UNIVERSITY OF VIRGINIA

DOCTORAL THESIS

A State-based Methodology for Engineering Portfolio Risk and Improving Efficiency

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A Dissertation

Presented to

the Faculty of the School of Engineering and Applied Science

University of Virginia

In Partial Fulfillment

of the requirements for the Degree

Doctor of Philosophy (Systems and Information Engineering)

by

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May 2019

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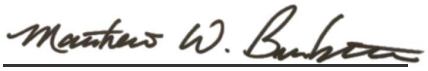
Author Signature: 

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"Study the past if you would define the future."

Confucius

Abstract

This dissertation joins the body of work in finance and analytics examining the methods for characterizing financial markets and improving asset allocation decisions. It explores the existing body of literature to discuss how research in the domain of finance and investment management has incorporated the use of discrete states into decision making, particularly with applications aimed at investment portfolio construction and design. An innovative approach is presented that derives a feature set incorporating return and risk characteristics from the market and segments the market into definable, discrete states. By segmenting the population into discrete states, this research can (1) define and analyze the transitional dynamics between states, (2) create state-specific asset allocation models based on a risk-return portfolio metric, (3) calculate the probability of state membership for any observation in a holdout period, and (4) create an asset allocation strategy that is able to adapt to changing market conditions. The research examines various optimization-based approaches for deriving the state-specific asset allocations and compare the results. Several techniques are presented for evaluating the quality of the state definition scheme presented, such as whether the transitional dynamics between states hold statistically across multiple time periods. The preliminary results show that this method produces promising risk-adjusted portfolio results; however, a great deal of work remains that can build upon these initial findings.

Acknowledgements

It's been said that it takes a village to raise a child. Well, it took the support of a rather large city to bring this dissertation to fruition...

Whit: I know it has been hard dealing with me throughout this process, but I am grateful beyond words for your support and patience.
Mom: Your unwavering support and encouragement helped me stay the course, and I am forever grateful.
WTS: It's been a long road that began in '98. Thanks for all the guidance, confidence, support and friendship along the way.
My Committee: I am extremely fortunate to have had the committee I did. A group composed of professors whose intellect I hold in high esteem and whose company I greatly enjoy. Thank you for taking this journey with me.
Andrew: I look back fondly at our times debating ideas at the whiteboard in our office or at the pub over a pint or two. Thanks for all the help and friendship along the way.
VRFC: To the family I never expected but am so grateful to have. Thanks for all the bruises, beers and friendship.
Lt. Bill Golden , USN: Our Friday coffee talks on the Lawn were a quintessential UVA experience for me, and I will miss them.
Ryan Boyer: Thanks for the help framing my thoughts with the topic modeling. It was greatly appreciated.
Stephen Adams: Your MATLAB wizardry was much appreciated as I worked through my portfolio optimization. Many thanks.
Jayne Weber: Thanks for enduring what must have seemed like an endless string of administrative questions. I appreciated all of your help and support.

There are many other people who are deserving of my thanks whether it be for helping me along the way with this doctoral research or making my time at the University of Virginia the truly wonderful experience that it has been. Thank you!

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List of Abbreviations

CAPM	**C**apital **A**sset **P**ricing **M**odel
EMH	**E**fficient **M**arket **H**ypothesis
GARCH	**G**eneralized **A**utor**R**gressive **C**onditional **H**eteroskedasticity
GDP	**G**ross **D**omestic **P**roduct
GNP	**G**ross **N**ational **P**roduct
HMM	**H**idden **M**arkov **M**odel
LDA	**L**atent **D**irichlet **A**llocation
MPT	**M**odern **P**ortfolio **T**heory
MVO	**M**ean **V**ariance **O**ptimization
NAV	**N**et **A**sset **V**alue

For My Family

Chapter 1

An Introduction

"*First comes thought; then organization of that thought, into ideas and plans; then transformation of those plans into reality. The beginning, as you will observe, is in your imagination.*"

Napoleon Hill

1.1 Motivation for Research

Early in my finance career as a young research analyst for the investment firm, LOC Advisers, I was exposed to a wide-range of theories as part of my introduction to the financial services profession. The multitude of theories and approaches ranged in style, purpose, indicator selection, and timeframe just to name a few. Some theories were aimed at providing general insights in the direction and health of the economy in hindsight, such as the Dow Theory. Others, like the efficient market hypothesis (EMH), provided guidance on how to navigate the financial markets. For EMH it seemed that less, much less, was more. Many to of these traditional theories that I spent time learning and understanding early on in my career seemed to be rarely, if ever, actually used in practice. It seemed that the value in learning and understanding many of the well-established theories of finance was to learn the language and mindset of the finance and investment management profession.

Many mutual fund managers subscribed to a 'fundamental' approach to security selection in which managers attempted to determine the underlying value of an asset and

those deemed to be the most undervalued relative to market price were assembled in an investment portfolio. This approach seemed reasonable given that if the analysis were performed correctly, then undervalued assets would be most likely to trend towards their fair market price thereby making undervalued assets the most likely to increase in value.

Another school of thought when it came to the practical application of security selection in financial involved individuals known as 'market technicians' or 'chartists' due to their tendencies analyze market price patterns in charts as opposed to company 'fundamentals' of valuation. Both schools of thought - fundamental and technical - identify either buying or selling opportunities for individual assets. While I found the ideas presented in such theories interesting, I did not view them as an investment management solution for creating a portfolio. Coming from a systems engineering background, I was interested in the overall system, which in this case was the overall portfolio. I viewed an investment management solution as one the addressed the needs of the portfolio as a whole.

As I continued to learn more about the investment management profession, I discovered the seminal work of Harry Markowitz presented in the historic paper Portfolio Selection. His presentation of an efficient frontier as part of what became known as Modern Portfolio Theory (MPT) possessed elements of systems thinking in a formalized framework for portfolio design and asset allocation. I was immediately interested in the theory and spoke to a portfolio manager in the firm about it. "The problem," he said, "is that what you see in the theoretical research are long-term results. In practice there are periods of time, sometimes extensive periods, when the markets deviated from the long-term trends, and the Markowitz allocation is not always ideal during those periods[1]." I was then directed to the graphic shown in Figure 1.1, which shows the efficient frontiers for stock-bond allocations for each decade since 1960. The portfolio manager pointed to how different each frontier was and by extension how different each ideal allocation was. He went on to say that you never really knew which frontier you were in until afterwards at which point it is too late to act. If you knew which frontier, or state, the markets were in, more insightful allocation decisions could be made.

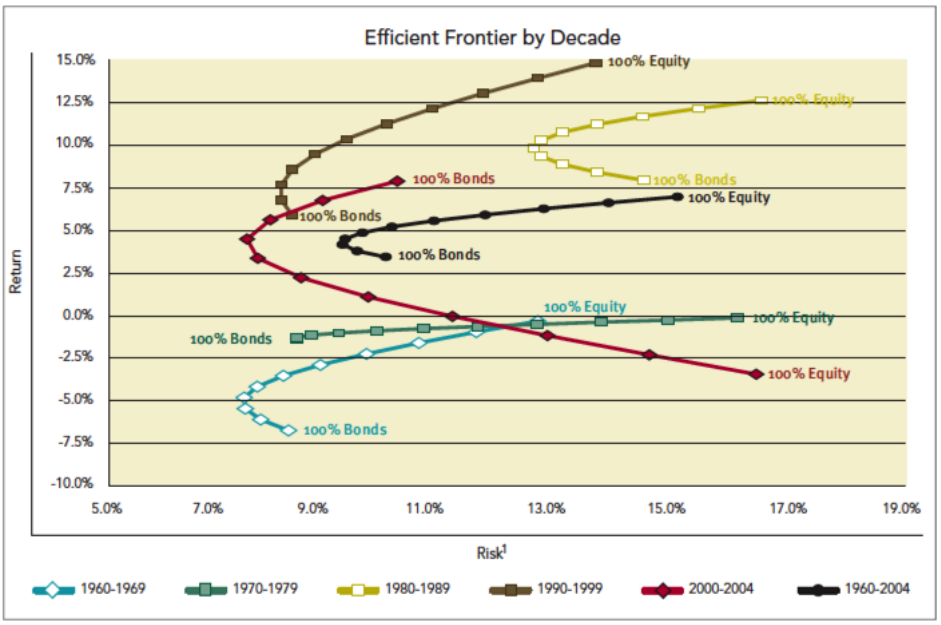


FIGURE 1.1: Stock-bond efficient frontiers organized by decade. Example of the shifting nature of frontiers and financial markets.

This closing thought about knowing what state the markets were in would give insight into asset allocation decisions has stayed with me and served as the motivation for pursuing this course of research.

1.2 Thesis Statement

The research contained within this dissertation is founded on the belief that not all periods of time in the financial markets are created equal and should not be treated as if they are in the name of convenience or simplicity. This research document presents the thesis that there exist discrete states within the financial markets defined by similarities between elements of a feature set, and knowledge of the state to which a particular time period belongs enables more informed decisions to be made regarding the construction and dynamic management of investment portfolios.

[1]I was also reminded of the John Maynard Keyes' quote that "[i]n the long run, we are all dead. But this long run is a misleading guide to current affairs."

The goal of this research effort is to subdivide the financial markets into states in a manner that provides insight into how to combine investment assets in a manner that improves the risk-return profile of portfolios. The achievement of this overarching goal will be supported by the following objectives:

1. Develop a minimum spanning feature set that can accurate capture market characteristics and differentiate between time periods
2. Assigned time periods into discrete states through unsupervised cluster analysis
3. Determine the appropriate metrics to evaluate the efficacy of cluster results and state assignment
4. Analyze state transition behavior and develop model to represent transition dynamics of the system
5. Implement a heuristic for constructing investment portfolio that combines state assignment results, evaluation of state assignment clustering results and state transition dynamics

1.3 Systems Thinking & Finance

1.3.1 Motivation

The financial markets and investment management industry is a highly profitable, intellectually rigorous profession. Entering into finance from a 'non-finance' background, specifically a systems engineering background, I found myself frequently questioning aspects of the financial industry wondering why it was structured as it was. The following section contains an essay co-authored with my advisor, William Scherer, who also has experience in the financial industry having come from a systems engineering background. The catalyst for the essay were conversations focused on the need for systems thinking in financial industry. It was our belief that all areas of the industry would benefit by incorporating a systems approach ranging from the design of products, regulation of markets, and even the selection process of an investor in determining the products to include in a

retirement portfolio. The following is a 'think piece' intended to motivate discussions on the structure of the financial services industry.

1.3.2 A Call for Systems Thinking in Financial Services

The financial crisis of 2007-2008 and the subsequent tightening of lending standards have had a continuing and significant impact on both investment accounts and the financial services industry. Devastating losses in equity markets worldwide resulted in depleted savings accounts for pension funds and individuals alike. Less available credit due to stricter lending standards, increased unemployment, and slowing economic activity affected firms globally, which in some cases necessitated government assistance and "bailouts," particularly in the automobile and financial services industries. In response, regulators tightened existing policies and enacted new ones, such as the central bank policy of quantitative easing, in hopes of stimulating the economy and preventing future financial crises.

Such "black swan" events coupled with numerous so-called "hundred- year storms" are examples of serious failings within the financial system to protect stakeholders and oversee detrimental behavior - intentional or otherwise - on behalf of the financial system. The common regulatory response of tightening existing policy or creating new policies often does not address the source of key weaknesses within the financial system: conflicting objectives among the multiple agents in the system or poorly chosen measures of performance and evaluation. As market dynamics and regulations evolve so must the skill sets needed to participate in and oversee them. The prevailing tools used in analyzing many financial market problems fall under the umbrella of statistics. Unfortunately, these tools alone may not be sufficient. Pate-Cornell (2012) discusses how existing statistical methods do not support the epistemic and random nature of uncertainties and calls for an "engineering risk analysis" perspective on risk management [122]. Such an approach involves monitoring signals and feedback strategies to proactively address both precursors and "near misses" as opposed to post-event reactions, such as tightening regulations. In light of the most recent financial crisis, it is clear a new paradigm is needed in designing and maintaining the system. The engineering risk analysis approach put

forward by Pate-Cornell is inspired by the methods and techniques used in systems engineering – a fundamental component of which is systems thinking. Systems Thinking has many definitions, but a specific definition is hard to pin-down. A typical definition is "a framework for seeing inter-relationships rather than things, for seeing patterns rather than static snapshots. It is a set of general principles spanning fields as diverse as physical and social sciences, engineering and management" [137]. We expand on this idea below and apply specific dimensions, or attributes, of systems thinking to key concepts.

There have been numerous calls for systems thinking as a potential savior for problems found in industries such as energy, healthcare, business strategy, education, transportation, national defense, as well as numerous other domains [71]. Bishai (2014) postulates that applying systems thinking to healthcare provides the opportunity to improve the decision-maker's understanding of the overall system and couple it with achieving specific objectives, such as an improvement in health [15]. Recently, there has also been a call to apply systems thinking to other complex global problems, an issue addressed in this piece [47]. Capra and Luigi (2014) make an extensive and elegant case for systems thinking in general and particularly in finance, arguing the need for a move from the outdated Newtonian scientific worldview, which is deeply rooted in a linear approach to problem solving [29]. Such a move in perspective is necessary in the complex and interconnected world of finance where systems interact in a highly non-linear fashion. Take the example of the way consumer lending interacts with the investment world. Mortgages were sold to consumers in order to finance their properties. When pricing mortgage loans, one of the primary objectives is pricing that is aligned with the risks involved with offering such a loan. A higher interest rate charge will result in lower take-up, whereas a lower charge may result in lower profit. However, when setting the interest rate charged to the consumer, originators do not explicitly consider the objectives of the eventual buyers of these mortgage loans at the end of the securitization process (i.e., pension funds). The financial services industry is a system of systems with multiple agents, resulting in multiple objectives. In such systems, information asymmetry negatively affects many stakeholders. Those with information have no reason to give up information that may result in another stakeholder's monetary benefit. This type of interaction between multiple agents and

systems requires a new mode of thinking and analysis.

While the concept can be defined in an a general sense, the label systems thinking is subject to wide variation among domains and researchers. It might be said that there is some mystery and ambiguity as to what systems thinking actually is. It can be perceived as anything from a simulation methodology (e.g., system dynamics [62] to a business process [137] to an engineering process (traditionally associated with the systems engineering community [e.g., INCOSE]) to a more philosophical basis of general systems thinking (examples include Von Bertalanffy (1956) [154]; Churchman (1968) [37]). Systems thinking appears to fall under the colloquial expression, "I know it when I see it," famously used by U.S. Supreme Court Justice Potter Stewart when asked to describe pornography.

This article takes a preliminary look at defining what practical attributes might define good systems thinking and applies that framework to challenge the current perspectives on the global financial services industry. Of the twenty (20) dimensions associated with systems thinking, nine (9) particularly apropos dimensions are presented in this paper and used to make the case that the financial services and investment industry is in dire need of a systems perspective, perhaps more so than any other domain listed above (Whitehead (2015)[157], Boyer (2017)[25]). Table 1.1 lists all twenty dimensions of systems thinking, and designates those used as examples in this paper with an asterisk (*).

This need is based on the failure of the financial services industry to deliver a system that exhibits even the most basic attributes of good systems thinking. This rather serious, challenging, and controversial conclusion is based on the absolute lack of objective success for most stakeholders engaged in current financial system.

Taking the problem as creating a financial services industry (a.k.a. the "system"), this paper examines several concepts of systems thinking that need to be considered if the goal is to develop a well-designed, resilient system that can benefit all of the system stakeholders by employing systems thinking. This paper uses a very broad definition of "financial

TABLE 1.1: Dimensions of Systems Thinking [157]

Descriptive Scenario*	Normative Scenario
System Boundaries*	Objectives*
System Stakeholders*	Indices of Performance*
Scope of Analysis	Develop Alternatives
Type of System	Outscope*
State of System	Evaluate & Rank Alternatives*
Life Cycle of System*	Interactions
Metrics	Iterate Analysis
Axiological Components*	Leverage Points
Observer Effects	Recommendations

services" that includes any transaction or regulation of a financial good, including real estate, investment firms, banks of all types, insurance companies, and can be applied to any individual who uses financial instruments, such as equities, debt, derivatives, currency, etc.

To demonstrate this approach, several critical dimensions of systems engineering are discussed and illustrated using examples from a variety of fields and then related to the financial services industry. Keep in mind that these dimensions are presented with regard to a particular system of interest and therefore have numerous interactions among them. While these dimensions are presented in a stand-alone format, they are meant to be part of an iterative process (see Gibson, Scherer, Gibson Smith (2016) [70] for more information). First, the importance of understanding the current scenario is described, followed by the need to understand all the stakeholders impacted by the system and their respective objectives and values, and how these objectives are measured. And lastly, consideration is given to axiological components, system life cycle, an understanding of system boundaries and ranking alternatives. The selected nine attributes are those systems thinking dimensions that are the most directly aligned with the failings of the financial system.

Dimensions to Consider

Descriptive Scenario

This is the process of describing the current state of the system. Creating the descriptive scenario is a simple yet fundamental idea in systems thinking – observe the system and understand what the current situation is [66, 156]. Consider an automobile-based transportation system where for years the cars were unreliable, but automobile manufacturers made a fortune. Or a TV-based entertainment system where an expensive TV worked perfectly for one year and sporadically decreased in quality soon after. Would these be considered successful systems? In other words, do they achieve the objectives of all the stakeholders in the system? The answer to this question is obviously no. The objectives of every stakeholder are not and cannot be achieved, particularly when objectives are often competing or conflicting. Even a well-designed system cannot always meet the objectives of every stakeholder; however, it does consider multiples stakeholders and the different ways in which the system impacts them. The current state of the financial system cannot be considered a successful system for many stakeholders, nor surprisingly, many stakeholders are not aware of the current state of financial services, especially the performance of the system for its participants. The goal of a descriptive scenario in systems thinking is to describe the "as is." Take the example of a typical investor in the investment world and consider what this important stakeholder has realized in terms of his or her objectives (i.e., maximize returns) since WWII? Investor returns, if invested in the S & P 500 from 1994 to 2013[2]would have an inflation-adjusted return of less than 5% per year [42]. Whether this is considered a good or bad investment outcome depends on the investor's goals. Honest attempts at calculating actual, realized returns are often ignored, refuted on spurious grounds, or result in outright attacks on the authors.

For example, the book *The Hedge Fund Mirage* [97] made a claim that hedge funds' returns over the period 1998-2010 were slightly less than Treasury bills. An online discussion group of the book by hedge fund and other industry professionals vigorously disputed this point, claiming that hedge funds had actually made slightly more than T-bills over

[2]Performance result is based on what Dalbar terms a "systematic equity investor" whose performance is represented by the SP 500 from 1994 to 2013 using data provided by Standard & Poor's. Results do not consider any fees or expenses. The investor continues investing at a constant rate regardless of price levels.

this period[3]. This line of debate misses the point completely. Should the focus of discussion on what many regards as the most sophisticated investment vehicle be on fractions of percentage points in either over- or under-performance relative to short-term Treasuries? Or should the focus be on investors' return expectation when investments were made 12 years prior and whether these expectations were realistic?

Systems Stakeholders

The stakeholders in a system are all the people impacted by the system and should be considered. In the design of an aircraft, for example, the stakeholders include the aircraft manufacturers and the passengers; however, other stakeholders include the airlines, flight attendants, aircraft mechanics, food service workers, pilots, gate agents, ground crews, on-line reservation system businesses, and the airports themselves, etc. If all these participants are not considered in the design of an aircraft, then success is unlikely. Boeing, as an example, understands this and has all stakeholders represented on an aircraft integrated product team [51]. This act of engaging all stakeholders has largely being been ignored in the evolution of financial systems. Perhaps it is more accurate to say that the major powerful stakeholders were considered in the design of various subsystems within the financial system. For example, designers of mortgage-backed securities considered the profit objectives of major stakeholders, such as investment banks, but was the financial health of sub-prime homeowners considered in designing such a system? Were the risks associated with these products clearly considered when mortgage-backed securities (MBS's) were designed? In another example, consider a university endowment fund. Do university hedge funds consider alumni, faculty and students in their private equity investments [61]? An alumni's objectives might be the continuation of the school, whereas a current student's objective might be to increase funding available for financial aid to current student body. Identifying stakeholders is particularly important when there are conflicting objectives, allowing for the tradeoffs between objectives to be clearly identified and all stakeholders made aware of the explicit and/or often implicit tradeoffs.

A favorite example within the systems thinking community is the redesign of the routes

[3]The online group featuring this discussion can be found at the Amazon book page: http://www.amazon.com/The-Hedge-Fund-Mirage-Illusion/dp/1118164318

for a bus system where the consultants did not involve the bus drivers in the process, resulting in non-feasible and extremely inefficient routes that the drivers would have immediately identified as problematic. Typically, tradeoffs must be made in the system design; however, not identifying stakeholders can often result in systems that are inferior on all attributes. The same group also conducted an on-bus, in-transit survey of the current routes to judge their efficacy – think about that one. We have seen similar issues in financial systems. Are hedge fund managers and clients happy with their results [60]? How should the welfare of one stakeholder be compared to the welfare of another in the system? There is a negative correlation between asset growth and returns in the hedge fund industry, suggesting that as hedge funds grow larger and, therefore make more, those who invest in these investment vehicles are making less [97]. Are all the stakeholders in the system considered when hedge fund size and compensation are used as metrics of hedge fund success?

Objectives

This domain involves identifying all the goals found in the system. Here the objectives of the system are described, e.g., what is the objective of the financial system? Clear objectives, typically hierarchical, are fundamental to the proper design of any system [70]. For example, the primary objective of a car (in a simple system) might be "to provide reliable transportation to and from work," and this system's achievement of its goal might be measured (the index of performance [IP]) by the number of working days per year (e.g., out of 252 days) that the car gets us to and from work. In similar fashion, what are the objectives of the stakeholders in a financial system? For an investor in the financial system, is an objective to "get SP 500-like returns?" Why has it become common to benchmark against indices, e.g. achieve SP 500-like returns or to outperform such an index? The Capital Asset Pricing Model and the notion of a market portfolio may have propagated this approach. However, haven't researchers and academics conceptualized more sophisticated models that describe the financial markets? Such objectives remind us of ridiculous conventional wisdom, such as "Don't wear white after Labor Day." A set of objectives should be complete, and the achievement of the objectives (see IPs below) means that the system has achieved its goal[70]. Are these really the goals that an

investor wants? Consider the case of a benchmark that beats the SP 500 by 5%. If the stock market (as commonly measured by the S&P 500) is up 45% in a given year, does the investor want to be up 50%? Would the investor be happy with this performance? Conversely, if the market declines by 35% the following year, would the investor be happy with returns of -30%? Given the two-year scenario described above, an investor would have a cumulative annualized return of 2.5% on an initial investment with no addition investments. The goal of beating the S&P 500 by 5% was achieved, but would an investor be satisfied with a 2.5% cumulative return after investing for two years? Perhaps the notion of objectives in such a system need to be redefined.

Many investments and investment managers are measured against benchmarks that do not necessarily achieve the objectives the investors want or need, i.e. their actual objectives have not been clearly and completely thought out and described. The industry has made simplistic attempts at objectives through the creation of so-called target date funds, such as a 2020 plan designed for individuals planning to retire in 2020 with risk/return assumptions baked in; however, this approach is not reflective of good systems thinking and is an incomplete and simplistic look at achieving investing goals. It was also shown to be inadequate for protecting investor capital during times of extreme market stress. Consider an individual investing in the early 2000's and planning to retire in 2010. Such an individual could choose to invest in 2010 target date funds, such as the three listed in Table 1.2.

TABLE 1.2: Target Date Fund Performance in 2008.
Source: Morningstar

Fund Name	**Ticker**	**2008 Performance**
Vanguard Target Retirement 2010	VTENX	-20.67
Fidelity Freedom 2010	FFFCX	-25.32
JHancock Retire Living through 2010	JLAAX	-29.82

When the financial crisis of 2007-2008 occurred, individuals with investments in such products lost between 20% and 30% of their investments just as they were planning to retire. These examples were selected because they provide comparable performance to

the family of funds as a whole , which had an average return of -24%, in addition to being provided by three of the larger and more prestigious financial services firms operating in the market today. One could argue that nobody could see the crash coming; however, it seems that the wrong objectives and associated IPs were developed by these funds and financial services industry as a whole [4]. Would the preservation of wealth not be a better objective for investors on the verge retirement than capital growth?

IPs [Indices of Performance]

This concept focuses on how to measure objectives. How should investor returns be measured, i.e., what is the index of performance? For example, if the goal is to have an effective Small Business Administration (SBA), then an effective measure of that goal is not the dollar amount of loans granted by the SBA, which was the measure (IP) used for years in evaluating the SBA. Perhaps a better IP would involve the return on the investment (ROI) measured by jobs created and the value of those jobs (this was eventually mandated; see, for example, OIG Audit Report 1-01, Results Act Performance Measurement for the 7(a) Business Loan Program). When considering investments in a financial system, should the IP be time-weighted versus dollar- or value-weighted for measuring returns? Time-weighted returns reflect little of the actual achievement of individual investors (stakeholders) goals. Many funds present great returns over long periods using a time-weighted return; however, actual investors (dollar-weighted) have lost money over that period. Take the example of the CGM Focus Fund, which at the end of 2009 was named Morningstar Fund of the Decade. The fund had amassed nearly $3.7 billion in assets and rose more than 18% annually out-performing its closest competitor by nearly 3% per year[5]. By this measure, the fund was considered an overwhelming success. However, during the same period, the typical investor in the fund experienced an annual loss of 11% [6][54, 98]. How is it that the "Fund of the Decade" lost the typical investor so much wealth? Was the fund advertised and measured using the wrong yardstick or IP?

[4]There are sixteen (16) funds in the Morningstar universe that: 1) have 2010 in the fund name, 2) are the oldest member of their respective share class to avoid double-counting, and 3) have performance in the year 2008. The average return for these funds in 2008 was -24.44%.

[5]The results are time-weighted or as Morningstar refers "industry returns" and do not consider the flow of capital or amount under management at any given time. Performance is examined over a 10-year period ending November 30, 2009.

[6]These results account for deposits and withdraws into the fund and are based on an internal rate of return calculation that Morningstar refers to as "investor returns."

Consider also how risk is managed – most investors/people are risk averse, and there is considerably more pain for an investor to lose a dollar than to make a dollar. How is this concept considered in financial investments? Typically, it is not. Well-considered objectives and associated IPs are necessary to effectively design a system that addresses these stakeholders' goals. As a simple consideration, the IPs must be meaningful (to stakeholders such as an investors; a Sharpe ratio as an IP is not likely meaningful to many investors), measurable (e.g., marked-to-market as opposed to some non-measurable statement of value), objective (can't be "good returns" or "outstanding performance" – interpretations of these depend on the individual and are not objective), and non-relativistic (i.e., typically not "outperform some benchmark by 5%"). Someday investors may be able to clearly articulate their investment goals and IPs to both themselves and the service providers in the financial industry. Should investors be asked to decide between various gambles to understand their risk appetite?

Axiological component

What are the values of the stakeholders? A well-designed system based on the principles of systems thinking must consider the values of all stakeholders involved in the systems and the impact of the system on those values. In an environmental system, for example, Greenpeace and Exxon have very different underlying value systems. Neither value set is right or wrong, just different. Systems are aware of these differences and allow for them in the design of systems, i.e., a good system design accommodates the differing values and clearly identifies the tradeoffs being made. Many system failures have resulted from the omission of this consideration. A historical but clear example of this was (and is) inappropriate large-scale technology that was deployed in many developing nations where the more appropriate technology was smaller scale, a case that has been well documented by Schumacher and others [134]. In financial markets the key stakeholders have disparate and non-specified value systems, e.g., the values of a large firm, like Goldman Sachs, versus the values of an individual investor versus the values of a government regulator.

A clear fiduciary responsibility should exist in relationships within the financial system; however, the data that show extreme concentrations of wealth in the financial advisory industry [33, 149] and the limited returns of actual investors (less than 4% on average over the last 30 years), demonstrate that the values of all the stakeholders in the current financial systems have not been properly considered.

Life-cycle of the systems

What is the full life cycle that should be considered when designing a system? A well-designed system considers the system from concept to retirement or replacement. The nuclear industry may not have fully considered the retirement notion regarding the disposal of radioactive waste where half-lives are measured in thousands of years. The United States has lagged in the consideration of the life cycle of consumer electronics, creating massive (and toxic) disposal problems with old TVs, appliances, etc. Many European countries have done a considerably better job in this regard requiring manufacturers to have cradle-to-grave management plans for all consumer electronics, i.e. they must take back and recycle all expired electronics [23]. What about the life cycle of financial investments – have they been considered in the design and use of financial products? As investors transition through various phases of life, their needs change. Are these changing needs properly addressed, particularly in regard to the changing environment? Herb Simon, Noble Prize-winning author, noted that many investors do not consider the appropriate time-horizon, nor the relative risk associated with investments (Simon, 1991). Have investment managers considered methods to elicit the appropriate time-horizons of investments from their clients or is their primary objective to simply accumulate assets to manage?

Outscoping and System boundaries

Have all alternatives been considered and was the right system defined, i.e., is the right problem being solved? Within this section two dimensions of systems thinking are considered: Outscoping and System Boundaries. The proper design of a system requires defining the appropriate boundaries of the problem being considered. Different system

boundary definitions can result in different and possibly poor decisions. These dimensions ask whether the problem has been considered broadly enough. For example, if airport security is improved to the point that security inspections discover all possible weapons then the problem has moved elsewhere. Such was the case in Russia where terrorists, unable to circumvent the security checkpoints at the airport, simply detonated explosives in the baggage area where there were no security and many people had gathered since they could not get into the security area. Acting in a similar fashion another terrorist targeted the exit at a concert in Manchester, England, where concert attendees and the parents of concertgoers had gathered as the show ended [113]. If the goal of the system is to protect the system users, then the appropriate boundary must be determined.

Likewise, the problem should be outscoped to ensure that the appropriate problem is being answered. Consider a recent Insurance Institute for Highway Safety (IIHS) study that recommended that teenagers, who tend to be involved in accidents at a higher rate than other classes of drivers, should drive heavier vehicles that fare well in accidents. But is this solving the wrong problem? Putting teenagers in bigger vehicles will likely result in more deaths since accidents involving teenagers with heavier vehicles may result in greater fatalities for those with lighter vehicles. Perhaps a better solution involves increasing the driving age or improving driver education?

In financial investment advising, the mix of stocks, bonds, and cash is often examined; and advice on different combinations is given based on current market conditions and forecasts. Perhaps this should be further outscoped? Should the average investor participate in financial markets at all? Would a more prudent decision be to invest in the real estate market? A fundamental consideration is also the compensation structure for investment managers and the impact such compensation has on risk. Are investment managers being compensated in the manner that encourages the behavior that an investor wants to see [89]? While many of these questions are considered in the investment world, not all players always do so because of conflicting objectives.

Evaluate and Rank Alternatives

Once alternatives have been identified and quantified, they must then be evaluated and compared. Modeling is one of the key tools in systems design, and model-based design has been the new paradigm in systems engineering. Models are used to develop scenarios and rank alternatives; however, the model is typically the starting point of an analysis where the subsequent and most critical work involves testing the validity of the assumptions on which the model was based and examining the sensitivity of the model output to the parameter's values. Sensitivity analysis involves ideas, such as determining the breakeven points (where the ranking of decisions changes) and the key lift points (leverage points), i.e., what parameter modification can be made for the lowest "cost" to generate the best system performance improvement? The performance of portfolio management science (the financial models), including modern portfolio theory (MPT), the efficient market hypothesis (EMH), and Capital Asset Pricing Model (CAPM) has proven to be dismal. Clearly the assumptions of the models are not valid and the sensitivity of the models to parameter values makes them of little value. For example, concepts like the non-stationarity of covariance and the complete lack of the validity of the growth-rate assumptions make the results of Markowitz portfolios for a particular period of time of very little value for investing future periods. Academic research is investigating improvements and generalizations of these models; however, little usable (for practice) results exist. Consider an efficient frontier illustrating the risk/return tradeoffs between equities and bonds from 1960 to 2010. Now, consider each decade as a stand-alone frontier as shown in Figure 1 below. No two decades exhibit similar risk/return profiles, and the decade from 2000-2010 was the only period to exhibit an inverted profile in which it was more profitable to take less risk, e.g. investing more heavily in bonds rather than equities. How meaningful would it be to rank alternatives on the previous decade's results if history has shown that no two are alike [131]? What is needed is a science of portfolio management that works – likely a combination of the ideas from Shiller and Fama [4]. Most models and theories built in finance are based on historical data, thus regulatory activities, such as Basel III, are reactive in nature as opposed to proactive and will therefore, only "fix some of the problems of yesterday but not the crisis of tomorrow" [139].

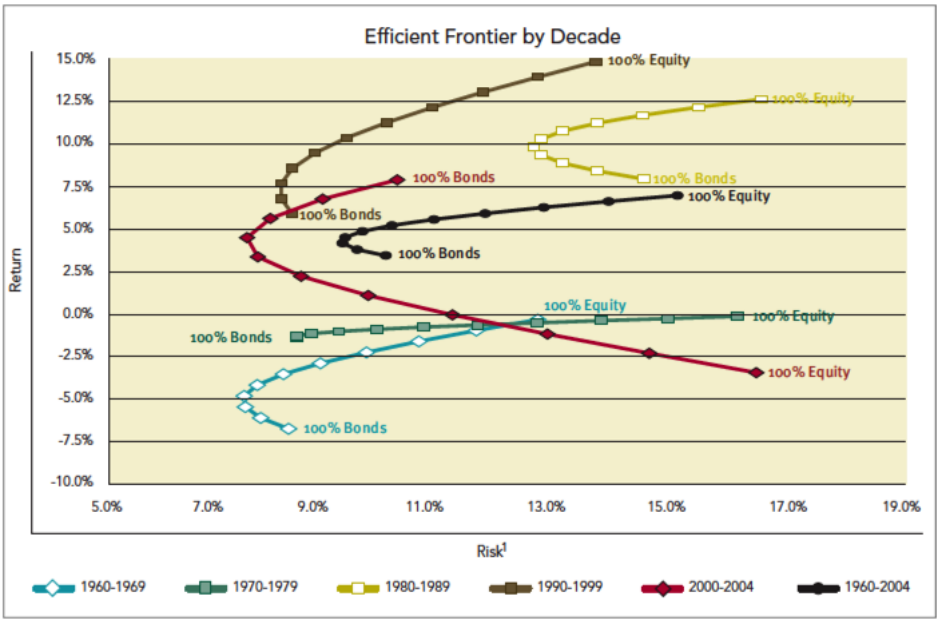


FIGURE 1.2: Organized by decade of efficient frontier of stocks and bonds. Source: Rydex Corporation [131]

Conclusions

This paper presents some basic principles of systems thinking as they relate to system design with attention paid to examining how these principles could be used to design and analyze the financial markets. By discussing the principles of good system design, this paper aims to illustrate the belief that the U.S. financial system is seriously flawed in that many systems thinking aspects have been ignored. Systems thinking in the financial service arena can benefit large players, such as regulators, as well as small, retail investors. The concepts of systems thinking can be applied to the financial system in hopes of developing a financial system that achieves the goals of multiple stakeholders, including those of the future stakeholders, and not just those of large players.

While this is a call for various stakeholders to consider systems thinking and incorporate its ideas when making decisions, it is not merely left to practitioners and regulators. This call also extends to educators and educational institutions who act as the gatekeepers for future generations of players. Universities and professional societies, such as the CFA Institute, wield an enormous amount of influence on the mindsets of future generations

and have an opportunity to provide leadership to those designing financial systems of tomorrow. The authors implore them to consider systems thinking when designing syllabi for the next generation.

1.4 Organization of Dissertation

This research document organized into the following sections:

- Chapter 1: Provides insight into the motivating factors that acted as catalyst behind this research endeavor. The thesis statement of this research in addition to the goals and objectives are stated. Additionally, an essay on the need for systems thinking in the field of financial services and investment management is provided to serve as an additional source of motivation for this research.
- Chapter 2: Background information on traditional finance and investment theory that act as the foundations for this research or derivative theories, which were developed to address short-comings in well-established financial theory in the same manner that this research incorporates engineering and probabilistic techniques in conjunction with traditional financial theory in order to create a new methodology for creating investment portfolios. This section goes on to provide a review of existing financial research academic literature which incorporates the idea of discrete states. It demonstrates the uniqueness of the techniques described as part of this research and how it fits into the existing body of research.
- Chapter 3: Offers an explanation of the various tasks and sub-processes, which are performed as part of this methodology. The section begins with a discussion of the feature set by which observations are described. It goes on to explain the process by which points are assigned to a cluster and state characteristics are determined. The determination of efficient frontiers within each nearly defined state is explained followed by the techniques developed to evaluate the efficacy of the newly created efficient frontiers. Transition dynamics as well as fund creation and methods for

evaluated clustered results. The optimization approaches used to assign weights and design the investment portfolios are discussed.

- Chapter 4: Presents the findings of the research through a series of case studies. This approach to presenting the results was selected due to the large number of permutations in the combinations of possibles parameters that can influence the findings of the research. The aim of the Results sections is show the potential of the proposed methodology.
- Chapter 5: Provides a summary of the research results and discusses their significance. It also details the limitations of this research and offers suggestions for paths and topics of future research efforts.

Chapter 2

Literature Review & Background Material

"If I have seen further than others, it is by standing upon the shoulders of giants."

Isaac Newton

2.1 Traditional Financial Theory

2.1.1 Introduction

Finance represents a field of research in which many of seminal theories were founded on or at least derived inspiration from previous research. This section provides an overview of several of the essential works in financial research. One of these core theories - Modern Portfolio Theory (MPT) - has acted as a motivating work for the research outlined in the thesis, and the core tenants of this theory are found through this research effort. The research of Markowitz represents a masterpiece of research on which numerous researchers have devoted time and energy trying to expand upon it. This research aspires to be such a research effort.

2.1.2 Markowitz Model & Modern Portfolio Theory (MPT)

History & Significance

"Put all your eggs in one basket – and watch that basket!"

Mark Twain, *The Tragedy of Pudd'nhead Wilson*

Mathematical Model

The model assumes that there are n assets in the investment universe. The mean, or expected, rate of return for each asset are $\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_n$, and the corresponding covariances are denoted as σ_{ij} [106]. Each portfolio is defined in terms of allocation, or weights, w to each asset, i = 1, which traditionally must sum to 1; however, research has been conducted into the effect of using leverage in Markowitz portfolios [5].

$$\begin{aligned} \text{minimize} \quad & \sum_{i,j=1}^{n} w_i w_j \sigma_{i,j} \\ \text{subject to} \quad & \sum_{i=1}^{n} w_i \bar{r}_i = \bar{r} \\ & \sum_{i=1}^{n} w_i = 1 \end{aligned} \tag{2.1}$$

Criticism

MPT received considerable attention and acclaim by proposing a novel approach to portfolio allocation in addition to serving as the foundation for other research efforts; however, there was also significant criticism. Markowitz made many assumptions about the behavior of investors and markets in developing his theory and it was argued that the underlying assumptions of MPT were unrealistic. Some of the common areas cited in critiques of MPT include [95]:

- *Investors are approaching portfolio allocation as a mean-variance optimization problem.* The reality is that investors have utility functions on which allocation decisions are

based. For investors to base decisions on the mean-variance optimization, one must assume that the combination of utility function and returns makes the problem similar to a mean-variance optimization one.

- *Investors are rational and risk-averse. This assumption comes from the efficient-market hypothesis.* Behavioral finance has demonstrated that investors are not always rational. It has also shown that decisions can be based on emotions, or "herd behavior", causing varying levels of risk-aversion among individuals. Research in behavioral finance has also shown that the risk-aversion of an individual is dynamic and varies based on the level of gains and losses [90, 151]. Robert Shiller researched market inefficiencies due in part to emotional influences for which he received the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel in 2013 (sometimes referred to as the Nobel Prize in Economics) [140, 141].

- *Correlations between assets are fixed.* Correlations are based on relationships between the underlying assets, and change when these relationships change. Underlying assets do not move in a prescribed and constant manner relative to one another [43]. During periods of financial stress, correlations between assets tend to increase because they decline together [72]. In other words, MPT breaks down precisely when investors are most in need of protection from risk events.

For the purposes of this research, the most relevant critique of Modern Portfolio Theory is the assumption that correlations between assets are fixed and constant. A commonly accepted view on financial markets is that "when the markets are experiencing large negative movements, returns are more highly correlated than returns during more normal times [28]. This phenomenon has been well documented by other researchers and does not require a prolonged discussion; however, periods of market stress are not the only times when market dynamics shift. Underlying asset prices are constantly changing, and as the prices of assets change so do the correlations between assets and the growth expectations for each asset class. As these features change so does the effect of portfolio diversification on risk management.

2.1.3 Capital Asset Pricing Model (CAPM)

The capital asset pricing model (CAPM) is a financial model used to estimate the theoretically required rate of return of an asset and has applications in making decisions about the addition of assets to investment portfolios. The model takes as inputs the following three parameters:

- an asset's sensitivity to systematic, market risk that cannot be removed through diversification often represented by the variable β in the financial industry
- the expected return of the market, $E(R_m)$
- the expected return of a risk-free asset, r_f

When applied to the pricing on individual assets, the security market line (SML) is used to demonstrate how the market should price individual securities in relation to their security risk. The SML enables individuals to calculate the reward-to-risk ratio of a security relative to the overall market. The SML shows the relationship between beta and expected return. As the dependent variable in the relationship, $E(R_i)$ is a function of β.

The intercept is the risk-free rate interest rate available to the market, slope of the SML is the market premium, $E(R_m) - r_f$. The x-axis represents the risk in terms of beta, and the y-axis represents the expected return. The slope of the SML represents the market risk premium. The SML can be viewed as a single-factor asset pricing model, where Beta represents exposure to changes in value of the market.

The relationship can be expressed by the following equation:

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

In 1972 Fischer Black developed another version of the CAPM, called Black CAPM or zero-beta CAPM, which does not consider the existence of a risk-free asset. Black's version was more robust against empirical testing. Fama and French criticized CAPM writing that "the failure of [it] in empirical tests implies that most applications of the model

are invalid" [59].

2.1.4 Efficient-Market Hypothesis (EMH)

Efficient-market hypothesis (EMH) is a theory in financial economics that states that an asset's prices fully reflects all available information. This implies that it is impossible to "beat the market" consistently on a risk-adjusted basis since market prices should react to new information or changes in discount rates because all other information that influences financial markets has already been incorporated into the prices of financial assets.

Professor Eugene Fama, developer of the EMH, argues that stocks always trade at fair value thereby making it impossible for investors to either purchase undervalued stocks or sell stocks overpriced ones. Therefore, it should be impossible for investors to outperform the overall market via superior stock selection techniques or market timing. The only way for investors to obtain higher returns than the overall market is by chance or by purchasing riskier investments.

There are three different forms of the hypothesis that suggest the amount of information incorporated into the pricing of financial assets.

- *Weak:* This version of the EMH states that asset prices reflect all past publicly available information. future prices cannot be predicted by analyzing prices from the past. Excess returns cannot be earned in the long run by using investment strategies based on historical share prices or other historical data. Share prices exhibit no serial dependencies, meaning that there are no "patterns" to asset prices. This implies that future price movements are determined entirely by information not contained in the price series. Hence, prices must follow a random walk.
- *Semi-strong:* implies that the market is efficient, reflecting all publicly available information. This hypothesis assumes that stocks adjust quickly to absorb new information. The semi-strong form EMH also incorporates the weak-form hypothesis.

Given the assumption that stock prices reflect all new available information and investors purchase stocks after this information is released, an investor cannot benefit over and above the market by trading on new information.

The Semi-Strong Form of the EMH argues that, in addition to the historical price data available in the weak form of EMH, public information about a company is available to investors and is incorporated into the current price of the stock. As information becomes publicly available, traders price the underlying immediately to reflect the new information. It's used as an argument against fundamental analysis, and suggests that any public information does not give predictive power on an underlying's future price. This means that performing fundamental analysis or watching the news can't give traders a better ability to predict an underlying's future price.

- *Strong:* The most inclusive variant, it states that all information - market, non-market and inside information - is reflected in asset prices. As such an investor cannot achieve a competitive advantage no matter what information he has and therefore serves as a deterrent to insider trading. This interpretation of market efficiency states that returns in excess of market returns cannot be achieved regardless of an investor's access to information.

Further Work on the Topic

Andrew Lo proposed an extension of this theory known as the Adaptive Market Hypothesis, which attempts to efficient market economic theory with behavioral economics by applying evolutionary principles, such as adaptation, competition and natural selection [102, 104].

2.1.5 French-Fama Factor Models

The French-Fama three factor model is a quantitative model in asset pricing and portfolio management for describing the returns of stocks. The model was developed by University of Chicago professors Eugene Fama and Kenneth French. The three factors considered by the model are 1) market risk, 2) relative performance of large companies

to small ones, and 3) the relative performance of low valuation companies to high ones.

The capital asset pricing model (CAPM) uses one variable, r_m (the returns of the market as a whole), to explain the returns of a stock or a portfolio of stocks. The French-Fama model builds on the CAPM foundation by adding two additional variables, SMB and HML, that attempt to account for a stock's market capitalization and valuation, respectively.

The expected return of a stock, r, is described as

$$r = R_f + \beta(R_m - R_f) + b_s \cdot SMB + b_v \cdot HML + \alpha$$

R_f is the risk-free rate of return, and R_m is the return of the market portfolio. The "three factor" coefficient β is analogous to the traditional coefficient of the same symbol used to describe market risk of a security in CAPM; however, the values are not equal as the French-Fama variant uses to additional factors. The SMB stands for "Small Minus Big" and is a measure of the historic excess returns of small firms over large firms. The HML stands for "High Minus Low" and is a measure of the historic excess returns of high-value stocks over low-value, growth stocks according to the stock, or company's book value. The coefficients b_s and b_v are associated with SMB and HML factors, respectively [56–59].

2.1.6 Two-Fund Theorem

How many assets are needed to include an investment universe? The Two-Fund Theorem addresses this question and suggests that any efficient portfolio can be replicated in terms of mean and variance using only two funds. Specifically, the theorem states that:

> *Two efficient funds or portfolios can be established so that any efficient portfolio can be duplicated, in terms of mean and variance, as a combination of these two. All*

> *investors seeking efficient portfolios need only invest in combinations of these two funds.[106]*

The implication of this theorem are significant. According to this theorem, allocating between two funds, typically described as mutual funds[1], could provide all of the investment services that any investor would ever require. Investors would not need to purchase any individual stocks or bonds, only a portfolio of shares in two mutual funds.

Unfortunately, this conclusion is based on several assumptions, which bring the efficacy of the two-fund theorem into question. First, it assumes that investors are only concerned with mean and variance. Next, the assumption is made that everyone has the same methodology for assessing and evaluating mean, variance and covariance.

2.1.7 Black-Litterman Model

In finance, the Black–Litterman model is a mathematical model for portfolio allocation developed in 1990 at Goldman Sachs by Fischer Black and Robert Litterman, and published in 1992. It seeks to overcome problems that institutional investors have encountered in applying modern portfolio theory in practice. The model starts with the equilibrium assumption that the asset allocation of a representative agent should be proportional to the market values of the available assets, and then modifies that to take into account the 'views' (i.e., the specific opinions about asset returns) of the investor in question to arrive at a bespoke asset allocation [31, 85].

In principle Modern Portfolio Theory (the mean-variance approach of Markowitz) offers a solution to this problem once the expected returns and covariances of the assets are known. While Modern Portfolio Theory is an important theoretical advance, its application has universally encountered a problem: although the covariances of a few assets can

[1] Mutual funds are investment strategies that allow you to pool your money together with other investors to purchase a collection of stocks, bonds, or other securities that might be difficult to recreate on your own. This is often referred to as a portfolio. The price of the mutual fund, also known as its net asset value (NAV), is determined by the total value of the securities in the portfolio, divided by the number of the fund's outstanding shares. *Source: Fidelity*

be adequately estimated, it is difficult to come up with reasonable estimates of expected returns.

Black–Litterman overcame this problem by not requiring the user to input estimates of expected return; instead it assumes that the initial expected returns are whatever is required so that the equilibrium asset allocation is equal to what we observe in the markets. The user is only required to state how his assumptions about expected returns differ from the markets and to state his degree of confidence in the alternative assumptions. From this, the Black–Litterman method computes the desired (mean-variance efficient) asset allocation[16, 17].

2.2 Use of Discrete States in Finance

2.2.1 Introduction

Financial markets are complex systems representing the global interaction of millions of individuals with widely varying objectives whose actions influence the behavior of the financial market system as a whole and by extension the reactionary future behavior of others participation in the system. The complex nature of such a system can result in abrupt changes in behavior, which can have significantly impact participants. For example, consider the drastic change in return, volatility and correlation patterns exhibited in the global stock markets during the global financial crisis of 2008. Many participants suffered financial ruin during this period because they neither anticipated nor understood the scale to which the market behavior would change. Alternatively, others profited because they were able to anticipate the changes in market behavior and respond to the prevailing market conditions. The impact that market behavior can have on the well being of those participating in these market has led people to want to understand and to describe the nature of markets better.

One approach that is becoming increasingly popular to classify, model and describe the behavior of financial markets is the idea of "state". While in finance it goes by many different names, such as regime, cycle, phase or market, the concept is the same - a subset of

the greater whole than is described in terms of key characteristics the values of which differentiate one state from others. There are several reasons that "states" are well suited for applications in the financial markets. First, the notation is natural and intuitive. Individuals already think of the financial market and economy in terms of states whether they are aware of it or not. One of the most well-know examples of state-based applications in the economy pertains to the business cycle. A robust economy is known as an *expansion*; however, a declining or contracting economy is considered to be a *recession*, a distinction made ex post facto [2]. Whether the economy is classified as a recession or not is an example of defining a state as shown in Figure 2.1. Many individuals and researchers think of the equity markets as either advancing or declining, or to use the parlance of the financial markets, to be either a "bull market" or a "bear market." To classify market periods in a binary fashion is to already consider financial markets in terms of discrete states, and as will be shown later in this survey, a two-state structure for identifying states is a very popular research approach.

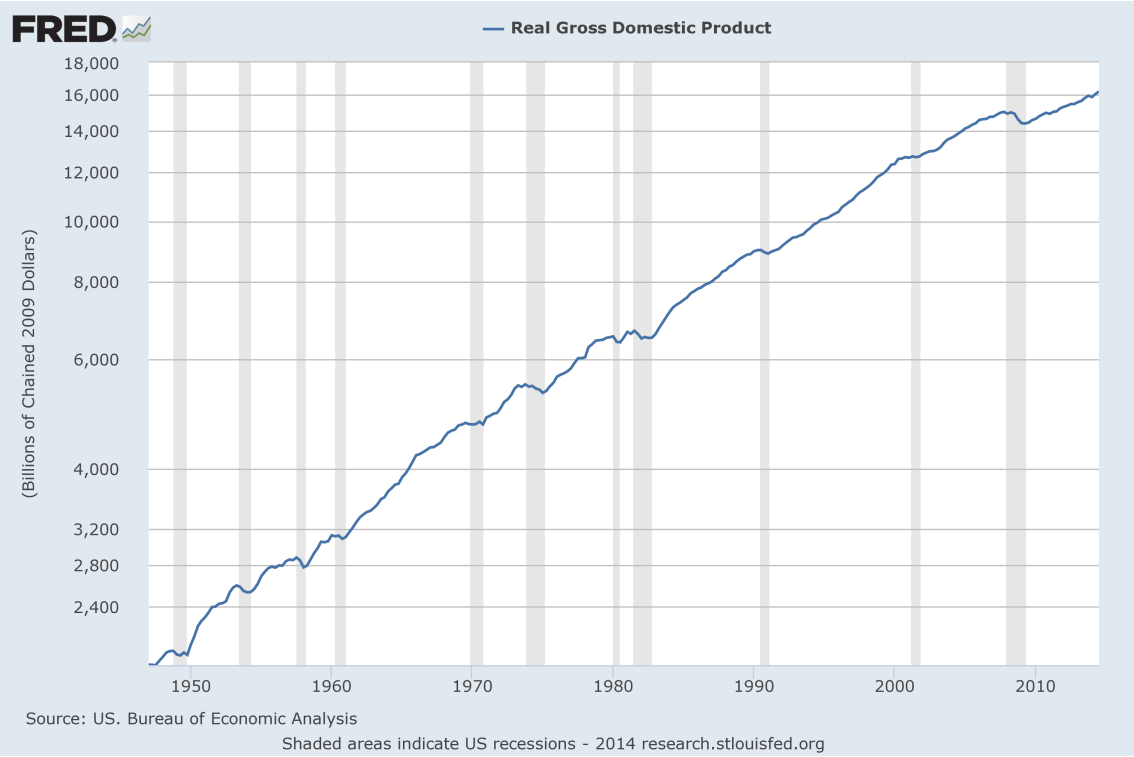


FIGURE 2.1: Periods of recession

[2]The National Bureau of Economic Research (NBER) Business Cycle Dating Committee defines periods of contraction (recessions), as "...a significant decline in economic activity spread across the economy, lasting more than a few months."

Secondly, the use of states can be incorporated into modeling approaches that can capture the dynamic behavior of underlying market characteristics. Figure 2.2 shows the changing relationship between equities and bonds over time. This relationship can significantly impact the amount of risk and volatility assumed when creating a portfolio of assets.

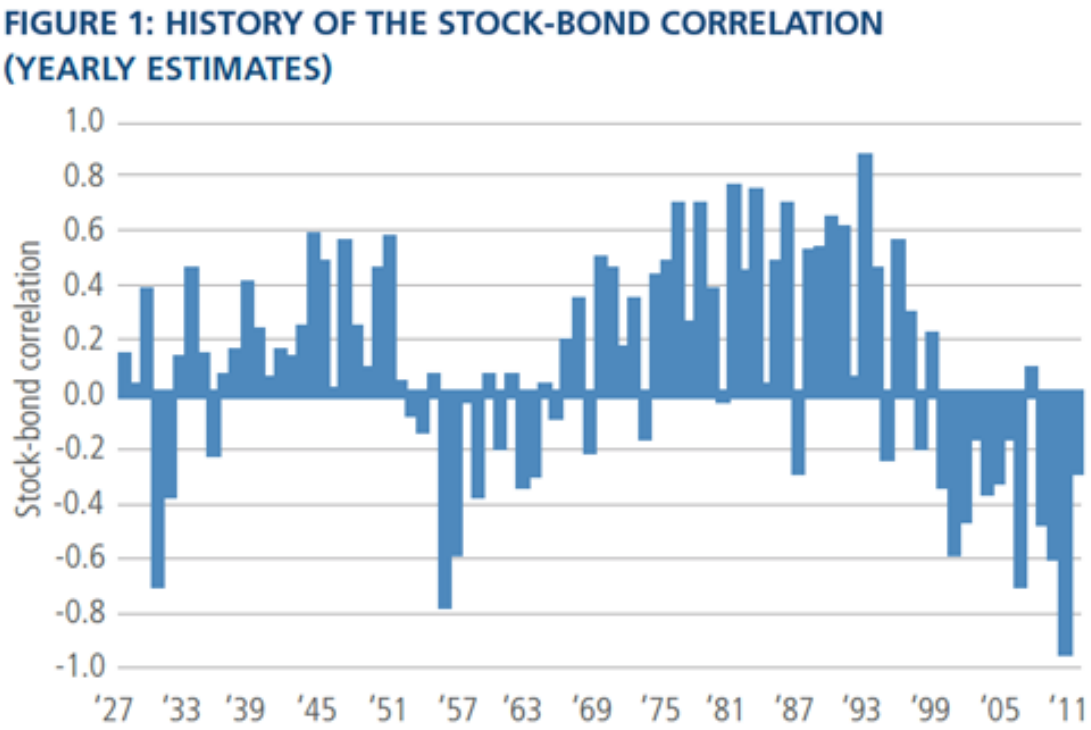


FIGURE 2.2: Annual stock-bond correlations

A great deal of time and effort has been dedicated to studying the financial markets from an analytical and statistical perspective so it would impossible to provide an exhaustive survey of all areas that fall within this domain. For this reason, a subset of the broader domain must be selected as the focus of financial survey papers. Lo (2000) surveyed three of the most well-known and influential ideas in finance: efficient markets, derivative pricing (Black-Scholes), and random walks[3][103]. Atsalakis (2009) reviewed forecasting techniques [6]. Thaler and Barberis (2003) examined the intersection of psychology and finance in the survey of behavioral finance [9]. Creal (2012) reviewed applications for simulation, particularly Monte Carlo methods [41]. Macdonald (1992) focused on economics of exchanges rates while Lane (2001) reviewed macroeconomic factors [99, 108]. Madhaven (2000) looked at the dynamics of exchanges and market microstructure[109].

Cheung (2001) and Ito (1988) reviewed research related to exchange rates[32, 86]. Some financial surveys focused on applied topics for professional publications, such as Swinkel (2004) and Chow (2011) who surveyed momentum strategies and alternatives to traditional indexes, respectively [36, 146]. Phalippou (2007) examined aspects of private equity, and Menkoff (2005) summarizes variations in fund manager strategies [115, 125]. Vanstone (2003) researched the software used to navigate and examine the financial markets [153]. Karpoff (1987) surveyed price discovery and trading research [94]. Carter (1990) reviewed work focused on individual security analysis and portfolios [30].

This survey differentiates itself from the surveys mentioned above by focusing on a smaller subset of literature from a variety of academic journals of varying disciplines with the common element being the use of the notion of a discrete "state," which can be applied in the fields of finance, economics and investment management. There are a wide range of modeling approaches that incorporate discrete states. There are also a wide range of applications to which these techniques can be applied.

2.2.2 Overview of Topic Modeling

The taxonomy of literature with regards to how discrete states are applied in the field of finance and investment management can take many forms. Segmentation may be made on the basis of the models used or modeling methodologies employed. Classifications could also be made based on the financial market to which the models are being applied. Both represent valid and common approaches; however, this survey utilizes a technique known as topic modeling, a sub-specialty from the field of text analysis and natural language processing [18]. Topic modeling applies unsupervised machine learning techniques to organize large corpora (collections of text documents, such as journal articles) and determine themes. This approaches makes organizational decisions without the external influence or bias as to how a collection should be organized [24]. The field of finance has not adopted topic modeling as a method to understand the organization

[3] In his survey Lo presents both sides of the argument for the random walk hypothesis in asset prices; however, the results of Lo and MacKinlay (1988) are presented to reject the hypothesis to a statistically significant degree.

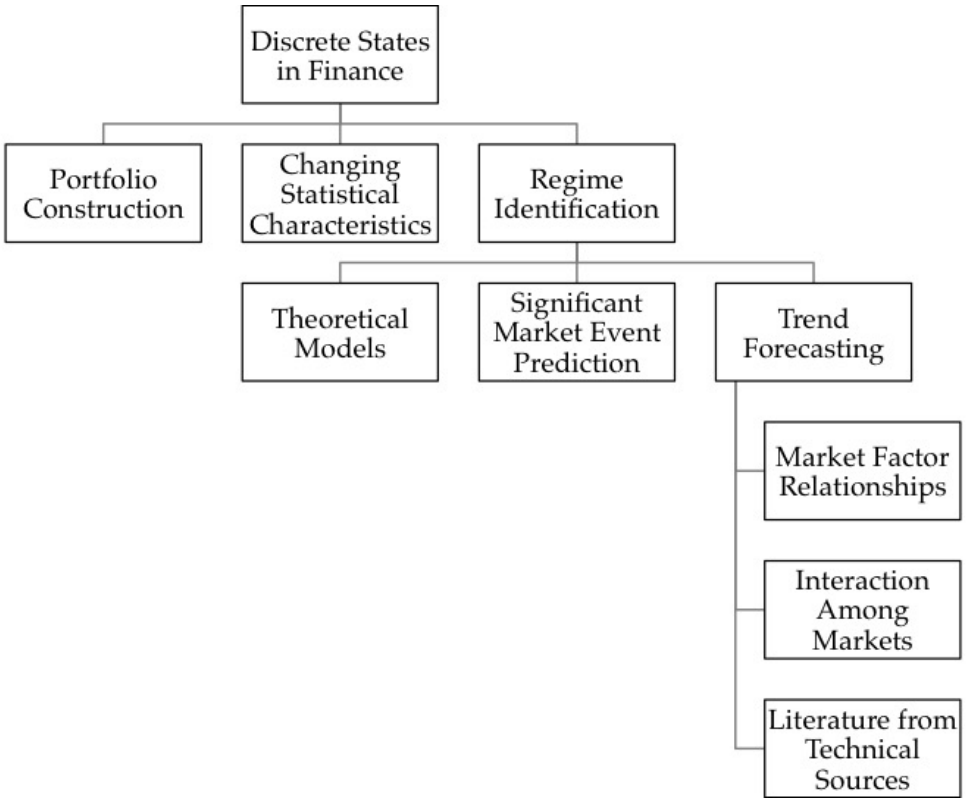


FIGURE 2.3: Organization discrete state literature with applications in finance and economics

of research, but it does offer a unique perspective on the differences and commonalities between various research pursuits.

This survey uses the most common topic modeling technique, latent Dirichlet allocation (LDA), to organize the financial literature [19, 126]. LDA is a probabilities model that takes the text of each document as an input and generates topics based on the frequency and associations of words. Each document is characterized in terms of the words and collection of words it contains. Topics are collections of words weighted or organized by frequency [24]. LDA does not produce the naming conventions. Naming is left to the user to determine based on 1) the most common words associated with a given theme and 2) a probability of each document in the corpus of documents belonging to each topic.

The two inputs required when utilizing a LDA topic modeling approach are the corpus, or collection, of documents to analyze and the number of topics. There are no restrictions on the number of topics, and the decision of the number should be based on the research

objectives for using LDA and the level of detail needed to be observed. This study uses a three topic structure for the LDA. The decision to use fewer topics allows for broader themes in the research literature to be identified.

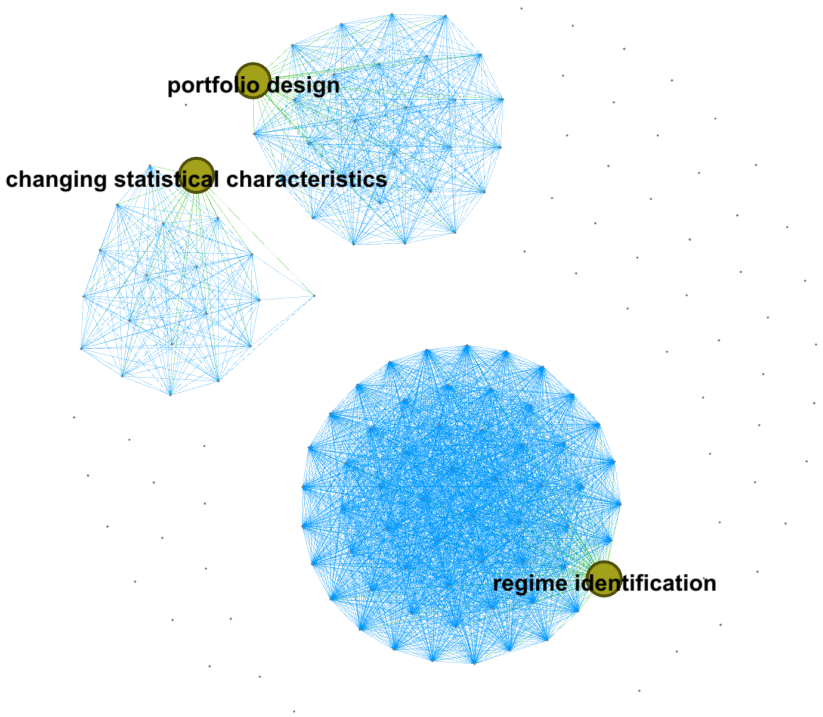


FIGURE 2.4: A three (3) topic LDA model of literature relating to the use states and regimes in finance. A corpus of 167 journal papers was used in the creation of the model.

Portfolio Design

Topic modeling identified *Portfolio Design* as one area of academic interest and focus within the world of discrete states in financial literature. Documents from corpus that lie near the center of this topic involve research, which examines issues of how to best assemble and manage a portfolio of investment assets giving consideration to the market conditions. Common terms in both academic and professional literature for referring to the market conditions are either the "market regime" or "state of the market". Some documents addresses the asset allocation and the proportion of available assets that should be allocated to particular investments. Many of articles near the center of this node come from industry, peer-reviewed publications and present research that is more applied than

theoretical in nature. One such example is Harry Markowitz's work, "Portfolio Selection" (1952), which falls close to the epicenter of this topic. Within this seminal work, Markowitz presents the mean-variance optimization framework for determining asset allocations resulting in portfolios that possess the lowest expected variability for given levels of expected return [114].

The principles of mean-variance optimization in connection with portfolio design presented by Markowitz do not explicitly utilize the concept of a market state or regime; however, it does show that portfolio design should consider two important inputs: a measure of expected growth and a measure of interaction between the assets in the investable universe, e.g. covariance matrix. Differences in these keys features can significantly impact investment decisions in terms of how much to allocate to each asset in the investment universe, which in turn affect the level of expected return and expected risk inherent in an investment portfolio. Discrete markets states offered an adaptable analytical framework for considering different underlying market conditions when constructing an investment portfolio.

Ang and Bekaert (2003) examine the use of discrete states in modeling market regimes for the purposes of making more informed asset allocation decision in the paper "How Do Market Regimes Affect Asset Allocation?". Ang and Bekaert address a known phenomenon in which equity returns, both domestically and international, are more correlated with one another in periods of market decline, often referred to as "bear markets", than at other times through dynamic asset allocation. This statistically significant asymmetry in correlations between market assets during different periods can have a significant impact on the amount of risk inherent in a portfolio [105]. Erb and Harvey (1994) consider the asymmetry of correlations between the equity indices of various countries during different periods of time. Erb and Harvey define states, or regimes, according to business cycles stating that "[e]quity cross-correlations are related to the coherence between business cycles in the respective countries...[c]orrelations are higher during recessions than during growth periods...low when two countries' business cycles are out of phase[53]."

Exploiting market states, or regimes as tends to be the term most often used in financial literature, with different expected returns and correlations is a difficult pursuit; however, Ang and Bekaert employ a regime-switching strategy, which they states that "presence of regimes should be exploitable in an active asset allocation program" and argue that such approaches dominate traditional, static strategies that do not account for regimes. Ang and Bekaert used a popular method in financial literature to account for the periodic asymmetries in market conditions - a regime switching (RS) or Markov switching models. This class of model allows data to be elicited from more than one distribution, which is associated with a particular market state or regime [2]. Their model considers six international, developed markets (insert footnote with specifics) as the investment universe. To build the model, expected excess returns are calculated using a variation on Black's (1972) zero-beta CAPM [87].

$$y_t^w = \mu^w + \sigma^w \epsilon_t^w \tag{2.2}$$

$$\begin{aligned} y_t^j &= (1-\beta^j)\mu^z + \beta^j \mu^w + \beta^j \sigma^w \epsilon_t^w + \bar{\sigma}^j \epsilon_t^j \\ &= \mu^z + \beta^j(\mu^w - \mu^z) + \beta^j \sigma^w \epsilon_t^w + \bar{\sigma}^j \epsilon_t^j \end{aligned} \tag{2.3}$$

Ang and Bekaert introduce a regime variable to their model, which indicates the state of the world market's business cycle - expansion or recession. It is assume that the portfolio manager knows which regime is realized at point in time t but does not know the regime at time $t+1$. It is only known that the regime variables follow a Markov process with constant transition probabilities p and q. With this change to the model, expected returns and variances now dynamically change over time. Expected returns for each of the six markets vary according to their betas relative to regime-based changes to world market expected growth. Conditional variance of the individual assets is more complex and is dependent on three factors: the degree of exposure to systematic risk as defined

by the asset's beta value in conjunction with the world market's regime value, an idiosyncratic volatility term unrelated to its systematic exposure and a "jump component", which arises because the conditional means differ across the regimes [2].

In a previous paper Ang, Bek created a model that also consisted of two market regimes: a normal, baseline regime and a bear market regime differentiated from the other by higher correlations among assets and lower returns [3]. Clarke and de Silva (1998) also employed a two regime model to demonstrate the impact asset allocation [39].

$$p\Sigma_i + (1-p)\Sigma_o \tag{2.4}$$

Chow (1999) also employs a two regime model framework to aid in the portfolio design process. Motivated by a desire to understand the risk inherent in investment portfolios caused by unstable risk parameters. Chow cites emerging market investments and hedge funds as industries that have failed to appreciate the level of risk assumed in portfolios and have experienced volatility and periods of loss as a result. This in response to what he deems "serious limitation of the typical risk-estimation procedure(s)", which treat all observations equally. Estimating parameters in this fashion may produce reasonable estimates for a long-term model, it misrepresents a portfolio's risk attributes during periods of turbulence noting that the "diversification that characterizes the sample on average disappears when it is most needed." Chow cautions long-term investors from using this method because "their portfolios may not survive to generate long-term performance if the portfolios cannot withstand exceptional periods of market turbulence".

To address the issue of accounting for parameter instability during periods of stress and turbulence, Chow segments the population into outliers and usual observations. Outliers are identified as observation that fall outside a multivariate "tolerance ellipse" based on the return distributions of the universe of investible assets. (insert a footnote on the idea

of unequal variances and circle vs ellipse). The author makes separate parameter estimates for the usual and outlier populations and then combines, or "blends", the results. Specifically, portfolios are created using a Markowitz mean-variance optimization (MVO) approach so the parameter estimates consist of (1) a vector of expected returns and (2) a covariance matrix. Estimates for the expected return vector remain unchanged since the focus of the research is on controlling risk not returns. The covariance matrix parameter is modified by combined the matrices from the two regimes - outliers and usual - into a single covariance matrix, which incorporates the probability of each regime occurring by weighing the parameter estimates accordingly. The original expected return estimates and modified covariance matrix serve as the inputs to the mean-variance optimization algorithm in a similar fashion to the technique presented in Chow (1995) [34]. Unfortunately no guidance or insight is offered for calculating the probability of a regime occurring, and the reader should select values that "reflect one's view about the likelihood of each regime and one's attitude toward each regime"[35].

Concerned with the mentality of many investors for considering only the average value of asset returns and correlation between them, Kritzman et al (2010) researched the characteristics of and applications for segmenting periods of time in the financial markets on the basis of "financial turbulence," which is an indication of an asset behaving in an uncharacteristic fashion given historical price behavior [143]. These periods of turbulence can include: extreme price movements, the decoupling of historically correlated assets or the convergence of historically uncorrelated assets. Kritzman employs a financial turbulence calculation developed by Chow (1999), a derivative of the "Mahalanobis distance" [110, 111][35].

$$d = \sqrt{(y-\mu)\Sigma^{-1}(y-\mu)'} \tag{2.5}$$

Mahalanobis intended to use this measure for a very different application than Chow

and Kritzman. Instead of determining periods when financial markets were behaving in an uncharacteristic fashion, Mahalanobis used it to analyze the characteristics of skulls, such as head length and nasal breadth, among members of various tribes and castes in India. At first glance an approach that seems ill-suited for an application to financial markets; however, consider the input and output instead of the domain. The measurement takes an n-dimensional feature set as the input from which population averages and correlations are derived and used to classify group members based on relative positioning within the population's collective feature set. Instead of assigning observations to tribes as Mahalanobis did, Kritzman deems an observation "quiet", making up 75% of the population, or "turbulent", which serve as the state definition methodology in this two-state model. In the paper, Kritzman shows that observations from the turbulent state coincide with periods of turmoil and stress in the financial markets such as Black Monday (1987), the Gulf War (1990-1991), the Russian Default (1998), the Technology ("Dot com") Bubble (2000-2001), 9/11 (2001), and the Financial Crisis (2008)[143].

Kritzman proposes three application for implementing the results of this state definition schema. First, considering the characteristics of turbulent regime allows investors to build portfolios that are better able to resist the turbulence that accompanies such periods. Kritzman employs a variation of mean-variance optimization in which key parameters - expected return vector and covariance matrix - are blended.

Next, the observations from the turbulent state can be used as a compliment to popular value at risk (VaR) method. VaR provides the largest loss a portfolio may sustain within a level of confidence; however, the traditional approach for calculating it uses the full sample covariance matrix to derive the portfolio standard deviation. Kritzman argues that a portfolio's value is most at risk in uncharacteristic, turbulent periods and estimates for the covariance matrix should be made using observations from the turbulent state, when losses are most likely to occur. The consideration for asymmetry in correlations between different regimes echoes the previous sentiments of Ang and Longin [2, 105]. And lastly, the market turbulence measure can act as standalone risk matrix. The paper uses the example of a 30-day smoothing of turbulence, which is assigned to one of five

risk levels. Portfolio risk exposure is then adjusted in accordance with the risk level [143].

Numerous other have used discrete states to research portfolio design applications. Ammann and Verhofen (2006) used Markov Chain Monte Carlo methods to estimate a regime-switching model to be used in a buy-and-hold portfolio choice problem making considerations to investors with varying degrees of risk aversion and investment horizons [1]. Graflund and Nilsson (2003) investigates intertemporal hedging of portfolio and the importance regime plays in the effectiveness of this strategy [73].

Changing Statistical Characteristics

A second subset of research and academic literature identified by the LDA topic model in Figure 2.4 focuses on the study of the differences in parameter values associated with various states and how these differences can lead to notable behavior of that state. This subset is being termed the Changing Statistical Characteristics of regimes. There is, of course, overlap between documents in this subset and those in the previously discussed subset on Portfolio Design. In order to design a portfolio with a certain end in mind, the feature values that influence the state variables must be understood and measure. One such example of this is the correlation among assets and the non-stationary, dynamic nature of them across regimes. Authors previously discussed, such as Chow and Ang [3, 34], understood the impact correlation can have on the volatility of a portfolio and modeled these dynamic features with the explicit goal of improving the construction of portfolios. Research associated with this subset does not focus on applications of portfolio design but on factors that affected the change in value.

In the article "Regime Switching for Dynamic Correlations", Pelletier presents a new multivariate volatility model known as the regime switching dynamic correlation (RSDC) model. Pelletier was motivated by the problems encountered by other multivariate volatility models, such as the necessity for both the variances be positive as well as the variance matrix to be positive semi-definite (PSD) at all points in time [124]. A popular multivariate modeling approach for volatility is Bollerslev's (1990) constant conditional correlation

(CCC) model, which decomposes the covariances of a returns vector into standard deviation and correlation [22]. As the name of the model suggests, Bollerslev created a model based on the assumption that correlations were constant . Under the assumptions of the CCC model, the problems experienced in many multivariate models, such the variance matrix being positive semi-definite, are more easily managed; however, the belief that correlations are constant through time is not an assumption that is supported by empirical data [52].

Pelletier expanded on Bollerslev's CCC theory by introducing the idea of a market state as a means to address the non-stationarity of market correlations. Like Bollerslev, Pelletier decomposes the covariance matrix into standard deviations and correlations; however, a regime-switching model determines the correlation matrices in Pelletier's RSDC model. Correlations vary given the regime, and transitions between regimes are governed by a Markov chain [124]. The CCC model can be considered a single regime version of the RSDC model.

In the paper "Monetary Policy Regime Shifts and the Unusual Behavior of Real Interest Rates", Huizinga et al (1985) employed the idea of discrete states and regimes in a unique way that could be considered counter to many of the methodologies presented in the statistical characteristics of regimes topic area. Instead of attempting to infer the characteristics of a known regime and perform analysis on those characteristics, Huizinga attempted to define the regimes based on the thesis that regimes exhibit similar characteristics and behaviors [84].

Motivated by the rapid climb in interest rates seen in early 1980's, the research investigates changes in the real interest rate processes by identifying "breakpoints", which represent regime boundaries. Huizinga approximates the ex post real interest rates[4]using linear regression equation. Within this framework, the consistency of the linear regression parameters, β, over several samples is used to determine whether a shift in the

stochastic process governing the real interest rates has occurred. The actual determination is made using a conventional F-test [117]. The method for determining the "breakpoints" of the interest rate process, or regime, uses a procedure introduced by Quandt. The approach involves finding the "breakpoints" that maximize the likelihood of the ex post real rate regression. Specifically, for each combination of breakpoints, a calculation is made of the log of the Quandt likelihood ratio, which is the maximized likelihood assuming no breaks divided by the maximized likelihood conditioned on the current set of breaks [127, 128].

A change in the regression parameters represent either 1) a change in the underlying information used for prediction or 2) a change in the relationship between variables in the information set. Either one could result in a change in the underlying stochastic process. Huizinga found "significant shifts in the stochastic process" indicating the presences of multiple regimes in real interest rates. Additionally, "a striking correspondence between the monetary policy regime change and the shift in the real rate process" was observed citing October 1979 and October 1982 as examples, which were also the most likely breakpoints in the regimes [84]. Boivin (2005) also examines monetary policy shifts in response to inflation using regime models [20]. Davig and Gerlach (2006) examined monetary policy in regards to stock market reactions to changes in state [45].

Bear markets represent a well-known, commonly-used means of a state application in the financial markets. Davis (2003) examines the two most recent bear markets in terms of not only the decline in market price but also in macroeconomic trends, financial development, trends within various sectors of the economy, and broader implications of falling market values in the developed markets of G-7 countries[5][46].

[4]The ex post real interest rate is the actual interest rate that occurs over a period of time. It takes into account the purchasing power of the time and is calculated by taking nominal interest rate and subtracting the inflation rate. It differs from the ex ante real interest rate, which serves as a forward-looking figure.

[5]The Group of 7 (G-7) is the group of countries consisting of the United States, Canada, France, Germany, Italy, Japan, and the United Kingdom. These countries represent the seven most advanced economies according to the International Monetary Fund [65]. They are a common surrogate for the developed global economies.

The research comes shortly after the so-called Technology ("Dot com") Bubble and aims at analyzing the underlying economic conditions, which may be viewed as features or state variables, and put the state of price decline in historical perspective with the most recent predecessor. Bear markets are commonly defined as price declines in excess of 20 percent over a period of several years accompanied by pessimism about future prospects. As previously noted, such periods also include an increase in the correlation among assets [105]. Davis examined two bear markets as the subjects of the study: 1973/1974 and 1999/2000[6]. Much of the analysis is description in nature quantifying feature values, such as the degree of decline and peak to trough durations of decline across various markets, in order to develop a historical reference point for the 1999/2000 period.

Davis examines the levels of risk observed within each of period using the econometric modeling framework - generalized autoregressive conditional heteroskedastic (GARCH) [21]. Volatility is the common metric for defining risk, and Davis examines both actual volatility and the expected volatility as determined by the pricing of options in the futures market according to the Black-Scholes Formula, terms to which Davis refers to as unconditional and conditional volatility, respectively. Davis argues that when conditional volatility exceeds unconditional, it indicates the broad market pessimism regarding future prospects, and all of the markets examined exhibited result. Unfortunately, Davis does not mention the lookback period for the calculation of unconditional volatility or the duration into the future (maturity date) of the expected conditional volatility. Davis also examined the levels of inflation, GDP growth, wealth–income ratios, debt levels, and interest rates. Davis concluded that the 1999/2000 bear was "less severe" than the period in mid-1970's because the "macroeconomic background was clearly less severe, with growth broadly maintained and inflation under control" and goes on to warn that "a further leg of share price falls is conceivable... and some features of the current bear market may yet lead to economic and financial difficulties.[46]"

[6]The Market Crash of 1987, or "Black Monday", was excluded from the study because it did not fit the definition of a bear market. Though it (October 19, 1987) was the largest one-day market crash in US stock market history in which the Dow Jones Industrial Average lost 22.6% of its value, or $500 billion dollars of shareholder value, the decline occurred in in a short, concentrated period of time, and the trend was rapidly reversed and the losses recovered.

Schwert (1989) examines monthly stock standard deviation from 1857-1987 and attempts to characterize it using macroeconomic factors, economic activity, financial leverage, and stock trading activity [136]. Like many other, Schwert identifies the Great Depression (1929-1939) as a period of "unusually high" variability, according to Officer (1973), that cannot be fully explained by his model [120]. High financial leverage, as suggested by Merton (1980), explains some of volatility [116]. Other scholars have posited explanations for this excess variability. French and Fama (1988) suggest "mean reversion" arguments, and Shiller (1981) suggests psychological reasons; however, Schwert is "hesitant to cede all this unexplained behavior to social psychologists as evidence off fads or bubbles [57, 58, 136, 142]." In his summary, Schwert acknowledges that a fundamental shift in behavior occurred during the Great Depression and goes on to discuss the notion of regimes in this area of research and potential benefit when he states that

> *Uncertainty about whether the "regime" had changed adds to the fundamental uncertainty reflected in past and future volatility of macroeconomic data. Hamilton's(1988) regime-switching model formalizes this notion [77, 136]*

In subsequent research, Schwert incorporated a regime-switching model to explain the changing nature of volatility and the connection to business cycles and regimes [135].

Regime Identification

The third area identified by the LDA topic modeling falls under the heading of *Regime Identification*. As shown by the network diagram in Figure 2.4, the majority of research surveyed resembled this topic more than either Portfolio Construction or Changing Statistical Characteristics. One of the most well-known and influential articles in the research domain of discrete states in finance lies near the center of this topic area. Hamilton (1989) developed a probability model to determine the future state, or regime, of the U.S. business cycle. Hamilton states very clearly the purpose and methodology of his research. The abstract begins that "[t]his paper proposes a very tractable approach to

modeling changes in regime. The parameters ... are viewed as the outcome of a discrete-state Markov process.[77]"

Hamilton was not the first researcher to study the relationship between the Gross National Product (GNP) and the business cycle. Several researchers studied the prevailing trends using autoregressive moving average (ARMA) models, such as Beveridge and Nelson (1981) and Campbell and Mankiw (1987) [13] [26] [27]. Other researchers searched for unobserved linear components, such as Clark (1987), Harvey (1985), and Stock and Watson (1986) [38] [81] [145]. All of these studies were founded on the assumption that the log difference in GNP was governed by a linear process. Hamilton proposed an alternative approach that based on a stationary, nonlinear process. More specifically, Hamilton was "concerned [with nonlinearties that] arise if the process is subject to discrete shifts in regime - episodes across which the dynamic behavior of the series is markedly different." Discrete states offered a mechanism to model the nonlinear dynamics of the process. Hamilton use a Markov regime switching model to describe the changes to the parameters within the process [77].

A GNP time series dataset from 1952 to 1984 was used to determine the change in economic productivity, and the log difference in quarterly GNP at time t is denoted as y_t. Since the process is assumed to be stationary, first-order Markov model structure as described in Figure 2.5, a maximum likelihood estimator (MLE) was used to estimate the parameters for a four-period autoregressive model (AR(4)), which would be used to calculate the conditional probability of being in a particular regime given a priori knowledge of state membership and the regression inputs, y_t.

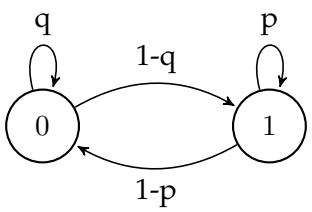


FIGURE 2.5: A first-order Markov representation of the business cycle. $S = 0$ indicates *recession*, and $S = 1$ indicates *expansion*.

The decision rule for determining whether the economy was in a recession at time t is

$P[S_t = 0] > 0.50$. The rule did not provide a substantial amount of insight since, as Hamilton described the results, "the algorithm is usually arriving at a fairly strong conclusion about whether the economy is in a recession" and rarely produced probabilities between 0.30 and 0.70. Hamilton said his algorithm that it "[formalized] the statistical identification of 'turning points'" in time series data[77].

The results of Hamilton's recession classification can be seen in Table 2.1. The estimates were made without the benefit of hindsight and used only information available at time t.

NBER		**Hamilton Est.**[7]	
Peak	**Trough**	**Peak**	**Trough**
1953 Q3	1954 Q2	1953 Q3	1954 Q2
1957 Q3	1958 Q2	1957 Q1	1958 Q1
1960 Q2	1961 Q1	1960 Q2	1960 Q4
1969 Q4	1970 Q4	1969 Q3	1970 Q4
1973 Q4	1975 Q1	1974 Q1	1975 Q1
1980 Q1	1980 Q3	1979 Q2	1980 Q3
1981 Q3	1982 Q4	1981 Q2	1982 Q4

TABLE 2.1: NBER and Hamilton (1989) Dating of U.S. Business Cycle

Previously Hamilton (1988) employed a similar discrete state, Markovian approach to investigate the existence of regimes in interest rates [79]. Later, a new EM algorithm for parameter estimation was introduced [78]. The causes for large shocks to the equity markets, such as the October 1987, was investigated using a regime perspective [80].

Like Hamilton Maheu and McCurdy (2000) applied a regime-switching model to describe the structure of stock returns. This class of model was selected because it "allows for a time-varying conditional mean and variance." Maheu and McCurdy extended the model presented by Hamilton and incorporated a duration component[112].

The duration component, D, is the length of the consecutive occurrences of state S_t, or self transitions, and is defined as

[7] The U.S. economy is classified as being in a recession when the the probability, according to the Hamilton full-sample smoothing method, is greater than 0.50.

$$D(S_t) = \begin{cases} D(S_{t-1}) + 1, & \text{if } S_t = S_{t-1}; \\ 1, & \text{otherwise;} \end{cases}$$

A two-state model was chosen because this practice "accords with a tradition of classifying stock markets into two regimes labeled bull and bear markets." With the state specification chosen, the state transition probabilities are using the logistic regression approach shown in Equation 2.6.

$$P(S_t = i|S_{t-1} = i, D(S_{t-1}) = d) = \frac{exp(\gamma_1(i) + \gamma_2(i)\tau)}{1 + exp(\gamma_1(i) + \gamma_2(i)\tau)} \tag{2.6}$$

where $i = 1, 2$ represents an index of the state, d represents the current duration, $\gamma_1(i)$ and $\gamma_2(i)$ are parameters, and τ is the maximum duration considered in the analysis.

Applying this two-state duration dependent model to a dataset of monthly returns of the U.S. equity index from 1834 to 1995. Results found nonlinear behavior in the market returns and existence of a high return, lower volatility state and a low return, high volatility state, which can be interpreted as a bull and bear state, respectively. It was found that the cause of duration dependence in the transition probabilities came from both the conditional mean and conditional variance values. This dependency led to higher gains in the early stages of bull markets as well as an increase in volatility in bears as the duration increases [112].

Given the number of documents that lie closer to the epicenter of this topic area than others, a more detailed analysis was performed on this topic to examine the subtopic organization in essence creating a hierarchical structure. Using a subset of the overall corpus consisting of those documents which lie closer to the epicenter of Regime Identification topic than the other two, another iteration of LDA topic modeling was performed continuing to use the three topic approach. Yet again, the majority of documents most resembled one topic and two smaller, niche topics were also identified. The results can

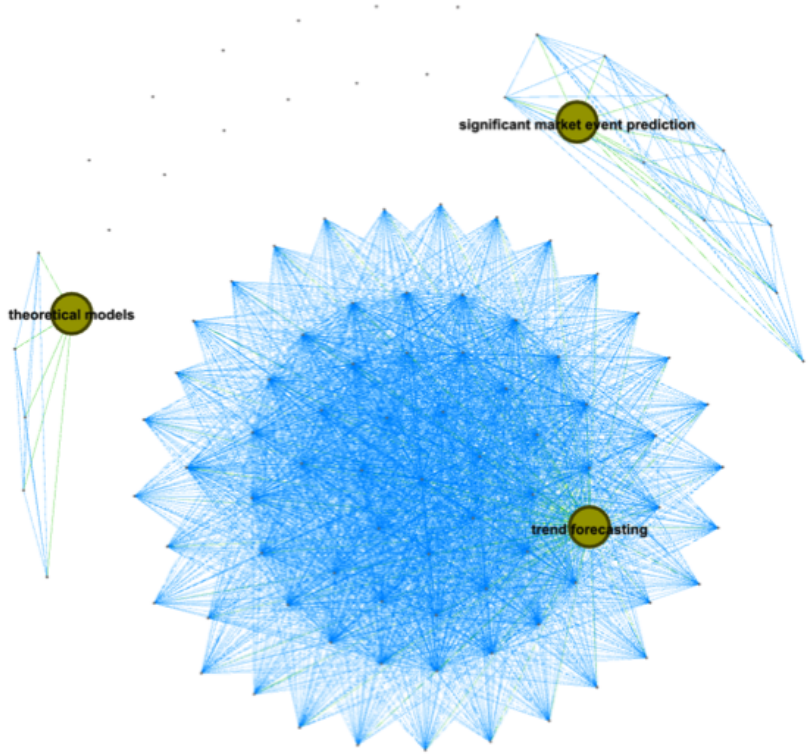


FIGURE 2.6: Subtopic diagram of Regime Identification section

be seen in Figure 2.6.

The first of the two niche topics - *Theoretical Models* - contains documents that are more theoretical in nature and focus less of the application of the research described. One example of such a document is Genon-Catalot et al (2000), which focuses on how stochastic volatility models can be structured as a hidden Markov model (HMM). While there are a wide range of applications for volatility models in the domain of finance, the example cited pertained to theoretical martingale problems [67].

The second of the smaller niche topics - *Significant Market Event Prediction* - contains literature that aims at using states to predict rare but significant events. Van Noreen (1993) employs a regime-switching framework using data from the Toronto Stock Exchange to examine whether deviations from market fundamentals could offer insight into the likelihood of rapid price increases and subsequent price collapse[119]. The model provided both the probability of a regime shift, which indicates the probability of a collapse, and the magnitude of the collapse price collapse.

Yang (2008) used a four-regime double-threshold DTGARCH model to examine the effects of daily movements in the currency and foreign exchange markets on both the equity and currency markets in Taiwan, Singapore, South Korea, Japan and the USA. Linear and nonlinear models are used to examine the reasons for asymmetries in market reactions [159]. The double threshold variables consider the effect of both positive and negative changes in currency, and the various levels of interaction between domestic and international events were considered. The results showed that all markets exhibited an asymmetric response in terms of stock market volatility to information from domestic and foreign markets. Stock markets responded more significantly to negative news from both domestic and foreign exchange markets.

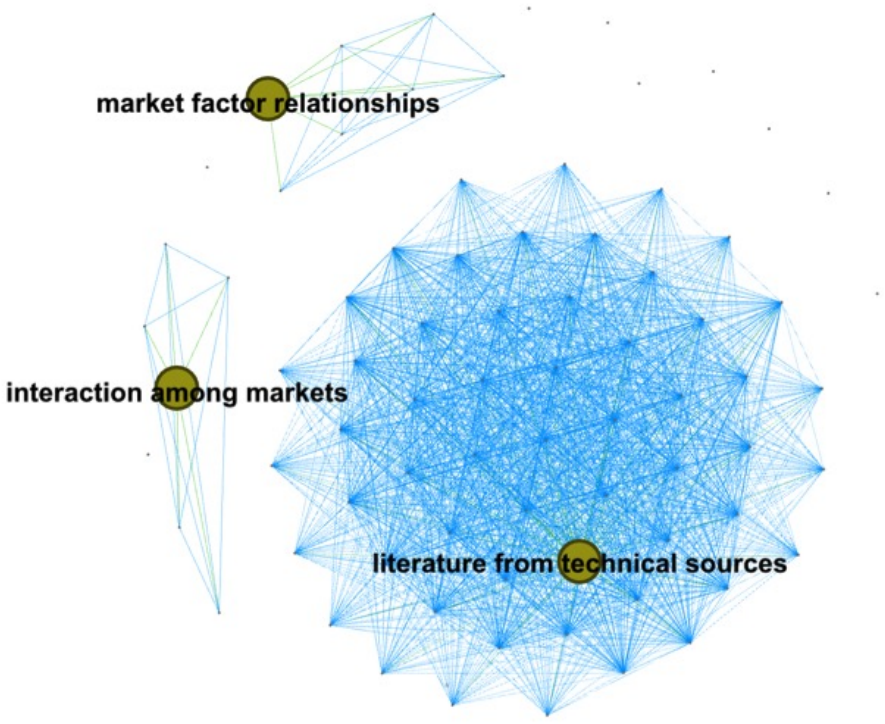


FIGURE 2.7: Subtopic diagram of Trend Forecasting section

Another topic area identified by the LDA modeling revealed a subset of documents that included research from technical sources not traditionally known for research on financial and economic topics; therefore, this topic area is labeled *Literature from Technical Sources*. The technical professional society IEEE (Institute of Electrical and Electronics Engineers)

publishes several technical journals, which on occasion publish the findings of research with application to the financial markets. For example, Yin and Zhou (2004) published in *IEEE Transaction Automated Controls* on portfolio selection with regime switching models. Yin considers a discrete-time version of Markowitz's portfolio selection problem where the markets parameters - vector of asset expected returns and covariance matrix - vary dependent on the regime, or *mode* to use engineering terminology. As the level of granularity and number of regimes increase so does the complexity so a complexity reduction technique is proposed that decomposes transition matrices and uses convergence methods to derive continuous-time mean-variance portfolio problem [160]. From the IEEE Intelligent Systems Design and Applications, Hassan (2005) used a hidden Markov model (HMM) to forecast prices for airline stocks. While the application of HMMs in price forecasting is not new, a variation from traditional techniques is that for various features of interest, the value is forecast and then nearest neighbors are drawn from an empirical dataset to act as the forecast [82]. Tu (2010) examined Bayesian techniques for parameter estimation to be used with regime switching models [150]. Ghezzi (2003) developed a dividend pricing model, which incorporated a Markov chain as the underlying analytical framework [68].

Another area identified through the LDA process, *Interaction Among Markets*, had a subtopic epicenter which consists of several documents that examined the interaction between various markets and impact that such interaction had on the markets. One such paper in this subtopic is Kanas (2003) investigates the validity of and factors affecting the relative purchasing power of a currency, or purchasing power parity (PPP)[8]. The majority of countries exhibited periods during which the real exchange rate is stationary and parity exists; however, there are also periods in which exchange rates are non-stationary, and parity does not hold suggesting that stationarity is regime dependent [91]. A formal statistical test can be performed to determine if Markov regime switching is occurring by using the non-standard Likelihood Ratio bounds test presented in Davies (1987), which examines a null hypothesis of no switching between regimes and an alternative hypothesis of regime switching [44].

The final subtopic is labeled Market Factor Relationships because many of the documents that are most similar to this topic tend to be explanatory in nature and aim to describe the relationship between various features using the regime concept as the underlying analytical framework. One example is Walid (2011) in which the connection between stock market volatility and foreign exchange (FX) markets is investigated. Using weekly data from 1994-2009 from four emerging market countries (Hong Kong, Singapore, Malaysia, Mexico), a Markov switching model is created, which identifies two distinct regimes in each country. In each country instance, the regimes were thematically similar: a high mean, low volatility regime and a low mean, high volatility regime. The characteristics of the two regimes are similar to results previous cited in this paper corresponding to the idea of a bull and bear states. Walid et al also found that the "relationship between the stock and FX markets is regime dependent" and that "stock market volatility reacts asymmetrically to events in the FX market." Lastly, FX changes played a significant role in identifying the stock market regimes[155].

[8]Purchasing power parity (PPP) is a theory that states that exchange rates between currencies are in equilibrium when their purchasing power is the same in each of the two countries. This implies that the exchange rate between two countries should equal the ratio of the two countries' price level of a fixed basket of goods and services. When a country's domestic price level is increasing (i.e., a country experiences inflation), that country's exchange rate must depreciated in order to return to PPP [152].

TABLE 2.2: Taxonomy of State-based Applications in Financial Literature

Decision Scope	Model Employed	Feature Set	Markets	Example Source
Portfolio Construction	Regression	Interest	Real Interest Rates (U.S.)	Huizinga [84]
		Growth	Fixed Income (U.S.)	Doe, Jane [49]
Volatility	HMM	Price	–	Genon-Catalot [67]
Business Cycle	RS	GNP	U.S.	Hamilton [77]
Price Prediction	HMM	Price	Equities (airlines)	Hassan [82]
		Price, Moving Avg.	Equities (U.S.)	Lin [101]
		Price (HLOC)	Equities (U.S.)	Gupta [76]

2.3 Contribution to the Literature

From an LDA perspective this research is most similar to papers in the "Changing Statistical Characteristics" corpus as identified by the MB icon in Figure 2.8. Given the nature and specific application of the technique proposed in this research, it could be assumed that it identify most closely with papers from the "Portfolio Design" corpus; however, the approach to portfolio design and the techniques employed by this research more closely resemble the papers from this subset of academic research.

The largest contribution to the existing body of research is that this research presents an approach to combine a long-stand theory from traditional financial with more contemporary techniques from field of machine learning and from the field of applied math by introducing a Markovian component. The marrying of techniques from various disciplines in an attempt to address the long-standing question of "how to improve investment portfolio returns."

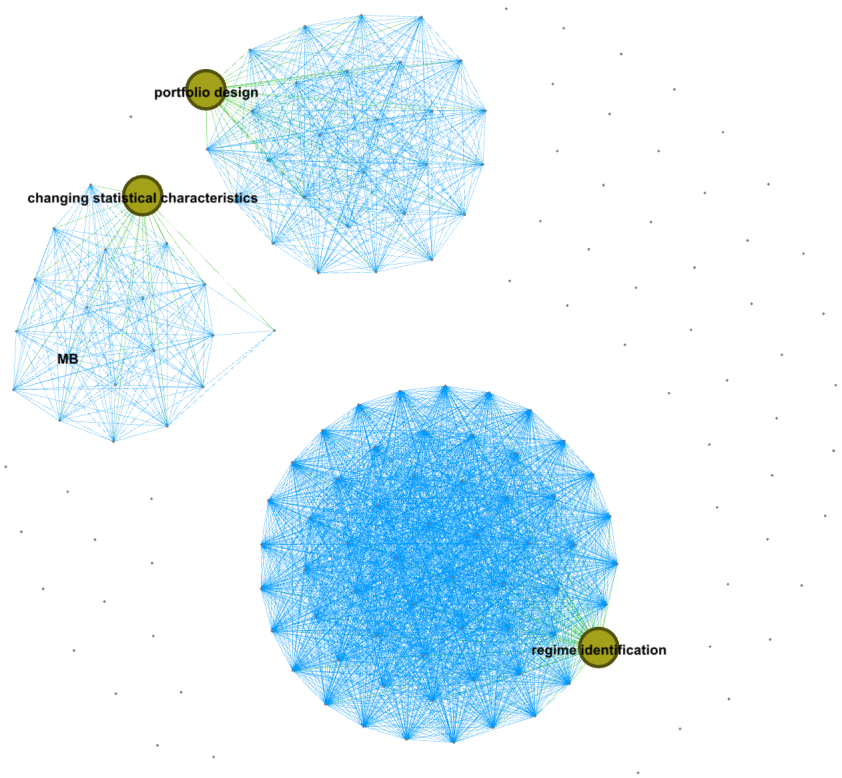


FIGURE 2.8: Topic Model of discrete state models used in financial

The existing paper and approach that resembles this approach most is the work by Guidolin [74], which presents portfolio design conditional on the market regime. That work differs from this research in that 1) it does not take a mean-variance optimization approach to making regime-specific asset allocation decisions, 2) introduce transitional dynamics into the optimization equation, and 3) make regime designations through unsupervised machine learning clustering techniques.

Chapter 3

Methodology

"The true method of knowledge is experiment."

William Blake

3.1 Overview

Financial markets are complex systems representing the global interaction of millions of individuals with widely varying objectives whose actions influence the behavior of the financial market system as a whole and by extension the reactionary future behavior of others participation in the system. The complex nature of such a system can result in abrupt changes in behavior, which can have significantly impact participants. Consider as an example the drastic change in return, volatility and correlation patterns exhibited in the global stock markets during the global financial crisis of 2008. Many participants suffered financial ruin during this period because they neither anticipated nor understood the scale to which the market behavior would change. Alternatively, others profited because they were able to anticipate the changes in market behavior and respond to the prevailing market conditions. The impact that market behavior can have on the well being of those participating in these market has led people to want to understand and to describe the nature of markets better.

One approach that remains popular to classify, model and describe the behavior of financial markets is the idea of "state". While in finance it goes by many different names, such

as regime, cycle, phase or market, the concept is the same - a subset of the greater whole than is described in terms of key characteristics the values of which differentiate one state from others. There are several reasons that "states" are well suited for applications in the financial markets. First, the notation is natural and intuitive. Individuals already think of the financial market and economy in terms of states whether they are aware of it or not. One of the most well-know examples of state-based applications in the economy pertains to the business cycle. A robust economy is known as an *expansion*; however, a declining or contracting economy is considered to be a *recession*, a distinction made ex post facto. Whether the economy is classified as a recession or not is an example of defining a financial market state. Many individuals and researchers think of the equity markets as either advancing or declining, or to use the parlance of the financial markets, to be either a "bull market" or a "bear market." To classify market periods in a binary fashion is to already consider financial markets in terms of discrete states, and as will be shown later in this survey, a two-state structure for identifying states is a very popular research approach.

The highly influential work on the subject, Hamilton (1989) introduces a new approach for modeling the business cycle by examining changes in regime through the use of a discrete-state Markov process. Using a four state model, Hamilton showed the change in underlying regime parameters as an economy shifted from one discrete state to another [77]. When modeling and analyzing a system using a discrete state-based framework, there are several important questions that must be addressed and answered. One of the most important of these asks whether the discrete states of this system recurrent and persistent? This paper attempts to provide some insight into this question by proposing a potential feature set for use in segmented periods of time into discrete financial market states, discussing techniques for segmenting periods into market states using the aforementioned feature set, and examining methods for comparing states to determining whether similar periods of time recur in market history.

3.2 Data

The analysis and examples incorporated into this section of the paper is based on daily price and yield data for seven investment assets representing the major US equity indices and bonds from 1979 to 2018. Four domestic equity indices of various market capitalization are included: S& P 500, NASDAQ Composite, Dow Jones Industrial Average, and the Russell 2000. The three bond products, which represent both public and corporate bonds, are: 10-year Treasury, Moody's AAA, and Moody's BAA [1].

3.3 Market State Measures

The intended use of any state-based model must be a primary consideration when selecting the feature set from which states will be created and the associated feature set must be selected with this objective in mind [132]. It needs to capture the key characteristics that differentiate states from one another in a meaningful way. Since the intended use of this methodology is to offer insight into the intelligent design of investment portfolios, the choice of features must support the intended use by providing information about the metrics of interest. Investment portfolios are commonly evaluated in terms of how well the portfolio balances the tradeoff between potential risk and potential reward. Harry Markowitz (1952) proposed a formal, scientific approach to address this problem in the seminal work, *Portfolio Selection* [114]. Markowitz introduces a mean-variance optimization (MVO) model that yields the optimal portfolio for every expected return level possible for a given universe of assets. Markowitz believes that the optimal portfolio for any level of expected return is the one with the least risk, which is commonly measured in term of volatility, specifically standard deviation. His MVO model has two parameters - a growth vector of the expected returns, r, of the assets in the universe of investments and the covariance matrix of asset returns, Σ. The growth vector affects the expected return aspect, and the covariance matrix affects the expected risk aspects represented in terms of volatility, or uncertainty, because the risk of a portfolio is a function of both the amount allocated to each asset and co-movement of the assets.

[1] The source of the data is the Federal Reserve Economic Data (FRED) https://fred.stlouisfed.org

The parameters of Markowitz's MVO model serve as the foundation for our features selected to define market states; therefore, covariance acts as a measure of risk, and expected growth acts as a measure of return. Clustering methods use this feature set to segment the financial markets into states. These features - expected growth vector and covariance matrix - are multidimensional measures that each need to be reduced to a single number to be used by the clustering algorithms. The intended application of a state-based model determines the features by which states are defined. Researchers have used a variety of features in the state definition process: Neftci (1984) used unemployment rates [118], Hamilton (1989) used GNP data [77], Ghezzi and Picardi (2003) applied [69], Badge (2012) examined macro-economic factors [7], Owyang and Ramey (2004) used elements of monetary policy [121], Guidolin and Timmerman (2007) uses equity and bond returns to classify market cycles as 'crash', 'slow growth', 'bull' and 'recovery' states [75], and Whitelaw (2000) uses expected returns and volatility [158]. This research presents a unique perspective in terms of the feature set used to define states by considering elements of the covariance matrix. The covariance matrix reflects the co-movement of assets and, as Markowitz's MVO model illustrates, impacts the volatility characteristics of a portfolio. Given that this research aims to support more intelligent portfolio design and the influence on overall portfolio volatility that the covariance matrix can have was the reason for using a covariance-based feature in the state definition process.

The first feature measures the expected return, or growth, vector over n periods of time and condenses the aggregate growth into a single number by considering the expected returns of all the individual component assets and representing the expected growth vector as a magnitude of growth for all the assets. The look-back parameter n can be adjusted at the discretion of the researcher based on the level of data granularity and the intended use of the analysis results. Equation 3.1 shows that the magnitude of growth is calculated using the Euclidean distance from the zero-return origin and represents the intensity of asset returns over the period of observation.

$$m(\mathbf{r}) = \sqrt{\sum_{i=1}^{n} \mathbf{r_i}^2} \tag{3.1}$$

As shown in Equation 3.1, the magnitude measure, m, does not reflect if the assets are moving in an increasing, positive manner or in a decreasing, negative one so a directional component, d, is introduced to account for the market direction. The addition of the component d is intended to capture the overall directional trend of asset returns in $\mathbf{r}$. To capture the prevailing directional trend of prices in $\mathbf{r}$, the mean of returns of $\mathbf{r}$ is determined. Equation 3.2 shows that the variable d is directional flag indicating whether the mean return is positive or negative by assuming a value of 1 or -1, respectively.

$$d(\mathbf{r}) = \begin{cases} 1 & \text{if } \frac{1}{n}\sum_{i=1}^{n} \mathbf{r_i} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.2}$$

The product of results from Equations 3.1 and 3.2 lead to the return growth measure, $g(\mathbf{r})$, shown in Equation 3.3. The growth measure is a discontinuous function that experiences 'jumps' in the plot of the function when the sign of d changes.

$$g(\mathbf{r}) = d(\mathbf{r}) * m(\mathbf{r}) \tag{3.3}$$

Each of the sub-components of the expected growth measure are shown in Figure 3.1. The magnitude value is plotted, and the value of the directional flag is overlaid in red. The additional of the directional flag results in gap between the elements of the sub-populations which have different values as shown in Figure 3.2.

The second feature, which measures the covariance matrix of assets in the investment universe, reduces the n by n covariance matrix to a single number that can differentiate between periods of time in terms of the co-movement of assets. A common approach for

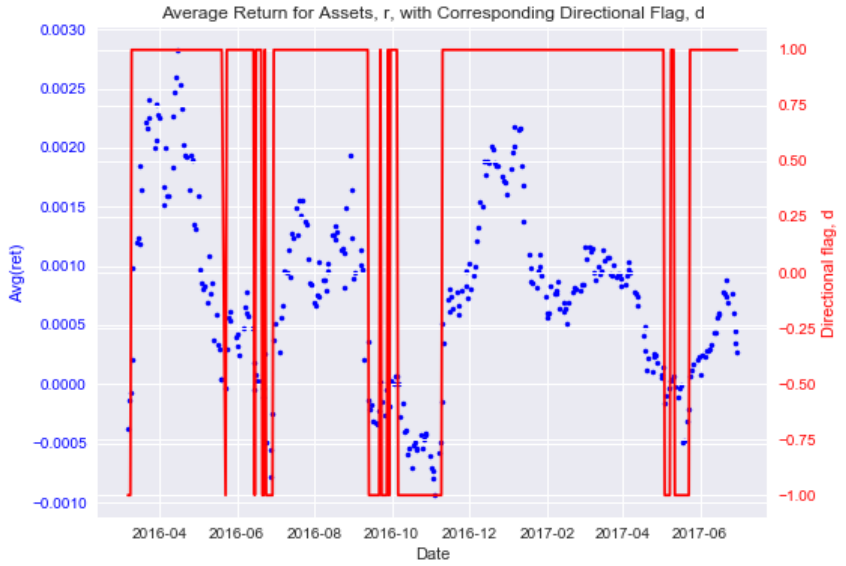


FIGURE 3.1: Expected Return Feature, $g(\mathbf{r})$: Average return for assets in r. The directional flag, d, indicates whether the average return is greater than zero or not.

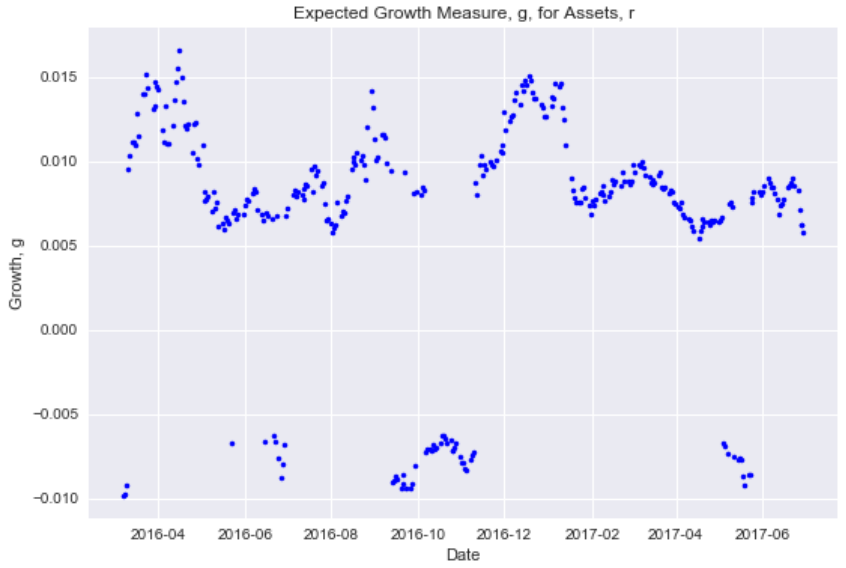


FIGURE 3.2: The feature $g(\mathbf{r})$ incorporates a directional component $d(\mathbf{r})$ and a magnitude vector $m(\mathbf{r})$.

quantifying a matrix, particularly a covariance matrix, is by calculating the distance between a base convariance matrix, $\mathbf{A}$, acting as a reference point and a covariance matrix of interest, $\mathbf{B}$. The base matrix may be selected for a variety of reasons, such as representing values of interest or an initial, starting state or values simply selected at random. This research applies a technique presented by Förstner (1999), which also calculates the

distance to a base matrix, and is presented in Equation 3.4 [63].

$$d_{\mathrm{F}}(\mathbf{A},\mathbf{B}) = \sqrt{\sum_{i=1}^{n} \ln^2 \lambda_i(\mathbf{A},\mathbf{B})} \tag{3.4}$$

where $\lambda_i(\mathbf{A},\mathbf{B})$ is the joint eigenvalue of the matrices[2].

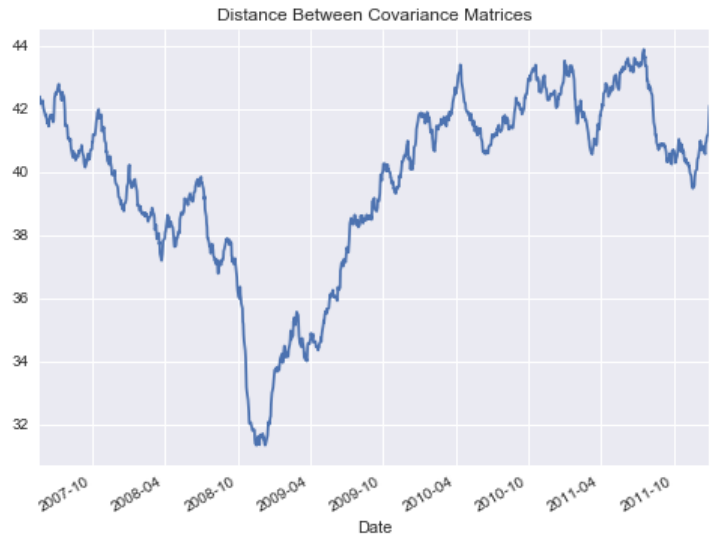


FIGURE 3.3: An example of the covariance distance described in Equation 3.4.

As mentioned previously the calculation for d_{F} is made relative from a matrix of interest, $\mathbf{A}$, to a static, base covariance matrix, $\mathbf{B}$. Since all distances are affected by the choice of baseline matrix $\mathbf{A}$, there is a concern regarding the degree to which this selection affects the distance calculation. This question was examined by taking two periods [3]that represent distinctive periods of market behavior and plotting the distance between the same $\mathbf{A}$ matrix. Two baseline matrices, $\mathbf{B_1}$ and $\mathbf{B_2}$, are calculated using all observations from Q1-Q2 2008 and Q3-Q4 2008, respectively. The $\mathbf{A}$ matrix incorporates a 22-day lookback period. This means that for each point in time t from the examination period (2013-2014)

[2]The covariance distance calculation described in Equation 3.4 requires that matrix B be positive definite in order for the calculation to be made. If this is not the case, the calculation cannot be made. Situation in which this is most common are when 1) a large number of assets are used to create the matrix and 2) a short look back period is used to calculate the matrix.

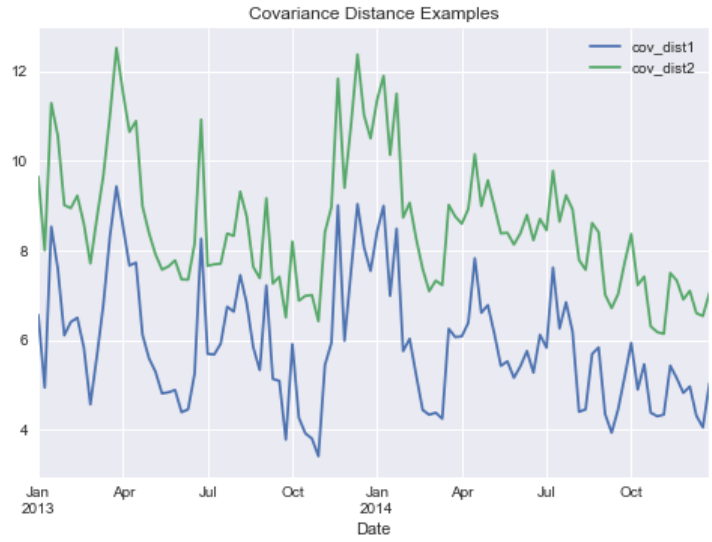


FIGURE 3.4: Impact of different baseline matrices (2008: Q1-Q2, Q3-Q4) in covariance distance calculation.

the previous 22 observations are used to calculate the covariance matrix. The distance between $\mathbf{A_t}$ and $\mathbf{B_1}$ and then between $\mathbf{A_t}$ and $\mathbf{B_2}$ is calculated. The examination window shifts one period, and $\mathbf{A}$ is recalculated using the observation from time *t+1* and the previous 22 periods. The distances between $\mathbf{A_t}$ and $\mathbf{B_2}$ are then recalculated and plotted. As shown in Figure 3.4, the selection of a base covariance matrix does affect the distance calculation; however, this effect is constant between points and can also be viewed as a vertical shift of distance plots. The constant difference in distance reduces the uncertainty and variability associated with this calculation as a quantifiable measure of the covariance matrix.

3.4 Determining Market States

3.4.1 Assignment of the Training Set and Cluster Creation

The overall dataset must be subdivided into two smaller subsets: a training set which will be used to create the market states and a holdout, testing set which will be used to

[3]The first half and second half of 2008 represent distinctly different periods of time in financial market history. Following the decline of global equity markets in Q4 2007, the many markets did not experience large gain or losses in the first half (Q1, Q2) of 2008; however, in the second half (Q3, Q4), markets worldwide experienced dramatic losses.

evaluate the state schema created on the training data. The selection of training and testing periods is a key step in the modeling process. Bailey et al. analyzed the problem of overfitting back-tested results in the context of investment returns. The paper presents that traditional statistical techniques designed to prevent or at least minimize regression overfitting are ill-suited and unreliable when applied to investment models and suggests the need to apply non-standard methods [8]. For the clustering methodology to be robust enough to be applied in periods beyond the training period and thereby prevent overfitting, the range of values across the list of features in the training set should encompass the range of values in the testing set. Figure 3.5 illustrates the observations from the training and testing sets to show whether the training region is representative of the testing region, which speaks to the robustness of the modeling approach. For this example, which includes observations of domestic market equities, 43% of all observations from the testing region (12/7/2007 - 3/6/2009) fall within the range of values from the training set (12/6/2006 - 9/25/2007). Differences are also examined based on the gaps created by the d component of the growth vector, g, calculated in Equation 3.2, which determines the sign of the vector. The percent of testing values that fall within the range of training values for positive and negative g values is 45% and 29%, respectively. The boundaries of the training and testing regions shown in this example should be reconsidered because observations with low covariance distance values and large growth magnitudes in the testing set fall outsides the boundaries of observation within the testing set. The modeling schema presented does not consider data points within this range of values and is therefore not correctly calibrated to address such data points properly.

Each observation can be quantified in terms of the two features of interest described in the previous section. Using these features, states within the financial markets can be created by grouping comparable observations. Many techniques exist for creating groups, but this research applies unsupervised clustering algorithms from the field of machine learning. Differences in the groups created by the various algorithms is a function of the methodology applied by the algorithm as well as the inputs and even randomness [130]. Many unsupervised learning approaches could be used to create states, such as DBSCAN, spectral methods, or Birch's; however, this section of research presents results

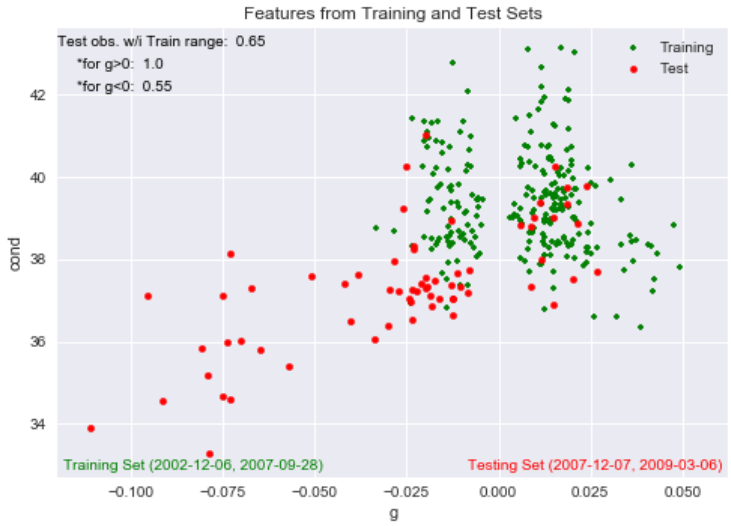


FIGURE 3.5: A comparison of observations from the training and test sets. Periods from the test region should fall within the bounds of training set observations. If an insufficient number of observations fall within these bounds then the training and testing regions should be reconsidered.

created using the k-means algorithm. A selection of algorithms considered is listed in Table G.1. Some of the algorithms take as a parameter the number of clusters, or states, into which the observations are assigned. The number of groups is commonly referred to by the variable k. Other algorithms do not take k as a parameter and make a determination of the number of cluster as part of the clustering process. The methods that receive k as an input are listed first and denoted in the column titled *Parameter* k in Table G.1. The k-means algorithm was selected because it is a well-known and commonly used algorithm that produced similar clustering results to other potential methodologies as shown in Figures 3.6, 3.7, 3.8, and 3.9.

Prior to the application of a clustering algorithm, the issue of scale must addressed by placing all points on the same scale. Features exist on a variety of size scales, and cluster assignments are biased towards the feature with the larger scale. To account for the differences in scale of the growth and covariance features, the data used to create the clusters must first be normalized. A common method for normalizing data sets is to calculate the number of standard deviations an observation is from the mean and represent the value in those terms according to Equation 3.5.

Method	Parameter k	Description
k-Means	Yes	• Separated samples into k groups by minimizing a the intra-cluster sum-of-squares.
Spectral	Yes	• Observations considered nodes in an inter-connected graph. Clusters are created by partitioning the graph into subgraphs based on its spectral decomposition.
Birch	Yes	• A hierarchical clustering approach well-suited for a large data-sets. Creates cluster trees from which features are created. Useful in dynamic, incremental decision-making processes.
Agglomerative	Yes	• Performs a bottoms-up, hierarchical clustering using a bottom up approach. Each observations initiates a cluster, and clusters are then successively merged. The metric for determining connection between clusters taken as a parameter.
DBSCAN	No	• Density-based clustering algorithm that groups together points in high-density regions and views observations in low-density areas as outliers. Takes the minimum number of clusters and a density concentration factor as inputs.
Mean Shift	No	• A centroid-based algorithm that iterates through candidates for the centroid of observations for a given region. A subsequent step removes duplicate or near-duplicate centroids prior to producing a final centroid set.

TABLE 3.1: Summary of Clustering Techniques [123]

$$x^* = \frac{x - \bar{x}}{s_x} \tag{3.5}$$

where $\bar{x}$ and s_x are the sample mean and standard deviation, respectively, of the population of the feature x.

Once the data has been normalized, a clustering algorithm can be applied without bias to feature size. Examples of results from four (4) algorithms are presented in Figure **??** to show a sample range of results based on technique. All four methods required k as an input so a value of $k = 4$ was provided to each.

The optimal numbers of clusters k will vary based on the problem. For the purposes of this article, a value of $k = 4$ was selected to provide a clustering solution that is general enough demonstrate the overall concept while providing enough detail as to illustrate pertinent differences in the clustering solutions presented. The methodology for selecting the value of k falls outside the scope of this article; however, this question lies at the center of creating an effective clustering methodology. Comparable results were achieved by three of the four algorithms; however, one method - Agglomerative Method - produced a single, dominant cluster comprising the vast majority of observations with the remaining clusters being sparsely populated (Figure 3.9). The results for the Agglomerative method show that it is not well-suited in this case and was removed from consideration; however, given the similarities in results of the other three methods, one can be used as an illustrative example. The k-means algorithm is a well-known, commonly-used method that produced results like those of the other two methods and therefore will be used as the example algorithm.

Applying a clustering algorithm to the training set, each daily observation consisting of the n-day growth vector magnitude and n-day covariance matrix distance pair is assigned to a specific state. Figure 3.10 shows the results of this sample clustering schema, which indicates the state to which each observation from the training set was assigned. Daily observations from the training set are assigned to one specific state because these cluster assignments define the framework for the overall state-based schema and subsequently acts as the foundation on which all future clustering decisions are based. Observations from the training set are not assigned absolutely to a cluster. Assignments from

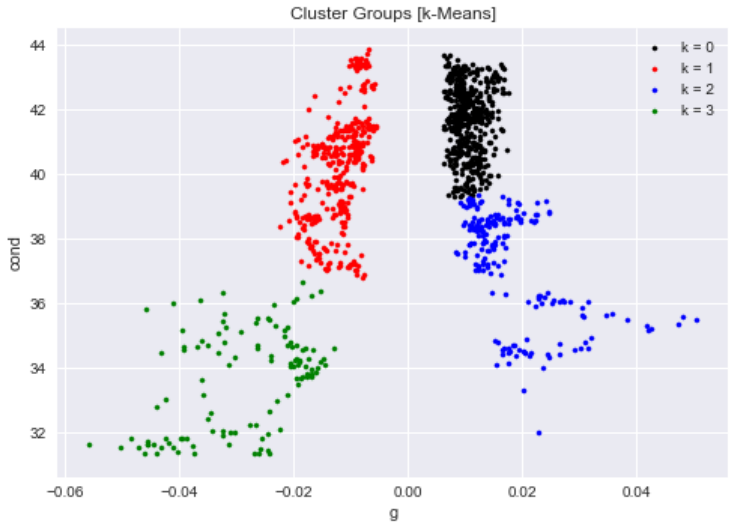


FIGURE 3.6: Results for k-Means. Similar to those in 3.7 and 3.8.

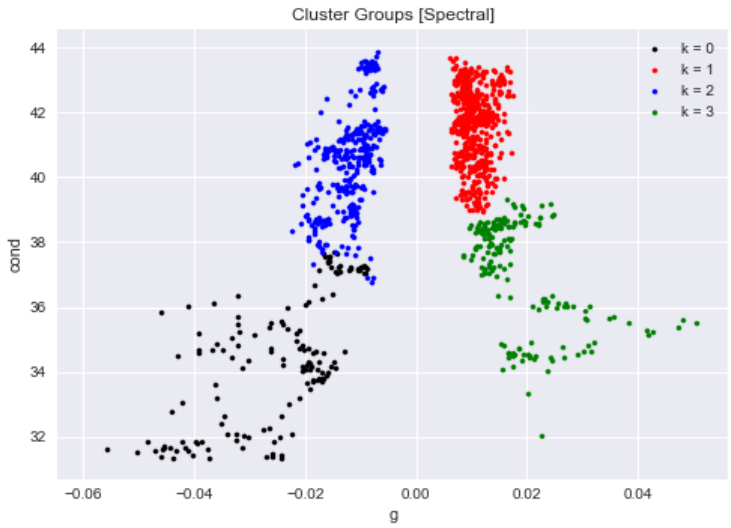


FIGURE 3.7: Spectral clustering results. Similar to those seen in 3.6 and 3.8.

observation in this set are made probabilistically by considering the likelihood of membership in each of the various clusters.

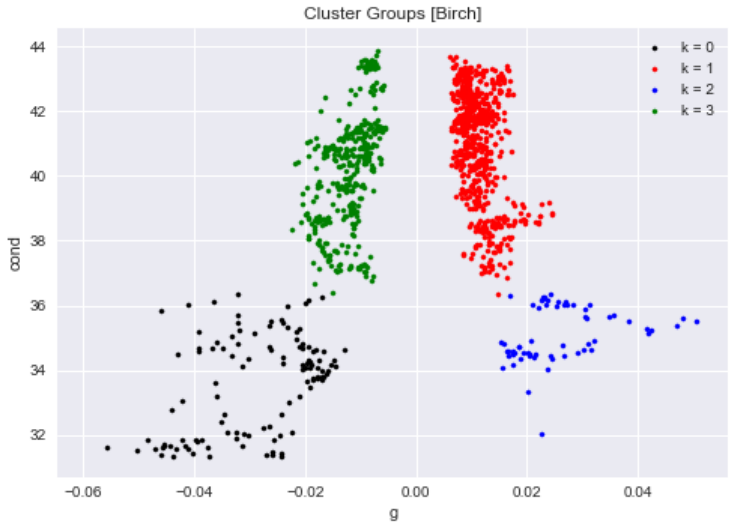


FIGURE 3.8: Results for Birch's Method. Similar to results seen in 3.6 and 3.7.

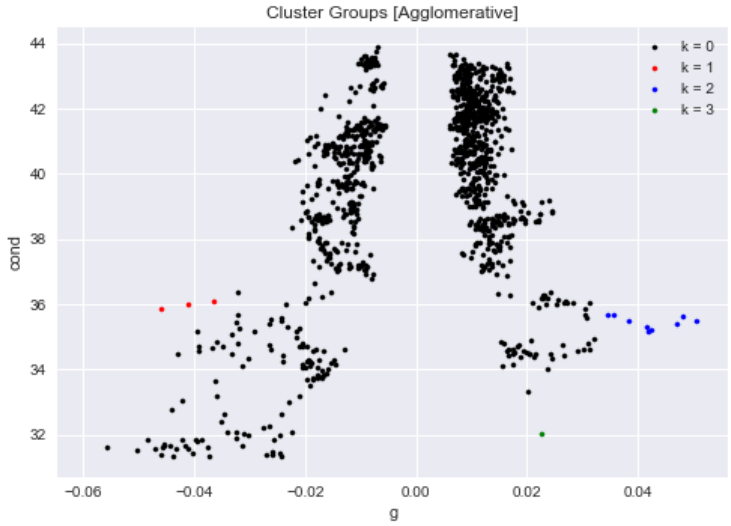


FIGURE 3.9: Agglomerative Method results are unlike others. Algorithm created one dominant cluster.

Probabilistic Assignment of Observations in the Testing Set

Unlike observations in the training set, which are assigned to a specific cluster (see Figure 3.10), the observations in the testing set are not explicitly assigned membership in a particular cluster. Instead observations from the testing set are given a likelihood of membership a state based on similarities to the 'typical' state member, which is expressed as the state's centroid. Therefore the distance between an observation x^* and the centroid[4]for

FIGURE 3.10: The assignment of observations to states in the training using the k-means algorithm.

cluster k, $x^{*(k)}$, can be calculated by the formula:

$$d_k(x^*, x^{*(k)}) = \sqrt{\sum_{j=0}^{n_j} (x_j^* - x_j^{*(k)})^2} \tag{3.6}$$

where j is the feature, and n_j is the total number of features used to describe the state.

Throughout this paper, two features - growth magnitude vector (g) and covariance distance (cov) - are used as the features of interest to create states.

The specific distance formula for this example is:

$$d_k(x^*, x^{*(k)}) = \sqrt{(x_g^* - x_g^{*(k)})^2 + (x_{cov}^* - x_{cov}^{*(k)})^2} \tag{3.7}$$

[4]The centroid is a way to represent the point at the center of a cluster. It is a measure of central tendency of each feature for the members of a cluster. It can be derived by calculating the arithmetic mean of each feature. [96]

The distance measure d_k from Equation 3.7 serves as the sole input parameter for determining state probability membership; however, the probability that an observation is a member of any given state is inversely related to its distance from the cluster's centroid. The closer an observation is to a cluster centroid the greater the degree of similarity to the typical member of the cluster. In terms of the variable d_k, an observation with a lower distance value to cluster k has more in common with the members of that cluster than an observation with a higher d_k and therefore has a higher probability of membership in that state. The inverse d_k ratio must be normalized as shown in Equation 3.8 to transform the ratio into a membership probability. The relationship between distance to a cluster can be expressed as:

$$p_k = \frac{d_k^{-1}}{\sum_{i=0}^{n} d_i^{-1}} \tag{3.8}$$

Using the relationship from Equation 3.8, the probability of membership in each state for observations in the testing set is shown in Figure 3.11.

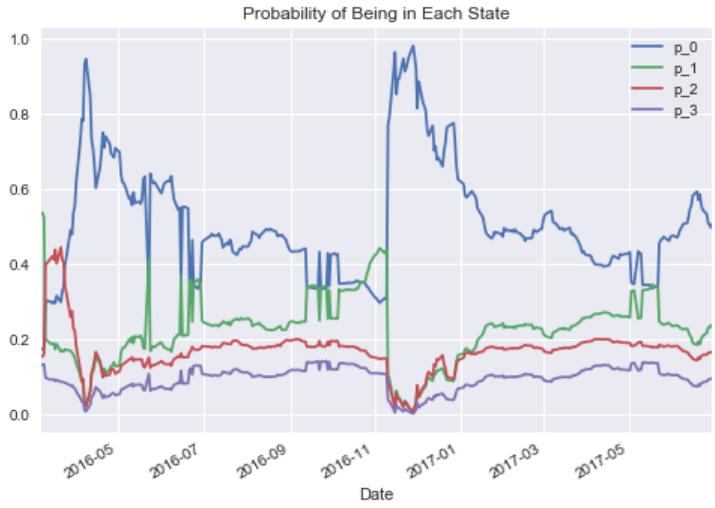


FIGURE 3.11: Using the clusters created using training data, a probabilistic estimate of cluster membership is made for the observations from the holdout, test set.

Using this methodology, each observation is given a probability of membership to every state. While such an approach has its benefits, chief among them is consideration to low probability events; however, this can also introduce noise into the analysis in the form of low probability membership events. One approach to address this issue is through the introduction of a level of confidence for state membership. This can be accomplished through the introduction of confidence ellipsoids, which is the multivariate (in this example, a bivariate case is presented) analogue to a statistical confidence interval. Like a confidence interval, confidence ellipsoids relate to the underlying population parameters. For example, a 95% confidence ellipsoid would not necessarily be expected to encircle 95% of the state observations; however, if repeated sampling from the underlying distribution were performed, the population mean would be expected to fall within the ellipsoid 95% of the time. For the purposes of this research, the ellipsoid acts as a quantifiable threshold for deciding whether an observation is eligibility for considerable as a member for a state [50, 64].

The confidence ellipsoid is a function of four variables: 1) the magnitude of the major axis, 2) the magnitude of the minor axis, 3) the scale of the ellipsoid (the level of confidence the ellipsoid represents), and 4) angle of orientation about the axis. The first two variables - magnitude of the major and minor axis, or height and width - are a function of the variance. High variance indicates that the data is widely distributed, and the mean is not well established, so the ellipsoid will be extended further in the direction of the high variance distribution. The basic equation in Equation 3.9 for an ellipsoid is applied to generate a shape with the appropriate proportions given the variance of each feature, a and b. The a and b axis represent features, which in the case of this example would be the growth vector and covariance distance, respectively. Large ellipsoids represent states with a wide range of values in one or both features, which would indicate that the state is more general than a state with a smaller ellipsoid and more concentrated feature set of its members.

$$f(a,b) = \left(\frac{a}{\sigma_a}\right)^2 + \left(\frac{b}{\sigma_b}\right)^2 = s \tag{3.9}$$

The third variable - confidence scaling factor of the ellipsoid - increases the size and by extension the level of confidence the results represent; however, the estimations about confidence need to be made with respect to a specific, underlying statistical distribution. A Chi-Squared distribution was selected because the terms $(\frac{a}{\sigma_a})^2 + (\frac{b}{\sigma_b})^2$ in Equation 3.9 represents an independent sum of squares distribution, and the sum of squared Gaussian observations is distributed according to a Chi-Squared distribution[88]. Using this distribution assumes of normality within the feature set, which make not hold true for all instances; however, the goal of this application is to provide a heuristic for assigning observations and therefore an approximation of statistical confidence is sufficient for this purpose. This scaling factor can be introduced by replacing s in Equation 3.9 with the appropriate value from the Chi Squared distribution that represents the appropriate level of confidence. Three commonly used values (df = 2) are listed in Table 3.2.

Confidence Level	S **Value**
90%	4.605
95%	5.991
99%	9.210

TABLE 3.2: Key Values from Chi-Squared Distribution (df=2).

An increase in the confidence level corresponds to a larger S value, which in turn corresponds to a larger ellipsoid. All points which satisfy Equation 3.10 meet the criteria for membership in the state at the given level of confidence.

$$\left(\frac{a}{\sigma_a}\right)^2 + \left(\frac{b}{\sigma_b}\right)^2 \leq S \tag{3.10}$$

The first three variables defined the shape and size of the ellipsoid. The fourth variable specifies the orientation of the ellipsoid. Just as the variance of each of feature affects the magnitude of the major and minor axis of the ellipse, the direction of the variance

as defined by the covariance matrix impacts the orientation of the ellipsoid as well. The eigenvalues of the covariance matrix - $v(a)$ and $v(b)$ - represent the variance of data in the direction of the eigenvectors. Using Equation 3.11, the angle of orientation, θ, between the major axis and the x-axis can be calculated.

$$\theta = \arctan\left(\frac{v(a)}{v(b)}\right) \tag{3.11}$$

Deriving the orientation angle to the x-axis from Equation 3.11 and applying it to the ellipsoid created through Equation 3.10 can be used to create confidence ellipsoids for each example result shown in Figure 3.6. The resulting cluster assignments with corresponding confidence ellipsoid overlays are illustrated in Figure 3.12.

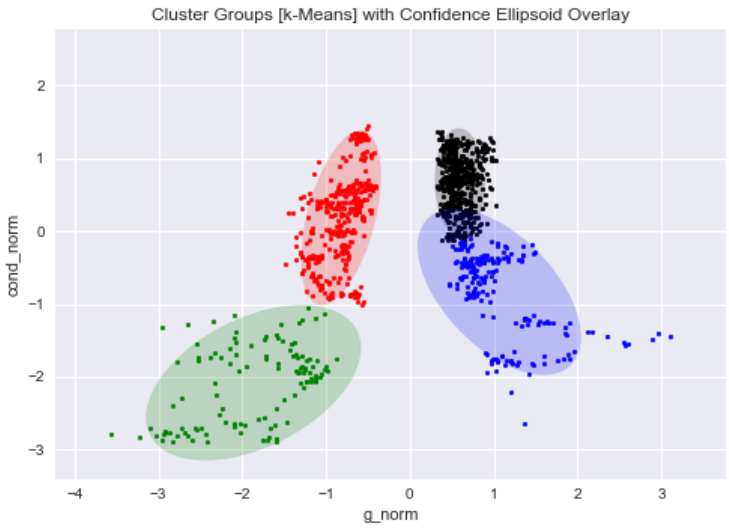


FIGURE 3.12: Cluster results from a $k = 4$ schema with an overlay of 2 standard deviation confidence ellipsoids.

The ellipsoid boundaries calculated in Equation 3.10 and shown in Figure 3.12 can be applied to the test set to determine the threshold for which states an observation can be probabilistically assigned and which states are removed from consideration. Figure 3.13 show the observations from the test set with the confidence ellipsoids based on the states from the training set. Observations that only fall within a single ellipsoid would

be assigned to a single state, observations that reside in overlapping ellipsoids would be probabilistically assigned to states as described in Equation 3.8. In the next section, we discuss how to evaluate the states created using this approach.

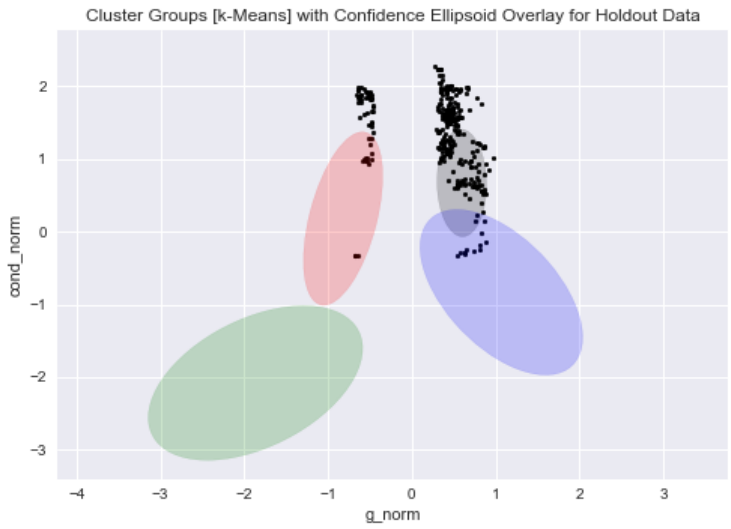


FIGURE 3.13: Observations from the holdout, test set are plotted against the 2 standard deviation confidence ellipsoids for clusters to estimate the cluster membership for the holdout observations.

Methods of Evaluating Market States

Having outlined a methodology for measuring key market characteristics, defining market states based on those key characteristics, and probabilistically assigning future periods to each of these market states, the results of this methodology must be evaluated to determine whether they offer value through the application of this state-based approach. The evaluation criteria revolve around the ideas of recurrence and persistent behavior. Both are essential to the practical application of any results derived from this method. First, for any insight derived from this approach to be useful, the state must recur. Otherwise, there would be not opportunity to use any insight gained. Secondly, knowing that a market state will recur at some time in the future is a necessary but not sufficient condition for adding value. There must also be a degree of insight into the behavior of the market while in that state.

Visual inspection of the transition behavior in Figure 3.10 suggests that observations are capable of 1) leaving a state indicating that it is *non-absorbing* in nature and 2) return to any state after transitioning from it indicating *recurrent* behavior. Both are necessary characteristics for this research because to examine the transition dynamics in a scientific and objective manner, the system of market states should be modeled as a Markov process. Such a model determines the probability of transitioning to future states j is governed only by the current state i, and any state visited prior, denoted as k, does not affect the transition dynamics of the system [12].

A transition probability matrix is a method of representing the transition behavior of a system. The matrix **P** in Figure 3.14 represents the probability a system being in state j conditional on visiting i in the previous time stage, $p_{i,j}$, for all combinations of states (0, n), where $p_{i,j} = P(x_{t+1} = j | x_t = i)$.

$$\mathbf{P} = \begin{pmatrix} p_{0,0} & p_{0,1} & \cdots & p_{0,n} \\ p_{1,0} & p_{1,1} & \cdots & p_{1,n} \\ \vdots & & & \\ p_{n,0} & p_{n,1} & \cdots & p_{n,n} \end{pmatrix}$$

FIGURE 3.14: Generalized transition probability matrix

$$\mathbf{P} = \begin{pmatrix} 0.91 & 0.00 & 0.09 & 0.00 \\ 0.00 & 0.95 & 0.00 & 0.05 \\ 0.15 & 0.02 & 0.84 & 0.00 \\ 0.01 & 0.09 & 0.00 & 0.90 \end{pmatrix}$$

FIGURE 3.15: Matrix for training set example

Visual inspection of Figure 3.10 demonstrates that each state is visited more than once suggesting that if a system leaves any state, it can return to that state. This finding of recurrent behavior can be reinforced by examining the system's transition probability matrix from which the long-run, steady state probability matrix can be calculated[5]. Using the example transition matrix shown in Figure 3.15, steady state probabilities for the system are [0.098 0.544 0.053 0.304]. All probabilities are greater than zero showing that all states can be reached at some point in future. If a state were not recurrent, the steady

state for it would be zero [12]. This characteristic is important in modeling the financial markets from a state-based perspective because the value added from such a model is predicated by the expectation that the system return to a given state, which in turn provides insight into the behavior of the system. The transition probability matrix and data used to create it can be used to examine the existence of the Markov property.

The existence of the Markov property can be examined using a frequency-based approach to determine if being in state k prior i had an effect on the transition probabilities to state j. The χ^2 test was used to determine if there is a statistically significant difference in transition probabilities based on the prior state k. All i, j combinations should be examined to determine the degree to which the system as a whole exhibits the Markov property [48, 133].

Table 3.3 shows how to structure the χ^2 test for examining the Markov property. This test uses the matrix P, only the p_{ij} values where i=1 and j=1 are included. Separate tests must be performed on each i, j pair. The sample is divided into subsets depending on the state k visited prior to being in i and is represented by a row in the table. For a subset to be eligible for the χ^2 test, the expected number of observations must be equal to at least five (5) records. Otherwise, the row should be excluded from consideration [48]. In this example, two rows ($k = 1, 3$) meet the minimum expected number of observation requirements and may be included in the calculation.

k	$p_{1,1}\|k$	$O_{j=1}$	$O_{j\neq1}$	$n_{1\|k}$	E[Obs]	Eligible?
0	0.00	0	0	0	0.00	
1	0.96	574	24	598	564.83	X
2	1.00	1	0	1	0.94	
3	0.70	23	10	33	31.22	X

TABLE 3.3: Result Summary for χ^2 Test of Markov Property: $p_{1,1}$

[5]The steady-state probability of a Markov chain represents the long-term probability of being in a state at any point in time. It can also be thought of as the percentage of time spent in each state in the long-term. Given that the steady-state is based on a long-term view of the system, the steady-state values are independent of the initial starting state. [12]

The χ^2 test is structured to examine the null hypothesis that equality exists among the transition probabilities given prior k state values, i.e. there is evidence of the Markov property. The statistical test found a p-value of 0.13 for the transitions for $p_{1,1}$; therefore, the test fails to reject the hypothesis that the Markov property exists for any level of significance less than 0.87. The testing should be repeated for all p_{ij} values in the same manner.

A second method to evaluate the efficacy of states created is to examine the persistence of the state characteristics between the training set on which the states were created and the testing set on which the states are evaluated. Specifically, the two features used to create the states - growth vector and covariance distance - are examined on a state-by-state basis using membership from the training set and distributions of feature characteristics are created. These distributions are then compared to those from the training set to determine if the features are statistically similar. Since observation from the testing set are probabilistically assigned to each state, testing set observations are assumed to be members of the state to which it has the highest probability of membership. The two-sample Kolmogorov-Smirnov (K-S) test compares two datasets to determine if they are statistically different. The K-S test was selected because it is non-parametric and makes no normality assumptions regarding the distribution of the datasets. The distributions of feature values for each feature-state pair are compared using the K-S test statistic, which is based on the point of maximum deviation between the cumulative distribution function of the training and test sets. Take as an example the growth vector for State 1. The distribution of values for both sets are shown in Figure 3.16, and the associated cumulative distribution from which the K-S test statistics is derived is shown in Figure 3.17.

$$D_{n,m} = \sup_z |F_1(z) - F_2(z)| \tag{3.12}$$

where $F_1(z)$ and $F_2(z)$ are the cumulative distribution being compared, and $\sup$ is the supremum function.

Using the cumulative distribution, the test statistics, D, is calculated to be 0.508 using

Equation 3.12, which equates to a p-value of 0.0001. The null hypothesis that the two samples come from the same underlying distribution is rejected [83]. This statistical result is supported by a visual examination of the distributions in Figure 3.16. The distributions of feature values, and by extension the behavior of the population in terms of this particular feature of interest, are statistically different between the training and test sets.

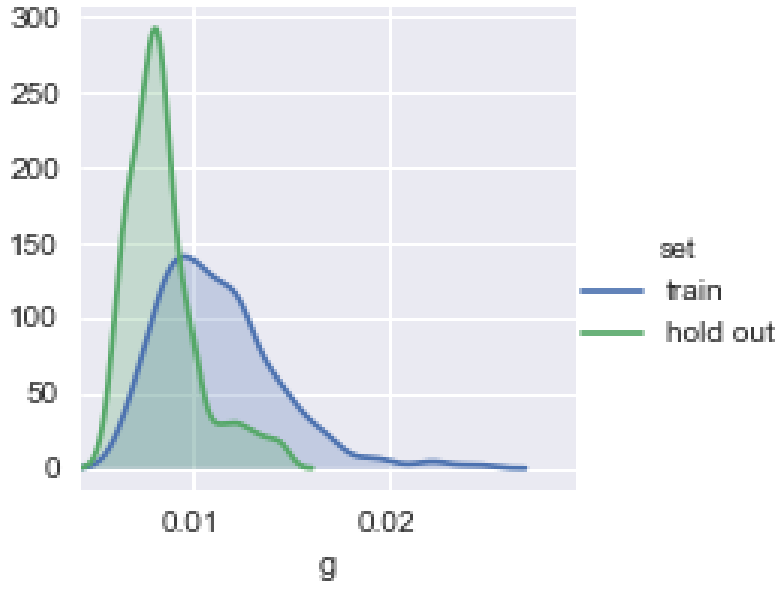


FIGURE 3.16: The distribution of growth vector values for training and hold out testing set for one state.

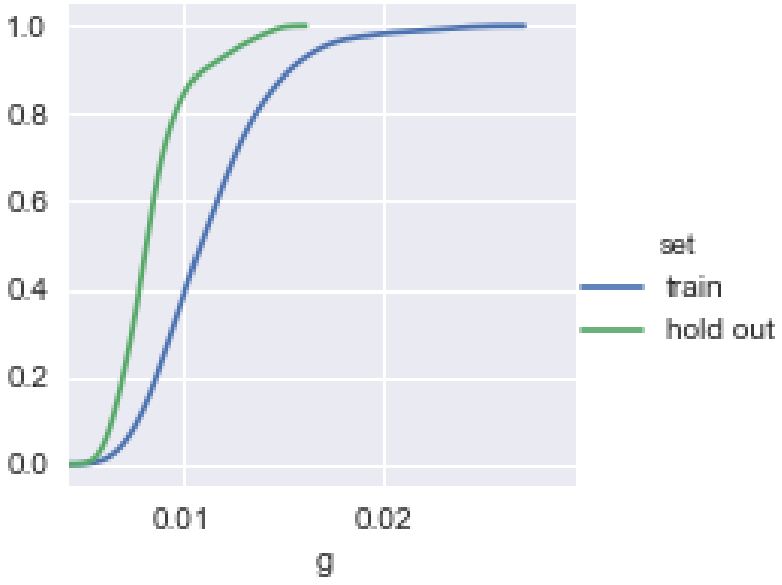


FIGURE 3.17: Cumulative distribution of growth values, which is used to create the K-S test statistic.

A third method for evaluating the results of clustering examines whether the transitional dynamics of the schema are robust. More specifically, it tests whether the transition behavior of the states from the training set as quantified by a transition probability matrix persists to states in the testing, hold-out set. As previously discussed, observations from the training set are members of states to a certain degree of probability as contrasted to states in the training set from which observations are assigned to states absolutely. To address this issue, observations are assigned to the nearest, highest-probability state from which the testing transition probability matrix is calculated. The comparison between the two matrices is made using the χ^2 test ([11], [14]). This approach uses the state transition probabilities, p_{ij}, from the training set combined with the number of actual observations in the state i from the testing set to determine the expected number of observations in state j. A statistical test is conducted on the differences between the expected number of observations and the actual number of observations to determine whether the differences are statistically significant.

3.5 Probability of State Membership

The process presented in this research assigns observations to a finite, discrete state, which when applied to all observations from the training dataset results in a sequence of states describing how a financial system transitions between various states through time. Given the information provided by these sequential state memberships, the system can be modeled as a stochastic process using a Markov chain such that the probability p_{ij} of a random variable X, which would represent the state of the financial system, being in a state j at time $t+1$ depends on only the state i in which it had been in at t. The process would be memoryless and thereby not consider state membership prior to time t. This version of a Markov chain is known as a first-order model such that

$$\begin{aligned} p_{ij} &= P(X(t+1) = j | X(0) = i_0, \ldots, X(t-1) = i_{t-1}, X(t) = i) \\ &= P(X(t+1) = j | X(t) = i) \end{aligned} \tag{3.13}$$

A transition probability formula shown in Equation 3.13 describe the state transitional dynamics of the process. All N^2 transition probabilities from the N-state system are summarized in this matrix. Modeling the state transitions of the system as a Markov chain enables probabilistic estimates to be made of the state in which the system will be n periods in the future such that

$$\begin{aligned} p_{ij}^{(n)} &= PX(t+n) = j, X(t+n-1) = j_{t+N-1}, \ldots, X(t+1) = j_{t+1} | X(t) = i \\ &= P(X(t+n) = j | X(t) = i) \end{aligned} \tag{3.14}$$

Raising the single step matrix described in Equation 3.13 to the nth power can derive the association n-step transition probability matrix shown in Equation 3.14.

A distribution of initial probabilities can be incorporated into the analysis describing the initial starting states. For this analysis, the distribution of initial starting conditions is represented as the probability of state membership p_k described in Equation 3.8. Given that p_k is used in the transition probability calculation shown in Equation 3.15 as the initial state distribution, the notation for this value is changed to a convention commonly used in the domain of Markov chain analysis. Therefore the initial probability that the system is in state k is written as $p_0^{(k)}$. Combining the probability of state membership from Equation 3.8 with the n-step transition matrix from Equation 3.14 results in the equation

$$p_0^{(k)} \Pi^n \tag{3.15}$$

which provides a probabilistic estimate of the state in which the system is in n-periods

given the probability distribution of current state membership.

A more complex model of the system can be created using a second-order Markov chain such that the probability p_{ij} of being in a state j at time t+1 depends on the states i_t and i_{t-1} in which it had been in at t and $t-1$, respectively, such that

$$\begin{aligned} p'_{i_t i_{t-1} j} &= P(X(t+1) = j | X(0) = i0, \ldots, X(t-1) = it-1, X(t) = i) \\ &= P(X(t+1) = j | X(t) = i) \end{aligned} \tag{3.16}$$

The choice to use a second-order Markov chain to model the system enables more information to be captured; however, the information captured may not offer additional insights if the first-order model is memoryless, and the state at time $t-1$ does not affect the transitions. Another concern is that by introducing another dimension, the state at time t-1, increases the number of pairwise transition probabilities. In an N-state system, the number of transition probabilities increases by a factor N going from N^2 probabilities in a first-order model to N^3 in a second order model. Depending on the amount of data available from which to derive the transition matrix, many transition probability matrices may be sparsely populated. Considerations should be given to the underlying data prior to using a higher order model and should only be applied to system models with a small number of states N in order to avoid this problem.

3.6 Local Optimization

The primary motivation for the development of the state definition methodology described in this research is that it could support the decision making when selecting assets to incorporate into an investment portfolio. More specifically, an investment framework is needed that indicates the asset allocation of a portfolio dependent on the market state.

This research incorporates the seminal work of Markowitz in several ways. First, in terms of the feature selection-using attributes utilized by Markowitz in research that became known as Modern Portfolio Theory (MPT) to quantify aspects of portfolio risk and return and cluster observations into market states. Second, the principles Markowitz employed to define investment frontiers are employed to make the asset allocation decisions given membership in each state. The mean-variance optimization (MVO) approach serves as the basis for making these asset allocation selections in what is referred to as the local optimization approach. This approach is referred to as local because the asset allocation decisions for each state are made considering only the characteristics of observations belonging to the particular state. Considerations for the probability of state membership and state transitional dynamics are made after the determining the optimal portfolio for each state.

One of the key concepts presented by Markowitz was the efficient frontier, which presents a Pareto optimal frontier of the optimal expected investment portfolios. The frontier is created using an optimization approach that determines the combination of assets delivering the highest level of expected return for various levels of expected risk assumed. Conversely the frontier could be constructed by minimizing the level of risk assumed for every level of expected risk current. This research utilizes this method of portfolio design; however, the universe of observations on which the frontier is created has been limited to only members of each state. This approach creates state-specific frontiers characterized by unique asset allocations and risk-return characteristics.

The state-specific frontier provides multiple 'optimal' portfolios from which investor may select. However, each point on the frontier represents the greatest level of expected return per level of risk that an investor can expect to receive. Any rational investor would select a point, which fall on frontier; however, the decision between points which fall on the frontier appeal to a rational investor. Therefore, a method is needed to select between points on the frontier. This research considers the optimal portfolio on the investment frontier to be the point with the greatest Sharpe Ratio. This ratio is a measure of excess returns achieved per unit of risk assumed [6].

This point also corresponds to point known as the Tangency Portfolio, which refers to the point at which a tangent line is formed between the risk-free rate of return and the frontier.

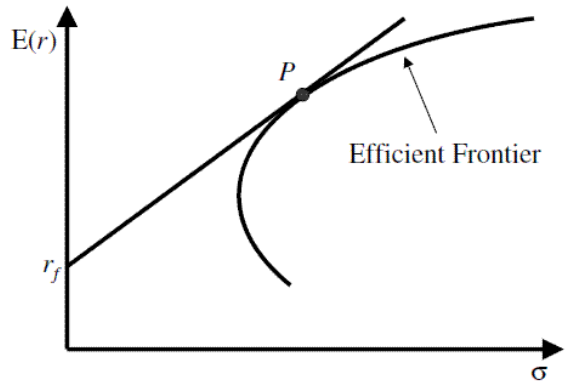


FIGURE 3.18: Tangency portfolio point, P, determined by tangent line from risk-free asset, r_f, and efficient frontier. *Source: CFA Institute.*

For each of the k_n states created in the schema, a 1-by-w_n vector of weights is created indicating the 'optimal' allocation to each of the w_n assets in the investment universe when an investment has transition into state k. The allocations specific to each state are combined into a k_n-by-w_n matrix, w^*_{local}, which summarizes the state-specific asset allocations. The Portfolio Design Sections details a strategy for creating a portfolio using this weight matrix.

3.7 Creating a Portfolio

The two key questions when approaching asset allocation from a state-based perspective are: 1) In which state is the system? and 2) How should a portfolio be allocated when in each state? Answers to both of these pivotal questions have been provided above and are combined to create a solution to the overall asset allocation question.

[6] As the Sharpe Ratio measures excess return per unit of standard deviation. 'Excess return' is defined as returns in excess of the risk-free rate, r_f, which is assumed to be 0 in this research. Therefore, the Sharpe Ratio is defined as $\frac{E(r)-r_f}{\sigma}$. A risk-free rate of 0% is assumed in the calculation of the Sharpe Ratio.

Using the test data set, a distribution of state membership probabilities can be calculated for any observation based on its relative distance to the center of each market state, i.e. the centroid, which can be combined with the n-step transition probability matrix Π^n as shown in Equation 3.15 to make a probabilistic estimation of the states in a which the system will find itself n periods in the future. The estimation takes into account the current probabilities of being in each state and the transitional dynamics of the system. Values for n, the number of transition steps into the future, can take any value from the range $0, \inf$. A step size of 0 considers only the probability of being in the current state and dismisses any state transition dynamics. An infinite step size would reach a steady-state distribution, assuming the transition matrix is not periodic in nature, making the initial probabilities obsolete because the probabilities of being in any state would be same regardless of the starting state. Regardless of the value selected as the step size parameter, the resulting output from the calculation is a vector of probabilities indication membership in a state at some point of interest.

The probability vector can be multiplied by the state-specific asset allocation matrix w^*_{local} to create a weighted-average vector of weights in which the amount to which each state-specific weighting scheme is considered in the overall allocation, written as W^*_{local}, is based on the probability of being in each state. No frictional or transactional costs are incurred as asset are re-allocated and portfolios rebalanced between periods.

$$W^*_{local} = p_0^{(k)} \Pi^n w^*_{local} \tag{3.17}$$

3.8 Global Optimization

A second optimization approach to derive the optimal set of state-specific asset allocation weights takes into consideration the probability of state membership, transitional

dynamics, expected return and expected risk in the optimization equation. Since this approach considers all states and the transitions between them when assigning the optimal allocations, it is referred to as global optimization approach, and the optimization output is, like the local optimization, a k_n-by-w_n matrix designated as w^*_{global}.

The objective function for the global optimization is the Sharpe Ratio calculated in terms of annualized values, which the optimization algorithm seeks to maximize using the training data set. The practice of using the Sharpe Ratio as a metric of interest in both optimizations is consistent and reinforces the premium placed on a risk-adjusted investment portfolio; however, the optimization for the local approach occurs at the state and frontier level while the optimization for the global occurs at the portfolio level.

This approach can be adapted for use on a data series of various levels of granularity, e.g. daily, weekly, or monthly prices[7]. Accounting for varying types of data granularity is made by adjusting the number of periods per year when making the conversion to annual values. As previously discussed, the Sharpe Ratio is the ratio of excess expected returns per unit of risk. More specifically, it is the expected return less the risk-free rate of return divided by the standard deviation. The formulas used to convert returns and standard deviation require two inputs – r, the returns for each period, and n_{per}, the number of periods occurring annually for the level of data granularity.

The returns vector r is derived by weighing the state-specific asset allocation by the probability of being in each state and then multiplying the period return vector by the asset allocation weight vector as shown in Equation 3.18.

$$r = p_0^{(k)} \Pi^n W^*_{global} R \tag{3.18}$$

where R is the return of each asset in the investment universe during each period [8].

[7] The number of periods per level of data granularity is as follows. Note that the number of daily periods is an estimate of the number of annual trading days. Daily: 252, Weekly: 52, Monthly: 12.

[8] As with the Local optimization-based portfolio, no transactional costs are incurred when asset allocations are changed between periods.

The formula to derive the annual return for a series of periodic returns is:

$$R_{ann} = (1 + \bar{r})^{n_{per}} - 1 \tag{3.19}$$

where $\bar{r}$ is the sample average of portfolio returns.

The formula for converting standard deviation is more computationally involved than that of that ones for returns as shown below.

$$S_{ann} = \sqrt{(s^2 + (1 + \bar{r})^2)^{n_{per}} - (1 + \bar{r})^{(2*n_{per})}} \tag{3.20}$$

where s is the sample standard deviation.

It is a common practice in the investment industry to convert to annual standard deviation using the linear transformation $S_{ann} = \sqrt{n_{per}} * s$; however, this methodology does not properly account for the nature of cumulative returns in an investment product, which expand and contract in accordance with a geometric series. The linear transformation introduces a bias into the annualized figure.

The combination of the two conversion equations plus the risk-free rate of return value results in the objective function for the annual Sharpe Ratio such that

$$\begin{aligned} SR_{ann} &= \frac{(R_{ann} - r_f)}{S_{ann}} \\ &= \frac{((1 + \bar{r})^{n_{per}} - 1) - r_f}{\sqrt{(s^2 + (1 + \bar{r})^2)^{n_{per}} - (1 + \bar{r})^{(2*n_{per})}}} \end{aligned} \tag{3.21}$$

3.9 Heuristic Overlays

3.9.1 Market Risk Level

The objective function used in an optimization-based portfolio design strategy is of critical importance. It clearly defines what is the goal in designing a portfolio and drives the asset allocation process in pursuit to that end. For this research, the risk-adjusted performance is of paramount importance. Specifically the goal is to optimize the Sharpe Ratio. This is accomplished two ways: 1) finding the tangent line to each of the efficient frontiers in the local optimization-based approach or by optimizing weights to maximize the Sharpe Ratio of the overall portfolio as applied in the global optimization-based approach. However, the ideal results achieve through optimization may not meet the realistic expectations and goals of an investor in practice and needs to be augmented by the use of a heuristic overlay.

One instance in which this can be applied in the context of this research is the observation that the maximization of Sharpe Ratio may result in a conservatively allocated portfolio that does not result in a sufficient level of returns may not be achieved because too little risk is assumed. There are several approaches that could be used to remedy this shortcoming. One would be to apply additional constraints to the optimization establishing a minimum return requirements or redefining the objective function to achieve another risk-adjusted measure of return. Yet another approach, which is presented in this research, is to apply a leverage[9]to the portfolio. Leveraged is applied such that the standard deviation of per period portfolio returns, i.e. daily or weekly, is equal to the standard deviation of the market returns during the training period. The leverage factor, l, that achieves this level of portfolio risk is the ratio of market risk to initial portfolio risk

such that

$$l = \frac{\sigma_{mkt}}{\sigma_{port}} \tag{3.22}$$

The leverage factor l is multiplied by the return vector r[10]to create the leveraged portfolio returns r_{lev}

$$r_{lev} = l * r \tag{3.23}$$

In situations where the σ_{mkt} is greater than σ_{port}, portfolio exposure to the market needs to be reduced by proportionally reducing allocations in all positions. Practically, this would be accomplished by selling the assets and investing the proceeds in cash.

[9]For the purposes of this research, a borrowing rate of 0% is assumed when borrowing assets to create a leveraged portfolio. While interest rates remain low at the time of this publication, a borrowing rate of 0% is a conservative estimate. This parameter value was selected to illustrate the concept and potential benefit. Future research should use a value realistic to rates offered to investment practitioners, i.e. Fed Funds or LIBOR plus (+) 0.50%.

[10]The portfolio return variable r is a generic representation denoting either r_{local} or r_{global}.

Chapter 4

Results

"Have no fear of perfection - you'll never reach it."

Salvador Dalí

4.1 Results Overview

4.1.1 Structure of Results Section

The results for this research project are presented in the form of a series of case studies. This format was selected given the number of permutations of inputs that can affect the results. Therefore a selection of case studies has been assembled to illustrate the notable results achieved over the course of the research.

4.2 Case Study: Initial Model Evaluation

4.2.1 Motivation

The case study presents an overall look at the state-based approach presented in the research and illustrate the how the results compare to market returns as well as to a one-state, Markowitz model.

4.2.2 Model Parameters & Specifications

The key features examined include:

- Use the **Macro** signal set described in Appendix F.
- Training period: (12/31/2004 - 12/31/2014); Testing period: (12/31/2014 - 2/28/2019). Weekly data.
- Feature lookback period: 7 Weeks.
- One (1) period step size in Markov model.
- k-Means Clustering Algorithm.

4.2.3 Comparison of Results

This example begins with the end in mind showing in Figure 4.1 that the various state-based models create portfolios with risk-return profiles better than market (S&P 500) measured in terms of Sharpe Ratios for the holdout test period in all eight (8) models with the number of clusters ranging from 2 to 9. The state-based approach also demonstrated better risk-adjusted performance than the single-state model representing Markowitz's MPT method in 4 of 8 portfolios using the local optimization method and 6 of 8 using the global optimization method. The models that employed a two-cluster and four-cluster design exhibited the best performance and are examined in greater detail in this case study.

Figure 4.2 displays the Sarpe Ratios achieved during the training and test periods. There is a noticeable decline in all models from the results in training to those in testing. This can be attributed to a certain degree of over-fitting. The global method achieved higher training results for all models, which is to be expected given that the optimization is more complex than its local counterpart; however, the global models were also able to maintain superior out-of-sample performance to the local method in all cases as well.

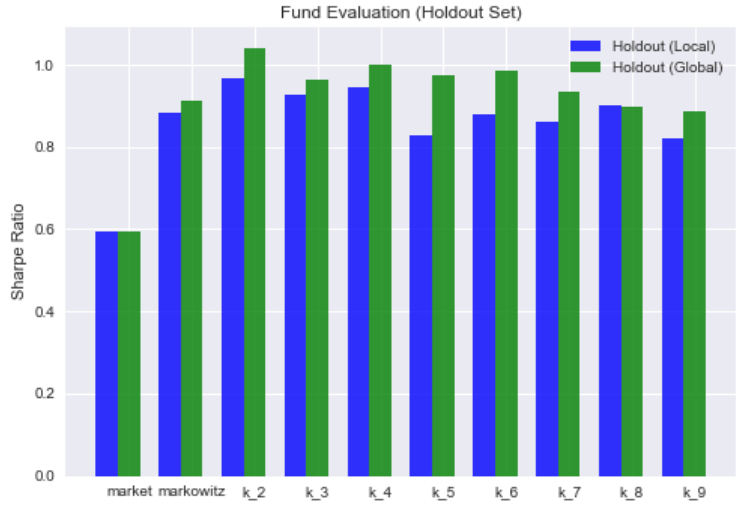


FIGURE 4.1: Risk-return profile (Sharpe Ratio) in the test set.

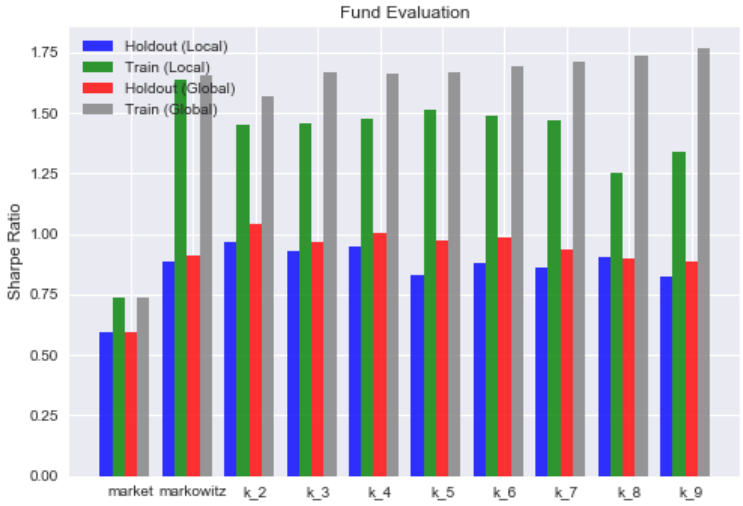


FIGURE 4.2: Risk-return profile (Sharpe Ratio) in the training and test sets.

One factor that contributes to the success of the models is that the selection of observations on which to formulate the states is representative of observations from the testing set on which estimations of state membership are to be made. Figure 4.3 shows observations from the training set (green) and those from the test set (red). Ninety-eight percent of the observations from the testing set fall within bounds of observations from the training indicating that the model was designed with consideration to this scale of observations. Were a large percentage of the testing set observations to fall outside the bounds of training set observations, then the model validity could be questioned given that it may not have been calibrated correctly to the data on which was being applied.

FIGURE 4.3: Observations from the training and testing sets.

The k-Means clustering algorithm is applied to the training set to assign observations to states. The two-state model shown in Figure 4.4 segments the population mainly on the basis of the normalized growth vector, g. With a select number of exceptions, observations with positive g_{norm} values are assigned to State 0 and those with negative values are assigned to State 1. In this version of normalized data, a positive value indicates an observation that lies above the sample mean while a negative values lies below. This assignment point is reinforced with Figure 4.5, which examines the aggregate performance of the S&P 500 during periods in the state. State 0 exhibits strong, bullish performance while State 1 shows negative, bearish performance. Note that while the state performance is plotted using a normal distribution in Figure 4.5, the returns are not necessarily

Gaussian in nature. This figures serves only as a general indication of market behavior while in the state.

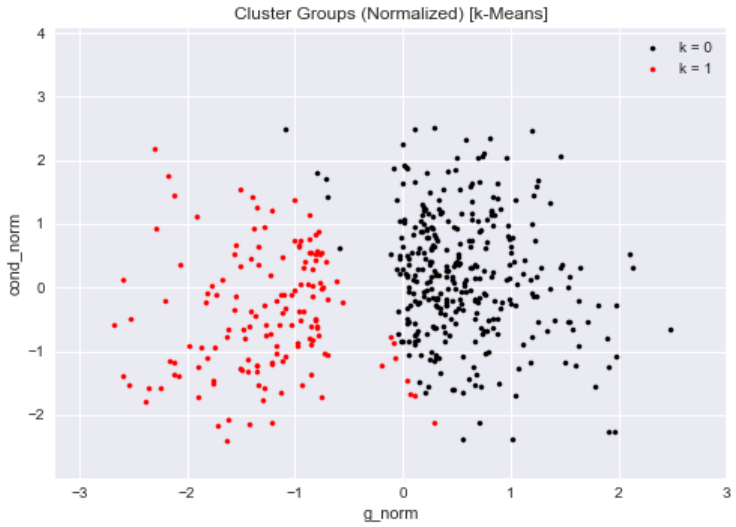


FIGURE 4.4: Cluster assignments for a two-state model.

The observations with negative g_{norm} values in State 0 were assigned to this state because of their high covariance distance value. The centroid for State 0 is comprised of positive g_{norm} and cov distance values therefore making it closer than the centroid for State 1 to observations with small negative g_{norm} and high covariance distances. The opposite is true for the select observations with positive g_{norm} values assigned to State 1.

The clustering results for the four-state model divides the population into quadrants of (high g, high cov), (high g, low cov), (low g, low cov), and (low g, high cov) shown in Figure 4.6. The market performance comparison in Figure 4.7 reveals that 2 market states - 0 and 2 - are connected with strong, bullish market behavior. State 1'with the high covariance distance and negative g exhibits bearish market behavior while State 1 shows even stronger bearish behavior.

The holdout period for this case extends from mid-2015 to early 2019. Examining the probabilities of being in each state shown in Figure 4.8, the model fluctuated between States 0 and 1 from mid-2015 until mid-2016 at which point the model probabilistically

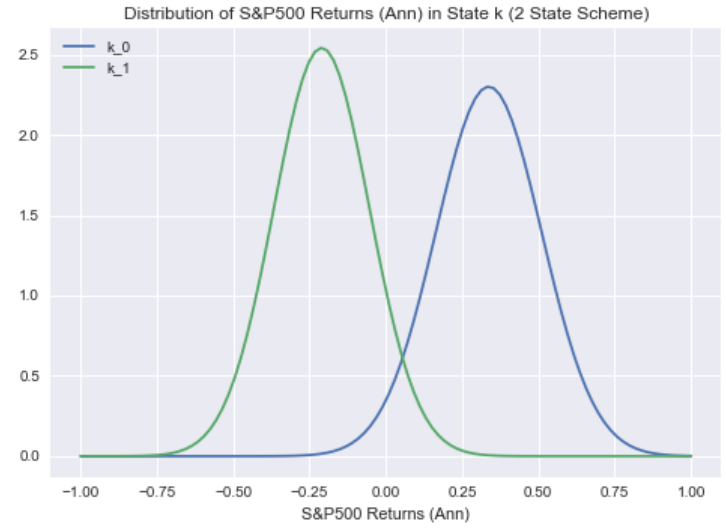


FIGURE 4.5: Representation of market performance while in states for two-state model.

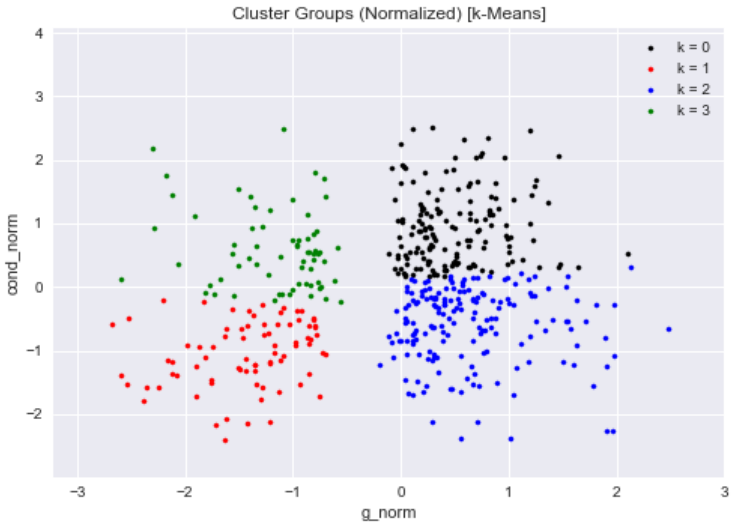


FIGURE 4.6: Cluster assignments for a four-state model.

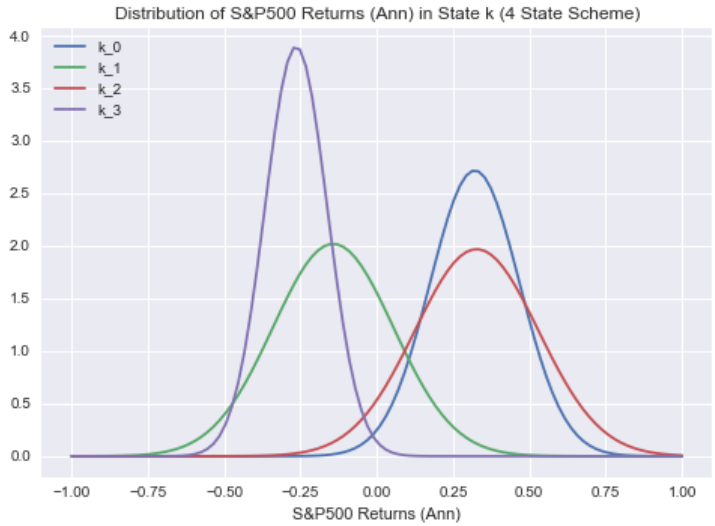


FIGURE 4.7: Representation of market performance while in states for four-state model.

favored State 0, which as was previously discussed is the bullish market state. This period of probabilistically favoring State 0 coincides with a strong bullish advance in the domestic equities market that persisted into early 2018. The period from mid-2015 through mid-2016 when the model was fluctuating between states corresponded to sideways movements in the domestic market consisting in short-term advances and short-term declines.

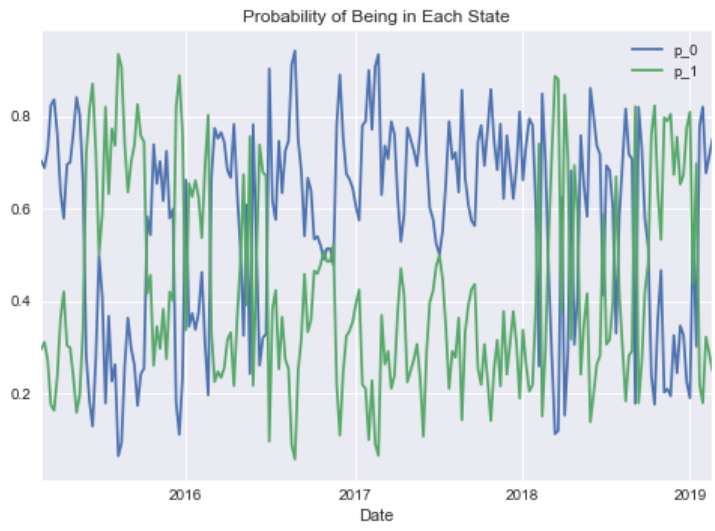


FIGURE 4.8: Probability of state membership in a two-state model for data in the holdout period.

Figure 4.9 shows the states into which observations were placed during the training period. While the ten-year period makes it difficult to examine state transitions at a tactical level, there are several high-level observations to make, particularly those pertaining to the turning points - inflection points before and after the 2007 to 2009 bear market also known as the 2008 Financial Market Crisis. Observations in the year leading up to the beginning of the bear market (October 2007) were predominantly in the State 0. At the time that the market entered into what became known as a bear market, the observations were fluctuating between membership in both states. This fluctuating behavior persists throughout the bear market. In early 2009 the model ceases its fluctuations and places observation in State 1 for a prolonged period of time. This extended allocation to State 1 corresponds to the ending of the bear market in March 2009.

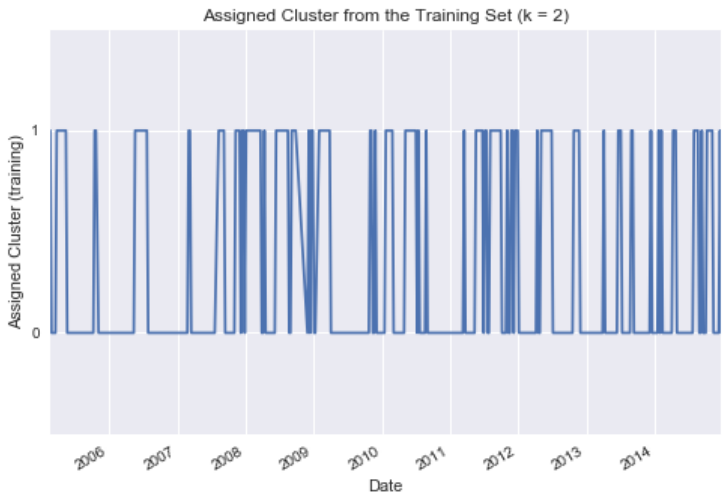


FIGURE 4.9: Probability of state membership in a two-state model for data in the training period.

Figure 4.10 shows confidence ellipsoids of two standard deviations placed around the clusters for the two-state model using data from the training period. A visual inspection of the results shows that the majority of observations fall within the confidence ellipsoid indicating few outliers that could skew state definition results. Projecting the confidence ellipsoids over observations from the test set as in Figure 4.11 shows that the majority of observation also fall within the bounds of an ellipsoid. Falling within the bounds of an ellipse suggests similarity with the state definition schema, and a model that is well

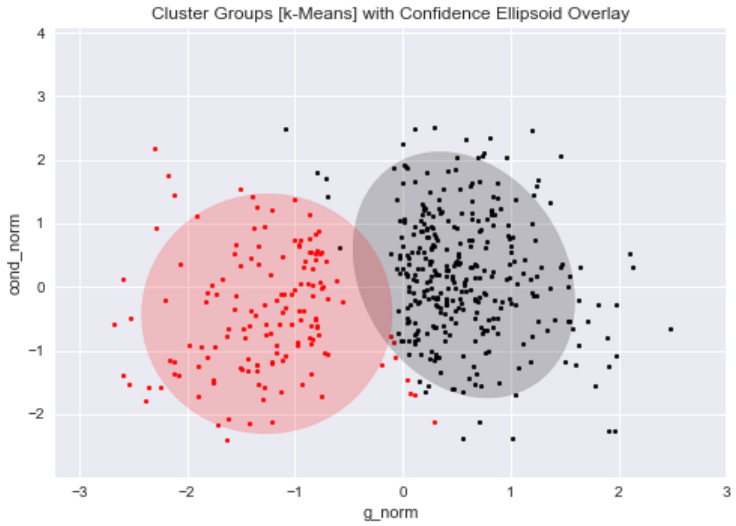


FIGURE 4.10: Confidence ellipsoids (2σ) for two-state model using training period data.

calibrated to the data.

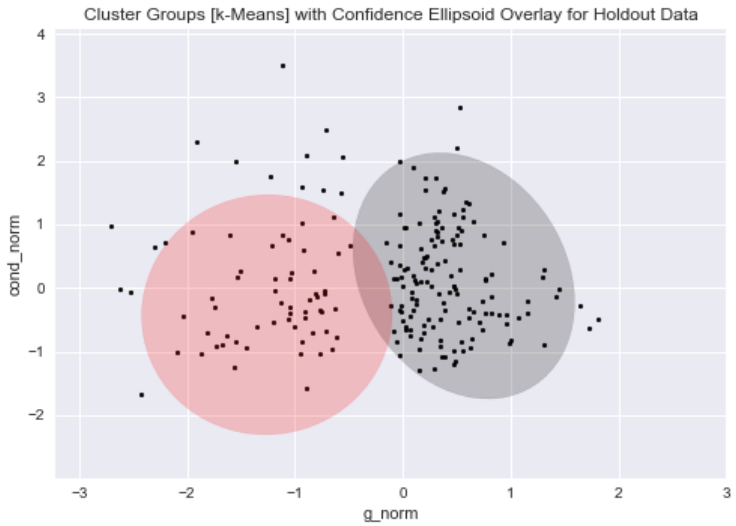


FIGURE 4.11: Confidence ellipsoids (2σ) for two-state model using test period data.

A visual inspection of an overlay of confidence ellipsoids for a four-sate model in Figure 4.12 shows that the majority of observations fall within the bounds of an ellipsoid; however, several observation fall within the overlapping regions between more than one

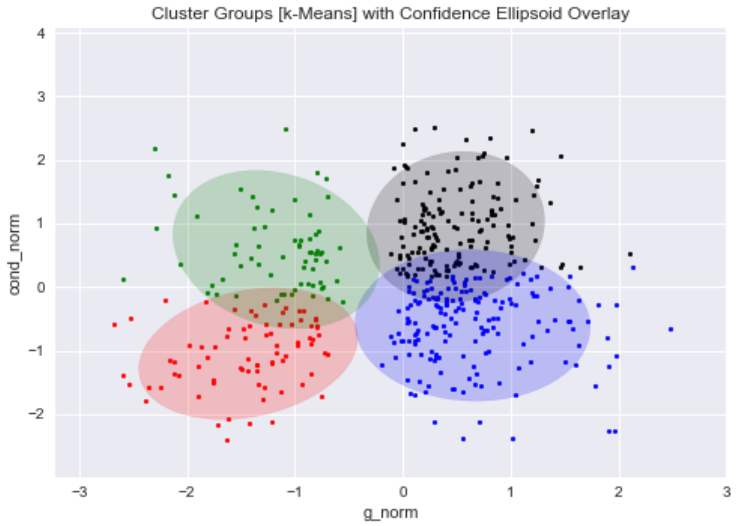


FIGURE 4.12: Confidence ellipsoids (2σ) for four-state model using training period data.

state. Such points would have similarities between both states even though they are definitely assigned to only one state.

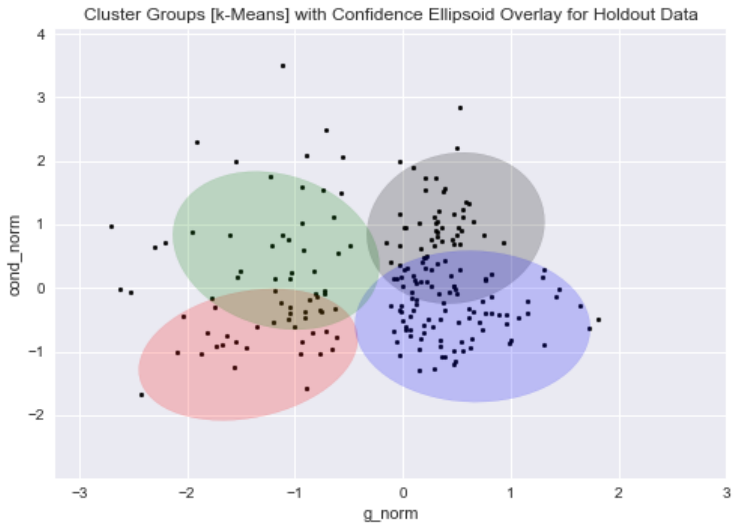


FIGURE 4.13: Confidence ellipsoids (2σ) for four-state model using test period data.

Like observations from the training set, confidence ellipsoids capture the majority of observations from the test set. And also like the training set, there exist instances in which observation fall within the ellipsoids of more than one state. One potential solution to

this situation is to only consider an observation for membership in a state if it falls within the bounds of a confidence ellipsoid. An observation can only have membership in more than one state if and only if it falls within the boundaries of overlapping confidence ellipsoids.

$$P_{train} = \begin{pmatrix} 0.88 & 0.12 \\ 0.28 & 0.72 \end{pmatrix}$$

FIGURE 4.14: Transition probability matrix for training set in two-state model.

$$P_{test} = \begin{pmatrix} 0.87 & 0.13 \\ 0.29 & 0.71 \end{pmatrix}$$

FIGURE 4.15: Transition probability matrix for testing set in two-state model.

Another point of similarity to compare when evaluating the effectiveness of the clustering model and the robustness of the model when transitioning between the training data set and the testing set is to determine if the state transition dynamics hold suggesting that the model is robust across time periods and applicable before the training set on which it was built. Figures 4.14 and 4.15 display the transition probability matrices for the training and testing periods. The frequency-based Chi Squared test looks for statistical evidence to reject the hypothesis that the matrices are alike, and as shown in Figure 4.16 by the red line, there is no statistical evidence to reject the hypothesis that the matrices are statistically comparable.

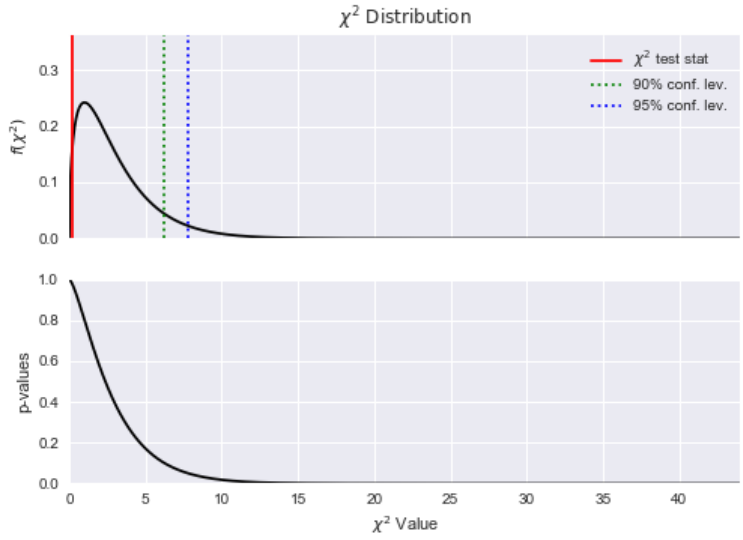


FIGURE 4.16: χ^2 Test results comparing transition probability matrices for a two-state model.

$$P_{train} = \begin{pmatrix} 0.58 & 0.01 & 0.31 & 0.10 \\ 0.05 & 0.55 & 0.19 & 0.21 \\ 0.27 & 0.09 & 0.60 & 0.03 \\ 0.21 & 0.27 & 0.09 & 0.42 \end{pmatrix}$$

FIGURE 4.17: Transition probability matrix for training set in four-state model.

$$P_{test} = \begin{pmatrix} 0.56 & 0.02 & 0.35 & 0.07 \\ 0.08 & 0.50 & 0.22 & 0.19 \\ 0.22 & 0.11 & 0.64 & 0.04 \\ 0.06 & 0.26 & 0.13 & 0.55 \end{pmatrix}$$

FIGURE 4.18: Transition probability matrix for testing set in four-state model.

The transition probability matrices shown in Figures 4.17 and 4.18 are less statistically comparable than the matrices from the two-state model; however, with a p-value of 0.51 the null hypothesis that the matrices are the same cannot be reject at any reasonable level of significance.

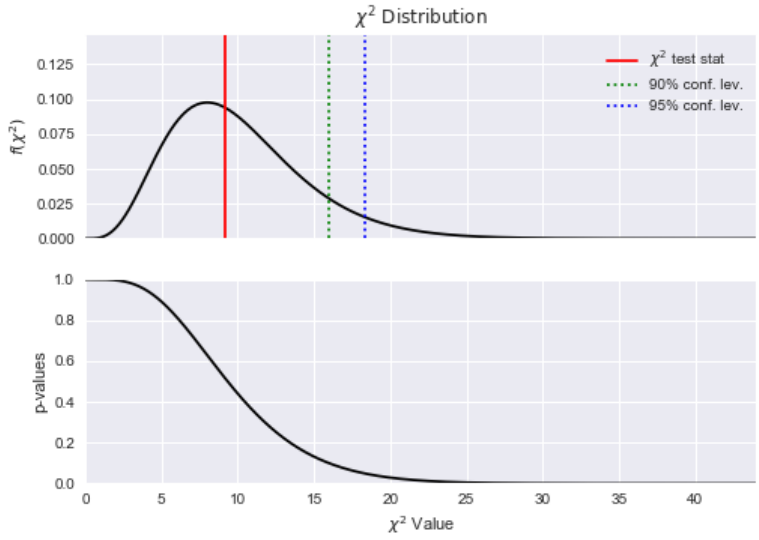


FIGURE 4.19: χ^2 Test results comparing transition probability matrices for a four-state model.

The next area of comparison examines the structures of the portfolios created and the resulting performance of the portfolios. The weights presented in Table 4.1 show the asset allocations given membership in each state broken down by optimization approach.

Method	State	EFA	EEM	AGG	RUT	IXIC	GSPC
Local	0	0.07	0.02	0.68	0.00	0.22	0.00
	1	0.00	0.00	1.00	0.00	0.00	0.00
Global	0	0.00	0.00	0.76	0.00	0.24	0.00
	1	0.00	0.00	1.00	0.00	0.00	0.00

TABLE 4.1: Portfolio weights for each market state based on both optimization approaches in a two-state model.

Recall that portfolios are created using the portfolio set referred to as the "Macro" set, which consists of six (6) indexes or ETFs designed to replicate an index. The Local optimization approach allocates heavily in both states to the asset, AGG, which receives a 0.68 and 1.00 allocation in States 0 and 1, respectively. Such a heavily is understandable when the nature of each asset is taken into consideration. The asset AGG is an aggregate bond ETF and also the least volatile asset in the investment universe. Given that the portfolio allocation selected for each state is the one with the highest Sharpe Ratio then portfolios highly allocated to the least-volatile asset thereby reducing the risk and

improving the overall risk profile would be expected. Examining the allocation for State 1, which is the bearish market state, shows complete allocation to AGG indicating that in declining market conditions, all equities would be sold and assets allocated to bonds.

The Global optimization results show slightly more conservative allocations than those of its Local counterpart. The Global allocation is described as 'more conservative' because a higher allocation is made to AGG even in the bullish State 0. There is approximately a 0.08 difference between the allocations to AGG. The Local weights allocated the difference to EFA and EEM, which are ETFs that replicate performance of developed and emerging foreign countries, respectively. Such markets are more volatile than bond markets.

FIGURE 4.20: The portfolio returns for a two-state model in the training period with weights assigned via Local optimization.

Figures 4.20 and 4.21 show the performance for the Local and Global portfolios for data in the training period, respectively. While both portfolios under perform the market from an absolute performance basis, they exhibit much better Sharpe Ratios than market. See Table 4.2 for the statistics on all portfolios.

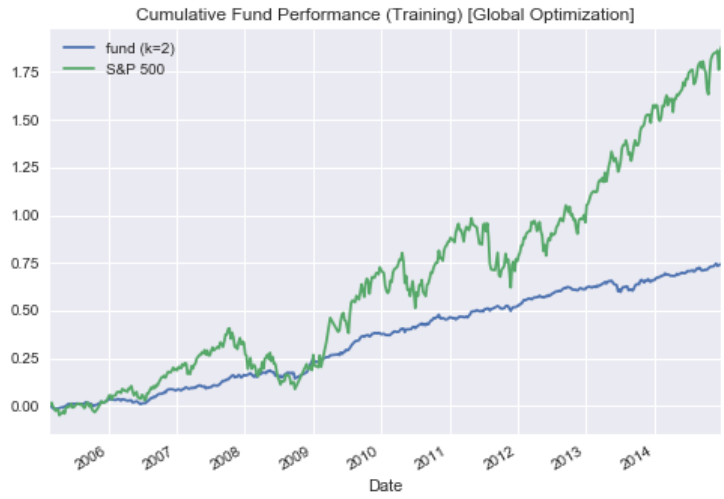


FIGURE 4.21: The portfolio returns for a two-state model in the training period with weights assigned via Global optimization.

		Training			**Testing**			
	k	r_a	σ_a	SR_a	r_a	σ_a	SR_a	**Lev.**
Market	–	0.122	0.165	0.738	0.085	0.144	0.593	1
Local	1	0.058	0.035	1.639	0.029	0.032	0.883	4.265
	2	0.057	0.039	1.45	0.035	0.036	0.968	3.963
	3	0.056	0.039	1.457	0.033	0.036	0.927	3.985
	4	0.058	0.039	1.476	0.035	0.036	0.946	3.96
	5	0.056	0.037	1.513	0.028	0.034	0.828	4.13
	6	0.058	0.039	1.486	0.031	0.035	0.878	3.956
	7	0.057	0.039	1.47	0.029	0.034	0.862	4.012
	8	0.062	0.05	1.249	0.039	0.043	0.902	3.192
	9	0.062	0.046	1.339	0.033	0.04	0.821	3.407
Global	1	0.056	0.034	1.655	0.029	0.032	0.913	4.473
	2	0.055	0.035	1.567	0.034	0.033	1.042	4.383
	3	0.059	0.035	1.668	0.032	0.034	0.964	4.345
	4	0.059	0.035	1.665	0.034	0.033	1.002	4.394
	5	0.059	0.035	1.670	0.033	0.034	0.974	4.400
	6	0.060	0.036	1.694	0.034	0.034	0.987	4.356
	7	0.061	0.035	1.715	0.03	0.032	0.935	4.251
	8	0.061	0.035	1.738	0.028	0.032	0.899	4.251
	9	0.063	0.036	1.769	0.029	0.032	0.886	4.191

TABLE 4.2: Portfolio performance for all state models.

One potential approach used in the investment management profession is to apply leverage to a portfolio in such a way to raise the portfolio risk, or volatility, to the level of market. For the portfolios discussed in this section, the level of respective leverage needed to raise the Local and Global portfolios to market level risk is 4.38 and 3.96. Portfolios are created for data in the testing period using both the original weights and the leveraged portfolio. The results are shown in Figures 4.22 and 4.23.

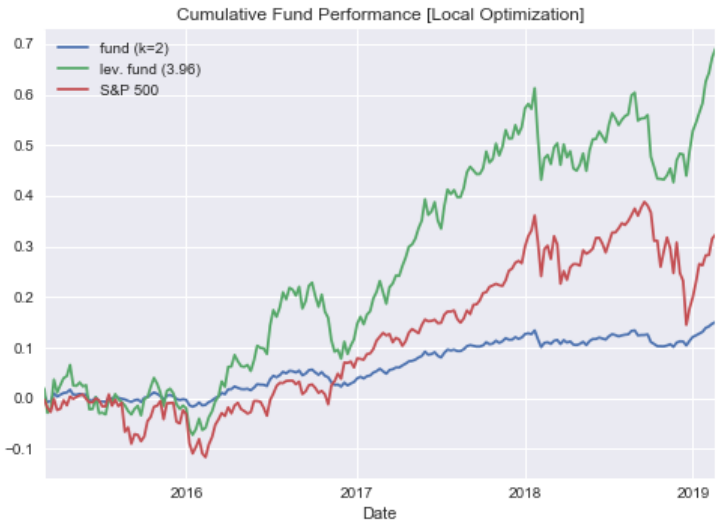


FIGURE 4.22: Fund performance in testing period using Local allocation weights.

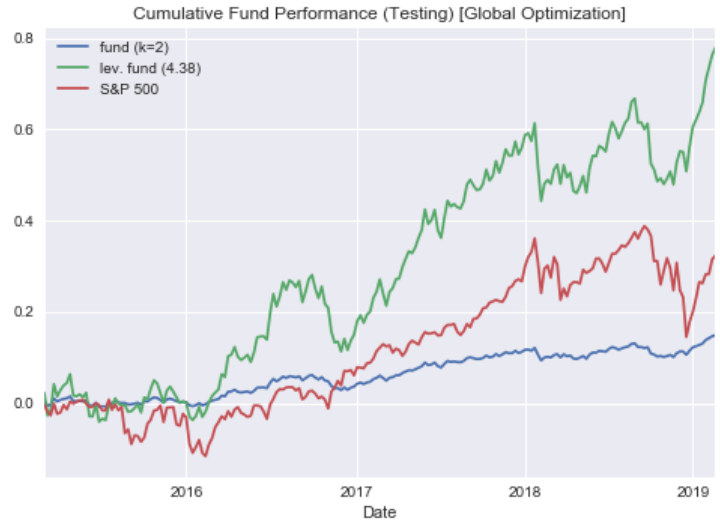


FIGURE 4.23: Fund performance in testing period using Global allocation weights.

Portfolios for both the two and four state model exhibit similar results. The unleveraged portfolios underperform the market market in absolute terms; however, when leveraged, the portfolios exhibit superior returns for comparable levels of portfolio risk.

The weights for the four-state model are shown in Table 4.3. The Local optimization treat both bearish states - 1 and 3 - the same by allocating all assets to the conservative bond security, AGG. For the same reasons discussed regarding the two-state model, a large percentage of assets are allocated to AGG in the bullish states as well. The Global results show the same allocation to 3 of the 4 states placing all assets in the security AGG. This result suggests that constraints should be added to the optimization limit the allocation to a single asset in order to create a portfolio design that would be more diversified and appealing to a live portfolio manager.

Method	State	EFA	EEM	AGG	RUT	IXIC	GSPC
Local	0	0.03	0.08	0.74	0.00	0.12	0.04
	1	0.00	0.00	1.00	0.00	0.00	0.00
	2	0.03	0.01	0.64	0.00	0.32	0.00
	3	0.00	0.00	1.00	0.00	0.00	0.00
Global	0	0.00	0.00	0.49	0.00	0.51	0.00
	1	0.00	0.00	1.00	0.00	0.00	0.00
	2	0.00	0.00	1.00	0.00	0.00	0.00
	3	0.00	0.00	1.00	0.00	0.00	0.00

TABLE 4.3: Portfolio weights for each market state based on both optimization approaches in a four-state model.

A final aspect of the portfolio design examined in this case study is the number of periods into the future to incorporate into the state probability calculation. The results presented consider looking one period, which in this case means one week, in the future by raising the transition probability matrix to the nth power for the Local optimization funds. First, the overall cumulative for performance for portfolios evaluated in the training period are examined to determine the step-size that produces the highest cumulative returns. As shown in Figures 4.24 and 4.25, both model (two- and four-state) reach steady-state at $n = 4$ at which point not further performance improvements are achieved.

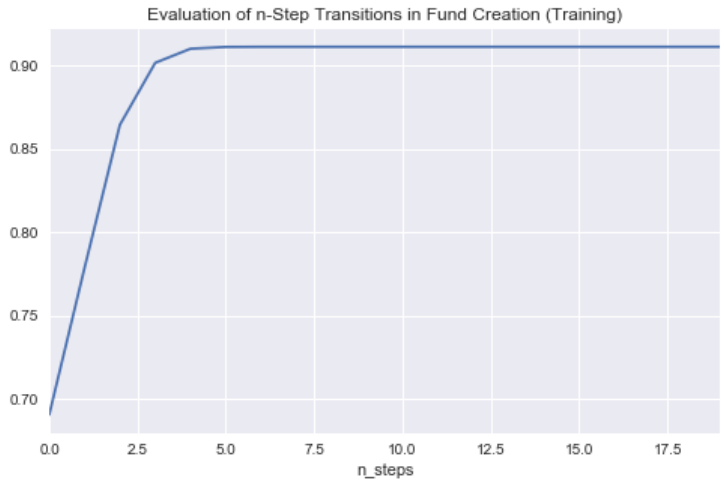


FIGURE 4.24: Step-size impact on cumulative market returns in training period for two-state, Local optimization solution.

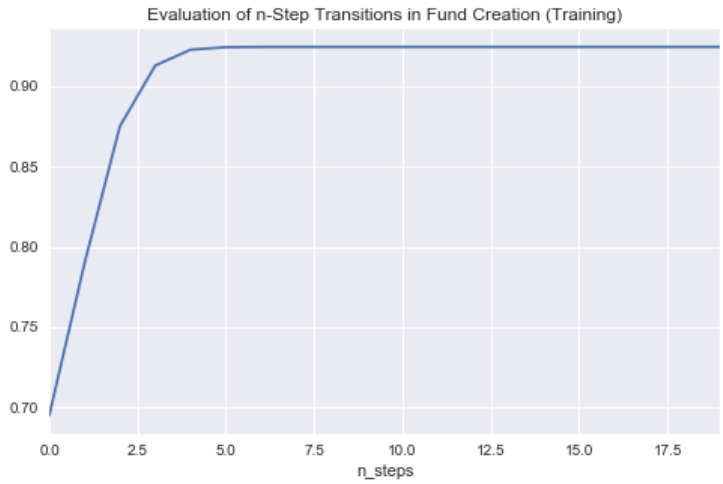


FIGURE 4.25: Step-size impact on cumulative market returns in training period for four-state, Local optimization solution.

When the same analysis is performed on the data from the test period, conflicting results are produced. The two-state model shown in Figure 4.26 changes its performance profile and performs better when a zero step, or no transitional dynamics, are taken into considerations. This suggests that only the initial p_0 probabilities would be used in the calculation of portfolio weights. Practically speaking, this decision would not have been undertaken since the portfolio with a step-size of 4 performed better in the training set would have been selected for use in the testing period.

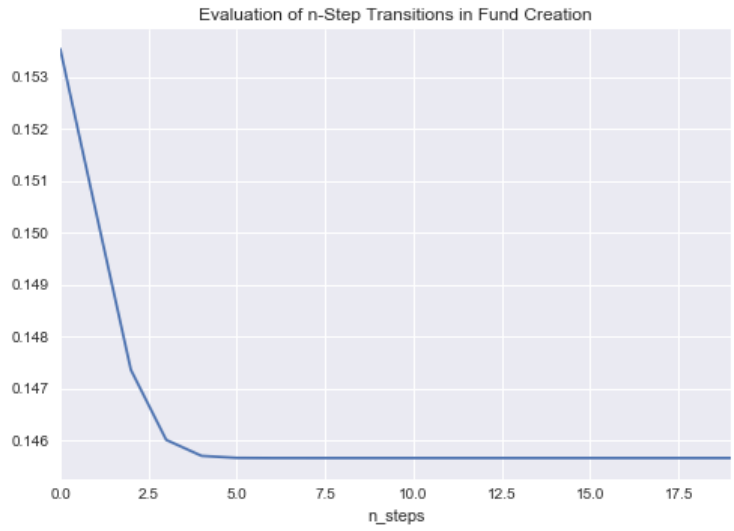


FIGURE 4.26: Step-size impact on cumulative market returns in test period for two-state, Local optimization solution.

The four-state model maintain its performance profile and achieved a maximum performance at the same level in the testing set as it did in the training set, $n = 4$.

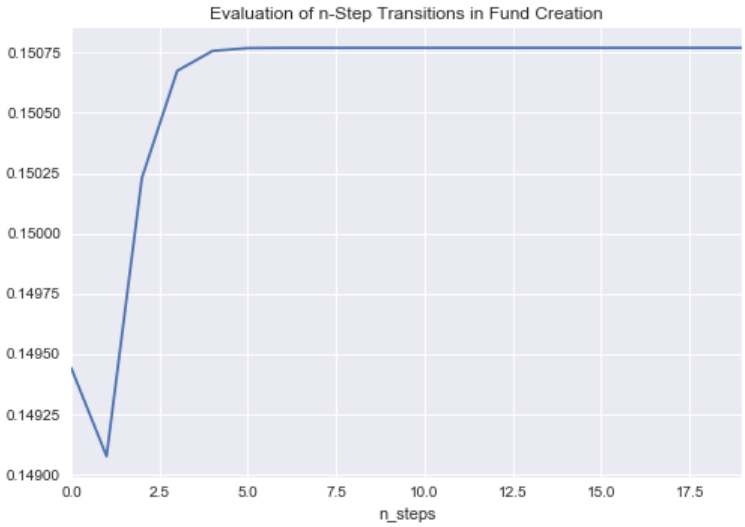


FIGURE 4.27: Step-size impact on cumulative market returns in test period for four-state, Local optimization solution.

4.3 Case Study: Bad Clustering, Algorithm Removed

4.3.1 Motivation

Several of the case studies presented in this section compare the results of generated using different machine learning algorithms to determine the best approach for future use. This case study singles out an algorithm that does not produce results well-suited for this data and approach.

4.3.2 Model Parameters & Specifications

The key features examined include:

- Both signal sets are used to illustrate how the results generated are not useful for this approach. The two signal sets include the Dow Jones Industrial Average (DJIA) set defined in Appendix E and the macro signal set defined in Appendix F.
- Training period: (12/31/2004 - 12/31/2014); Testing period: (12/31/2014 - 2/28/2019). Daily data.
- Feature lookback period: 44 days.

- One (1) period step size in Markov model.
- Clustering assignments created using the **Agglomerative** clustering algorithm using k values from 1 to 9.

4.3.3 Comparison of Results

The results derived through the Agglomerative approach to clustering yield results that were not beneficial to this application of an unsupervised machine learning. A key element this approach is intended to capture is identifying the underlying populations in a larger overall population and gain insight into the characteristics of these underlying populations. This insight is where the value in this methodology lies. The results from this method identified two large, dominant populations that, regardless of the number of clusters designated in the approach,dominated the overall results. Result such as this do not enable the user to identify smaller sub populations and would not be beneficial in this particular application. The results shown in Table 4.4 as well as a visual inspection of the two instances presented in Figures 4.28 and 4.29.

For this reason, this method is removed from consideration for future use in this application.

		Observations Clustered to k								
Signal	k	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
DJIA	0	2299	2298	610	1688	607	1684	4	1678	1674
	1		1	1	3	4	4	1678	3	3
	2			1688	607	1684	604	604	604	604
	3				1	1	1	1	2	4
	4					3	3	3	2	2
	5						3	3	3	3
	6							6	6	6
	7								1	1
	8									2
Macro	0	2306	2304	2297	2285	12	2284	2282	2276	8
	1		2	2	12	2284	8	8	8	2264
	2			7	7	7	7	7	7	7
	3				2	2	2	2	6	12
	4					1	1	1	1	1
	5						4	4	4	4
	6							2	2	2
	7								2	2
	8									6

TABLE 4.4: Distribution of observations as part of the clustering by Agglomerative method. Highly concentrated clusters.

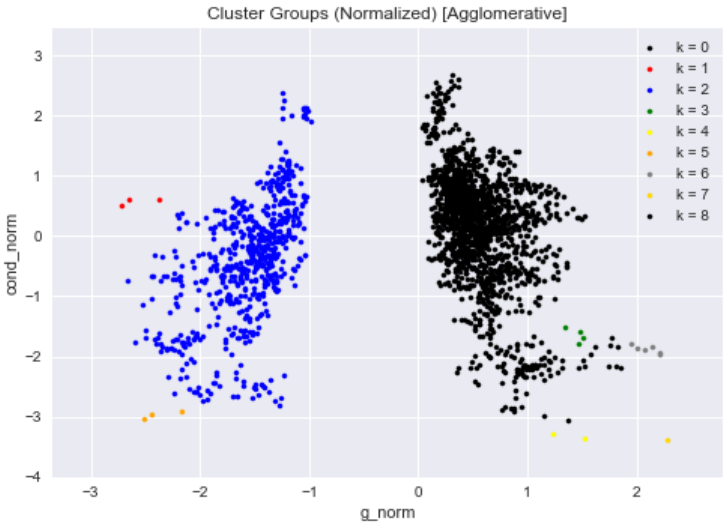


FIGURE 4.28: Allocation of observations in a k=9 schema built on the DJIA signal set.

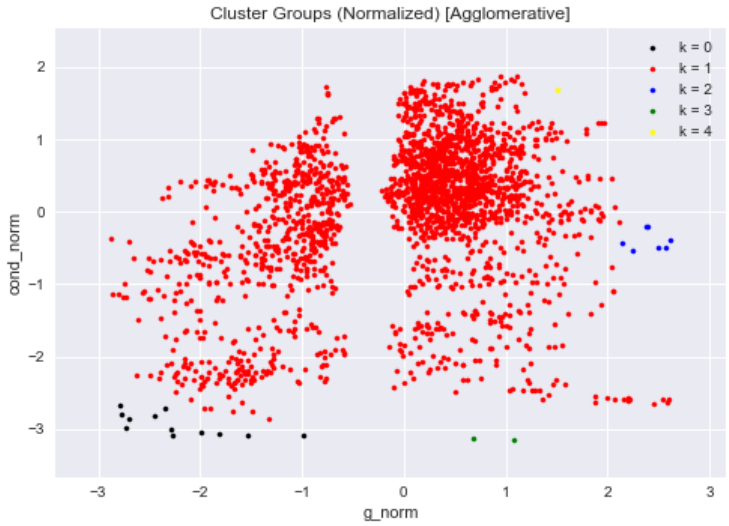


FIGURE 4.29: Allocation of observations in a k=5 schema built on the Macro signal set.

4.4 Case Study: Adaptive Model, Multiple Periods

4.4.1 Motivation

Many investment models perform well in specific environments but are unable to maintain superior performance once the market transitions from conditions favorable to the market. This case study examines the ability of this methodology to adapt to changing market conditions according to the following scenario.

The model is designed using a five-year sample of training data and then evaluated on a 1-year holdout sample. In the first iteration of model, the training and test sets are 2005-2009 and 2010, respectively. The next iteration of model evaluation acts as if it is the end of the holdout period, and the model is reassessed. The training set expands to included what was the testing set (2005-2010) and the next calendar year becomes the holdout set for Iteration 2. For a full schedule of iteration dates, see Table 4.5 below. Note that the testing set begins prior to the holdout calendar year in order to allow the 44-day lookback period for the states measure to be calculated and allow for a full calendar year of results for the model evaluation.

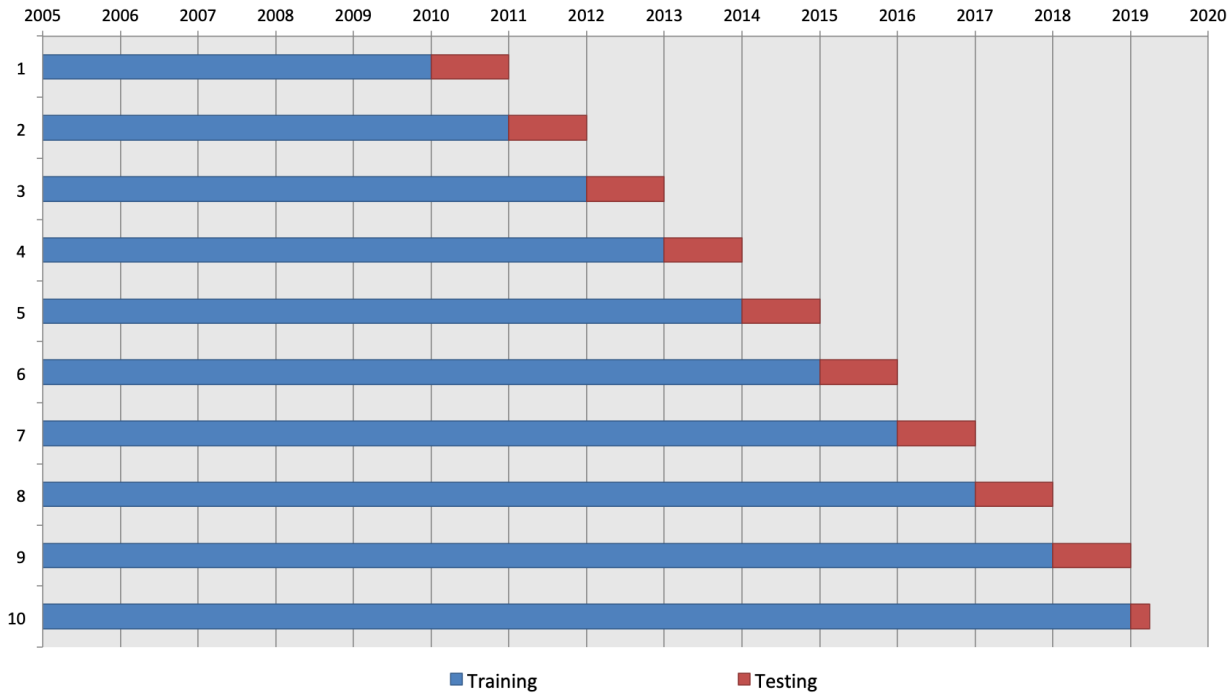


FIGURE 4.30: Summary of training and testing periods for each iteration of evaluation in this case study.

		Iteration of Model Evaluations									
	Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
	2005	O	O	O	O	O	O	O	O	O	O
	2006	O	O	O	O	O	O	O	O	O	O
	2007	O	O	O	O	O	O	O	O	O	O
	2008	O	O	O	O	O	O	O	O	O	O
	2009	O	O	O	O	O	O	O	O	O	O
	2010	X	O	O	O	O	O	O	O	O	O
	2011		X	O	O	O	O	O	O	O	O
	2012			X	O	O	O	O	O	O	O
	2013				X	O	O	O	O	O	O
	2014					X	O	O	O	O	O
	2015						X	O	O	O	O
	2016							X	O	O	O
	2017								X	O	O
	2018									X	O
	2/2019										X
Train	Start	12/31/04	12/31/04	12/31/04	12/31/04	12/31/04	12/31/04	12/31/04	12/31/04	12/31/04	12/31/04
	End	11/6/09	11/12/10	11/11/11	11/9/12	11/8/13	11/7/14	11/6/15	11/11/16	11/10/17	11/9/18
Test	Start	11/6/09	11/12/10	11/11/11	11/9/12	11/8/13	11/7/14	11/6/15	11/11/16	11/10/17	11/9/18
	End	12/31/10	12/31/11	12/31/12	12/31/13	12/31/14	12/31/15	12/31/16	12/31/17	12/31/18	2/28/19

TABLE 4.5: Summary of rolling training and testing periods. "O" represents a training period; "X" represents the holdout test period.

4.4.2 Model Parameters & Specifications

The key features examined include:

- The signal sets include is the **Macro** set defined in Appendix F.
- Feature lookback period: 7 weeks.
- One (1) period step size in Markov model.
- Clustering by k-Means Algorithm using k values from 1 to 9.

4.4.3 Comparison of Results

This case study is aimed at showing the adaptability of the state-based model and how it can change with shifting financial market conditions and environments. It is also intended to show the value of evaluating more data within the training set in order to better understand the diverse group of states and model the states accordingly. In the first instance, Iteration 1, the results are shown in Figure 4.31.

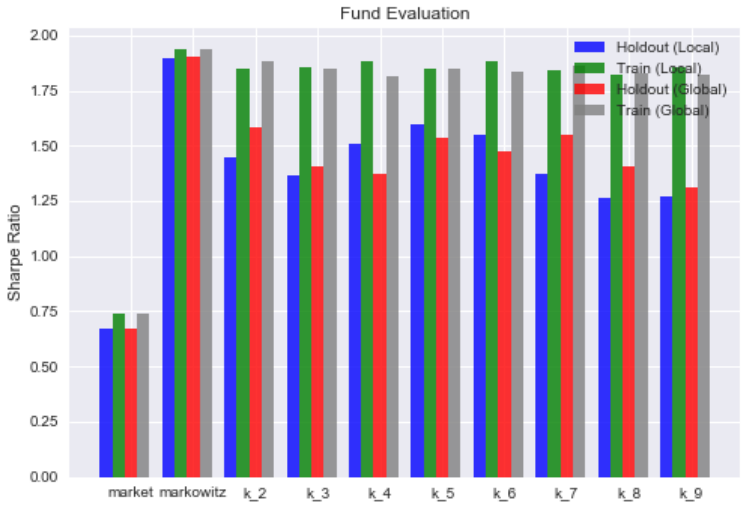


FIGURE 4.31: Sharpe Ratio results for Iteration 1.

All of the optimization-based models exhibit better risk-adjusted performance than market; however, the one-state Markowitz model outperformed all multi-state models in this period. A comparable result is found to exist in the Iteration 2 in Figure 4.32 results as

well- all optimization approaches outperform market with higher risk-adjusted performance, and the Markowitz results are superior for both the local and global optimization than all of the multi-state models.

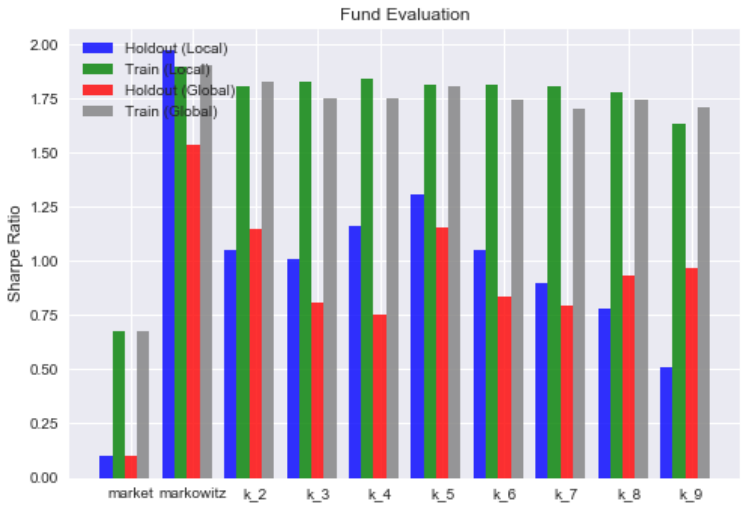


FIGURE 4.32: Sharpe Ratio results for Iteration 2.

The results for Iteration 3 shown in Figure 4.33 display results in which all multi-state strategies perform better than both the market and a Markowitz, single-state solution. The results for this iteration are also interesting in that performance results for the testing region out perform results from the training set. In prior iterations we saw slippage from the results achieved in the initial training set used to fit the allocations when the model was applied to data from the test period. The reduction in Sharpe Ratios from the training set to the testing was attributed to overfitting in the initial model; however, the out performance in the testing set cannot be attributed to this reason.

Additionally, the local solution results exhibited superior results to the Global results. In particular the nine-state model, which represents the most complex model, demonstrated the best Sharpe Ratio performance. In addition to a better risk-return Sharpe Ratio performance than the single-state Markowitz portfolio, all of the multi-state portfolio demonstrated higher returns as well.

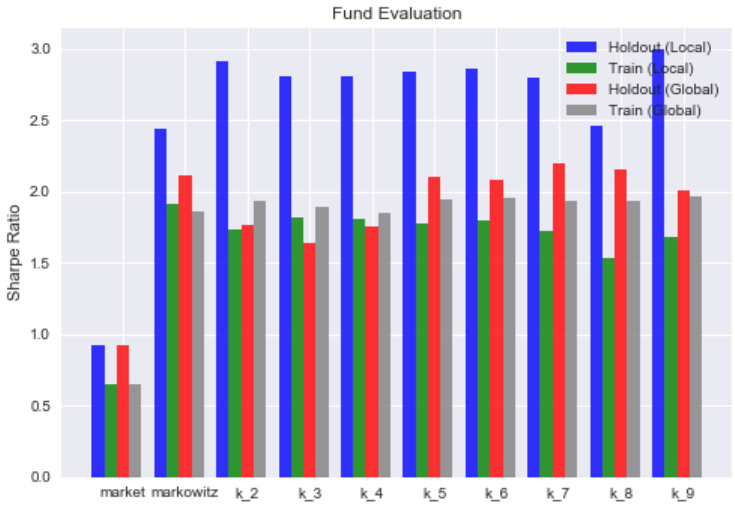


FIGURE 4.33: Sharpe Ratio results for Iteration 3.

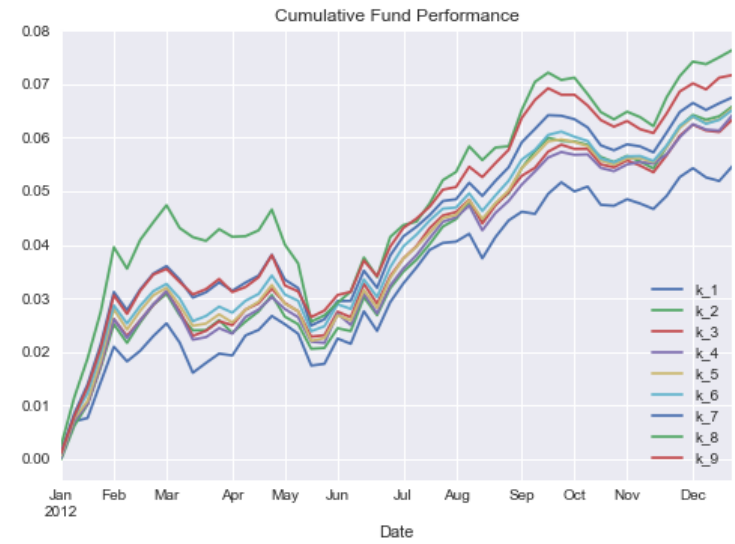


FIGURE 4.34: Fund performance in testing region for all portfolios during Iteration 3.

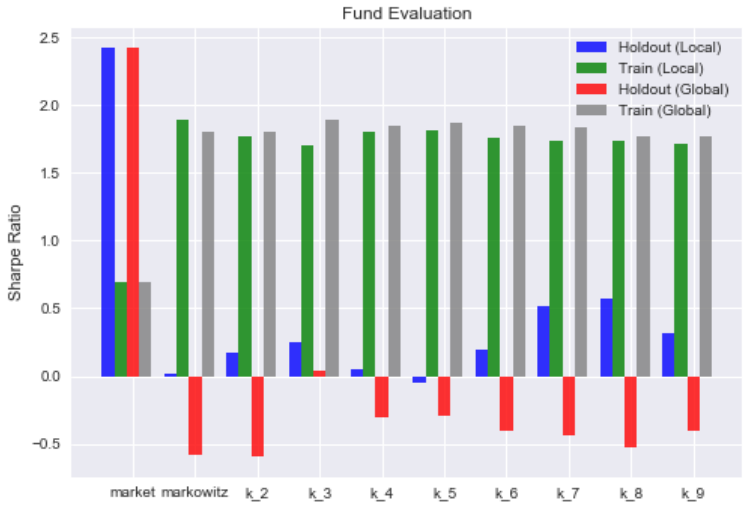


FIGURE 4.35: Sharpe Ratio results for Iteration 4.

Iteration 4 shows an extreme drop off in performance for all optimization-based portfolio results in the testing set, particularly with respect to performance relative to market. In diagnosing the cause for this under performance relative to market, the first area of investigation is the state specific asset allocation's. These results are shown in Table 4.7. For both the Local and the Global solutions a substantial portion of the portfolio is dedicated to the bond asset, AGG. This asset is the least volatile on the 6 securities available in the investment universe. For this reason the optimization procedure attempting to reduce the Sharpe Ratio is motivated to allocate heavily to this asset when optimizing the objective function.

Method	State	EFA	EEM	AGG	RUT	IXIC	GSPC
Local	0	0.04	0.14	0.75	0.00	0.03	0.03
	1	0.00	0.00	1.00	0.00	0.00	0.00
	2	0.10	0.01	0.75	0.00	0.23	0.00
	3	0.00	0.00	1.00	0.00	0.00	0.00
Global	0	0.00	0.44	0.56	0.00	0.00	0.00
	1	0.00	0.00	0.98	0.00	0.02	0.00
	2	0.00	0.00	1.00	0.00	0.00	0.00
	3	0.00	0.00	1.00	0.00	0.00	0.00

TABLE 4.6: Portfolio weights for each market state based on both optimization approaches in a four-state model from Fig. 4.35. No allocation restrictions.

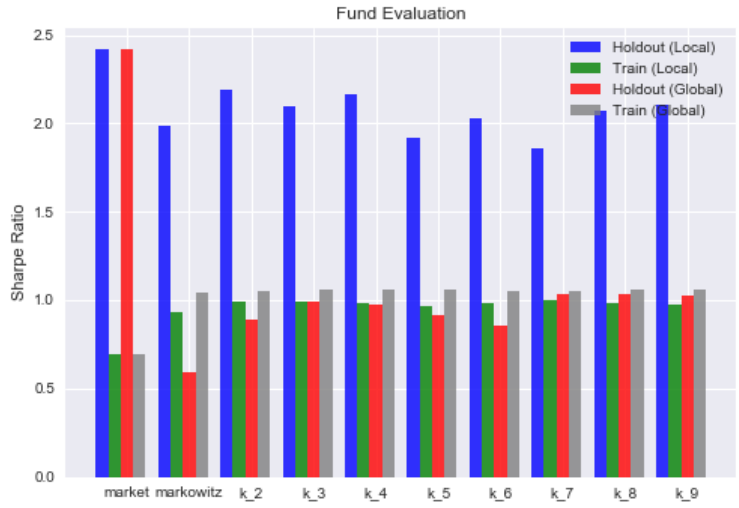


FIGURE 4.36: Sharpe Ratio results for Iteration 4 with a 40% maximum allocation constraint imposed.

During this test period, the asset AGG declined in a manner not seen in the training data. This declining behavior in the AGG asset was not observed in training and therefore was not properly incorporated in the state definitions. This outcome prompted the issue of overlays to the optimization process which make a portfolio more realistic to a real-world investor offering a more practical solution to the theoretical result.

Method	State	EFA	EEM	AGG	RUT	IXIC	GSPC
Local	0	0.00	0.00	0.40	0.00	0.20	0.40
	1	0.14	0.00	0.40	0.00	0.13	0.32
	2	0.00	0.00	0.40	0.20	0.00	0.40
	3	0.04	0.06	0.40	0.00	0.18	0.32
Global	0	0.00	0.00	0.40	0.20	0.40	0.00
	1	0.00	0.33	0.40	0.00	0.27	0.00
	2	0.00	0.40	0.40	0.00	0.20	0.00
	3	0.00	0.40	0.40	0.00	0.20	0.00

TABLE 4.7: Portfolio weights for each market state based on both optimization approaches in a four-state model from Fig. 4.36. 40% max allocation.

More specifically, a real-world portfolio manager would not invest so heavily in a single asset, therefore a 'diversification constraint' is applied to the asset allocation selection optimization imposing a 40% maximum allocation to any individual asset. The value of 40% was selected based on industry experience that this would be a reasonable maximum value for a possible portfolio allocation. The results of imposing this constraint are shown

in Figure 4.36 and the updated portfolio weights are shown in Table 4.7.

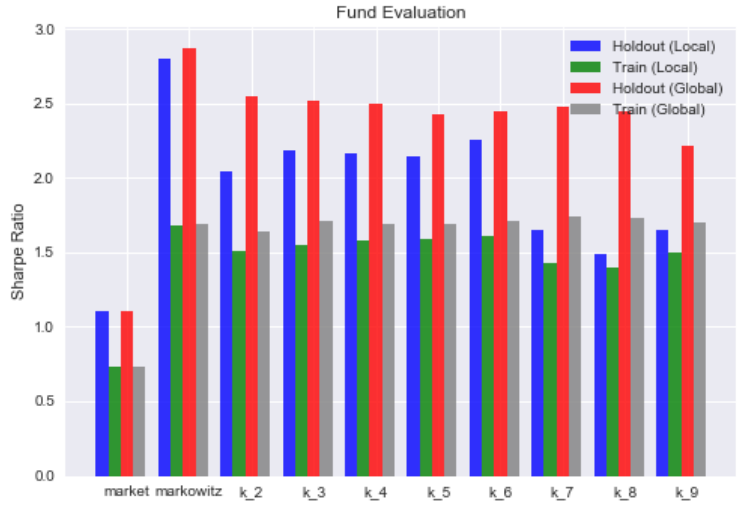


FIGURE 4.37: Sharpe Ratio results for Iteration 5.

The results for Iteration 5 show better risk-adjusted performance in the test region from the results achieved in the training set. In the single-state Markowitz portfolio both the Local and Global solutions achieve similar risk-adjusted performance results, which outperformed all of the multi-state models. In this example, the Global solution for the multi-state models produced superior performances to the Local solution in the less complex models, such as the two and three-state model variants.

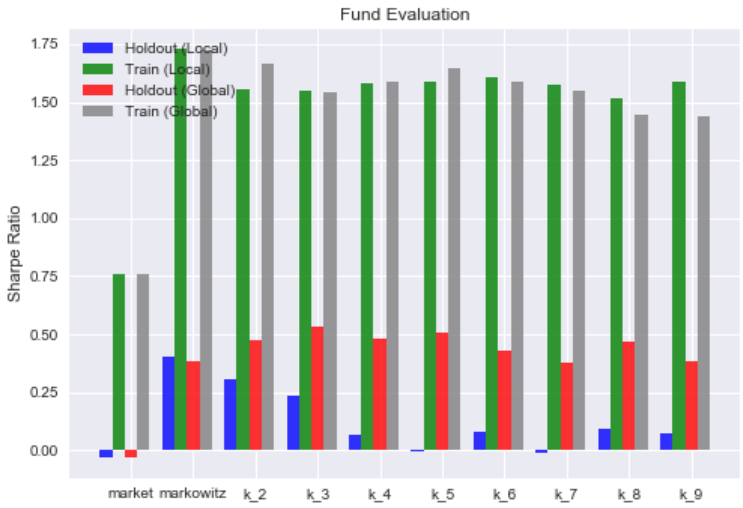


FIGURE 4.38: Sharpe Ratio results for Iteration 6.

Iteration 6 results show results slip noticeably from the training set to the test set. The test set for this particular iteration saw a noticeable decline in the risk-return characteristics of the market. In the training set the market had a Sharpe Ratio of approximately 0.75 as compared to the ratio in the testing center which is exhibited a slightly negative Sharpe Ratio. The optimization-based models also saw a drop in their risk-return profiles from the training set to the testing set. The single-state Markowitz portfolio produced superior performance in the training sets relative to the multi-state models; however, the single-state model saw a greater decline in results as the model was applied to the testing set. All of the multi-state models produced superior risk-return profiles to both market and the single-state model.

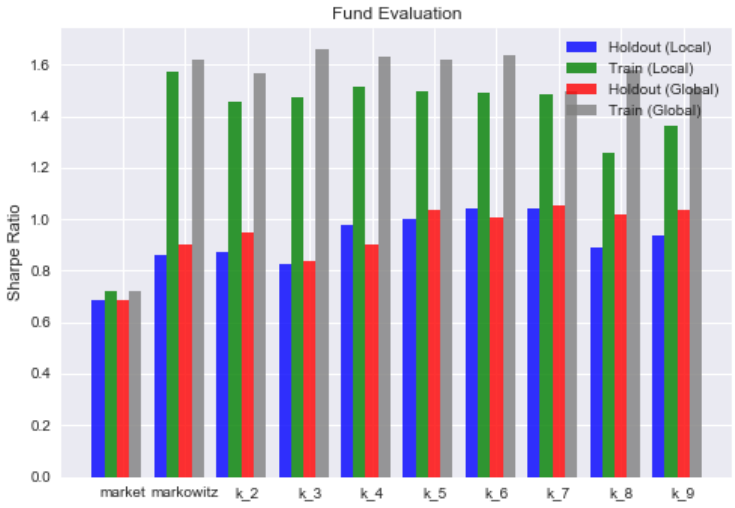


FIGURE 4.39: Sharpe Ratio results for Iteration 7.

The market produced similar risk-return characteristics in the training set and within the test period. All of the optimization models saw a drop off in the risk-return characteristics for both the Local and Global solutions from the results created on the data from the training set and then apply to the hold out sample from the testing set. This could be attributed to an element of overfitting the model parameters. In this iteration the multi-state models produced superior performance to the single-state Markowitz portfolio as well as to the market. More complex models, such as the six and seven-state to model variants performed better than less complex models. The superior performance of the six and seven state models apply to both Local and Global solution results.

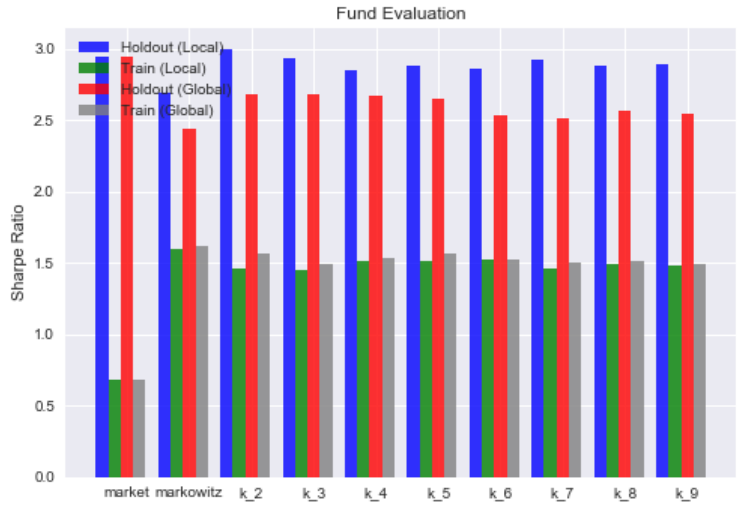


FIGURE 4.40: Sharpe Ratio results for Iteration 8.

In a situation akin to the results achieved in Iteration 5, the results from Iteration 8 also display a superior risk-return profiles in the holdout, test set than those achieved during the training period. Instances such as this appear to be connected with strong advances in the equity markets. These periods are characterized by high returns and low volatility as reflected by a Sharp Ratio of approximately three. The multi-state models, particularly the solutions created by the Local optimization, outperform the single-state Markowitz model results. The Local results for the two-state model offered the best risk-return profile. All of the Local solutions for the multi-state model outperform their Global solution counterparts. In spite of the global solutions for the multi-state models failing to outperform the local versions, the Global solutions still offer higher risk return characteristics than the Markowitz model.

Iteration 9 experienced a drastic drop in risk-adjusted performance from the training set to the testing set like the previous example, Iteration 4. This under performance coincides with a period of decline in the equity markets. Local optimization solutions did not achieve positive results in this time period. The global solutions were able to achieve positive Mark portfolio performance. The most effective model was a complex version, specifically an eight-state model. The differentiating characteristics between the Local and Global optimization solution in this case was not the failure of the model to adapt to

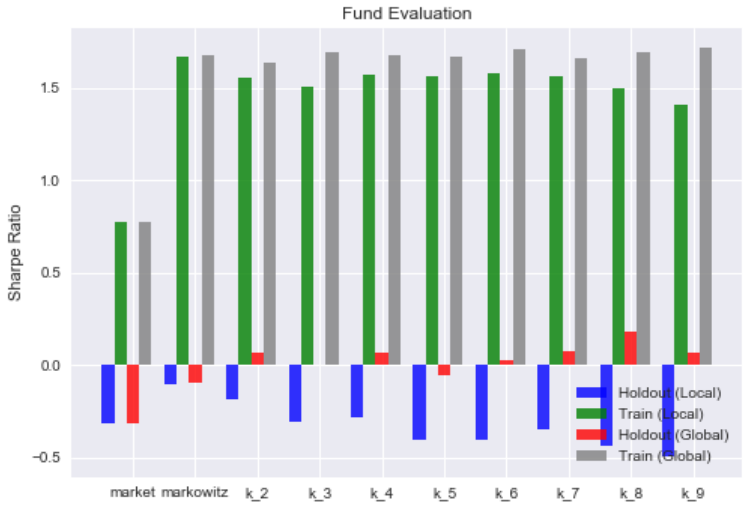


FIGURE 4.41: Sharpe Ratio results for Iteration 9.

the 'market crash' state experience from October to December 2018. Both versions of the model identify the abrupt market decline and assume a conservative allocation position. The difference in model performance came earlier in the year when the Local optimization models failed to capture positive returns when the equity markets were experiencing a period of advance.

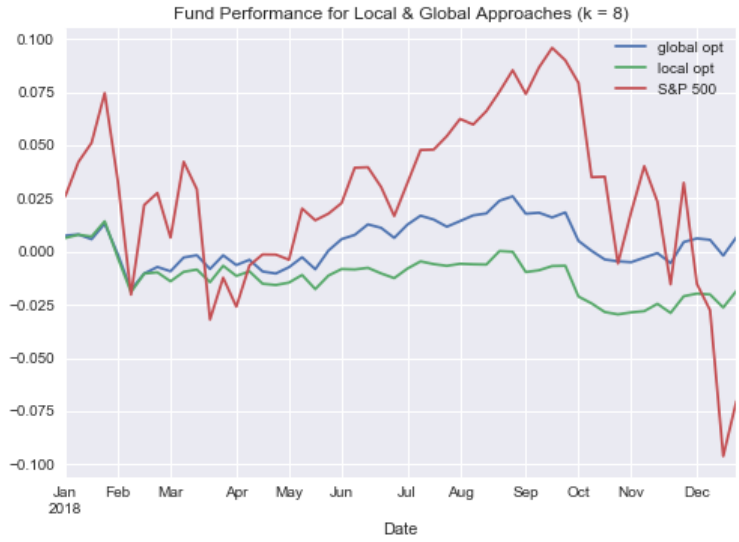


FIGURE 4.42: Unleverd performance of both Local and Global portfolios in the testing set for Iteration 9. Local solution fails to capture gains during market advance.

Figure 4.42 shows that the Global model was advancing with the market while the Local portfolio was moving sideways and not achieving portfolio gains. From October through the end of December as the equity markets were experiencing a large decline, both the Global and Local portfolios experienced small declines and ultimately begin to advance. The key difference, as previously stated, in the portfolio performance is due to the failure of the Local portfolio to capture the gains in the late spring and early summer. In examining the portfolio allocations shown in Figure 4.43, this is due in large part to the allocation differences to the NASDAQ Composite security (IXIC).

Method	**State**	**EFA**	**EEM**	**AGG**	**RUT**	**IXIC**	**GSPC**
Local	0	0.04	0	0.83	0.07	0.06	0
	1	0.14	0.25	0.6	0	0	0.01
	2	0.04	0.02	0.58	0	0.36	0
	3	0	0	1	0	0	0
	4	0	0	1	0	0	0
	5	0	0	1	0	0	0
	6	0.01	0.05	0.59	0	0.35	0
	7	0.11	0	0.76	0	0.09	0.03
Global	0	0	0	1.00	0	0	0
	1	0	0	1.00	0	0	0
	2	0	0	1.00	0	0	0
	3	0	0	1.00	0	0	0
	4	0	0	1.00	0	0	0
	5	0	0	1.00	0	0	0
	6	0	0	0	0	1.00	0
	7	0	0	0.55	0	0.45	0

TABLE 4.8: .

Global states six and seven take large positions in the IXIC as shown in Table 4.8. During the periods of market advance the market was largely in these two states. While the Local portfolios have allocations in the NASDAQ, the Global portfolios have much larger allocations to this product, which allowed the Global portfolio to benefit more from the broad-based market advance.

The performance of the Global portfolio is shown in Figure 4.44. The leveraged portfolio is highly correlated to markets through the first 10 months of 2019 at which point the Global portfolio disconnects from market and advances while the broader markets are

	Global						Local						Difference (Global minus Local)					
Date	EFA	EEM	AGG	RUT	IXIC	GSPC	EFA	EEM	AGG	RUT	IXIC	GSPC	EFA	EEM	AGG	RUT	IXIC	GSPC
1/5/18	0.00	0.00	0.72	0.00	0.28	0.00	0.06	0.06	0.75	0.01	0.11	0.01	-0.06	-0.06	-0.03	-0.01	0.17	-0.01
1/12/18	0.00	0.00	0.84	0.00	0.16	0.00	0.06	0.06	0.76	0.01	0.11	0.01	-0.06	-0.06	0.08	-0.01	0.06	-0.01
1/19/18	0.00	0.00	0.81	0.00	0.19	0.00	0.07	0.06	0.75	0.01	0.10	0.01	-0.07	-0.06	0.07	-0.01	0.08	-0.01
1/26/18	0.00	0.00	0.74	0.00	0.26	0.00	0.06	0.06	0.74	0.01	0.12	0.01	-0.06	-0.06	0.00	-0.01	0.14	-0.01
2/2/18	0.00	0.00	0.79	0.00	0.21	0.00	0.06	0.05	0.74	0.01	0.13	0.01	-0.06	-0.05	0.05	-0.01	0.08	-0.01
2/9/18	0.00	0.00	0.73	0.00	0.27	0.00	0.06	0.06	0.75	0.01	0.12	0.01	-0.06	-0.06	-0.02	-0.01	0.15	-0.01
2/16/18	0.00	0.00	0.86	0.00	0.14	0.00	0.04	0.04	0.84	0.01	0.07	0.01	-0.04	-0.04	0.02	-0.01	0.07	-0.01
2/23/18	0.00	0.00	0.69	0.00	0.31	0.00	0.07	0.06	0.74	0.00	0.12	0.01	-0.07	-0.06	-0.06	0.00	0.20	-0.01
3/2/18	0.00	0.00	0.77	0.00	0.23	0.00	0.06	0.06	0.77	0.01	0.10	0.01	-0.06	-0.06	0.00	-0.01	0.13	-0.01
3/9/18	0.00	0.00	0.83	0.00	0.17	0.00	0.04	0.03	0.85	0.00	0.07	0.01	-0.04	-0.03	-0.02	0.00	0.10	-0.01
3/16/18	0.00	0.00	0.85	0.00	0.15	0.00	0.04	0.04	0.83	0.01	0.07	0.01	-0.04	-0.04	0.02	-0.01	0.08	-0.01
3/23/18	0.00	0.00	0.91	0.00	0.09	0.00	0.03	0.03	0.90	0.00	0.04	0.00	-0.03	-0.03	0.02	0.00	0.05	0.00
3/30/18	0.00	0.00	0.88	0.00	0.11	0.00	0.04	0.03	0.87	0.00	0.05	0.01	-0.04	-0.03	0.02	0.00	0.06	-0.01
4/6/18	0.00	0.00	0.82	0.00	0.18	0.00	0.07	0.07	0.76	0.01	0.10	0.01	-0.07	-0.07	0.06	-0.01	0.09	-0.01
4/13/18	0.00	0.00	0.88	0.00	0.12	0.00	0.04	0.04	0.86	0.01	0.06	0.01	-0.04	-0.04	0.02	-0.01	0.06	-0.01
4/20/18	0.00	0.00	0.85	0.00	0.15	0.00	0.04	0.04	0.83	0.01	0.07	0.01	-0.04	-0.04	0.02	-0.01	0.08	-0.01
4/27/18	0.00	0.00	0.78	0.00	0.22	0.00	0.06	0.06	0.77	0.01	0.10	0.01	-0.06	-0.05	0.01	-0.01	0.12	-0.01
5/4/18	0.00	0.00	0.84	0.00	0.16	0.00	0.04	0.04	0.83	0.01	0.07	0.01	-0.04	-0.04	0.01	-0.01	0.09	-0.01
5/11/18	0.00	0.00	0.82	0.00	0.18	0.00	0.04	0.04	0.83	0.00	0.07	0.01	-0.04	-0.04	-0.01	0.00	0.10	-0.01
5/18/18	0.00	0.00	0.71	0.00	0.29	0.00	0.06	0.06	0.74	0.01	0.12	0.01	-0.06	-0.06	-0.03	-0.01	0.17	-0.01
5/25/18	0.00	0.00	0.65	0.00	0.35	0.00	0.07	0.07	0.70	0.00	0.14	0.01	-0.07	-0.07	-0.06	0.00	0.21	-0.01
6/1/18	0.00	0.00	0.76	0.00	0.24	0.00	0.06	0.05	0.76	0.01	0.11	0.01	-0.06	-0.05	0.00	-0.01	0.13	-0.01
6/8/18	0.00	0.00	0.66	0.00	0.34	0.00	0.07	0.06	0.74	0.00	0.12	0.01	-0.07	-0.06	-0.07	0.00	0.22	-0.01
6/15/18	0.00	0.00	0.69	0.00	0.31	0.00	0.06	0.06	0.74	0.00	0.12	0.01	-0.06	-0.06	-0.05	0.00	0.19	-0.01
6/22/18	0.00	0.00	0.71	0.00	0.29	0.00	0.06	0.06	0.74	0.01	0.12	0.01	-0.06	-0.06	-0.03	0.00	0.17	-0.01
6/29/18	0.00	0.00	0.70	0.00	0.30	0.00	0.07	0.06	0.73	0.00	0.13	0.01	-0.07	-0.06	-0.03	0.00	0.18	-0.01
7/6/18	0.00	0.00	0.85	0.00	0.15	0.00	0.03	0.02	0.88	0.00	0.05	0.01	-0.03	-0.02	-0.03	0.00	0.10	-0.01
7/13/18	0.00	0.00	0.81	0.00	0.19	0.00	0.07	0.08	0.73	0.01	0.09	0.01	-0.07	-0.08	0.07	-0.01	0.10	-0.01
7/20/18	0.00	0.00	0.69	0.00	0.31	0.00	0.07	0.06	0.73	0.00	0.13	0.01	-0.07	-0.06	-0.04	0.00	0.18	-0.01
7/27/18	0.00	0.00	0.82	0.00	0.17	0.00	0.06	0.06	0.76	0.01	0.10	0.01	-0.06	-0.06	0.06	-0.01	0.08	-0.01
8/3/18	0.00	0.00	0.84	0.00	0.16	0.00	0.05	0.04	0.82	0.01	0.07	0.01	-0.05	-0.04	0.02	-0.01	0.08	-0.01
8/10/18	0.00	0.00	0.81	0.00	0.19	0.00	0.06	0.06	0.78	0.01	0.09	0.01	-0.06	-0.06	0.03	-0.01	0.10	-0.01
8/17/18	0.00	0.00	0.75	0.00	0.25	0.00	0.06	0.05	0.77	0.01	0.10	0.01	-0.06	-0.05	-0.01	-0.01	0.14	-0.01
8/24/18	0.00	0.00	0.76	0.00	0.24	0.00	0.06	0.06	0.76	0.01	0.11	0.01	-0.06	-0.06	0.01	-0.01	0.13	-0.01
8/31/18	0.00	0.00	0.79	0.00	0.21	0.00	0.06	0.06	0.76	0.01	0.10	0.01	-0.06	-0.06	0.03	-0.01	0.11	-0.01
9/7/18	0.00	0.00	0.80	0.00	0.20	0.00	0.06	0.06	0.76	0.01	0.10	0.01	-0.06	-0.06	0.04	-0.01	0.11	-0.01
9/14/18	0.00	0.00	0.88	0.00	0.12	0.00	0.04	0.04	0.86	0.01	0.06	0.01	-0.04	-0.04	0.02	-0.01	0.06	-0.01
9/21/18	0.00	0.00	0.79	0.00	0.21	0.00	0.07	0.06	0.75	0.01	0.10	0.01	-0.07	-0.06	0.03	-0.01	0.11	-0.01
9/28/18	0.00	0.00	0.75	0.00	0.25	0.00	0.06	0.06	0.76	0.01	0.11	0.01	-0.06	-0.06	-0.02	-0.01	0.15	-0.01
10/5/18	0.00	0.00	0.82	0.00	0.18	0.00	0.06	0.06	0.77	0.01	0.09	0.01	-0.06	-0.06	0.05	-0.01	0.09	-0.01
10/12/18	0.00	0.00	0.79	0.00	0.21	0.00	0.05	0.04	0.80	0.01	0.09	0.01	-0.05	-0.04	-0.01	0.00	0.12	-0.01
10/19/18	0.00	0.00	0.85	0.00	0.15	0.00	0.04	0.04	0.84	0.01	0.07	0.01	-0.04	-0.04	0.01	-0.01	0.08	-0.01
10/26/18	0.00	0.00	0.86	0.00	0.14	0.00	0.04	0.04	0.84	0.01	0.07	0.01	-0.04	-0.04	0.02	-0.01	0.07	-0.01
11/2/18	0.00	0.00	0.83	0.00	0.17	0.00	0.04	0.04	0.83	0.01	0.07	0.01	-0.04	-0.04	0.00	-0.01	0.09	-0.01
11/9/18	0.00	0.00	0.81	0.00	0.19	0.00	0.05	0.04	0.81	0.01	0.09	0.01	-0.05	-0.04	0.00	-0.01	0.11	-0.01
11/16/18	0.00	0.00	0.89	0.00	0.11	0.00	0.04	0.03	0.86	0.01	0.06	0.01	-0.04	-0.03	0.03	-0.01	0.05	-0.01
11/23/18	0.00	0.00	0.89	0.00	0.11	0.00	0.04	0.03	0.86	0.01	0.06	0.01	-0.04	-0.03	0.03	-0.01	0.05	-0.01
11/30/18	0.00	0.00	0.86	0.00	0.14	0.00	0.04	0.04	0.84	0.01	0.07	0.01	-0.04	-0.04	0.02	-0.01	0.08	-0.01
12/7/18	0.00	0.00	0.85	0.00	0.15	0.00	0.05	0.04	0.82	0.01	0.08	0.01	-0.05	-0.04	0.04	-0.01	0.07	-0.01
12/14/18	0.00	0.00	0.85	0.00	0.15	0.00	0.04	0.04	0.84	0.01	0.07	0.01	-0.04	-0.04	0.01	-0.01	0.08	-0.01
12/21/18	0.00	0.00	0.87	0.00	0.13	0.00	0.04	0.04	0.83	0.01	0.07	0.01	-0.04	-0.04	0.04	-0.01	0.06	-0.01
12/28/18	0.00	0.00	0.87	0.00	0.13	0.00	0.04	0.04	0.84	0.01	0.07	0.01	-0.04	-0.04	0.03	-0.01	0.06	-0.01

FIGURE 4.43: Comparison between Local and Global models of overall portfolio allocations for eight-state model. Red values represent lower allocation in Local allocation.

declining. This break from the correlation with market was due to a shift in the probability of state membership which led to larger positions taken in the bond product, AGG.

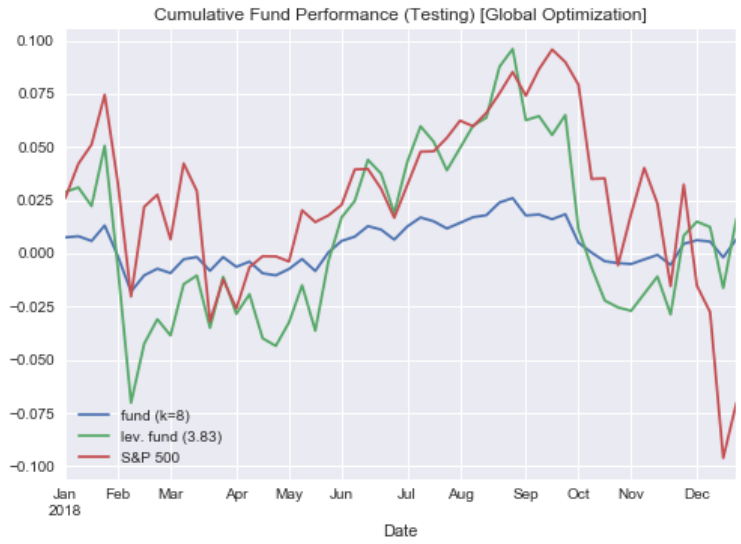


FIGURE 4.44

Iteration 10 is a short term snapshot of the first two months of 2019. During this period the market experienced a steady advance, which is in turn resulted in an exceptionally high Sharpe Ratio. It is extremely unlikely that this risk-return profile will persist throughout the year; however, the portfolio risk-return characteristics shown in Figure 4.45 and the overall performance in Figure 4.46 were included to illustrate the current structure of the portfolio.

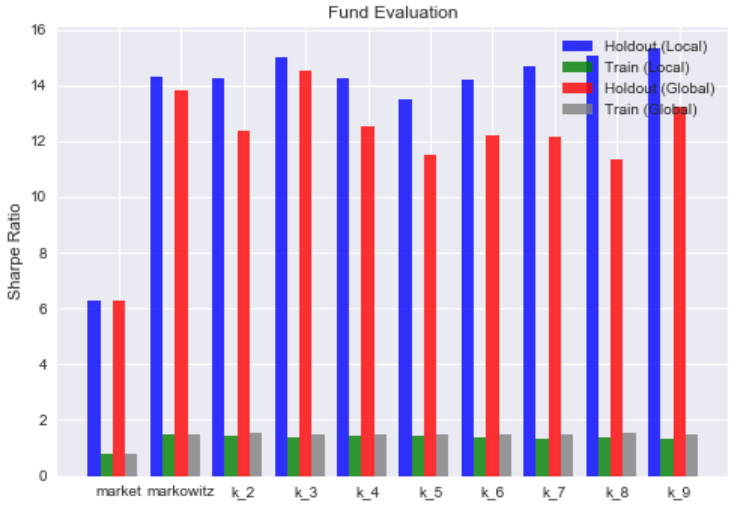


FIGURE 4.45: Sharpe Ratio results for Iteration 10.

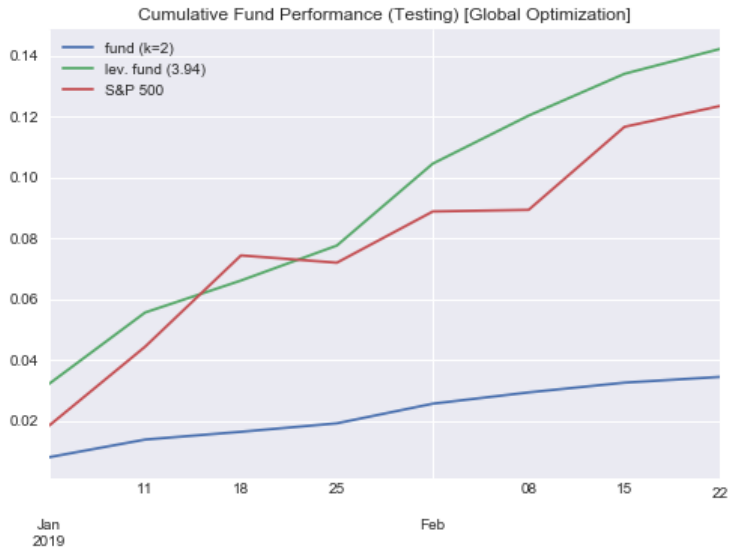


FIGURE 4.46: Performance of a two-state Local model for Iteration 10.

4.5 Case Study: Impact of Allocation Constraints

4.5.1 Motivation

To determine the impact of the imposing a restriction on the amount allocated to an asset.

4.5.2 Model Parameters & Specifications

The key features examined include:

- The signal sets include is the **Macro** set defined in Appendix F.
- Feature lookback period: 7 weeks.
- One (1) period step size in Markov model.
- Clustering by k-Means Algorithm using k values from 1 to 9.

4.5.3 Comparison of Results

Iteration 4

This case study takes a closer look at the impact of applying a 40% maximum allocation constraint to any asset in the optimization-based weight assignment to the investment portfolios. This constraint is motivated by the results found in Case Study **??**: Iteration 4 in which a noticeable performance drop off occurred from the training period to the testing as a result of a performance change in an integral asset incorporated in the portfolio set. As a result of the objective function aiming to maximize the risk-return profile of the portfolio, Sharpe Ratio, the algorithm allocates assets to the less volatile assets.

Heavy allocations were made to this asset in the training region, which led to the lower Sharpe Ratio for numerous portfolios. Unfortunately the heavy allocation is susceptible to variations in the behavior of an individual asset. For this reason diversification of assets is a desirable attribute in portfolio management and should be considered in more detail. In this case study we are examining means to alleviate the effect and impact of this

situation by limiting the allocation allowed to any single asset by imposing a diversification constraint. It is expected to have a negative impact on the objective function value in the training set; however, it can be considered an attempt to alleviate the diversification risk experienced in this example.

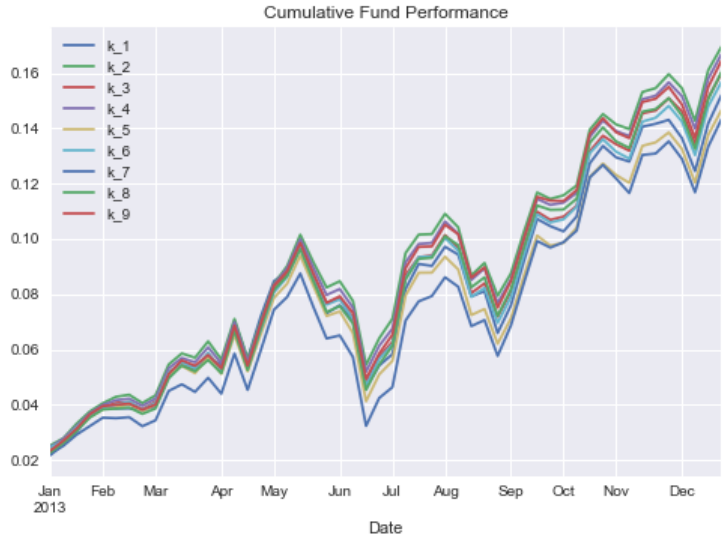


FIGURE 4.47: Performance of optimization-based models with 40% ceiling on allocation to any asset in test period (Iteration 4).

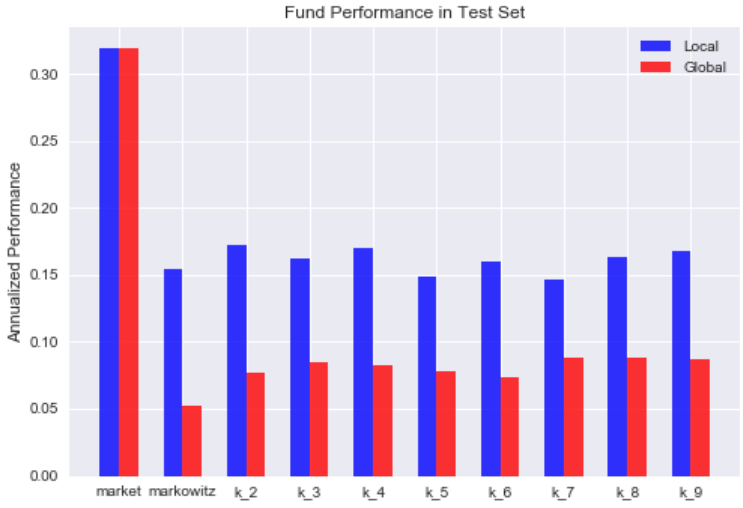


FIGURE 4.48: Summary performance of optimization-based models with 40% ceiling on allocation to any asset in test period.

Consider the results in Table 4.9. There is a decline in the Sharpe Ratio and returns experienced in the training center due to the fact that the asset AGG experience negative

returns. The impact of this negative performance is amplified in comparison to the superior performance of the equities markets, which represent the benchmark asset. By imposing the 40% constraint, assets are diversified more investment portfolio, which in turn leads to improved risk return profiles, particularly for the global optimization solution.

Iteration 4

		No Constraints				**Max 40% Allocation**			
		Training		**Testing**		**Training**		**Testing**	
	k	SR_a	r_a	SR_a	r_a	SR_a	r_a	SR_a	r_a
Market		0.695	0.134	2.421	0.319	0.695	0.134	2.421	0.319
Local	1	1.886	0.077	0.016	0.001	0.934	0.106	1.984	0.155
	2	1.773	0.076	0.178	0.008	0.995	0.119	2.190	0.172
	3	1.699	0.08	0.254	0.011	0.993	0.115	2.098	0.162
	4	1.806	0.076	0.054	0.002	0.986	0.115	2.165	0.169
	5	1.817	0.074	-0.049	-0.002	0.963	0.114	1.917	0.149
	6	1.755	0.079	0.196	0.009	0.980	0.116	2.032	0.159
	7	1.733	0.079	0.514	0.022	1.000	0.121	1.862	0.146
	8	1.736	0.076	0.577	0.025	0.986	0.116	2.069	0.163
	9	1.71	0.077	0.318	0.014	0.978	0.115	2.107	0.167
Global	1	1.803	0.069	-0.577	-0.023	1.038	0.161	0.59	0.053
	2	1.804	0.068	-0.592	-0.024	1.052	0.154	0.892	0.077
	3	1.896	0.073	0.036	0.001	1.058	0.155	0.993	0.084
	4	1.851	0.078	-0.299	-0.013	1.058	0.155	0.970	0.083
	5	1.868	0.078	-0.289	-0.012	1.055	0.154	0.910	0.077
	6	1.846	0.075	-0.402	-0.017	1.049	0.154	0.858	0.073
	7	1.838	0.073	-0.438	-0.018	1.053	0.153	1.033	0.088
	8	1.764	0.069	-0.524	-0.021	1.056	0.153	1.035	0.088
	9	1.764	0.067	-0.407	-0.016	1.056	0.152	1.027	0.087

TABLE 4.9: Comparison of model performance in Iteration 4 when 40% max allocation constraint imposed.

A second instance, Iteration 8, was examined to observe the impact of imposing the 40% on allocation weights. The iteration was selected because the state-based models showed positive performance in excess of market in addition to having the state-based models outperform the single-state Markowitz models. As shown in Table 4.10 by imposing the 40% constraint on the allocation amounts, the Sharpe Ratio achieved in the testing time

period outperforms the results achieved in the training. Unfortunately, the multi-state strategies did not achieve Sharpe Ratios as high as the results achieved in the single-state Markowitz model. As in the previous example for Iteration 4, the maximum possible allocation was made to the low-volatility, bond fund AGG in most cases; however, the constraint forces a diversification of assets, which is representative of a robust model that is less likely to be overfit to the training set of data, which can be applied to multiple time periods with promising results.

Iteration 8

		No Constraints				**Max 40% Allocation**			
		Training		**Testing**		**Training**		**Testing**	
	k	SR_a	r_a	SR_a	r_a	SR_a	r_a	SR_a	r_a
Market		0.675	0.113	2.95	0.196	0.675	0.113	2.95	0.196
Local	1	1.594	0.064	2.697	0.076	0.873	0.086	3.732	0.14
	2	1.456	0.061	3.001	0.086	0.948	0.094	3.352	0.146
	3	1.452	0.062	2.936	0.085	0.934	0.092	3.541	0.144
	4	1.507	0.06	2.853	0.081	0.915	0.090	3.625	0.143
	5	1.513	0.061	2.883	0.079	0.903	0.089	3.667	0.141
	6	1.526	0.061	2.858	0.078	0.905	0.092	3.216	0.132
	7	1.456	0.061	2.927	0.081	0.922	0.095	3.167	0.136
	8	1.494	0.06	2.879	0.079	0.934	0.095	3.567	0.150
	9	1.476	0.06	2.893	0.079	0.925	0.096	3.178	0.140
Global	1	1.618	0.059	2.435	0.065	0.984	0.103	3.15	0.163
	2	1.57	0.059	2.677	0.075	0.984	0.103	3.15	0.163
	3	1.489	0.059	2.68	0.078	0.984	0.103	3.15	0.163
	4	1.529	0.06	2.675	0.077	0.984	0.103	3.147	0.163
	5	1.560	0.061	2.651	0.075	0.981	0.103	3.139	0.161
	6	1.524	0.058	2.53	0.070	0.977	0.103	3.126	0.159
	7	1.497	0.056	2.517	0.07	0.979	0.104	3.03	0.160
	8	1.516	0.058	2.571	0.071	0.979	0.104	3.055	0.160
	9	1.492	0.057	2.546	0.071	0.982	0.104	3.108	0.162

TABLE 4.10: Comparison of model performance in Iteration 8 when 40% max allocation constraint imposed.

The annualized performance of each configuration during the holdout period is shown in Figure 4.50, and the aggregate performance is displayed again in Figure 4.49 but in this view it shows the overall returns relative to the market. The performance during this

period by market outperforms the state-based strategies but with more market risk and volatility. If the leverage factor were applied to these results, returns in excess of market could be achieved. See the example of the Global 8-state model shown in Figure XX. A leverage factor of 1.55 is applied to the portfolio.

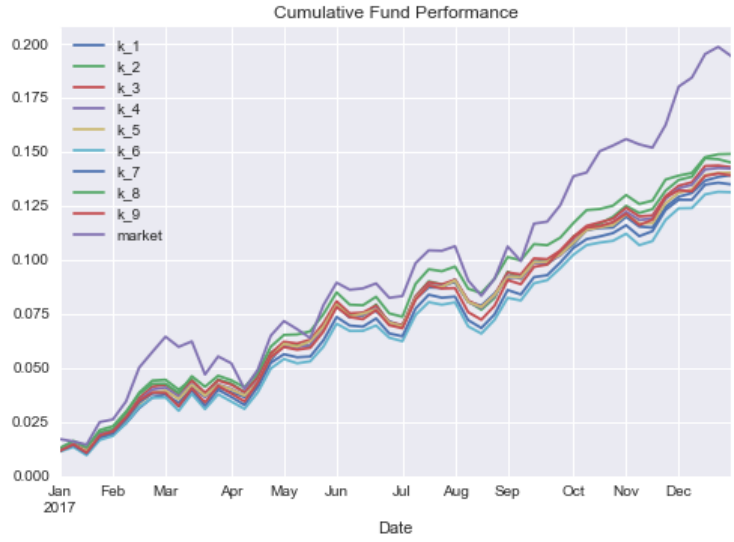


FIGURE 4.49: Performance of optimization-based models with 40% ceiling on allocation to any asset in test period.

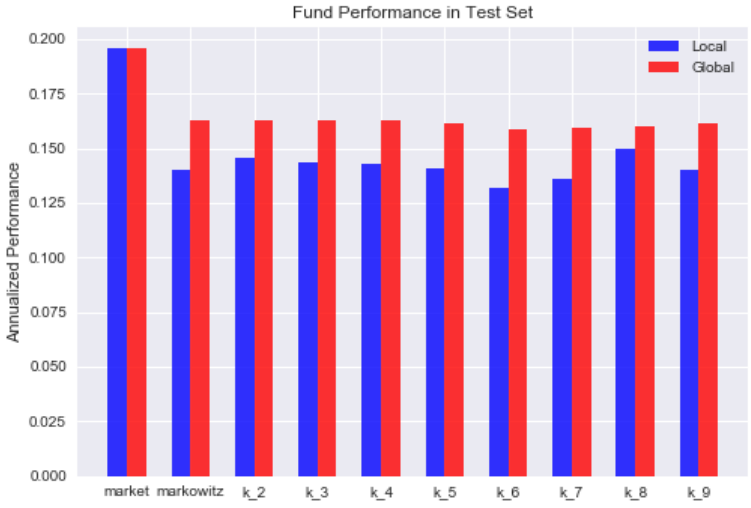


FIGURE 4.50: Summary performance of Local optimization-based models with 40% ceiling on allocation to any asset in test period with Iteration 4 time frame.

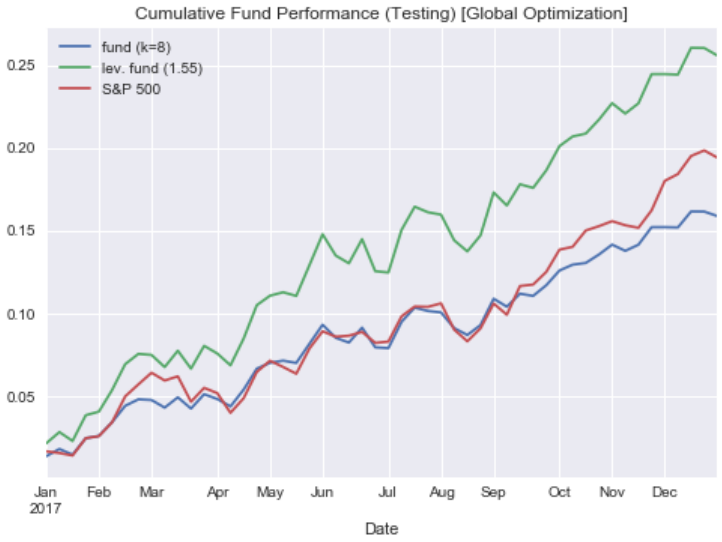


FIGURE 4.51: Performance of leveraged (1.55) Global 8-state model in test set relative to market and unleveraged portfolio.

4.6 Case Study: Features as Market Risk Indicator

4.6.1 Motivation

Other case studies in this section exam the value added by the clustering methodology described in this dissertation. This study examines whether if there is stand alone value provided in the feature set. More specifically, an examination is performed to determine if there predictive power in the pairwise feature set. This question of value is examined by looking to determine if data values or ranges can be connected with future equity market performance.

Several combinations of data granularity and look back are selected and future market performance is indicated by the color gradient of the pairwise data point.

4.6.2 Comparison of Results

First, a near-term look ahead period of 22-days, which is approximately one month, is examined using with 22 and 44 day lookback period to generate the features. The 22-day results, shown in Figure 4.52, suggests a relationship between cov_{norm} and future

returns. Observations with low normalized cov values are associated with low market performance, and the associated market performance is higher for observations with high cov values.

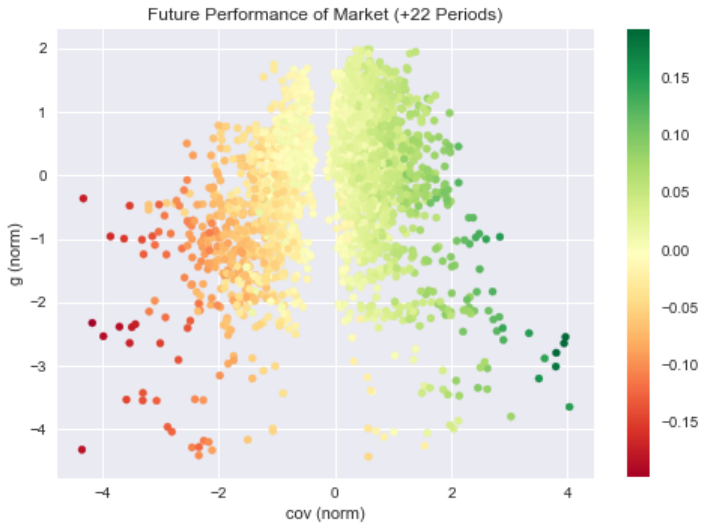


FIGURE 4.52: 22-day future performance plot.
Lookback period: 22 Days.

In this instance, market performance is connected to cov and not g. Figure 4.53 shows similar short-term findings in which low cov values are associated with lower future market returns; however, these results derived using a 44-day lookback period are not as clear cut and compelling.

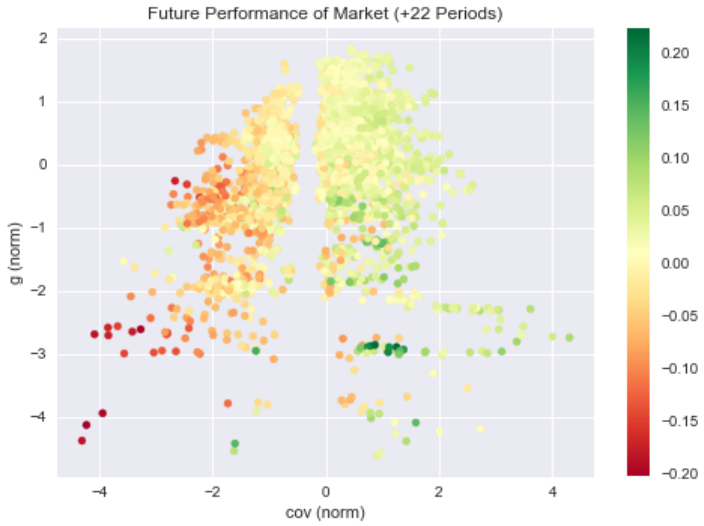


FIGURE 4.53: 22-day future performance plot.
Lookback period: 44 Days.

Next, a 44-day (approximately two months) look ahead is examined using a 44-day look back for calculating the features. The 44-day results suggested a relationship between *cov* and future 44-day performance in which the observations with the lowest *cov* values tended to exhibit low market performance. Observations with higher *cov* values were associated with higher market returns over this period that .

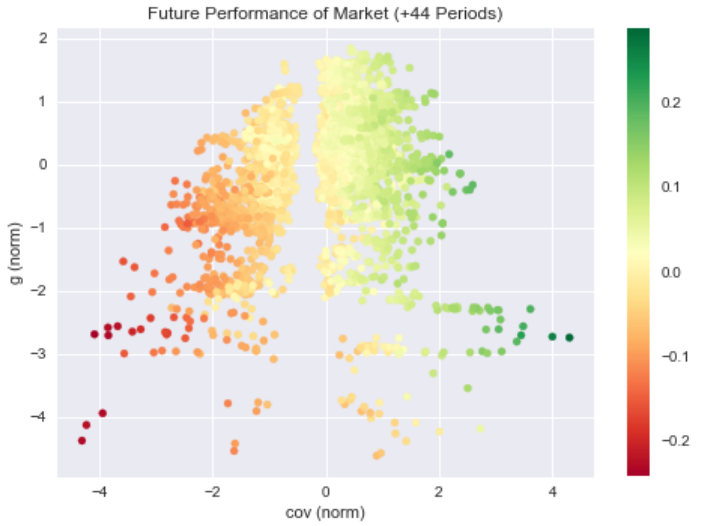


FIGURE 4.54: 44-day future performance plot.
Lookback period: 44 Days.

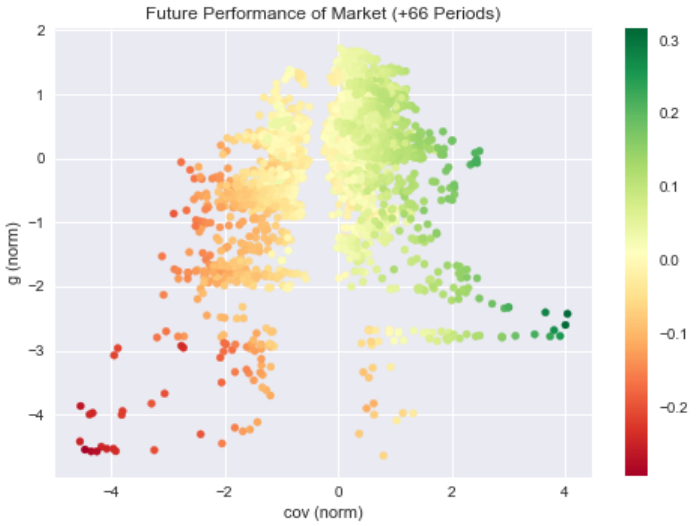


FIGURE 4.55: 66-day future performance plot.
Lookback period: 66 Days.

Next, a 66-day look ahead period is examined to see if there exists a relationship over that period of time using a 66-day look back for calculating the features used and, like previous versions 22 and 44 days future performance plots shown in Figure 4.52 and Figure 4.54, there appears to be a relationship between the *cov* and the performances 66-days into the future. The persistence of this relationship is examined in another way by using a weekly version of this illustration. A 12-week future performance viewpoint is taken using a 12-week look back for calculating the features as shown in Figure 4.56. Visual inspection of the two plots reveal that comparable results are found regardless of the level of data granularity used in the calculation of the features.

FIGURE 4.56: 12-week future performance plot.
Lookback period: 12 Weeks.

Both plots represent comparable look back periods for calculating feature values and look ahead periods for calculating market performance. Both plots also show that the increase in cov value is associated with observations that have increasing future performance values. The relationship between future performance of 66 days in both plots appears to be independent of the value of g.

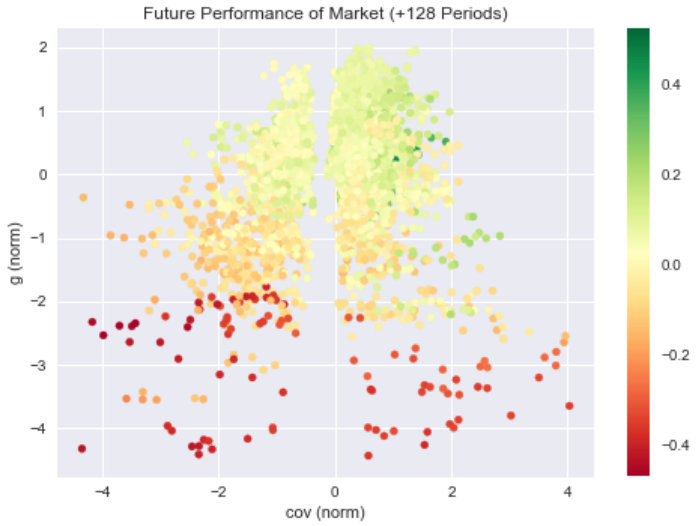


FIGURE 4.57: 128-day future performance plot.
Lookback period: 22 Days.

Lastly, the relationship between the feature set and a longer look ahead period is examined.What is considered a longer look ahead in this context is 128-day, or half a year, and 250-day, or one full year, period. A change in the relationship between superior future performance and the feature set is observed between the long-term and near-term periods.

FIGURE 4.58: 252-day future performance plot.
Lookback period: 22 Days.

What is shown in the long-term plots as contrasted with those in the previously discussed, short term plots is that there appears to be a relationship between the features and future performance related to the g value. This relationship did not exist in the near-term look ahead graphics but rather the future performance was connected to the cov value.

4.7 Summary of Results

Below is a summary of some of the key results presented as part of the Results section case studies.

- The example shown in Case 4.2 demonstrated multiple portfolio (k = 2, 3, 4, 6) with Sharpe Ratios higher than both market and single-state Markowitz portfolios in the holdout set. The two-state model showed robust and recurrent transition probability matrices between the training and hold out set, which were found to be statistically similar at 95% significance level. Global and Local portfolios produced similar asset allocations; however, the Global portfolios outformed the Local portfolios in terms of Sharpe Ratio in the majority of cases. The optimal number of steps into the future, n, was not consistent across models depending on the number states. Steady states was achieved in all states; however, higher returns were achieved with a step size of 0 in some instances, i.e. two-state model, while higher returns were achieved with larger step size, i.e. four-state model. This aspect needs to be investigated further.

- Case 4.3 showed that certain algorithms are not well-suited for this application. The example presented is the Agglomerative method, which resulted in one or two dominant states. Such results do not capture the state-based characteristics of various populations of the data.

- The example in Case 4.4 examines evolving training and testing regions. In the first instance, the multi-state models outperform market; however, the single-state Markowitz portfolio produces a better Sharpe Ratio. Subsequent instances show

superior returns and Sharpe Ratio for the multi-state models compared to the market and single-state Markowitz portfolios. Additionally, Iteration 4 showed a noticeable decline in performance from training to the holdout due to shift in the performance of the fixed income asset, AGG, which received heavy allocations due to the low volatility of the asset. An allocation constraint of 40% was applied to the asset in the portfolio weighting optimization, which resulted in superior performance relative to market and Markowitz in terms of both returns and Sharpe Ratio. This concept is explored further in Case 4.5.

- The final example examines how the feature set can be used to independent of clustering and portfolio design. The examples shows the relationship between the future performance and cov value in the short-term (22 days). This relationship changes as the future performance period is extended to 44, 66 days. The future performance becomes more dependent on the g magnitude feature.

Chapter 5

Conclusions & Future Work

"It is by no means an irrational fancy that, in a future existence, we shall look upon what we think our present existence, as a dream."

Edgar Allan Poe

5.1 Discussion of Results

The state definition and portfolio creation technique presented within this research effort demonstrated promising results; however, it also showed that further research is needed to develop this model into a workable methodology appropriate for the management of actual capital.

The state-based models produced portfolios with Sharpe Ratios that exceeded those of both market and single-state, Markowitz portfolios *at times*. The underlying contributing factors to this performance have yet to be identified and requires further research to better understand. It was also found that portfolios possessing high Sharpe Ratios may not achieve the required rate of return necessary for the portfolio to be used in a practical investment management setting, and therefore assets need to be leveraged to reach adequate return levels. The degree of leverage needed to generate a portfolio with market-level risk is a function of the basket of securities used to create the portfolios. Considerations must also be made to the risk profile of the investor and determine if a portfolio with market-level risk is suitable. It was also observed that low volatility

assets, such as fixed income mutual funds or ETFs, are highly utilized in portfolio allocation strategies due to the objective function that allocation asset weights maximizing the Sharpe Ratio by allocating to assets with low volatility.

Given that the objective function strove to maximize the Sharpe Ratio, some low-volatility assets received heavy allocations. In some instances this lead to situations in which the model was overfit and excessive allocations were made to the low-volatility fixed income products. When states are not stationary, particularly with respect to the returns of assets in the investment set, this could lead to poor portfolio performance. In order to avoid this overfitting and create a more practical portfolio, it was found that allocation constraints should be applied to the optimization algorithm limiting the maximum allocation to any given asset. This constraint is useful in several ways. Firstly, it creates a practical portfolio that could potentially be used by a practitioner in a live trading environment. Secondly, it could also reduce the possibilities of overfitting the model by having an excessively concentrated portfolio.

It was also found that using the two features examined in this research - growth magnitude and the covariance distance - produced value independent of that achieved through clustering and applied to asset allocation decisions. More specifically, it was found that a pairwise examination of the future values of each point exhibited predictive qualities. It was also observed that the relationship between the predictive power in the short term as compared to the medium and longer-term shifted from a dependency on covariance distance to a dependency on the growth magnitude, respectively.

5.2 Future Works

Research within the financial analysis domain is iterative in nature. In spite of the promising preliminary results from this research effort, there is still much more work needed in all aspects of this research pursuit. Below is a summary of several areas that would benefit from further work and associated research questions.

- Signal Set
  - Inspiration was taken from the underlying parameters of MPT - expected growth and convariance - and used to define the state space and create states. What calculations could be performed on these measures to create a feature space by which to define states?
  - Extending the feature set beyond expected growth and covariance, what attributes could contain information and be used to create discrete states for use in designing investment portfolios?
    - Economic Indicators
      - Many economic indicators are published on a monthly basis and would not be suitable for a tactical, small-term investment strategy; however, some indicators in this space can offer insight into the changing nature of the economy and by extension the financial markets.
      - Indicators of interest for possible inclusion in the signal set include: slope of interest rate curve, manufacturing indexes, PPI (producer price index), CPI (consumer price index), OECD leading indicators, price of gold relative to dollar, dollar index, year-over-year change in GDP.
    - Fundamental Indicators
      - Similar to economic indicators, fundamental indicators are not published on a daily basis and would not be suitable for short-term, tactical portfolios, but such indicators are useful in analyzing the value proposition of markets and can be useful in forecasting large-scale markets shifts.
      - Indicators of interest for possible inclusion in the signal set include: P/E (price-to-earnings) ratios, P/Div (price-to-dividend) ratio, P/B (price-to-book) ratio, P/S (price-to-sales) ratio.
    - Technical and Quantitative Indicators

· Technical and quantitative indicators can quantify price moments and trends. They are available in all levels of price granularity (daily, weekly, monthly, quarterly).

· Indicators of interest for possible inclusion in the signal set include: volatility index (VIX), momentum (MACD, exponential and arithmetic moving averages), relative strength index (RSI), commodity channels, sentiment (AAII bull-bear ratio), trading volume, open interest, new highs, new lows, percentage of S&P 500 stocks above key moving averages, average correlations for index component units, i.e. median correlation for S&P 500 stocks.

– Feature selection

* Given the small number of features in the feature set (2), a detailed analysis of the significance and contributions of each feature was not necessary. If the signal set is expanded to include some of the features specified in the sections above, the significance of the various features should be evaluated to determine whether all of the features are adding value to the state-definition process and should remain in the signal set.

* Principle component analysis (PCA) is one technique used to assess the contribution of features to the overall result; however, other approaches should be evaluated as well.

* What metric should be used to determine if a feature is valuable to the state definition process?

- Determination of state-specific portfolios

– This research made asset allocation decisions using the risk-return profile of a portfolio, specifically the Sharpe Ratio, as the primary metric; however, there are other metrics that can be used in determining the weights for each state-specific portfolio.

– The asset allocation decisions for each state-specific portfolio are critical decisions within this research and drive the portfolio performance as much or

more than any other element.

- Incorporate the use of the French-Fama 3 factor model as a basis for constructing the state-specific portfolios[1]. A portfolio composed of French-Fama factors would be able to identify the sources of state-specific portfolio returns.
- Asset allocation decisions are based on the idea of maximizing the Sharpe Ratio. In a recent paper, French and Fama discuss how maximizing the Sharpe Ratio can be used in constructing asset pricing models[55]. "Roll's Critique" of CAPM suggests that finding the maximum Sharpe Ratio is indicative of CAPM being valid [129]. Therefore, finding the maximum Sharpe Ratio in each state is the equivalent of finding portfolios that support CAPM. Future work should examine various asset pricing models to be used in the calculation of Sharpe Ratio to be used as the objective function when creating state-specific model portfolios.
- Use an 'expert system' to make the state-specific asset assignments instead of an optimization-based approach. This 'expert system' should employ the judgement of a domain expert to asset the characteristics of the sate, such as the time periods of state membership, expected return, expected future return and correlation of assets.

- Evaluation of State Quality
  - A more in-depth look at states created as part of the clustering process in required and further criteria is needed to define the criteria for what constitutes a 'good' and 'bad' state.
  - Once criteria is set in place for defining the characteristics of a 'good' and 'bad' state, subsequent actions much be defined to determine how such states should be handled.
  - For example, if a state is deemed 'bad', should the allocation for that state be placed entirely in cash given the lack of predictive power the state is determined to have? What other actions should be taken for such states?

- Evaluation of Portfolio Performance

- A robust testing plan needs to be developed to provide results to the finance community that the approach presented in this research offers promising results.

- Baily et al. states that the returns of investment portfolios cannot be tested in the same way as other models and presents a new approach that examines the probability of overfitting results[8]. Can this technique be incorporated into the evaluation of the model results?

- Barillas et al. examines methods for comparing trading model performance stating that

  > *In contrast, we show that for models with traded factors, when the comparison is framed appropriately in terms of success in pricing both the test-asset and factor returns, the extent to which each model is able to price the factors in the other model is what matters for model comparison .[10]*

  A detailed examination of how this approach to comparing trading performance between the multi-state based model relative to both market and Markowitz could be conducted to determine a methodology and factor set.

- In addition to the other performance comparison metrics incorporated into this research path, the French-Fama factors should be included in an analysis of the portfolio performance to examine whether such factors played a part in the portfolio performance. More specifically, the returns for 1) each multi-state model, 2) the single-state Markowitz model and 3) the market should used as the dependent variable in a regression analysis using the French-Fama factors as the underlying independent variable to examine the degree to which these factors influence portfolio performance. All variants of the French-Fama factor models should be considered.

[1]Data for the French-Fama 3 factor model can be found at the following: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data$_{l}ibrary.html$

Appendix A

Markov Property Test

Within this research the chi-squared test, also written as Chi Square (χ^2) test, is used to test for the existence of the Markov property in a system modeled as a Markov chain as shown in Figure A.1. The χ^2 test is a statistical test where the test statistics is sampled from a chi-squared distribution when the null hypothesis is assumed to be true. This test makes the determination of significant difference between the expected frequencies and observed frequencies.

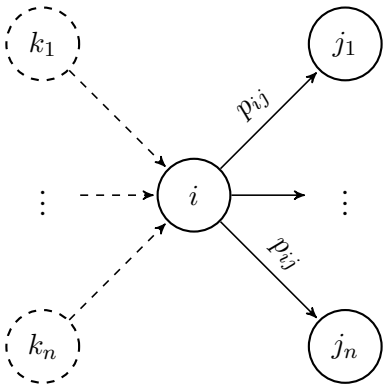


FIGURE A.1: A illustration of a Markov Property

When using this approach to test for the existence of the Markov property, you begin with the belief that the property exists and attempt to discover evidence to reject this belief.

Hypothesis Test

H_0: Markovian property exists

H_a: Does not exist

The test evaluates each cell in the transition probability matrix separately meaning that each pairwise combination between an i (from) state and j (to) state is evaluated. Each pairwise combination is subdivided into groups based on the state visited prior to arriving in state i. The state that preceded i is denoted as state k.

A system that possesses the Markov property experiences transitions between states based solely on the state in which it currently exists. Differences between groups based on the state prior to i. If the Markov property existed, significant differences would not exist between each of the groups. What constitutes a 'significant difference' depends on the level of significance deserved for the test. Only groups, or rows, with an expected number of observations greater than 5 should be included in the test.

Test Statistic:

The χ^2 test statistic is calculated as follows:

$$\chi^2 = \sum_{\text{all } k} \frac{(\text{observed} - \text{estimated expected})^2}{\text{estimated expected}} = \sum \frac{(n_{ij} - n_i p_{i,j})^2}{n_i p_{i,j}} \tag{A.1}$$

Observations are subdivided into two populations - observations transitioned as expected into state j and those that did not. As the test statistic gets larger, there is increasing amount of cumulative difference between the expected and actual observations. This is indicative of a greater degree of difference between the groups and thereby more evidence that the null hypothesis of the presences of the Markov property could be rejected.

The criteria for rejecting the null hypothesis at a significance level of α and accepting the alternative hypothesis is for χ^2 to fall within the rejection region defined as:

$$\chi^2 > \chi^2_{\alpha,k-1} \tag{A.2}$$

The rejection criteria can be viewed in terms of an observed level of significance or p-value such that the null hypothesis, H_0, should be rejected if the p-value falls below the desired level of significance as follows:

$$\text{p-value} < \alpha \tag{A.3}$$

A low p-value would suggest that it is unlikely that a χ^2 test statistic of this level would be observed if the null hypothesis (the existence of the Markovian property) were true.

Consider the two following examples which examines cells within the same transition probability matrix for a $k = 6$ state structure. In one instance, the Markov property is found and in the other it is not. The statistical test is made on the the follow transition probability matrix

$$\mathbf{P} = \begin{pmatrix} 0.58 & 0.01 & 0.02 & 0.09 & 0.30 & 0.00 \\ 0.03 & 0.67 & 0.05 & 0.00 & 0.21 & 0.04 \\ 0.02 & 0.05 & 0.61 & 0.11 & 0.14 & 0.07 \\ 0.07 & 0.00 & 0.22 & 0.64 & 0.07 & 0.00 \\ 0.18 & 0.10 & 0.07 & 0.04 & 0.60 & 0.00 \\ 0.00 & 0.09 & 0.15 & 0.00 & 0.00 & 0.76 \end{pmatrix}$$

FIGURE A.2: Example transition probability matrix.

A.1 Example 1: No Evidence to Reject Null, i.e. Existence of Markovian Property

The first case examines the transition from 1-to-1, denoted as (1, 1), which has a transition probability, $p_{i,j}$, of 0.67 (highlighted above). As described in the section, in order to test for the existence of the Markov property, all of the observation that fell into State 1 (i) are collected as well as data on the previous state (k) and the subsequent state (j). The population of i observations is subdivided into groups based on the previous state, k.

If a system exhibits the Markov property, there would be statistical evidence of observations transitioning from i to j with similar frequency regardless of the prior state k. Transitions are considered as either transitioning to the state j of interest or not. In Example 1 the state of interest j is 1 so observations in Table A.1 are counted in terms of transitioning from 1-to-1, denoted as $O_{j=1}$, or transitioning to some other state, denoted as $O_{j\neq 1}$.

The other columns in used in Table A.1 are: k - the previous state, $p_{i,j}|k$ - the transition probability given the prior state k, $\mathrm{n}_{i|k}$ - the total number of state i observations which were previously in state k, E[Obs] - the expected number of transitions given the Markovian Property $p_{i,j}\mathrm{n}_{i|k}$, and Eligible? - a flag indicating the rows that qualify for inclusion in the calculation of the test statistic given the rule of $E[\text{Obs}] > 5$.

k	$p_{1,1}\|k$	$\mathrm{O}_{j=1}$	$O_{j\neq 1}$	$\mathrm{n}_{i\|k}$	E[Obs]	Eligible?
0	1.00	2	0	2	1.34	
1	0.68	127	59	186	125.13	✓
2	0.61	11	7	18	12.40	✓
3	0.00	0	0	0	0.00	
4	0.64	36	20	56	37.67	✓
5	0.69	9	4	13	8.75	✓

TABLE A.1: Result Summary for χ^2 Test of Markov Property: $p_{1,1}$

Applying Equation A.1 to the eligible rows (1, 2, 4, 5) yields a χ^2 test statistic of 0.21, which does not fall within even the liberal rejection region for a confidence level of $\alpha = 0.20$. The lower bound for this rejection region is 4.64. This result corresponds to a p-value of 0.98, which does not offer statistical evidence to reject the hypothesis that the Markovian property is present.

A visual inspection confirms this finding. The eligible observations transition from 1-to-1 with probabilities of 0.68, 0.61, 0.64, and 0.69. The behavior of the subgroups given the various k values are similar to the aggregate transitions $p_{1,1}$ of 0.67. The results of this test support the belief that the transitions from 1-to-1 are not influenced to a statistically significant degree by the prior state.

A.2 Example 2:
Evidence to Reject Null, i.e. No Markovian Property

The second case examines the transition from 3-to-2, denoted as (3, 2), which has a transition probability, $p_{i,j}$, of 0.22 (highlighted above). The same process used in the example above is applied to this example. The tabular results from the statistical test are included in Table A.2.

k	$p_{3,2}\|k$	$O_{j=2}$	$O_{j\neq 2}$	$n_{i\|k}$	E[Obs]	Eligible?
0	0.26	6	23	29	6.44	✓
1	0.00	0	1	1	0.22	
2	0.41	15	22	37	8.22	✓
3	0.18	28	128	156	34.67	✓
4	0.25	5	15	20	4.44	
5	0.00	0	0	0	0.00	

TABLE A.2: Result Summary for χ^2 Test of Markov Property: $p_{3,2}$

Applying Equation A.1 to the eligible rows (0, 2, 3, 4) yields a χ^2 test statistic of 6.89, which is greater than critical value of 5.99 for a significance level of $\alpha = 0.05$. This result corresponds to a p-value of 0.037, which offers statistical evidence to reject the hypothesis

that the Markovian property is present.

A visual inspection of the tabular results supports the statistical findings in this case. While the overall transition probability $p_{3,2} = 0.22$, the transitions for each subgroup seem to vary based on the previous state k ranging from 0.18 to 0.41. This transition behavior from 3-to-2 is inconsistent and does not reflect the Markovian property.

The following snippet of Python (.py) code creates the tabular results used to calculate the test statistic.

```
for i_ in range(0, k):
        for j_ in range(0, k):
            #print(i_, j_)
            ts = np.zeros((k, 6))
            if p[i_, j_] > 0:
                df_O = df2.get_group((i_, j_))
                p_ = p_tran[i_, j_]
                for k_ in range(0, k):
                    df = df_O[df_O['k'] == k_]
                    o_ = len(df)
                    df_E = df1[(df1['i'] == i_)
                        & (df1['k'] == k_)]
                    n_ = len(df_E)
                    e_ = n_*p_

                    # Write iteration for each a priori
                    # state, k_, to results matrix
                    ts[k_, 0] = k_
                    ts[k_, 1] = n_
                    ts[k_, 2] = o_
                    if n_ > 0:
                        ts[k_, 3] = o_/n_
                    else:
                        ts[k_, 3] = 0
                    ts[k_, 4] = n_ - o_
                    ts[k_, 5] = e_
                df_ts = pd.DataFrame(ts)

                # only use records where exp > 5
                df_ts = df_ts[df_ts['exp'] > 5]
```

And the test statistics are calculated using the following code.

```
import scipy.stats as stats

chi_squared_stat = (((obs-exp)**2)/exp).sum()
p_value = 1-stats.chi2.cdf(
    x = chi_squared_stat,
    df = degf
)
```

Appendix B

Global Portfolio Optimization - Implementation Script

Within this Appendix there will be sample code of the python code used to create the portfolios. Examples should both local and global variants

glob_opt_script.m

```
%% Initial Setup
% Parameters
ret_in_flag = 0;

% Read in static files from .csv
filename0 = 'Data/parameter_input.csv';
filename1 = 'Output/N_returns_results_DJ_train.csv';

% Read in .csv files
x = csvread(filename0, 9, 1,[9 1 14 1]);
k = csvread(filename0, 16, 1,[16 1 17 1]);
r_ = csvread(filename1, 1, 1);
n = csvread(filename0, 13, 1,[13 1 13 1]);
data = readtable(filename0);

% Assign parameters from .csv input
```

```
17 m = size(r_,2);                          % Number of assets
18 rf=double(x(1));
19 lb_val = double(x(3));
20 ub_val = double(x(4));
21 mkt_exp = double(x(6));
22 k_min = k(1);
23 k_max = k(2);
24 t_str = string(data{15,{'Value'}});
25
26 % Determine matrix size
27 a = size(r_,1);
28
29 % Determine the annual periods for level of granularity
30 switch t_str
31     case 'D'
32         t = 252;
33     case 'M'
34         t=12;
35     otherwise
36         % default to weekly
37         t=52;
38 end
39
40
41 %% Loop specific to the value of k
42 for k = k_min:k_max
43     % Define filename based on k value
44     filename2 = sprintf('Output/k_%d_prob_S_train.csv', k);
45     filename3 = sprintf('Output/matlab_k_%d_glob_wts.csv', k);
46     filename4 = sprintf('Output/k_%d_tran_prob.csv', k);
47
```

```
48      % Read in each k-specific .csv file
49      p0 = csvread(filename2, 1, 1);
50      p1 = csvread(filename4, 0, 0);
51      b = size(p0,1);
52      pn = p1^n;
53      p = p0*pn;
54
55      % Only read in one instance of returns, r, matrix
56      if ret_in_flag == 0
57          % Adjust return, r, matrix - remove excess rows in r
58          first_row = a-b+1;
59          r = r_(first_row:a,:);
60          a2 = size(r,1);
61          ret_in_flag = 1;        % flip flag that r has been read
                in
62      end
63
64      % Create the initial weights matrix
65      w0 = (1/m)*ones(1,m*k);
66
67      %% Optimization procedure
68  % define constraints
69
70  % Build Aeq matrix
71  Aeq = zeros(k, k*m);
72  for i = 1:k
73      temp = zeros(k, k*m);
74      temp(i,(i-1)*m+1:i*m)=1;
75      Aeq = Aeq+temp;
76  end
77
```

```
78 beq = ones(k,1);
79 lb = lb_val*ones(1,k*m);
80 ub = ub_val*ones(1,k*m);
81
82 % Optimization function
83 f = @(w)sharpe_ratio2(w, r, p, k, m, rf,t);
84 options = struct('MaxFunEvals', 50000);
85 w_ = fmincon(f,w0,[],[],Aeq,beq,lb,ub,[],options);
86 w = reshape(w_,m,k)';
87
88 % Optimal Sharpe Ratio
89 sharpe = -1*sharpe_ratio2(w_, r, p, k, m, rf,t)
90
91 % Output results
92 csvwrite(filename3, w);
93 end
```

Appendix C

Global Portfolio Optimization - Function

This Appendix contains the MATLAB code that defines the global portfolio optimization function used in determining the asset allocation. The parameters of this function are as follows.

Function Parameters

k: the number of states in the clustering schema

m: the number of assets in the investment universe

w: a matrix of weights assigned to each assets with dimensions of m by k

r: a vector of returns for assets in the investment universe

p: probability of being in a state at any given time

rf: the risk-free rate of return used in the calculation of the Sharpe Ratio

t: the number of time periods in year for the level of data granularity, i.e. for monthly return, t=12

glob_opt_script.m

```
function ratio = sharpe_ratio2(w, r, p, k, m, rf, t)

w = reshape(w,m,k)';
rf=0;
```

```
5
6  x = p*w;
7  y = x.*r;
8  z = sum(y, 2);
9
10 mu = mean(z);
11 s = std(z);
12
13 mu_ann = (1+mu)^t-1;
14
15 s_D = (s^2+(1+mu)^2)^t;
16 s_Q = (1+mu)^(2*t);
17 std_ann = sqrt(s_D - s_Q);
18
19 ratio = -(mu_ann-rf)/std_ann;
```

Appendix D

Standard Deviation Conversion

This Appendix contains an essay discussing the bias contained in the commonly-used method converting the standard deviation measures of financial instruments from a more granular version used to make the initial calculations, such as monthly or weekly, into the more understood annual version. The underlying mathematics behind the differences in the conversion methodologies is outlined, and the circumstances surrounding the periods when the differences are greatest.

A Fundamental Misunderstanding of Risk:
The Bias Associated with the Annualized Calculation of Standard Deviation

Introduction

Risk lies at the heart of modern finance and investing. Within this world risk comes from not knowing what the future holds for the performance of financial instruments, and therefore a great deal of effort has been dedicated to developing methods to quantify it. When a wide-range of outcomes are possible, the likelihood of negative outcomes increases, which is the reason for expressing financial risk in terms of volatility, such as standard deviation or probability of negative outcome as determined by models based on volatility measures like Value at Risk (VaR). Uncertainty is very important in pricing models. Investors will only invest in bonds that are priced in a manner to compensate investors for the possibility of the bond defaulting, and the seminal Black-Scholes formula

for pricing options uses standard deviation to account for market volatility. Investment portfolio decisions are largely based on balancing return against risk, and more specifically, volatility measures. Markowitz (1952) introduced a ground-breaking approach for constructing portfolios by determining the proportional combination of assets that produced the lowest level of risk (in terms of standard deviation) for various levels of portfolio return [114]. Sharpe (1966) presented a measure for evaluating the performance of portfolios called the "reward-to-variability ratio" that has since become known as the Sharpe Ratio, the proportion of return in excess of the risk-free option expected per unit of risk (standard deviation) [138].

The essential role that the risk plays in finance is undeniable; however, the manner by which to quantify and represent this risk has been the subject of much research and debate because the consequences of failing to accurately represent the amount of risk in a particular scenario can be dire. Returns are most commonly represented by Gaussian distributions and discussed in terms of means and standard deviations. Many researchers have taken issue with this practice, such as Mandelbrot (1963) who wrote that models based on such distributions misrepresented the likelihood of events in the 'fat tails' that represent the most significant outcomes.

But what if there is a misunderstanding of the level of risk at even a more basic level? What if the value that is most commonly used as a measure of risk and uncertainty is being misrepresented?

Explanation/Math

Consider how a series of periodic returns translates into the aggregate return represented on a timescale with which people are able to relate, or more specifically how a series of monthly returns can be expressed as the annual performance[1]. Let r_i represent the percentage change at time i in the price of a financial instrument, p_i, calculated by the

equation

$$r_i = \frac{p_i}{p_{i-1}} - 1 \tag{D.1}$$

The cumulative performance after period i, denoted as R_i, is the product of two factors: the return in period i, r_i, and the cumulative return accumulated prior to i represented as R_{i-1}.

$$R_i = (1 + R_{i-1})(1 + r_i) - 1 \tag{D.2}$$

$$R_a = (1 + r_1)(1 + r_2) \dots (1 + r_n) - 1 \tag{D.3}$$

If this expression were to be simplified where $\bar{r}$ represents the expected monthly return during the n periods of interest. Note that the $\bar{r}$ is calculated using the geometric mean and not the arithmetic. The choice of means was made because the geometric variant accounts for the compounding effect observed in investment returns.

$$R_a = \prod_{i=1}^{n} (1 + r_i) - 1 \tag{D.4}$$

$$\bar{r} = \sqrt[n]{R_a + 1} - 1 \tag{D.5}$$

$$R_a = (\bar{r} + 1)^n - 1 \tag{D.6}$$

[1]Monthly returns were selected as the granular level for the the periodic returns because it is a commonly used practice although the principle being discussed is applicable to more granular data, such as weekly or daily.

Using the expected monthly return result from Equation D.5, the standard deviation on a monthly basis, σ_M, can be calculated by

$$\sigma_M = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(r_i - \bar{r})^2} \tag{D.7}$$

Knowing that the variance of any random variance can be represented as the second moment about the mean, and the standard deviation is the square root of this second moment then the standard deviation of any random variable can be expressed as

$$S[X] = \sqrt{E[X^2] - E[X]^2} \tag{D.8}$$

Combining the aggregate return calculations from Equation D.6 with Equation D.8, an expression for the annualized standard deviation of returns is created

$$\begin{aligned} \sigma_A &= S[R_a] = \sqrt{E[R_a^2] - E[R_a]^2} \\ &= S[R_a + 1] = \sqrt{E[(R_a + 1)^2] - E[R_a + 1]^2} \\ &= \sqrt{(\sigma_M^2 + (1 + \bar{r})^2)^n - (1 + \bar{r})^{(2n)}} \end{aligned} \tag{D.9}$$

This is the same result derived in a 1965 article by James Tobin[2][148] and has been applied on occasion in academic and professional literature. One such example is Levy and Gunthorpe (1993)[100]; however, this approach has failed to be universally accepted as the methodology for making the rather common and extremely critical calculation within the financial world. Levy (2012, 2013) presented the problem and also derives the formula shown in Equation D.9[92, 93]. Many academic publications, such as MacKinlay (2000)[40] and Lustig (2011)[107], have chosen to use an alternative methodology that introduces a bias into the derived annual value. The commonly-used, alternative method is a function of only the period standard deviation and the level of granularity but not the expected return. As such, this method assumes a linear relationship between, in the case of this example, the monthly and annual standard deviation in which the monthly value is multiplied by the square root of twelve[3], the number of periods in a year, as shown in

Equation D.10.

$$\sigma_A = \sqrt{n} * \sigma_M \tag{D.10}$$

This linear approach shown in Equation D.10 has its foundations in basic probability theory. Consider a series of identical, independent (i.i.d.) outcomes, X_i, that represent incremental measures of performance within a larger time period. The cumulative performance for the entire time period, Y, can be expressed as the summation of all the n incremental measurements

$$Y = \sum_{i=1}^{n} X_i \tag{D.11}$$

Basic axioms of probability show that the relationship between the standard deviations of the random variables X and Y is linear according to

$$S[Y] = \sqrt{n} * S[X] \tag{D.12}$$

The rationale described in Equations D.11 and D.12 is correct and represents mathematical fact when applied to a collection of random variables the cumulative total of which is the sum of all variables in the series; however, it cannot be extended to a series of investment returns. The reason for this is that the cumulative return of an investment security is the result of a compounding, geometric series of returns as described in Equation D.4 and not the summation of returns. Equation D.11 would hold true for the cumulative returns of an investment product if the returns were converted to a logarithmic scale. Unfortunately many investors do not understand the underlying reasoning for this, which causes them to either over-estimate or under-estimate the level of risk assumed. does not hold for higher order moments, such as standard deviation, so Equation D.12 cannot be

[3]James Tobin was an American economist who developed the foundational elements of Keynesian economics, served on the Board of Governors of the Federal Reserve System, and taught at Harvard and Yale University. He received the Nobel Memorial Prize in Economic Sciences in 1981 "for his analysis of financial markets and their relations to expenditure decisions, employment, production and prices." [147]

[3]The level of granularity is represented by the number of periods, n, in a year, e.g. 12 months, 52 weeks, and 255 days (average number of trading days in a year).

adapted to investment returns by converting returns to a logarithmic scale.

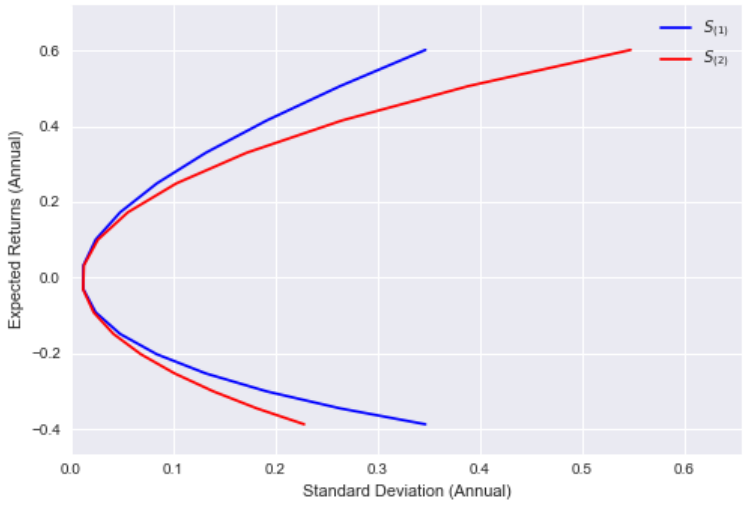


FIGURE D.1: Annual efficient frontiers created using both σ_A conversion methods.

To illustrate the bias that comes with the linear method from Equation D.10 review the following example which utilizes the idea of the efficient frontier from Markowitz's Modern Portfolio Theory (MPT). A theoretical, pareto optimal frontier is presented in Table D.1 showing the minimum amount of risk at each level of portfolio return. The monthly frontier is then converting into the more interpretable annualized form using both of the conversion methods discussed and displayed in Figure D.1. The difference between the methods, which is the bias referenced throughout this article, is shown in Figure D.2.

Figure D.2 shows that the bias is greatest when the expected periodic returns are highest thereby underestimating the amount of risk in a portfolio and fueling the complacency inherent during bullish growth periods in the markets. Conversely, the bias is lowest when the expected returns are lowest during bearish markets causing investors to overestimate the level of volatility and risk in the markets.

[4] $S_{(1)}$ represents the method described in this article, which incorporates r in the calculation to created an annualized figure. $S_{(2)}$ is the linear transformation method, $S = \sqrt{n} * s_n$.

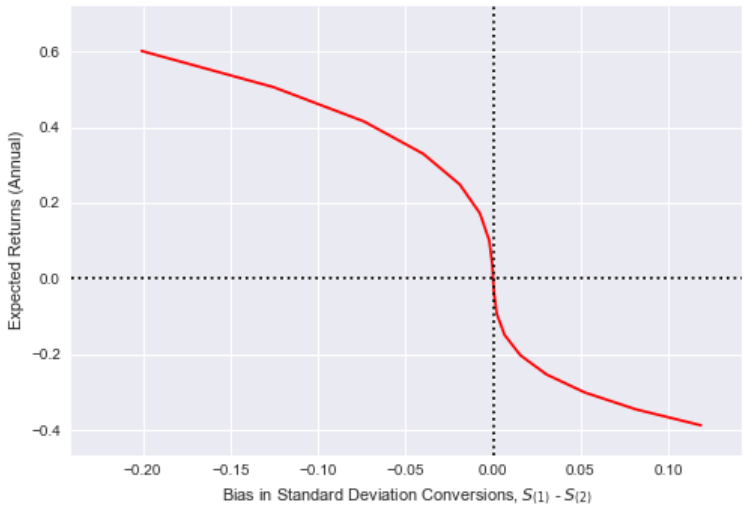


FIGURE D.2: Difference in annual standard deviation values at variation expected return, $\bar{r}$, levels.

Monthly		**Annual**[4]			
$\bar{r}$	σ_M	R	$S_{(1)}$	$S_{(2)}$	$\Delta(S)$
-0.040	0.100	-0.387	0.346	0.227	0.118
-0.034	0.075	-0.345	0.262	0.181	0.081
-0.029	0.055	-0.300	0.191	0.138	0.052
-0.024	0.037	-0.252	0.131	0.101	0.030
-0.018	0.024	-0.202	0.083	0.068	0.015
-0.013	0.013	-0.148	0.047	0.041	0.006
-0.008	0.006	-0.091	0.023	0.021	0.002
-0.002	0.003	-0.031	0.011	0.011	0.000
0.002	0.003	0.032	0.011	0.012	-0.001
0.008	0.006	0.100	0.023	0.026	-0.002
0.013	0.013	0.172	0.047	0.055	-0.007
0.018	0.024	0.248	0.083	0.102	-0.019
0.024	0.037	0.329	0.131	0.171	-0.039
0.029	0.055	0.414	0.191	0.264	-0.073
0.034	0.075	0.505	0.262	0.388	-0.125
0.040	0.100	0.601	0.346	0.547	-0.200

TABLE D.1: Comparison of Annual Standard Deviation Values Using Two Methods and the Associated Bias

The effects of the bias increase as the granularity becomes finer, such as going from monthly data in which there are 12 observation periods per year to daily data with 255 observations per annum. The effects become particularly noticeable for periods of high expected returns. See the efficient frontier created using daily data in Figure D.3.

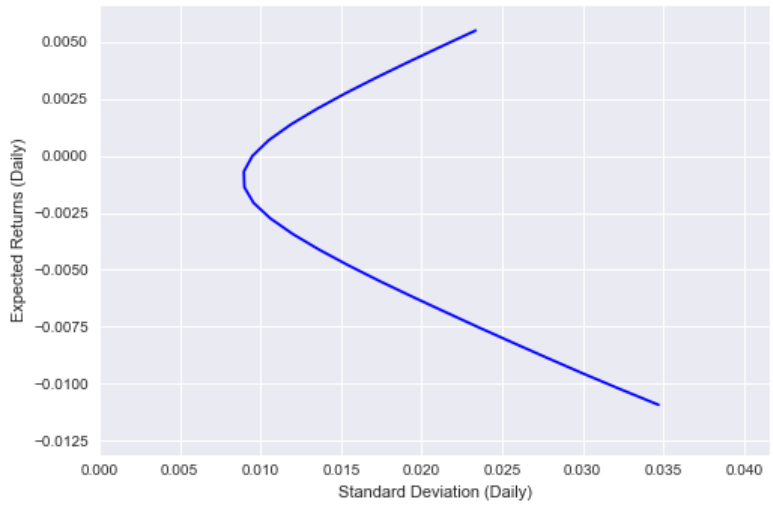


FIGURE D.3: Efficient frontier created using daily returns. Reflect single day risk and return.

When this frontier is converted to annual version as shown in Figure D.4, an interesting phenomenon occurs - at the inflection point, the frontier begins to grow instead of decay exponentially. An efficient frontier should exhibit diminishing marginal returns per unit of risk. This result mean that more risk must be assumed into order to achieve an additional unit of expected return.

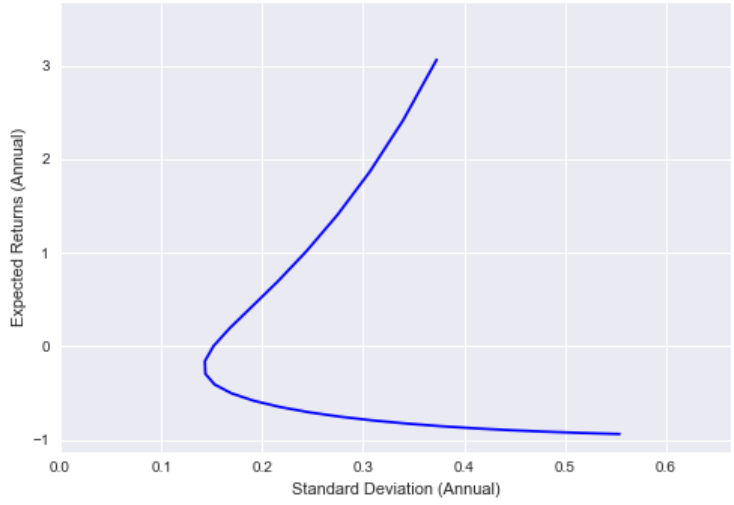


FIGURE D.4: Annualized version of efficient frontier using daily data. Note increasing marginal returns.

The annual efficient frontier in Figure D.4 shows the opposite effect. This outcome suggests that there exists a point beyond which more expected returns can be achieved by assuming less potential risk. The relationship between risk and return becomes skewed in the transformation process because returns increases exponentially while risk increases linearly. An investor who elects to behave rationally would continue to move further along the exponentially increasing frontier indefinitely because the risk-return profile of the position would continue to improve. Such a representation of the relationship between expected return and risk misrepresents the balance between these two factors and could lead investors into imprudent investment decisions.

Conclusion

The topic presented in paper in not a new arrival to the world of finance. Tobin first presented it in 1965, subsequent white papers in 2012 and 2013 revisited the topic, and financial information provider MorningStar even used it for a period before reverting to the linear method [144]. The objective of this paper is not to introduce a ground-breaking piece of original research but rather to reiterate the need for understanding the way risk is quantified in terms of the volatility of assets and portfolios.

The various techniques used for calculating risk can perceive distinct levels of risk which in turn can affect the criteria by which decisions are made. This paper elects to use the risk-return characteristics of an investment portfolio as an illustrative example. The relationship between risk and return determine the 'optimal' portfolio, which determines the asset allocation choices for the portfolio. What is of concern more so than two approaches for calculating the same measure of risk arriving at different values is the circumstances under which these differences are greatest. The commonly-used linear method for calculating risk has its greatest negative bias relative to the quadratic risk calculation approach when returns are high consequently underestimating the level of risk. Such a situation would exist near market peaks after a period of prolonged investment advances thereby making an investor less likely to exit. Conversely, the linear conversion method exhibits

its greatest positive bias when returns are low, such as at a market bottom. Overestimating the level of risk in a portfolio would make it more difficult for an investor to reenter the market by making asset allocation choices to assume more risk.

Investors who make decisions based on statistical measures need to understand the differences in the two methods presented in this paper and the circumstances under which they differ most and thereby become more informed decision makers.

Appendix E

Securities - Signal Set (DJIA)

Symbol	Name	Product Type
MMM	3M	Equity
AXP	American Express	Equity
AAPL	Apple	Equity
BA	Boening	Equity
CAT	Caterpillar	Equity
CVX	Chevron	Equity
CSCO	Cisco Systems	Equity
KO	Coca-Cola	Equity
DIS	Disney	Equity
DWDP	DowDuPont	Equity
XOM	ExxonMobile	Equity
GE	General Electric	Equity
GS	Goldman Sachs	Equity
HD	Home Depot	Equity
IBM	IBM	Equity
INTL	Intel	Equity
JNJ	Johnson & Johnson	Equity
JPM	JP Morgan Chase	Equity
MCD	McDonald's	Equity
MRK	Merck	Equity
MSFT	MicroSoft	Equity
NKE	Nike	Equity
PFE	Pfizer	Equity
PG	Procter & Gamble	Equity
TRV	Travelers	Equity
UTX	United Technology	Equity
UNH	UnitedHealth Group	Equity
VZ	Verizon	Equity
WMT	WalMart	Equity

TABLE E.1: List of securities included in the Dow Jones Industrial Average signal set.

Appendix F

Securities - Signal Set (Macro)

The list of securities listed below provide a macro-level signal set by which to define states. As opposed to the signal set outlined in Appendix E, which is composed of the equities that comprise the Dow Jones Industrial Average (DJIA), this set is 1) made up of fewer securities, 2) uses indexes and ETFs instead of equities and 3) incorporates an international component to the analysis. This set is referred to as *macro* because it uses securities that reflect the performance of broad markets as opposed to individual companies.

Symbol	Name	Product Type
^GSPC	S&P 500	Index
^IXIC	NASDAQ	Index
^RUT	Russell 2000	Index
AGG	iShares Aggregate Bond	ETF
EEM	iShares MSCI Emerging Markets	ETF
EFA	iShares MSCI EAFE	ETF

TABLE F.1: List of securities included in the macro signal set.

Appendix G

Securities - Portfolio Set

The list of securities listed below make up the universe of investment assets used to create the investment portfolios illustrated in this research. These securities represent popular mutual funds used in the investment management industry. This portfolio of assets represents a variety of categories of mutual funds including balanced funds, fixed income (or bond) funds, and inverse funds.

Symbol	Name	Fund Type [1]
FPACX	FPA Crescent	Equity
DODBX	Dodge Cox Balanced	Equity
SGENX	First Eagle Global	World Allocation
MALOX	BlackRock Global Allocation	World Allocation
GRRZX	Grizzly Short	Bear Market
DODIX	Dodge Cox Income	Intermediate-Term Bond
FPNIX	FPA New Income	Short-Term Bond
VEIEX	Vanguard Emerging Mkts Stock	Diversified Emerging Mkts
MACSX	Matthews Asian Growth Inc	Pacific/Asia ex-Japan Stk
PAUIX	PIMCO All Asset All Authority	Tactical
FLSAX	Fidelity Advisor Leveraged	Mid-Cap Blend

TABLE G.1: List of securities included in the investment set.

[1]The data source for the "Fund Type" classifications is Yahoo! Finance and Morningstar. The full "Fund Type" classification for FPACX and DODBX is "Allocation–50% to 70% Equity" but has been abbreviated as "Equity."

Appendix H

Rydex - Inefficient Frontier

A motivating factor for this research was the 'Inefficient Frontier' piece published by the financial services firm, Rydex. Included in this Appendix is the research note that discusses the non-stationary nature of efficient frontiers and the difficulty in using a single-frontier approach when investing.

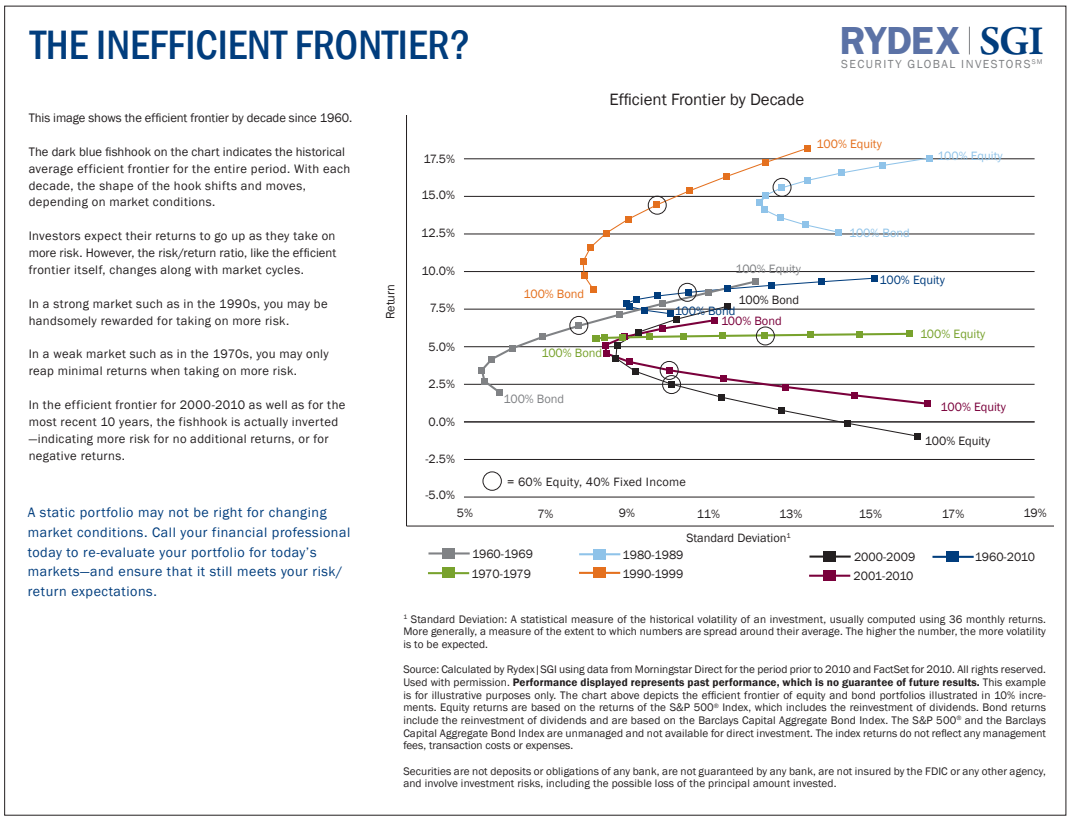


FIGURE H.1: 'Inefficient Frontier', page 1

THE INEFFICIENT FRONTIER?

RYDEX | SGI
SECURITY GLOBAL INVESTORS℠

This image shows the efficient frontier by decade since 1960.

The dark blue fishhook on the chart indicates the historical average efficient frontier for the entire period. With each decade, the shape of the hook shifts and moves, depending on market conditions.

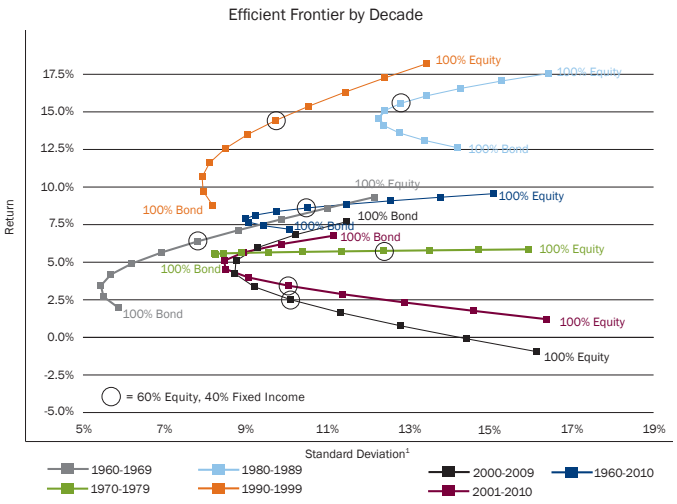
Investors expect their returns to go up as they take on more risk. However, the risk/return ratio, like the efficient frontier itself, changes along with market cycles.

In a strong market such as in the 1990s, you may be handsomely rewarded for taking on more risk.

In a weak market such as in the 1970s, you may only reap minimal returns when taking on more risk.

In the efficient frontier for 2000-2010 as well as for the most recent 10 years, the fishhook is actually inverted —indicating more risk for no additional returns, or for negative returns.

A static portfolio may not be right for changing market conditions. Call your financial professional today to re-evaluate your portfolio for today's markets—and ensure that it still meets your risk/return expectations.



[1] Standard Deviation: A statistical measure of the historical volatility of an investment, usually computed using 36 monthly returns. More generally, a measure of the extent to which numbers are spread around their average. The higher the number, the more volatility is to be expected.

Source: Calculated by Rydex | SGI using data from Morningstar Direct for the period prior to 2010 and FactSet for 2010. All rights reserved. Used with permission. **Performance displayed represents past performance, which is no guarantee of future results.** This example is for illustrative purposes only. The chart above depicts the efficient frontier of equity and bond portfolios illustrated in 10% increments. Equity returns are based on the returns of the S&P 500® Index, which includes the reinvestment of dividends. Bond returns include the reinvestment of dividends and are based on the Barclays Capital Aggregate Bond Index. The S&P 500® and the Barclays Capital Aggregate Bond Index are unmanaged and not available for direct investment. The index returns do not reflect any management fees, transaction costs or expenses.

Securities are not deposits or obligations of any bank, are not guaranteed by any bank, are not insured by the FDIC or any other agency, and involve investment risks, including the possible loss of the principal amount invested.

FIGURE H.2: 'Inefficient Frontier', page 2

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