From Neutrality to Locality

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Fintan Horan

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On my honor as a University Student, I have neither given nor received unauthorized aid on this assignment as defined by the Honor Guidelines for Thesis-Related Assignments.

Advisor: William F. Stafford Jr., Ph.D.
Department of Engineering and Society

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FINTAN HORAN, University of Virginia

> *"In an election, the voters choose their politicians; but in redistricting, the politicians choose their voters."*
>
> — Indiana Chamber of Commerce, Outlook, Volume 1, Issue 3

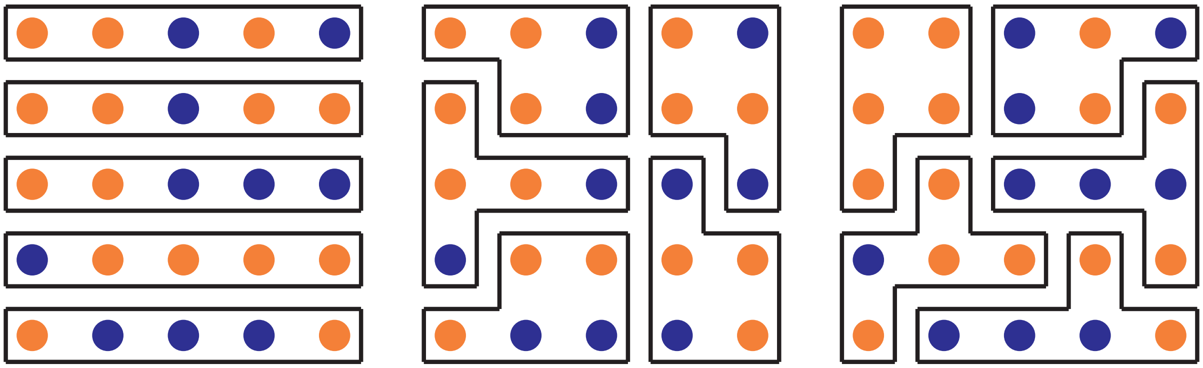
1 INTRODUCTION

The United States is a representative democracy composed of *single-member districts*[1], and every ten years, electoral district boundaries are redrawn to account for population changes in a process known as redistricting. Redistricting involves clustering the geographic census units of a state into districts of roughly equal populations from which these representatives are elected. Ideally, redistricting would proportionally reflect the political views of the population, but historically, the process has been carried out by state legislatures in a highly partisan manner, where the controlling party both commissions and approves the new district maps. Because of the high-stakes nature of single-member districts, redistricting is a fierce battleground for partisan and racial gerrymandering.

Gerrymandering is the manipulation of district boundaries towards some political end, and the practice is nearly as old as the United States itself. The term "Gerry-mander" comes from an 1812 illustration by the *Salem Gazette* depicting the unnatural shape of a district signed into law by then Massachusetts Governor Elbridge Gerry. There are two primary

Figure 1: "Gerry-mander" illustration.

[1]Where the candidate who wins the most votes is elected to be the representative of the district.



(a) Proportional representation. **(b)** All seats won by the Orange Party. **(c)** 60% of seats won by the Blue Party.

Figure 2: Proportional representation versus gerrymandered districts.

techniques employed in gerrymandering. The first is known as *cracking*, where a voting group is excessively dispersed such that they fail to achieve a majority in one or more districts, while the second is known as *packing*, where a voting group is stuffed into few districts to dilute their voting power (Duchin & Walch, 2022). Both techniques can be used to gain huge margins in the outcome of an election.

Consider the trivial example of 25 people in a 5×5 grid where the goal is to draw 5 districts with 5 people in each district. Suppose 15 of the people are *Orange Party* voters, and the remaining 10 are *Blue Party* voters. In an ideal outcome, there would be proportional representation such that 60% of the seats were won by the Orange Party and 40% were won by the Blue Party. But suppose the Orange Party was in control of the process. *Cracking* could be leveraged to manipulate the district boundaries such that the Blue Party won no seats, as in Figure 2b. On the other hand, as Figure 2c illustrates, if the Blue Party was in control, they could *pack* Orange Party voters into two districts, ensuring 60% of the seats despite having only 40% of the votes.

1.1 Redistricting in Virginia

Virginia has had a long and fraught history of gerrymandering. The state first experienced a "Gerry-mander" before the eponymous label was used to describe the 1812 Massachusetts State Senate district, when in 1789, Patrick Henry designed a districting plan with the intention of denying James Madison a seat in the United States House of Representatives (Altman & McDonald, 2012). As Virginia saw an increase in urbanization following World War II, redistricting shifted towards urban-rural conflict. Prior to 1970, it was customary for Virginia to appoint a redistricting commission composed of state legislators with the goal of providing recommendations on the redistricting process. But once the commission reported its recommendations, the General Assembly would seize back control of the process, ignore the commission's advice, and draw districts that limited representation in urban areas (Hardy et al., 1981).

Despite the establishment of the Voting Rights Act of 1965 (VRA), which was designed to outlaw racial discrimination in voting, Virginia had four decades of racial gerrymandering, where the General Assembly drew districts that diluted the voting power of Black Virginians. Most recently, Virginia had multiple legal challenges to maps from the 2011 redistricting cycle reach the Supreme Court of the United States, and after deeming the districts to be racial gerrymanders, the maps were eventually re-drawn by federal courts (DeFord & Duchin, 2019). In the wake of this litigation, which put Virginia under a national spotlight, interest groups formed amid public outcry and began to galvanize support for redistricting reform. The rapid membership growth of groups such as OneVirginia2021 in the intervening years emphasized the growing shift in public opinion that there is a conflict of interest when the General Assembly draws its own districts (Green, 2017). The goal of redistricting reform is thus to democratize the process such that it is no longer the case that politicians pick their voters instead of the other way around.

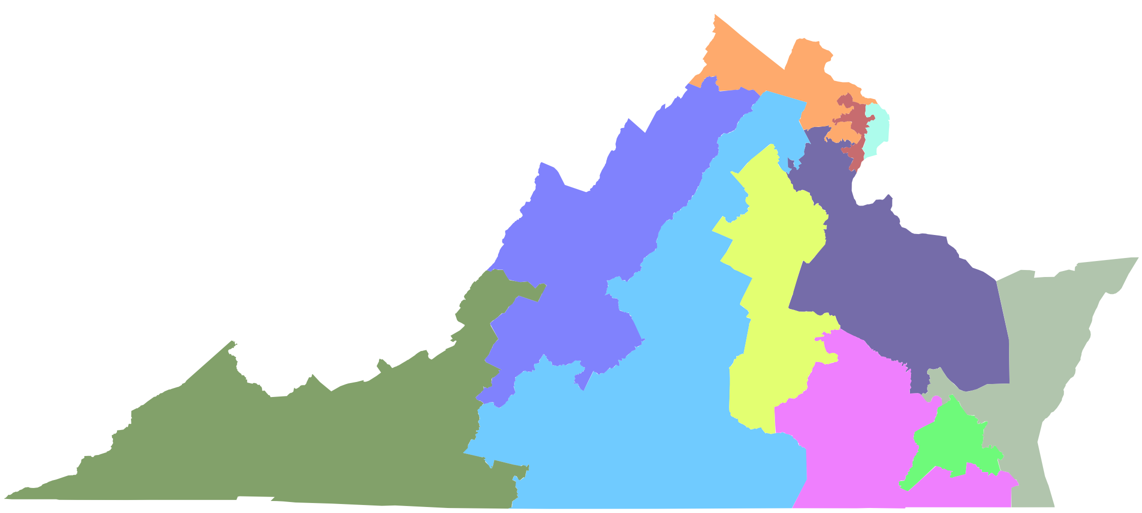


Figure 3: 2011 Gerrymandered Congressional Districts

1.2 The Virginia Redistricting Commission

At the forefront of redistricting reform is the *redistricting commission*. In 2019, in response to reform advocates, the General Assembly approved a reading of an amendment that would create a type of bipartisan redistricting commission. The Virginia Redistricting Commission (VRC) would consist of sixteen members, half of whom would be state legislators, equally divided between the two major political parties, while other half would be citizens recommended by legislative leaders and selected by retired judges. However, the commission would not have full autonomy, requiring approval of districting plans by the General Assembly—language which was opposed by various anti-gerrymandering groups. In the event of deadlock, the redistricting process would be assumed by the Supreme Court of Virginia (SCOVA) (Keena, 2022). The role of SCOVA in the process led to opposition from Black legislators, in part due to the fact that justices on the court were appointed by a Republican-controlled General Assembly. There were also arguments that the commission itself was designed to fail, due to the ease of deadlock made possible by the high supermajority threshold required for approval (Moomaw, 2020). But despite these oppositions, the amendment was sent to voters who approved it with a nearly two-to-one ratio (“Live Election Results; Constitutional Amendment #1”, 2020).

The success of a redistricting commission is often directly tied to its structure (Imamura, 2022), and there are several different types of redistricting commissions that currently exist. *Advisory commissions* are those which do not have autonomy in the redistricting process and require approval of district plans by the legislative body. Some states also employ *Backup commissions*, which have autonomy in the redistricting process only once the legislative body fails to agree on a set of maps. There are also *Politician commissions*, in which members are mostly elected officials or their appointees. Finally, the model commission structure is the *Independent commission*, which has full autonomy in the process and whose members are neither legislators nor elected officials (Cain, 2011).

But Virginia’s structure is unique, as it is the only *hybrid* redistricting commission in the nation—one with both citizens and legislators as members. Furthermore, the VRC does not have autonomy, requiring approval from the General Assembly in what can only influence the actions of the legislator members. While the hybrid structure was meant to promote bipartisanship, the VRC ultimately failed to produce any maps due to partisan disputes in its inaugural redistricting cycle, resulting in the new districts being drawn by SCOVA (Keena, 2022). Thus if the VRC is to have success in the future, the Commission must overcome some of the obstacles that plagued it in the 2021 redistricting cycle.

One of the key issues that led to the Commission's deadlock was its inability to agree on a neutral mapmaker to provide technical expertise in the redistricting process, with one legislator going as far to say that no such entity exists. The VRC instead hired two sets of partisan mapmakers with the goal of melding their work in a move that some members argued was setting the Commission up for failure (Commission, 2021b). Thus in the remaining sections of this paper, I explore whether neutrality can exist in redistricting by examining the technical and human-elements of the mapmaking process. I then propose a solution to the neutral mapmaker problem through the formation of an independent redistricting lab in Virginia to provide technical expertise for future iterations of the VRC.

> *"There is only one way to do reapportionment—feed into the computer all the factors except political registration."*
>
> — Ronald Reagan, Los Angeles Times, January 21, 1972, at A3

2 NEUTRALITY OF PROCESS

As the VRC illustrates, the process of drawing electoral boundaries is one that is inherently political. Because of this, many have questioned why we aren't removing politics from the equation by having a computer draw the maps instead? This is a sentiment that has been echoed since the beginning of the computer revolution. As Vickrey writes:

> If there is thus no available criterion of substantive fairness, it is necessary, if there is to be any attempt at all to purify the electoral machinery in this respect, to resort to some kind of procedural fairness. This means, in view of the subtle possibilities for favoritism, that the human element must be removed as completely as possible from the redistricting process (Vickrey, 1961).

2.1 A Brief Technical Introduction to Redistricting

Redistricting involves the clustering of census units known as *blocks*[2] in such a way that satisfies federal and state requirements. After decades where the rural vote carried significantly more weight due to districts largely being composed of counties, constitutional requirements were put in place that mandate that districts be drawn with populations as equal as practicable in what is known as *one person, one vote* (Altman & McDonald, 2012). For Virginia's Congressional districts, this practicability has been defined to be exact, such that the maximum deviation of a district is *one person*. But for General Assembly districts, the population deviation is generally more flexible, with codified language that there must be no more than a ±5% deviation[3] (Commission, 2021a).

Virginia also has language concerning the contiguity of districts. We can generally think of contiguity as requiring that a district be one singular and connected shape, but Virginia extends this definition to require that no district is connected only by bodies of water. Additionally, in response to gerrymandering which has notoriously resulted in extraordinary shapes that can sometimes span hundreds of miles, there is also language that the districts must be compact (Commission, 2021a). But compactness lacks a clear definition and is not a sufficient criteria by which to judge the good intention of a district, as it is possible to have compact but gerrymandered districts (Duchin & Walch, 2022).

District maps must also comply with the VRA as well as the Equal Protection Clause of the 14th Amendment. That is, districts should provide the opportunity for minority voters to elect their candidate of choice, but they should also not be drawn such that race is used as the predominant factor. Virginia also requires the preservation of *Communities of Interest* (COI), which are groups of people living in an area who share social, economic, or political interests. Finally, Virginia requires *political neutrality* such that maps do not favor or disfavor one political party over another (Commission, 2021a), and there are also often considerations of whether or not mapmakers should prevent incumbent legislators from being placed into the same district.

[2]Census blocks are small geographic areas–often bounded by roads, rivers, and other nonvisible boundaries–with associated demographic data.

[3]Though the VRC advised mapmakers to strive for a ±2% deviation, and the court drew maps following a ±2.5% rule.

Despite some early optimism that computers could solve redistricting by selecting the objectively best map from the set of legal maps (Weaver & Hess, 1963), we now know redistricting to be an *NP-Hard* problem (Kueng et al., 2019). That is, finding the *best possible* redistricting plan is *impossible*. As Moon Duchin illustrates in *Political Geometry*, suppose we wanted to try to understand just *how* difficult it is to find the best redistricting plan by partitioning a 4×4 grid into 4 districts where the only requirement is that the districts are contiguous. In this case, there are exactly 117 permutations we would need to evaluate in order to identify the best option. But by the time we expanded the grid to 10×10 with 10 districts, simply enumerating the permutations is beyond the reach of our current computing capabilities (Duchin & Walch, 2022). Thus we would need infinitely more time than the ten years between each census cycle to find the objectively best plan when partitioning Virginia's 163,490 census blocks into 100 House of Delegates districts.

Yet despite the computational complexity of the redistricting problem, there are trillions upon trillions of legal maps that satisfy this set of criteria (DeFord & Duchin, 2019). While it is infeasible to identify the objectively best plan, or the global optimum, heuristics can be leveraged to identify local optimums, or otherwise plans which are *good enough*. To this end, computational redistricting has the promise to, "elevate the legislative redistricting debate from a battle over line drawing to a discussion of representational goals" (Browdy, 1990, p. 1381).

2.2 A Computational Approach

Optimization is the mathematical or computational problem of finding the best solution from a set of alternatives, where the *best* solution is determined based on some objective function that needs to be minimized or maximized subject to some set of constraints (Duchin & Walch, 2022). One of the first computational approaches to redistricting was a heuristic optimization model published by Weaver and Hess (1963). The authors proposed their work as an option in the event that district maps needed to be redrawn by the courts, or when legislatures deadlocked due to political impasse. "One way of accomplishing this end," they argued, "could be to adopt a mechanical formula which makes the actual drafting of district lines non-discretionary once general principles of representation have been determined" (Weaver & Hess, 1963, p. 288).

Weaver and Hess recognized redistricting to be analogous to the *warehouse location-allocation* optimization problem of operations research. In the location-allocation problem, the objective is to identify the number, location, and size of the warehouses that will most efficiently serve a set of customers with goods (Cooper, 1963). Weaver and Hess formulate the problem such that districts are the warehouses and population units are the customers. In order to minimize the allocation cost, a compactness measure is proposed based on the physics principle of *moment of inertia*, which is the sum of squared distances from each unit to its axis of rotation, as this measure is smallest when the units are concentrated at the center (Weaver & Hess, 1963).

The allocation of each unit to a district is done using a subroutine that uses linear programming to solve what is known as the *transportation problem*, which seeks to minimize the distribution cost of transporting M goods to N locations (Bradley et al., 1977), where the cost is defined to be the moment of inertia (Weaver & Hess, 1963). Rather than solving the allocation problem using linear programming, George et al. (1997) models the problem as one using minimum-cost network flow, where the goal is to minimize the population-weighted distance of each geographic unit to each district while satisfying population constraints imposed by flow (George et al., 1997).

In the technical report of this thesis, I took an approach similar to that of George et al. (1997), leveraging network flow to assign geographic units to district centers while optimizing for population equality and compactness. I approached the actual generation of district maps in a race-blind manner and evaluated the districts post-hoc on the basis of population equality and minority representation. Despite not considering race in the drawing of the districts, I found that a network-based clustering approach produced districts with low population deviations and strong minority representation opportunities, particularly in the generated House of Delegates map in Figure 4 (Horan, 2024).

2.3 The Nascent Machine

While optimization approaches remain an active research area, it is rare for an optimization approach to be employed in real-world redistricting. The primary role of

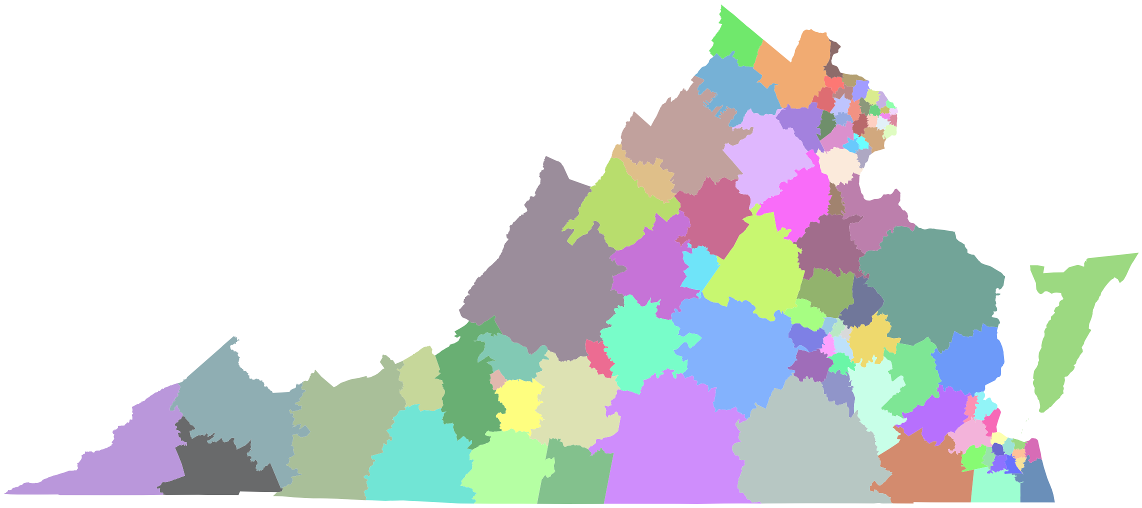


Figure 4: Generated House of Delegates Districts

computational methods in the current redistricting process is through statistical outlier detection. Much of the recent work involves the use of *Markov-Chain Monte Carlo* methods which generate *ensembles* of district plans for evaluation. Because there are *trillions* of legal plans that satisfy the basic requirements of redistricting, ensembles of thousands or even millions of plans can be generated to help paint a picture of what the *universe* of district plans looks like for a particular locality. These techniques can thus be used to detect statistical outliers, or otherwise district plans that deviate excessively from what would be considered to be a *normal* legal plan. Tools such as this have been successfully utilized in the litigation of district maps as racial gerrymanders (DeFord & Duchin, 2019; Duchin & Walch, 2022).

However, there are arguments against the use of computers to fully automate the redistricting process. The majority of those involved in redistricting are not mathematicians or computer scientists, thus there is a level of trust that the end users of the computer algorithms and tools must place in those who develop them (Duchin & Walch, 2022). Futhermore, algorithmic approaches are not void of the potential for bias or abuse. For example, an algorithmic approach may be able to satisfy a set of criteria, but it also may be written such that the districts it produces are *unintentionally gerrymandered* (Duchin & Spencer, 2022). Many algorithms are also *stochastic* in nature, and an end-user could leverage this randomness by simply re-running the algorithm until it produces a result that satisfies some agenda. Thus it is possible for a user to act under the guise of procedural neutrality despite cherry-picking the results (Duchin & Walch, 2022). But while computational redistricting may not offer an immediate solution to the problem of redistricting, computers remain a ubiquitous component of the map-making process.

> *"Until a machine can do human work, it is best to limit its use strictly and also limit the use of machine-like theories that try to organize society. American society is not a collection of faceless particles. It is composed of highly diverse and yet interconnected sets of people. A political theory suggesting that people are interchangeable like nuts and bolts is likely to be both fallacious and detrimental to the personal happiness of the citizenry."*
>
> — Alfred De Grazia, Public and Republic: Political Representation in America

3 FAIRNESS OF OUTCOME

While many computational approaches optimize for criteria such as population equality, contiguity, and compactness, there is insufficient development in the more complex and nuanced aspects of redistricting. Computational models may be able to objectively produce maps in a neutral manner, but the map-making process is as much of an art as it is a science.

3.1 Communities, Race, and Partisan Fairness

Computational redistricting models seek to solve the *general* problem of redistricting. However, redistricting is a *local* problem that requires an understanding of the communities affected by the redistricting process. In Virginia, there are well-established regions which share social, economic, and political interests, including the Blue Ridge Mountains, the Shenandoah Valley, Appalachia, Southside Virginia, and the Piedmont. These regions were all explicitly considered when SCOVA drew the maps in the 2021 redistricting cycle (Grofman & Trende, 2021), and highlights the importance of understanding the political geography and history of a state. Further, in the 2021 redistricting cycle, the VRC received 66 instances of public-defined COI. These included things such as historically black neighborhoods; areas with schools, hospitals, and shopping corridors; suburban and rural areas; commuter regions affected by transportation funding; metropolitan areas of cities; farmland and agri-tourism; locations home to large groups of immigrants; and university areas ("Community Links", 2021). And these communities were vocal throughout the redistricting process. Virginia has had a redistricting process that has disenfranchised many groups throughout its history, thus having vibrant public participation is essential to ensuring equitable representation for all Virginians (Altman & McDonald, 2012), and further emphasizes the insufficiency of a generic model which does not take into account the local context of a state.

As previously noted, districts must also provide an opportunity for racial and ethnic communities to elect their candidate of choice. Language in the VRA requires that a minority group is sufficiently large, geographically clustered, politically cohesive, and that there exists *politically polarized voting* in the surrounding area. Because VRA compliance frequently results in litigation, a *racially polarized voting analysis* can be needed to identify areas which must comply with the criteria of the VRA (Palmer & Schneer, 2021). Mapmakers have historically created *majority-minority* districts to satisfy this criterion, which are districts which have a minority population greater than 50%. However, these types of districts have also been used to *pack* minority voters into few districts, diluting their voting strength. As Lublin et al. (2020) argue, due to an increase in politically polarized voting, recent electoral data suggests that minority groups now have a higher probability of representation in districts without a majority. Thus *minority opportunity* districts are becoming more favorable, which are districts where a minority group has a population in the 40-50% range (Lublin et al., 2020).

In the discussion on gerrymandering, Figure 2a illustrated the idea of *proportional representation*, where the seats won by a party were proportional to the number of party voters. This idea of *partisan fairness* is also fundamental in redistricting, and one which voters unanimously believe most clearly denotes a fair district plan (Duchin & Walch, 2022). Unlike racial gerrymandering, which is outlawed by the VRA, there are no federal statutes protecting against partisan gerrymandering (Cox, 2004), and the result is that districts are increasingly drawn where the majority of seats are not competitive, oftentimes disproportionally so ("Dubious Democracy 2022", n.d.). While it has been argued that political competition does not necessarily produce better electoral outcomes or lead to greater levels of voter approval (Adams et al., 2010; Brunell, 2010), proponents of competitive districts argue that partisan skew leaves voters underrepresented ("Dubious Democracy 2022", n.d.), and that a lack of competitive seats makes it such that a few number of primary voters ultimately decide general elections (Li & Leaverton, 2022).

These three criteria—COI, racial fairness, and partisan fairness—are all essential to ensuring that a redistricting plan is fair. However, they are also in tension with one another. For example, a map that is racially fair may not be politically fair, and a map that is politically fair may not be fair to COI. This is the challenge of redistricting: to balance these competing interests in a way that is fair to all.

3.2 The Well-Intentioned Mapmaker

Section 3.1 detailed some of the more complex and nuanced aspects of redistricting which are rarely considered in computational models but are essential for fair representation. Thus neutrality of process through computational redistricting does not necessarily guarantee fairness of outcome. Just as redistricting is a local problem, it is also a *human* problem. Understanding and evaluating the tradeoffs that go into redistricting, as well as the communities that are affected by it, requires a level of human understanding that is difficult to replicate in a computational model. Because of this, maps continue to be drawn by humans, and even with future advances in computational approaches, the human element will always be needed (Duchin & Walch, 2022). But it is not without issue.

The redistricting process is inherently political, and the human mapmaker is not immune to the political pressures that come with the job. However, the issue of transparency in the process is possibly of greater concern. Most mapmakers use proprietary Geographic Information System (GIS) tools to draw maps. Mapmakers today are armed with finely-grained data on the geographic units that make up a state, including demographic data and election results that can be leveraged in real-time to draw maps that achieve specific political outcomes. Institutional experience in redistricting and a sufficient knowledge of a state can thus enable a mapmaker to gerrymander with *surgical precision*.

Thus the promise that computers could *remove* politics and bias from the redistricting process has instead led the reality that computers are currently being used as a tool to *enhance* these biases (Altman & McDonald, 2010).

4 THE NEUTRALITY OF TRANSPARENCY

If computational redistricting is not yet capable of producing fair maps, and human mapmakers are not immune to political pressures or bias, then the goal should be to make the map drawing process as transparent as possible. Redistricting commissions are a step towards transparency (Lamar et al., n.d.), and the VRC does well in this regard as it publishes all of its data and submitted maps. However, the process by which maps are drawn remains opaque, and once the map drawing gets assumed by SCOVA, public input in the process is diminished. Furthermore, there are steep barriers to entry due to the cost of the proprietary GIS tools used to draw maps, thus making public participation at any point of the process difficult.

4.1 A Localized Approach

Rather than developing computational models that solve the *general* problem of redistricting, we should strive to develop models that solve the *local* problem of redistricting. That is, we should extend models beyond optimizing for population equality, contiguity, and compactness to include criteria such as COI as well as racial and partisan fairness. This would allow the procedural neutrality offered by computational redistricting to take a step towards practicality.

In my technical report, I geographically constrained Virginia's Eastern Shore, which is connected to mainland Virginia by a single bridge, in order to satisfy the contiguity criterion (Horan, 2024). Similar techniques can be employed to constrain regions of a state which share social, economic, and political interests. Furthermore, while my work did not consider racial or partisan fairness in the model, future work could consider the addition of these constraints through techniques such as weight penalization with the goal of developing a model that is able to produce fair maps for Virginia and act as a neutral starting point for the VRC.

4.2 Overcoming Inexperience

The VRC tasked its two sets of partisan legal counsels to identify candidates who would act as impartial entities to provide technical assistance to the Commission in the redistricting process, but only one was ever presented. The counsel representing the partisan interests of the Democratic party recommended that the University of Richmond's Spatial Analysis Lab (SAL) be the neutral party that does the map drawing. While the lab had no direct experience with redistricting, their research requires expertise in the GIS tools used in the map drawing process, and the lab had recently done significant analysis and mapping of census demographics. However, the Republican counsel argued that time constraints were an issue, and that the lab's lack of experience in redistricting would outweigh its technical expertise. Futher, the Republican counsel expressed concerns that the public would believe SAL to have a partisan lean (Commission, 2021b).

However, such an entity is likely the closest that the Commission can get to a neutral entity in the map drawing process. Thus a solution to this problem is to form an *independent redistricting lab* in Virginia to provide technical expertise to future iterations of the VRC. Such a lab could be focused on transparency and ethics, and the lab could also strive to better understand the political geography, history, and communities that make up Virginia. Furthermore, the lab and could leverage current and future literature in computational redistricting for use as an effective tool in the redistricting process, and work towards the development of models such as those proposed in 4.1. A redistricting lab could also work towards the development and maintenance of free and open-source software that could be used by the public to draw maps, as well as engaging in public outreach to mobilize interest and knowledge of redistricting, lowering the barrier to entry for public participation and helping move towards a more transparent and equitable redistricting process.

5 CONCLUSION

In this paper, I have highlighted how redistricting is a politically charged process that has been historically used to disenfranchise minority groups. I have also shown how computational redistricting can be used to draw maps that are procedurally neutral, but that the fairness of these maps is still up for debate. I have argued that the goal of redistricting should be to make the process as transparent as possible, and that the development of a redistricting lab in Virginia could help to achieve this goal. I have also argued that computational models should be developed that solve the local problem of redistricting, rather than the general problem, such that they can take into account the more complex and nuanced aspects of redistricting that are essential for fair representation but are currently best achieved by human mapmakers.

Computational models may not yet be a viable solution to the problem of neutrality in redistricting, but there need not be an *all or nothing* approach to their use. In George et al. (1997), the authors described a computational model that was utilized alongside a redistricting commission in New Zealand to draw maps in an iterative manner. Thus in the right hands, a computational approach can be leveraged as an effective tool to create fair maps based on commission feedback. However, for a truly fair process, Virginia should consider changes to the system by which we elect our representatives.

In *social choice theory*, *Duverger's Law* states that plurality-rule elections tend to favor a two-party system (Riker, 1982). What this means is that when voters only have one choice on a ballot, they are more likely to vote for one of the two major parties. But there are electoral systems that overcome this limitation. *Ranked Choice Voting* (RCV) allows voters to rank candidates in order of preference, and is touted as "a way to make democracy work better by giving voters more say in their representation, by allowing ballots that better describe their preferences and policy views" (Duchin & Walch, 2022, p. 416). RCV is itself a complex system, with many algorithms that can be used to determine the winner of an election (Olson, 2017), but the idea is that it provides opportunities for more representative outcomes. The formation of the VRC was a step in the right direction, but the structure of the Commission is such that it is still susceptible to political influence. Thus a more radical change to the electoral system in Virginia may be needed to ensure fair representation for all Virginians.

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