HETEROGENEITY OF YOUNG AND OLD INDIVIDUALS: MACROECONOMIC EFFECTS AND POLICY IMPLICATIONS

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Heterogeneity of Young and Old Individuals: Macroeconomic Effects and Policy Implications

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(ABSTRACT)

This dissertation consists of two chapters which study the macroeconomic effects and policy implications of the heterogeneity of young and old individuals, especially in the process of population aging.

The first chapter examines the effects of (heterogeneous) household sentiment on business cycles and finds its implications for fiscal policy. Empirical analyses show that individuals, especially the young, can have more pessimistic or optimistic views about the future economy than the data-generating measure. This household irrationality stems from the fact that individuals, especially the young, place more weight on recent observations when they form expectations. The life-cycle learning model incorporating the household weighting schemes demonstrates that the household sensitivity to recent observations amplifies the effects of technology shocks. However, amplification effects become less extensive as the population ages since older households have lower sensitivity to recent shocks and thus, they have less pessimistic or optimistic expectations than younger households. Simulation results also show that a 10%p increase in the old population ratio leads to about a 16% decrease in the output volatility. Moreover, this chapter provides some fiscal policy implications. First, the government spending multiplier declines about 10% when the old population ratio rises by 10%p. Furthermore, welfare analyses find that sensitive reactions to recent observations amplify the effects of government spending and improve the welfare of the population. However, their welfare from government spending deteriorates as the population ages since the amplification effects become weak in an aging society.

Also, the second chapter examines how industrial restructuring induced by population aging affects the effectiveness of monetary policy. Using the euro area panel data, I estimate that a 1%p increase in the proportion of the population 65 years or over raises (lowers) the share of the service (manufacturing) industries by 1.07%p (1.21%p). This is attributed mainly to the heterogeneous consumption patterns of the young and old. Chapter 2 also finds that identified monetary policy shocks in the euro area have less significant impacts on service industries' output than on manufacturing industries' output due to the weak cost of capital channel in the service sector. As a result, the output effects of monetary policy decrease by up to 33% as the old population ratio rises by 10%p since population aging leads to an increase in the share of the service sector. Lastly, the theoretical model that combines overlapping generations and a new Keynesian framework with two sectors provides the mechanism for a decline in the output effects of monetary policy in an aging society. That is, the cost of capital channel of monetary policy does not operate well as the population ages due to the expansion of the service industries.

JEL Classification: D40, D83, E32, E52, E62, J11

Keywords: Learning, Sentiment, Population Aging, Business Cycle, Fiscal Policy, Industrial Structure, Monetary Policy

Dedication

FOR MY BELOVED HAERI, SIWOO, AND SOON-TO-BE-BORN BABY

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Chapter 1

Household Sentiment and Population Aging: Macroeconomic Effects and Implications for Fiscal Policy

1.1 Introduction

The sentiment of economic agents has been considered as one of the main drivers that induce the fluctuations in economic activities. For example, the consumer sentiment channel indicates the path that a positive (negative) economic shock, for example expansionary (contractionary) fiscal or monetary policy, improves (deteriorates) consumer sentiment or expectations about the future economy, causing an additional increase (decrease) in private consumption and corporate investment. So far, such academic interest has led to many studies on how the sentiment of economic agents affects the aggregate economy. Nonetheless, to my knowledge, there have been a limited number of studies on the macroeconomic effects of *heterogeneous* sentiment between young and old individuals. Consequently, this chapter mainly aims to examine how heterogeneous household sentiment or expectations about the future economy impacts the fluctuations in business cycles and the effects of economic policies.

Surveys of Consumers conducted by the University of Michigan have been often used as an indicator of consumer sentiment. Especially, Figure 1.1 plots the index of consumer expectations (ICE) by age group during two recent recessions: the Great Recession and the COVID-19 Recession. This index focuses on consumers' prospects for their financial situation, the general economy over the near-term, and the long-term. Overall, consumers' prospects about the future economy worsen during downturns. However, their reactions to the shock are different by age group. To be specific, older individuals seem to respond to the recent shock less sensitively than younger individuals. In other words, the ICE of younger individuals decreases sharply whereas the ICE of older individuals does not decrease as much during recessions. For example, the lower panel demonstrates that the expectations of the oldest group have the smallest deterioration during the COVID-19 Recession, and then the expectations of the next oldest group are slightly worse than that. As this pattern continues, expectations are the worst in the youngest group. This stylized fact suggests that sentiment or expectations can differ across age groups, which is originated from their different sensitivities to recent shocks.

Based on the findings above, my study examines how household sentiment about the future economy affects the fluctuations in economic activities, and then finds its implications for fiscal policy. In particular, expectations are heterogeneous between young and old individuals, which is created by the different rules for expectations of young and old individuals. Specifically, this research answers the following questions. First, "How does household sentiment, especially heterogeneous sentiment between young and old households, affect the dynamics of macroeconomic variables?" A related question is "To what extent does heterogeneous household sentiment contribute to reduced macroeconomic volatility in recent decades when the population has aged fast?". Lastly, "What are the fiscal policy implications of (heterogeneous) household sentiment for an aging society?".

This chapter provides empirical findings that show the properties of household sentiment and verify whether the household sentiment channel operates in the data.¹ First, a belief wedge, which is defined as the difference between mean one-year-ahead unemployment forecast from the Michigan survey and vector autoregression (VAR) unemployment forecast, demonstrates that household expectations can be more optimistic or pessimistic than the data generating measure. Also, young individuals have a higher degree of sentiment than old individuals and thus, their sentiment is more volatile and associated with macro-variables. Literature finds that the difference in expectations is originated from the fact that young people place more weight on recent observations when they form expectations. Moreover, a counterfactual experiment using a structural VAR model shows that the shutdown of the household sentiment channel significantly reduces the effects of macroeconomic shocks on GDP growth. This result implies that household sentiment has an additional predictive power for the future economy.

I construct the life-cycle learning model that combines the real business cycle (RBC) learning and overlapping generations (OLG) framework based on Eusepi and Preston (2011) and Gertler (1999). To be more specific, this chapter assumes that households have an incomplete model of the economy. In other words, they update their beliefs about market clearing prices, such as a real wage and rental rate of capital, using a constant gain learning algorithm in which households place more weight on recent

¹The household sentiment channel refers to the process in which economic experiences impact household sentiment and then, the affected household sentiment has an influence on their economic activities.



Figure 1.1: Index of Consumer Expectations (ICE) by Age Group

Notes: The Index of Consumer Expectations focuses on three areas: how consumers view prospects for their own financial situation, how they view prospects for the general economy over the near-term, and their view of prospects for the economy over the long-term. The y-axis denotes the difference between ICE in a current period and average ICE throughout the history from 1978q1 to 2021q2. The shaded areas indicate NBER recession periods. *Source*: Surveys of Consumers conducted by University of Michigan

observations when they expect future market prices. This learning mechanism plays an important role in generating additional fluctuations in economic activities. In particular, young households have larger gain parameters than old households, which means young households place more weight on the recent data than old households. The gain parameters for young and old households are borrowed from Malmendier and Nagel (2016). The life-cycle learning model incorporating heterogeneous household weighting schemes presents novel macro-variable dynamics and the implications for business cycles and government policies.

Thus, this chapter compares the dynamics of macro-variables in response to a technology shock in the life-cycle (LC) learning model and LC rational expectation (RE) model. This approach helps in finding how the household learning mechanism differently affects the fluctuations in economic activities than rational expectations. In this analysis, the LC learning model demonstrates that the household sensitivity to recent observations amplifies the effects of the technology shock. However, these amplification effects become less extensive as the old population share rises since old households have relatively lower sensitivity to recent shocks.

In addition, this chapter examines how the household learning mechanism affects the volatility of business cycles. The household learning behavior causes additional fluctuations in economic activities, which creates almost the same variations of macroe-conomic variables as in the data. Furthermore, the LC learning model demonstrates that the fluctuations in business cycles decrease as the population ages due to the lower sensitivity of old households to recent experiences. To be specific, in the LC learning model, a 10% points increase in the old population share induces about a 16% decrease in the output volatility. In the U.S. data, the GDP volatility has declined about 67%, but the old population ratio has risen about 7% points from the

1990's to 2010's. These results suggest a meaningful role of population aging in the reduction of the GDP volatility. However, the representative agent (RA) learning model produces inordinately greater volatility than the LC learning model. This is because the RA does not consider the probability of death and losing a job and has a lager time discount factor. Thus, the higher weight on over-expectations in the RA learning model creates excessive amplification effects. This result shows why the life-cycle assumption is inevitable to match the data.

The life-cycle learning model provides several fiscal policy implications. To begin with, the household learning behavior also amplifies the responses of macro-variables to the government spending shock financed by a lump-sum tax. In particular, the young households' higher sensitivity significantly contributes to these over-responses. Therefore, output responses to the government spending shock become smaller as the proportion of old households increases due to their relatively lower sensitivity to recent policy shocks. Quantitatively, the government spending multiplier declines about 10% when the old population ratio rises by 10% points. Furthermore, a policy experiment finds that the earlier implementation of government spending in downturns helps the economy recover fast from recessions due to the household weighting scheme on stimulus policies under learning. Thus, it is necessary to reduce legislative or implementation lags in fiscal policy during the recessions. Instead of the lumpsum tax, this chapter also assumes that a capital or labor tax finances government spending. The main finding is that while the output effects of government spending are amplified in the learning model, not all government spending is useful to improve the economy. In other words, government spending financed by the capital tax is more effective in boosting the economy than that financed by the labor tax. This is because the labor tax, which is levied on young households or workers, causes a sharp decrease in their labor supply. However, both policies are ineffective to support the economy in the rational expectation model in which the household sentiment channel does not work.

Additionally, there are a few findings from welfare analyses of government spending. First, this chapter conducts a welfare experiment in the LC learning model and finds that government spending is not helpful to improve the welfare of households despite its positive output effects. Although the welfare of old households rises slightly due to government spending, the welfare of young households decreases sharply because of a tax increase. Also, as the proportion of old households increases, the welfare of both young and old households declines since the amplification of government spending effects becomes weak as the population ages. Nonetheless, the welfare of the young deteriorates more drastically when the population ages. Intuitively, if the government implements a stimulus policy, the economy performs relatively worse in an aging society due to the lower sensitivity of old households to recent shocks. Then, young individuals overreact to this relatively poor performance of the economy more than old individuals. Second, this chapter compares the welfare effects of government spending in the RE and learning model. The noteworthy finding is that the amplification of the effects of government spending under learning leads to an improvement in the welfare of the population. Third, this chapter employs the distortionary taxes instead of the lump-sum tax in the welfare experiment. The main finding is that using the labor tax to increase government spending is better in a welfare aspect by a slight margin than using the capital tax, even though using the capital tax is better in terms of the output effects. This is because the capital tax has negative direct impacts on the welfare of both the young and old, but the labor tax directly reduces only the welfare of young households (workers).

Related Literature. First of all, there is a growing literature about sentimentdriven business cycles. [e.g., Angeletos and La'O (2013), Benhabib, Wang, and Wen (2015), Benhabib, Liu, and Wang (2016), and Lorenzoni (2009)]. However, my study employs different mechanisms and frameworks than previous papers. Specifically, household sentiment is originated from their higher sensitivity to recent economic experiences although information frictions or noise shocks give rise to changes in household sentiment in the existing papers. In particular, this chapter studies heterogeneous household sentiment between young and old people, which has not been sufficiently dealt with in the literature.

Empirical findings in the previous papers show that individuals are more sensitive to recent experiences when they expect the future economy [e.g., Malmendier and Nagel (2011), Malmendier and Nagel (2016), and Malmendier and Shen (2018)]. In particular, Malmendier and Nagel (2016) and Malmendier and Shen (2018) claim that young individuals put more weight on recent observations than old individuals when they form expectations. Bhandari, Borovicka, and Horvitz (2019) also shows that individuals can be overly optimistic or pessimistic about the future economy than the data generating measure under rational expectations.

Learning literature demonstrates that household learning behavior can be a source of business cycle fluctuations. For instance, Eusepi and Preston (2011) introduces imperfect information and learning behavior in the real business cycle framework and show that the household learning rule with more weight on recent observations can amplify the fluctuations in business cycles. In particular, households in their model expect future market clearing prices using the minimum state variable (MSV) constant gain learning rule instead of rational expectations, which is commonly used in the learning literature [e.g., Mitra, G. W. Evans, and Honkapohja (2013), and

G. W. Evans and Honkapohja (2001)].

My study introduces a overlapping generation (OLG) framework in the RBC learning model of Eusepi and Preston (2011) on the basis of Gertler (1999), Blanchard (1985), and Gali (2021). A life-cycle assumption allows the model to capture young and old individuals' different learning rules. To be more specific, their different gain parameters represent the heterogeneous learning rules between the young and old. Branch and McGough (2009) and Honkapohja and Mitra (2005) also make use of the different gain parameters as a source of heterogeneity in household expectations.

This chapter relates to earlier researches explaining the stylized facts in an aging society. As an example, the reduction in the volatility of business cycles in recent decades is explained with the improved monetary policy [e.g., Stock and Watson (2003)] and the low volatility of old individuals' employment and hours worked [e.g., Jaimovich and Siu (2009)]. Furthermore, Basso and Rachedi (2021) and Honda and Miyamoto (2020) empirically and theoretically demonstrate that the government spending multiplier is low in an aging society. However, my paper using the life-cycle learning model can be a novel alternative to the papers above.

The structure of this chapter is as follows. Section 1.2 presents the empirical evidence for (heterogeneous) household sentiment and its correlation with macroeconomic variables. In particular, this chapter estimates a structural VAR model to verify the household sentiment channel. Then, Section 1.3 builds the life-cycle learning model in which young and old households have the different weighting schemes on the past data when they form expectation. After that, Section 1.4 explores the macroeconomic effects of (heterogeneous) household sentiment and Section 1.5 finds its fiscal policy implications for an aging society. Finally, Section 1.6 concludes.

1.2 Empirical Evidence for (Heterogeneous) Household Sentiment

This section calculates a belief wedge that is one of the household sentiment measures. After that, a structural VAR model is estimated to verify whether the household sentiment channel operates or not in the real world. Then, this chapter explores heterogeneous household sentiment between young and old individuals, and computes its correlation with macroeconomic variables.

1.2.1 Household Sentiment

Household sentiment can be suggested by belief wedges, which demonstrate that household sentiment is not just a proxy for other fundamental variables. In other words, households can perceive the future economic situation more optimistically or pessimistically than the current overall economic conditions.

Definition. The belief wedge is defined as the difference between mean one-yearahead unemployment rate forecast from the Michigan survey and VAR unemployment rate forecast (henceforth, VAR Wedge).

Belief Wedge = Expected Unemployment from Michigan Survey - VAR Forecast

Here, VAR forecasts are considered as the data generating measure under the rational expectations.

Methodologies. This chapter calculates the belief wedges based on Bhandari, Borovicka, and Horvitz (2019) and Mankiw (2003). First, VAR forecasts are estimated from a standard quarterly forecasting VAR model containing nine variables: CPI inflation, real GDP, unemployment rate, the relative price of investment goods, capital utilization rate, hours worked, consumption rate (=Consumption/GDP), investment rate (=Investment/GDP), and federal funds rate. The time lag is 2 and all time series are from FRED. Also, the sample period is from 1960Q1 to 2017Q1 due to data availability. For a robustness, my research uses the unemployment forecast from a survey of professional forecasters (SPF) conducted by federal reserve bank of Philadelphia instead of the VAR forecast (henceforth, SPF Wedge).²

Results. Figure 1.2 shows how two belief wedges have changed over time and the results are almost the same as in Bhandari, Borovicka, and Horvitz (2019). This figure shows a noticeable pattern that both VAR and SPF Wedges increase sharply during recession periods and decrease gradually after the end of the recession. These patterns suggest individuals' time-varying sentiment about the future economy. Therefore, in this chapter, the belief wedges will be used as the measures of the household sentiment about the future economy.



Figure 1.2: Belief Wedges

Notes: The constructions of the time series are based on Bhandari, Borovicka, and Horvitz (2019) and Mankiw (2003).

 $^{^2{\}rm Keane}$ and Runkle (1990), using a survey of professional forecasters, find that professional forecasters have rational expectations.

1.2.2 Household Sentiment Channel

My study estimates a structural VAR model to check whether the household sentiment channel operates or not in the real world. Specifically, Section 1.2.2 examines whether household sentiment has an *additional* effect on the economy by conducting a counterfactual experiment.

Model. The structural VAR model specification is as follows

$$\Psi Y_t = C + \sum_{j=1}^l \Phi_j Y_{t-j} + \varepsilon_t \text{ with } \varepsilon_t \sim i.i.d. \ N(0,\Omega) \text{ and } cov(\varepsilon_{i,t},\varepsilon_{j,t}) = 0 \ \forall \ i \neq j \ (1.1)$$

where C is a constant vector, Y_t is the vector containing six endogenous variables: (a) real stock prices growth, (b) household financial status, (c) belief wedge, (d) GDP growth, (e) inflation, and (f) federal funds rate, and ε_t is an innovation vector.³

The time lag (l) is set as seven based on several criteria for choosing the optimal lag length, such as AIC (Akaike information criterion). Moreover, the structural shock is identified by Cholesky decomposition method. The order of variables as above is determined based on the standard assumption that a delayed response of real variables and inflation to the monetary policy shock [Christiano, Eichenbaum, and C. L. Evans (1997)]. Although the literature provides no clear indication about the position of household sentiment, Leduc and Sill (2013) order the consumer sentiment index before real variables since respondents don't have exact information about the respondents of the same time when the surveys are conducted.⁴ However, since the respondents

 $^{^{3}}$ I add the real stock prices growth in the VAR model following Leeper (1992). Moreover, household financial status is the proportion of households who are better off minus worse off financially compared to a year ago, which are from Michigan survey.

⁴For example, respondents had interviews from Jan. 2 to Jan. 27, 2020 and survey results for Jan. 2020 were released on Jan. 31, 2020. However, inflation for Jan. 2020 was released on Feb. 13, 2020.

are aware of stock prices and their financial status when they participate in the survey, this chapter places the real stock prices growth and household financial status measure first. Therefore, in this setting of the variable order, the household sentiment channel indicates the path that the improved (worsened) household financial status decreases (increases) the belief wedge and then it finally leads to the rise (reduction) in GDP growth.

Data. The real stock prices growth rate is calculated using the Standard and Poor's 500 index deflated by the personal consumption expenditure (PCE) price index in the previous month. This methodology is based on Leeper (1992). Furthermore, the proxy variable for household sentiment is the belief wedge (i.e., VAR Wedge) that is estimated in Section 1.2 and the GDP growth is from U.S. Bureau of Economic Analysis (BEA). Lastly, the inflation indicates PCE inflation from U.S. Bureau of Labor Statistics (BLS) and the effective federal funds rate is from Federal Reserve Bank of New York. The sample period for the structural VAR estimation is from 1961Q3 to 2017Q1 due to the data availability.

Counterfactual Experiment. This chapter conducts the counterfactual experiment to verify the existence of the household sentiment channel in the real world. Specifically, in the counterfactual experiment, the household sentiment channel is shut down, which indicates that the belief wedge, i.e., household sentiment, is not affected by the innovations to the household financial status in the structural VAR model. To construct such an environment, the lag polynomials in the belief wedge equation are zero except for the own lags of the belief wedge. This assumption makes the belief wedge not to involve in the propagation of the household financial status shock. Moreover, the contemporaneous responses of the belief wedge are ruled out by imposing zero restrictions. **Results.** Figure 1.3 gives the impulse response results from the structural VAR estimation under shutdown or no shutdown of the household sentiment channel. First of all, Panel (a) plots the response of the belief wedge to the positive innovation to the household financial status. This panel indicates that the household sentiment improves after the positive household financial status shock when the household sentiment channel is not shut down. On the contrary, this effect disappears by construction when the channel is shut down. Moreover, Panel (b) provides the response of GDP growth to the positive innovation to the belief wedge and indicates that the high belief wedge, i.e., worsened household sentiment, significantly reduces the GDP growth rate regardless of shutdown or no shutdown of the channel. Lastly, Panel (c) shows that the shutdown of the household sentiment channel weakens the effects of the positive innovation to the household financial status on the GDP growth rate. In other words, the household financial status shock has an significant effect on the GDP growth rate in the case of no shutdown of the channel. However, if the channel is shutdown, its effect comes to be almost insignificant with the 90% confidence interval.

In summary, it can be concluded that the shutdown of the household sentiment channel significantly reduces the effects of positive macroeconomic shocks on GDP growth. To be more specific, even though households are better off financially, their sentiment or expectations about the future economy do not improve due to the shutdown of the channel. As a result, household sentiment cannot encourage their economic activities despite the positive shock. Accordingly, the impulse response results demonstrate the existence of the household sentiment channel in the real world.







(b) Response of GDP to Positive Innovation to Belief Wedge



(c) Response of GDP to Positive Innovation to Household Financial Status

Figure 1.3: Impulse Responses of Structural VAR Model Variables from Counterfactual Experiment

Notes: NO Shutdown denotes the household sentiment channel operates and conversely, Shutdown indicates the household sentiment channel does not operate. The shaded area indicates 90% confidence intervals.

Figure 1.4 indicates the response of the GDP level to a positive innovation to household financial status, and Figure 1.5 denotes the forecast error variance decomposition (FEVD) for GDP growth, especially the contribution of the household financial status shock. According to Figure 1.4, there is a significant gap between the responses of the GDP level under no shutdown and shutdown of the household sentiment channel although both economies start with the same GDP level=100 at time 0. This gap suggests that the household sentiment channel operates strongly in the real world. Also, as in Figure 1.5, the contribution of the household financial status shock to GDP growth under no shutdown of the channel is at least twice as much as that under shutdown of the channel. Hence, the FEVD results suggest that the shock to the household financial status become relatively less important factor for the GDP growth when the household sentiment channel is shut down.



Figure 1.4: Impulse Response of GDP Level to Positive Innovation to Household Financial Status

Notes: I calculate the response of the GDP level using the estimation results for Equation (1.1). NO Shutdown denotes the household sentiment channel operates and conversely, Shutdown indicates the household sentiment channel does not operate.



Figure 1.5: Forecast Error Variance Decomposition (FEVD) for GDP Growth: Contribution of Household Financial Status Shock

Notes: This chapter computes FEVD for GDP Growth based on Equation (1.1). NO Shutdown denotes the household sentiment channel operates and conversely, Shutdown indicates the household sentiment channel does not operate.

1.2.3 Heterogeneous Household Sentiment

The belief wedges also suggest that household sentiment is heterogeneous across age groups. For example, the belief wedge differences between age 35-44 and 65-97 and between age 45-54 and 65-97 surge during recessions and decrease gradually afterwards as in Figure 1.6. This pattern signifies that younger individuals tend to react to recent macro-shocks more sensitively than older individuals. Put differently, young individuals who are aged 35-44 and 45-54 have more pessimistic expectations about the future unemployment rate than old individuals who are of age 65-97 after they experience downturns.





Notes: The constructions of the time series are based on Bhandari, Borovicka, and Horvitz (2019) and Mankiw (2003).

Table 1.1 also shows that the belief wedges of older individuals are less closely related to economic activities. To be specific, correlation coefficients between both belief wedges and macroeconomic variables, i.e., GDP, consumption, and investment growth, have relatively smaller negative values as age increases. This pattern suggests that old individuals sentiment is less closely associated with economic conditions. On top of that, since old individuals respond to changes in economic conditions less sensitively than young individuals, their belief wedges vary less frequently. Table 1.2 provides relative standard deviations of belief wedges among age groups. According to this table, the standard deviations of the belief wedge are smaller than those of young individuals. For instance, the standard deviation of the SPF Wedge of the 35-44 cohort (σ_{35-44}^{SPF}) is 4.6% larger than the standard deviation of the aggregate SPF Wedge (σ_{Au}^{SPF}) whereas the standard deviation of the SPF Wedge of the 65-97 cohort (σ_{65-97}^{SPF})

	(1) GDP	(2) Consumption	(3) Investment
VAR Wedge	-0.433	-0.375	-0.381
(i) VAR Wedge (18-34)	-0.445	-0.387	-0.382
(ii) VAR Wedge (35-44)	-0.438	-0.379	-0.393
(iii) VAR Wedge $(45-54)$	-0.432	-0.371	-0.387
(iv) VAR Wedge $(55-64)$	-0.413	-0.358	-0.365
(v) VAR Wedge (65 & Over)	-0.412	-0.356	-0.357
SPF Wedge	-0.483	-0.462	-0.372
(i) SPF Wedge (18-34)	-0.481	-0.464	-0.353
(ii) SPF Wedge (35-44)	-0.482	-0.459	-0.391
(iii) SPF Wedge (45-54)	-0.490	-0.462	-0.397
(iv) SPF Wedge $(55-64)$	-0.439	-0.425	-0.336
(v) SPF Wedge (65 & Over)	-0.444	-0.428	-0.324

is 1.5% lower than the standard deviation of the aggregate SPF Wedge (σ_{All}^{SPF}) .⁵

 Table 1.1: Correlations Between Belief Wedges and Macro-variables

Notes: All macro-variables are detrended with the HP-filter. Also, () denote age groups. For example, VAR Wedge (18-34) is the VAR Wedge of the 18-34 age group.

Table 1.2: Relative Standard Deviations of Belief Wedges among Age Group

$\sigma^{SPF}_{18-34}/\sigma^{SPF}_{All}$	$\sigma^{SPF}_{35-44}/\sigma^{SPF}_{All}$	$\sigma^{SPF}_{45-54}/\sigma^{SPF}_{All}$	$\sigma^{SPF}_{55-64}/\sigma^{SPF}_{All}$	$\sigma^{SPF}_{65\& ext{Over}}/\sigma^{SPF}_{All}$
1.038	1.046	1.025	0.992	0.985
$\sigma^{VAR}_{18-34}/\sigma^{VAR}_{All}$	$\sigma^{VAR}_{35-44}/\sigma^{VAR}_{All}$	$\sigma^{VAR}_{45-54}/\sigma^{VAR}_{All}$	$\sigma^{VAR}_{55-64}/\sigma^{VAR}_{All}$	$\sigma^{VAR}_{65\&\mathrm{Over}}/\sigma^{VAR}_{All}$
0.991	1.011	1.023	0.998	1.004

Notes: σ 's are the standard deviations of each belief wedge denoted by superscripts and subscripts indicate age groups.

In Section 1.2.2, we found that household sentiment additionally affects the economic activities through the household sentiment channel. This chapter also claims that

⁵The expectations by an older group are also less volatile in the Michigan survey data. For example, during the same period as the belief wedge, the standard deviations of the index of expected unemployment by age 18-34, 35-44, 45-54, 55-64, and 65-97 are 15.4, 18.2, 17,2, 16.7, and 15.0 respectively.
the magnitudes of these additional effects can be different by age group due to their different sentiment. In other words, since older individuals have a relatively lower degree of sentiment, their sentiment has a smaller impact on the economy. For instance, old individuals become less optimistic about the future economy than young individuals after positive shocks take place and so increase their consumption less. The existing papers, such as Malmendier and Shen (2018), support this claim with empirical findings. In Malmendier and Shen (2018), they provide regression results suggesting that after controlling for income, wealth, age, and other controls, economic shocks have a smaller *additional* effect on consumption expenditure as individual's age increases.⁶ They state that the beliefs-based channel is weaker in an old cohort than in a younger cohort. Besides, these additional effects have nothing to do with whether the macro-shocks are positive or negative.

1.2.4 Micro-Foundations for Irrational Households

Literature presents the micro-foundation for households irrational behavior that we find in the previous sections. To be specific, existing papers argue that household irrationality is originated from their learning processes which put more weight on recent observations when they expect the future economy. In particular, young individuals place more weight on recent experiences than old individuals. Literature supports this claim with various empirical findings. First of all, Malmendier and Nagel (2011) find that higher recently experienced stock returns are related to more optimistic beliefs about future stock returns since individuals put more weight on recent returns when they form expectations. Moreover, Malmendier and Nagel (2016) claim that

⁶They regress the log change in consumption from the Nielsen Homescan Data (2004-2013) on the interaction of age with the log change in unemployment conditions, controlling the controls.

individuals overweight inflation realized during their lifetimes, in particular, more recent data when they expect future inflation. Also, beliefs are heterogeneous between the young and old since young individuals place more weight on the recent data than older individuals. This is because recent experiences make up a more substantial part of young individuals' lifetimes. Furthermore, this learning process provides a microfoundation for models of perpetual learning such as constant-gain learning. Besides, Malmendier and Shen (2018) show that personal experiences of unemployment affect consumption decisions in the long-run. Households who have lived through times of high unemployment, or who have experienced more personal unemployment, spend significantly less on consumption. Especially, these effects are weaker for older than younger cohorts. In their analysis, measures of unemployment experiences are constructed with linearly declining weights, which indicates individuals' fading memory.

1.3 Model

The model of this chapter is constructed on the basis of Eusepi and Preston (2011). In the model, households have an incomplete model of the economy. To be specific, they update their beliefs about market clearing prices such as a real wage and rental rate of capital by extrapolating historical patterns in observed data. In particular, when households form expectations about future market prices, they put more weight on recent observations. This learning mechanism is a key factor to generate the *amplification* of the fluctuations in real economic activities.

Moreover, I introduce a life-cycle (LC) assumption, i.e., young households or workers and old households or retirees, in the RBC learning model following Gertler (1999), Blanchard (1985), and Gali (2021). As mentioned in Section 1.2.4, Malmendier and Nagel (2016) find that young individuals place more weight on the recent data than old individuals. The heterogeneous agents who have different weighting schemes on the past data will present novel macro-variables dynamics and implications for business cycles and government policies.

1.3.1 Firms

There are identical competitive firms of mass one. Each firm produces goods using capital K_t and labor H_t . The production function is

$$Y_t = (K_t)^{\alpha} (X_t H_t)^{1-\alpha}$$
(1.2)

where $0 < \alpha < 1$. X_t denotes the aggregate labor-augmenting technical progress which evolves via

$$\ln(\frac{X_{t+1}}{X_t}) = \ln(\chi_{t+1}) = \ln(\overline{\chi}) + u_{\chi,t+1}$$
(1.3)

where $u_{\chi,t}$ indicates an i.i.d. random variable with zero mean and standard deviation $\sigma_{u_{\chi}}$. The stochastic process for the evolution of the technological shock is assumed known to the agents.

Firms maximize their profits with factor prices, real wage W_t and return to capital R_t^k , as given. The optimality conditions are

$$W_t = (1 - \alpha) K_t^{\alpha} X_t^{1 - \alpha} H_t^{-\alpha} = (1 - \alpha) \frac{Y_t}{H_t}$$
(1.4)

$$R_t^k = \alpha K_t^{\alpha - 1} (X_t H_t)^{1 - \alpha} = \alpha \frac{Y_t}{K_t}$$
(1.5)

From now on, lowercase letters denote variables normalized with the technology X_t . Then, the normalized production function and optimality conditions are

$$y_t = (k_t)^{\alpha} (\chi_t)^{-\alpha} (H_t)^{1-\alpha}$$
(1.6)

$$w_t = (1 - \alpha)(k_t)^{\alpha} (\chi_t)^{-\alpha} H_t^{-\alpha} = (1 - \alpha) \frac{y_t}{H_t}$$
(1.7)

$$R_t^k = \alpha k_t^{\alpha - 1} (\chi_t)^{-\alpha} (H_t)^{1 - \alpha} = \alpha \frac{y_t}{k_t}$$
(1.8)

1.3.2 Households

This chapter assumes an economy with overlapping generations following Gertler (1999), Gali (2021), and Blanchard (1985). The size of the population is constant and normalized to one. Each individual has the constant probability γ of surviving into the following period, independently of his/her age and economic status. Moreover, each worker faces the constant probability 1-v of becoming retired permanently. This probability is also independent of his/her age and economic status. Consequently, the size of young individuals (workers) at any time is the constant $\phi = \frac{1-\gamma}{1-v\gamma} \in (0, 1]$ and that of old individuals (retirees) is $1 - \phi = \frac{\gamma(1-v)}{1-v\gamma}$.

Moreover, this chapter assumes a perfect annuity market which insures agents against the risk of death as in Gertler (1999). To be specific, households have an annuity contract with a perfectly competitive insurance company. On the contract, the company issues a payment proportional to households' financial wealth. Also, their wealth will be transferred to the insurance company when households die. Then, households who survive to the next period receive all the returns in this market. I also introduce an insurance market that reduces the risk of loss of income from retirement.

1.3.2.1 Old Households (Retirees)

'Retired' agent of retired cohort 'a' is one individual who retired 'a' quarters ago.⁷ Each agent maximizes the following Bellman equation⁸

$$V^{o}(A^{o}_{a,t}, A^{o}_{t}, A_{t}, K_{t}) = \operatorname{Max}\{\ln C^{o}_{a,t} + \gamma \beta \tilde{E}^{o}_{t} V^{o}(A^{o}_{a+1,t+1}, A^{o}_{t+1}, A_{t+1}, K_{t+1})\}$$
(1.9)

and the budget constraint for old households is

$$C_{a,t}^{o} + \gamma A_{a+1,t+1}^{o} = R_t A_{a,t}^{o} + S_t \tag{1.10}$$

where \tilde{E}_t indicates *subjective* expectations for the future, $C_{a,t}^o$ is consumption of old households, and $A_{a,t}^o$ is the asset that old households hold at the beginning of time t. My research assumes the perfect annuity market in which households can be insured against the risk of death. Thus, only survivors receive all the returns and households who die are paid nothing. In addition, due to the absence of arbitrage between loans and capital, the real interest rate R_t satisfies $R_t = R_t^k + 1 - \delta$ where δ is the depreciation rate. Moreover, S_t is the social security benefit that old households receive. Furthermore, the state variables $\{A_t^o, A_t, K_t\}$ are necessary when households expect the future wages and returns to capital, which will be discussed in Section 1.3.3 in detail. Lastly, this chapter assumes that households and firms know only their own objectives, constraints, and beliefs, which is also the main assumption of Eusepi and Preston (2011).

⁷I borrow notations from Baksa and Munkacsi (2019).

⁸Log utility for consumption is necessary for steady-state labor supply along a balanced growth path [see King, Plosser, and Rebelo (1988)].

Then, Euler equation is

$$(C_{a,t}^{o})^{-1} = \beta \tilde{E}_{t}^{o}[(C_{a+1,t+1}^{o})^{-1}R_{t+1}]$$
(1.11)

Also, the Euler equation and budget constraint can be normalized with technology as follows

$$(c_{a,t}^{o})^{-1} = \beta \tilde{E}_{t}^{o}[(c_{a+1,t+1}^{o})^{-1}\chi_{t+1}^{-1}R_{t+1}]$$
(1.12)

$$c_{a,t}^{o} + \gamma a_{a+1,t+1}^{o} = R_t a_{a,t}^{o} \chi_t^{-1} + s_t \tag{1.13}$$

To obtain aggregate consumption of old households, this chapter derives the intertemporal budget constraint (IBC) from the one-period budget constraint (i.e., Equation (1.13)). After that, the IBC is log-linearized around a balanced growth path (BGP) and then I aggregate each cohort's consumption using Euler equation (i.e., Equation (1.12)). Finally, we obtain log-linearized aggregate consumption of old households

$$\hat{c}_{t}^{o} = \eta_{ay}(1-v)\hat{a}_{t-1}^{y} + \eta_{ao}\hat{a}_{t}^{o} + \eta_{r\chi}(\hat{R}_{t} - \hat{\chi}_{t}) + \eta_{s}\hat{s}_{t}$$

$$-\eta_{r}^{e} \underbrace{\tilde{E}_{t}^{o}}_{h=0} \sum_{h=0}^{\infty} (\gamma\beta)^{h} \hat{R}_{t+1+h} + \eta_{s}^{e} \tilde{E}_{t}^{o} \sum_{h=0}^{\infty} (\gamma\beta)^{h} \hat{s}_{t+1+h}$$
(1.14)
$$\underbrace{= \frac{1}{2} \sum_{h=0}^{\infty} (\gamma\beta)^{h} \hat{R}_{t+1+h} + \eta_{s}^{e} \tilde{E}_{t}^{o} \sum_{h=0}^{\infty} (\gamma\beta)^{h} \hat{s}_{t+1+h}$$

where η 's consist of primitive model parameters.⁹ Note that Equation (1.14) shows how current and expected variables, such as real interest rates and social security benefits, affect the consumption decision of old households.

⁹The derivation of Equation (1.14) is in Appendix A.

1.3.2.2 Young Households (Workers)

'Young' agent of cohort 'b' is one individual of its cohort who started to work (was born) 'b' quarters ago. Each agent maximizes the following Bellman equation

$$V^{y}(A^{y}_{b,t}, A^{y}_{t}, A_{t}, K_{t}) = \operatorname{Max}\{\ln C^{y}_{b,t} + \theta \ln(1 - H_{b,t}) +$$

$$\gamma \beta \tilde{E}^{y}_{t}[vV^{y}(A^{y}_{b+1,t+1}, A^{y}_{t+1}, A_{t+1}, K_{t+1}) + (1 - v)V^{o}(A^{yo}_{b+1,t+1}, A^{y}_{t+1}, A_{t+1}, K_{t+1})]\}$$

$$(1.15)$$

and the budget constraint for young households is

$$C_{b,t}^{y} + v\gamma A_{b+1,t+1}^{y} + (1-v)\gamma A_{b+1,t+1}^{yo} = R_{t}A_{b,t}^{y} + W_{t}H_{b,t} - T_{t}$$
(1.16)

where $C_{b,t}^{y}$ is consumption of young households and since there are two possible future states, young households save $A_{b+1,t+1}^{y}$ for staying young and $A_{b+1,t+1}^{yo}$ for retiring next period. Complete asset markets insure young households against the risk of retirement. Also, W_t is the real wage, $H_{b,t}$ is the labor supply, and T_t is the lump-sum tax. Lastly, the state variables $\{A_t^y, A_t, K_t\}$ are necessary when young households expect the future wages and returns to capital, which will be discussed in detail in Section 1.3.3.

Young households solve the maximization problem, Equation (1.15), subject to the budget constraint, Equation (1.16). Then, the normalized Euler equations, labor supply condition, and budget constraint of young households are as follows

$$(c_{b,t}^y)^{-1} = \beta \tilde{E}_t^y [(c_{b+1,t+1}^y)^{-1} \chi_{t+1}^{-1} R_{t+1}]$$
(1.17)

$$(c_{b,t}^y)^{-1} = \beta \tilde{E}_t^y [(c_{b+1,t+1}^o)^{-1} \chi_{t+1}^{-1} R_{t+1}]$$
(1.18)

$$\frac{\theta c_{b,t}^{y}}{1 - H_{b,t}} = w_t \tag{1.19}$$

$$c_{b,t}^{y} + v\gamma a_{b+1,t+1}^{y} + (1-v)\gamma a_{b+1,t+1}^{yo} = R_{t}a_{t}^{y}\chi_{t}^{-1} + w_{t}H_{b,t} - \tau_{t}$$
(1.20)

Finally, this chapter obtains log-linearized aggregate consumption of young households applying the same way as used to get aggregate consumption of old households

where ψ 's consist of primitive model parameters.¹⁰ Note that Equation (1.21) shows how current and expected variables such as the real interest rates, real wages, and taxes affect the consumption decision of young households. In particular, the last two terms in Equation (1.21) appear since young households (workers) also consider that they can lose their jobs and become retirees at any time in the future with the probability of 1 - v.

¹⁰The derivation of Equation (1.21) is in Appendix A.

1.3.3 Household Beliefs

Households are assumed to use an economic model to forecast future wages and returns to capital. The model relates the wage and return to capital to the aggregate capital and the distribution of the wealth between young and old households, which are two minimum state variables (MSV) in the model

$$\hat{R}_t^k = \mu_r + \mu_{rk}\hat{k}_t + \mu_{r\lambda}\lambda_t + e_t^r \tag{1.22}$$

$$\hat{w}_t = \mu_w + \mu_{wk}\hat{k}_t + \mu_{w\lambda}\lambda_t + e_t^w \tag{1.23}$$

$$\hat{k}_{t+1} = \mu_k + \mu_{kk}\hat{k}_t + \mu_{k\lambda}\lambda_t + e_t^k \tag{1.24}$$

$$\lambda_{t+1} = \mu_{\lambda} + \mu_{\lambda k} \hat{k}_t + \mu_{\lambda \lambda} \lambda_t + e_t^{\lambda} \tag{1.25}$$

where $\lambda_t = \frac{\hat{a}_t^o}{\hat{a}_t}$ represents the distribution of the wealth between young and old households, and e_t denotes a regression error or households consider e_t as a idiosyncratic disturbance (i.e., a perceived white noise unobserved shock). Equation (1.22)-(1.25) can be rewritten in a matrix form as Equation (1.26)

$$z_{t}' = \begin{pmatrix} \hat{R}_{t}^{k} \\ \hat{w}_{t} \\ \hat{k}_{t+1} \\ \lambda_{t+1} \end{pmatrix}, \ x_{t-1} = \begin{pmatrix} 1 \\ \hat{k}_{t} \\ \lambda_{t} \end{pmatrix}, \ \zeta_{t} = \begin{pmatrix} \mu_{r,t} & \mu_{w,t} & \mu_{\lambda,t} \\ \mu_{rk,t} & \mu_{wk,t} & \mu_{\lambda k,t} \\ \mu_{r\lambda,t} & \mu_{w\lambda,t} & \mu_{\lambda\lambda,t} \end{pmatrix}, \ e_{t}' = \begin{pmatrix} e_{t}^{r} \\ e_{t}^{w} \\ e_{t}^{k} \\ e_{t}^{\lambda} \end{pmatrix}$$

$$z_t = x'_{t-1}\zeta_t + e_t. (1.26)$$

Under rational expectations, $\mu_{r,t} = \mu_{w,t} = \mu_{k,t} = \mu_{\lambda,t} = 0$, and the coefficients of the

model are time-invariant (i.e., $\mu_{ij,t} = \overline{\mu}_{ij}$ where $i \in \{r, w, k, \lambda\}$ and $j \in \{k, \lambda\}$). Also, $e_t^r = \overline{\mu}_{r\chi} \hat{\chi}_t$, $e_t^w = \overline{\mu}_{w\chi} \hat{\chi}_t$, $e_t^k = \overline{\mu}_{k\chi} \hat{\chi}_t$, and $e_t^\lambda = \overline{\mu}_{\lambda\chi} \hat{\chi}_t$ in the rational expectation model.¹¹ However, under learning, households update the coefficients every period as they observe new data. The updating algorithm can be expressed with constant gain recursive least squares estimates which adaptive learning literature commonly makes use of

$$\zeta_t^i = \zeta_{t-1}^i + g^i (Q_t^i)^{-1} x_{t-1} \underbrace{(z_t - x'_{t-1} \zeta_{t-1}^i)}_{\text{Forecast Error}}$$
(1.27)

$$Q_t^i = Q_{t-1}^i + g^i (x_t x_t' - Q_{t-1}^i)$$
(1.28)

where $i \in \{y, o\}$ and Q_t^i is an estimate of the second moment matrix of regressors.

Equations (1.27) and (1.28) imply that the coefficients estimated in the previous period are updated according to the forecast error it produces for the current period. In addition, the gain parameter g allows the deviation from rational expectations. That is, the larger g is, the higher is the relative weight put on more recent data.¹²

$$\zeta_{k,t} = \left[\sum_{i=1}^{t} (1-g)^{i-1} x_{t-i} x_{t-i}'\right]^{-1} \left[\sum_{i=1}^{t} (1-g)^{i-1} x_{t-i} k_{t-i+1}\right]$$

¹¹Eusepi and Preston (2011) document that if the technology shock is included in the households' forecasting model under learning, they learn quickly since the technology shock is the only disturbance in the model. To be specific, the initial coefficients of equation (1.22) - (1.25) are from the rational expectation model. Thus, agents correctly expect market prices, wages and returns to capital, if the technology shock is used in the forecasting model. The intuition for leaving technology out of the forecasting model is as follows. Households are not aware of other agent's technologies. Even though they know there is a stochastic shock, they don't know its determinants (e.g., population growth or aggregate labor supply) and that firms are identical. Thus, households do not understand the exact aggregate production function and the relationship between market prices and the aggregate technology shock.

 $^{^{12}}$ Equation (1.27) and (1.28) are derived from following constant gain least squares estimator. Derivations are in Carceles-Poveda and Giannitsarou (2007).

Also, the model nests rational expectations since the model converges to the model under rational expectations as g goes to zero.

Moreover, this chapter assumes that young and old households have different gain parameters, i.e., g^y and g^o , based on Malmendier and Nagel (2016)'s empirical findings.¹³ ¹⁴ In particular, since young individuals put more weight on recent data than old individuals, $g^y > g^o$. Collin-Dufresne, Johannes, and Lochstoer (2017) also assume that the gain parameter of the young agents is five times greater than that of the old agents.¹⁵ As young and old households have different weighting schemes on past data, their adaptive expectations about factor prices can be distinct from each other. Therefore, this difference finally produces different dynamics of macro-variables than those in existing papers assuming a representative agent.

There is one more assumption worth mentioning. In my model, agents do not know how other agents expect. This is because if young households are aware of old households' expectations or vice versa, they can internalize the impacts of updating their beliefs on the economy. So, they quickly learn and find the true evolution of wages and interest rates since household learning behavior cannot create forecast errors.

1.3.4 Government

The government levies lump-sum taxes T_t on young households (workers) and consumes G_t . Also, it pays retirees social security benefits S_t each period. Thus, the government satisfies the following budget constraint

$$\phi T_t = G_t + (1 - \phi)S_t \tag{1.29}$$

 $^{^{13}\}mathrm{Malmendier}$ and Nagel (2016) find that as people get older, they place less weight on recent data.

 $^{^{14}}$ According to Honkapohja and Mitra (2006), agents' different degrees of responsiveness to the updating function, i.e., different gain parameters, can be a source of heterogeneity in the learning model.

 $^{^{15}2.5\%}$ for the young and 0.5% for the old. See Collin-Dufresne, Johannes, and Lochstoer (2017)

Furthermore, this chapter assumes that the government maintains its policy variables at steady-state levels, which is also announced by the government in advance. The policy announcement is assumed to be credible and so agents believe that there will be no changes in the government policies in the future. These assumptions are on the basis of Mitra, G. W. Evans, and Honkapohja (2013).

1.3.5 Market Clearing

Goods and asset market clearing conditions are as follows:

$$Y_t = C_t + I_t + G_t \tag{1.30}$$

$$C_t = C_t^y + C_t^o \tag{1.31}$$

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{1.32}$$

$$A_t = K_t \tag{1.33}$$

$$A_t = A_t^y + A_t^o \tag{1.34}$$

1.3.6 Model Parameterization

Table 1.3 presents parameter and steady-state values in the model. First of all, the RBC model parameters are set as commonly used values in literature such as King and Rebelo (2000), etc. Moreover, the probability of death (γ) and the probability of losing job (v) are calculated based on Gali (2021)'s calibaration. Most importantly, gain parameters for old and young households, g^o and g^y , are borrowed from Malmendier and Nagel (2016) in which they estimate gain parameters by age using

inflation expectation data. For robustness checks, this chapter also employs various gain parameter values and it will be discussed in Section 1.4.1.

Parameter	Description	Value
α	Capital Share	1/3
eta	Time Discount Factor	0.995
ϕ	Proportion of Young Households	0.8
γ	Probability of Death	0.9959
v	Probability of Losing Job	0.9989
heta	Weight on Utility from Leisure	3.39
g^o	Gain Parameter of Old Households	0.010
g^y	Gain Parameter of Young Households	0.018
σ_{u_χ}	Standard Deviation of Technology Shock	0.0078
$ar{\chi}$	Growth of Productivity in Steady-State	1.0053
Ē	Labor in Steady-State	1/4

Table 1.3: Parameter and Steady-State Values

Notes: Parameter values are calibrated based on King and Rebelo (2000), Gali (2021), and Malmendier and Nagel (2016).

1.4 Effects of (Heterogeneous) Household Sentiment on Business Cycles

This section compares the impulse response results from the life-cycle (LC) model under learning and the ones from the LC model under rational expectations. This approach helps in finding how the learning mechanism affects business cycles differently than rational expectations (RE). My study also compares the impulse response results from the LC model under learning and the ones from a representative agent (RA) model under learning to check what difference the life-cycle assumption creates. Finally, this chapter studies how the household learning behavior impacts the volatility of business cycles and then finds the implications for an aging society.

1.4.1 LC Model under Learning vs LC Model under RE

The simulation process for obtaining the impulse response results is as follows. The 2000-period simulation under RE provides initial steady-state coefficients for the households' forecasting model, Equation (1.22) - (1.25), and the simulation data are discarded. The coefficients are updated every period in the learning model but keep their initial values in the RE model. Then, a N-period impulse response is obtained with a 1% permanent technology shock in the period 2001. This simulation is repeated 2,000 times. Finally, I report the median responses of model variables to the technology shock. Simulation methodologies are on the basis of Eusepi and Preston (2011) and Mitra, G. W. Evans, and Honkapohja (2013).

Aggregate Effects. The impulse response results show that the household sensitivity to recent observations creates amplification effects on business cycles. Figure 1.7 plots the responses of output, consumption, investment, and hours to a 1% permanent technology shock. The upper left panel demonstrates that the output response to the positive shock is amplified under learning rather than rational expectations since households place more weight on recent experiences under learning. Intuitively, households become more optimistic about the future economy after the positive economic shock due to their sensitivity to recent shocks and improved sentiment increases output sharply through the household sentiment channel. The upper right panel shows that households consume less under learning than under rational expectations during the early period after the shock since they have more optimistic expectations about the future returns to capital and increase investment sharply in the learning model. However, households eventually consume more under learning than under rational expectations in the future. Bottom panels also show the overshooting of investment and labor supply under learning after the positive technology shock hits the economy. Additionally, output, investment, and hours responses in Figure 1.7 exhibit a hump-shaped profile as in Eusepi and Preston (2011). These results are also in line with Cogley and Nason (1995) which demonstrate that the response of output to a technology shock is hump-shaped.



Figure 1.7: Impulse Responses of Macrovariables to 1% Permanent Technology shock

Notes: The solid line denotes the median impulse responses from the model under learning. The dotted line indicate the 25th and 75th percentile impulse response under learning. Also, the dashed line indicates the impulse responses under rational expectations. Moreover, unshocked BGP signifies the initial balanced growth path before the permanent technology shock occurs. **Mechanisms.** The mechanisms of (heterogeneous) household sentiment in the learning model can be explained with Figure 1.8, which shows young and old household's expected present value of returns to capital and labor in the learning and rational expectation model. For example, the expected present values of returns to capital and labor for young households under learning and RE are

Under Learning:
$$\tilde{E}_t^y \sum_{h=0}^{\infty} (v\gamma\beta)^h \hat{R}_{t+1+h}$$
 and $\tilde{E}_t^y \sum_{h=0}^{\infty} (v\gamma\beta)^h \hat{w}_{t+1+h}$
Under RE: $E_t \sum_{h=0}^{\infty} (v\gamma\beta)^h \hat{R}_{t+1+h}$ and $E_t \sum_{h=0}^{\infty} (v\gamma\beta)^h \hat{w}_{t+1+h}$

where these terms appear in young households' aggregate consumption decision rule under learning and RE. Then, this chapter finds noticeable differences between the RE and learning model. In the RE model, the expected returns to capital increases sharply but the expected wage rises gradually due to the positive technology shock. However, after households in the learning model observe the positive technology shock, they raise the expected returns to capital overly but the expectation about the wage becomes even flatter because of their overweight on the recent observations. Figure 1.8 demonstrates that the expected present value of returns to capital in the learning model is overly higher than in the rational expectation model whereas the expected present value of returns to labor under learning is lower than under rational expectations. In particular, young households have more optimistic expectations about the future real interest rate than old households due to young households' relatively higher sensitivity to recent shocks. However, both young and old cohorts' expectations are almost the same in the rational expectation model in which no different weighting schemes exist.



Figure 1.8: Expected Present Value of Returns to Capital and Labor

Notes: The legend in the upper panel indicates (a) Young under Learning: $\tilde{E}_t^y \sum_{h=0}^{\infty} (v\gamma\beta)^h \hat{R}_{t+1+h}$, (b) Old under Learning: $\tilde{E}_t^o \sum_{h=0}^{\infty} (\gamma\beta)^h \hat{R}_{t+1+h}$, (c) Young under RE: $E_t \sum_{h=0}^{\infty} (v\gamma\beta)^h \hat{R}_{t+1+h}$, and (d) Old under Learning: $E_t \sum_{h=0}^{\infty} (\gamma\beta)^h \hat{R}_{t+1+h}$. And the legend in the lower panel denotes (a) Young under Learning: $\tilde{E}_t^y \sum_{h=0}^{\infty} (v\gamma\beta)^h \hat{w}_{t+1+h}$ and (b) Young under RE: $E_t \sum_{h=0}^{\infty} (v\gamma\beta)^h \hat{w}_{t+1+h}$.

Consumption, Investment, and Hours. The mechanism above explains the dynamics of macro-variables in Figure 1.7 with more details. Due to overly optimistic expectations about the future returns to capital, households consume less and increase investment sharply right after the shock and then raise their consumption in the future. These dynamics are based on the intertemporal decision rule. In other words, they smooth their consumption through their investment. Furthermore, young households supply more labor today to raise their investment and future consumption.

Oscillations in Expectations The households learning behavior that places more weight on the recent observations when they form expectations leads to oscillations in expectations as in Figure 1.8. And this oscillations in expectations create more fluctuations in economic activities. This chapter discusses the effects of (heterogeneous) household sentiment on the volatility of business cycles in Section 1.4.3.

Persistence. One important difference between in the learning and RE model is whether persistent effects exist or not. Figure 1.7 shows that the reactions under learning are more persistent than under RE. This persistence is originated from the household learning behavior as in Figure 1.8. Since households place more weight on recent data, it takes quite long time for expectations under learning to reach expectations under RE. In other words, the persistence in the learning model is due to the adjustments in beliefs. Nonetheless, if there is no more technology shock, household irrationality disappears in the end as forecasting errors go to zero.

Effects by Age Group. The impulse response results lead to a conclusion that young households sentiment after the economic shocks is the critical factor for the amplification of the responses of the macroeconomic variables. Upper panels in Figure 1.9 plot the responses of consumption by age group in the LC learning and rational expectation model. The left response of consumption shows that under learning, young households over-react to the technology shock more than old households. This result can be explained by the findings in the literature that young individuals put more weight on the recent experiences than old individuals [e.g., Malmendier and Nagel (2016)]. However, the right response of consumption indicates no significant difference between consumption of young and old households under rational expectations. Bottom panels also show that young households save more than old households after the technology shock takes place due to their relatively higher sensitivity to the shock. One thing to note is that the consumption gap between young and old households is persistent since the permanent technology shock causes the changes in the wealth distribution between them. To be specific, the young's overly high expectations about future interest rates have them raise their saving sharply in response to the permanent technology shock in the learning model, which results in the change in the distribution of wealth between the young and old. As a result, young individuals come to possess relatively more assets than before, and this divergence leads to the persistent gap in consumption between the two cohorts.



Figure 1.9: Impulse Responses of Consumption and Saving to 1% Permanent Technology Shock by Age Group

Moreover, Figure 1.10 denotes the response of output in the LC learning model by the proportion of young households in the economy (ϕ): 0.7 (=70%), 0.8 (=80%), and 0.9 (=90%). In line with the previous results, if the proportion of young households decreases, which means the old population share rises, then the response of output becomes smaller. Intuitively, population aging increases the proportion of old households who have relatively lower sensitivity to recent shocks. Consequently, the amplification effects of household sentiment become weak in an aging society.



Figure 1.10: Impulse Response of Output under Learning by Proportion of Young Households (ϕ)

Notes: The parameter ϕ denotes the share of young households in the economy and 0.8 (=80%) is the baseline value for this parameter.

Gain Parameters. For a robustness, this chapter checks how the response of output to the technology shock varies according to gain parameters [Figure 1.11]. First, if the gain parameters of young and old households, i.e., g^y and g^o , are zero, then the response comes to be identical to the response of output in the rational expectation model. Since zero gain parameters imply that household sentiment does not play a role in the economy, household expectations under learning match rational expectations. Furthermore, the upper panel indicates that the response of output to the shock becomes larger as the gain parameter gets bigger since larger gain parameters imply that households over-react to a recent shock more sensitively. Last but not least, according to the lower panel, the increase in the parameter of young households and the decrease in the parameter of old households produce a bigger response than the opposite case since young households account for a large portion in the population and also they have optimistic views about not only future returns to capital but also future wages.



Figure 1.11: Impulse Response of Output under Learning by Gain Parameter

Notes: In the model, the baseline values for young and old households' gain parameters, g^y and g^o , are 0.018 and 0.010 respectively.

1.4.2 LC Model under Learning vs RA Model under Learning

The sentiment of economic agents has been considered as a critical factor to cause economic fluctuations. Thus, many studies related to household sentiment or expectations have been conducted so far. Nevertheless, previous studies generally employ representative agent models [e.g., Eusepi and Preston (2011)]. However, in contrast to existing papers, this chapter assumes heterogeneous agents, especially young and old households. Therefore, Section 1.4.2 compares the life-cycle learning model and the representative agent learning model to check what roles the life-cycle assumption plays in the learning model.

1.4.2.1 Representative Agent Learning Model

Section 1.4.2.1 modifies some parts of the life-cycle learning model to obtain the representative agent learning model.

Firms. The firm sector in the representative agent learning model is assumed to be the same as in the life-cycle learning model.

Households. An infinitely-lived representative agent maximizes the following Bellman equation

$$V(A_t^{RA}) = \max\{\ln C_t^{RA} + \theta \ln(1 - H_t^{RA}) + \beta \tilde{E}_t^{RA}[V(A_{t+1}^{RA})]\}$$
(1.35)

and his/her budget constraint is

$$C_t^{RA} + A_{t+1}^{RA} = R_t^{RA} A_t^{RA} + W_t^{RA} H_t^{RA} - T_t^{RA}$$
(1.36)

where \tilde{E}_t^{RA} is the subjective expectation or belief of the representative agent, C_t^{RA} is consumption, A_t^{RA} is the asset, W_t^{RA} is the real wage, H_t^{RA} is the labor supply, and T_t^{RA} is the lump-sum tax.

Then, this chapter obtains log-linearized aggregate consumption of the representative agent using the intertemporal budget constraint and Euler equation

$$\begin{split} \bar{c}\hat{c}_{t}^{RA} &= \frac{(1-\beta)\bar{a}}{(1+\theta)\beta}\hat{a}_{t}^{RA} + \left[\bar{c} - \frac{\bar{w}}{(1+\theta)}\right](\hat{R}_{t}^{RA} - \hat{\chi}_{t}^{RA}) \tag{1.37} \\ &+ \frac{(1-\beta)\bar{w}}{(1+\theta)}\hat{w}_{t}^{RA} - \frac{(1-\beta)\bar{\tau}}{(1+\theta)}\hat{\tau}_{t}^{RA} \\ &- \frac{\beta(\bar{w}^{-}\bar{\tau})}{(1+\theta)}\tilde{E}_{t}^{RA}\sum_{h=0}^{\infty}\beta^{h}[\hat{R}_{t+1+h}^{RA} - \hat{\chi}_{t+1+h}^{RA}] \\ &+ \frac{(1-\beta)\beta\bar{w}}{(1+\theta)}\tilde{E}_{t}^{RA}\sum_{h=0}^{\infty}\beta^{h}\hat{w}_{t+1+h}^{RA} \\ &- \frac{(1-\beta)\beta\bar{\tau}}{(1+\theta)}\tilde{E}_{t}^{RA}\sum_{h=0}^{\infty}\beta^{h}\hat{\tau}_{t+1+h}^{RA} \end{split}$$

For the comparison, log-linearized aggregate consumption in the life-cycle learning model, which is the summation of Equation (1.14) and (1.21), is as follows

$$\bar{c}\hat{c}_{t} = \left[\frac{(1-\gamma\beta)(1-v)}{\beta} + \psi'v\right]\bar{a}^{y}\hat{a}_{t}^{y} + \frac{(1-\gamma\beta)}{\beta}\bar{a}^{o}\hat{a}_{t}^{o} \qquad (1.38)$$

$$+ \left[\bar{c} - \frac{\psi'\phi\beta\bar{w}}{(1-v\gamma\beta)}\right](\hat{R}_{t} - \hat{\chi}_{t}) + \psi'\phi\beta\bar{w}\hat{w}_{t} - \psi'\phi\beta\bar{\tau}\hat{\tau}_{t}$$

$$- \left[\frac{\psi'\phiv\gamma\beta^{2}(\bar{w}-\bar{\tau})}{(1-v\gamma\beta)}\right]\tilde{E}_{t}^{y}\sum_{h=0}^{\infty}(v\gamma\beta)^{h}[\hat{R}_{t+1+h} - \hat{\chi}_{t+1+h}]$$

$$+ \psi'\phiv\gamma\beta^{2}\bar{w}\tilde{E}_{t}^{y}\sum_{h=0}^{\infty}(v\gamma\beta)^{h}\hat{w}_{t+1+h}$$

$$- \psi'\phiv\gamma\beta^{2}\bar{\tau}\tilde{E}_{t}^{y}\sum_{h=0}^{\infty}(v\gamma\beta)^{h}\hat{\tau}_{t+1+h}$$

where

$$\psi^{'} = \frac{(1 - v\gamma\beta)(1 - \gamma\beta)}{(1 + \theta)\beta(1 - \gamma\beta) + (1 - v)\gamma\beta^{2}}$$

Belief Updating. The representative agent is assumed to use an economic model to forecast future wages and returns to capital. The model relates the wage and return to capital to the aggregate capital which is the unique state variable in the representative agent model

$$\hat{R}_{t}^{k,RA} = \mu_{r}^{RA} + \mu_{rk}^{RA} \hat{k}_{t}^{RA} + e_{t}^{r,RA}$$
(1.39)

$$\hat{w}_t^{RA} = \mu_w^{RA} + \mu_{wk}^{RA} \hat{k}_t^{RA} + e_t^{w,RA}$$
(1.40)

$$\hat{k}_{t+1}^{RA} = \mu_k^{RA} + \mu_{kk}^{RA} \hat{k}_t^{RA} + e_t^{k,RA}$$
(1.41)

where e_t denotes a regression error.¹⁶

1.4.2.2 Main Differences between Two Learning Models: LC and RA Learning Model

Differences in Model. Both aggregate consumption equations in the LC and RA learning model, Equation (1.37) and (1.38), are overall similar to each other but there are some differences. For instance, there is one more state variable in the LC learning model other than aggregate capital (\hat{k}_t) . To be specific, the distribution of wealth between young and old households $(\hat{a}_t^y \text{ and } \hat{a}_t^o)$ is necessary to determine aggregate consumption in the LC learning model. Moreover, coefficients on the variables in the consumption decision rules are distinct between two learning models due to their differences. In particular, time discount factors create the main differences.

¹⁶For the forecasting model in the LC learning model, see Equation (1.22)-(1.25).

Table 1.4 compares the coefficients and time discount factors on the expected terms from both learning models and implies that the differences in consumption behavior between two models are mostly originated from the presence or absence of the probabilities of death (γ) and losing a job (v). In the life-cycle learning model, households consider that they cannot survive into the next period or retire in the following period. Therefore, they put less weight on the future (i.e., smaller coefficients and time discount factor on the expected terms). Conversely, the representative household has relatively larger weights on the expected wealth (\hat{w}_{t+1+h} and \hat{R}_{t+1+h}) since they do not consider the case of the death and retirement in the future.

 Table 1.4: Coefficients and Time Discount Factors on Expected Terms in Aggregate

 Consumption

	(A) \hat{w}_{t+1+h}
Representative Agent Model	Life-cycle Model
$\frac{(1-\boldsymbol{\beta})\boldsymbol{\beta}\boldsymbol{\bar{w}}}{(1+\boldsymbol{\theta})}\tilde{E}_{t}^{RA}\sum_{h=0}^{\infty}\boldsymbol{\beta}^{\boldsymbol{h}}\hat{w}_{t+1+h}^{RA}$	$\phi_{\frac{(1-v\gamma\beta)v\gamma\beta\bar{w}}{(1+\theta)+(1-v)\frac{\gamma\beta}{(1-\gamma\beta)}}}\tilde{E}_t^y\sum_{h=0}^{\infty}(v\gamma\beta)^h\hat{w}_{t+1+h}$
	(B) \hat{R}_{t+1+h}
Representative Agent Model	Life-cycle Model
$\frac{\boldsymbol{\beta}(\boldsymbol{\bar{w}}-\boldsymbol{\bar{\tau}})}{(\mathbf{1+\theta})}\tilde{E}_{t}^{RA}\sum_{h=0}^{\infty}\boldsymbol{\beta}^{\boldsymbol{h}}\hat{R}_{t+1+h}^{RA}$	$\phi_{\frac{v\gamma\beta(\bar{w}-\bar{\tau})}{(1+\theta)+(1-v)\frac{\gamma\beta}{(1-\gamma\beta)}}}\tilde{E}_t^y\sum_{h=0}^{\infty}(v\gamma\beta)^h\hat{R}_{t+1+h}$

Differences in Results. Figure 1.12 gives the impulse response results from the LC and RA learning model. Main difference between two models is that the responses in the RA learning model are 1.5 to 2 times larger than the ones in the LC learning model. This is because the representative household puts much more weight on over-optimistically expected wealth after the positive shock than households in the LC model due to their larger time discount factor. In other words, their irrationality dominates that of households in the LC model. The problem is that the extensive

over-responses of the representative household do not match the data in the real world. This problem will be addressed in Section 1.4.3 in detail. Furthermore, in my LC and RA learning model, gain parameters are calibrated as $g^y = 0.018(1.8\%)$, $g^o = 0.010(1.0\%)$, and $g^{RA} = 0.0164(1.64\%)$ based on the estimations of Malmendier and Nagel (2016). However, Eusepi and Preston (2011) use the small gain parameter 0.002 (0.2%) that is much below the normal range for it, 0.007 (0.7%) - 0.05 (5.0%), which is given by the literature. This small gain parameter is necessary to reduce the agents' extensive over-responses in their RA learning model and to match the data.



Figure 1.12: Impulse Responses of Output, Consumption, Investment, and Hours to 1% Permanent Technology Shock in LC and RA Learning Model

Notes: The solid line is the response in the RA learning model and the dashed line is the response in the LC learning model.

1.4.3 Effects on Volatility of Business Cycles

The life-cycle learning model is able to match the volatility of business cycles in the data well. This property suggests that the LC learning model is appropriate to study the causes of the changes in the volatility of business cycles in recent years. Therefore, this chapter shows how much population aging contributes to the recent reduction in the fluctuations of business cycles using the LC learning model.¹⁷

There are several findings from relative standard deviations of macro-variables provided by the data and model simulations in Table 1.5. First of all, Row A presents the relative standard deviations of macro-variables in the data. Then, the Row B shows that when the parameter ϕ , the proportion of young households, is 0.8 (80%), the amplification effects derived from the households learning behavior in the LC learning model produce almost the same variations of macroeconomic variables as in the data.¹⁸ However, the RA learning model creates huge fluctuations in macro-variables due to the extensive over-responses to the shocks of the representative household who has the larger time discount value.¹⁹ This is why the life-cycle assumption is inevitable to match the data well. Second, the LC learning model demonstrates that the fluctuations in business cycles decrease as the population ages. In other words, if the proportion of young households in the economy (ϕ) decreases from 90% (0.9) to 70% (0.7), then the relative standard deviation of output decreases from 1.77 to 1.49 as in Row B. Lastly, according to Row C, rational expectation models regardless of the RA or LC assumption can explain only half of the volatility of macro-variables in the data.

¹⁷The 2000-period simulation under RE presents initial steady-state coefficients for the households' forecasting model, and the simulation results are discarded. The 300-period simulation is then conducted to match the sample size for the U.S. data from 1947Q1 to 2019Q4.

 $^{^{18}}$ Eusepi and Preston (2011) show that a learning model fits the data better.

 $^{^{19}}$ See Section 1.4.2.2

	(1) σ_Y/σ_{Pr}	(2) σ_C/σ_Y	(3) σ_I/σ_Y	(4) σ_H/σ_Y
A. Data	1.64	0.82	4.33	1.20
B. Learning				
(i) Life-Cycle				
$\phi = 0.7$	1.49	0.75	4.12	1.66
$\phi = 0.8$ (Baseline)	1.64	0.74	4.32	1.88
$\phi = 0.9$	1.77	0.85	4.61	2.11
(ii) Representative	2.59	0.75	4.63	3.26
C. Rational Expectation				
(i) Life-Cycle				
$\phi = 0.8$	0.88	0.47	1.96	0.32
(ii) Representative	0.90	0.49	1.93	0.36

Table 1.5: Relative Standard Deviations of Macro-Variables from Model Simulations

Notes: All macro-variables are logged and detrended with the HP-filter. Also, Pr stands for productivity.

The simulation results from the LC learning model imply that population aging is one of the factors reducing the fluctuations of business cycles in recent decades. To begin with, Table 1.6 indicates that the volatility of macro-variables decreases sharply in recent past years. To be specific, the standard deviations of GDP, Consumption, and Investment declines 67.2%, 44.4%, and 20.3% respectively from the 1990s to 2010s. Also, Figure 1.13 plots the U.S. population aging trend and shows the surge in the old population ratio and median age in the U.S. during the period when the volatility of macro-variables decreases. For example, the proportion of the population who are 60 years or over in the U.S. rises from 21.4% in 1999 to 28.2% in 2019. These stylized facts suggest that population aging is related to the reduction in the fluctuations of macro-variables. Literature lists the improved monetary policy, regulatory changes and financial market innovation, and so on for the causes of the decline in the macroeconomic volatility since the 1990s. Furthermore, Jaimovich and Siu (2009)

claim that demographic changes account for 1/5 to 1/3 of the moderation of the US economy. They mainly argue that the low volatility of old individuals' employment and hours worked leads to the moderation. In contrast to their statement, this chapter claims that the increase in the share of old individuals who are less sensitive to recent observations also contributes in part to the lower volatility of business cycles. The life-cycle learning model simulations demonstrate that a 10% points increase in the old population ratio leads to about a 16% decrease in output volatility.²⁰ As above, in the U.S. data, the GDP volatility has declined about 67%, but the old population ratio has risen about 7%p from the 1990's to 2010's. These findings imply a meaningful role of population aging in the moderation of business cycle fluctuations. Table 1.6: Standard Deviations of Annual Growth Rates of Macro-Variables from

U.S. Data

(1) Standard Deviations					
Period	A. GDP	B. Consumption	C. Investment		
1960Q1-2020Q3	2.38	2.10	9.52		

(2) Standard Deviations, Relative to 1960Q1-2020Q3					
Period	A. GDP	B. Consumption	C. Investment		
(i) 1960Q1-1969Q4	0.86	0.81	0.85		
(ii) 1970Q1-1979Q4	1.14	1.07	1.19		
(iii) 1980Q1-1989Q4	1.12	0.97	1.32		
(iv) 1990Q1-1999Q4	0.64	0.72	0.69		
(\mathbf{x}) 200001 200004	0.91	0.90	0.99		
(1) 2000&1-2009&4	(0.68)	(0.61)	(0.79)		
(vi) 2010Q1-2019Q4	0.21	0.40	0.55		

Notes: Table 1.6 is the extended version of the table in Stock and Watson (2003). Furthermore, () indicates the calculations in which the data for the Great Recession (2007Q4-2009Q2) are excluded.

²⁰Output volatility is calculated using the growth rate of output from the model simulations.



Figure 1.13: U.S. Population Aging Trend Sources: OECD.Stat and UN World Population Prospects 2019

1.5 Fiscal Policy Implications of (Heterogeneous) Household Sentiment

This section studies how aggregate household sentiment and heterogeneous household sentiment affects the macroeconomic and welfare effects of government spending. First, government spending is financed by a lump-sum tax. After that, distortionary taxes, i.e., a labor and capital tax, are utilized to raise government spending.

1.5.1 Government Spending Financed by Lump-Sum Tax

Section 1.5.1 details the effects of government spending financed by the lump-sum tax. First of all, this chapter checks how government spending affects the dynamics of aggregate macro-variables in the learning and rational expectation model differently. Then, heterogeneous household sentiment explains lower government spending multipliers in an aging society. After that, a policy experiment demonstrates that the earlier implementation of government spending is more effective in helping the economy recover faster during a recession. Last but not least, this chapter performs welfare analyses to verify how government spending impacts young and old individuals' welfare.

1.5.1.1 Aggregate Effects of Government Spending under Learning

My study obtains the impulse responses of the macroeconomic variables to government spending equal to 5% of the steady-state output level, which is financed by the same amount of the lump-sum tax increase. This government spending can be considered as a *one-time surprise* shock. To be specific, government spending evolves via

$$g_t = \begin{cases} \bar{g} + 0.05\bar{y}, & \text{for } t = 1\\ \bar{g}, & \text{for } t = 2, 3, 4, \cdots \end{cases}$$

Figure 1.14 gives the impulse response results from the surprise government spending shock. Overall, the household learning mechanism produces the over-responses of the macro-variables as in the case of the technology shock in Figure 1.7. Moreover, while the responses are persistent under learning, agents under RE react to the transitory government spending shock only in a short period. These results are in line with the findings of the previous papers that the output multipliers for government purchases are significantly higher under learning, which contrasts with the implausibly low values under rational expectations [e.g., Mitra, G. W. Evans, and Honkapohja (2013)].



Figure 1.14: Impulse Responses of Output, Consumption, Investment, and Hours to Government Spending (Equal to 5% of Steady-State Output Level)

Notes: The solid line denotes the median impulse responses from the simulation under learning. And the dotted line indicate the 25th and 75th percentile impulse response under learning, and the dashed line is the responses under rational expectations.

1.5.1.2 Heterogeneous Household Sentiment and Effects of Government Spending by Age Group

Figure 1.15 plots young and old households' impulse responses of consumption and saving to government spending (equal to 5% of the steady-state output level) funded

by the same amount increase in the lump-sum tax. First, heterogeneous responses in this figure are similar to the results from the technology shock in Figure 1.9. Second, the heterogeneity between young and old households' reaction is also caused by their different weighting schemes on recent observations.



Figure 1.15: Impulse Responses of Consumption and Saving to Government Spending (Equal to 5% of Steady-State Output Level) by Age Group

Moreover, the responses of output to the government spending shock become smaller as the proportion of old households increases (e.g., $\phi: 0.9 \rightarrow 0.7$) [Figure 1.16]. This is because old households have relatively lower sensitivity to the government spending shock and so the amplification effects become weak in an aging society.



Figure 1.16: Impulse Response of Output to Government Spending Shock by Proportion of Young Households (ϕ)

Furthermore, Figure 1.17 provides the cumulative government spending multipliers in the first, second, and third year after the shock using the impulse response results in Figure 1.16. The cumulative government spending multiplier (M) is defined as

$$M_T = \frac{\sum_{i=0}^{T} (\beta \gamma)^i (y_{t+i} - \bar{y})}{g_t - \bar{g}}$$
(1.42)

where T is the cumulation period and \bar{y} and \bar{g} are the steady-state values for output and government spending. In general, the government spending multiplier declines about 10% when the old population ratio rises 10% points. This result is in agreement with existing papers that empirically and theoretically show that the government spending multiplier is low in an aging society [e.g., Basso and Rachedi (2021) and Honda and Miyamoto (2020)]. However, my model suggests a novel reason why population aging reduces the government spending multiplier: the heterogeneous learning behavior between young and old individuals.



Figure 1.17: Cumulative Government Spending Multiplier by Proportion of Young Households (ϕ)

1.5.1.3 Policy Experiment: Timing Effects of Government Spending under Learning during Recession

In Section 1.5.1.3, a policy experiment is conducted to derive the practical implications of the learning behavior for government expenditure. Specifically, this chapter studies the effects of government spending with different implementation times during a recession and examines whether the earlier implementation of a stimulus policy helps the economy recover faster from the recession.²¹

Policy Experiment. In the experiment, government expenditures are implemented after a negative technology shock occurs, but the implementation timing is different.²²

²¹According to G. W. Evans, Honkapohja, and Mitra (2009), it is well-known that there are long lags when implementing new fiscal policy. For instance, after a new policy is proposed, it needs to be signed into law and takes some time to be effective. In some cases, the process of changes in fiscal policy can take more than two years.

²²Chari, Kehoe, and McGrattan (2007) demonstrate that input financing frictions causing inefficiency in the usage of input factors can be observationally equivalent to negative productivity shocks. So, Mitra, G. W. Evans, and Honkapohja (2019) consider negative innovations to productivity as a convenient shortcut for modeling distortions related to the financial crisis. Also, the technology shock does not play a critical role in the experiment since this experiment intends to check the effects of government spending at different timings.

To be specific, this chapter considers three scenarios on the basis of Mitra, G. W. Evans, and Honkapohja (2019). First of all, the government spending shock takes place simultaneously with the negative technology shock. Secondly, the government implements the stimulus policy one year later after the negative technology shock hits the economy. Lastly, the government spending shock occurs two years later after the negative technology shock takes place.

Timing Effects of Government Spending in Learning Model. The upper left panel in Figure 1.18 gives the responses of output to the 1% negative technology shock at time t and the government spending shocks (equal to 5% of steady-state output) at time t, t+4, and t+8 which are all surprise shocks. Especially, the upper right panel plots the difference between output with the government spending shock at time t and t+4 when the negative technology shock hits the economy at time t. This panel indicates that an earlier stimulus policy is more effective in boosting the economy. The mechanism can be explained with the lower left panel that shows how the weights on the policies change over time. If the stimulus policy is implemented early, the high weight for that policy leads to a better performance of the economy at an early stage. Even if the areas of these two yellow spaces are the same in the end, it is better to obtain concentrated positive effects early than getting dispersed effects later. As a result, the rapid policy implementation helps in a swift economic recovery. Furthermore, although the population ages ($\phi \downarrow$), earlier policies are still better than later ones as in the upper right panel.

Timing Effects of Government Spending in RE Model. The lower right panel in Figure 1.18 indicates the difference between output responses to the government spending shocks at time t and t + 4 when the negative technology shock occurs at time t in the rational expectation model. This chapter finds that there is no
significant difference in the effects of government spending at different timings under rational expectations. This is because households in the rational expectation model do not have the weighting schemes as in the learning model, and thus they produce no forecast errors when expecting the future economy. Consequently, the earlier government spending shocks cannot have persistent effects, which leads to the absence of the timing effects of government policies in the rational expectation model.



Figure 1.18: Effects of Government Spending Shock at Different Timings

Notes: The steady-state (SS) in the y-axis label indicates the new BGP after the permanent technology shock takes place. The upper right panel shows the difference between the solid and dashed line in the upper left panel. The lower left panel shows young individuals' weights on government spending shocks at time t and t+4.

1.5.1.4 Welfare Analyses of Government Spending Financed by Lumpsum Tax

Section 1.5.1.4 studies how government spending financed by the lump-sum tax impacts the welfare of young and old households in the RE and LC learning model by conducting welfare experiments. In particular, my study estimates the welfare effects of government spending using equivalent variations on the basis of the methodology in literature.

Welfare Experiments Using Lump-Sum Tax. This chapter conducts the welfare experiments under the following scenarios to study the welfare effects of government spending. Specifically, during 500 periods, a random technology shock with mean zero and standard deviation σ^u takes place every period in the RE and LC learning model.²³ Then, I assume two economies in each RE and LC learning model. In the first economy, the government counteracts negative technology shocks by increasing its spending.²⁴ However, in the second economy, the government does not respond to the technology shocks. By comparing the welfare of the population in these two economies, this chapter can estimate the welfare effects of the government spending policy.

Equivalent Variations. Following Hunt (2021), this chapter estimates the welfare effects of the government spending policy using the equivalent variations (EV) which

²³This chapter still considers negative innovations to technology as a shortcut for modeling distortions related to the crisis. This assumption does not have a great impact on main findings since this experiment intends to check the welfare effects of government spending and compares the economy with and without the government spending shock.

 $^{^{24}}$ For instance, the government increases its spending by 5% of the steady-state output level in response to a -1% technology shock.

are calculated by:

Under RE,
$$\sum_{t=1}^{T} (\beta \gamma)^{t-1} U(C_{RE,t}^{NonGov}(1+\Delta_{RE}^{Gov})) = \sum_{t=1}^{T} (\beta \gamma)^{t-1} U(C_{RE,t}^{Gov})$$
 (1.43)

Under Learning,
$$\sum_{t=1}^{T} (\beta\gamma)^{t-1} U(C_{Learning,t}^{NonGov}(1+\Delta_{Learning}^{Gov})) = \sum_{t=1}^{T} (\beta\gamma)^{t-1} U(C_{Learning,t}^{Gov})$$
(1.44)

where C_t^{NonGov} and C_t^{Gov} indicate consumption in the economy without and with the government spending policy, respectively. Also, the duration of the welfare experiments T is 500. The interpretation of Equation (1.43) is that the EV equates the utility of household in the first economy with the government spending policy to the utility of household in the second economy without the government spending plus a fraction Δ . In other words, for example, household in the second economy without government spending needs to consume $\Delta^{Gov}\%$ more every period to have the same welfare as household has in the first economy with government spending. As a result, $\Delta_{RE}^{Gov}\%$ and $\Delta_{Learning}^{Gov}\%$ demonstrate the welfare effects of government spending in the RE and LC learning model.

Welfare Effects of Government Spending in LC Learning Model. Table 1.7 details the welfare effects of government spending in the LC learning model $(\Delta_{Learning}^{Gov}\%)$ by the proportion of young households (ϕ). There are several findings from the welfare analyses. First of all, government spending is not helpful to improve the welfare of households. Although the welfare of old households improves slightly due to government spending, the welfare of young households deteriorates sharply since the increase in the lump-sum tax lowers the young's consumption and the rise in labor supply for future consumption raises disutility (see Column B. $\phi = 0.8$). Second, as the proportion of old households increases ($\phi \downarrow$), the welfare of both young and old households deteriorates since old households have relatively lower sensitivity to the government spending shock and so the amplification effects become weak in an aging society (see Last Row). However, the welfare of young households deteriorates more drastically compared to the welfare of old households (see Column A-C (%p)). The intuition for the last finding is as follows: the increase in the proportion of old households in the LC learning model indicates that fewer people have a high degree of optimism induced by government spending. As a result, after the government spending shock takes place, the economy performs relatively worse. Meanwhile, since young households over-react to this relatively poor performance of the economy, they consume less and supply labor more. However, old households do not respond to the relatively poor economy as much as young households. As a result, the welfare of young households deteriorates to more extent than the welfare of old households as the population ages.

Cohort	EV from	Welfare Effe	$\Lambda C (\% n)$		
		A. $\phi = 0.7$	B. $\phi = 0.8$	C. $\phi = 0.9$	- A-C (70p)
Old	Consumption	0.0009	0.0014	0.0064	-0.0055
Young	Labor Supply	-3.0553	-2.6538	-2.3468	-0.7085
	Consumption	-0.9306	-0.7154	-0.5764	-0.3542
	Sum	-3.9859	-3.3692	-2.9232	-1.0627
Total Population		-2.7898	-2.6951	-2.6303	-0.1596

Table 1.7: Equivalent Variations in LC Learning Model by Proportion of Young Households (ϕ): Lump-Sum Tax

Notes: Parameter ϕ indicates the proportion of young households in the economy and the baseline value for ϕ is 0.8.

Comparing Welfare Effects under RE and under Learning. Table 1.8 provides the welfare effects of government spending in both the RE (Δ_{RE}^{Gov} %) and LC learning ($\Delta_{Learning}^{Gov}$ %) model. There are several notable things. Firstly, as in the LC learning model, government spending in the RE model also has a negative impact on the welfare of households, especially young households' welfare due to the tax increase. Thus, the welfare of the total population in the RE model deteriorates after the government spending shock takes place (see Column A. RE). Secondly, even though the overall welfare effects of government spending are still negative in the LC learning model, those are greater than the welfare effects in the RE model (see Last Row). This is because household optimism from government spending amplifies the government spending shock effects and improves the overall welfare of households. Nonetheless, heterogeneous household sentiment differently affects the welfare of young and old households: it reduces the welfare of old households but increases the welfare of young households (see Column B-A(%p)). The mechanism for the last finding is as follows. Government spending crowds out private consumption, and old households in the LC learning model reduce their consumption more than in the RE model due to their over-reaction. On the other hand, young households (workers) in the LC learning model greatly increase the labor supply to increase future consumption. Accordingly, despite the disutility caused by the increase in labor supply, the welfare of young households in the LC learning model is greater than that in the RE model due to overly increased consumption.

1.5.2 Government Spending Financed by Distortionary Taxes

In Section 1.5.2, the distortionary taxes, i.e., the labor and capital tax, finance government spending. First of all, some modifications to the LC learning model are provided, and then the effects of government spending financed by each distortionary tax are verified. Finally, this chapter explores the welfare effects of these government policies.

		Welfare effects	Learning vs RE	
Cohort	EV from	A. RE	B. Learning	$\mathbf{D} \wedge (\mathcal{O}_{\mathbf{n}})$
		$(\Delta^{Gov}_{RE}\%)$	$(\Delta^{Gov}_{Learning}\%)$	D-A (70 p)
Old	Consumption	0.0168	0.0014	-0.0154
	Labor Supply	-2.5715	-2.6538	-0.0824
Young	Consumption	-2.1436	-0.7154	1.4281
	Sum	-4.7150	-3.3692	1.3458
Total Population		-3.7687	-2.6951	1.0736

Table 1.8: Equivalent Variations in Rational Expectation and Life-Cycle Learning Model: Lump-Sum Tax

Notes: Parameter ϕ has the baseline value 0.8 and the last column in bold shows the welfare effects of household sentiment in the life-cycle learning model relative to the rational expectation model.

1.5.2.1 Model Modifications

There are some modifications to the LC learning model due to the usage of the distortionary taxes instead of the lump-sum tax.

(a) Firms

Firms have the same production function, Equation (1.2), using the capital and labor. Also, the labor-augmenting productivity follows the same process, Equation (1.3).

(b) Old Households

Old households have the same utility function and belief updating rule. However, their budget constraints change because of the distortionary taxes. The normalized budget constraint for old households is

$$c_{a,t}^{o} + \gamma a_{a+1,t+1}^{o} = R_t a_{a,t}^{o} \chi_t^{-1} + s_t \tag{1.45}$$

where $R_t = R_t^k (1 - \tau_t^k) + 1 - \delta$.

Thus, log-linearized aggregate consumption of old households changes from Equation (1.14) to Equation (1.46)

$$\hat{c}_{t}^{o} = \eta_{ay}^{'}(1-v)\hat{a}_{t-1}^{y} + \eta_{ao}^{'}\hat{a}_{t}^{o} - \eta_{\tau^{k}}^{'}\hat{\tau}_{t}^{k} + \eta_{s}^{'}\hat{s}_{t}$$

$$+ \eta_{\tau^{k}\chi}^{'}\left[\frac{\bar{R}^{k}(1-\bar{\tau}^{k})}{\bar{R}}\hat{R}_{t}^{k} - \hat{\chi}_{t}\right] + \eta_{\tau^{k}}^{e'}\tilde{E}_{t}^{o}\sum_{h=0}^{\infty}(\gamma\beta)^{h}\hat{\tau}_{t+1+h}^{k}$$

$$- \eta_{\tau^{k}}^{e'}\frac{\bar{R}^{k}(1-\bar{\tau}^{k})}{\bar{R}}\underbrace{\tilde{E}_{t}^{o}\sum_{h=0}^{\infty}(\gamma\beta)^{h}\hat{R}_{t+1+h}^{k}}_{h=0} + \eta_{s}^{e'}\tilde{E}_{t}^{o}\sum_{h=0}^{\infty}(\gamma\beta)^{h}\hat{s}_{t+1+h}$$

$$= PV \text{ of Returns to Capital}$$

$$(1.46)$$

where $\eta^{'}$ consists of the primitive model parameters.

(c) Young Households

Young households also have the same utility function and belief updating rule. However, the normalized budget constraint for young households is transformed into

$$c_{b,t}^{y} + v\gamma a_{b+1,t+1}^{y} + (1-v)\gamma a_{b+1,t+1}^{y} = R_{t}a_{b,t}^{y}\chi_{t}^{-1} + (1-\tau_{t}^{H})w_{t}H_{b,t}$$
(1.47)

where $R_t = R_t^k (1 - \tau_t^k) + 1 - \delta$.

Due to alterations in the young households' budget constraint, the labor supply condition is also modified into Equation (1.48) from Equation (1.19)

$$\frac{\theta c_{b,t}^y}{1 - H_{b,t}} = (1 - \tau_t^H) w_t \tag{1.48}$$

Also, log-linearized aggregate consumption of young households changes from Equa-

tion (1.21) to Equation (1.49)

$$\begin{split} \hat{c}_{t}^{y} &= \psi_{ay}^{'} \hat{a}_{t}^{y} + \psi_{rk\chi}^{'} \left[\frac{\bar{R}^{k} (1 - \bar{\tau}^{k})}{\bar{R}} \hat{R}_{t}^{k} - \hat{\chi}_{t} \right] + \psi_{w}^{'} \bar{w} (1 - \bar{\tau}^{H}) \hat{w}_{t} - \psi_{\tau^{H}}^{'} \hat{\tau}_{t}^{H} - \psi_{\tau^{k}}^{'} \hat{\tau}_{t}^{k} \quad (1.49) \\ &+ \psi_{w}^{e'} \bar{w} (1 - \bar{\tau}^{H}) \underbrace{\tilde{E}_{t}^{y}}_{h=0} \sum_{h=0}^{\infty} (v\gamma\beta)^{h} \hat{w}_{t+1+h} - \psi_{\tau^{H}}^{e'} \tilde{E}_{t}^{y} \sum_{h=0}^{\infty} (v\gamma\beta)^{h} \hat{\tau}_{t+1+h}^{H} \\ &- \psi_{rk}^{e'} \frac{\bar{R}^{k} (1 - \bar{\tau}^{k})}{\bar{R}} \underbrace{\tilde{E}_{t}^{y}}_{h=0} \sum_{h=0}^{\infty} (v\gamma\beta)^{h} \hat{R}_{t+1+h}^{k} + \psi_{\tau^{k}}^{e'} \tilde{E}_{t}^{y} \sum_{h=0}^{\infty} (v\gamma\beta)^{h} \hat{\tau}_{t+1+h}^{k} \\ &- (1 - v)\gamma\psi_{rko}^{e'} \frac{\bar{R}^{k} (1 - \bar{\tau}^{k})}{\bar{R}} \underbrace{\tilde{E}_{t}^{y}}_{h=1} \sum_{h=1}^{\infty} (\frac{1 - v^{h}}{1 - v}) (\gamma\beta)^{h} \hat{R}_{t+1+h}^{k} \\ &- (1 - v)\gamma\psi_{rko}^{e'} \tilde{E}_{t}^{y} \sum_{h=1}^{\infty} (\frac{1 - v^{h}}{1 - v}) (\gamma\beta)^{h} \hat{\tau}_{t+1+h}^{k} \\ &+ (1 - v)\gamma\psi_{rko}^{e'} \tilde{E}_{t}^{y} \sum_{h=1}^{\infty} (\frac{1 - v^{h}}{1 - v}) (\gamma\beta)^{h} \hat{s}_{t+h} \end{split}$$

where ψ' consists of the primitive model parameters.

(d) Government

The distortionary taxes revise the government budget constraint from Equation (1.29) to Equation (1.50)

$$\tau_t^H W_t H_t + \tau_t^k R_t^k K_t = G_t + (1 - \phi) S_t \tag{1.50}$$

Also, this chapter still assumes that the government maintains its policy variables, i.e., τ_t^H, τ_t^k , and s_t , at the steady-state levels, which is announced by the government in advance. The policy announcement is assumed to be credible and so agents believe that there will be no changes in the government policies in the future.

1.5.2.2 Effects of Government Spending Financed by Distortionary Taxes

This chapter obtains the impulse responses of the macroeconomic variables to government spending financed by the distortaionary taxes (i.e., the labor and capital tax). As in the case of the lump-sum tax, government spending is a *one-time surprise exogenous* shock. Here, the government increases its spending by 2% of the steady-state output level at time t=1 and maintains it at the steady-state level since time t=2. Thus, the evolution of government spending is as follows

$$g_t = \begin{cases} \bar{g} + 0.02\bar{y}, & \text{for } t = 1 \\ \bar{g}, & \text{for } t = 2, 3, 4, \cdots \end{cases}$$

/

Government Spending Financed by Labor Tax. If government spending is financed by the labor tax, the labor tax rate increases from 25.0% to 29.7% at time t=1. After that, it decreases to the steady-state level. To be specific, the labor and capital tax evolve via

$$\tau_t^H = \begin{cases} 0.2973, & \text{for } t = 1\\ 0.2500 \ (=\bar{\tau}^H), & \text{for } t = 2, 3, 4, \cdots \end{cases} \text{ and } \tau_t^k = 0.1500 \ (=\bar{\tau}^k) \text{ for } t = 1, 2, 3, \cdots$$

Government Spending Financed by Capital Tax. If the capital tax finances government spending, the capital tax rate increases from 15.0% to 21.0% at time t=1 and returns to the initial level next period. Specifically, the labor and capital tax evolve via

$$\tau_t^H = 0.2500 \ (=\bar{\tau}^H) \text{ for } t = 1, 2, 3, \cdots \text{ and } \tau_t^k = \begin{cases} 0.2104, & \text{for } t = 1\\ 0.1500 \ (=\bar{\tau}^k), & \text{for } t = 2, 3, 4, \cdots \end{cases}$$

Output Responses.²⁵ Figure 1.19 shows the difference between the response of output under learning and rational expectations by the proportion of young households (ϕ) when the labor or capital tax finances government spending. Due to the households sensitivity to recent shocks, the effects of government spending are amplified as in the case of the lump-sum tax. Moreover, the amplification effects weaken (i.e., the difference, $\hat{y}_t^{Learning} - \hat{y}_t^{RE}$, decreases) as the proportion of old households rises $(\phi \downarrow)$ since they have relatively lower sensitivity to recent government spending shocks.



Figure 1.19: Difference in Response of Output to Government Spending Shock between under Learning and RE by Proportion of Young Households (ϕ)

 $^{^{25}}$ Here, this chapter shows only the output responses. But Appendix B describes the responses of other macroeconomic variables to government spending financed by the labor and capital tax.

1.5.2.3 Comparing Effects of Government Spending Financed by Labor and Capital Tax

Figure 1.20 compares the effects of government spending financed by the labor and capital tax. First of all, the upper panels plot the cumulative responses of output to government spending funded by the labor and capital tax. Although government spending is not effective to stimulate the economy in the RE model, its effects are amplified in the learning model due to households' sensitivity to recent shocks. Nonetheless, government spending financed by the capital tax is more helpful to boost the economy than that funded by the labor tax as in the upper panels. The mechanism for this finding is as follows. Government spending crowds out investment, which increases the real interest rate and lowers the real wage. Consequently, while the higher capital tax with the increased real interest rate gives moderate effects on both young and old households' saving, the higher labor tax with the decreased real wage creates intensive effects especially on young households' labor supply. The lower panels plot the responses of hours to government spending financed by the labor and capital tax. And this chapter finds that the labor tax levied on young households (workers) causes a sharp decrease in their labor supply whereas no such effect can be found in the case of the capital tax.

1.5.2.4 Welfare Analyses of Government Spending Financed by Distortionay Taxes

Section 1.5.2.4 studies how government spending financed by distortionary taxes, i.e., the capital and labor tax, affects young and old households' welfare in the presence of heterogeneous household sentiment. To tackle this question, this chapter conducts welfare experiments and calculates equivalent variations (EV) as in Section 1.5.1.4. Finally, my study examines the welfare effects in the LC learning model and then compares the welfare effects in the RE and LC learning model.



(a) Cumulative Responses of Output to Government Spending Shock



(b) Responses of Hours to Government Spending Shock

Figure 1.20: Impulse Responses of Output and Hours to Government Spending Financed by Distortionary Taxes

Welfare Experiments Using Distortionary Taxes. During 500 periods, a random technology shock with mean zero and standard deviation σ^u takes place every period in the RE and LC learning model. Then, this chapter assumes two economies in each RE and LC learning model: In the first economy, the government counteracts negative technology shocks.²⁶ However, in the second economy, the government does not change its spending responding to the technology shock. By comparing the welfare of the population in these two economies, this chapter can estimate the welfare effects of government spending using the distortionary taxes in each RE and LC learning model.

Welfare Effects of Government Spending in LC Learning Model. Table 1.9 shows the welfare effects of government spending, which is financed by the capital and labor tax, in the RE and LC learning model. There are a few things worth noting. Since both young (worker) and old (retiree) households hold assets, changes in the capital tax has a direct impact on the welfare of both young and old households. However, changes in the labor tax directly reduces only the welfare of young (worker) households. Consequently, for old households, using the labor tax when increasing government spending is more beneficial than using the capital tax. According to Column B. Learning in Table 1.9, government spending financed by the capital tax deteriorates the welfare of old households, using the labor tax improves the welfare of the old. Moreover, for young households, using the labor tax when raising government spending is still slightly favorable than using the capital tax. This is because the labor tax improves the young's welfare by lowering their labor supply sharply even if it deteriorates their welfare from consumption. To summarize, using

 $^{^{26}}$ For instance, the government increases its spending by 2% of the steady-state output level in response to a -1% technology shock. Raised government spending is financed by the increase in the capital or labor tax rate. In other words, the capital tax rate rises from 25.0% to 29.7% or the labor tax rate increases from 15.0% to 21.4%.

the labor tax when the government increases its spending is better in terms of the welfare effects although Section 1.5.2.3 finds that using the capital tax is better in terms of the output effects. Moreover, in an aging society, using the capital tax is still more effective to increase output but more old individuals need to put up with the decrease in their welfare.

Comparing Welfare Effects under RE and under Learning. The last columns in Table 1.9 indicate the effects of household sentiment on the welfare of young and old households since it compares the welfare effects in the RE and learning model. According to the results in the last columns, household sentiment greatly improves the welfare of young and old households when the government uses the labor tax to finance its spending. In particular, young households' sentiment plays a significant role in improving the total welfare of the population. The mechanism for this finding is as follows: after a one-time government spending shock occurs, the labor tax drastically reduces the young's labor supply, which improves their welfare from leisure. Also, since the labor tax rate goes back to the initial steady-state level in the next period, young households supply more labor than before, which increases their consumption sharply. All in all, young households' higher sensitivity to the shock greatly improves their welfare over the level in the RE model.

1.6 Conclusion

The sentiment of economic agents has been considered as one of the main drivers of economic fluctuations. Accordingly, many studies associated with the effects of sentiment on the economy have been conducted so far. This chapter also deals with the impacts of household sentiment on business cycles and, in addition, studies how

(1) Capital Tax							
		Welfare Effect	s of Gov Spending	Learning vs RE			
Cohort	EV from	A. RE	B. Learning	$\mathbf{D} \wedge (0' \mathbf{n})$			
		$(\Delta^{Gov}_{RE}\%)$	$(\Delta^{Gov}_{Learning}\%)$	D- A (70 p)			
Old	Consumption	-0.0014	-0.0073	-0.0058			
	Labor Supply	-0.7389	-0.7551	-0.0162			
Young	Consumption	-0.5436	-0.4307	0.1130			
	Sum	-1.2825	-1.1858	0.0968			
Total Population		-1.0263 -0.9501		0.0762			
		(2) Lab	or Tax				
		Welfare Effec	ts of Gov Spending	Learning vs RE			
Cohort	EV from	A. RE	B. Learning	$B_{-} \Lambda (\% p)$			
		$(\Delta^{Gov}_{RE}\%)$	$(\Delta^{Gov}_{Learning}\%)$	D- A (70 p)			
Old	Consumption	-0.0065	0.0015	0.0080			
	Labor Supply	0.1348	0.2454	0.1106			
Young	Consumption	-2.3994	-1.4297	0.9697			
	Sum	-2.2646	-1.1843	1.0803			
Total Population		-1.8130	-0.9471	0.8658			

Table 1.9: Equivalent Variations in Rational Expectation and Life-Cycle Learning Model: Capital and Labor Tax

Notes: Parameter ϕ has the baseline value 0.8 and the last column shows the welfare effects of household sentiment in the LC learning model relative to the RE model.

household sentiment influences the effects of government policies. In particular, in contrast to literature using the representative agent assumption, this chapter assumes heterogeneous agents, i.e., young and old households, who have distinct sentiment or expectations due to their different sensitivity to recent experiences.

There are two main empirical findings in this chapter. First of all, the belief wedge that is one of the measures of household sentiment demonstrates that household expectations can be more optimistic or pessimistic about the future economy than the data generating measure. Especially, the young have a higher degree of sentiment and the old. The mechanism for these findings is that individuals, especially the young, put more weight on recent observations when they form expectations.

The life-cycle learning model incorporating heterogeneous household weighting schemes shows that the household sensitivity to recent observations amplifies the effects of economic shocks. However, these amplification effects become less extensive as the old population share rises since old people have relatively lower sensitivity to recent experiences. Therefore, simulation results of the LC learning model suggest that a 10% points increase in the old population ratio leads to about a 16% decrease in output volatility. In the U.S. data, the GDP volatility declines about 67%, but the old population ratio rises about 7% from the 1990s to 2019s.

Fiscal policy implications from the life-cycle learning model are as follows. Firstly, the government spending multiplier declines about 10% when the old population ratio rises by 10% points since the amplification effects decline in an aging society. Secondly, although the output effects of government spending are amplified in the learning model, government spending financed by the capital tax is more effective in boosting the economy than that funded by the labor tax. This is because the labor tax reduces young households' labor supply sharply.

In addition, there are a few findings from the welfare analyses of fiscal policy. First of all, as the old households share increases, the welfare of young and old households from government spending deteriorates since the economy performs relatively worse in response to the government spending shock in an aging society due to old households' lower sensitivity to the shocks. Secondly, using the labor tax to increase government spending is better in a welfare aspect than using the capital tax. This is because the capital tax has a negative direct impact on the welfare of both young and old households, but the labor tax directly deteriorates only the welfare of young (worker) households.

Finally, some economic policy suggestions come up based on the results of this study. First, economic policies should be decided and implemented from a long-term perspective in an aging society since population aging reduces the economic volatility and hinders the short-term effects of government policies. Also, the government needs to consider how its policies can affect young and old individuals differently given their heterogeneous sentiment. For instance, the capital and labor tax have distinct impacts on two cohorts, which leads to different economic results. Moreover, this chapter implies that a faster and larger fiscal stimulus policy is necessary in an aging society to support a swift recovery during recessions.

Chapter 2

Population Aging Effects on Impacts of Monetary Policy in Euro Area: Through Industrial Restructuring

2.1 Introduction

Population aging is one of the most challenging social problems that many countries are facing now, and even worse the population aging trend is expected to be stronger in the forthcoming decades. In particular, European countries have already experienced population aging, and they are anticipated to become even older than now in the near future. Figure 2.1 plots population aging projections around the world. Most regions, especially the European Union, show significant upward trends in population aging. Due to these demographic changes, many politicians and economists are concerned about how population aging will affect our society and economy in the future.

In that context, this chapter examines the impacts of population aging on the economy especially focusing on the heterogeneous consumption bundles of young and old individuals. Specifically, older individuals spend a larger portion of their income or wealth in services such as housing and health services but less on manufactured goods like vehicles and clothes compared to younger individuals. As a result, population aging can cause industrial restructuring due to the young and old's distinct consumption patterns. For example, the service (manufacturing) industries share has sharply increased (decreased) in aged countries in recent decades. One related problem is that monetary policy can be less effective when the proportion of the service industries rises since the conventional interest rate or cost of capital channel weakly operates in the service sector.¹ In that vein, this chapter mainly answers following questions. First, "How does population aging change industrial structures?" and second, "How do changes in industrial structures induced by population aging affect the effectiveness of monetary policy?".



Figure 2.1: Population Aging Projection around the World

Notes: Time series show the proportion of population aged 65 years or over by region. *Sources*: OECD.Stat

¹Conventional interest rate or cost of capital channel: a contractionary monetary policy leads to an increase in real interest rates due to the nominal rigidity. Firms find that their real cost of borrowing has increased and so cut back on their investment expenditures [Ireland (2005)].

To tackle these questions, this chapter firstly estimates how much population aging affects the size of the service or manufacturing industries. Then, using these results, this study also estimates to what extent population aging reduces the output effects of monetary policy through industrial restructuring. Lastly, I build the model that combines overlapping generations and a new Keynesian framework with two sectors to demonstrate the mechanism through which monetary policy comes to be less effective in an aging society.

To begin with, analyzing the statistics of the euro area provides following stylized facts. First, the old population ratio has been increasing since the 1990s and its upward trend has been even stronger since the mid-2000s. Second, old individuals in the euro area and also in the U.S. spend more (less) money on goods that are produced in the service (manufacturing) sector than young individuals. Specifically, people who 60 years old or over spend a larger portion of their income or wealth on services such as health and housing services whereas they spend less on manufacturing goods such as vehicles and clothes. Third, the proportion of the service (manufacturing) industries has risen (declined) in the euro area while the population has aged fast, especially since the 1990s. Given the stylized facts above, it can be concluded that population aging has contributed in part to the rise (decline) of the proportion of the service (manufacturing) industries due to the young and old's heterogeneous consumption baskets.

Then, this chapter estimates the effects of population aging on the industrial structure in the euro area using the balanced panel data. The panel data includes ten countries that are selected based on the GDP level and eleven industries in the euro area. The main finding is that a 1% point increase in the proportion of the population who are 65 years old or over raises the proportion of the service industries by 1.071% points but lowers the manufacturing industries share by 1.210% points. Moreover, among the service industries, the sizes of the industries related to *real estate* and *human health* increase significantly due to population aging. These results are consistent with the stylized facts that old individuals spend a larger proportion of their income or wealth on housing and healthcare services.

Furthermore, the monetary policy shocks in the euro area need to be identified in order to estimate the cross-industry impacts of monetary policy. Therefore, this chapter estimates a structural VAR model to obtain the structural shocks to monetary policy in the euro area. All the methodologies are based on Peersman and Smets (2001) in which they use a recursive restriction and short and long run restriction when estimating the structural VAR model.

Using identified monetary policy shocks in the euro area, this study estimates the output effects of monetary policy on eleven industries. The main finding is that the service industries are less sensitively affected by the monetary policy shock than the manufacturing industries. To be specific, a 1% point increase in the benchmark interest rate two quarters ago causes the growth rate of the total, manufacturing, and service industries' output at a current quarter to decrease by 0.19%p, 0.52%p, and 0.11%p, respectively.² Consequently, the output effects of monetary policy are weak in a country which has the higher service industries share. Existing papers provide mechanisms behind these results. For instance, Dedola and Lippi (2005) find that the effects of monetary policy are stronger in industries that produce durable goods and have greater working capital.³ ⁴ Other than these factors, Peersman and Smets (2002)

 $^{^{2}}$ The result that monetary policy is less effective in the service industries is in agreement with Ganley and Salmon (1997).

³Working capital mainly consists of trade receivables and inventories including raw materials, semi-finished goods, and final goods.

⁴Barth and Ramey (2000) state that interest rates and credit conditions affect firms' long-run ability to produce (by investing in fixed capital), they can also be expected to affect firms' short-run

also find some indications that the capital intensity of production and the degree of openness have an impact on the average monetary policy sensitivity. Since the service industries do not produce durable goods and do not have working capital and the capital intensity as much as manufacturing industries, the conventional interest rate or cost of capital channel of the monetary policy transmission does not operate well in the service sector.

Finally, this chapter estimates how the output effects of monetary policy in the euro area would change due to population aging using the estimation results above. To be specific, a 1% point contractionary monetary policy shock reduces the quarterly growth rate of total output of all industries by 0.162% points in the second quarter after the monetary policy shock hits the economy. However, the growth rate of total output declines only by 0.108% points when the population aging ratio rises by 10% points since population aging raises the proportion of the service industries in the economy. In other words, the output effects of monetary policy decrease by up to 33% due to a 10% points increase in the old population ratio.

Based on the empirical findings of this chapter, I build a model to study the mechanism that shows how the cost of capital channel becomes weak in an aging society. This model combines overlapping generations and a new Keynesian framework in which two sectors, the manufacturing and service sector, exist on the basis of Gali (2021) and Lee (2021). In the model, young households supply labor and earn a wage whereas old households do not. Also, young and old households have distinct consumption baskets. Specifically, they have different consumption weights on manufacturing goods and services. Due to this difference, population aging changes the economic size of each sector. Besides, this chapter assumes that firms in the man-

ability to produce (by investing in working capital).

ufacturing industries are more capital-intensive than those in the service industries. Therefore, changes in the proportion of each sector can affect the impacts of monetary policy since these changes influence the cost of capital channel of monetary policy.

The impulse response results from the theoretical model demonstrate that output of the manufacturing industries responds to the monetary policy shock more sensitively than output of the service industries. Also, the output effects of monetary policy declines by about 30% when the old population ratio increases by 10% points. This result is in agreement with the empirical findings that the output effects of monetary policy decreases by about 33% due to a 10% points increase in the population aging ratio. The key mechanism which the model provides is that the conventional interest rate or cost of capital channel of monetary policy transmission comes to be weak in an aging society due to a rise in the proportion of the service sector. To be more specific, as the population ages, the consumption expenditures on services, which are non-durable and produced by less capital-intensive firms, account for the larger proportion of household spending. As a result, the interest rate changes by the central bank is harder to affect the consumption and investment demand in an aging society. Moreover, firms have less abilities to adjust their production capacities through capital investment according to the interest rate changes as the share of less capital-intensive firms rises.

Related Literature. There are three strands of literature which are related to this chapter: 1) population aging effects on the industrial structure, (2) the cross-industry heterogeneity of the impacts of monetary policy, and 3) the effects of monetary policy in an aging society. To my knowledge, there are a limited number of analyses combining these topics. Therefore, my paper bridges this gap.

First of all, existing studies such as Cravino, Levchenko, and Rojas (2019) point out

that the aging of the population changes the industrial structure, and in particular, the service industry grows significantly in an aging society, which is also one of the main claims of this chapter.

Moreover, previous papers show significantly different effects of monetary policy shocks across manufacturing industries [e.g., Barth and Ramey (2000) and Loo and Lastrapes (1998)]. Raddatz and Rigobon (2003) find some heterogeneity in the sectoral responses to monetary policy. Especially, consumption of durables is highly interest-rate sensitive. Besides using the U.S. data, Ganley and Salmon (1997) and Hayo and Uhlenbrock (1999) show the cross-industry heterogeneity of the effects of monetary policy using the U.K. and Germany data, respectively. Dedola and Lippi (2005) demonstrate sizable and significant cross-industry differences in the effects of monetary policy using disaggregated industry data from five OECD countries. In particular, Dedola and Lippi (2005) reveal that the impacts of monetary policy are more substantial in the industries that produce durable goods and have greater working capital. Moreover, Peersman and Smets (2005) estimate the effects of monetary policy on output growth of eleven industries in seven euro area countries. They find that the monetary policy effects vary according to the durability of the goods produced in the sector and this result is the evidence for the conventional interest rate or cost of capital channel of monetary policy transmission. Peersman and Smets (2002) also show some indications that the capital intensity of production and the degree of openness have an effect on the monetary policy sensitivity. The papers mentioned above estimate the cross-industry effects of monetary policy mostly using manufacturing industries data. However, my study mainly compares the effects of monetary policy on the service industries with the effects on the manufacturing industries. This approach intends to verify the contention that population aging increases the

proportion of the service sector, which is less sensitive to interest rate changes.

Furthermore, existing papers study how the effects of monetary policy changes as the population ages. To begin with, Jones (2018) argues that population aging explains one-third of the gap between log output per capita and its trend in 2015 since demographics lowered real rates, which causes the zero lower bound to bind between 2009 and 2015. Also, Imam (2013) provides the empirical evidence for that the demographic shift to an older society causes changes in monetary policy to have a more benign impact on the economy. Moreover, Kantur (2013) constructs a model that is the merger of an overlapping generation setup and New Keynesian framework and shows monetary policy is less effective as the population ages due to declines in the interest rate sensitivity of real economic activity. Also, Wong (2016) empirically and theoretically demonstrates that the consumption of young people is significantly more responsive to interest rate shocks than older people. She argues that the refinancing channel of monetary policy can explain the heterogeneity between the young and old's reactions to monetary policy. Despite the papers introduced here, there are a limited number of papers that explore the industrial restructuring induced by population aging and its effects on the impacts of monetary policy together. Therefore, this chapter contributes to fill the gap.

The structure of this chapter is as follows. Section 2.2 summarizes changes in the old population ratio and industrial structures over time in the euro area. Then, this chapter presents the empirical evidence for how population aging affects the industrial structures in the euro area. Section 2.3 identifies monetary policy in the eurozone and demonstrates the cross-industry effects of the monetary policy. Moreover, this study forecasts changes in the effectiveness of monetary policy in the future caused by population aging. After that, Section 2.4 describes the model, and Section 2.5 details

the population aging effects on the impacts of monetary policy and its mechanism. Finally, Section 2.6 concludes.

2.2 Population Aging and Industrial Restructuring in Euro Area

This section shows the population aging trend and industrial structure changes in the euro area in recent decades. After that, empirical analyses using the euro area panel data estimate the effects of population aging on the industrial structures based on value-added and employment.

2.2.1 Population Aging Trend and Changes in Industrial Structures

According to OECD and Eurostat data, the old population ratio has escalated in the euro area since the 1990s. During that period, the size of the service (manufacturing) industries has also increased (decreased). Given that young and old individuals have the different consumption baskets, population aging seems to contribute in part to the rise (decline) of the service (manufacturing) sector in the euro area.

2.2.1.1 Population Aging Trend

Population aging has progressed at fast pace in the euro area in the last decades. Figure 2.1 shows that European countries have already experienced population aging and thus, their old population ratio, the proportion of the population who are 65 years old or over, is the highest among economic regions around the world. Moreover, Figure 2.2 gives the historical population aging trend with more details. The figure provides the box plot of the population aging ratio of the euro area member countries and indicates that the old population ratios of the eurozone countries have been surging since the 1990s.⁵ In particular, their upward trends become even stronger since the mid-2000s.



Figure 2.2: Population Aging Trend in Euro Area (Box Plot)

Notes: The proportion of people who are 65 years or over (% of total population) in sixteen member countries of the eurozone. *Sources*: OECD.Stat

2.2.1.2 Heterogeneous Consumption Bundles by Age Group

Table 2.1 and 2.2 provide the structures of consumption expenditure by age group in the euro area and the U.S. in 2019, respectively. According to two tables, old individuals in both regions have the similar consumption expenditure structures. Specifically, old individuals consume the service (manufacturing) goods more (less) than young

⁵The euro area consists of 16 countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Netherlands, Portugal, Slovak Republic, Slovenia, and Spain.

individuals.⁶

Heterogeneous Consumption Bundles in Euro Area. Table 2.1 supports the claim that young and old individuals have the different consumption baskets. In general, old individuals who are 60 years old or over spend a larger proportion of their income or wealth on the services, for example, *housing* and *health services*. However, their expenditure on manufacturing goods, such as *vehicles* and *clothes*, is much lower than that of young individuals.

	Under	30 - 44	45 - 59	60 years
	30 years	years	years	or over
Housing	23.9	22.9	22.7	27.1
Food and Non-Alcoholic Beverages	12.1	13.6	14.4	15.5
Transport	14.5	13.6	13.8	9.8
Water, Electricity, Gas and Other Fuels	6.1	6.4	6.8	8.6
Recreation and Culture	7.8	7.7	7.7	7.0
Furnishings, Household Equipment, and Routine Household Maintenance	4.1	4.6	4.5	4.9
Health	2.1	2.6	3.1	4.8
Restaurants and Hotels	7.9	7.1	6.5	4.6
Clothing and Footwear	5.4	5.5	4.8	3.4
Communications	3.4	2.8	2.8	2.4
Alcoholic Beverages, Tobacco, and Narcotics	2.3	2.0	2.3	2.0
Education	1.5	1.1	1.4	0.3
Miscellaneous Goods and Services	9.1	10.0	9.2	9.6

Table 2.1: Structure of Consumption Expenditure by Age Group in Euro Area (%)

Note: The consumption structure is based on the data in 2019. Sources: Eurostat

Heterogeneous Consumption Bundles in U.S. Table 2.2 shows that consumers

 $^{^{6}}$ Cravino, Levchenko, and Rojas (2019) also find that the portion of expenditures devoted to services rises with household age, using the U.S. household level data.

in the U.S. also have the similar consumption structures as the consumers in the euro area. For instance, old individuals spend more money on *housing* and *health care* but less on *transportation* and *apparel* than young individuals.

	Under	25-34	35-44	45-54	55-64	65 years
	25 years	years	years	years	years	or over
Housing	32.4	35.9	33.0	30.9	30.5	34.8
Transportation	21.1	18.0	18.3	17.3	16.4	14.9
Food	14.9	12.9	13.0	13.0	12.3	13.1
Personal Insurance and Pensions	8.5	12.7	12.8	13.4	13.2	5.7
Healthcare	3.8	5.5	6.4	6.9	8.6	13.6
Entertainment	3.5	4.2	4.9	5.0	5.7	4.7
Cash Contributions	1.1	1.5	2.9	2.7	3.4	5.1
Apparel and Services	3.6	3.3	3.3	3.1	2.6	2.6
Education	7.6	2.1	1.6	3.5	2.6	0.7
Miscellaneous	0.8	1.2	1.3	1.4	1.7	1.6
Personal Care Products and Services	1.2	1.1	1.2	1.3	1.2	1.4
Alcoholic Beverages	0.8	0.9	0.7	0.9	1.0	1.0
Tobacco Products and Smoking Supplies	0.5	0.6	0.5	0.5	0.6	0.4
Reading	0.1	0.1	0.1	0.1	0.1	0.3

Table 2.2: Structure of Consumption Expenditure by Age Group in U.S. (%)

Note: The consumption structure is based on the data in 2019. *Sources*: U.S. Bureau of Labor Statistics

2.2.1.3 Changes in Industrial Structures

Table 2.3 details the changes in the industrial structures based on value-added and employment in the euro area. According to the table, the proportion of the service industries has risen, but the proportion of the manufacturing industries has declined while population aging has progressed at fast pace from 1995 to 2019. To be specific, the proportion of the service industries based on value-added and employment has increased from 68.1% to 73.6% and 66.9% to 76.5%, respectively. However, the proportion of the manufacturing industries based on value-added and employment has decreased from 19.8% to 16.4% and 18.5% to 13.2%, respectively.

All in all, given the population aging trend and the changes in the industrial structures in the euro area, it can be claimed that population aging has raised the size of the service industries but has lowered that of the manufacturing industries. This chapter provides the empirical evidence for this claim in Section 2.2.2.

2.2.2 Effects of Population Aging on Industrial Structures

Section 2.2.2 estimates how population aging affects the industrial structures in the euro area using the balanced panel data.

Model. In order to check the effects of population aging on the industrial structures in the euro area, this chapter regresses each industry share based on value-added or employment on a population aging variable and controls. The panel data consists of ten countries and eleven industries in the euro area.⁷ The specification of the panel regression model for a country i and industry j is as follows⁸

$$Y_{i,t}^{j} = \alpha^{j} + \beta^{j} Aging_{i,t} + \delta^{j} Controls_{i,t} + \psi_{i}^{j} + e_{i,t}^{j}$$

$$(2.1)$$

where $Y_{i,t}^{j}$ is the share of an industry j based on value-added or employment in a

⁷Top ten countries among eurozone members are selected based on GDP level (i.e., Austria, Belgium, Finland, France, Germany, Ireland, Italy, Netherlands, Portugal, and Spain).

⁸The model specification is based on Rowthorn and Ramaswamy (1997), Kang (2017), and Cravino, Levchenko, and Rojas (2019).

	Value-Added		Emplo	oyment
	1995Q4	2019Q4	1995Q4	2019Q4
Services	68.1	73.6	66.9	76.5
Manufacturing	19.8	16.4	18.5	13.2
Agriculture	2.6	1.7	5.5	3.0
Construction	6.1	5.4	7.5	6.1
Service 1	19.0	19.0	23.9	25.0
Service 2	3.9	5.0	2.2	2.9
Service 3	5.0	4.4	2.9	2.4
Service 4	9.6	11.3	0.8	1.0
Service 5	9.1	11.7	8.0	14.0
Service 6	18.1	18.9	23.0	24.4
Service 7	3.4	3.4	6.0	6.7

Table 2.3: Changes in Industrial Structure Based on Value-Added and Employment in Euro Area (%)

Notes: Agriculture contains forestry and fishing. Service 1 = Wholesale and retail trade, transport, accommodation and food service activities, Service 2 = Information and communication, Service 3 = Financial and insurance activities, Service 4 = Real estate activities, Service 5 = Professional, scientific and technical activities; administrative and support service activities, Service 6 = Public administration, defence, education, human health and social work activities, Service 7 = Arts, entertainment and recreation; other service activities; activities of household and extra-territorial organizations and bodies. *Sources*: Eurostat

country i, $Aging_{i,t}$ is the population aging variable that is measured as the proportion of the population who 65 years or over, $Controls_{i,t}$ is the vector containing controls, ψ_i^j is the country fixed effect, and $e_{i,t}^j$ is the error term.⁹

Controls. The stylized facts in Section 2.2.1 suggest that population aging raises the proportion of the service industries and reduces the manufacturing industries' share due to old individuals' high consumption of services. In addition to population

 $^{^{9}}$ Hausman test rejects the null hypothesis that the random effect model is appropriate at a 1% significance level in all eleven industries' estimation.

aging, literature points out some other factors which can affect the industrial structures. First, Baumol, Blackman, and Wolff (1989) argue that labor productivity is the critical determinant for industrial structure changes. Moreover, Rowthorn and Ramaswamy (1997) state that changes in trade patterns can impact the country's industrial structures rather than domestic expenditures. Besides, an exchange rate that measures the price competitiveness in trade can also change the trade patterns and industrial structures. In addition to these factors, the GDP per capita level, GDP growth, and population density are controlled to estimate the effects of population aging on the industrial structures.¹⁰

Data. This chapter uses the following data set. The eleven industry shares based on value-added or employment are calculated using the data from Eurostat and the population aging ratio, the proportion of population 65 years or over, is from OECD.Stat. Also, the labor productivity is the real labor productivity per person from Eurostat and the population density consists of two time series, the country's population and its land cover, which Eurostat provides. Moreover, this chapter employs real export, import, and GDP data in the national account from Eurostat to calculate each country's trade share. Besides, the real effective exchange rate is compiled by Bank for International Settlements. Lastly, sample period is from 1995Q1 to 2019Q4 due to data availability and all time series are seasonally adjusted.

Results. Table 2.4 and 2.5 provides the results from the panel analysis. To begin with, the first row in tables denotes dependent variables, the industry shares based on value-added of each industry. Also, the first column indicates all explanatory

¹⁰Therefore, the control vector $Controls_{i,t}$ consists of following variables: $Productivity_{i,t}$ which is the labor productivity, $PopulationDensity_{i,t}$ which is the population of each country divided by its land cover, $GDPpercapita_{i,t}$ which is the total GDP divided by the population, $TradeShare_{i,t}$ which is the sum of export and import divided by GDP, $REER_{i,t}$ which is the real effective exchange rate, $GDPgrowth_{i,t}$ which is the GDP growth rate.

variables. This chapter mostly focuses on the coefficient on the population aging variable at the second row. According to Table 2.4, the population aging raises the size of the service and construction industries but reduces the proportion of the manufacturing and agriculture industries. To be specific, a 1% point increase in the proportion of the population who are 65 years old or over raises the proportion of the service industries by 1.071% points but lowers the manufacturing industries' share by 1.210% points. Moreover, Table 2.5 shows the effects of population aging on the service industry structures. First of all, population aging raises the sizes of most service industries given the coefficients on these industries are positive and significant. In particular, a 1% point rise in the population aging ratio increases the size of service 4, real estate activities, by 0.636% points and service 6, public administration, defence, education, human health and social work activities, by 0.293% points. These results in Table 2.5 is consistent with the fact that old individuals spend the larger portion of their income or wealth on housing and healthcare services as in Table 2.1.

For a robustness check, value-added, the basis of the industry share, is replaced with employment and the results are provided in Table 2.6 and 2.7. In general, the results based on the employment share are similar to the ones based on the value-added share. For example, according to Table 2.6, a 1% point increase in the proportion of the population who are 65 years old or over raises the proportion of the service industries based on employment by 0.968% points and reduces the proportion of the manufacturing industries by 0.618% points.

All things considered, it can be concluded that population aging enlarges the size of the service industries but contracts the size of the manufacturing industries as suggested in Section 2.2.1.¹¹

¹¹This conclusion is in agreement with Cravino, Levchenko, and Rojas (2019).

	(1)	(2)	(3)	(4)
	Services	Manufacturing	Agriculture	Construction
Aging	1.071^{***}	-1.210^{***}	-0.060^{***}	0.113***
	(0.050)	(0.051)	(0.011)	(0.025)
Productivity	0.305***	-0.058^{***}	-0.024^{***}	-0.262^{***}
	(0.022)	(0.022)	(0.005)	(0.011)
Popul.Density	0.142***	-0.165^{***}	0.003	0.036***
	(0.011)	(0.011)	(0.002)	(0.005)
GDP per capita	-0.003^{***}	0.001***	-0.000^{***}	0.003***
	(0.000)	(0.000)	(0.000)	(0.000)
TradeShare	-0.013^{*}	0.096***	-0.010^{***}	-0.072^{***}
	(0.008)	(0.008)	(0.002)	(0.004)
REER	0.188***	-0.151^{***}	-0.057^{***}	-0.006
	(0.013)	(0.014)	(0.003)	(0.007)
GDP growth	-0.438^{***}	0.450***	0.055***	-0.024
	(0.045)	(0.046)	(0.010)	(0.022)
Constant	YES	YES	YES	YES
Obs.	992	992	992	992
R-squared	0.699	0.583	0.663	0.604

Table 2.4: Effects of Population Aging on Industrial Structures (Value-Added)

Notes: Agriculture contains forestry and fishing. Standard errors in parentheses and *** p < 0.01, ** p < 0.05, * p < 0.1.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Service 1	Service 2	Service 3	Service 4	Service 5	Service 6	Service 7
Aging	0.058***	-0.128^{***}	-0.027	0.636***	0.132^{***}	0.293^{***}	0.106***
	(0.020)	(0.016)	(0.018)	(0.027)	(0.014)	(0.025)	(0.005)
Productivity	0.057^{***}	0.044^{***}	-0.043^{***}	0.128^{***}	0.069***	0.024^{**}	0.029***
	(0.009)	(0.007)	(0.008)	(0.012)	(0.006)	(0.011)	(0.002)
Popul.Density	-0.010^{**}	-0.010^{***}	0.025^{***}	-0.008	0.053***	0.091***	0.003***
	(0.004)	(0.003)	(0.004)	(0.006)	(0.003)	(0.005)	(0.001)
GDP per capita	-0.001^{***}	0.000***	-0.000	-0.000^{***}	0.000*	-0.000^{***}	-0.000^{***}
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
TradeShare	0.016***	0.011***	0.008***	-0.026^{***}	0.003	-0.017^{***}	-0.009^{***}
	(0.003)	(0.002)	(0.003)	(0.004)	(0.002)	(0.004)	(0.001)
REER	0.000	-0.019^{***}	0.039***	0.089***	0.007^{*}	0.055***	0.014^{***}
	(0.005)	(0.004)	(0.005)	(0.007)	(0.004)	(0.007)	(0.001)
GDP growth	-0.033^{*}	-0.007	-0.054^{***}	-0.082^{***}	-0.085^{***}	-0.151^{***}	-0.029^{***}
	(0.018)	(0.014)	(0.016)	(0.024)	(0.013)	(0.022)	(0.004)
Constant	YES						
Obs.	992	992	992	992	992	992	992
R-squared	0.406	0.506	0.264	0.530	0.822	0.497	0.557

Table 2.5: Effects of Population Aging on Service Industry Structures (Value-Added)

Notes: Service 1 = Wholesale and retail trade, transport, accommodation and food service activities, Service 2 = Information and communication, Service 3 = Financial and insurance activities, Service 4 = Real estate activities, Service 5 = Professional, scientific and technical activities; administrative and support service activities, Service 6 = Public administration, defence, education, human health and social work activities, Service 7 = Arts, entertainment and recreation; other service activities; activities of household and extra-territorial organizations and bodies. Standard errors in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

	(1)	(2)	(3)	(4)
	Services	Manufacturing	Agriculture	Construction
Aging	0.968***	-0.618^{***}	-0.307^{***}	-0.043^{*}
	(0.043)	(0.028)	(0.016)	(0.024)
Productivity	0.517***	-0.181^{***}	-0.038^{***}	-0.298^{***}
	(0.019)	(0.012)	(0.007)	(0.011)
Popul. Density	0.019**	-0.072^{***}	0.029***	0.024***
	(0.009)	(0.006)	(0.004)	(0.005)
GDP per capita	-0.003^{***}	0.000***	-0.000^{***}	0.003***
	(0.000)	(0.000)	(0.000)	(0.000)
TradeShare	0.058***	0.004	-0.008^{***}	-0.054^{***}
	(0.007)	(0.004)	(0.003)	(0.004)
REER	0.181***	-0.112^{***}	-0.074^{***}	0.004
	(0.012)	(0.007)	(0.004)	(0.007)
GDP growth	-0.195^{***}	0.147^{***}	0.072***	-0.024
	(0.039)	(0.025)	(0.015)	(0.022)
Constant	YES	YES	YES	YES
Obs.	992	992	992	992
R-squared	0.854	0.825	0.728	0.641

Table 2.6: Effects of Population Aging on Industrial Structures (Employment)

Notes: Agriculture contains forestry and fishing. Standard errors in parentheses and *** p < 0.01, ** p < 0.05, * p < 0.1.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Service 1	Service 2	Service 3	Service 4	Service 5	Service 6	Service 7
Aging	0.006	0.070***	-0.087^{***}	0.017^{***}	0.724^{***}	-0.033^{*}	0.279^{***}
	(0.023)	(0.004)	(0.005)	(0.002)	(0.025)	(0.020)	(0.008)
Productivity	0.173***	0.021***	0.013***	0.004^{***}	0.057***	0.241***	0.010***
	(0.010)	(0.002)	(0.002)	(0.001)	(0.011)	(0.009)	(0.004)
Popul.Density	-0.0773^{***}	0.00445^{***}	-0.0242^{***}	-0.000839	0.0680***	0.0389***	0.00502***
	(0.005)	(0.001)	(0.001)	(0.001)	(0.005)	(0.004)	(0.002)
GDP per capita	-0.001^{***}	0.000***	-0.000^{***}	-0.000	0.000***	-0.002^{***}	-0.000^{***}
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
TradeShare	0.046***	-0.003^{***}	0.009***	0.001**	-0.012^{***}	0.035***	-0.016^{***}
	(0.004)	(0.001)	(0.001)	(0.000)	(0.004)	(0.003)	(0.001)
REER	0.074^{***}	-0.007^{***}	0.013***	0.007***	0.017***	0.060***	0.008***
	(0.006)	(0.001)	(0.001)	(0.001)	(0.007)	(0.005)	(0.002)
GDP growth	-0.005	-0.013^{***}	-0.017^{***}	0.001	-0.093^{***}	-0.046^{***}	-0.019^{**}
	(0.020)	(0.004)	(0.005)	(0.002)	(0.022)	(0.018)	(0.008)
Constant	YES	YES	YES	YES	YES	YES	YES
Obs.	992	992	992	992	992	992	992
R-squared	0.488	0.628	0.552	0.362	0.820	0.770	0.593

Table 2.7: Effects of Population Aging on Service Industry Structures (Employment)

Notes: Service 1 = Wholesale and retail trade, transport, accommodation and food service activities, Service 2 = Information and communication, Service 3 = Financial and insurance activities, Service 4 = Real estate activities, Service 5 = Professional, scientific and technical activities; administrative and support service activities, Service 6 = Public administration, defence, education, human health and social work activities, Service 7 = Arts, entertainment and recreation; other service activities; activities of household and extra-territorial organizations and bodies. Standard errors in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

2.3 Cross-Industry Impacts of Monetary Policy and Population Aging Effects on Impacts of Monetary Policy in Euro Area

This section verifies the cross-industry heterogeneity of the impacts of monetary policy in the euro area. First of all, monetary policy shocks in the euro area are identified following the methodologies of existing papers. Then, this chapter estimates how monetary policy impacts each industry differently using the balanced panel data. Finally, my study explores how population aging affects the output effects of monetary policy in the euro area.

2.3.1 Identification of Monetary Policy Shocks

In order to check the cross-industry impacts of monetary policy, the first step is to identify monetary policy shocks in the euro area. This chapter estimates a structural VAR model to obtain the structural shocks to monetary policy in the euro area. All the methods are based on Peersman and Smets (2001).

Model. The structural VAR model specification is as follows

$$Z_t = C + \sum_{j=1}^l A_j Z_{t-j} + \sum_{j=1}^m B_j X_{t-j} + \varepsilon_t \quad \text{with } \varepsilon_t \sim i.i.d. \ N(0, \Omega)$$
(2.2)

where C is the constant vector, Z_t is the vector containing four endogenous variables: (a) the real GDP, (b) the consumer prices, (c) the domestic nominal short-term interest rate, and (d) the real effective exchange rate, X_t is the vector including three exogenous variables: (a) the world commodity price index, (b) the U.S. real GDP, and (c) the U.S. nominal short-term interest rate, and ε_t is the innovation vector. Also, the time lags (*l* and *m*) are determined based on the several criteria for choosing the optimal lag length, such as AIC (Akaike information criterion) and SIC (Schwarz information criterion).

Restrictions. Monetary policy is measured as the structural shocks to the nominal short-term interest rate. For a robustness check, this chapter employs two restrictions for the identification of monetary policy shocks: 1) the recursive restriction and 2) the short and long run restriction. First of all, the structural shocks are identified by the Cholesky decomposition method, i.e., the recursive restriction. The order of variables as above is determined based on the standard assumption that a delayed response of real variables and inflation to the monetary policy shock [Christiano, Eichenbaum, and C. L. Evans (1997)]. Moreover, the short and long run restriction are used. In this assumption, only supply shocks have permanent effects on output, but demand, monetary policy, and exchange rate shocks cannot affect output in the long-term. Moreover, in the short-term, the monetary policy and exchange rate have no contemporaneous impacts on output, and the monetary policy is also not affected by the exchange rate shocks in the same period. All these assumptions, i.e., the ordering of variables in the recursive restriction and the short and long run restriction, are the same as the assumptions in Peersman and Smets (2001).

Data. The data set is as follows. First of all, real GDP, consumer price index, and 3-month interbank rates in the euro area are from the Eurostat. Also, the real effective exchange rate is compiled by Bank for International Settlements. Moreover, the global price index of all commodities from International Monetary Fund is used as the world commodity price index in the structural VAR model. Besides, the U.S. real GDP is from the U.S. Bureau of Economic Analysis, and the U.S. nominal short-

term interest rate is the 3-month treasury bill yield from Board of Governors of the Federal Reserve System. The data are seasonally adjusted and the frequency of all data is quarterly. Lastly, the sample period for the structural VAR estimation is from 1995Q1 to 2019Q4 due to the data availability.

Results. Figure 2.3 plots two monetary policy measures, i.e., the structural shocks to the nominal short-term interest rate, in the euro area. Overall, there were three large negative shocks after 2000. The periods when these large negative shocks occur are in line with the periods when European Central Bank (ECB) dropped its policy interest rate. To be specific, ECB reduced key rates by 2.75%p from May 10th, 2001 to Jun. 5th, 2003, by 3.00%p from Oct. 8th, 2008 to May 7th, 2009, and by 1.25%p from Nov. 3rd, 2011 to Nov. 7th, 2013. Also, two measures of monetary policy, based on the recursive and short and long run restriction, are closely related. In other words, the correlation coefficient between two monetary policy measures is 0.7609.



Figure 2.3: Identification of Monetary Policy Shocks in Euro Area

Notes: This chapter uses the recursive and short and long run restriction when monetary policy shocks in the euro area are identified based on Peersman and Smets (2001).

2.3.2 Cross-Industry Effects of Monetary Policy

Section 2.3.2 estimates the effects of monetary policy on output growth of eleven industries in ten countries of the euro area.¹² The methodologies are based on Peersman and Smets (2005).

Model. The dynamic panel-data model is estimated to check the cross-industry effects of monetary policy

$$\Delta Y_{i,t}^{j} = c^{j} + \sum_{k=1}^{p} \phi_{k}^{j} \Delta Y_{i,t-k}^{j} + \sum_{k=1}^{q} \gamma_{k}^{j} M P_{t-k} + \psi_{i}^{j} + \mu_{i,t}^{j}$$
(2.3)

where $\Delta Y_{i,t}^{j}$ is the growth rate of an industry j output based on value-added in a country i, MP_{t} is the monetary policy indicator, ψ_{i}^{j} is the country fixed effect, and $\mu_{i,t}^{j}$ is the error term.

Since the country fixed effect ψ_i^j is correlated with the lagged dependent variables $\Delta Y_{i,t-k}^j$, standard estimators are inconsistent. To solve this problem, this chapter employs a consistent generalized method of moments (GMM) estimator as in Arellano and Bond (1991). Also, p and q are set as two and four respectively, which is based on the lags in Peersman and Smets (2005). In this specification, the coefficients of our interest are γ_k^j for k = 1, 2, 3, 4 which demonstrate the cross-industry effects of monetary policy.

Data. The output growth of each industry based on value-added is calculated using the data from Eurostat. In addition, the monetary policy indicators are the two measures identified in Section 2.3.1. Lastly, the sample period is from 1995Q1 to 2019Q4 due to the data availability.

¹²Ten countries are the same as in Section 2.2.2: Germany, France, Italy, Spain, Netherlands, Belgium, Ireland, Austria, Finland, and Portugal.

Results. Table 2.8 and 2.9 provide the estimation results from the panel analysis in which the recursive restriction assumption is used when monetary policy shocks in the euro area are identified. In particular, Table 2.8 presents the effects of monetary policy on main industries, i.e., total, manufacturing, services, agriculture, and construction. In general, monetary policy at two quarters ago is significantly effective on output of total, manufacturing, and service industries at a current quarter. Numerically, this chapter finds that a 1% point increase in the interest rate two quarters ago causes the growth rate of total, manufacturing, and service industries' output at a current quarter to decrease by 0.19%p, 0.52%p, and 0.11%p, respectively (see the third row in bold in Table 2.8). The main finding in these results is that the service industries are less sensitively affected by the monetary policy shock than the manufacturing industries. Specifically, the coefficient for the service industries is five times less than that for the manufacturing industries. Furthermore, Table 2.9 shows the effects of monetary policy on the service industries. In agreement with the previous results, monetary policy at two quarters ago significantly affects most service industries at a current quarter. Still, the coefficients for service industries are much smaller than those for the manufacturing industries. The result that monetary policy is less effective in the service industries is consistent with the findings in the literature, for example Ganley and Salmon (1997).

The panel analysis using the monetary policy shocks identified with the short and long run restriction is also conducted for a robustness check. Table 2.10 and 2.11 present the effects of monetary policy on the main and service industries, respectively. In general, the estimation results are consistent with the results above where the recursive restriction is used. To be specific, a 1% point contractionary monetary policy shock two quarters ago decreases the growth rate of total, manufacturing, and service industries' output by 0.13%p, 0.29%p, and 0.07%p, respectively (see the third row in bold in Table 2.10). Here, the coefficient for the service industries is still much smaller than that for the manufacturing industries. Furthermore, my research finds that the monetary policy shock is less effective in most service industries regardless of whether it employs the different monetary policy measure (see the third row in bold in Table 2.11).

Mechanisms. Previous papers provide several factors to explain the more (less) effective monetary policy in the manufacturing (service) sector. For instance, Dedola and Lippi (2005) find that the effects of monetary policy is stronger in industries that produce durable goods and have greater working capital. Moreover, Peersman and Smets (2005) explain the differences in the monetary policy effects with the durability of the goods produced in each sector. They also argue that the capital intensity of production and the degree of openness have an impact on the monetary policy sensitivity. Since the service industries do not produce durable goods and do not have working capital and the capital intensity as much as the manufacturing industries, the conventional interest rate or cost of capital channel of the monetary policy transmission does not operate well in the service sector.

	(1)	(2)	(3)	(4)	(5)
	Total	Manufacturing	Services	Agriculture	Construction
MP_{t-1}	0.0000	-0.0002	0.0001	0.0014	0.0000
	(0.0005)	(0.0017)	(0.0003)	(0.0019)	(0.0009)
MP_{t-2}	-0.0019^{***}	-0.0052^{***}	-0.0011^{***}	-0.0010	0.0012
	(0.0005)	(0.0017)	(0.0003)	(0.0019)	(0.0009)
MP_{t-3}	-0.0008	-0.0021	-0.0005^{*}	-0.0035^{*}	-0.0010
	(0.0005)	(0.0017)	(0.0003)	(0.0019)	(0.0008)
MP_{t-4}	-0.0001	-0.0003	0.0003	-0.0022	0.0001
	(0.0005)	(0.0017)	(0.0003)	(0.0019)	(0.0008)
ΔY_{t-1}	-0.0525	-0.1718^{***}	0.1812***	-0.0814^{**}	0.1559^{***}
	(0.0330)	(0.0329)	(0.0326)	(0.0330)	(0.0322)
ΔY_{t-2}	0.0902***	-0.0328	0.2179***	-0.0551^{*}	0.1707***
	(0.0330)	(0.0329)	(0.0327)	(0.0328)	(0.0308)
Constant	0.0047***	0.0071***	0.0031***	0.0037**	0.0006
	(0.0005)	(0.0016)	(0.0003)	(0.0017)	(0.0008)
Obs.	928	928	928	928	928
Wald χ^2 (6)	24.52	37.38	119.82	13.92	68.56

Table 2.8: Effects of Monetary Policy on Main Industries (Recursive Restriction)

Notes: Agriculture contains forestry and fishing. Standard errors in parentheses and *** p < 0.01, ** p < 0.05, * p < 0.1.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Service 1	Service 2	Service 3	Service 4	Service 5	Service 6	Service 7
MP_{t-1}	0.0003	0.0001	0.0000	-0.0004	0.0009	-0.0004	0.0006
	(0.0004)	(0.0010)	(0.0011)	(0.0004)	(0.0006)	(0.0004)	(0.0007)
MP_{t-2}	-0.0011^{**}	-0.0013	-0.0024^{**}	-0.0006^{*}	-0.0019^{***}	-0.0001	-0.0013^{*}
	(0.0004)	(0.0010)	(0.0011)	(0.0004)	(0.0006)	(0.0004)	(0.0007)
MP_{t-3}	-0.0011^{**}	-0.0013	-0.0009	0.0000	-0.0002	-0.0001	-0.0004
	(0.0004)	(0.0009)	(0.0011)	(0.0004)	(0.0006)	(0.0004)	(0.0007)
MP_{t-4}	0.0004	-0.0010	0.0021**	-0.0007^{**}	-0.0003	0.0003	-0.0002
	(0.0004)	(0.0009)	(0.0011)	(0.0003)	(0.0006)	(0.0004)	(0.0006)
ΔY_{t-1}	0.1426^{***}	-0.0818^{**}	-0.1135^{***}	0.1600***	0.1363***	-0.0812^{**}	-0.2005^{***}
	(0.0323)	(0.0328)	(0.0328)	(0.0326)	(0.0325)	(0.0330)	(0.0328)
ΔY_{t-2}	0.1791***	0.0622^{*}	0.0745**	0.1661***	0.1594^{***}	-0.0527	-0.0173
	(0.0320)	(0.0330)	(0.0328)	(0.0321)	(0.0325)	(0.0327)	(0.0327)
Constant	0.0031***	0.0134***	0.0051***	0.0032***	0.0057***	0.0034***	0.0048***
	(0.0004)	(0.0011)	(0.0010)	(0.0004)	(0.0007)	(0.0004)	(0.0006)
Obs.	928	928	928	928	928	928	928
Wald χ^2 (6)	76.83	15.07	29.78	72.86	58.79	9.86	42.91

Table 2.9: Effects of Monetary Policy on Service Industries (Recursive Restriction)

Notes: Service 1 = Wholesale and retail trade, transport, accommodation and food service activities, Service 2 = Information and communication, Service 3 = Financial and insurance activities, Service 4 = Real estate activities, Service 5 = Professional, scientific and technical activities; administrative and support service activities, Service 6 = Public administration, defence, education, human health and social work activities, Service 7 = Arts, entertainment and recreation; other service activities; activities of household and extra-territorial organizations and bodies. Standard errors in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

	(1)	(2)	(3)	(4)	(5)
	Total	Manufacturing	Services	Agriculture	Construction
MP_{t-1}	0.0007	0.0019	0.0003	0.0044**	0.0010
	(0.0005)	(0.0018)	(0.0003)	(0.0020)	(0.0009)
MP_{t-2}	-0.0013^{**}	-0.0029^{*}	-0.0007^{***}	0.0026	0.0010
	(0.0005)	(0.0018)	(0.0003)	(0.0020)	(0.0009)
MP_{t-3}	-0.0002	-0.0001	-0.0004	0.0009	-0.0014
	(0.0005)	(0.0018)	(0.0003)	(0.0020)	(0.0009)
MP_{t-4}	0.0010*	0.0029	0.0008***	0.0011	0.0002
	(0.0005)	(0.0018)	(0.0003)	(0.0020)	(0.0009)
ΔY_{t-1}	-0.0461	-0.1673^{***}	0.1900***	-0.0832^{**}	0.1607***
	(0.0335)	(0.0334)	(0.0328)	(0.0335)	(0.0331)
ΔY_{t-2}	0.0949***	-0.0311	0.2128***	-0.0612^{*}	0.1913***
	(0.0335)	(0.0334)	(0.0330)	(0.0336)	(0.0330)
Constant	0.0047***	0.0070***	0.0031***	0.0036**	0.0008
	(0.0005)	(0.0016)	(0.0003)	(0.0018)	(0.0008)
Obs.	900	900	900	900	900
Wald χ^2 (6)	19.01	31.51	112.11	14.40	74.88

Table 2.10: Effects of Monetary Policy on Main Industries (Short and Long Run Restriction)

Notes: Agriculture contains forestry and fishing. Standard errors in parentheses and *** p < 0.01, ** p < 0.05, * p < 0.1.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Service 1	Service 2	Service 3	Service 4	Service 5	Service 6	Service 7
MP_{t-1}	0.0002	0.0014	0.0010	-0.0003	0.0008	-0.0005	0.0001
	(0.0005)	(0.0010)	(0.0012)	(0.0004)	(0.0007)	(0.0004)	(0.0007)
MP_{t-2}	-0.0005	-0.0011	-0.0020^{*}	-0.0008^{**}	-0.0008	0.0001	-0.0005
	(0.0005)	(0.0010)	(0.0012)	(0.0004)	(0.0007)	(0.0004)	(0.0007)
MP_{t-3}	-0.0006	-0.0001	-0.0015	0.0002	-0.0004	-0.0004	-0.0004
	(0.0005)	(0.0010)	(0.0012)	(0.0004)	(0.0007)	(0.0004)	(0.0007)
MP_{t-4}	0.0014^{***}	-0.0003	0.0028**	-0.0005	0.0003	0.0004	0.0008
	(0.0005)	(0.0010)	(0.0012)	(0.0004)	(0.0007)	(0.0004)	(0.0007)
ΔY_{t-1}	0.1553^{***}	-0.0812^{**}	-0.1119^{***}	0.1703***	0.1429^{***}	-0.0818^{**}	-0.2100^{***}
	(0.0329)	(0.0332)	(0.0332)	(0.0329)	(0.0327)	(0.0332)	(0.0331)
ΔY_{t-2}	0.1708***	0.0624^{*}	0.0704^{**}	0.1625***	0.1531^{***}	-0.0484	-0.0291
	(0.0329)	(0.0336)	(0.0331)	(0.0329)	(0.0326)	(0.0331)	(0.0331)
Constant	0.0030***	0.0132***	0.0049***	0.0032***	0.0057***	0.0035***	0.0047^{***}
	(0.0004)	(0.0011)	(0.0010)	(0.0004)	(0.0007)	(0.0004)	(0.0006)
Obs.	900	900	900	900	900	900	900
Wald χ^2 (6)	70.76	13.59	28.77	74.57	51.70	10.97	42.88

Table 2.11: Effects of Monetary Policy on Service Industries (Short and Long Run Restriction)

Notes: Service 1 = Wholesale and retail trade, transport, accommodation and food service activities, Service 2 = Information and communication, Service 3 = Financial and insurance activities, Service 4 = Real estate activities, Service 5 = Professional, scientific and technical activities; administrative and support service activities, Service 6 = Public administration, defence, education, human health and social work activities, Service 7 = Arts, entertainment and recreation; other service activities; activities of household and extra-territorial organizations and bodies. Standard errors in parentheses and *** p<0.01, ** p<0.05, * p<0.1.

Impulse Response by Industry. Figure 2.4 plots the response of each industry's output growth to a 1% point contractionary monetary policy shock using the estimation results in Table 2.8 and 2.9. The unit of the y-axis of all panels is a quarterly growth rate in percentages. In the upper left panel, the response of the manufacturing industries is almost five times larger than that of the service industries. Numerically, the 1% point contractionary monetary policy shock reduces the growth rate of the manufacturing industries by 0.52% p in the second quarter after the shock takes place. However, it lowers the growth rate of the service industries only by 0.16% p. Also, panels show that almost other industries except the manufacturing are not strongly affected by the monetary policy shock.

Impulse Response by Country Group. A revised dynamic panel model, Equation (2.4), is estimated to check the difference in the effects of monetary policy between the countries with the high and low share of the service sector

$$\Delta Y_{i,t} = c + \sum_{k=1}^{p} \phi_k \Delta Y_{i,t-k} + \mathbb{1}_i \sum_{k=1}^{q} \gamma_k^{high} M P_{t-k} + (1 - \mathbb{1}_i) \sum_{k=1}^{q} \gamma_k^{low} M P_{t-k} + \psi_i + \mu_{i,t} \quad (2.4)$$

where $\Delta Y_{i,t}$ is the growth rate of total industry output based on value-added in a country *i*, and $\mathbb{1}_i$ is a dummy variable indicating whether a country *i* has the high share of the service sector or not. Specifically, a country is regarded to have the high share of the service sector if its historic average from 1995Q1 to 2019Q4 of the service industry share exceeds the sample median of 71.3%. Furthermore, this chapter uses the data from the eighteen countries of the euro area to increase the sample size.¹³

¹³Countries with the high proportion of service industries include Luxembourg, Malta, Cyprus, Greece, France, Belgium, Netherlands, Portugal, and Italy. Also, Countries with the low proportion of service industries consist of Estonia, Latvia, Spain, Finland, Austria, Germany, Lithuania, Ireland, and Slovenia. Moreover, this chapter excludes Slovakia among the nineteen member countries of the euro area due to the data availability.



Figure 2.4: Impulse Response of Output in Each Industry to 1%p Contractionary Monetary Policy Shock

Notes: Service 1 = Wholesale and retail trade, transport, accommodation and food service activities, Service 2 = Information and communication, Service 3 = Financial and insurance activities, Service 4 = Real estate activities, Service 5 = Professional, scientific and technical activities; administrative and support service activities, Service 6 = Public administration, defence, education, human health and social work activities, Service 7 = Arts, entertainment and recreation; other service activities; activities of household and extra-territorial organizations and bodies. Using the results from the estimation of Equation (2.4), Figure 2.5 plots the output response of the total industry in countries with the high or low share of the service sector to a 1% point contractionary monetary policy shock. The main finding is that the output effects of monetary policy are more significant in the countries with the lower share of the service sector. In other words, the 1% point contractionary monetary policy shock reduces the growth rate of the total industry in the countries with the low share of the service industries by 0.29% points in the second quarter after the monetary policy shock takes place. However, the output growth rate declines only 0.11% points in the countries with the high share of the service industries.



Figure 2.5: Impulse Response of Total Industry Output to 1%p Contractionary Monetary Policy Shock by Country Group

2.3.3 Population Aging Effects on Impacts of Monetary Policy: Findings from Empirical Analyses

Section 2.3.3 examines how the effects of monetary policy in the euro area would change due to population aging in the future. To tackle this question, I use the estimation results in Section 2.2.2 and Section 2.3.2 (i.e., Table 2.4 and 2.8). Specifically, my study assumes to what extent the old population ratio increases in the future with the euro area data and then expects new industrial structures induced by population aging using Table 2.4. Finally, I calculate changes in the effects of monetary policy due to the industrial restructuring using Table 2.8.

Population Aging Forecasts. Using the population projection of each country which Eurostat provides, Figure 2.6 shows the box plot of the projections of the old population ratio in the euro area countries from 2020 to 2050. According to this figure, population aging is expected to progress at faster pace in the future. Specifically, the median old population ratio of the eurozone countries (65 years old and over, %) rises from 20% in 2020 to 28% in 2050. Nonetheless, some countries are anticipated to experience even severer population aging in the future. As an example, the old population ratio in Spain, Portugal, and Italy are expected to increase by 13.1%p (from 19.6% in 2020 to 32.7% in 2050), 11.5%p (from 22.7% to 33.7%), and 10.6%p (from 23.1% to 33.7%), respectively. Thus, this chapter assumes a 10% points increase in the old population ratio to estimate how the effects of monetary policy change due to population aging.

Expected Changes in Industrial Structures. If other factors do not change, i.e., *ceteris paribus*, the predicted changes in each industry share due to population aging can be calculated using the empirical results in Table 2.4. Given the population aging prospects in the euro area above, it is supposed that the population aging ratio increases by 10% points. Table 2.12 shows how much the main industry shares increase or decrease due to the change in the old population ratio. For instance, the manufacturing and service industry share are expected to decrease by 12.1% points and increase 10.7% points due to the 10% points increase in the old population ratio.

Finally, this chapter can estimate the response of total industry output to a 1% point contractionary monetary policy shock when the old population ratio rises 10% points using the estimation results in Table 2.8 and Table 2.12.



Figure 2.6: Population Aging Projection in Euro Area (Box Plot)

Notes: The proportion of the population who are 65 years old or over among ten member countries of the eurozone (Austria, Belgium, Finland, France, Germany, Ireland, Italy, Netherlands, Portugal, and Spain). *Sources*: Eurostat

Table 2.12: Predicted Changes in Main Industry Shares due to 10%p Increase in Population Aging Ratio

Manufacturing	Services	Agriculture	Construction
-12.1%p	+10.7%p	-0.6%p	+1.1%p
$(17.0\% \rightarrow 4.9\%)$	$(73.0\% \rightarrow 83.7\%)$	$(1.7\% \to 1.1\%)$	$(5.4\% \to 6.5\%)$

Notes: The changes in the industry shares are calculated using the empirical results in Table 2.4. The parentheses indicates (Industry Share in $2019 \rightarrow Predicted$ Industry Share). Lastly, Agriculture includes Forestry and Fishing.

Population Aging Effects on Impacts of Monetary Policy. Figure 2.7 shows the response of total industry output to a 1% point contractionary monetary policy shock. From this figure, it can be concluded that the effects of monetary policy

decrease when the population ages. To be specific, the 1% point contractionary monetary policy shock reduces the quarterly growth rate of the industry's total output by 0.162%p in the second quarter after the monetary policy shock hits the economy. However, the monetary policy shock lowers the growth rate of total industry output only by 0.108%p when the old population ratio rises by 10% points. In other words, the effects of monetary policy decrease by 33.1% in the second quarter after the monetary policy shock takes place due to the 10% points increase in the old population ratio.

In summary, population aging raises the size of the service industries and lowers the size of the manufacturing industries on account of the different consumption baskets between young and old individuals. As a result, the effects of monetary policy are diminished since the conventional interest rate or cost of capital channel of the monetary policy transmission does not operate well in the service sector.



Figure 2.7: Impulse Response of Total Industry Output to 1%p Contractionary Monetary Policy Shock by Proportion of Old Population

Notes: In Baseline, the industry shares and old population ratio are the same as in 2019. Also, the old population denotes the population who 65 years old or over.

2.4 Model

In order to explore the mechanism that shows how population aging weakens the conventional interest rate or cost of capital channel of monetary policy transmission, my study constructs a model that combines overlapping generations and a new Keynesian framework in which two sectors, i.e., the manufacturing (M) and service (S) industries, exist. This model is mainly based on Gali (2021) and Lee (2021).¹⁴

2.4.1 Households

This chapter assumes an economy with overlapping generations following Gali (2021), Blanchard (1985), and Gertler (1999). The size of the population is constant and normalized to one. Each individual has the constant probability γ of surviving into the following period, independently of his/her age and economic status. Also, each worker faces the constant probability 1 - v of becoming retired permanently. This probability is also independent of his/her age. Consequently, the size of young individuals (workers) at any time is the constant $\phi = \frac{1-\gamma}{1-v\gamma} \in (0,1]$ and that of old individuals (retirees) is $1 - \phi = \frac{\gamma(1-v)}{1-v\gamma}$.

Moreover, this chapter assumes a perfect annuity market which insures agents against the risk of death as in Blanchard (1985). To be specific, households have an annuity contract with a perfectly competitive insurance company. On the contract, the company issues a payment proportional to households' financial wealth. Also, their wealth will be transferred to the insurance company when households die. Then, households who survive to the next period receive all the returns in this market. Furthermore, complete securities markets mitigate the risk of loss of income from retirement.

 $^{^{14}\}mathrm{All}$ non-linear equilibrium conditions are provided in Appendix C.

2.4.1.1 Young Households (Workers)

'Young' agent of cohort 'b' is one individual of its cohort who started to work (was born) 'b' quarters ago.¹⁵ They maximize expected lifetime utility

$$E_t \sum_{t=s}^{\infty} (\beta \gamma)^{t-s} \log C_{b,t}^Y$$
(2.5)

subject to the budget constraint

$$P_t^Y C_{b,t}^Y + \gamma P_{M,t} A_{b,t}^Y = P_{M,t} A_{b-1,t-1}^Y (1+i_{t-1}) + P_{M,t} W_t N_{b,t}^Y$$
(2.6)

where P_t^Y , $C_{b,t}^Y$, and $A_{b,t}^Y$ are the price index, consumption, and asset of young households. Also, $P_{M,t}$ is the price index of manufacturing goods, which are the numeraire. In contrast to old households, young households supply labor $N_{b,t}^Y = N_{M,b,t}^Y + N_{S,b,t}^Y$ which is determined by the labor demand of firms in each sector, and they earn a wage W_t .¹⁶

Based on the findings in Section 2.2.1.2, young and old households are assumed to have heterogeneous consumption baskets. To be specific, consumption of young households is defined with their own consumption weights on manufacturing goods and services, i.e., ω_M^Y and $1 - \omega_M^Y$, which have different values from old households' consumption weights ω_M^O and $1 - \omega_M^O$. Then, young households' consumption is

$$C_{b,t}^{Y} \equiv \left[\omega_{M}^{Y^{\frac{1}{\eta}}} C_{M,b,t}^{Y^{\frac{\eta-1}{\eta}}} + (1 - \omega_{M}^{Y})^{\frac{1}{\eta}} C_{S,b,t}^{Y^{\frac{\eta-1}{\eta}}}\right]^{\frac{\eta}{\eta-1}}$$
(2.7)

¹⁵I borrow notations from Baksa and Munkacsi (2019).

¹⁶This chapter assumes perfect mobility and labor supplies are perfect substitutes.

where $C_{q,b,t}^{Y} \equiv \left(\int_{L_q} \left(\frac{1}{J_q}\right)^{\frac{1}{\theta}} C_{q,b,t}^{Y}(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}, q \in \{M, S\}$ is consumption of the sectoral good q. Here, $i \in L_M = [0, J_M)$, and $i \in L_S = [J_S, 1]$ where the parameters J_M and J_S measure the economic size of each sector. Also, since two generations have heterogeneous consumption baskets, they face different price indices. For young households, P_t^Y is the consumer price index of young households' final consumption goods and this price index is defined as

$$P_t^Y = \left[\omega_M^Y P_{M,t}^{1-\eta} + (1-\omega_M^Y) P_{S,t}^{1-\eta}\right]^{\frac{1}{\eta-1}}$$
(2.8)

where $P_{q,t} = \left(\int_{L_q} \frac{1}{J_q} P_{q,t}(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}, q \in \{M, S\}$ are the price indices of the sectoral good q.

Then, this chapter obtains the consumption equation of young households using the Euler equation from the young households' optimization problem and their budget constraint, i.e., Equation (2.5) and Equation (2.6),

$$P_t^Y C_{b,t}^Y = (1 - \beta \gamma) \Big[P_{M,t} A_{b-1,t-1}^Y (1 + i_{t-1}) + \frac{1}{\phi} P_{M,t} E_t \sum_{k=0}^\infty (v\gamma)^k \frac{1}{\prod_{s=0}^{k-1} (1 + i_{t+s})} W_{t+k} N_{t+k} \Big]$$
(2.9)

where aggregate labor supply N_t is unformly allocated among young households, so $N_{b,t}^Y = \frac{N_t}{\phi} .^{17}$

Equation (2.9) can be rearranged using a new variable Ω_t^Y which denotes the present value of the lifetime labor income

$$P_t^Y C_{b,t}^Y = (1 - \beta \gamma) \left[P_{M,t} A_{b,t-1}^Y (1 + i_{t-1}) + \frac{1}{\phi} P_{M,t} \Omega_t^Y \right]$$
(2.10)

 $^{^{17}}$ See Gali (2021).

where

$$\Omega_t^Y = W_t N_t + \frac{v\gamma}{1+i_t} E_t \Omega_{t+1}^Y$$
(2.11)

Given the decision on $C_{b,t}^Y$, young households optimally allocate the expenditure on $C_{M,b,t}^Y$ and $C_{S,b,t}^Y$ by minimizing the total expenditure under constraints

$$C_{M,b,t}^{Y} = \omega_{M}^{Y} \left(\frac{P_{M,t}}{P_{t}^{Y}}\right)^{-\eta} C_{b,t}^{Y}$$

$$(2.12)$$

$$C_{S,b,t}^{Y} = (1 - \omega_{M}^{Y}) \left(\frac{P_{S,t}}{P_{t}^{Y}}\right)^{-\eta} C_{b,t}^{Y}$$
(2.13)

and similarly, given the decisions on $C_{M,b,t}^{Y}$ and $C_{S,b,t}^{Y}$, young households optimally allocate the expenditure on $C_{M,b,t}^{Y}(i)$ and $C_{S,b,t}^{Y}(i)$

$$C_{M,b,t}^{Y}(i) = \left(\frac{1}{J_M}\right) \left(\frac{P_{M,t}(i)}{P_{M,t}}\right)^{-\theta} C_{M,b,t}^{Y}$$

$$(2.14)$$

$$C_{S,b,t}^{Y}(i) = \left(\frac{1}{J_{S}}\right) \left(\frac{P_{S,t}(i)}{P_{S,t}}\right)^{-\theta} C_{S,b,t}^{Y}$$
(2.15)

Equation (2.10) and (2.12)-(2.15) can be aggregated by multiplying both left and right hand sides by the population of each cohort¹⁸

$$P_t^Y C_t^Y = (1 - \beta \gamma) \left[v P_{M,t} A_{t-1}^Y (1 + i_{t-1}) + P_{M,t} \Omega_t^Y \right]$$
(2.16)

$$C_{M,t}^{Y} = \omega_{M}^{Y} \left(\frac{P_{M,t}}{P_{t}^{Y}}\right)^{-\eta} C_{t}^{Y}$$

$$(2.17)$$

$$C_{S,t}^{Y} = (1 - \omega_{M}^{Y}) \left(\frac{P_{S,t}}{P_{t}^{Y}}\right)^{-\eta} C_{t}^{Y}$$
(2.18)

$$C_{M,t}^{Y}(i) = \left(\frac{1}{J_{M}}\right) \left(\frac{P_{M,t}(i)}{P_{M,t}}\right)^{-\theta} C_{M,t}^{Y}$$
(2.19)

¹⁸Detailed aggregation methods are in Baksa and Munkacsi (2019).

$$C_{S,t}^{Y}(i) = \left(\frac{1}{J_{S}}\right) \left(\frac{P_{S,t}(i)}{P_{S,t}}\right)^{-\theta} C_{S,t}^{Y}$$
(2.20)

2.4.1.2 Old Households (Retirees)

'Retired' agent of retired cohort 'a' is one individual who retired 'a' quarters ago. They maximize expected lifetime utility

$$E_t \sum_{t=s}^{\infty} (\beta \gamma)^{t-s} \log C_{a,t}^O$$
(2.21)

subject to the budget constraint

$$P_t^O C_{a,t}^O + \gamma P_{M,t} A_{a,t}^O = P_{M,t} A_{a-1,t-1}^O (1+i_{t-1})$$
(2.22)

where P_t^O , $C_{a,t}^O$, and $A_{a,t}^O$ are the price index, consumption, and asset of old households, respectively. Since old households have already retired, they do not earn income from labor.

Similar to young households, old households also have their own consumption baskets

$$C_{a,t}^{O} \equiv \left[\omega_{M}^{O^{\frac{1}{\eta}}} C_{M,a,t}^{O^{\frac{\eta-1}{\eta}}} + (1 - \omega_{M}^{O})^{\frac{1}{\eta}} C_{S,a,t}^{O^{\frac{\eta-1}{\eta}}}\right]^{\frac{\eta}{\eta-1}}$$
(2.23)

where $C_{q,a,t}^{O} \equiv \left(\int_{L_q} (\frac{1}{J_q})^{\frac{1}{\theta}} C_{q,a,t}^{O}(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}, q \in \{M, S\}$ is old households' consumption tion of the sectoral good q. And ω_M^O is the old households' consumption weight on manufacturing goods, which has a different value from the young households' consumption weight ω_M^Y . Furthermore, old households face their own price index which is defined as

$$P_t^O = \left[\omega_M^O P_{M,t}^{1-\eta} + (1-\omega_M^O) P_{S,t}^{1-\eta}\right]^{\frac{1}{\eta-1}}$$
(2.24)

Then, using the Euler equation from the old households' optimization problem and their budget constraint, i.e., Equation (2.21) and Equation (2.22), consumption of old households is determined by the condition

$$P_t^O C_{a,t}^O = (1 - \beta \gamma) P_{M,t} A_{a-1,t-1}^O (1 + i_{t-1})$$
(2.25)

Given the decision on $C_{a,t}^O$, old households optimally allocate the expenditure on $C_{M,a,t}^O$ and $C_{S,a,t}^O$ by minimizing the total expenditure under constraints

$$C_{M,a,t}^{O} = \omega_M^O \left(\frac{P_{M,t}}{P_t^Y}\right)^{-\eta} C_{a,t}^O$$
(2.26)

$$C_{S,a,t}^{O} = (1 - \omega_{M}^{O}) \left(\frac{P_{S,t}}{P_{t}^{Y}}\right)^{-\eta} C_{a,t}^{O}$$
(2.27)

and similarly, given the decisions on $C^{O}_{M,a,t}$ and $C^{O}_{S,a,t}$, old households optimally allocate the expenditure on $C^{O}_{M,a,t}(i)$ and $C^{O}_{S,a,t}(i)$

$$C_{M,a,t}^{O}(i) = \left(\frac{1}{J_{M}}\right) \left(\frac{P_{M,t}(i)}{P_{M,t}}\right)^{-\theta} C_{M,a,t}^{O}$$
(2.28)

$$C_{S,a,t}^{O}(i) = \left(\frac{1}{J_{S}}\right) \left(\frac{P_{S,t}(i)}{P_{S,t}}\right)^{-\theta} C_{S,a,t}^{O}$$
(2.29)

Equation (2.25) and (2.26)-(2.29) can be aggregated by multiplying both left and right hand sides by the population of each cohort

$$P_t^O C_t^O = (1 - \beta \gamma) P_{M,t} [A_{t-1}^O + (1 - v) A_{t-2}^Y] (1 + i_{t-1})$$
(2.30)

$$C_{M,t}^{O} = \omega_M^O \left(\frac{P_{M,t}}{P_t^Y}\right)^{-\eta} C_t^O \tag{2.31}$$

$$C_{S,t}^{O} = (1 - \omega_{M}^{O}) \left(\frac{P_{S,t}}{P_{t}^{Y}}\right)^{-\eta} C_{t}^{O}$$
(2.32)

$$C_{M,t}^{O}(i) = \left(\frac{1}{J_{M}}\right) \left(\frac{P_{M,t}(i)}{P_{M,t}}\right)^{-\theta} C_{M,t}^{O}$$
(2.33)

$$C_{S,t}^{O}(i) = \left(\frac{1}{J_S}\right) \left(\frac{P_{S,t}(i)}{P_{S,t}}\right)^{-\theta} C_{S,t}^{O}$$

$$(2.34)$$

2.4.2 Firms

Young households are endowed with the ability to produce differentiated goods when they are born. Thus, they set up and run firms until they retire or die, whichever comes first.¹⁹ Firm $i \in [0,1]$ in the sector $q \in \{M, S\}$ produces a differentiated good using labor (N) and capital (K). Their production functions are

$$Y_{M,t}(i) = Z_t Z_{M,t} N_{M,t}(i)^{1-\alpha} K_{M,t-1}(i)^{\alpha}$$
(2.35)

$$Y_{S,t}(i) = Z_t Z_{S,t} N_{S,t}(i)^{1-\zeta} K_{S,t-1}(i)^{\zeta}$$
(2.36)

where Z_t is the aggregate productivity and $Z_{M,t}$ and $Z_{S,t}$ are the sector specific productivity. Moreover, this chapter assumes $\alpha > \zeta$, which reflects the fact that manufacturing firms are more capital-intensive. In particular, ζ is assumed to be zero in this chapter for the tractability of the model.²⁰ The differences in the capital intensity between manufacturing and service firms allow population aging to cause the changes in the impacts of monetary policy by affecting the effectiveness of the cost of capital

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¹⁹See Gali (2021).

²⁰The zero capital share in the service industries, $\zeta = 0$, indicates that firms in the service sector only use labor when they produce goods whereas firms in the manufacturing sector use both labor and capital.

channel.

Based on young and old households' optimization conditions, firms in the manufacturing and service sector have different demand functions

$$Y_{M,t}(i) = \left(\frac{\omega_M^Y}{J_M}\right) \left(\frac{P_{M,t}}{P_t^Y}\right)^{-\eta} \left(\frac{P_{M,t}(i)}{P_{M,t}}\right)^{-\theta} C_t^Y + \left(\frac{\omega_M^O}{J_M}\right) \left(\frac{P_{M,t}}{P_t^O}\right)^{-\eta} \left(\frac{P_{M,t}(i)}{P_{M,t}}\right)^{-\theta} C_t^O \quad (2.37)$$
$$Y_{S,t}(i) = \left(\frac{1-\omega_M^Y}{J_S}\right) \left(\frac{P_{S,t}}{P_t^Y}\right)^{-\eta} \left(\frac{P_{S,t}(i)}{P_{S,t}}\right)^{-\theta} C_t^Y + \left(\frac{1-\omega_M^O}{J_S}\right) \left(\frac{P_{S,t}}{P_t^O}\right)^{-\eta} \left(\frac{P_{S,t}(i)}{P_{S,t}}\right)^{-\theta} C_t^O \quad (2.38)$$

Here, sectoral output is defined with a CES aggregate of differentiated goods in each sector, and sectoral labor employment is the summation of all firms' labor employment in each sector

$$Y_{M,t} \equiv \left(\int_{L_M} \left(\frac{1}{J_M}\right)^{\frac{1}{\theta}} Y_{M,t}(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$$
(2.39)

$$Y_{S,t} \equiv \left(\int_{L_S} \left(\frac{1}{J_S}\right)^{\frac{1}{\theta}} Y_{S,t}(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$$
(2.40)

$$N_{M,t} = \int_{L_M} N_{M,t}(i) \, di \tag{2.41}$$

$$N_{S,t} = \int_{L_S} N_{S,t}(i) \, di \tag{2.42}$$

Moreover, this chapter obtains input demand functions and marginal costs in both sectors by minimizing the total cost, $W_t N_{M,t} + r_t^k K_{M,t}$ (for firms in the manufacturing sector) or $W_t N_{S,t}$ (for firms in the service sector), under the constraint Equation (2.35) or Equation (2.36)

$$W_{t} = (1 - \alpha) m c_{M,t} \left(\frac{K_{M,t-1}}{N_{M,t}}\right)^{\alpha} Z_{t} Z_{M,t}$$
(2.43)

$$r_t^k = \alpha m c_{M,t} \left(\frac{N_{M,t}}{K_{M,t-1}}\right)^{1-\alpha} Z_t Z_{M,t}$$
(2.44)

$$mc_{M,t} = \frac{1}{Z_t Z_{M,t}} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t^k}{\alpha}\right)^{\alpha}$$
(2.45)

$$mc_{S,t} = \frac{W_t}{Z_t Z_{S,t}} \tag{2.46}$$

Based on Calvo (1983), this chapter assumes that firms in each sector q can set the prices of their products with the probability of $1 - h_q$ every period while they keep their prices with the probability of h_q . Consequently, the aggregate prices in each sector q are determined by the following equations

$$P_{M,t} = \left[(1 - h_M) P_{M,t}^{*^{-1-\theta}} + h_M P_{M,t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}$$
(2.47)

$$P_{S,t} = \left[(1 - h_S) P_{S,t}^{*}^{1-\theta} + h_S P_{S,t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}$$
(2.48)

where $P_{q,t}^*$ is the price set in the sector q in the period t which maximizes firms' expected profits.

For the price setting of firms in the manufacturing sector, they adjust their prices $P_{M,t}^*$ to maximize the expected sum of discounted profit

$$\max_{P_{M,t}^{*}(i)} E_{t} \sum_{k=0}^{\infty} h_{M}^{k} \Lambda_{t,t+k} [P_{M,t}^{*}(i)Y_{M,t+k}(i) - P_{M,t+k}mc_{M,t+k}(i)Y_{M,t+k}(i)]$$
(2.49)

where $\Lambda_{t,t+1} = \beta \frac{C_t^Y}{C_{t+1}^Y} \frac{P_t^Y}{P_{t+1}^Y}$ is the stochastic discount factor, which comes from the assumption that firms are owned by young households.

Then, the first-order condition is

$$E_t \sum_{k=0}^{\infty} h_M^k \Lambda_{t,t+k} \Big[P_{M,t}^*(i) - \frac{\theta}{\theta - 1} P_{M,t+k} m c_{M,t+k}(i) \Big] \frac{1}{J_M} \Big(\frac{P_{M,t}^*(i)}{P_{M,t}} \Big)^{-\theta} Y_{M,t+k} = 0 \quad (2.50)$$

All firms adjusting their prices in the period t face the same optimization problem and thus, they all choose the same price level. Accordingly, this chapter drops an individual index i. So, $P_{M,t}^*(i) = P_{M,t}^*$ and $mc_{M,t}(i) = mc_{M,t}$. Moreover, Equation (2.50) can be rearranged using new variables

$$P_{M,t}^* = \frac{\theta}{\theta - 1} \frac{A_{M,t}}{B_{M,t}} \tag{2.51}$$

$$A_{M,t} = P_{M,t}^{1+\theta} m c_{M,t} Y_{M,t} + h_M E_t \{ \Lambda_{t,t+1} A_{M,t+1} \}$$
(2.52)

$$B_{M,t} = P_{M,t}^{\theta} Y_{M,t} + h_M E_t \{ \Lambda_{t,t+1} B_{M,t+1} \}$$
(2.53)

In a similar way, the price in the service sector is also determined by following equations

$$P_{S,t}^* = \frac{\theta}{\theta - 1} \frac{A_{S,t}}{B_{S,t}} \tag{2.54}$$

$$A_{S,t} = P_{S,t}^{1+\theta} m c_{S,t} Y_{S,t} + h_S E_t \{ \Lambda_{t,t+1} A_{S,t+1} \}$$
(2.55)

$$B_{S,t} = P_{S,t}^{\theta} Y_{S,t} + h_S E_t \{ \Lambda_{t,t+1} B_{S,t+1} \}$$
(2.56)

2.4.3 Asset market

A stock price in individual firms, $Q_t(i)$, should satisfy the asset pricing Equation (2.57) in which the future stocks price are discounted by the factor $v\gamma$ since firms are owned by young households²¹

$$Q_t(i) = D_t(i) + v\gamma E_t\{\Lambda_{t,t+1}Q_{t+1}(i)\}$$
(2.57)

and it can be aggregated across firms

$$Q_t = D_t + v\gamma E_t \{\Lambda_{t,t+1} Q_{t+1}\}$$
(2.58)

where D_t is the aggregate dividend which is calculated by the equation

$$D_{t} = \int_{L_{M}} \left(\frac{P_{M,t}(i)}{P_{M,t}} - \frac{1}{Z_{t}Z_{M,t}} \left(\frac{W_{t}}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_{t}^{k}}{\alpha} \right)^{\alpha} \right) Y_{M,t}(i) \, di \qquad (2.59)$$
$$+ \int_{L_{S}} \left(\frac{P_{S,t}(i)}{P_{M,t}} - \frac{W_{t}}{Z_{t}Z_{S,t}} \right) Y_{S,t}(i) \, di$$

When iterating Equation (2.58) forward, this chapter can obtain

$$Q_t = \sum_{k=0}^{\infty} (v\gamma)^k E_t \{\Lambda_{t,t+k} D_{t+k}\}$$
(2.60)

and this equation indicates that stocks price in the period t is the expected sum of the discounted aggregate dividend.

In order to simplify Equation (2.59) further, this chapter introduces new variables that are the price dispersion in each sector

$$\Delta_{M,t} = \int_{L_M} \left(\frac{P_{M,t}(i)}{P_{M,t}}\right)^{-\theta} di$$
(2.61)

$$\Delta_{S,t} = \int_{L_S} \left(\frac{P_{S,t}(i)}{P_{S,t}}\right)^{-\theta} di$$
(2.62)

Then, Equation (2.59) can be rearranged

$$D_{t} = \left(1 - \frac{mc_{M,t}\Delta_{M,t}}{J_{M}}\right)Y_{M,t} + \left(\frac{P_{S,t}}{P_{M,t}} - \frac{mc_{S,t}\Delta_{S,t}}{J_{S}}\right)Y_{S,t}$$
(2.63)

where

$$\Delta_{M,t} = J_M (1 - h_M) \left(\frac{P_{M,t}^*}{P_{M,t}}\right)^{-\theta} + J_M h_M \left(\frac{P_{M,t-1}}{P_{M,t}}\right)^{-\theta} \Delta_{M,t-1}$$
$$\Delta_{S,t} = J_S (1 - h_S) \left(\frac{P_{S,t}^*}{P_{S,t}}\right)^{-\theta} + J_S h_S \left(\frac{P_{S,t-1}}{P_{S,t}}\right)^{-\theta} \Delta_{S,t-1}$$

2.4.4 Monetary Policy

To close the model, a central bank sets the nominal interest rate. Specifically, monetary policy is characterized by the Tayler rule

$$1 + i_t = \frac{1}{\beta} \left(\frac{1 + \pi_{M,t}}{1 + \pi_M} \right)^{\phi_{\pi_M}} \left(\frac{1 + \pi_{S,t}}{1 + \pi_S} \right)^{\phi_{\pi_S}} \left(\frac{Y_{M,t}}{Y_{M,t}^f} \right)^{\phi_{Y_M}} \left(\frac{Y_{S,t}}{Y_{S,t}^f} \right)^{\phi_{Y_S}} \exp\left(\nu_t\right)$$
(2.64)

where π_M and π_S are the steady-state inflation rates which are zero and $Y_{M,t}^f$ and $Y_{S,t}^f$ are the flexible price output levels. Also, ν_t is the monetary policy shock and follows AR(1) process in logarithm.

2.4.5 Market Clearing

Market clearing conditions are as follows:

(a) National accounts

$$Y_t = C_t + I_{M,t} (2.65)$$

where investment is determined by the law of motion of capital

$$I_{M,t} = K_{M,t} - (1-\delta)K_{M,t-1}$$
(2.66)

(b) Goods market

$$C_t = C_{M,t} + C_{S,t} (2.67)$$

$$Y_{M,t} = C_{M,t} + I_{M,t} (2.68)$$

$$C_{M,t} = C_{M,t}^O + C_{M,t}^Y (2.69)$$

$$Y_{S,t} = C_{S,t}$$
 (2.70)

$$C_{S,t} = C_{S,t}^O + C_{S,t}^Y (2.71)$$

where Equation (2.68) and (2.70) are based on the assumption that only manufacturing goods can be used for capital investment.

(c) Assets market

$$A_t = Q_t + K_{M,t} \tag{2.72}$$

$$A_t = A_t^O + A_t^Y \tag{2.73}$$

2.5 Population Aging and Decreased Effectiveness of Monetary Policy

This section provides detailed explanations about why monetary policy becomes less effective in an aging society, especially through the industrial restructuring. First of all, this chapter discuses model parameter values and shows the dynamics of macroeconomic variables induced by the monetary policy shock. Moreover, my study estimates to what extent population aging reduces the output effects of the monetary policy shock. Lastly, I present the mechanism of the decreased effectiveness of monetary policy when the population ages.

2.5.1 Model Parameterization

Table 2.13 provides the baseline model parameter values. First of all, basic parameters such as β , δ , and α are set based on commonly used values in the literature. Also, demographic parameters, i.e., ϕ , γ , and v, are chosen following Gali (2021). In particular, the baseline value for ϕ is 0.8 (=80%) but we replace it with 0.7 (=70%) or $0.9 \ (=90\%)$ to examine the population aging effects. Moreover, the elasticity of substitution across goods in each sector θ is 6 but the elasticity of substitution between manufacturing goods and services is close to but slightly larger than 1 because two goods are assumed to be weak substitutes [Acemoglu (2002)].²² And young (old) households' consumption weight on manufacturing goods $\omega_M^Y(\omega_M^O)$ is calibrated as 0.32 (0.22) based on their consumption structures in Section 2.2.1.2.²³ For Calvo parameters h_M and h_S , this chapter follows the empirical evidence that manufacturing goods prices are adjusted more frequently than services prices [e.g., Bils and Klenow (2004)]. Calvo parameters are also calibrated to match the fact that the average duration of goods prices in the euro area is 13 months, which is the empirical finding of Alvarez et al. (2005). Besides, the market share of the manufacturing (service) industries $J_M(J_S)$ is 0.3 (0.7) based on the industry structure data in the euro area. In addition, the persistence of productivity shocks such as ρ^Z , ρ^{Z_M} , and ρ^{Z_S} are 0.9. And the standard deviations of productivity shocks, i.e., σ^Z , σ^{Z_M} , and σ^{Z_S} , are 0.01. Furthermore, monetary policy rule parameters, ϕ_{π_M} , ϕ_{π_S} , ϕ_{Y_M} , and ϕ_{Y_S} , are set on the basis of the literature about the Taylor rule. Lastly, the persistence of the monetary policy shock ρ^{ν} is 0.5 and the standard deviation of the monetary policy shock σ^{ν} is

 $^{^{22}}$ Acemoglu (2002) states that two factors in the CES production function are gross substitutes if the elasticity of substitution is greater than 1 and gross complements if the elasticity of substitution is less than one.

²³Cravino, Levchenko, and Rojas (2019) find that households in their 60s have service expenditure shares 10-12 percentage points higher than households in their 30s.

Additionally, given the empirical findings that the size of service (manufacturing) industries increases (decreases) due to population aging, this chapter adjusts the parameter values of the industry shares $(J_M \text{ and } J_S)$ and consumption weights $(\omega_M^Y$ and $\omega_M^O)$ when the proportion of young households (ϕ) changes. Table 2.14 provides these parameter values.

Parameter	Description	Value				
	(A) Household					
β	Time discount factor	0.99				
ϕ	Proportion of young households	0.80				
γ	Probability of death	0.9959				
v	Probability of losing job	0.9989				
heta	EOS across sectoral goods	6				
η	EOS between M and S goods	1.5				
ω^o_M	Old HHs' consumption share on M goods	0.22				
ω_M^Y	Young HHs' consumption share on M goods	0.32				
(B) Firm						
α	Capital Share	0.33				
h_M	Calvo parameter in M sector	0.65				
h_S	Calvo parameter in S sector	0.82				
J_M	Manufacturing industry share	0.30				
J_S	Service industry share	0.70				
$ ho^Z, ho^{Z_M}, ho^{Z_S}$	Persistence of productivity shocks	0.90				
$\sigma^Z, \sigma^{Z_M}, \sigma^{Z_S}$	Standard deviations of productivity shocks	0.01				
	(C) Monetary Policy					
$\phi_{\pi_M}, \phi_{\pi_S}$	Reaction to inflation	0.75				
ϕ_{Y_M}, ϕ_{Y_S}	Reaction to output	0.25				
$ ho^{ u}$	Persistence of MP shock	0.50				
$\sigma^{ u}$	Standard deviation of MP shock	0.01				

Table 2.13: Model Parameter Values

	$\phi = 0.7$	$\phi = 0.8$	$\phi = 0.9$
J_M	0.20	0.30	0.40
J_S	0.80	0.70	0.60
ω_M^Y	0.23	0.32	0.41
ω_M^O	0.13	0.22	0.31

Table 2.14: Adjustments in Parameter Values according to Changes in Proportion of Young Households (ϕ)

Note: The sizes of both manufacturing and service industries and the consumption weights of young and old households are calibrated based on the results in Section 2.2 and existing papers' empirical findings [e.g., Cravino, Levchenko, and Rojas (2019)].

2.5.2 Mechanism for Less Effective Monetary Policy in Aging Society

Section 2.5.2 examines the effects of the monetary policy shock on the dynamics of macroeconomic variables in the model in which two types of households (i.e., young and old households) and two sectors (i.e., manufacturing and service sector) exist. In particular, this chapter shows to what extent the impacts of monetary policy weaken as the population ages and presents the mechanism behind the results.

2.5.2.1 Dynamics of Macroeconomic Variables Induced by Monetary Policy Shock

Figure 2.8 shows the impulse responses of macroeconomic variables such as output, consumption, investment, inflation, and real interest rate to a one-standard-deviation positive shock to the monetary policy rate. In general, the responses of variables are in agreement with the traditional effects of the monetary policy shock in literature. For example, the contractionary monetary policy shock raises the real interest rate, which reduces private consumption through the intertemporal consumption decision,

and lowers investment due to the high cost of capital. Finally, output decreases and the declines in aggregate demand also reduce inflation.



Figure 2.8: Impulse Responses of Macroeconomic Variables to One-Standard-Deviation Positive Shock to Monetary Policy Rate

Note: The one-standard-deviation positive shock indicates a 1%p increase in the monetary policy rate.

Figure 2.9 indicates the impulse responses of sectoral output and inflation to the one-standard-deviation positive shock to the monetary policy rate. The main finding is that output in the manufacturing industries responds to the contractionary monetary policy shock more sensitively than output in the service industries as in the left panel. In particular, these results are in accordance with the empirical findings of this chapter. Specifically, I find in Section 2.3.2 that a 1% point increase in the policy interest rate reduces manufacturing industries' output growth by 0.52% points in the second quarter after the shock. However, service industries' output growth decreases only by 0.11% points despite the same-size monetary policy shock. Additionally, the right panel shows that inflation also reacts to the monetary policy shock differently in each sector. Since the contractionary monetary policy shock reduces the demand for the manufacturing goods sharply in contrast to the services, inflation in the manufacturing industries also decreases more drastically than in the service industries. This chapter mainly focuses on the cross-industry output effects of monetary policy in an aging society, but examining the cross-industry inflation effects of monetary policy will be also worthwhile.



Figure 2.9: Impulse Responses of Sectoral Output and Inflation to One-Standard-Deviation Positive Shock to Monetary Policy Rate

2.5.2.2 Population Aging Effects on Impacts of Monetary Policy: Findings from Theoretical Model

The theoretical model demonstrates that the impacts of monetary policy weakens as the population ages due to the industrial restructuring. Figure 2.10 shows the impulse responses of output to the one-standard-deviation positive shock to the monetary policy rate when the proportion of young households (ϕ) decreases from 90% (=0.9) to 70% (=0.7). And Figure 2.10 indicates that the output effects of monetary policy decline by about 30% when the old population share rises by 10% points (i.e., ϕ decreases from 0.8 to 0.7). Here, this chapter assumes that the service (manufacturing) industries share increases (decreases) by 10% (10%) points when the proportion of old households increases by 10% points based on the empirical findings in Section 2.3.2, and these assumptions play a key role in the decreased effectiveness of monetary policy. Lastly, the results from the theoretical model are closely related to those from the empirical analyses in Section 2.3.3, which show that the effects of monetary policy decrease by about 33% when the old population share increases by 10% points.²⁴



Figure 2.10: Impulse Response of Output to One-Standard-Deviation Positive Shock to Monetary Policy Rate by Proportion of Young Households (ϕ)

²⁴Compare Figure 2.7 and Figure 2.10.
Mechanisms. The key mechanism for the less effective monetary policy in an aging society is that population aging causes the conventional interest rate or cost of capital channel of the monetary policy transmission to become less significant through the industrial restructuring. The detailed explanations are as follows. When the population ages, the consumption expenditures on services, which are non-durable and produced by less capital-intensive firms, account for the larger proportion of household spending. For that reason, the interest rate changes by the central bank is harder to affect the consumption and investment demand in an aging society. Moreover, firms have less abilities to adjust their production capacities through capital investment according to the interest rate changes as the share of less capital-intensive firms rises. Figure 2.11 shows the contribution of investment to output growth when the one-standard-deviation positive shock to the monetary policy rate hits the economy. This figure demonstrates that the contribution of investment keeps decreasing as the proportion of old households rises, which supports the less significant cost of capital channel in an aging society.



Figure 2.11: Contribution of Investment to Output Growth in Response to One-Standard-Deviation Positive Shock to Monetary Policy Rate (%p)

2.6 Conclusion

In recent years, as the population has aged fast around the world, especially in developed countries such as the euro area, economists and politicians become to have more interest in the impacts of population aging on our economy and society. Accordingly, many studies have been conducted in relation to population aging, e.g., various studies on the effects of population aging on the impacts of fiscal and monetary policy. However, in contrast to existing papers, this study focuses on the effects of the industrial restructuring caused by population aging on the effectiveness of monetary policy about which there have been a limited number of research so far.

The key empirical findings of this chapter using the euro area panel data are as follows. First, a 1% point increase in the proportion of the population who are 65 years old or over raises the proportion of the service industries by 1.071% points but lowers the manufacturing industries share by 1.210% points. Second, the service industries are less sensitive to the monetary policy shock than the manufacturing industries. Specifically, a 1% point increase in the monetary policy rate two quarters ago causes the growth rate of total, manufacturing, and service industries output at a current quarter to decrease by 0.19% points, 0.52% points, and 0.11% points, respectively. As a result, the output effects of monetary policy become weaker in a country which has a higher service industry share. Literature finds that the conventional interest rate or cost of capital channel of the monetary policy transmission does not operate well in the service sector since the service industries do not produce durable goods and do not have working capital and capital intensity of production as much as the manufacturing industries. Last but not least, the effects of monetary policy decrease by up to 33% due to a 10% points increase in the old population ratio since it raises the size of the service sector by about 11%p.

In addition to the empirical findings, the theoretical model also demonstrates that the output effects of monetary policy decline by about 30% when the old population ratio increases by 10% points. The main mechanism is that the conventional interest rate or cost of capital channel of monetary policy transmission comes to be weak in an aging society due to a rise in the size of the service sector. To be more specific, as the population ages, the consumption expenditures on services, which are nondurable and produced by less capital-intensive firms, account for the larger proportion of household spending. As a result, the interest rate changes by the central bank is harder to affect the consumption and investment demand in an aging society. Moreover, firms have less abilities to adjust their production capacities through capital investment according to the interest rate changes as the share of less capital-intensive firms rises.

In light of the results in this study that the output effects of monetary policy decrease due to population aging, it is necessary to establish and implement more drastic monetary policy in order to stabilize the economy in an aging society. Moreover, this study only deals with the effects of population aging on the output impacts of monetary policy, but further research needs to study the effects of population aging on the inflation impacts of monetary policy, too.

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Appendices

Appendix A

Log-linearized Aggregate Consumption of Young and Old Households in Life-Cycle Learning Model

Appendix A derives log-linearized aggregate consumption of young and old households in the life-cycle learning model. After that, solutions to the LC learning model are provided.

A.1 Old Households (Retirees)

Normalize the one-period budget constraint of old households

$$c_{a,t}^{o} + \gamma a_{a+1,t+1}^{o} = R_t a_{a,t}^{o} \chi_t^{-1} + s_t \tag{A.1}$$

Iterate forward Equation (A.1) and I obtain the intertemporal budget constraint of

old households

$$\underbrace{\tilde{E}_{t}^{o}\sum_{n=0}^{\infty}\prod_{h=0}^{n}\gamma^{n}\frac{\chi_{t+h}}{R_{t+h}}c_{a+n,t+n}^{o}}_{=(i)} = \underbrace{a_{a,t}^{o}}_{=(ii)} + \underbrace{\tilde{E}_{t}^{o}\sum_{n=0}^{\infty}\prod_{h=0}^{n}\gamma^{n}\frac{\chi_{t+h}}{R_{t+h}}s_{t+n}}_{=(iii)}$$
(A.2)

Log-linearize the intertemporal budget constraint using Euler Equation (1.12)

$$(i) = \frac{\beta \bar{c}_a^o}{1 - \gamma \beta} \hat{c}_{a,t}^o + \frac{\beta \bar{c}_a^o}{1 - \gamma \beta} (\hat{\chi}_t - \hat{R}_t)$$

 $(ii) = \bar{a}_a^o \hat{a}_{a,t}^o$

$$(iii) = \frac{\beta \bar{s}}{1 - \gamma \beta} (\hat{\chi}_t - \hat{R}_t) + \frac{\gamma \beta^2 \bar{s}}{1 - \gamma \beta} \tilde{E}_t^o \sum_{h=0}^\infty (\gamma \beta)^h (\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h})$$

$$+\beta\bar{s}\hat{s}_t + \gamma\beta^2\bar{s}\tilde{E}_t^o\sum_{h=0}^\infty (\gamma\beta)^h\hat{s}_{t+1+h}$$

Plug (i), (ii), and (iii) into Equation (A.2)

$$\bar{c}_{a}^{o}\hat{c}_{a,t}^{o} = \frac{(1-\gamma\beta)}{\beta}\bar{a}_{a}^{o}\hat{a}_{a,t}^{o} + (\bar{c}_{a}^{o}-\bar{s})(\hat{R}_{t}-\hat{\chi}_{t}) + (1-\gamma\beta)\bar{s}\hat{s}_{t} \qquad (A.3)$$

$$+ \gamma\beta\bar{s}\tilde{E}_{t}^{o}\sum_{h=0}^{\infty}(\gamma\beta)^{h}(\hat{\chi}_{t+1+h}-\hat{R}_{t+1+h})$$

$$+ \frac{(1-\gamma\beta)}{\gamma\beta}\bar{s}\tilde{E}_{t}^{o}\sum_{h=0}^{\infty}(\gamma\beta)^{h}\hat{s}_{t+1+h}$$

To aggregate consumption of old households, multiply both sides with $\sum_{a=0}^{\infty} N_{a,t}^{o}$ where

 $N^o_{a,t}$ is the number of old households who belong to cohort 'a'

$$\bar{c}^{o}\hat{c}_{t}^{o} = \frac{(1-\gamma\beta)}{\beta} [(1-v)\bar{a}^{y}\hat{a}_{t-1}^{y} + \bar{a}^{o}\hat{a}_{t}^{o}] + [\bar{c}^{o} - (1-\phi)\bar{s}](\hat{R}_{t} - \hat{\chi}_{t})$$
(A.4)
+ $(1-\phi)(1-\gamma\beta)\bar{s}\hat{s}_{t}$
+ $(1-\phi)\gamma\beta\bar{s}\tilde{E}_{t}^{o}\sum_{h=0}^{\infty}(\gamma\beta)^{h}(\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h})$
+ $(1-\phi)(1-\gamma\beta)\gamma\beta\bar{s}\tilde{E}_{t}^{o}\sum_{h=0}^{\infty}(\gamma\beta)^{h}\hat{s}_{t+1+h}$

In particular, Equation (A.4) is obtained using Baksa and Munkacsi (2019)'s following assumption

$$\sum_{a=o}^{\infty} N_{a,t}^{o} a_{a,t}^{o} = N_{0,t}^{o} a_{0,t}^{o} + \sum_{a=1}^{\infty} N_{a,t}^{o} a_{a,t}^{o} = (1-v)a_{t-1}^{y} + a_{t}^{o}$$
(A.5)

$$\left(\because N_{0,t}^{o} a_{0,t}^{o} \approx (1-v) N_{t-1}^{y} \frac{a_{t-1}^{y}}{N_{t-1}^{y}} = (1-v) a_{t-1}^{y} \quad \text{and} \quad \sum_{a=1}^{\infty} N_{a,t}^{o} a_{a,t}^{o} = \sum_{a=1}^{\infty} \gamma N_{a-1,t-1}^{o} a_{a,t}^{o} = a_{t}^{o} \right)$$

Therefore,

$$\sum_{a=o}^{\infty} N_{a,t}^{o} \bar{a}_{a}^{o} \hat{a}_{a,t}^{o} = (1-v) \bar{a}^{y} \hat{a}_{t-1}^{y} + \bar{a}^{o} \hat{a}_{t}^{o}$$
(A.6)

Rearrange Equation (A.4) and finally we can have the consumption decision rule of old households in the LC learning model

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$$\hat{c}_{t}^{o} = \frac{(1 - \gamma\beta)(1 - v)}{\beta} \frac{\bar{a}^{y}}{\bar{c}^{o}} \hat{a}_{t-1}^{y} + \frac{(1 - \gamma\beta)}{\beta} \frac{\bar{a}^{o}}{\bar{c}^{o}} \hat{a}_{t}^{o} \qquad (A.7)$$

$$+ \left[1 - (1 - \phi)\frac{\bar{s}}{\bar{c}^{o}}\right] (\hat{R}_{t} - \hat{\chi}_{t}) + (1 - \phi)(1 - \gamma\beta)\frac{\bar{s}}{\bar{c}^{o}} \hat{s}_{t} + (1 - \phi)\gamma\beta\frac{\bar{s}}{\bar{c}^{o}} \tilde{E}_{t}^{o} \sum_{h=0}^{\infty} (\gamma\beta)^{h} (\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h}) + (1 - \phi)(1 - \gamma\beta)(\gamma\beta)\frac{\bar{s}}{\bar{c}^{o}} \tilde{E}_{t}^{o} \sum_{h=0}^{\infty} (\gamma\beta)^{h} \hat{s}_{t+1+h}$$

A.2 Young Households (Workers)

We rearrange the budget constraint of young households using the labor supply condition, Equation (1.19)

$$(1+\theta)c_{b,t}^y + v\gamma a_{b+1,t+1}^y + (1-v)\gamma a_{b+1,t+1}^{yo} = R_t \chi_t^{-1} a_{b,t}^y + w_t - \tau_t$$
(A.8)

Iterate forward Equation (A.8) and I obtain the intertemperal budget constraint of young households

$$\underbrace{(1+\theta)\tilde{E}_{t}^{y}\sum_{n=0}^{\infty}\prod_{h=0}^{n}(v\gamma)^{n}\frac{\chi_{t+h}}{R_{t+h}}c_{b+n,t+n}^{y}}_{=(ii)} = \underbrace{a_{b,t}^{y}}_{=(ii)} + \underbrace{\tilde{E}_{t}^{y}\sum_{n=0}^{\infty}\prod_{h=0}^{n}(v\gamma)^{n}\frac{\chi_{t+h}}{R_{t+h}}w_{t+n}}_{=(iii)} - \underbrace{\tilde{E}_{t}^{y}\sum_{n=0}^{\infty}\prod_{h=0}^{n}(v\gamma)^{n}\frac{\chi_{t+h}}{R_{t+h}}\tau_{t+n}}_{=(iv)} - \underbrace{(1-v)\gamma\tilde{E}_{t}^{y}\sum_{n=0}^{\infty}\prod_{h=0}^{n}(v\gamma)^{n}\frac{\chi_{t+h}}{R_{t+h}}a_{b+1+n,t+1+n}}_{=(v)}$$
(A.9)

Log-linearize the intertemporal budget constraint of young households using Euler Equations (1.17) and (1.18)

$$(i) = \frac{(1+\theta)\beta\bar{c}_b^y}{(1-v\gamma\beta)}(\hat{\chi}_t - \hat{R}_t + \hat{c}_{b,t}^y)$$

$$(ii) = \bar{a}_b^y \hat{a}_{b,t}^y$$

$$(iii) = \frac{\beta \bar{w}}{1 - v\gamma \beta} (\hat{\chi}_t - \hat{R}_t) + \beta \bar{w} \hat{w}_t + \frac{v\gamma \beta^2 \bar{w}}{1 - v\gamma \beta} \tilde{E}_t^y \sum_{h=0}^{\infty} (v\gamma \beta)^h (\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h}) + v\gamma \beta^2 \bar{w} \tilde{E}_t^y \sum_{h=0}^{\infty} (v\gamma \beta)^h \hat{w}_{t+1+h}$$

$$(iv) = \frac{\beta\bar{\tau}}{1 - v\gamma\beta} (\hat{\chi}_t - \hat{R}_t) + \beta\bar{\tau}\hat{\tau}_t + \frac{v\gamma\beta^2\bar{\tau}}{1 - v\gamma\beta} \tilde{E}_t^y \sum_{h=0}^{\infty} (v\gamma\beta)^h (\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h}) + v\gamma\beta^2\bar{\tau}\tilde{E}_t^y \sum_{h=0}^{\infty} (v\gamma\beta)^h \hat{\tau}_{t+1+h}$$

$$\begin{aligned} (v) &= (1-v)\gamma \left[\frac{\beta^2 \bar{c}_b^y}{(1-\gamma\beta)(1-v\gamma\beta)} (\hat{\chi}_t - \hat{R}_t + \hat{c}_{b,t}^y) - \frac{\beta^2 \bar{s}}{1-\gamma\beta} (\hat{\chi}_t - \hat{R}_t) \right. \\ &- \frac{\beta^2 \bar{s}}{1-\gamma\beta} \tilde{E}_t^y \sum_{h=0}^{\infty} (v\gamma\beta)^h (\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h}) \\ &- \frac{\beta^2 \bar{s}}{1-\gamma\beta} \tilde{E}_t^y \sum_{h=1}^{\infty} (\frac{1-v^h}{1-v}) (\gamma\beta)^h (\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h}) \\ &- \frac{\beta^2 \bar{s}}{\gamma\beta} \tilde{E}_t^y \sum_{h=1}^{\infty} (\frac{1-v^h}{1-v}) (\gamma\beta)^h \hat{s}_{t+1+h} \right] \end{aligned}$$

Plug (i), (ii), (iii), (iv), and (v) into Equation (A.9) and rearrange it. After that,

multiply both sides with $\sum_{b=0}^{\infty} N_{b,t}^y$ where $N_{b,t}^y$ is the number of young households who belong to cohort 'b'

$$\begin{split} \bar{c}^y \hat{c}^y_t &= \left[\frac{(1-\gamma\beta)(1-v\gamma\beta)}{(1+\theta)\beta(1-\gamma\beta)+(1-v)\gamma\beta^2} \right] \left[v \bar{a}^y \hat{a}^y_t \qquad (A.10) \\ &+ \left(\left(\frac{(1+\theta)\beta}{1-v\gamma\beta} + \frac{(1-v)\gamma\beta^2}{(1-\gamma\beta)(1-v\gamma\beta)} \right) \bar{c}^y - \phi \left(\frac{\beta(\bar{w}-\bar{\tau})}{1-v\gamma\beta} + \frac{(1-v)\gamma\beta^2 \bar{s}}{1-\gamma\beta} \right) \right) (\hat{R}_t - \hat{\chi}_t) \\ &+ \phi \beta \bar{w} \hat{w}_t - \phi \beta \bar{\tau} \hat{\tau}_t + \phi \bar{w} v \gamma \beta^2 \tilde{E}^y_t \sum_{h=0}^{\infty} (v\gamma\beta)^h \hat{w}_{t+1+h} - \phi \bar{\tau} v \gamma \beta^2 \tilde{E}^y_t \sum_{h=0}^{\infty} (v\gamma\beta)^h \hat{\tau}_{t+1+h} \\ &+ \phi \left(\frac{(\bar{w}-\bar{\tau})v\gamma\beta^2}{1-v\gamma\beta} + \frac{(1-v)\gamma\beta^2 \bar{s}}{1-\gamma\beta} \right) \tilde{E}^y_t \sum_{h=0}^{\infty} (v\gamma\beta)^h (\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h}) \\ &+ \phi (1-v) \frac{\gamma\beta^2 \bar{s}}{1-\gamma\beta} \tilde{E}^y_t \sum_{h=1}^{\infty} \left(\frac{1-v^h}{1-v} \right) (\gamma\beta)^h (\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h}) \\ &+ \phi (1-v) \frac{\gamma\beta^2 \bar{s}}{\gamma\beta} \tilde{E}^y_t \sum_{h=1}^{\infty} \left(\frac{1-v^h}{1-v} \right) (\gamma\beta)^h \hat{s}_{t+h} \right] \end{split}$$

Rearrange Equation (A.10) and finally we can have the consumption decision rule of young households in the LC learning model

$$\begin{aligned} \hat{c}_{t}^{y} &= \psi v \bar{a}^{y} \hat{a}_{t}^{y} + \psi \phi \beta \bar{w} \hat{w}_{t} - \psi \phi \beta \bar{\tau} \hat{\tau}_{t} \end{aligned} \tag{A.11} \\ &+ \left[1 - \psi \phi \left(\frac{\beta (\bar{w} - \bar{\tau})}{1 - v \gamma \beta} + \frac{(1 - v) \gamma \beta^{2} \bar{s}}{1 - \gamma \beta} \right) \right] (\hat{R}_{t} - \hat{\chi}_{t}) \\ &+ \psi \phi \bar{w} v \gamma \beta^{2} \tilde{E}_{t}^{y} \sum_{h=0}^{\infty} (v \gamma \beta)^{h} \hat{w}_{t+1+h} - \psi \phi \bar{\tau} v \gamma \beta^{2} \tilde{E}_{t}^{y} \sum_{h=0}^{\infty} (v \gamma \beta)^{h} \hat{\tau}_{t+1+h} \\ &+ \psi \phi \left[\frac{(\bar{w} - \bar{\tau}) v \gamma \beta^{2}}{1 - v \gamma \beta} + \frac{(1 - v) \gamma \beta^{2} \bar{s}}{1 - \gamma \beta} \right] \tilde{E}_{t}^{y} \sum_{h=0}^{\infty} (v \gamma \beta)^{h} (\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h}) \\ &+ (1 - v) \gamma \psi \phi \frac{\beta^{2} \bar{s}}{1 - \gamma \beta} \tilde{E}_{t}^{y} \sum_{h=1}^{\infty} \left(\frac{1 - v^{h}}{1 - v} \right) (\gamma \beta)^{h} (\hat{\chi}_{t+1+h} - \hat{R}_{t+1+h}) \\ &+ (1 - v) \gamma \psi \phi \frac{\beta^{2} \bar{s}}{\gamma \beta} \tilde{E}_{t}^{y} \sum_{h=1}^{\infty} \left(\frac{1 - v^{h}}{1 - v} \right) (\gamma \beta)^{h} \hat{s}_{t+h} \end{aligned}$$

where

$$\psi = \frac{(1 - \gamma\beta)(1 - v\gamma\beta)}{\bar{c}^y[(1 + \theta)\beta(1 - \gamma\beta) + (1 - v)\gamma\beta^2]}$$

A.3 Solutions to Life-Cycle Learning Model

Appendix A.3 briefly shows the solutions to the LC learning model.¹ As in Equation (A.7) and (A.11), it is necessary to calculate the expected present value of returns to capital and labor in order to have the solutions of consumption of young and old households. Variables other than the market prices in the consumption decision rules are predetermined or perfectly foresighted.

From the forecasting model, Equation (1.22) to (1.25), we can calculate the expected evolution of the aggregate capital and wealth distribution between young and old households as follows

$$x'_{t} = \begin{pmatrix} 1 \\ \hat{k}_{t} \\ \lambda_{t} \end{pmatrix}, \quad \tilde{B} = \begin{pmatrix} 1 & 0 & 0 \\ \mu_{k,t-1} & \mu_{kk,t-1} & \mu_{k\lambda,t-1} \\ \mu_{\lambda,t-1} & \mu_{\lambda k,t-1} & \mu_{\lambda\lambda,t-1} \end{pmatrix},$$
$$x_{t+1} = \tilde{B}x_{t}$$
(A.12)

Iterating Equation (A.12) gives

$$x_{t+h} = \tilde{B}^h x_t$$

¹More details are in Mitra, G. W. Evans, and Honkapohja (2019).

Then, the expected present values of market prices by young households are

$$\tilde{E}_{t}^{y} \sum_{h=0}^{\infty} (v\gamma\beta)^{h} \hat{w}_{t+1+h} = \frac{1}{v\gamma\beta} \tilde{E}_{t}^{y} \sum_{h=1}^{\infty} (v\gamma\beta)^{h} \hat{w}_{t+h}$$

$$= \frac{1}{v\gamma\beta} \tilde{E}_{t}^{y} \sum_{h=1}^{\infty} (v\gamma\beta)^{h} (\mu_{w,t-1} \ \mu_{wk,t-1} \ \mu_{w\lambda,t-1}) \tilde{B}^{h} x_{t}$$

$$= \frac{1}{v\gamma\beta} (\mu_{w,t-1} \ \mu_{wk,t-1} \ \mu_{w\lambda,t-1}) (v\gamma\beta) \tilde{B} (I - v\gamma\beta\tilde{B})^{-1} x_{t}$$

$$= (\mu_{w,t-1} \ \mu_{wk,t-1} \ \mu_{w\lambda,t-1}) \tilde{B} (I - v\gamma\beta\tilde{B})^{-1} x_{t}$$

Also,

$$\tilde{E}_{t}^{y} \sum_{h=0}^{\infty} (v\gamma\beta)^{h} \hat{R}_{t+1+h} = (\mu_{r,t-1} \; \mu_{rk,t-1} \; \mu_{r\lambda,t-1}) \tilde{B} (I - v\gamma\beta\tilde{B})^{-1} x_{t}$$

Finally, we can have the solution to young households' consumption, and the solution to old households' consumption also can be obtained using the same way above.

Appendix B

Effects of Government Spending Financed by Distortionary Taxes

This section describes the impulse responses of macro-variables to government spending financed by the labor and capital tax.

B.1 Effects of Government Spending Financed by Labor Tax

Government spending, labor tax, and capital tax evolve via

$$g_t = \begin{cases} \bar{g} + 0.02\bar{y}, & \text{for } t = 1 \\ \bar{g}, & \text{for } t = 2, 3, 4, \cdots \end{cases}$$

$$\tau_t^H = \begin{cases} 0.2973, & \text{for } t = 1\\ 0.2500 \ (= \bar{\tau}^H), & \text{for } t = 2, 3, 4, \cdots \end{cases}$$

and $\tau_t^k = 0.1500 \ (= \bar{\tau}^k) \text{ for } t = 1, 2, 3, \cdots$

Aggregate Effects. Figure B.1 gives the impulse responses of output, consumption, investment, and hours to government spending (equal to 2% of the steady-state output level) funded by the same amount increase in the labor tax. Foremost, young households reduce their labor supply sharply in both the LC learning and rational expectation model due to the drastic increase in the labor tax from 25% to about 30%. This reduced labor supply lowers consumption, investment, and output at the time of the government spending shock. However, in contrast to the rational expectation model, household sentiment induced by the positive government spending shock in the LC learning model causes agents to increase investment and consumption overly after the shock.

Effects by Age Group. Figure B.2 plots the impulse responses of young and old households' consumption and saving to government spending shock financed by the labor tax. According to the lower left panel, young households decrease their saving sharply when the shock occurs since the labor tax reduces young households' labor supply. Nonetheless, young households under learning increase labor supply and saving overly after the labor tax rate returns to the initial steady-state level. Also, due to the drastic increase in saving, young people's consumption decreases as in the upper left panel. However, as old households do not adjust their labor supply, their saving and consumption fluctuate less. In contrast to the learning model, the young and old's responses for consumption and saving are similar to each other in the rational expectation model in which there exists no heterogeneity in the young and old's learning rule.



Figure B.1: Impulse Responses of Output, Consumption, Investment, and Hours to Government Spending (Equal to 2% of the Steady-State Output Level) Financed by Labor Tax

Notes: The increase in government spending by 2% of the steady-state output level requires the labor tax to be raised from 25.0% to 29.7%.



Figure B.2: Impulse Responses of Consumption and Saving to Government Spending (Equal to 2% of the Steady-State Output Level) Financed by Labor Tax by Age Group

Notes: The increase in government spending by 2% of the steady-state output level requires the labor tax to be raised from 25.0% to 29.7%.

B.2 Effects of Government Spending Financed by Capital Tax

Government spending, labor tax, and capital tax evolve via

$$g_t = \begin{cases} \bar{g} + 0.02\bar{y}, & \text{for } t = 1 \\ \bar{g}, & \text{for } t = 2, 3, 4, \cdots \end{cases}$$
$$\tau_t^H = 0.2500 \ (=\bar{\tau}^H) \text{ for } t = 1, 2, 3, \cdots$$
$$\text{and } \tau_t^k = \begin{cases} 0.2104, & \text{for } t = 1 \\ 0.1500 \ (=\bar{\tau}^k), & \text{for } t = 2, 3, 4, \cdots \end{cases}$$

Aggregate Effects. Figure B.3 gives the impulse responses of output, consumption, investment, and hours to government spending (equal to 2% of the steady-state output level) funded by the capital tax. Since the labor tax keeps its steady-state level, there is no sharp decline in hours when the government spending shock occurs. Still, households have more optimistic expectations in the learning model due to the positive government spending shock. Thus, output under learning increases additionally compared to output under rational expectations as in the upper left panel. However, only small positive effects of government spending exist in the rational expectation model in which the household sentiment channel does not operate.

Effects by Age Group. Figure B.4 plots the impulse responses of the young and old households' consumption and saving to government spending financed by the capital tax. Since the capital tax affects both cohorts through their budget constraints at the same time, there are no significant differences in the responses of consumption and

saving between young and old households in the rational expectation model (see upper and lower right panels). Nonetheless, as young and old households behave differently due to their distinct belief updating rules in the life-cycle learning model, young households' consumption and saving fluctuate more than those of old households (see upper and lower left panels).



Figure B.3: Impulse Responses of Output, Consumption, Investment, and Hours to Government Spending (Equal to 2% of the Steady-State Output Level) Financed by Capital Tax

Notes: The increase in government spending by 2% of the steady-state output level requires the capital tax to be raised from 15.0% to 21.0%.



Figure B.4: Impulse Responses of Consumption and Saving to Government Spending (Equal to 2% of the Steady-State Output Level) Financed by Capital Tax by Age Group

Notes: The increase in government spending by 2% of the steady-state output level requires the capital tax to be raised from 15.0% to 21.0%.

Appendix C

Non-linear Equilibrium Conditions to Life-Cycle New Keynesian Model with Two Sectors

(a) Households

$$P_t^Y C_t^Y = (1 - \beta \gamma) [v P_{M,t} A_{t-1}^Y (1 + i_{t-1}) + P_{M,t} \Omega_t^Y]$$
(C.1)

$$\Omega_t^Y = W_t N_t + \frac{v\gamma}{1+i_t} E_t \Omega_{t+1}^Y \tag{C.2}$$

$$P_t^O C_t^O = (1 - \beta \gamma) P_{M,t} [A_{t-1}^O + (1 - v) A_{t-2}^Y] (1 + i_{t-1})$$
(C.3)

$$P_t^O = \left[\omega_M^O P_{M,t}^{1-\eta} + (1-\omega_M^O) P_{S,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$$
(C.4)

$$P_t^Y = \left[\omega_M^Y P_{M,t}^{1-\eta} + (1-\omega_M^Y) P_{S,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$$
(C.5)

$$C_{M,t}^{O} = \omega_{M}^{O} \left(\frac{P_{M,t}}{P_{t}^{O}}\right)^{-\eta} C_{t}^{O}$$
(C.6)

$$C_{S,t}^{O} = (1 - \omega_{M}^{O}) \left(\frac{P_{S,t}}{P_{t}^{O}}\right)^{-\eta} C_{t}^{O}$$
(C.7)

$$C_{M,t}^{Y} = \omega_{M}^{Y} \left(\frac{P_{M,t}}{P_{t}^{Y}}\right)^{-\eta} C_{t}^{Y}$$
(C.8)

$$C_{S,t}^Y = (1 - \omega_M^Y) \left(\frac{P_{S,t}}{P_t^Y}\right)^{-\eta} C_t^Y \tag{C.9}$$

$$P_t^Y C_t^Y + P_{M,t} A_t^Y = v P_{M,t} A_{t-1}^Y (1+i_{t-1}) + P_{M,t} W_t N_t$$
(C.10)

$$P_t^O C_t^O + P_{M,t} A_t^O = P_{M,t} [A_{t-1}^O + (1-v) A_{t-2}^Y] (1+i_{t-1})$$
(C.11)

(b) Firms

$$mc_{M,t} = \frac{1}{Z_t Z_{M,t}} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t^k}{\alpha}\right)^{\alpha}$$
(C.12)

$$mc_{S,t} = \frac{W_t}{Z_t Z_{S,t}} \tag{C.13}$$

$$\log Z_t = \rho^Z \log Z_{t-1} + \sigma^Z e_t^Z \tag{C.14}$$

$$\log Z_{M,t} = \rho^{Z_M} \log Z_{M,t-1} + \sigma^{Z_M} e_t^{Z_M}$$
(C.15)

$$\log Z_{S,t} = \rho^{Z_S} \log Z_{S,t-1} + \sigma^{Z_S} e_t^{Z_S}$$
(C.16)

$$W_{t} = (1 - \alpha) m c_{M,t} \left(\frac{K_{M,t-1}}{N_{M,t}}\right)^{\alpha} Z_{t} Z_{M,t}$$
(C.17)

$$r_t^k = \alpha m c_{M,t} \left(\frac{N_{M,t}}{K_{M,t-1}}\right)^{1-\alpha} Z_t Z_{M,t}$$
(C.18)

$$Y_{S,t} = J_S \frac{Z_t Z_{S,t} N_{S,t}}{\Delta_{S,t}} \tag{C.19}$$

$$Y_{M,t} = J_M \frac{Z_t Z_{M,t} N_{M,t}^{1-\alpha} K_{M,t-1}^{\alpha}}{\Delta_{M,t}}$$
(C.20)

$$\Delta_{M,t} = J_M (1 - h_M) \left(\frac{P_{M,t}^*}{P_{M,t}}\right)^{-\theta} + J_M h_M \left(\frac{P_{M,t-1}}{P_{M,t}}\right)^{-\theta} \Delta_{M,t-1}$$
(C.21)

$$\Delta_{S,t} = J_S (1 - h_S) \left(\frac{P_{S,t}^*}{P_{S,t}}\right)^{-\theta} + J_S h_S \left(\frac{P_{S,t-1}}{P_{S,t}}\right)^{-\theta} \Delta_{S,t-1}$$
(C.22)

$$N_t = N_{M,t} + N_{S,t} \tag{C.23}$$

$$P_{M,t}^* = \frac{\theta}{\theta - 1} \frac{A_{M,t}}{B_{M,t}} \tag{C.24}$$

$$A_{M,t} = P_{M,t}^{1+\theta} m c_{M,t} Y_{M,t} + h_M E_t \{ \Lambda_{t,t+1} A_{M,t+1} \}$$
(C.25)

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$$B_{M,t} = P_{M,t}^{\theta} Y_{M,t} + h_M E_t \{ \Lambda_{t,t+1} B_{M,t+1} \}$$
(C.26)

$$P_{M,t} = [(1 - h_M)P_{M,t}^{*^{-1-\theta}} + h_M P_{M,t-1}^{1-\theta}]^{\frac{1}{1-\theta}}$$
(C.27)

$$P_{S,t}^* = \frac{\theta}{\theta - 1} \frac{A_{S,t}}{B_{S,t}} \tag{C.28}$$

$$A_{S,t} = P_{S,t}^{1+\theta} m c_{S,t} Y_{S,t} + h_S E_t \{ \Lambda_{t,t+1} A_{S,t+1} \}$$
(C.29)

$$B_{S,t} = P_{S,t}^{\theta} Y_{S,t} + h_S E_t \{ \Lambda_{t,t+1} B_{S,t+1} \}$$
(C.30)

$$P_{S,t} = [(1 - h_S)P_{S,t}^{*}{}^{1-\theta} + h_S P_{S,t-1}{}^{1-\theta}]^{\frac{1}{1-\theta}}$$
(C.31)

(c) Assets Market

$$\Lambda_{t,t+1} = \beta \frac{C_t^Y}{C_{t+1}^Y} \frac{P_t^Y}{P_{t+1}^Y}$$
(C.32)

$$P_{M,t}Q_t = P_{M,t}D_t + v\gamma E_t\{\Lambda_{t,t+1}P_{M,t+1}Q_{t+1}\}$$
(C.33)

$$D_t = \left(1 - \frac{mc_{M,t}\Delta_{M,t}}{J_M}\right)Y_{M,t} + \left(\frac{P_{S,t}}{P_{M,t}} - \frac{mc_{S,t}\Delta_{S,t}}{J_S}\right)Y_{S,t}$$
(C.34)

$$1 + i_t = (1 + r_t)E_t\{1 + \pi_{t+1}\}$$
(C.35)

(d) Monetary Policy

$$1 + i_t = \frac{1}{\beta} \left(\frac{1 + \pi_{M,t}}{1 + \pi_M} \right)^{\phi_{\pi_M}} \left(\frac{1 + \pi_{S,t}}{1 + \pi_S} \right)^{\phi_{\pi_S}} \left(\frac{Y_{M,t}}{Y_{M,t}^f} \right)^{\phi_{Y_M}} \left(\frac{Y_{S,t}}{Y_{S,t}^f} \right)^{\phi_{Y_S}} \exp\left(\nu_t\right)$$
(C.36)

$$1 + \pi_{M,t} = \frac{P_{M,t}}{P_{M,t-1}} \tag{C.37}$$

$$1 + \pi_{S,t} = \frac{P_{S,t}}{P_{S,t-1}} \tag{C.38}$$

$$(1 + \pi_t) = (1 + \pi_{M,t})^{J_M} (1 + \pi_{S,t})^{J_S}$$
(C.39)

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(e) Market Clearings

$$Y_t = C_t + I_{M,t} \tag{C.40}$$

$$I_{M,t} = K_{M,t} - (1 - \delta) K_{M,t-1}$$
(C.41)

$$C_t = C_{M,t} + C_{S,t} \tag{C.42}$$

$$C_{M,t} = C_{M,t}^{O} + C_{M,t}^{Y}$$
(C.43)

$$C_{S,t} = C_{S,t}^{O} + C_{S,t}^{Y}$$
(C.44)

$$Y_{M,t} = C_{M,t} + I_{M,t} (C.45)$$

$$Y_{S,t} = C_{S,t} \tag{C.46}$$

$$A_t = Q_t + K_{M,t} \tag{C.47}$$

$$A_t = A_t^O + A_t^Y \tag{C.48}$$