Supplemental Materials for "Missing Data in Discrete Time State-Space Modeling of Ecological Momentary Assessment Data: A Monte-Carlo Study of Imputation Methods"

ARTICLE HISTORY
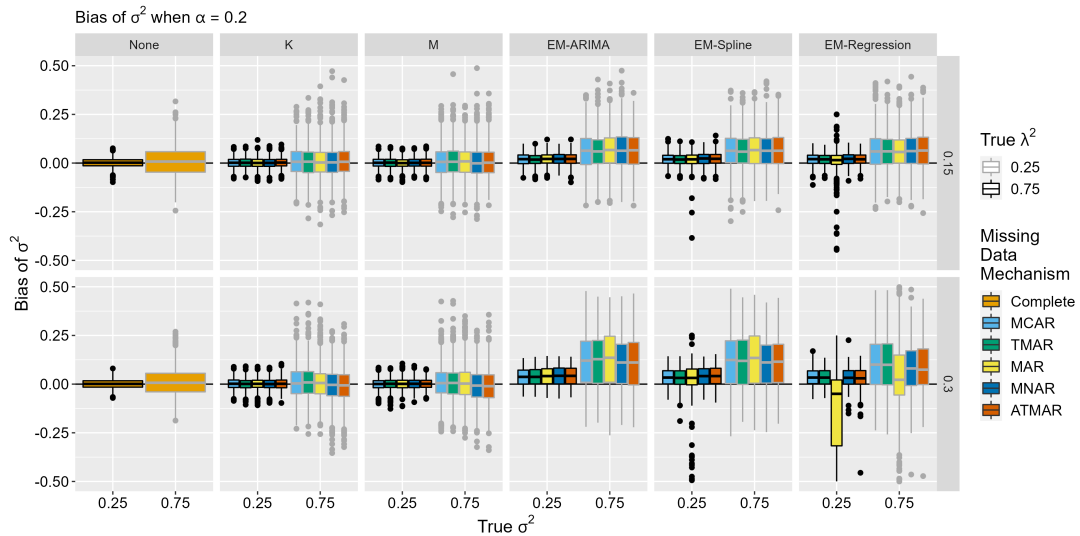
Compiled May 15, 2023

1. Measurement error: σ

Figure 1.: The above graphs shows the bias of the estimated σ^2 (on the y-axis) by the true σ^2 (on the x-axis for $\alpha = .2$. The respective missingness mechanisms are shown: complete data (orange), MCAR (sky blue), TMAR (green), MAR (yellow), MNAR (dark blue), and ATMAR (dark orange). The outlines on the box plots show the differing loading conditions: light grey for low loadings/high measurement error and black for high loadings/low measurement error. As the y-axis was restricted to range from -.5 to .5, 986 outliers were removed, primarily from the EM-Regression condition. For the box plots, the bottom of the box is the first quartile, the central line is the median, the top of the box is the third quartile, the whiskers extend 1.5(Interquartile Range), and the dots beyond the whiskers are outliers.

Figure 2.: The above graphs shows the bias of the estimated σ^2 (on the y-axis) by the true σ^2 (on the x-axis for $\alpha = .7$. The respective missingness mechanisms are shown: complete data (orange), MCAR (sky blue), TMAR (green), MAR (yellow), MNAR (dark blue), and ATMAR (dark orange). The outlines on the box plots show the differing loading conditions: light grey for low loadings/high measurement error and black for high loadings/low measurement error. As the y-axis was restricted to range from -.5 to .5, 2822 outliers were removed, primarily from the EM-Regression condition. For the box plots, the bottom of the box is the first quartile, the central line is the median, the top of the box is the third quartile, the whiskers extend 1.5(Interquartile Range), and the dots beyond the whiskers are outliers.
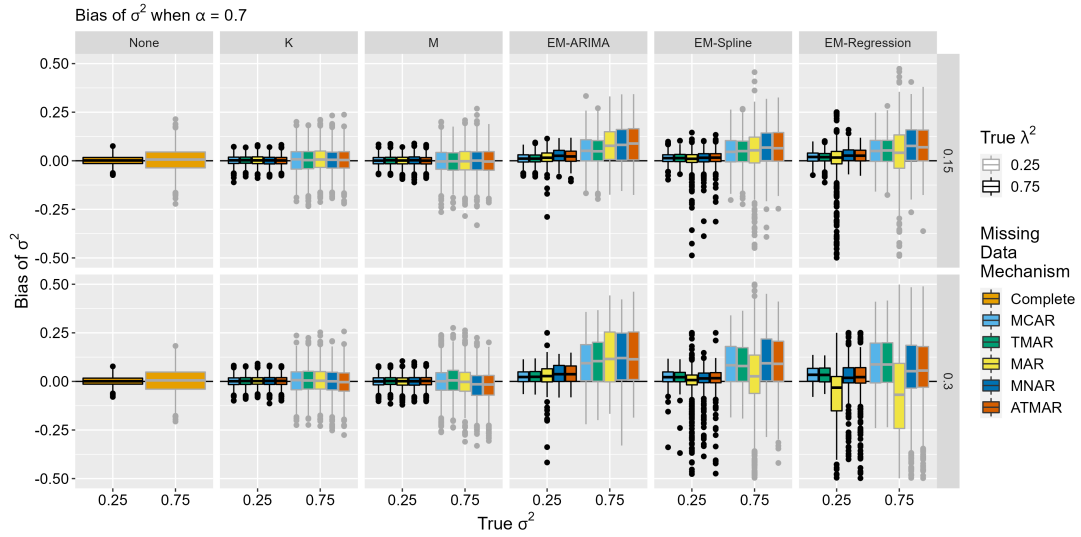
2. Loadings: λ

Figure 3.: The above graphs shows the bias of the estimated λ^2 (on the y-axis) by the true λ^2 (on the x-axis for $\alpha = .2$. The respective missingness mechanisms are shown: complete data (orange), MCAR (sky blue), TMAR (green), MAR (yellow), MNAR (dark blue), and ATMAR (dark orange). The outlines on the box plots show the differing cross-lagged conditions: light grey for no relation and black for a strong relation. As the y-axis was restricted to range from -.5 to .5, 1593 outliers were removed, primarily from the EM-Regression condition. For the box plots, the bottom of the box is the first quartile, the central line is the median, the top of the box is the third quartile, the whiskers extend 1.5(Interquartile Range), and the dots beyond the whiskers are outliers.
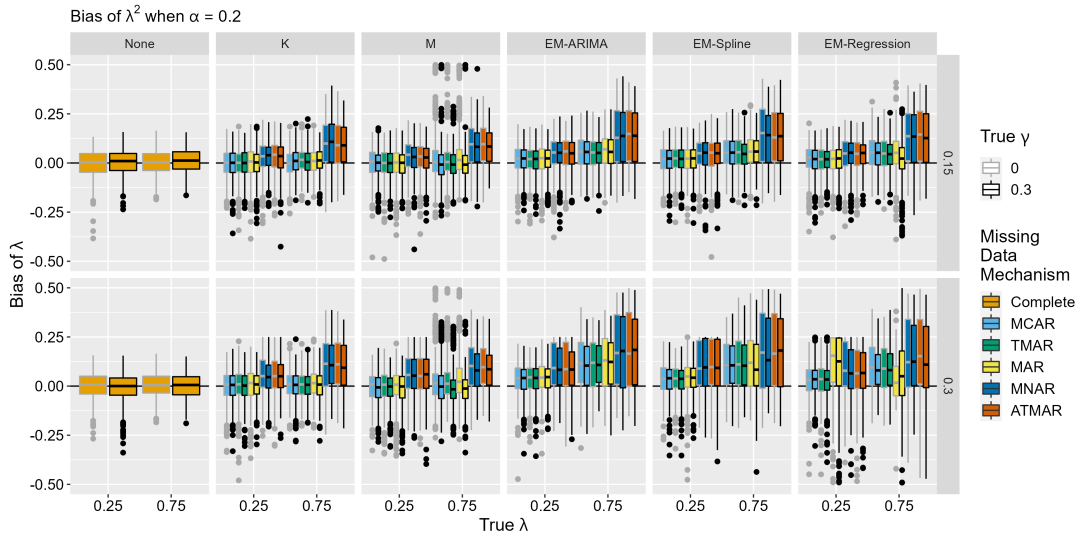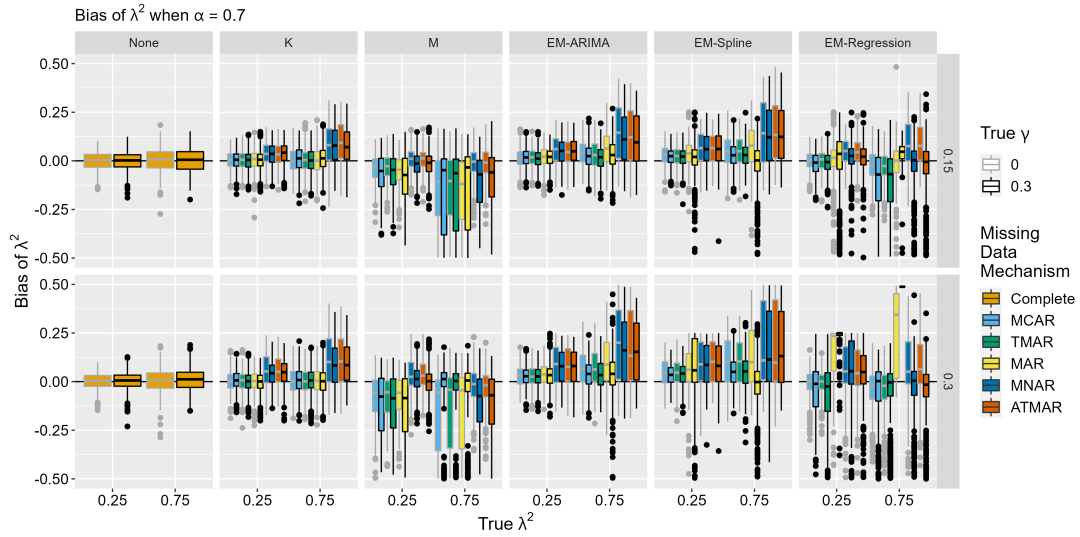
Figure 4.: The above graphs shows the bias of the estimated λ^2 (on the y-axis) by the true λ^2 (on the x-axis for $\alpha = .7$. The respective missingness mechanisms are shown: complete data (orange), MCAR (sky blue), TMAR (green), MAR (yellow), MNAR (dark blue), and ATMAR (dark orange). The outlines on the box plots show the differing cross-lagged conditions: light grey for no relation and black for a strong relation. As the y-axis was restricted to range from -.5 to .5, 8542 outliers were removed, primarily from the EM-Regression condition. For the box plots, the bottom of the box is the first quartile, the central line is the median, the top of the box is the third quartile, the whiskers extend 1.5(Interquartile Range), and the dots beyond the whiskers are outliers.

3. Standard errors

Missingness	Imputation	True α	True λ^2	α_{11}	σ_1	σ_2	σ_3	λ_{11}	λ_{12}	λ_{13}
MNAR	EM-ARIMA	0.2	0.75	0.046	0.097	0.097	0.098	0.028	0.028	0.028
MNAR	EM-ARIMA	0.2	0.25	0.062	0.08	0.079	0.106	0.037	0.038	0.04
MNAR	EM-ARIMA	0.7	0.75	0.032	0.093	0.094	0.092	0.027	0.027	0.027
MNAR	EM-ARIMA	0.7	0.25	0.04	0.076	0.076	0.076	0.039	0.039	0.039
MNAR	EM-Regression	0.2	0.75	0.047	0.099	0.11	0.097	0.03	0.03	0.03
MNAR	EM-Regression	0.2	0.25	0.064	0.086	1.471	1.562	0.038	0.038	0.037
MNAR	EM-Regression	0.7	0.75	0.039	0.107	0.099	0.113	0.04	0.039	0.033
MNAR	EM-Regression	0.7	0.25	0.085	0.211	0.091	1.516	0.056	0.071	0.072
MNAR	EM-Spline	0.2	0.75	0.046	0.097	0.097	0.097	0.028	0.028	0.028
MNAR	EM-Spline	0.2	0.25	0.065	0.08	0.081	0.077	0.041	0.04	0.039
MNAR	EM-Spline	0.7	0.75	0.03	0.092	0.094	0.094	0.027	0.027	0.027
MNAR	EM-Spline	0.7	0.25	0.034	0.079	0.076	0.075	0.037	0.037	0.037
MNAR	K	0.2	0.75	0.061	0.12	0.119	0.123	0.042	0.042	0.042
MNAR	K	0.2	0.25	0.11	0.107	0.109	0.108	0.073	0.072	0.073
MNAR	K	0.7	0.75	0.039	0.12	0.118	0.115	0.04	0.04	0.04
MNAR	K	0.7	0.25	0.06	0.1	0.098	0.099	0.063	0.062	0.063
MNAR	M	0.2	0.75	0.06	0.124	0.122	0.117	0.053	0.053	0.053
MNAR	M	0.2	0.25	0.225	1.324	0.129	0.132	0.144	0.144	0.147
MNAR	M	0.7	0.75	0.049	0.124	0.12	0.12	0.053	0.054	0.053
MNAR	M	0.7	0.25	0.111	0.11	0.11	0.104	0.105	0.106	0.103
TMAR	EM-ARIMA	0.2	0.75	0.048	0.101	0.1	0.1	0.031	0.031	0.031
TMAR	EM-ARIMA	0.2	0.25	0.086	0.11	0.117	0.108	0.059	0.061	0.06
TMAR	EM-ARIMA	0.7	0.75	0.034	0.096	0.096	0.097	0.031	0.031	0.031
TMAR	EM-ARIMA	0.7	0.25	0.051	0.087	0.086	0.086	0.042	0.042	0.042
TMAR	EM-Regression	0.2	0.75	0.049	0.101	0.102	0.101	0.032	0.032	0.032
TMAR	EM-Regression	0.2	0.25	0.12	0.132	0.118	0.119	0.064	0.064	0.063
TMAR	EM-Regression	0.7	0.75	0.042	0.099	0.099	0.097	0.035	0.035	0.035
TMAR	EM-Regression	0.7	0.25	0.064	0.09	0.089	0.09	0.047	0.046	0.046
TMAR	EM-Spline	0.2	0.75	0.049	0.101	0.101	0.102	0.031	0.031	0.031
TMAR	EM-Spline	0.2	0.25	0.087	0.109	0.12	0.109	0.06	0.061	0.059
TMAR	EM-Spline	0.7	0.75	0.031	0.095	0.094	0.095	0.03	0.03	0.03
TMAR	EM-Spline	0.7	0.25	0.04	0.084	0.084	0.083	0.038	0.038	0.037
TMAR	K	0.2	0.75	0.067	0.12	0.119	0.122	0.044	0.044	0.044
TMAR	K	0.2	0.25	0.122	0.136	0.136	0.124	0.086	0.087	0.084
TMAR	K	0.7	0.75	0.038	0.116	0.115	0.114	0.042	0.042	0.042
TMAR	K	0.7	0.25	0.056	0.101	0.101	0.102	0.057	0.057	0.057
TMAR	M	0.2	0.75	0.062	0.124	0.12	0.116	0.093	0.093	0.093
TMAR	M	0.2	0.25	0.11	0.147	0.609	0.907	0.09	0.09	0.089
TMAR	M	0.7	0.75	0.059	0.119	0.12	0.117	0.053	0.053	0.053
TMAR	M	0.7	0.25	0.091	0.116	0.111	0.115	0.071	0.07	0.073

Table 1.: Standard error for α_{11}, σ_1, σ_2, σ_3, λ_{11}, λ_{12}, and λ_{13} for the $\gamma = 0$ and 30% missingness condition. MCAR was excluded as it behaves similarly to TMAR, and ATMAR and MAR were excluded as they behave similarly to MNAR.

Missingness	Imputation	True α	True λ^2	α_{11}	σ_1	σ_2	σ_3	λ_{11}	λ_{12}	λ_{13}
MNAR	EM-ARIMA	0.2	0.75	0.044	0.099	0.096	0.097	0.028	0.028	0.028
MNAR	EM-ARIMA	0.2	0.25	0.062	0.077	0.078	0.084	0.038	0.039	0.04
MNAR	EM-ARIMA	0.7	0.75	0.029	0.093	0.093	0.094	0.027	0.027	0.027
MNAR	EM-ARIMA	0.7	0.25	0.034	0.077	0.077	0.078	0.036	0.036	0.036
MNAR	EM-Regression	0.2	0.75	0.046	0.1	0.1	0.097	0.03	0.03	0.03
MNAR	EM-Regression	0.2	0.25	0.102	0.085	0.082	0.083	0.044	0.041	0.043
MNAR	EM-Regression	0.7	0.75	0.083	1.019	0.409	0.133	1.813	0.285	0.349
MNAR	EM-Regression	0.7	0.25	0.239	0.658	0.144	0.123	0.136	0.154	0.146
MNAR	EM-Spline	0.2	0.75	0.044	0.097	0.098	0.097	0.028	0.028	0.028
MNAR	EM-Spline	0.2	0.25	0.054	0.074	0.076	0.077	0.033	0.035	0.036
MNAR	EM-Spline	0.7	0.75	0.027	0.097	0.093	0.096	0.029	0.029	0.028
MNAR	EM-Spline	0.7	0.25	0.029	0.079	0.777	0.078	0.035	0.192	0.038
MNAR	K	0.2	0.75	0.06	0.121	0.12	0.122	0.042	0.042	0.042
MNAR	K	0.2	0.25	0.123	0.11	0.109	0.114	0.075	0.075	0.08
MNAR	K	0.7	0.75	0.035	0.12	0.119	0.119	0.041	0.041	0.041
MNAR	K	0.7	0.25	0.053	0.101	0.102	0.101	0.054	0.054	0.054
MNAR	M	0.2	0.75	0.059	0.121	0.12	0.117	0.043	0.044	0.044
MNAR	M	0.2	0.25	0.205	0.119	0.127	0.131	0.129	0.129	0.135
MNAR	M	0.7	0.75	0.046	0.123	0.123	0.117	0.058	0.058	0.059
MNAR	M	0.7	0.25	0.083	0.107	0.102	0.105	0.092	0.091	0.092
TMAR	EM-ARIMA	0.2	0.75	0.047	0.1	0.101	0.1	0.031	0.031	0.031
TMAR	EM-ARIMA	0.2	0.25	0.082	0.101	0.105	0.105	0.055	0.055	0.056
TMAR	EM-ARIMA	0.7	0.75	0.03	0.095	0.095	0.096	0.032	0.032	0.032
TMAR	EM-ARIMA	0.7	0.25	0.041	0.084	0.082	0.083	0.038	0.037	0.037
TMAR	EM-Regression	0.2	0.75	0.048	0.1	0.101	0.1	0.032	0.032	0.032
TMAR	EM-Regression	0.2	0.25	0.109	0.147	0.113	0.105	0.059	0.058	0.057
TMAR	EM-Regression	0.7	0.75	0.044	0.102	0.098	0.098	0.045	0.046	0.045
TMAR	EM-Regression	0.7	0.25	0.1	0.093	0.091	0.091	0.048	0.047	0.048
TMAR	EM-Spline	0.2	0.75	0.047	0.099	0.101	0.103	0.031	0.031	0.031
TMAR	EM-Spline	0.2	0.25	0.084	0.109	0.099	0.103	0.056	0.055	0.055
TMAR	EM-Spline	0.7	0.75	0.028	0.095	0.095	0.095	0.032	0.032	0.032
TMAR	EM-Spline	0.7	0.25	0.035	0.081	0.082	0.083	0.034	0.035	0.035
TMAR	K	0.2	0.75	0.063	0.121	0.117	0.12	0.043	0.044	0.043
TMAR	K	0.2	0.25	0.11	0.126	0.12	0.122	0.077	0.076	0.076
TMAR	K	0.7	0.75	0.033	0.113	0.112	0.114	0.041	0.041	0.041
TMAR	K	0.7	0.25	0.046	0.097	0.099	0.098	0.049	0.049	0.05
TMAR	M	0.2	0.75	0.059	0.12	0.121	0.117	0.066	0.066	0.067
TMAR	M	0.2	0.25	0.105	0.127	0.131	0.126	0.082	0.084	0.081
TMAR	M	0.7	0.75	0.057	0.119	0.121	0.116	0.062	0.062	0.062
TMAR	M	0.7	0.25	0.085	0.112	0.11	0.11	0.066	0.066	0.067

Table 2.: Standard error for α_{11}, σ_1, σ_2, σ_3, λ_{11}, λ_{12}, and λ_{13} for the $\gamma = 0.3$ and 30% missingness condition. MCAR was excluded as it behaves similarly to TMAR, and ATMAR and MAR were excluded as they behave similarly to MNAR.

4. Median absolute relative bias

Missingness	Imputation	True α	True λ^2	α_{11}	σ_1	σ_2	σ_3	λ_{11}	λ_{12}	λ_{13}
MNAR	M	0.2	0.75	0.722	0.088	0.085	0.081	0.256	0.249	0.259
TMAR	M	0.2	0.75	0.459	0.088	0.091	0.071	0.08	0.078	0.08
MNAR	M	0.7	0.75	0.218	0.085	0.081	0.09	0.116	0.121	0.127
TMAR	M	0.7	0.75	0.476	0.076	0.082	0.097	0.578	0.602	0.606
MNAR	M	0.2	0.25	1.555	0.093	0.085	0.072	0.576	0.587	0.515
TMAR	M	0.2	0.25	0.495	0.076	0.061	0.082	0.212	0.229	0.289
MNAR	M	0.7	0.25	0.154	0.089	0.076	0.083	0.407	0.442	0.428
TMAR	M	0.7	0.25	0.437	0.077	0.088	0.075	0.636	0.589	0.634
MNAR	K	0.2	0.75	0.983	0.078	0.074	0.066	0.288	0.295	0.28
TMAR	K	0.2	0.75	0.238	0.071	0.071	0.073	0.08	0.071	0.064
MNAR	K	0.7	0.75	0.034	0.083	0.077	0.079	0.294	0.284	0.302
TMAR	K	0.7	0.75	0.041	0.088	0.056	0.078	0.059	0.059	0.062
MNAR	K	0.2	0.25	1.773	0.086	0.082	0.077	0.538	0.524	0.515
TMAR	K	0.2	0.25	0.494	0.074	0.093	0.082	0.175	0.207	0.252
MNAR	K	0.7	0.25	0.135	0.065	0.066	0.066	0.51	0.515	0.509
TMAR	K	0.7	0.25	0.058	0.07	0.068	0.06	0.129	0.142	0.136
MNAR	EM-Spline	0.2	0.75	1.308	0.312	0.319	0.32	0.49	0.498	0.498
TMAR	EM-Spline	0.2	0.75	0.287	0.263	0.275	0.303	0.305	0.305	0.299
MNAR	EM-Spline	0.7	0.75	0.132	0.168	0.204	0.229	0.565	0.559	0.559
TMAR	EM-Spline	0.7	0.75	0.096	0.2	0.171	0.182	0.271	0.259	0.265
MNAR	EM-Spline	0.2	0.25	3.493	0.27	0.272	0.265	0.934	0.927	0.923
TMAR	EM-Spline	0.2	0.25	0.38	0.303	0.306	0.314	0.308	0.322	0.318
MNAR	EM-Spline	0.7	0.25	0.287	0.311	0.3	0.299	0.812	0.823	0.807
TMAR	EM-Spline	0.7	0.25	0.142	0.244	0.247	0.239	0.365	0.375	0.406
MNAR	EM-Regression	0.2	0.75	1.155	0.283	0.265	0.278	0.457	0.455	0.456
TMAR	EM-Regression	0.2	0.75	0.317	0.266	0.296	0.277	0.258	0.247	0.263
MNAR	EM-Regression	0.7	0.75	0.074	0.312	0.293	0.314	0.316	0.337	0.318
TMAR	EM-Regression	0.7	0.75	0.33	0.289	0.295	0.251	0.138	0.125	0.115
MNAR	EM-Regression	0.2	0.25	2.915	0.211	0.219	0.228	0.872	0.916	0.987
TMAR	EM-Regression	0.2	0.25	0.404	0.274	0.262	0.265	0.331	0.315	0.323
MNAR	EM-Regression	0.7	0.25	0.259	0.271	0.272	0.255	0.737	0.718	0.707
TMAR	EM-Regression	0.7	0.25	0.266	0.26	0.271	0.264	0.198	0.178	0.186
MNAR	EM-ARIMA	0.2	0.75	1.251	0.323	0.319	0.337	0.491	0.48	0.482
TMAR	EM-ARIMA	0.2	0.75	0.213	0.306	0.298	0.303	0.292	0.29	0.301
MNAR	EM-ARIMA	0.7	0.75	0.073	0.33	0.338	0.322	0.48	0.493	0.49
TMAR	EM-ARIMA	0.7	0.75	0.043	0.218	0.206	0.235	0.188	0.198	0.186
MNAR	EM-ARIMA	0.2	0.25	3.145	0.25	0.275	0.276	0.923	0.881	0.861
TMAR	EM-ARIMA	0.2	0.25	0.488	0.296	0.321	0.286	0.325	0.292	0.361
MNAR	EM-ARIMA	0.7	0.25	0.228	0.321	0.33	0.325	0.694	0.711	0.701
TMAR	EM-ARIMA	0.7	0.25	0.08	0.288	0.284	0.269	0.219	0.229	0.244

Table 3.: Median absolute relative bias for α_{11}, σ_1, σ_2, σ_3, λ_{11}, λ_{12}, and λ_{13} for the γ = 0 and 30% missingness condition. MCAR was excluded as it behaves similarly to TMAR, and ATMAR and MAR were excluded as they behave similarly to MNAR.

Missingness	Imputation	True α	True λ^2	α_{11}	σ_1	σ_2	σ_3	λ_{11}	λ_{12}	λ_{13}
MNAR	M	0.2	0.75	0.722	0.096	0.079	0.074	0.22	0.214	0.219
TMAR	M	0.2	0.75	0.512	0.092	0.073	0.067	0.077	0.08	0.079
MNAR	M	0.7	0.75	0.135	0.086	0.089	0.096	0.266	0.288	0.311
TMAR	M	0.7	0.75	0.479	0.077	0.075	0.073	0.956	0.966	0.979
MNAR	M	0.2	0.25	1.565	0.079	0.107	0.077	0.523	0.545	0.542
TMAR	M	0.2	0.25	0.51	0.071	0.091	0.085	0.209	0.199	0.172
MNAR	M	0.7	0.25	0.105	0.086	0.099	0.097	0.264	0.265	0.3
TMAR	M	0.7	0.25	0.464	0.096	0.095	0.061	1.006	0.932	0.983
MNAR	K	0.2	0.75	0.984	0.094	0.087	0.074	0.281	0.276	0.274
TMAR	K	0.2	0.75	0.224	0.08	0.09	0.083	0.06	0.06	0.068
MNAR	K	0.7	0.75	0.038	0.073	0.064	0.103	0.222	0.231	0.235
TMAR	K	0.7	0.75	0.027	0.076	0.085	0.059	0.055	0.058	0.05
MNAR	K	0.2	0.25	1.461	0.078	0.074	0.071	0.395	0.424	0.388
TMAR	K	0.2	0.25	0.422	0.069	0.087	0.078	0.225	0.227	0.238
MNAR	K	0.7	0.25	0.084	0.063	0.065	0.062	0.337	0.312	0.356
TMAR	K	0.7	0.25	0.044	0.072	0.069	0.075	0.113	0.128	0.137
MNAR	EM-Spline	0.2	0.75	1.396	0.324	0.31	0.326	0.475	0.457	0.461
TMAR	EM-Spline	0.2	0.75	0.256	0.26	0.273	0.285	0.282	0.277	0.276
MNAR	EM-Spline	0.7	0.75	0.147	0.161	0.182	0.175	0.489	0.487	0.483
TMAR	EM-Spline	0.7	0.75	0.073	0.19	0.167	0.182	0.135	0.135	0.123
MNAR	EM-Spline	0.2	0.25	3.836	0.273	0.253	0.267	0.972	0.969	0.968
TMAR	EM-Spline	0.2	0.25	0.286	0.317	0.283	0.288	0.245	0.276	0.315
MNAR	EM-Spline	0.7	0.25	0.282	0.276	0.306	0.294	0.752	0.748	0.744
TMAR	EM-Spline	0.7	0.25	0.086	0.212	0.211	0.242	0.29	0.271	0.263
MNAR	EM-Regression	0.2	0.75	1.133	0.261	0.285	0.29	0.412	0.409	0.419
TMAR	EM-Regression	0.2	0.75	0.315	0.26	0.291	0.263	0.222	0.209	0.218
MNAR	EM-Regression	0.7	0.75	0.368	0.331	0.324	0.34	1.027	1.147	1.08
TMAR	EM-Regression	0.7	0.75	0.47	0.3	0.249	0.274	1.001	1.012	1
MNAR	EM-Regression	0.2	0.25	2.383	0.25	0.225	0.227	0.741	0.786	0.694
TMAR	EM-Regression	0.2	0.25	0.5	0.264	0.28	0.294	0.274	0.294	0.297
MNAR	EM-Regression	0.7	0.25	0.677	0.319	0.318	0.316	0.925	0.994	0.953
TMAR	EM-Regression	0.7	0.25	0.485	0.281	0.253	0.269	0.65	0.607	0.634
MNAR	EM-ARIMA	0.2	0.75	1.366	0.343	0.344	0.329	0.459	0.471	0.485
TMAR	EM-ARIMA	0.2	0.75	0.219	0.284	0.299	0.3	0.273	0.278	0.274
MNAR	EM-ARIMA	0.7	0.75	0.115	0.315	0.313	0.343	0.406	0.408	0.41
TMAR	EM-ARIMA	0.7	0.75	0.036	0.18	0.152	0.188	0.12	0.12	0.111
MNAR	EM-ARIMA	0.2	0.25	2.617	0.275	0.274	0.278	0.771	0.777	0.748
TMAR	EM-ARIMA	0.2	0.25	0.324	0.301	0.291	0.296	0.317	0.347	0.308
MNAR	EM-ARIMA	0.7	0.25	0.226	0.32	0.346	0.347	0.598	0.612	0.597
TMAR	EM-ARIMA	0.7	0.25	0.053	0.266	0.243	0.253	0.182	0.177	0.221

Table 4.: Median absolute relative bias for α_{11}, σ_1, σ_2, σ_3, λ_{11}, λ_{12}, and λ_{13} for the γ = 0.3 and 30% missingness condition. MCAR was excluded as it behaves similarly to TMAR, and ATMAR and MAR were excluded as they behave similarly to MNAR.