shock. A high relative price ($p^R_{k,t}$) results in a decline in both relative output ($y^R_{k,t}$) and relative consumption ($c^R_{k,t}$), which further decreases marginal costs. Consequently, adjusting firms in low-frequency sectors reduce their prices by more than they would in the absence of strategic complementarity in pricing setting.

The strategic complementarity thus renders prices in high-frequency sectors less responsive and prices in low-frequency sectors more responsive to aggregate shocks, thereby exerting countervailing forces on the aggregate price level. To put it differently, $y^R_{k,t}$ and $c^R_{k,t}$ of high-frequency sectors in Equation (2.10) and the variables of low-frequency sectors have the opposite signs after a shock. Therefore, at first pass, the overall effects on the shift terms, $\Theta_{y,t}$ and $\Theta_{c,t}$, in Equation (2.11) and thus on the rate of aggregate inflation, π_t, appear ambiguous.

The influence from high-frequency sectors, however, dominates. The reason is that a majority of price-adjusting firms are in high-frequency sectors. Therefore, pricing decisions in those sectors have disproportionately greater influences on the adjustments of the aggregate price level. In the aggregate Phillips curve (2.11), this property is captured by the 'weight parameter' λ_k, which is convexly increasing in the sectoral frequency of price changes, $1 - \alpha_k$. Therefore, both $\Theta_{y,t}$ and $\Theta_{c,t}$ rise while π_t declines in response to a contractionary monetary shock.

2.4 Numerical analysis

In this section, we illustrate the results under the three (stylized) asset market settings. Notice that the mechanism discussed in the previous section also applies to other aggregate shocks absent in this model – such as a shock to preference, production technology, and government spending. These shocks would generate greater deviations of output from the natural level when asset markets are incomplete. For the purpose of illustration, we focus exclusively on the effect of monetary shocks, which has been the source of significant debate in the literature.

2.4.1 Parameterization

The parameters are set to standard values in the literature. We start with the preference parameters. The frequency of the model is a month, and the discount factor, β, is set to 0.9967, implying a 4% annual steady-state interest rate. We have already assumed implicitly that the intertemporal elasticity of substitution equals 1 by using the log period utility function. The Frisch elasticity of labor supply, φ^{-1}, is also set to 1.

The within-sector elasticity of substitution between different varieties, θ, is set to 6, which implies a 20 percent steady-state mark-up for the firms. We set the across-sector elasticity of substitution, η, to 1 based on the estimate in Hobijn and Nechio (2019).

To parameterize the distribution of price rigidity across sectors, we use the price setting statistics in the U.S. economy reported by Bils and Klenow (2004). Specifically, we map the sectors in the model into the goods and services categories in their study, and set $\{1-\alpha_k\}_{k=1}^{K}$ to the monthly frequency of price changes of the corresponding categories. The sectoral weights $\{n_k\}_{k=1}^{K}$ are set to the CPI weights for these categories; we normalize the weights so that $\sum_{k=1}^{K} n_k = 1$. The number of sectors is 350: $K = 350$.

Regarding monetary policy parameters, we set ϕ_π to 2.55 and ϕ_π to 0.433/12 based on the estimates in Carvalho et al. (2021) The monetary shock follows an AR(1) process with the autoregressive coefficient set to $0.95^{1/3}$. We later consider alternative parameterization of monetary policy.

Finally, we assume $\Omega\left(B_{k,t}\right)=\frac{\epsilon}{PY}B_{k,t}^{2}$, and, following Heaton and Lucas (1996), interpret the cost term as a wedge between the lending and borrowing rates. We set ϵ to a sufficiently large number (10^3) to approximate financial autarky. The bond-only economy, in theory, is an intermediate case between the two extreme asset market setups: autarky and complete markets. Quantitatively, we find that the behavior of the bond-only economy is quite similar to that of the complete markets model when bond trading is fully frictionless (for example, when we set $\epsilon = 10^{-6}$).[12] We thus do not present this case to avoid cluttering the figures below. Instead, we report a case in which ϵ is set to 0.001. To put this value into perspective, one may consider the implied interest rate spread. When households lend and borrow 50% of their incomes, the implied spread is 0.2 percentage point when ϵ =0.001.[13]

The model is solved by the method of Sims (2002). Since the dimension of the multisector model is large, we follow the suggestion by Lee and Park (2020) and transform the model into a reduced form before applying Sims' method. This reduces the computation time by more than 90%.

2.4.2 Impulse responses and monetary non-neutrality

Figure 2.1 illustrates our main finding. It shows the impulse response functions (IRFs) of inflation and output to an expansionary monetary shock (a decrease in μ_t by 1/12 percentage point) under the three asset market assumptions: i) complete asset markets, ii) non-contingent bond-only market, and iii) financial autarky.

Based on the IRFs of output, we compute a measure of monetary non-neutrality over different time horizons. The measure is given by the relative cumulative IRFs (denoted by $CIRF_t^R$) – the ratio of the cumulative IRF of output to that obtained

[12]This finding – that temporal smoothing through a single bond is close enough for full insurance – is in fact ubiquitous in the business cycle literature. For example, see Heathcote and Perri (2002) who compare the same three asset market setups as our paper does.

Importantly, we only consider aggregate shocks in the numerical exercise, so the difference in labor income across sectors is entirely due to heterogenous price stickiness.

[13]Type-k household Euler equation is given as: $c_{k,t}=E_t\left[c_{k,t+1}\right]-\left(i_t-E_t\pi_{t+1}-2\epsilon b_{k,t}\right).$

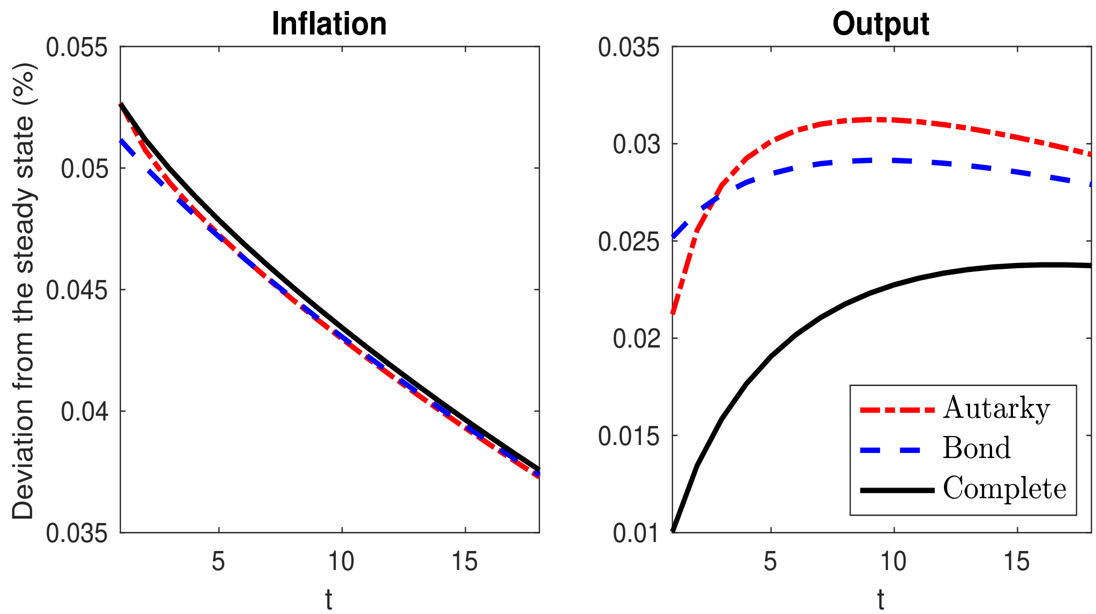


Figure 2.1: Response of inflation and output to a decrease in monetary shock.

under complete asset markets:

$$CIRF_t^R = \frac{\sum_{j=1}^{t} IRF_j \text{ of output}}{\sum_{j=1}^{t} IRF_j \text{ of output under complete markets}},$$

where IRF_t is the impulse response function at time t when the shock hits the economy at $t = 1$. Table 2.1 reports this measure associated with the alternative asset market setups, for $t = 1$, 6 and 12.

Table 2.1: Relative cumulative impulse responses of output

	$CIRF_t^R$		
	$t = 1$	$t = 6$	$t = 12$
Financial autarky	2.170	1.688	1.515
Bond economy	2.508	1.708	1.470

The figure and table indicate that imperfect risk-sharing increases the degree of monetary non-neutrality significantly. The cumulative responses of output over the 12-month horizon under financial autarky and in the bond-only economy are respectively 52% and 47% more than that under complete market benchmark. Moreover, on impact ($t = 1$), the initial responses of output under financial autarky and in the bond-only economy are respectively 117% and 151% more than that under complete markets. Interestingly, while the degree of monetary non-neutrality, overall, is greatest under financial autarky, output responds by more in the bond-only economy in the very short run. The reason is that households can smooth out the effects of a shock in the bond-only economy, so the second shift term, $\Theta_{c,t}$, can in fact decrease by more on impact when the shock is highly persistent – as shown in the second panel of Figure 2.2.

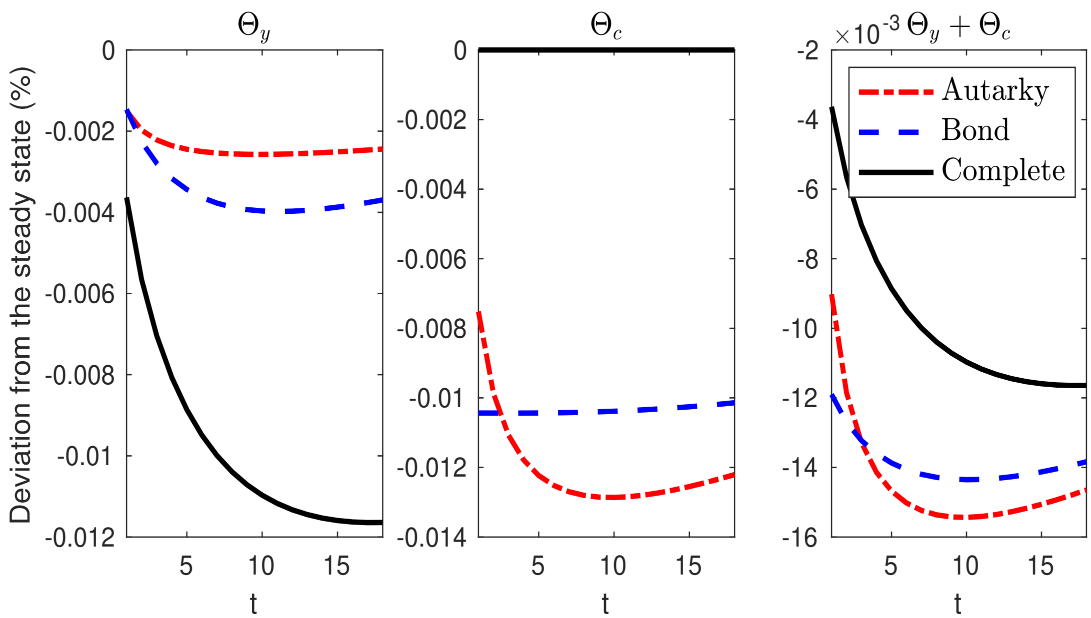


Figure 2.2: Response of the shift terms to a decrease in monetary shock.

To illustrate the mechanism discussed in section 2.3.2, Figure 2.2 shows the IRFs of the shift terms, $\Theta_{y,t}$ and $\Theta_{c,t}$. The last panel of the figure reveals that the sum of the shift terms, $\Theta_{y,t} + \Theta_{c,t}$, decreases by more as risk-sharing becomes harder (except for the first two periods). Such the behavior of $\Theta_{y,t} + \Theta_{c,t}$ is the reason that the responses of output differ under the alternative asset market assumptions, as explained in section 2.3.2. While the sum decreases by more under incomplete markets, $\Theta_{y,t}$ (shown in the first panel) decreases by less. The reason is the wealth effect which generates across-sector strategic complementarity. This produces stronger comovements of sectoral output.

We also report the results under alternative monetary policy rules. We consider two cases. First, nominal output, $m_t = p_t + y_t$, follows an exogenous stochastic process, as often assumed in the literature that studies monetary non-neutrality. In particular, we assume m_t follows an AR(1) in growth rates: $\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t$ with $\rho_m = 0.89$ as in Carvalho (2006). The value implies that shocks have a half-life of 6 months. Second, we consider a Taylor rule with an interest smoothing term. We set the coefficient on the lagged interest rate to $0.697^{1/3}$ based on the estimate in Carvalho et al. (2021) who estimate the rule in a model similar to ours. Introducing the lagged term reduces the estimate of the autoregressive coefficient in the shock process to $0.576^{1/3}$ according to their study.

Figure 2.3 and 2.4 show the IRFs of inflation, output, and the shift terms in the first case. The figures illustrating the second case are qualitatively similar to Figure 2.1-2.4, and thus are omitted. Table 2.2 quantifies the results on monetary non-neutrality. The results are the same as before qualitatively.

Quantitatively, the marginal contribution of imperfect risk-sharing to monetary non-neutrality, measured by $CIRF_t^R$, is less than before, yet is still non-negligible. The increases of output over the 12-month horizon under financial autarky and in the bond-only economy are respectively 35% and 20% more than those under complete market benchmark with the nominal output growth rule; they are 39% and 14% more with interest rate smoothing.[14] Including output growth in the Taylor rule, as in Coibion and Gorodnichenko (2011), makes little difference quantitatively in terms of the marginal contribution of imperfect risk-sharing to monetary non-neutrality.

[14] This does not imply that the *level* contribution of imperfect risk-sharing (i.e. the *difference* in the cumulative output responses across different asset market setups, rather than the ratio) is small as greater interest rate smoothing leads to greater output responses. We find that the level differences are in fact greater with interest rate smoothing.

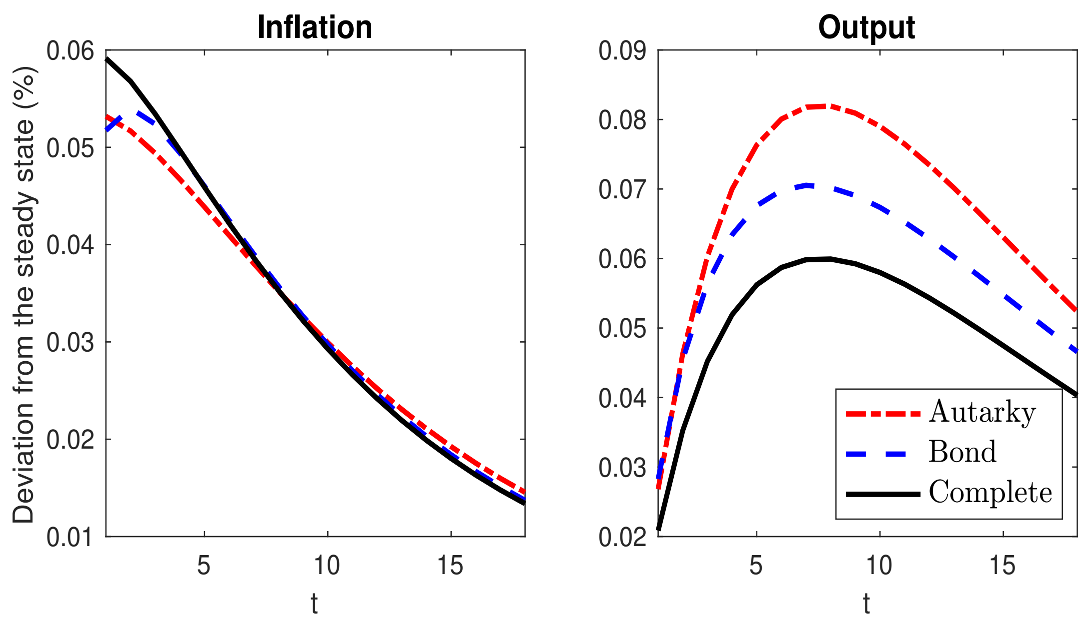


Figure 2.3: Response of inflation and output to a decrease in monetary shock.

Table 2.2: Relative cumulative impulse responses of output – different policy rules

		$CIRF_t^R$		
		$t = 1$	$t = 6$	$t = 12$
Nominal output growth rule	Financial autarky	1.358	1.345	1.351
	Bond economy	1.355	1.234	1.195
Interest rate smoothing	Financial autarky	1.294	1.407	1.390
	Bond economy	1.084	1.123	1.140

2.5 Comparison with firm-specific labor markets

This section compares our model to an analogous model with firm-specific labor markets that are popular in the literature (e.g., Woodford (2003)). Labor markets are now segmented not only across sectors (as before) but also across firms within each sector, and each household (type-ik household) works for a particular firm (firm i in sector k). Otherwise, the model in this section is the same as the previous model.

We consider two cases for risk-sharing. First, risk-sharing is perfect within each sector and imperfect across sectors, in which case we can still assume a representative

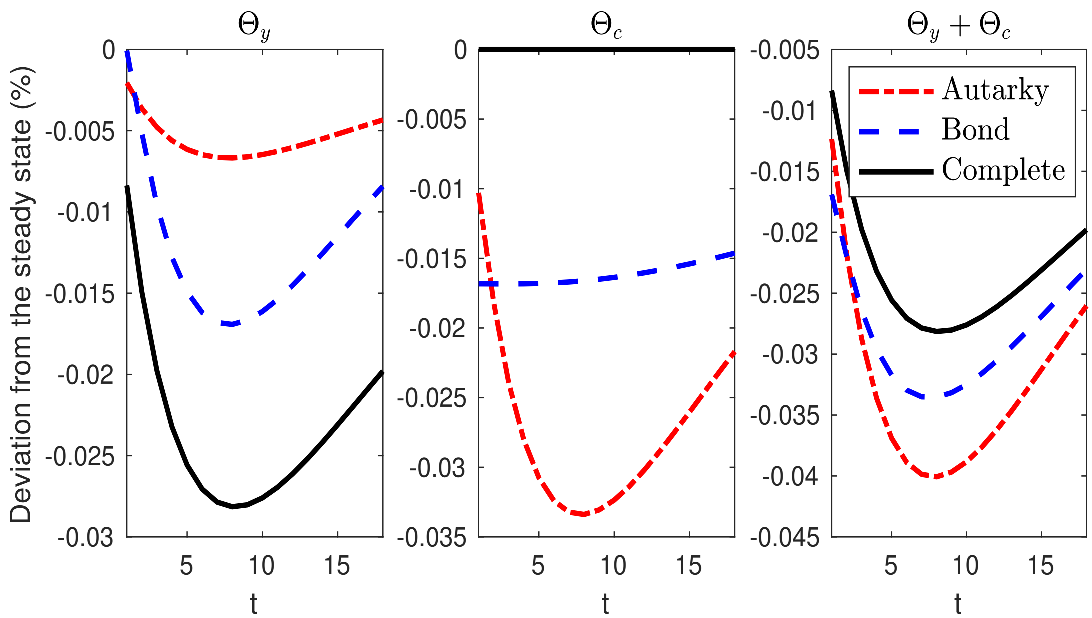


Figure 2.4: Response of the shift terms to a decrease in monetary shock.

household for each sector, as in the previous model. Second, risk-sharing is imperfect within and across sectors. We show below that firm-specific labor markets, regardless of the asset market assumptions, generate the *micro* type of real rigidities, which dampen price responses to all shocks. When it comes to the role of risk-sharing, incomplete asset markets, in the first case, still generate the *third* type of real rigidities leading to two-way pricing interactions. In contrast, incomplete markets, in the second case, give rise to the micro type of real rigidities which slow down price responses to all shocks.

For each case, we discuss the nature of pricing interactions using the frictionless optimal price equations and present the Phillips curve as in section 2.3, which is then followed by numerical illustrations. A full detail of the models is provided in the appendix.

2.5.1 Case I: Perfect risk-sharing within sectors

Firm-specific labor markets generate a new pricing interaction on top of what is shown in section 2.3.1. The reason is that marginal costs are now firm-specific, rather than sector-specific, as each firm pays its workers the firm-specific wage rate, $w_{k,t}(i)$. Since a change in $p_{k,t}(i)$ now influences $w_{k,t}(i)$ through a *within-sector* expenditure switching effect, $p_{k,t}(i)$ appears in marginal costs of the firm. Firms take that into account setting their prices.[15]

As mentioned, we first consider the case in which across-sector risk-sharing is imperfect but within-sector risk-sharing is perfect. A firm's frictionless optimal price is then given as:

$$\begin{aligned} p^{**}_{k,t}(i) &= \text{firm-}i\text{ nominal marginal costs} \\ &= p_t + (1+\varphi)\, y_t - (1+\varphi)\, a_{k,t} \\ &- \underbrace{\eta\varphi\,(p_{k,t} - p_t)}_{\text{Segmented labor markets across sectors}} + \underbrace{(c_{k,t} - y_t)}_{\text{Imperfect risk-sharing across sectors}} \\ &- \underbrace{\theta\varphi\left(p^{**}_{k,t}(i) - p_{k,t}\right)}_{\text{Segmented labor markets within a sector}} \end{aligned} \tag{2.12}$$

Firm i's nominal marginal costs in (2.12) are the same as that shown in (2.9) until the second line. The terms in the second line remain because labor is still immobile across sectors (as a direct implication of firm-specific labor) and across-sector risk-sharing is still imperfect, as in the previous model.[16]

The last term in the third line is introduced due to labor market segmentation within sectors. It reflects the *within-sector* expenditure switching effect: A rise in $p_{k,t}(i)$, ceteris paribus, decreases $y_{k,t}(i)$, $h_{k,t}(i)$, and $w_{k,t}(i)$, which leads to a decline in marginal costs. This term renders prices less responsive irrespective of shock types. Notice that the coefficient on $p_{k,t}$, given by $(\theta - \eta)\,\varphi$, is now positive to the extent that the within-sector elasticity is greater than the across-sector elasticity. Therefore, considering the role of labor market segmentation in isolation, pricing decisions are

[15] Firm i is a representative for good-i producing firms, so it is still a wage taker as in Woodford (2003). However, firms understand that all good-i producing firms that reoptimize make the same pricing decisions.

[16] We here use the demand relation, $y_{k,t}(i) - y_t = -\theta\varphi\,(p_{k,t}(i) - p_{k,t}) - \eta\varphi\,(p_{k,t} - p_t)$, which reflects two layers of expenditure switching effects. Notice that even if all firms in sector k happened to charge the same price (i.e., $p_{k,t}(i) = p_{k,t}$), eliminating any effects from firm-specific labor, labor market segmentation at the sector level would still operate to the extent that $p_{k,t} \neq p_t$ and hence $w_{k,t}(i) = w_{k,t} \neq w_t \equiv \sum_{k=1}^{K} \int_{\mathcal{I}_k} w_{k,t}(i)di$.

now strategic complements with respect to sector-specific shocks, in contrast to the previous model.

However, when it comes to the role of risk sharing, the result is the same as before. For the given labor market assumption, imperfect risk-sharing provides a mechanism that makes prices less responsive to aggregate shocks and more responsive to idiosyncratic shocks, as captured by the term, $c_{k,t} - y_t$.

The aggregate Phillips curve is given as

$$\pi_t = \beta E_t\left[\pi_{t+1}\right] + \kappa^I y_t + \Theta^I_{y,t} + \Theta^I_{c,t}, \tag{2.13}$$

where

$$\kappa^I \equiv \frac{1+\varphi}{1+\theta\varphi}\sum_{k=1}^{K} n_k\lambda_k, \quad \Theta^I_{y,t} \equiv \frac{\varphi+\eta^{-1}}{1+\theta\varphi}\sum_{k=1}^{K} n_k\lambda_k y^R_{k,t}, \quad \Theta^I_{c,t} \equiv \frac{1}{1+\theta\varphi}\sum_{k=1}^{K} n_j\lambda_k c^R_{k,t}.$$

The superscript I is used to denote the slope and shift terms of the Phillips curve in "*Case I*". Notice that $\kappa^I = (1+\theta\varphi)^{-1}\kappa$, $\Theta^I_{y,t} = (1+\theta\varphi)^{-1}\Theta_{y,t}$, and $\Theta^I_{c,t} = (1+\theta\varphi)^{-1}\Theta_{c,t}$. Therefore, labor market segmentation at the firm level reduces all the coefficients in the Phillips curve by a factor of $(1+\theta\varphi)$ compared to the case in which labor markets are only segmented at the sector level. On the other hand, the role of imperfect risk-sharing, as before, is captured by the last shift term $\Theta^I_{c,t}$; under complete asset markets, $\Theta^I_{c,t} = 0$.

We illustrate the responses of inflation and output in Figure 2.5. Table 2.3 quantifies the results on monetary non-neutrality. The additional mechanism created by the last term in (2.12) increases monetary non-neutrality under all three asset market setups. For example, the responses of output on impact (i.e. at $t = 1$) in Figure 2.5 are roughly six time greater than those in Figure 2.1.

Interestingly, the gap in the extent of monetary non-neutrality between the case of complete markets and the cases of incomplete markets widens. For example, the cumulative response of output over the 12-month horizon in the bond-only economy is now 83% more than that under complete market; this 83% in Table 2.3 is even greater than the 47% in Table 2.1. This indicates that imperfect risk-sharing plays a greater role under firm-specific labor markets than under sector-specific labor markets.

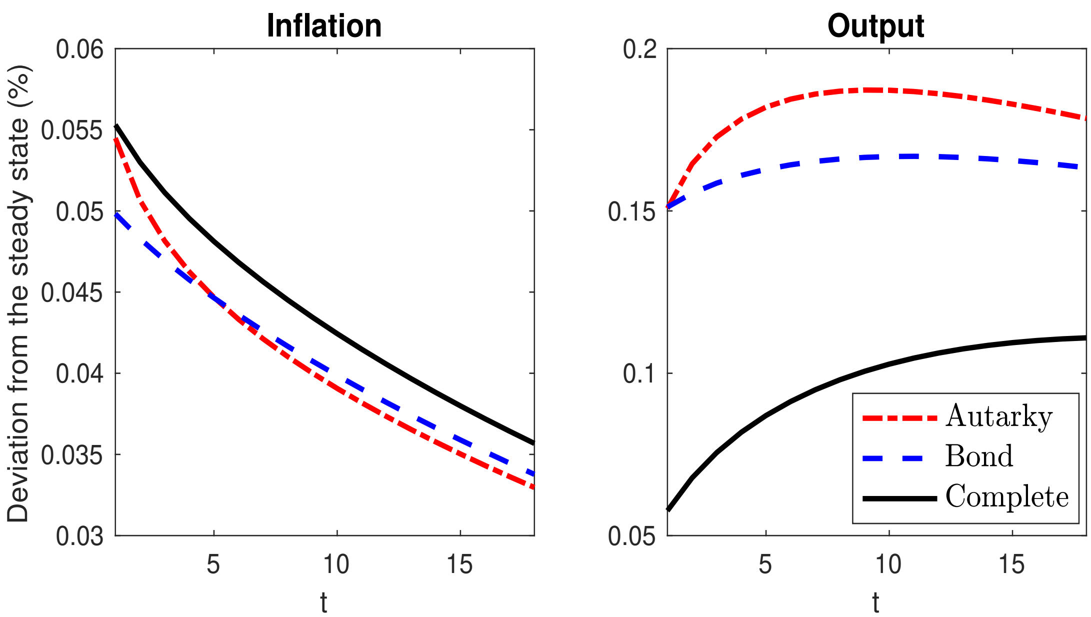


Figure 2.5: Response of inflation and output to a decrease in monetary shock in the first model with firm-specific labor markets.

Table 2.3: Relative cumulative responses of output – firm-specific labor and sector-specific households

	$CIRF_t^R$		
	$t = 1$	$t = 6$	$t = 12$
Financial autarky	2.619	2.227	2.011
Bond economy	2.623	2.067	1.826

2.5.2 Case II: Imperfect risk-sharing within sectors

We now turn to the second case in which risk-sharing is imperfect within each sector in addition to across sectors. A firm's frictionless optimal price in this case is given as:

$$\begin{aligned} p^{**}_{k,t}(i) &= \text{firm-}i \text{ nominal marginal costs} \\ &= p_t + (1+\varphi)\, y_t - (1+\varphi)\, a_{k,t} \qquad (2.14) \\ &- \underbrace{\eta\varphi\left(p_{k,t}-p_t\right)}_{\text{Segmented labor markets across sectors}} + \underbrace{\left(c_{k,t}-y_t\right)}_{\text{Imperfect risk-sharing across sectors}} \\ &- \underbrace{\theta\varphi\left(p^{**}_{k,t}(i)-p_{k,t}\right)}_{\text{Segmented labor markets within a sector}} + \underbrace{\left(c_{k,t}\left(i\right)-c_{k,t}\right)}_{\text{Imperfect risk-sharing within a sector}}, \end{aligned}$$

where the last term, $c_{k,t}\left(i\right)-c_{k,t}$, is new and reflects a within-sector wealth effect that is created by within-sector imperfect risk-sharing. This term, similar to the other term in the same line, renders prices less responsive irrespective of shock types. Whenever a firm raises its price (for whatever reasons), the demand for firm's output declines, which in turn decreases labor income and consumption of the household who work for the firm. This decreases the firm's marginal costs. Therefore, the firm does not raise its price as much.

The Phillips curve is obtained as:

$$\pi_t = \beta E_t\left[\pi_{t+1}\right] + \kappa^{II} y_t + \Theta^{II}_{y,t} + \Theta^{II}_{c,t}, \qquad (2.15)$$

where

$$\kappa^{II} \equiv \frac{1+\varphi}{1+\theta\varphi}\sum_{k=1}^{K} n_k \lambda^{II}_k\left(\epsilon\right), \ \ \Theta^{II}_{y,t} \equiv \frac{\varphi+\eta^{-1}}{1+\theta\varphi}\sum_{k=1}^{K} n_k \lambda^{II}_k\left(\epsilon\right) y^R_{k,t}, \ \ \Theta^{II}_{c,t} \equiv \frac{1}{1+\theta\varphi}\sum_{k=1}^{K} n_j \lambda^{II}_k\left(\epsilon\right) c^R_{k,t}.$$

The superscript II is used to denote the slope and shift terms in "*Case II*". Unlike the previous cases shown in (2.11) and (2.13), imperfect within-sector risk-sharing changes the Phillips curve in two dimensions. First, as before, it introduces the second shift term, $\Theta^{II}_{c,t}$. Second, it now reduces the slope of the Phillips curve. Under perfect within-sector risk-sharing, $\lambda_k^{II} = \lambda_k$, and the slope of the Phillips curve in (2.15) is identical to those in the previous Phillips curves. Otherwise, one can show that $\lambda_k^{II} < \lambda_k$, and moreover $\lambda_k^{II}(\epsilon)$ is decreasing in ϵ: $\frac{\partial \lambda_k^{II}(\epsilon)}{\partial \epsilon} < 0$. We prove this claim in the appendix.

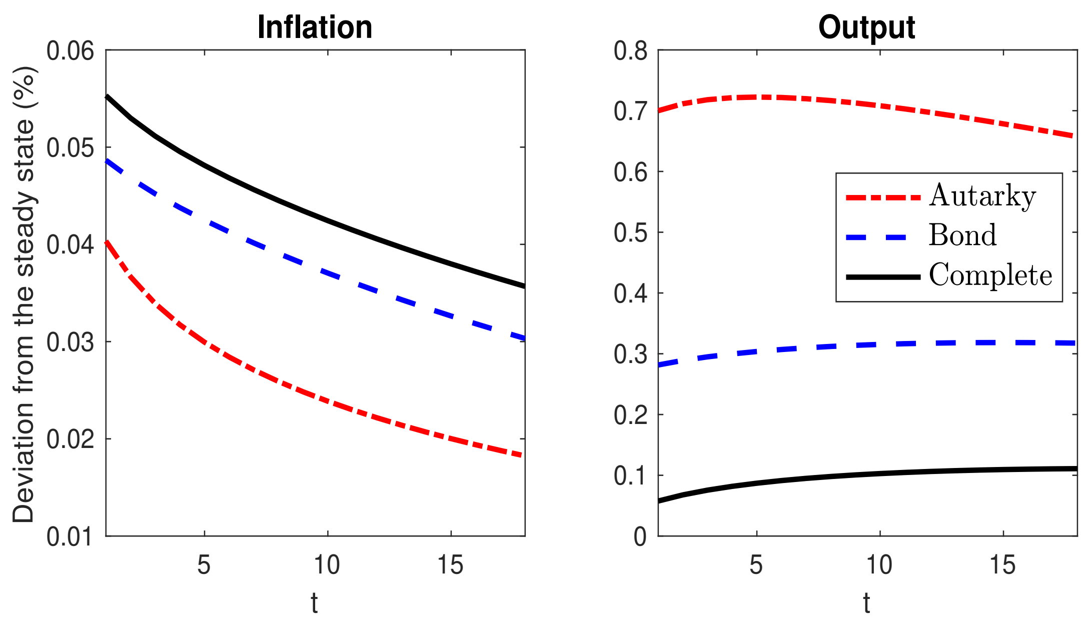


Figure 2.6: Response of inflation and output to a decrease in monetary shock in the second model with firm-specific labor markets.

We show numerical results in Figure 2.6 and Table 2.4. With the additional mechanism produced by within-sector imperfect risk-sharing, the model now generates even greater monetary non-neutrality than in the previous cases. Moreover, the gap in the extent of monetary non-neutrality between the model with complete markets and the models with incomplete markets widens even further. For example, the cumulative response of output over the 12-month horizon in the bond-only economy is now 243% more than that under complete market.

2.6 Sector-specific productivity shocks

Before concluding the paper, we return to the original model with sector-specific labor markets and illustrate the role of strategic substitutability in price setting in

Table 2.4: Relative cumulative responses of output – firm-specific labor and firm-specific households

	$CIRF_t^R$		
	$t = 1$	$t = 6$	$t = 12$
Financial autarky	11.810	9.146	7.897
Bond economy	4.884	3.851	3.427

amplifying price responses to sector-specific shocks. In particular, we compare the volatility of sectoral inflation and output, measured by their standard deviations, under incomplete asset markets to those under complete markets – when sectoral productivity shocks are the sole driving forces.

To this end, we shut down the monetary disturbances and instead let each sectoral productivity, $\{a_{k,t}\}$, follow an independent AR(1) process with the autoregressive coefficient set to $0.95^{1/3}$ and the standard deviation of its innovations set to 10%.[17] The model is then simulated for 100,000 time periods under complete and incomplete markets with the same realized shocks. The standard deviations – our volatility measure – are estimated on the simulated time series.

Figure 2.7 shows the cross-sectional distribution of the volatility of sectoral inflation under complete markets and under financial autarky.[18] The strategic substitutability induced by imperfect risk-sharing shifts the distribution to the right, increasing the mean of the distribution by 2.03 percentage points (from 3.88% to 5.91%).[19] Sectoral inflation is thus more volatile under incomplete markets, implying that sectoral prices tend to respond by more to the sector-specific shocks when households cannot share their labor income risks. So the exercise in this section confirm that our proposed mechanism, as discussed in section 2.3.1, does not produce the implausible micro-level implications outlined in the introduction.

[17] The estimates of the standard deviations of sectoral productivity shocks are substantially larger than those of aggregate shocks in the literature (e.g., Carvalho et al. (2021)). Midrigan (2011) shows that a monthly standard deviation of idiosyncratic shocks should be between 8% and 11.2% to explain highly volatile prices. Pasten et al. (2020) set the autoregressive coefficient to unity (i.e. a Random walk process for sectoral productivity shocks) and show that the mean of the estimated standard deviations of innovations is 9.92%.

[18] Here we omit the results from the bond-only economy.

[19] The strategic substitutability induced by imperfect risk-sharing increases the standard deviation of aggregate inflation by 0.3 percentage points.

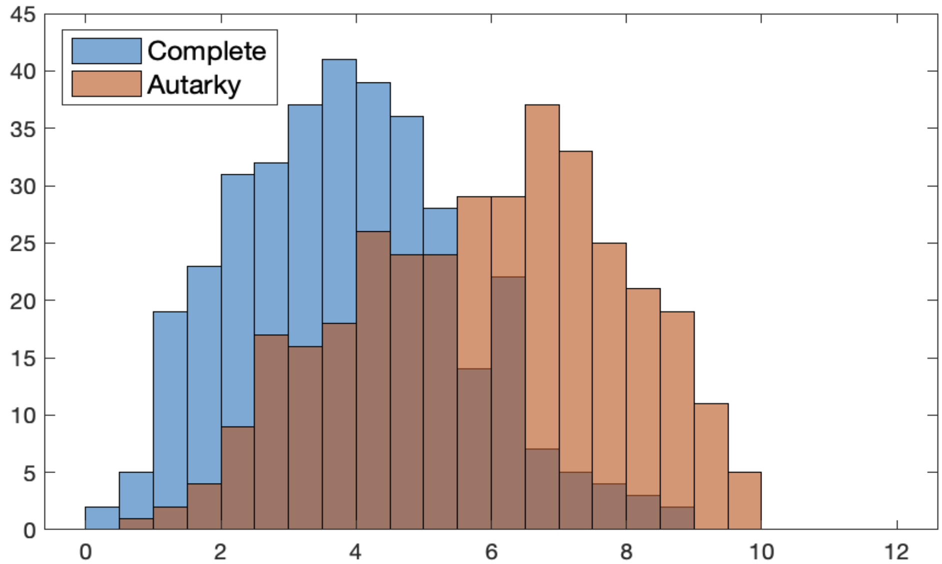


Figure 2.7: Cross-sectional distribution of volatility (%) of sectoral inflation

Finally, as a side note, we point out that sectoral outputs respond by more to sector-specific productivity shocks under imperfect risk-sharing as a consequence of greater price responses. Figure 2.8 reveals that imperfect risk-sharing shifts the cross-sectional distribution of the volatility of sectoral output to the right; that is, sectoral outputs are more volatile. This in turn leads to an increase in the volatility of aggregate output – although a large portion of sectoral output movements is averaged out. The standard deviation of aggregate output (driven by the idiosyncratic shocks) rises by 0.35 percentage points thanks to the strategic substitutability.

Clearly, we do not view that the current model provides an adequate framework for a quantitative analysis of the propagation of idiosyncratic shocks, and we do not parameterize the model to match any moments in the data. Our analysis, however, does suggest that the pricing interactions emphasized in this paper have a potential to generate an interesting propagation mechanism of such shocks.

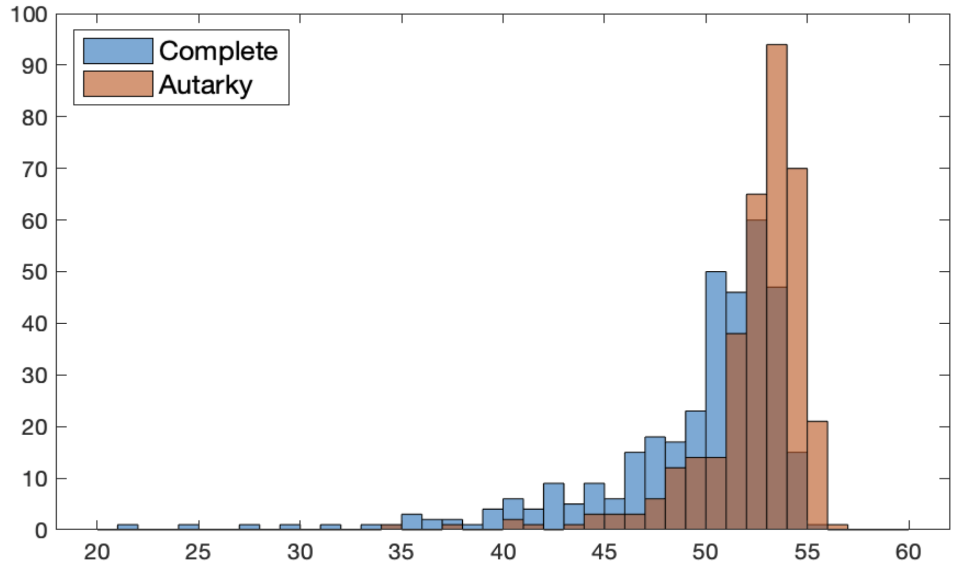


Figure 2.8: Cross-sectional distribution of volatility (%) of sectoral output

2.7 Conclusion

A majority of recent multisector sticky-price models used for quantitative analyses features segmented labor markets at the sectoral level (e.g., Carvalho et al. (2021); Pasten et al. (2019); Pasten et al. (2020); Smets et al. (2018)). Besides its plausibility, the feature has become popular also because the resulting sector-specific wages help the models generate dynamics more consistent with macro- and micro-evidence. However, these models either rely on the representative-household abstraction, or equivalently assume perfect risk-sharing among households of different sectors – despite ample evidence that labor income risk-sharing is not as ideal as complete-market models predict due to a myriad of financial frictions. Apart from tractability and simplicity, the representative-household abstraction would be justified if such household heterogeneity did not affect aggregate dynamics significantly.

Our study shows that this is not the case. Imperfect risk-sharing among households of different sectors generates pricing interactions that dampen price responses to aggregate shocks while allowing for large price responses to idiosyncratic shocks.

While this statement holds for any aggregate shocks, we have focused on monetary shocks in this paper. Our numerical analysis shows that the mechanism has a potential to produce significant monetary non-neutrality – without sacrificing volatile individual prices. That being said, our model is stylized, so more work is necessary to answer how such household heterogeneity matters in more elaborate settings.

In future work, it would be interesting to see the normative implications of risk-sharing. Aoki (2001), Benigno (2004), Mankiw and Reis (2003b), and Eusepi et al. (2011) study optimal monetary policy in a multisector framework similar to ours, and propose that the central bank should stabilize a price index weighted disproportionately toward low-frequency sectors instead of Consumer Price Index or Personal Consumption Expenditure Price Index. However, the weights on sectoral prices in the central bank's target price index would also have distributional implications for households of different sectors: Assigning a disproportionately large weight on certain sectors might benefit some households at the expense of other households. Consequently, the optimal weights would differ depending on asset market environment.

In this paper, we have focused on the propagation of aggregate shocks – in particular, monetary policy disturbances. However, our discussion in section 2.3.1 and in section 2.6 suggests that asset market assumptions matter for the propagations of idiosyncratic shocks: Idiosyncratic productivity shocks collectively may contribute more to aggregate output fluctuations with imperfect risk-sharing. A quantitative analysis with a multisector sticky-price model with detailed input-output production linkages and heterogeneous households seems promising. We leave this potentially interesting endeavor for future research.

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Appendix A

Optimal Monetary Policy under Heterogeneous Consumption Baskets

A.1 Proofs

A.1.1 Proof of Proposition 1.1

Proof. Assume that the efficient allocation can be supported by the market outcome. Then, we have

$$w_t = w_t^E \quad \text{and} \quad q_t = q_t^E$$

Substituting them into the labor market clearing condition and the labor supply schedule of type U households,

$$n_{U,t} = \frac{1-\sigma}{\sigma+\varphi}\left(w_t - q_t\right)$$

Combining the budget constraint and the labor supply schedules of type C households,

$$n_{C,t} = \frac{1-\sigma}{\sigma+\varphi} w_t$$

Labor supply from each type of households are different as long as $q_t^E = a_{1,t} - a_{2,t} \neq 0$

and $\sigma \neq 1$

$$n_{C_t} - n_{U_t} = -\frac{1-\sigma}{\sigma+\varphi} q_t^E$$

and the efficient condition does not hold. This contracts to the assumption. □

A.1.2 Proof of Proposition 1.2

Proof. 1) It is obvious by Eq.(1.30) and Eq.(1.31).
2) Assume that closing both output gaps is feasible, $\widetilde{y}_{1,t} = \widetilde{y}_{2,t} = 0$. Then by labor supply schedule of both types of households, type C households' budget constraint, and labor market clearing condition, we have

$$\begin{aligned}
\widetilde{n}_{C,t} &= \frac{1-\sigma}{\sigma}\frac{1}{1+\varphi}z_2 q_t^E \\
\widetilde{n}_{U,t} &= -\frac{1-\sigma}{\sigma}\frac{1}{1+\varphi}z_1 q_t^E \\
\widetilde{w}_t &= \frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}z_2 q_t^E \\
\widetilde{q}_t &= \frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}q_t^E \\
\widetilde{w}_t - \widetilde{q}_t &= -\frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}z_1 q_t^E
\end{aligned}$$

If there is no nominal friction in either sector, constant markup leads to $\widetilde{w}_t = 0$ or $\widetilde{w}_t - \widetilde{q}_t = 0$, which contradicts to the solution for $\widetilde{w}_t$ or $\widetilde{w}_t - \widetilde{q}_t$ derived above.

If nominal friction exists in both sectors, we have by the Phillips curve that

$$\begin{aligned}
\pi_{1,t} &= \beta E_t[\pi_{1,t+1}] + \kappa_1 \frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}z_2 q_t^E \\
&= \kappa_1 \frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}z_2 \sum_{s=0}^{\infty}\beta^s E_t q_{t+s}^E \\
&= \kappa_1 \frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}z_2 \frac{1}{1-\beta\rho_a} q_t^E
\end{aligned}$$

where we assume $\rho_{a_1} = \rho_{a_2} = \rho_a$. Similarly,

$$\pi_{2,t} = -\kappa_2 \frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}z_1 \frac{1}{1-\beta\rho_a} q_t^E$$

However, above solutions contradicts to the definition of relative price:

$$
\begin{aligned}
\widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E &= \frac{\sigma+\varphi}{\sigma(1+\varphi)}(q_t^E - q_{t-1}^E) \\
&\neq -\frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}\frac{1}{1-\beta\rho_a}(z_1\kappa_2 + z_2\kappa_1)q_t^E = \pi_{2,t} - \pi_{1,t}
\end{aligned}
$$

□

A.1.3 Proof of Proposition 1.3

Proof. We follow Woodford (2003) in deriving the welfare-theoretic loss function. Note first that under the assumptions on employment subsidy and government transfers, the steady state is efficient and equitable, $N_{C,t}=N_{U,t}=N=C_{C,t}=C_{U,t}=C=Y=1$ with the wage and the relative price being unity, $W=Q=1$. Thus we have

$$\frac{V_N(N_{U,t})}{U_C(C_{U,t})} = W = 1 = \frac{W}{Q} = \frac{V_N(N_{c,t})}{U_C(C_{C,t})}$$

Define $h_U \equiv 1-\lambda$ and $h_C \equiv \lambda$. Taking a second-order approximation to the equally weighted sum of both types of households' utilities around the efficient zero-inflation steady state,

$$\begin{aligned}
&\sum_{j=U,C} h_j \mathcal{U}(C_{h,t}, N_{h,t}) \\
&= \sum_{j=U,C} h_j \left[\begin{array}{c} U_c Y\{c_{j,t} + \frac{1-\sigma}{2} c_{j,t}^2\} \\ -V_N N\{n_{j,t} + \frac{1-\sigma}{2} n_{j,t}^2\} \end{array} \right] + t.i.p. + o(||\xi||^3) \\
&= U_c Y \left[\begin{array}{c} (1-\lambda)\{\widetilde{c}_{U,t} + \frac{1-\sigma}{2}\widetilde{c}_{U,t}^2 + (1-\sigma)c_{U,t}^E \widetilde{c}_{U,t}\} + \lambda\{\widetilde{c}_{C,t} + \frac{1-\sigma}{2}\widetilde{c}_{C,t}^2 + (1-\sigma)c_{C,t}^E\widetilde{c}_{C,t}\} \\ -(1-\lambda)\{\widetilde{n}_{U,t} + \frac{1+\varphi}{2}\widetilde{n}_{U,t}^2 + (1+\varphi)n_{U,t}^E\widetilde{n}_{U,t}\} - \lambda\{\widetilde{n}_{C,t} + \frac{1+\varphi}{2}\widetilde{n}_{C,t}^2 + (1+\varphi)n_{C,t}^E\widetilde{n}_{C,t}\} \end{array} \right] \\
&\quad + t.i.p. + o(||\xi||^3) \qquad \text{(A.1)}
\end{aligned}$$

Taking a second order approximation to the labor market clearing condition,

$$\begin{aligned}
&\omega(\widetilde{n}_{1,t} + \frac{1}{2}\widetilde{n}_{1,t}^2 + n_{1,t}^E\widetilde{n}_{1,t}) + (1-\omega)(\widetilde{n}_{2,t} + \frac{1}{2}\widetilde{n}_{2,t}^2 + n_{2,t}^E\widetilde{n}_{2,t}) \\
&= (1-\lambda)(\widetilde{n}_{U,t} + \frac{1}{2}\widetilde{n}_{U,t}^2 + n_{U,t}^E\widetilde{n}_{U,t}) + \lambda(\widetilde{n}_{C,t} + \frac{1}{2}\widetilde{n}_{C,t}^2 + n_{C,t}^E\widetilde{n}_{C,t}) + t.i.p. + o(||\xi||^3)
\end{aligned} \qquad \text{(A.2)}$$

Let us define $\hat{p}_{j,t}(i) \equiv p_{j,t}(i) - p_{j,t}$. Then, by a second order approximation,

$$\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{1-\theta} = e^{(1-\theta)\hat{p}_{j,t}(i)} = 1 + (1-\theta)\hat{p}_{j,t}(i) + \frac{(1-\theta)^2}{2}\hat{p}_{j,t}^2(i) + o(||\xi||^3) \qquad \text{(A.3)}$$

Since $\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{1-\theta}di = 1$ by the price aggregator, we integrate Eq.(A.3) to derive

$$E_i^j\{\hat{p}_{j,t}(i)\} = \frac{\theta-1}{2}E_i^j\{\hat{p}_{j,t}^2(i)\} \tag{A.4}$$

Similarly, taking a second order approximation, integrating the result, and substituting Eq.(A.4),

$$\begin{aligned}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} &= 1-\theta\hat{p}_{j,t}(i)+\frac{\theta^2}{2}\hat{p}_{j,t}^2(i)+o(||\xi||^3)\\ \frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} &= 1-\theta E_i^j\{\hat{p}_{j,t}(i)\}+\frac{\theta^2}{2}E_i^j\{\hat{p}_{j,t}^2(i)\}+o(||\xi||^3)\\ &= 1+\frac{\theta^2}{2}E_i^j\{\hat{p}_{j,t}^2(i)\}+o(||\xi||^3)\end{aligned} \tag{A.5}$$

Since $E_i^j\{\hat{p}_{j,t}^2(i)\} = \frac{1}{z_j}\int_{\mathcal{I}_j}\hat{p}_{j,t}^2(i)di = \frac{1}{z_j}\int_{\mathcal{I}_j}\left(p_{j,t}(i)-p_{j,t}\right)^2 di$, and we know that in the first order $p_{j,t} = E_i^j\{p_{j,t}(i)\}$, we derive that $E_i^j\{\hat{p}_{j,t}^2(i)\} = Var_i^j\{p_{j,t}(i)\}$. Substituting this into Eq.(A.5),

$$\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} = 1+\frac{\theta^2}{2}Var_i^j\{p_{j,t}(i)\}+o(||\xi||^3) \tag{A.6}$$

Thus we derive the second order approximation to the price dispersion in each sector as

$$d_{j,t} \equiv \log\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} = \frac{\theta^2}{2}Var_i^j\{p_{j,t}(i)\}+o(||\xi||^3) \tag{A.7}$$

We have $N_{j,t} = \int_{\mathcal{I}_j}\frac{Y_{j,t}(i)}{A_tA_{j,t}}di = \frac{1}{z_j}\frac{Y_{j,t}}{A_tA_{j,t}}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta}di$ by the relative demand function. Taking a second order approximation and substituting Eq.(A.7), we derive

$$
\begin{aligned}
n_{j,t} &= y_{j,t} - a_t - a_{j,t} + \log \frac{1}{z_j} \int_{\mathcal{I}_j} \left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} di + o(||\xi||^3) \\
&= y_{j,t} - a_t - a_{j,t} + \frac{\theta}{2} Var_i^j\{p_{j,t}(i)\} + o(||\xi||^3) \\
n_{j,t}^2 &= y_{j,t}^2 + a_t^2 + a_{j,t}^2 - 2(a_t + a_{j,t}) y_{j,t} + 2 a_t a_{j,t} + o(||\xi||^3) \\
\Rightarrow\ \widetilde{n}_{j,t} &= \widetilde{y}_{j,t} + \frac{\theta}{2} Var_i^j\{p_{j,t}(i)\} + o(||\xi||^3) \qquad \text{(A.8)} \\
\widetilde{n}_{j,t}^2 + 2 n_{j,t}^E \widetilde{n}_{j,t} &= \widetilde{y}_{j,t}^2 + 2 y_{j,t}^E \widetilde{y}_{j,t} - 2(a_t + a_{j,t}) \widetilde{y}_{j,t} + t.i.p. + o(||\xi||^3) \qquad \text{(A.9)}
\end{aligned}
$$

Substituting Eqs.(A.2), (A.8) and (A.9) into Eq.(A.1), and canceling out the cross terms,

$$
\text{(A.1)} = -\frac{U_c Y}{2} \left[\begin{array}{c} \omega \theta Var_i^1\{p_{1,t}(i)\} + (1-\omega) \theta Var_i^2\{p_{2,t}(i)\} \\ +\sigma(1-\lambda) \widetilde{c}_{U,t}^2 + \sigma \lambda \widetilde{c}_{C,t}^2 \\ +\varphi(1-\lambda) \widetilde{n}_{U,t}^2 + \varphi \lambda \widetilde{n}_{C,t}^2 \\ +\frac{1}{\eta} \omega(1-\omega)(\widetilde{y}_{1,t} - \widetilde{y}_{2,t})^2 \end{array} \right] + t.i.p. + o(||\xi||^3) \qquad \text{(A.10)}
$$

Let us define $\Delta_t^j \equiv Var_i^j\{P_{j,t}(i)\}$. According to Woodford (2003),

$$
\begin{aligned}
\Delta_t^j &= \alpha_j \Delta_{t-1}^j + \frac{\alpha_j}{1-\alpha_j} \pi_{j,t}^2 + o(||\xi||^3) \\
&= \underbrace{\alpha_j^{t+1} \Delta_{-1}^j}_{t.i.p.} + \sum_{k=0}^{t} \alpha_j^{t-k} \frac{\alpha_j}{1-\alpha_j} \pi_{j,t}^2 + o(||\xi||^3)
\end{aligned}
$$

and the present valued sum of the cross-sectional price dispersion can be rewritten in terms of present valued sum of squared inflation as

$$
\sum_{t=0}^{\infty} \beta^t \Delta_t^j = \frac{\alpha_j}{(1-\alpha_j)(1-\alpha_j \beta)} \sum_{t=0}^{\infty} \beta^t \pi_{j,t}^2 + t.i.p. + o(||\xi||^3) \qquad \text{(A.11)}
$$

Substituting Eq.(A.11) into Eq.(A.10), and summing up the present valued utilities,

$$E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=U,C} h_j \mathcal{U}(C_{h,t}, N_{h,t}) = -\frac{U_c Y}{2} E_0 \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} \omega \frac{\theta}{\kappa_1} \pi_{1,t}^2 + (1-\omega) \frac{\theta}{\kappa_2} \pi_{2,t}^2 \\ +\sigma(1-\lambda) \widetilde{c}_{U,t}^2 + \sigma \lambda \widetilde{c}_{C,t}^2 \\ +\varphi(1-\lambda) \widetilde{n}_{U,t}^2 + \varphi \lambda \widetilde{n}_{C,t}^2 \\ +\frac{1}{\eta} \omega (1-\omega)(\widetilde{y}_{1,t} - \widetilde{y}_{2,t})^2 \end{bmatrix}$$
$$+ t.i.p. + o(||\xi||^3) \tag{A.12}$$

By the final good market clearing condition,

$$\widetilde{y}_t = (1-\lambda)\widetilde{c}_{U,t} + \lambda \widetilde{c}_{C,t} + o(||\xi||^2)$$
$$\Rightarrow (1-\lambda)\widetilde{c}_{U,t}^2 + \lambda \widetilde{c}_{C,t}^2 = \widetilde{y}_t^2 + \lambda(1-\lambda)(\widetilde{c}_{U,t} - \widetilde{c}_{C,t})^2 + t.i.p. + o(||\xi||^3) \tag{A.13}$$

By the output aggregator and the labor market clearing condition,

$$\widetilde{y}_t = \omega \widetilde{y}_{1,t} + (1-\omega)\widetilde{y}_{2,t} = \omega \widetilde{n}_{1,t} + (1-\omega)\widetilde{n}_{2,t} = (1-\lambda)\widetilde{n}_{U,t} + \lambda \widetilde{n}_{C,t} + o(||\xi||^2)$$
$$\Rightarrow (1-\lambda)\widetilde{n}_{U,t}^2 + \lambda \widetilde{n}_{C,t}^2 = \widetilde{y}_t^2 + \lambda(1-\lambda)(\widetilde{n}_{U,t} - \widetilde{n}_{C,t})^2 + t.i.p. + o(||\xi||^3) \tag{A.14}$$

By the price aggregator,

$$p_{1,t} - p_t = -(1-\omega) q_t - \frac{1-\eta}{2} \omega (1-\omega) q_t^2 + o(||\xi||^3) \tag{A.15}$$
$$p_{2,t} - p_t = \omega q_t - \frac{1-\eta}{2} \omega (1-\omega) q_t^2 + o(||\xi||^3) \tag{A.16}$$

Since we have exact relative demand functions in terms of relative price and aggregate output,

$$y_{1,t} = -\eta(p_{1,t} - p_t) + y_t \tag{A.17}$$
$$y_{2,t} = -\eta(p_{2,t} - p_t) + y_t \tag{A.18}$$

Substituting Eqs.(A.15)-(A.16) into Eqs.(A.17)-(A.18),

$$y_{1,t} = y_t + (1-\omega)\eta q_t + \omega(1-\omega)\frac{\eta(1-\eta)}{2}q_t^2 + o(||\xi||^3) \tag{A.19}$$

$$y_{2,t} = y_t - \omega\eta q_t + \omega(1-\omega)\frac{\eta(1-\eta)}{2}q_t^2 + o(||\xi||^3) \tag{A.20}$$

Subtracting Eq.(A.20) from Eq.(A.19), and rewriting in terms of gaps,

$$(\widetilde{y}_{1,t}-\widetilde{y}_{2,t})^2 = \eta^2\widetilde{q}_t^2 + o(||\xi||^3) \tag{A.21}$$

substituting Eqs.(A.13)-(A.14) and (A.21) into Eq.(A.12),

$$\text{(A.12)} = -\frac{U_cY}{2}E_0\sum_{t=0}^{\infty}\beta^t\begin{bmatrix} \omega\frac{\theta}{\kappa_1}\pi_{1,t}^2 + (1-\omega)\frac{\theta}{\kappa_2}\pi_{2,t}^2 \\ +(\sigma+\varphi)\widetilde{y}_t^2 \\ +\sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t})^2 \\ +\varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t})^2 \\ +\frac{1}{\eta}\omega(1-\omega)(\widetilde{y}_{1,t}-\widetilde{y}_{2,t})^2 \end{bmatrix} + t.i.p. + o(||\xi||^3)$$

$$= -\frac{U_cY}{2}E_0\sum_{t=0}^{\infty}\beta^t\begin{bmatrix} \omega\frac{\theta}{\kappa_1}\pi_{1,t}^2 + (1-\omega)\frac{\theta}{\kappa_2}\pi_{2,t}^2 \\ +(\sigma+\varphi)\widetilde{y}_t^2 \\ +\sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t})^2 \\ +\varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t})^2 \\ +\eta\omega(1-\omega)\widetilde{q}_t^2 \end{bmatrix} + t.i.p. + o(||\xi||^3) \tag{A.22}$$

We can simplify the loss function further by deriving the first order relations between distributional variables and aggregate variables using the equilibrium conditions on the household side,

$$\begin{aligned}(1-\lambda)\widetilde{c}_{U,t} + \lambda\widetilde{c}_{C,t} &= (1-\lambda)\widetilde{n}_{U,t} + \lambda\widetilde{n}_{C,t} + o(||\xi||^2) \\ \varphi\widetilde{n}_{U,t} + \sigma\widetilde{c}_{U,t} &= \widetilde{w}_t - (1-\omega)\widetilde{q}_t + o(||\xi||^2) \\ \varphi\widetilde{n}_{C,t} + \sigma\widetilde{c}_{C,t} &= \widetilde{w}_t - (1-\omega)\widetilde{q}_t + o(||\xi||^2) \\ \widetilde{c}_{C,t} &= \widetilde{n}_{C,t} + \widetilde{w}_t - (1-\omega)\widetilde{q}_t + o(||\xi||^2)\end{aligned}$$

from which we derive the following relations in the first order:

$$\widetilde{n}_{U,t} = \frac{1-\lambda(1-\sigma)}{(1-\lambda)(1-\sigma)}\widetilde{n}_{C,t} + o(||\xi||^2) \tag{A.23}$$

$$\widetilde{c}_{U,t} = \frac{1-\lambda(1+\varphi)}{(1-\lambda)(1+\varphi)}\widetilde{c}_{C,t} + o(||\xi||^2) \tag{A.24}$$

Since $\widetilde{y}_t = (1-\lambda)\widetilde{c}_{U,t} + \lambda\widetilde{c}_{U,t} = (1-\lambda)\widetilde{n}_{U,t} + \lambda\widetilde{n}_{U,t}$ in the first order,

$$\widetilde{c}_{C,t} = (1+\varphi)\widetilde{y}_t \tag{A.25}$$

$$\widetilde{c}_{U,t} = \frac{1-\lambda(1+\varphi)}{1-\lambda}\widetilde{y}_t \tag{A.26}$$

$$\widetilde{n}_{C,t} = (1-\sigma)\widetilde{y}_t \tag{A.27}$$

$$\widetilde{n}_{U,t} = \frac{1-\lambda(1-\sigma)}{1-\lambda}\widetilde{y}_t \tag{A.28}$$

substituting Eqs.(A.25)-(A.28) into Eq.(A.22), we finally have that

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=U,C} h_j \mathcal{U}(C_{h,t}, N_{h,t}) \\ &= -\frac{U_c Y}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \omega \frac{\theta}{\kappa_1} \pi_{1,t}^2 + (1-\omega) \frac{\theta}{\kappa_2} \pi_{2,t}^2 \\ +(\sigma+\varphi) \frac{1-\lambda(1-\sigma\varphi)}{1-\lambda} \widetilde{y}_t^2 \\ +\eta\omega(1-\omega)\widetilde{q}_t^2 \end{array} \right] + t.i.p. + o(||\xi||^3) \end{aligned} \tag{A.29}$$

Note that $\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}$ is increasing in λ, implying that as the share of the financially constrained households increases, output stabilization becomes relatively more important than price stabilization. If $\alpha_1 = \alpha_2 = \alpha$ and thus $\kappa_1 = \kappa_2 = \kappa$, we can rewrite Eq.(A.29) as

$$(\text{A.29}) = -\frac{U_c Y}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \frac{\theta}{\kappa} \pi_t^2 + \omega(1-\omega) \frac{\theta}{\kappa} (\pi_{2,t} - \pi_{1,t})^2 \\ +(\sigma+\varphi) \frac{1-\lambda(1-\sigma\varphi)}{1-\lambda} \widetilde{y}_t^2 \\ +\eta\omega(1-\omega)\widetilde{q}_t^2 \end{array} \right] + t.i.p. + o(||\xi||^3) \tag{A.30}$$

□

A.1.4 Proof of Proposition 1.4

Proof. We solve a Ramsey problem of the utilitarian central bank when the price of sector 1 is flexible and the price of sector 2 is sticky, $\alpha_1 = 0$, under **HomCB**. The opposite case will be exactly symmetric under homogeneous consumption baskets. We set up the Lagrangian as:

$$\mathcal{L}_t = \frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} (1-\omega)\frac{\theta}{\kappa_2}\pi_{2,t}^2 + (\sigma+\varphi)\widetilde{y}_t^2 + \eta\omega(1-\omega)\widetilde{q}_t^2 \\ +\sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t})^2 + \varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t})^2 \end{array} \right]$$
$$+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{1,t} \left\{ \pi_{2,t} - \beta\pi_{2,t+1} + \kappa_2 \widetilde{q}_t \right\}$$
$$+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{2,t} \left\{ \widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E - \pi_{2,t} + \pi_{1,t} \right\}$$
$$+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{3,t} \left\{ \omega\widetilde{y}_{1,t} + (1-\omega)\widetilde{y}_{2,t} - (1-\lambda)\widetilde{n}_{U,t} - \lambda\widetilde{n}_{C,t} \right\}$$
$$+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{4,t} \left\{ \varphi\widetilde{n}_{U,t} + \sigma\widetilde{c}_{U,t} + (1-\omega)\widetilde{q}_t \right\}$$
$$+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{5,t} \left\{ \varphi\widetilde{n}_{C,t} + \sigma\widetilde{c}_{C,t} + (1-\omega)\widetilde{q}_t \right\}$$
$$+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{6,t} \left\{ \widetilde{c}_{C,t} - \widetilde{n}_{C,t} - \widetilde{w}_t + (1-\omega)\widetilde{q}_t \right\}$$
$$+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{7,t} \left\{ \widetilde{y}_{1,t} - (1-\lambda)\widetilde{c}_{U,t} - \lambda\widetilde{c}_{C,t} - \eta(1-\omega)\widetilde{q}_t \right\}$$
$$+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{8,t} \left\{ \widetilde{y}_{2,t} - (1-\lambda)\widetilde{c}_{U,t} - \lambda\widetilde{c}_{C,t} + \eta\omega\widetilde{q}_t \right\}$$
$$+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{9,t} \left\{ \widetilde{y}_t - \omega\widetilde{y}_{1,t} - (1-\omega)\widetilde{y}_{2,t} \right\}$$

where $\{\psi_{1,t}\}, \cdots, \{\psi_{9,t}\}$ are the Lagrange multipliers.

First order conditions are as follows:

$$
\begin{aligned}
\partial\pi_{1,t} &: 0 = \psi_{2,t} \\
\partial\pi_{2,t} &: 0 = (1-\omega)\theta\pi_{2,t} + \kappa_2(\psi_{1,t} - \psi_{1,t-1}) - \kappa_2\psi_{2,t} \\
\partial\widetilde{y}_{1,t} &: 0 = \omega\psi_{3,t} + \psi_{7,t} - \omega\psi_{9,t} \\
\partial\widetilde{y}_{2,t} &: 0 = (1-\omega)\psi_{3,t} + \psi_{8,t} - (1-\omega)\psi_{9,t} \\
\partial\widetilde{c}_{U,t} &: 0 = \sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t}) + \sigma\psi_{4,t} - (1-\lambda)\psi_{7,t} - (1-\lambda)\psi_{8,t} \\
\partial\widetilde{c}_{C,t} &: 0 = -\sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t}) + \sigma\psi_{5,t} + \psi_{6,7} - \lambda\psi_{7,t} - \lambda\psi_{8,t} \\
\partial\widetilde{n}_{U,t} &: 0 = \varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t}) - (1-\lambda)\psi_{3,t} + \varphi\psi_{4,t} \\
\partial\widetilde{n}_{C,t} &: 0 = -\varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t}) - \lambda\psi_{3,t} + \varphi\psi_{5,t} - \psi_{6,t} \\
\partial\widetilde{q}_t &: 0 = \eta\omega(1-\omega)\widetilde{q}_t + \kappa_2\psi_{1,t} + \psi_{2,t} - \beta E_t[\psi_{2,t+1}] + (1-\omega)\psi_{4,t} + (1-\omega)\psi_{5,t} + (1-\omega)\psi_{6,t} \\
&\quad - \eta(1-\omega)\psi_{7,t} + \eta\omega\psi_{8,t} \\
\partial\widetilde{y}_t &: 0 = (\sigma+\varphi)\widetilde{y}_t + \varphi_{9,t}
\end{aligned}
$$

Simplifying first order conditions, they reduce down to two equations:

$$
(1-\omega)\theta\pi_{2,t} + \kappa_2(\psi_{1,t} - \psi_{1,t-1}) = 0 \tag{A.31}
$$

$$
\kappa_2\psi_{1,t} = -\eta\omega(1-\omega)\widetilde{q}_t + (1-\omega)\left(\widetilde{y}_t - \frac{\sigma\varphi\lambda}{\sigma+\varphi}(\widetilde{c}_{U,t}-\widetilde{c}_{C,t}-\widetilde{n}_{U,t}+\widetilde{n}_{C,t})\right) \tag{A.32}
$$

By using Lagrangian constraints, we rewrite Eq.(A.32) in terms of $\widetilde{q}_t$,

$$
\kappa_2\psi_{1,t} = -(1-\omega)\left(\eta\omega + \frac{1-\omega}{\sigma+\varphi}\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}\right)\widetilde{q}_t \tag{A.33}
$$

Substituting Eq.(A.33) into Eq.(A.31), and using the Phillips curve in sector 2, we derive a second-order difference equation where $\phi \equiv \frac{1}{\theta}\left(\eta\omega + \frac{1-\omega}{\sigma+\varphi}\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}\right) > 0$ in this proof:

$$
E_t[\widetilde{q}_{t+1}] - \left(1 + \frac{1}{\beta} + \frac{\kappa_2}{\beta\phi}\right)\widetilde{q}_t + \frac{1}{\beta}\widetilde{q}_{t-1} = 0
$$

Solving the equation, we find $\widetilde{q}_t = \lambda_2\widetilde{q}_{t-1}$ where the two eigenvalues satisfies $0 < \lambda_2 <$

$1 < \lambda_1$.

Assuming that all the variables are in the steady state initially including $\widetilde{q}_{-1} = 0$, the dynamics under optimal monetary policy achieves efficiency as follows:

$$\widetilde{q}_t^{OMP} = \widetilde{w}_t^{OMP} = \pi_{2,t}^{OMP} = \widetilde{y}_t^{OMP} = \widetilde{y}_{1,t}^{OMP} = \widetilde{y}_{2,t}^{OMP} = \widetilde{c}_{U,t}^{OMP} = \widetilde{c}_{C,t}^{OMP} = \widetilde{n}_{U,t}^{OMP} = \widetilde{n}_{C,t}^{OMP} = 0$$
$$\pi_{1,t}^{OMP} = -q_t^E + q_{t-1}^E$$

□

A.1.5 Proof of Proposition 1.5

Proof. We solve a Ramsey problem of the utilitarian central bank when the prices of both sectors are sticky to the same degree, $0 < \alpha_1 = \alpha_2 = \alpha$, under **HomCB**. The set-up of Lagrangian is the same as that in the proof of Proposition 1.6 except that we have $\kappa_1 = \kappa_2 = \kappa$ now.

Rewriting Lagrangian constraints corresponding to $\{\psi_{1,t}\}, \cdots, \{\psi_{3,t}\}$,

$$\pi_{1,t} - \beta E_t[\pi_{1,t+1}] = \kappa \widetilde{w}_t \tag{A.34}$$

$$\pi_{2,t} - \beta E_t[\pi_{2,t+1}] = \kappa \widetilde{w}_t - \kappa \widetilde{q}_t \tag{A.35}$$

$$\widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E = \pi_{2,t} - \pi_{1,t} \tag{A.36}$$

Aggregating Eqs.(A.34)-(A.35) with sector size,

$$\pi_t - \beta E_t[\pi_{t+1}] = \kappa(\widetilde{w}_t - (1-\omega)\widetilde{q}_t) \tag{A.37}$$

Substituting Eqs.(A.48)-(A.51) into eq.(A.37),

$$\pi_t - \beta E_t[\pi_{t+1}] = \kappa(1+\varphi)\widetilde{y}_t \tag{A.38}$$

Rewriting Eqs.(A.52)-(A.53),

$$\pi_t = -\frac{1}{\theta}\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}(\widetilde{y}_t - \widetilde{y}_{t-1}) \tag{A.39}$$

$$\begin{aligned} 0 = &-(1-\omega)\theta\pi_{2,t} + \kappa\psi_{3,t} + A(L)\psi_{3,t} - \beta A(L)E_t[\psi_{3,t+1}] + \eta\omega(1-\omega)A(L)\widetilde{q}_t \\ &- (1-\omega)\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t \end{aligned} \tag{A.40}$$

Substituting Eq.(A.39) into Eq.(A.38), we derive a second order difference equation where $\phi \equiv \frac{1}{\theta}\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda} > 0$ in this proof:

$$E_t[\widetilde{y}_{t+1}] - \left(1 + \frac{1}{\beta} + \frac{\kappa(1+\varphi)}{\beta\phi}\right)\widetilde{y}_t + \frac{1}{\beta}\widetilde{y}_{t-1} = 0$$

Solving the equation, we find $\widetilde{y}_t = \lambda_2 \widetilde{y}_{t-1}$ where the two eigenvalues satisfies $0 < \lambda_2 <$

$1 < \lambda_1$. Assuming that all the variables are in the steady state initially including $\widetilde{y}_{-1}=0$,

$$\widetilde{y}_t^{OMP} = \pi_t^{OMP} = 0$$

Subtracting Eq.(A.34) from Eq.(A.35),

$$\pi_{2,t} - \pi_{1,t} = \beta\big(E_t[\pi_{2,t+1}] - E_t[\pi_{1,t+1}]\big) - \kappa\widetilde{q}_t \tag{A.41}$$

Substituting Eq.(A.36) into Eq.(A.41), we derive a second order difference equation:

$$E_t[\widetilde{q}_{t+1}] - \left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta}\right)\widetilde{q}_t + \frac{1}{\beta}\widetilde{q}_{t-1} = \left(\frac{1}{\beta} + 1 - \rho\right)q_t^E - \frac{1}{\beta}q_{t-1}^E$$

Solving the equation,

$$\widetilde{q}_t = -q_t^E + \frac{(\lambda_1 - 1)(1 - \lambda_1)}{\lambda_1 - \rho}\sum_{k=0}^{\infty}\lambda_2^k q_{t-k}^E$$

where the two eigenvalues satisfies $0 < \lambda_2 < 1 < \lambda_1$. The central bank loses control over $\widetilde{q}_t$ if $\alpha_1 = \alpha_2$, because it is affected only by exogenous shocks, q_t^E, moving independently from other variables. Note that this is derived by using only Phillips curves in both sectors and the definition of relative price.

To summarize, the dynamics under optimal monetary policy are given as follows:

$$
\begin{aligned}
\pi_t^{OMP} &= \widetilde{y}_t^{OMP} = \widetilde{c}_{U,t}^{OMP} = \widetilde{c}_{C,t}^{OMP} = \widetilde{n}_{U,t}^{OMP} = \widetilde{n}_{C,t}^{OMP} = 0 \\
\widetilde{q}_t^{OMP} &= -q_t^E + \frac{(\lambda_1 - 1)(1-\lambda_1)}{\lambda_1 - \rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E \\
\widetilde{w}_t^{OMP} &= -(1-\omega) q_t^E + (1-\omega) \frac{(\lambda_1 - 1)(1-\lambda_1)}{\lambda_1 - \rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E \\
\widetilde{y}_{1,t}^{OMP} &= -\eta(1-\omega) q_t^E + \eta(1-\omega) \frac{(\lambda_1 - 1)(1-\lambda_1)}{\lambda_1 - \rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E \\
\widetilde{y}_{2,t}^{OMP} &= \eta\omega q_t^E - \eta\omega \frac{(\lambda_1 - 1)(1-\lambda_1)}{\lambda_1 - \rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E
\end{aligned}
$$

where achieving efficiency is infeasible. Note that as the price converges to flexible price, $\alpha \to 0$, we have $\lambda_1 \to \infty$ and $\lambda_2 \to 0$. Thus relative price, wage and sectoral output gap converge to efficient levels, $\widetilde{q}_t \to 0$, $\widetilde{w}_t \to 0$, $\widetilde{y}_{1,t} \to 0$ and $\widetilde{y}_{2,t} \to 0$. □

A.1.6 Proof of Proposition 1.6

Proof. We solve a Ramsey problem of the utilitarian central bank when the prices of both sectors are sticky, but to different degrees, $0 < \alpha_1 < \alpha_2$, under **HomCB**. The opposite case will be exactly symmetric under homogeneous consumption baskets. We set up the Lagrangian as:

$$
\begin{aligned}
\mathcal{L}_t = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t & \left[\begin{array}{c} \omega \frac{\theta}{\kappa_1} \pi_{1,t}^2 + (1-\omega) \frac{\theta}{\kappa_2} \pi_{2,t}^2 + (\sigma+\varphi) \widetilde{y}_t^2 + \eta\omega(1-\omega) \widetilde{q}_t^2 \\ +\sigma\lambda(1-\lambda)(\widetilde{c}_{U,t} - \widetilde{c}_{C,t})^2 + \varphi\lambda(1-\lambda)(\widetilde{n}_{U,t} - \widetilde{n}_{C,t})^2 \end{array} \right] \\
+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{1,t} & \left\{ \pi_{1,t} - \beta\pi_{1,t+1} - \kappa_1 \widetilde{w}_t \right\} \\
+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{2,t} & \left\{ \pi_{2,t} - \beta\pi_{2,t+1} - \kappa_2 \widetilde{w}_t + \kappa_2 \widetilde{q}_t \right\} \\
+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{3,t} & \left\{ \widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E - \pi_{2,t} + \pi_{1,t} \right\} \\
+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{4,t} & \left\{ (1-\lambda)\widetilde{c}_{U,t} + \lambda \widetilde{c}_{C,t} - (1-\lambda)\widetilde{n}_{U,t} - \lambda \widetilde{n}_{C,t} \right\} \\
+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{5,t} & \left\{ \varphi \widetilde{n}_{U,t} + \sigma \widetilde{c}_{U,t} - \widetilde{w}_t + (1-\omega)\widetilde{q}_t \right\} \\
+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{6,t} & \left\{ \varphi \widetilde{n}_{C,t} + \sigma \widetilde{c}_{C,t} - \widetilde{w}_t + (1-\omega)\widetilde{q}_t \right\} \\
+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{7,t} & \left\{ \widetilde{c}_{C,t} - \widetilde{n}_{C,t} - \widetilde{w}_t + (1-\omega)\widetilde{q}_t \right\}
\end{aligned}
$$

where $\{\psi_{1,t}\}$, $\cdots$, $\{\psi_{7,t}\}$ are the Lagrange multipliers.

First order conditions are as follows:

$$
\begin{aligned}
\partial\pi_{1,t} &: 0 = \omega\theta\pi_{1,t} + \kappa_1(\psi_{1,t} - \psi_{1,t-1}) + \kappa_1\psi_{3,t} \\
\partial\pi_{2,t} &: 0 = (1-\omega)\theta\pi_{2,t} + \kappa_2(\psi_{2,t} - \psi_{2,t-1}) - \kappa_2\psi_{3,t} \\
\partial\widetilde{c}_{U,t} &: 0 = (\sigma+\varphi)(1-\lambda)\widetilde{y}_t + \sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t}) + (1-\lambda)\psi_{4,t} + \sigma\psi_{5,t} \\
\partial\widetilde{c}_{C,t} &: 0 = (\sigma+\varphi)\lambda\widetilde{y}_t - \sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t}) + \lambda\psi_{4,t} + \sigma\psi_{6,t} + \psi_{7,t} \\
\partial\widetilde{n}_{U,t} &: 0 = \varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t}) - (1-\lambda)\psi_{4,t} + \varphi\psi_{5,t} \\
\partial\widetilde{n}_{C,t} &: 0 = -\varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t}) - \lambda\psi_{4,t} + \varphi\psi_{6,t} - \psi_{7,t} \\
\partial\widetilde{w}_t &: 0 = -\kappa_1\psi_{1,t} - \kappa_2\psi_{2,t} - \psi_{5,t} - \psi_{6,t} - \psi_{7,t} \\
\partial\widetilde{q}_t &: 0 = \eta\omega(1-\omega)\widetilde{q}_t + \kappa_2\psi_{2,t} + \psi_{3,t} - \beta E_t[\psi_{3,t+1}] + (1-\omega)\psi_{5,t} + (1-\omega)\psi_{6,t} + (1-\omega)\psi_{7,t}
\end{aligned}
$$

Simplifying first order conditions, they reduce down to four equations where $A(L) \equiv 1-L$:

$$0 = \omega\theta\pi_{1,t} + \kappa_1 A(L)\psi_{1,t} + \kappa_1\psi_{3,t} \tag{A.42}$$

$$0 = (1-\omega)\theta\pi_{2,t} + \kappa_2 A(L)\psi_{2,t} - \kappa_2\psi_{3,t}) \tag{A.43}$$

$$0 = \kappa_1\psi_{1,t} + \kappa_2\psi_{2,t} - \widetilde{y}_t + \frac{\sigma\varphi\lambda}{\sigma+\varphi}(\widetilde{c}_{U,t}-\widetilde{c}_{C,t}-\widetilde{n}_{U,t}+\widetilde{n}_{C,t}) \tag{A.44}$$

$$0 = \eta\omega(1-\omega)\widetilde{q}_t + \kappa_2\psi_{2,t} + \psi_{3,t} - \beta E_t[\psi_{3,t+1}] - (1-\omega)\left(\widetilde{y}_t - \frac{\sigma\varphi\lambda}{\sigma+\varphi}(\widetilde{c}_{U,t}-\widetilde{c}_{C,t}-\widetilde{n}_{U,t}+\widetilde{n}_{C,t})\right) \tag{A.45}$$

Pre-multiplying Eqs.(A.44)-(A.45) by $A(L)$, and substituting Eqs.(A.42)-(A.43) into them,

$$0 = \omega\theta\pi_{1,t} + (1-\omega)\theta\pi_{2,t} + \kappa_1\psi_{3,t} - \kappa_2\psi_{3,t} + A(L)\left(\widetilde{y}_t - \frac{\sigma\varphi\lambda}{\sigma+\varphi}(\widetilde{c}_{U,t}-\widetilde{c}_{C,t}-\widetilde{n}_{U,t}+\widetilde{n}_{C,t})\right) \tag{A.46}$$

$$
\begin{aligned}
0 = &-(1-\omega)\theta\pi_{2,t} + \kappa_2\psi_{3,t} + A(L)\psi_{3,t} - \beta A(L)E_t[\psi_{3,t+1}] + \eta\omega(1-\omega)A(L)\widetilde{q}_t \\
&- (1-\omega)A(L)\left(\widetilde{y}_t - \frac{\sigma\varphi\lambda}{\sigma+\varphi}(\widetilde{c}_{U,t}-\widetilde{c}_{C,t}-\widetilde{n}_{U,t}+\widetilde{n}_{C,t})\right)
\end{aligned} \tag{A.47}
$$

By using Lagrangian constraints corresponding to $\{\psi_{4,t}\}, \cdots, \{\psi_{7,t}\}$, the definition of aggregate output gap, goods market clearing condition and labor market clearing condition, we write distributional variables in terms of $\widetilde{w}_t-(1-\omega)\widetilde{q}_t$ or $\widetilde{y}_t$,

$$\widetilde{c}_{C,t} = \frac{1+\varphi}{\sigma+\varphi}(\widetilde{w}_t-(1-\omega)\widetilde{q}_t) = (1+\varphi)\widetilde{y}_t \tag{A.48}$$

$$\widetilde{c}_{U,t} = \frac{1-\lambda(1+\varphi)}{(1-\lambda)(\sigma+\varphi)}(\widetilde{w}_t-(1-\omega)\widetilde{q}_t) = \frac{1-\lambda(1+\varphi)}{1-\lambda}\widetilde{y}_t \tag{A.49}$$

$$\widetilde{n}_{C,t} = \frac{1-\sigma}{\sigma+\varphi}(\widetilde{w}_t-(1-\omega)\widetilde{q}_t) = (1-\sigma)\widetilde{y}_t \tag{A.50}$$

$$\widetilde{n}_{U,t} = \frac{1-\lambda(1-\sigma)}{(1-\lambda)(\sigma+\varphi)}(\widetilde{w}_t-(1-\omega)\widetilde{q}_t) = \frac{1-\lambda(1-\sigma)}{1-\lambda}\widetilde{y}_t \tag{A.51}$$

Substituting Eqs.(A.48)-(A.51) into Eqs.(A.46)-(A.47),

$$0 = (\kappa_2-\kappa_1)\psi_{3,t} - \theta\pi_t - \frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t \tag{A.52}$$

$$\begin{aligned} 0 = &-(1-\omega)\theta\pi_{2,t} + \kappa_2\psi_{3,t} + A(L)\psi_{3,t} - \beta A(L)E_t[\psi_{3,t+1}] + \eta\omega(1-\omega)A(L)\widetilde{q}_t \\ &- (1-\omega)\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t \end{aligned} \tag{A.53}$$

Substituting Eq.(A.52) into Eq.(A.53), we derive a targeting rule

$$\begin{aligned} &\frac{1}{\kappa_2-\kappa_1}\left[\begin{array}{c} \kappa_2\{\theta\pi_t+\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t\} + A(L)\{\theta\pi_t+\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t\} \\ -\beta A(L)\{\theta E_t[\pi_{t+1}]+\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)E_t[\widetilde{y}_{t+1}]\} \end{array}\right] \\ &= (1-\omega)\theta\pi_{2,t}-\eta\omega(1-\omega)A(L)\widetilde{q}_t + (1-\omega)\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t \end{aligned} \tag{A.54}$$

□

A.1.7 Proof of Proposition 1.7

Proof. We follow Woodford (2003) in deriving the welfare-theoretic loss function. Note first that under the assumptions on employment subsidy and government transfers, the steady state is efficient and equitable, $N_{C,t}=N_{U,t}=N=C_{C,t}=C_{U,t}=C=Y=1$ with the wage and the relative price being unity, $W=Q=1$. Thus we have

$$\frac{V_N(N_{U,t})}{U_C(C_{U,t})} = W = 1 = \frac{W}{Q} = \frac{V_N(N_{c,t})}{U_C(C_{C,t})}$$

Define $h_U \equiv 1-\lambda$ and $h_C \equiv \lambda$, and note that we assumed $z_1 \equiv \lambda$ and $z_2 \equiv 1-\lambda$. Taking a second-order approximation to the equally weighted sum of both types of households' utilities around the efficient zero-inflation steady state,

$$\begin{aligned}
&\sum_{j=U,C} h_j \mathcal{U}(C_{h,t}, N_{h,t}) \\
&= \sum_{j=U,C} h_j \begin{bmatrix} U_c Y\{c_{j,t} + \frac{1-\sigma}{2} c_{j,t}^2\} \\ -V_N N\{n_{j,t} + \frac{1-\sigma}{2} n_{j,t}^2\} \end{bmatrix} + t.i.p. + o(||\xi||^3) \\
&= U_c Y \begin{bmatrix} z_1\{\widetilde{c}_{C,t} + \frac{1-\sigma}{2}\widetilde{c}_{C,t}^2 + (1-\sigma)c_{C,t}^E \widetilde{c}_{C,t}\} + z_2\{\widetilde{c}_{U,t} + \frac{1-\sigma}{2}\widetilde{c}_{U,t}^2 + (1-\sigma)c_{U,t}^E \widetilde{c}_{U,t}\} \\ -z_1\{\widetilde{n}_{C,t} + \frac{1+\varphi}{2}\widetilde{n}_{C,t}^2 + (1+\varphi)n_{C,t}^E \widetilde{n}_{C,t}\} - z_2\{\widetilde{n}_{U,t} + \frac{1+\varphi}{2}\widetilde{n}_{U,t}^2 + (1+\varphi)n_{U,t}^E \widetilde{n}_{U,t}\} \end{bmatrix} \\
&\quad + t.i.p. + o(||\xi||^3) \qquad \text{(A.55)}
\end{aligned}$$

Taking a second order approximation to the labor market clearing condition,

$$\begin{aligned}
&\omega(\widetilde{n}_{1,t} + \frac{1}{2}\widetilde{n}_{1,t}^2 + n_{1,t}^E \widetilde{n}_{1,t}) + (1-\omega)(\widetilde{n}_{2,t} + \frac{1}{2}\widetilde{n}_{2,t}^2 + n_{2,t}^E \widetilde{n}_{2,t}) \\
&= (1-\lambda)(\widetilde{n}_{U,t} + \frac{1}{2}\widetilde{n}_{U,t}^2 + n_{U,t}^E \widetilde{n}_{U,t}) + \lambda(\widetilde{n}_{C,t} + \frac{1}{2}\widetilde{n}_{C,t}^2 + n_{C,t}^E \widetilde{n}_{C,t}) + t.i.p. + o(||\xi||^3)
\end{aligned} \qquad \text{(A.56)}$$

Let us define $\hat{p}_{j,t}(i) \equiv p_{j,t}(i) - p_{j,t}$. Then, by a second order approximation,

$$\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{1-\theta} = e^{(1-\theta)\hat{p}_{j,t}(i)} = 1 + (1-\theta)\hat{p}_{j,t}(i) + \frac{(1-\theta)^2}{2}\hat{p}_{j,t}^2(i) + o(||\xi||^3) \qquad \text{(A.57)}$$

Since $\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{1-\theta}di = 1$ by the price aggregator, we integrate Eq.(A.57) to derive

$$E_i^j\{\hat{p}_{j,t}(i)\} = \frac{\theta-1}{2}E_i^j\{\hat{p}_{j,t}^2(i)\} \tag{A.58}$$

Similarly, taking a second order approximation, integrating the result, and substituting Eq.(A.58),

$$\begin{aligned}
\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} &= 1-\theta\hat{p}_{j,t}(i)+\frac{\theta^2}{2}\hat{p}_{j,t}^2(i)+o(||\xi||^3)\\
\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} &= 1-\theta E_i^j\{\hat{p}_{j,t}(i)\}+\frac{\theta^2}{2}E_i^j\{\hat{p}_{j,t}^2(i)\}+o(||\xi||^3)\\
&= 1+\frac{\theta^2}{2}E_i^j\{\hat{p}_{j,t}^2(i)\}+o(||\xi||^3)
\end{aligned} \tag{A.59}$$

Since $E_i^j\{\hat{p}_{j,t}^2(i)\} = \frac{1}{z_j}\int_{\mathcal{I}_j}\hat{p}_{j,t}^2(i)di = \frac{1}{z_j}\int_{\mathcal{I}_j}\left(p_{j,t}(i)-p_{j,t}\right)^2 di$, and we know that in the first order $p_{j,t} = E_i^j\{p_{j,t}(i)\}$, we derive that $E_i^j\{\hat{p}_{j,t}^2(i)\} = Var_i^j\{p_{j,t}(i)\}$. Substituting this into Eq.(A.59),

$$\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} = 1+\frac{\theta^2}{2}Var_i^j\{p_{j,t}(i)\}+o(||\xi||^3) \tag{A.60}$$

Thus we derive the second order approximation to the price dispersion in each sector as

$$d_{j,t} \equiv \log\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} = \frac{\theta^2}{2}Var_i^j\{p_{j,t}(i)\}+o(||\xi||^3) \tag{A.61}$$

We have $N_{j,t} = \int_{\mathcal{I}_j}\frac{Y_{j,t}(i)}{A_tA_{j,t}}di = \frac{1}{z_j}\frac{Y_{j,t}}{A_tA_{j,t}}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta}di$ by the relative demand function. Taking a second order approximation and substituting Eq.(A.61), we derive

$$\begin{aligned}
n_{j,t} &= y_{j,t} - a_t - a_{j,t} + \log\frac{1}{z_j}\int_{\mathcal{I}_j}(\frac{P_{j,t}(i)}{P_{j,t}})^{-\theta}di + o(||\xi||^3)\\
&= y_{j,t} - a_t - a_{j,t} + \frac{\theta}{2}Var_i^j\{p_{j,t}(i)\} + o(||\xi||^3)\\
n_{j,t}^2 &= y_{j,t}^2 + a_t^2 + a_{j,t}^2 - 2(a_t + a_{j,t})y_{j,t} + 2a_t a_{j,t} + o(||\xi||^3)\\
\Rightarrow\ \widetilde{n}_{j,t} &= \widetilde{y}_{j,t} + \frac{\theta}{2}Var_i^j\{p_{j,t}(i)\} + o(||\xi||^3) \qquad (A.62)\\
\widetilde{n}_{j,t}^2 + 2n_{j,t}^E\widetilde{n}_{j,t} &= \widetilde{y}_{j,t}^2 + 2y_{j,t}^E\widetilde{y}_{j,t} - 2(a_t + a_{j,t})\widetilde{y}_{j,t} + t.i.p. + o(||\xi||^3) \qquad (A.63)
\end{aligned}$$

Substituting Eqs.(A.56), (A.62) and (A.63) into Eq.(A.1), and canceling out the cross terms,

$$(A.55) = -\frac{U_cY}{2}\left[\begin{array}{c} z_1\theta Var_i^1\{p_{1,t}(i)\} + z_2\theta Var_i^2\{p_{2,t}(i)\}\\ +z_1\sigma\widetilde{y}_{1,t}^2 + z_2\sigma\widetilde{y}_{2,t}^2\\ +z_1\varphi\widetilde{n}_{C,t}^2 + z_2\varphi\widetilde{n}_{U,t}^2 \end{array}\right] + t.i.p. + o(||\xi||^3) \quad (A.64)$$

where $\widetilde{y}_{1,t}\equiv\widetilde{c}_{C,t}$ and $\widetilde{y}_{2,t}\equiv\widetilde{c}_{U,t}$ by goods market clearing condition.

Let us define $\Delta_t^j\equiv Var_i^j\{P_{j,t}(i)\}$. According to Woodford (2003),

$$\begin{aligned}
\Delta_t^j &= \alpha_j\Delta_{t-1}^j + \frac{\alpha_j}{1-\alpha_j}\pi_{j,t}^2 + o(||\xi||^3)\\
&= \underbrace{\alpha_j^{t+1}\Delta_{-1}^j}_{t.i.p.} + \sum_{k=0}^{t}\alpha_j^{t-k}\frac{\alpha_j}{1-\alpha_j}\pi_{j,t}^2 + o(||\xi||^3)
\end{aligned}$$

and the present valued sum of the cross-sectional price dispersion can be rewritten in terms of present valued sum of squared inflation as

$$\sum_{t=0}^{\infty}\beta^t\Delta_t^j = \frac{\alpha_j}{(1-\alpha_j)(1-\alpha_j\beta)}\sum_{t=0}^{\infty}\beta^t\pi_{j,t}^2 + t.i.p. + o(||\xi||^3) \qquad (A.65)$$

Substituting Eq.(A.65) into Eq.(A.64), and summing up the present valued utilities,

$$E_0\sum_{t=0}^{\infty}\beta^t\sum_{j=U,C}h_j\mathcal{U}(C_{h,t},N_{h,t})=-\frac{U_cY}{2}E_0\sum_{t=0}^{\infty}\beta^t\left[\begin{array}{c}\frac{z_1\theta}{\kappa_1}\pi_{1,t}^2+\frac{z_2\theta}{\kappa_2}\pi_{2,t}^2\\+z_1\sigma\widetilde{y}_{1,t}^2+z_2\sigma\widetilde{y}_{2,t}^2\\+z_1\varphi\widetilde{n}_{C,t}^2+z_2\varphi\widetilde{n}_{U,t}^2\end{array}\right]+t.i.p.+o(||\xi||^3) \tag{A.66}$$

Deriving the relation between labor supply gap of type C households and output gap 1 by the relation between consumption and labor supply of the constrained households,

$$\widetilde{n}_{C,t}=\frac{1-\sigma}{1+\varphi}\widetilde{y}_{1,t}+\frac{1-\sigma}{\sigma}\frac{1}{1+\varphi}z_2q_t^E \tag{A.67}$$

$$\widetilde{n}_{C,t}^2=\left(\frac{1-\sigma}{1+\varphi}\right)^2\widetilde{y}_{1,t}^2+2\left(\frac{1-\sigma}{1+\varphi}\right)^2\frac{z_2}{\sigma}q_t^E\widetilde{y}_{1,t}+t.i.p. \tag{A.68}$$

Substituting Eq.(A.67) into the labor market clearing condition, we derive the relation between labor supply gap of the unconstrained households and output gaps:

$$\widetilde{n}_{U,t}=\frac{z_1}{z_2}\frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t}+\widetilde{y}_{2,t}-\frac{1-\sigma}{\sigma}\frac{1}{1+\varphi}z_1q_t^E+o(||\xi||^2) \tag{A.69}$$

$$\begin{aligned}\widetilde{n}_{U,t}^2=&\frac{z_1^2}{z_2^2}\left(\frac{\sigma+\varphi}{1+\varphi}\right)^2\widetilde{y}_{1,t}^2+\widetilde{y}_{2,t}^2+2\frac{z_1}{z_2}\left(\frac{\sigma+\varphi}{1+\varphi}\right)\widetilde{y}_{1,t}\widetilde{y}_{2,t}\\&-2\frac{1-\sigma}{\sigma}\frac{1}{1+\varphi}\left(\frac{z_1}{z_2}\frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t}+\widetilde{y}_{2,t}\right)z_1q_t^E+t.i.p.+o(||\xi||^3)\end{aligned} \tag{A.70}$$

substituting Eqs.(A.68) and (A.70) into Eq.(A.66), we can rewrite the loss function in terms of inflation and output gaps only:

$$
\begin{aligned}
&-\frac{U_cY}{2}E_0\sum_{t=0}^{\infty}\beta^t
\left[\begin{array}{c}
\underbrace{\frac{z_1\theta}{\kappa_1}}_{\equiv\,\Gamma_{\pi_1}}\ \pi_{1,t}^2+\ \underbrace{\frac{z_2\theta}{\kappa_2}}_{\equiv\,\Gamma_{\pi_1}}\ \pi_{2,t}^2 \\
+\ \underbrace{z_1\left[\sigma+\left(\frac{1-\sigma}{1+\varphi}\right)^2\varphi+\frac{z_1}{z_2}(\frac{\sigma+\varphi}{1+\varphi})^2\varphi\right]}_{\equiv\,\Gamma_{y_{11}}}\widetilde{y}_{1,t}^2 \\
+\ \underbrace{2z_1\varphi\frac{\sigma+\varphi}{1+\varphi}}_{\equiv\,\Gamma_{y_{12}}}\widetilde{y}_{1,t}\widetilde{y}_{2,t}\ +\ \underbrace{z_2(\sigma+\varphi)}_{\equiv\,\Gamma_{y_{22}}}\widetilde{y}_{2,t}^2 \\
+\ \underbrace{2\varphi\frac{1-\sigma}{1+\varphi}\frac{z_1z_2}{\sigma}\left(\frac{1-\sigma}{1+\varphi}-\frac{z_1}{z_2}\frac{\sigma+\varphi}{1+\varphi}\right)q_t^E}_{\equiv\,\Gamma_{y_1}}\widetilde{y}_{1,t}\ \underbrace{-\ 2\varphi\frac{1-\sigma}{1+\varphi}\frac{z_1z_2}{\sigma}q_t^E}_{\equiv\,\Gamma_{y_2}}\widetilde{y}_{2,t}
\end{array}\right] \\
&\qquad + t.i.p. + o(||\xi||^3) \\
&= -\frac{U_cY}{2}E_0\sum_{t=0}^{\infty}\beta^t
\left[\begin{array}{c}
\Gamma_{\pi_1}\pi_{1,t}^2+\Gamma_{\pi_2}\pi_{2,t}^2 \\
+\Gamma_{y_{11}}(\widetilde{y}_{1,t}-x_{1,t}^*)^2+\Gamma_{y_{12}}(\widetilde{y}_{1,t}-x_{1,t}^*)(\widetilde{y}_{2,t}-x_{2,t}^*)+\Gamma_{y_{22}}(\widetilde{y}_{2,t}-x_{2,t}^*)^2
\end{array}\right] \\
&\qquad +\ t.i.p. + o(||\xi||^3) \qquad\qquad (A.71)
\end{aligned}
$$

$$
\left(\begin{array}{l}
x_{1,t}^*\equiv\dfrac{2\Gamma y_{22}\Gamma y_1-\Gamma y_{12}\Gamma y_2}{\Gamma y_{12}{}^2-4\Gamma y_{11}\Gamma y_{22}}=\dfrac{1-\sigma}{\sigma}\dfrac{\varphi}{\sigma+\varphi}\dfrac{(\sigma-z_2)z_2}{\sigma\varphi+z_2}q_t^E \\
x_{2,t}^*\equiv\dfrac{2\Gamma y_{11}\Gamma y_2-\Gamma y_{12}\Gamma y_1}{\Gamma y_{12}{}^2-4\Gamma y_{11}\Gamma y_{22}}=\dfrac{1-\sigma}{\sigma}\dfrac{\varphi}{\sigma+\varphi}\dfrac{z_1z_2}{\sigma\varphi+z_2}q_t^E
\end{array}\right.
$$

where $\Gamma_{y_{12}}^2-4\Gamma_{y_{11}}\Gamma_{y_{22}}<0$ holds implying that the contour of the loss function is elliptical with its center being $(x_{1,t}^*,\ x_{2,t}^*)$.

Note that target output gaps shifts according to relative productivity shock, q_t^E, and their directions depends on the value of σ that measures the relative size of the income effect compared to the substitution effect in labor supply and households' preference on consumption smoothing,

$$
x_{1,t}^*\begin{cases}>0, & \text{if }\ z_2<\sigma<1\\ \leq 0, & \text{otherwise}\end{cases}\quad\text{and}\quad x_{2,t}^*\begin{cases}>0, & \text{if }\ \sigma<1\\ \leq 0, & \text{otherwise}\end{cases}
$$

□

A.1.8 Proof of Proposition 1.8

Proof. We solve a Ramsey problem of the utilitarian central bank when the price of sector 1 is flexible and the price of sector 2 is sticky, $\alpha_1 = 0$, under **HetCB**. We set up the Lagrangian as:

$$\mathcal{L}_t = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left[\begin{array}{c}\frac{z_2\theta}{\kappa_2}\pi_{2,t}^2\\ +\Gamma_{y_{11}}(\widetilde{y}_{1,t}^N - x_{1,t}^*)^2 + \Gamma_{y_{12}}(\widetilde{y}_{1,t}^N - x_{1,t}^*)(\widetilde{y}_{2,t} - x_{2,t}^*) + \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x_{2,t}^*)^2\end{array}\right]$$
$$+E_0\sum_{t=0}^{\infty}\beta^t\psi_{1,t}\left\{\pi_{2,t} - \beta\pi_{2,t+1} + \kappa_2\widetilde{q}_t\right\}$$
$$+E_0\sum_{t=0}^{\infty}\beta^t\psi_{2,t}\left\{\widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E - \pi_{2,t} + \pi_{1,t}\right\}$$
$$+E_0\sum_{t=0}^{\infty}\beta^t\psi_{3,t}\left\{z_1\widetilde{y}_{1,t}^N + z_2\widetilde{y}_{2,t} - z_2\widetilde{n}_{U,t} - z_1\widetilde{n}_{C,t}^N\right\}$$
$$+E_0\sum_{t=0}^{\infty}\beta^t\psi_{4,t}\left\{\varphi\widetilde{n}_{U,t} + \sigma\widetilde{y}_{2,t} + \widetilde{q}_t\right\}$$

where $\{\psi_{1,t}\}$, $\cdots$, $\{\psi_{4,t}\}$ are the Lagrange multipliers.

First order conditions are as follows:

$$\partial\pi_{1,t} : 0 = \psi_{2,t}$$
$$\partial\pi_{2,t} : 0 = \frac{z_2\theta}{\kappa_1}\pi_{2,t} + \psi_{1,t} - \psi_{1,t-1} - \psi_{2,t}$$
$$\partial\widetilde{q}_t : 0 = \kappa_2\psi_{1,t} + \psi_{2,t} - \beta E_t[\psi_{2,t+1}] + \psi_{4,t}$$
$$\partial\widetilde{y}_{2,t} : 0 = \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x_{2,t}^*) + \frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{1,t}^N - x_{1,t}^*) + z_2\psi_{3,t} + \sigma\psi_{4,t}$$
$$\partial\widetilde{n}_{U,t} : 0 = -z_2\psi_{3,t} + \varphi\psi_{4,t}$$

Simplifying first order conditions into one equation:

$$\pi_{2,t} = \frac{z_1}{\theta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}(q_t^E - q_{t-1}^E) - \frac{1}{\theta}\widetilde{y}_{2,t} + \frac{1}{\theta}\widetilde{y}_{2,t-1} \tag{A.72}$$

By using Lagrangian constraints corresponding to $\{\psi_{1,t}\}$, $\{\psi_{3,t}\}$, and $\{\psi_{4,t}\}$,

$$\pi_{2,t} - \beta E_t[\pi_{2,t+1}] - \kappa_2(\sigma+\varphi)\widetilde{y}_{2,t} + \kappa_2\frac{1-\sigma}{\sigma}\varphi z_1 q_t^E = 0 \tag{A.73}$$

Substituting Eq.(A.72) into Eq.(A.73), we derive a second-order difference equation where $\gamma_1 \equiv \frac{z_1}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\big(1+\beta-\beta\rho+\kappa_2\theta(\sigma+\varphi)\big)$ and $\gamma_2 \equiv \frac{z_1}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}$ in this proof:

$$E_t[\widetilde{y}_{2,t+1}] - \left(1 + \frac{1}{\beta} + \frac{\kappa_2\theta(\sigma+\varphi)}{\beta}\right)\widetilde{y}_{2,t} + \frac{1}{\beta}\widetilde{q}_{t-1} = -\gamma_1 q_t^E + \gamma_2 q_{t-1}^E$$

Solving the equation,

$$\begin{aligned}\widetilde{y}_{2,t} = \lambda_2\widetilde{y}_{2,t-1} &+ \frac{z_1}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\left(1+\beta-\beta\rho-\frac{1}{\lambda_1}+\kappa_2\theta(\sigma+\varphi)\right)\frac{1}{\lambda_1-\rho}q_t^E \\ &- \frac{z_1}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{1}{\lambda_1}q_{t-1}^E\end{aligned} \tag{A.74}$$

where the two eigenvalues satisfies $0<\lambda_2<1<\lambda_1$. Simplifying further with $\lambda_1+\lambda_2 \equiv 1+\frac{1}{\beta}+\frac{\kappa_2\theta(\sigma+\varphi)}{\beta}$ and $\lambda_1\lambda_2 \equiv \frac{1}{\beta}$,

$$\widetilde{y}_{2,t}^{OMP} = z_1\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}q_t^E = \widetilde{y}_{2,t}^N \tag{A.75}$$

Substituting Eq.(A.75) into Eq.(A.72),

$$\widetilde{\pi}_{2,t}^{OMP} = 0 \tag{A.76}$$

Solving for the rest variables,

$$\begin{aligned}\widetilde{n}_{U,t}^{OMP} &= -z_1\frac{1-\sigma}{\sigma+\varphi}q_t^E \\ \widetilde{q}_t^{OMP} &= 0 = \widetilde{q}_t^N \\ \widetilde{\pi}_{1,t}^{OMP} &= -q_t^E + q_{t-1}^E\end{aligned}$$

We find that optimal policy achieves flexible price (natural) allocation. □

A.1.9 Proof of Proposition 1.9

Proof. We solve a Ramsey problem of the utilitarian central bank when the price of sector 2 is flexible and the price of sector 1 is sticky, $\alpha_2 = 0$, under **HetCB**. We set up the Lagrangian as:

$$
\begin{aligned}
\mathcal{L}_t = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t & \left[\begin{array}{c} \frac{z_1\theta}{\kappa_1}\pi_{1,t}^2 \\ +\Gamma_{y_{11}}(\widetilde{y}_{1,t} - x_{1,t}^*)^2 + \Gamma_{y_{12}}(\widetilde{y}_{1,t} - x_{1,t}^*)(\widetilde{y}_{2,t} - x_{2,t}^*) + \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x_{2,t}^*)^2 \end{array} \right] \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{1,t} \left\{ \pi_{1,t} - \beta \pi_{1,t+1} - \kappa_1 \widetilde{w}_t \right\} \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{2,t} \left\{ \widetilde{w}_t - \widetilde{q}_t \right\} \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{3,t} \left\{ \widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E - \pi_{2,t} + \pi_{1,t} \right\} \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{4,t} \left\{ z_1 \widetilde{y}_{1,t} + z_2 \widetilde{y}_{2,t} - z_2 \widetilde{n}_{U,t} - z_1 \widetilde{n}_{C,t} \right\} \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{5,t} \left\{ \varphi \widetilde{n}_{U,t} + \sigma \widetilde{y}_{2,t} \right\} \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{6,t} \left\{ \varphi \widetilde{n}_{C,t} + \sigma \widetilde{y}_{1,t} - \widetilde{w}_t \right\} \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{7,t} \left\{ \widetilde{y}_{1,t} - \widetilde{w}_t - \widetilde{n}_{C,t} + \frac{1-\sigma}{\sigma} z_2 q_t^E \right\}
\end{aligned}
$$

where $\{\psi_{1,t}\}, \cdots, \{\psi_{7,t}\}$ are the Lagrange multipliers.

First order conditions are as follows:

$$\begin{aligned}
\partial\pi_{1,t} &: 0 = \frac{z_1\theta}{\kappa_1}\pi_{1,t} + \psi_{1,t} - \psi_{1,t-1} + \psi_{3,t} \\
\partial\pi_{2,t} &: 0 = -\psi_{3,t} \\
\partial\widetilde{w}_t &: 0 = -\kappa_1\psi_{1,t} + \psi_{2,t} - \psi_{6,t} - \psi_{7,t} \\
\partial\widetilde{q}_t &: 0 = -\psi_{2,t} + \psi_{3,t} - \beta E_t[\psi_{3,t+1}] \\
\partial\widetilde{y}_{1,t} &: 0 = \Gamma_{y_{11}}(\widetilde{y}_{1,t} - x^*_{1,t}) + \frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{2,t} - x^*_{2,t}) + z_1\psi_{4,t} + \sigma\psi_{6,t} + \psi_{7,t} \\
\partial\widetilde{y}_{2,t} &: 0 = \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x^*_{2,t}) + \frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2\psi_{4,t} + \sigma\psi_{5,t} \\
\partial\widetilde{n}_{U,t} &: 0 = -z_2\psi_{4,t} + \varphi\psi_{5,t} \\
\partial\widetilde{n}_{C,t} &: 0 = -z_1\psi_{4,t} + \varphi\psi_{6,t} - \psi_{7,t}
\end{aligned}$$

Simplifying first order conditions into one equation:

$$\pi_{1,t} = -\frac{k_0}{z_1\theta}(\widetilde{y}_{1,t} - \widetilde{y}_{1,t-1}) + \frac{k_0}{z_1\theta}(x^*_{1,t} - x^*_{1,t-1}) \tag{A.77}$$

where $k_0 \equiv \frac{1+\varphi}{\sigma+\varphi}z_1[\sigma + (\frac{1-\sigma}{1+\varphi})^2\varphi + \frac{z_1}{z_2}\frac{(\sigma+\varphi)\sigma\varphi}{(1+\varphi)^2}]$ and $k_1 \equiv \sigma + (\frac{1-\sigma}{1+\varphi})^2\varphi + \frac{z_1}{z_2}\frac{(\sigma+\varphi)\sigma\varphi}{(1+\varphi)^2}$.

Simplifying Lagrangian constraints corresponding to $\{\psi_{5,t}\}, \cdots, \{\psi_{7,t}\}$,

$$\widetilde{w}_t = \frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + z_2\frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}q^E_t \tag{A.78}$$

Substituting Eq.(A.78) into the labor market clearing condition,

$$\widetilde{y}_{2,t} = -\frac{z_1}{z_2}\frac{\varphi}{\sigma+\varphi}\widetilde{w}_t + z_1\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}q^E_t \tag{A.79}$$

Substituting Eq.(A.78) into the Phillips Curve in sector 1,

$$\pi_{1,t} - \beta E_t[\pi_{1,t+1}] - \kappa_1\frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} - \kappa_1 z_2\frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}q^E_t = 0 \tag{A.80}$$

Substituting Eq.(A.77) into Eq.(A.80), we derive a second-order difference equation where $\gamma_1 \equiv \frac{z_2}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}[(1+\beta-\beta\rho)\frac{\sigma-z_2}{\sigma\varphi+z_2} - \frac{\theta\kappa_2}{k_1}(\frac{\sigma+\varphi}{1+\varphi})^2]$ and $\gamma_2 \equiv \frac{z_2}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{\sigma-z_2}{\sigma\varphi+z_2}$ in this proof:

$$E_t[\widetilde{y}_{1,t+1}] - \left(1 + \frac{1}{\beta} + \frac{\kappa_1\theta}{\beta k_1}\left(\frac{\sigma+\varphi}{1+\varphi}\right)^2\right)\widetilde{y}_{2,t} + \frac{1}{\beta}\widetilde{y}_{1,t-1} = -\gamma_1 q_t^E + \gamma_2 q_{t-1}^E$$

Solving the equation,

$$\begin{aligned}\widetilde{y}_{1,t} = \lambda_2\widetilde{y}_{1,t-1} &+ \frac{z_2}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{1}{\lambda_1-\rho}\left[\left(1+\beta-\beta\rho-\frac{1}{\lambda_1}\right)\frac{\sigma-z_2}{\sigma\varphi+z_2}-\frac{\kappa_1\theta}{k_1}\left(\frac{\sigma+\varphi}{1+\varphi}\right)^2\right]q_t^E \\ &- \frac{z_2}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{1}{\lambda_1}\frac{\sigma-z_2}{\sigma\varphi+z_2}q_{t-1}^E \end{aligned} \quad \text{(A.81)}$$

where the two eigenvalues satisfies $0<\lambda_2<1<\lambda_1$. Simplifying further with $\lambda_1+\lambda_2 \equiv 1+\frac{1}{\beta}+\frac{\kappa_1\theta}{\beta k_1}(\frac{\sigma+\varphi}{1+\varphi})^2$ and $\lambda_1\lambda_2 \equiv \frac{1}{\beta}$,

$$\begin{aligned}\widetilde{y}_{1,t}^{OMP} &= \underbrace{z_2\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{\sigma-z_2}{\sigma\varphi+z_2}q_t^E}_{=x_{1,t}^*} - z_2\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{\sigma\varphi+\sigma}{\sigma\varphi+z_2}\frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho}\sum_{k=0}^{\infty}\lambda_2^k q_{t-k}^E \\ &\xrightarrow{\text{as } \alpha_1\to 0} -z_2\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}q_t^E = \widetilde{y}_{1,t}^N\end{aligned} \quad \text{(A.82)}$$

Note that as $\alpha_1 \to 0$, $\lambda_1 \to \infty$ and $\lambda_2 \to 0$.

Substituting Eq.(A.82) into Eq.(A.77),

$$\widetilde{\pi}_{1,t}^{OMP} = \frac{k_0}{z_1\theta}z_2\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{\sigma\varphi+\sigma}{\sigma\varphi+z_2}\frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho}\left(q_t^E-(1-\lambda_2)\sum_{k=0}^{\infty}\lambda_2^k q_{t-k}^E\right)$$

Solving for the rest variables,

$$
\begin{aligned}
\widetilde{y}_{2,t}^{OMP} &= \underbrace{z_1 \frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} \frac{z_2}{\sigma\varphi+z_2} q_t^E}_{=x_{2,t}^*} + z_1 \frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} \frac{\sigma\varphi}{\sigma\varphi+z_2} \frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E \\
&\xrightarrow{\text{as } \alpha_1 \to 0} z_1 \frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} q_t^E = \widetilde{y}_{2,t}^N \\
\widetilde{n}_{U,t}^{OMP} &= -z_1 \frac{1-\sigma}{\sigma+\varphi} \left(\frac{z_2}{\sigma\varphi+z_2} q_t^E + \frac{\sigma\varphi}{\sigma\varphi+z_2} \frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E \right) \\
&\xrightarrow{\text{as } \alpha_1 \to 0} -z_1 \frac{1-\sigma}{\sigma+\varphi} q_t^E = \widetilde{n}_{U,t}^N \\
\widetilde{n}_{C,t}^{OMP} &= z_2 \frac{1-\sigma}{\sigma+\varphi} \left(\frac{\varphi+z_2}{\sigma\varphi+z_2} q_t^E - \frac{\varphi-\sigma\varphi}{\sigma\varphi+z_2} \frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E \right) \\
&\xrightarrow{\text{as } \alpha_1 \to 0} z_2 \frac{1-\sigma}{\sigma+\varphi} q_t^E = \widetilde{n}_{C,t}^N \\
\widetilde{w}_t^{OMP} &= \widetilde{q}_t^{OMP} = z_2 \frac{1-\sigma}{\sigma} \frac{\varphi}{1+\varphi} \frac{\sigma\varphi+\sigma}{\sigma\varphi+z_2} \left(q_t^E - \frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E \right) \\
&\xrightarrow{\text{as } \alpha_1 \to 0} 0 = \widetilde{w}_t^N = \widetilde{q}_t^N
\end{aligned}
$$

Thus the market outcome under optimal monetary policy fails to obtain efficiency. Note that under optimal monetary policy we have

$$
\frac{\widetilde{y}_{2,t} - x_{2,t}^*}{\widetilde{y}_{1,t} - x_{1,t}^*} = -\frac{z_1}{z_2} \frac{\varphi}{1+\varphi}
$$

Rearranging the terms,

$$
\varphi z_1 (\widetilde{y}_{1,t} - x_{1,t}^*) + (1+\varphi) z_2 (\widetilde{y}_{2,t} - x_{2,t}^*) = 0 \tag{A.83}
$$

It is trivial to prove that flexible price allocation is achievable, thus the latter is sub-optimal.

□

A.1.10 Proof of Proposition 1.10

Proof. We solve a Ramsey problem of the utilitarian central bank when the prices of both sectors are sticky to the same degree, $0 < \alpha_1 < \alpha_2 = \alpha$, under **HetCB**. The set-up of Lagrangian is the same as that in the proof of Proposition 1.11 except that we have $\kappa_1 = \kappa_2 = \kappa$ now:

$$
\begin{aligned}
\mathcal{L}_t = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t & \left[\begin{array}{c} \frac{z_1\theta}{\kappa_1}\pi_{1,t}^2 + \frac{z_2\theta}{\kappa_2}\pi_{2,t}^2 \\ +\Gamma_{y_{11}}(\widetilde{y}_{1,t} - x_{1,t}^*)^2 + \Gamma_{y_{12}}(\widetilde{y}_{1,t} - x_{1,t}^*)(\widetilde{y}_{2,t} - x_{2,t}^*) + \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x_{2,t}^*)^2 \end{array} \right] \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{1,t} \left\{ \pi_{1,t} - \beta \pi_{1,t+1} - \kappa_1 \widetilde{w}_t \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{2,t} \left\{ \pi_{2,t} - \beta \pi_{2,t+1} - \kappa_2 \widetilde{w}_t + \kappa_2 \widetilde{q}_t \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{3,t} \left\{ \widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E - \pi_{2,t} + \pi_{1,t} \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{4,t} \left\{ z_1 \widetilde{y}_{1,t} + z_2 \widetilde{y}_{2,t} - z_2 \widetilde{n}_{U,t} - z_1 \widetilde{n}_{C,t} \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{5,t} \left\{ \varphi \widetilde{n}_{U,t} + \sigma \widetilde{y}_{2,t} - \widetilde{w}_t + \widetilde{q}_t \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{6,t} \left\{ \varphi \widetilde{n}_{C,t} + \sigma \widetilde{y}_{1,t} - \widetilde{w}_t \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{7,t} \left\{ \widetilde{y}_{1,t} - \widetilde{w}_t - \widetilde{n}_{C,t} + \frac{1-\sigma}{\sigma} z_2 q_t^E \right\}
\end{aligned}
$$

where $\{\psi_{1,t}\}, \cdots, \{\psi_{7,t}\}$ are the Lagrange multipliers.

First order conditions are as follows:

$$\begin{aligned}
\partial\pi_{1,t} &: 0 = \frac{z_1\theta}{\kappa_1}\pi_{1,t} + \psi_{1,t} - \psi_{1,t-1} + \psi_{3,t} \\
\partial\pi_{2,t} &: 0 = \frac{z_2\theta}{\kappa_1}\pi_{2,t} + \psi_{2,t} - \psi_{2,t-1} - \psi_{3,t} \\
\partial\widetilde{w}_t &: 0 = -\kappa_1\psi_{1,t} - \kappa_2\psi_{2,t} - \psi_{5,t} - \psi_{6,t} - \psi_{7,t} \\
\partial\widetilde{q}_t &: 0 = \kappa_2\psi_{2,t} + \psi_{3,t} - \beta E_t[\psi_{3,t+1}] + \psi_{5,t} \\
\partial\widetilde{y}_{1,t} &: 0 = \Gamma_{y_{11}}(\widetilde{y}_{1,t} - x^*_{1,t}) + \frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{2,t} - x^*_{2,t}) + z_1\psi_{4,t} + \sigma\psi_{6,t} + \psi_{7,t} \\
\partial\widetilde{y}_{2,t} &: 0 = \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x^*_{2,t}) + \frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2\psi_{4,t} + \sigma\psi_{5,t} \\
\partial\widetilde{n}_{U,t} &: 0 = -z_2\psi_{4,t} + \varphi\psi_{5,t} \\
\partial\widetilde{n}_{C,t} &: 0 = -z_1\psi_{4,t} + \varphi\psi_{6,t} - \psi_{7,t}
\end{aligned}$$

Rewriting Lagrangian constraints corresponding to $\{\psi_{1,t}\}, \cdots, \{\psi_{3,t}\}$,

$$\pi_{1,t} - \beta E_t[\pi_{1,t+1}] = \kappa\widetilde{w}_t \tag{A.84}$$

$$\pi_{2,t} - \beta E_t[\pi_{2,t+1}] = \kappa\widetilde{w}_t - \kappa\widetilde{q}_t \tag{A.85}$$

$$\widetilde{q}_t - \widetilde{q}_{t-1} + q^E_t - q^E_{t-1} = \pi_{2,t} - \pi_{1,t} \tag{A.86}$$

Subtracting Eq.(A.90) from Eq.(A.85),

$$\pi_{2,t} - \pi_{1,t} = \beta\big(E_t[\pi_{2,t+1}] - E_t[\pi_{1,t+1}]\big) - \kappa\widetilde{q}_t \tag{A.87}$$

Substituting Eq.(A.86) into Eq.(A.87), we derive a second order difference equation:

$$E_t[\widetilde{q}_{t+1}] - \left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta}\right)\widetilde{q}_t + \frac{1}{\beta}\widetilde{q}_{t-1} = \left(\frac{1}{\beta} + 1 - \rho\right)q^E_t - \frac{1}{\beta}q^E_{t-1}$$

Solving the equation,

$$\widetilde{q}_t = -q^E_t + \frac{(\lambda_1 - 1)(1 - \lambda_1)}{\lambda_1 - \rho}\sum_{k=0}^{\infty}\lambda_2^k q^E_{t-k}$$

where the two eigenvalues satisfies $0 < \lambda_2 < 1 < \lambda_1$. The central bank loses control over $\widetilde{q}_t$ if $\alpha_1 = \alpha_2$, because it is affected only by exogenous asymmetric shocks, q_t^E independently from other variables. Note that this is derived by using only Phillips curves in both sectors and the definition of relative price.

Rewriting Eqs.(A.96)-(A.97),

$$z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x_{1,t}^*) + z_2A(L)(\widetilde{y}_{2,t} - x_{2,t}^*) = 0 \qquad \text{(A.88)}$$

$$\kappa\psi_{3,t} + A(L)\psi_{3,t} - \beta A(L)E_t[\psi_{3,t+1}] = z_2\theta\pi_{2,t} + \frac{z_1\varphi}{1+\varphi}A(L)(\widetilde{y}_{1,t} - x_{1,t}^*) + z_2A(L)(\widetilde{y}_{2,t} - x_{2,t}^*) \qquad \text{(A.89)}$$

A targeting rule Eq.(A.88) closes the model, and Eq.(A.89) only determines $\psi_{3,t}$ if $\alpha_1 = \alpha_2$. □

A.1.11 Proof of Proposition 1.11

Proof. We solve a Ramsey problem of the utilitarian central bank when the prices of both sectors are sticky, but to different degrees, $0 < \alpha_1 < \alpha_2$, under **HetCB**. We set up the Lagrangian as:

$$
\begin{aligned}
\mathcal{L}_t = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t & \left[\begin{array}{c} \frac{z_1\theta}{\kappa_1}\pi_{1,t}^2 + \frac{z_2\theta}{\kappa_2}\pi_{2,t}^2 \\ +\Gamma_{y_{11}}(\widetilde{y}_{1,t} - x_{1,t}^*)^2 + \Gamma_{y_{12}}(\widetilde{y}_{1,t} - x_{1,t}^*)(\widetilde{y}_{2,t} - x_{2,t}^*) + \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x_{2,t}^*)^2 \end{array} \right] \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{1,t} \left\{ \pi_{1,t} - \beta\pi_{1,t+1} - \kappa_1 \widetilde{w}_t \right\} \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{2,t} \left\{ \pi_{2,t} - \beta\pi_{2,t+1} - \kappa_2 \widetilde{w}_t + \kappa_2 \widetilde{q}_t \right\} \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{3,t} \left\{ \widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E - \pi_{2,t} + \pi_{1,t} \right\} \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{4,t} \left\{ z_1 \widetilde{y}_{1,t} + z_2 \widetilde{y}_{2,t} - z_2 \widetilde{n}_{U,t} - z_1 \widetilde{n}_{C,t} \right\} \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{5,t} \left\{ \varphi \widetilde{n}_{U,t} + \sigma \widetilde{y}_{2,t} - \widetilde{w}_t + \widetilde{q}_t \right\} \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{6,t} \left\{ \varphi \widetilde{n}_{C,t} + \sigma \widetilde{y}_{1,t} - \widetilde{w}_t \right\} \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \psi_{7,t} \left\{ \widetilde{y}_{1,t} - \widetilde{w}_t - \widetilde{n}_{C,t} + \frac{1-\sigma}{\sigma} z_2 q_t^E \right\}
\end{aligned}
$$

where $\{\psi_{1,t}\}, \cdots, \{\psi_{7,t}\}$ are the Lagrange multipliers.

First order conditions are as follows:

$$\begin{aligned}
\partial\pi_{1,t} &: 0 = \frac{z_1\theta}{\kappa_1}\pi_{1,t} + \psi_{1,t} - \psi_{1,t-1} + \psi_{3,t} \\
\partial\pi_{2,t} &: 0 = \frac{z_2\theta}{\kappa_1}\pi_{2,t} + \psi_{2,t} - \psi_{2,t-1} - \psi_{3,t} \\
\partial\widetilde{w}_t &: 0 = -\kappa_1\psi_{1,t} - \kappa_2\psi_{2,t} - \psi_{5,t} - \psi_{6,t} - \psi_{7,t} \\
\partial\widetilde{q}_t &: 0 = \kappa_2\psi_{2,t} + \psi_{3,t} - \beta E_t[\psi_{3,t+1}] + \psi_{5,t} \\
\partial\widetilde{y}_{1,t} &: 0 = \Gamma_{y_{11}}(\widetilde{y}_{1,t} - x^*_{1,t}) + \frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{2,t} - x^*_{2,t}) + z_1\psi_{4,t} + \sigma\psi_{6,t} + \psi_{7,t} \\
\partial\widetilde{y}_{2,t} &: 0 = \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x^*_{2,t}) + \frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2\psi_{4,t} + \sigma\psi_{5,t} \\
\partial\widetilde{n}_{U,t} &: 0 = -z_2\psi_{4,t} + \varphi\psi_{5,t} \\
\partial\widetilde{n}_{C,t} &: 0 = -z_1\psi_{4,t} + \varphi\psi_{6,t} - \psi_{7,t}
\end{aligned}$$

Simplifying first order conditions, they reduce down to four equations where $A(L) \equiv 1 - L$:

$$0 = z_1\theta\pi_{1,t} + \kappa_1 A(L)\psi_{1,t} + \kappa_1\psi_{3,t} \tag{A.90}$$

$$0 = z_2\theta\pi_{2,t} + \kappa_2 A(L)\psi_{2,t} - \kappa_2\psi_{3,t}) \tag{A.91}$$

$$\begin{aligned}
0 = \kappa_1\psi_{1,t} + \kappa_2\psi_{2,t} - \left(\frac{z_2}{\varphi} - z_1\right)\frac{\varphi}{z_2(\sigma+\varphi)}\left[\frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{1,t} - x^*_{1,t}) + \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x^*_{2,t})\right] \\
- \frac{1+\varphi}{\sigma+\varphi}\left[\Gamma_{y_{11}}(\widetilde{y}_{1,t} - x^*_{1,t}) + \frac{\Gamma_{y_{22}}}{2}(\widetilde{y}_{2,t} - x^*_{2,t})\right]
\end{aligned} \tag{A.92}$$

$$0 = \kappa_2\psi_{2,t} + \psi_{3,t} - \beta E_t[\psi_{3,t+1}] - \frac{1}{\sigma+\varphi}\left[\frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{1,t} - x^*_{1,t}) + \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x^*_{2,t})\right] \tag{A.93}$$

Pre-multiplying Eqs.(A.92)-(A.93) by $A(L)$, and substituting Eqs.(A.90)-(A.91) into them,

$$0 = z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \kappa_1\psi_{3,t} - \kappa_2\psi_{3,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t}) \tag{A.94}$$

$$0 = -z_2\theta\pi_{2,t} + \kappa_2\psi_{3,t} + A(L)\psi_{3,t} - \beta A(L)E_t[\psi_{3,t+1}] - \frac{z_1\varphi}{1+\varphi}A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) - z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t}) \tag{A.95}$$

Simplifying further,

$$\psi_{3,t} = \frac{1}{\kappa_2 - \kappa_1}\left[z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t})\right] \tag{A.96}$$

$$\kappa_2\psi_{3,t} + A(L)\psi_{3,t} - \beta A(L)E_t[\psi_{3,t+1}] = z_2\theta\pi_{2,t} + \frac{z_1\varphi}{1+\varphi}A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t}) \tag{A.97}$$

Substituting Eq.(A.96) into Eq.(A.97), we derive a targeting rule

$$\begin{aligned}
&\frac{\kappa_2}{\kappa_2 - \kappa_1}\left[z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t})\right] \\
&+\frac{1}{\kappa_2 - \kappa_1}\left[\begin{array}{c} \left[z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t})\right] \\ -\left[z_1\theta\pi_{1,t-1} + z_2\theta\pi_{2,t-1} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t-1} - x^*_{1,t-1}) + z_2 A(L)(\widetilde{y}_{2,t-1} - x^*_{2,t-1})\right] \end{array}\right] \\
&-\frac{\beta}{\kappa_2 - \kappa_1}\left[\begin{array}{c} E_t\left[z_1\theta\pi_{1,t+1} + z_2\theta\pi_{2,t+1} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t+1} - x^*_{1,t+1}) + z_2 A(L)(\widetilde{y}_{2,t+1} - x^*_{2,t+1})\right] \\ -E_{t-1}\left[z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t})\right] \end{array}\right] \\
&= z_2\theta\pi_{2,t} + \frac{z_1\varphi}{1+\varphi}A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t})
\end{aligned}$$

□

A.2 Heterogeneous Consumption Baskets

We provide the system of equations and some derivations of the equilibrium in the efficient allocation and the decentralized model under **HetCB** ($\omega_U = 0,\ \omega_C = 1$).

A.2.1 Efficient Allocation

We derive the economy's efficient allocation by solving a social planner's problem that maximizes the weighted sum of utility of both types of households, subject to the resource and technology constraints

$$\max_{\{C_{h,t}, N_{h,t}, Y_{j,t}(i)\}} \left\{ \varpi_U (1-\lambda) \left[\frac{C_{U,t}^{1-\sigma}}{1-\sigma} - \frac{N_{U,t}^{1+\varphi}}{1+\varphi} \right] + \varpi_C \lambda \left[\frac{C_{C,t}^{1-\sigma}}{1-\sigma} - \frac{N_{C,t}^{1+\varphi}}{1+\varphi} \right] \right\}$$

$$\begin{aligned} s.t. \qquad \lambda C_{C,t} &= \left(\int_{\mathcal{I}_1} \left(\frac{1}{z_1} \right)^{\frac{1}{\theta}} Y_{1,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \\ (1-\lambda) C_{U,t} &= \left(\int_{\mathcal{I}_2} \left(\frac{1}{z_2} \right)^{\frac{1}{\theta}} Y_{2,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \\ (1-\lambda) N_{U,t} + \lambda N_{C,t} &= \int_{\mathcal{I}_1} \frac{Y_{1,t}(i)}{A_t A_{1,t}} di + \int_{\mathcal{I}_2} \frac{Y_{2,t}(i)}{A_t A_{2,t}} di \end{aligned}$$

where $\{\varpi_h\}$ denotes Pareto weights. First order conditions with respect to $C_{h,t}$, $N_{h,t}$, and $Y_{j,t}(i)$ are given by

$$\varpi_C C_{C,t}^{-\sigma} = \mu_1 \tag{A.98}$$

$$\varpi_U C_{U,t}^{-\sigma} = \mu_2 \tag{A.99}$$

$$\varpi_C N_{C,t}^{\varphi} = \mu_3 \tag{A.100}$$

$$\varpi_U N_{U,t}^{\varphi} = \mu_3 \tag{A.101}$$

$$\mu_1 Y_{1,t}^{\frac{1}{\theta}}\ z_1^{-\frac{1}{\theta}} Y_{1,t}(i)^{-\frac{1}{\theta}} = \mu_3 \frac{1}{A_t A_{1,t}} \tag{A.102}$$

$$\mu_2 Y_{2,t}^{\frac{1}{\theta}}\ z_2^{-\frac{1}{\theta}} Y_{2,t}(i)^{-\frac{1}{\theta}} = \mu_3 \frac{1}{A_t A_{2,t}} \tag{A.103}$$

where μ_1, μ_2 and μ_3 are Lagrange multipliers. According to the last two conditions, $Y_{j,t}(i)$ should have a common value, $Y_{j,t}(i) = \frac{Y_{j,t}}{z_j}$, implying no output dispersion within sector in the efficient allocation.

Simplifying further, the efficient allocation is characterized by

$$N_{C,t}^{E\ \varphi} = C_{C,t}^{E\ -\sigma} A_t A_{1,t} \tag{A.104}$$

$$N_{U,t}^{E\ \varphi} = C_{U,t}^{E\ -\sigma} A_t A_{2,t} \tag{A.105}$$

$$\frac{N_{C,t}^E}{N_{U,t}^E} = \left(\frac{\varpi_C}{\varpi_U}\right)^{-\varphi} \tag{A.106}$$

$$\lambda C_{C,t}^E = Y_{1,t}^E \tag{A.107}$$

$$(1-\lambda) C_{U,t}^E = Y_{2,t}^E \tag{A.108}$$

$$(1-\lambda) N_{U,t}^E + \lambda N_{C,t}^E = \frac{Y_{1,t}^E}{A_t A_{1,t}} + \frac{Y_{2,t}^E}{A_t A_{2,t}} \tag{A.109}$$

where E stands for "*Efficient*".

Since the efficient allocation is affected by relative Pareto weights, $\frac{\varpi_C}{\varpi_U}$, we assume that a social planner is utilitarian ($\varpi_U = \varpi_C$). Then, the log-linearized system of equations of the efficient allocation around the deterministic efficient zero-inflation steady state is given by

$$n_{C,t}^E = n_{U,t}^E (\equiv n_t^E) \tag{A.110}$$

$$\varphi n_{C,t}^E + \sigma c_{C,t}^E = a_t + a_{1,t} \tag{A.111}$$

$$\varphi n_{U,t}^E + \sigma c_{U,t}^E = a_t + a_{2,t} \tag{A.112}$$

$$c_{C,t}^E = y_{1,t}^E \tag{A.113}$$

$$c_{U,t}^E = y_{2,t}^E \tag{A.114}$$

$$n_t^E = z_1 (y_{1,t}^E - a_t - a_{1,t}) + z_2 (y_{2,t}^E - a_t - a_{2,t}) \tag{A.115}$$

The dynamics of variables expressed in terms of exogenous processes are given by

$$n_t^E = n_{C,t}^E = n_{U,t}^E = \underbrace{\frac{1-\sigma}{\sigma+\varphi}}_{+/-}\left(a_t + n_1 a_{1,t} + n_2 a_{2,t}\right) \tag{A.116}$$

$$y_{1,t}^E = c_{C,t}^E = \underbrace{\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)a_t}_{+/+} \underbrace{+\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1\right)a_{1,t}}_{+/+} \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2\, a_{2,t}}_{-/+} \tag{A.117}$$

$$y_{2,t}^E = c_{U,t}^E = \underbrace{\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)a_t}_{+/+} \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1\, a_{1,t}}_{-/+} \underbrace{+\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2\right)a_{2,t}}_{+/+} \tag{A.118}$$

$$n_{1,t}^E = \underbrace{\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)a_t}_{+/-} \underbrace{+\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1\right)a_{1,t}}_{+/-} \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2\, a_{2,t}}_{-/+} \tag{A.119}$$

$$n_{2,t}^E = \underbrace{\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)a_t}_{+/-} \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1\, a_{1,t}}_{-/+} + \underbrace{\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2\right)a_{2,t}}_{+/-} \tag{A.120}$$

where the signs are when $\sigma<1$ and $\sigma>1$, respectively. The implied wage and relative price are derived as $w_t^E = a_t + a_{1,t}$ and $q_t^E = a_{1,t} - a_{2,t}$, so we identify heterogeneous real wages, $w_{C,t}^E (= w_t^E) = a_t + a_{1,t}$ and $w_{U,t}^E (= w_t^E - q_t^E) = a_t + a_{2,t}$, in the efficient allocation.

Steady State

By assuming $A = A_1 = A_2 = 1$, we have symmetric steady state as follows.

$$C_C = C_U = N_C = N_U = 1$$

$$Y_1 = N_1 = \lambda \text{ and } Y_2 = N_2 = 1 - \lambda$$

A.2.2 Sticky-Price Allocation

We present the system of equations that characterize the first-order approximation of the equilibrium of the model under sticky-price.

- Consumption baskets (Goods market clearing condition)

$$\widetilde{c}_{C,t} = \widetilde{y}_{1,t} \tag{A.121}$$

$$\widetilde{c}_{U,t} = \widetilde{y}_{2,t} \tag{A.122}$$

- Euler equation

$$\widetilde{y}_{2,t} - E_t[\widetilde{y}_{2,t+1}] = -\frac{1}{\sigma}\left(\widetilde{i}_t - E_t[\pi_{2,t+1}] - r_t^E\right) \tag{A.123}$$

where $r_t^E \equiv \sigma(E_t[y_{2,t+1}^E] - y_{2,t}^E)$

- Labor supply schedule of type U households

$$\varphi\widetilde{n}_{U,t} + \sigma\widetilde{y}_{2,t} = \widetilde{w}_t - \widetilde{q}_t \tag{A.124}$$

- Labor supply schedule of type C households

$$\varphi\widetilde{n}_{C,t} + \sigma\widetilde{y}_{1,t} = \widetilde{w}_t \tag{A.125}$$

- Budget constraint of type C households

$$\widetilde{w}_t + \widetilde{n}_{C,t} = \widetilde{y}_{1,t} + \frac{1-\sigma}{\sigma} z_2 q_t^E \tag{A.126}$$

- Labor market clearing condition

$$z_1\widetilde{y}_{1,t} + z_2\widetilde{y}_{2,t} = z_1\widetilde{n}_{C,t} + z_2\widetilde{n}_{U,t} \tag{A.127}$$

- Phillips curve in sector 1

$$\pi_{1,t} = \beta E_t[\pi_{1,t+1}] + \frac{(1-\alpha_1\beta)(1-\alpha_1)}{\alpha_1}\widetilde{w}_t \tag{A.128}$$

- Phillips curve in sector 2

$$\pi_{2,t} = \beta E_t[\pi_{2,t+1}] + \frac{(1-\alpha_2\beta)(1-\alpha_2)}{\alpha_2}\big(\widetilde{w}_t - \widetilde{q}_t\big) \tag{A.129}$$

- Real marginal cost in sector 1

$$\widetilde{w}_t = \frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + z_2\frac{\varphi}{1+\varphi}\frac{1-\sigma}{\sigma}q_t^E \tag{A.130}$$

- Real marginal cost in sector 2

$$\widetilde{w}_t - \widetilde{q}_t = \varphi\frac{z_1}{z_2}\frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + (\sigma+\varphi)\widetilde{y}_{2,t} - z_1\frac{\varphi}{1+\varphi}\frac{1-\sigma}{\sigma}q_t^E \tag{A.131}$$

- Relative price

$$\widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E = \pi_{2,t} - \pi_{1,t} \tag{A.132}$$

- Monetary policy

$$\widetilde{i}_t = \phi_{\pi_1}\pi_{1,t} + \phi_{\pi_2}\pi_{2,t} + \phi_{y_1}y_{1,t} + \phi_{y_2}y_{2,t} + \nu_t \tag{A.133}$$

- Exogenous processes

$$a_t = \rho_a a_{t-1} + \sigma_a\varepsilon_t^a \tag{A.134}$$

$$a_{1,t} = \rho_{a_1}a_{1,t-1} + \sigma_{a_1}\varepsilon_t^{a_1} \tag{A.135}$$

$$a_{2,t} = \rho_{a_2}a_{2,t-1} + \sigma_{a_2}\varepsilon_t^{a_2} \tag{A.136}$$

$$\nu_t = \rho_\nu a_{t-1} + \sigma_\nu\varepsilon_t^\nu \tag{A.137}$$

Steady State

Steady state is symmetric due to fiscal specifications. Note that the steady state is efficient.

$$
\begin{aligned}
&C_C = C_U = N_C = N_U = A = A_1 = A_2 = 1\\
&Y_1 = N_1 = \lambda \text{ and } Y_2 = N_2 = 1-\lambda\\
&W = Q = 1\\
&D = \frac{1}{\theta} \text{ and } T_U = -\frac{1}{\theta(1-\lambda)}
\end{aligned}
$$

A.2.3 Flexible-Price Allocation

The system of equations of the equilibrium of the model under flexible-price is the same except that sectoral Phillips curves are replaced with constant markup or zero real marginal cost gap, $\widetilde{w}_t^N = \widetilde{w}_t^N - \widetilde{q}_t^N = 0$, where N stands for *natural* or flexible-price allocation. Thus we present the first-order approximation to the solutions under flexible-price as functions of exogenous processes or $q_t^E = a_{1,t} - a_{2,t}$.

$$\widetilde{y}_{1,t}^N = -\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi} z_2 q_t^E \tag{A.138}$$

$$\widetilde{y}_{2,t}^N = \frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi} z_1 q_t^E \tag{A.139}$$

$$\widetilde{n}_{C,t}^N = \frac{1-\sigma}{\sigma+\varphi} z_2 q_t^E \tag{A.140}$$

$$\widetilde{n}_{U,t}^N = -\frac{1-\sigma}{\sigma+\varphi} z_1 q_t^E \tag{A.141}$$

$$\widetilde{w}_t^N = 0 \tag{A.142}$$

$$\widetilde{q}_t^N = 0 \tag{A.143}$$

Steady State

Steady state is the same as in the model sticky-price, and thus efficient.

$$
\begin{aligned}
&C_C = C_U = N_C = N_U = A = A_1 = A_2 = 1\\
&Y_1 = N_1 = \lambda \text{ and } \; Y_2 = N_2 = 1-\lambda\\
&W = Q = 1\\
&D = \frac{1}{\theta} \text{ and } \; T_U = -\frac{1}{\theta(1-\lambda)}
\end{aligned}
$$

A.2.4 Asymmetric redistribution of inflationary pressure

The effects of sectoral output gaps and adjustment terms on dynamics of sectoral inflation are asymmetric as shown in the Phillips curves rewritten in terms of sectoral output gaps:(1) inflation in sector 1 is affected only by output gap 1, while (2) inflation in sector 2 is affected by both output gaps; (3) a relative productivity shock q_t^E has the opposite consequences in each sector. (1) and (2) imply the redistribution of inflationary pressure across sectors as the labor demand is redistributed across households, and (3) is due to the lack of risk-sharing. We analyze the asymmetry in inflation dynamics in order.

$$
\begin{aligned}
\pi_{1,t} &= \beta E_t[\pi_{1,t+1}] + \kappa_1 \left(\frac{\sigma+\varphi}{1+\varphi} \widetilde{y}_{1,t} + z_2 \frac{\varphi}{1+\varphi} \frac{1-\sigma}{\sigma} q_t^E \right)\\
\pi_{2,t} &= \beta E_t[\pi_{2,t+1}] + \kappa_2 \left(\frac{z_1}{z_2} \varphi \frac{\sigma+\varphi}{1+\varphi} \widetilde{y}_{1,t} + (\sigma+\varphi) \widetilde{y}_{2,t} - z_1 \frac{\varphi}{1+\varphi} \frac{1-\sigma}{\sigma} q_t^E \right)
\end{aligned}
$$

The real wages in the Phillips curves are those in equilibrium that explain how demand or output gaps affect marginal costs through the labor market and eventually inflation. Thus the relations of real wages and outputs in equilibrium are derived by labor supply relations and budget constraints of households, goods market clearing conditions, labor market clearing condition and production function.

We begin with sector 1 in which the real marginal cost coincides with type C households' real wage gap, $\widetilde{w}_t$. According to type C households' labor supply schedule, their real wage is the ratio between marginal disutility of labor supply and marginal utility of consumption, or in log, real wage is marginal disutility of labor supply less marginal utility of consumption in equilibrium. Since their labor supply has a perfect correlation with their consumption, $n_{C,t} = \frac{1-\sigma}{1+\varphi} y_{1,t}$ considering the redistribution of labor demand by $\frac{\sigma+\varphi}{1+\varphi} y_{1,t}$, we find that the real wage w_t is associated only with output 1. This is expressed in gaps by

$$\begin{aligned}\widetilde{w}_t &= \varphi \widetilde{n}_{C,t} - (-\sigma \widetilde{c}_{C,t}) = \varphi\left(1 - \frac{\sigma+\varphi}{1+\varphi}\right)\widetilde{y}_{1,t} + \sigma \widetilde{y}_{1,t} + (adjustment\ term) \\ &= \frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + (adjustment\ term)\end{aligned}$$

Next, consider sector 2 in which the real marginal cost coincides with type U households' real wage gap, $\widetilde{w}_t - \widetilde{q}_t$. Analogously, type U households' real wage is their marginal disutility of labor supply less marginal utility of consumption. Since labor demand is redistributed from type C to type U, labor supply of the latter is affected by type C households consumption, $y_{1,t}$, as well as their own consumption, $y_{2,t}$. Thus the real wage $w_t - q_t$ is associated with both output 1 and 2.This is expressed in gaps by

$$\begin{aligned}\widetilde{w}_t - \widetilde{q}_t &= \varphi \widetilde{n}_{U,t} - (-\sigma \widetilde{c}_{U,t}) = \varphi\big(\widetilde{y}_{2,t} - \frac{z_1}{z_2}(\widetilde{n}_{C,t} - \widetilde{y}_{1,t})\big) - (-\sigma c_{U,t}) \\ &= \frac{z_1}{z_2}\varphi \frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + (\sigma+\varphi)\widetilde{y}_{2,t} + (adjustment\ term)\end{aligned}$$

Suppose an increase in output gap 1. On the one hand, HtM households' real wage should increase to support a higher consumption in equilibrium, which is in turn associated with an increase in labor hours assuming $\sigma < 1$. As marginal utility of consumption decreases and marginal disutility of labor increases, their real wage would increase in equilibrium, leading to higher inflation in sector 1. On the other hand, type U households' labor hours also increase as labor demand is redistributed. This raises their marginal disutility of labor and real wage in equilibrium, with inflation in sector 2 also increasing. Now suppose an increase in output gap 2. Since HtM households are not affected by sector 2, and the labor demanded by sector 2 is filled, in effect, by type U households, their marginal disutility of labor increases, inducing inflation in sector 2 to rise.

We summarize the analysis as follows:

- Each output gap poses inflationary pressure by a factor "$\sigma + \varphi$": φ and $-\sigma$ reflects marginal disutility of labor gap and marginal utility of consumption gap, with the difference between them being the real wage gap in equilibrium, through which inflationary pressure is created in each sector.

- (*Marginal utility of consumption channel: "σ"*) An increase in each sectoral output gap lowers marginal utility of consumption gap of households who consume goods from that sector intensively, creating inflationary pressure on its own sector: output gap 1 (output gap 2) affects type C (type U) households' marginal utility of consumption gap creating inflationary pressure on sector 1 (sector 2) by a factor σ.[1]

- (*Marginal disutility of Labor supply channel: "φ"*) Since outputs are produced by labor hours that is a source of disutility, an increase in each sectoral output gap can raise marginal disutility of labor supply of each type of households creating inflationary pressure. How much each inflationary pressure is distributed to each sector is determined by how labor demanded by each sector is distributed to each type of household. As labor demand is redistributed from sector 1 to sector 2 by $\frac{\sigma+\varphi}{1+\varphi} y_{1,t}$, inflationary pressure is also redistributed by a factor φ adjusted by sector size.

Lastly, note that inefficient distribution of inflation occurs, which is represented by the adjustment terms in the Phillips curves that show up as a result of the impossibility. They are similar to cost-push shocks in that they add stochasticity to inflation dynamics even under zero output gaps and hence divine coincidence no longer holds, but different in that the former always disappears as we aggregate sectoral inflation with the economic size of each sector, because they put inflationary pressure on each sector in the opposite direction but in the same size as much as the consequences of

[1] Unlike the marginal disutility of labor supply channel, there is no redistibution of inflationary pressure, because consumption sectors of each type of household is completely differennt. But if we introduce the general case of heterogeneous consumption baskets where households have common share of consumption, there will be redistribution of inflationary pressure across households depending on who consumes more intensively.

the lack of risk-sharing (before adjusted for sectoral size). Let us take as an example the case of a positive shock on sector-specific productivity $a_{1,t}$ as seen in Section 1.2.5 and Section 1.2.5. Due to financial constraints, type C households have to work more and type U households have to work less than under efficient allocation. Since marginal disutility of labor supply gap is higher (lower) for type C (type U) households, their real wage gap that equals to real marginal cost, $\widetilde{w}_t$ ($\widetilde{w}_t - \widetilde{q}_t$), and inflation in the sector of consumption, $\pi_{1,t}$ ($\pi_{2,t}$), are higher (lower) in equilibrium in the absence of risk-sharing, implying that inefficient distribution of labor supply translates to inefficient distribution of inflationary pressure across sectors. As a result, inflation dynamics in both sectors are amplified if $\sigma < 1$, or subdued if $\sigma > 1$, considering that the shock leads to a negative output gap in sector 1 and a positive output gap in sector 2 due to nominal rigidity.

A.2.5 Wage Elasticity of Labor Hours

Using the example of a household that makes a static decision on consumption and labor supply given the wage with utility function and budget constraint below in Section 1.3.3, we derive the wage elasticity of labor hours as below:

$$\varepsilon_{N,W} \equiv \frac{\partial N}{\partial W}\frac{W}{N} = \frac{1-\sigma(\frac{WN+\frac{\partial M}{\partial W}W}{WN+M})}{\varphi+\sigma(\frac{WN}{WN+M})} = \frac{1-\sigma(\frac{WN+\varepsilon_{M,W}M}{WN+M})}{\varphi+\sigma(\frac{WN}{WN+M})}$$

Since HtM households depend entirely on wage income ($M = 0,\ \varepsilon_{M,W} < 0$), their wage elasticity of labor hours would be $\frac{1-\sigma}{\varphi+\sigma}$. However, the unconstrained households have other sources of income, dividend, which is countercyclical ($M > 0,\ \varepsilon_{M,W} < 0$). If $\sigma > \frac{WN+M}{WN+\varepsilon_{M,W}M}(> 1)$, the unconstrained households' wage elasticity of labor hours would be smaller in absolute terms than that of HtM ($|\varepsilon_{N,W,\text{type C}}| = |\frac{1-\sigma}{\varphi+\sigma}| > |\varepsilon_{N,W,\text{type U}}|$). If not, wage elasticity of labor hours is higher for the unconstrained households. But in this case, consumption volatility gets more important as labor hours gets relatively less volatile than that of consumption with $\varepsilon_{C,W} > \varepsilon_{N,W}$.

A.3 Homogeneous Consumption Baskets

We provide the system of equations and some derivations of the equilibrium in the efficient allocation and the decentralized model under **HomCB** ($\omega_U = \omega_C = \frac{1}{2}$).

A.3.1 Efficient Allocation

As both types of households are of the same preference consuming homogeneous consumption baskets and under the same economic constraints, the first-best is that consumption and labor supply are equalized across all the households as if there is a representative household:

$$c_t^E \equiv c_{C,t}^E = c_{U,t}^E \tag{A.144}$$

$$c_{1,t}^E \equiv c_{C,1,t}^E = c_{U,1,t}^E \tag{A.145}$$

$$c_{2,t}^E \equiv c_{C,2,t}^E = c_{U,2,t}^E \tag{A.146}$$

$$n_t^E \equiv n_{C,t}^E = n_{U,t}^E \tag{A.147}$$

The log-linearized system of equations of the efficient allocation around the deterministic efficient zero-inflation steady state is given by

$$\omega(c_{1,t}^E - c_t^E) + (1-\omega)(c_{2,t}^E - c_t^E) = 0 \tag{A.148}$$

$$\varphi n_t^E + \sigma c_t^E + \frac{1}{\eta}(c_{1,t}^E - c_t^E) = a_t + a_{1,t} \tag{A.149}$$

$$\varphi n_t^E + \sigma c_t^E + \frac{1}{\eta}(c_{2,t}^E - c_t^E) = a_t + a_{2,t} \tag{A.150}$$

$$n_t^E + a_t + \omega a_{1,t} + (1-\omega) a_{2,t} = c_t^E \tag{A.151}$$

The dynamics of variables expressed in terms of exogenous processes are given by

$$c_t^E = \frac{1+\varphi}{\sigma+\varphi}\big(a_t + \omega a_{1,t} + (1-\omega)a_{2,t}\big) \tag{A.152}$$

$$c_{1,t}^E = c_t^E + (1-\omega)\eta(a_{1,t} - a_{2,t}) \tag{A.153}$$

$$c_{2,t}^E = c_t^E - \omega\eta(a_{1,t} - a_{2,t}) \tag{A.154}$$

$$n_t^E = \frac{1-\sigma}{\sigma+\varphi}\big(a_t + \omega a_{1,t} + (1-\omega)a_{2,t}\big) \tag{A.155}$$

Steady State

We assume $A = A_1 = A_2 = 1$, and have symmetric steady state as follows.

$$C_C = C_U = N_C = N_U = 1$$
$$Y_1 = N_1 = C_1 = C_{C,1} = C_{U,1} = \omega$$
$$Y_2 = N_2 = C_2 = C_{C,2} = C_{U,2} = 1-\omega$$

A.3.2 Sticky-Price Allocation

We present the system of equations that characterize the first-order approximation of the equilibrium of the model under sticky-price.

- Euler equation

$$\widetilde{c}_{U,t} - E_t[\widetilde{c}_{U,t+1}] = -\frac{1}{\sigma}\Big(\widetilde{i}_t - (\omega E_t[\pi_{1,t+1}] + (1-\omega)E_t[\pi_{2,t+1}]) - r_t^E\Big) \tag{A.156}$$

  where $r_t^E \equiv \sigma(E_t[c_{U,t+1}^E] - c_{U,t}^E)$

- Labor supply schedule of type U households

$$\varphi\widetilde{n}_{U,t} + \sigma\widetilde{c}_{U,t} = \widetilde{w}_t - (1-\omega)\widetilde{q}_t \tag{A.157}$$

- Labor supply schedule of type C households

$$\varphi\widetilde{n}_{C,t} + \sigma\widetilde{y}_{1,t} = \widetilde{w}_t - (1-\omega)\widetilde{q}_t \tag{A.158}$$

- Budget constraint of type C households

$$\widetilde{w}_t - (1-\omega)\widetilde{q}_t + \widetilde{n}_{C,t} = \widetilde{c}_{C,t} \tag{A.159}$$

- Labor market clearing condition

$$\omega\widetilde{y}_{1,t} + (1-\omega)\widetilde{y}_{2,t} = \lambda\widetilde{n}_{C,t} + (1-\lambda)\widetilde{n}_{U,t} \tag{A.160}$$

- Phillips curve in sector 1

$$\pi_{1,t} = \beta E_t[\pi_{1,t+1}] + \frac{(1-\alpha_1\beta)(1-\alpha_1)}{\alpha_1}\widetilde{w}_t \tag{A.161}$$

- Phillips curve in sector 2

$$\pi_{2,t} = \beta E_t[\pi_{2,t+1}] + \frac{(1-\alpha_2\beta)(1-\alpha_2)}{\alpha_2}\left(\widetilde{w}_t - \widetilde{q}_t\right) \tag{A.162}$$

- Real marginal cost in sector 1

$$\widetilde{w}_t = (\sigma+\varphi)\widetilde{y}_t + (1-\omega)\widetilde{q}_t \tag{A.163}$$

- Real marginal cost in sector 2

$$\widetilde{w}_t-\widetilde{q}_t = (\sigma+\varphi)\widetilde{y}_t - \omega\widetilde{q}_t \tag{A.164}$$

- Relative price

$$\widetilde{q}_t - \widetilde{q}_{t-1} + q^E_t - q^E_{t-1} = \pi_{2,t} - \pi_{1,t} \tag{A.165}$$

- Monetary policy

$$\widetilde{i}_t = \phi_{\pi_1}\pi_{1,t} + \phi_{\pi_2}\pi_{2,t} + \phi_{y_1}y_{1,t} + \phi_{y_2}y_{2,t} + \nu_t \tag{A.166}$$

- Exogenous processes

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a \tag{A.167}$$

$$a_{1,t} = \rho_{a_1} a_{1,t-1} + \sigma_{a_1} \varepsilon_t^{a_1} \tag{A.168}$$

$$a_{2,t} = \rho_{a_2} a_{2,t-1} + \sigma_{a_2} \varepsilon_t^{a_2} \tag{A.169}$$

$$\nu_t = \rho_\nu a_{t-1} + \sigma_\nu \varepsilon_t^\nu \tag{A.170}$$

Steady State

Steady state is symmetric due to fiscal specifications. Note that the steady state is efficient.

$$\begin{aligned}
&C_C = C_U = N_C = N_U = A = A_1 = A_2 = 1 \\
&Y_1 = N_1 = C_1 = C_{C,1} = C_{U,1} = \omega \\
&Y_2 = N_2 = C_2 = C_{C,2} = C_{U,2} = 1 - \omega \\
&W = Q = 1 \\
&D = \frac{1}{\theta} \text{ and } T_U = -\frac{1}{\theta(1-\lambda)}
\end{aligned}$$

A.3.3 Flexible-Price Allocation

The system of equations of the equilibrium of the model under flexible-price is the same except that sectoral Phillips curves are replaced with constant markup or zero real marginal cost gap, $\widetilde{w}_t^N = \widetilde{w}_t^N - \widetilde{q}_t^N = 0$, where N stands for *natural* or flexible-price allocation. Unlike the **HetCB** case, flexible-price allocation under **HomCB** achieves the efficient allocation closing both output gaps and labor supply gaps.

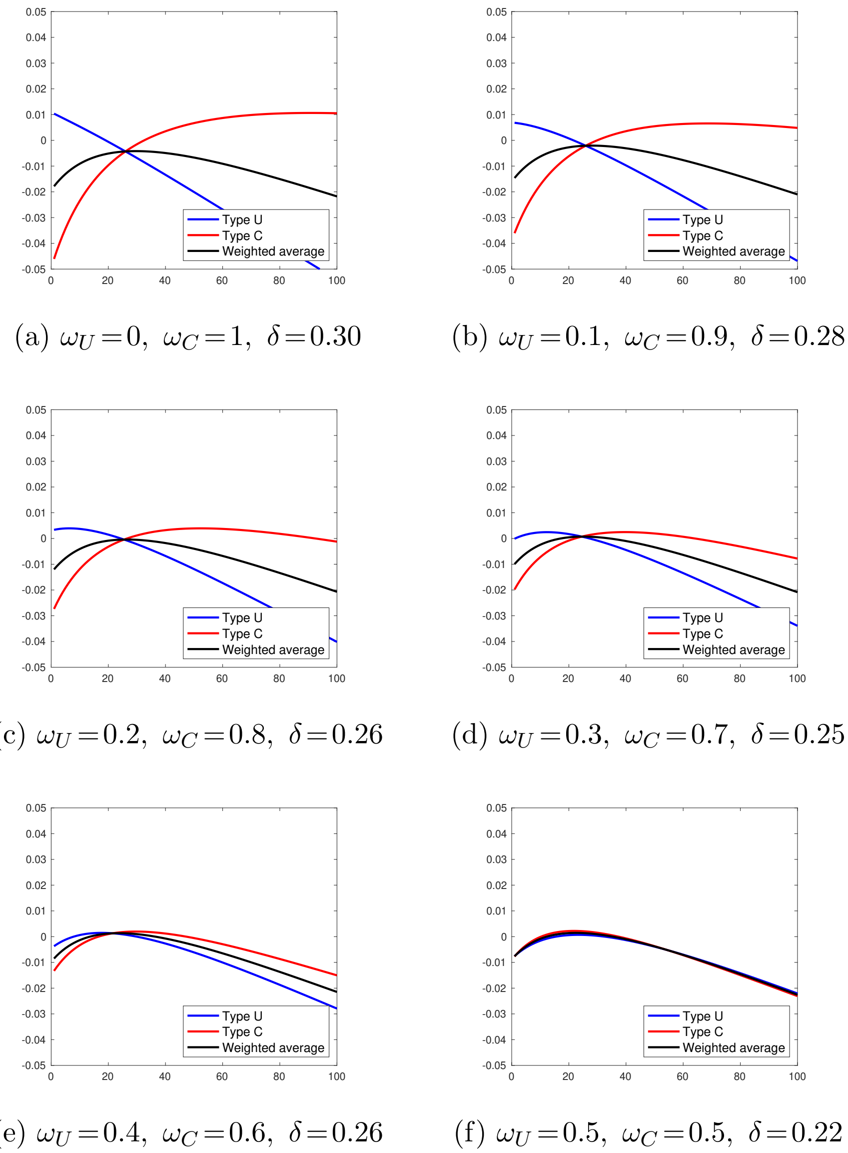
$$\widetilde{y}_{1,t}^N = \widetilde{y}_{2,t}^N = \widetilde{n}_{C,t}^N = \widetilde{n}_{U,t}^N = \widetilde{w}_t^N = \widetilde{q}_t^N = 0 \tag{A.171}$$

Steady State

Steady state is the same as in the model sticky-price, and thus efficient.

$$C_C = C_U = N_C = N_U = A = A_1 = A_2 = 1$$
$$Y_1 = N_1 = \lambda \text{ and } Y_2 = N_2 = 1 - \lambda$$
$$W = Q = 1$$
$$D = \frac{1}{\theta} \text{ and } T_U = -\frac{1}{\theta(1-\lambda)}$$

A.4 Figures



(a) $\omega_U = 0,\ \omega_C = 1,\ \delta = 0.30$

(b) $\omega_U = 0.1,\ \omega_C = 0.9,\ \delta = 0.28$

(c) $\omega_U = 0.2,\ \omega_C = 0.8,\ \delta = 0.26$

(d) $\omega_U = 0.3,\ \omega_C = 0.7,\ \delta = 0.25$

(e) $\omega_U = 0.4,\ \omega_C = 0.6,\ \delta = 0.26$

(f) $\omega_U = 0.5,\ \omega_C = 0.5,\ \delta = 0.22$

Figure A.1: Redistributive effects of inflation targeting policy (under no inequality)

Appendix B

Monetary Non-Neutrality in a Multisector Economy: The Role of Risk-Sharing

B.1 The baseline model

This section gives a full detail of the model considered in section 2-4 of the main text. The mode features sector-specific labor markets and incomplete markets. Households working in different sectors fail to insure perfectly against their labor income risk. We present the equilibrium conditions, the steady state, the log-linearized equilibrium conditions and the derivation of the Phillips curve.

B.1.1 Households

Households who work in sector k (or type-k households) seek to maximize

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\log C_{k,t} - \omega_k \frac{H_{k,t}^{1+\varphi}}{1+\varphi} \right) \right],$$

subject to the flow budget constraint

$$P_t C_{k,t} + B_{k,t} + \Omega(B_{k,t}) = R_{t-1} B_{k,t-1} + W_{k,t} H_{k,t} + P_t T_t + \Pi_t$$

where $\Omega(B_{k,t}) = \frac{\epsilon}{PY} B_{k,t}^2$. Type-$k$ household's first order conditions are

$$1 + \Omega'(B_{k,t}) = \beta R_t \mathbb{E}_t \left[\left(\frac{C_{k,t}}{C_{k,t+1}} \right) \left(\frac{P_t}{P_{t+1}} \right) \right],$$
$$\frac{W_{k,t}}{P_t} = \omega_k H_{k,t}^{\varphi} C_{k,t}.$$

B.1.2 Firms

The final consumption good, Y_t, is produced by perfectly competitive firms with a production technology

$$Y_t = \left(\sum_{k=1}^{K} n_k^{1/\eta} Y_{k,t}{}^{(\eta-1)/\eta} \right)^{\eta/(\eta-1)},$$

The appropriate price index for the final consumption good is

$$P_t = \left(\sum_{k=1}^{K} n_k P_{k,t}^{1-\eta} \right)^{1/(1-\eta)}, \tag{B.1}$$

where $P_{k,t}$ is the sectoral price index associated with $Y_{k,t}$. Given Y_t, $P_{k,t}$ and P_t, the optimal demand for sector-k good minimizes total expenditure, $P_t Y_t$, and is given by

$$Y_{k,t} = n_k \left(\frac{P_{k,t}}{P_t} \right)^{-\eta} Y_t \quad \forall k.$$

Each sectoral good is a composite of $\{Y_{k,t}(i)\}_{i \in \mathcal{I}_k}$ that are produced by firms in sector k

$$Y_{k,t} = \left(\left(\frac{1}{n_k} \right)^{1/\theta} \int_{\mathcal{I}_k} Y_{k,t}(i)^{(\theta-1)/\theta} di \right)^{\theta/(\theta-1)} \quad \forall k,$$

The corresponding price index for a sectoral good is given by

$$P_{k,t} = \left(\frac{1}{n_k} \int_{\mathcal{I}_k} P_{k,t}(i)^{1-\theta} di \right)^{1/(1-\theta)} \quad \forall k.$$

Given $Y_{k,t}$, $P_{k,t}(i)$ and $P_{k,t}$, the optimal demand for good i is given by

$$Y_{k,t}(i) = \frac{1}{n_k}\left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\theta} Y_{k,t}.$$

Firm i in sector k uses a linear production function to produce $Y_{k,t}(i)$

$$Y_{k,t}(i) = A_{k,t}H_{k,t}(i), \tag{B.2}$$

Firms in sector k adjust their prices with probability $1-\alpha_k$ each period. The sectoral price, $P_{k,t}$, evolves as

$$P_{k,t} = \left[\frac{1}{n_k}\int_{\mathcal{I}_k^*} P_{k,t}^*(i)^{1-\theta}di + \alpha_k P_{k,t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}},$$

where $P_{k,t}^*(i)$ is an optimal price chosen by firm i when $i \in \mathcal{I}_k^*$. The set $\mathcal{I}_k^* \subset \mathcal{I}_k$, with measure $n_k(1-\alpha_k)$, is a randomly chosen subset in which firms adjust their prices.

A firm that adjusts its price at time t choose $P_{k,t}^*(i)$ that maximizes its expected discounted profits

$$\max_{P_{k,t}^*(i)} \mathbb{E}_t \sum_{j=0}^{\infty} \alpha_k^j q_{k,t,t+j} \frac{\Pi_{k,t+j}(i)}{P_{t+j}},$$

where $q_{k,t,t+j}$ is the real stochastic discount factor between time t and $t+j$. We assume that firms discount future profits using the real interest rate, in which case, $q_{k,t,t+j} = \prod_{z=0}^{j} R_{t+z}^{-1}\frac{P_{t+z+1}}{P_{t+z}}$.

$\Pi_{k,t+j}(i)$ is the firm's nominal profit at time $t+j$ given that the price is chosen at time t

$$\Pi_{k,t+j}(i) = P_{k,t}(i)Y_{k,t+j}(i) - W_{k,t+j}H_{k,t+j}(i).$$

The first order condition is given by

$$0 = \mathbb{E}_t \sum_{j=0}^{\infty} \alpha_k^j q_{k,t,t+j} Y_{t+j} \left(\frac{P_{k,t}^*(i)}{P_{k,t+j}}\right)^{-\theta} \left(\frac{P_{k,t+j}}{P_{t+j}}\right)^{-\eta} \left\{ \left(\frac{P_{k,t}^*(i)}{P_{t+j}}\right) - \left(\frac{\theta}{\theta-1}\right) \underbrace{\frac{W_{k,t+j}}{A_{k,t+j}P_{t+j}}}_{\equiv \frac{MC_{k,t+j}}{P_{t+j}}} \right\}, \tag{B.3}$$

where $MC_{k,t+j} = \frac{W_{k,t+j}}{A_{k,t+j}}$ denotes sector-k nominal marginal costs at $t+j$.

B.1.3 Government policy

The government budget constraint is:

$$\frac{B_t - R_{t-1}B_{t-1}}{P_t} + \sum_{k=1}^{K} n_k \Omega(B_{k,t}) = T_t + G_t,$$

where we assume $B_t = G_t = 0$ for simplicity.

Monetary policy is characterized by a Taylor-type rule

$$R_t = \beta^{-1} \left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \exp(\mu_t),$$

B.1.4 Steady state

For simplicity, we make two assumptions that deliver a symmetric steady state: i) the steady-state levels of sector-specific productivities are the same across sectors: specifically, $A_k = 1$ for all k, without loss of generality;[1] ii) $\omega_k = n_k^{-\varphi}$ for all k. The second assumption relates the relative disutilities of labor to the size of the sectors, and equalizes steady-state sectoral wages.

We solve for $\left\{Y, C, H, \frac{W}{P}, \frac{\Pi}{P}\right\}$: the steady state values of aggregate output, aggregate consumption, aggregate hours, real wage, and real profits. Once we obtain these aggregate variables, it is trivial to characterize the steady-state values for sectoral and micro variables using the symmetric nature of the steady state (i.e. $C_k = C$,

[1]Similary, we fix the steady-state level of all other exogenous processes at unity.

$H_k = H$, $Y_k = n_k Y_k(i) = n_k Y$, $H_k(i) = H$, $\Pi_k(i) = \Pi$, $\frac{W_k}{P} = \frac{W}{P}$, and $\frac{P(i)}{P} = \frac{P_k}{P} = 1$).

By exploiting the symmetry, the system of equilibrium conditions can be reduced to the following six equations:

$$\text{Household Budget Constraint} \quad C = \left(\frac{W}{P}\right) H + \left(\frac{\Pi}{P}\right) \tag{B.4}$$

$$\text{Aggregate Labor Supply:} \quad \left(\frac{W}{P}\right) = H^{\varphi} C \tag{B.5}$$

$$\text{Aggregate Technology:} \quad Y = H \tag{B.6}$$

$$\text{Aggregate Resource Constraint:} \quad Y = C \tag{B.7}$$

$$\text{Aggregate Profit:} \quad \left(\frac{\Pi}{P}\right) = Y - \left(\frac{W}{P}\right) H \tag{B.8}$$

$$\text{Mark-up:} \quad 1 = \left(\frac{\theta}{\theta - 1}\right)\left(\frac{W}{P}\right), \tag{B.9}$$

First, it is trivial to obtain the real wage from (B.9):

$$\left(\frac{W}{P}\right) = \frac{\theta - 1}{\theta}.$$

Combining (B.5), (B.6) and (B.7),

$$Y = C = H = \left(\frac{\theta - 1}{\theta}\right)^{\frac{1}{1+\varphi}}$$

Substituting into (B.4),

$$\left(\frac{\Pi}{P}\right) = \frac{1}{\theta}\left(\frac{\theta - 1}{\theta}\right)^{\frac{1}{1+\varphi}}.$$

B.1.5 Equilibrium conditions in log-linear approximations

Here, we show the system of linear difference equations.

CES Aggregates, market clearing, and definitions

- Aggregate price level

$$p_t = \sum_k n_k p_{k,t}$$

- Sectoral price level

$$p_{k,t} = \frac{1}{n_k} \int_{\mathcal{I}_k} p_{k,t}(i) di$$

- Aggregate consumption

$$c_t = \sum_k n_k c_{k,t}$$

- Bond market clearing

$$0 = \sum_k n_k b_{k,t}$$

- Sectoral labor demand

$$h_{k,t} = \frac{1}{n_k} \int_{\mathcal{I}_k} h_{k,t}(i) di$$

- Sectoral output

$$y_{k,t} = \frac{1}{n_k} \int_{\mathcal{I}_k} y_{k,t}(i) di$$

- Aggregate output

$$y_t = c_t$$

- Aggregate wage

$$w_t = \sum_k n_k w_{k,t}$$

- Aggregate hours

$$h_t = \sum_k n_k h_{k,t}$$

Demand functions

$$y_{k,t} - y_t = -\eta\left(p_{k,t} - p_t\right) \tag{B.10}$$

$$y_{k,t}(i) - y_{k,t} = -\theta\left(p_{k,t}(i) - p_{k,t}\right) \tag{B.11}$$

Household's additional FOCs

- Euler equation of type-k household

$$c_{k,t} = E_t\left[c_{k,t+1}\right] - \left(i_t - E_t\pi_{t+1}\right) + 2\epsilon b_{k,t}$$

- Labor supply of type-k household

$$w_{k,t} - p_t = \varphi h_{k,t} + c_{k,t} \tag{B.12}$$

$$c^R_{k,t} = -\psi_1 b_{k,t} + \frac{1}{\beta}\psi_1 b_{k,t-1} + \psi_2 y^R_{k,t} - \psi_2 a^R_{k,t}, \quad \Big(\psi_1 \equiv \theta, \quad \psi_2 \equiv (\theta - 1)(1 + \varphi)\Big)$$

where relative sectoral consumption, relative sectoral output, and relative sector-specific productivity are defined by $c^R_{k,t} \equiv c_{k,t} - y_t$, $y^R_{k,t} \equiv y_{k,t} - y_t$, and $a^R_{k,t} \equiv a_{k,t} - \sum_k n_k a_{k,t}$.

Firms

- Production function

$$y_{k,t}(i) = a_{k,t} + h_{k,t}(i) \tag{B.13}$$

- Nominal marginal cost

$$mc_{k,t} = w_{k,t} - a_{k,t} \tag{B.14}$$

- Optimality condition for the re-optimizing firms

$$p^*_{k,t} = (1 - \alpha_k\beta)E_t\sum_{s=0}^{\infty}\alpha^s_k\beta^s mc_{k,t+s}$$

- Sectoral Phillips curve

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\left((1+\varphi)y_t + (\varphi+\eta^{-1})y^R_{k,t} + c^R_{k,t} - (1+\varphi)a_{k,t}\right)$$

Log-linear approximate model

By aggregating proper equations at the disaggregated level, we can obtain the system of equations that determines the equilibrium of the variables of interest:

$$\{y_t, \pi_t, i_t, h_t\} \text{ and } \left\{c^R_{k,t}, y^R_{k,t}, b_{k,t}, \pi_{k,t}\right\}_{k=1}^{K}.$$

The following $4 + (4 \times K)$ equations determine the equilibrium dynamics of those variables:

$$y_t = E_t\left[y_{t+1}\right] - (i_t - E_t\pi_{t+1}) \tag{B.15}$$

$$y_t = \sum_k n_k a_{k,t} + h_t \tag{B.16}$$

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\left((1+\varphi)y_t + (\varphi+\eta^{-1})y^R_{k,t} + c^R_{k,t} - (1+\varphi)a_{k,t}\right) \tag{B.17}$$

$$\pi_t = \sum_k n_k \pi_{k,t} \tag{B.18}$$

$$\Delta y^R_{k,t} = -\eta\left(\pi_{k,t} - \pi_t\right) \tag{B.19}$$

$$c^R_{k,t} = E_t c^R_{k,t+1} + 2\epsilon b_{k,t} \tag{B.20}$$

$$c^R_{k,t} = -\psi_1 b_{k,t} + \frac{1}{\beta}\psi_1 b_{k,t-1} + \psi_2 y^R_{k,t} - \psi_2 a^R_{k,t} \tag{B.21}$$

$$\begin{aligned} i_t &= \phi_\pi \pi_t + \phi_c y_t + \mu_t \\ &\textit{or} \\ m_t &= p_t + y_t = \textit{exogenous stochastic process} \end{aligned} \tag{B.22}$$

where $\psi_1 \equiv \theta$ and $\psi_2 \equiv (\theta-1)(1+\varphi)$. The first equation (B.15) is the aggregate Euler equation for all the households, often referred to as the intertemporal IS equation;

(B.16) the aggregate resource constraint is obtained by integrating the production functions over all firms; (B.17) gives the sectoral Phillips curves and (B.18) delivers aggregate inflation; the demand function for sectoral consumption goods is given by (B.19); the results of imperfect risk-sharing across K types of households are shown in type-k household's Euler equation (B.20) and their budget constraint (B.21); the last equation, (B.22), characterizes monetary policy and closes the model.

B.1.6 Derivation of the Phillips Curve

Log-linearizing the first-order conditions of the price-readjusting firms (2.6),

$$p^*_{k,t} = (1 - \alpha_k \beta) E_t \sum_{s=0}^{\infty} \alpha_k^s \beta^s mc_{k,t+s} \tag{B.23}$$

Rearranging (B.23), we have

$$p^*_{k,t} = (1 - \alpha_k \beta) mc_{k,t} + \alpha_k \beta E_t p^*_{k,t+1} \tag{B.24}$$

Log-linearizing the relation that determines the sectoral price level (B.1), we obtain

$$p_{k,t} = (1 - \alpha_k) p^*_{k,t} + \alpha_k p_{k,t-1} \tag{B.25}$$

Combining (B.24) and (B.25), we derive the sectoral Phillips Curve (PC)

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + \frac{(1 - \alpha_k)(1 - \alpha_k \beta)}{\alpha_k} (mc_{k,t} - p_{k,t}) \tag{B.26}$$

Now we show how marginal cost is determined. Integrating the production function (B.13) over $\mathcal{I}_k$, we have

$$y_{k,t} = a_{k,t} + h_{k,t} \tag{B.27}$$

Combining (B.14) with (B.12) and (B.27), we obtain

$$\begin{aligned} mc_{k,t} &= w_{k,t} - p_t - a_{k,t} + p_t \\ &= \varphi h_{k,t} + c_{k,t} - a_{k,t} + p_t \\ &= \varphi y_{k,t} + c_{k,t} - (1+\varphi)a_{k,t} + p_t \end{aligned} \tag{B.28}$$

Plugging (B.28) into (B.26), we derive the sectoral PC as

$$\begin{aligned} \pi_{k,t} &= \beta E_t \pi_{k,t+1} + \lambda_k (mc_{k,t} - p_{k,t}) \\ &= \beta E_t \pi_{k,t+1} + \lambda_k \Big(\varphi y_{k,t} + c_{k,t} - (1+\varphi)a_{k,t} + p_t - p_{k,t} \Big) \end{aligned}$$

where $\lambda_k \equiv \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}$. Substituting (B.10) to express relative price in terms of sectoral output,

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + \lambda_k \Big((\varphi + \eta^{-1}) y_{k,t} + c_{k,t} - \eta^{-1} y_t - (1+\varphi)a_{k,t} \Big)$$

Rewriting the sectoral PC in terms of relative sectoral output and relative consumption of type-k households, we finally have

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + \lambda_k \Big((1+\varphi) y_t + (\varphi + \eta^{-1}) y^R_{k,t} + c^R_{k,t} - (1+\varphi)a_{k,t} \Big)$$

Aggregate PC is derived by aggregation of sectoral inflation

$$\begin{aligned} \pi_t &= \sum_{k=1}^{K} n_k \pi_{k,t} \\ &= \beta E_t \pi_{t+1} - (1+\varphi) \sum_{k=1}^{K} n_k \lambda_k a_{k,t} \\ &\quad + \underbrace{(1+\varphi) \sum_{k=1}^{K} n_k \lambda_k}_{\kappa} \, y_t + \underbrace{(\varphi + \eta^{-1}) \sum_{k=1}^{K} n_k \lambda_k y^R_{k,t}}_{\equiv \Theta_{y,t}} + \underbrace{\sum_{k=1}^{K} n_k \lambda_k c^R_{k,t}}_{\equiv \Theta_{c,t}} \end{aligned}$$

B.2 The model with perfect risk-sharing within sectors in firm-specific labor markets (Case I)

We now give a full description of the model considered in section 5.1 of the main text. We focus on the differences from the baseline specification.

B.2.1 Households

Households who work in sector k (or type-k households) seek to maximize

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\log C_{k,t}-\omega_k\int_{\mathcal{I}_k}\frac{H_{k,t}(i)^{1+\varphi}}{1+\varphi}di\right)\right],$$

subject to the flow budget constraint

$$P_tC_{k,t}+B_{k,t}+\Omega(B_{k,t})=R_{t-1}B_{k,t-1}+\int_{\mathcal{I}_k}W_{k,t}(i)H_{k,t}(i)di+P_tT_t+\Pi_t$$

Type-k household's first order conditions are

$$\begin{aligned}1+\Omega'(B_{k,t})&=\beta R_t\mathbb{E}_t\left[\left(\frac{C_{k,t}}{C_{k,t+1}}\right)\left(\frac{P_t}{P_{t+1}}\right)\right],\\ \frac{W_{k,t}(i)}{P_t}&=\omega_kH_{k,t}(i)^{\varphi}C_{k,t}.\end{aligned}$$

B.2.2 Firms

The firm ik's nominal profit at time $t+j$ given that the price is chosen at time t is given by

$$\Pi_{k,t+j}(i)=P_{k,t}(i)Y_{k,t+j}(i)-W_{k,t+j}(i)H_{k,t+j}(i).$$

The first order condition of the firm's profit maximization problem is given by

$$0 = \mathbb{E}_t \sum_{j=0}^{\infty} \alpha_k^j q_{k,t,t+j} Y_{t+j} \left(\frac{P_{k,t}^*(i)}{P_{k,t+j}} \right)^{-\theta} \left(\frac{P_{k,t+j}}{P_{t+j}} \right)^{-\eta} \left\{ \left(\frac{P_{k,t}^*(i)}{P_{t+j}} \right) - \left(\frac{\theta}{\theta - 1} \right) \underbrace{\frac{W_{k,t+j}(i)}{A_{k,t+j} P_{t+j}}}_{\equiv \frac{MC_{k,t+j}(i)}{P_{t+j}}} \right\}, \tag{B.29}$$

where $MC_{k,t+j}(i) = \frac{W_{k,t+j}(i)}{A_{k,t+j}}$ denotes the nominal marginal costs of the firm ik at $t + j$.

B.2.3 Government policy

The behavior of the government is the same as in the baseline model.

B.2.4 Steady state

Steady sate equilibrium is the same as the one in the baseline model.

B.2.5 Equilibrium conditions in log-linear approximations

CES Aggregates, market clearing, and definitions

- Sectoral wage

$$w_{k,t} = \frac{1}{n_k} \int_{\mathcal{I}_k} w_{k,t}(i) di$$

- Aggregate wage

$$w_t = \sum_k n_k w_{k,t} = \sum_k \int_{\mathcal{I}_k} w_{k,t}(i) di$$

Household's additional FOCs

- Labor supply of type-k household

$$w_{k,t}(i) - p_t = \varphi h_{k,t}(i) + c_{k,t} \tag{B.30}$$

Integrating over sector k,

$$\frac{1}{n_k}\int_{\mathcal{I}_k} w_{k,t}(i)di - p_t = \varphi h_{k,t} + c_{k,t}$$

The aggregate labor supply relation is derived as

$$w_t - p_t = \varphi h_t + y_t$$

Firms

- Nominal marginal cost
$$mc_{k,t}(i) = w_{k,t}(i) - a_{k,t} \tag{B.31}$$
- Optimality condition for the re-optimizing firms
$$p^*_{k,t} = (1-\alpha_k\beta)E_t\sum_{s=0}^{\infty}\alpha_k^s\beta^s mc_{k,t+s}(i)$$
- Sectoral Phillips curve
$$\pi_{k,t} = \beta E_t\pi_{k,t+1} + \frac{\lambda_k}{1+\theta\varphi}\left((1+\varphi)y_t + (\varphi+\eta^{-1})y^R_{k,t} + c^R_{k,t} - (1+\varphi)a_{k,t}\right)$$
where $\lambda_k \equiv \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}$

Log-linear approximate model

By aggregating proper equations at the disaggregated level, we can obtain the system of equations that determines the equilibrium of the variables of interest:

$$\{y_t, \pi_t, i_t, h_t\} \text{ and } \left\{c^R_{k,t}, y^R_{k,t}, b_{k,t}, \pi_{k,t}\right\}^K_{k=1}.$$

The following $4 + (4 \times K)$ equations determine the equilibrium dynamics of those variables:

$$y_t = E_t\left[y_{t+1}\right] - (i_t - E_t\pi_{t+1}) \tag{B.32}$$

$$y_t = \sum_k n_k a_{k,t} + h_t \tag{B.33}$$

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k} \frac{1}{1+\theta\varphi} \left((1+\varphi) y_t + (\varphi+\eta^{-1}) y^R_{k,t} + c^R_{k,t} - (1+\varphi) a_{k,t} \right) \tag{B.34}$$

$$\pi_t = \sum_k n_k \pi_{k,t} \tag{B.35}$$

$$\Delta y^R_{k,t} = -\eta \left(\pi_{k,t} - \pi_t \right) \tag{B.36}$$

$$c^R_{k,t} = E_t c^R_{k,t+1} + 2\epsilon b_{k,t} \tag{B.37}$$

$$c^R_{k,t} = -\psi_1 b_{k,t} + \frac{1}{\beta} \psi_1 b_{k,t-1} + \psi_2 y^R_{k,t} - \psi_2 a^R_{k,t} \tag{B.38}$$

$$\begin{aligned} i_t &= \phi_\pi \pi_t + \phi_c y_t + \mu_t \\ &\quad or \\ m_t &= p_t + y_t = exogenous\ stochastic\ process \end{aligned} \tag{B.39}$$

B.2.6 Derivation of the Phillips Curve

We first show how the marginal cost is determined. Suppose that firm ik sets its price at $p^*_{k,t}$ and cannot readjust the price again. Combining (B.31) with (B.30), (B.13), and (B.11),

$$\begin{aligned} mc_{k,t+s}(i) &= w_{k,t+s}(i) - p_{t+s} - a_{k,t+s} + p_{t+s} \\ &= \varphi h_{k,t+s}(i) + c_{k,t+s} - a_{k,t+s} + p_{t+s} \\ &= \varphi y_{k,t+s}(i) + c_{k,t+s} - (1+\varphi) a_{k,t+s} + p_{t+s} \\ &= \varphi y_{k,t+s} + c_{k,t+s} - \theta\varphi(p^*_{k,t} - p_{k,t+s}) - (1+\varphi) a_{k,t+s} + p_{t+s} \end{aligned}$$

Let

$$mc_{k,t+s}(i) = -\theta\varphi p^*_{k,t} + (1+\theta\varphi) p_{k,t+s} + \widetilde{mc}_{k,t+s} \tag{B.40}$$

where

$$\widetilde{mc}_{k,t+s} = \varphi y_{k,t+s} + c_{k,t+s} - (1+\varphi)a_{k,t+s} - (p_{k,t+s} - p_{t+s}) \tag{B.41}$$

Log-linearizing the first-order conditions of the price-readjusting firms (B.29),

$$p^*_{k,t} = (1-\alpha_k\beta)E_t \sum_{s=0}^{\infty} \alpha_k^s \beta^s mc_{k,t+s}(i) \tag{B.42}$$

Plugging (B.40) and (B.41) into (B.42),

$$\begin{aligned} p^*_{k,t} &= (1-\alpha_k\beta)E_t \sum_{s=0}^{\infty} \alpha_k^s \beta^s \Big(-\theta\varphi p^*_{k,t} + (1+\theta\varphi)p_{k,t+s} + \widetilde{mc}_{k,t+s} \Big) \\ &= -\theta\varphi p^*_{k,t} + (1-\alpha_k\beta)E_t \sum_{s=0}^{\infty} \alpha_k^s \beta^s \Big((1+\theta\varphi)p_{k,t+s} + \widetilde{mc}_{k,t+s} \Big) \end{aligned} \tag{B.43}$$

Rearranging (B.43),

$$\begin{aligned} p^*_{k,t} &= \frac{1-\alpha_k\beta}{1+\theta\varphi} E_t \sum_{s=0}^{\infty} \alpha_k^s \beta^s \Big((1+\theta\varphi)p_{k,t+s} + \widetilde{mc}_{k,t+s} \Big) \\ &= (1-\alpha_k\beta)p_{k,t} + \frac{1-\alpha_k\beta}{1+\theta\varphi}\widetilde{mc}_{k,t} + \alpha_k\beta E_t p^*_{k,t+1} \end{aligned} \tag{B.44}$$

Log-linearizing the relation that determines the sectoral price level (B.1), we obtain

$$p_{k,t} = (1-\alpha_k)p^*_{k,t} + \alpha_k p_{k,t-1} \tag{B.45}$$

Combining (B.44) and (B.45), we have

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k} \frac{1}{1+\theta\varphi} \widetilde{mc}_{k,t} \tag{B.46}$$

Plugging (B.41) into (B.46), we derive the sectoral PC as

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + \lambda_k \frac{1}{1+\theta\varphi} \Big(\varphi y_{k,t} + c_{k,t} - (1+\varphi)a_{k,t} - (p_{k,t} - p_t) \Big) \tag{B.47}$$

where $\lambda_k \equiv \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}$. Substituting (B.10) to (B.47) to express relative price in terms of sectoral output, and rewriting the sectoral PC in terms of relative sectoral output and relative consumption of type-k households,

$$\begin{aligned}\pi_{k,t} &= \beta E_t\pi_{k,t+1} + \lambda_k \frac{1}{1+\theta\varphi}\Big((\varphi+\eta^{-1})y_{k,t} + c_{k,t} - \eta^{-1}y_t - (1+\varphi)a_{k,t}\Big)\\ &= \beta E_t\pi_{k,t+1} + \lambda_k \frac{1}{1+\theta\varphi}\Big((1+\varphi)y_t + (\varphi+\eta^{-1})y^R_{k,t} + c^R_{k,t} - (1+\varphi)a_{k,t}\Big)\end{aligned}$$

Aggregate PC is derived by aggregation of sectoral inflation

$$\begin{aligned}\pi_t &= \sum_{k=1}^{K} n_k\pi_{k,t}\\ &= \beta E_t\pi_{t+1} - \frac{1+\varphi}{1+\theta\varphi}\sum_{k=1}^{K} n_k\lambda_k a_{k,t}\\ &\quad + \underbrace{\frac{1+\varphi}{1+\theta\varphi}\sum_{k=1}^{K} n_k\lambda_k}_{\equiv\kappa^I} y_t + \underbrace{\frac{\varphi+\eta^{-1}}{1+\theta\varphi}\sum_{k=1}^{K} n_k\lambda_k y^R_{k,t}}_{\equiv\Theta^I_{y,t}} + \underbrace{\frac{1}{1+\theta\varphi}\sum_{k=1}^{K} n_k\lambda_k c^R_{k,t}}_{\equiv\Theta^I_{c,t}}\end{aligned}$$

B.3 The model with imperfect risk-sharing within sectors in firm-specific labor markets (Case II)

We here provide a full description of the model considered in section 5.2 of the main text. We focus on the differences from the baseline specification. In addition, the claims made for the aggregate Phillips curve in that section are proved here.

B.3.1 Households

Households who work at firm i in sector k (or type-i households) seek to maximize

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\log C_{k,t}(i) - \omega_k \frac{H_{k,t}(i)^{1+\varphi}}{1+\varphi} \right) \right],$$

subject to the flow budget constraint

$$P_t C_{k,t}(i) + B_{k,t}(i) + \Omega(B_{k,t}(i)) = R_{t-1} B_{k,t-1}(i) + W_{k,t}(i) H_{k,t}(i) + P_t T_t + \Pi_t$$

Type-i household's first order conditions are

$$1 + \Omega'(B_{k,t}(i)) = \beta R_t \mathbb{E}_t \left[\left(\frac{C_{k,t}(i)}{C_{k,t+1}(i)} \right) \left(\frac{P_t}{P_{t+1}} \right) \right],$$

$$\frac{W_{k,t}(i)}{P_t} = \omega_k H_{k,t}(i)^{\varphi} C_{k,t}(i).$$

B.3.2 Firms

The behavior of firms is the same as in the previous model.

B.3.3 Government

The behavior of the government is the same as in the previous model.

B.3.4 Steady state

Steady state equilibrium is the same as the one in the previous model.

B.3.5 Equilibrium conditions in log-linear approximations

Household's additional FOCs

- Labor supply of type-k household

$$w_{k,t}(i) - p_t = \varphi h_{k,t}(i) + c_{k,t}(i) \tag{B.48}$$

Integrating over sector k,

$$\frac{1}{n_k}\int_{\mathcal{I}_k} w_{k,t}(i)di - p_t = \varphi h_{k,t} + c_{k,t}$$

The aggregate labor supply relation is derived as

$$w_t - p_t = \varphi h_t + y_t$$

Firms

- Sectoral Phillips curve

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + g(\alpha_k, \epsilon)\Big((1+\varphi)y_t + (\varphi + \eta^{-1})y^R_{k,t} + c^R_{k,t} - (1+\varphi)a_{k,t}\Big)$$

where $g(\alpha_k, \epsilon) \equiv \left\{\frac{(1-\alpha_k\beta)(1-\alpha_k)}{\alpha_k}\right\}\left[\frac{(1-\alpha_k\beta\delta)^2}{(1+\theta\varphi+\psi_2)(1-\alpha_k\beta\delta)^2-\psi_2(1-\alpha_k)^2\beta\delta}\right] = \frac{1}{1+\theta\varphi}\lambda^{II}_k(\epsilon)$.

Log-linear approximate model

By aggregating proper equations at the disaggregated level, we can obtain the system of equations that determines the equilibrium of the variables of interest:

$$\{y_t, \pi_t, i_t, h_t\} \text{ and } \left\{c^R_{k,t}, y^R_{k,t}, b_{k,t}, \pi_{k,t}\right\}^K_{k=1}.$$

The following $4 + (4 \times K)$ equations determine the equilibrium dynamics of those variables:

$$y_t = E_t\left[y_{t+1}\right] - (i_t - E_t\pi_{t+1}) \tag{B.49}$$

$$y_t = \sum_k n_k a_{k,t} + h_t \tag{B.50}$$

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + g(\alpha_k, \epsilon)\Big((1+\varphi)y_t + (\varphi + \eta^{-1})y^R_{k,t} + c^R_{k,t} - (1+\varphi)a_{k,t}\Big) \tag{B.51}$$

$$\pi_t = \sum_k n_k \pi_{k,t} \tag{B.52}$$

$$\Delta y^R_{k,t} = -\eta\left(\pi_{k,t} - \pi_t\right) \tag{B.53}$$

$$c^R_{k,t} = E_t c^R_{k,t+1} + 2\epsilon b_{k,t} \tag{B.54}$$

$$c^R_{k,t} = -\psi_1 b_{k,t} + \frac{1}{\beta}\psi_1 b_{k,t-1} + \psi_2 y^R_{k,t} - \psi_2 a^R_{k,t} \tag{B.55}$$

$$\begin{aligned} & i_t = \phi_\pi \pi_t + \phi_c y_t + \mu_t \\ & \quad or \\ & m_t = p_t + y_t = exogenous\ stochastic\ process \end{aligned} \tag{B.56}$$

B.3.6 Derivation of the Phillips Curve

Since the dynamics of relative consumption and bond holding play important roles in firms' pricing decisions, we first present the household optimality conditions. Log-linearizing the household Euler equation and budget constraint, and then expressing them in terms of relative consumption, relative bond holding, and relative price yields

$$\begin{aligned} c^R_{k,t}(i) &= E_t\left[c^R_{k,t+1}(i)\right] + 2\epsilon b^R_{k,t}(i) \\ c^R_{k,t}(i) &= -\psi_1 b^R_{k,t}(i) + \beta^{-1}\psi_1 b^R_{k,t-1}(i) - \psi_2 p^R_{k,t}(i), \end{aligned}$$

where $\psi_1 \equiv \theta$ and $\psi_2 \equiv \psi_1\left(\theta - 1\right)\left(1+\varphi\right)$.[2]

Combining the first and the second equations, we can substitute out type-i household's relative consumption $c^R_{k,t}(i)$, which gives an equation that describes the dynam-

[2] $x^R_{k,t}(i)$ denotes a percentage deviation of $X^R_{k,t}(i)$ from its steady state (which is equal to zero). Therefore it must be that $c^R_{k,t}(i) = c_{k,t}(i) - c_{k,t}$, $b^R_{k,t}(i) = b_{k,t}(i) - b_{k,t}$, and $p^R_{k,t}(i) = p_{k,t}(i) - p_{k,t}$.

ics of a household's relative bond holding given the relative price:

$$E_t \left[b^R_{k,t+1}(i) + \left(\beta^{-1} - 1 - \frac{2\epsilon}{\psi_1} \right) b^R_{k,t}(i) + \beta^{-1} b^R_{k,t-1}(i) \right] = \frac{\psi_2}{\psi_1} E_t \left[p^R_{k,t+1}(i) + p^R_{k,t}(i) \right] \tag{B.57}$$

Turning to firms, the log-linearized first order condition of a firm that sets its price at time t is

$$\hat{E}^i_t \sum_{s=0}^{\infty} (\alpha_k \beta)^s \left\{ p^*_{k,t}(i) - p_{t+s} \right\} = \hat{E}^i_t \sum_{s=0}^{\infty} (\alpha_k \beta)^s \, mc_{k,t+s}(i).$$

The expectation operator, $\hat{E}^i_t$ is distinct from E_t as emphasized in Woodford (2005): $\hat{E}^i_t$ is type-i firm's expectation at time t conditioned on its own price remaining unchanged for the entire future period from time t onwards. Because households and firms are so small in size, they cannot affect aggregate or sectoral level variables. Thus distinguishing the two expectation operators is important only for micro level variables. After substituting out relative consumption from marginal cost $mc_{k,t+s}(i)$ and then replacing $\hat{E}^i_t \left[p^R_{k,t+s}(i) \right]$ by $p^{*R}_{k,t}(i) - \sum_{j=1}^{s} E_t \pi_{k,t+j}$, the firm's log-linearized first order condition can be written as

$$\begin{aligned} p^{*R}_{k,t}(i) = & \left(\frac{1 - \alpha_k \beta}{1 + \varphi\theta + \psi_2} \right) \sum_{s=0}^{\infty} (\alpha_k \beta)^s E_t \left[V_{k,t+s} \right] + \sum_{s=1}^{\infty} (\alpha_k \beta)^s E_t \left[\pi_{k,t+s} \right] \\ & - \psi_1 (1 - \alpha_k) \left(\frac{1 - \alpha_k \beta}{1 + \varphi\theta + \psi_2} \right) \sum_{s=0}^{\infty} (\alpha_k \beta)^s \hat{E}^i_t \left[b^R_{k,t+s}(i) \right] \\ & + \beta^{-1} \psi_1 \left(\frac{1 - \alpha_k \beta}{1 + \varphi\theta + \psi_2} \right) b^R_{k,t-1}(i), \end{aligned} \tag{B.58}$$

where

$$V_{k,t} \equiv (1 + \varphi) \, y_t + \left(\varphi + \eta^{-1} \right) y^R_{k,t} + c^R_{k,t} - (1 + \varphi) \, a_{k,t}$$

is the common factor across all the firms within a sector. The operator, E_t is used in the first two summations on the right hand side of (B.58) in place of $\hat{E}^i_t$ since those terms have only aggregate and sector-level variables.

Finally, the expected value of the firm's next-period price must be a weighted average of the current price and next-period's *optimal* price:

$$E_t\left[p^R_{k,t+1}(i)\right] = \alpha_k\left[p^R_{k,t}(i) - E_t\pi_{k,t+1}\right] + (1-\alpha_k)\,E_t\left[p^{*R}_{k,t+1}(i)\right]. \tag{B.59}$$

The three equations, (B.57), (B.58), and (B.59) together characterize the dynamics of micro level variables $\{b^R_{k,t}(i), p^R_{k,t}(i), p^{*R}_{k,t}(i)\}$, given the time path of the aggregate and sector level variables, $\{V_{k,t}, \pi_{k,t}\}$. The system of linear difference equations is, however, hard to solve analytically. We thus take the undetermined coefficient method as in Woodford (2005). From equation (B.57), we posit that the time path of relative bond holding follows

$$b^R_{k,t}(i) = \delta b^R_{k,t-1}(i) + \upsilon p^R_{k,t}(i), \tag{B.60}$$

where δ and υ are some functions of the structural parameters. From (B.58) and (B.60), it then follows that a firm's optimal price satisfies:

$$p^{*R}_{k,t}(i) = p^{*R}_{k,t} + \lambda b^R_{k,t-1}(i), \tag{B.61}$$

where λ is again a function of the parameters, and $p^{*R}_{k,t}$ denotes the common component of optimal prices of the firms who set prices anew in sector k, which is a function of the aggregate and sector variables only. If the set of parameters, $\{\lambda, \delta, \upsilon\}$ and the common component, $p^{*R}_{k,t}$ were known, one could easily construct the Phillips curve.

The first step to determine $\{\lambda, \delta, \upsilon\}$ and $p^{*R}_{k,t}$ is substituting (B.61) into (B.59) to obtain:

$$E_t\left[p^R_{k,t+1}(i)\right] = \alpha_k p^R_{k,t}(i) + \lambda(1-\alpha_k) b^R_{k,t}(i). \tag{B.62}$$

Note that (B.60), the posited time path of bond holdings, should satisfy the difference equation (B.57) after $E_t\left[p^R_{k,t+1}(i)\right]$ is substituted out using (B.62). This is true if and only if $\{\lambda, \delta, \upsilon\}$ satisfy the following conditions:

$$\upsilon = \frac{(1-\alpha_k)\,\psi_2\delta}{\alpha_k\psi_1\delta - \beta^{-1}\psi_1} \tag{B.63}$$

$$\lambda = \frac{\beta^{-1} - \alpha_k\delta}{(1-\alpha_k)\,\psi_2}\left[\frac{2\epsilon}{\beta^{-1}-\delta} - \frac{\psi_1\,(1-\delta)}{\delta}\right]. \tag{B.64}$$

Note we have expressed υ and λ as a functions of δ. One more relation is needed to determine $\{\lambda, \delta, \upsilon\}$ and the firm's first order condition (B.58) provides that additional relation. Based on (B.60), $\hat{E}_t^i\left[b_{k,t+s}^R(i)\right]$ can be expressed as

$$\begin{aligned}
&\hat{E}_t^i\left[b_{k,t+s}^R(i)\right] \\
&= \delta\hat{E}_t^i\left[b_{k,t+s-1}^R(i)\right] + \upsilon\hat{E}_t^i\left[p_{k,t+s}^R(i)\right] = \delta\hat{E}_t^i\left[b_{k,t+s-1}^R(i)\right] + \upsilon\left[p_{k,t}^{*R}(i) - \sum_{j=1}^{s} E_t\pi_{k,t+j}\right],
\end{aligned}$$

which implies the following equation:

$$\begin{aligned}
&\sum_{s=0}^{\infty}(\alpha_k\beta)^s\,\hat{E}_t^i\left[b_{k,t+s}^R(i)\right] \\
&= \left(\frac{\delta}{1-\delta\alpha_k\beta}\right) b_{k,t-1}^R(i) + \frac{\upsilon}{(1-\alpha_k\beta)(1-\delta\alpha_k\beta)}\left[p_{k,t}^{*R}(i) - \sum_{s=1}^{\infty}(\alpha_k\beta)^s E_t\left[\pi_{k,t+s}\right]\right].
\end{aligned}$$

Plugging this expression into the firm's first order condition, (B.58), we obtain:

$$\Psi p_{k,t}^{*R}(i) = \left(\frac{1-\alpha_k\beta}{1+\varphi\theta+\psi_2}\right)\sum_{s=0}^{\infty}(\alpha_k\beta)^s E_t\left[V_{k,t+s}\right] + \Psi\sum_{s=1}^{\infty}(\alpha_k\beta)^s E_t\left[\pi_{k,t+s}\right] + \Phi b_{k,t-1}^R(i), \tag{B.65}$$

where

$$\begin{aligned}
\Psi &\equiv 1 - \frac{\psi_2\left(1-\alpha_k\right)^2\delta}{\left(1+\varphi\theta+\psi_2\right)\left(1-\alpha_k\beta\delta\right)\left(\beta^{-1}-\alpha_k\delta\right)} \\
\Phi &\equiv \frac{\psi_1\left(1-\alpha_k\beta\right)\left(\beta^{-1}-\delta\right)}{\left(1+\varphi\theta+\psi_2\right)\left(1-\alpha_k\beta\delta\right)}.
\end{aligned}$$

Comparing (B.65) and (B.61), one can solve for $p_{k,t}^{*R}$:

$$p_{k,t}^{*R} = \Psi^{-1}\left(\frac{1-\alpha_k\beta}{1+\varphi\theta+\psi_2}\right)\sum_{s=0}^{\infty}(\alpha_k\beta)^s E_t\left[V_{k,t+s}\right] + \sum_{s=1}^{\infty}(\alpha_k\beta)^s E_t\left[\pi_{k,t+s}\right], \tag{B.66}$$

and the coefficient λ satisfies the following equation:

$$\Psi\lambda = \Phi. \tag{B.67}$$

The three equations, (B.63), (B.64), and (B.67) jointly determine the coefficients $\{\lambda, \delta, \upsilon\}$ if a solution exists. The system of equations is nonlinear in $\{\lambda, \delta, \upsilon\}$, and thus there could be more than one solution. Following Woodford (2005), we only consider a solution that would make the joint dynamics of relative price and relative bond holdings convergent so that the means and the variances remain bounded. We can rewrite equations (B.60) and (B.62) as the following system:

$$\begin{pmatrix} E_t\left[p^R_{k,t+1}(i)\right] \\ b^R_{k,t}(i) \end{pmatrix} = \begin{pmatrix} \alpha_k + (1-\alpha_k)\,\lambda\upsilon & (1-\alpha_k)\,\lambda\delta \\ \upsilon & \delta \end{pmatrix} \begin{pmatrix} p^R_{k,t}(i) \\ b^R_{k,t-1}(i) \end{pmatrix}. \tag{B.68}$$

The system is stable if and only if the eigenvalues of the coefficient matrix are inside the unit circle.

Lemma B.0.1. *If* $\alpha_k\beta^{-1} \leq 1$*, then the system (B.68) is stable if and only if* $0 < \delta < \beta^{-1}$.

See the following subsections for the proof of Lemma 1 and 2. Based on Lemma 1, we focus only on the values of δ on the interval $(0, \beta^{-1})$, and α_k on $(0, \beta)$ in what follows. A natural question to ask at this point might be if there exists such a $\{\lambda, \delta, \upsilon\}$ that solve (B.63), (B.64), and (B.67) while satisfying the stability condition, $0 < \delta < \beta^{-1}$. Lemma 2 shows that there indeed exists a unique set of $\{\lambda, \delta, \upsilon\}$ as long as ϵ is positive.

Lemma B.0.2. *There exists a unique set of* $\{\lambda, \delta, \upsilon\}$ *that satisfies (B.63), (B.64), and (B.67), and* $0 < \delta < \beta^{-1}$ *if* $\epsilon > 0$.

As mentioned above, once we find the solution for $\{\lambda, \delta, \upsilon\}$, the generalized NK Phillips curve can be constructed by combining (B.65) which determines a firm's relative optimal price $p^{*R}_{k,t}(i)$ and (B.69) which determines the dynamics of the sector price level $p_{k,t}$.

$$p_{k,t} = \frac{1}{n_k}\int_{\mathcal{I}^*_k} p^*_{k,t}(i)di - \alpha_k p_{k,t-1}. \tag{B.69}$$

Substituting (B.61) into the equation above, one obtains

$$\alpha_k \pi_{k,t} = \frac{1}{n_k}\int_{\mathcal{I}^*_k} \left(p^{*R}_{k,t} + \lambda b^R_{k,t-1}(i)\right) di,$$

implying

$$p_{k,t}^{*R} = \frac{\alpha_k}{1-\alpha_k}\pi_{k,t}, \tag{B.70}$$

because $\int_{\mathcal{I}_k^*} b_{k,t-1}^R(i)di = 0$ holds due to the assumption of time-dependent pricing. Note that time-dependent pricing is a crucial assumption that allows me to avoid keeping track of distributions of household wealth. Substituting (B.70) into (B.66) gives the "sector-level Phillips curve":

$$\pi_{k,t} = \beta E_t\left[\pi_{k,t+1}\right] + g\left(\alpha_k,\epsilon\right)V_{k,t}, \tag{B.71}$$

where

$$g\left(\alpha_k,\epsilon\right) \equiv \left\{\frac{\left(1-\alpha_k\beta\right)\left(1-\alpha_k\right)}{\alpha_k}\right\}\left[\frac{\left(1-\alpha_k\beta\delta\right)^2}{\left(1+\varphi\theta+\psi_2\right)\left(1-\alpha_k\beta\delta\right)^2-\psi_2\left(1-\alpha_k\right)^2\beta\delta}\right] \tag{B.72}$$

We have made explicit in (B.72) g's dependence on the financial friction parameter, ϵ, since δ is a function of this parameter.

To summarize the results obtained so far, the sectoral Phillips curve, for each sector k, is given by

$$\pi_{k,t} = \beta E_t\left[\pi_{k,t+1}\right] + g\left(\alpha_k,\epsilon\right)\left[\left(1+\varphi\right)y_t + \left(\varphi+\eta^{-1}\right)y_{k,t}^R + c_{k,t}^R\right] - \zeta_{k,t}, \tag{B.73}$$

where $g\left(\alpha_k,\epsilon\right)$ is given by (B.72), and $\{\lambda,\delta,\upsilon\}$ satisfy (B.63), (B.64), (B.67) and $0<\delta<\beta^{-1}$. The disturbance term $\zeta_{k,t}$ consists of exogenous shocks:

$$\zeta_{k,t} \equiv g\left(\alpha_k,\epsilon\right)\left[\left(1+\varphi\right)a_{k,t}\right].$$

The Phillips curve for aggregate inflation π_t is consequently obtained by taking a weighted sum of sectoral Phillips curves:

$$\pi_t = \beta E_t\left[\pi_{t+1}\right] + \kappa y_t + \Theta_{y,t} + \Theta_{c,t} - \zeta_t,$$

where

$$\Theta_{y,t} \equiv \left(\varphi + \eta^{-1}\right) \sum_{k=1}^{K} n_k g\left(\alpha_k, \epsilon\right) y_{k,t}^{R}, \qquad \Theta_{c,t} \equiv \sum_{k=1}^{K} n_k g\left(\alpha_k, \epsilon\right) c_{k,t}^{R},$$

$$\kappa \equiv (1+\varphi) \sum_{k=1}^{K} n_k g\left(\alpha_k, \epsilon\right), \qquad \zeta_t \equiv \sum_{k=1}^{K} n_k \zeta_{k,t}.$$

B.3.7 Comparison of Phillips curve slope

This subsection shows that the slope of the Phillips curve is smaller in the Case II model than in the Case I model. For this purpose, we compare $\lambda_k^{II}(\epsilon)$ to λ_k. We omitted the subscript k for brevity.

$$\begin{aligned}
\lambda_k &\equiv \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \\
\lambda_k^{II}(\epsilon) &\equiv (1+\varphi\theta) g^{\mathcal{HH}}(\alpha, \epsilon) \\
&= (1+\varphi\theta) \left\{ \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \right\} \left[\frac{(1-\alpha\beta\delta)^2}{(1+\varphi\theta+\psi_2)(1-\alpha\beta\delta)^2 - \psi_2 (1-\alpha)^2 \beta\delta} \right].
\end{aligned}$$

Taking the ratio of $\lambda_k^{II}(\epsilon)$ to λ_k, we obtain

$$\begin{aligned}
\frac{\lambda_k^{II}(\epsilon)}{\lambda_k} &= \frac{(1-\alpha\beta\delta)^2 (1+\varphi\theta)}{(1+\varphi\theta+\psi_2)(1-\alpha\beta\delta)^2 - \psi_2 (1-\alpha)^2 \beta\delta} \\
&= \frac{(1+\varphi\theta)(1-\alpha\beta\delta)^2}{(1+\varphi\theta)(1-\alpha\beta\delta)^2 + \psi_2 (1-\alpha\beta\delta)^2 - \psi_2 (1-\alpha)^2 \beta\delta} \\
&= \frac{(1+\varphi\theta)(1-\alpha\beta\delta)^2}{(1+\varphi\theta)(1-\alpha\beta\delta)^2 + \psi_2 \left\{1 - 2\alpha\beta\delta + (\alpha\beta\delta)^2 - \beta\delta + 2\alpha\beta\delta - \alpha^2\beta\delta\right\}} \\
&= \frac{(1+\varphi\theta)(1-\alpha\beta\delta)^2}{(1+\varphi\theta)(1-\alpha\beta\delta)^2 + \underbrace{\psi_2 \left\{(1-\beta\delta) + \alpha^2\beta\delta(1-\beta\delta)\right\}}_{>0}} < 1.
\end{aligned}$$

B.3.8 Proof of Lemma 1

The eigenvalues of the system are the roots of the following equation:

$$f(X) = (\alpha + (1-\alpha)\lambda\upsilon - X)(\delta - X) - (1-\alpha)\lambda\delta\upsilon = X^2 - \{\alpha + \delta + (1-\alpha)\lambda\upsilon\} X + \alpha\delta$$

The two roots are given by

$$X_1 = 0.5\left\{\alpha + \delta + (1-\alpha)\lambda\upsilon + \sqrt{(\alpha + \delta + (1-\alpha)\lambda\upsilon)^2 - 4\alpha\delta}\right\},$$
$$X_2 = 0.5\left\{\alpha + \delta + (1-\alpha)\lambda\upsilon - \sqrt{(\alpha + \delta + (1-\alpha)\lambda\upsilon)^2 - 4\alpha\delta}\right\}.$$

The term inside the root $\left\{(\alpha + \delta + (1-\alpha)\lambda\upsilon)^2 - 4\alpha\delta\right\}$ is always positive, implying that X_1 and X_2 are two real roots with $X_1 \geq X_2$. Consequently, for the system to be stable, the following two conditions must hold:

$$(i)\ X_1 < 1 \text{ and } (ii)\ X_2 > -1.$$

(i) Note that the first condition, $X_1 < 1$, holds if and only if

$$\sqrt{(\alpha + \delta + (1-\alpha)\lambda\upsilon)^2 - 4\alpha\delta} < 2 - \{\alpha + \delta + (1-\alpha)\lambda\upsilon\},$$

which holds if and only if the following two conditions are met:

$$\text{(a)} : (\alpha + \delta + (1-\alpha)\lambda\upsilon)^2 - 4\alpha\delta < (2 - \{\alpha + \delta + (1-\alpha)\lambda\upsilon\})^2$$
$$\text{(b)} : 2 - \{\alpha + \delta + (1-\alpha)\lambda\upsilon\} > 0$$

(a) and (b) can be simplified as

$$\text{(a)} : \alpha + \delta + (1-\alpha)\lambda\upsilon < 1 + \alpha\delta$$
$$\text{(b)} : \alpha + \delta + (1-\alpha)\lambda\upsilon < 2$$

Suppose $\alpha\delta \geq 1$. Then condition (a) becomes irrelevant. And, (b) can be written as:

$$\delta < 1 + (1-\alpha)(1-\lambda\upsilon).$$

Suppose $\alpha\delta \leq 1$. Then condition (b) becomes irrelevant, and (a) can be written as:

$$(1-\alpha)(\delta + \lambda\upsilon - 1) < 0,$$

which can be simplified to

$$\delta < 1 - \lambda\upsilon$$

assuming $0 < \alpha < \beta$ (which will be the case throughout the paper). In sum, the first condition, $X_1 < 1$, holds if and only if

$$\delta < 1 + (1-\alpha)(1-\lambda\upsilon) \quad \text{and} \quad \alpha\delta \geq 1 \tag{B.74}$$

or

$$\delta < 1 - \lambda\upsilon \quad \text{and} \quad \alpha\delta \leq 1 \tag{B.75}$$

(ii) The second condition, $X_2 > -1$, holds if and only if

$$\delta > -\left(1 + \frac{1-\alpha}{1+\alpha}\lambda\upsilon\right), \tag{B.76}$$

Note

$$\lambda\upsilon = (1-\delta) - \frac{2\epsilon\delta}{\psi_1\left(\beta^{-1} - \delta\right)},$$

which leads to

$$1 - \lambda\upsilon = \delta + \frac{2\epsilon\delta}{\psi_1\left(\beta^{-1} - \delta\right)}$$

The condition (B.74) cannot be true because

$$\begin{aligned} &\delta < 1 + (1-\alpha)(1-\lambda\upsilon) \leq 1 + (1-\alpha)\delta \qquad (\because 1 - \lambda\upsilon \leq \delta \text{ when } \alpha\delta \geq 1) \\ &\iff \alpha\delta < 1, \end{aligned}$$

which contradicts $\alpha\delta \geq 1$. Therefore the stability conditions are summarized by (B.75) and (B.76). Consider (B.75) first:

$$\begin{aligned}
\delta < 1 - \lambda\upsilon = \delta + \frac{2\epsilon\delta}{\psi_1\left(\beta^{-1} - \delta\right)} \quad &\text{and} \quad \alpha\delta \leq 1 \\
\iff 0 < \frac{2\epsilon\delta}{\psi_1\left(\beta^{-1} - \delta\right)} \quad &\text{and} \quad \alpha\delta \leq 1 \\
\iff 0 < \delta < \frac{1}{\beta} \quad &\text{and} \quad \delta \leq \frac{1}{\alpha} \iff 0 < \delta < \frac{1}{\beta}.
\end{aligned} \tag{B.77}$$

Now let us consider (B.76). From (B.75), it can be shown that $\lambda\upsilon < 1 - \delta < 1$. Then we have

$$-\left(1 + \frac{1-\alpha}{1+\alpha}\lambda\upsilon\right) < -\left(1 + \frac{1-\alpha}{1+\alpha}\right) < -1 < \delta, \tag{B.78}$$

which shows that (B.75) implies (B.76). Therefore the inequality (B.77) alone gives the stability condition, and this proves Lemma 1.

B.3.9 Proof of Lemma 2

The system of nonlinear equations for $\{\lambda, \delta, \upsilon\}$ is given by the following three equations:

$$\frac{\beta^{-1} - \alpha_k\delta}{\left(1-\alpha_k\right)\psi_2}\left(1 - \frac{\psi_2\left(1-\alpha_k\right)^2\delta}{\left(1+\varphi\theta+\psi_2\right)\left(1-\alpha_k\beta\delta\right)\left(\beta^{-1}-\alpha_k\delta\right)}\right)\left[\frac{2\epsilon}{\beta^{-1}-\delta} - \frac{\psi_1\left(1-\delta\right)}{\delta}\right]$$

$$= \frac{\psi_1\left(1-\alpha_k\beta\right)\left(\beta^{-1}-\delta\right)}{\left(1+\varphi\theta+\psi_2\right)\left(1-\alpha_k\beta\delta\right)} \tag{B.79}$$

$$\upsilon = \frac{\left(1-\alpha_k\right)\psi_2\delta}{\alpha_k\psi_1\delta - \beta^{-1}\psi_1} \tag{B.80}$$

$$\lambda = \frac{\beta^{-1} - \alpha_k\delta}{\left(1-\alpha_k\right)\psi_2}\left[\frac{2\epsilon}{\beta^{-1}-\delta} - \frac{\psi_1\left(1-\delta\right)}{\delta}\right]. \tag{B.81}$$

Given δ (and other parameters), λ and υ are uniquely determined by (B.80) and (B.81). Therefore it remains to show if there exist δ that satisfy (B.79) and $0 < \delta < \frac{1}{\beta}$.

Rewrite (B.79) as:

$$2\epsilon = \underbrace{\frac{\psi_1 (1-\delta)(\beta^{-1}-\delta)}{\delta} + \frac{\psi_1\psi_2 (1-\alpha_k\beta)(1-\alpha_k)(\beta^{-1}-\delta)^2}{\left[(1+\varphi\theta+\psi_2)(1-\alpha_k\beta\delta)^2 \beta^{-1} - \psi_2 (1-\alpha_k)^2 \delta\right]}}_{\equiv K(\delta)}, \tag{B.82}$$

where $K(\delta)$ is a continuous function on $\delta \in \left(0, \frac{1}{\beta}\right)$. In the two limiting cases where $\epsilon = 0$ and $\epsilon = \infty$, $\delta = \beta^{-1}$ and $\delta = 0$ respectively satisfy (B.82). Moreover, it is tedious yet straightforward to show $\frac{\partial K(\delta)}{\partial \delta} < 0$ for $\delta \in \left(0, \frac{1}{\beta}\right)$, which implies that for each value of $\epsilon \in (0, \infty)$, there exists one value of δ that satisfies (B.82) and that δ is decreasing in ϵ with the following properties:

$$\lim_{\epsilon \to 0} \delta = \frac{1}{\beta} \quad \text{and} \quad \lim_{\epsilon \to \infty} \delta = 0.$$

This proves Lemma 2.