Deterioration Modeling of Subordinate Elements and Element Interaction for Bridge Management Systems

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ABSTRACT

Today, transportation agencies face aging infrastructure, increasing traffic demands and funding limitations, making efficient allocation of maintenance, rehabilitation, and replacement (MR&R) resources increasingly difficult. Many State Departments of Transportation (DOTs) use asset management principles, such as deterioration modeling and future condition prediction, to manage and schedule maintenance for thousands of bridges in an inventory or network. Deterioration modeling is an essential bridge management tool for predicting future condition and for helping allocate MR&R funds. However, existing deterioration modeling methods fail to account for interaction between bridge elements, particularly subordinate deterioration.

Subordinate deterioration occurs when element deterioration is impacted by the deterioration of a separate element, for example, bridge joints affecting the condition of bearings, pier caps, beam ends and abutments. Bridge engineers recognize subordinate deterioration exists for certain bridge elements, but it is ignored in current deterioration modeling. This study provides an investigation into the effect of deck joint deterioration on the deterioration of steel bearings and reinforced concrete pier caps for bridges in the Commonwealth of Virginia. However, the techniques developed in this research are general and can be applied to investigate the subordinate interaction of other elements.

State DOTs are required by federal law to inspect and assess the condition of bridges by visual inspection. This is captured through element-level inspections, where the condition of individual members and components of a bridge are rated on a numeric scale based on condition definitions. Element-level data from Virginia’s inspection database were used to develop datasets of bridge inspection reports with steel bearing and pier cap elements. First, exploratory statistical analysis, using categorical data methods, was conducted to determine the significance of subordinate deterioration. The investigation then explored different statistical models to predict the condition of the subordinate element. Finally, this research proposed a method to develop the transition probability matrices for elements that have a subordinate relationship to be used in Markov Chain deterioration modeling.

In this research, a statistically significant association between joints and condition of subordinate elements was found. This showed that subordinate deterioration existed in the inspection data and could be incorporated into bridge management practices. Multi-category logistic regression models were developed but failed the global goodness-of-fit test, suggesting the models did not accurately reflect the inspection data. The proposed Markov Chain approach to incorporate subordinate deterioration provided useful results and was calibrated using minimization of the squared error and the goodness-of-fit test.

Keywords: Deterioration Modeling, Bridge Management, Element Interaction, Subordinate Deterioration

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1.0 Introduction

Bridges represent major investments for the owner as well as for the community of users. In any transportation system, a bridge acts as a primary element of the community network because "[i]t likely controls the capacity; it is the highest cost per mile; and if the bridge fails, the system fails" (Barker and Puckett 2013). Currently there are over 21,000 bridges and culverts in the State of Virginia; over 60% of these structures are older than forty years (VDOT 2014). To manage maintenance and repairs for this bridge inventory, the Virginia Department of Transportation (VDOT) uses element-level inspection techniques to describe and maintain data on the in-service condition of these bridges. The collected inspection data are used for many purposes, including the development of deterioration models to predict future condition of bridge elements and to assist in decision-making and optimal resource allocation to maintain Virginia's infrastructure (Reardon and Chase 2015).

1.1 Background

Maintenance and repair actions due to bridge deterioration accounts for a significant portion of DOT budget expenditures (VDOT 2014). Deterioration is caused by many factors, including repeated and increased loading, environmental factors and lack of maintenance. Today, transportation agencies face aging infrastructure, increasing traffic demands and funding limitations and are mandated to use asset-based management for the allocation of maintenance, rehabilitation, and replacement (MR&R) resources. To assist maintenance engineers, bridge management systems (BMS) have been developed and have the capability to generate predictive models of bridge deterioration based on inspection report data. These systems help optimize MR&R decision-making and fund allocation for bridge networks under financial constraint. "The quality of these decisions depends, to a great extent, on the ability to predict the future condition of bridges" (Agrawal and Kawaguchi 2009).

Despite this, current modeling and analysis of deterioration do not account for the interaction of elements or subordinate deterioration (Sianipar and Adams 1997, LeBeau and Wadia-Fascetti 2000, Wild et al. 2013). Subordinate deterioration occurs when the deterioration of one specific element is related to, or affected by, the deterioration of another distinct element. The inclusion of this interaction in bridge management would provide greater efficiency in the allocation of funds for maintaining bridge inventories and provide greater functionality of bridge management programs to capture in-service conditions.

1.2 Subordinate Deterioration

As described, the concept of subordinate deterioration occurs when deterioration of a specific element is dependent on the deterioration of other elements. A common example known to maintenance engineers is the deterioration of elements associated with the deterioration (and eventual failure) of bridge joints. The primary function of joints in a bridge is to provide discontinuities in the deck to accommodate expansion and contraction due to temperature change. The joint is typically sealed by a flexible membrane or includes a material to protect superstructure and substructure elements below from water and debris. Bridge joint seals can be neoprene rubber, silicone adhesive or assemblies. Open joints also exist and may include a channel or catch system to divert water and debris. However, as bridge joints deteriorate and fail, water, deicing chemicals, salts and debris accumulate on bridge bearings, beam ends, abutments, and pier caps located under the joints, thus affecting the condition and accelerating the deterioration of those elements. In this application, the joints act as the primary element while bearings, beam ends, abutments and pier caps are subordinate elements.

1.3 Objectives

The concept of subordinate deterioration is known to bridge maintenance engineers; however, no method or study quantifying this interaction has been developed and consequently this relationship is largely ignored in current modeling. This research investigates subordinate deterioration between bridge elements by statistically describing the relationships between elements and developing more descriptive predictive models that account for the interaction for bridge management systems through the use of historic bridge inspection reports. This research focuses on the interaction of joint elements with bearing and pier cap elements. Specifically, this research looks to answer the following fundamental questions:

- Do subordinate relationships exist between these bridge elements?
    - Are deterioration rates for bearing and pier cap elements different with and without the presence of subordinate interaction?
    - Are these relationships statistically significant?
- Can subordinate deterioration relationships be modeled?
    - If a joint is in a certain condition, can the condition of the subordinate element be predicted?
    - Can this be incorporated in deterioration modeling for current bridge management systems?

To answer these questions, this study identified bridges with and without subordinate interactions for the different elements and evaluated the condition and deterioration in each case. It was hypothesized that the presence of joints would have a profound effect by increasing the deterioration of the subordinate elements as compared to the deterioration process absent of joints. It was further hypothesized, if joints were present, joints in poorer condition would result in a higher level of deterioration for the subordinate elements than joints in good or fair condition.

The importance of deterioration models is recognized in the bridge maintenance community. This investigation provided more detailed insight concerning the in-service operating conditions of bridges and developed enhanced deterioration models. This, in turn, will help state maintenance engineers plan and allocate resources in the most efficient way possible by accounting for subordinate relations between elements and increasing the usefulness of modeling bridge deterioration.

2.0 Literature Review

Federal bridge inspection requirements were implemented as a result of the collapse of the Ohio River Bridge in Point Pleasants, West Virginia on December 15, 1967. The “Silver Bridge”, as it was called, was a suspension bridge that used a steel eyebar chain and hanger system to support the deck. The collapse occurred due to a fracture in one of the eyebars “as a result of the joint action of stress corrosion and corrosion fatigue” (NTSB 1970). This collapse led to the creation of the National Bridge Inspection Standards (NBIS) that requires all bridges on public roads under a state’s jurisdiction be inspected a least once every two years (Barker and Puckett 2013). Also, bridges with spans greater than twenty feet must be cataloged and reported in the National Bridge Inventory (NBI) (Federal Register 2004). Two other bridge collapses contributed to the formalization of bridge inspection standards: the Mianus River Bridge in Greenwich, Connecticut and the Schoharie Creek Bridge in Amsterdam, New York. These incidents led to the primary inspection and documentation requirements that are common practice today and are reliant on visual inspection of condition to identify deterioration of the structure.

2.1 Bridge Inspection Types

Resulting from the implementation of the NBIS requirements, State DOTs are required to collect and report inspection data on the condition of bridges to the Federal Highway Administration (FHWA). This inspection requirement is based on the General Condition Rating (GCR) of bridge components using visual inspection. The rating is based on a scale of 0-9, with 9 being excellent condition and 0 being the failed condition and apply to four major bridge subsystems: deck, superstructure, substructure and culverts (FHWA 1995). These data are used by FHWA to appropriate funding to State DOTs.

The NBI ratings provide a broad overall description of bridge condition and in the 1980s, concerns whether the NBI rating system provided the best representation of condition led to the development of the element-level inspection method. Element-level data, still based on visual inspection, differ from the GCR approach in that data are collected on individual elements instead of the broad NBI components (VDOT 2007). Element-level data are more descriptive of a bridge because each element has a prescribe unit of quantity and inspectors are required to specify quantities of an element that are in the different condition states.

This inspection approach was developed for use with bridge management systems. BMS is a tool to assist in the management and allocation of maintenance funding and resources for a network of bridges as opposed to a single bridge. “A good BMS is a comprehensive database of bridge, traffic cost and safety data and an ongoing program for data collection and an analytical tool to systematically yield a network-level analysis and optimization of bridge data” (VDOT 2007). One of the most attractive features of these systems to State DOTs is the ability to capture and analyze the effects of the deterioration process and to develop deterioration models to predict future condition for bridge elements using historic inspection data. Passed in 1991, the

Intermodal Surface Transportation Efficiency Act (ISTEA) required the use of BMS by State DOTs (Wells 1994). As described, the goal of these systems was to manage a network of bridges instead of a single bridge. Currently, Pontis is the most common BMS system used by State DOTs in the United States. However, the recently developed AASHTOWare BrM system will replace the Pontis system in the near future.

2.2 Pontis Bridge Management System

In the United States, the Pontis software system, developed by the American Association of State Highway and Transportation Officials (AASHTO), serves as the primary asset-based management system used by State DOTs to collect bridge condition data and manage bridge inventories. Pontis acts as a database to store inspection data as wells as an optimization tool to perform deterioration modeling and asset management analysis. The system has the capability to analyze a single bridge, a group of bridges or an entire inventory of bridges by simulating different maintenance and deterioration scenarios as well as determining optimal maintenance plans and feasible actions (Gutkowski and Arenella 1998). The feasible action and optimization capabilities of Pontis are based on deterioration modeling from inspection data.

To standardize element-level inspections for use with Pontis, AASHTO identified and defined Commonly Recognized (CoRe) elements as important elements that exist in the main bridge types across the United States (VDOT 2007). State DOTs also have the ability to include agency defined elements, in addition to the CoRe elements, for inspection reports. Each element is rated on a scale of 1 to 3, 4, or 5, depending on the element. In this system, one is the excellent condition state and the highest value (3, 4 or 5) is the worst condition state as described by the condition state definitions. In addition to the CoRe and agency defined elements, Pontis incorporates Smart Flags, which are similar to elements but are identifiers of specific issues and defects such as fatigue and section loss. Smart Flags are an important feature of Pontis because they provide additional detail beyond the scope of the defined elements. In the State of Virginia, VDOT defines 111 inspection elements and 19 Smart Flags.

This research investigates the interaction of joint elements with bearing and pier cap elements. The CoRe guidelines define five joint elements, six bearing elements and pier cap elements for steel, timber, reinforced and prestressed concrete. The quantity for joint and pier cap elements are described in units of length while bearing quantities are described as each. These elements are listed in Appendix A with the CoRe condition state definitions.

2.3 Markov Deterioration Modeling

Current deterioration modeling by BMS is based on Markov Chains. This approach is a stochastic method that captures the randomness of a system and is well accepted for modeling bridge deterioration (Agrawal and Kawaguchi 2009). The Markov process determines the probability of a quantity transitioning from one finite state to another, based only on the current state (Norris 1997). Because of this, Markov Chains are termed as memoryless; the future probability does not rely on the past states of the element. This method is suited for use with bridge deterioration because the finite states of the Markov Chain are the condition states used in visual inspections. Based on the number of discrete states, this method develops a transition probability matrix (TPM) that describes the probability of an element in one state changing to the next state. An arbitrary TPM is shown in Eq. 1.

$$TPM = \begin{bmatrix} P_{11} & \cdots & P_{1k} \\ \vdots & \ddots & \vdots \\ P_{k1} & \cdots & P_{kk} \end{bmatrix} \qquad \text{Eq.1}$$

Here, P_{ij}, is the probability of an element in the i^{th} state transitioning to the j^{th} state where $i = 1...k$ and $j = 1...k$, with k being the total number of states. Thus, the diagonal values are the transition probabilities of an element remaining in the same state, called the retaining probabilities. As described by Thompson (2011), TPMs used for deterioration modeling must be square, only have values above the diagonal, and have positive values. These requirements are based on the assumption that elements do not spontaneously improve in condition (maintenance is not incorporated) and elements do not change more than a single condition state between transition periods. This creates zero values for probabilities of transitioning to a condition state not directly following the current state. Also, the sum of the probabilities in a row must equal to one (100 percent), as all quantity of an element must be accounted for as either remaining in the same state or transitioning to a new state. The final condition state is referred to as the terminal state and the probability is one (100 percent) because quantity can no longer transition to another state. This creates a simplified TPM for bridge deterioration as shown in Eq.2; a transition probability matrix for an element with three condition states is shown.

$$TPM = \begin{bmatrix} P_{11} & 1 - P_{11} & 0 \\ 0 & P_{22} & 1 - P_{22} \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{Eq.2}$$

The TPMs developed from Markov Chains can be used to predict the distribution of an element in each condition state after a transition period. This is done through matrix multiplication of a row matrix containing the current distribution of an element quantity in each condition state by the transition probability matrix. This is represented in Equation 3, where C_o, is the current distribution of element quantities in each condition state and C_t is the final distribution of quantities after a transition period.

$$C_t = C_o(TPM) \qquad \text{Eq.3}$$

This can be extended to additional transitions simply by repeating the multiplication with the final distribution (Keshavarzrad et al. 2014). Thus, the distribution after the nth transition is presented as

$$C_n = C_o(TPM)^n \quad \text{Eq. 4}$$

In the application of bridge deterioration, the transition period is taken as one year. For most bridges, minimum requirements state that a bridge must be inspected once every two years; however, a one year transition period is still applicable. The benefit of Markov Chains is the ability to show the distribution of quantity after each transition or at a future time of interest.

2.4 Multi-category Logistic Regression

The primary method of deterioration modeling for bridges is Markov Chains. However, other disciplines use logistic regression for deterioration models. Logistic regression is a statistical method for determining the probability of a dependent variable being assigned to a category through the relationship with independent variables. The most common form is binary logistic regression, where two possible categories exist for the dependent variable, usually described as success or failure. However, extensions of binary logistic regression are used to model a dependent variable with more than two possible categories. An introduction to multi-category logistic regression is provided in Appendix B.

Because multi-category logistic regression describes the probability of a dependent variable, the method can be utilized for deterioration modeling where the condition states are defined as the categories of the dependent variable. Salman and Salem (2012) provided a study of applying logistic regression to model deterioration of sewer lines for the Metropolitan Sewer District of Greater Cincinnati. Three methods, ordinal, multinomial and binary models, were used “to predict probabilities associated with future condition levels of individual wastewater collection pipes” (Salman and Salem 2012). Data were collected and included 11,373 inspection reports characterizing the condition of the sewer lines on a scale of 1-5. The lower value indicates better condition and is based on the National Association of Sewer Service Companies

(NASSC) inspection standards. The research used various independent variables to model the condition rating of the sewer lines. Results indicated that the ordinal method, based on the proportional odds assumption, violated the assumption and could not be applied. However, deterioration models were developed using the multinomial and binary logistic regression techniques to create predictive models for sewer line condition.

A separate study (Tran et al. 2009) was conducted to develop alternative deterioration models to Markov Chains for storm water pipes in Australia. Here, pipe condition was based on a categorical scale of 1 to 3. The study used independent variables including pipe size, location, age and other variables to predict the condition state assignment using multi-category logistic regression as well as a probabilistic neural networks model. The Chi-Squared Goodness-of-Fit Test was utilized to test the relative performance of the models for a sample of inspection data for 417 pipe sections for the City of Greater Dandenong, Australia. The results of the study found that the multi-category logistic regression model did not provide good fit based on the Chi-Squared Test; however, this study did demonstrate the applicability of multi-category logistic regression for deterioration modeling.

2.5 Element Interaction

Multi-category logistic regression has been used to model deterioration of pipe networks. Additionally, numerous studies (Agrawal et.al 2010, Keshavarzrad, et al. 2014, Wellalage, et al. 2015) have been conducted on the use of Markov Chains for deterioration modeling of bridge and infrastructure assets as well as to improve and calibrate the transition probabilities. However, very few studies address the impact of element interaction or subordinate deterioration for deterioration modeling techniques which is ignored in current bridge management practices.

One study that does investigate element interaction was conducted by Sianipar and Adams (1997). The study described element interaction as the process where the deterioration of one or more elements is related to the deterioration of another element in the system. This study investigated the use of fault-tree models to determine the probability that deck deterioration will accelerate based on element interaction. "Fault trees and event trees are logic diagrams consisting of a top event and a structure delineating the ways in which the top event can occur" (Wood 1985). In their research, different interaction scenarios were presented with the main focus placed on the interaction of bearings and expansion joints on the top event of accelerated deterioration of concrete decks. The joints and bearings were primary elements and the concrete deck was the subordinate element. Bearing malfunctions and expansion joint malfunctions were used as intermediate events, each composed of simple events such as worn bearings and joint damage, to describe the probability of the top event. The researchers used expert elicitation to provide the probabilities of the simple events to quantify the fault-tree. Results showed that the interaction between "transverse flexure cracks and damage to joint seals has the largest contribution to the occurrence of the top event" (Sianipar and Adams 1997). This research considered the interaction of elements for the study of element deterioration by showing that deterioration of a primary element can be affected by secondary elements. A second study (LeBeau and Wadia-Fascetti 2000) continued the use of fault trees to investigate bridge deterioration interaction and provided similar results. Similar to fault trees, impact trees incorporate logic statements to characterize the interaction of elements for deterioration models. A recent study (Wild et al. 2013) investigated the use of this method to account for element interaction in bridge deterioration modeling and prediction.

Currently, no formal method accounting for element interaction in deterioration modeling is implemented. The fault tree and impact tree methods acknowledge the effect an element can have on the deterioration of another element but focus on the use of probability theory to investigate the change in probability of failure. Also, these methods are not based on bridge inspection or condition state assignments.

A feature in Pontis does allow element deterioration rates to be modified based on four different environment states (Cambridge 2005). These states are used to describe elements in harsher environments, which tend to deteriorate quicker. These different deterioration environments can be used to represent element interaction informally, but are inadequate to fully describe subordinate deterioration (Sianipar and Adams 1997). The four environment states given in the Pontis system (Cambridge 2005) are the following:

1. Benign- no environmental condition affecting deterioration
2. Low- environmental conditions create no adverse impacts or are mitigated by past non-maintenance actions or highly effective protective systems
3. Moderate- typical level of environmental influence on deterioration
4. Severe- environmental factors contribute to rapid deterioration

For example, a bearing could be assigned a harsher Pontis environmental state if it is known that a joint above the element has failed, thus increasing the deterioration rate for the bearing. However, the environmental states are inadequate to represent interaction because 1) they are developed from “external factors such as traffic volumes, traffic loads, and operating practice”, 2) they may be previously used to represent other conditions, and 3) implementation is subjective (Sianipar and Adams 1997). This identified an area of needed study and can improve deterioration modeling for the application of bridge management and asset-based resource allocation.

2.6 Previous Research

By a previous request from VDOT, the Markov deterioration models for all 111 Virginia CoRe bridge elements were developed in a separate project. These models were developed for use with the Virginia Pontis BMS to characterize the independent deterioration of the elements and did not account for subordinate deterioration. The deterioration models developed included the models for steel fixed bearing and reinforced concrete pier cap elements and provided the transition probability matrices and Markov Models. The TPMs for these elements are included in Appendix D. The transition probabilities were obtained by using the Solver optimization tool in Microsoft Excel to optimize the TPM values through minimizing the squared error between the modeled quantity distribution and observed quantity distribution from historic inspection reports in the Pontis database from 1994 to 2012. These models do not incorporated subordinate deterioration and could be used as a comparison to the results of this study.

3.0 Methodology

The objectives of this research were to investigate if any relationship exists for subordinate deterioration of two related bridge elements through the use of inspection data and to develop methods to predict the deterioration caused by interaction. The main focus for this research was the relationship between joint condition and the deterioration of bearing elements and the relationship between joint condition and the deterioration of pier caps elements. Here, joints are defined as the primary element, the element whose deterioration affects the condition of another element, and bearings and pier caps are defined as the subordinate elements whose condition can be affected by the deterioration of the primary element.

Data for this research were obtained from the VDOT Pontis database. The database provided the historical inspection reports from 1994 to 2012 for all bridges in Virginia. The data were exported to a Microsoft Access database to simplify data analysis. The development of the Microsoft Access database is described in a previous project (Johnston 2013). The relational database included tables for element-level inspection data as well as all reporting fields required by the NBI guidelines. Both the element-level data and NBI data were used to generate data samples that characterize bridges with and without subordinate deterioration.

3.1 Bridge Types Investigated

In bridge design, bearings support the girders and transfer load from the superstructure to the substructure. The importance of bearings is often disregarded; however, properly functioning bearings are critical to avoid freezing or locking of movement, which can lead to high stresses and even failure of the structure (Zhao and Tonias 2012). For simply-supported designs, bearings are placed at the end of each span, either at the end abutments or intermediate piers, and allow longitudinal movement and/or rotation due to loading and temperature effects. Typically, bridge joints are located above the bearings to allow the deck to expand and contract as a result of the same effects. Because of this, the deterioration of the joint causes the potential for water, road salts and debris to contact and corrode the bearing and pier cap, leading to significant deterioration. However, simply-supported bridges, can be retrofitted to eliminate the joints, thus protecting the bearing for water and debris. Figure 1 shows typical joint-bearing layouts for two span simply-supported bridges. This is similar for single span bridges with the exception of the intermediate support location.

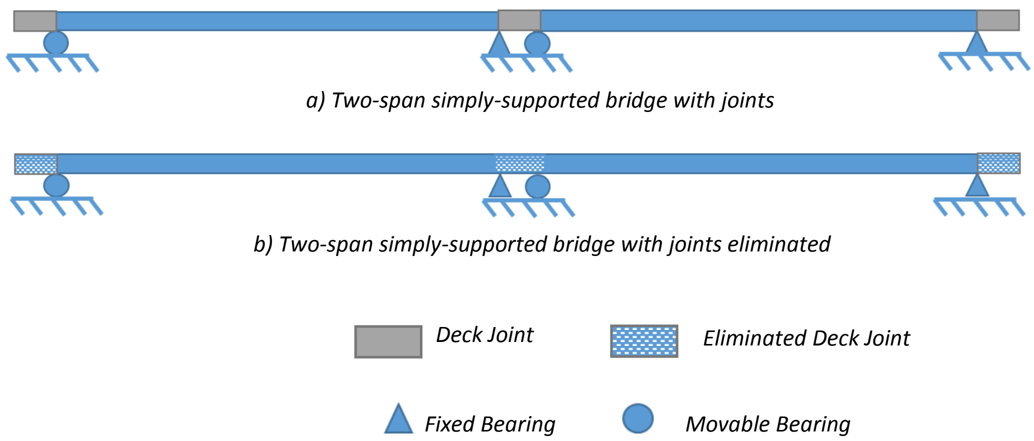

Figure 1- Typical simply-supported bridge layouts

For continuous bridge designs, a single girder can extend multiple spans with bearings located at the ends of the span as well as in between the ends. This design eliminates the need for joints within the deck. At the end of the spans, joints may or may not be present over the bearings or also may be eliminated. In a continuous span, there always exists a bearing line that is protected by the deck as shown in Figure 2.

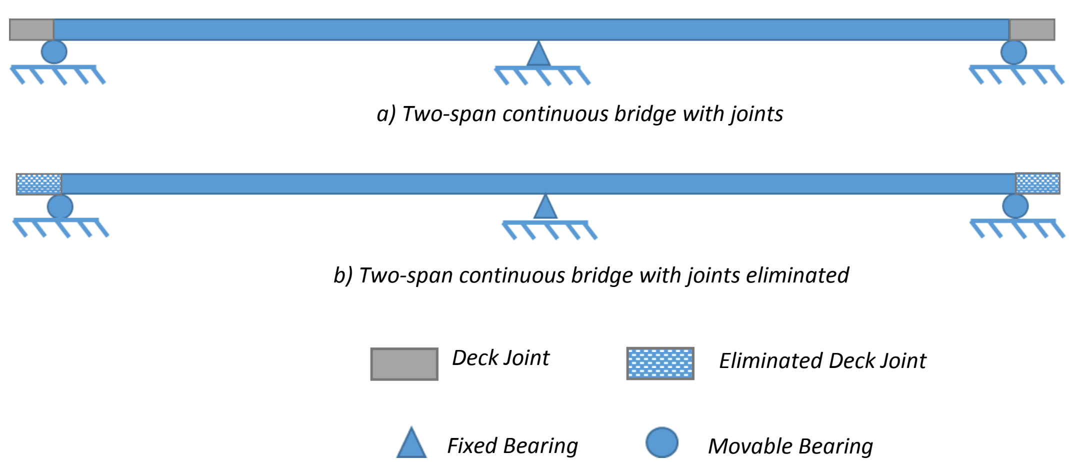

Figure 2- Typical continuous bridge layouts

Because of the large number of bridges in Virginia and the multitude of bearing and joint configurations, this research only investigates one- and two-span bridges. One- and two-span bridges account for the majority of bridges and facilitate the ability to identify the configuration of joint, bearing and pier cap elements and whether subordinate relationships are present. The Pontis data were analyzed to identify which bridges, and more specifically, which bearing and pier cap elements, are subjected to subordinate deterioration from those elements that are not subjected to subordinate deterioration. The objective was to develop a sample of bridges with joints over the bearings and pier caps and to compare the interaction to bearings and pier caps covered by the deck of a continuous span bridge. This was accomplished using the element-level inspection data and NBI data.

3.2 Data Processing and Cleaning

As part of the NBI standards, DOTs are required to report the material and configuration of bridges. Using *Federal Item 43- Structure Type* of the NBI coding guide, the classification of each bridge can be determined (FHWA 1995). *Federal Item 43* is given in Table 1.

Table 1- NBI bridge types

Federal Item 43	Structure Type
Materialmain 1	Concrete- Simply-Supported
Materialmain 2	Concrete- Continuous
Materialmain 3	Steel- Simply-Supported
Materialmain 4	Steel- Continuous
Materialmain 5	Prestressed Concrete- Simply-Supported
Materialmain 6	Prestressed Concrete- Continuous

All bridges are identified by a unique Federal Identification number called the bridge key. By using the NBI coding for structure type, the bridge samples could be obtained by creating a query in the Microsoft Access database to include only inspection records for each bridge type.

The inspection data for each structure type was exported to Microsoft Excel, where the samples were further filtered by eliminating bridges with more than two spans.

It was necessary to remove bridges where joint elimination retrofits occurred from the data samples. Additional data processing and filtering were necessary to select a sample that would accurately include the desired interaction of elements. To investigate the subordinate effects of joint deterioration on bearing deterioration, the investigation was limited to steel bearings, the most common and most susceptible bearing type to corrosion and deterioration. Typically, steel bearings are only applicable to bridges that also have steel superstructures. Therefore, to identify bridges for this sample, the bearing portion of the investigation was limited to bridges coded as Materialmain 3 and Materialmain 4, steel superstructures.

Once steel, one- and two-span bridges were identified, quantity and condition of joint elements for each bridge inspection were identified. The quantity of joint elements used in inspection reports is linear feet; therefore, the actual number of joints had to be calculated based on the deck width to determine the configuration of joints over the bearings. A major factor in this computation was the inclusion of the skew angle. The skew of a bridge is the angle between the centerline of the abutment/pier and a line perpendicular to the direction of the roadway. For bridges with large skew angles, the linear footage of a single joint would be much greater than the deck width. The total number of joints present on a bridge was determined by dividing the quantity of joint by the deck width adjusted for the skew angle. This is given in Eq. 5.

$$Number\ of\ Joints = \frac{Quantity\ of\ Joint}{Deck\ width \div \cos(skew)} \qquad \text{Eq. 5}$$

The calculation was necessary to identify the configuration of joints on different bridges and whether joints had been eliminated. For typical bridges, the maximum number of joints possible is the number of spans plus one; that is, joints can occur at the beginning and end of each span. Therefore, single-span bridges can have a maximum of two joints and two-span bridges can have three joints. In the case of continuous bridges, the number of joints would be less than or equal to the number of spans. This was important to identify whether bearings were subjected to subordinate deterioration, especially for the case of simply-supported bridges since, as described, a bridge could be coded as a simply-supported superstructure, but the deck could be made continuous by retrofits that eliminate the deck joints. These bridges must not be included in the sample of bridges with joints since subordinate deterioration no longer exists. By subtracting the number of joints present on a bridge from the maximum possible number of joints, the number of bearing lines that do not have joints present, was identified. Simply-supported bridges that had bearings without joints present and bridges with erroneous number of joints were removed so only simply-supported bridges with joints above all bearings were used to populate the sample of bearings with joints. For continuous bridges, the calculation described should result in one bearing line without joints and bridges that did not meet this requirement were removed. Furthermore, the most typical bearing configuration for continuous bridges is movable-fixed-movable where the middle fixed bearing does not have a joint above. Only bridges in this configuration were used to populate the sample of bearings without joints. This was accomplished by eliminating bridges that did not have twice as many movable bearings (Element 311) than fixed bearings (Element 313). Because of this configuration, only fixed bearings existed without element interaction. To remain consistent, this study only focuses on the subordinate deterioration of fixed bearing elements (Element 313); however, a complimentary dataset of movable bearings was developed and could be investigated separately.

The investigation of subordinate deterioration for joint-pier cap relationships only focused on Element 234- Reinforced Concrete Pier Cap because reinforced concrete is the most common material for pier cap design. To develop the sample of pier caps with and without joints, similar procedures used for categorizing the bearing samples were applied. However, the sample of bridges was not limited to bridges with steel superstructures but was extended to all superstructure types (Materialmain 1-6) as deterioration of pier caps is significant regardless of the material type of the superstructure. Data processing was simplified for the pier cap samples since only two-span bridges needed to be investigated.

3.3 Element Condition Improvements

In any standard maintenance program, joints are replaced, steel bearings are repainted and concrete is repaired to improve condition. Elements that had major improvements in the condition states were identified and removed from the sample. Major improvements for joints, bearings and pier caps were identified by comparing the average of condition state for the element from one inspection report to the next. If the average condition state increased greater than ten percent, it was assumed that a repair or replacement to the element had occurred. Also, for each dataset, bridges older than sixty-five years at the time of inspection were removed because improvements or repairs likely occurred and may not be included in the inspection reports. Also, this age was selected based on the 50-100 year design life common for most bridges in Virginia (VDOT 2014). The purpose of these checks is to limit the sample to include only condition data for continued deterioration processes because repairs can affect future deterioration rates.

3.4 Data Errors

As expected with any large set of field data, inconsistencies and errors exist. Common errors included missing data, erroneous values and erroneous element coding. The most prominent error occurred as a result of calculating the number of joints present and the number of bearing lines without joints. This calculation was used to identify the profile of bridges and whether joints exist over all bearings or whether bearings did not have joints present above. However, many instances occurred where the calculated number of joints exceeded the possible number of joints, resulting in negative number of bearings without joints. To investigate the cause of these errors, a random sample of bridges with this error were investigated using the element-level data, NBI data, bridge design plans and visual images. An example of this is the twin bridges carrying northbound and southbound traffic of North Sycamore Street over Four Mile Run in Northern Virginia. The bridges are identified by bridge key 22 for the southbound bridge and the bridge key 23 for the northbound bridge and both have identical inspection reports. Inspection reports are available for the years 1996, 2000, 2010 and 2012. The first three inspection reports show quantities of 28 for Element 301- Pourable Joint Seal and 24 for Element 302- Compression Joint Seal. The last inspection report only shows quantity of 24.384 for Element 302. The bridges are coded as single span with a skew angle of 25 degrees. The deck width is 11.3 meters (37 feet) with length of 19.5 meters (64 feet). The number of protected bearing lines calculated was -2.

Using Google Maps, the location was found using the latitude and longitude in the NBI coding. The bridges have two lanes of traffic with a sidewalk on one side of each bridge. This supports the deck measurement in the Pontis database assuming twelve foot lanes. The calculated length of joint based on the deck width and skew angle is 12.46 meters (41 feet).

Figure 3- NBL and SBL N. Sycamore Street over Four Mile Run (Google Maps 2015)

There are two possible reasons that these bridges did not pass the protected bearing check: 1) the joint units are in feet and the deck units are in meters or 2) the quantities are double counted. Calculating the length of joint using the skew was 12.46 meters, thus two joints would require 24.92 meters of joints. This is close to the values given in either quantity of joint, (28 for Element 301 and 24 for Element 302). Thus one of the values may be erroneous. For the final inspection report in 2012, however, there is only 24.384 quantity for Element 302, which would be accurate. This suggests that the 28 quantity in Element 301 for the first inspection report is an error.

Investigations of three other bridges found similar issues. The conclusion of the investigation was that the majority of erroneous values from calculating the number of joints was due to inconsistent units of measure, where the deck width was in meters and the joint length was in feet. However, other errors and field conditions exist, leading to erroneous calculations. It was also noted that documentation or plans of repair or retrofits commonly do not exist with other data for a bridge. As was previously described, bridges with quantity errors were not included in the samples of data.

3.5 Field Inspection

As part of this research, a field visit with VDOT officials to inspect the condition of bearings and pier caps for bridges with and without joints was performed. The investigation identified highway bridges in the Staunton district where joint eliminations had been performed and bridges where joints were still present. This provided a direct comparison of condition for bearing and pier cap elements where joints are present and where joints had been eliminated. The investigation found that bridges where joints had been eliminated showed less or slowed deterioration of bearings, pier caps and abutments than bridges where joints were still present.

The field investigation also included a continuous span bridge where a line of bearings and a pier cap were not subjected to joint interaction, as was the focus of this study. The bridge carries Route 654, White Hill Road over I-81 in Augusta County and is a four-span bridge built in 1967. The two end spans of the bridge are simply-supported and the middle two spans over I-81 are continuous. The bridge is shown in Figure 4.

Figure 4- Rte. 654 White Hill Road over I-81 (Google Maps 2015)

The joints between the simply-supported spans and the continuous spans recently had been eliminated. The bridge girders as well as the bearings had been repainted. Because of this, the bearings under the continuous span and the bearings where joints previously existed showed no significant difference in condition. However, the condition of the pier caps at the location of the joints before elimination showed significant patching and repairs, while the pier cap located under the continuous span where no joints ever existed was in near-perfect condition. The condition of the pier cap under the continuous span is shown in Figure 5 and the pier cap located where joints previously existed is shown in Figure 6.

Figure 5- Pier cap with no joints present

Figure 6- Pier cap where joints existed before elimination

This field investigation supported the hypothesis that element interaction of joints with bearings and pier caps create a subordinate deterioration effect where the deterioration of the bearings and pier caps are dependent on the deterioration of the joint element. This field visit also supported the methodology of this study to remove bridges where joint elimination retrofits have occurred and when improvements to the elements have occurred.

4.0 Analysis of Data and Results

With the development of the datasets of inspection reports for bearings and pier caps with and without joints, data analysis was conducted. The analysis began by investigating if a statistically significant difference in condition existed between the samples for elements with subordinate deterioration and elements without subordinate deterioration. Next, statistical modeling was conducted to develop a method to predict the condition of the subordinate elements based on the primary joint element condition using multi-categorical logistic regression. Finally, a method to develop transition probabilities for subordinate deterioration in Markov Chains using conditional probabilities was proposed and calibrated.

4.1 Exploratory Data Analysis

As previously described, the quantity for joint elements is linear feet (see Appendix A for condition state definitions); thus, inspectors may assign portions of the total length of joints to the three condition states defined under the CoRe element definitions. Because of this, a method was developed to categorize joint condition based on the average condition state. Classification of the joint elements is shown in Table 2.

Table 2- Joint average condition classification

Good	Fair	Poor
1-1.67	1.67-2.33	2.33-3

The condition of a given joint type present on the bridge was classified as good, fair or poor condition based on this calculation. For example, brkey 7 identifies a bridge coded with Element 302 – Compression Joint Seal and has 13.6 feet in CS1, 3 feet in CS2, and 7.4 feet in CS3. The average joint condition state for this bridge is 1.74 and is classified as fair condition. This provided a method for further analysis by dividing the samples of bearings and pier caps with joints into subsamples based on the condition of the joint elements.

For each category of joint condition, the mean and standard deviation of bearing conditions was calculated. Results are shown in Table 3.

Table 3- Average bearing condition by joint category

Joint Category	# of Inspection Reports with Bearing Element 313	Bearing AVE CS	Bearing Std Dev
No Joint	1268	1.09	0.281
Good	1630	1.21	0.398
Fair	658	1.39	0.473
Poor	283	1.57	0.543

Similar analysis for the subordinate deterioration of pier caps due to the presence of joints was conducted. For pier cap elements, four condition states are defined by the CoRe Definitions and can be found in Appendix A. The mean and standard deviation, given each joint category, is provided in Table 4.

Table 4- Average pier cap condition by joint category

Joint Category	# Inspection Reports with Pier Cap 234	Pier Cap AVE CS	Pier Cap Std Dev
No Joint	1654	1.03	0.124
Good	796	1.12	0.289
Fair	256	1.36	0.704
Poor	219	1.57	0.626

This initial analysis was based on the average condition of the joints and as well as the average condition of the subordinate elements and showed that average condition state increases for the subordinate elements and when the joints were classified in poorer condition. However, additional methods using categorical data analysis further investigated the relationships and more fully utilized the data.

Significant work has been done to develop visualization techniques to develop "insightful graphical display[s]… to reveal some aspects of the data" (Friendly 2000). To further investigate the samples, R software for statistical analysis was used to visualize the relationships in the data. Using the vcd package for visualizing categorical data, a ternary plot (also known as a trilinear plot) was generated (Meyer et al. 2015). The ternary plot specifically applies to a dataset with three variables where the coordinate points are plotted based on the percentages of data in each category and the vertices of the triangle represent the extreme location for each category. "Each profile point is a weighted average, or centroid, of the vertices" (Greenacre 2007). A ternary plot for bearing inspection reports was developed to show the effect of the presence or absence of a joint on bearing condition.

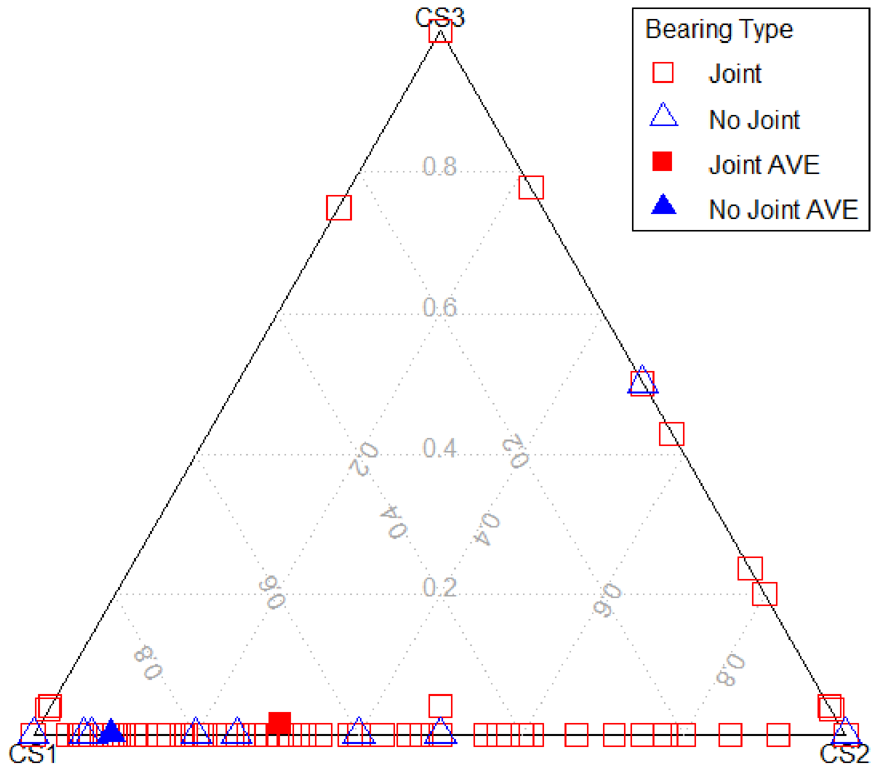

Figure 7- Ternary plot for bearing samples

The ternary plot shows the distribution of the percentage of bearing quantity, with and without joints, in each condition state for each inspection report. The overall average condition state distribution was also determined and is shown by the solid square and triangle in Figure 7. These two points are significant because they are the centroid of the samples (Greenacre 2007). The ternary plot shows for the majority of inspections, bearing quantity is in CS1 and CS2 with less quantity in CS3. Also, the dispersion of bearing condition states is greater for elements with joints present, as indicated by the open red squares.

The use of a ternary plot was extended to investigate the relation between joint classification and the condition state distribution of bearings.

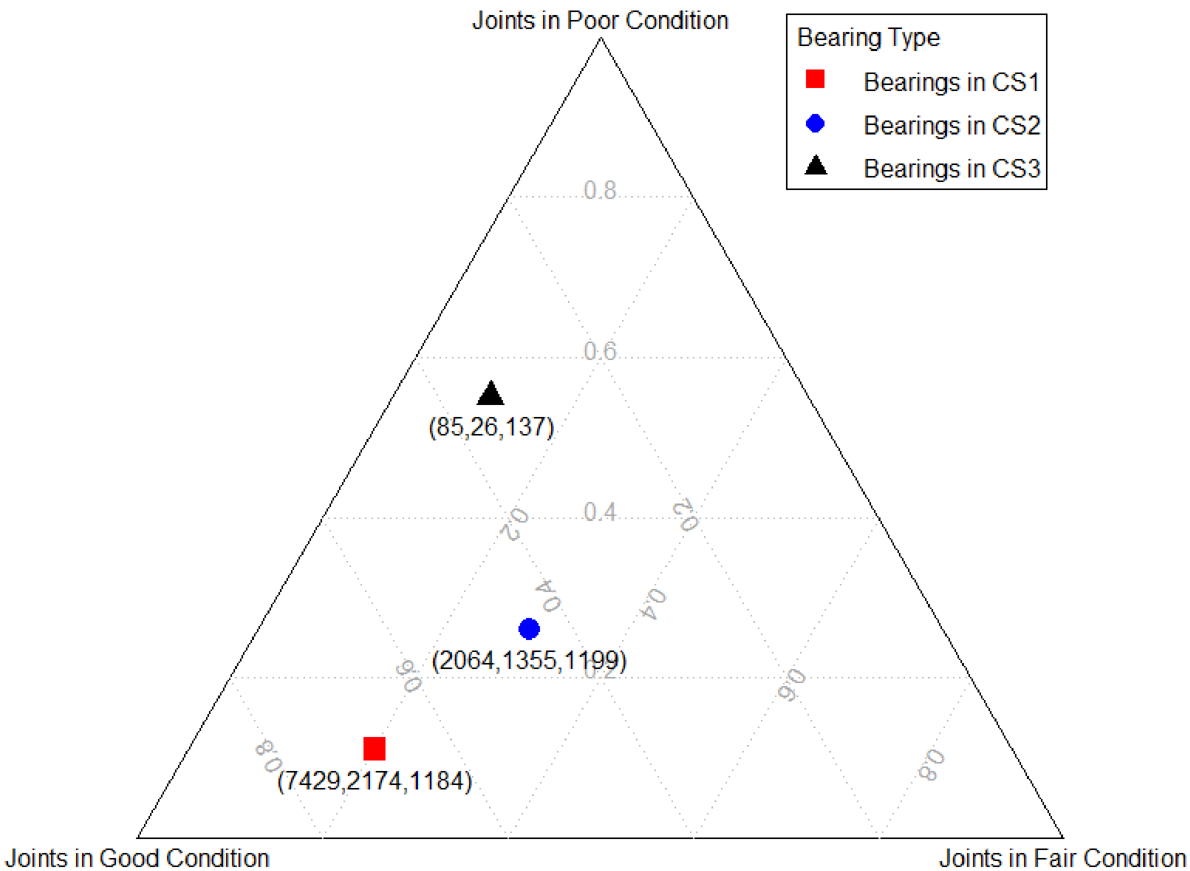

Figure 8- Distribution of bearing condition states based on joint condition classification

Figure 8 relates the joint type present when bearings are coded for the three different condition states. The vertices are the joint condition classifications and the coordinate points are based on the quantity of joint in each classification state. For inspection reports of bearings coded as CS1, approximately 70% have joints classified as good; whereas for bearings coded as CS3, approximately 35% of joints are classified as good and nearly 55% are classified as poor. Again, only steel fixed bearings were investigated based on the standard continuous bridge configuration.

Because reinforced concrete pier cap elements are defined with four condition states by the CoRe definitions (see Appendix A), a trilinear plot cannot visually express the data. However, a similar approach was taken using a percent stack chart to show the distribution of quantity for each condition state and is shown in Figure 9. Note the vertical scale begins at 50% because the majority of quantity for both samples are assigned to the first condition state.

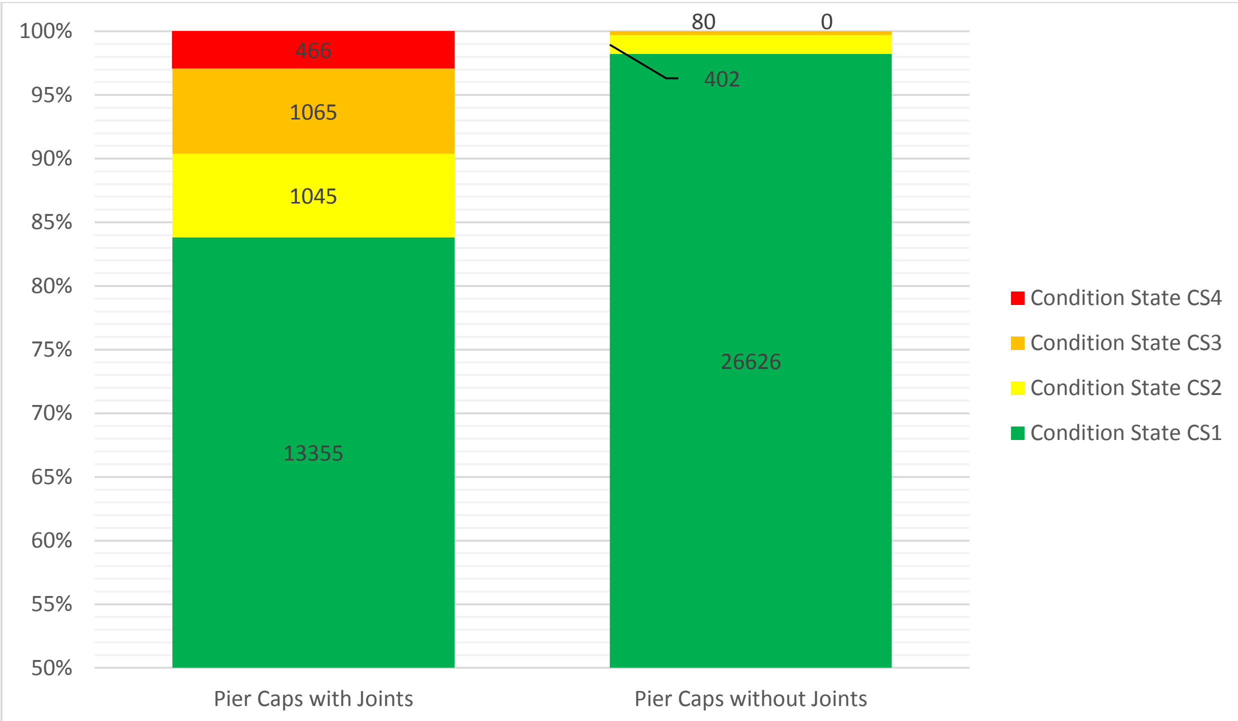

Figure 9- Pier cap quantity condition state profile

Here, a much larger portion of quantity is assigned to CS2, CS3, and CS4 when joints are above the pier caps. When joints are not present, 98% of the quantity is assigned to CS1 with no quantity in CS4. Similar to the bearing investigation, the sample of pier caps with joints can be further investigated for the classification of the joint that is present. Figure 10 shows the association between pier cap condition and the joint present, as classified in the three categories.

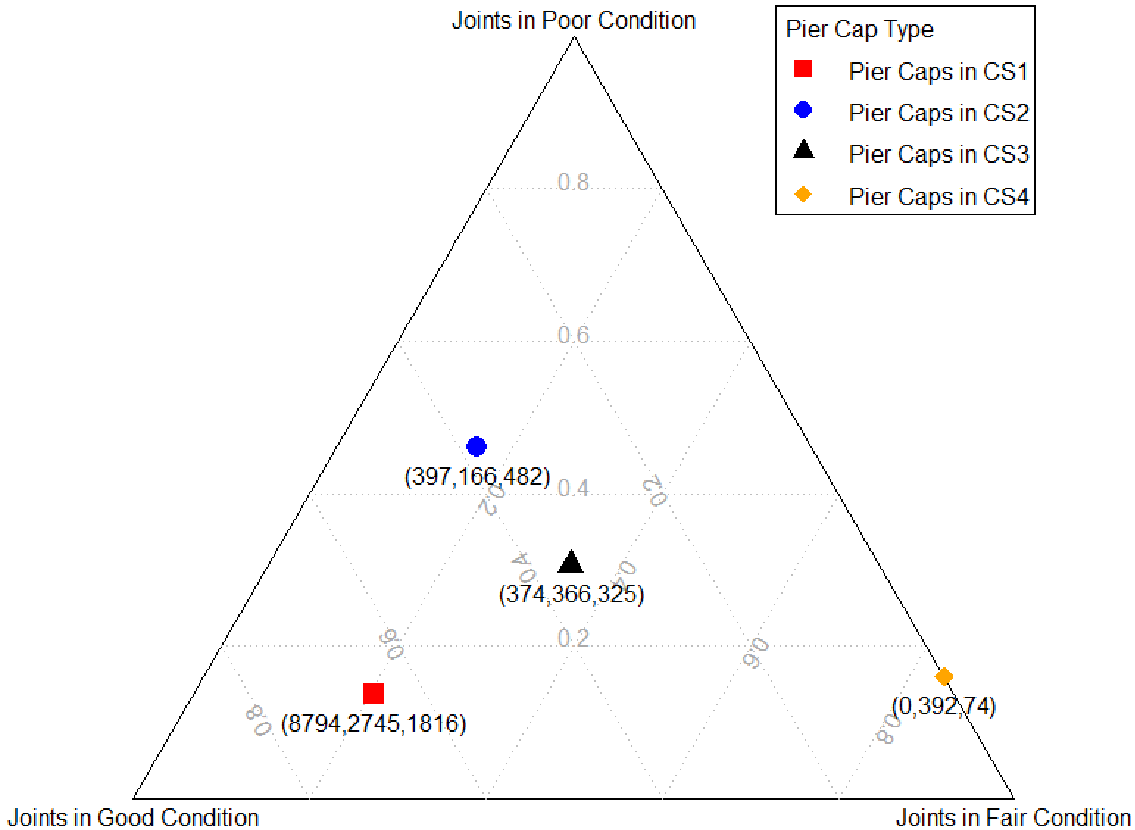

Figure 10- Distribution of pier cap condition states based on joint condition classification

Here, the ternary plot shows similar results to the bearing sample with joints. Of note, the plot shows that in all cases of pier caps coded with quantity in condition state four, the joint present above the element is in either fair or poor condition, but never in good condition.

4.2 Statistical Significance of Categorical Data

The initial exploratory data analysis supported the hypothesis that element interaction affects the condition of subordinate elements. However, a primary goal of this research was to determine whether the association between the joint and subordinate element deterioration is statistically significant. By assigning element quantity to condition states, the data are categorical, and therefore, the data cannot be analyzed as continuous variables with typical tests such as the z-test or t-test. Thus, categorical methods, primarily based on the Chi-Squared statistic, were used to identify statistical significance.

For the analysis of statistical significance, contingency tables were developed to cross tabulate data based on two variables: joint presence and the subordinate element condition states. Table 5 shows the contingency table for bearing condition with and without joints.

Table 5- Contingency table for bearing condition states and joints

Bearing Class	Condition State			Col Profile
	CS1	CS2	CS3	(Row Sum)
313 Bearings with Joints	10787	4618	248	15653
313 Bearings without Joints	6981	739	2	7722
Row Profile (Column Sum)	17768	5357	250	23375

To investigate if there is a statistical relationship between the condition state of bearings with joints and bearings without joints, the Chi-Squared Test for Independence was utilized. This test is based on the null hypothesis (H_o) that the variables are independent and the alternative hypothesis (H_A) that the variables are not independent. In a two-way contingency table as presented, the cell frequency can be presented as n_{ij} and the cell proportions can be presented as p_{ij}. To test the hypothesis, the observed frequencies are compared to expected frequencies calculated assuming independence of the samples. If two variables are statistically independent, the joint probability distribution of a cell is the marginal probability of the first variable times the marginal probability of the second variable (Walpole et al. 2007). Thus, the expected frequency of a certain cell can be calculated as

$$Exp_{ij} = \frac{n_{i+}n_{+j}}{n} \qquad \text{Eq. 6}$$

Here, the plus sign (+) indicates a summation over the indices and n is the grand total of the contingency table (Agresti 2007). This equation can be described as the row sum multiplied by the column sum divided by the grand total of the contingency table. As defined earlier and using the same notation for the test of independence, the null and alternative hypothesis infer

$$H_O: p_{ij} = p_{i+}p_{+j} \ and \ H_A: p_{ij} \neq p_{i+}p_{+j} \qquad \text{Eq.7}$$

The expected value can be compared to the observed value using the Pearson Chi-Squared statistic:

$$\chi^2 = \sum \frac{(Obs_{ij} - Exp_{ij})^2}{Exp_{ij}} \qquad \text{Eq.8}$$

The Chi-Squared statistic was compared to the value of the Chi-Squared distribution at a designated level of significance to determine whether the null hypothesis should be rejected. For standard hypothesis testing procedure, the designated level of significance is normally α = 0.05. Thus, if the Chi-Squared Statistic is greater than the value of the Chi-Squared distribution at α = 0.05, then the null hypothesis is rejected.

To determine whether the distribution of bearing condition state is independent of whether a joint is present or not present above the bearing, the R software chisq.test code performs the Test for Independence. The R output is shown below.

```
X-squared = 1328.1, df = 2, p-value < 2.2e-16

> xsq$observed
                Condition
Bearings           CS1   CS2 CS3
  With Joints    10787  4618 248
  Without Joints  6981   739   2
> xsq$expected
                Condition
Bearings                CS1      CS2       CS3
  With Joints    11898.289 3587.299 167.41176
  Without Joints  5869.711 1769.701  82.58824
```

Figure 11- R output for Chi-Squared Test for Independence of bearings and joints

The observed and expected values are given under the headings xsq$observed and xsq$expected, respectively. The value of the Chi-Squared distribution at α = 0.05 on two degrees of freedom, found using standard Chi-Squared Distribution tables, is 5.991. The test statistic value is much greater than the evaluation of the distribution (1328.1>>5.991), indicating that the null hypothesis of independence should be rejected, supporting the claim that there is an association between the two variables. The associated p-value of the hypothesis is also shown.

The same procedure was conducted for the samples of pier cap elements. The contingency table was developed to include the four condition states defined for pier caps and is shown in Table 6.

Table 6- Contingency table for pier cap condition states and joints

Pier Cap Class	Condition State				
	CS1	CS2	CS3	CS4	Row Sum
Pier Caps with Joints	13355.5	1045.2	1064.7	465.9	15931.3
Pier Caps without Joints	26626.0	402.4	80.0	0	27108.4
Column Sum:	39981.5	1447.6	1144.7	465.9	43039.7

The Chi-Squared Statistic was calculated using R software and the output is given in Figure 12.

```
        Pearson's Chi-squared test

data:  piercap
X-squared = 3324.8, df = 3, p-value < 2.2e-16

> xsq$observed
                Condition
Bearings               CS1      CS2      CS3     CS4
  With Joints     13355.49 1045.209 1064.673 465.906
  Without Joints  26625.99  402.397   79.971   0.000
> xsq$expected
                Condition
Bearings               CS1      CS2      CS3      CS4
  With Joints     14799.29 535.8366 423.6941 172.4568
  Without Joints  25182.19 911.7694 720.9499 293.4492
```

Figure 12- R output for Chi-Squared Test for Independence for pier caps and joints

The value of the Chi-Squared distribution at $\alpha = 0.05$ on three degrees of freedom is 7.815. Again, the statistic value is much greater than the evaluation of the distribution (3324.8>>7.815), indicating that the null hypothesis of independence should be rejected.

To further investigate the deviation from independence, an association plot was developed using R. Association plots graphically display the Chi-Squared Test by visually representing deviations from independence for each variable. In the association plot, cell height is proportional to the residual value (observed value minus expected value), thus the baseline represents independence or no deviation (Friendly 2000). Positive deviations are show above the baseline and negative deviations are below the baseline. The cell width is proportional to the number of observations for each category.

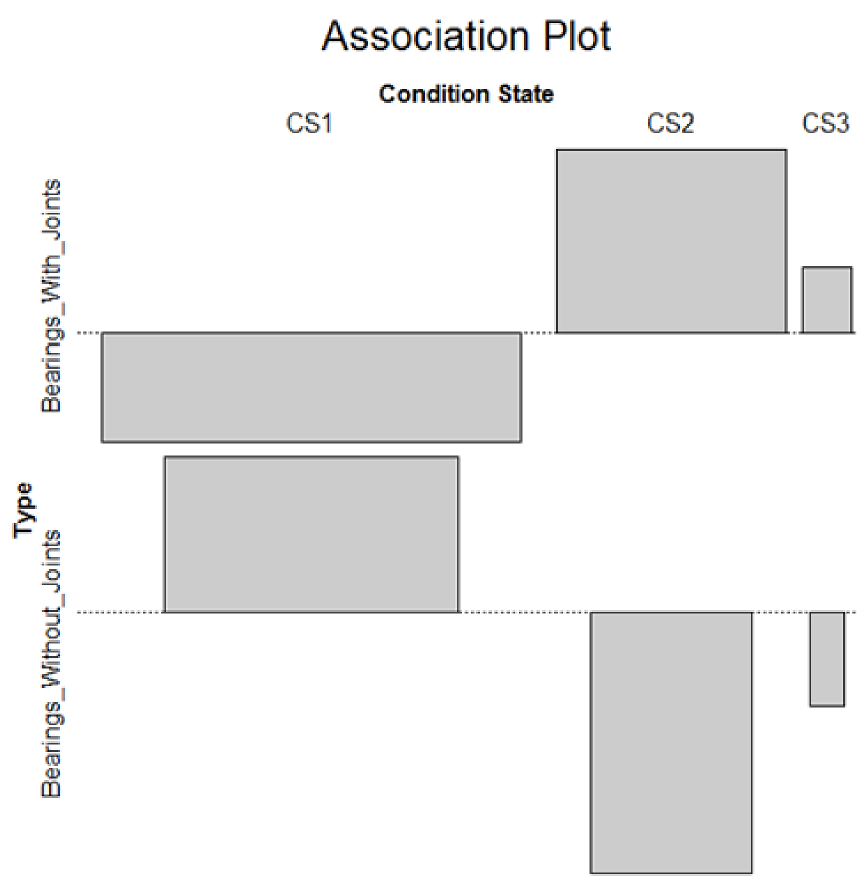

Figure 13- Association plot for bearing samples

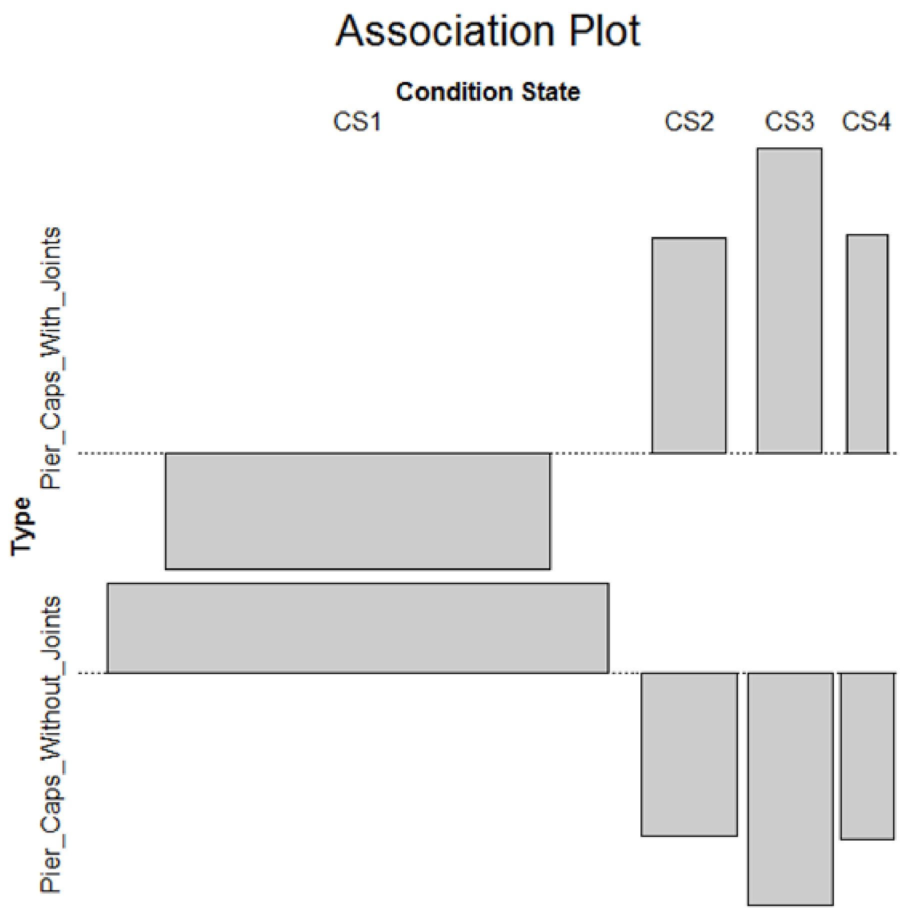

Figure 14- Association plot for pier cap samples

As shown, there is deviation from the baseline in each case, which is the reason for the rejection of independence from the Chi-Squared Test. Also, it is significant to note that in each case the samples with joints are approximately mirror images to the samples without joints. For the sample of bearings with joints present, the deviation is negative for CS1 and positive for CS2 and CS3, indicating that for CS1, less bearings are observed than expected and for CS2 and CS3 more bearings are observed than expected (based on independence). The exact opposite occurs for the sample of bearings without joints; if independence between variables existed, more bearings than expected occur in CS 1 and less than expected occur in CS 2 and CS 3. This suggests an association that bearings with joints tend to have more quantity in CS 2 and CS 3 and bearings without joints tend to have more quantity in CS 1. The results for the pier cap samples exhibit the same mirrored pattern of deviation from independence.

To further quantify and visualize the Chi-Squared statistic, correspondence analysis was utilized. Correspondence analysis (CA) is a method of visualizing the associations in contingency tables and is closely related to the Chi-Squared value calculated in the Test for Independence (Friendly 2000). In normal application of data representation, scatterplots can be used to show data points based on two variables. The distance between these points is based on the coordinates of the data point and can be calculated as the Euclidean or straight line distance where the distance between two points is the square root of the squared differences between coordinates (Greenacre 2007). Correspondence analysis can be used to represent the distances between data points as the Chi-Squared distance. This provides a "graphical method to represent the structure of cross tabulations to shed light on underlying mechanisms" (Yelland 2014). To plot distances, correspondence analysis reduces the dimensions of the data. The number of dimensions for a contingency table is the smaller of the number of rows or columns. "CA identifies dimensions along which there is very little dispersion of the profile points and eliminates these low-information directions of spread" (Greenacre 2007). However, for the data in this study, two-way contingency tables were used, already providing low dimensional data.

Using the R software ca package, a correspondence analysis was performed for the developed contingency tables (Nenadic and Greenacre 2007). The results of the correspondence analysis showed the relative location of the points and were plotted based on the Chi-Squared distances. The plot is one-dimensional and the column categories act as the direction of the axes.

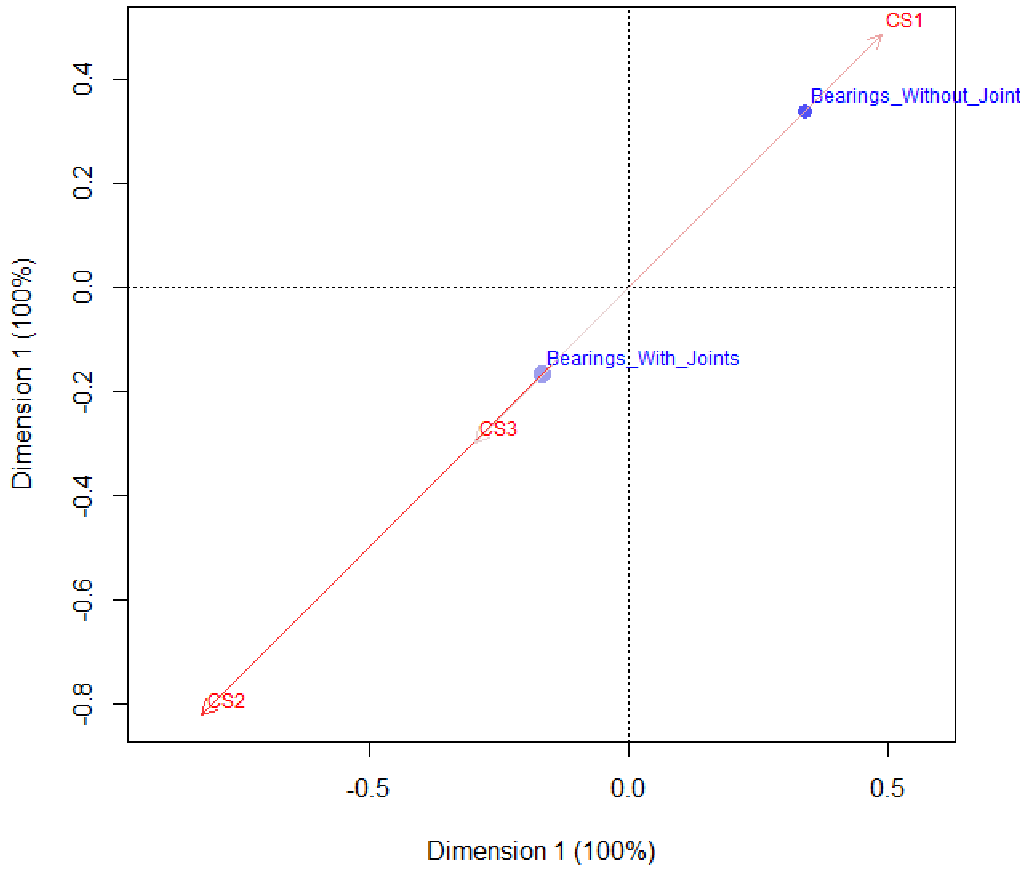

Figure 15- Correspondence analysis for bearing samples

Figure 15 shows that samples of bridges without joints are more closely associated to CS1 while bridges with joints are more associated with CS2 and CS3.

In addition to the visual map, the output of correspondence analysis provides the total and principle inertia values. Total inertia is calculated as the Chi-Squared statistic divided by the grand total of the contingency table. This provides a measure of dispersion and can be used to determine a correlation coefficient (Greenacre 2007). The principle inertia is the contribution of each principle axis and sum to the total inertia. Because the contingency table for bearing data is one dimensional, one principle inertia accounts for the total inertia for the contingency table of bearing data and is given by R as 0.0568 or 5.68%. By taking the square root of the inertia, a correlation coefficient can be interpreted (Greenacre 2007). In this case, the square root of the inertia is 0.238 or 23.8%. “As a rule of thumb, any value of this correlation coefficient in excess of 0.2 [20%] indicates significant dependency” (Bendixen 2003).

The same procedures were applied with the samples using pier cap data. Again, one dimension can be used to fully visualize the relationship between variables. This is shown in the following figure.

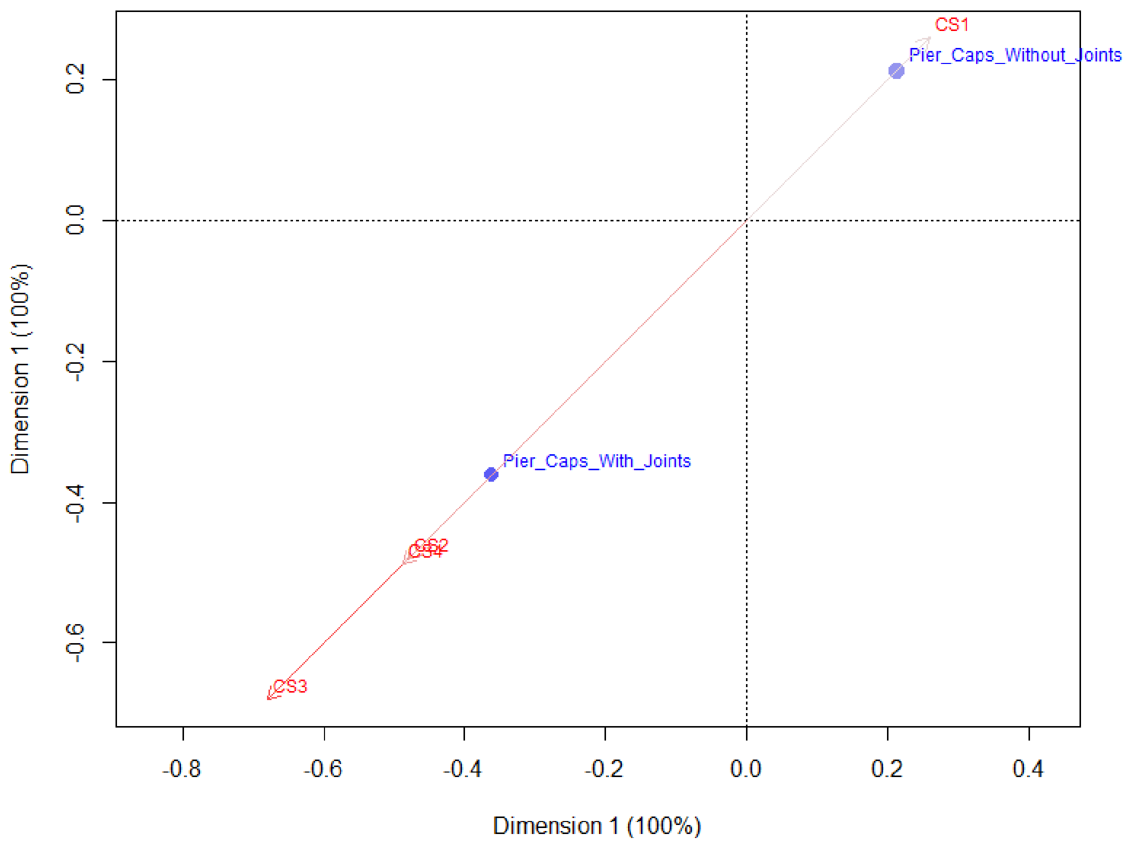

Figure 16- Correspondence analysis for pier cap samples

The plot of the Chi-Squared distances shows how the two samples are associated with the condition states. The sample without joints is more associated with quantities in CS1 and the sample with joints is more associated with CS2, CS3 and CS4. Again to provide quantification of the relationships, the inertia is calculated as 0.0772 or 7.72%. To quantify the correlation, the square root of the total inertia is calculated as 0.278 (27.8%).

4.3 Multi-category Logistic Regression Modeling

The previous section investigated the association and relationship between a primary element and two subordinate elements. The second research objective of this study was to investigate methods to account for element interaction by predicting the condition state distribution of subordinate elements based on the primary element. Models accounting for the relationship of each subordinate element were developed to attempt to predict the condition of the subordinate element as a dependent variable using the condition of the primary element as an independent, explanatory variable. To do this, multi-categorical logistic regression was used to develop generalized linear models (GLMs). A brief introduction to multi-categorical logistic regression is provide in Appendix B.

Multi-category logistic regression is an extension of the binary logistic regression technique and can be used to model cases with three or more categories (Agresti 2007). Two types of multi-category logistic regression models exist. Multinomial models do not consider order between the categories whereas ordinal models can be used for data with hierarchal categories. The models provided the probability of bearing or pier cap element quantity being assigned to a certain condition state based on an explanatory variable using the log odds or logit link function. The explanatory variable chosen for the logit model was the average condition state of the joint element as calculated for the joint classification.

Using the datasets for bearing and pier cap elements with joints, the data was inputted into R software. Using the mlogit package and referencing Katchova (2013), multinomial logistic regression models using base-line category logits were fit to the data (Croissant 2013). The independent variable selected to develop the probability distribution of condition states was the joint average condition. This was used because it provided an average of the joint condition. The logit model for fixed steel bearings is shown in Figure 17.

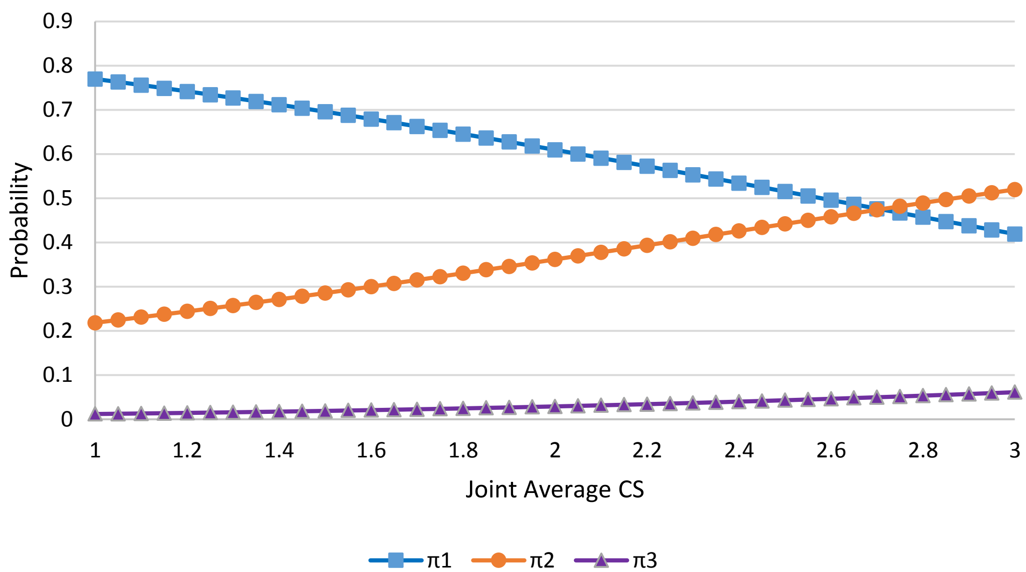

Figure 17- Multinomial logit model for bearings condition states

The probability for each condition state is given as π_1 for probability of CS1, π_2 for probability of CS2 and π_3 for probability of CS3. The logit model shows when all joint quantity is in CS1, there is an 80% probability of the bearings being in CS1, 20% probability of the bearings being in CS2 and negligible probability of being in CS3. However, as the deterioration of the joint progresses, the probability of bearings in CS1 decreases, while probabilities of CS2 and CS3 increase. For the fitted data, however, the probability of quantity in CS3 remains under ten percent for the range of joint condition, indicating quantity in CS3 is a rare occurrence.

A multinomial logit model using base-line category logits was also developed for pier cap elements based on joint average condition state. This is shown in Figure 18 and shows similar results as for the bearing model. The slope and intercept (α and β) parameters developed from the logistic regression equations are given in Appendix C.

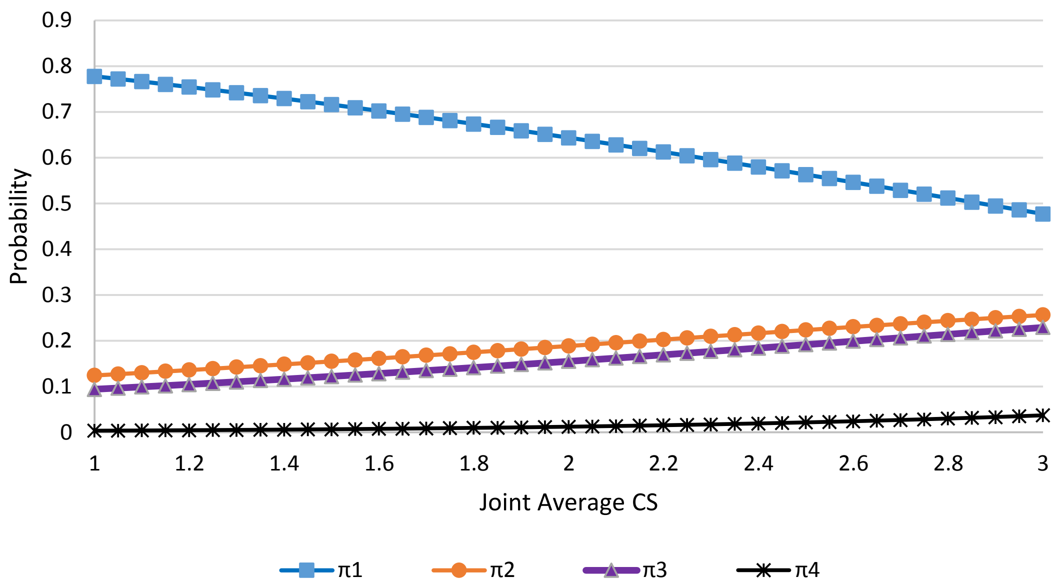

Figure 18- Multinomial logit model for pier cap condition states

Using the probabilities developed by the logit models, a predicted condition state profile for bearings or pier caps for each inspection report was calculated by multiplying the condition state probability by the total quantity of bearings or pier caps for a certain inspection report. The expected distribution of quantity based on the model was calculated for each inspection report and compared to the observed data. The observed data and the predicted quantities using the logit model probabilities are shown below.

Table 7- Observed quantity distribution for bearings

Bearing Condition State	Joint AVE Condition			Row Sum
	Good	Fair	Poor	
CS1	7429	2174	1184	10787
CS2	2064	1355	1199	4618
CS3	85	26	137	248
Column Sum	9578	3555	2520	15653

Table 8- Predicted quantity distribution for bearings from multinomial model

Bearing Condition State	Joint AVE Condition			Row Sum
	Good	Fair	Poor	
CS1	7217	2152	1112	10481
CS2	2231	1297	1263	4791
CS3	131	106	144	382
Column Sum	9578	3555	2520	15653

Table 9- Observed quantity distribution for pier caps

Pier Caps Condition State	Joint AVE Condition			Row Sum
	Good	Fair	Poor	
CS1	8757.2	2771.7	1816.5	13345.4
CS2	397.1	166.1	482.0	1045.2
CS3	373.5	366.2	324.9	1064.7
CS4	0	391.5	74.4	465.9
Column Sum	9527.8	3695.5	2697.8	15921.2

Table 10- Predicted quantity distribution for pier caps from multinomial model

Pier Cap Condition State	Joint AVE Condition			Row Sum
	Good	Fair	Poor	
CS1	7298.0	2371.9	1325.6	10995.4
CS2	1243.0	701.0	677.5	2621.6
CS3	946.6	576.3	601.8	2124.8
CS4	40.2	46.3	93.0	179.5
Column Sum	9527.8	3695.5	2697.8	15921.2

To test the fit of the models to the actual data, the Chi-Squared Goodness-of-Fit test was employed. The test is similar to the Test for Independence utilized earlier, except the expected values are determined from the logit model and not calculated based on independence. The Chi-Squared values for the bearing model and pier cap models were 106.7 and 4749, respectively. As before, small Chi-Squared values indicate an accurate model while large Chi-Squared values indicate poor fit.

To attempt to develop better fit, the models were refined by using ordinal logit models that accounted for the inherent hierarchal ordering of the categories or condition states of the data. This method was based on the cumulative logit model assuming proportional odds. Appendix B also provides a brief introduction to ordinal logit models. The proportional odds model accounts for the order of the categories by relating the probability of each category to the cumulative probability of each category. “This results in models that have simpler interpretations and potentially greater power than baseline-category logit models” (Agresti 2007). To test the proportional odds assumption, a Likelihood Ratio Test was performed to compare the hypothesis that the slope coefficient is the same for each category against the alternative hypothesis that the coefficients are different. R code was developed referencing Bilder and Loughin (2015) to develop the hypothesis test. The test produced a p-value of 0.685 and 1 for the bearing model and the pier cap model, respectively. This indicates that the null hypothesis should not be rejected, supporting the use of the proportional odds assumption. Despite this, it should be noted that this does not confirm the proportional odds assumption is true, but “it does offer some assurance that a proportional odds model provides a reasonable approximation…” (Bilder and Loughin 2015). The probabilities were obtained and the expected quantity for each condition state was calculated as before. The ordinal models are shown in Figures 19 and 20 with the predicted quantity distribution shown in Tables 11 and 12. The ordinal models were developed using R and referencing Bilder and Loughin (2015).

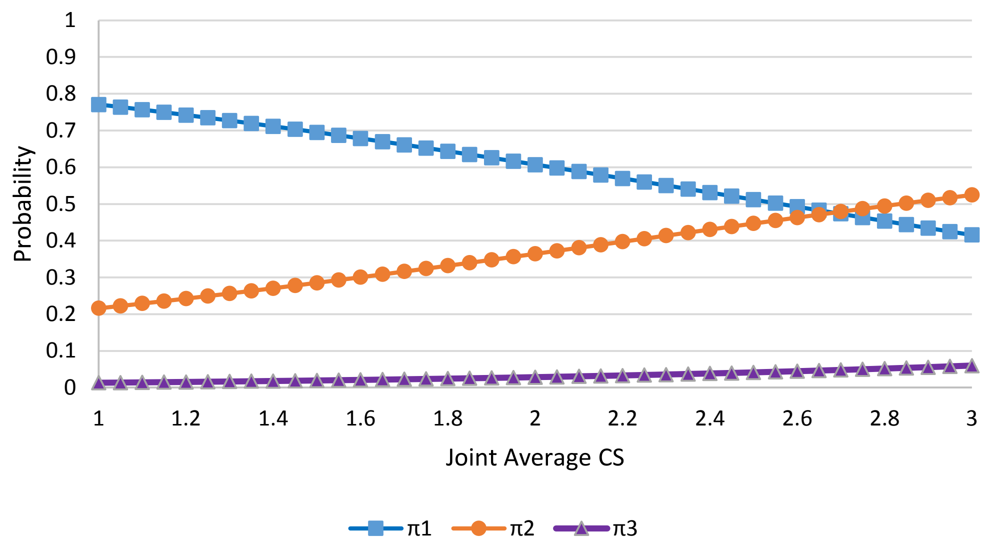

Figure 19- Ordinal logit model for bearings

Table 11- Predicted quantity distribution for bearings from ordinal model

Bearing Condition State	Joint AVE Condition			Row Sum
	Good	Fair	Poor	
CS1	7226	2145	1104	10475
CS2	2213	1308	1276	4797
CS3	139	103	140	381
Column Sum	9578	3555	2520	15653

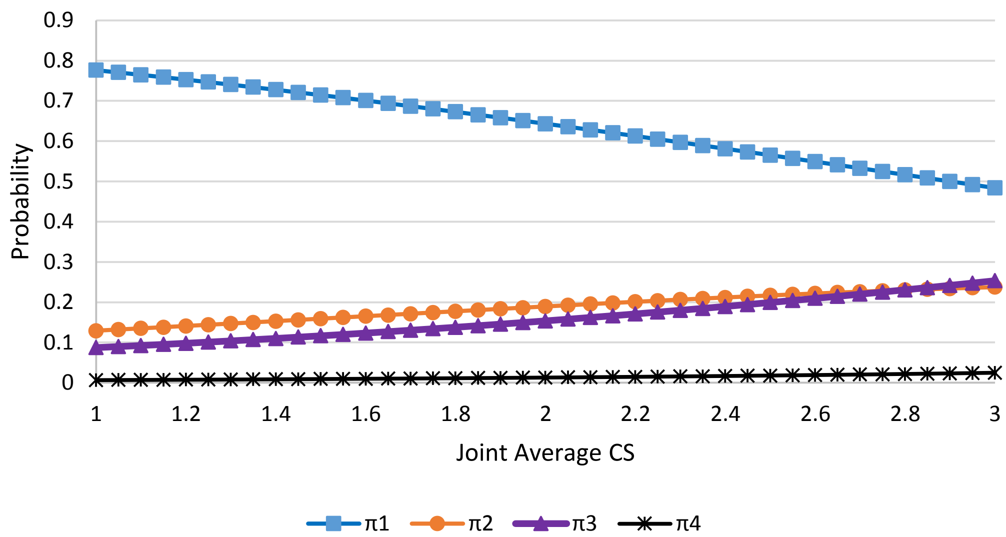

Figure 20- Ordinal logit model for pier caps

Table 12- Predicted quantity distribution for pier caps from ordinal model

Pier Cap Condition State	Joint AVE Condition			Row Sum
	Good	Fair	Poor	
CS1	7286.3	2371.8	1342.3	11000.4
CS2	1285.7	702.2	631.5	2619.4
CS3	885.8	572.5	659.6	2117.9
CS4	70.1	49.0	64.4	183.5
Column Sum	9527.9	3695.6	2697.8	15921.2

The ordinal method provided slightly different models than the multinomial models. The Chi-Squared values for the cumulative logit models with proportional odds were 106.7 and 4596. For the bearing model, the Chi-Squared value was unchanged, however, the ordinal model reduced the Chi-Squared value compared to the baseline category logit model for the pier cap element. Despite this, the values remained very large.

4.4 Subordinate Deterioration using Markov Chains

A more powerful and widely accepted method for modeling deterioration is Markov Chains. As described, Markov Chains are used for deterioration modeling in most current bridge management applications and many different studies have investigated the use and calibration of Markov Chains; however, no method has attempted to include element interaction. To account for element interaction, a method to calculate the diagonal transition probabilities for the subordinate element (bearing and pier cap elements) was developed and calibrated based on the joint element condition.

The developed method calculates the diagonal transition probabilities, the probabilities of quantity remaining in a certain condition state, based on the percent of quantity of the joint element in each condition state multiplied by conditional probabilities given each condition state of the joint. The overall concept of conditional probabilities were used to relate the quantity of joint in each condition state to the rate of deterioration for the subordinate element. This is shown in by the following equation.

$$\begin{bmatrix} P_{11} \\ P_{22} \\ P_{33} \end{bmatrix} = \begin{bmatrix} CS_1 \cdot TP_{1_1} + CS_2 \cdot TP_{2_1} + CS_3 \cdot TP_{3_1} \\ CS_1 \cdot TP_{1_2} + CS_2 \cdot TP_{2_2} + CS_3 \cdot TP_{3_2} \\ CS_1 \cdot TP_{1_3} + CS_2 \cdot TP_{2_3} + CS_3 \cdot TP_{3_3} \end{bmatrix} \qquad \text{Eq. 9}$$

$$TP|_{joint=CS1} = \begin{pmatrix} TP_{1_1} \\ TP_{1_2} \\ TP_{1_3} \end{pmatrix} \qquad TP|_{joint=CS2} = \begin{pmatrix} TP_{2_1} \\ TP_{2_2} \\ TP_{2_3} \end{pmatrix} \qquad TP|_{joint=CS3} = \begin{pmatrix} TP_{3_1} \\ TP_{3_2} \\ TP_{3_3} \end{pmatrix}$$

Here, CS_i is the percentage of quantity in each condition state for the joint (primary element) and the column matrices, $TP|_i$, are the conditional probabilities for the subordinate element (bearings or pier caps) given the quantity of joint in each condition state. To develop the full transition probability matrix for a subordinate element, the transition probabilities of quantity moving to the next state was calculated by subtracting the diagonal (retaining) probability from one, as was earlier described in Equation 2.

This method was implemented using Microsoft Excel Visual Basic for Applications (VBA). VBA code was necessary to implement the matrix multiplication to account for different number of transition periods and is given in Appendix F. To arrive at the conditional probability matrices, the values were calibrated based on minimizing the squared error between calculated quantity distribution and the actual quantity distribution observed for each inspection report. The Chi-Squared Goodness-of-Fit test was also used to characterize the global fit. For this study, calibration was based on manual trial and error adjustment to improve fit. The calibration found that the conditional probabilities for bearing elements shown in Table 13 approached a minimum in the squared error as well as passed the Chi-Squared Test to a level of significance of 0.05.

Table 13- Bearing conditional probabilities from calibration

Bearing State	$TP\|_{joint=CS1}$	$TP\|_{joint=CS2}$	$TP\|_{joint=CS3}$
CS1	0.989	0.982	0.977
CS2	0.996	0.997	0.997
CS3	1	1	1

The bearing conditional probabilities created a minimum for the squared error of the calculated and actual condition state profiles. Using the Chi-Squared Goodness-of-Fit Test, the calculated Chi-Squared value was 5.64 and compared to the Chi-Squared distribution on two degrees of freedom of 5.991 at an $\alpha = 0.05$. This shows a statistically accepted level of fit.

The same technique was employed to calibrate the conditional probabilities for pier caps. The calibration yielded values presented in Table 14.

Table 14- Pier cap conditional probabilities from calibration

Pier Cap State	$TP\|_{joint=CS1}$	$TP\|_{joint=CS2}$	$TP\|_{joint=CS3}$
CS1	0.997	0.992	0.990
CS2	0.963	0.923	0.965
CS3	0.998	0.966	0.981
CS4	1	1	1

The pier cap conditional probabilities showed a convergence to a minimum of the squared error between calculated and observed condition state distribution for the inspection report data. Despite this, the Chi-Squared value calculated was 107.3 and was compared to the Chi-Squared distribution of 7.815 for three degrees of freedom at $\alpha = 0.05$.

To illustrate the developed method, the data for bearings were filtered and only inspection data for bridges with all joint quantity in CS1 remain. In this set of bridges, the conditional transition probabilities $TP|_{joint=CS1}$ are multiplied by one and become the diagonal, retaining transition probabilities. Using these transition probabilities, the Markov deterioration model can be developed. This is shown in Figure 21.

Figure 21- Subordinate Markov deterioration model for fixed bearings with joints in CS1

Similarly, the subordinate deterioration model for bearings when joints are in CS3 were developed. In this case the conditional probabilities $TP|_{joint=CS3}$ in Table 13 become the overall transition probabilities for the model. This is shown in Figure 22.

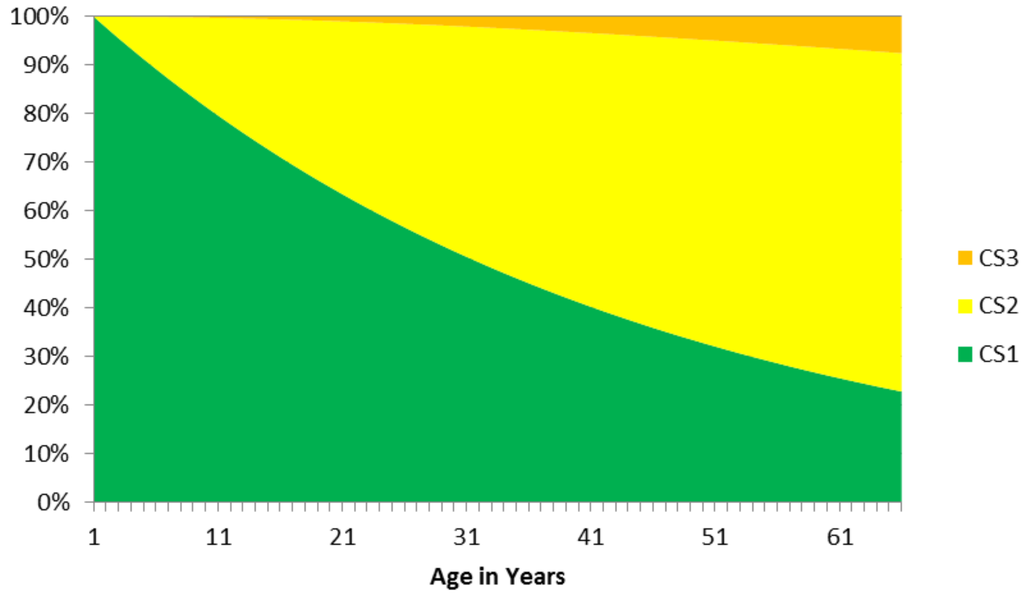

Figure 22- Subordinate Markov deterioration model for fixed bearings with joints in CS3

These deterioration models show that when all the joint quantity is assigned to CS1, the deterioration of the bearings is less than when all the joint quantity is assigned to CS3. After a predicted life of 65 years, 50 percent of the bearings with joints are predicted to remain in CS1 while for the same period, if the joints are in CS3, only 25 percent of the bearings are predicted to remain in CS1. This would be expected given the previous discussion of results showing the statistical significance of association between joints and subordinate elements. These models accounting for subordinate deterioration are more dynamic and were compared to the static CoRe element deterioration model that does not incorporate subordinate deterioration developed in a previous study (Section 2.6). The CoRe deterioration model for the fixed bearing element is shown in Figure 23.

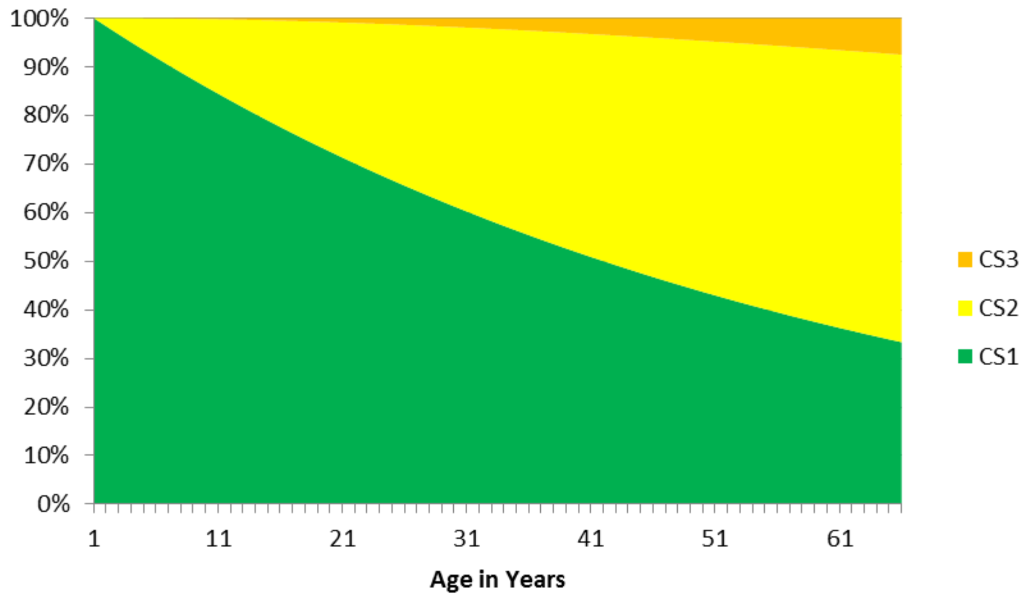

Figure 23- CoRe deterioration model for fixed bearings

When comparing the CoRe model to the subordinate deterioration models when joint elements are in CS1 and CS3, it is shown that if joints are in good condition, the subordinate deterioration model shows less deterioration than the CoRe model, however, when the joints are in poor condition, the subordinate deterioration model shows greater deterioration than the CoRe model. This is expected since the CoRe model does not incorporate subordinate deterioration but includes all cases of bearing condition regardless of the condition or presence of a joint element. The transition probability matrices for the CoRe element and subordinate deterioration models are given in Appendix D.

Similar results were obtained for the pier cap element data. When all joint quantity is coded in CS1, the deterioration of the pier cap is shown in Figure 24.

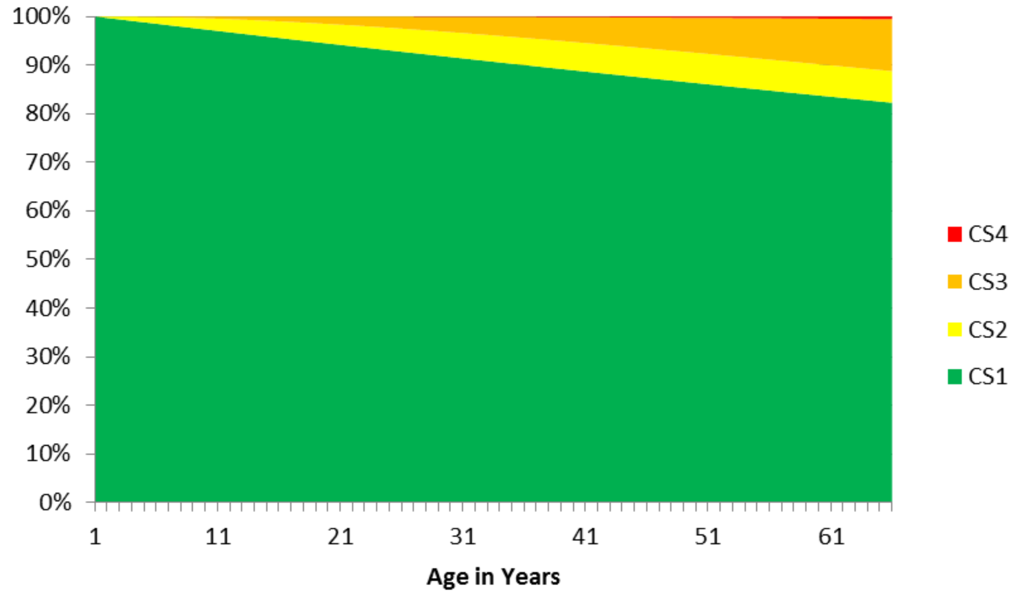

Figure 24- Subordinate Markov deterioration model for pier cap with joint in CS1

As deterioration of the joint progresses, the deterioration of pier cap also progresses. The subordinate deterioration model when the joint is coded as CS3 is shown in Figure 25.

Figure 25- Subordinate Markov deterioration model for pier cap with joint in CS3

Again, the developed deterioration models capture the influence of joint deterioration on the deterioration of the subordinate element and can be compared to the CoRe model for Reinforced Concrete Pier Cap when subordinate deterioration is not considered. The CoRe Model is shown in Figure 26.

Figure 26- CoRe deterioration model for reinforced concrete pier cap

The comparison between the CoRe model and the subordinate deterioration models shows that the CoRe model exhibits a deterioration rate greater than the subordinate deterioration model when joints are in CS1, but significantly less deterioration than the subordinate deterioration model when joints are in CS3. Again, the subordinate deterioration models provide a dynamic deterioration model based on the effect of joint deterioration as compared to the static CoRe model currently used in BMS.

5.0 Conclusions

This research provides an in-depth investigation of the element deterioration interaction of two bridge elements using data analysis of element-level inspection data. The main contributions of this study to bridge management include:

- A general framework to investigate element interaction
- A statistical investigation of two subordinate interactions using categorical data techniques
- An investigation of two modeling methods to incorporate subordinate interaction into current deterioration modeling practices

The investigation began with statistical comparison of elements with interaction to those not subjected to interaction. Predictive models using logistic regression were investigated as a method to determine subordinate condition state distribution based on the primary element. Finally, a new method was proposed to include element interaction in Markov Chain deterioration modeling using calibrated conditional probabilities. Statistical tests and methods were employed at each stage of the study.

5.1 Exploratory Data Analysis

The initial exploratory analysis of data included statistical methods and visualization techniques to characterize the relation of subordinate element condition to joint elements. The investigation showed for bridges without joints or for well-performing joints, bearing elements have a lower mean condition state, thus, are in better condition, whereas bearings associated with joints exhibiting deterioration show a higher mean condition, indicating worse condition. A similar trend is shown for pier cap elements. This suggests there is significant impact due to element interaction.

Through the use of statistical methods developed specifically for analysis of categorical data, the associations between joints and subordinate elements were presented. The use of categorical data analysis methods further explained the findings of the initial exploratory analysis. The ternary plot for bearings and the percent stack chart for pier caps supported the hypothesis that joints affect subordinate element condition. As shown, for the majority of inspections, subordinate elements are in CS1 with smaller percentages in CS2 and even less in CS3 (for bearings) and CS4 (for pier caps). However, in both data samples for subordinate elements, when joints are present, significantly higher deterioration with greater quantity in CS3 and CS4 occurs. This is further supported by the ternary plots showing the relation between joint classifications and element condition states. The subordinate elements rated as CS1 are located closer to the vertex of joints being in good condition, and subordinate elements in poorer condition states tend toward condition of joints in poorer condition.

The Chi-Squared Test for Independence provided a test of statistical significance that confirms the difference in condition when joints are present and when joints are not present. Association plots were developed to characterize the relation between the expected and observed quantities for each subordinate element. Correspondence analysis provided a further investigation of the Chi-Squared statistic. Visual maps were developed and correlation coefficients were calculated showing significant dependency of the subordinate element condition to the presence and condition of joint elements.

5.2 Multi-categorical Logistic Regression Modeling

The overall findings of the previous sections support the hypothesis of element interaction and subordinate deterioration. Because of this, modeling techniques to characterize and predict subordinate element condition were investigated. Multi-category logistic regression models were developed to model the probability of quantity being assigned to a certain condition state based on joint average condition state as the explanatory variable. The global goodness-of-fit tests showed that the models do not adequately represent the actual data for both bearing and pier cap elements when only joint average condition is used as the explanatory variable.

5.3 Subordinate Deterioration using Markov Chains

The current and most popular method of modeling deterioration of bridge elements uses Markov Chains. This research incorporated the effects of subordinate deterioration into Markov Chains by developing a method to determine the transition probabilities for the subordinate element based on the condition states of the primary element. The calibration of the conditional probabilities for the subordinate elements resulted in a convergence to a minimum of the squared error between the observed and modeled data for both steel fixed bearings and reinforced concrete pier caps. The global fit was tested using the Chi-Squared Goodness-of-Fit Test and showed the conditional probabilities for the bearing subordinate deterioration passed at a five percent level of significance. The conditional probabilities for pier cap elements did not pass the Chi-Squared Test. However, the presented method shows useful results to incorporate subordinate deterioration in Markov deterioration modeling and further optimization and calibration methods can be applied in future research.

5.4 Discussion of Age

Age is a factor when considering deterioration and future condition prediction. Older bridges are typically expected to exhibit greater deterioration than younger bridges. In the initial analysis describing the difference of condition between subordinate elements with joints and elements without joints, the age of each population was investigated. The average age of the bridges in the bearing sample with joints was 25.9 years with a standard deviation of 14.7 years while the age of bridges in the bearing sample without joints was 18.7 years with a standard deviation of 10.8 years. Similarly for the pier cap samples, the average age for the sample with joints was 28 years and the sample without joints was 18 years. A non-pooled t-test showed that the average age of the two populations are different at a five percent level of significance for each subordinate element. Age was not included in the investigation because the goal of this research was to investigate the current status of the bridges with element interaction. Also, many other variables affect deterioration including atmospheric conditions, location, industrial region, as well as protective systems such as paint and maintenance. Also, element-level inspection reports began in 1994, thus any maintenance or repair action prior to this date was not captured in the data. Because of this, older bridges may exhibit better than expected condition for their age or even better condition compared to newer bridges.

This research does not attempt to "uncouple" age from the deterioration process but instead attempts to relate condition of the subordinate element to a primary element, given the historic inspection data. In the development of the logit models, the use of the joint average condition as the explanatory variable provides an association to age through information of the joint condition as it is expected that joints in good condition represent younger age while joints in poor condition are characteristic of older bridge. Again however, other variables significantly affect joint deterioration such as weather, application of salt and deicing chemicals, level of expansion and contraction as well as other effects.

A brief exploratory investigation used age as the explanatory variable in the logistic regression model to predict the distribution of condition states of the bearings. The method used the same procedures described earlier. The results found that age does not accurately predict the distribution of condition state for bearing elements. A second regression model was developed and incorporated age and joint average condition as explanatory variables in a baseline multi-category logistic regression model. Despite the additional variable, incorporating age into the logistic regression model still did not provide a better fitting model when compared to the logit models with only joint average condition as the explanatory variable.

For the application of the Markov Models, age is incorporated by the number of transition periods. In bridge deterioration modeling, the transition period is taken as one year. The calibration of the transition probabilities for the model are based on minimizing the squared error between the model and observed quantity distribution in each condition state for each inspection report which occurs at a specific age of the bridge at the time of the inspection. Because of this, age is already incorporated as a factor in the developed models.

5.5 Future Research

An area of future research is the refinement of the logistic regression models. The multi-category logistic regression models developed in this study using a single independent variable did not provide satisfactory global fit to the data. This may be due to lack of data; however, additional variables, beyond the average joint condition state, may have an impact on the deterioration that is not captured by the current models. The inclusion of a combination of additional explanatory variables, such as VDOT district, age, average daily traffic, etc. may create better fitting models. A second possible method of creating better fit for the logit models is to include random effects. In the data, bridges are inspected multiple times in their lifespan and are grouped based on district, possibly causing correlation between responses. Random effect models include parameters to account for “clustered” data or repeated measurements (Bilder and Loughin 2015). However, with each refinement, the complexity of the models increase.

The method to account for subordinate deterioration using Markov Chains minimized the squared error between the model and observed data. However, the conditional probabilities for the pier cap model did not provide adequate fit based on the Chi-Squared Goodness-of-Fit Test. This may be caused by obtaining conditional probability values for a local minimum of the squared error rather than a global minimum. More rigorous optimization techniques are available and can be utilized to increase the fit to the data (Wellalage et.al. 2015). However, the presented method shows useful results to incorporate subordinate deterioration in Markov deterioration modeling and further optimization and calibration methods can be applied in future research.

5.6 Future Application

By investigating joint to bearing and joint to pier cap interactions, this research provided a framework that can be applied to other elements. The methods and procedures described in this study are general and can be extended to additional interactions that include:

- Influence of joints on the deterioration of abutments
- Influence of joints on the deterioration of columns
- Influence of joints on the deterioration of pier walls

Also, to improve inspection procedures and bridge management practices, AASHTO has updated element-level inspection by modifying inspection standards, consolidating inspection elements and redefining element condition states (Reardon and Chase 2015). The newly developed AASHTOWare BrM software will incorporated these changes and will supersede the Pontis system. With these changes, element-level data inspection and collection will remain similar, except the number of condition states will be standardized to four (CS1-4) and inspection elements will be consolidated to provide greater detailed data. Unlike the Pontis CoRe definitions, the new inspection definitions are created for each element and describe all possible deterioration paths related to the element (AASHTO 2015). In addition to multipath deterioration, new inspection elements have been created that separate protective elements and underlying elements. An example of this is Element 107 - Steel Girder and Element 515 - Steel Protective Coating, where Element 107 describes the underlying steel and Element 515 describes the paint or coating (AASHTO 2015). This provides an additional opportunity for the application of element interaction. This is an important development because it introduces significant concern for modeling with regards to the deterioration of the underlying subordinate element and the protective coating element. Two significant subordinate interactions that can be investigated with this method include:

- Influence of protective coatings on the deterioration of steel elements
- Influence of wearing surfaces on the deterioration of deck/slab elements

The AASHTOWare BrM software will continue the use of Markov Chains but will incorporate the Weibull Distribution to determine the transition of an element from CS1 to CS2, after which the Markov Chains will be used (Johnson 2013). This modification addresses calls for more accurate models to describe the initiation phase of deterioration (O'Conner et al. 2013). The Weibull Distribution differs from Markov Chains because it is a time-based model and not a state-based model. The Weibull Distribution relies on the duration an element remains in the condition state to model the transition to the next state (Agrawal et al. 2010). This improves modeling of the first transition and is significant since the majority of element quantity usually resides in CS 1 (AASTHO 2012). Despite these advances, AASHTO currently provides no guidance to account for element interaction and is still ignored in deterioration modeling.

5.7 Recommendations

Subordinate Deterioration was shown to have a significant effect on the deterioration of bridge elements. Because of this and the future application relating to the development of the AASHTOWare BrM system, it is recommended VDOT begin implementation of a method to account for element interaction in bridge management practices. As shown, this can increase the efficiency and quality of maintenance and repair decision-making.

Also as a result of this research, data errors and inconsistencies were shown to be abundant in the Pontis database. Typical issues found by this research included double-counted quantities, inconsistent units and absence of data. This suggested that the data quality controls currently implemented by the software are inadequate. Data quality may be increased by providing addition control measures when data is inputted into the systems. A possible method to address input errors would be to provide a check that compares the data inputted by an inspector to data already in the system. If the new data are outside a given range of error from the previous data, the inspector would be prompted to override the error by providing reasoning for the discrepancy or a description of any maintenance or repair action that occurred to cause the change of quantity. This can lead to the development of a unified method to track and document maintenance and repair actions to improve data quality and help inspectors provide more accurate data. Currently, no method to record maintenance actions such as joint eliminations, repainting, or other repairs exists with the current inspection database. A method to unify inspection data with documented MR&R actions can further increase the quality and effectiveness of data for future data analysis, which will lead to enhanced modeling and condition prediction.

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Appendix A- Select Pontis CoRe Elements and Definitions

Adopted from VDOT *Element Data Collection Manual*, 2007

Joint Elements

Element 300	Strip Seal Expansion Joint
Element 301	Pourable Joint Seal
Element 302	Compression Joint Seal
Element 303	Assembly Joint/Seal
Element 304	Open Expansion Joint

Joint Condition States (quantity: linear feet)

CS1	The element shows minimal deterioration The seal shows no leakage at any point along the length Gland is secure and has no defects Debris in joint is not causing any problems The adjacent deck and/or header are sound The armored joint anchorage shows no signs of looseness Fingers are not broken or misaligned Welds exhibit no problems The coating system is functioning as intended
CS2	The element shows moderate deterioration and/or minor cohesion failures The seal shows signs of leakage along the joint Gland shows signs of abrasion or minor tearing or is partially pulled out of extrusion Significant debris is in all or part of the joint and is affecting joint performance The adjacent deck and/or header exhibit no spalls The armored joint anchorage is loose Fingers are bent or misaligned Welds exhibit minor cracking The coating system is beginning to fail with the element beginning to corrode
CS3	The element has failed The seal shows signs of leakage along the length Gland has failed from abrasion or tearing or has pulled out of extrusion The adjacent deck and/or header exhibits spalls The armored joint anchorage has failed Fingers are missing or broken Welds are failing The coating system has failed with the element exhibiting advanced corrosion

Bearing Elements

Element 310	Elastomeric Bearing
Element 311	Movable Bearing
Element 312	Enclosed/Concealed Bearing or System
Element 313	Fixed Bearing
Element 314	Pot Bearing
Element 315	Disk Bearing

Bearing Condition States (quantity: each)

CS1	The element shows little or no deterioration and has minimal debris and corrosion The coating system is sound and functioning as intended Vertical and horizontal alignments are within limits Bearing support member is sound. There is no cracking of support members Any lubrication system is functioning properly The supported member is stable under traffic
CS2	The coating system has failed and exposed metal may show moderated to heavy corrosion with some pitting but still functions as intended The assemblies have moved causing minor cracking in the supporting concrete Debris buildup is affecting bearing movement Bearing alignment and/or load carrying capacity is still tolerable
CS3	Section loss sufficient to warrant supplemental supports or load restrictions Bearing alignment may be beyond tolerable limits Shear keys and the lubrication system have failed

Pier Cap Elements

Element 230	Steel Pier Cap- Uncoated
Element 231	Steel Pier Cap- Coated
Element 233	Prestressed Concrete Pier Cap
Element 234	Reinforced Concrete Pier Cap
Element 235	Timber Pier Cap

Reinforced Concrete Pier Cap Condition States (quantity: linear feet)

CS1	Little or no deterioration There may be discoloration, efflorescence and/or superficial cracking without effect on strength and/or ability to function as intended
CS2	Minor deterioration Minor cracks, and spalls may be present but there is no exposed reinforcing or surface evidence of rebar corrosion
CS3	Moderate deterioration Some delaminations and/or spalls may be present and some reinforcing may be exposed Corrosion of rebar may be present but loss of section is incidental and does not warrant structural analysis
CS4	Advanced deterioration Corrosion of reinforcement and/or loss of concrete section are sufficient to warrant structural analysis

Appendix B- Introduction to Multi-category Logistic Regression

Many are familiar with binary logistic regression but fewer are familiar with the extension of logistic regression to multiple categories. An excellent reference for an introduction to categorical data and multi-category logistic regression is *An Introduction to Categorical Data Analysis* (Agresti 2007). A brief review based on this work is provided along with reference to other works to provide an introduction to multiple category data analysis and logistic regression.

Logistic regression is a statistical method for determining the probability of a dependent variable through the relationship with independent variables. The most common form is binary logistic regression, where two possible categories exist for the dependent variable, usually described as success or failure. However, extensions of the binary logistic regression can be extended to model a dependent variable with more than two possible categories based on the multinomial distribution. Logistic regression is based on the logit link function as given below (Friendly 2000).

$$Logit[\pi(x)] = log\left(\frac{\pi(x)}{1-\pi(x)}\right) = \alpha + \beta x \qquad \text{Eq. D-1}$$

Here, $\pi(x)$ is the probability of an independent variable, given an explanatory variable x and can be simplified to an intercept parameter (α) and slope parameter (β). The term in the parentheses is referred to as the odds of success and the logit function may be referred to as the log odds function (Friendly 2000). Eq. D-1 can be rearranged to give the probability (Friendly 2000):

$$\pi(x) = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}$$

Eq. D-2

To apply logistic regression to multiple categories, the types of categorical data must be acknowledged. Categorical data can be classified as nominal when the categories have no specific order or ordinal when each category does have a specific ordered. Based on this, two types of multi-categorical logistic regression models exist: multinomial logistic regression and ordinal logistic regression.

The most common multinomial logit model is the baseline-category logit. “Logit models for nominal response variables pair each category with a baseline category… the choice of the baseline category is arbitrary” (Agresti 2007). In effect, the model is developed based on multiple binary regressions between the selected baseline category and the remaining categories. Using the relationships of probability and the logit link function described, the probability of the j^{th} category of a total of h categories is defined as

$$\pi_j = \frac{e^{\alpha_j+\beta_j x}}{\sum e^{\alpha_h+\beta_h x}}$$

Eq. D-3

The sum of the probabilities of each category is one (100%). “Software for multicategory logit models fits all the equations simultaneously” (Agresti 2007).

For cases where the categories of data are in a specific order, the ordering can be used to enhance a logistic model. These models are ordinal logistic regression and rely on cumulative probability.

"Cumulative probability for Y is the probability that Y falls at or below a particular point" (Agresti 2007):

$$P(Y \leq j) = \sum_{i=1}^{j} \pi_i$$

Eq. D-4

The most common ordinal model also assumes proportional odds. The proportional odds model "assumes that the logit of these cumulative probabilities changes linearly as the explanatory variable changes, and also that the slope of this relationship is the same regardless of the category j" (Bilder and Loughin 2015). This simplifies the model to have only one slope parameter (β). The probability of a single category is the cumulative probability minus the probability of the lower categories.

$$\pi_j = P(Y = j) = P(Y \leq j) - P(Y \leq j - 1) = \frac{e^{\alpha_j + \beta x}}{\sum e^{\alpha_h + \beta x}} - \frac{e^{\alpha_{j-1} + \beta x}}{\sum e^{\alpha_h + \beta x}}$$

Eq. D-5

Other cumulative multiordinal logit models that do not assume proportional odds exist, however, become increasingly complex. These models have additional parameters and include partial proportional odds models, adjacent-categories models and non-proportional odds models that consider separate effects (Agresti 2010, Bilder and Loughin 2015). These models were not explored for this study.

Appendix C- Multi-category Logistic Regression Model Parameters

This appendix provides the intercept (α) and slope (β) parameters for the four multi-category logistic regression models shown in Figures 17-20 developed using R software. The parameters are applied to Eq. D-3 for the baseline category model and Eq. D-5 for the ordinal model.

C.1 – Baseline Category Model

Parameter	Variable	Bearing Model	Pier Cap Model
α: intercept	CS1	0	0
	CS2	-1.99593	-2.43681
	CS3	-5.25927	-2.80317
	CS4	-	-6.76048
β: slope	CS1:Joint Ave CS	0	0
	CS2:Joint Ave CS	0.73726	0.605685
	CS3:Joint Ave CS	1.11406	0.690323
	CS4:Joint Ave CS	-	1.401229

C.2 – Ordinal Proportional Odds Model

Parameter		Bearing Model	Pier Cap Model
α: intercept	CS1|CS2	1.9878	1.9001
	CS2|CS3	5.0865	2.9165
	CS3|CS4	-	5.6284
β: slope	Joint Ave CS	-0.7762	-0.6548

Appendix D- Markov Deterioration Transition Probability Matrices

D.1 - Fixed Bearings Transition Probability Matrices

CoRe Element 313- Fixed Bearing

TP	Total	CS1	CS2	CS3
0.983	CS1	0.983	0.017	0.000
0.997	CS2	0.000	0.997	0.003
1	CS3	0.000	0.000	1.000

Subordinate Deterioration Model for Fixed Bearings: Joints in CS1

TP	Total	CS1	CS2	CS3
0.989	CS1	0.989	0.011	0.000
0.996	CS2	0.000	0.996	0.004
1	CS3	0.000	0.000	1.000

Subordinate Deterioration Model for Fixed Bearings: Joints in CS2

TP	Total	CS1	CS2	CS3
0.982	CS1	0.982	0.018	0.000
0.997	CS2	0.000	0.997	0.003
1	CS3	0.000	0.000	1.000

Subordinate Deterioration Model for Fixed Bearings: Joints in CS3

TP	Total	CS1	CS2	CS3
0.978	CS1	0.978	0.022	0.000
0.997	CS2	0.000	0.997	0.003
1	CS3	0.000	0.000	1.000

D.2 - Reinforced Concrete Pier Caps Transition Probability Matrices

CoRe Element 234- Reinforced Concrete Pier Cap

TP	Total	CS1	CS2	CS3	CS4
0.994	CS1	0.994	0.006	0.000	0.000
0.969	CS2	0.000	0.969	0.031	0.000
0.998	CS3	0.000	0.000	0.998	0.002
1	CS4	0.000	0.000	0.000	1.000

Subordinate Deterioration Model for Fixed Bearings: Joints in CS1

TP	Total	CS1	CS2	CS3	CS4
0.999	CS1	0.999	0.001	0.000	0.000
0.959	CS2	0.000	0.959	0.041	0.000
0.999	CS3	0.000	0.000	0.999	0.001
1	CS4	0.000	0.000	0.000	1.000

Subordinate Deterioration Model for Fixed Bearings: Joints in CS2

TP	Total	CS1	CS2	CS3	CS4
0.994	CS1	0.994	0.006	0.000	0.000
0.919	CS2	0.000	0.919	0.080	0.000
0.966	CS3	0.000	0.000	0.966	0.034
1	CS4	0.000	0.000	0.000	1.000

Subordinate Deterioration Model for Fixed Bearings: Joints in CS3

TP	Total	CS1	CS2	CS3	CS4
0.989	CS1	0.989	0.011	0.000	0.000
0.965	CS2	0.000	0.965	0.035	0.000
0.957	CS3	0.000	0.000	0.957	0.043
1	CS4	0.000	0.000	0.000	1.000

Appendix E- Multi-category Logistic Regression R Code

This appendix provides the R code used to develop the multi-category logistic regression models.

E.1 Bearing Code

```
######Multicategory Logit Models for Bearing Data########

###Multinomial Logit Model###
#reference: Katchova (2013)
library(package=mlogit)
A<-read.csv("Logit_data.csv")
attach(A)
table(state)
mltable<-mlogit.data(A,choice="state",shape="wide")

mlogit.model.bearing<-mlogit(state~0|Joint.Ave.CS,data=mltable,reflevel="CS1")
summary(mlogit.model.bearing)

###ORDINAL MODEL FOR BEARING DATA
##Cumulative Logit Model with Proportional Odds
#reference: Bilder and Loughin (2015)

A<-read.csv("Logit_data.csv")
levels(A$state)
A$state.order <- factor(A$state, levels = c("CS1","CS2", "CS3"))
levels(A$state.order)
library(package=MASS)

mod.fit.ord <- polr(formula = state.order ~ Joint.Ave.CS, data = A, method =
"logistic")
summary(mod.fit.ord)

##NOTE##
#Signs from output must be reverse to state estimated model for the beta coefficients

##TEST PROPORTIONAL ODDS ASSUMPTION
#Use VGAM package to remake proportional odds model
library(package=VGAM)
prop.odd.model<-vglm(formula=state.order~Joint.Ave.CS,
family=cumulative(parallel=TRUE),data=A)
summary(prop.odd.model)

##Nonproportional Odds Model
nonprop.odd.model<-vglm(formula=state.order~Joint.Ave.CS,
family=cumulative(parallel=FALSE),data=A)
summary(nonprop.odd.model)

#Likelihood Ratio Test
LRT<-deviance(prop.odd.model)-deviance(nonprop.odd.model)
df<-prop.odd.model@df.residual-nonprop.odd.model@df.residual
p.value<-1-pchisq(q=LRT,df=df)
data.frame(LRT,df,p.value)
```

E.2 Pier Cap Code

```
#####Multicategory logit models for Pier Cap data#######
#Predict probabilities of CS using count data

###Multinomial mlogit
library(mlogit)
piercap<-read.csv("Pier_Cap.csv")
attach(piercap)
table(State)
mtable<-mlogit.data(piercap,choice="State",shape="wide")

mlogit.model.piercap<-mlogit(State~0|J.Ave.CS,data=mtable,reflevel="CS1")
summary(mlogit.model.piercap)

###Ordinal Logit model for pier cap data
##Cumulative Logit Model with Proportional Odds
ord<-read.csv("Pier_Cap.csv")
attach(ord)
levels(ord$State)
ord$State.order<-factor(ord$State,levels=c("CS1","CS2","CS3","CS4"))
levels(ord$State.order)
library(package=MASS)
mod.fit.ordinal<-polr(formula=State.order~J.Ave.CS,data=ord,method="logistic")
summary(mod.fit.ordinal)
#Note: sign must be reversed for slope (beta coefficient)

##TEST PROPORTIONAL ODDS ASSUMPTION
#Use VGAM package to remake proportional odds model
library(package=VGAM)
prop.odd.model.pc<-vglm(formula=State.order~J.Ave.CS,
family=cumulative(parallel=TRUE),data=ord)
summary(prop.odd.model.pc)

##Nonproportional Odds Model
nonprop.odd.model.pc<-vglm(formula=state.order~Joint.Ave.CS,
family=cumulative(parallel=FALSE),data=A)
summary(nonprop.odd.model.pc)

#Likelihood Ratio Test
LRT.pc<-deviance(prop.odd.model.pc)-deviance(nonprop.odd.model.pc)
df.pc<-prop.odd.model.pc@df.residual-nonprop.odd.model.pc@df.residual
p.value.pc<-1-pchisq(q=LRT.pc,df=df.pc)
data.frame(LRT.pc,df.pc,p.value.pc)
```

Appendix F- Subordinate Markov Chains VBA Code

This appendix provides the Microsoft Excel VBA code to implement the matrix multiplication for the transition periods of the Markov process to calibrate the conditional probabilities for the subordinate deterioration models.

F.1 Bearing VBA Code

```
Option Base 1
Sub TPM_Multiplication()

Dim rng1 As Range
Dim rng2 As Range
Dim rng3 As Range
Dim rng4 As Range
Dim rng5 As Range
Dim rng6 As Range
Dim rng7 As Range
Dim rng8 As Range
Dim rng9 As Range
'Dim rng10 As Range
'Dim rng11 As Range
'Dim rng12 As Range

Dim TPM(3, 3) 'Dimension Transition Probability Matrix
Dim SCS(1, 3) 'Subordinate Element initial Condition State row matrix (1 0 0)
Dim BCS() 'Bearing Condition State row matrix after n generations

Dim i As Integer    'index variable
Dim x As Integer    'index variable

Set rng1 = Range("Y:Y") 'Transition Probability 11
Set rng2 = Range("Z:Z") 'Transition Probability 12
Set rng3 = Range("AA:AA") 'Transition Probability 22
Set rng4 = Range("AB:AB") 'Transition Probability 23
Set rng5 = Range("AC:AC") 'Transition Probability 33 (terminal)

Set rng6 = Range("AH:AH")   'Expected Quantity for CS1
Set rng7 = Range("AI:AI")   'Expected Quantity for CS2
Set rng8 = Range("AJ:AJ")   'Expected Quantity for CS3

Set rng9 = Range("X:X") 'Age (number of generations)

'Set rng10 = Range("D:D")    'Primary Element (Joint) Percent CS1
'Set rng11 = Range("E:E")    'Primary Element (Joint) Percent CS2
'Set rng12 = Range("F:F")    'Primary Element (Joint) Percent CS3

'NumRows is number of rows of data/inspection reports
NumRows = Range("Y1", Range("Y1").End(xlDown)).Rows.Count
Start = 2   'initialize at 2 to skip col headings

For x = Start To NumRows
```

```
'TPM is the transition probability matrix developed based on the conditional
probabilities and joint condition matrix
TPM(1, 1) = rng1.Cells(x).Value 'TP11
TPM(1, 2) = rng2.Cells(x).Value 'TP12
TPM(1, 3) = 0                   'TP13
TPM(2, 1) = 0                   'TP21
TPM(2, 2) = rng3.Cells(x).Value 'TP22
TPM(2, 3) = rng4.Cells(x).Value 'TP23
TPM(3, 1) = 0                   'TP31
TPM(3, 2) = 0                   'TP32
TPM(3, 3) = rng5.Cells(x).Value 'TP33

'SCS is the INITIAL subordinate element (bearing) condition matrix
'SCS matrix is (1 0 0); begins in first CS and transitions by TPM
SCS(1, 1) = 1 'Bearing CS1
SCS(1, 2) = 0 'Bearing CS2
SCS(1, 3) = 0 'Bearing CS3

n = rng9.Cells(x).Value 'age/number of generations to raise TPM

    BCS() = Application.WorksheetFunction.MMult(SCS, TPM) 'BCS is Bearing Condition
State Matrix based on joint CS

i = 1 ' initialize the number of generations to 1

'If n is greater than 1 then:
'Do loop to loop through number of generations
'raises the TPM to the power of n
'Cf=Co*(TPM)^n
If n > 1 Then
    Do
        BCS() = Application.WorksheetFunction.MMult(BCS, TPM)
        i = i + 1
    Loop Until i = n
    'Output condition state profile
    rng6.Cells(x) = BCS(1) 'Expected Qty CS1
    rng7.Cells(x) = BCS(2) 'Expected Qty CS2
    rng8.Cells(x) = BCS(3) 'Expected Qty CS3
End If
'If n equals 1 then use BCS before Do Loop
'which is after one generation
If n = 1 Then
    'Output condition state profile
    rng6.Cells(x) = BCS(1) 'Expected Qty CS1
    rng7.Cells(x) = BCS(2) 'Expected Qty CS2
    rng8.Cells(x) = BCS(3) 'Expected Qty CS3
End If

Next

End Sub
```

F.2 Pier Cap VBA Code

```
Option Base 1
Sub TPM_Multiplication()

Dim rng1 As Range
Dim rng2 As Range
Dim rng3 As Range
Dim rng4 As Range
Dim rng5 As Range
Dim rng6 As Range
Dim rng7 As Range
Dim rng8 As Range
Dim rng9 As Range
Dim rng10 As Range
Dim rng11 As Range
Dim rng12 As Range

Dim TPM(4, 4) 'Dimension Transition Probability Matrix
Dim SCS(1, 4) 'Subordinate Element initial Condition State row matrix (1 0 0)
Dim PCS() 'Pier Cap Condition State row matrix after n generations

Dim i As Integer    'index variable
Dim x As Integer    'index variable

Set rng1 = Range("AD:AD") 'Transition Probability 11
Set rng2 = Range("AE:AE") 'Transition Probability 12
Set rng3 = Range("AF:AF") 'Transition Probability 22
Set rng4 = Range("AG:AG") 'Transition Probability 23
Set rng5 = Range("AH:AH") 'Transition Probability 33
Set rng6 = Range("AI:AI") 'Transition Probability 34
Set rng7 = Range("AJ:AJ") 'Transition Probability 44

Set rng8 = Range("AL:AL") 'Expected Quantity for CS1
Set rng9 = Range("AM:AM") 'Expected Quantity for CS2
Set rng10 = Range("AN:AN") 'Expected Quantity for CS3
Set rng11 = Range("AO:AO")  'Expected Quantity for CS4

Set rng12 = Range("AC:AC") 'Age (number of generations)

'NumRows is number of rows of data/inspection reports
NumRows = Range("AD1", Range("AD1").End(xlDown)).Rows.Count

Start = 2   'initialize at 2 to skip col headings

For x = Start To NumRows

'TPM is the transition probability matrix developed based on the conditional
probabilities and joint condition matrix
TPM(1, 1) = rng1.Cells(x).Value 'TP11
TPM(1, 2) = rng2.Cells(x).Value 'TP12
TPM(1, 3) = 0                   'TP13
TPM(1, 4) = 0                   'TP14
TPM(2, 1) = 0                   'TP21
TPM(2, 2) = rng3.Cells(x).Value 'TP22
```

```
TPM(2, 3) = rng4.Cells(x).Value 'TP23
TPM(2, 4) = 0                   'TP24
TPM(3, 1) = 0                   'TP31
TPM(3, 2) = 0                   'TP32
TPM(3, 3) = rng5.Cells(x).Value 'TP33
TPM(3, 4) = rng6.Cells(x).Value 'TP34
TPM(4, 1) = 0                   'TP41
TPM(4, 2) = 0                   'TP42
TPM(4, 3) = 0                   'TP43
TPM(4, 4) = rng7.Cells(x).Value 'TP44

'SCS is the INITIAL subordinate element (bearing) condition matrix
'SCS matrix is (1 0 0 0); begins in first CS and transitions by TPM
SCS(1, 1) = 1 'Bearing CS1
SCS(1, 2) = 0 'Bearing CS2
SCS(1, 3) = 0 'Bearing CS3
SCS(1, 4) = 0 'Bearing CS4

n = rng12.Cells(x).Value 'age/number of generations to raise TPM

    PCS() = Application.WorksheetFunction.MMult(SCS, TPM) 'PCS is Pier Cap Condition
State Matrix based on joint CS

i = 1 ' initialize the number of generations to 1

'If n is greater than 1 then:
'Do loop to loop through number of generations
'raises the TPM to the power of n
'Cf=Co*(TPM)^n
If n > 1 Then
    Do
        PCS() = Application.WorksheetFunction.MMult(PCS, TPM)
        i = i + 1
    Loop Until i = n
    'Output condition state profile
    rng8.Cells(x) = PCS(1) 'Expected Qty CS1
    rng9.Cells(x) = PCS(2) 'Expected Qty CS2
    rng10.Cells(x) = PCS(3) 'Expected Qty CS3
    rng11.Cells(x) = PCS(4) 'Expected Qty CS4
End If
'If n equals 1 then use BCS before Do Loop
'which is after one generation
If n = 1 Then
    'Output condition state profile
    rng8.Cells(x) = PCS(1) 'Expected Qty CS1
    rng9.Cells(x) = PCS(2) 'Expected Qty CS2
    rng10.Cells(x) = PCS(3) 'Expected Qty CS3
    rng11.Cells(x) = PCS(4) 'Expected Qty CS4
End If

Next

End Sub
```