PREPARING FOR ALGEBRA:

THE ROLE OF ALGEBRAIC THINKING IN CURRICULUM AND INSTRUCTION

A Capstone Project

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ABSTRACT

Algebra is known as a gatekeeper to upper-level math, college, and careers (Carraher & Schliemann, 2019). When students take algebra matters in relation to their access to additional math courses, because it is the foundation before further study in science, technology, engineering, and math. Algebra plays a powerful role in students' preparation for their future. Many school districts restrict students' enrollment in algebra in middle school grades. This was the case in Whispering Falls School District in Virginia. Students can take algebra as early as seventh grade, but very few are enrolled. This exploratory case study examined how students were prepared to take algebra by analyzing the components of algebraic thinking in the elementary and middle school math curriculum and instruction within the school district. Through document analysis, curriculum mapping, interviews, and observations, this study investigated how these components converged or diverged to support students' algebraic thinking.

Keywords: algebra, algebraic thinking, curriculum, instructional practices

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APPROVAL OF THE CAPSTONE

This capstone, “Preparing for Algebra: The Role of Algebraic Thinking in Curriculum and Instruction,” has been approved by the Graduate Faculty of the School of Education and Human Development in partial fulfillment of the requirements for the degree of Doctor of Education.

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DEDICATION

To Shane,

Without whom this never would have been finished.

Thank you for always believing in me.

Love you more.

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Chapter 1

Introduction

Statement of the Problem

Algebraic thinking is a critical component of mathematics because algebra is the foundation of most mathematical ideas, concepts, and courses (Wettergren, 2022). Twenty-four years ago, in the publication *Principles and Standards for School Mathematics,* the National Council for Teachers of Mathematics (NCTM) recommended that students engage in algebraic thinking starting in elementary grades and continuing throughout their high school math courses (NCTM, 2000). The recommendation states that students must engage in rich mathematical experiences steeped in conceptual understanding (Margiera et al., 2017). In 2008, the National Mathematics Advisory Panel (NMAP) reiterated the importance of integrating algebraic thinking into the elementary grades to explore arithmetic concepts through recognizing patterns, structure, and relationships between concepts, the crux of algebraic thinking (NMAP, 2008). Research by Blanton et al. (2018) demonstrated that integrating algebraic thinking into the elementary math curriculum in early grades can lessen many students' difficulty transitioning from elementary arithmetic to secondary math concepts.

Algebra is a well-known gatekeeper course due to its connection with advanced math and science classes, which opens doors to college opportunities and careers in science, technology, and math. (STEM) (Carraher & Schliemann, 2019). In most school systems, algebra is treated as a solitary course or a point in time during the secondary math sequence (Domina et al., 2016). However, algebraic thinking, which encourages students to think about numbers through relationality, helps students make meaning by focusing on the connections among numbers, patterns, and operations (Venenciano et al., 2020). In the elementary grades, algebraic thinking builds students' capacity to seek patterns, relationships, and structures of numbers (Knuth et al., 2018). Blanton et al. (2018) demonstrated that students could engage in algebraic thinking in the elementary grades, using arithmetic concepts, noting that early algebraic thinking promotes the engagement of thinking across concepts that transcend grade levels and supports students' ability to think about the interconnectedness of math. Opponents of early algebra believe that students are not developmentally prepared to think abstractly, based on Piaget's theory of student development (Hornburg et al., 2022). However, interventions such as Blanton et al. (2018) and Carraher and Schliemann (2019) have demonstrated that students can grapple with algebraic thinking in the context of elementary-level mathematics and succeed.

Teachers' knowledge of algebraic thinking and their ability to teach it are vital considerations when supporting the integration and recognition of algebraic thinking in elementary classrooms (Blanton & Kaput, 2005). Demonty et al. (2018) examined elementary teachers' content knowledge in addition to their pedagogical content knowledge regarding algebraic thinking, and most teachers were unable to recognize the connections between arithmetic and algebraic thinking. To engage students in algebraic thinking, which requires teachers to make connections across concepts, teachers must have a basic understanding of algebra and how to teach it (Blanton & Kaput, 2005). During instruction, teachers must capitalize on students' responses, incorporate algebraic thinking, and engage with student discussions to promote a deeper understanding of mathematics.

Because algebra is an entry point for higher-level math and careers in STEM, it plays a powerful role in mathematics education (Matthew & Fuchs, 2020). Therefore, it is crucial to identify and implement the factors that support students' access to and success in algebra

(Matthews & Fuchs, 2020). One of these factors is algebraic thinking in elementary and middle school.

Problem of Practice

In Whispering Falls School District[1] (WFD), students can take Algebra I as early as seventh grade based on their test scores on the Measures of Academic Progress (MAP) Assessment and the Virginia Standards of Learning (SOLs) year-end assessments (WFD, 2023). In WFD, the policy states that students must meet specific criteria, depending on their grade level, to be placed into Algebra 1 in seventh or eighth grade (Appendix A). Because of the restrictions on who can take algebra in middle school, only 10% of students at Boxwood Middle School take the course in seventh grade, and about 40% of the students take the course in eighth grade. Additionally, the assessment data from the SOLs and the MAP assessments utilized by WFD to place students into algebra does not explicitly measure algebraic skills or the ability to think algebraically. Additionally, very few students of color are enrolled in either class. For the 2023 - 2024 school year, one black student was in the seventh-grade algebra class.

Algebraic thinking supports students' future success in algebra and their overall conceptual understanding of mathematics; its integration into elementary and middle school curricula is tantamount to student success (Carraher & Schliemann, 2019). NCTM (2000) recommends integrating algebraic thinking into the elementary through middle school mathematics curriculum to support long-term understanding of math concepts.

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[1] All names are pseudonyms to preserve anonymity.

Although the components of algebraic thinking have been highlighted as essential for algebra readiness by NCTM and NMAP, it is unclear how these components are currently represented in the curriculum, instruction, or assessment of WFD in grades three through eight to adequately prepare students for algebra placement. This study aimed to examine how the WFD math curricula prepare students to take algebra. Using a curriculum mapping strategy, this study analyzed the knowledge and skills of algebraic thinking represented in the WFD mathematics curriculum documents and the degree to which the curriculum supports algebraic thinking across grades three through eight. Additionally, classroom visits occurred using an observation protocol to observe instructional practices related to algebraic thinking for grades three to eight and how the teachers translated the written curriculum into instructional practices. Lastly, interviews were conducted with mathematics specialists and the division math coordinator to understand the process of embedding algebraic thinking into the local curriculum and instruction. The research questions for this study were:

Research Question 1: To what extent are key ideas associated with algebraic thinking introduced and reinforced across the elementary and middle school math curricula in WFD?

Research Question 2: What instructional practices related to algebraic thinking are observable in WFD's grades three through eight mathematics classes?

Key Terms and Definitions

Algebraic thinking: Refers to the processes and reasoning skills that help students make sense of math (Chimoni et al., 2018). For this study, algebraic thinking encapsulates the meaning-making processes of mathematics, which help students make connections between concepts, understand the underlying structure of numbers and operations, and represent these relationships using words, numbers, and symbols.

Algebra readiness: The degree to which students have mastered certain skills, concepts, and thinking processes in preparation for taking the high school credit course, Algebra 1.

Chapter 2

Literature Review

The Importance of Algebraic Thinking

Algebra is a crucial component in the teaching of mathematics. Although it primarily exists as a single secondary course, the elements of algebraic thinking can and should be developed over time, beginning as early as elementary school (NCTM, 200). Most elementary mathematics curricula focus on computation and arithmetic, making transitioning from elementary and middle school math to high school algebra difficult for many students (Demonty et al., 2018). The research focused on this approach, referred to as "arithmetic-then-algebra," indicates it has not improved students' math achievement, particularly for marginalized students (Blanton et al., 2015, p. 40). Recommendations from literature and organizations like NCTM have called for more explicitly integrating algebraic thinking skills into the elementary curriculum (NCTM, 2014; Warren et al., 2016).

Algebraic Thinking Through the Years

Twenty-four years ago, in the publication *Principles and Standards for School Mathematics,* the National Council for Teachers of Mathematics (NCTM) recommended that students engage in algebraic thinking starting in elementary grades and continuing throughout their high school math courses (NCTM, 2000). The recommendation states that students must engage in rich mathematical experiences steeped in conceptual understanding (Margiera et al., 2017). Soon after, in 2004, former NCTM president Cathy Seeley declared the professional development focus for the organization to be "Developing Algebraic Thinking: A Journey from Preschool to High School" (Seeley, 2004). Her address to members highlighted the need to integrate algebraic thinking skills across grade levels and courses (Seeley, 2004). The algebraic thinking skills she highlighted in her address included the analysis of patterns, drawing conclusions in the form of generalizations, and understanding how things in math change (Seeley, 2004).

In a report issued by the United States Department of Education (2008), the National Mathematics Advisory Panel (NMAP) reiterated the importance of integrating algebraic thinking into elementary grades by incorporating pattern recognition, number structure, and relationships between concepts into the curriculum. NMAP stated the need to establish a solid foundational understanding of algebraic concepts by instituting a coherent curriculum (NMAP, 2008). Further, the panel recommended that students master specific concepts before engaging with algebra, such as equality, mathematical properties, fluency with whole numbers, and problem-solving skills (NMAP, 2008).

Then, in 2010, the National Governors Association adopted the Common Core State Standards (CCSS) for Mathematics, which embedded algebraic thinking in the mathematics standards and curricula from kindergarten onward (CCSS, 2010). The adoption restated the need for including algebraic thinking in elementary and middle school math to engage students in conceptual understanding and deeper mathematics thinking (CCSS, 2010).

In 2014, NCTM released a research brief on the same topic: algebraic thinking and its development from elementary to secondary school (NCTM, 2014). The brief highlighted the need to integrate algebraic thinking throughout grade-level math. During the decade between the two statements, three themes about algebraic thinking had come to the surface from research in the field: thinking relationally about equality, thinking about pattern generalization, and thinking about relationships in problem-solving situations, as ways of embedding algebraic thinking in elementary arithmetic topics (NCTM, 2014). Each movement highlights the importance of algebraic thinking throughout students' mathematical experiences. Yet, very few curricula in the United States integrate algebraic thinking skills into the elementary school mathematics curriculum (Blanton et al., 2018).

Algebraic Thinking Defined

James Kaput's Contributions

Although many scholars have attempted to refine, examine, and interpret the definition of algebraic thinking, the most common references stem from Dr. James Kaput of Dartmouth University. His 2008 seminal piece, "What is Algebra? What is Algebraic Thinking?" is the opening chapter of a book called *Algebra in the Early Grades.* In his chapter, he defines the two "core aspects" of thinking within algebra and the three content "strands" of algebraic thinking, which are the basis for many other definitions of algebraic thinking (Kaput, 2008, p. 11; Kieran, 2022).

Kaput's Content Strands. Kaput defined three content strands of algebra that are explored throughout mathematics. The first strand defines algebra as the study of structures and systems in arithmetic (Kaput, 2008). This strand includes skills such as generalizing about arithmetic, reasoning about the properties of numbers, and the relationships between numbers (Kaput, 2008). Kaput considered this strand the "heart of algebra," as represented by generalized arithmetic (Kaput, 2008, p. 12). The second strand focuses on functional thinking and examines variations of expressions in search of patterns (Kaput, 2008). This strand explores covarying quantities in search of patterns and extends to tables, graphs, and other algebraic symbols, including functions and expressions. The third strand is algebra as a language for modeling

(Kaput, 2008). This strand includes understanding the variable as an unknown quantity and its representation in different contexts (Chimoni et al., 2018).

Early Algebraic Thinking. Kaput is a "key figure" in the literature about algebraic thinking, and his definition is often cited as the essential explanation of the thinking and content included in algebra (Kieran, 2022, p. 1133). He is said to have coined "early algebra" and is credited with conceptualizing algebraic thinking in elementary school (Kieran, 2022, p. 1133). Kaput believed in and studied the introduction of algebraic thinking in early grades. He wrote about the need to "rework" algebra so that it is not seen as a course to be taken in the path of secondary mathematics but as a unifying concept bringing coherence to the K-8 curriculum (Kaput, 2008, p. 6). He saw the integration of algebra into elementary math as a thread that could be woven into each grade through reasoning and problem-solving. He is often cited as the leader of early algebra discussions and its initial conceptualization (Kieran, 2022).

Other Notable Definitions of Algebraic Thinking

Chimoni et al. (2018) define algebraic thinking as not just about traditional algebra content but also the processes and reasoning that help students make sense of math. Algebraic thinking is an amalgamation of thinking processes that help students understand the underlying connections between numbers, number systems, and mathematical syntax (Chimoni et al., 2018). Engagement in algebraic thinking in elementary math encourages students to explore relationships between numbers and think about arithmetic by examining patterns, changes, and relationships between mathematical processes (Chimoni et al., 2018). Kieran (2022) describes algebraic thinking as "multi-dimensional" (p. 1133). By that, she means that there is not one singular action that defines algebraic thinking. However, many curricular and instructional elements support algebraic thinking, and together, they create a multi-dimensional definition that highlights the complexity of the concept. Kieran (2022) says algebraic thinking can be used with and without the symbolic notation of algebra and includes specific skills like problem-solving, modeling, predicting, and proving, which she says can be incorporated into early math as ways of thinking. Sun et al. (2023) expanded on this idea by stating that children use a series of processes to summarize and generalize about number structure, patterns, and quantitative reasoning. Commonalities across these definitions show that the ability to generalize, examine relationships between numbers, and understand the structure of numbers and arithmetic are critical skills associated with algebraic thinking.

Algebraic Thinking and Algebra Readiness

Algebra is the entry point for high school mathematics; all students must funnel through this class as a gateway to further high school credit courses (NMAP, 2008). Algebra has been dubbed the linchpin of mathematics education because of its foundational role in the sequence of secondary math (Knuth et al., 2016). Secondary math courses are taken in sequence, which means students must complete one course before being promoted to the next. The linear progression starting with Algebra 1 dictates when and how many math classes students can take in preparation for their post-high school plans (Reyes & Domina, 2017). Success in the first algebra class is linked to students' future success in math, college, and careers (Knuth et al., 2016). So, ensuring students are well prepared to take algebra is paramount to their success.

According to the NMAP (2008), the crucial math concepts of algebra readiness include fluency with operations and a conceptual understanding of rational numbers. Additionally, students must understand equality and equivalence (Ketterlin-Geller et al., 2019). These content components fit into the strands of algebraic thinking. Generalized arithmetic supports students understanding of the number system and relationships between numbers. Students' ability to make connections between the concepts of magnitude and place value supports their capacity to think flexibly about numbers, an essential algebraic reasoning skill (Ketterlin-Geller et al., 2015).

Rational numbers, including fractions, are strongly connected to algebra readiness (Knuth et al., 2016). Operations with fractions lead to more complex fraction concepts like magnitude, ratios, proportions, and rates (Barbieri et al., 2021). Manipulating fractions and algebraic thinking involve the ability to follow steps and translate complex symbols into mathematical applications (Barbieri et al., 2021). Working with fractions requires students to think and reason abstractly, like algebra (Knuth et al., 2016).

The equal sign is ubiquitous in algebra. In elementary grades, students experience it as a signal of computation (Knuth et al., 2016). Left-to-right computation using the equal sign reinforces the misconception that equations are unidirectional (Hornburg et al., 2022). A more sophisticated understanding, which supports algebraic thinking, is to approach the equal sign as a relation between the values on either side (Knuth et al., 2016). Focusing on equality instead of the operation of the equal sign supports students' relational thinking and the big idea of equality; a concept students apply in Algebra (Hornburg et al., 2022).

Three Key Content Components of Algebraic Thinking

Three key content areas are consistently referenced throughout algebraic thinking research: generalized arithmetic, functional thinking, and equations and equivalence. This section will explore each component and examine the associated instructional practices.

Generalized Arithmetic

Kaput (2008) named generalized arithmetic the “heart of algebra” (p. 12). Generalized arithmetic involves noticing and naming the underlying structures and relationships between numbers, operations, and properties (Sun et al., 2023; Blanton et al., 2015). Kaput (2008) describes generalized arithmetic as the ability to generate generalizations from arithmetic by reasoning about the relationships between numbers and operations. More specifically, generalized arithmetic is understanding and using the properties of numbers and the laws of operations to transform numbers (Sun et al., 2023).

Generalized arithmetic includes concepts such as number properties, operations, place value, and forms of numbers (Blanton et al., 2015). For example, the generalized arithmetic strand includes the properties of equality, such as the commutative property. Students explore the relationality of the equal sign to express that $a + b$ and $b + a$ represents an equivalent expression (Blanton et al., 2015). When students engage in generalized arithmetic, they also begin to recognize patterns from numbers and operations to generalize. A typical example shows that doubling an odd number always results in an even product (Sun et al., 2023).

Instructional Practices for Generalized Arithmetic. Blanton and Kaput (2005) conducted a case study about the teacher practices that promote algebraic thinking. For this study, Blanton and Kaput (2005) observed a third-grade teacher over a school year to look for ways in which the teacher was able to incorporate algebraic thinking into her instructional practices. One pertinent finding of this review is that the generalized arithmetic strand offers multiple entry points for students to apply algebraic thinking (Blanton & Kaput, 2005). For example, throughout the 204 observations, over half of them involved properties and relationships of whole numbers, operations, missing values, and patterns (Blanton & Kaput, 2005). The most common strategies incorporated into her instruction about generalized arithmetic were using “tools” to support student thinking, like tables, charts, diagrams, number lines, and graphs (Blanton & Kaput, 2005, p. 432). Using multiple representations of mathematical relationships supported students’ algebraic reasoning by helping them make connections between different forms of numbers or operations. Students were able to use these representations to make claims about patterns they saw or create an argument for their reasoning about a generalization.

Another teaching method documented in this case study was the teachers' use of conversations in the classroom to engage students in algebraic discourse (Blanton & Kaput, 2005). During the observation period, the teacher began to infuse algebraic thinking questions into her teaching practices through "spontaneous" and "planned" algebraic reasoning conversations with students (Blanton & Kaput, 2005, p. 418). The third-grade curriculum's focus on operations and properties in this study enabled the teacher to engage students in discussions about the relationships between quantities and procedures, encouraging them to identify patterns and make generalizations. Utilizing planned and spontaneous questions during instruction, which probe students to look beyond arithmetic and justify their thinking, encourages students to think about more than computation (Blanton & Kaput, 2005).

Functional Thinking

Functional thinking starts with analyzing patterns in their simplest forms (Kaput, 2008). Algebraic thinking related to patterns encourages students to investigate patterns, generalize from them, and express them in words, numbers, or symbols (Afonso & McAuliffe, 2019). In elementary school, students begin to explore patterns by examining geometric forms, identifying similarities and differences, and then making generalizations about subsequent items in the sequence (Kieran, 2022). Next, students delve into arithmetic patterns, which assist them in inferring terms in the pattern using operations. As they progress to middle school, they start to explore ratios and proportions, examining the relationships between co-varying quantities, and considering how changes in one variable impact the other (Chimoni et al., 2018). As students' algebraic thinking becomes more advanced, they begin to investigate relationships between a term's position in a pattern and the term itself, develop rules or generalizations about the pattern, and articulate the pattern in terms of variables (Kaput, 2008).

Concepts embedded in functional thinking also include patterns within variables, expressions, and equations (Chimoni, 2018). Here, students are not looking for the underlying rule or universal generalization; instead, they seek to understand the relationship between co-varying quantities and how those relationships can be expressed using variables, expressions, or equations (Kieran, 2022). As students' functional thinking evolves, so do their representations of the relationships between covarying quantities. In elementary school, students are exposed to drawing, tables, charts, and graphs, and as they enter middle grades, they start to explore tables of values, function machines, coordinate planes, and equations. (Blanton et al., 2015; Sun et al., 2023; Kieran, 2022).

Instructional Practices for Functional Thinking. Carraher and Schliemann (2018) conducted a longitudinal study of third through fifth-grade students using classroom activities that connect arithmetic concepts to algebraic thinking, specifically functional thinking. As a result of the study, Carraher and Schliemann (2018) were able to pinpoint specific teaching strategies that helped students bridge the gap from arithmetic to algebra. First, the premise of functional thinking is the exploration of relationships, so when teaching with algebraic thinking in mind, students must be guided toward looking for and articulating how one quantity affects another (Carraher & Schliemann, 2018). Additionally, these relationships should be expressed using real-world contexts when applicable. In this study, Carraher and Schliemann (2018) focused on open-ended problems that represented simple but realistic situations for students to discuss, represent, and solve. Another strategy that was used to support students' thinking and discussions was collaborative work (Carraher & Schliemann, 2018). When students worked collaboratively to solve problems, they could learn from each other by engaging in mathematical discourse. Like generalized arithmetic, using multiple representations of functions, including drawings, number lines, graphs, and tables, helps students recognize and justify relationships between quantities (Carraher & Schliemann, 2018).

Equations and Equivalence

It is worth noting that some authors include equations and equivalence in the generalized arithmetic category (Kaput, 2008; Chimoni et al., 2018). According to Stephens et al. (2017), the ability to express generalizations about properties relies upon students' understanding of the equals sign and equivalence because equations are based on properties. Hence, equality is part of generalized arithmetic. Kaput (2008) also includes the equal sign in the generalized arithmetic strand. For this literature review, and because of the importance of equality across all mathematics, this review will focus on equality outside of generalized arithmetic to highlight its value in thinking algebraically.

Students in early elementary are taught to see the equal sign from an operational point of view; that is, the equal sign is a symbol to compute and find an answer (Stephens et al., 2013). Whereas the relational perspective, the more advanced algebraic concept, means that students understand the equal sign means "the same as" and denotes equivalence or balance (NCTM, 2014; Blanton et al., 2018; Chimoni et al., 2018; Stephens et al., 2013, p. 174). Equality is crucial to understanding algebra because of its omnipresence throughout the mathematics curriculum (Stephens et al., 2013). An early misunderstanding of the equal sign, or failure to develop a relational understanding, can have long-term implications as students move toward secondary math courses when they must manipulate equations (Stephens et al., 2013; Utami &

Prabawanto, 2023). Blanton et al. (2018) define this core area of algebraic thinking as "equivalence, expressions, equations, and inequalities" (p. 31). Their description of this component includes representing and reasoning with the symbolic forms of equations and expressions and a relational understanding of equality (Blanton et al., 2015).

Developing an Understanding of Equality. A student's understanding of the equal sign evolves as their algebraic thinking develops. Stephens et al. (2013) observed three stages of students' thinking regarding the equal sign. First, students who view the equal sign from an "operational" point of view hold the most simplistic understanding (Stephens et al., 2013, p. 174). Students in this stage see the equal sign as a signal to operate or compute, as they work from left to right. The second stage is "relational-computational" because students understand that the values on either side of the equal sign are related. However, they must calculate or compute to justify their reasoning (Stephens et al., 2013, p. 174). Lastly, students enter the "relational-structural" stage, which is the deepest level of understanding and demonstrates the most flexibility in thinking (Stephens et al., 2013, p. 174). Students in this phase understand the relational view of the equal sign and can use the structure of equations and numbers to support their reasoning. For example, when presented with the equation 23 + 45 = 22 + 46, a student in the relational-computational stage would add the values on each side to prove they are the same. A student in the relational-structural phase would be able to explain that they are the same because taking one from the 23 on the left side of the equation is the same as adding one to the 45 on the right side of the equation, using the structure of numbers to justify their solution in place of computation (Stephens et al., 2013). Similarly, Blanton (2015) described three stages of thinking about equality using similar phrasing: operational, computational, and relational.

Matthews and Fuchs (2020) also studied the impact of the relational understanding of the equal sign. Their study tested second-grade students with open equations that had a missing number represented by a blank in the equation. This test aimed to measure students' understanding of the equal sign by examining their ability to make both sides of the equation the same (Matthews & Fuchs, 2020). In fourth grade, the students were assessed again using a test that included solving equations and completing function tables (Matthews & Fuchs, 2020). This test was used to measure students' algebraic knowledge. Using a direct regression, equal sign knowledge in second grade was the most significant predictor of fourth-grade equation solving (Matthews & Fuchs, 2020). The finding supports the idea that the equal sign is vital for mathematical and algebraic thinking, and students' ability to manipulate numbers in equations.

Instructional Practices that Support Equality and Equivalence. Blanton et al. (2015) designed and implemented a framework for early algebra intervention to support students' algebraic thinking, explicitly targeting students' understanding of the equal sign. In the longitudinal study, teachers used eighteen lessons over the course of one school year on various topics starting with lessons on the "relational understanding of the equal sign" (Blanton et al., 2015, p. 32). The instructional components of the lessons focused on investigations and open-ended tasks grounded in real-world scenarios. Students were asked to represent their ideas in different ways, including drawings, written explanations, variables, and graphs (Blanton et al., 2015). Teachers also employed mathematical discourse to support the articulation of ideas and the exchange of information. Students in the intervention groups outperformed control group students on post-test measures for the study and maintained their advantage one year after the intervention ended (Blanton et al., 2015).

Algebraic Thinking in Math Curriculum

Integrating algebraic thinking into math curricula has yet to be a common practice. However, substantial data has been collected that indicates students can grapple with the thinking skills of algebra and be successful (Afonso & McAuliffe, 2019; Blanton et al., 2018; Chimoni et al., 2018). Algebraic thinking should be incorporated into elementary math curricula by connecting the content of algebraic thinking to the concepts of current elementary math curricula.

The Development of Algebraic Thinking

Developing Algebraic Thinking through Arithmetic. In 2004, Kieran analyzed multiple math curricula and student thinking levels. Her findings showed that some curriculum resources focus on computation and do not include elements of algebraic thinking (Kieran, 2004). To infuse arithmetic with algebraic thinking, she found that the curricula must include five focus areas: relationality, operations and their inverses, representing and solving problems, utilizing numbers and letters with operations, and understanding the meaning of the equal sign (Kieran, 2004). Focusing on these skills moves students from the calculation of numbers into a relational perspective aimed at making connections across concepts (Kieran, 2004).

Stephens et al. (2017) view the integration of algebraic thinking and computation skills as a natural fit. The elementary math curriculum emphasizes numbers, computation, and operations, which form the foundation of generalized arithmetic. Encouraging students to consider the relationships among these concepts and explore the underlying structure of numbers helps to bridge the gap between arithmetic and algebraic thinking (Pitta-Pantazi et al., 2020). By introducing algebraic thinking in elementary school, students engage in thought processes that foster a deeper understanding of mathematics (Kieran, 2004).

Developing Early Algebraic Thinking. Traditionally, algebra has been introduced to students after they have had substantial experience with arithmetic. This supports the idea that mastery of arithmetic is a necessary foundation for engaging in algebraic thinking (Warren et al., 2016). Advocates of early algebra believe this to be untrue; early algebra studies have focused on embedding specific types of tasks and thinking processes in elementary math curricula to demonstrate that students can engage in algebraic thinking from a young age (Blanton et al., 2018). The purpose of algebraic thinking is not to introduce algebra concepts to students at an earlier age or grade level but to expose them to the connections among numbers, operations, and properties of numbers (Afonso & McAuliffe, 2019). Proponents of early algebra argue that separating arithmetic and algebra makes the transition to secondary math more challenging for students (Warren et al., 2016). By using numbers and computation as an entry point, teachers can help students expand their thinking by observing and generalizing the connections they notice between numbers (Stephens et al., 2017). Algebraic thinking nurtures students' natural curiosity about how things work by exploring the underlying structure of numbers and operations through patterns (Sibgatullin et al., 2022)

In a 2022 study of students in grades two through four, teachers and researchers designed lessons that would support students' relational thinking, the structure of numbers, and general patterns (Wettergren, 2022). Teachers used contextual problems representing real-life situations and supported their teaching with multiple representations and manipulatives. The lessons were observed and videotaped to code both in the moment and afterward. Across grade levels, students demonstrated algebraic thinking skills about equal signs and equations (Wettergren, 2022). Students also demonstrated improvements in their understanding of variables and in their overall problem-solving abilities with real-life scenarios using various mathematical representations (Wettergren, 2022). This research indicates that students in elementary grades can develop algebraic thinking skills through teacher-mediated lessons.

Early Algebra Intervention Curriculum. One approach examined over iterative trials is the implementation of an "early algebra intervention" in elementary grades (Blanton et al., 2015, p. 40). This section will highlight the work of Blanton et al. (2019). This study is an extension of previous work by Blanton et al. (2018), wherein they developed a curricular framework for algebraic thinking in grades three through five. In this study, the curriculum framework was implemented starting in third grade, and students' data was tracked over three years to measure the impact of the intervention on their readiness for algebra in middle school (Blanton et al., 2019). This study is significant because other researchers have examined the data from different perspectives, primarily focusing on teachers' instructional techniques.

Blanton et al. (2019) implemented the early algebra framework to engage students in algebraic thinking through existing arithmetic concepts in the local curriculum for grades three through five. An instructional sequence was developed to include algebraic thinking practices: generalizing, representing, justifying, and reasoning relative to the "Big Ideas" of elementary math, which were identified as generalized arithmetic, equivalence, expressions, equations, and inequalities, and functional thinking (Blanton et al., 2019, p. 1935). The framework included 18 one-hour lessons throughout the school year, a minimal amount of time compared to the whole year. Teachers participating in the intervention received professional development (PD) to support the implementation each year. The purpose of the PD was to develop the teachers' understanding of algebraic thinking, how students communicate their own algebraic thinking, and, most importantly, to strengthen teaching practices related to algebraic concepts (Blanton et al., 2019).

Students participated in the intervention for three years, from third through fifth grade. At the end of each year, students in the intervention group showed an advantage over those in the control group in two areas: their understanding of algebraic concepts (e.g., solving equations) and their use of structural strategies (e.g., noticing fundamental properties). The data from this study show that strengthening students' algebraic thinking skills in elementary school lasts through their transition to middle school (Blanton et al., 2019). Additionally, the intervention served to support math teaching because teachers received professional development as part of the implementation.

Developing Algebraic Thinking in Middle School

Using Kaput's framework from 1998, Pitta-Pantazi et al. (2020) examined students' algebraic thinking abilities through four content strands: generalized arithmetic, functional thinking, modeling languages, and algebraic proof (Kaput, 1998). (It should be noted that in 2008, Kaput updated this framework to include the three strands that were mentioned previously, omitting algebraic proof). Pitta-Pantazi et al. (2020) assessed students in grades eight and nine on 23 tasks that reflected the four content strands. They found that students in middle school were first able to complete functional thinking tasks, then generalized arithmetic (Pitta-Pantazi et al., 2020). Once those skills were mastered, students moved into modeling language and, finally, algebraic proofs (Pitta-Pantazi et al., 2020). These findings demonstrate potential differences between younger learners and middle school students in algebraic thinking. However, these results also indicate that students move through stages of algebraic thinking, which can impact planning for curriculum and instruction (Pitta-Pantazi et al., 2020).

Developing Algebraic Thinking Through a Progression of Thinking Skills. Chimoni et al. (2018) tested a specific sequence of learning that improved students' ability to think algebraically while increasing the difficulty of the tasks they were able to complete. The study explored fourth through seventh-grade algebraic tasks across three content strands: generalized arithmetic, functional thinking, and modeling (Chimoni et al., 2018). Researchers took note of the processes and reasoning students engaged in with each task and then categorized them into three groups based on their performance (Chimoni et al., 2018). Results showed that students could solve the tasks in a specific progression: generalized arithmetic, then functional thinking, and lastly, modeling tasks (Chimoni et al., 2018). This finding is important because it helps teachers better understand how students' algebraic thinking evolves through tasks that connect arithmetic and algebra. This information is helpful to educators who can design learning experiences that mimic this progression.

These findings also highlight differences between later middle school learners (Pitta-Pantazi et al., 2020) and students transitioning from elementary to middle school (Chimoni et al., 2018). The emphasis on arithmetic in elementary school may be reflected in the performance of students in grades four through seven in Pitta-Pantazi et al.'s (2020) study, as compared to the eighth and ninth graders in Chimoni et al.'s (2018) research, who demonstrated greater skills in functional thinking due to their focus on proportionality in middle school

Enactment of Math Curriculum

Between the written curriculum and student outcomes lies the enactment of the curriculum, which includes the interpretation of the written curriculum, the transmission of information to students, and the interactions between teachers and students (Remillard & Heck, 2014). Remillard and Heck (2014) created a framework to outline the concept of curriculum enactment by math teachers. The process starts when teachers receive the "official curriculum," national or state standards delineating student objectives (Remillard & Heck, 2014, p. 708). The official curriculum is then translated into the “designated curriculum,” which is done by a local agency, like a school district, to include instructional materials and guidance about how the curriculum should be used (Remillard & Heck, 2014, p. 710). Now, the curriculum lies in the hands of the teacher, who must transform the curriculum into daily plans that reflect their interpretation of the resources and resonate with the students they engage with. Remillard and Heck (2014) refer to this as the “teacher-intended curriculum” (p. 711). The next step for the teacher is to enact the curriculum, which includes both the scripted and unscripted interactions among teachers and students, interactions among students themselves, and students engaging with the math tasks presented to them.

During the implementation of the curriculum, Remillard and Heck (2014) identified four dimensions of focus that can be observed during instruction. They are mathematics (e.g., content, topics, practices), instructional interactions (e.g., how teachers, students, math, and resources connect), teacher pedagogical moves (e.g., how math is represented and engaged with), and tools and resources (e.g., instructional resources, technology, physical tools).

The implementation of the curriculum varies from teacher to teacher. Since teachers must interpret the written curriculum and transmit it to students, there is room for each teacher to create their own meaning and communicate the message in their own distinct way. Otten and Soria (2014) showed this in their study of three teachers implementing the same lesson sequence by measuring the time each teacher spent during the three phases of the lesson and the level of cognitive demand required of the students. Although they were using the same instructional resources, the time spent on each segment and the level of cognitive demand varied widely from class to class (Otten & Soria, 2014). The teacher-intended curriculum and enactment are unique to the teacher and the students she works with at a particular time (Remillard & Heck, 2014).

Identifying Algebraic Thinking in Instruction

Teachers must take specific instructional actions to develop algebraic thinking in students (Carraher & Schliemann, 2019). This section highlights the role of the teacher in recognizing and interacting with students' algebraic thinking. Ristroph et al. (2022) analyzed instructional interactions when elementary teachers chose whether to engage with students' algebraic thinking. In another study, Lee et al. (2023) categorized algebraic thinking by examining the rigor of discourse between teachers and students in elementary classrooms.

Acting on Algebraic Thinking

Blanton and Kaput (2005) described elementary teachers' algebra "eyes and ears" in terms of their ability "to spot opportunities for algebraic thinking" (p. 76). Using the data from Blanton et al. (2018), Ristroph et al. (2022) examined teachers' interactions with students during mathematical discussions to pinpoint algebraic thinking. The study examined videotaped lessons and identified opportunities for teachers to notice and act upon students' algebraic thinking (Ristroph et al., 2022). Prior to watching the lessons, researchers identified segments of the lessons that called for algebraic thinking and defined those as "anticipated moments" (Ristroph et al., 2022, p. 274). Conversely, while they watched the videotaped lessons, any algebraic thinking that could happen in the moment to support understanding an algebraic concept initiated by either teachers or students was labeled "spontaneous" (Ristroph et al., 2022, p. 275).

The results showed that teachers engaged with more than 70% of the anticipated algebraic thinking segments of the lessons (Ristroph et al., 2022). If the lesson included connections to algebraic thinking, teachers would likely address the content during its enactment. Teachers also acted upon 77% of the spontaneous opportunities to engage with algebraic thinking outside the planned lesson (Ristroph et al., 2022). This is potential evidence that

Blanton et al.'s (2018) algebra intervention's educative nature supported teachers' understanding of early algebraic thinking (Ristroph et al., 2022).

Algebraic Thinking Through Discourse

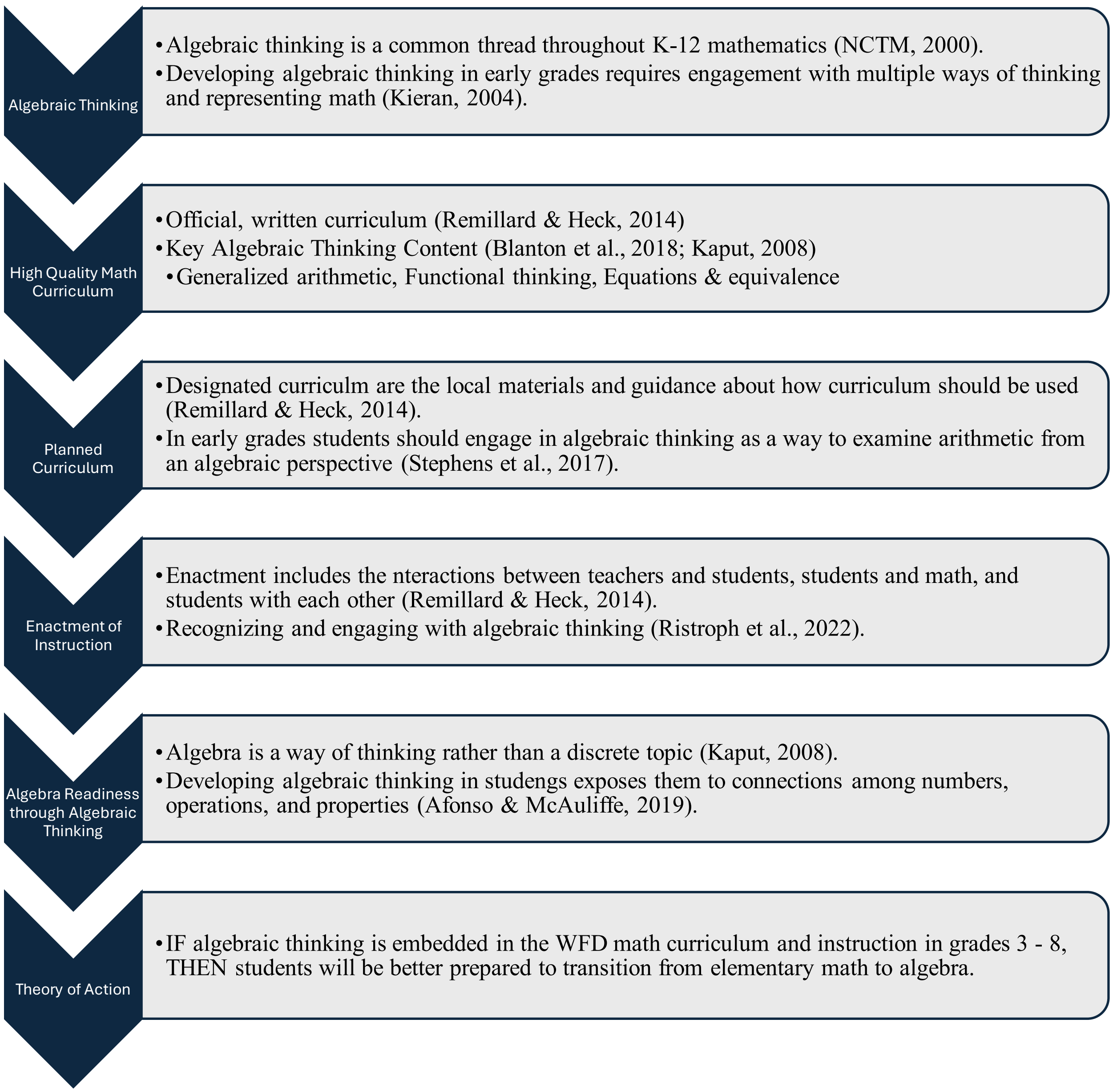
Lee et al. (2023) also used data from the Blanton et al. (2018) study to code the interactions between teachers and students using mathematical discourse. Lee et al. (2023) counted and coded teacher and student interactions, determining whether they were anticipated or spontaneous. Of the spontaneous moments, they coded the dialogue into "response" categories that defined the extent of the mathematical conversation (Lee et al., 2023, p. 244). They found that as teachers engaged with the early algebra intervention curriculum, their responses to students evolved from "acknowledging" student responses as correct or incorrect to "extending" responses by pressing for additional justifications and building on other students' answers (Lee et al., 2023, p. 246). Using linear regression analysis, Lee et al. (2023) found that when teachers engaged in discourse with students using thinking processes like justification and generalization, students generally showed higher gains on the algebra posttest. Early algebra activities implemented by elementary teachers can support algebraic thinking when teachers understand the algebraic nature of the problem and can guide students through discourse that connects arithmetic and algebraic thinking (Lee et al., 2023).

Conceptual Framework

A conceptual framework comprises multiple parts demonstrating how they intersect, inform, and influence each other (Ravitch & Carl, 2021). The problem of practice in this study focuses on readiness for algebra and the specific content and instructional practices that support students' algebraic thinking. For students to engage in algebraic thinking, it must be integrated into the curriculum and recognized by teachers through their instructional practices. In this way, algebra becomes a way of thinking about math, its connections, and representations rather than an isolated course in time. This conceptual framework begins with algebra readiness as a key feature that influences the curriculum, instruction, and student outcomes.

Figure 1

Conceptual Framework



Chapter 3

Methods

Study Design

The design for this problem of practice is an exploratory case study that examined where and how algebraic thinking concepts and key ideas existed in the WFD curriculum. Additionally, this study included classroom observations of the teaching practices related to algebraic thinking and interviews with math specialists in WFD elementary and middle schools. The data collected from examining the district curriculum documents, observing math instruction, and interviewing math leaders informed the recommendations to the school district regarding future practices in elementary and middle school curriculum and instruction to prepare students for algebra.

Study Context

The site for this study was Whispering Falls School District (WFD), a small urban school district located in Virginia. WFD comprises nine schools and three alternative centers, serving approximately 4,400 students and employing over 400 teachers (WFD, 2022). WFD is a diverse district, with the population comprising 40.2% white, 26.2% black, 13.6% Hispanic/Latino, 6.1% Asian, and 13.8% identifying as multiple races (Virginia Department of Education [VDOE], 2023). More than 30 languages are spoken among the student population, with Spanish, Dari, and Pashto being the most common (WFD, 2022). The city where WFD is situated hosts an International Rescue Committee (IRC) Center for Refugees, which places many students into WFD schools (IRC, 2023). Recently, the percentage of English language learners has grown to 19.4% from 16%, and according to the VDOE (2023), 63.3% of students qualify as economically disadvantaged.

There are six elementary schools in WFD, all of which receive Title I funding. Two schools in WFD cater to middle-grade students. Willow Upper School serves grades five and six, while Boxwood Middle School accommodates seventh and eighth grade. Elementary teachers cover all subject areas. At Willow and Boxwood, teachers specialize in subjects, and together, there are 20 math teachers across both schools (WFD Staff Directory, 2024). Each school in WFD has a math specialist who connects with the district math coordinator to facilitate communication between the schools. The math specialists liaise with the district's math coordinator to design and implement the district-wide curriculum utilizing district resources and state standards of learning (SOLs). Math specialists convene weekly with teacher teams to plan math instruction and analyze student data.

Changes to the State Standards

In the 2024-2025 school year, Virginia implemented new math SOLs, which will be assessed for the first time in the spring of 2025. This version of the SOLs highlights the importance of algebra, stating, "Algebra is the gateway to higher education and promising careers" (Virginia Mathematics SOLs, 2023, p. 2). The standards include a strand for each grade level from kindergarten through eighth grade called "Patterns, Functions, and Algebra" (Virginia Mathematics SOLs, 2023). Although this strand is identified explicitly as the algebra strand, the skills and concepts related to algebraic thinking live in multiple strands, including "Number and Number Sense" and "Computation and Estimation" (Virginia Mathematics SOLS, 2023, p. 5).

Sampling

A purposeful typical sampling method was employed in this study, choosing two elementary schools of similar size and population for the observations (Nyimbili & Nyimbili, 2024). Typical case sampling focuses on the average population within the study's context; in this instance, selecting two schools with comparable size and student demographics represented the general populations of all six elementary schools in WFD (Nyimbili & Nyimbili, 2024). The target population for this survey consisted of all math teachers at the four schools in grades three and above. Since the observations were voluntary, I relied on volunteers to participate in the study. Consequently, the teachers involved formed a convenience sample as they had to agree to the observation, which needed to occur within a specific time frame. Regarding the interview participants, I met with the entire sample, which included the four math specialists from the study schools and the coordinator.

Selected Schools

The study occurred in two elementary schools and the two middle grades schools of WFD. The first school is Cedar Elementary. Cedar's K-4 enrollment was around 260 students in 2024 (VDOE School Quality Profiles, 2024). The student population was 21.8% white, 30.8% black, 17.9% Hispanic, 16.8% multiple races, and 13% Asian (VDOE School Quality Profiles, 2024). The population was 99% economically disadvantaged, and 21.1% of students were English language learners (VDOE School Quality Profiles, 2024). The second elementary school was Maple Elementary. Maple enrolled about 285 students in 2024: 56.5% white, 14.7% black, 13.3% Hispanic, 13.3% multiple races, and 2.1% Asian (VDOE School Quality Profiles, 2024). Unlike Cedar, only 31.6% of students at Maple were economically disadvantaged, and 11.6% of the population were identified as English Language learners (VDOE School Quality Profiles, 2024).

The study included both middle-grade sites as typical cases because only two middle-grade schools are in the district. Willow Upper Elementary serves fifth and sixth-grade students with an enrollment of around 600. The student population was 44% white, 26.2% black, 10%

Hispanic, 14.3% multiple races, and 5.4% Asian. Willow does not receive Title I funding, but 50% of the students were economically disadvantaged, and 21.5% of the students were English learners. Lastly, Boxwood Middle School had 560 seventh and eighth-grade students. Boxwood's students were 36.5% white, 27.5% black, 17.1% Hispanic, 13% multiple races, and 5.9% Asian. Like Willow, Boxwood does not receive Title I funds, but 56.9% of the students were economically disadvantaged, and 26.3% were English learners.

Table 1

Demographics of Selected WFD Schools

School	Total Pop.	White	Black	Hispanic/ Latino	Multiple Races	Asian	ELL%	Econ. Dis. %
Cedar	260	21.8%	30.8%	17.9%	16.8%	13%	21.1%	99%
Maple	285	56.5%	14.7%	13.3%	13.3%	2.1%	11.6%	31.6%
Willow	600	44%	26.2%	10%	14.3%	5.4%	21.5%	50%
Boxwood	560	36.5%	27.5%	17.1%	13%	5.9%	26.3%	56.9%

Participants

Teachers. In WFD elementary schools, teachers cover all subject areas. At Cedar Elementary, a third-grade teacher took part in the observations. Miss P., the third-grade teacher, has been at Cedar for seven years. At Willow, two fifth-grade teachers who instruct both math and science were observed. The first teacher at Willow, Miss S., is in her second year and started in 2023. Miss L. also teaches fifth grade. She has taught in another state previously and has six years of teaching experience.

Two teachers were observed teaching only math in sixth grade. Miss F has taught at Willow in fifth and sixth grade for about six years. Miss W. is an experienced teacher with over 20 years in the school district. She previously taught at the high school but has been at Willow for more than a decade.

Math Specialists and Math Coordinator. Maple has one math specialist who works with all kindergarten through fourth-grade teachers. Brooke has been a teacher for about 20 years. Cedar has two math specialists for the 2024-2025 school year. One specialist works with kindergarten through second-grade teachers and the other works with third- and fourth-grade teachers. Jane, the specialist who participated in the interviews, has been a teacher for 14 years, and this is her second year as a math specialist. Willow and Boxwood also have one math specialist who works with all math teachers at the site. The math specialist at Willow is Brian, who previously taught fourth grade in another school, and Kevin is at Boxwood, who is new to the school this year. A summary of the specialist's experience is in Table 1. Lastly, the math coordinator will be a participant. There is one math coordinator for the school district, Melinda, and she works with the math teachers and specialists at each school. This is her first year in WFD as the coordinator.

Table 2

Interview Participants Background Information

Name	School/Role	Years of experience	Math credentials	Previous Work Experience
Miranda	District/Coordinator	20+	Math Specialist Certification	Coordinator in neighboring district, math teacher at middle and high school
Brooke	Maple Elem/Specialist	20+	Math Specialist Certification	Specialist in neighboring district, 2nd year at Maple

Name	School/Role	Years of experience	Math credentials	Previous Work Experience
Jane	Cedar Elem/Specialist	15	None	Special education teacher, 2nd year as a math specialist
Brian	Willow Upper School/Specialist	14	None	Elementary teacher, 2nd year as a math specialist
Kevin	Boxwood Middle	25+	Master's Degree in Math	Secondary teacher, math interventionist in another district, 1st year at Boxwood

Note. All names are pseudonyms.

Timeline and Procedures

After receiving approval from UVA IRB-SBS and the WFD school district, emails were sent to individual school principals to notify them of the approved study. In January 2025, I emailed all the principals at the site schools to request permission to observe their teachers and interview math specialists. Each principal approved, and I contacted individual math teachers in grades three through eight. The email provided basic information about the study and an information sheet. Since the observations were voluntary, I relied on teachers to respond to my request. Due to inclement weather during February, all the observations had to be rescheduled multiple times.

All the observations took place in February 2025. Two of the teachers who responded were from Maple Elementary, one was from Cedar, and four were from Willow. After multiple emails with teachers and the math specialist, there were no responses to observation requests at Boxwood Middle School. Two teachers from Maple Elementary could not reschedule their observations due to a pre-planned Field Trip. Interviews were scheduled with participants via email and conducted over Zoom. Each interview lasted between 30 and 60 minutes.

Data Sources

Documents

The district documents had to be sent to me by the math coordinator for me to access them. The requested documents were the district curriculum guides, pacing guides, lesson plans for the weeks of the observation window, and associated assessments. The documents that were received are summarized in Table 3.

The Virginia Math SOLs were also included in the document analysis and accessed through the Virginia DOE website. Using the READ process the documents were organized for analysis following four steps. (Dalgish et al., 2020). First, the materials were "Readied," which in this case means they were received from WFD via a shared Google folder and placed into UVABOX. The initial organization was to categorize the documents according to type. Second data was "Extracted" from each document and added to a spreadsheet. Gross (2018) refers to this critical information as the document "demographics" (p. 546). The demographics collected for these curriculum documents included the title, authors, audience, document type, content, and purpose. The third step was the "Analysis" phase in which the data was compared for similarities and differences across all the shared documents. The fourth step, addressed in chapter 4, was to "Distill" findings.

Table 3

WFD Documents by Type, Purpose, Quantity, and Title

Document Type	Purpose	Quantity	Document Titles
District Pacing Guide	Guide for teachers to plan from, including SOLs and days of instruction for each unit	4	3rd Grade 24-25 Math Pacing Guide Overview 4th Grade 24-25 Math Pacing Guide Overview 24-25 Math 5 Investigations Pacing Guide 24-25 Math 6 Division Pacing Guide
Grade/School level planning document	Plans for daily instruction including resources and activities	4	24-25 5th Math Walker Team Planning Document by Unit 24-25_6th Math Walker Team Planning by Unit 2024-2025 Math 7 Pacing Guide and Calendar 2024-2025 Math 8 Pacing Guide and Calendar
Unit of Instruction	Unit created by the school district to supplement for new SOLs	5	24-25 WFD 3rd Grade Fractions Unit 24-25 WFD Fourth Grade Fractions Unit 3rd WFD Patterns, Functions, and Algebra Unit 3rd Grade WFD Adapted Unit 3 Inv. 3.1-3.5 4th Grade - Line Graphs Unit V2
District Assessment	Quarterly assessments for each grade level	6	Grade 3 Q2 Grade 4 Q2 (part 1 and part 2) Grade 5 Q2 (calculator and non calculator) Grade 6 Q2 (calculator and non-calculator) Grade 7 Q2 (calculator and non-calculator) Grade 8 Q2

Note. In grades four through seven, there are specific SOLs where students may use a calculator, and the assessments are divided into two parts to accommodate this.

Observations

To answer RQ2, WFD math classes were observed to capture evidence of teachers implementing strategies for algebraic thinking. Observations aim to understand the setting and phenomenon being studied from the participants' perspective (Hatch, 2022). Data were collected using the observation protocols found in Appendix B. Before the observation, I asked each teacher to discuss the lesson and what I should expect to observe. This was a short informal interview with two questions in the observation protocol.

The protocol included a table to collect information about the day, time, location, teacher, grade level, space for field notes, and pre- and post-interview questions. The protocol incorporated a list of three key content areas of algebraic thinking from Blanton et al.'s (2018) early algebra intervention framework: generalized arithmetic, functional thinking, and equations and equivalence. A second table included the three content areas and related instructional practices for algebraic thinking from the literature review. Running notes were collected during the observation to capture as much detail about the lesson and teacher/student interactions as possible.

The protocol concluded with two questions that facilitated a short debrief with the teachers following the observations. This allowed for clarifications or unresolved questions to be addressed. The informal interviews enabled the teachers to share any insights from their perspectives that might have been relevant to the study (Hatch, 2002). Lastly, a space was included for the researcher to capture any notes or questions that may have been sparked during the observation.

Interviews

Interviews with the math specialists and district math coordinator were semi-structured to allow participants to engage in a formal yet flexible conversation about math curriculum and instruction in the school district (Hatch, 2022). The open-ended interview questions allowed participants to share their perspectives in detail and allowed me, the interviewer, to ask follow-up questions as needed (Hatch, 2002). The interviews were an opportunity to explore what WFD math specialists believed to be the priorities for curriculum and instruction in preparation for algebra (see protocol in Appendix C). The district math coordinator was also a participant in the interviews. The interview protocol for the coordinator (see Appendix D) focused on how the curriculum is developed across the grade levels to prepare students to take algebra.

Data Collection

Observations

Each observation lasted between 45 and 60 minutes. The design of the observation protocol (Appendix B) allowed data to be captured in two areas: content and instructional practices. In the content area, the focus of the lesson was coded. In instructional practices, the strategies were coded relative to the content area. Additionally, running notes were kept of teacher actions, language, students' responses, and types of problems being posed and solved. During and after each observation, I added my questions, and reflections to the form to connect the observation and my understanding of algebraic thinking.

Interviews

Each specialist in the sample was interviewed for approximately 45 minutes. The interviewees were one math specialist from each school, and the district math coordinator (Table 2). I also conducted one follow-up interview with Kevin from Boxwood Middle School to ask additional questions about the curriculum documents the coordinator provided. The recordings and transcripts were stored in UVABOX.

The interviews provided insight into how the curriculum documents were developed and their purpose. The interview protocol questions (Appendix C, Appendix D) were open-ended. They included questions about curriculum resources, curriculum development, the 2023 SOLs, and how teachers think about algebraic thinking. During each interview, I asked follow-up questions to probe for details or clarify.

Data Analysis

Document Analysis

Document analysis aimed to identify gaps in the curriculum, areas of redundancy, and patterns that demonstrated algebraic thinking to answer RQ1. Once the documents were organized in the spreadsheet, they were sorted by type to look for similarities and differences across each grade level. Descriptive data was also collected for each document to make note of items in each document that made them unique or elements that were the same across documents.

Curriculum Mapping

Curriculum mapping is a process that shows the alignment between the components of a written curriculum, such as state standards, learning objectives, lesson activities, and assessments (Lam & Tsui, 2013). Curriculum maps can be utilized for various purposes, including evaluating a program, comparing the written curriculum to the enacted one, and examining gaps or redundancies across a program (Cooper et al., 2024). The curriculum mapping process was used to investigate RQ1 by examining the intersections of key algebraic thinking skills, state standards, and the local curriculum.

The curriculum mapping process was done in two stages. Stage one included a review of the 2023 Virginia mathematics SOLs for grades three through eight to look for key algebraic thinking content in three areas: generalized mathematics, functional thinking, and equations and equivalence (Blanton et al., 2018). The data was input in an Excel spreadsheet to create a matrix with the key concepts as column headings and grade-levels as labels for each row. The individual SOLs and topics were added to the spreadsheet, categorized into the three content areas of algebraic thinking. Then the cells were color-coded to display which key skills were included at each grade level (See Figure 2).

Figure 2

Screenshot from SOL Coding Spreadsheet

	Generalized Arithmetic			Functional Thinking		
	Number Properties	***Properties of Operations***	***Relationships***	***Patterns***	***Co-varying quantities***	***Linear Functions***
Grade 3	3.NS.1 place value	3.CE.1 add/subtract	3.CE.2 multiply/divide	3.PFA.1 patterns with additon and subtraction		
	3.NS.2 base 10	3.CE.2 multiply/divide				
	3.NS.3 fractions					
Grade 4	4.NS.1 place value	4.CE.1 add/subtract	4.NS.5 fractions and decimals	4.PFA.1 patterns with addition, subtraction, multiplication		
	4.NS.2 base 10	4.CE.2 multiply/divide				
	4.NS.3 fractions	4.CE.3 add/subtract fractions				
	4.NS.4 decimlas	4.CE.4 add/subtract decimals				
Grade 5	5.NS.2 prime & composite	5.CE.1 add, subtract, multiply, divide	5.NS.1 fractions and decimals	5.PFA.1 patterns including fractions and decimals		
		5.CE.2 add/subtract fractions				
		5.CE.3 operations with decimals				
		5.CE.4 order of operations				
Grade 6	6.NS.2 integers	6.CE.1 operations with fractions	6.NS.1 fraction, decimal, percent		6.PFA.1 ratios	
	6.NS.3 exponents	6.CE.2 operations with integers			6.PFA.2 proportions	
Grade 7	7.NS.1 scientific notation	7.CE.1 operations with rational numbers	7.NS.3 square roots/perfect squares		7.CE.2 solve proportions	
	7.NS.2 rational numbers				7.PFA.1 represent proportions	
Grade 8			8.NS.1 real number relationships		8.CE.1 proportional reasoning	8.PFA.2 relations
			8.NS.2 real number system			8.PFA.3 linear functions

Note. Colors represent content strands from the 2023 Math SOLs: Blue is Number and Number Senses, green is Computation and. Estimation, and orange is Patterns, Functions, and Algebra.

Stage two of the mapping process included reviewing the local curriculum against the results of part one. This process showed the intersection of the key algebraic skills highlighted in the SOLs with those in the WFD curriculum documents. To compare these documents against the coding from phase one, I created a matrix in Excel for each grade level. I listed the SOLs for each unit from the grade level curriculum documents, then color coded them according to the three content areas of algebraic thinking. The matrix shows where algebraic thinking content is introduced and reinforced, or not (See Figure 3).

Figure 3

Screenshot from Curriculum Map

	Unit 1	**Unit 2**	**Unit 3**	**Money Unit**	**Unit 5**	**Geometry Unit**	**Unit 4**	**Fractions**
Grade 3	3.NS.1	3.PS.1	3.NS.1	3.NS.4	3.CE.2	3.MG.4	3.MG.1	3.NS.3
	3.CE.2	3.MG.1	3.NS.2				3.MG.2	
	3.MG.1		3.CE.1*					
	Unit 1	**Line graphs**	**Unit 3**	**Elapsed Time**	**Unit 4**	**Unit 5**	**Fractions**	**Unit 6**
Grade 4	4.CE.2	4.PS.1	4.CE.2	4.MG.2	4.MG.1	4.NS.1	4.NS.3	4.NS.4
					4.MG.3	4.NS.4	4.NS.4	4.NS.5
					4.MG.4	4.CE.1	4.CE.3	
					4.MG.5			
					4.MG.6			
	VA Unit 1	**VA Unit 2**	**VA Unit 9**	**Unit 1**	**Unit 3**	**Unit 4**	**Unit 6**	**Unit 7**
Grade 5	5.PS.1	5.NS.2	5.PFA.1	5.CE.1	5.NS.1	5.CE.1	5.CE.3	5.CE.2
	5.PS.2	5.CE.4*	5.PFA.2		5.CE.2		5.NS.1	5.CE.3
		5.PS.3						
		5.PFA.1						
	Unit 1	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**
Grade 6	6.PS.1	6.NS.2	6.MG.3	6.CE.2	6.NS.1	6.PFA. 1	6.NS.3	6.PS.1
		6.PFA.4	6.MG.4	6.PFA.3	6.CE.1	6.PFA.2	6.MG.2	6.PS.2
						6.MG.1		
	Unit 1	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**
Grade 7	7.PS.2	7.MG.3	7.NS.1	7.PS.1	7.CE.2	7.PFA.1	7.PFA.2	7.PFA.2d*
			7.NS.2				7.PFA.3	7.MG.1
			7.NS.3				7.PFA.4	
			7.CE.1					
			7.PFA.2					
	Unit 1	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**
Grade 8	8.PS.2	8.PFA.1	8.CE.1	8.NS.1	8.MG.1	8.PFA.5	8.PS.3	8.MG.3
		8.MG.2	8.MG.1	8.NS.2	8.MG.2		8.PFA.2	8.PFA.3
		8.MG.5			8.MG.4			
					8.PFA.4			

Note. The colors in this figure are coded according to the algebraic thinking content strands (Blanton et al., 2018). Blue represents generalized arithmetic. Orange represents functional thinking and green represents equations and equivalence.

The documents provided by WFD included unit plans for supplemental instruction not addressed in the curriculum resource (i.e., textbook) for third and fourth grade. The unit plans were read and coded for algebraic thinking content and instructional strategies. Those data were entered into a spreadsheet to look for commonalities across the codes. Once the data were entered into an Excel spreadsheet, color coding was used to identify when algebraic thinking content was introduced and reinforced. Reviewing the spreadsheet's coded data should yield big ideas or themes that show connections within and across grade levels (Gross, 2018).

Classroom Observations

The purpose of the classroom observations was to observe the enactment of the written curriculum and, more specifically, the instructional practices related to algebraic thinking in response to research question two. During the observation, I documented the content strand the lesson was grounded in and the instructional practices that were observed during the class. Notes were added about the concept or activity students worked on and the teacher's strategies were highlighted on the protocol. After the observation, the running notes were coded. The running notes were coded according to the instructional strategies that occurred during the lesson. Each coded line was added to a spreadsheet, which allowed for sorting by code after all the observations had occurred. The initial code refers to the algebraic thinking content area, and the secondary code refers to the concept or strategy used. An excerpt from the spreadsheet is shown in Figure 4.

Figure 4

Observation Data Coding Examples

Data source	Grade level	Line	Code	Secondary code	Data
Miss P Observation	3	5a	Generalized Arithmetic Instructional Strategy	Tools to support student thinking	Students used fraction tiles, bar models of fractions to color in, and circle models
Miss P Observation	3	5c	Equality Istructional Strategy	Multiple representations	writing equations for composing and decomposing fraction
Miss P Observation	3	11	Generalized Arithmetic Instructional Strategy	questioning	How did you know it should be 8?

Note. This is a screenshot from the spreadsheet.

Interviews

The interviews were initially coded using *a priori* codes (Appendix E) derived from the literature on algebraic thinking content and instructional practices. The interviews were read a second time for additional data that was related to math teaching, curriculum development, or instructional practices not captured by the *a priori* codes. Then, by comparing the content of the highlighted quotes, emergent codes began to develop from commonalities across interviews. The emergent codes were foundational skills, curriculum resources, assessment, and professional learning. Evidence from the interviews was then added to a spreadsheet with these headers for each column: data source, line(s) of text, code, data (quotes from the participants), notes about the context of the quote, and memos (See Figure 5). After inputting the data, they were read through again to ensure the coding was consistent across the codes from all five interviews. Lastly, the sheet was sorted by column C, the codes, to examine the data for each code and determine trends and themes.

Figure 5

Interview Coding Example

Data source	Page number	Line	Code	Data	Data collection notes
Brian interview	16	142	Algebra readiness Algebraic thinking content Curriculum development	If if algebra readiness is the goal, what are the things that are the biggest priorities? And so that's also then, too, when we, when we think about our goals for tier 3, that's the things that we're working on in tier 3. So we have the ideas there. The structure isn't. The structure is not quite there yet, but in terms of the the focus. You know the focus for each of those you know, tier 2 and tier 3 times the ideas are there, and that was the the big goal was algebra readiness.	Preparing for algebra explicitly
Brooke Intervew	p. 11 - 12	105-110	Algebra readiness Algebraic thinking content	honestly, I think that we that teachers miss the functions and algebra piece of it. The standard is very specific in its language around the patterning piece. And so. yeah, I think that sometimes, as I reflect, even just in this moment, I think that we there tends to be a honing in on like. And I well, let me finish the sentence, and I'll say more. We hone in on. Okay, we have this unit on patterns. We're going to show kids various patterns, and we're going to look at how they grow. They, you know, increase, or. you know, decrease or repeat	Connections between patterns, functions, and algebra
Kevin interview part 2		29 - 31	Algebra readiness Algebraic thinking content	Because, you know, as you know, moving into 7th grade from 6th grade. You're having much more algebraic reasoning. And you know, having that type of representation is is significant, and especially moving into our our 8th grade kind of pieces so very excited about that. been working with them.	representation meaning algebra tiles here

Note. This is a screenshot from the original spreadsheet to demonstrate organization of the data.

Positionality

As the previous math coordinator in WFD, my relationship to this problem of practice was a crucial factor in this study. When the capstone process began, I was still the coordinator in the school district; I transitioned into a new role but maintained professional connections with the new coordinator and math specialists in WFD. Given this previous experience, I had some biases to consider when exploring this problem.

During the observation process, I had to suspend judgment regarding which teachers volunteered or did not volunteer to be observed. Having previously observed most math teachers in the district, I had predisposed notions of their teaching experience. However, using the observation protocol kept me focused on the intent of the observation and the data to collect. I was also very clear in my communication with teachers that my observations served no evaluative purpose. As a graduate student, I reminded them that the observations were only to collect data relative to the problem of practice and shared a study information sheet with each of them before scheduling the visits. As a former employee, teachers could have perceived my observation notes would be shared with administrators, but that was not the intent of the time in their classes, and I communicated that in writing and through the IRB protocol.

Lastly, my career experience includes 10 years of teaching math and 12 years being a math leader at the school and district levels. My knowledge and understanding of the K-12 math curriculum are extensive, allowing me to see the elements of the observations or interviews through my experience in hundreds of math classes over two decades. The protocols for observations and interviews included a space for me to capture thoughts that I could review later and eliminate if they are not related to the research questions. It was helpful for me to keep reflections in that space of the protocol and be aware of the lens of my personal/professional experience.

Trustworthiness

Multiple data collection methods allowed me to triangulate the data across documents, observations, and interviews to increase the study's credibility (Merriam & Tisdell, 2015). During each coding round, I kept analytic memos of the potential connections among data sources (Bazeley, 2013). Because this study involved multiple data sources over two months, it was vital to employ memos to capture any thoughts, revelations, or questions that arose over time (Merriam & Tisdell, 2015). Memos were captured during and after each coding round for documents, observations, and interviews. Each protocol included a space for reflective comments to be completed for each interview or observation, which reminded me to capture my immediate thoughts after each instance. Additionally, I kept a notebook with questions, ideas, and potential areas to further explore as the study went on. A sample of those notes can be seen in Appendix F.

Chapter 4

Findings and Reflections

This study of the algebraic thinking in the local curriculum and instruction of Whispering Falls Schools was focused on two research questions:

RQ 1: To what extent are key ideas associated with algebraic thinking introduced and reinforced across the elementary and middle school math curricula in WFD?

RQ 2: What instructional practices related to algebraic thinking are observable in WFD's grades three through eight mathematics classes?

This inquiry into WFD's curriculum and instruction practices included analyzing the district's documents, interviewing math specialists, and conducting classroom observations. This chapter explores the findings of each analysis and looks for commonalities and differences across all three. It ends with the researcher's reflections on the connections between the findings and the literature.

Document Analysis

The purpose of the document analysis was to examine where key algebraic thinking content was represented in the SOLs and in the WFD curriculum documents. The analysis happened in two phases. The first phase was the comparison of three algebraic thinking content areas (generalized arithmetic, functional thinking, and equations and equivalence) against the 2023 Virginia Mathematics SOLs. The second phase compared the coded Math SOLs from phase one to the WFD curriculum documents. The matrices show the intersection of algebraic thinking content and the local curriculum throughout the school year in each grade level of math.

Generalized Arithmetic in the Mathematics SOLs

Coding the Math SOLs in terms of algebraic thinking content created the visual display in Figure 1, which shows how the content develops over time. The concepts embedded in generalized arithmetic involve understanding numbers, operations, and the relationships between different forms of numbers (Sun et al., 2023). Compared to the Virginia Math SOLs, this placed the strands of Number and Number Sense (NS) and Computation and Estimation (CE) in the generalized arithmetic columns, with only a few exceptions. Table 3 displays the generalized arithmetic strand and the related SOLs by grade level. In terms of skill evolution through the lens of algebraic thinking, elementary students focus on learning about the properties of numbers (i.e., place value, magnitude, factors) and how to compute with them. Then, in the middle grades, students explore the relationships between real numbers and how those systems are interconnected. Table 4 illustrates the concentration of NS and CE standards that form the focus of elementary and middle school math curricula through number properties, properties of operations, and the relationships between types of numbers and number systems.

Table 4

Generalized Arithmetic in the 2023 Math SOLs

	Generalized Arithmetic		
	Number Properties	Properties of Operations	Relationships
Grade 3	3.NS.1 place value	3.CE.1 add/subtract	
	3.NS.2 base 10	3.CE.2 multiply/divide	
	3.NS.3 fractions		
Grade 4	4.NS.1 place value	4.CE.1 add/subtract	4.NS.5 fractions and decimals
	4.NS.2 base 10	4.CE.2 multiply/divide	
	4.NS.3 fractions	4.CE.3 add/subtract fractions	
	4.NS.4 decimals	4.CE.4 add/subtract decimals	

	Number Properties	Properties of Operations	Relationships
Grade 5	5.NS.2 prime & composite	5.CE.1 add, subtract, multiply, divide 5.CE.2 add/subtract fractions 5.CE.3 operations with decimals 5.CE.4 order of operations	5.NS.1 fractions and decimals
Grade 6	6.NS.2 integers 6.NS.3 exponents	6.CE.1 operations with fractions 6.CE.2 operations with integers	6.NS.1 fraction, decimal, percent
Grade 7	7.NS.1 scientific notation 7.NS.2 rational numbers	7.CE.1 operations with rational numbers	7.NS.3 square roots/perfect squares
Grade 8			8.NS.1 real number relationships 8.NS.2 real number system

Functional Thinking in the Mathematics SOLs

The development of functional thinking skills in the Math SOLs encourages students to engage with arithmetic patterns in third and fourth grades before introducing input and output tables in fifth grade. In sixth grade, the emphasis shifts to ratios and proportions, highlighting the connection between ratios and various representations of proportionality. The focus continued in seventh grade with proportional relationships, particularly within the context of rate of change, which transitioned into linear functions in eighth grade. Table 5 illustrates the progression from third to eighth grade in the functional thinking strand of algebraic thinking. The Math SOL strand represented in functional thinking is Patterns, Functions, and Algebra (PFA) in the Math SOLs.

Table 4 also shows two Math SOLs from computation and estimation. In seventh grade, students solve problems involving proportional relationships, and in eighth grade, they apply proportional reasoning to solve contextual problems. These are applications of co-varying quantities and were coded as functional thinking along with the other standards for ratios and proportions to show the relationships between the standards.

Table 5

Functional Thinking in the 2023 Math SOLs

	Functional Thinking		
	Patterns	Co-varying quantities	Linear Functions
3rd Grade	3.PFA.1 patterns with addition and subtraction		
4th Grade	4.PFA.1 patterns with addition, subtraction, multiplication		
5th Grade	5.PFA.1 patterns including fractions and decimals		
6th Grade		6.PFA.1 ratios 6.PFA.2 proportions	
7th Grade		7.CE.2 solve proportions 7.PFA.1 represent proportions	
8th Grade		8.CE.1 proportional reasoning	8.PFA.2 relations 8.PFA.3 linear functions

Equations and Equivalence in the Math SOLs

Students in third and fourth grade explore the equals sign by working with equations, and in fifth grade, they are introduced to variables. Throughout middle school, students engage with a variety of expressions, equations, and inequalities by solving, graphing, and representing them in contextual situations.

The content area of algebraic thinking, which includes equivalence, equations, expressions, inequalities, and variables, was primarily represented in the Math SOLs through the PFA strand. The concepts in this strand, including solving equations and inequalities, would traditionally be referred to as algebra content by many teachers. However, the patterns, functions, and algebra strand present a divergence between the Math SOLs and algebraic thinking content strands. The content strands separate functions and equations, whereas the PFA Math SOLs group all these concepts together. Neither approach is incorrect; however, the connection between patterns, functions, and algebra becomes unclear when they are lumped together. This will be discussed further in the interview analysis. In Table 6, the shift from equality to forms of equations can be observed.

Table 6

Equations and Equivalence in the 2023 Math SOLs

	Equations and Equivalence				
	Equal sign as relational	Equations	Inequalities	Expressions	Variables
Grade 3	3.CE.1 (d) equal sign	3.CE.2 (g) equations			
Grade 4	4.CE.2 (d) equal sign	4.CE.2 (c) equations			
Grade 5					5.PFA.2 variables

	Equal sign as relational	Equations	Inequalities	Expressions	Variables
Grade 6		6.PFA.3 one step equations	6.PFA.4 graph inequalities		
Grade 7		7.PFA.3 two-step equations	7.PFA.4 two step inequalities	7.PFA.2 evaluate expressions	
Grade 8		8.PFA.4 multistep linear equations	8.PFA.5 multistep linear inequalities	8.PFA.1 equivalent expressions	

Note. In grades three and four, two specific standards address equality, not the overall standard; hence, the notation is different.

Equality. Given the importance of understanding the relational aspect of the equal sign in algebraic thinking, it is curious how the concept is narrowly used and described in the elementary standards. The equal sign is introduced in first grade:

> 1.CE.1.i Describe the equal symbol (=) as a balance representing an equivalent relationship between expressions on either side of the equal symbol (e.g., 6 and 1 is the same as 4 and 3; 6 + 1 is balanced with 4 + 3; 6 + 1 = 4 + 3), (VDOE Math SOLs, 2023).

In second grade, students use the "not equal" symbol to represent relationships where expressions do not hold the same value (VDOE Math SOLS, 2023, p. 19). In third and fourth grades, students utilize either the equal or not equal symbols to compare expressions. Then, students apply equality to solve equations using the properties of real numbers and equalities in grades five through eight. Algebraic thinking homes in on the aspect of relationality with the equal sign, so it was surprising not to see this emphasis in the Math SOLs throughout the grade levels.

SOLs and Algebraic Thinking

It is important to note that the language of the Math SOLs offers numerous examples of concepts related to algebraic thinking. Across the grade levels, the Math SOLs require students to examine multiple representations, express co-varying quantities in various forms, apply properties of real numbers, and generalize about patterns, among other things (VDOE Math SOLs, 2023). Analyzing the language of the standards in the context of algebraic thinking reveals similarities. These similarities should then translate into instruction, assuming that teachers also grasp the content of algebraic thinking and the related instructional practices. This will also be addressed in the reflection.

Curriculum Mapping

The curriculum map represents the coding of the Math SOLs, from Phase 1 of the document analysis, in comparison to the WFD curriculum documents. Specifically, the documents that were most useful for this exercise were the pacing guides because they included the standards and concepts that were addressed through the school year, and the time spent on each one.

Generalized Arithmetic in the WFD Curriculum

The curriculum map shows the generalized arithmetic content area of algebraic thinking against the pacing guide provided by WFD (Figure 6). The matrix shows how standards are chunked together and how they iterate, or do not, throughout the school year. The color coding shows the algebraic thinking content area for each Math SOL.

In third and fourth grade, there is a pattern of returning to major topics throughout the year. For example, in third grade 3.NS.1, place value, is repeated three separate times throughout the year. Similarly, in fourth grade, over the course of the year, 4.CE.2, solving multiplication and division problems, is included four different times. However, each Math SOL is only listed once for every course after fourth grade. In fact, these standards are confined to one unit at each level for generalized arithmetic in seventh and eighth grade. It would seem the adopted curriculum resource supports reinforcement of skills over time, whereas the other pacing guides do not.

Figure 6

Excerpt from Curriculum Map

Grade 4													
Unit	**Unit 1**	**WFD unit**	**Unit 3**	**WFD unit**	**Unit 4**	**Unit 5**	**WFD unit**	**Unit 6**	**WFD unit**	**Unit 7**	**WFD unit**	**Unit 8**	
Topic	Multiplication and divison	Line graphs	Multiplication and division 2	Elapsed time	Area and perimeter	Addition, subtraction, and the number system	Fractions	Fraction cards and decimals	Decimals	Multiplication and division 3	Patterns and probability	Patterns, tables, equations	
Standards	4.CE.2	4.PS.1	4.CE.2	4.MG.2	4.MG.1	4.NS.1	4.NS.3	4.NS.4	4.NS.4	4.CE.2*	4.PFA.1	4.CE.1	
					4.MG.3	4.NS.4	4.NS.4	4.NS.5	4.NS.5	4.MG.1	4.PS.2	4.CE.2	
					4.MG.4	4.CE.1	4.CE.3		4.CE.4			4.PFA.1	
					4.MG.5								
					4.MG.6								
Grade 5													
Unit	**VA Unit 1**	**VA Unit 2**	**VA Unit 9**	**Unit 1**	**Unit 3**	**Unit 4**	**Unit 6**	**Unit 7**	**VA Unit 6**	**VA Unit 8**			
Topic	Data cycle	Order of operations, number system	Patterns, variables, expressions	Multiplication and division strategies	Fractions and mixed numbers	Multiplication and division algorithms	Decimals	Multiplying fractions and mixed numbers	Measuremen t	Geometry			
Standards	5.PS.1	5.NS.2	5.PFA.1	5.CE.1	5.NS.1	5.CE.1	5.CE.3	5.CE.2	5.MG.1	5.MG.2			
	5.PS.2	5.CE.4*	5.PFA.2		5.CE.2		5.NS.1	5.CE.3		5.MG.3			
		5.PS.3											
		5.PFA.1											

Note. The color coding is associated with the algebraic thinking content areas from Blanton et al.'s (2018) early algebra intervention framework. Blue indicates generalized arithmetic content, orange shows functional thinking, and green represents equations and equivalence.

Functional Thinking in WFD Curriculum

Within the WFD curriculum, this strand is unique in that the curriculum resource that is used in elementary, *Math Investigations*, does not address the PFA standards. At the elementary level, the units addressing the PFA standards are written by the specialists or adapted from the Math Forward website resources. The curriculum map (Appendix G) shows that these standards are taught once during the year at each grade level. In a more detailed version of the curriculum map (Figure 7), the duration of each unit of instruction is included. In third grade, students spend five days learning about patterns, functions, and algebra. In fourth grade, the standard is divided over two units.

In middle school, students learn to apply proportionality to contextual situations, which begs the question of how these topics are taught and through what strategies. Without the context of proportionality, these standards, 7.CE.2 and 8.CE.1, could be taught through computation and calculation. But the connection to functional thinking brings purpose and a connection between ratios, proportions, and real-world contexts. Without evidence from Boxwood, this remains in question.

Figure 7

Excerpt from Curriculum Map with Duration Included

Grade 5										
Unit	**VA Unit 1**	**VA Unit 2**	**VA Unit 9**	**Unit 1**	**Unit 3**	**Unit 4**	**Unit 6**	**Unit 7**	**VA Unit 6**	**VA Unit 8**
Topic	Data cycle	Order of operations, number system	Patterns, variables, expressions	Multiplication and division strategies	Fractions and mixed numbers	Multiplication and division algorithms	Decimals	Multiplying fractions and mixed numbers	Measurement	Geometry
Standards	5.PS.1	5.NS.2	5.PFA.1	5.CE.1	5.NS.1	5.CE.1	5.CE.3	5.CE.2	5.MG.1	5.MG.2
	5.PS.2	5.CE.4*	5.PFA.2		5.CE.2		5.NS.1	5.CE.3		5.MG.3
		5.PS.3								
		5.PFA.1								
Duration (days)	18	14	12	14	20	18	17	18	11	21
Grade 6										
Unit	**Unit 1**	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**		
Topic	Data cycle	Integers and inequalities	Coordinate plane and congruence	Operations with integers and equations	Operations with fraction, decimal, percent	Proportional reasoning	Exponents and applications	Statistics		
Standards	6.PS.1	6.NS.2	6.MG.3	6.CE.2	6.NS.1	6.PFA. 1	6.NS.3	6.PS.1		
		6.PFA.4	6.MG.4	6.PFA.3	6.CE.1	6.PFA.2	6.MG.2	6.PS.2		
						6.MG.1				
Duration (days)	10	15	20	30	30	25	20	10		

Note. Colors represent the algebraic thinking content areas. Blue indicates generalized arithmetic. Orange represents functional thinking. And green represents equations and equivalence.

Equations and Equivalence in WFD Curriculum

The curriculum map notates exactly where the equality instruction is placed in the pacing guides. In third grade, it is near the start of the school year, but in fourth grade, it is much later.

Given its importance in algebraic thinking, this is problematic if students are not working with equality throughout the year. The concentration of the equations and equivalence strand is much greater in the middle school levels (Figure 8). Students are solving and graphing equations and inequalities and working with expressions in the middle grade levels, culminating with linear functions in eighth grade.

In terms of the research questions, there is a concern about preparing students for algebra in this strand. Students' first opportunity to take algebra comes in the seventh grade. Based solely on the Math SOLs students are exposed to solving multistep equations in the seventh-grade standards, which is a significant skill related to algebra. So, for students taking algebra in seventh grade, and only heterogeneous groupings of students throughout the preceding grade levels, are teachers differentiating in a way that might prepare those who are ready for algebra by infusing these skills into the curriculum? This will be discussed in the last section of this chapter, researchers' reflections.

Figure 8

Excerpt of Curriculum Map for Middle School Grades

Grade 6									
Unit	**Unit 1**	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**	
Topic	Data cycle	Integers and inequalities	Coordinate plane and congruence	Operations with integers and equations	Operations with fraction, decimal, percent	Proportional reasoning	Exponents and applications	Statistics	
Standards	6.PS.1	6.NS.2	6.MG.3	6.CE.2	6.NS.1	6.PFA. 1	6.NS.3	6.PS.1	
		6.PFA.4	6.MG.4	6.PFA.3	6.CE.1	6.PFA.2	6.MG.2	6.PS.2	
						6.MG.1			
Grade 7									
Unit	**Unit 1**	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**	**Unit 9**
Topic	Data cycle	Quadrilaterals	Rational numbers	Probability	Proportional reasoning	Slope	Expressions, equations and inequalities	Surface area and volume	Proportional reasoning with geometry
Standards	7.PS.2	7.MG.3	7.NS.1	7.PS.1	7.CE.2	7.PFA.1	7.PFA.2	7.PFA.2d*	7.MG.1
			7.NS.2				7.PFA.3	7.MG.1	7.MG.2
			7.NS.3				7.PFA.4		7.MG.4
			7.CE.1						
			7.PFA.2						
Grade 8									
Unit	**Unit 1**	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**	**Unit 9**
Topic	Data cycle	Equivalent expressions	Consumer math	Real and irrational numbers	Geometry	Solving Inequalities	Scatterplots and data	Linear equations	Independent and dependent events
Standards	8.PS.2	8.PFA.1	8.CE.1	8.NS.1	8.MG.1	8.PFA.5	8.PS.3	8.MG.3	8.PS.1
		8.MG.2	8.MG.1	8.NS.2	8.MG.2		8.PFA.2	8.PFA.3	
		8.MG.5			8.MG.4				
					8.PFA.4				

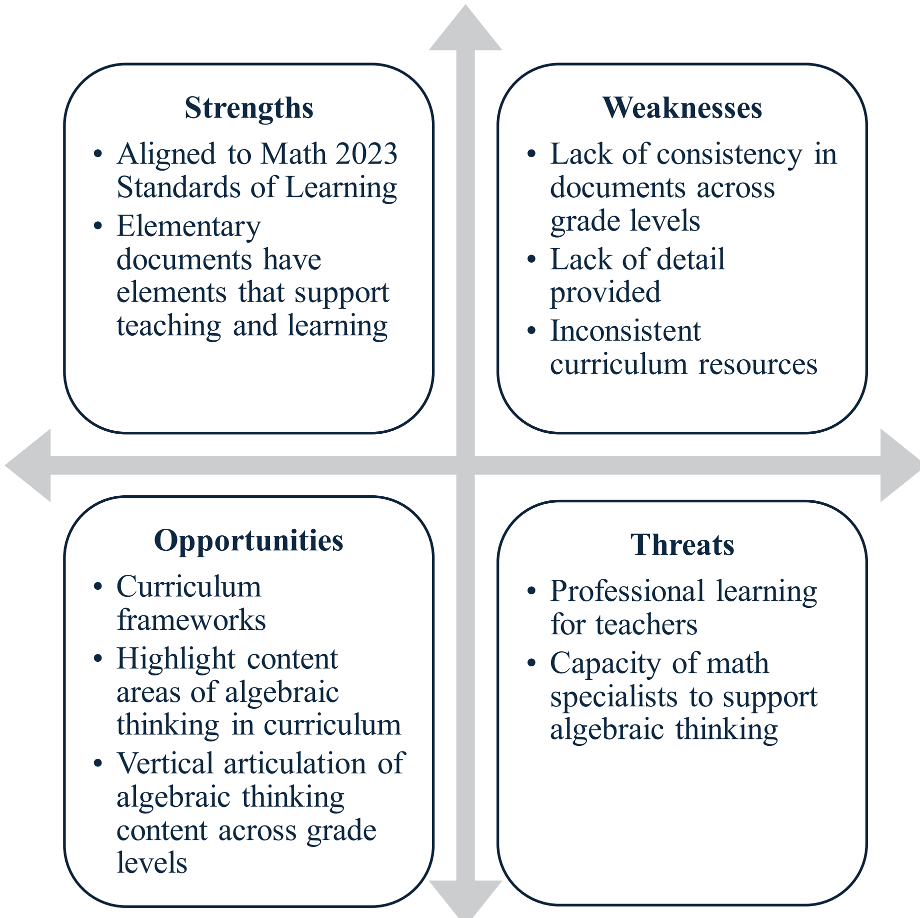
Note. The algebraic thinking content strand of equations and equivalence is color coded green. Generalized arithmetic is coded blue and functional thinking is orange.

SWOT Analysis of WFD Curriculum Documents

A SWOT analysis was used to summarize findings from the document analysis and curriculum map (Figure 9). A SWOT analysis highlights areas of strength, weakness, opportunities for improvement, and threats or barriers to making changes (Lin et al., 2023). Using the SWOT analysis allowed for synthesis across all the documents that led to the recommendations in chapter 5.

Figure 9

SWOT Analysis Visual



Strengths

Each math course across grades three through eight was aligned to the 2023 Virginia Mathematics SOLs. Given that this is the first year of implementation for the new standards, this was a significant strength of the curriculum documents. Each grade level included every standard for the course and, in some cases, addressed a standard more than once.

The elementary documents were thorough and included more detail than the other grade levels. The documents that were provided for the analysis included elements such as learning targets, visual representations of concepts, advice for modeling mathematics, and multiple strategies for teachers to implement for difficult concepts. These elements support teaching and learning across contexts (school buildings) and students with varying levels of readiness.

Weaknesses

The most pronounced weakness was the lack of consistency across the documents, and the lack of detail provided to the users. Based on the samples that were given, there was no consistent district-wide framework for each course. Pacing guides do not provide the guidance needed for teachers to enact the curriculum. The pacing guides only communicate the order and time spent on each unit, without any guidance for how to teach, and how the concepts will be assessed. The documents lacked critical details that support the teaching and learning of math.

Additionally, the use of curriculum resources was inconsistent across grade levels. The middle grades seem to create their own curriculum resources, while the elementary grades used an adopted resource that guided their documents. When teachers are given the capability to use inconsistent resources, students have disparate experiences that may not align with the goals of the math department.

Opportunities

There is a substantial opportunity to overcome these weaknesses by designing a district-wide template for curriculum frameworks that addresses the inconsistencies and lack of detail in the current documents. The creation of a curriculum framework template would bring alignment to the grade levels because it would require that each course would have similar components, including learning goals with specified outcomes. The template could serve many purposes. The first being the alignment across grade levels; it would also allow for the articulation of algebraic thinking content areas. Teachers would be able to see the connections within and across grade levels that support algebraic thinking for students and highlight the importance of preparation for algebra. Additionally, the frameworks would outline appropriate resources and instructional practices. The opportunities will be explored in depth in chapter 5.

Threats

One obstacle to changing curriculum documents or adding an emphasis in the curriculum like algebraic thinking, is that it requires teacher professional learning. Professional learning for teachers is an investment in time and resources, human and capital, that must be considered. Additionally, it would be the responsibility of the math coordinator and math specialists to carry out such training, and one must consider the capacity of those people when planning for professional learning.

Algebraic Thinking and WFD Curriculum

The document analysis and curriculum mapping aimed to identify where algebraic thinking concepts are introduced and reinforced, to answer RQ 1. In phase 1 of the document analysis, the SOLs were coded into the three algebraic thinking content strands, as seen in Figure 2. Then, the second phase of the analysis involved cross-referencing the coded standards against the units of instruction for each grade level (Figures 6, 7, 8). A second version of the curriculum map in Appendix H shows where the three content strands of algebraic thinking appear in the course sequence according to the pacing guides. Here, one can see how long students spent in each content strand, and if any of the SOLs were repeated throughout the year. The algebraic thinking skills, by way of the Mathematics SOLs are being introduced at each grade level, insofar as all the math SOLs are addressed for each level. There is insufficient evidence to support that the math SOLs are being reinforced in individual grade levels, beyond third and fourth grade. To understand which skills are being reinforced during each course would require more detail than currently documented in the district pacing guides.

Interviews with Math Leaders

During the interview process, four math specialists and the math coordinator for WFD agreed to participate. Each specialist has a unique background in math education that is worth including because it adds a layer of understanding to each of their perspectives on math and algebra readiness (See Table 2). The specialist interview questions were focused on curriculum resources, the new Mathematics SOLs, how and if they see algebraic thinking manifest in their schools, and their perception of algebra readiness in the school district. The coordinator was interviewed last, the questions in the protocol for the coordinator included clarifications about topics that received different answers from the specialists. The focus of her interview also helped to see the problem of practice from a K – 12 perspective.

Math Specialists' Perspective on Curriculum

When asked about the development of local curriculum in WFD, each math specialist agreed that the focus this year was on the implementation of the new standards. Melinda, the district coordinator said, "I think right now we're focused on teaching the standards in every grade level," (2/19/25, p. 19). Similarly, Jane who is in her second year as an elementary specialist, said that the new standards have "…expanded our conversations of like, oh, so this relates to this, and that's why they're doing this and like seeing, like all of the pieces come together. I feel like [that] has happened a lot more" (2/6/25, p. 13).

During the interviews, curriculum and instruction discussions focused on the standards, however, the approach to curriculum varied significantly across grade levels. There was consensus that the elementary curriculum was built from the adopted resource, *Investigations* (2016) for kindergarten through fifth grade. After that, however, the curriculum becomes "teacher-constructed," (Kevin, 2/19/25, p. 12). The adopted curriculum resource in the middle grades is *Envision* (2021), but Brian, Melinda, and Kevin mentioned that teachers rarely used it, if at all. According to Brian, “Sixth grade continues to develop most of their own materials. I mean, officially, we have *Envision* adopted, but it gets used pretty scarcely,” (2/7/25, p. 12). Regarding the middle school curriculum, Melinda states:

> When we get to 5th and above things, just get a little wild west for lack of a better analogy… They got away from using it [*Envisions*] pretty much at all. So secondary went to basically, teacher constructed curriculum and cobbled together. And… the math

specialists were, you know, still working on trying to make sure that there was a solid, pacing guide. There's unit plans at some grade levels depending on which teacher worked on it, or what you know, which math specialists, whether you were at Willow or Boxwood, you had different levels [of completion] (2/19/25, p. 12).

Kevin’s experience at Boxwood confirmed Melinda’s statement. He noted, “So the curriculum was kind of pieced together. So it was like just an Excel spreadsheet. It was going off of what Math Forward created,” (2/7/25, p. 10). Math Forward is a website created by the math supervisors and teachers in Virginia to create a common repository of scope and sequences for each grade level, and related activities based on the 2023 Mathematics SOLs (Mathforward.com, 2023). Brian from Willow Upper School stated they started with the document from Math Forward as well to create the pacing guide for sixth grade math. Brian remarked on the math six curriculum documents:

> And so we've used the Math Forward pacing as the initial framework for the work that we're doing, and for the most part have kept with that pulling activities from Math Forward when they were available, as well as filling in with other activities from previous years, as well as co-developing some of it (2/7/25, p. 12).

One of the drawbacks of the Math Forward site is that many of the units were left unfinished, and so the repository is incomplete. According to Melinda, “The plan this year was for 6th grade and up to follow math forward pacing and depending on the grade level, there are lessons for some grade level(s), some teams didn't get as far. So teachers are still having to create the actual lessons,” (2/19/25, p. 13-14). Kevin thought the lack of commonality in the secondary grades was a barrier, “But when it comes to, you know, again having the new standards thrust upon us, not having a curriculum that kind of binds us together, has been a big challenge,” (2/7/25, p. 3).

When it came to basic resources, elementary schools demonstrated greater uniformity in the documents they utilize. Everyone agreed that the *Investigations* resource was the main component of the curriculum. Jane explained, “So ours [curriculum] mainly uses *Investigations* as the primary source. However, we have a lot of supplemental units that were often created by the math specialists when the *Investigations* did not meet the current math SOLs that just changed in 2023,” (2/6/25, p. 3). Brooke concurred, stating that there are benefits to the *Investigations* curriculum that support student learning, “And I think that there are parts of that curriculum that are good, and that encourage kids to think conceptually and solve problems,” (2/14/25, p. 3). Melinda mentioned that the goal for the elementary team was to utilize the *Investigations* resource as much as possible and create materials for situations when the resource does not align with the standards. Melinda shared, “So, they [specialists] just sort of work together or split up the grade levels to supplement; this past year was a bigger lift because they were trying to realign everything to the new standards,” (2/19/25, p. 11).

Inconsistency Between Grade Levels. There was a lack of continuity between the elementary grade documents and the middle school documents. Although only two of the six elementary math specialists were interviewed, Melinda indicated the curriculum documents were used consistently across all six elementary schools. When asked about this she replied, “I think that the math specialist team was very strong. And so then they could kind of say, this is what we’re doing. And they could be more consistent,” (2/19/25, p. 17). In her example, she went on to explain that, because six elementary specialists were working collaboratively at that level, it led to a more robust set of curriculum documents in elementary, and uniformity across schools in the use of the materials. While at the middle school buildings, where teachers were not using the adopted resources, there was less documentation of how teachers were planning and preparing for lessons, and only one specialist at each level.

Algebraic Thinking in the Curriculum

RQ 1 asks how algebraic thinking concepts were introduced and reinforced in the math curriculum in WFD. To answer this question, the specialists were asked how they see algebraic thinking manifested in the curriculum, and some specialists were able to address the question directly, while others were not. Both Brooke and Brian discussed how algebraic thinking is or is not visible in the curriculum. For Brooke, she understood the connections that should be made between algebraic thinking and computation, but she did not see it showing up in the curriculum or instruction at her school. Brooke explained:

> It's particularly elementary teachers who, I think, would say they don't teach algebra. They would just say, it's about X's, and you know, like X equals. And that kind of thing, …which is that generalizing, conjecturing piece of it, and that can cross so many of the strands that we teach, but we sometimes miss that (2/14/25, p. 9).

Brian was able to describe in depth what he had developed for his school to reinforce skills that support students’ algebra readiness at Willow Upper School. This was significant because it is after sixth grade that students have their first opportunity to take algebra. Brian stated:

> The other piece of our curriculum is that we have that dedicated tier two time… 25-30 min of every class period is spent spiral reviewing skills, you know, previously taught skills… the strands that were more focused on algebra readiness. So, looking at you know, multiplication and division in 5th grade…the algebraic thinking strands, and then with 5th grade, really thinking about fractions and what we need to do with fractions… Fractions, decimals, percents, integers, and integer operations, and then ratios like the proportional reasoning pieces, are the kind of big focuses in 6th grade in terms of that spiral review (2/7/25, p.13).

Willow Upper School is incorporating time into each math class for students to practice skills, primarily from the general arithmetic strand of algebraic thinking, ensuring they master skills related to algebra readiness. Brian described when the teachers worked on the pacing guide last summer, using Math Forward resources as a guide, they decided to move the algebra units to the beginning of the year. "One of the things that both of our teams decided to do, just in terms of algebra, is to move their algebra units earlier in the year, in order to use kind of the algebraic thinking as a framework," Brian explained (2/7/25, p. 18). In the curriculum documents, the fifth-grade pacing showed that the variables and expressions unit was the third unit they taught this school year, and in the sixth grade it was the second unit of the year. Brian said, "We're definitely prioritizing those topics in terms of pacing," (2/7/25, p. 19).

Although the work at Willow sounded promising, it seemed to be happening in isolation. Based on the evidence from the other specialists, Willow was the only school with a targeted plan for preparing students to take algebra. At Boxwood, Kevin stated that there is no intentional focus outside of the math blocks. When asked how teachers were preparing students for algebra who are currently in eighth grade, Kevin replied, “Because [I] talked about that before, getting that linear understanding of slope, Y intercept, graphing, how it ideally, how it relates contextually to the real world. You know, and how it relates to solving linear equations, how it relates to solving linear inequalities. That's probably been the biggest focus,” (2/19/25, p. 10). According to Kevin, students in Math 8 focused on mastering linear equations so they can enter Algebra 1 in ninth grade with a foundational understanding of linear relationships and graphing. While in Math 7, teachers were focused on solving multi-step equations. Kevin was hopeful that this skill would support students so that more qualify to take Algebra 1 in eighth grade.

Table 6

Algebraic Thinking and WFD Curriculum According to Math Specialists

Participant	Statement Regarding Algebraic Thinking in WFD Curriculum
Brooke	We have not had a lot of conversations at the elementary level about algebra readiness… I think we have had conversations about like, yeah, student thinking and conjecturing. And where we are getting kids to think about relationships among numbers and all those kinds of things but naming that as algebra readiness is not something that we have had as much conversation with at the elementary level (2/14/25, p. 18).
Jane	I cannot say that we've used like those words as a topic in in conversation. Now, I'm not saying that we don't talk about algebraic thinking, but I don't. I don't feel like we tend to have that as a focal point of a conversation (2/6/25, p. 10).
Brian	If algebra readiness is the goal, what are the things that are the biggest priorities? And so that's also then, too, when we, when we think about our goals for tier 3, that's the things that we're working on in tier 3. So we have the ideas there. The structure isn't. The structure is not quite there yet, but in terms of the focus. You know the focus for each of those you know, tier 2 and tier 3 times the ideas are there, and that was the big goal was algebra readiness (2/7/25, p. 16)
Melinda	So we’re focused on teaching the state standards in every grade level and not necessarily thinking about where we are going (2/19/25, p. 19)

Jane specifically mentioned that she did not see algebraic thinking reflected in the elementary curriculum. This should not be surprising because of Jane's background in elementary special education. She explained how concepts evolve over time in the elementary curriculum by initially detailing how students learn to break numbers apart (decomposing) and reassemble them (composing) for multiplication, before introducing the algorithm later in the year. She explained it like this, "So they understand the components of why the algorithm works, instead of just memorizing the steps like understanding why," (2/6/25, p. 9). In terms of algebraic thinking, this would be an example of students making connections or examining multiple representations. However, it was not labeled or discussed as such with specialists or teachers.

As a former secondary teacher, Melinda understood the importance of thinking of curriculum through an algebraic lens, but she also named some barriers that were preventing her from meeting that goal with the specialist team. In her interview, when asked how students were being prepared for algebra, she responded:

> So like if we have to…you have to make choices. We only have so many hours in a day. There's only so many minutes for intervention… but like things that should be solid and get the most attention, get the most scrutiny, the most reteaching, whatever we want to call it, should be anything that builds a foundation for algebra (2/19/25, p. 19).

Although time in the school day has been a barrier, she also saw the lack of resources for secondary to be an even larger hurdle. "But we shouldn't be creating from the ground up," she said, referring to the curriculum development process (2/19/25, p.13). She wanted the schools to go through a curriculum adoption process so that everyone can be working from the same baseline: "Once we go through an adoption, they're going to get an aligned curriculum that they'll be expected to use and supported to use," (2/19/25, p. 13).

Supporting teachers to think algebraically

One barrier to algebraic thinking based on the conversations with specialists, was the teachers themselves. Kevin was very clear about the barriers he faced with Boxwood's teachers: "And we also have teachers who, you know, need some help with their math. Who are not mathy and need to know how that can relate to instruction as well. So that's the challenge that we are in," (2/7/25, p.25). Kevin said many of the teachers in the school lacked experience and some teachers he referred to as "career switchers" (2/7/25, p. 18). The lack of experience teaching math also provided some challenges. Kevin described, "Teachers, in the world of math, teachers are very unwilling to go, 'I got a question about the math and I got a question about the instruction.' And sometimes they will say instruction. But it's really the math," (2/7/25, p. 18).

Brian shared a similar concern regarding teachers' ability to make connections in the math content. Brian stated, "I think that's one of the things that we're working on. I think that particularly teachers that have more of the elementary background or have kind of a harder time articulating where we are going with it…Sometimes math is not always even, you know, something they're as comfortable teaching," (2/7/25, p. 20). Brian continued to explain that during group conversations he tried to expose teachers to the connections between concepts, or highlighted where the current topic will lead to in the next grade. He was also trying to reinforce algebraic language with teachers so that when students transition to algebra, they have the vocabulary to go with it in whatever grade suits them. He gave an example, "Why is it so important that we're talking about multiplying by the reciprocal when we divide fractions? Well, it's because that that carries us into algebraic thinking, and that carries us into where they're going," (2/7/25, p. 21).

In the elementary space, Brooke agreed that not all teachers were especially skilled at teaching math, “Particularly elementary teachers don't necessarily love or feel confident with math, and it's because we were all taught or not, all of us, I'll say. But a lot of us were taught in a very procedural way. I know I was,” (2/14/25, p. 8). Brook went on to say, that she wants to support teachers to support students in ways that make sense for both the student and the teacher, and she felt like teachers were amenable to that idea. Brooke noted that she took time to show teachers examples in small settings to improve their math content knowledge for concepts like fractions. She explained, “It's key to be able to do those like, micro PLs [Professional Learning] with teachers in professional, in like PLCs [Professional Learning Community] and things where you can really show like, here's what we're trying to get kids to think about. And here's a way that we can do that so that it makes sense, and that there's a reason behind it,” (2/14/25, p. 9).

The other elementary specialist, Jane, said that in her school it depended on the teachers she worked with. She described how the third-grade teachers talked about the way concepts were taught, and how students grappled with math ideas. In other grade levels, the conversations were more surface level, and the teachers discussed how the lesson should be presented instead of the connections students were making with the content. Melinda saw the disconnect, as well, when it comes to teacher content knowledge:

> And I think the thing about the fractions and the thing about equality and inequalities, and all of that comes back to the teacher content knowledge, if the teachers don't understand it and feel comfortable with it, when they're never going to teach it at the depth that it needs to be taught at. And they're or they're going to oversimplify things, or they're going to try to get just to the algorithm or something they feel comfortable with. And the students keep missing the conceptual piece (2/19/25, p. 32).

Patterns, Functions, and Algebra. One of the questions that was asked as a follow-up in the interviews was whether teachers understood the connections between patterns, functions, and algebra in the new math SOLs. The consensus across the specialists was that teachers do not understand how the three ideas are interconnected, which is a largely an algebraic concept. When asked directly if teachers can make the connection between the three concepts, Kevin replied, “I would say the majority [of teachers], no. And when I mean majority probably as high as 70%, 80%,” (2/7/25, p. 21). Brooke stated it this way:

> Honestly, I think that we, that teachers, miss the functions and algebra piece of it. The standard is very specific in its language around the patterning piece. And so yeah, I think that sometimes, as I reflect, even just in this moment, I think that there tends to be a honing in on like…Okay, we have this unit on patterns. We're going to show kids various patterns, and we're going to look at how they grow. They, you know, increase, or you know, decrease or repeat (2/14/25, p. 11-12).

It is worth noting that in this instance Brooke was describing her experience discussing the standards with teachers. Teachers were not thinking beyond what they were required to teach in the standard. And Melinda agreed that teachers saw the concepts as siloed and not interconnected. She discussed teachers’ understanding of the learning progression for fractions:

> Obviously, patterns, functions, and algebra. It says it right there in the name, but I think some of it goes back to proportional reasoning and fraction understanding, rational numbers. Because I think if we, if we don't understand the concept of that relationship, and how that builds, and where we end up going with fractions and rates, unit rates. … I think they see it as a thing, like this is fractions, that's a thing, and then in a couple of

years somebody sees something else. This is a thing, and it's like, No, no, it's like it's all the same thing (2/19/25, p. 26).

The disconnect was not that the teachers were not making the connections for students; they do not make it for themselves, thus they do not know how to make connections for students. As Melinda stated earlier, the teachers were focused on teaching their grade level standards, and nothing more. Kevin made this point in his interview, “So we are missing valuable opportunities to build algebraic reasoning and thinking with our teachers, that hopefully, that then can be transitioned [to students],” (2/7/25, p. 25).

Classroom Observations

Observations occurred in five classrooms across three grade levels. The purpose of the observations was to capture data for RQ 2: What instructional practices related to algebraic thinking are observable in WFD’s grades three through eight mathematics classes? The classroom observations tracked the content in terms of the three algebraic thinking content strands identified by Blanton et al., (2018). Additionally, the protocol included instructional practices related to each content strand, and the researcher took running notes during the observation. In this analysis, the focus was on how the teacher carried out the instructional practices to engage students in algebraic thinking, as described in the literature review.

Third Grade with Miss P

On the day Miss P.’s third grade class was observed students practiced composing and decomposing fractions using models. Students also wrote equations for the fractions to show how they could be added together or subtracted apart. This lesson was coded as both generalized arithmetic and equations and equality. However, primarily students were working with a variety of fraction models to show multiple representations of composition and decomposition.

Miss P. used numerous instructional practices quickly to keep her third graders thinking about fractions and the different forms they can be shown. Throughout the warm-up and mini lesson (approximately 20 minutes) she showed students multiple representations of fractions: circle fraction models, a set model, bar models, and equations. She also utilized tools to support students' thinking by having similar models at students' tables. Miss P. also used discourse by asking students to talk with their neighbor. She asked 12 questions about fractions in that same 20 minutes. Some examples of the questions Miss P asked are below:

- Who can tell me what the denominator should be?
- How do you know it should be eight?
- How many pieces are in the whole circle?
- How do we write this as a mixed number?
- What are all the ways to represent five sixths?
- What is a different equation for the same picture?

Table 8

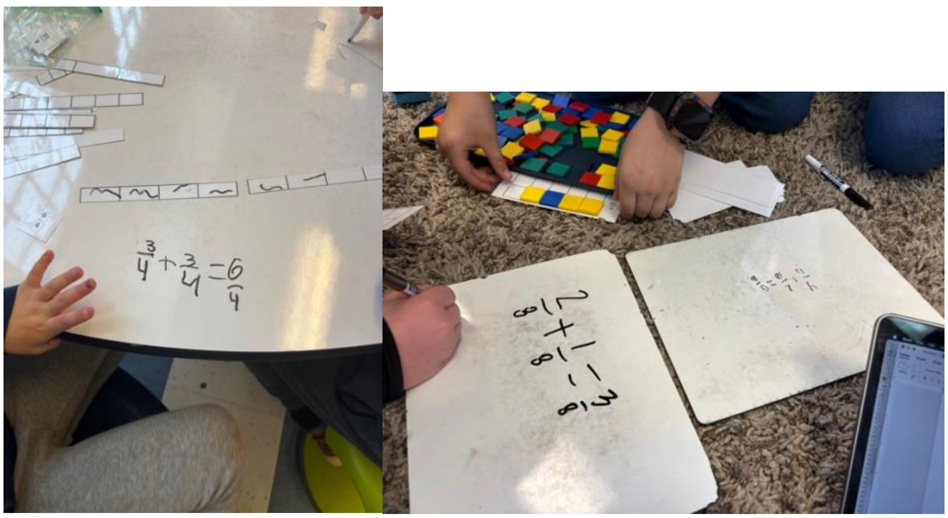
Fraction Stations in Miss P.'s Class

Station	Model	Task
1: Decompose with tiles	Fraction tiles	Break proper fractions apart into unit fractions
2: Fraction build it	Bar models	Build proper fractions from parts; write the equation
3: Fraction wall break apart	Area tiles	"Break" the bar model into parts and write an equation

After the mini lesson, students worked at stations in small groups. Each station had a fraction model, and a task related to composing or decomposing fractions (See Table 8). Samples of students' work can be seen in Figure 10. Students worked steadily while they were at the stations. Miss P. visited each station multiple times to ensure students were working and understanding the tasks. She asked questions that probed students' thinking or helped them get started on the task. Some of the questions she asked were: Which part is the denominator? How many parts are colored in? How many parts are in the whole? What is another way you can represent that?

Figure 10

Student Work Samples from Miss P.'s Class



Note. The photo on the left shows the Fraction build it station with bar models and an equation. The photo on the right shows the Fraction wall break apart station and equation written by a student.

Regarding algebraic thinking, Miss P.'s lesson provided evidence that students understood the structure of fractions, insofar as they could break them apart (decompose), put them together (compose), and write equations that represented their compositions of fractions. She also showed evidence of four instructional practices related to generalized arithmetic: tools to support student thinking, multiple representations (of fractions), use of discourse, and questioning.

Fifth Grade Fractions and Decimals

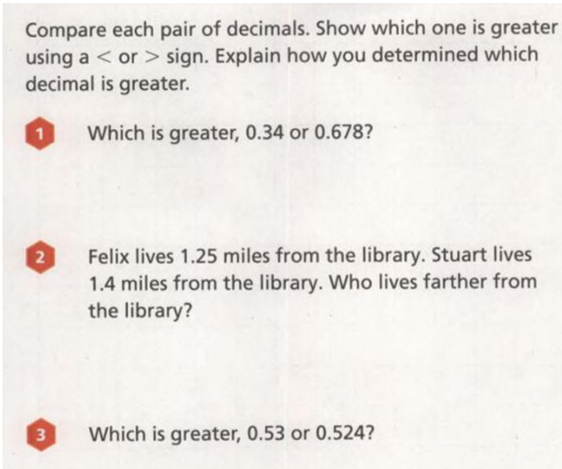
Two teachers were observed in fifth grade at Willow Upper School. According to the pacing guide, the fifth-grade unit they were working on during the observation week was Unit 6: Comparing and Ordering Decimals. Although the teachers were in the same unit, they were working on different lessons during the observations.

Both lessons were coded as generalized arithmetic. Miss S.'s class compared decimals, and Miss L.'s class converted fractions into decimals. Thus, both lessons dealt with the structure of numbers and the relationships between numbers. Both lessons involved models, shown by videos, and students were able to see multiple representations of decimals. Both classes also utilized tools to support student thinking.

The enactment of each lesson was where the classes diverged. Miss S. walked students through a Brainpop video, pausing at points to discuss and complete the check for understanding questions as a whole class. For example, she stopped the video to ask students to compare 0.17 and 0.71. Students used an area model at their desks to shade in the two different numbers so they could compare them, then compared their shading to the models in the video. The video also showed different methods for comparing decimals such as number lines and place value comparisons. Afterwards, the students worked in pairs on worksheet problems for comparing decimals (See Figure 11).

Figure 11

Examples from Miss S.'s Independent Practice Worksheet



Compare each pair of decimals. Show which one is greater using a < or > sign. Explain how you determined which decimal is greater.

1. Which is greater, 0.34 or 0.678?

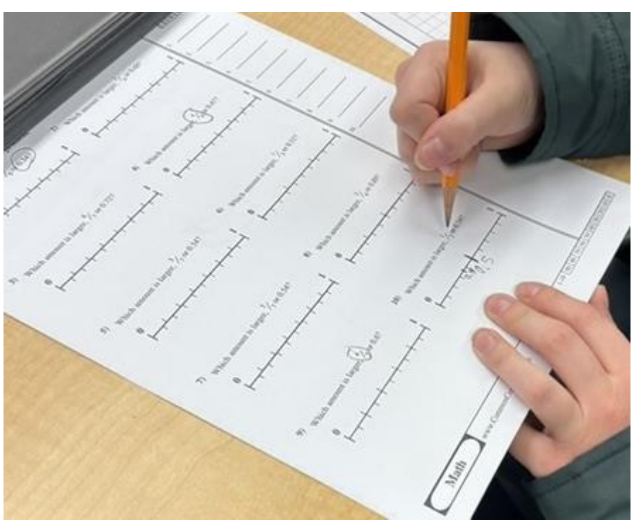
2. Felix lives 1.25 miles from the library. Stuart lives 1.4 miles from the library. Who lives farther from the library?

3. Which is greater, 0.53 or 0.524?

In Miss L.'s class, students watched a video from Math Antics that explained how to use long division to convert fractions to decimals. She paused the video to explain the models in the video, and she recreated the problems on the whiteboard while students worked along with her at their desks. They converted three fractions to decimals together. When the video was over, she gave them a worksheet to complete with a partner. The worksheet had comparison problems with fractions, decimals, and a number line (Figure 12).

Figure 12

Independent Practice Worksheet in Miss L.'s Class



In both classes, students were observed completing the practice problems with varying degrees of success. In Miss S.'s class, students did not have access to tools, like number lines, or area models, to support their learning. They had decimals to compare, but no strategies to help them make sense of the values, which was a main point of the videos. Students practiced with area models of fractions and number lines during the mini lesson, but during the independent practice, the tools were not used or referred to by the teacher. Also, the directions on the worksheet asked students to "Explain" how they got the answer, which Miss S. did not require students to complete. As I watched students finish the problems, students were guessing, or copying from other students, without much effort or thinking.

In Miss L.'s class there was general confusion about the practice tasks students were given. The worksheet asked students to compare a fraction and a decimal with a number line provided for each problem (Figure 12). All three components, fraction, decimal, and number line, had not been part of the video lesson they had watched. Miss L. visited many pairs of students explaining how to continue converting the fractions into decimals so that students could compare them, but no one knew what to do with the number line. Although she provided the tool for thinking, she had not modeled how students should use the lesson's elements together.

In both classrooms, the content was applicable to algebraic thinking, but the execution lacked connections that would support students' independent thinking through the problems. Teachers failed to connect representations to the numbers using models when it was clear that students did not yet have the mental schema to visualize the size of decimals or the appropriate strategies to make sense of the numbers. In this case, teachers did not integrate algebraic thinking strategies into students' independent work time, causing a disconnect between the instruction and the independent practice they were expected to complete.

Proportions in Sixth Grade

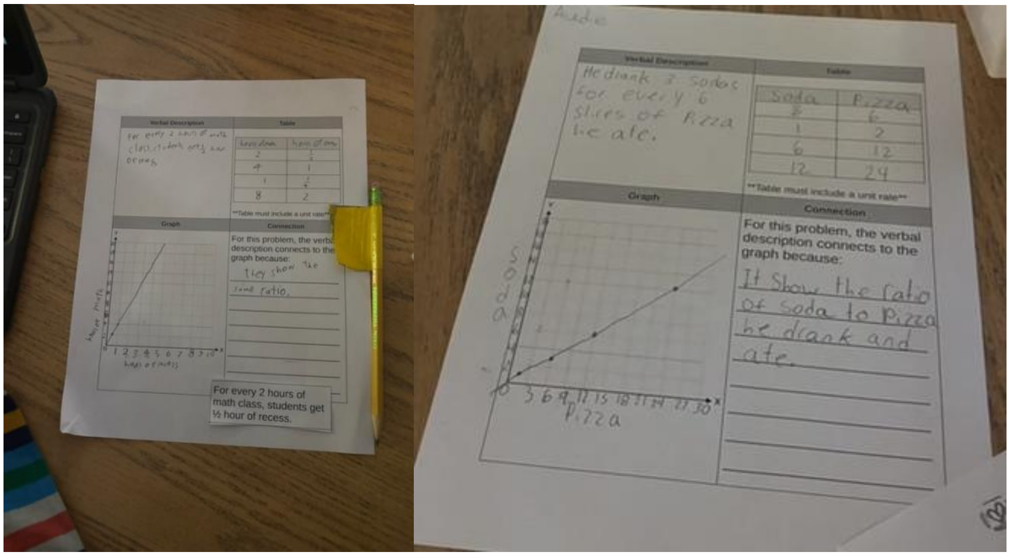
The two sixth-grade observations occurred during consecutive class periods, so the teachers had similar learning objectives for their classes. Miss F.'s class worked on making connections between different representations of proportional relationships and Miss W.'s class determined whether a graph or table showed a proportional relationship. In both classes, students examined ratio tables, graphs, and verbal scenarios to determine if the scenario showed a proportional relationship. This content falls under functional thinking using co-varying quantities.

In both classes, the teacher began by modeling an example of what the students would be doing independently. As Miss F. worked through her example with the students, she used language that oversimplified the process. For example, she said "Is four over ten the unit rate? Why not?" And the students replied, "Because the denominator isn't one." Although this fact is true, it is not the full definition of unit rate. Then, when they found the unit rate, she wrote it on the board as 1:2.5; which she then "flipped" it over so the one was in the denominator. As she worked through the example, she asked closed questions like, "What goes on the x-axis? How do we make the scale, count by one or two? What number do I choose next, anything?"

Finally, she asked the class, "How do I know, based on the table, that it is proportional?" One student answered, "They are all multiples of each other." Again, this is partially true. The missed opportunity was to refer to the unit rate and urge students to examine co-varying quantities. Students were then sent to complete their own problems and share them with another group (See Figure 13).

Figure 13

Student Work Samples from Miss F.'s Class

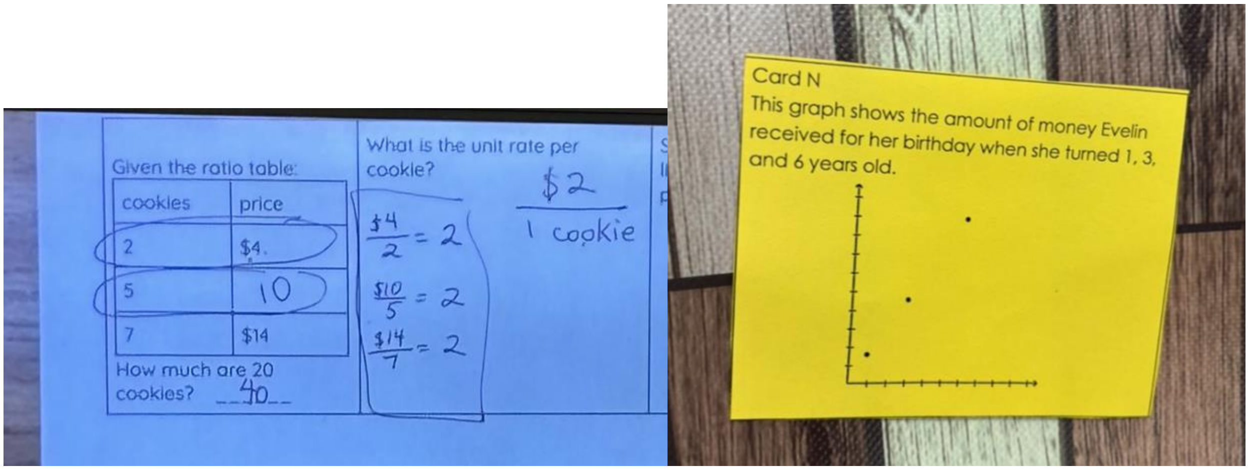


Note. Students were given a scenario and asked to represent it in words, a table, and a graph, then describe the connection between the representations.

Miss W.'s class worked on a similar topic, where they evaluated tables, graphs, and scenarios to determine if the relationship displayed was proportional (See Figure 14). Both classes employed collaborative work and multiple representations of proportional relationships. Miss W.'s instruction was different because she encouraged students to determine the unit rate for each example. She said, "If we make a ratio and simplify it, we should always get the unit rate." As students moved around the room to work on different problems that were posted, she asked students, "Is this a multiplicative relationship?" and "What is the unit rate?" and "How do you know it's proportional?"

Figure 14

Examples from Miss W.'s Class



Note. The photo on the left demonstrates the example Miss W. completed as a model for students. The photo on the right shows one scenario where students needed to determine whether it showed proportionality.

Each lesson presented content aligned with algebraic thinking content in functional thinking. Each classroom also employed multiple representations of co-varying quantities, an instructional strategy that supports algebraic thinking (Carraher & Schliemann, 2018). However, in Miss F.'s class, students struggled to connect the three representations of proportionality. Her instruction lacked the connections and specificity that would help students deepen their thinking. In Miss W.'s class the students worked to determine if the different representations were proportional, but most could not complete the task independently. The enactment in each class lacked the depth that would have supported students making connections across the representations.

Overall analysis

After reviewing and analyzing the documents, interviews, and observations for patterns and trends related to algebraic thinking content and instructional practices, I looked across all three sources for themes that could be discerned in terms of overall findings with implications for recommendations in chapter 5. Here, I present overarching themes with evidence that supports them.

Different expectations for each grade level

There is a disconnect in the curriculum documents created for elementary grades compared to the middle school documents. The elementary specialists described the curriculum development expectations with clarity and certainty. Jane and Brooke separately described using the *Math Investigations* curriculum as the starting point, and supplemental plans were added to meet the expectations of the math SOLs in areas not aligned to the standards. For example, in third grade, the supplemental fractions units were inserted into the pacing guide documents, and the specialists created daily plans for teachers to follow. I also witnessed these plans in action in a third-grade classroom and they were aligned to algebraic thinking content.

Brain shared that Willow had also adopted the *Investigations* curriculum in fifth grade. And, according to the planning documents for fifth grade, they were working on Investigation 1, from Unit 6 during the time of the observations. However, that was not the instruction that was observed. Teachers utilized videos from other resources as instruction, and the only part of *Investigations* that was included was the workbook page students completed for practice. This is not to say that a resource is the only answer; but utilizing a resource that includes questioning, multiple representations, tools to support student thinking, and discourse routines would bring teachers closer to practices that evoke algebraic thinking.

In the middle school grades, both Brian and Kevin articulated that the curriculum was constructed by teachers who did not use the adopted resource as a baseline for instruction. For sixth grade, the planning document resembled a list of activities teachers might complete. When observed on the same day, two teachers were completing different tasks with different learning targets, and neither was using the target that was in the planning document. Additionally, one of the classes, Miss F., used an activity that was not on the planning form. Which also showed that each grade level, and in this instance each classroom, was following a unique plan, as opposed to the one that was developed collaboratively, as Brian described.

The lack of data from Boxwood made it hard to infer anything about curriculum and instruction, however from Boxwood's math specialist, Kevin's descriptions, teachers were generally working in siloes; "You have seventh grade that is trying to be a little bit unified. And in eighth grade we're in three different places," (2/7/25, p. 11). He described the planning process this way, "We also have teachers that they feel…I'm teaching slope with 2 points today. That's all I need to do. And I'm going to download a sheet, or, you know, write up some problems and go… we have a lot of teachers who live day to day" (2/7/25, p. 22). The implication was that teachers were not planning ahead or planning units of instruction collaboratively. Based on the documents provided for the study, very little is known about the algebraic thinking content and strategies used in grades seven and eight. The only references to algebraic thinking in the curriculum documents are the 2023 Math SOLs, which merely list the skills; there are no implications for instruction practices.

Throughout the interviews, although the specialists understood that algebraic thinking and algebra readiness were important goals for students, they voiced concerns about teachers being able to see the connections between the standards and algebraic thinking. Particularly in middle school, when students were on the cusp of taking algebra, the curriculum documents focused solely on the standards, and there was no evidence of integrating algebraic thinking practices. Kevin, the specialist for Boxwood Middle, said, "So we are missing valuable opportunities to build algebraic reasoning and thinking with our teachers, that hopefully, that then can be transitioned [to students],” (2/7/25, p. 25).

Researcher Reflection on Findings

The findings and related data in this chapter are the evidence in response to the problem of practice for WFD. Students can take algebra as early as seventh grade in WFD, how is the math curriculum preparing students to engage in that course? To answer the research questions, I examined the standards and curriculum documents, interviewed math leaders in the district, and observed math classes. In this section, I will synthesize those findings in conjunction with the literature on the topic that will inform the recommendations in chapter 5.

Foremost, WFD math specialists acknowledged that algebra, algebra readiness, and algebraic thinking are critical components of the district’s math program. In her interview Brooke said, “Because, yeah, we certainly know that algebra readiness is, for lack of a better word, a problem… so it shouldn't be left as a conversation,” (2/14/25, line 194). However, the math coordinator also acknowledged that they were not doing enough about it. Teachers lack the time, resources, and capacity to make connections between their standards and algebraic thinking without support. Carraher and Schliemann (2019) noted that teachers must take specific instructional actions to develop algebraic thinking in students using strategies that help them bridge the gap between arithmetic and algebra. For example, in the sixth-grade classroom observations, Miss W. and Miss F. unknowingly used many of the strategies Carraher and Schliemann (2018) found beneficial: using real-world contexts, open-ended problems, multiple representations, and collaborative work. But they both missed one vital piece, which was guiding the students towards looking for and articulating how one quantity affects another. This part of the lesson, making connections between representations, was left for the students to determine when their work indicated that they required additional guidance.

In fifth-grade classrooms, I witnessed a similar occurrence, students unable to connect direct instruction to independent practice. In this case, students used fractions and decimals from the generalized arithmetic strand to explore comparison. Both classes were comparing numbers; decimals in one class and fractions and decimals in the other. Blanton and Kaput (2005) found that most of the interactions in elementary classes are through generalized arithmetic and offer multiple entry points to apply algebraic thinking. In the fifth-grade classrooms, very little evidence indicated teachers were making connections between numbers, magnitude, and place value. Although the teachers utilized tools to support student thinking during direct instruction: area models, place value chart, and number lines, students did not connect the concept of comparison with the tools during independent practice. So, when students worked independently, the tools and representations were not used, leaving the students without strategies for making sense of the comparisons. The inconsistency in the use of instructional practices held students back and stopped them short of engaging in thinking about the concept.

Both examples demonstrate complex answers to RQ 2. There was evidence of algebraic thinking instructional practices in each classroom. However, they were not being implemented in ways that cultivate algebraic thinking. The connections between the strategies, content, and algebraic thinking were lacking in that students could not complete work without the teacher's support.

WFD curriculum documents all contained evidence of the "official curriculum" or the state standards defining the student objectives for each grade level (Remillard & Heck, 2014, p. 708). In the elementary grades, there was also evidence of the "designated curriculum" that was designed using the curriculum resources and local curricular materials (Remillard & Heck, 2014, p. 710). What seemed to be missing was the “teacher-intended” curriculum, which is how teachers make sense of the lessons and transform them into plans that the teacher has interpreted for use with his/her students (Remillard & Heck, 2014, p. 711). In his interview, Kevin referred to this as preparing to teach. He said, “The vast majority of teachers are really good at doing this [planning]. What they struggle with is the preparation,” (2/7/25, p. 21). He went on to explain that teachers know what to do according to the pacing guide, but do not invest time into preparing the teacher-intended curriculum.

Curriculum enactment requires teachers to prepare for a multitude of classroom interactions; they must consider the interactions between themselves and the students, the students and each other, and most importantly the students and the math (Remillard & Heck, 2014). Although enactment can look different for each teacher, they should not differ in the mathematics they deliver to students, nor in the pedagogical moves supporting the content being taught. And, at Boxwood especially, if there is no guarantee that teachers are using the same curriculum resources, then one cannot assume they are enacting the same or similar curriculum.

NCTM recommends that students engage in algebraic thinking starting in elementary school and continuing through high school courses (NCTM, 2000). NCTM highlights ways in which algebraic thinking can be integrated into elementary and middle school mathematics: thinking relationally about the equal sign, thinking about pattern generalization, and thinking about relationships in problem solving scenarios (NCTM, 2014). The purpose of algebraic thinking is not to introduce algebra concepts early; rather, it should support students to make connections among the topics they are learning throughout elementary and middle school (Afonso & McAuliffe, 2019). With support from their teachers, students can seek to understand the underlying connections between numbers and operations, which will ease the transition from arithmetic in elementary school to algebra in middle school (Warren et al., 2016). Arithmetic in elementary can serve as a link between computation and algebraic thinking, if the instructional practices help to highlight the connections. As Brooke explained, that is often missed in elementary math: "We totally miss it when we think about all the pieces of computation like, there's so much conjecturing and generalizing and sort of justifying that kids can do in computation. But we don't name it as such… It can be missed that, like there's all that thinking and developing of algebraic thinking can happen across the standards," (2/14/25, p. 13).

A concern raised by the specialists was the lack of conversations and planning that took place between grade levels and school buildings. Brian noted, "We don't have enough of those vertical conversations. I do feel like we're starting to have more of those which is a positive, you know…but definitely something that we're continuing to think about in terms of the professional learning and development of our staff," (2/7/25, p. 22). Chapter five explores recommendations for creating a cohesive strand of algebraic thinking through the math curriculum and instruction in WFD.

Limitations

The primary limitation of this study was the resources I could access. Although I was given digital copies of many district documents, most links embedded in the copies were unavailable because I could not access district shared drives. Additionally, time was a factor for this study. The observation window planned for was interrupted by winter weather delays and closings, causing some observations to be cancelled and some that needed to be rescheduled. I could not observe classes for each grade level. This limited the sample size. However, the data from this study could support further work in the area of algebraic thinking at the observed grade levels.

The February observations had the potential to limit the content observable in the classroom according to the district's scope and sequence. However, as the literature indicates, algebraic thinking should be embedded in multiple contexts throughout the elementary and middle school curriculum.

Staff turnover in leadership at the school level and as a math specialist team at each school also led to inconsistencies in expectations around planning with and implementing the local curriculum. And at each of the middle schools, the math specialists had only been in place for one or two years. Additionally, Cedar and Willow had new principals in 2023, and all the schools had at least one new assistant principal within the last two years. These staffing changes can lead to changes in procedures and expectations within the buildings, particularly as it applies to math, the expectations for using district curriculum documents or district-approved resources were inconsistent according to the specialists.

Lastly, 2024-2025 was the implementation year for the new Virginia Mathematics, but the local curriculum had not yet been fully updated to reflect the changes. The pacing guides for the middle school grades at Boxwood were unfinished for grades seven and eight and did not reflect the new math SOLs for the second semester. Fourth grade pacing guides were also incomplete.

Delimitations

The delimitations of this study helped narrow the focus and constrain the number of observations and interviews I conducted by only working within four schools in the district. I opted to focus on grades three through eight because the transition into and out of middle school is unique in WFD. Students typically transition into sixth grade, but in WFD, they transition twice: once into Willow for fifth and sixth grade and once into Boxwood for seventh and eighth grade. One element considered was how this impacts the curriculum and instruction.

Chapter 5

Recommendations

Recommendation 1: Develop a consistent curriculum framework template for all grade levels and math courses.

Throughout the document analysis, interviews, and classroom observations, there is evidence of inconsistent documentation of what teachers are doing and how. Math specialists and the coordinator refer to curriculum documents as “the wild west” and “pieced together” which is not supportive of the coherent teaching and learning that make a high-quality curriculum effective (Leinwand, 2014). Documents in the middle grades lack cohesion; there are no common resources, daily plans are lists of activities, and the instructional strategies, or best practices for teaching specific concepts, are not referenced. The lack of coherence does not support the unified introduction and reinforcement of algebraic thinking content across grade levels.

Curriculum Frameworks

One strategy to ensure that all students access algebraic thinking to support algebra readiness is to design a curriculum framework that emphasizes the content, instructional practices, and levels of thinking that facilitate algebraic understanding. A curriculum framework featuring common elements emphasizes the math department's priorities, ensuring everyone understands the focus across grade levels. Commonality across grade levels allows teachers to focus on translating the designated, or local, curriculum to meet their students' needs to enact the content (Remillard & Heck, 2014). See Appendix I for an example that was designed with the elements described here.

A curriculum framework ensures that teachers are aligned in the purpose and outcomes of each course. The framework includes a logical scope and sequence, that defines the time and breadth of the topics to be covered. A framework designed around elementary and middle school math's major ideas and concepts, includes learning goals with well-defined outcomes so that each teacher is teaching towards the same targets. A framework also allows for the integration of specific teaching methods that are aligned with the content. In the case of algebraic thinking, a framework guides teachers to use instructional strategies that are most effective for the algebraic thinking content strand, explored further in Recommendation 1A.

WFD maintains unleveled or heterogeneous classes for math through middle school. Since students with varying skills and interests are grouped in courses, teachers should be prepared to provide opportunities for students to engage in differentiated tasks that support their readiness, interests, and learning preferences. Providing space in the curriculum framework will help teachers consider how students will be engaged and supported throughout the learning process.

Consistent documents across grade levels ensure that the vertical articulation of concepts reinforces students' understanding as they progress through courses. Vertical articulation supports teaching and learning math because it connects concepts across grade levels and connects teachers to each other through conversations about what and how skills are taught (Leinwand, 2014). Vertical articulation should be a component of developing curriculum frameworks to create a coherent and comprehensive curriculum (Leinwand, 2014). In the sample framework, an element titled "Vertical Connections" encourages teachers review what students have learned in a previous course and what they will learn in the following course.

Melinda, Brian, and Kevin expressed concerns during their interviews regarding the teacher-created curriculum implemented in the middle school grades. A curriculum framework allows the math coordinator and specialists to outline the appropriate teaching resources at each grade level. Whether that is an adopted textbook resource or open-sourced tasks, a framework delineates what teachers use and how they use it to provide equitable learning experiences across classes and schools. A benefit of a consistent framework is that it also sets consistent expectations for staff, so they know what and how to teach.

Recommendation 1a: Highlight algebraic thinking skills and instructional strategies in the curriculum framework to support algebra readiness.

To ensure that all students access algebraic thinking to support algebra readiness, algebraic thinking connections must be included in the curriculum framework and planning documents. By utilizing the 2023 SOLs, WFD can capitalize on the connections between the standards and algebraic thinking content as an area of focus in the frameworks.

Mathematics SOLs and Algebraic Thinking Content. The Mathematics SOLs align with the content strands of algebraic thinking, as shown in the SOL mapping exercise (Figure 15). Categorizing the SOLs by algebraic thinking content area shows how the skills in the SOLs connect with the content areas. Integrating algebraic thinking as a common thread throughout the curricula reflects the idea that algebra is not a topic but a way of thinking that supports math learning (Kaput, 2008; NCTM, 2000).

Figure 15

Section of Curriculum Framework Highlighting Algebraic Thinking

Learning Plan			
Algebraic Thinking Content Connections	Generalized Arithmetic	Functional Thinking	Equations and Equivalence
Algebraic Thinking Instructional Practices	Generalized Arithmetic	Functional Thinking	Equations and Equivalence
Learning Progression			
Learning Experiences and Instruction	Day 1	Curriculum Resources	
	Day 2	Curriculum Resources	
	Day 3	Curriculum Resources	

Blanton et al. (2018) created a framework for early algebra demonstrating that integrated algebraic thinking lessons did not overburden teachers' or students' curricular expectations. By integrating lessons into the original scope and sequence, the study's lessons enhanced concepts already addressed in the curriculum by supplementing them with specific algebraic thinking skills and teaching practices. Using arithmetic as an entry point helps students think about and apply the interconnectedness of mathematical operations and symbols (Blanton et al., 2018). As Brooke described, these connections are currently missing in the WFD curriculum. She said "We totally miss it when we think about all the pieces of computation like, there's so much conjecturing and generalizing and sort of justifying that kids can do in computation. But we don't name it as such… It can be missed that, like there's all that thinking and developing of algebraic thinking that can happen across the standards," (2/14/25, p. 13). Highlighting the connections between the algebraic thinking content and the math SOLs, WFD can make a step towards improving these skills with students.

Recommendation 2: Support teachers' understanding of the development of algebraic thinking and the instructional practices that support students' algebraic thinking.

To integrate algebraic thinking into the curriculum, teachers must understand the content and its connections to algebraic thinking. For most teachers this will require professional learning. During the interviews with specialists, they discussed the need to support teachers' content knowledge in this domain. Brooke said succinctly, "Particularly, elementary teachers don't necessarily love or feel confident with math," (Interview, 2/14/25). Teachers need content and instructional support to make the shift towards incorporating algebraic thinking into their current teaching.

Most elementary teachers do not recognize the connections between arithmetic and algebraic thinking, however, to interact with students, their teachers must recognize and respond to students when they engage in algebraic thinking content (Demonty et al., 2018). This requires a specific type of content knowledge, which many teachers are not trained for. To engage students in algebraic thinking, which requires teachers to make connections across concepts, teachers must have a basic understanding of algebra and how to teach it (Blanton & Kaput, 2005). The second recommendation is a professional learning plan to integrate algebraic thinking into teaching and learning.

Professional Learning Plan

It is critical that the math coordinator, in partnership with the math specialists, define the purpose of integrating algebraic thinking into the curriculum to support teachers' learning. Because students begin to access algebra in seventh grade, the team should share how algebraic thinking supports algebra readiness for all students. Algebraic thinking strengthens students' ability to make connections across concepts and their capacity to think flexibly about numbers (Ketterlin-Geller et al., 2015). Early algebra activities implemented by elementary teachers can support algebraic thinking when teachers understand the algebraic nature of the problem and can guide students through discourse that connects arithmetic and algebraic thinking (Lee et al., 2023). Teachers must solidify their content knowledge to make connections between concepts while teaching.

The professional learning (PL) plan should include the content components of algebraic thinking and how they relate to the current standards and newly proposed curriculum frameworks (See Appendix J). The first strategy of the PL plan will engage teachers in learning about the components of algebraic thinking and how they connect to the 2023 Math SOLs. Teachers will also be introduced to the instructional strategies that support algebraic thinking. In the data collected during classroom observations, each teacher had evidence of alignment to the 2023 Math SOLs, the missing piece is the connection to algebraic thinking. The PL plan will connect the algebraic thinking content, the SOLs, and the instructional practices that align to both.

Figure 16

Excerpt from Sample Professional Learning Plan for WFD

<table>
<tr><th colspan="6">Identify the professional learning strategies, related details, and steps you will take to implement the strategies in your school division.</th></tr>
<tr><th colspan="2">Professional learning strategies (choose from below)</th><th rowspan="2">Grade(s) targeted</th><th rowspan="2">Learning Experiences</th><th rowspan="2">Resources and supports</th><th rowspan="2">Next steps</th></tr>
<tr><th>• Examining student work and thinking
• Demonstration lessons
• Action research</th><th>• Coaching
• Mentoring
• Study groups
• Workshops or seminars
• Other</th></tr>
<tr><td colspan="2">Strategy 1:

Workshop/Seminar</td><td>Grades 3 – 8</td><td>Goal: Teachers will explore the algebraic thinking content areas and related instructional practices in relation to their grade level curriculum.</td><td>Resources:
• Video samples of teaching
• Algebraic thinking curriculum map</td><td>Teachers will bring curriculum documents (plans, frameworks, etc.) for next session.</td></tr>
</table>

The second strategy of the PL plan will engage teachers in exploring the vertical articulation of algebraic thinking content across grade levels. Understanding what students have learned and how they learned it helps teachers iterate and expand on concepts from elementary to middle school. As Brian said, “And there's been like conversations about like, where are they going… … how is this, what we're doing right now, supporting the algebra readiness going forward?” (2/7/25, p. 19). Supporting teachers in understanding how current learning influences the future, particularly how algebraic thinking enhances math learning both now and, in the future, strengthens the overall math program through transparency and collaboration.

The third strategy of the plan involves teachers examining student work for evidence of algebraic thinking. This involves teachers preparing for and teaching using algebraic thinking instructional practices. Including this as a stage of the plan holds teachers accountable for engaging in the teaching practices and looking for evidence that students are exploring relationships, making connections, and discussing math through algebraic thinking. It also allows teachers to look at examples of student work across grade levels and see how concepts change over time, which supports teachers’ content knowledge and understanding of vertical articulation.

Although this plan is divided into three stages, these are not intended to be one-time sessions. It will take several sessions to accomplish each goal. Because this would be a district-wide PL plan, it is feasible that the math team would chunk the curriculum into parts for teachers to understand, implement, and then collect evidence of teaching specific concepts throughout the year. Because time is a finite resource, this plan would best be carried out over the course of several months, emphasizing its importance by keeping algebraic thinking the focus of teachers’ time during the year.

One significant concern for this PL plan is whether all math specialists can support a strategy to emphasize algebraic thinking at both the district and school levels. Since not all math specialists were interviewed for this study, it cannot be determined if everyone is prepared or able to endorse such a plan. Among the small sample of four specialists, two are not certified math teachers, raising questions about their expertise and understanding of the full spectrum of K-12 mathematics, or at least K–algebra. This factor should be considered before proceeding with a districtwide plan.

Conclusion

The purpose of this study was to examine the curriculum and instruction of WFD as it pertains to algebraic thinking to support algebra readiness. Overall, the data collected showed that WFD math leaders understand that algebraic thinking is a component of a complete math curriculum but have yet to actualize that goal. One barrier seems to be the lack of consistency across the district's curriculum documents and resources. By establishing a curriculum framework that establishes goals and emphasizes algebraic thinking, the math department can focus on alignment to the frameworks across schools and grade levels. With the implementation of new frameworks, teachers will need to be trained. Creating a district-wide math teacher professional learning plan that supports the implementation of the frameworks, along with algebraic thinking content and pedagogy development, further supports teachers to align with the district's goals of algebra readiness.

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Appendix A

WFD Algebra Placement Chart

WFD Mathematics Placement Criteria 2023 - 2024			
Grade	Previous Class	Pathways	Criteria
7th Grade	Math 6	Pre-Algebra 7 Unleveled	All Students
		Algebra I	Winter or Spring MAP ≥ 90%tile AND SOL Score ≥ 500 (Pass Advance)
8th Grade	Pre-Algebra 7	Pre-Algebra 8 Unleveled	Spring MAP < 230 AND SOL < 450
		Algebra I	Spring MAP ≥ 230 OR SOL Pass ≥ 450
	Algebra I	Geometry	SOL Pass > 400
9th Grade	Pre-Algebra 8	Algebra I, Part I	Must be written into IEP
		Algebra I Double Blocked	SOL Score < 400
		Algebra I	SOL Pass > 400
	Algebra I	Algebra I	SOL Score < 400
		Geometry	SOL Pass > 400
	Geometry	Algebra II	Algebra I SOL Score <500
		Algebra II Honors	Algebra I SOL Score >500

Appendix B

Classroom Observation Protocol

Research Questions	Research Question 2: What instructional practices related to algebraic thinking are observable in CCS's grades three through eight mathematics classes?
Teacher	
Date/Time	
Location	
Lesson Focus (Standard, Learning Objective, Topic)	

Date: **Time:**
Pre-observation Reflection Questions
Tell me about the lesson I'm going to see.
Is there anything you want me to look for?

Key Algebraic Thinking Content	**Evidence**
Generalized Arithmetic - Structure of numbers - Properties of operations - Relationships between numbers	
Functional Thinking - Patterns - Co-varying quantities	
Equations and Equality - Relational understanding of the equal sign - Equations - Expressions	

- Inequalities	

Instructional Practices by Content Area	
Generalized Arithmetic - Tools to support student thinking - Multiple representations - Use of discourse - Questioning	
Functional Thinking - How one quantity affects another - Open-ended problems - Collaborative work - Multiple representations	

Post-observation Teacher Reflection Questions
How do you think the lesson went?
Is there anything you want to tell me that I might not have captured?

Post-observation Researcher Reflection:

Appendix C

Interview Protocol for Math Specialists

Research Questions	Research Question 1: To what extent are key ideas associated with algebraic thinking introduced and reinforced across the elementary and middle school math curricula in CCS?
Participant	
Date/Time	
Location	ZOOM
Linked Recording	
Purpose	Gather information about curriculum and instructional planning in CCS regarding algebra, algebra readiness, and algebraic thinking.
Script for Consent	This interview is to collect data on the Mathematics Program in the CCS school district. This meeting is being recorded so that it can be quickly transcribed. The data may help inform any recommendations to the school district. Names will be redacted, and the school district will be referred to by pseudonyms. Do you consent to being interviewed today?
Consent Agreement	

Introduction:

The focus of this study is algebra readiness through algebraic thinking. I'm going to define those terms for you, in relation to this study, so that we use similar terminology while discussing this topic. Algebra readiness is the set of skills and concepts students should master to be prepared for Algebra. Algebraic thinking helps students understand the underlying connections between numbers, number systems, and mathematical symbols (Chimoni et al., 2018). For this study, I am investigating three content components of algebraic thinking: generalized arithmetic, functional thinking, and equality and equivalence and related instructional strategies.

Interview Questions

1. Please tell me about your experience in CCS, and any other pertinent experiences with math curriculum and instruction.
2. Talk me through what the curriculum looks like in CCS.
    a. What are the resources that you use for planning?
3. What role does algebra readiness play in CCS?
4. Considering the aspects of algebraic thinking I mentioned earlier, where do you see those ideas manifest in the curriculum in CCS?
5. Are there any instructional strategies being used that relate to the algebraic thinking content?

Post-Interview Reflection:

Appendix D

Interview Protocol for Math Coordinator

Research Questions	RQ1: To what extent are key ideas associated with algebraic thinking introduced and reinforced across the elementary and middle school math curricula in CCS?
Participant	
Date/Time	
Location	Zoom
Linked Recording	
Purpose	
Script for Consent	This interview is to collect data on the Mathematics Program in the CCS school district. This meeting is being recorded so that it can be quickly transcribed. The data may help inform any recommendations to the school district. Names will be redacted, and the school district will be referred to by pseudonyms. Do you consent to being interviewed today?
Consent Agreement	

Interview Questions

1. Please tell me about your experience in CCS, and any other pertinent experience with math curriculum and instruction.
2. Let's talk a little about curriculum development. What's the process in the city? (what resources are used, how do you determine priorities)
3. The purpose of this study is to examine how content areas and instructional strategies related to algebra readiness show up in the C&I of CCS. From your perspective, how is the math department in CCS preparing kids for Algebra through C&I? (interventions, concentrated review, etc)
4. One thing I have discussed with the specialists is the connection between algebraic thinking and content at each grade level. From your point of view what does that look like holistically – across grade levels and schools?

Post-Interview Reflection:

Appendix E

Codebook for Interview Data

***A priori* codes**

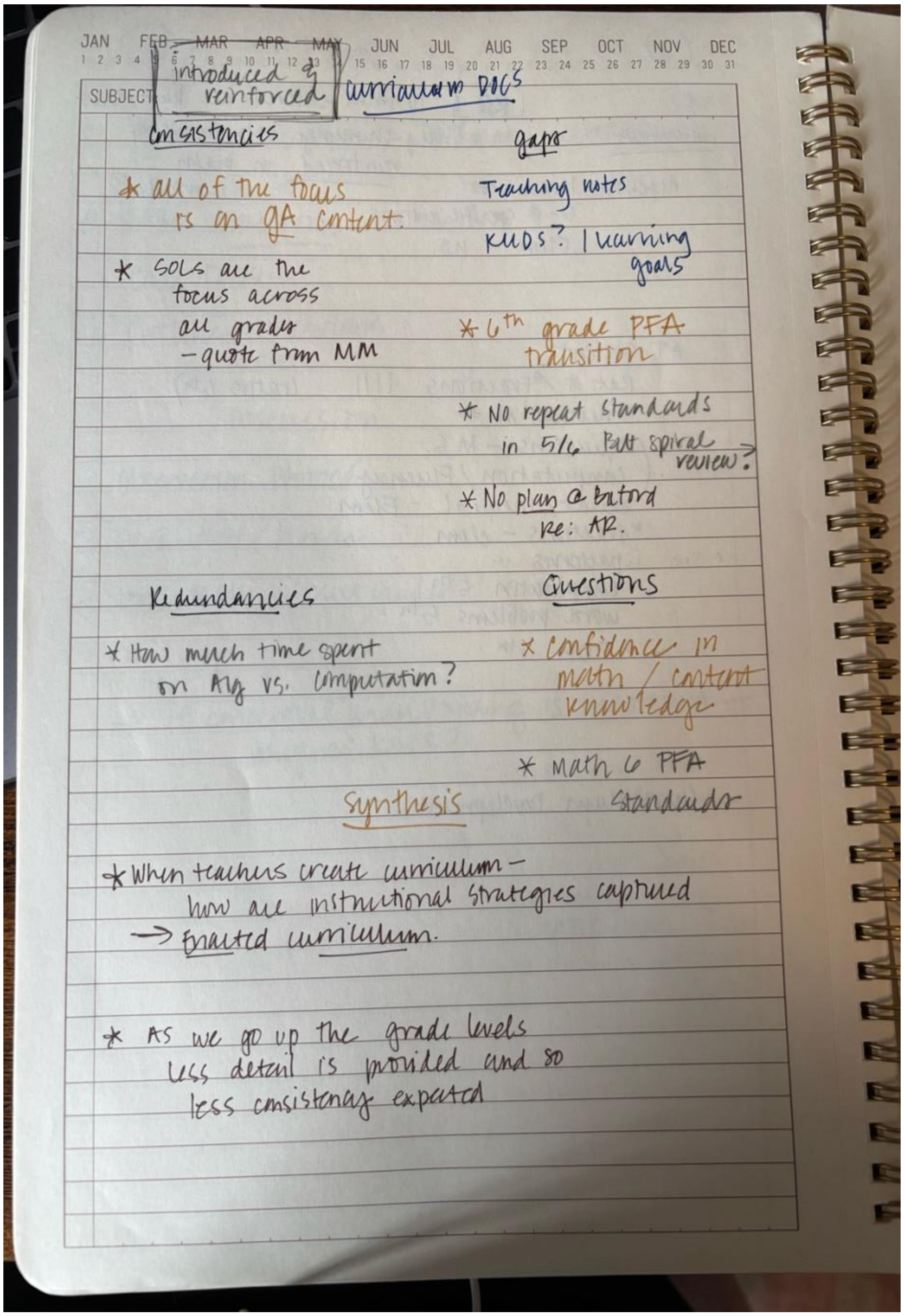
Code	Definition	Inclusionary Criteria	Exclusionary Criteria
Algebraic thinking content	The content areas of algebraic thinking, generalized arithmetic, functional thinking, and equality and equivalence	Teachers' perceptions of these topics and how they manifest in grade level curriculum	Other content not related to algebraic thinking
Instructional practices	Teacher actions with students to promote algebraic thinking	Any practices associated with instruction that are connected to skills/process of algebraic thinking	Behavior management practices School policies
Curriculum development	Process of planning curriculum in the context of local planning with algebraic thinking in mind	Local curriculum development in the context of planning for units/lessons to be used in this district	

Emergent codes

Code	Definition	Inclusionary Criteria	Exclusionary Criteria
Foundational skills	Skills students have, or have not, mastered prior to algebra	Basic facts, fractions, operations	Standards that come in other grade levels
Curriculum resources	Resources (physical, commercial, or web-based) available to teachers for instructional purposes	Textbooks State documents Computer programs Physical manipulatives Videos	Resources unrelated to math
Assessment	Tests, quizzes, formative or summative	Any opportunity to capture what students have or have not learned	State tests
Professional learning	Opportunities for teachers to learn and improve practice	Professional development, learning from others	Opportunities unrelated to math

Appendix F

Sample of Researcher Notes



JAN FEB MAR APR MAY JUN JUL AUG SEP OCT NOV DEC
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

SUBJECT introduced & reinforced | curriculum DOCS

consistencies

* all of the focus is on GA content.

* SOLs are the focus across all grades
  - quote from MM

gaps

Teaching notes

KUDS? / learning goals

* 6th grade PFA transition

* No repeat standards in 5/6 But spiral review?

* No plan @ Buford re: AR.

Redundancies

* How much time spent on Alg vs. computation?

Questions

* confidence in math / content knowledge

* Math 6 PFA standards

Synthesis

* When teachers create curriculum - how are instructional strategies captured → Enacted curriculum.

* As we go up the grade levels less detail is provided and so less consistency expected

Appendix G

Curriculum Map

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1		**Unit 1**	**Unit 2**	**Unit 3**	**Money Unit**	**Unit 5**	**Geometry Unit**	**Unit 4**	**Fractions**	**PFA unit**	**Fractions**	**Unit 7**	**Unit 8**	**Measurement**
2		3.NS.1	3.PS.1	3.NS.1	3.NS.4	3.CE.2	3.MG.4	3.MG.1	3.NS.3	3.PFA.1	3.NS.3	3.NS.1	3.CE.2	3.MG.1
3	**Grade 3**	3.CE.2	3.MG.1	3.NS.2				3.MG.2				3.CE.1		3.MG.3
4		3.MG.1		3.CE.1*										
5														
6														
7		**Unit 1**	**Line graphs**	**Unit 3**	**Elapsed Time**	**Unit 4**	**Unit 5**	**Fractions**	**Unit 6**	**Decimals**	**Unit 7**	**Patterns anc**	**Unit 8**	
8		4.CE.2	4.PS.1	4.CE.2	4.MG.2	4.MG.1	4.NS.1	4.NS.3	4.NS.4	4.NS.4	4.CE.2*	4.PFA.1	4.CE.1	
9	**Grade 4**					4.MG.3	4.NS.4	4.NS.4	4.NS.5	4.NS.5	4.MG.1	4.PS.2	4.CE.2	
10						4.MG.4	4.CE.1	4.CE.3		4.CE.4			4.PFA.1	
11						4.MG.5								
12						4.MG.6								
13														
14		**VA Unit 1**	**VA Unit 2**	**VA Unit 9**	**Unit 1**	**Unit 3**	**Unit 4**	**Unit 6**	**Unit 7**	**VA Unit 6**	**VA Unit 8**			
15		5.PS.1	5.NS.2	5.PFA.1	5.CE.1	5.NS.1	5.CE.1	5.CE.3	5.CE.2	5.MG.1	5.MG.2			
16	**Grade 5**	5.PS.2	5.CE.4*	5.PFA.2		5.CE.2		5.NS.1	5.CE.3		5.MG.3			
17			5.PS.3											
18			5.PFA.1											
19														
20		**Unit 1**	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**					
21	**Grade 6**	6.PS.1	6.NS.2	6.MG.3	6.CE.2	6.NS.1	6.PFA. 1	6.NS.3	6.PS.1					
22			6.PFA.4	6.MG.4	6.PFA.3	6.CE.1	6.PFA.2	6.MG.2	6.PS.2					
23							6.MG.1							
24														
25		**Unit 1**	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**	**Unit 9**				
26		7.PS.2	7.MG.3	7.NS.1	7.PS.1	7.CE.2	7.PFA.1	7.PFA.2	7.PFA.2d*	7.MG.1				
27	**Grade 7**			7.NS.2				7.PFA.3	7.MG.1	7.MG.2				
28				7.NS.3				7.PFA.4		7.MG.4				
29				7.CE.1										
30				7.PFA.2										
31														
32		**Unit 1**	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**	**Unit 9**				
33		8.PS.2	8.PFA.1	8.CE.1	8.NS.1	8.MG.1	8.PFA.5	8.PS.3	8.MG.3	8.PS.1				
34	**Grade 8**		8.MG.2	8.MG.1	8.NS.2	8.MG.2		8.PFA.2	8.PFA.3					
35			8.MG.5			8.MG.4								
36						8.PFA.4								

Appendix H

Curriculum Map with Duration of Units

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**Grade 3**													
2	Unit	**Unit 1**	**Unit 2**	**Unit 3**	**WFD Unit**	**Unit 5**	**WFD Unit**	**Unit 4**	**WFD unit**	**WFD unit**	**WFD unit**	**Unit 7**	**Unit 8**	**WFD unit**
3	Topic	Multiplication and Division	Model with data	Addtion, Subtraction, Number system	Money	Multiplication and division 2	Geometry	Geometry and measurement	Fractions	Patterns, functions, and algebra	Fractions 2	Addition, subtraction, and number system 2	Multiplication and division 3	Volume, weight, and time
4	Standards	3.NS.1	3.PS.1	3.NS.1	3.NS.4	3.CE.2	3.MG.4	3.MG.1	3.NS.3	3.PFA.1	3.NS.3	3.NS.1	3.CE.2	3.MG.1
5		3.CE.2	3.MG.1	3.NS.2				3.MG.2				3.CE.1		3.MG.3
6		3.MG.1		3.CE.1*										
7	Duration (days)	23	14	25	5	11	4	9	9	5	7	6	4	5
8														
9	**Grade 4**													
10	Unit	**Unit 1**	**WFD unit**	**Unit 3**	**WFD unit**	**Unit 4**	**Unit 5**	**WFD unit**	**Unit 6**	**WFD unit**	**Unit 7**	**WFD unit**	**Unit 8**	
11	Topic	Multiplication and divison	Line graphs	Multiplication and division 2	Elapsed time	Area and perimeter	Addition, subtraction, and the number system	Fractions	Fraction cards and decimals	Decimals	Multiplication and division 3	Patterns and probability	Patterns, tables, equations	
12	Standards	4.CE.2	4.PS.1	4.CE.2	4.MG.2	4.MG.1	4.NS.1	4.NS.3	4.NS.4	4.NS.4	4.CE.2*	4.PFA.1	4.CE.1	
13						4.MG.3	4.NS.4	4.NS.4	4.NS.5	4.NS.5	4.MG.1	4.PS.2	4.CE.2	
14						4.MG.4	4.CE.1	4.CE.3		4.CE.4			4.PFA.1	
15						4.MG.5								
16						4.MG.6								
17	Duration (days)	14	8	20	3	21	19	27	3	17	20	8	14	
18														
19	**Grade 5**													
20	Unit	**VA Unit 1**	**VA Unit 2**	**VA Unit 9**	**Unit 1**	**Unit 3**	**Unit 4**	**Unit 6**	**Unit 7**	**VA Unit 6**	**VA Unit 8**			
21	Topic	Data cycle	Order of operations, number system	Patterns, variables, expressions	Multiplication and division strategies	Fractions and mixed numbers	Multiplication and division algorithms	Decimals	Multiplying fractions and mixed numbers	Measurement	Geometry			
22	Standards	5.PS.1	5.NS.2	5.PFA.1	5.CE.1	5.NS.1	5.CE.1	5.CE.3	5.CE.2	5.MG.1	5.MG.2			
23		5.PS.2	5.CE.4*	5.PFA.2		5.CE.2		5.NS.1	5.CE.3		5.MG.3			
24			5.PS.3											
25			5.PFA.1											
26	Duration (days)	18	14	12	14	20	18	17	18	11	21			
27														
28	**Grade 6**													
29	Unit	**Unit 1**	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**					
30	Topic	Data cycle	Integers and inequalities	Coordinate plane and congruence	Operations with integers and equations	Operations with fraction, decimal, percent	Proportional reasoning	Exponents and applications	Statistics					
31	Standards	6.PS.1	6.NS.2	6.MG.3	6.CE.2	6.NS.1	6.PFA. 1	6.NS.3	6.PS.1					
32			6.PFA.4	6.MG.4	6.PFA.3	6.CE.1	6.PFA.2	6.MG.2	6.PS.2					
33							6.MG.1							
34	Duration (days)	10	15	20	30	30	25	20	10					
35														
36	**Grade 7**													
37	Unit	**Unit 1**	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**	**Unit 9**				
38	Topic	Data cycle	Quadrilaterals	Rational numbers	Probability	Proportional reasoning	Slope	Expressions, equations and inequalities	Surface area and volume	Proportional reasoning with geometry				
39	Standards	7.PS.2	7.MG.3	7.NS.1	7.PS.1	7.CE.2	7.PFA.1	7.PFA.2	7.PFA.2d*	7.MG.1				
40				7.NS.2				7.PFA.3	7.MG.1	7.MG.2				
41				7.NS.3				7.PFA.4		7.MG.4				
42				7.CE.1										
43				7.PFA.2										
44	Duration (days)	8	12	18	12	18	18	22	12	18				
45														
46	**Grade 8**													
47	Unit	**Unit 1**	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**	**Unit 9**				
48	Topic	Data cycle	Equivalent expressions	Consumer math	Real and irrational numbers	Geometry	Solving Inequalities	Scatterplots and data	Linear equations	Independent and dependent events				
49	Standards	8.PS.2	8.PFA.1	8.CE.1	8.NS.1	8.MG.1	8.PFA.5	8.PS.3	8.MG.3	8.PS.1				
50			8.MG.2	8.MG.1	8.NS.2	8.MG.2		8.PFA.2	8.PFA.3					
51			8.MG.5			8.MG.4								
52						8.PFA.4								
53	Duration (days)	18	18	15	8	15	14	19	24	14				

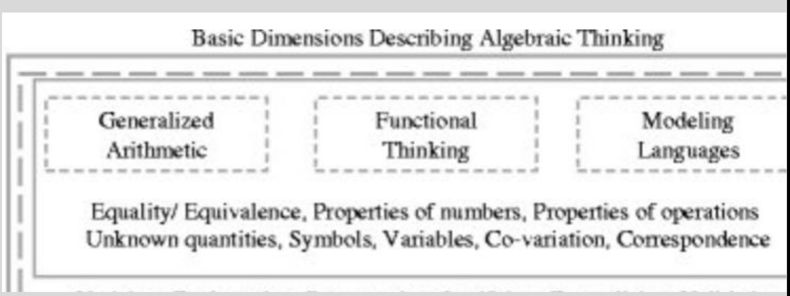
Appendix I

Sample Curriculum Framework Template

Content Area	Mathematics	Grade Level	
Course Name/Course Code			

Strand	2023 Virginia Math Standards of Learning	Code
1. Number and Number Sense		
2. Computation and Estimation		
3. Measurement and Geometry		
4. Probability and Statistics		
5. Patterns, Functions, and Algebra		

Algebraic Thinking



(From Chimoni et al., 2018)

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Titles	Length of Unit	Sequence

Unit Title		**Length of Unit**	
Understandings	*Students will understand that…* •		
Essential Questions	•		
Concepts			
Vertical Connections	Previous grade level	Following grade level	
Knowledge	*Students will know…* •	**Skills**	*Students will be able to…* •
Opportunities for Differentiation	Interest	Learning Preference	Readiness
Assessment	Summative Formative		
Rubric			
Learning Plan			
Algebraic Thinking Content Connections	**Generalized Arithmetic**	**Functional Thinking**	**Equations and Equivalence**

Algebraic Thinking Instructional Practices	**Generalized Arithmetic**	**Functional Thinking**	**Equations and Equivalence**
Learning Progression			
Learning Experiences and Instruction	Day 1	Curriculum Resources	
	Day 2	Curriculum Resources	
	Day 3	Curriculum Resources	
	Day 4	Curriculum Resources	
	Day 5	Curriculum Resources	

Appendix I

Sample Professional Learning Plan

Part 1		
District-wide mathematics professional learning goal		
The district will work towards.... *Embedding algebraic thinking content connections and associated algebraic thinking instructional practices in the curriculum framework to support students' algebra readiness.*		
Define how you will integrate attention to one or more of the Guiding Principles for School Mathematics[1]:	**Describe which of the following Effective Mathematics Teaching Practices[1] will be in the foreground of this Professional Learning Model Plan:**	
• Teaching and learning • Access and equity • Curriculum • Tools and technology • Assessment	• Establish mathematics goals to focus learning • Implement tasks that promote reasoning and problem solving • Use and connect mathematical representations • Facilitate meaningful mathematical discourse	• Pose purposeful questions • Build procedural fluency from conceptual understanding • Support productive struggle in learning mathematics • Elicit and use evidence of student thinking
This plan focuses on the integration of algebraic thinking content and teaching practices into grades three through eight mathematics curriculum.	• This PL will use representation of algebraic thinking concepts to demonstrate connections for teachers. • Teachers will also use planned for questioning and prompts to facilitate math discourse and elicit student thinking.	

Part 2				
Identify the professional learning strategies, related details, and steps you will take to implement the strategies in your school division.				
Professional learning strategies (choose from below) • **Examining student work and thinking** • **Demonstration lessons** • **Action research** • **Coaching** • **Mentoring** • **Study Groups** • **Workshops/Seminar**	**Grade(s)**	**Learning Experiences**	**Resources and supports**	**Next steps**

Strategy 1: **Workshop/Seminar**	Grades 3 – 8	**Goal:** Teachers will explore the algebraic thinking content areas and related instructional practices in relation to their grade level curriculum.	Resources: • Video samples of teaching • Algebraic thinking curriculum map	Teachers will bring curriculum documents (plans, frameworks, etc.) for next session.
Strategy 2: **Study Groups**	Grades 3 – 8	**Goal:** Teachers work together to examine the vertical articulation of concepts and skills related to algebraic thinking.	Resources: • Curriculum documents • Algebraic thinking curriculum map	Teachers will implement a lesson with algebraic thinking before the next session and collect evidence of student learning.
Strategy 3: **Examining Student Work and Thinking**	Grades 3 – 8	**Goal**: Teachers will evaluate students' work in terms of algebraic thinking content.	Resources: • Student work • Algebraic thinking curriculum map	Teachers will continue to plan and prepare for engaging in algebraic thinking with students.