Leader Following Consensus of Discrete-Time Second-Order Multi-Agent Systems: A Simulation Study

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To my parents, Nalan and Zeynel Cebeci,

for their love and support throughout my life.

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Abstract

In this research, we study the leader following consensus problem for discrete-time multi-agent systems. The multi-agent systems we consider are composed of informed follower agents, uninformed follower agents and the virtual leader agent. A distributed consensus algorithm is proposed to realize our control objectives: position and velocity tracking of the virtual leader moving at constant speed. A comprehensive simulation study is presented to investigate system performance and to illustrate that the control algorithm achieves the desired objectives. In particular, under our proposed control algorithm, all informed follower agents will track the virtual leader. And, if an uninformed follower agent is connected to an informed follower agent from time to time, it will also be able to track the virtual leader. Numerical simulations demonstrate that even a small percentage of informed follower agents can drive a large portion of follower agents to track the virtual leader. It is also shown that the consensus rate increases as the percentage of informed follower agents increases, as the sensing radius increases, or as the total number of the follower agents increases.

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Chapter 1

Introduction

Decision-making and group behavior in biological systems has long been of interest to researchers in physics, biology and social sciences [1-10]. Most well-known group behaviors in the nature appear in the form of flocking of birds, herding of sheep and cattle, packs of wolves, schooling of fishes, and swarming of bats, insects and bacteria. Humans also work in groups. Many different groups form highly complex societies. Societies need group decisions and cannot function properly without consensus. Collective behaviors have been of great interest to computer scientists as well [10-13]. Helbing et al. [10] analyzes a specific case of flocking in humans. By using a pedestrian behavior model they simulate life threatening scenarios such as fire in a crowded building and propose optimal escape strategies that integrate both individualistic inclinations and flocking instincts of the man. In 1985, Amkraut et al. [11] created an animated video of a flock of birds using computer graphics methods. In this work the trajectories of birds were predetermined with each bird and each static object in the environment having rejection forces around their boundaries. Clearly, it was not intended for flock modeling. In [12] Reynolds proposed the first computer animation of flocking. Basis of the animation depends on three flocking rules: 1) Separation: avoid collision with neighboring agents; 2) Alignment: match velocities with neighboring agents; 3) Cohesion: stay within close proximity to neighboring agents. Reynolds' flocking is considered to be groundbreaking work and has been used in Hollywood movies and computer games. Vicsek et al. [13] have designed a discrete-time model of a self-driven first order particle system. All autonomous agents move in the Euclidean space at the same speed but with different heading angles. The heading angle of each agent is individually updated by averaging its own angle and neighboring agents' angles.

Pioneering work of physicists and computer scientists have attracted the attention of control theory researchers and engineers around the world. Engineering implementations of multi-agent systems cover cooperative control of unmanned aerial and underwater vehicles, robots and sensor networks [14-21]. Such applications could be used in robot-assisted search and rescue in hazardous environments; exploration, surveillance and combat missions; swarm of medical microbots; and traffic systems.

Over the past two decades, control scientists have provided many different control algorithms, mathematical tools and theorems for flocking, consensus and alignment problems for multi-agent systems [22-38]. The classical flocking model of Reynolds has been a milestone for many flocking research papers. In 2006, Olfati-Saber [22] proposed a theoretical framework based on the flocking rules of Reynolds. He proposed three algorithms. The first algorithm embodies the three rules of Reynolds but leads to fragmentation of agents and therefore does not exhibit actual flocking behavior. The second algorithm combines the first algorithm with a navigational feedback term. Each individual agent is able to track the leader when provided its location and velocity information. As a result, flocking is guaranteed and all agents remain cohesive and move with the same velocity in free space following the leader. The third algorithm enables obstacle avoidance while flocking. In Olfati-Saber's algorithm it is assumed that all follower agents can access the information of the leader but both in nature and practical examples this may not always be realistic. In 2009, Su et al. [23] provided a modified version of Olfati-Saber's second algorithm. With only a fraction of the follower agents having information of the leader, the majority of the agents can still track the virtual leader. Both the algorithms in [22] and the algorithms in [23] contain an attractive/repulsive potential function to realize the rule of cohesion, i.e., to keep the flock members at an ideal distance to each other. It is a non-linear gradient term with symmetrical properties. Each agent has a sensing radius and all the agents inside the radius are considered its neighbor agents. If two agents within the same neighborhood get too close, the potential function applies as a repelling force. If agents tend to move further away, attractive forces apply. When at the ideal distance the potential function reaches the unique minimum. In [22] and [23], the controllers and the dynamic multi-agent systems operate in continuous-time. Both papers use the Lyapunov approach [36] to prove the asymptotic behavior of the system. Attractive/repulsive potential function is a quite commonly used tool in flocking problems in the literature and has also been applied in [24-29]. In such systems [24-28], agents form distance based switching networks. Other switching networks have also been pursued [31-34].

In some of the consensus problems in literature [30-34], graph theory and stochastic matrix properties are provided as stability analysis tools. Jadbabaie et al. [30] points out that, there does not exist a common quadratic Lyapunov function to show that discrete-time Vicsek model is stable. Using dynamical systems and graph theory methods such as the Wolfowitz lemma [35], they provide a mathematical explanation to emergence behavior in discrete-time Vicsek model and prove that a consensus value in alignment can be reached. In [31,32], the consensus problem under time-delay is studied. Qin et al. [33] investigate the consensus of discrete-time second-order multi-agents under switching topologies and consider the coordination of four mobile robots as an application. However, the distance-based connectivity approach for discrete-time second-order multi-agent system consensus has been lacking in literature. The aim of this thesis is to fill that gap. We design a consensus algorithm for position and velocity tracking of the virtual leader moving at a constant speed. Then we carry out a comprehensive simulation study to investigate system performance and to illustrate that the control algorithm achieves the desired objectives. Under our proposed control algorithm, the simulation results show that all informed follower agents will track the virtual leader. And, if an uninformed follower agent is connected to an informed follower agent from time to time, it will also be able to track the virtual leader. Numerical simulations demonstrate that even a small percentage of informed follower agents can drive a large portion of follower agents to track the virtual leader. It is also shown that the consensus rate increases as the percentage of informed follower agents increases, as the sensing radius increases, or as the total number of the follower agents increases.

The remainder of this thesis is organized as follows. In Chapter II, we present our dynamic system model, introduce the fundamental background in graph theory and describe our algorithm. In Chapter III, we report on our simulation study and make a detailed system performance analysis. Chapter IV concludes the thesis.

Chapter 2

Backgrounds and Algorithm Description

Our multi-agent system is composed of informed agents and uninformed agents. Informed agents have access to the virtual leader's position and velocity information whereas the uninformed hold no information regarding the leader. The virtual leader agent is referred to as the gamma agent and determines the group speed and direction. In this section, we present our agent model, provide theoretical background for graph theory concepts we refer to, and describe our control algorithm.

2.1 System Dynamics

Our multi-agent system consists of a group of N dynamic agents that are moving in an n dimensional Euclidean space. The equations of motion have been discretized using the forward Euler approximation method and the resulting system dynamics is represented as,

$$\begin{aligned} x_i(k+1) - x_i(k) &= Tv_i(k), \\ v_i(k+1) - v_i(k) &= Tu_i(k), \quad i = 1, 2, ..., N, \end{aligned} \tag{2.1}$$

where $x_i, v_i \in \mathcal{R}^n$ are respectively the position and velocity vectors of agent i, $u_i \in \mathcal{R}^n$ is the control input acting on agent i, and T is the sampling time. For notational convenience, we define vectors $x_i, v_i \in \mathcal{R}^n$ as,

$$x_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ \vdots \\ x_{in} \end{bmatrix}, \quad v_i = \begin{bmatrix} v_{i1} \\ v_{i2} \\ \vdots \\ v_{in} \end{bmatrix}.$$

Agents can only communicate with other agents that are within their neighborhoods. Each agent has limited communication capability which is constrained by the sensing radius R. We define the neighborhood of agent i at time instant k as:

$$\mathcal{N}_i(k) = \{j : \|x_i - x_j\| < R, j = 1, 2, ..., N, j \neq i\}, \tag{2.2}$$

where $\|.\|$ is the Euclidean norm. The communication is undirected, that is, if agent i has access to the information of agent j, agent j would also have access to the information of agent i. Relative distances between agents might vary during their evolution, thus neighbors of agent i might change. Agents form switching networks based on the distance information.

It is assumed that the virtual leader, referred to as the γ-agent, moves along the free space with a constant velocity v_d. There is no control input applied on the γ-agent and its dynamics is described by,

$$\begin{aligned} x_\gamma(k+1) - x_\gamma(k) &= T v_d(k), \\ v_\gamma(k+1) - v_\gamma(k) &= 0, \end{aligned} \tag{2.3}$$

with initial conditions $(x_\gamma(0), v_\gamma(0)) = (x_d, v_d)$. We will construct a control algorithm that enables both the position and velocity of the follower agents to converge to those of the γ-agent and achieve consensus.

2.2 Graph Theory

Before proceeding to the description of our control algorithm, we introduce some basic concepts of graph theory. Graphs are mathematical objects consisting of a set of nodes and edges. In this thesis, we denote a neighboring graph at time instant k by $\mathcal{G}(k) = \{\mathcal{V}, \mathcal{E}(k)\}$, where the set of nodes (vertices) $\mathcal{V} = \{\nu_1, \nu_2, ..., \nu_N\}$ represents the agents in the group, and $\mathcal{E}(k) = \{(i,j) \in \mathcal{V} \times \mathcal{V} : i \sim j\}$ represents the set of edges containing unordered pairs of vertices. Here, i and j indicates that agent i and j are neighbors. In our multi-agent network, the edges do not have an orientation, and thus the graphs are undirected. Because of the fact that neighbors of each agent can change during the course of evolution, the network may switch until consensus is reached. This means that the neighboring graphs may vary in time. Let us illustrate the concept of undirected graphs with an example.

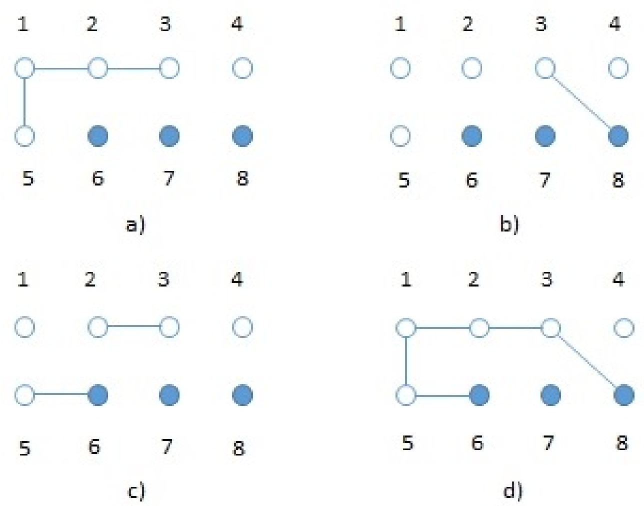

Figure 2.1: Neighboring graphs at given time instants $k_i, i = 1, 2, 3, 4$.

In Figure 2.1, we present a series of neighboring graphs at given time instants

$k_i, i = 1, 2, 3, 4$, from a) through d). In this example, the size of the whole agent group $N = 8$. Thus the set of nodes in all graphs is $\mathcal{V} = \{\nu_1, \nu_2, \nu_3, \nu_4, \nu_5, \nu_6, \nu_7, \nu_8\}$. Uninformed agents are denoted by circles and the informed agents are denoted by dots. In our example, neighbors of agents change in time. The neighboring relations for each graph is represented by their edges:

$$
\begin{aligned}
\mathcal{E}(k_1) &= \{(1,2),(1,5),(2,3)\}\,, \\
\mathcal{E}(k_2) &= \{(3,8)\}\,, \\
\mathcal{E}(k_3) &= \{(2,3),(5,6)\}\,, \\
\mathcal{E}(k_4) &= \{(1,2),(1,5),(2,3),(3,8),(5,6)\}\,.
\end{aligned}
$$

Another important concept in graph theory is adjacency. Nodes i and j are said to be adjacent at time k if the pair (i, j) is an element of the edge set $\mathcal{E}(k)$. $A(k) = (a_{ij}(k))$ is called the adjacency matrix and can be used to represent the graph $\mathcal{G}(k)$. That is $a_{ij}(k) = 1$ if agents i and j are adjacent, $a_{ij}(k) = 0$ otherwise. For instance, the adjacency matrix of the graph at time instant k_4 is given as

$$
A(k_4) = \begin{bmatrix}
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

If a pair of agents (i, j) are adjacent, then there exists a link (path) between the two. A path is defined as a sequence of distinct vertices such that consecutive vertices are adjacent. There can be multiple paths between two nodes i and j. To see this, we can look at graph $\mathcal{G}(k_4)$ once again. It is clear that there exists a path between ν_1 and ν_8: $P(\nu_1, \nu_8) = \nu_1\nu_2\nu_3\nu_8$. An uninformed agent can be directly or indirectly connected to an informed agent through such a path. Here ν_1 is indirectly connected to ν_8 through ν_2 and ν_3, where ν_3 is directly linked to the informed node ν_8.

2.3 The Control Algorithm

As stated earlier, the main goal of this thesis is to achieve virtual-leader tracking of multi-agents in discrete-time. Both [22] and [23] provide continuous-time virtual leader tracking algorithms. The control input in [23] is given by,

$$\begin{aligned} u_i &= -\sum_{j\in\mathcal{N}_i(t)} \nabla_{x_i}\psi_\alpha(||x_i - x_j||_\sigma) + \sum_{j\in\mathcal{N}_i(t)} a_{ij}(k)(v_j - v_i) \\ &\quad -h_i\left[c_1(x_i - x_\gamma) + c_2(v_i - v_\gamma)\right]. \end{aligned} \tag{2.4}$$

The algorithm consists of three main components. First component is the nonlinear gradient term which contains $\psi_\alpha(||x_i - x_j||_\sigma)$, the attractive/repulsive function that acts as the cohesive force to keep agents at ideal distance d_α. Function ψ_α reaches its maximum as $||x_i - x_j||_\sigma$ goes to 0, is at its unique minimum if i and j are at the ideal distance d_α, and is constant for $||x_i - x_j||_\sigma \geq R_\sigma$. The σ-norm $||_\sigma||$ of a vector is a map $\mathcal{R}^n \to \mathcal{R}_+$ defined as,

$$||z||_\sigma = \frac{1}{\varepsilon}\left[\sqrt{1 + \varepsilon||z||^2} - 1\right]$$

with a parameter $\epsilon > 0$. Unlike the norm $||z||$, which may not be differentiable at $z = 0$, the map $||z||_\sigma$ is differentiable everywhere. This property of σ-norm enables the construction of attractive/repulsive function ψ_α. The function ψ_α (see Figure 2.2) is a nonnegative smooth pairwise potential function of the distance

$z = (||x_i - x_j||_\sigma)$ that regulates the position between agent i and its flockmates.

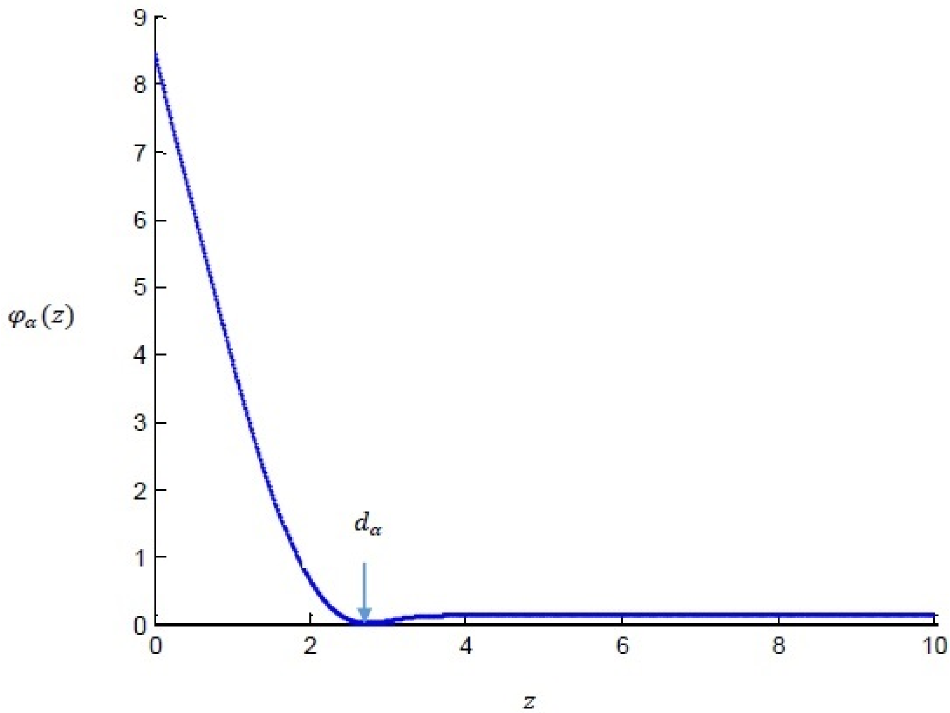

Figure 2.2: An attractive-repulsive function

Second component of the algorithm provides the velocity alignment between neighboring agents, where $A(t) = (a_{ij}(t))$ is the adjacent matrix which is defined as,

$$a_{ij}(t) = \begin{cases} 0, & \text{if } i = j, \\ \rho_h(||x_j - x_i||_\sigma / ||r||_\sigma), & \text{if } i \neq j, \end{cases}$$

with the bump function $\rho_h(z), h \in (0,1)$, being

$$\rho_h(z) = \begin{cases} 1, & \text{if } z \in [0,h), \\ \frac{1}{2}[1 + cos(\pi(\frac{z-h}{1-h})], & \text{if } z \in [h,1], \\ 0 & \text{otherwise.} \end{cases}$$

The third component of the algorithm is called the navigational feedback term where $h_i = 1$ if agent i is informed and $h_i = 0$ otherwise. Both analytical and numerical results of [23] show that not only all the informed agents but also some uninformed agents track the virtual leader. Indeed, the simulation results show that with a very small group of informed agents, the algorithm can cause most of the agents to move with the desired velocity, i.e., reach velocity consensus with the virtual leader. Motivated by these results, we discretized the algorithm for leader following consensus of discrete-time multi-agent systems. However, in our simulations with the discretized algorithm we have observed that the consensus is achieved only with a very small sampling time and with a very restrictive set of control algorithm parameters. Here, we take a simpler connectivity approach and design our discrete-time consensus algorithm,

$$\begin{aligned} u_i &= p_1 \sum_{S_j \in \mathcal{N}_i(k)} a_{ij}(k)\left(x_j(k) - x_i(k)\right) + p_2 \sum_{S_j \in \mathcal{N}_i(k)} a_{ij}(k)\left(v_j(k) - v_i(k)\right) \\ &\quad - h_i\left[c_1\left(x_i(k) - x_\gamma(k)\right) + c_2\left(v_i(k) - v_\gamma(k)\right)\right], \end{aligned} \tag{2.5}$$

where $p_1, p_2 > 0$ and $A(k) = (a_{ij}(k))$ is the adjacency matrix which is defined by,

$$a_{ij}(k) = \begin{cases} 1, & \text{if } \|x_i - x_j\| < R, \\ 0, & \text{if } \|x_i - x_j\| \geq R. \end{cases}$$

Agents i and j will be considered connected if their distance is less than R, and $a_{ij}(k)$ will be set to 1. Otherwise $a_{ij}(k)$ will be set to 0. We assume that first M agents are informed agents ($1 \leq M \leq N$). That is $h_i = 1$ for $i = 1, 2, 3, ..., M$, and $h_i = 0$ for $i = M+1, M+2, M+3, ..., N$.

There are two different types of uninformed agents: Type I and Type II. Denote the union of all neighboring graphs across a nonempty finite time interval $[k_i, k_{i+1}), k_{i+1} > k_i$ as $\hat{\mathcal{G}}(k_i, k_{i+1})$, whose edges are the union of the edges of those neighboring graphs. For an uninformed agent, if there is a path between itself and one informed agent in the union $\hat{\mathcal{G}}(k_i, k_{i+1})$, then we say that there exists a joint path between the uninformed agent and the informed agent during the finite time interval $[k_i, k_{i+1})$. An uninformed agent is called a Type I uninformed agent if there exists an infinite sequence of contiguous, nonempty and uniformly bounded time-intervals $[k_i, k_{i+1}), i = 1, 2, \cdots$, such that across each time interval there exists a joint path between this agent and one informed agent. Otherwise, it is called a Type II uninformed agent. It is important to note that a Type I uninformed agent is not required to stay in touch with an informed agent all the time. It only requires to be linked with an informed agent through a joint path during each time interval $[k_i, k_{i+1})$. Such a joint link is established as long as this Type I uninformed agent is connected with an informed agent at any given time instant during the time interval $[k_i, k_{i+1})$, or it is connected at a time instant during $[k_i, k_{i+1})$ with another Type I uninformed agent which is connected with an informed agent at a different time instant during $[k_i, k_{i+1})$. In other words, a Type I uninformed agent is only required to get in touch, directly or indirectly, with an informed agent from time to time.

As the intervals $[k_i, k_{i+1}), i = 1, 2, \cdots$, are uniformly bounded, a Type I uninformed agent will not stay out of touch with an informed agent for a period longer than the bound on these intervals. Thus, there exists a sufficiently large $K > 0$ such that, for all $k \geq K$, there does not exist any joint path between any Type II uninformed agent and any informed agent. This assumption implies that an uninformed agent that is disconnected from all informed agents for a long enough period of time will stay disconnected from them forever. Under this assumption, all the informed agents and Type I uninformed agents cannot be influenced by Type II uninformed agents directly or indirectly for all $k \geq K$. Once the consensus is reached the interaction network will not switch and all Type I uninformed agents will have a path to an informed agent in the fixed interaction network.

Our extensive simulation study indicates that not only all the informed agents but also some uninformed agents will track the virtual leader. Indeed, the simulation results show that even a small percentage of informed follower agents can drive a large portion of follower agents to track the virtual leader. It is also shown that the consensus rate increases as the percentage of informed follower agents increases, as the sensing radius increases, or as the total number of the follower agents increases.

Chapter 3

Simulation Study

The demonstration of simulation results in this thesis is organized as follows. First we investigate the consensus performance of follower agents. We observe the trends in consensus rate and understand the factors effecting it. Second, we make a statistical analysis by further investigating the consensus rate for different sizes of agent groups with varying values of sensing radius and varying percentage of informed agents.

3.1 Consensus Performance

In this section, we investigate the consensus performance for $N = 20$ agents moving in the $n = 2$ dimensional Euclidean plane. The initial positions and velocities of each agent is generated randomly from boxes $[0, 10] \times [-10, 0]$ and $[0, 4] \times [-4, 0]$, respectively. The velocity of the virtual leader is set to $v_\gamma = [1, 1]^T$. The controller parameters are set to be $c_1 = c_2 = 2$ and $p_1 = p_2 = 0.02$.

Using these parameters, we simulate four different scenarios. In the first case, we choose the percentage of informed agents to be 10% and the sensing radius to be $R = 5$. We plot both the tracking errors and the trajectories of all agents to observe the convergence behavior. Then we proceed to the second scenario. This time we change the percentage of informed agents from 10% to 30% and compare the results with the previous scenario. In the third scenario, we change the percentage of informed agents from 30% to 50% and again compare the results. For the last case, keeping the percentage of informed agents at 50%, we increase the sensing radius from $R = 5$ to $R = 7$.

In all our scenarios, we also measure the consensus rate for a more accurate comparison. Our aim is to understand the effects of the percentage of informed agents and the sensing radius on the rate of consensus.

3.1.1 $N = 20$, $M = 2$, $R = 5$

The simulation results shown in Figures 3.1-3.4, indicate that the position and velocity tracking errors in all dimensions go to zero for informed agents only. The uninformed agents do not track the virtual leader.

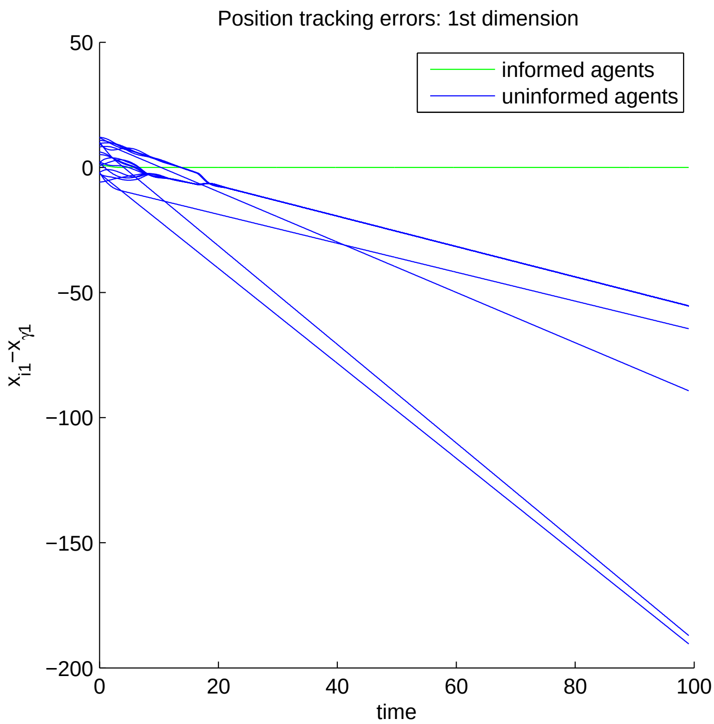

Figure 3.1: $N = 20$, $M = 2$, $R = 5$, $T = 0.1$s, $t = 100$s

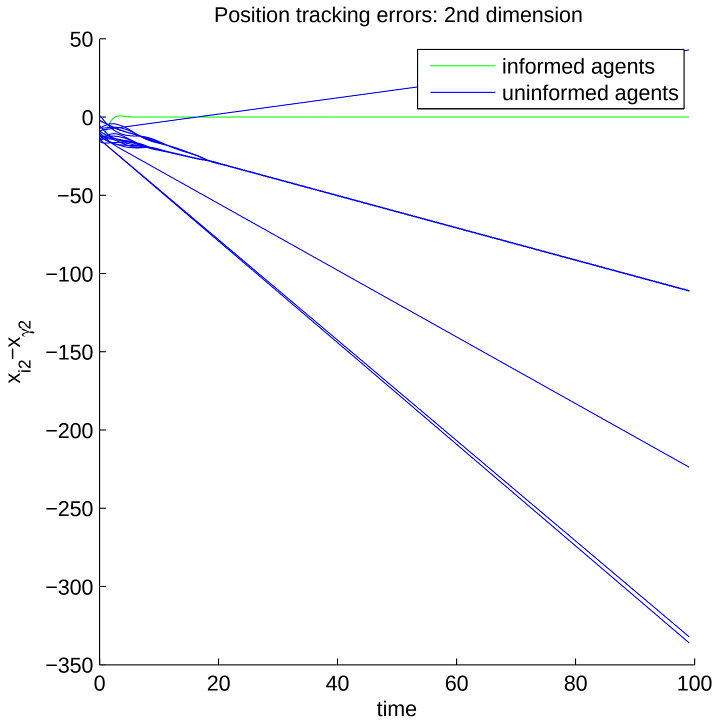

Figure 3.2: $N = 20$, $M = 2$, $R = 5$, $T = 0.1$s, $t = 100$s

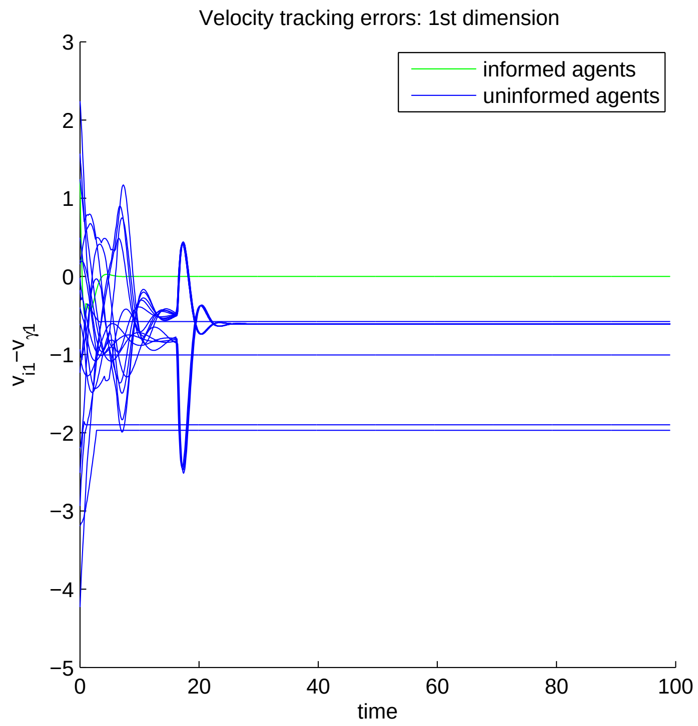

Figure 3.3: $N = 20$, $M = 2$, $R = 5$, $T = 0.1$s, $t = 100$s

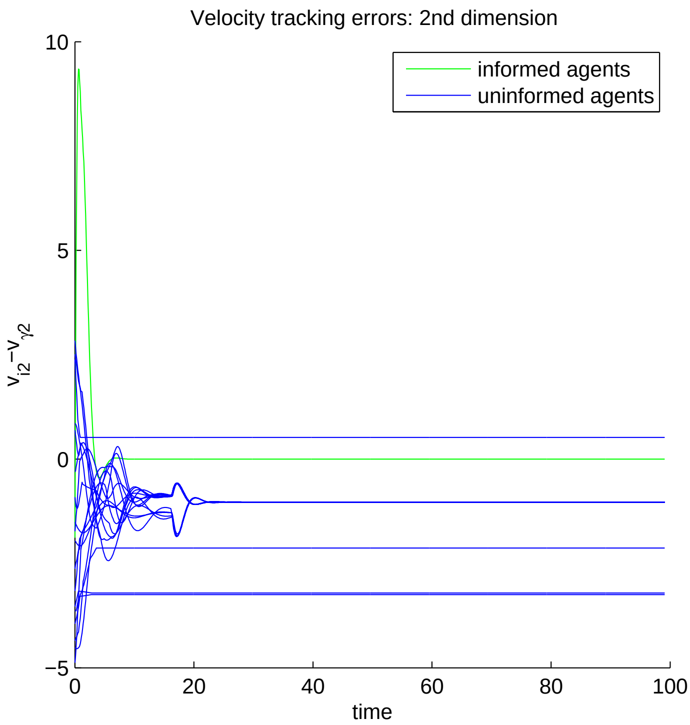

Figure 3.4: $N = 20$, $M = 2$, $R = 5$, $T = 0.1$s, $t = 100$s

The simulation results shown in Figures 3.5-3.8 indicate that the informed agents reach position and velocity consensus with the virtual leader while uninformed agents lose track of it. Velocities of all informed agents converge to the $v_\gamma = [1, 1]^T$. The rate of consensus for our first scenario is only 10% since we have only $M = 2$ informed agents which are the only agents that reach consensus with the virtual leader.

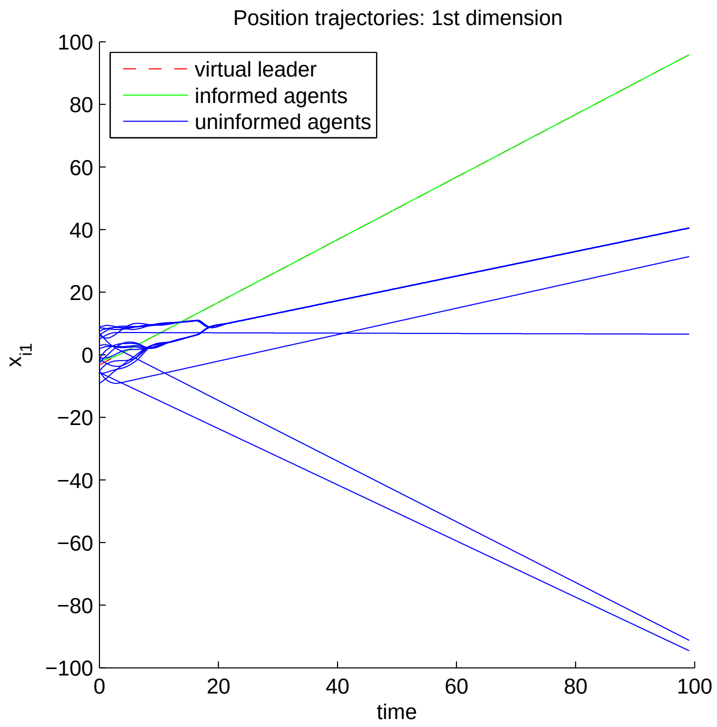

Figure 3.5: $N = 20$, $M = 2$, $R = 5$, $T = 0.1$s, $t = 100$s

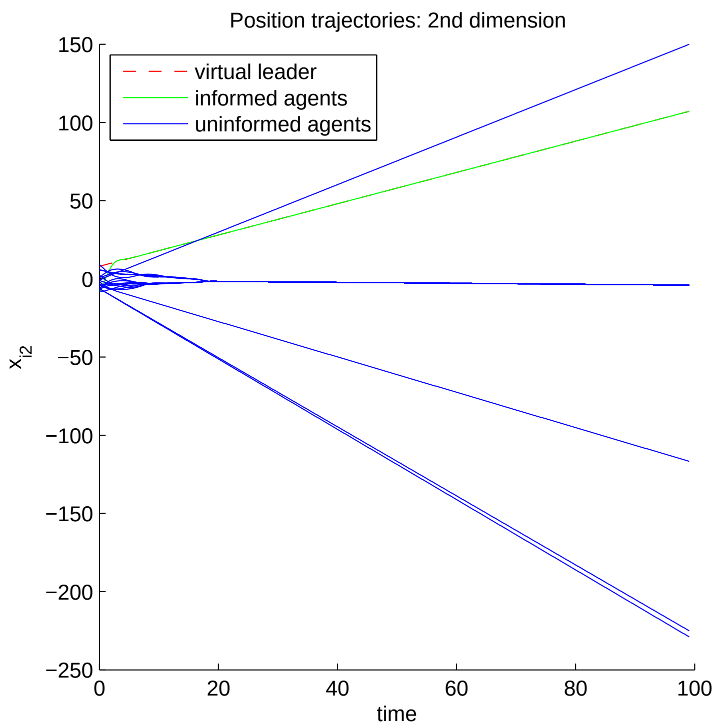

Figure 3.6: $N = 20$, $M = 2$, $R = 5$, $T = 0.1$s, $t = 100$s

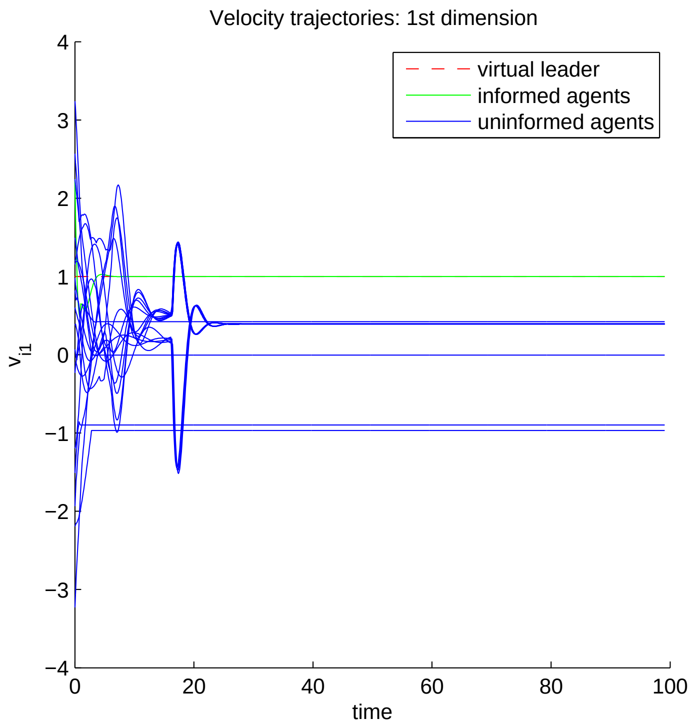

Figure 3.7: $N = 20$, $M = 2$, $R = 5$, $T = 0.1$s, $t = 100$s

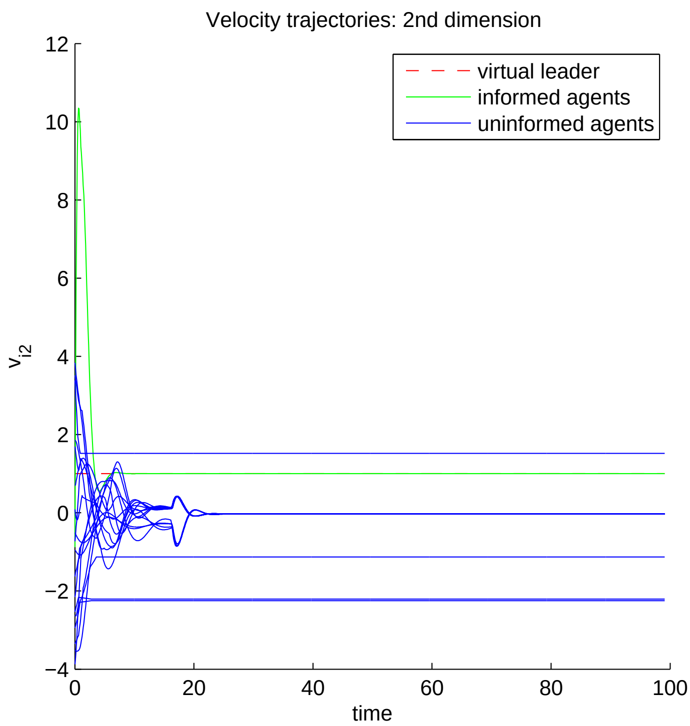

Figure 3.8: $N = 20$, $M = 2$, $R = 5$, $T = 0.1$s, $t = 100$s

3.1.2 $N = 20,\ M = 6,\ R = 5$

In our second scenario, we have increased the number of informed agents in the group. From Figures 3.9-3.12, we see that position and velocity tracking errors go to zero not only for all informed agents but also for some uninformed agents this time.

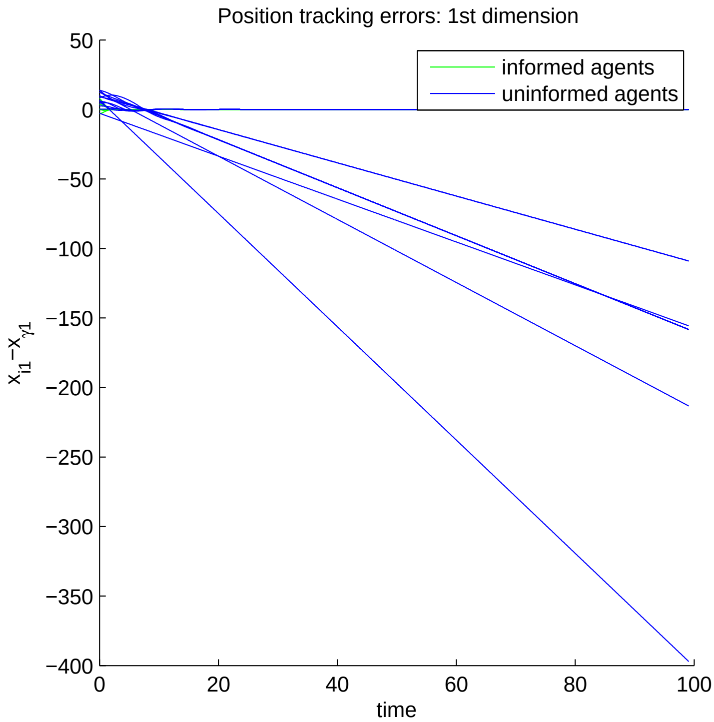

Figure 3.9: $N = 20,\ M = 6,\ R = 5,\ T = 0.1\text{s},\ t = 100\text{s}$

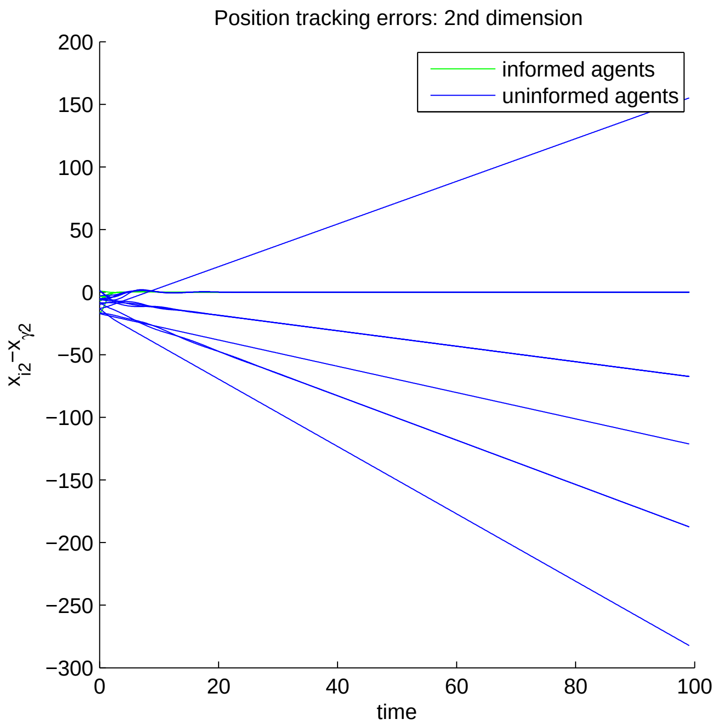

Figure 3.10: $N = 20$, $M = 6$, $R = 5$, $T = 0.1$s, $t = 100$s

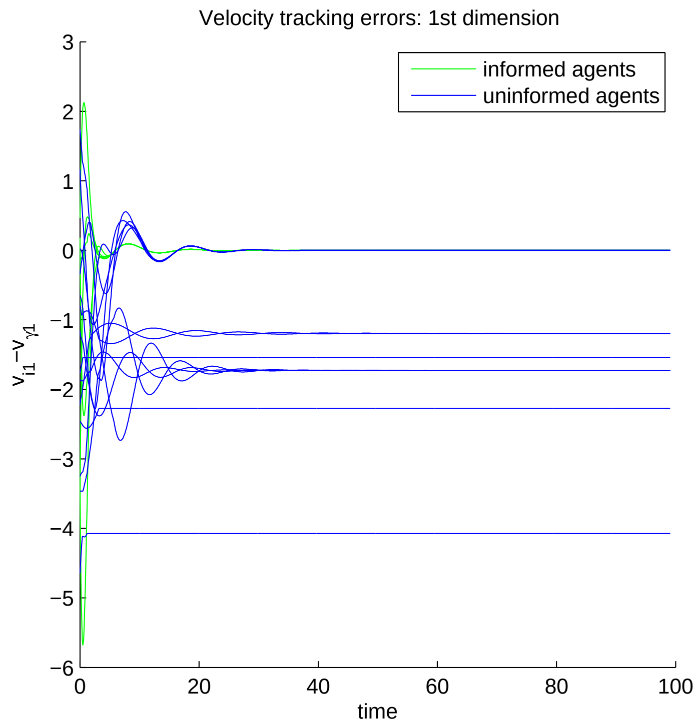

Figure 3.11: $N = 20$, $M = 6$, $R = 5$, $T = 0.1$s, $t = 100$s

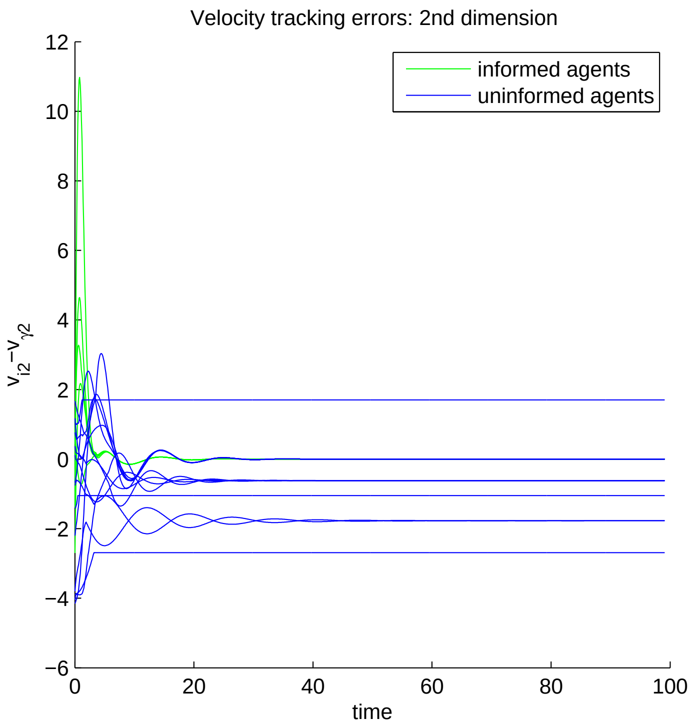

Figure 3.12: $N = 20$, $M = 6$, $R = 5$, $T = 0.1$s, $t = 100$s

All informed agents and some uninformed agents reach position and velocity consensus with the virtual leader (see Figures 3.13-3.16). The rate of consensus for our second example is 55%.

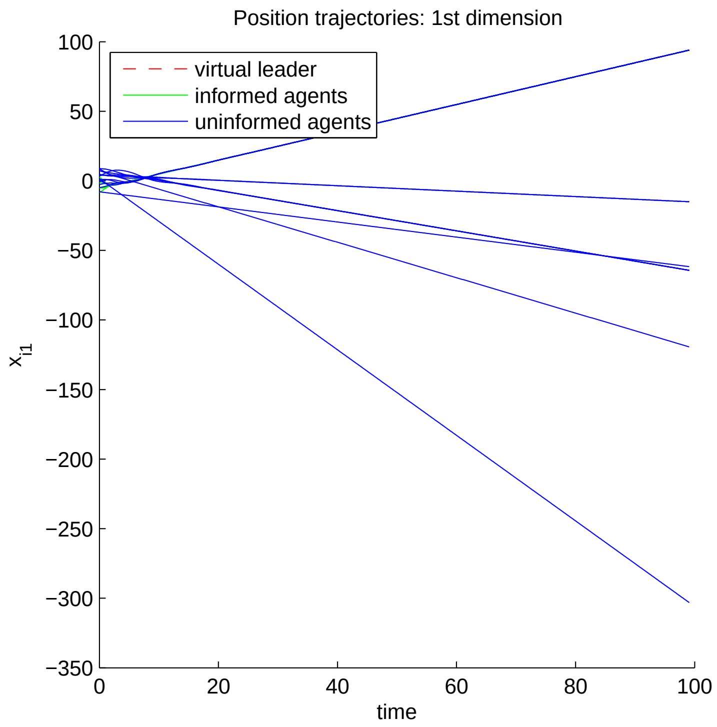

Figure 3.13: $N = 20,\ M = 6,\ R = 5,\ T = 0.1\mathrm{s},\ t = 100\mathrm{s}$

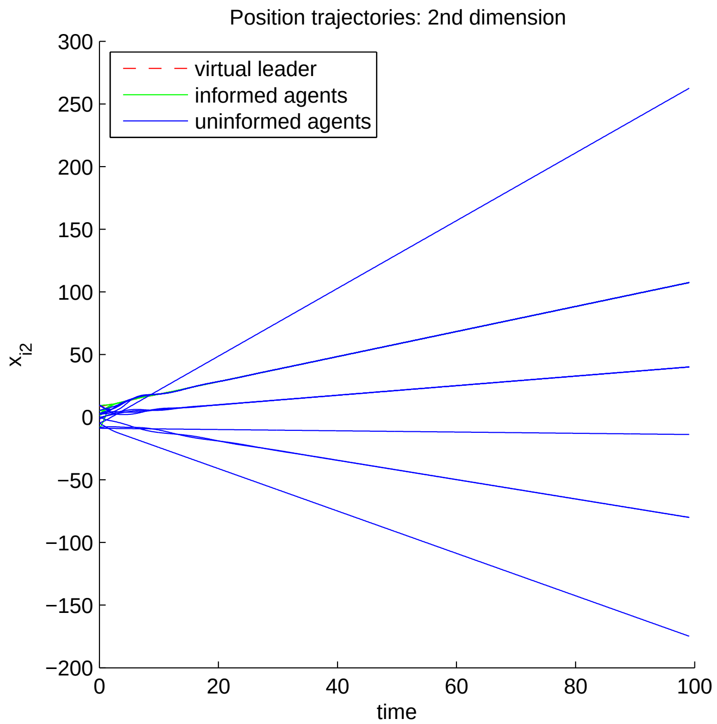

Figure 3.14: $N = 20,\ M = 6,\ R = 5,\ T = 0.1\mathrm{s},\ t = 100\mathrm{s}$

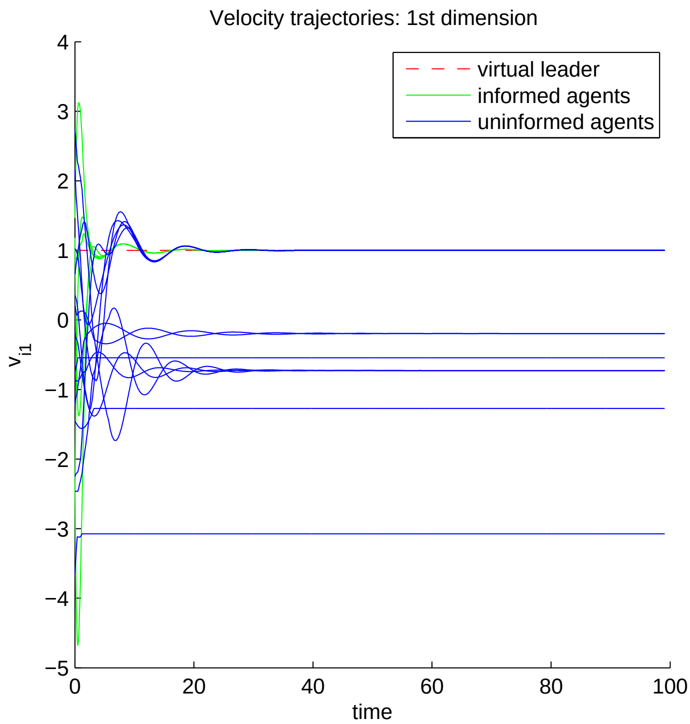

Figure 3.15: $N = 20$, $M = 6$, $R = 5$, $T = 0.1$s, $t = 100$s

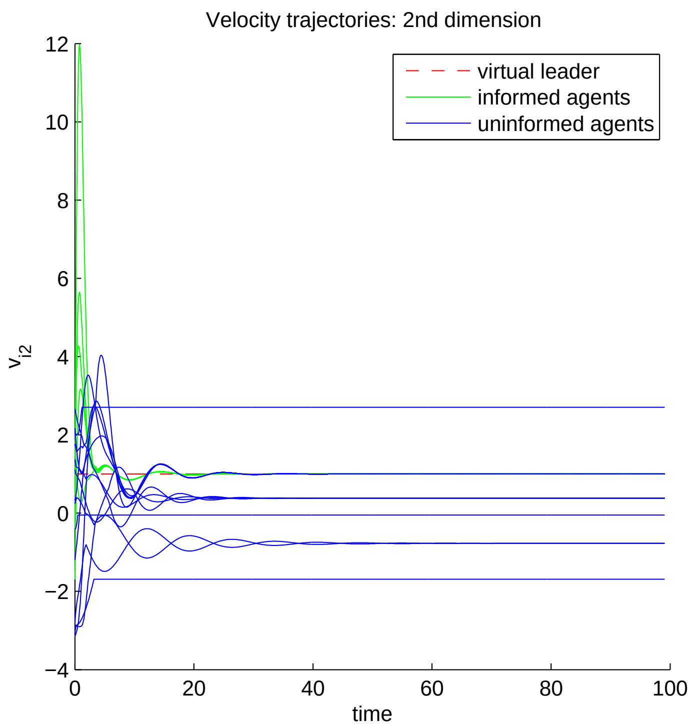

Figure 3.16: $N = 20$, $M = 6$, $R = 5$, $T = 0.1$s, $t = 100$s

3.1.3 $N = 20,\ M = 10,\ R = 5$

In our third scenario, we have 50% of all agents informed ($M = 10$). From Figures 3.17-3.20, we see that position and velocity tracking errors for all informed agents and a large number of uninformed agents go to zero.

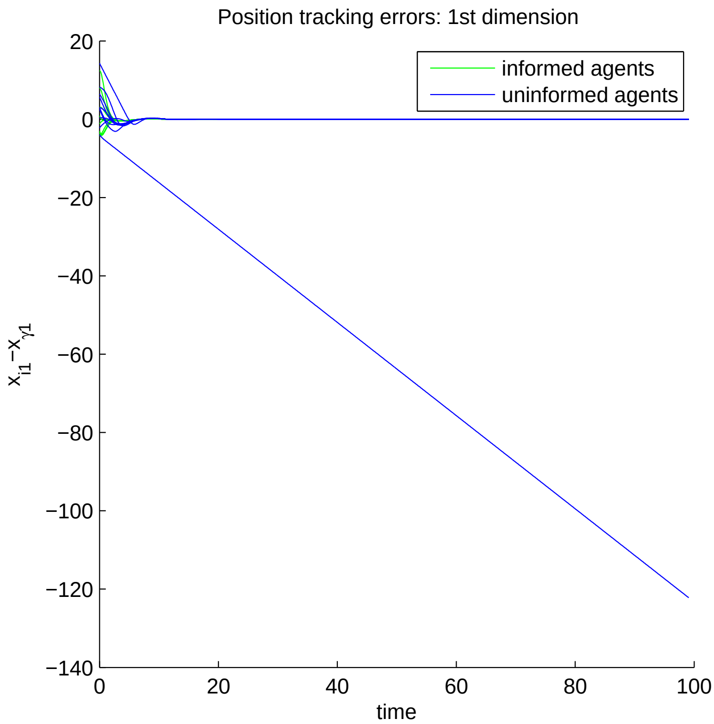

Figure 3.17: $N = 20$, $M = 10$, $R = 5$, $T = 0.1$s, $t = 100$s

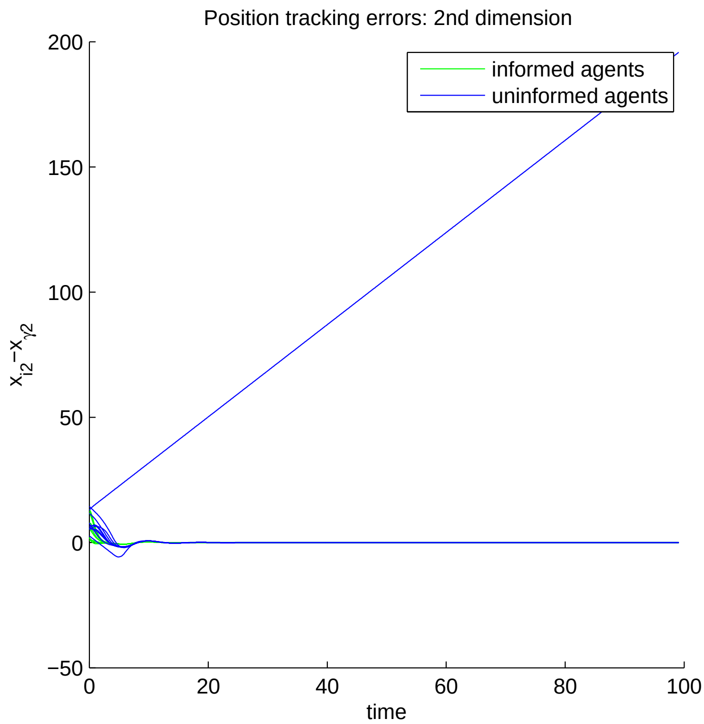

Figure 3.18: $N = 20$, $M = 10$, $R = 5$, $T = 0.1$s, $t = 100$s

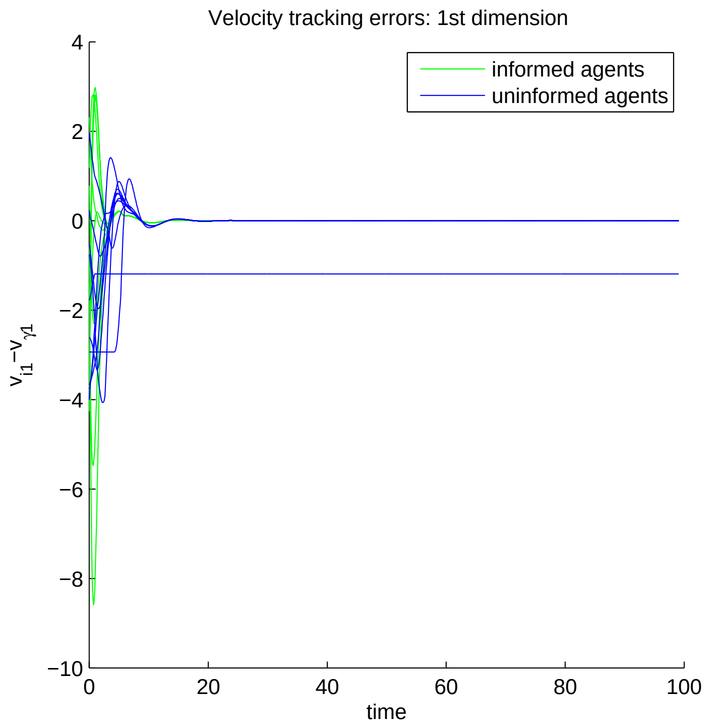

Figure 3.19: $N = 20$, $M = 10$, $R = 5$, $T = 0.1$s, $t = 100$s

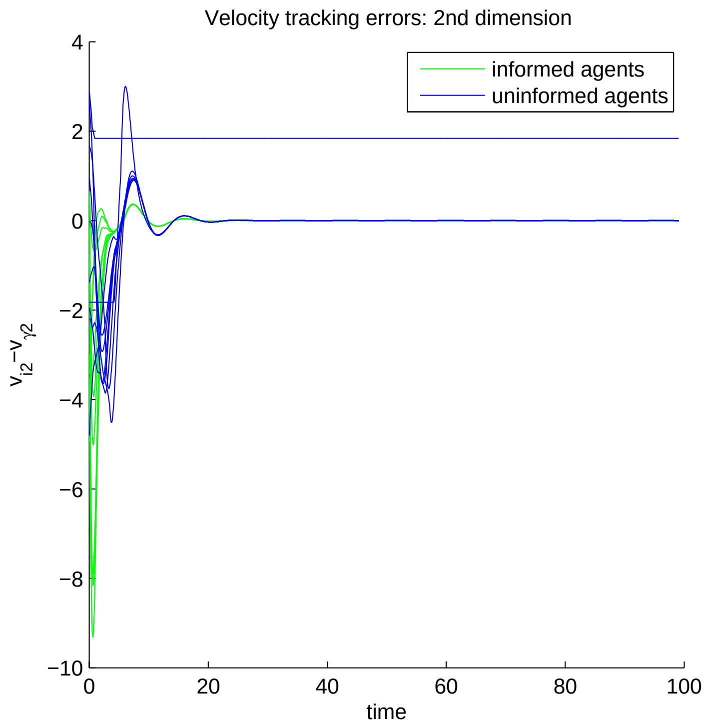

Figure 3.20: $N = 20$, $M = 10$, $R = 5$, $T = 0.1$s, $t = 100$s

Simulation results shown in Figures 3.21-3.24 indicate that all informed agents and a large number of uninformed agents reach position and velocity consensus with the virtual leader. The rate of consensus is measured as 95%.

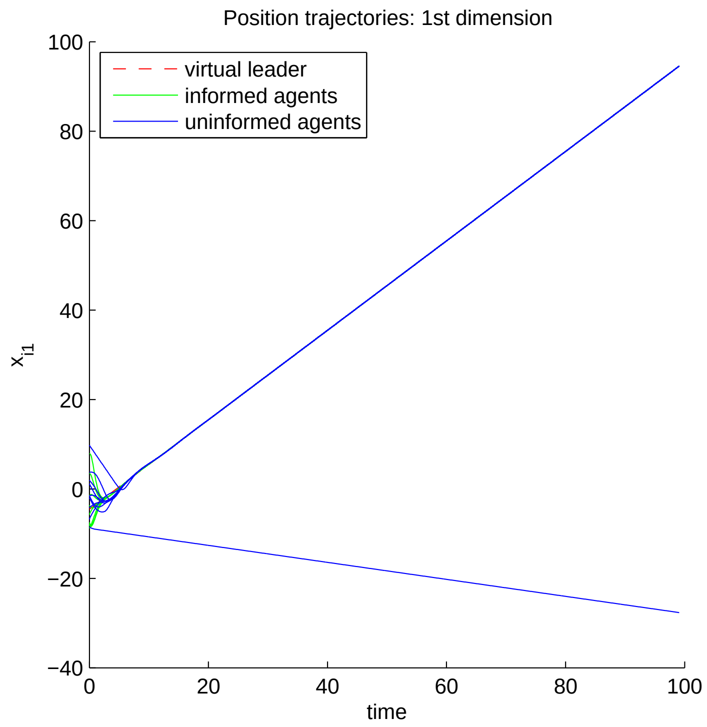

Figure 3.21: $N = 20,\ M = 10,\ R = 5,\ T = 0.1\text{s},\ t = 100\text{s}$

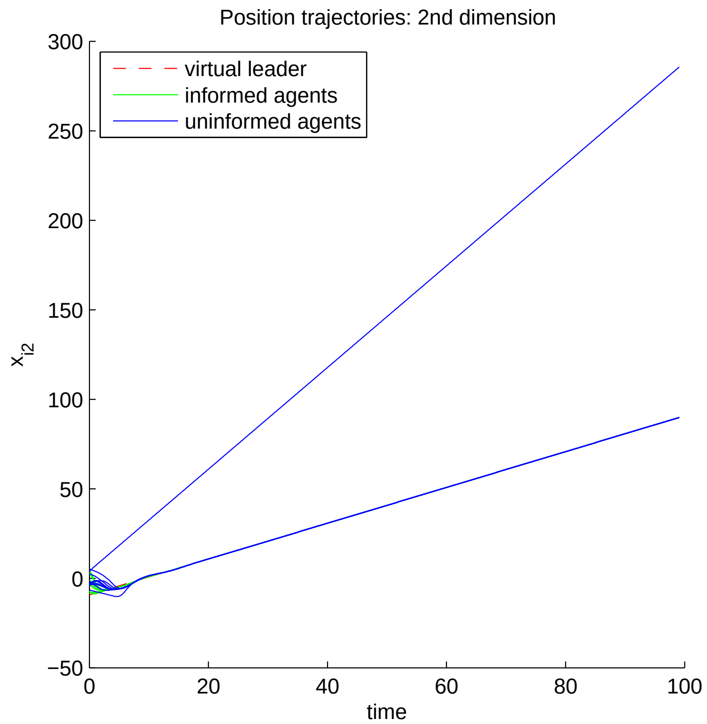

Figure 3.22: $N = 20,\ M = 10,\ R = 5,\ T = 0.1\text{s},\ t = 100\text{s}$

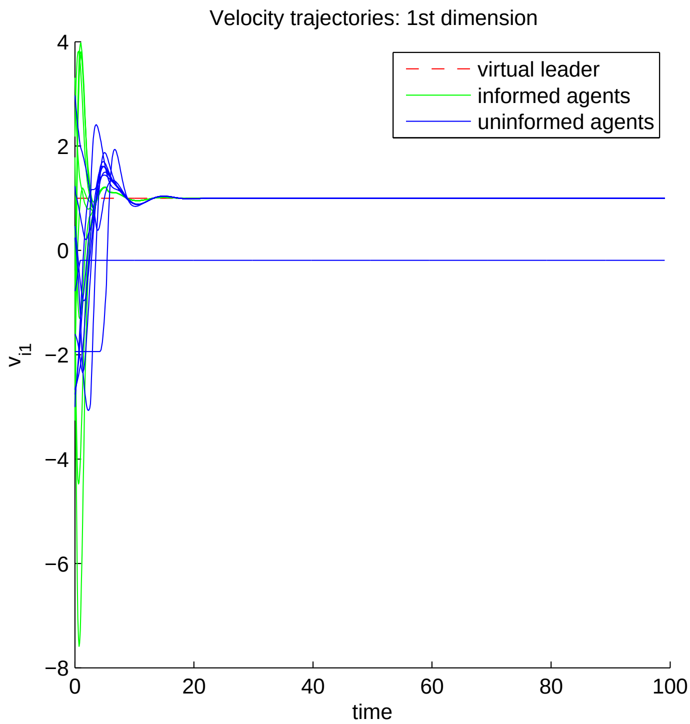

Figure 3.23: $N = 20$, $M = 10$, $R = 5$, $T = 0.1$s, $t = 100$s

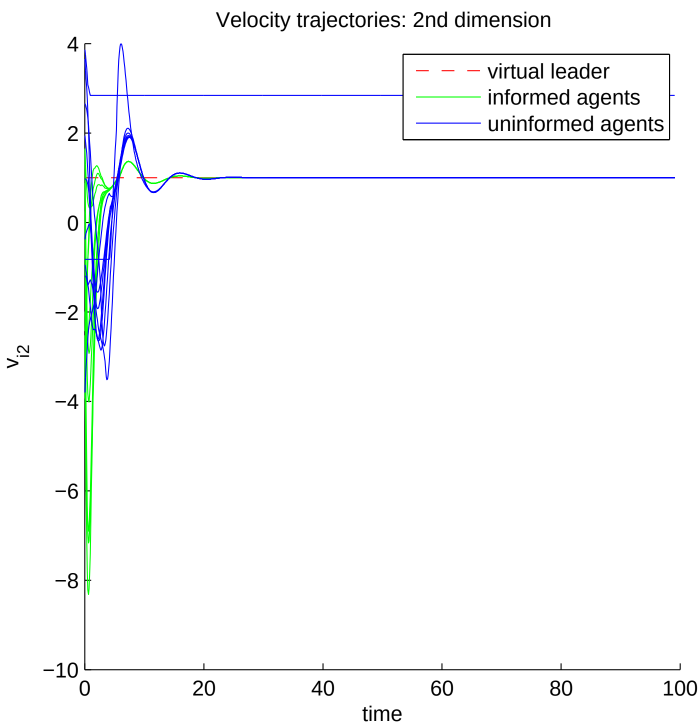

Figure 3.24: $N = 20$, $M = 10$, $R = 5$, $T = 0.1$s, $t = 100$s

3.1.4 $N = 20,\ M = 10,\ R = 7$

In our final scenario, we increase the sensing radius to $R = 7$. We see from Figures 3.25-3.28 that the position and velocity tracking errors of all agents both informed and uninformed, go to zero.

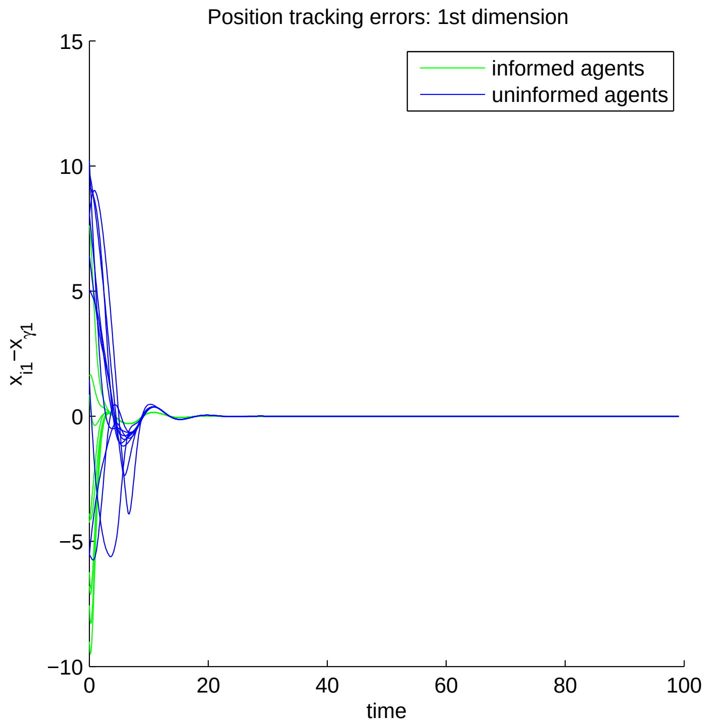

Figure 3.25: $N = 20$, $M = 10$, $R = 7$, $T = 0.1$s, $t = 100$s

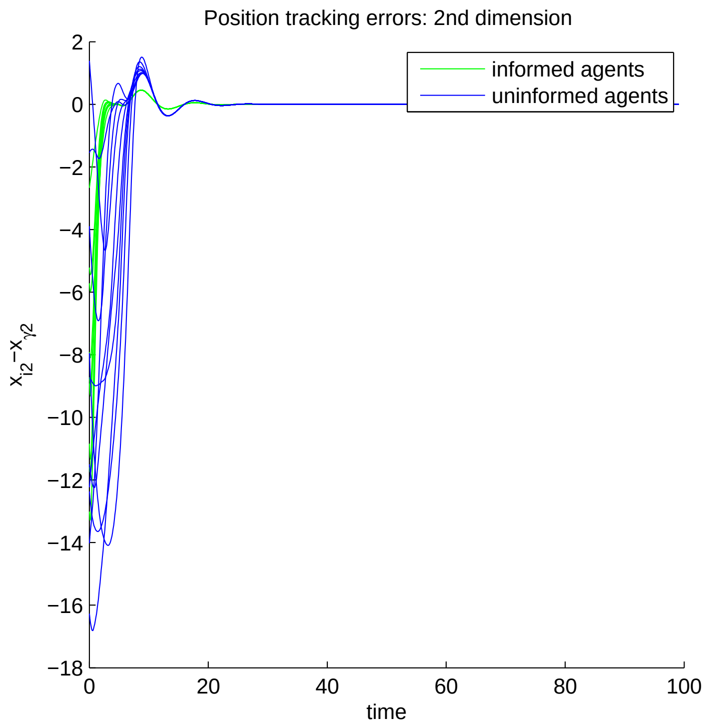

Figure 3.26: $N = 20$, $M = 10$, $R = 7$, $T = 0.1$s, $t = 100$s

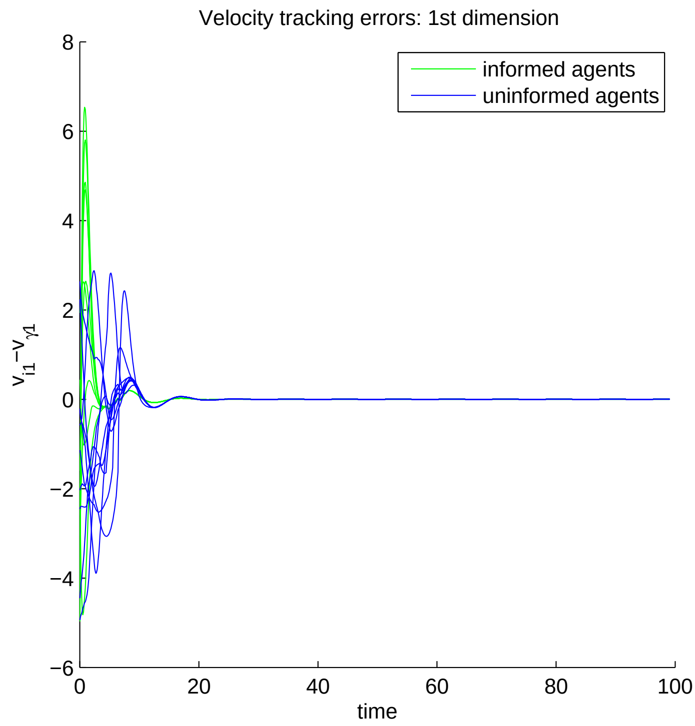

Figure 3.27: $N = 20$, $M = 10$, $R = 7$, $T = 0.1$s, $t = 100$s

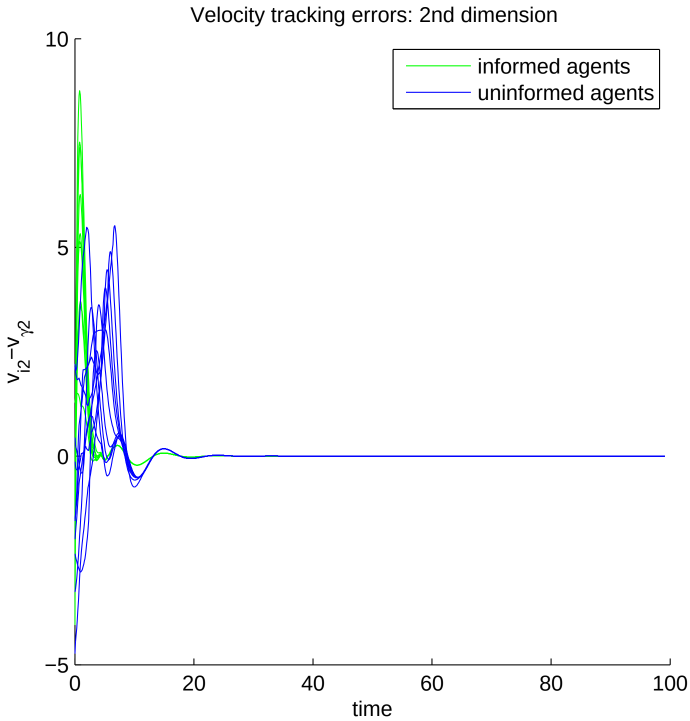

Figure 3.28: $N = 20$, $M = 10$, $R = 7$, $T = 0.1$s, $t = 100$s

As demonstrated in Figures 3.29-3.32, all agents reach position and velocity consensus with the virtual leader. The consensus rate is 100%.

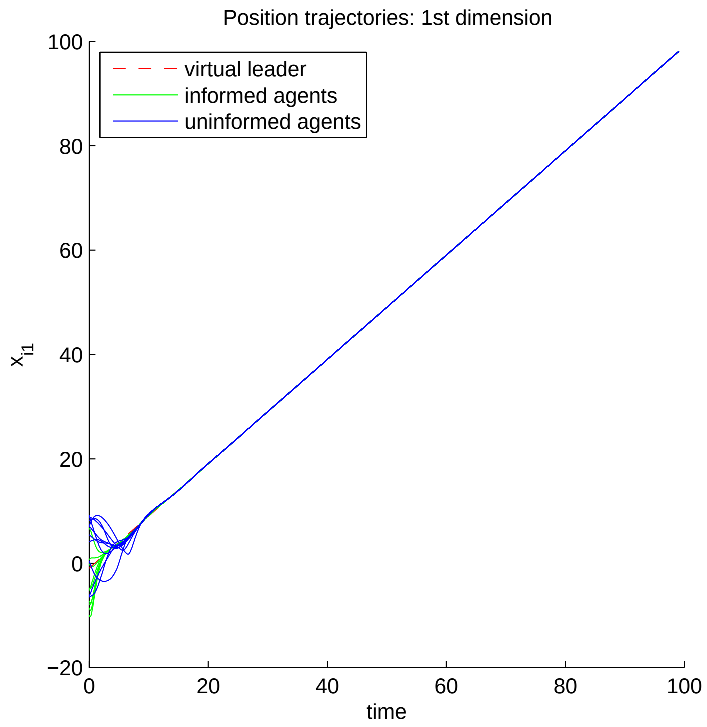

Figure 3.29: $N = 20,\ M = 10,\ R = 7,\ T = 0.1\text{s},\ t = 100\text{s}$

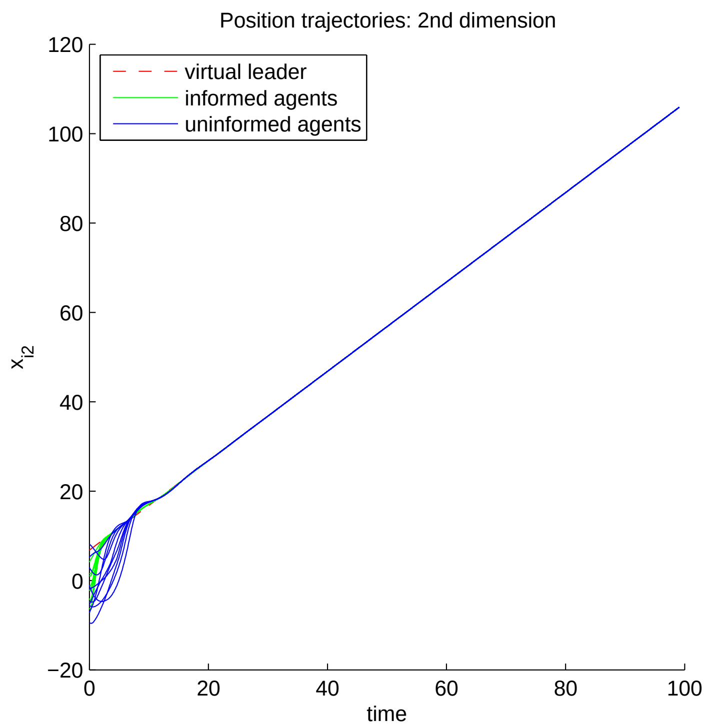

Figure 3.30: $N = 20,\ M = 10,\ R = 7,\ T = 0.1\text{s},\ t = 100\text{s}$

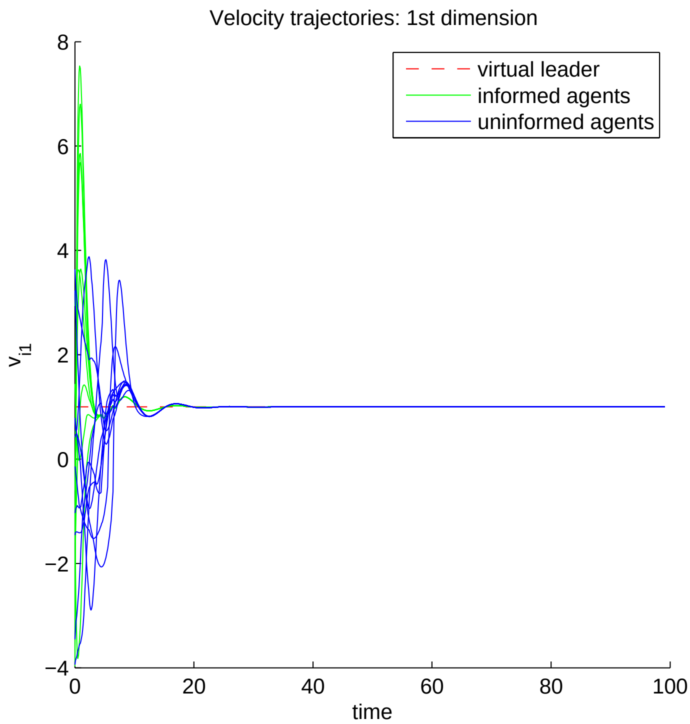

Figure 3.31: $N = 20$, $M = 10$, $R = 7$, $T = 0.1$s, $t = 100$s

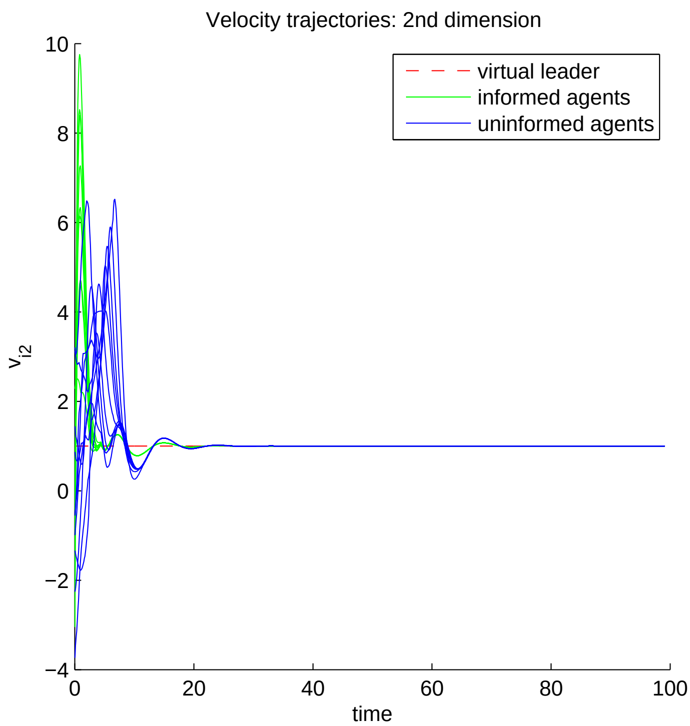

Figure 3.32: $N = 20$, $M = 10$, $R = 7$, $T = 0.1$s, $t = 100$s

The consensus performance simulation results show that the rate of informed agents and sensing radius R affects the consensus rate dramatically. The trend we observe is that the rate of consensus will increase as the percentage of informed agents and sensing radius increase but we need a comprehensive simulation study that covers a larger scale of scenarios for further investigation. Therefore, we make our statistical analysis in the next section.

3.2 Statistical Analysis

In our statistical analysis, we measure the rate of consensus with respect to varying sensing radius values that are $R = 5, 7, 10$ and 20 for agent groups with sizes $N = 20, 50, 100, 300, 500$ and 1000. In these measurements first we consider only 10% of whole agent groups are informed agents. Then we consider $30\%, 50\%$ and 100% are informed agents and repeat our measurements with all N and R that are given.

Let us recall that all simulation results depend on the randomly generated initial conditions of positions and velocities of the agents. The consensus rate might vary each time the same case is simulated thus for each consensus measurement in this section, the consensus rate is the averaging of 50 realizations.

First, we present the tabular demonstration of our statistical analysis results. The rate of informed agents is fixed to 10% in Table 3.1. It is fixed to 30% and 50% in Tables 3.2 and 3.3, respectively. We present the consensus rate measurements with respect to varying sensing radius values for agent groups with different sizes.

Second, we present the graphical illustration of our results and the impact of variables on rate of consensus is discussed. In Figure 3.33, we illustrate the effect of increasing the percentage of informed agents from 10% to 100%. For instance, with $R = 5$ and 10% of $N = 100$ agents are informed agents, only about 38% of all follower agents reach consensus with the leader. If the percentage of informed

10% Informed				
N	$R = 5$	$R = 7$	$R = 10$	$R = 20$
20	18.00%	26.90%	28.50%	30.70%
50	30.48%	65.72%	67.48%	69.40%
100	38.56%	67.58%	99.74%	100.00%
300	38.57%	76.90%	99.79%	100.00%
500	38.81%	80.16%	99.81%	100.00%
1000	37.38%	86.02%	99.85%	100.00%

Table 3.1: Rate of Consensus With 10% Informed Agents

30% Informed				
N	$R = 5$	$R = 7$	$R = 10$	$R = 20$
20	53.80%	82.20%	98.80%	100.00%
50	55.84%	85.52%	99.44%	100.00%
100	58.62%	88.06%	99.50%	100.00%
300	58.27%	88.36%	99.73%	100.00%
500	58.58%	89.96%	99.78%	100.00%
1000	58.06%	89.98%	99.75%	100.00%

Table 3.2: Rate of Consensus With 30% Informed Agents

50% Informed				
N	$R = 5$	$R = 7$	$R = 10$	$R = 20$
20	71.30%	86.80%	99.20%	100.00%
50	71.84%	89.56%	99.56%	100.00%
100	73.00%	91.22%	99.68%	100.00%
300	73.25%	91.09%	99.70%	100.00%
500	73.04%	91.80%	99.72%	100.00%
1000	73.25%	90.67%	99.67%	100.00%

Table 3.3: Rate of Consensus With 50% Informed Agents

agents is increased to 50%, this time 73% of the group reach consensus with the virtual leader. Thus, we see that the rate of consensus increases as the percentage of informed agents increase. Furthermore, the larger the group, the smaller the percentage of informed agents is needed to guide the majority of the group. For example, for $R = 7$, in order for 80% of the agents to reach consensus with the leader, approximately 30% of agents should be informed follower agents when the group size is $N = 100$ but only about 10% of the agents need to be the informed agents when the group size is $N = 1000$. Thus, for sufficiently large groups only a small fraction of informed agents will guide the majority of agents.

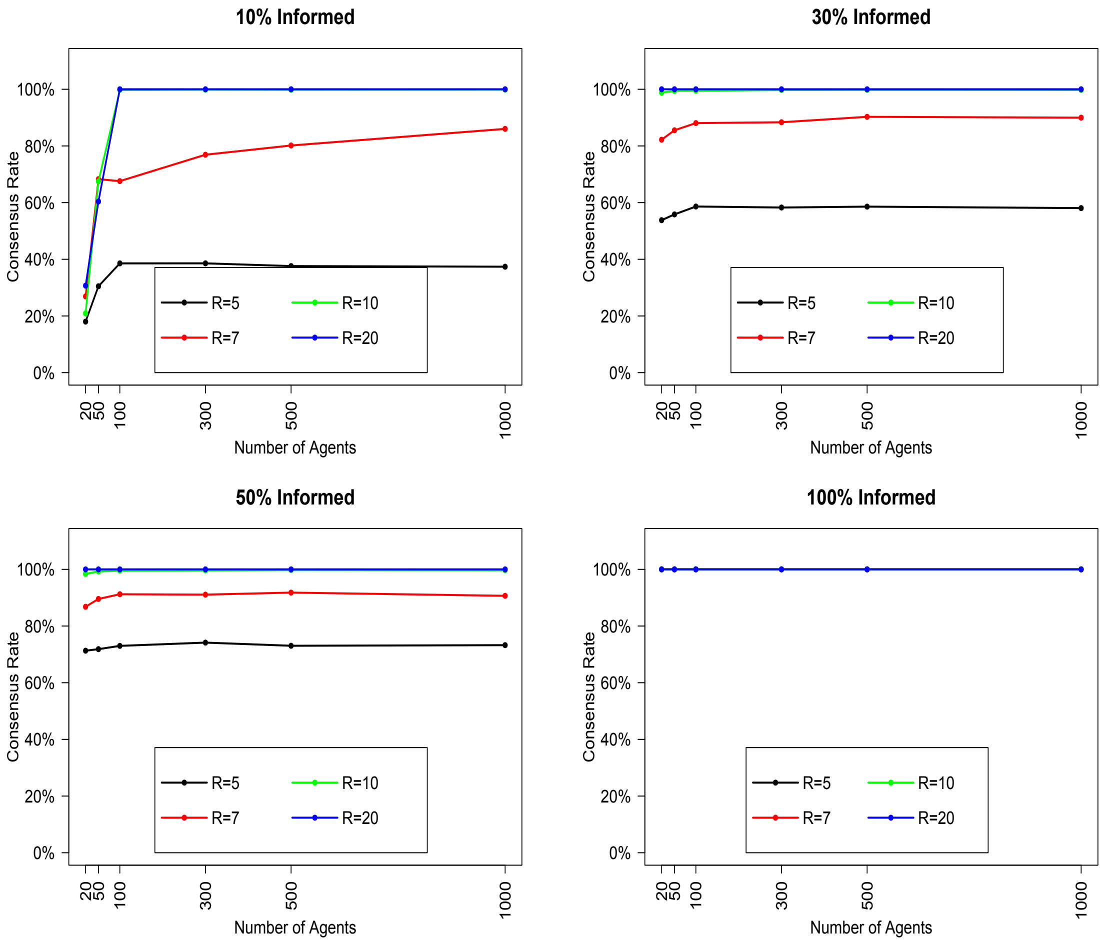

Figure 3.33: Effect of the Percentage of Informed Agents on the Consensus Rate

The communication capability is also an important factor affecting the consensus rate (see Figure 3.34). If the percentage of informed agents is 30%, for $N = 50$ agents the consensus rate is about 55% when $R = 5$. If the sensing radius $R = 7$ then the consensus rate becomes 85.52%. For $R = 10$, 99.44% consensus is achieved.

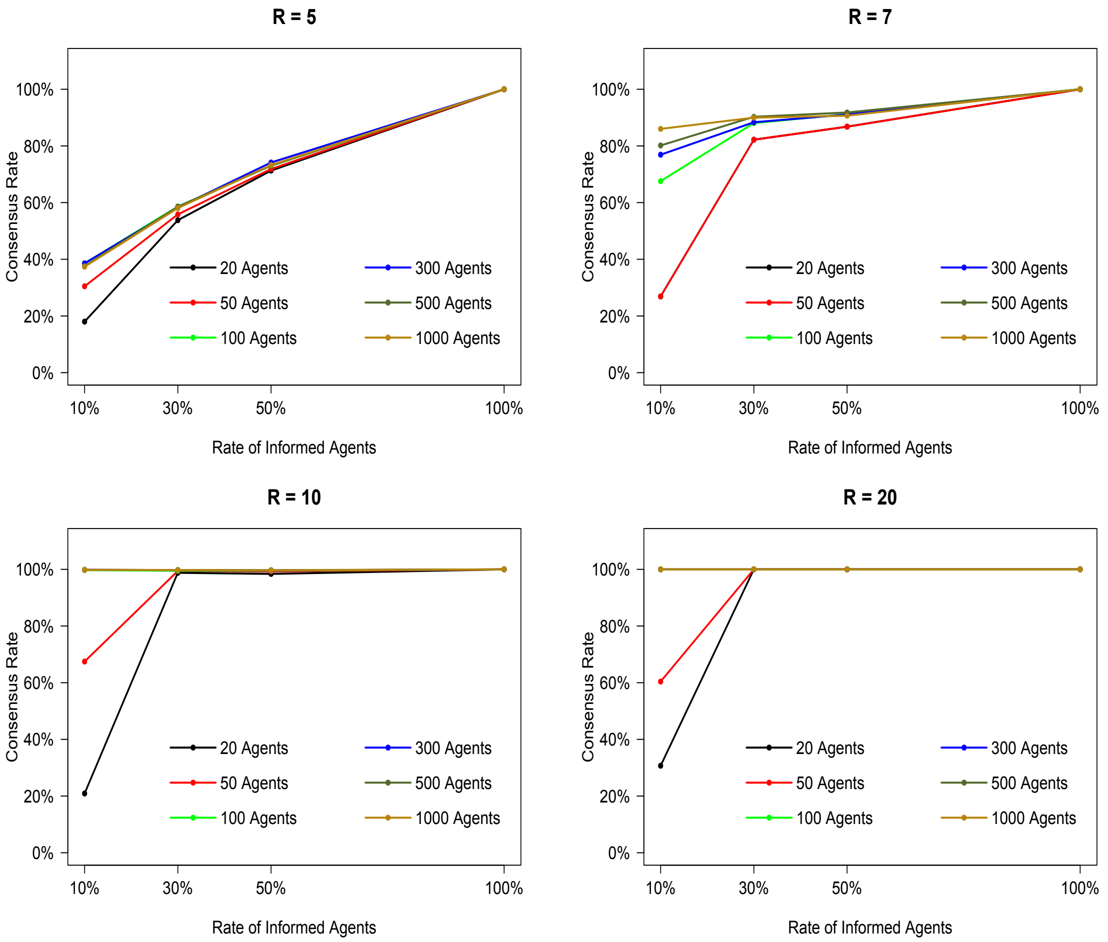

Figure 3.34: Effect of the Sensing Radius on the Consensus Rate

Chapter 4

Conclusion

In this work we have designed a control algorithm for leader following consensus of discrete-time second-order multi-agent systems. A comprehensive simulation study is presented to investigate system performance and to illustrate that the control algorithm achieves the consensus. The simulation results show that all informed follower agents will track the virtual leader. And, if an uninformed follower agent is connected to an informed follower agent from time to time, it will also be able to track the virtual leader. Numerical simulations demonstrate that even a small percentage of informed follower agents can drive a large portion of follower agents to track the virtual leader. The larger the size of the multi-agent system the bigger majority of the follower agents will track the virtual leader. Simulation results also show that when only a fraction of the follower agents are informed follower agents, both sensing radius of the follower agents and percentage of informed follower agents affects the consensus rate significantly. That is, the consensus rate increases as the percentage of informed follower agents increases or as the sensing radius increases.

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