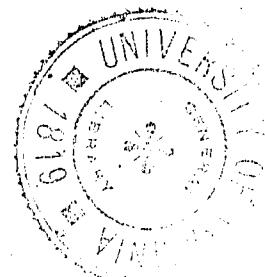


DEFINITIVE DETERMINATION OF COMET 1887 IV.

PRESENTED TO THE FACULTY OF THE UNIVERSITY OF VIRGINIA ON APPLYING FOR
THE DEGREE OF DOCTOR OF SCIENCE,

BY

FRANK MULLER,
OF VIRGINIA.



U. Va.
U. Va. Doctoral
Dissertation

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Nos. 174-5.

DEFINITIVE DETERMINATION OF THE ORBIT OF COMET 1887 IV,

BY FRANK MULLER.

This comet was discovered by Mr. E. E. BARNARD at the observatory of Vanderbilt University, Nashville, Tenn., on May 12. It was then 0'.5 in diameter, with a stellar nucleus of the 11-12 magnitude; about perihelion it was of the 9-10 magnitude, with a tail 2' long; soon after it became diffuse and elongated in the direction north and south, and was last seen by Mr. BARNARD on August 11, when its theoretical relative brightness was 0.3.

1. *Preliminary elements, perturbations, and ephemeris.*— Several sets of elements were published, not resting however upon sufficient data to indicate any deviation from a parabolic path. Finally, Mr. CHANDLER (*Astronomical Journal*, No. 160) noted that the observations could not be satisfied by parabolic elements. From normal places for May 14, June 12, and July 12, he computed the following elements, which represent the observations closely, and are practically definitive:

$$\begin{aligned} T &= \text{June } 16.66108 \text{ Greenwich M.T.} \\ \omega &= 15^\circ 8' 3".7 \\ \Omega &= 245.13.16.8 \\ i &= 17^\circ 32' 53.4'' \\ \log q &= 0.1441634 \\ \log a &= 2.5009248 \\ e &= 0.9956014 \end{aligned}$$

Upon these elements I have based the ephemeris for the preparation of the normal places. That the differences between the ephemeris and observation might contain only the errors of observation, the perturbations were computed and applied to the ephemeris. All the planets from *Mercury* to *Saturn* were considered; the disturbances were small, and were chiefly caused by *Jupiter*, and by the *Earth*, with which the comet was nearly in conjunction during the whole apparition. The perturbations of the rectangular ecliptic coordinates were computed for every eight days, and the corresponding changes in right-ascension and declination were as follows:

1887 Gr. M.T.	$\Delta\alpha$	$\Delta\gamma$	$\Delta\zeta$	$\Delta\alpha$	$\Delta\delta$
May 10.0	-242	-73	-5	-6.44	+3.49
18.0	167	49	6	5.17	1.98
26.0	-106	-31	-5	-3.72	+0.81

1887 Gr. M.T.	$\Delta\alpha$	$\Delta\gamma$	$\Delta\zeta$	$\Delta\alpha$	$\Delta\delta$
June 3.0	— 59	— 17	— 3	— 2.27	+ 0.14
11.0	26	8	2	1.06	— 0.08
19.0	7	2	1	0.27	0.05
27.0	0	0	0	0.00	0.00
July 5.0	6	2	1	0.25	0.07
13.0	25	9	2	0.94	0.27
21.0	57	20	5	1.96	0.56
29.0	99	36	9	3.17	0.89
Aug. 6.0	153	57	15	4.48	1.21
14.0	— 216	— 84	— 21	— 5.78	— 1.52

To avoid introducing any irregularity in the interpolation of the ephemeris, $\Delta\alpha$ and $\Delta\delta$ were separately obtained for every day. The masses and places of the disturbing planets were taken from the Berlin *Jahrbuch*.

The reduction to apparent place was computed, by means of the independent star-numbers of the *American Nautical Almanac*, for every four days. By interpolation this correction also was obtained for every day.

These corrections being applied, the following is the resulting ephemeris for Greenwich mean noon and midnight of each day:

1887	α	δ	$\log \Delta$
May 12.0	15 ^h 9 ^m 40.94	— 30° 56' 47.5"	9.6845
12.5	10 27.95	30 41 52.6	
13.0	11 15.34	30 26 40.8	9.6790
13.5	12 3.12	30 11 11.9	
14.0	12 51.30	29 55 26.1	9.6737
14.5	13 39.86	29 39 23.4	
15.0	14 28.82	29 23 3.7	9.6685
15.5	15 18.18	29 6 27.3	
16.0	16 7.94	28 49 34.2	9.6634
16.5	16 58.09	28 32 24.4	
17.0	17 48.64	28 14 58.0	9.6585
17.5	18 39.58	27 57 15.3	
18.0	19 30.93	27 39 16.4	9.6537
18.5	20 22.67	27 21 1.3	
19.0	21 14.82	27 2 30.3	9.6491
19.5	22 7.36	26 43 43.7	
20.0	23 0.30	26 24 41.5	9.6447
20.5	23 53.66	25 5 24.0	
21.0	24 47.41	25 45 51.5	9.6404
21.5	15 25 41.56	— 25 26 4.2	

	1887	α	δ	log Δ
July	25.0	17 ^h 43 ^m 35.24	+8° 41' 30.4"	9.7872
	25.5	44 35.56	8 43 42.8	
	26.0	45 35.76	8 45 45.4	9.7929
	26.5	46 35.84	8 47 38.7	
	27.0	47 35.81	8 49 22.7	9.7987
	27.5	48 35.66	8 50 57.7	
	28.0	49 35.40	8 52 23.8	9.8044
	28.5	50 35.02	8 53 41.2	
	29.0	51 34.52	8 54 50.0	9.8103
	29.5	52 33.92	8 55 50.6	
	30.0	53 33.19	8 56 43.0	9.8161
	30.5	54 32.36	8 57 27.4	
	31.0	55 31.41	8 58 4.0	9.8219
	31.5	56 30.35	8 58 33.0	
	Aug.	1.0	57 29.18	9.8277
		1.5	58 27.89	
		2.0	17 ^h 59 26.50	9.8335
		2.5	18 0 25.00	
		3.0	1 23.38	9.8393
		3.5	2 21.66	
		4.0	3 19.83	9.8451
		4.5	4 17.90	
		5.0	5 15.86	9.8509
		5.5	6 13.71	
		6.0	7 11.46	9.8567
		6.5	8 9.11	
		7.0	9 6.65	9.8625
		7.5	10 4.10	
		8.0	11 1.44	9.8684
		8.5	11 58.70	
		9.0	12 55.85	9.8742
		9.5	13 52.90	
		10.0	14 49.87	9.8800
		10.5	15 46.74	
		11.0	16 43.52	9.8858
		11.5	17 40.21	
		12.0	18 36.82	9.8916
			+8 40 26.0	

Comparison of part of this ephemeris with an ephemeris computed by Mr. CHANDLER showed small differences, which Mr. CHANDLER explained by stating that in the reduction to apparent place he had used the *British Nautical Almanac*, in which nutation-terms of short period are neglected.

2. *Comparison-Stars.*—The stars used lie between 30° south declination, and 8° north declination. The positions of all stars brighter than the ninth magnitude, between —2° and +5°, have been kindly furnished by Prof. KORTAZZI of Nicolajew, and Prof. Boss of Albany, from their as yet unpublished A.G. Zone observations. The positions of the stars north of +5° had not been reduced, but I am indebted to Prof. BRUNS of Leipsic, for the apparent places of these stars. With a few exceptions, all the positions from the A.G. zones are the means of two or more observations. I am

also indebted to Mr. SKINNER of the U.S. Naval Observatory for places from various catalogues.

South of —2°, the southern limit of the *Astronomische Gesellschaft* zones, I have collected nearly all the observations of the stars used; this appears desirable for the detection of proper motion, and the elimination of accidental errors. While a complete discussion is not warranted, some system is necessary in combining these miscellaneous observations. The weights given below were assigned, taking into consideration the accuracy of observation and the chance of the effect of a proper motion too small to be detected. These weights were used when the places depended on one or two observations; for three to six observations the weights were increased by one-half; for seven or more observations they were doubled, the systematic error of the catalogues being then probably in excess of the accidental errors.

For the sake of brevity, I have given only the adopted mean place and the authorities upon which it rests. To indicate the various catalogues the following abbreviations are used:

- A. = Oeltzen's Argelauder. Weight $\frac{1}{4}$.
- A.G. = *Astronomische Gesellschaft* zones. Abbreviations are attached to show the places of observations; Albany, Leipsic and Nicolajew. Weight $\frac{3}{4}$.
- A N. = Star-places in *Astronomische Nachrichten*.
- Ar. = Second Armagh Catalogue. Weight $\frac{1}{2}$.
- B = Bonn Observations, Vol. VI. Weight $\frac{1}{4}$.
- Br. = Brisbane, Observations at Paramatta. Weight $\frac{1}{8}$.
- C, CZ = Argentine General or Zone Catalogue. Weight 1.
- Cin. = Cincinnati Zone Catalogue. Weight $\frac{1}{2}$.
- Cp. = Catalogues, for the epochs 1840, '50, '60, '80, of stars observed at the Cape of Good Hope. 1840–60, weight $\frac{1}{2}$; 1880, weight 1.
- D. = Observations made at Dunsink, 1885. Weight 1.
- G. = Göttingen. Weight $\frac{1}{2}$.
- Gr. = Greenwich. Weight $\frac{1}{2}$.
- L. = Lamont. Weight $\frac{1}{4}$.
- P. = Pulkowa. Weight $\frac{1}{2}$.
- R. = Radcliffe, 1884–85–86. Weight 1.
- Si. = Santini. Weight $\frac{1}{4}$.
- Sj. = Schjellerup. Weight $\frac{3}{4}$.
- T. = Tacchini (Washburn Obs. publications). Weight $\frac{1}{4}$.
- W.Z. = Washington Zones. Weight $\frac{1}{8}$.
- Y. = Yarnall. Weight $\frac{1}{2}$.
- Comp. indicates that the places were obtained by a comparison with a known star.

The Washington zones were only used when no other authority was available.

MEAN PLACES FOR 1887.0 OF THE COMPARISON-STARS.

No.	α	δ	Authorities.	No.	α	δ	Authorities.
1	15 ^h 10 ^m 57.34	—29° 43' 55.8"	A, C, Cp, '40, '50, '60, '80, Y	4	15 12 30.07	—28° 33' 37.8"	A, C
2	11 3.17	30 3 38.6	A, CZ, Y	5	12 51.25	29 29 22.9	CZ
3	15 11 45.84	—30 47 40.8	A, C, Cp '40, '80	6	15 13 4.02	—29 41 39.4	A, CZ

No.	α	δ	Authorities	No.	α	δ	Authorities
123	16 58 57.16	+2 55 27.4	AG Alb	145 ¹	17 25 8.04	+6 51 3.0	B
124	17 4 30.38	5 2 48.3	AG Leip	146	26 36.76	7 36 0.6	AG Leip
125	5 7.35	5 31 19.2	"	147	28 4.41	7 15 29.3	"
126	7 2.80	5 2 43.4	"	148	28 27.14	8 11 5.0	"
127	7 18.84	6 11 1.4	"	149	34 29.64	7 51 50.0	"
128	8 11.42	5 56 32.6	"	150	35 12.11	8 32 10.7	"
129	8 47.07	5 35 2.0	Comp	151	36 25.76	8 16 31.4	"
130	10 55.07	5 15 15.5	AG Leip	152	37 32.46	8 6 49.8	$\frac{1}{2}$ (W+LL)
131	11 21.54	5 58 23.9	Comp	153	38 6.16	8 29 15.4	B
132	13 21.41	6 12 17.2	AG Leip	154	38 20.62	8 39 59.5	AG Leip
133	14 18.38	6 40 4.2	"	155	42 8.30	8 35 22.7	"
134	15 18.14	6 32 59.1	"	156	45 39.37	8 33 41.9	"
135	15 21.47	6.48 24.4	"	157	47 29.06	8 50 7.2	B
136	15 54.89	6 32 35.2	L, W	158	47 30.14	8 43 5.4	B
137	16 48.80	6 46 8.7	AG Leip	159	17 49 59.13	8 55 15.5	A.G. Leip
138	17 50.19	6 43 46.0	Comp	160	18 8 29.90	8 56 44.1	W
139	17 54.30	7 11 59.9	B	161	8 55.59	8 51 4.3	A.G. Leip
140	18 55.62	6 37 26.7	AG Leip	162	9 30.02	8 56 43.1	"
141	19 44.46	6 31 54.8	"	163	9 45.65	8 45 20.4	Sj, W
142	20 2.40	6 45 49.0	"	164	12 41.25	8 47 39.4	Comp
143	20 51.72	7 41 42.6	"	165	15 36.34	8 34 40.9	A.G. Leip
144	23 4.19	7 21 45.7	"	166	18 17 17.22	+8 43 45.4	"
145	17 24 8.62	+7 36 36.2	"				

REMARKS.

3. The Cape Catalogue of 1880 gives a proper motion of $+0^{\circ}.010$ deduced from comparison with the Catalogue of 1840, but the other observations do not sustain the determination.

7. A provisional proper motion of $-0''.08$ in δ was used.

18. Prof. Kortazzi used a proper motion for this star and the preceding one determined by comparison with Lacaille; Lacaille not being available I have used a provisional proper motion of $+0^{\circ}.010$ in α .

20. The S.P.D. of BRISBANE 5382 requires a correction of $-5''$.

28. The declination of YARNALL 6425 is incorrect. Prof. FRISBY has kindly given the correct declination for 1860, $-21^{\circ} 39' 9''$.

29. Lalande not being available, a provisional proper motion of $+0^{\circ}.005$ in α was used.

3. *Observations of the Comet.* — On collecting all the observations published, 313 were found available. The right-ascensions and declinations given below are the observed values corrected for parallax and for the adopted places of the comparison-stars; $\Delta\alpha$ and $\Delta\delta$ are the differences (O—C) between these values and the values given by the ephemeris. The observations are arranged alphabetically with reference to the place of observation. The name of the observer, the

39. Prof. Boss gives the proper motion as $-0^{\circ}.009$ and $-0''.50$ (A.J. 157).

44. A proper motion, given in the Cincinnati Zone Catalogue, of $-0^{\circ}.0075$ and $-0''.111$ was used.

55. There appears to be a small negative proper motion in right ascension.

66. The proper motion of $+0^{\circ}.0112$ and $-0''.514$, given in the Pulkowa Catalogue, was used.

92. The right-ascensions given in LL, W, L, Sj, C, D, and A.G. Nic. are well satisfied with a proper motion of $+0^{\circ}.028$. The declinations are very discordant but are best satisfied with a proper motion of $-0''.09$.

aperture of the instrument used, and the place of the publication of the observations, are given. In many cases the character of the instrument used was not stated; in these cases it has been assumed in accordance with the known equipment of the observatory in question. *R* denotes that a ring-micrometer, and *Mer.* that a meridian circle, was used, in other cases a filar-micrometer was employed.

Gr. M.T.	α	δ	$\Delta\alpha$	$\Delta\delta$	*	Gr. M.T.	α	δ	$\Delta\alpha$	$\Delta\delta$	*	
May						Algiers.	RAMBAUD.	A.N. 2788 ; B.A. Oct., Nov.				
13.67719	15 12 20.43	-30 5 44.0	+0.28	-5.3	2	May	15 22 2.87	-26 45 47.4	+1.11	-3.5	14	
15.71082	15 38.40	28 59 26.3	(-0.71	-4.1)	9	20.37656	23 40.76	26 10 19.6	0.31	-8.5	17	
18.72316	20 46.43	27 12 52.6	+0.54	-5.2	13	21.36892	25 27.82	25 31 16.7	.49	+ 0.1	18	
23.71682	29 46.85	23 55 30.0	+0.35	+ 0.2	27	23.37552	29 9.03	24 9 50.2	.74	- 8.0	24	
23.75353	15 29 50.83	-23 53 58.9	+0.21	- 0.9	38	24.37556	31 0.84	23 27 51.0	.05	6.2	23	
May 15.	Observation doubtful.					25.41920	33 0.70	22 43 4.6	.83	2.6	37	
Algiers.	RAMBAUD.	50 cm.	A.N. 2788 ; B.A. Oct., Nov.			26.40731	34 54.24	22 0 0.7	+0.06	5.7	40	
May	16.46698	15 16 55.30	-28 33 44.5	+0.53	-11.6	10	28.40828	15 38 50.08	-20 30 36.3	-0.16	6.3	39
18.39274	15 20 12.44	-27 25 5.3	+0.81	- 7.8	11							

ADDENDUM.

BY FRANK MULLER.

The following observations were omitted in preparing the copy for the printer, and the omission was not discovered until too late to insert them in their proper place on p. 49.

I take this opportunity of acknowledging the receipt, since the completion of my work, of observations of the comet made at Vienna, Nicolajew, and Cincinnati. I regret the omission of these observations, and had hoped that my request published in the *Astronomische Nachrichten* of June 8 would have insured their earlier publication.

Algiers.	TRÉPIED.	50 cm.	<i>A.N.</i> 2788; <i>B.A.</i> , Oct., Nov.		
Gr. M.T.	α	δ	$\Delta\alpha$	$\Delta\delta$	*
June 9.37068	16 ^h 4 ^m 5.95 ^s	—11° 7' 24.4"	+0.47	— 8.6	59
10.43735	6 27.64	10 17 34.7	+0.83	11.6	62
15.42039	17 35.50	6 32 19.0	+0.66	5.7	72
16.39474	19 47.13	5 50 22.5	+0.63	11.9	86
20.39425	28 48.44	3 6 41.0	—0.03	21.2	94
22.45514	33 27.28	1 48 16.9	+0.16	13.3	101
23.38888	16 35 34.74	— 1 16 37.4	(—0.02	—152.0)	107

1888 July 14.

Gr. M.T.	α	δ	$\Delta\alpha$	$\Delta\delta$	*	Gr. M.T.	α	δ	$\Delta\alpha$	$\Delta\delta$	*						
Orwell Park. PLUMMER. 10 in. M.N. Nov.																	
July 21.47249	16 36 26.53	+ 8 20 58.1	+ 0.25	- 9.1	154	May 14.43434	15 13 34.11	- 29 41 38.7	+ 0.59	- 7.9	1						
24.46698	42 30.92	8 39 7.7	.10	+ 9.1	156	15.41784	15 15 9.90	- 29 9 14.2	- 0.14	- 2.0	9						
27.50462	48 36.83	8 51 12.9	.62	14.4	157	July 8.30730	17 8 53.19	+ 5 35 11.1	- 0.02	- 5.5	125						
28.50153	17 50 35.48	+ 8 53 51.6	+ 0.28	+ 10.2	157	10.38359	13 14.42	6 11 16.2	+ .33	- 6.8	132						
Padua. ABETTI. 7 in.? A.N. 2823.																	
May 14.51634	15 13 42.21	- 29 38 59.3	+ 0.76	- 7.7	1	12.39276	17 31.91	6 43 11.3	+ .28	- 3.0	140						
18.47724	20 20.97	27 21 55.5	.66	4.0	11	13.36287	19 35.56	6 57 10.5	+ .44	- 1.7	142						
20.49040	23 53.01	26 5 55.1	.38	8.7	16	14.37836	21 43.64	7 10 52.4	- .16	+ 0.5	144						
20.49040	23 53.24	26 5 51.9	.61	5.5	17	15.35634	23 47.53	7 23 11.7	+ .40	+ 4.5	144						
21.45571	25 37.57	25 28 54.2	.83	5.2	18	16.42534	26 1.56	7 35 26.8	.24	- 5.5	146						
24.46965	31 7.84	23 24 48.3	.39	3.2	32	17.38343	28 1.35	7 45 41.7	.32	(-78.1)	146						
24.46965	15 31 7.84	- 23 24 49.9	+ 0.39	- 4.8	33	21.36128	36 12.84	8 20 23.0	.18	+ 3.0	151						
June 7.37704	15 39 43.45	- 12 41 33.7	+ 0.39	- 4.9	51	22.36046	38 15.28	8 27 24.8	.44	20.0	154						
8.37370	16 1 54.37	11 54 21.4	.53	7.1	58	24.41052	17 42 23.98	+ 8 38 57.4	+ 0.00	+ 15.6	155						
8.37370	1 54.13	1 54 28.3	.29	14.0	58 ¹	Aug. 6.32891	18 7 49.20	+ 8 56 3.7	(-0.19	+ 22.5)	162						
23.47807	16 35 47.63		+ 0.69		97	7.32986	18 9 44.21	+ 8 53 58.0	- 0.35	+ 10.6	162						
23.48737		- 1 10 44.8		- 11.1	97	May 14, 15. MILLOSEVICH, observer.											
Palermo. AGNELLO. 25 cm. A.N. 2788, 2790.																	
May 15.41719	15 15 10.75	- 29 9 24.7	+ 0.77	- 11.2	9	August 6. Observation doubtful.											
21.44685	23 34.27	25 29 19.7	(-1.51	68.6)	18	Scarborough. LOHSE. 15.5 in. M.N. 47.											
28.38754	38 48.50	20 31 33.0	+ 0.76	6.0	47	May 20.47065	15 23 50.73	- 26 6 38.1	+ 0.21	- 5.7	17						
30.37860	42 49.57		1.33		50	21.51326	25 42.98	25 25 36.9	-.02	4.4	18						
31.35450	15 44 49.30	- 18 15 0.6	+ 1.21	- 18.0	49	29.52798	15 41 5.08	- 19 39 30.9	+ 0.29	- 6.4	49 ¹						
May 15. ZONA, observer.																	
Prague. WEINEK and GRUSS. 6 in. R. A.N. 2788.																	
May 27.39249	15 36 50.42	- 21 16 20.2	+ 0.76	- 6.5	37	Washington. FRISBY. 9.6 in. A.J. 157.											
27.41442	15 36 52.94	- 21 15 16.5	+ 0.70	- 1.6	37	May 14.69101	15 13 59.30	- 29 33 11.0	+ 0.93	+ 0.1	1						

ADDITIONAL OBSERVATIONS.

After the solution of the normal equations there appeared, in A.N. 2835, observations made on ten nights by Herr KAMMERMANN at Geneva, and in A.N. 2837, observations on eight nights by M. STUYVAERT at Brussels.

These observations are referred chiefly to anonymous stars, and being distributed among six groups, the only effect, probably, would be slightly to increase the weights.

4. *Errors of Observation.*—In order to assign appropriate weights to a series of observations it is necessary to consider the mean, or probable, error of the series. This depends only upon the accidental errors; to determine which, however, the differences between the observed and computed places must first be freed from systematic errors.

Each difference consists of

I. Systematic errors;

ε_e , the error of the preliminary ephemeris
 ε_p , the personal equation of the observer.

II. Accidental errors;

ε_a , the accidental error of observation.
 ε_s , the error of the star-place.

I. Systematic errors.

a. Error of the preliminary ephemeris.

This error may be determined with sufficient accuracy for purposes of weighting by dividing the observations into

groups, and taking the means of the differences with reference to the number of observations. The results are:

Mean Date	$\Delta\alpha$	$\Delta\delta$	No. Obs	Adopted Date.	Wt.
May 22	+ 0.54	" - 5.2	92	1887	6
June 12	.51	5.8	48	162	3
20	.42	9.3	66	172	4
July 11	.38	- 1 2	31	192	2
21	+.27	+ 3.9	27	202	2
Aug. 8	- 0.13	+ 0.0	6	222	½

Solving by least squares the six equations of condition furnished, the errors of the ephemeris at any time t days from June 1 are, for right-ascension and declination respectively,

$$\begin{aligned} \varepsilon_e &= 0^s.52 + 0^s.00112 t + 0^s.0000631 t^2 \\ \varepsilon_e &= -5''.5 - 0''.0123 t - 0''.001373 t^2 \end{aligned} \quad (1)$$



0°.51 and 7°.5, given by the mean of all the observations. These corrections are given for every tenth day, in the following table :

	$\Delta(J\alpha)$	$\Delta(J\delta)$
May 12	-0.51	+7.20
22	0.51	7.48
June 1	0.51	7.50
11	0.493	7.24
21	0.463	6.70
July 1	0.419	5.90
11	0.364	4.82
21	0.296	3.46
31	0.215	+1.83
Aug. 10	-0.123	-0.07

The corrections from this table being added to the differences of a given observer, the mean of the resulting differences will with changed sign, be the personal error of that observer.

Let v_1, v_2, \dots, v_i be the residuals of the individual differences from this mean, then

$$v^2 = \varepsilon_o^2 + \varepsilon_s^2$$

If ε_1 be the mean error of observation in an observation depending upon a single comparison,

$$\varepsilon_1^2 = \frac{[n\varepsilon_o\varepsilon_s]}{i-1}$$

where $[n\varepsilon_o\varepsilon_s] = n_1\varepsilon_{o1}^2 + n_2\varepsilon_{o2}^2 + \dots + n_i\varepsilon_{oi}^2$

Substitution gives

$$(2) \quad \varepsilon_1^2 = \frac{[nvv] - [n\varepsilon_o\varepsilon_s]}{i-1},$$

and the mean error of an observation depending upon a number of comparisons n is

$$(3) \quad \varepsilon = \sqrt{\left(\frac{\varepsilon_1^2}{n} + \varepsilon_s^2\right)}$$

When the star-places are carefully determined, ε_s will usually be small in comparison with ε_o and the probable error will then be approximately given by

$$(4) \quad r = 0.6745 \sqrt{\frac{[nvv]}{n(i-1)}},$$

which is the formula commonly used.*

There is however an objection to weighting strictly in accordance with this, or any other similar formula. On each night there are undetermined instrumental and personal errors peculiar to that night; such errors are not diminished by an increased number of comparisons, hence the insufficiency of a formula in which the probable error depends solely upon the number of comparisons. I have used (4)

* The accurate formula is, however, not much more difficult in application, and when the error of observation is small, the discrepancies arising from the use of the approximate formula may become appreciable. The mean error would be determined from (2) by assigning to each observation a value of ε_s in accordance with the mean error of the catalogue on the authority of which the place of the comparison-star rested; then, for each observation,

$$r = 0.6745 \sqrt{\frac{\varepsilon_1^2}{n} + \varepsilon_s^2},$$

ε_s being assigned as before.

when the number of comparisons did not exceed sixteen, but I have assumed that the probable error was not diminished by a number of comparison greater than this. This limit is probably too great, while the comet was strongly condensed.

The calculated probable errors of a single comparison, arranged in order of the size of the instrument, are :

Place and Observer	Probable Error	Aperture
Algiers, Rambaud	1.02	6.6 cm
“ Trépied	0.92	10.8
Cambridge, Wendell	0.50	6.3
Nice, Charlois	0.18	{ 1.4 5.5
Bordeaux, Courty	1.03	10.8
“ Flamme	0.45	9.9
“ Rayet	0.79	{ 7.6 12.3
Marseilles, Borrelly	0.31	6.7
Orwell Park, Plummer	0.39	{ 15.0 17.3
Geneva, Kammermann	0.82	8.7
Rome, Cerulli	0.47	6.8
Besançon, Gruey	0.94	(*28.0) 20
“ Herique	0.67	11.9
Padua, Abetti	0.44	5.5
Gohlis, Winkler	0.65	26.0
Kremsmünster, Schwab	0.61	9.3
Nashville, Barnard	0.61	8.4
		15 R

* Adopted probable error, 20''.

When two errors are given, the latter refers to the observations made in July and August when the comet was more difficult to observe in declination; in most cases there was no difference. All the Rome observations were made during these months. As a guide to assigning probable errors in accordance with the size of the instrument used, I have grouped these results.

Aperture	Mean Aper.	Prob. Error	No. of Obs.
15 — 18 cm	16 cm	0.58 12.3	4
20 — 26	24	0.60 10.1	5, 4
36 — 38	37	0.59 8.1	5

The solution of the equations of condition, taken with equal weights, shows that the probable error in declination may be nearly represented by the equation

$$r = 10''.3 + (25-a) 0''.20, \quad (5)$$

for apertures between 15^{cm} and 40^{cm}, a being the aperture in centimeters. The probable error in right-ascension appears to be independent of the aperture.

The probable error of each observer was calculated by (4) when the number of observations was seven or more. In other cases it was, the aperture of instrument being the only available guide, assigned by (5) and taken as 0°.60 in right-ascension. The weight was then calculated by

$$p = \frac{r_0^2}{r^2} = \frac{(5''.5)^2}{r^2}$$

(12)

$5''$.5 being assumed as the probable error of a single comparison in a standard observation.

5. *Formation of Normal Places.* — Weights being given to each observation, and personal equation applied, in accordance with the preceding results, the residuals were divided into groups, and their weighted means taken.

Group	Mean Date	(O-C) $\Delta\alpha$ $\Delta\delta$	No. of Obs.	Weight
May 12-21	17.8	+ 0.620 — 6.65	48	152 180
May 22-31	26.0	.503 6.80	42	146 168
June 1-16	12.9	.502 7.70	63 62	214 136
June 17-30	21.8	.385 8.57	68	206 135
July 1-14	10.5	.377 — 1.32	34 33	104 75
July 15-31	22.2	.303 + 2.04	27 26	115 43
Aug. 1-10	8.4	+ 0.141 + 4.20	6	25 14

The corresponding normal places are:—

Date	α	δ
1887 May 18.0	229° 52' 47.92"	-27° 39' 28.87"
	233 31 43.66	22 17 58.58
June 13.0	243 2 12.91	8 19 54.68
	248 6 29.18	- 2 5 11.54
July 10.0	258 5 2.11	+ 6 4 43.29
	264 22 33.85	8 24 39.87
Aug. 8.0	272 45 11.59	+ 8 52 17.07

6. *Formation of Normal Equations and Determination of Definitive Elements.* — The coefficients of the variations of the elements were computed by the formulas given by OPIOLZER.* Whence were formed the following

* Lehrbuch zur Bahnbestimmung. Zweiter Band, pp. 405-406.

EQUATIONS OF CONDITION (coefficients logarithmic).

Right-Ascension.

$$0.9158 \Delta\alpha \cos \delta = 9.1014 \delta i' + n9.7910 \sin i' \delta\Omega' + 0.5091 \delta\pi' + 0.6943 \delta \log q + n8.6336 \delta T + n9.4702 \delta e$$

0.8439	9.3201	n9.7866	0.5296	0.6382	n8.6698	n9.3811
0.8722	9.5725	n9.6509	0.5267	0.3633	n8.6929	n8.6523
0.7601	9.6108	n9.4889	0.4993	0.0396	n8.6719	8.8019
0.7465	9.5663	n8.5065	0.4145	n9.9454	n8.5856	9.8627
0.6557	9.4678	9.0244	0.3502	n0.2324	n8.5105	9.4675
0.3323	9.2351	+ 9.3578	+ 0.2622	+ n0.3689	+ n8.3957	+ 9.5169

Declination.

n0.8228 $\Delta\delta$	= n0.5027	+ n9.7115	+ 9.0392	+ 9.9016	+ n6.7537	+ 7.5274
n0.8325	n0.5168	n9.9688	9.3296	9.4857	n7.4650	n8.1708
n0.8865	n0.4630	n0.2538	9.7602	n0.1533	n7.9423	n7.9174
n0.9330	n0.3902	n0.3102	9.8723	n0.8483	n8.0196	8.1229
n0.1399	n0.1846	n0.3282	9.9686	n0.4771	n8.0424	8.6720
0.3032	n0.0168	n0.3278	9.9782	n0.4762	n7.9908	8.7276
0.6149	n9.7208	+ n0.2965	+ 9.9526	+ n0.4242	+ n7.9062	+ 8.6984

After multiplying each equation by the square root of its weight, and rendering all the equations homogeneous by the introduction of the factors

$$\begin{aligned}x &= 1.6303 \delta i' & t &= 1.7852 \delta \log q \\y &= 1.3754 \delta\Omega' \sin i & u &= 9.8581 \delta T \\z &= 1.6919 \delta\pi' & w &= 0.7159 \delta e\end{aligned}$$

residual unit = 2.0374

the normal and elimination equations were obtained and checked in the usual way.

NORMAL EQUATIONS (natural numbers).

+3.2495 x	+2.4826 y	+0.0344 z	+0.6054 t	-0.0712 u	+0.0262 w	= +3.2908
+2.4326	+3.5101	-1.5012	+0.3476	+1.3998	+0.3285	= +1.3221
+0.0344	-1.5012	+3.8590	+1.8300	-3.7560	-0.3742	= +3.2762
+0.6054	+0.3476	+1.8300	+2.8651	-1.7499	+1.5851	= +2.5187
-0.0712	+1.3998	-3.7560	-1.7499	+3.6648	+0.3049	= -3.1962
+0.0262	+0.3285	-0.3742	-1.5851	+0.3049	+1.5846	= -0.5268

ELIMINATION EQUATIONS (coefficients logarithmic).

+0.51181 x	+0.38607 y	+8.53656 z	+9.78204 t	+n8.85248 u	+8.41830 w	= 0.51730
0.22763	n0.18384	n9.02366	0.16230	9.48982	= n0.05748	
	0.39412	0.23756	n0.38766	n8.97864	= 0.34428	
	0.18772	8.75435	n0.17734	n0.17734	= 9.46761	
		7.74036	7.14613	= 8.38021		
			7.69897	= 8.32015		

Direct solution of these equations gave

$$\begin{aligned}\log x &= 0.3653 & \delta i' &= + 5.92 \\ \log y &= 0.4387 & \delta\Omega' &= -33.13 \\ \log z &= 0.1488 & \delta\pi' &= + 3.12\end{aligned}$$

$$\begin{aligned}\log t &= 0.6181 & \delta \log q &= + 0.0000360 \\ \log u &= 0.5185 & \delta T &= + 0.002418 \\ \log w &= 0.6212 & \delta e &= + 0.0004249\end{aligned}$$

(13)

The small coefficients in the equations involving u and w alone, show that the values obtained by direct solution are highly uncertain. An attempt was made to diminish this uncertainty by expressing the other unknown quantities in terms of w and an absolute term. Accordingly,

$$\begin{aligned}x &= 9.93648 + 0.16283 w \\y &= 9.39094 + n0.47594 w \\z &= 0.71350 + n0.57532 w \\t &= 8.47124 + 0.61496 w \\u &= 0.63985 + n0.02695 w\end{aligned}$$

the coefficients being logarithmic.

Substitution in the equations of condition and solution of the fourteen resulting equations for w gave

$$(1) \quad \log w = 0.6604;$$

the corresponding changes of the elements are given in the column numbered I in the table below. Comparison with the normal places gave $-9''.86$ as the sum of the weighted residuals, showing that the uncertainty of solution had not been wholly eliminated. To determine by trial what variation of the eccentricity would best distribute the residuals, an assumption was made, —

$$(2) \quad \log w = 0.6814,$$

which gave the changes of the elements numbered II. Comparison with the normal places gave the residuals numbered II. Interpolation between the values (1) and (2) gave

$$(3) \quad \log w = 0.6788$$

The results of this assumption are given in the columns numbered III. They show no improvement on the results of the second hypothesis. The residuals given by direct comparison being so nearly equal, it was considered safe to interpolate between them, on the assumption that the sum of the weighted residuals should be zero. The weighted values are numbered (IV) and the sum of their squares being slightly smaller than for the other assumed variations of the eccentricity, the value

$$(4) \quad \log w = 0.6801$$

was taken as the definitive value.

CHANGES OF ELEMENTS.

	I	II	III
δT	+ 0.002344	+ 0.002301	+ 0.002307
$\delta \pi'$	+ $''$.33	+ 1.88	+ 1.94
$\delta i'$	+ 6.27	+ 6.47	+ 6.45

	I	II	III
$\delta \Omega'$	$''$ -36.59	$''$ -38.55	$''$ -38.33
$\delta \log q$	+ 0.0000393	+ 0.0000413	+ 0.0000411
δe	+ 0.0004650	+ 0.0004880	+ 0.0004852

RESIDUALS (Normal—Computed).

Unweighted.		Weighted		
II	III	II	(IV)	III
Right Ascension.				
+ $''$.08	$''$ +0.91	$''$ +1.33	$''$ +1.23	$''$ +1.12
-0.59	-0.57	-0.72	-0.71	-0.70
+0.21	+0.17	+0.31	+0.28	+0.25
-0.89	-0.94	-1.28	-1.31	-1.35
+0.61	+0.53	+0.62	+0.58	+0.54
0.72	0.64	0.77	0.73	0.68
+0.80	+0.76	+0.40	+0.39	+0.38
Declination.				
-0.60	-0.84	-0.80	-0.96	-1.13
+0.59	+0.25	+0.76	+0.54	+0.32
+0.75	+0.41	+0.88	+0.68	+0.48
-1.43	-1.77	-1.66	-1.86	-2.05
+0.81	+0.53	+0.70	+0.58	+0.46
+0.75	+0.34	+0.50	+0.36	+0.22
-1.27	-1.67	-0.47	-0.54	-0.62
Sum +1.54	-1.25	+1.36	-0.01	-1.40

All the weights have been divided by 100 for convenience. The sums of the squares of the residuals are :

	II	(IV)	III
Unweighted	$''$ 10.02	$''$ 10.11	$''$ 10.59
Weighted	10.87	10.72	10.96

Transforming the preliminary elements from the ecliptic to the equator, and adding the changes interpolated between II and III, the resulting definitive elements of Comet 1887 IV are :

$$\begin{aligned}T &= \text{June } 16.663384 \text{ Gr. M.T.} \\ \pi' &= 257^\circ 4' 4''.38 \\ \Omega' &= 313^\circ 54' 12.14 \\ i' &= 22^\circ 20' 9.02 \\ \log q &= 0.1442046 \\ e &= 0.9960879\end{aligned}\} \text{ Mean Equator } 1887.0$$

CONSTANTS FOR THE EQUATOR.

$$\begin{aligned}x &= r [9.9830763] \sin (349^\circ 18' 5''.82 + v) \\ y &= r [9.9843699] \sin (254^\circ 50' 26.90 + v) \\ z &= r [9.5798234] \sin (303^\circ 9' 52.24 + v)\end{aligned}$$