Endogenous Network Formation: Theory and Applications

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Abstract

The first chapter of this dissertation describes a new model of endogenous economic network formation, and the two chapters that follow describe applications of this model. The model described in the first chapter features a finite set of economic agents who choose to form relationships with one another. As part of this model, I present a new definition of equilibrium networks. I describe the model and discuss an illustrative example. In addition to demonstrating the power of a network formation model, I present algorithms for computing equilibrium networks.

In the next chapter, I apply this model to firms choosing input suppliers and forming a production network. I analyze the effect of a single firm losing its equilibrium input supplier. I show that when one of these firms loses its input supplier, aggregate output may actually increase. Simulations of the model indicate that this increase in output is more likely when (1) the firm that loses its supplier has fewer customers prior to losing its supplier and more customers after losing its supplier, and (2) the production network as a whole is relatively less interconnected prior to the input removal and more interconnected after the input removal.

In the final chapter, I again apply the model to the formation of production networks. However, in this chapter, I use it to investigate the effect of the application of an ad valorem tariff to a single product in the production network. Simulations of this model indicate that even when the tariff is applied to only a single product, on average, prices paid by consumers increase for all goods in production. I find that consumer prices are likely to be higher after the tariff is applied when (1) the production economy is less interconnected, and (2) when the product to which the tariff is applied is used as an input by many other firms.

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Chapter 0

Introduction

This dissertation presents my research on economic networks. The relationships between economic agents and the networks they form are a powerful tool in helping us to better understand the economic world around us. In the following chapters, I describe the research I have conducted in this area. Chapter 1 describes a new model of economic network formation and the following chapters apply this model to two important economic questions.

Chapter 1 describes a model of endogenous network formation in which the links of an economic network are formed by individual economic agents choosing to form, or not to form, relationships with one another. In the model, the economic agents' utility is affected by which relationships they form. I present a new definition of an equilibrium network, a coordination-proof network. This definition considers a wider set of deviations than the definition predominantly discussed in the literature, pairwise-stable networks. I describe algorithms to compute both types of equilibrium networks. The model features the applicability of a finite set of agents, the power of an endogenously formed equilibrium network, and the tractability of computability.

Chapter 2 applies the model described in Chapter 1 to the context of firms choosing input suppliers to form a production network. I use the model to investigate the effect of a firm no longer having access to an input supplier. I find that, contrary to our intuition and existing model results, when a firm loses access to its input supplier, aggregate output may increase. This happens because the removal of access to an input supplier can lead to the creation of new network equilibria and sometimes these new equilibria produce higher output. The model of network formation allows us to understand this seemingly strange result.

Chapter 3 applies the model to the creation of a production network and uses it to investigate the effect of imposing an ad valorem tariff on a single product in the network. To my knowledge this is the first use of a network model to analyze the effects of tariffs. I use this model to understand the role that the production network structure plays in the change to consumer prices that results from levying the tariff. I find that prices will be higher if, after the tariff is applied, the production economy is relatively less interconnected but the tariff product is used as an input by relatively many firms. This chapter uses this model of network formation to understand how the structure of the production economy affects the prices that consumers face.

This research represents not only a contribution to economics, but also to network science. The new equilibrium network definition described in Chapter 1 is applicable to contexts outside of economics. Additionally, to my knowledge no algorithm previously existed for computing pairwise-stable networks. Chapter 1 outlines such an algorithm, as well as an algorithm for computing coordination-proof networks. The associated Matlab code is included in the Appendix.

Networks have been used as modeling tools in many disciplines outside of economics for some time. They are used widely in operations research to model router communication, air travel, shipping routes, and industrial security. They are used in neuroscience to model the neural pathways in the brain. They are used in biology to model gene regulatory networks. Computer science has made powerful use of network-based data structures, such as recurrent neural networks.

Despite their popularity in other disciplines, networks have only recently begun to be used in economics. The research in this dissertation contributes to a growing literature that uses network science to answer important economic questions. The time it has taken for networks to become popular in economics is surprising, particularly considering how well suited networks are to economic contexts. The relationships and links between individuals, firms, and countries play a large role in economic outcomes and can be modeled well using the nodes and links of a network. The chief barriers to using networks in economics are data and computation. The research presented here helps to break down the latter barrier directly. Furthermore, as high resolution data become available, the models presented here are well equipped to make use of these data.

The applications discussed in this dissertation focus on production networks and macroeconomic questions. The potential applications to economics of networks in general, and the model presented in Chapter 1 in particular, extend far beyond the work presented here. Networks have already been used in macroeconomics, trade, industrial organization, development economics, and many other areas. They can be used to model supply chains, information diffusion, trade relationships, and even publication networks.

The reason that networks make such good modeling tools, especially in a field such as economics, is that they couple a large degree of granularity with comprehensive conclusions. The nodes and links of networks allow us to model the *individual* firm, country, farmer in a developing country, or publication. But taken as a whole, the network can represent the universe of firms (an entire production network), the universe of countries (a global trade network), or the universe of farmers (an entire social network in a developing country). As a result, networks can capture the effect of micro level changes - removing a single input from the production network, teaching a single farmer a new sustainable technique, publishing a single new paper - on the other nodes of the network, individually and in the aggregate.

Despite the modeling power offered by networks, using them without modeling their formation removes much of the economics from economic networks. The relationships and connections that economic agents form are the result of some strategic or optimizing decision. Furthermore, these decisions can affect and be affected by every other agent's decision. To treat these decisions as fixed or exogenous fails to take advantage of the ability to model the entire network of relationships. For example, previous production network research has often held the network of input relationships as fixed. When an input is removed, the other links remain fixed, and any increase in price or decrease in efficiency propagates across these fixed links. This implicitly assumes that firms do not act to mitigate any losses they face. To allow firms to change their choices in the face of a new economic environment requires the formation of a new network.

Including this network formation aspect in models can help explain seemingly counterintuitive results. For example, in Chapter 2, I find that when a firm's input is removed from the production network, the total output produced by the network as a whole can actually increase, contrary to existing model results. The reason this happens is that, when the input is removed as an option, new input relationships become profitable that were not profitable before. The firm that lost its input chooses a new input. This changes the decisions of other firms. New choices become optimal. More efficient supply chains become possible. Without a model of network formation, it would not be possible to investigate this outcome.

Additionally, with a model of network formation, it becomes possible to ask questions that could not even be asked before. There is research that indicates that network characteristics, such as connectivity, clustering, and centrality, affect individual and aggregate economic outcomes. Using a model of network formation, it becomes possible to understand the effect of economic changes on the network structure. Now it is possible to ask not just how the network affects the economy, but rather how the economy affects the network.

The research in this dissertation contributes to the ability to use network science to better understand the economy. I look forward to contributing to this further in the future.

Chapter 1

A Theory of Endogenous Network Formation

1.1 Introduction

I present a model of endogenous network formation. The nodes of the network represent economic agents and the weighted and directed edges between them represent economic relationships between the agents. Equilibrium networks are networks for which the agents do not have utility-improving deviations. I discuss a specific definition of an equilibrium economic network that has been used in the literature previously and then I define a new equilibrium definition of economic networks. Finally, I outline algorithms to compute each of these equilibrium networks.

Networks have become a popular modeling tool in economic literature. They have been used in many different fields to address interesting questions. In international trade, trade networks have been used to study aggregate sales volatility, for example, in diGiovani, Levchenko, and Mejean (2014). Social networks have been used in the study of economic development to understand the diffusion of information. See, for example, Banerjee, Chandrasekhar, Dulfo, and Jackson (2010). In macroeconomics, networks have been used to understand the effect that the linkages between firms and sectors of the economy have on the growth of the economy, and to understand how the linkages between banks can amplify or mitigate financial shocks. See Oberfield (2013), Acemoglu et al. (2012), and Acemoglu et al. (2015), respectively. But in most economic network research until this point, these economic networks are taken as given and held fixed. For example, when networks have previously been used to understand the effect of a firm losing access to a chosen input, the method of modeling this is to take the network of inputs as given, remove the specific input, and hold the rest of the edges of the network fixed. Thus the increase in price or decrease in efficiency propagates throughout the rest of the network along these fixed edges. However, in a model such as this, the firms are confined to the inputs they chose before this change occurred. It would be reasonable to assume firms would act to mitigate their losses by switching input suppliers. As a result, new edges of the network would form and old edges would disappear. An entirely new network would emerge. Modeling the formation of the network is an important aspect of answering some economic questions.

This chapter presents a model in which individual economic agents form an equilibrium network by choosing to form relationships with each other. Specifically, an equilibrium network will be a network without any utility-improving deviations of a specific type. The type of deviations considered determines the type of equilibrium definition. A pairwise-stable network, for example, is a network with no pairwise utility-improving deviations. That is, no two agents that *could* form a relationship would be made better off by forming that relationship, holding the rest of the network fixed. This concept of equilibrium has been discussed previously in economic research. It is an attractive definition in economics because an edge exists in a pairwise-stable network only if both agents involved want the relationship to exist; it makes them both better off. This makes sense for many economic applications. For example, we would not expect firms to form input contracts, or countries to form trade relationships, unless both parties wanted to do so. At the end of this chapter, I present code to compute a pairwise-stable network. To my knowledge, this is the first algorithm to compute a pairwise-stable network.

Pairwise stability is, however, not a very strict standard; it rules out only deviations by exactly one edge of the network. It is reasonable to expect groups of agents to switch relationships jointly to make themselves jointly better off. This is not a type of deviation considered under the definition of pairwise stability. In this chapter, I define a new equilibrium network concept that does check for - and rule out - these group deviations. I call such a network a *coordination-proof* network. It is a network such that no group of agents can see an improvement in their utility by jointly switching relationships. This definition also has the attractive feature that no edge exists unless both agents want it, as under pairwise-stability. In fact, any coordination-proof network is pairwise-stable. However, it also rules out group deviations, making it a stricter definition. That is, it is a refinement of pairwise-stable equilibrium.

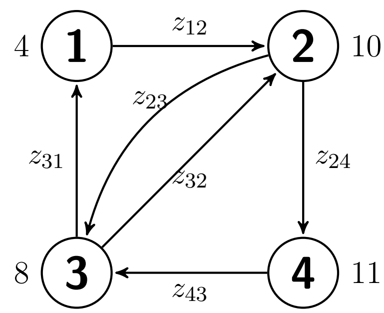
Networks are a remarkably widely-applicable modeling tool. As described above, they have been used in many different fields and contexts. Their popularity is only beginning to grow due to their high level of detail along with their powerful conclusions. The model described in this chapter can be applied to any context in which there is a finite set of agents and the utility of each agent depends on the relationships he forms. The Matlab code to find the set of pairwise-stable and coordination-proof networks for the specific context of firms choosing input suppliers under constant returns to scale production, which is used as an example throughout this chapter, is provided in the Appendix.

1.2 Economic Networks

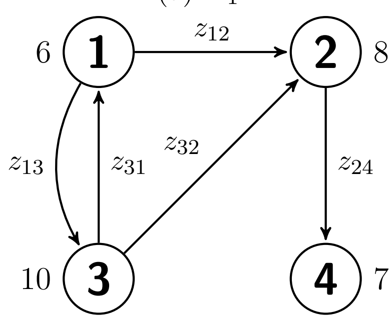
A *network* consists of a set, S, of nodes and a set, E, of directed, weighted, edges between the nodes. Specifically, the elements that compose the set E consist of ordered pairs of nodes, paired with weights: $\{(s_1, s_2), w_{s_1 s_2}\}$ where $s_1, s_2 \in S$ and $w \in \mathbb{R}$. If these nodes represent economic agents and the edges between the nodes represent economic relationships between the agents, then the network in question is an *economic network*. In such an economic context, there are different relationships available to each agent and the utility or payoffs of each agent depends on which of these relationships is in use. That is, the utility of a given economic agent in an economic network depends on which edges exist in the network. Furthermore, this utility depends not only on the edges connected to a given agent, but the edges connected to the other agents in the network as well.

Example: Production Networks

In a production network, the nodes represent firms which are choosing from among other firms' products to use as inputs in their own production process. The edges between them represent these input relationships and the weights on these edges represent productivity match values. The profits of each firm depend on their choice of inputs, as well as their inputs' choice of inputs, and so on. In Figure 1.1 each node represents a firm, and an edge from one firm to another indicates that the latter firm is buying the former firm's input to use in production. Figure 1.1 compares two different production networks, N_1 and N_2. The numbers to the side of the nodes represent profits. In N_1, Firm 3 is using two inputs - Firm 2's product and Firm 4's product - and earns a profit of \$8. In N_2, Firm 3 switches to using only one input - Firm 1's product - and earns a higher profit of \$10.



(a) N_1



(b) N_2

Figure 1.1: Production Networks

1.3 Equilibrium Networks

When the edges that exist in a network are the result of optimizing decisions of economic agents, we can define equilibrium networks. I will describe two equilibrium network concepts, one that has been discussed in the literature before and one that I developed as a refinement of the previous one.

The set of networks which could be equilibrium networks depends on the set of edges, or relationships, available to each economic agent. The *potential network* includes every edge available to each agent. This delimits all of the possible equilibrium networks.

Example: Potential Production Network

Figure 1.2 shows an example of a potential production network. Firm 2 has three options for inputs: Firm 1's product, Firm 3's product, and Firm 4's product. Firm 3, on the other hand, has two options: Firm 2's product and Firm 4's product.

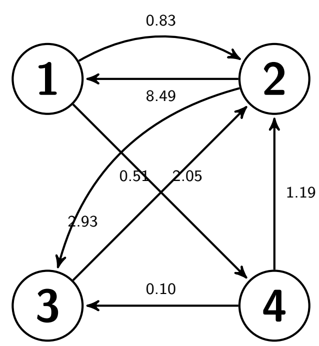


Figure 1.2: Potential Production Network

The set of feasible networks, $\mathscr{F}$, is the set of all networks that could be equilibrium networks. These are subnetworks of the potential network. Which subnetworks are feasible networks depends on the economic context and on the utility generating process. For example, if firms in a production network face a production function defined for each possible input e, with the form $y_j = \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}} z(e)x(e)^\alpha l_j^{1-\alpha}$, where α is common to all firms and l_j is the amount of labor used by firm j, then, in equilibrium, each firm will use only one input. Thus the set of feasible networks will consist of subnetworks of the potential network such that every node has exactly one edge pointing to it.

Example: Feasible Production Networks

Suppose the firms in the production network face the production function specified in the previous paragraph. As discussed, they will only choose one input in equilibrium and the set of feasible networks will be all of the subnetworks of the potential network such that every node has one edge pointing to it. Figure 1.3 enumerates all of the feasible networks for the potential production network in Figure 1.2, when the production function exhibits constant returns to scale. The edge weights are suppressed for clarity.

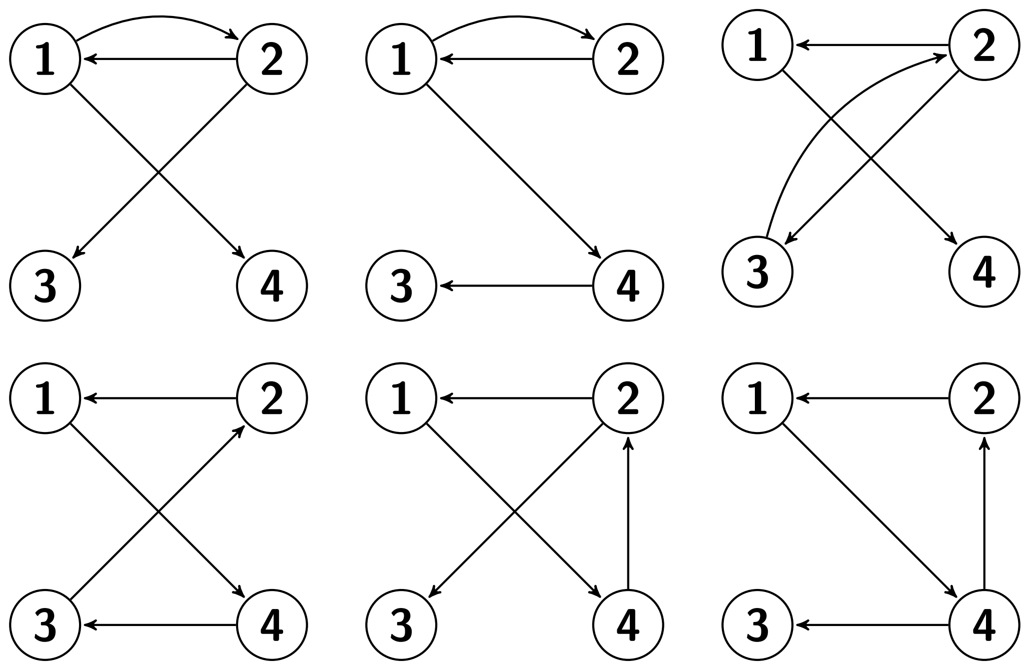


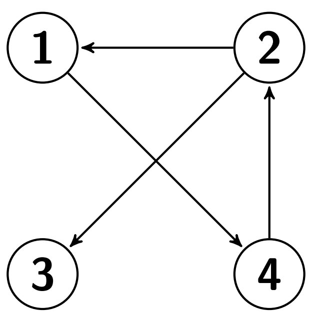
Figure 1.3: Feasible Production Networks with CRS Production Functions

Note that the potential network could be the complete network, that is, the network such that there is an edge from every node to every node. However, by specifying a potential network, this model allows for contexts in which the complete network is not available. For example, in a production network, the complete network would not be a reasonable potential network. We would not expect to see an edge from a firm that produces green beans to a firm that produces truck tires.

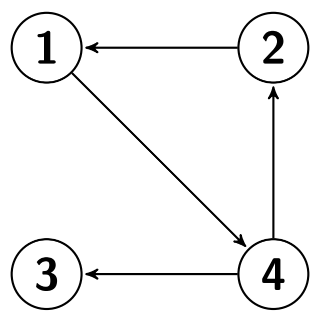
To define an equilibrium network requires well defined deviations. If there are no deviations from a given network that would be utility-improving, such a network is an equilibrium network. The deviations which must be ruled out determine the equilibrium network definition. For a given network, $N \in \mathscr{F}$, an i-adjacent network is another network, $\widetilde{N} \in \mathscr{F}$ that differs by exactly i edges. Such a network is a deviation from N. Note that the set of feasible networks is by definition closed under deviations; deviations can be made only to networks in the set of feasible networks.

Example: Adjacent Production Networks

Figure 1.4 depicts two feasible production networks, N_1 and N_2, that are 1-adjacent to each other. Firm 3 is using Firm 2 as a supplier in network N_1 and switches to using Firm 4 as a supplier in N_2.



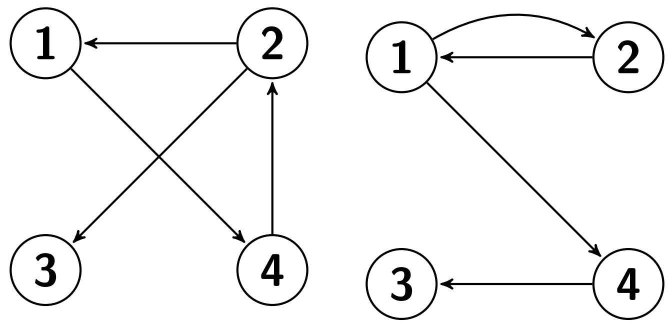
(a) N_1



(b) N_2

Figure 1.4: 1-Adjacent Production Networks

Figure 1.5 depicts two feasible production networks, N_1 and N_3, that are 2-adjacent to each other. Both Firms 2 and 3 switch suppliers.



(a) N_1 (b) N_3

Figure 1.5: 2-Adjacent Production Networks

For a given network, N, let $\widetilde{N}^{A,S_A}$ denote the $|A|$-adjacent network to N associated with the agents in set A switching from the relationships in use in N to the relationships specified by the agents in the ordered set, $S_A = \{S(f) : f \in A\}$. Using the example in Figure 1.5, $N_3 = \widetilde{N_1}^{\{2,3\},\{1,4\}}$.

The first definition of network equilibrium is that of a pairwise-stable network. This is a definition that has been used in the literature previously. See Jackson (1988) and Oberfield (2013), for example. Let $I_j = \{s \in S : \{(s,j), w_{sj}\} \in E\}$ be the set of nodes which have edges pointing to node j in the potential network. That is, I_j is the set of economic agents with which j may form a relationship. Let $\{u_j^N\}_{j \in S}$ be the set of utilities for each agent in S for a given network N. A pairwise-stable network is a network, $N^* \in \mathscr{F}$, such that no agent j, along with any potential relative $i \in I_j$, would be made better off by moving to the 1-adjacent network defined by j and i. A pairwise-stable network is any network with no utility-improving pairwise deviations. A pairwise deviation from a network N is any network that differs from N by one edge (formed by a pair of agents). To find a pairwise-stable network, all feasible networks that differ by exactly one edge must be ruled out as utility-improving. The set of deviations that must be considered and ruled out is the set of all 1-adjacent networks to N^*.

Example: Pairwise Deviations from a Feasible Production Network

Figure 1.6 depicts all of the pairwise deviations from the feasible production network, N_1.

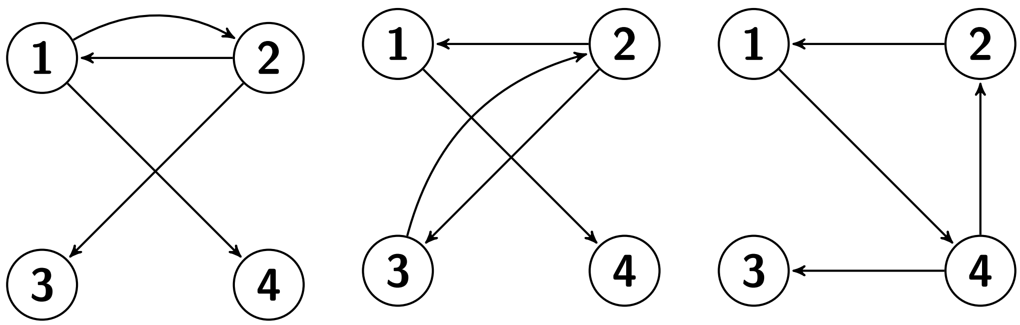
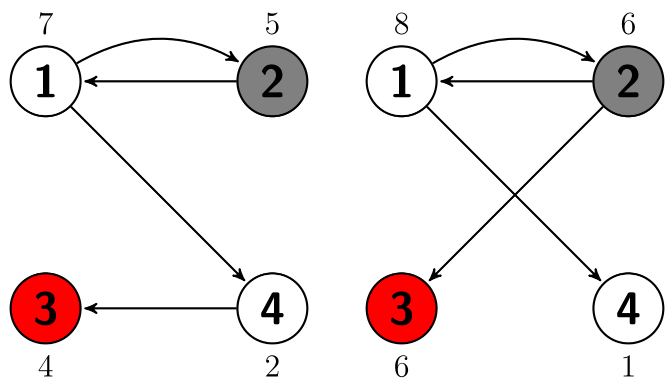


Figure 1.6: Pairwise Deviations from Production Network N_1

Any network for which there are no profitable pairwise deviations is a pairwise-stable network. For a given potential network, set of feasible networks, and associated utilities, there may be no pairwise-stable networks or multiple pairwise-stable networks. A pairwise-stable network is any network that is stable to deviations by one edge only.

Example: A Production Network that is Not Pairwise-Stable

Figure 1.7 shows a network, N_3, that is not pairwise-stable. When Firm 3 switches suppliers - from Firm 4 in N_3 to Firm 2 in N_4 - both Firm 3 and the alternative supplier, Firm 2, see higher profits. Because this profitable deviation exists, N_3 is not pairwise-stable.



(a) N_3 (b) N_4

Figure 1.7: N_3 is not Pairwise-Stable

Consider a network that is pairwise-stable. After checking every network that differs by exactly one edge, and comparing all of the appropriate utilities, there exist no pair of agents that would be made better off by switching exactly one edge. But suppose that by switching two edges, that is, if two agents change their relationships simultaneously, those two agents and their new relatives saw higher utilities. This is a reasonable deviation to consider in many contexts. However, in checking for a pairwise-stable network, this deviation is not considered. Next I describe a new definition of an equilibrium network that is stable to deviations by all possible numbers of edges.

Let C_S^m be the set of all combinations of size m of the nodes in S. E.g., if $S = \{1, 2, 3, 4\}$, then $C_S^2 = \big\{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\}\big\}$. Let $C_S = \big\{C_S^m\big\}_{m=1}^{|S|}$. For a given feasible network, $F \in \mathscr{F}$, let I_j^F be the set of edges pointing to j in F. A coordination-proof network, N^*, is a network such that no set of firms of any size - from 1 to the number of firms - can be made better off by jointly deviating to another feasible network. Formally, coordination-proof networks are $\{N^* \in \mathscr{F} : \forall C_S^m \in C_S, \forall j \in C_S^m, \forall I \in \mathscr{P}(I_j \setminus I_J^{N^*}), \neg\exists(j,k)k \in I, s.t. u_j^{\widetilde{N}_{jk}} > u_j^N \text{ and } u_k^{\widetilde{N}_{jk}} > u_k^N\}$.

That is, it is a feasible network such that for every possible combination of agents and possible relatives, no such combination would see higher utility by deviating to the associated adjacent network. A coordination-proof network is a feasible network that does not have any utility-improving group deviations, where a group can consist of any combination of nodes. To find a coordination-proof network, all feasible networks that differ by any number of edges must be ruled out as utility-improving. This requires checking networks that differ by 1 edge, 2 edges, 3 edges, and so on. The set of deviations that must be considered is the set of all i-adjacent networks for $i = 1, 2, 3, \ldots M$, where M is the maximum number of edges that can be changed.

Example: Group Deviations from a Feasible Production Network

Figure 1.8 depicts all of the group deviations for the set of firms $\{2, 3\}$ for production network N_1.

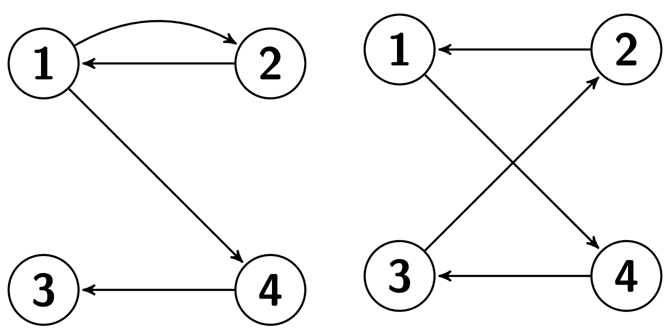


Figure 1.8: Group Deviations of $\{2, 3\}$ from N_1

As with pairwise-stable networks, for a given potential network, set of feasible networks, and associated utilities, there may be zero or multiple coordination-proof networks. Any coordination-proof network is also pairwise-stable but not necessarily vice-versa. Therefore, the number of coordination-proof networks will be less than or equal to the number of pairwise-stable networks. In the case of the production network example discussed throughout this chapter, about one third of the pairwise-stable networks are coordination-proof.

1.4 Computation of Equilibrium Networks

To compute a pairwise-stable or coordination-proof equilibrium network requires a set of feasible networks and, for each such feasible network, a list of utilities for each agent. Networks are typically represented as matrices for the purposes of computation. The edges are represented as an *adjacency matrix*, $A = [a_{ij}]$. If there is an edge from node j to node i, $a_{ij} = 1$, otherwise $a_{ij} = 0$. The edge weights are stored in an associated matrix, $W = [w_{ij}]$, where w_{ij} is the weight on the edge from j to i if there is one, and zero otherwise. See Figure 1.9 for an example.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$W = \begin{bmatrix} 0 & 8.49 & 0 & 0 \\ 0 & 0 & 2.93 & 0 \\ 0 & 2.95 & 0 & 0 \\ 0.51 & 0 & 0 & 0 \end{bmatrix}$$

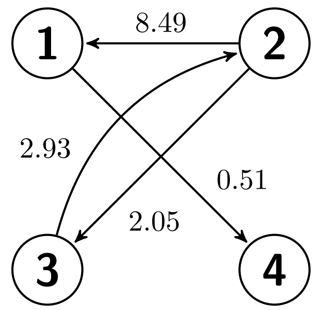


Figure 1.9: Adjacency Matrix, Weight Matrix, and Network

A potential network can be used to enumerate every feasible network. As mentioned above, what constitutes a feasible network will depend on the economic context at hand. In the example discussed throughout this chapter, the set of feasible networks is the set of all networks such that every node has exactly one edge pointing to it. The code to recursively build this set from a given potential network is included in the Appendix.

The utility generating process is also context-specific. For the purposes of computing equilibrium networks, all that is required is a set of utilities for each agent associated with each feasible network. To find the set of equilibrium networks requires checking if each feasible network meets the conditions of the particular equilibrium definition. To do this, the possible deviations from the given feasible network are enumerated. For pairwise stability, this requires enumerating all the possible alternative relationships for each agent. For coordination proof networks, this requires enumerating all the possible alternative relationships for every *combination* of agents. For each such relationship or set of relationships, the alternative feasible network which represents that deviation is constructed. This is done by copying the current feasible network's adjacency matrix and switching the appropriate edges.

Once the alternative feasible network is found, the utilities can be compared. If both the agents and their potential alternative relatives have higher utility in the alternative feasible network, the original feasible network is not an equilibrium network. That feasible network is rejected and the next one is checked. If at least one of the agents or potential alternative relatives do not have higher utility in the alternative feasible network, the next possible deviation is checked. If no deviations are utility-improving, the current feasible network is an equilibrium network. This network is added to the set of equilibrium networks and the next feasible network is checked. The result is a set of equilibrium networks that may be the empty set or may contain multiple elements.

Here I present an outline for algorithms to compute the set of pairwise-stable equilibrium networks and to compute the set of coordination-proof networks. It takes as input a potential network and returns a set of networks which satisfy the conditions of pairwise-stable and coordination-proof networks, respectively. Note that this set may be the null set or it may contain multiple elements.

1.4.1 An Algorithm for Computing Pairwise-Stable Networks

1. Initialize the set of pairwise-stable networks as empty.

2. Enumerate the set of feasible networks.

3. For each feasible network:

    (a) Enumerate the pairwise deviations.

    (b) For each deviation:

        i. Check if both firms in the pair are made better off.

        ii. If they are, stop.

iii. If they are not, check the next deviation.

(c) If any of the deviations are profitable, this feasible network is not pairwise-stable; stop.

(d) If there are no profitable deviations, this feasible network is pairwise-stable; add it to the set of pairwise-stable networks.

4. Return the set of pairwise-stable networks.

1.4.2 An Algorithm for Computing Coordination-Proof Networks

1. Initialize the set of coordination-proof networks as empty.
2. Enumerate the set of feasible networks.
3. For each feasible network:
   (a) Enumerate the group deviations.
   (b) For each deviation:
      i. Check if all of the firms in the group are made better off.
      ii. If they are, stop.
      iii. If they are not, check the next deviation.
   (c) If any of the deviations are profitable, this feasible network is not coordination-proof; stop.
   (d) If there are no profitable deviations, this feasible network is coordination-proof; add it to the set of coordination-proof networks.

4. Return the set of coordination-proof networks.

Appendix 1 contains the Matlab code for computing the set of pairwise-stable networks and coordination-proof networks for the production network context discussed in the examples throughout this chapter.

1.5 Conclusion

This chapter presents a model for the endogenous formation of equilibrium networks. As networks become a prominent modeling tool in economic studies, the ability to build these networks as a result of agents' optimizing decisions will become increasingly useful. The model presented here can apply to any context with a finite set of agents whose utility is affected by the relationships they choose to form with one another. As such, there is a great deal of opportunity for future work that makes use of this model. The following chapters include two different applications of the model described here.

Chapter 2

Application: Production Networks and Input Exclusion

2.1 Introduction

When the US automobile industry was failing, the president of Ford supported the bailout of his competitors, General Motors and Chrysler. He did this because if GM and Chrysler failed, their upstream input suppliers would fail, and Ford would no longer have access to those suppliers. (Baqaee, 2013) Recent economic literature has begun investigating how the interconnectedness of agents determine aggregate outcomes. Acemoglu et al. (2012) explore how the sector level input-output network leads to aggregate fluctuations. di Giovanni, Levchenko, and Mejean (2014) analyze what percentage of aggregate volatility can be attributed to network linkages between firms. However, most of the existing literature on economic networks takes the networks themselves as given. When faced with an economic shock, economic agents adapt. They act to mitigate their losses or to improve their outcomes. Their decisions, and thus the links of the economic network, change. As a result, for some economic questions it is necessary to model the formation of the network.

I describe a model of endogenous network formation wherein a finite set of individual economic agents choose to form relationships with one another and thereby form the links of an equilibrium economic network. This model allows me to ask and answer new questions. The endogeneity of the network formation allows for individual agents to react to economic shocks and for these reactions to determine a new network. I apply this model to the context of individual firms choosing intermediate input suppliers and thereby forming an equilibrium production network. Then, I use it to find the effect on the production network when an individual firm loses its equilibrium input supplier.

In the model, each agent chooses whether to form a relationship with a set of available other agents, thereby forming the links of an equilibrium network. I define three network allocations: a solution to the planner's problem, a pairwise-stable equilibrium, and a new refinement of the pairwise-stable equilibrium, a coordination-proof equilibrium. The planner considers all feasible networks and chooses the one that maximizes a measure of consumer welfare. The solution to the planner's problem is used to provide a baseline comparison for the efficiency of the equilibrium network definitions. The equilibrium definition used predominantly in the literature, a pairwise-stable equilibrium, is a network in which no possible pair of potentially-related firms would be made better off by deviating to a network in which that relationship is chosen. This is a restrictive definition; it does not allow for the consideration of more than one pair of agents deviating to a different potential relationship at a time. As such, I define a coordination-proof equilibrium as a network such that no *set* of potentially-related pairs can be made better off by a multi-lateral deviation to a different network. That is, considering all possible combinations of agents - of size 1, 2, up to the entire set of agents and each of their alternative relationships not in use - no set of potentially-related pairs would be made better off by switching to the network in which those links are active.

In this chapter, I describe the model in the context of firms choosing intermediate input suppliers. That is, the agents are firms and the relationships being chosen are the use of one firm's product by another firm in production. I analyze the effect of a firm losing its equilibrium input supplier on the network as a whole and on the aggregate output produced by the firms that make up the network. Note that the model is more general than this particular contextualization.

We have evidence that firm choices are driven by the production network in which they are placed. As in the case of Ford, GM, and Chrysler, the links between a firm and its suppliers, as well as the links between other firms, play a role in the choices of firms. Furthermore, firms may lose input suppliers in many ways. It may be the result of a natural disaster, as in the case of American Toyota factories, when an earthquake in Japan prevented the factories from getting necessary parts for production. It may be the result of a cyberattack, as in the case of many Ukranian businesses when the Ukranian shipping infrastructure was shut down due to a ransomware attack. It may even be policy driven. Protectionist trade policies may prevent the use of oil from Saudi Arabia or avocados from Mexico. Health and safety regulations led to the loss of asbestos as a major construction input. Finally, these changes at the firm level do affect the macroeconomy. di Giovanni, Levchenko and Mejean (2014) find that a majority of aggregate volatility is driven by changes at the firm level and that this percentage is growing over time.

I show that, contrary to results of existing models, when a firm loses its equilibrium input supplier, aggregate output may actually increase. In the solution to the planner's problem, output will always decrease when a firm loses an input supplier. However, I show that there exist parameters of the model such that when an edge is deleted from a pairwise-stable equilibrium, it is possible for output to be greater in the new pairwise-stable equilibrium. In fact, this possibility survives in the coordination-proof equilibrium.

Output increases after an edge is deleted when that edge is the only edge preventing a particularly high-output network from being an equilibrium. In most cases, when an edge is deleted, the set of new equilibrium networks is a proper subset of the previous set of equilibrium networks and thus output is lower. However, there are situations in which the edge that is deleted was the only potentially-related pair preventing a new network from being an equilibrium. When that edge is deleted, the new network becomes an equilibrium network and if that new network has a higher output than the original equilibrium network, then output will increase. Economically, this occurs when one buyer-supplier pair is being made better off at the expense of lower output in the economy as a whole. When this relationship is no longer possible, the higher-output network can be sustained as an equilibrium.

I simulate the model, and the results of this simulation suggest network characteristics which lead to output increasing when an input is removed. The level of connectivity in an economic network affects the aggregate outcomes of that network.[1] I measure the connectivity of a given network using the average distance of the shortest paths from each node to every other node. The average shortest path distance is the average distance of the shortest undirected path from each node of the network

[1] Acemoglu et al. (2015) analyzes the role of connectivity in the fragility of financial networks, for example.

to every other node. The results of the simulations indicate that the probability that output increases is higher when the production network is less connected - that is, has a longer average shortest path distance - before the input is removed and more connected after the input is removed.

In addition to investigating the role of the connectivity of the network as a whole, I also investigate the role of the connectivity of the individual firm that loses its input supplier. The results of the simulation indicate that aggregate output is more likely to increase when the firm that loses its input has more alternative suppliers from which to choose a new input. The increase in output is also more likely when the firm has relatively few customers before the input is removed and relatively many customers after the input is removed. These results taken together indicate that output is more likely to increase after an input is removed when the production economy is less interconnected before the input is removed and more interconnected after the input is removed.

2.2 Network Model

The input suppliers available to each firm define a *potential production network*. The potential network is made up of a finite set J of firms and directed edges between them. An edge pointing from firm a to firm b in the potential network indicates that firm a's output *can* be used by firm b in production. When an equilibrium network is determined, the set of firms will remain the same, but the set of edges will be a subset of the edges in the potential network. An edge from a to b in an equilibrium network will mean that firm a's output *is* used in firm b's production.

Each firm $j \in J$ produces a single good. This good can be consumed in two ways: either as an input in another firm's production as described by the potential network or as a final consumption good by a representative consumer. That is, if y_j is the amount of good j which firm j produces, then y_j is partitioned in the following manner:

$$y_j = \sum_{e \in C_j} x(e) + y_j^0$$

where C_j is the set of edges pointing away from firm j to each of firm j's customers, $x(e)$ is the amount of good j used by each such customer as an input in their own production, and y_j^0 is the amount of good j consumed as a final good by the representative consumer.

Let S_j denote the set of edges pointing to firm j; this describes the set of inputs available to firm j. Each such input, $e \in S_j$, defines a different production function:

$$y_j(e) = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z(e) x(e)^\alpha l_j^{1-\alpha}$$

where $x(e)$ is the amount of the associated input good, l_j is the amount of labor used by firm j, and $z(e)$ is an edge-specific productivity parameter. The production parameter α is the same across all firms. I allow for the possibility of firm j using multiple intermediate input goods. However because the production functions exhibit constant returns to scale, only one of the intermediate inputs will be used in equilibrium, unless two edges offer the same marginal cost which occurs with zero probability.

The representative consumer has preferences over the products produced by the firms in J according to

$$U(y_1^0, y_2^0, \ldots, y_{|J|}^0) = \Big(\sum_{j \in J} (y_j^0)^{\frac{\epsilon-1}{\epsilon}}\Big)^{\frac{\epsilon}{\epsilon-1}}$$

and she supplies L units of labor, inelastically.

2.2.1 Possible Equilibrium Networks

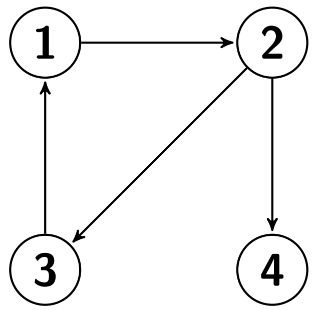
The set of networks which could be equilibrium networks is the set of all subnetworks of the potential network such that each firm has exactly one edge pointing to it. Label this set of possible equilibrium networks, henceforth referred to as PENs for simplicity, $\mathscr{N}$. The size of this set - the number of PENs - for a given potential network is determined by the number of potential input suppliers available to each firm. Let $m_j = |S_j| =$ the in-degree of j for each $j \in J$.

Theorem 1. *The number of PENs for a given potential network is given by the product of the number of suppliers available to each firm. That is,* $|\mathscr{N}| = \prod_{j \in J} m_j$.

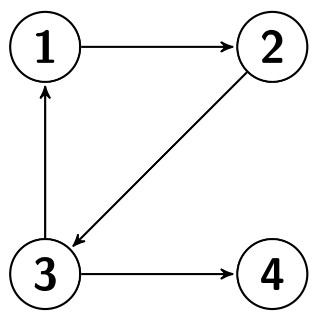
Proof. For each $N \in \mathscr{N}$, each firm has exactly one edge pointing to it. For each firm, j, the number of ways to pick one edge from the m_j edges available is m_j. The number of ways to pick one for every $j \in J$ is the number of ways to do so for the first firm, multiplied by the number of ways to do so for the second firm, multiplied by the number of ways to do so for the third firm, and so on for every firm. Therefore the number of ways to pick one edge for each firm is $m_1 \cdot m_2 \cdot m_3 \cdot \cdots \cdot m_{|J|} = \prod_{j \in J} m_j$. □

In order to define an equilibrium network from which agents do not wish to deviate, I first define how a deviation manifests. For a given network, $N \in \mathscr{N}$, an i-adjacent network is another network, $\widetilde{N} \in \mathscr{N}$ that differs by exactly i edges. See Figures 2.1 and Figure 2.2.

Figure 2.1: Network N_1 is 1-adjacent to N_2 and vice versa.

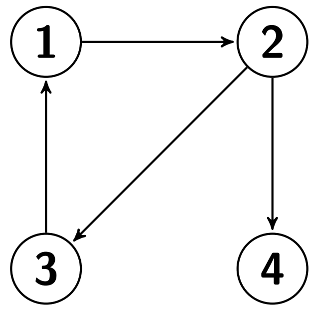


(a) N_1

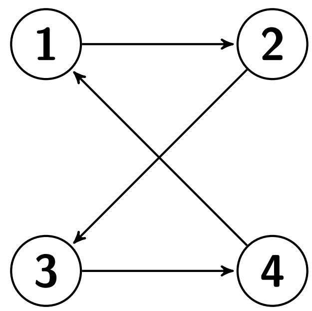


(b) N_2

Figure 2.2: Network N_3 is 2-adjacent to N_4 and vice versa.



(a) N_3



(b) N_4

In the context of the model described in this chapter, a firm switching from one supplier to another defines a 1-adjacent network, two firms switching from each of their suppliers to others defines a 2-adjacent network and so on. For a given network, N, let $\widetilde{N}^{F,S_F}$ denote the $|F|$-adjacent network to N associated with the firms in set F switching from the suppliers in use in N to the suppliers specified in the associated ordered set $S_F = \{S(f) : f \in F\}$.

2.3 The Planner's Problem

Here I describe the planner's problem both to provide a basis for comparison for other equilibrium outcomes and to build intuition for the model. The planner considers all of the PENs, and for each solves a standard consumer utility maximization problem.

$$\max_{\left\{\{y_j^0, x(e_j), l_j\}_{j\in J}\right\}_{N\in\mathscr{N}}} \Big(\sum_{j\in J}(y_j^0)^{\frac{\epsilon-1}{\epsilon}}\Big)^{\frac{\epsilon}{\epsilon-1}} \equiv Y^0$$

s.t.

$$y_j^0 + \sum_{e\in\hat{D}_j} x(e) \leq \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}} z(e_j)x(e_j)^\alpha l_j^{1-\alpha} \qquad \forall j\in J$$

$$\sum_{j\in J} l_j = L$$

Each $N \in \mathscr{N}$ defines a different edge pointing to each firm j, labeled e_j, and a different set of edges pointing away from firm j to each of its customers, labeled $\hat{D}_j$. The first constraint is the technology constraint: the consumer and the customers of firm j cannot consume more than firm j produces using the input defined by N. The second constraint is the labor constraint: all of the firms in J use only the labor supplied by the representative consumer.

Each of the PENs in $\mathscr{N}$ defines a different maximization problem across $\{y_j^0, x(e_j), l_j\}_{j\in J}$, each of which the planner solves. The planner then selects the network and choice variables corresponding to the largest Y^0. The Lagrangian for each PEN is:

$$\mathscr{L} = Y^0 + \sum_{j\in J}\lambda_j\Big[\frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}} z(e_j)x(e_j)^\alpha l_j^{1-\alpha} - y_j^0 - \sum_{e\in\hat{D}_j} x(e)\Big] + \mu\Big[L - \sum_{j\in J} l_j\Big].$$

Define the individual efficiency of each firm, $q_j \equiv \frac{\mu}{\lambda_j}$. This will be useful in characterizing individual and aggregate outcomes, as defined in the following theorems.

Theorem 2. *In the solution to the planner's problem, the efficiency of a given firm is a function of the efficiency of the input supplier of that firm. That is,* $q_j = z(e_j)q^{\alpha}_{s(e_j)}$*, where* $s(e_j)$ *is the input supplier used by* j*.*

Proof. The technology constraint gives

$$\lambda_j = \frac{1}{z(e_j)}\lambda^{\alpha}_{s(e_j)}\mu^{1-\alpha}.$$

Using the definition of q_j and then rearranging,

$$\frac{\mu}{q_j} = \frac{1}{z(e_j)}\Big(\frac{\mu}{q_{s(e_j)}}\Big)^{\alpha}\mu^{1-\alpha}$$

$$q_j = \mu\Big(z(e_j)\Big(\frac{q_{s(e_j)}}{\mu}\Big)^{\alpha}\frac{1}{\mu^{1-\alpha}}\Big)$$

$$q_j = z(e_j)q^{\alpha}_{s(e_j)}$$

□

Theorem 3. *The measure of aggregate output,* Y^0*, is a function of the efficiencies of all of the individual firms.*

$$Y^0 = \Big(\sum_{j\in J} q_j^{\epsilon-1}\Big)^{\frac{1}{\epsilon-1}} \cdot L$$

See Appendix 2 for proof.

2.4 Equilibrium Definitions

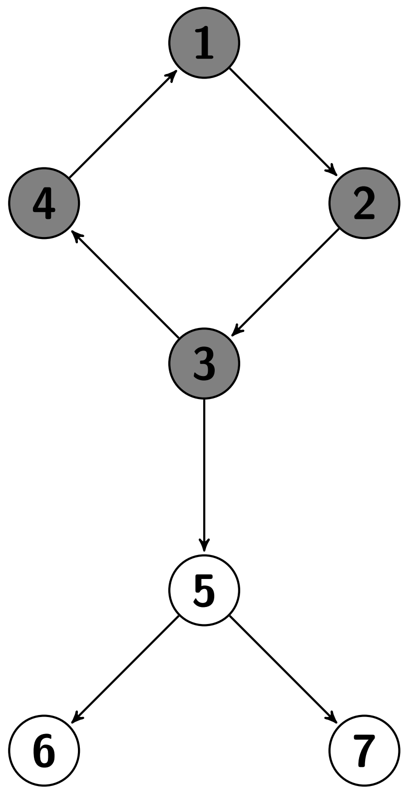
Here I define a pairwise-stable equilibrium network and a refinement of it, a coordination-proof equilibrium network. The definition of these require a list of payoffs for each firm for each PEN, $\left\{\{\pi\}_{j\in J}\right\}_{N\in\mathscr{N}}$. The optimal derivation of these will be described in the following section. A pairwise-stable network is a network, $N \in \mathscr{N}$, such that no firm j, along with any potential supplier of j, would be made better off by moving to the 1-adjacent network defined by j and the potential supplier. Formally, it is a network $\left\{N \in \mathscr{N} : \forall j, \forall k \in S_j \setminus \{i_j^N\}, \neg\exists(j,k) \text{ s.t. } \pi_j^{\widetilde{N}^{j,k}} > \pi_j^N \text{ and } \pi_k^{\widetilde{N}^{j,k}} > \pi_k^N\right\}$, where i_j^N is the supplier of firm j in N. Note that this restricts the potential firm deviations considered. It does not allow for the individual firms to consider the possibility of the other firms simultaneously deviating in their decision. A pairwise-stable equilibrium network merely needs to be better than a 1-adjacent network for one pair of firms at a time. Next I define an equilibrium network that needs to be better than i-adjacent network for i firms at a time.

A coordination-proof equilibrium network is a network, $N \in \mathscr{N}$, such that not only would no *one* pair of firm and alternate supplier be made better off by moving to the 1-adjacent network defined by the pair, but no two pairs would be made better off, no three pairs, and so on up to the number of firms. See Section 1.3 of Chapter 1 for a formal definition of a coordination-proof network.

2.5 Prices and Profit Maximization

When exactly one edge is pointing to each firm, there are only two possible network shapes that can make up each connected component of the entire network. These are cycles and branches. A cycle is a set of nodes such that the in-degree and out-degree of each node is exactly one. A branch is a set of nodes such that the in-degree of each node is one but the out-degree is unrestricted. See Figure 2.3.

Figure 2.3: The gray nodes form a cycle; the white nodes form a branch.



Any connected component must contain exactly one cycle and any branch in that connected component must have its root on the cycle.[2] These two shapes are critical in calculating the prices.

Firms set prices for both the portion of their output consumed by the representative consumer and the portion consumed by each of their network customers - the

[2]A brief outline of the proof by contradiction: If there was no cycle, then there would need to be a node with no supplier. If there was more than one cycle, then there would exist some node with more than one supplier.

other firms which use their good as an input. Label the price of y_j^0 as p_j^0. For each network customer of firm j, j charges a two-part tariff. That is, j sets $\{p(e), \tau(e)\}_{e \in \hat{D}_j}$, for each $\hat{D}_j$ defined by each $N \in \mathscr{N}$. Here $\tau(e)$ is a fixed fee and $p(e)$ is a price per unit of product j.

Theorem 4. *The per-unit price, $p(e)$, that firm j charges the customer to which edge e points is firm j's marginal cost of production.*

See Appendix 4 for proof. As a result of this, the per-unit price firm j charges is a function of the marginal cost of the input supplier used by firm j, $p(e) = MC_j = \frac{1}{z(e_j)} MC_{s(e_j)} w^{1-\alpha}$, where w is the price of labor to all firms. Because the price charged by each firm can be written in terms of the supplier's marginal cost, all of these prices can be calculated using only the network structure and $z(e_j)$'s. The price charged by any firm on a cycle can be traced back through each supplier until it is expressed in terms of itself, thus there is a closed form solution for any price on a cycle. The price charged by any firm on a branch can be traced up to the root node of the cycle, which must be on a cycle, thus any such price can be calculated. See Appendix 4 for formal derivation.

In equilibrium, the fixed fee, $\tau(e)$, is the surplus between the demand for product j and the marginal cost of producing j. For a given demand curve, when firm j buys a more expensive input, the marginal cost increases and, therefore, decreases the fixed fee.

The profit maximization problem each firm j solves is

$$\max_{p_j^0, y_j^0, x(e_j), l_j} p_j^0 y_j^0 + \sum_{e \in \hat{D}_j} [p(e)x(e) + \tau(e)] - [p(e_j)x(e_j) + \tau(e_j)] - w l_j$$

s.t.

$$y_j^0 + \sum_{e \in \hat{D}_j} x(e) \leq \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z(e_j) x(e_j)^\alpha l_j^{1-\alpha}$$

and all firms are jointly subject to the labor constraint, $\sum_{j \in J} l_j = L$.

Each $N \in \mathscr{N}$ defines a different profit maximization problem for each firm and the solutions to these problems produce a set of payoffs for each firm for each PEN. These payoffs are what determine the pairwise-stable and the coordination-proof equilibria, as defined in the previous section.

Just as the efficiency of an individual firm in the planner's problem is defined as the ratio of the two shadow costs μ and λ_j, define the efficiency of the individual firms in this case as, $\widetilde{q}_j \equiv \frac{w}{MC_j}$. As in the case of the planner's problem, this can be written in terms of the efficiency of firm j's input supplier.

Theorem 5. *The efficiency of a given firm is a function of the efficiency of the input supplier of that firm,* $\widetilde{q}_j = z(e_j)\widetilde{q}^\alpha_{s(e_j)}$*, where* $s(e_j)$ *is the identity of the input supplier used by* j*.*

Proof. Cost minimization and the result above,

$$MC_j = \frac{1}{z(e_j)} MC^\alpha_{s(e_j)} w^{1-\alpha}.$$

The definition of $\widetilde{q}_j$ gives,

$$\frac{w}{\widetilde{q}_j} = \frac{1}{z(e_j)} \Big(\frac{w}{\widetilde{q}_{s(e_j)}}\Big)^\alpha w^{1-\alpha}$$

$$\widetilde{q}_j = z(e_j)\widetilde{q}_{s(e_j)}.$$

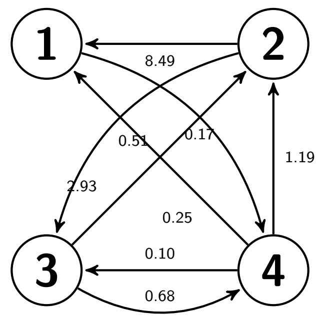
□

2.6 Input Removal

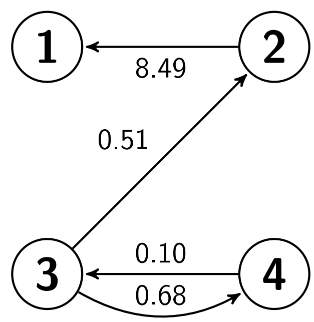
I compare the aggregate output generated by an equilibrium network before and after an edge of the network is removed. Let e^* be the deleted edge and j^* be the firm to which e^* points. That is, j^* uses e^* in its production. Deleting this edge creates a new potential network, and thus a new set of PENs. This new set of PENs is a proper subset of the original set. The new equilibrium network is determined from among this new set of PENs.

An intuitive analysis of the result from this edge deletion may suggest a path-dependent cascade of the drop in j^*'s efficiency along all of the edges emanating from j^* and its customers, holding the other equilibrium network links fixed. However, this is not necessarily an equilibrium. See Figure 2.4 for an example.

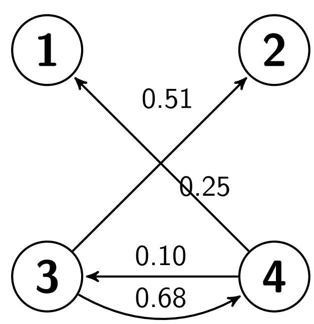
Figure 2.4: Deleting an edge and holding the other edges fixed is not neccessarily an equilibrium



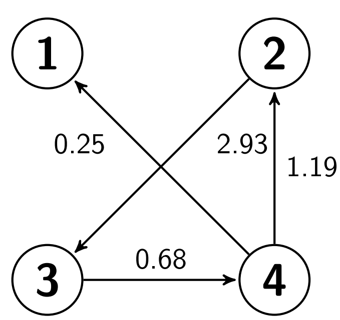
(a) Potential Network



(b) Original Equilibrium Network



(c) Holding other edges fixed



(d) The equilibrium network when the edge is deleted

Figure 2.4(a) shows the potential network and Figure 2.4(b) shows the original coordination-proof equilibrium. Note that this is also then a pairwise-stable equilibrium. If the edge from firm 2 to firm 1 is deleted and the other edges are held fixed, while firm 1 chooses the lowest marginal cost supplier available to it - firm 4 - then the network will be as shown in Figure 2.4(c). However, this network is neither pairwise-stable nor coordination-proof. The coordination-proof equilibrium that results from deleting the edge from firm 2 to firm 1 is shown in Figure 2.4(d).

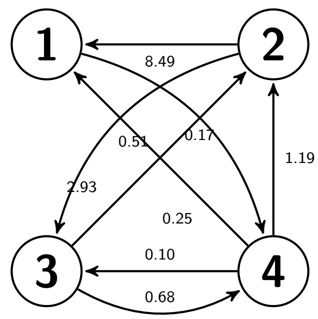
While in the case of a solution to the planner's problem, the new output will be lower than the original output, this is not necessarily true in the case of a new pairwise-stable equilibrium, and, in fact, this result survives the equilibrium refinement of the coordination-proof equilibrium.

Result 1. *There exist parameters of the model such that the output produced by a pairwise-stable equilibrium or by a coordination-proof equilibrium increases when an edge is deleted and a new equilibrium of the same type is determined.*

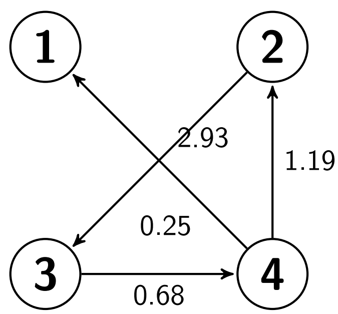
This will occur when the edge that is deleted is the only edge preventing a higher-output network from being pairwise stable or coordination proof. If, when an edge is deleted the set of new equilibrium networks is a proper subset of the original set of equilibrium networks, then output will not increase.[3] However, this need not be the case. There are situations when deleting an edge makes it possible for a new network to be pairwise stable or coordination proof. If the output produced by such a new network is higher than the output in the original network, then output will increase. See Figure 2.5 for an example.

[3]It will be the same output with a probability of zero

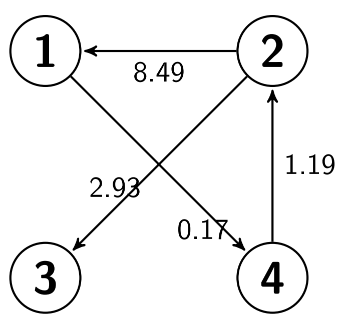
Figure 2.5: Output increases when the edge from firm 4 to firm 3 is deleted.



(a) Potential Network



(b) Original Equilibrium Network, Output = 0.1430



(c) New Equilibrium, Output= 0.1904

There are three coordination-proof networks corresponding to the potential network shown in Figure 2.5(a). Of those, the one that offers the highest output, 0.1430, is depicted in Figure 2.5(b). When the edge from firm 4 to firm 1 is deleted, the new set of coordination-proof equilibria consists of five networks. From those, the highest possible output is now 0.1904. The network that produces this output is depicted in

Figure 2.5(c).

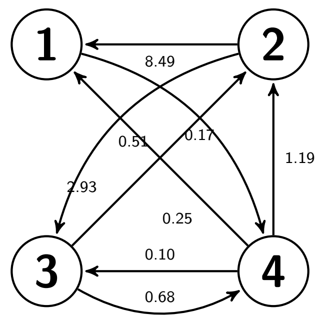
2.6.1 Network Connectivity and Firm Centrality

I restrict my focus to the situation in which output falls when an edge is deleted. These results hold in the case that output rises, as well. The connectivity of the equilibrium network as a whole plays a role in how far aggregate output falls after an edge is deleted. I use the average shortest path distance to measure connectivity. This measures the average number of undirected links it takes to get from one firm to any other firm. The longer the average shortest path distance, the higher the connectivity of the network. The intuitive relationship may be that the more connected an equilibrium production network is, the harder each firm will be hit when j^* loses its input supplier and has to choose a different one, and this aggregate output will drop by more when a network is more connected. While the simulations described in the next section show that on average this is the case, the opposite may also occur.

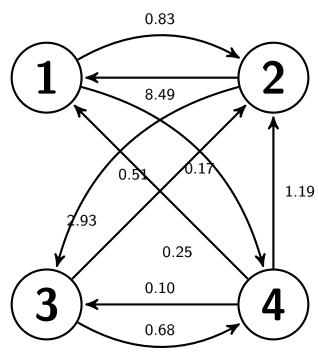
Result 2. *There exist parameters of the model such that higher connectivity in an original equilibrium network can lead to a smaller decrease in aggregate output.*

See Figures 2.6 and 2.7 for an example.

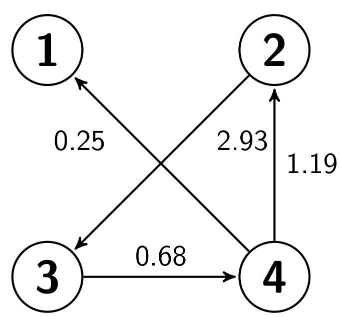
Figure 2.6: Output decreases by more when firm 2 has more alternative suppliers.



(a) Potential Network #1, Firm 2 has two available suppliers.

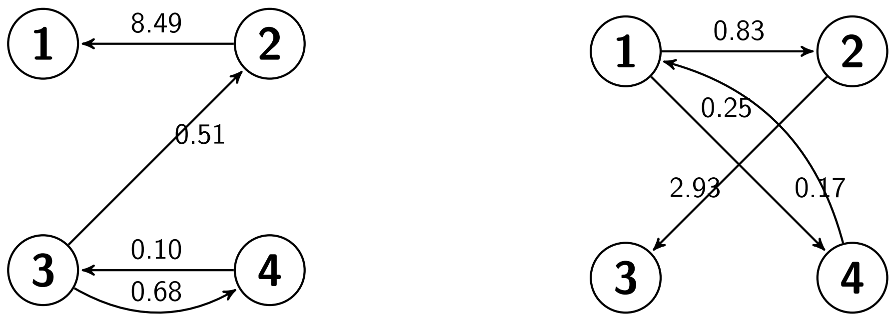


(b) Potential Network #2, Firm 2 has three available suppliers.



(c) Original Equilibrium Network, Output = 0.1430

Figure 2.7: Output decreases by more when firm 2 has more alternative suppliers.



(a) New Equilibrium for Potential Network #1, Output= 0.0371

(b) New Equilibrium for Potential Network #2, Output= 0.0356

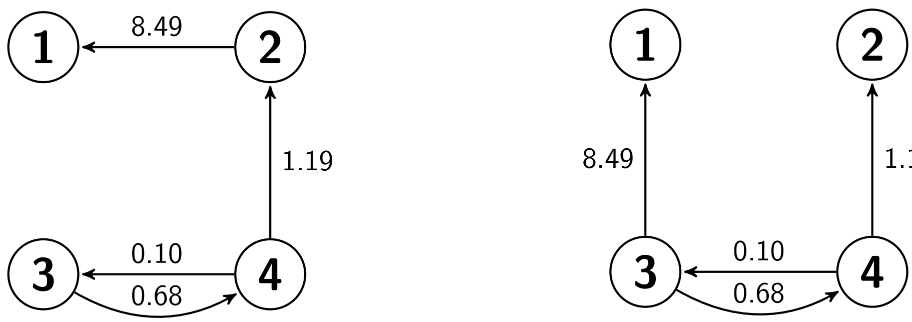
In the first potential network, shown in Figure 2.6(a), firm 2 has two available suppliers and in the second potential network, Figure 2.6(b), firm 2 has three available suppliers. Figure 2.6(c) shows the coordination-proof equilibrium they have in common, in which output is 0.1430. When the edge from firm 4 to firm 2 is deleted from both of them, the resulting new coordination-proof equilibria are shown in Figure 2.7(a) and Figure 2.7(b), respectively. The output in the former is 0.0371, while the output in the latter is 0.0356.

Just as the connectivity of the network as a whole affects the drop in output, the level of connectedness, or centrality, of j^* in the potential network, as measured by the number of alternative suppliers available to j^*, plays a role as well. As one might expect, the average outcome is that, if j^* has more alternative suppliers to choose from when it loses its equilibrium input supplier, the drop in aggregate output will be smaller than if j^* had fewer alternative suppliers. However, the opposite effect is possible.

Result 3. *There exist parameters of the model such that a higher number of alternative suppliers for j^* can lead to a larger drop in aggregate output when j^* loses access* *to its supplier.*

See Figures 2.8 and 2.9 for an example.

Figure 2.8: Output decreases by less for the more connected network.



(a) Original Equilibrium #1, Output= 0.0452, Avg. Path Distance= 1.6875

(b) Original Equilibrium #2, Output= 0.0445, Avg. Path Distance= 1.5

Figure 2.9: Output decreases by less for the more connected network.

(a) New Equilibrium #1, Output= 0.0371

(b) New Equilibrium #2, Output= 0.1854

The network in Figure 2.8(a) is less connected than the network in Figure 2.8(b); however when the edge from firm 4 to firm 2 is deleted from both of them, the output in the less connected network decreases more.

2.7 Simulation Results

I simulate this model by generating potential production networks and then finding a solution to the planner's problem, all of the pairwise-stable equilibrium networks, and all of the coordination-proof equilibrium networks. I create the potential network by drawing the number of possible input suppliers for each firm from a Poisson(3) distribution. The identity of each supplier is drawn uniformly with replacement from the other firms. The productivity parameter, $z(e)$, for each edge e is drawn from a Pareto$(0.2, -1.8)$ distribution. This parameterization is motivated by the Carvalho (2012) survey on Input-Output analysis. These parameterizations of these distributions are estimated to correspond with the network characteristics observed in current Input-Output networks, including out-degree (number of customers) and path distance. Note that drawing the supplier identities with replacement allows for multiple edges from a given firm. However, because the productivity parameters are realizations of continuous random variables, the edge with the higher z will always be chosen.[4] These simulations consisted of the creation of $1,000$ potential networks, in 554 of which a solution to all three problems was found.

2.7.1 Removal of an Input Supplier

For each successful solution to the planner's problem, pairwise-stable equilibrium, and coordination-proof equilibrium determination, I delete each edge in use in equilibrium. I do this by removing that edge from the potential network and then find a new solution of each type. In $2,691$ of these edge deletion experiments, a new solution to all three allocations is found. I take the ratio of output after the edge is deleted

[4]Ties occur with a probability of zero.

to output before the edge is deleted. Label this relative output for each allocation: planner, pairwise-stable, and coordination-proof.

I measure the connectivity of each original equilibrium using the average shortest path distance. I do this by calculating the length of the undirected path from each node to every other node and taking the average across all such paths. A larger average shortest path distance corresponds to a less connected network. For each edge deletion, i, I regress the relative output of each allocation on the average shortest path distance of the corresponding original equilibrium network. That is, I estimate the following regression equation.[5]

$$\widehat{\text{relative output}}^{E} = \beta(\text{avg. shortest path distance})$$

for each $E \in$ {Planner's Solution, Pairwise-Stable, Coordination-Proof} using ordinary least squares. The results are reported in Table 1.

Table 1: Output and Connectivity

E	$\hat{\beta}$
Planner's Solution	0.3907
Pairwise-Stable	0.2812
Coordination-Proof	0.3803

Each of the three estimated regression coefficients is positive, indicating that a larger average shortest path distance is correlated with a larger relative output. This means

[5]The constant is omitted because an average shortest path distance of zero indicates that there is only one firm. Without an intermediate input to use, this firm produces 0.

that more connected equilibrium networks are correlated with larger drops in output after an edge is deleted, and this result holds over all three allocations.

Label the number of input suppliers available to j^* in the potential network as $\#\text{sup}_i$ and the number of customers j^* has in the original equilibrium network $\#\text{cust}_i$. The observation is one edge-deletion experiment and each such edge-deletion experiment defines a j^*. For each equilibrium type,
$E \in \{$Planner's Solution, Pairwise-Stable,Coordination-Proof$\}$, I estimate

$$\widehat{\text{relative output}}^{E} = \gamma_1(\#\text{sup}) + \gamma_2(\#\text{cust})$$

using ordinary least squares.[6] The results are reported in Table 2.

Table 2: Output and Centrality

E	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Planner's Solution	0.2615	0.0325
Pairwise-Stable	0.2014	-0.0175
Coordination-Proof	0.2518	0.0178

The estimated regression coefficients on the number of available suppliers are positive for all three equilibrium definitions. This indicates that a larger number of available input suppliers is correlated with a smaller drop in output after an input is deleted.

The estimated regression coefficients on the number of customers in the original equilibrium network are all positive except in the pairwise stable case. These positive

[6]The constant is omitted because a firm with no suppliers and no customers would indicate that there is only one firm. With no intermediate input, this firm produces 0.

coefficients indicate that the more customers j^* has when it loses its input supplier, the higher output will be afterwards. However, in the case of the pairwise-stable and coordination-proof equilibria, the confidence intervals of $\hat{\gamma}_2$ include zero, and this was true for every size of simulation. While it may be the case that in the planner's problem a larger number of customers is correlated with a smaller drop in aggregate output, this cannot be concluded for pairwise-stable nor for coordination-proof networks.

2.7.2 Increased Output

I investigate the role that network connectivity and firm centrality play in the probability that output increases after an input is removed from the production network. In doing this, I restrict my focus to the coordination-proof edge deletion experiments because any coordination-proof equilibrium is also pairwise stable. To understand the role of network connectivity, I consider the average shortest path distance of the potential network, the original equilibrium production network, and the new equilibrium production network that results after the input is removed. I also include characteristics of j^*. Specifically, I consider the number of suppliers from which j^* can choose a new input, the number of firms that buy j^*'s product in the original equilibrium production network, and the number of firms that buy j^*'s product in the new equilibrium production network. These six characteristics are the explanatory variables in a binary Logistic regression for which the dependent variable is the probability that relative output is greater that one; that is, that output increased after the edge was removed. The marginal effects of the explanatory variables in this regression are reported in Table 3.

Table 3: Logit Regression Marginal Effects

Characteristic	ME
Potential Network Average Shortest Path Distance	−0.0280
Original Equilibrium Average Shortest Path Distance	0.0671
New Equilibrium Average Shortest Path Distance	−0.0260
Number of Possible Suppliers	0.0073
Original Number of Customers	−0.0317
New Number of Customers	0.0253

The results regarding the connectivity of the production network indicate the following. First, the more connected the potential network is, the higher the likelihood that output will increase. A one-link increase in the average shortest path distance of the potential network is associated with a 2.8 percentage point decrease in the likelihood that output increases. Second, the less connected the original equilibrium production network and the more connected the new equilibrium production network, the higher the likelihood that output will increase. A one-link increase in the average shortest path distance of the original equilibrium is associated with a 6.71 percentage point increase in the likelihood, while a one link increase in the average shortest path distance of the new equilibrium is associated with a 2.6 percentage point decrease in the likelihood.

The firm characteristic results indicate the following. First, the more suppliers available to j^*, the higher the likelihood that output will increase. One more supplier is associated with a 0.73 percentage point increase in the probability that output increases. Second, the fewer firms buying from j^* in the original equilibrium production network and the more firms buying from j^* in the new equilibrium production network, the higher the likelihood that output increases. One more customer in the original equilibrium is associated with a 3.17 percentage point decrease in the probability that output will increase, while one more customer in the new equilibrium is associated with a 2.53 percentage point increase in the probability.

2.8 Multiple Inputs: A Case Study

Consider a production function that specifies two inputs for firm j, e_j^1 and e_j^2,

$$y_j = z(e_j^1)z(e_j^2)\Big[\big(\frac{x(e_j^1)}{\gamma}\big)^{\gamma}\big(\frac{x(e_j^2)}{1-\gamma}\big)^{1-\gamma}\Big]^{\alpha} l_j^{1-\alpha}.$$

The set of feasible networks expands to include not just networks in which every firm has one input but networks in which every firm has either one or two inputs. If in a given feasible network, firm j has one input, it solves the profit maximization problem described previously in this chapter. If instead, a firm has two inputs, it solves the following profit maximization problem.

$$\max_{p_j^0, y_j^0, x(e_j^1), x(e_j^2), l_j} p_j^0 y_j^0 + \sum_{e \in \hat{D}_j} [p(e)x(e) + \tau(e)] - [p(e_j^1)x(e_j^1) + \tau(e_j^1)] - [p(e_j^2)x(e_j^2) + \tau(e_j^2)] - w l_j$$

s.t.

$$y_j^0 + \sum_{e \in \hat{D}_j} x(e) \leq z(e_j^1)z(e_j^2)\Big[\big(\frac{x(e_j^1)}{\gamma}\big)^{\gamma}\big(\frac{x(e_j^2)}{1-\gamma}\big)^{1-\gamma}\Big]^{\alpha} l_j^{1-\alpha}$$

I run an edge deletion experiment on a specific potential network as a case study in the robustness of this model with respect to the assumption of constant returns to scale. I find initial equilibrium networks for the two different production functions, and then compare the outcomes when edges are deleted. The potential network is depicted below, in Figure 2.10, with the edge weights specified in matrix W, below.

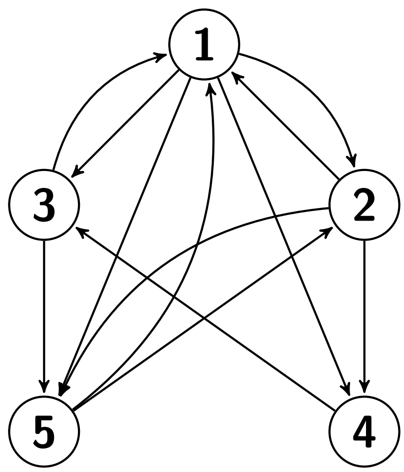


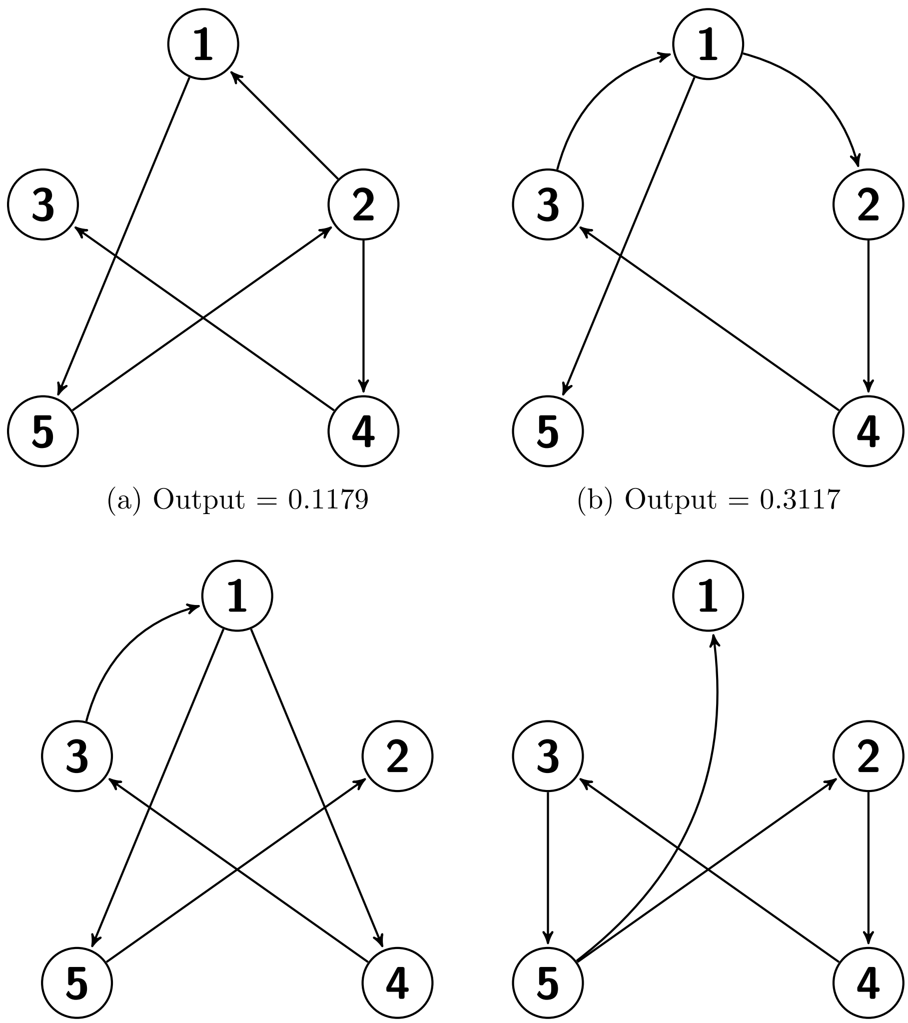
Figure 2.10: Potential Network

$$W = \begin{bmatrix} 0 & 0.16 & 2.84 & 0 & 0.08 \\ 0.33 & 0 & 0 & 0 & 0.98 \\ 0.21 & 0 & 0 & 17.31 & 0 \\ 0.10 & 1.26 & 0 & 0 & 0 \\ 5.03 & 0.25 & 0.07 & 0 & 0 \end{bmatrix}$$

Firms 1 and 5 have three available suppliers while the rest of the firms have two available suppliers. There are $3 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 72$ feasible networks in which every firm has only one input supplier. There are $[3 + \binom{3}{2}] \cdot [2 + \binom{2}{2}] \cdot [2 + \binom{2}{2}] \cdot [2 + \binom{2}{2}] \cdot [3 + \binom{3}{2}] = 972$ feasible networks in which firms have either one or two input suppliers.

2.8.1 Initial Equilibrium

When firms are restricted to one input, there are four coordination-proof networks, shown in Figure 2.11. These four constitute 5.6% of the 72 feasible networks. The mean output across these is 0.1586. The network with the highest output, 0.3117, is depicted in Figure 2.11(b).



(a) Output = 0.1179

(b) Output = 0.3117

(c) Output = 0.1374

(d) Output = 0.0680

Figure 2.11: Coordination-Proof Networks for the 1 - Input Model

When firms can have either one or two inputs, there are 53 coordination-proof networks, constituting 5.5% of the 972 feasible networks. This percentage is very similar to the one-input case. Furthermore, the network with the highest output in the multiple-input case is the same network as in the one-input case, the network shown in Figure 2.11(b), with an output of 0.3117. The mean output across the 53 coordination-proof networks is 0.0895, lower than the mean output in the one-input case.

I delete two edges from this equilibrium - using both the one-input model and the two-input model - and compare the outcomes. I delete Firm 1 and Firm 5's input suppliers, one at a time. Both Firm 1 and Firm 5 have two more suppliers to choose from, but they differ in that Firm 1 has two network customers in the original equilibrium and Firm 5 has none.

2.8.2 The Removal of Firm 1's Input

First, I delete the edge from Firm 3 to Firm 1 from the original potential network and find the coordination-proof networks under both the one-input assumption and allowing for two inputs. When only one input is possible, the new set of coordination-proof networks consists of only two networks, both of which are in the original set of one-input, coordination-proof networks. They are the networks depicted in Figure 2.11(a) and (d). No new one-input equilibrium networks are created by deleting this edge. The mean output across the two fell to 0.0930. The highest output given by either of these is 0.1179. The network depicted in Figure 2.11(a) produces this output. In this case, when an edge is deleted, the resulting equilibrium network is the same whether firms are restricted to one input or whether they can have two inputs.

When two inputs are possible, the new set of equilibrium networks consists of nine networks, more than one of which has firms using two inputs. The mean across these fell to 0.0384. However, the network that produces the highest output in this case is the same network as in the one-input case, the network depicted in Figure 2.11(a), with an output of 0.1179.

2.8.3 The Removal of Firm 5's Input

Next, I delete the edge from Firm 1 to Firm 5 from the original potential network and find the coordination-proof networks for the one-input case and two-input case. When only one input is possible, there are three coordination proof networks. The mean across these is 0.0543. Two new coordination-proof networks are created when this edge is removed, but the network that produces the highest output is one of the four original coordination-proof networks, depicted in Figure 2.11(d). This network produces an output of 0.0680.

When two inputs are possible, the new set of coordination-proof networks consists of 41 networks. The mean output across these is 0.0383, which is very similar to the mean output when the previous edge is deleted and the two-input equilibrium is found, 0.0384. The maximum output created by any of these 41 networks is 0.1349. Note that this is a smaller decrease from the initial equilibrium than when firms are restricted to one input. The network that produces this output is depicted in Figure 2.12.

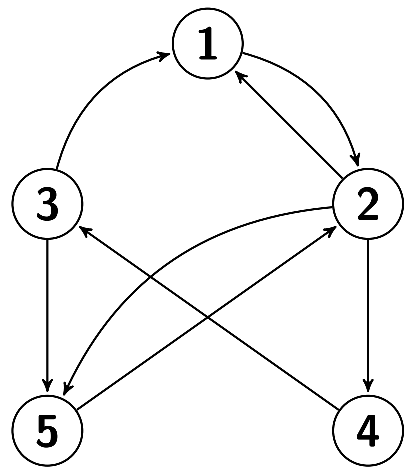


Figure 2.12: Highest Output Network, 2nd Edge Deletion, Two Inputs Possible

While further research is needed to understand the distributional effects of allowing for more than one input, this case study illuminates several facts. First, even when multiple inputs are allowed, the maximal output network may still have every firm using only one input. Second, despite dramatically increasing the number of feasible networks, the percentage of feasible networks that are coordination-proof does not necessarily dramatically change. Finally, allowing for two inputs may lead to a smaller drop in output when an edge is removed.

2.9 Conclusion

The key contributions of this chapter are a new network model which features a finite number of firms and endogenous network determination, a refinement of the standard network equilibrium, and a better understanding of the role network connectivity and firm centrality play in the determination of aggregate outcomes. I apply the model to investigate the effect of a firm losing an input supplier in the production network and find that when this happens, the resulting aggregate output can be higher. Simulation results indicate the following. First, on average, the more connected a production network is, the smaller the decrease in aggregate output will be when a firm loses an input supplier. Second, on average, the more alternative suppliers that firm has, the smaller the drop in output will be when that firm loses its input supplier. Finally, it is more likely that output will increase when (1) the firm that loses its supplier goes from having fewer customers before it lost its supplier to having many customers afterwards and (2) the network as a whole goes from less connected before this input is removed to more connected afterwards. A case study on the robustness of this model to the assumption of constant returns to scale indicates that, were multiple inputs an option, the network that produces the most output may still have all firms using only one input.

Chapter 3

Application: Production Networks and Tariffs

3.1 Introduction

On March 8, 2018, US President Donald Trump announced a 25% tariff on imported steel and a 10% tariff on imported aluminum. (Horsley, 2018) The effect of these tariffs on the production economy and on the prices consumers pay are topics of heated political debate. In this chapter, I investigate the effect of ad valorem tariffs on the firm-to-firm production network. Recent economic literature has begun investigating how the interconnectedness of agents determines aggregate outcomes. Acemoglu et al. (2012) explore how the sector level input-output network leads to aggregate fluctuations. di Giovanni, Levchenko, and Mejean (2014) analyze what percentage of aggregate volatility can be attributed to network linkages between firms. However, most of the existing literature on economic networks takes the networks themselves as given. When faced with an economic shock, economic agents adapt. They act to mitigate their losses or to improve their outcomes. Their decisions, and thus the links of the economic network, change. As a result, for some economic questions it is necessary to model the formation of the network.

I present a model of endogenous network formation wherein a finite set of individual economic agents choose to form relationships with one another and thereby form the links of an equilibrium economic network. This model allows me to ask and answer new questions. The endogeneity of the network formation allows for individual agents to react to economic shocks and for these reactions to determine a new network. I apply this model to the context of individual firms choosing intermediate input suppliers and thereby forming an equilibrium production network. Then, I use it to find the effect on the production network when a tariff is levied on a particular product.

In the model, each agent chooses whether to form a relationship with a set of available other agents, thereby forming the links of an equilibrium network. I define two network allocations: a pairwise-stable equilibrium and a new refinement of the pairwise-stable equilibrium, a coordination-proof equilibrium. The equilibrium definition predominantly used in the literature, a pairwise-stable equilibrium network, is a network in which no possible pair of potentially-related firms would be made better off by deviating to a network in which that relationship is chosen. This is a restrictive definition; it does not allow for the consideration of more than one agent deviating to a different potential relationship at a time. As such, I define a coordination-proof equilibrium as a network such that no *set* of potentially-related pairs can be made better off by a multi-lateral deviation to a different network. That is, considering all possible combinations of agents - of size 1, 2, up to the entire set of agents and each of their alternative relationships not in use - no set of potentially-related pairs would be made better off by switching to the network in which those links are active.

As in the previous chapter, I describe the model in the context of firms choosing intermediate input suppliers. Here, I use the model to analyze the effect of an ad valorem tariff levied on a particular product produced by a firm in the production network. I compare the prices paid by consumers before and after the tariff is applied.

We have evidence that firm choices are driven by the production network in which they are placed. For example, when the US automobile industry was failing, the president of Ford supported the bailout of his direct competitors, General Motors and Chrysler. He did this because if GM and Chrysler failed, their upstream suppliers would fail, and Ford would no longer have access to these suppliers. (Baqaee, 2013) Tariffs effect the choices of the firms in the production network and are a common component in international trade policy. In addition to the steel and aluminum tariffs, in January 2018, the Trump administration implemented tariffs on solar panels and large residential washing machines. (Gonzales, 2018) Changes at the firm level, such as tariffs, do affect the macroeconomy. di Giovanni, Levchenko and Mejean (2014) find that a majority of aggregate volatility is driven by changes at the firm level and that this percentage is growing over time.

The prices that firms charge to the final consumer depend on the inputs used by each firm and on the prices they must pay for these inputs. The prices firms pay for their inputs depend on the inputs their inputs use, and on the inputs of those inputs, and so on. Therefore, the prices that consumers face depend on the structure of the production economy as a whole. When a tariff is applied to a single product, the effect on the prices that consumers pay depends on the producer of that product and on the network structure of production.

To understand the effect of a tariff on the production network and on the prices faced by consumers, I simulate the model presented in this chapter. I find that, on average, when a 15% tariff is applied, the prices paid by consumers increase by 19%. I also investigate the effect of the level of interconnectedness in the production economy and of the producer of the tariff product on the post-tariff prices consumers face. When the economy has the potential to be very interconnected and when the post-tariff equilibrium production network is very interconnected, the post-tariff prices are, on average, lower than when there is less interconnectedness in the production economy. Additionally, when the producer of the product to which the tariff is applied is used as an input by fewer firms after the tariff is applied, the post-tariff prices will be lower than if it is used as an input by many firms.

3.2 Network Model

The model I use to investigate the effect of a tariff is the same as that described in Section 2.2 of Chapter 2.

Each firm $j \in J$ produces a single good. This good can be consumed in two ways: either as an input in another firm's production as described by the potential network or as a final consumption good by a representative consumer.

S_j denotes the set of edges pointing to firm j; this describes the set of inputs available to firm j. Each such input, $e \in S_j$, defines a different production function:

$$y_j(e) = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z(e) x(e)^\alpha l_j^{1-\alpha}$$

where x(e) is the amount of the associated input good, l_j is the amount of labor used by firm j, and $z(e)$ is an edge-specific productivity parameter. The production parameter α is the same across all firms. The representative consumer has preferences over the products produced by the firms in J according to

$$U(y_1^0, y_2^0, \ldots, y_{|J|}^0) = \left(\sum_{j \in J} (y_j^0)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

and she supplies L units of labor, inelastically.

3.3 Equilibrium Definitions

I use the same definitions of equilibrium networks as in Chapter 2. The definitions of these require a list of payoffs for each firm for each PEN, $\left\{\{\pi\}_{j \in J}\right\}_{N \in \mathcal{N}}$. The optimal derivation of these will be described in the following section. A pairwise-stable network is a network, $N \in \mathcal{N}$, such that no firm j, along with any potential supplier of j, would be made better off by moving to the 1-adjacent network defined by j and the potential supplier. A coordination-proof equilibrium network is a network, $N \in \mathcal{N}$, such that not only would no *one* pair of firm and alternate supplier be made better off by moving to the 1-adjacent network defined by the pair, but no two pairs would be made better off, no three pairs, and so on up to the number of firms. See Section 1.3 for a formal definition of both pairwise-stable networks and coordination-proof networks.

3.4 Prices and Profit Maximization

As in the previous chapter, firms set prices for both the portion of their output consumed by the representative consumer and the portion consumed by each of their network customers - the other firms which use their good as an input. The price of y_j^0 is p_j^0. For each network customer of firm j, j charges a two-part tariff. The per-unit price that firm j charges to each network customer is firm j's marginal cost of production. See Theorem 4 in Chapter 2. As a result of this, the per-unit price firm j charges is a function of the marginal cost of the input supplier used by firm j, $p(e) = MC_j = \frac{1}{z(e_j)} MC_{s(e_j)} w^{1-\alpha}$, where w is the price of labor to all firms. Because the price charged by each firm can be written in terms of the supplier's marginal cost, all of these prices can be calculated using only the network structure and $z(e_j)$'s.

The profit maximization problem each firm j solves is

$$\max_{p_j^0, y_j^0, x(e_j), l_j} p_j^0 y_j^0 + \sum_{e \in \hat{D}_j} [p(e)x(e) + \tau(e)] - [p(e_j)x(e_j) + \tau(e_j)] - wl_j$$

s.t.

$$y_j^0 + \sum_{e \in \hat{D}_j} x(e) \leq \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z(e_j) x(e_j)^\alpha l_j^{1-\alpha}$$

and all firms are jointly subject to the labor constraint, $\sum_{j \in J} l_j = L$.

3.5 Tariffs

I investigate the effect of levying an ad valorem tariff, t_0, on a single firm in the network. Label this firm j^T. Every network customer of j^T must pay $100 \times t_0\%$ more per unit of j^T's product. The new profit maximization problem faced by each firm in the network is the following:

$$\max_{p_j^0, y_j^0, x(e_j), l_j} p_j^0 y_j^0 + \sum_{e \in \hat{D}_j} [p(e)x(e) + \tau(e)] - [(1+t)p(e_j)x(e_j) + \tau(e_j)] - wl_j$$

s.t.

$$y_j^0 + \sum_{e \in \hat{D}_j} x(e) \leq \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z(e_j) x(e_j)^\alpha l_j^{1-\alpha}$$

where $t = t_0$ for customers of j^T and $t = 0$ otherwise. The tariff revenue, $\sum_{e \in C_{j^T}} t_0 p(e)x(e)$, is returned to the representative consumer.

Holding edges and allocations fixed, all firms buying j^T's product, that is $j \in C_{j^T}$ will see their costs increase - and thus profits decrease - by $t_0 p(e_j)x(e_j)$. Because the tariff increases the cost to other firms of buying j^T's product, these firms may switch input suppliers, changing the edges and allocations of goods across the network. The prices that firms charge to the representative consumer are affected by the inputs they chose and indirectly by the input choices of the other firms. When a tariff is implemented, the prices that firms must pay for their inputs are likely to increase, either directly or indirectly. As a result, the price charged to the representative consumer are also likely to increase so as to mitigate the increase in cost and potential decrease in profit. In the following section, I simulate the model and compare output and prices before and after the tariff is levied.

3.6 Simulation Results

I simulated this model using the same method and parameterization as in Section 2.7 of Chapter 2. I first find an initial equilibrium network. I did this by finding the set of coordination-proof networks for a given potential network, and setting the network that produces the highest aggregate output to be the equilibrium network. Then, I apply a 15% ad valorem tariff to the products of each firm, one at a time, and find the new equilibrium network. This simulation consisted of 280 tariff experiments with a network consisting of five firms.

In 44% of these tariff experiments, the new equilibrium network is different from the original equilibrium network. That is, 44% of the time, the application of the tariff leads to firms changing input suppliers in the face of the new prices.

On average, the price that the consumer pays for each good increased after the tariff was applied. The average price increase across all goods is 18.92%. To examine the role network and firm characteristics play in the change to consumer prices, I regress the average post-tariff consumer price on the connectivity of the potential network, of the original equilibrium network and of the new equilibrium network and on the number of suppliers, initial number of network customers, and final number of network customers of j^T. As in the previous chapter, I measure the connectivity of a network using the average shortest path distance. The results of this regression are presented in Table 1.

Table 1: Consumer Prices	
Variable	$\hat{\beta}$
Potential Network Avg. Shortest Path	0.0194
Original Eq. Network Avg. Shortest Path	−0.0089
New Eq. Network Avg. Shortest Path	0.0030
Number of Potential Suppliers	0.0016
Original Number of Network Customers	−0.0020
New Number of Network Customers	0.0035

On average, the less connected the potential network is, the higher the post-tariff consumer prices are. The more interconnected the economy could be, the lower the prices faced by the consumer will be. Higher post-tariff prices are associated with a more connected original equilibrium network and with a less connected new equilibrium network. When the production economy is less connected before the tariff is applied and more connected after the tariff is applied, we would expect the prices faced by consumers to be lower after the tariff is applied.

Finally, when the producer of the product to which the tariff is applied has fewer network customers beforehand, and more network customers afterward, this is associated with higher post-tariff consumer prices. The more other firms that buy j^T's product after the tariff is applied, the higher we expect consumer prices to be.

When a tariff is levied against a single product, the structure of the production network, as well as characteristics of the producer of that product, play a role in the change to the prices consumers pay for all of the goods in the economy. On average, when the production economy has the capacity to be very interconnected, the post-tariff prices will be lower. When the firm to which the tariff is applied has more firm customers, we would expect the post-tariff prices to be higher.

3.7 Conclusion

This chapter applies a model of endogenous network formation to the context of firms choosing input suppliers to form a production network. I use this model to analyze the effect of a tariff applied to a product produced by a firm in the production network. I find that, on average, the prices that consumers face increase after the tariff is applied. If the firm-to-firm production network is more interconnected after the tariff is applied, the prices that consumers pay will be lower than if the network is less interconnected. Additionally, after the tariff is applied, if the tariff product is an input to relatively many firms, the prices that consumers pay will be higher than if the tariff product is an input to fewer firms.

Chapter 4

Conclusion and Future Work

This dissertation describes my contribution to economics through my research in economic networks. I present a new model of endogenous economic network formation and explore two applications of the model to interesting and important economic questions. I find that there are often counterintuitive results and that economic networks can help us understand these.

There exist many avenues for future work stemming from the research presented here. As discussed previously, the model described in Chapter 1 can apply to a myriad of different contexts, applications, and fields. I intend to use the model described in Chapter 2 to investigate the possibility that there is some parameterization of the model such that when an input is removed, output increases *on average*. The tariff model described in Chapter 3 could be used to investigate the effect of ad valorem tariffs on aggregate output, input prices, consumer demand, input substitution, and any number of other interesting economic measures. I look forward to the opportunity to pursue these questions using the skills I have developed producing the research contained herein.

Chapter 5

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Chapter 6

Appendices

6.1 Appendix 1: Matlab Code

```
%This function takes as input a potential network adjacency matrix, P, and
%returns a matrix, Pot_Eq, containing all the possible lists of suppliers
(that
%would make up an equilibrium network's adjacency matrix). It calls the
%recursive function rec_matrix.

function [Pot_Eq] = Find_Potentials(P)

%get function variables
Potential = P;

%How many firms?
num_firms = size(Potential,1);

%Construct the matrix of potential supplier indices
sup_matrix = zeros(num_firms, num_firms - 1);
%And count how many rows you'll need for the storing matrix.
num_rows = 1;
for i = 1:num_firms
    %What are the indices of the nonzero elements (suppliers)?
    index_list = find(P(i,:));
    %How many are there?
    num_options = length(index_list);
    temp_rows = num_rows;
    num_rows = temp_rows*num_options;
    %For each of those options put the index in the supplier matrix
    for s = 1:num_options
        index = index_list(s);
        sup_matrix(i,s) = index;
    end
end
%sup_matrix

%construct the temporary storing matrix
temp_store = zeros(num_rows,num_firms);

%construct initial vector
init_vector = zeros(num_firms,1);

%call the function for firm 0
return_mat = rec_matrix(0,init_vector,sup_matrix,temp_store);

Pot_Eq = return_mat;

end


%This is a recurseive function that enumerates all of the different
%   combinations of suppliers possible for a given potential network
%   adjacency matrix.

%It takes as input
%   - firm (row) we're on
%   - a working vector naming the suppliers used
%   - the potential adjacency matrix
```

```
%   - matrix to store the finished list for each possible eq. network

function [matrix] = rec_matrix(f, sup_vec, sup_mat, mat)

firm = f; %which firm are we on
supplier_vector = sup_vec; %list of suppliers we're building
supplier_matrix = sup_mat; %Matrix describing the suppliers available to
                            %each firm
matrix = mat;   %matrix where we're storing the final product

%How many firms are we working with?
num_firms = size(matrix,2);

%RETURN CONDITION

%If we're on the last firm
if firm == num_firms

    %Print the list of supplier's we've created on this round
    %supplier_vector

    %What's the next open spot in the matrix?
    next_spot = find(matrix(:,1)==0,1,'first');

    %Store it in the matrix
    %Only make changes to this matrix if you're on the last firm and have
    %completed a list of suppliers
    matrix(next_spot,:) = supplier_vector;

    %Return

else
%Otherwise, for each potential supplier of the next firm, call the function


    %What are the supplier options for the next firm?
    firm_suppliers = nonzeros(supplier_matrix(firm+1,:));
    %How many?
    n = length(firm_suppliers);

    %For each such supplier, add him to the vector of suppliers and call the
    %function with it.
    for i = 1:n
        %create a temp vector so you don't ruin it
        temp_vec = supplier_vector;
        %put potential supplier i in the supplier vector
        temp_vec(firm+1) = firm_suppliers(i);

        %call the function with the new working vector for the next firm
        matrix = rec_matrix(firm+1,temp_vec,supplier_matrix,matrix);

    end

end
```

```
%This function determines which of the potential equilbrium networks are
%pairwise stable equilbria.

%It takes as input:
%   - The list of Potential Equilibrium networks, P
%   - The Payoffs for each firm for each such network, Pay
%   - The possible suppliers for each firm, Sup

%It returns a matrix describing the equilbrium, E
%   -This is a list of all equilibria


function [E] = Pick_Equilibrium_Mult(P,Pay,Sup)

%Get the function values
Pot_Matrix = P; %lists the potential eq networks
Payoffs = Pay;  %lists the payoffs for each firm for each network
Suppliers = Sup; %lists the suppliers for each firm

dim = size(Pot_Matrix);

%Create a boolean indicating whether you found a profitable deviation
found_dev = 0;


%How many options for eq network are there?
num_options = dim(1);

%How many firms are there?
num_firms = dim(2);

%initialize E as a matrix of zeros num_options by num_firms
E = zeros(num_options, num_firms);
%create a counter to store where the next available row is.
storage_counter = 1;

%create vectors to hold the current and other eq network options
current = zeros(num_firms,1);
other = zeros(num_firms,1);

%create vectors to hold the payoffs for the two networks
current_payoffs = zeros(num_firms,1);
other_payoffs = zeros(num_firms,1);

%Starting from the first option, check if each matrix is pairwise stable
o = 1;
while o <= num_options
    %Get the current network
    current = Pot_Matrix(o,:);

    %Get the payoffs for the current network
    current_payoffs = Payoffs(o,:);

    %set the indicator to 0
    found_dev = 0;
```

```
    %For each firm, check to see if there is a deviation they would take
    f = 1;
    while f <= num_firms
        %What other suppliers does this firm have
        sup = nonzeros(Suppliers(f,:));
        num_sup = length(sup);


        %For each supplier that isn't the one being used right now,
        %construct a row describing the alternative matrix
        s = 1;
        while s <= num_sup
            %only if the supplier is different from the current one
            %current(f)
            %other = zeros(1,num_firms);

            if sup(s) ~= current(f)
                %copy current
                other = current;

                %put the alternative supplier option in for firm f in the
                %alternative network
                other(f) = sup(s);

                %find the index of the other vector in the potential
                %network matrix
                other_index = find(ismember(Pot_Matrix,other,'rows'));

                %get the payoffs associated with that network
                other_payoffs = Payoffs(other_index,:);
                Payoffs(other_index,:);
                %Payoffs

                %Are both the current firm and the potential new
                %supplier made better off?
                if other_payoffs(f) > current_payoffs(f) &&
other_payoffs(sup(s)) > current_payoffs(sup(s))
                    %set the indicator to 1, break the supplier loop
                    found_dev = 1;
                    %disp('found dev')
                    break;
                else
                    %otherwise move to the next supplier
                    %disp('did not find dev')
                    temp_s_2 = s;
                    s = temp_s_2 + 1;
                end
            else
                %If it IS the current supplier, just go to the next
                %supplier
               temp_s = s;
               s = temp_s + 1;
            end
        end
```

```
        %If you found a deviation, break the firm loop, you don't need to
keep
        %looking
        if found_dev == 1
            break
        else
            %otherwise go on to the next firm
            temp_f = f;
            f = temp_f + 1;
        end
    end

    %If you found a deviation, increase o becuase the one you tried wasn't
    %an eq.
if found_dev == 1
    temp_o = o;
    o = temp_o + 1;
 %Otherwise you didn't find a deviation and this is a pairwise equilbirum
   %     set the next row of E to be current, incremenet the storage
   %     counter and increment o.
else
    E(storage_counter,:) = current;
    %increment storage counter
    temp_count = storage_counter;
    storage_counter = temp_count + 1;
    %increment o
    temp_o_2 = o;
    o = temp_o_2 + 1;
end

end

%trim E
%find the index of the first 0
zero_index = find(~E(:,1),1);
%trim E from the zero_index to num_options
if zero_index > 1
    E(zero_index:num_options,:) = [];
end
end
```

```
%This function returns the coalition-proof equilibrium network (if one
%exists and a list of -1's if one does not). It takes as input:
%   - The list of possible equilibrium matrices
%   - The list of associated payoffs
%   - The sup matrix that lists the possible suppliers for each firm

%It returns a network, e, that lists the supplier of each firm (or -1's)

function [E] = Find_CP_Eq_Mult(Pot_Mat, Pay, Sup)

%get the function variables
Pot = Pot_Mat; %The list of possible eq matrices
Payoffs = Pay; %The list of payoffs associated with each such matrix
Suppliers = Sup; %Lists the possible suppliers for each firm

%get the dimensions of the pot matrix
pot_dim = size(Pot);
num_opt = pot_dim(1);
num_firms = pot_dim(2);

%Initialize E as a list of zero vectors
%allowing for every network to be CP
%trim at the end
E = zeros(num_opt,num_firms);
%create a storage counter
storage_counter = 1;

%create a counter to count the matrices visited
m = 1;
%while there are still matrices to check
%EQ MATRIX LOOP
while m <= num_opt

    %for each matrix, check if it is coalition proof
    %   --> check each coaliton size
    %get the current possible eq network we're trying
    poss_eq = Pot(m,:);

    %COALITION SIZE LOOP
    %check each coalition size
    %if any of them have a profitable deviation, set coaliton_flag = 1 and
found_dev = 1 and end
    coalition_size = num_firms;
    coalition_flag = 0;
    found_dev = 0;
    while coalition_flag == 0
        %check each size
        %check the coalitions of the current coalition_size
        %coalition_size
        found_dev = check_coalition(poss_eq, m, Pot, Payoffs, Suppliers,
coalition_size);
        %coalition_size

        %if there was a profitable deviation, give up on this network
        if found_dev == 1
```

```
            coalition_flag = 1;
        else
            %decrease the coalition size
            %if it's 1, set the flag to 1 to to end the loop
            if coalition_size == 1
                coalition_flag = 1;
            else
                temp_co_size = coalition_size;
                coalition_size = temp_co_size - 1;
                %coalition_size
            end
        end
    %END COALITION CHECKING LOOP
    end

    %if found_dev = 1, you found a deviation and need to increase the m
    %counter to go to the next network
    if found_dev == 1
        %go to next poss eq network
        temp_m = m;
        m = temp_m + 1;
    else
        %if you didn't find a deviation that means you got through all the
        %coalition sizes and no profitable deviations
        %   --> this is a coalition proof network
        %store it in E
        E(storage_counter,:) = poss_eq;
        %increment the storage vector
        temp_counter = storage_counter;
        storage_counter = temp_counter + 1;

        %increment m
        temp_m = m;
        m = temp_m + 1;
    end
    %If you never find a deviation, you never store anything, and E is just
    %a thing of zeros

%END OF MATRIX LOOP
end

%trim E
%find the first 0
zero_index = find(~E(:,1),1);

%trim E from the zero_index to num_options
if zero_index > 1
    E(zero_index:num_opt,:) = [];
end

end
```

6.2 Appendix 2: The Planner's Problem

6.2.1 Aggregate Output and Efficiency

Each firm j uses a number of supply chains in the production process. A supply chain is a string of firms, each producing intermediate goods for the next firm in the chain. Label the set of supply chains which lead to firm j as $\mathcal{S}_j$. Partition firm j's final output, y_j^0, by final output produced by each supply chain, $s \in \mathcal{S}_j$. That is, $\sum_{s\in\mathcal{S}_j} y_j^0(s) = y_j^0$. To make $y_j^0(s)$, firm j uses labor $l_j^0(s) \leq l_j$ and intermediate input $x_j^0(s) \leq x_j$. This $x_j^0(s)$ is produced by firm j's supplier using $l_j^1(s)$ and $x_j^1(s)$, this $x_j^1(s)$ is produced using $l_j^2(s)$ and $x_j^2(s)$, and so on up the supply chain. In general, I write $l_j^{k+1}(s)$ and $x_j^{k+1}(s)$ are the labor and intermediate input amounts used to make $x_j^k(s)$, along supply chain s to make firm j's output for final consumption. The lower subscript describes the good at the end of the supply chain and the superscript describes the step up the supply chain,

Let $\lambda_j^0(s)$ be the marginal social cost of producing $x_j^0(s)$ and let μ be the marginal social cost of labor, following the earlier notation. The optimal choices of $x_j^0(s)$ and $l_j^0(s)$ give:

$$\frac{\lambda_j^k(s)x_j^k(s)}{\alpha} = \frac{wl_j^k(s)}{1-\alpha}$$
$$\frac{\lambda_j^{k+1}(s)x_j^{k+1}(s)}{\alpha} = \frac{wl_j^{k+1}(s)}{1-\alpha}.$$

Using the technological constraint,

$$\lambda_j = \frac{1}{z(e_j)}\lambda_{s(e_j)}^{\alpha}\mu^{1-\alpha}$$

where $s(e_j)$ is the supplier of edge e_j. This means that the marginal social cost of producing product j is determined by the marginal social cost of firm j's supplier. This will be necessary for connecting efficiency across the entire network. The network structure gives $\lambda_j^{k+1}(s)x_j^{k+1}(s) = \alpha\lambda_j^k(s)x_j^k(s)$, so the optimality condition for the $(k+1)$th step up the supply chain becomes $\lambda_j^k(s)x_j^k(s) = \frac{wl_j^{k+1}(s)}{1-\alpha}$. Substituting this into the optimality condition for the kth step gives $l_j^{k+1}(s) = \alpha l_j^k(s)$. That is, the labor used in each step along the supply chain to make intermediate goods for $y_j^0(s)$ is a constant share of the labor used in the previous step.

Let $l_j(s)$ be the total labor used along supply chain s to make $y_j^0(s)$ such that $l_j(s) = \sum_{k=1}^{\infty} l_j^k(s) = \sum_{k=1}^{\infty} \alpha^k l_j^0(s) = \frac{l_j^0(s)}{1-\alpha}$. From the optimality condition for the final step in the supply chain,

$$\lambda_j y_j^0(s) = \frac{wl_j^0(s)}{1-\alpha} = wl_j(s).$$

Finally, write $y_j^0 = \sum_{s\in\mathcal{S}_j} y_j^0(s) = \sum_{s\in\mathcal{S}_j} \frac{w}{\lambda_j} l_j(s) = \frac{w}{\lambda_j}\sum_{s\in\mathcal{S}_j} l_j(s) = \frac{w}{\lambda_j} l_j$. From the definition of q_j, I can now write $y_j^0 = q_j l_j$.

From here, I use the solution to this planner's problem to show that $Y^0 = L\left[\sum_{j=1}^{J} q_j^{\epsilon-1}\right]^{\frac{1}{\epsilon-1}}$. Let $\left[\sum_{j=1}^{J_t} q_j^{\epsilon-1}\right]^{\frac{1}{\epsilon-1}} \equiv Q$. First, the first order condition with respect to firm y_j^0 is $\left(Y^0\right)^{\frac{1}{\epsilon}}(y_j^0)^{-\frac{1}{\epsilon}} = \lambda_j$. Rearranging this gives $\lambda_j = \left(\frac{y_j^0}{Y^0}\right)^{-\frac{1}{\epsilon}}$. This expression can be used to show that $\sum_{j=1}^{J_t} \lambda_j^{1-\epsilon} = 1$.

Rewriting the definition of firm efficiency for q_j allows me to write $\sum_{j=1}^{J_t}\left(\frac{w}{q_j}\right)^{1-\epsilon} = 1$. Then, using the definitions of q_j and Q, I can write $\frac{y_j^0}{Y^0} = \left(\frac{q_j}{Q}\right)^{\epsilon}$.

Finally, I use the labor constraint to show $L = \sum_{j=1}^{J_t}\left(l_j\right) = \sum_{j=1}^{J_t}\left(\frac{y_j^0}{q_j}\right) = Y^0 Q^{-\epsilon}\sum_{j=1}^{J_t} q_j^{\epsilon-1} = Y^0 Q^{-\epsilon} Q^{\epsilon-1} = \frac{Y^0}{Q}$. Rewritten, this is the expression I need: $Y^0 = QL$.

6.3 Appendix 3: Equilibrium

6.3.1 Existence of a Solution to Each Agents' Problem

I use the Theorem of the Maximum to show that the individual agents' problems each have a solution. Recall that the optimization process proceeds as follows: taking the set of inputs that are used and their associated two-part prices as given, firms maximize profits by choosing the price for final output, p_j^o, the amount of final output, y_j^o, the amount of the intermediate good they use, x_j, and the labor they use, l_j. Then the firms choose their optimal input and the two-part prices they charge their potential customers, that is, the set of edges that are used in the production network, $\hat{E}$, and prices for each edge, $\{p(e), \tau(e)\}_{e \in \hat{E}}$.

The objective function is certainly continuous, so I focus here on the compactness and continuity of the constraint correspondence. Each firm must choose prices for each technique for which they are a supplier. These prices are drawn from compact and continuous sets. These prices in turn continuously determine the budget sets for the firms. Note that for each possible input choice, there is a set of four choice variables, p_j^o, y_j^o, x_j, and l_j, each of which is chosen from a compact and continuous correspondence determined by the budget sets.

The constraint correspondence is therefore the finite Cartesian product of compact and continuous correspondences and is therefore compact and continuous. Thus, the Theorem of the Maximum applies and each agent's problem has a solution.

Given that each agent has a solution to her maximization problem, and that in each time period there is a finite number of agents, Kakutani's Fixed Point Theorem applies and there exists a Mixed Strategy Nash Equilibrium in each time period.

6.4 Appendix 4: Prices

6.4.1 Pricing at Marginal Cost

Proposition: The per-unit price, $p(e)$, that firm j charges to firm i along edge e is the marginal cost of firm j, MC_j.

Proof: Following Oberfield (2013), suppose for the purpose of contradiction that firm j charges some other price, $p(e) = \hat{p} \neq MC_j$. Label the associated contract $(\hat{p}, \hat{\tau})$.

Consider the deviation from $(\hat{p}, \hat{\tau})$, $p(e) = \widetilde{p} = MC_j$ and $\tau(e) = \widetilde{\tau} = \hat{\tau} + (\hat{p} - MC_j)x(e) + K$, where

$$K = \frac{1}{2}\Big\{(\widetilde{p_i^0} - \widetilde{c_i})\widetilde{y_i^0} - (\hat{p_i^0} - \widetilde{c_i})\hat{y_i^0} + \sum_{e \in D_i}(p(e) - \widetilde{c_i})[x_{b(e)}(\widetilde{y_i^0}) - x_{b(e)}(\hat{y_i^0})]\Big\}$$

and $\widetilde{c_i} = \frac{1}{z(e)} MC_j^{\alpha} w^{1-\alpha}$, the marginal cost of i given the deviation. I will show that both firm j and firm i are made better off by this deviation and thus $(\hat{p}, \hat{\tau})$ is not optimal.

First, $\widetilde{\pi}_j - \hat{\pi}_j = K > 0$.

Next, $\widetilde{\pi}_i - \hat{\pi}_i = \widetilde{p_i^0}\widetilde{y_i^0} - \hat{p_i^0}\hat{y_i^0} + \sum_{e \in D_i} p(e)[x_{b(e)}(\widetilde{y_i^0}) - x_{b(e)}(\hat{y_i^0})] - [\widetilde{c_i}\widetilde{y_i} - \hat{c_i}\hat{y_i}] - [\widetilde{\tau} - \hat{\tau}]$. By using, (i) $p(e)x(e) = \alpha c_i y_i$ and (ii) $\frac{c_i}{\widetilde{c_i}} = \left(\frac{p(e)}{c_j}\right)^{\alpha}$, I can write:

$$\widetilde{\pi}_i - \hat{\pi}_i =$$

$$(\widetilde{p_i^0} - \widetilde{c_i})\widetilde{y_i^0} - (\hat{p_i^0} - \widetilde{c_i})\hat{y_i^0} + \sum_{e \in D_i}(p(e) - \widetilde{c_i})[x_{b(e)}(\widetilde{y_i^0}) - x_{b(e)}(\hat{y_i^0})] - K$$

$$+\big[\big((1-\alpha)+\frac{MC_j}{p(e)}\alpha\big)\big(\frac{p(e)}{MC_j}\big)^{\alpha}-1\big]\widetilde{c}_i\hat{y}_i.$$

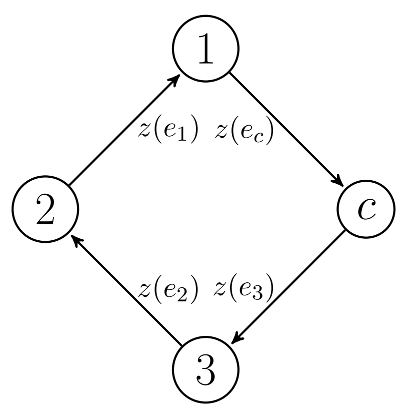
By the definition of K, the first line of the equation is positive. Jensen's Inequality gives that $[(1-\alpha)+x\alpha]x^{-\alpha}\geq 1$ for $\alpha\in[0,1]$. Applying this to the second line of the equation gives that $[(1-\alpha)+\frac{MC_j}{p(e)}\alpha](p(e)/MC_j)^{\alpha}\geq 1$, so the second line of the equation is non-negative. As a result, $\widetilde{\pi}_i-\hat{\pi}_i$ is postive and both firm i and firm j are made better off by firm j charging MC_j.

6.4.2 Derivation of Price Expressions

Both of the following derivations are driven by the fact that the price that each firm pays for each unit of the input they use is the marginal cost of the supplier of that input, as proved above.

1. Price Paid by Firms on a Cycle

   For a cycle of length c, there are c firms and c edges. Label these firms $1,\ldots,c$. Without loss of generality, we find the price of firm c and label the supplier c uses as 1, the supplier 1 uses as 2 and so on.



   The price that c pays for its input is $p(e_c)=\frac{1}{z(e_1)}p(e_1)^{\alpha}w^{1-\alpha}$, where $p(e_1)$ is the price firm 1 pays for its input. This price is $p(e_1)=\frac{1}{z(e_2)}p(e_2)^{\alpha}w^{1-\alpha}$, where

$p(e_2)$ is the price firm 2 pays for its input. Continuing in this way I can write the price that firm $c-1$ pays as $p(e_{c-1}) = \frac{1}{z(e_c)} p(e_c)^{\alpha} w^{1-\alpha}$. Substituting each price expression into the previous one gives:

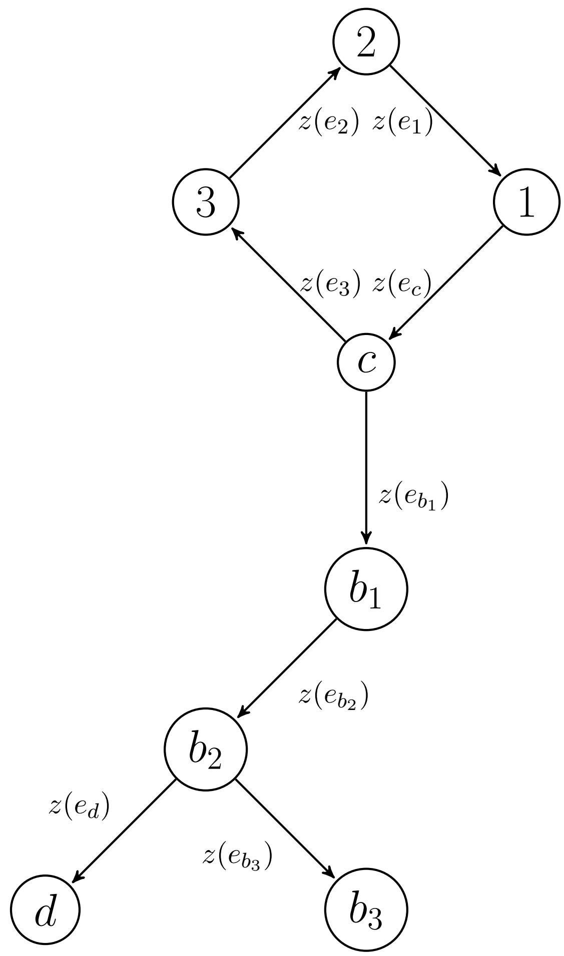
$$p(e_c) = p(e_c)^{\alpha^c} \left[\frac{1}{z(e_1)} \frac{1}{z(e_2)^{\alpha}} \cdots \frac{1}{z(e_c)^{\alpha^{c-1}}} \right] w^{(1-\alpha)+\alpha(1-\alpha)+\ldots\alpha^{c-1}(1-\alpha)}.$$

Solving for $p(e_c)$ gives:

$$\begin{aligned} p(e_c) &= \left(\left[\frac{1}{z(e_1)} \frac{1}{z(e_2)^{\alpha}} \cdots \frac{1}{z(e_c)^{\alpha^{c-1}}} \right] w^{1-\alpha^c} \right)^{\frac{1}{1-\alpha^c}} \\ &= w \prod_{i=1}^{c} \left(\frac{1}{z(e_i)} \right)^{\frac{\alpha^{i-1}}{1-\alpha^c}}. \end{aligned}$$

2. Price Paid by Firms on a Branch

Each connected component of the network has one cycle, and potentially many branches emanating from that cycle. Thus, each branch has a root node on the cycle. Because each firm pays the marginal cost of its supplier, the cost of any branch firm can be traced back and written in terms of the price of this root node. Let firm d be d edges down the branch from the node where $d > 1$.



The price that d pays for its input is the marginal cost of its supplier. The price the supplier pays is the marginal cost of his supplier and so on up to the root node, whose price was found in the above derivation. Label $MC_r = \frac{1}{z(e_r)} p_r^\alpha w^{1-\alpha}$. Then the price that firm d pays is

$$p(e_d) = MC_r^{\alpha^{d-1}} \left[\prod_{i=1}^{d-1} \frac{1}{z(e_{b_i})^{\alpha^{d-i-1}}} \right] w^{(1-\alpha)\sum_{k=0}^{d-2} \alpha^k}$$

$$= MC_r^{\alpha^{d-1}} \left[\prod_{i=1}^{d-1} \frac{1}{z(e_{b_i})^{\alpha^{d-i-1}}} \right] w^{(1-\alpha)\frac{\alpha^d - 1}{\alpha - 1}}$$

$$= MC_r^{\alpha^{d-1}} \Big[\prod_{i=1}^{d-1} \frac{1}{z(e_{b_i})^{\alpha^{d-i-1}}} \Big] w^{1-\alpha^d}.$$

For a firm that is only one edge away, for example firm b_1 in the figure, the price that firm pays is the marginal cost of the root node, MC_r.