Asset Returns and Executive Compensation under Earnings Management

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Abstract

Executives' desire to use financial reports, especially bottom-line earnings, to pursue their own financial interests gives rise to the phenomenon of *earnings management*, which is defined as intentional manipulation of reported earnings by knowingly choosing accounting methods and estimates that do not accurately reflect the firm's underlying fundamentals. Empirical research documents that earnings management behavior is pervasive in the corporate world. This dissertation analyzes the economic implications of earnings management along two dimensions. The first chapter examines the effects of earnings management on executive compensation, and the second chapter investigates asset return dynamics under earnings management.

The first chapter develops the optimal contract in a principal-agent model with financial reporting and moral hazard. The optimal contract is characterized both analytically and numerically, and the necessary and sufficient condition for earnings management to occur is explicitly derived. The model provides an explanation for the positive association between earnings management and incentive pay observed in both time series and cross section. The model's predictions regarding the changes of earnings management behavior and compensation structure in response to corporate governance legislations are also consistent with empirical observations. Further, this contract model serves as a micro-foundation for the asset pricing model in Chapter 2.

The second chapter investigates stock return dynamics in an environment where executives have an incentive to maximize their compensation by artificially inflating earnings. The principal-agent model with financial reporting, developed in Chapter 1, is embedded in a Lucas asset-pricing model with periodic revelations of the underlying profitability. The return process generated from the model is consistent with a range of financial regularities observed in the return data, namely volatility clustering, asymmetric volatility, and increased idiosyncratic volatility. By incorporating earnings management in an otherwise standard asset-pricing model, this study presents a mechanism through which corporate misconduct provides a unifying cause for these stylized financial facts. In addition, the calibrated model indicates that earnings management by individual firms may not only deliver the observed features in their own stock returns, but also be powerful enough to generate market-wide patterns.

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Contents

Chapter 1

Earnings Management and Compensation Structure under Moral Hazard

This chapter investigates the optimal structure of executive compensation with the possibility of information manipulation. I characterize the optimal compensation contract analytically and numerically, and establish necessary and sufficient conditions for earnings management to occur. The model shows that taking earnings management into consideration leads to a compensation structure that is more responsive to reported earnings. The model provides an explanation for the observed positive association between earnings management and incentive compensation in both time series and cross section. In addition, the model predictions regarding the changes of earnings management behavior and executive compensation structure in response to corporate governance legislations are consistent with empirical observations.

1.1 Introduction

[1] The primary role of financial reporting is to provide corporate executives with a credible means of communicating their private information on firms' performance to external shareholders. This role can become entangled with executives' desire to use financial reports, especially bottom-line earnings, to pursue their own financial interests. Such motives give rise to the phenomenon of *earnings management*, defined as intentional manipulation of reported earnings by knowingly choosing accounting methods and estimates that do not accurately reflect the firm's underlying fundamentals.

Empirical evidence suggests that earnings management behavior is pervasive. Burgstahler and Dichev (1997) estimate that 8-12 percent of the firms with small pre-managed earnings decreases manipulate earnings to achieve earnings increases, and 30-44 percent of the firms with small pre-managed losses manage earnings to create positive earnings. Grahama et al. (2005) document that 78 percent of executives in their survey admit to manipulating earnings to maintain predictability in earnings. General Accounting Office (2006) report that 6.8 percent of publicly listed companies made financial restatement announcements in 2005 to correct previously reported financial results, and over the period of January 1, 2002 through September 30, 2005, the total number of restating companies (1,084) represents 16 percent of the average number of listed companies in their sample.

In the wake of the massive financial scandals of the early 2000s, the integrity of financial reporting and consequences of corporate misreporting have received increased academic attention and regulatory scrutiny. The economic costs due to distorted information flow can be substantial. On average, stock prices fall by 10% on the days

[1] I thank Albert Choi, Maxim Engers, Nathan Larson, Leo Martinez, Leonard Mirman, Edward Simpson Prescott, and John Weinburg for their helpful comments.

around earnings restatement announcements.[2] It is also observed that earnings management distorts the allocation of capital.[3] In order to minimize the economic costs from misreporting, academics, practitioners and regulators have called for corporate governance reforms to strengthen shareholder power. However, these efforts do not render the study of financial misreporting irrelevant. Earnings management still runs rampant in the world of business. The accounting irregularities at Freddie Mac and American International Group that precipitated the economic downturn of 2008 indicate that such behavior can engender significant economic consequences.[4]

In an attempt to better understand the causes and consequences of earnings management, this paper adopts an optimal contracting approach to identify how the possibility of earnings management affects the structure of managerial compensation, to analyze the responses of earnings management and compensation structure to corporate governance legislations, and to illustrate the underlying economic tradeoff that gives rise to earnings management. The model results underline the central importance of earnings management in accounting for time-series and cross-sectional variations in compensation structure and add to the ongoing debate on regulations

[2]Turner et al. [2001] report negative market adjusted returns of -12.3% over an eight-day window. Palmrose et al. [2004] document an average abnormal returns of about -9% over a two-day announcement window. Wu [2002] shows that the market reacts negatively with over -11% cumulative abnormal returns during a three-day window.

[3]See Burns and Kedia (2006) and Kedia and Philippon (2007).

[4]Morgan Stanley determined the accounting tactics, while legal, enabled Freddie Mac, and to a lesser extent Fannie Mae, to overstate the value of their reserves. Both companies also pushed inevitable losses into future by sharply curtailing their repurchase of soured mortgages out of the securitizations they have guaranteed. "Fannie Mae and Freddie Mac were 'playing games with their accounting' to meet reserve requirements, prompting the government to seize control of the companies," U.S. Senator Richard Shelby said (Bloomberg (September 9, 2008)). In the case of AIG, PricewaterhouseCoopers prompted an announcement about the material accounting weaknesses related to the valuation of AIG's derivatives holdings. Prosecutors insisted that five former executives from the American International Group deliberately mounted a fraud to manipulate its financial statements, after a string of AIG scandals early this decade."Accounting flaws at American International Group significantly understated the insurance giant's losses on complex financial instruments linked to mortgages and corporate debt," AIG Said in an official public statement (The New York Times (February 12, 2008)).

of executive compensation.

Specifically, this model yields three main implications for executive compensation: First, the optimal pay-performance sensitivity is positively associated with the possibility of manipulation. Second, managerial compensation becomes less sensitive to performance in response to tightened corporate governance legislations. Finally, earnings management arises, in part, because of the substitutability of financial manipulation for managerial effort in enhancing reported performance and hence executive pay. We discuss each of these in greater detail below.

The first implication of the model is that variations in executive compensation are linked to variations in the possibility of earnings management. The model suggests that taking financial reporting incentive into consideration leads to a greater pay-performance sensitivity than otherwise. The possibility of artificially inflating performance serves as an insurance for the manager against a low compensation payoff and thus weakens incentives to exert effort. Specifically, when earnings management is possible, the manager has an incentive to slack off at work and subsequently overstate earnings. In order to overcome this motive and incentivize effort, shareholders need to provide stronger financial incentives in the compensation contract. Aggarwal and Samwick (2003) report that managers with divisional responsibilities have lower pay-performance sensitivities than chief executives. They attribute the substantial incentives of the top management team to a less precise individual-specific signal of effort than those of the managers with lower managerial responsibilities. While not mutually exclusive with their explanation, we present earnings management as an additional factor that generates noises in the performance-valuation process of chief executives and thereby magnifies pay-performance sensitivities.

The positive association between manipulation and incentive pay suggested by the model is consistent with empirical observations in both time series and cross section. In time series, research documenting the trend in earnings management indicates that companies' management of earnings increased steadily from the late 1980s until the passage of the Sarbanes-Oxley Act (SOX) (Brown 2001; Bartov et al. 2002; Lopez and Rees 2002; Cohen et al. 2004). Meanwhile, the explosive growth of performance-based compensation was a notable characteristic of the late 20th century economic scene. Stock options replaced base salaries, becoming the single largest component of compensation during the 1990s (Hall and Liebman 1998; Murphy 1999; Bergstresser and Philippon 2006). The cross-sectional variations of earnings management and executive compensation also follow the predicted pattern. Gao and Shrieves (2002) show that earnings management intensity, as measured by the absolute value of discretionary (or "unexplained") current accruals scaled by asset size, is related to the managerial compensation contract. They find that the amounts of stock options and bonuses, and the incentive intensity of stock options, are positively related to earnings management intensity, whereas salaries are negatively related. Bergstresser and Philippon (2006) find evidence that companies with a higher level of earnings management are the companies with more incentive pay.[5]

A second implication of the model is that in response to tightened corporate governance legislations, executive compensation will be less responsive to performance. In this analysis, the cost managers incur when manipulating earnings is interpreted as a policy parameter determined by exogenous regulatory authorities. When the manipulation cost increases due to more stringent corporate governance, in an environment where managerial effort positively influences output, the mananager has stronger incentives to exert productive effort in order to avoid unfavorably high manipulation

[5] The conventional wisdom is that performance pay is the *reason* for manipulation. While not necessarily contradicting this widespread perception, our model provides an alternative and admittedly counter-intuitive theory that accounts for the observed correlation. Causality test remains a task for future work.

cost in financial reporting, and therefore less financial incentive is required in the compensation contract to motivate effort. At the aggregate level, in an economy with more effective corporate governance, or analogously, at the individual level, in the firms with stronger internal control system over financial reporting, shareholders can induce productive effort with less usage of performance-based compensation.

The model is compatible with the empirical finding that firms respond to the additional liabilities SOX imposes on executives by increasing the proportion of fixed compensation (and hence reducing the proportion of incentive compensation). SOX requires that chief management certify the veracity of their financial reports and return incentive-based compensation in the case of restatements. Cohen et al. (2005) show that the ratio of incentive compensation to fixed salary declined significantly subsequent to the passage of SOX. Narayanan and Seyhun (2005) also show that the average grant size has steadily declined since the passage of SOX, and the total number of options granted shows a downward trend in the post-SOX period as well. Similarly, Bayer and Burhop (2008) document a significant decrease in pay-performance sensitivities of executive pay after a reform in legal rules of corporate governance in Germany. Another clear implication derived from the model is that earnings management will be less widespread in a more litigious corporate governance environment. Cohen et al. (2004) and Narayanan and Seyhun (2005) present evidence that earnings management has diminished abruptly after the passage of SOX.

Finally, this theory implies that earnings management arises, in part, because of a tension between inducing managerial effort and motivating truthful reporting: A relatively sensitive payment schedule generates an incentive to misreport, whereas a compensation schedule that is not responsive enough fails to motivate the desired level of effort. If the cost of manipulating earnings is small compared to the cost of exerting effort, it is prohibitively difficult to elicit the truth while maintaining the manager's incentive to put forth effort, and earnings management emerges endogenously under the optimal contract. The model further indicates that since productive effort and earnings management are substitutes to the manager in the sense they both improve reported peformance and hence executive compensation, any factor that may affect one managerial decision will also have a considerable impact on the other.

This adds a new perspective to the current debate on regulations of managerial compensation: the aforementioned conflict presents a tradeoff between weakened firm-wide efficiency versus improved economy-wide efficiency. On one hand, the restrictions such as the current cap imposed on executive compensation and the ban on golden parachute add additional constraints on the compensation contract design problem, and as a standard principal-agent framework suggests, make it more difficult to induce effort, resulting in weaker corporate performances. On the other hand, the model suggests that regulations can be optimal from an efficiency viewpoint. Suppose that the companies inflating their performance show seemingly stronger financial statements than honest firms and hence attract more capital from investors. The restrictions on executive compensation discourage manipulation and are hence likely to improve capital allocation across firms, helping direct the resources in the economy to the most efficient use.

The paper examines a contracting system in which a representative shareholder hires a manager to manage a comany. Manager exerts an unobserved effort that affects the firm's earnings, and the earnings are privately observed by the manager. The manager may take a costly action to distort the firm's performance in order to maximize his compensation, and whether the manager has such an option is stochastic. Our model of managerial reporting under moral hazard is built upon the modelling device provided by Dye (1988). The message space is limited to a single-dimensional signal while the privately-informed agent receives multiple dimensions of information, and therefore the Revelation Principle is not applicable. Dye (1988) considers external demand for earnings management induced by current shareholders' attempts to alter prospective investors' perceptions of the firm's value. Similar to Dye (1988), Evans and Sridhar (1996) further study the coordination of a financial reporting system and a contracting system in a principal-agent model where the manager privately observes both economic earnings and the available reporting discretion. Truthful reporting occurs in the equilibrium only when the financial reporting system dictates it. Arya et al. (1998) introduce a model where allowing a manager to manipulate earnings serves as a commitment device. In an environment where the ability of shareholders to make binding commitments is constrained and the implementation of contracts depends entirely on the willingness of both parties to remain in the contract, earnings management is strictly preferable because it reduces owner intervention. These studies do not derive the compensation contract explicitly to analyze the impications of of financial misrepresentation for executive compensation, which is the primary goal of this paper. One strength of the current model is that the optimal contract is characterized completely and analytical tractability is kept as much as possible.

A recent paper by Crocker and Slemrod (2007) considers an alternative environment where the Revelation Principle can be applied. In solving the model, they assume a monotonically increasing reporting function, and actual earnings can be unambiguously recovered by inverting the reporting function. However, in our problem the principal faces uncertainty over whether earnings management occurs. There is a second dimension of private information the manager possesses in our model, and hence the reporting function is no longer invertible and the principal cannot perfectly infer the true outcome. Similar to Crocker and Slemrod (2007), Goldman and Slezak (2006) also consider an agency model in which incentive compensation is determined so as to strike a balance between effort and manipulation. As a model that constructs a rationale for earnings management, our model can be viewed as complementary to theirs. However, our model challenges the main results in Goldman and Slezak (2006) on how pay-performance sentivities vary with the potential for manipulation and the change of public policies. In particular, they find that pay-performance sensitivities are lower with the possibility of manipulation, while our model produces the opposite results. The key difference is that we view earnings management as a phenomenon that fundamentally reflects the information asymmetry between shareholders and managers, and therefore analyze an environment in which earnings management is not fully unravelled by a rational and sophisticated principal; however, they focus on the equilibria with symmetric information where investors can perfectly discount the inflated performance and hence assign less performance pay. We argue that our model provides comparative static insights that are more closely aligned with the empirical literature.

The rest of the paper proceeds as follows. Section 2 lays out the principal-agent model. Section 3 shows general results and relevant properties of the optimal pay contract with manipulation. In Section 4, the model is extended to continuous earnings. Section 5 illustrates model implications for analysis and interpretations of empirical observations. Section 6 presents concluding remarks.

1.2 Model

1.2.1 Model setup

A risk-neutral principal (shareholders) hires a risk-averse agent (manager) for one period. The utility function of the manager is represented by a strictly increasing

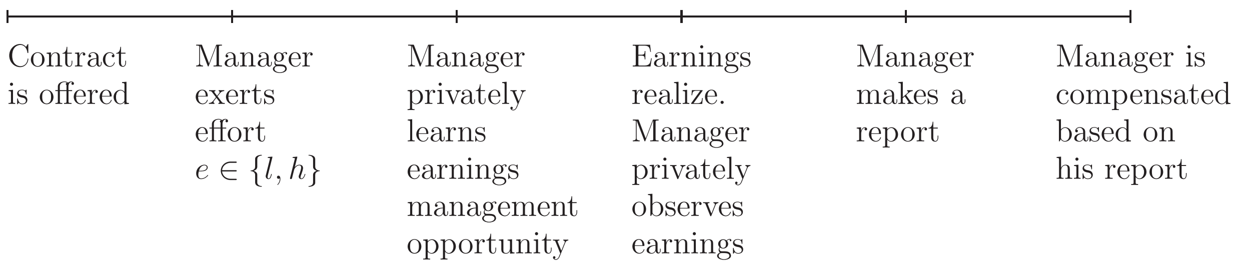

Figure 1.1: Model Timeline

and strictly concave function $U(\cdot)$. The firm's earnings are stochastic and influenced by the manager's effort. The unobserved effort level of the manager e can take two values, low (l) and high (h), that is, $e \in \{l, h\}$ where $l < h$. The manager incurs disuility from exerting effort, denoted by the cost function $a(e)$. In particular, high effort is associated with a cost of $a(h) = c$, while low effort involves no cost: $a(l) = 0$. Earnings y take two possible values, $y \in \{L, H\}$, where $L < H$. Let p_e be the probability that the earnings are equal to H when the effort is e, where $e \in \{l, h\}$ with $p_h > p_l$.

The time line of Figure 1 chronicles the sequence of events in the model. After the manager accepts the take-it-or-leave-it contract offered by the principal, he decides whether to exert managerial effort. After exerting effort, the manager also privately learns whether he has an opportunity to manage earnings. With probability x, he has discretion over how much earnings to report.[6] With probability $(1 - x)$, the manager is prohibited from manipulating earnings. Thus, in an economy where there

[6]This analysis does not make a distinction between earnings management and fraud. While the accounting choices that explicitly violate Generally Accepted Accounting Principles (GAAP) clearly constitute both earnings management and fraud, according to the SEC, systematic choices made within the boundries of GAAP can constitute earnings management as long as they are used to obscure the true performance of a firm and will lead to adverse consequences for the firm in the same way as fraud.

Following this notion, there is no economic difference between fraud and earnings management in the model: in both cases the reported number is different from the true amount, and the manager bears costs for such behavior.

are a large number of such shareholders and managers, x represents the fraction of managers who are able to manipulate earnings. Then the manager privately observes the earnings and makes an earnings announcement.

If the manager produces an inaccurate report, the manager incurs a personal cost, denoted by $\phi(\cdot)$. ϕ is a function of the discrepancy between true earnings and reported earnings. Reporting honestly incurs no cost, that is, $\phi(0) = 0$.[7] When the manager overstates earnings, there is a positive cost $\phi(H - L) = \psi > 0$.[8] We also assume that there is no cost associated with underreporting, and we will see that there is no incentive to understate earnings in this model. We define that earnings management occurs when the reported earnings differ from true earnings. More specifically, earnings management emerges in this environment if the manager announces that high earnings (H) have been achieved when the actual realization of earnings is low (L).[9]

As the contract must be designed based on mutually observed variables, the manager's compensation must be based on his report. As long as the manager's reported earnings fall in the set $\{L, H\}$, the principal cannot directly detect whether or not the manager has misstated earnings and must pay the manager before the revelation of

[7] There are two frictions in the model that restrain earnings management: earnings management opportunity that realizes with probability x and the cost involved in misstating earnings ϕ. This model can be also considered with only one friction: the cost of manipulation with a simple stochastic structure. The manipulation cost now in the model follows a binary distribution with two possible realizations ∞ and ψ.

[8] The cost of manipulating earnings includes the educational cost of learning how to modify certain components of earnings without getting detected, the costs involved in bribing auditors not to report a discrepancy in the earnings report, and expected reputation damage in case of being caught.

[9] This paper has a central focus on upward manipulation. The reason to focus on misreporting on upside is that overstatement of earnings is more widespread than understatement in the data and more problematic in general. Empirical work on SEC enforcement actions aimed at violations of Generally Accepted Accounting Principles suggests that over-reporting is the more frequent source of firmwide financial misrepresentation (Feroz et al. [1991]). The average amount of restated earnings is hugely negative, and over 75% of restating firms restated their earnings downwards, indicating a strong drive to appear more productive than they actually are. Burgstahler and Dichev [1997] also estimate that 8-12 percent of the firms with small pre-managed earnings decreases manipulate earnings to achieve earnings increases, and 30-44 percent of the firms with small pre-managed losses manage earnings to create positive earnings.

underlying true performance. In fact, the *stochastic* opportunity to manipulate earnings breaks down a direct mapping between reported earnings and actual earnings, obscuring the true performance under the optimal contract.[10] It is also assumed that the manager is essential to the operation of the firm, so the contract must be such that the manager (weakly) prefers to work for the principal regardless of whether he ends up with the opportunity to engage in earnings management.

One key feature of our framework is that shareholders are uncertain about whether the earnings report is inflated. If all managers were able to exaggerate the firm's performance and the degree of manipulation was homogeneous,[11] a rational principal would actually gauge the true state of the firm and a correct assessment would be reflected in the equilibrium incentive contract, for example, in Goldman and Slezak's (2006) signal-jamming model. In reality, however, shareholders are often faced with a significant degree of information asymmetry as to whether and how much financial results are biased. It is natural to think that managers are not all equally versed in manipulating financial records: It depends on their ethical integrity and individual capability to portray an unjustified success. In capturing the additional source of uncertainty on both shareholders and managers, we do not model detection technology in this framework.[12] We show in the next section that manipulation uncertainty has

[10]Here, whether the manager has the opportunity of managing earnings is assumed to be a random event, and the outcome is the manager's private information. Generally Accepted Accounting Principles (GAAP) provide guidelines on how to record and summarize each type of economic transaction, and hence define the accounting latitude available to senior management in financial reporting. In practice, certain economic activities, those where there is no hard-and-fast rule for which accounting method to use, lead to more discretion than others. In any particular period, economic transactions of this type may or may not take place. By virtue of being closer to the operations process, only the manager knows the extent of these activities and hence the degree of reporting latitude available.

In modelling language, the stochastic opportunity to manage earnings adds an additional noise in financial reports that principal cannot perfectly filter out. Due to the additional uncertainty, the principal needs to make inferences as to whether earnings management occurs, and earnings management is not fully unravelled in the equilibrium.

[11]An environment where the opportunity to manipulate reports realizes with certainty while the size of manipulation is stochastic would be essentially identical to the current model.

[12]There is a vast literature on costly state verification, which is an element we do not consider

systematic impact on the characteristics of the optimal incentive contract.

1.2.2 The optimal contract

The contract between the risk-neutral principal and the risk-averse manager includes a set of wages contingent on the reports, which can be alternatively characterized as a set of contingent utilities. The utility level corresponding to compensation level w_i, $i \in \{L, H\}$, is denoted as $U(w_i) = u_i$. Let $U^{-1}(\cdot) = V(\cdot)$. Then $V(u_i)$ is the cost to the principal of providing the agent with utility u_i. Since $U(\cdot)$ is a strictly increasing and strictly concave function, $V(\cdot)$ is a strictly increasing and strictly convex function.

The model presented in this section places restrictions on the ability of the manager to communicate the truth. That is, the manager observes multiple dimensions of information, including the value of earnings and the realization of misreporting opportunity. However, he is only permitted to communicate a single-dimensional signal, which is an earnings announcement. Communication is restricted in that the manager cannot fully communicate the full dimensionality of his private information because of a limited message space, and hence the Revelation Principle is not applicable. As a result, the reporting function is not invertible and true earnings cannot be unambiguously backed out from the reports.

In this environment, the contract must not only induce effort but also control for the manager's reporting incentive. In this paper, we assume that the difference in the earnings is large enough that the principal always wants to implement high effort. The objective of the manager is to maximize his utility by choosing a level of effort and a reporting strategy represented by $R(y)$, subject to the contract he is offered.

in this paper. In the current environment, the inability of the principal to implement a contract contingent on detection of manipulation reflects enforcement frictions observed in reality, such as imperfect auditing, inadequate and delayed SEC investigations, and CEO departure before detection and liquidation.

The manager's utility is of the form $U_m(e, R(y)) = xE[u_{R(y)} - \phi(R(y) - y) - a(e)] + (1 - x)E[u_y - a(e)]$. The principal chooses the utility values u_i, $i \in \{L, H\}$, and recommended reporting choice $R(y)$ for each realization of earnings that minimize the expected cost of inducing effort.[13]

Formally, the optimal contract solves

$$\min_{u_H, u_L, R(H), R(L)} \quad E[V(u)|h]$$
$$= x[p_h V(u_{R(H)}) + (1 - p_h)V(u_{R(L)})] + (1 - x)[p_h V(u_H) + (1 - p_h)V(u_L)]$$

subject to

$$h = \arg\max_{e \in \{l,h\}} xE[u_{R(y)} - \phi(R(y) - y) - a(e)] + (1 - x)E[u_y - a(e)], \qquad \forall y \in \{L, H\}. \tag{1.1}$$

$$E[u|h] = xE[u_{R(y)} - \phi(R(y) - y) - a(e)|h] + (1 - x)E[u_y - a(e)|h] \geq \bar{U}. \tag{1.2}$$

The objective function is the expected cost for the principal to motivate effort. The first term is the cost to the principal of implementing effort when the manager has an opportunity to misreport, and the second term is the cost if he does not have such an opportunity. The first constraint is the incentive constraint for the manager's choice of effort—here, we assume that the principal wants to induce high effort. The second is the participation constraint, with $\bar{U}$ being the manager's outside option. In addition to these constraints, when the manager has an opportunity to misstate earnings, the principal faces another constraint. As the reporting decision has been necessarily delegated to the manager, the "recommended reporting strategy" has to

[13]Same as in the standard principal-agent model, the principal is the residual claimant, and hence entitled to receive the firm's earnings. The one-step departure from the standard model here is that the principal in this model does not observe the earnings when he has to compensate the manager.

be voluntarily followed by the manager:[14]

$$R(y) = \arg\max_{r \in \{L,H\}} u_r - \phi(r - y) \qquad \forall y \in \{L, H\}. \tag{1.3}$$

The optimal contract includes a set of utility promises $\{u_H, u_L\}$ and the recommended action $\{e^*, R(y)\}$.[15] Following convention, we assume that the principal wants to induce high effort, that is, $e^* = h$. There are 4 possible reporting strategies the manager may take:

Strategy 1: Report truthfully, that is, $\{R(H) = H, R(L) = L\}$.

Strategy 2: Report high earnings no matter which level of earnings is realized, that is, $\{R(H) = H, R(L) = H\}$.

Strategy 3: Report low earnings no matter which level of earnings is realized, that is, $\{R(H) = L, R(L) = L\}$.

Strategy 4: Report high earnings if low earnings are realized and report low earnings if high earnings are realized, that is, $\{R(H) = L, R(L) = H\}$.

It is straightforward to see that strategy 3 cannot be achieved without sacrificing effort, and strategy 4 is not possible to implement. The contracting problem is solved by characterizing the optimal payment schedule that implements high effort as well as each of strategy 1 and strategy 2, and then calculating the cost the principal incurs. The recommended reporting choice is the strategy that enables the principal to motivate high effort at the least cost, and the set of utility promises associated

[14] This incentive constraint captures the inapplicability of the Revelation Principle in this setting: the manager is merely allowed to convey his private information on both true earnings and manipulation opportunity using only one single-dimensional signal and must report a mixture of these two in the earnings report. If the Revelation Principle applied, we would have two truth-telling incentive constraints, one for the earnings and another for the realization of manipulation opportunity.

[15] We acknowledge the possibility that the optimal level of effort may change in the presence of manipulation when extending the model to include multiple levels of effort, especially a continuum of effort. For the purpose of this paper, we would focus on the conditional output distributions given effort, specifically the speed at which the output distributions change with effort level, such that the highest effort remains optimal for the principal.

with the recommended reporting choice is the compensation schedule in the optimal contract.

Below we will see that in some situations it is impossible to satisfy (2.1) and (2.3) simultaneously with a truthful report. In such a case, the principal has to endure a falsified report if he wants to implement high effort.

1.3 Results

The principal-agent model is a natural one to study executive compensation; unfortunately, analytical solutions do not readily emerge from an environment where the agent is entitled to make multiple decisions. The current model, however, yields an analytical solution that enlightens the causes of earnings management, and provides testable patterns in compensation structure and reporting practices. To appreciate both the scope and limitations of numerical calculations in Section 4, it helps to first understand the basic incentive structure inherent in the optimal contract.

Figure 2.3 summarizes the main results. The optimal contract is described as the curve ABC. It depicts how the wedge between promised utilities assigned to reports of high and low earnings varies with different values of manipulation cost ψ.

Lemma 1 *If $\psi < c/(p_h - p_l)$, truthful reporting is not implementable.*

Proof: Incentive compatibility constraint on truthful reporting (2.3) is

$$\begin{aligned} u_H - u_L &\leq \psi. \quad \text{If low earnings are realized, and} \\ u_H - u_L &\geq 0. \quad \text{If high earnings are achieved.} \end{aligned}$$

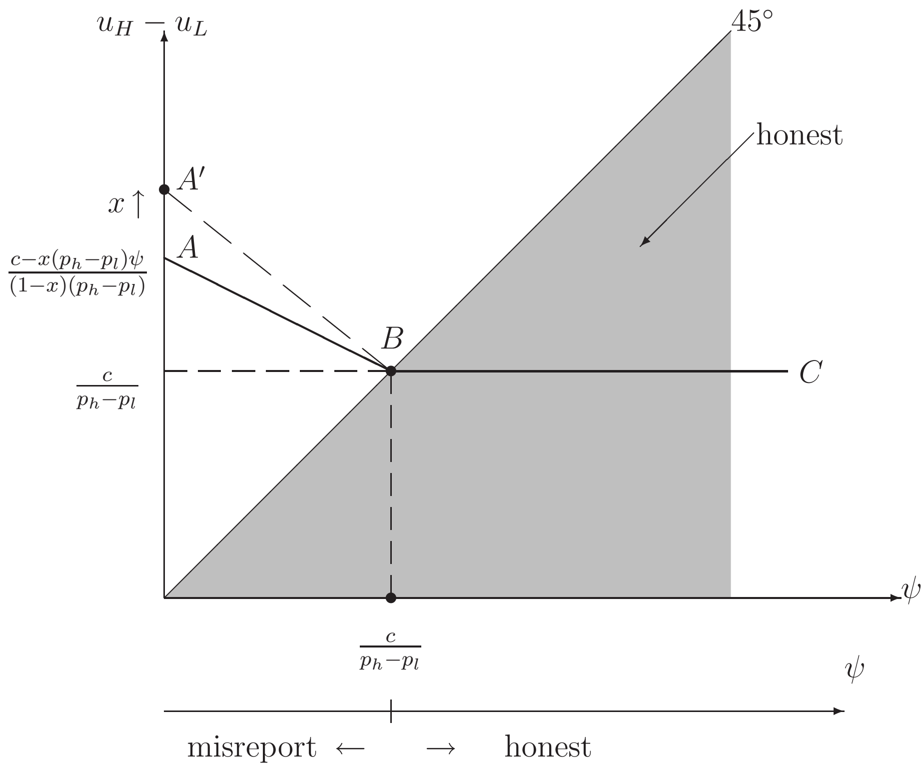

Figure 1.2: Main results

Combining these two, we obtain

$$0 \leq u_H - u_L \leq \psi \tag{1.4}$$

Incentive compatibility constraint on exerting high effort (2.1) is

$$p_h(u_H - c) + (1 - p_h)(u_L - c) \geq p_l u_H + (1 - p_l)u_L,$$

which implies

$$u_H - u_L \geq \frac{c}{p_h - p_l}. \tag{1.5}$$

If $\psi < c/(p_h - p_l)$, incentive constraints on reporting choice (1.4) and effort decision (1.5) can not be satisfied simultaneously, therefore truthful reporting is not feasible.

□

In Figure 2.3, the inequality (1.4) is represented by the shaded area below the 45° line. The inequality (1.5) is represented by the area above the horizontal line at $c/(p_h - p_l)$. For $\psi < c/(p_h - p_l)$, we can see that these areas do not overlap: If the cost of manipulating earnings is small compared to the cost of exerting effort, it is impracticable to implement truthful reporting. When the principal governs the manager's effort and reporting incentives, a sharp conflict arises between the desire of the principal to implement high effort and motivate truthful reporting. The low-output manager will choose to report honestly when the payoff differential, that is, the difference between the payoff following high and low reported earnings, is sufficiently small, in particular, smaller than the cost of manipulation. However, the agent will exert high effort only if the compensation wedge is sufficiently large compared to the cost of effort. The conflict makes it impossible to motivate effort while obtaining truthful reporting in this case, and the manager will always falsify the report when he has a chance to do so. Earnings management thus emerges under the optimal contract.

Lemma 2 *If $\psi < c/(p_h - p_l)$ holds, the optimal contract satisfies*

$$u_H - u_L = \frac{c - x(p_h - p_l)\psi}{(1-x)(p_h - p_l)}. \tag{1.6}$$

Proof: If $\psi < c/(p_h - p_l)$, from Lemma 1, truth-telling is not implementable. The only implementable reporting strategy is $\{R(H) = H, R(L) = H\}$. The incentive compatibility constraint on reporting choice (2.3) becomes

$$\begin{aligned} u_H - u_L &\geq \psi. \quad \text{If low earnings are realized, and} \\ u_H - u_L &\geq 0. \quad \text{If high earnings are achieved.} \end{aligned}$$

Combining these two, we obtain

$$u_H - u_L \geq \psi \tag{1.7}$$

The incentive compatibility constraint on effort decision (2.1) in this case becomes

$$\begin{aligned} &x[p_h(u_H - c) + (1 - p_h)(u_H - \psi - c)] + (1 - x)[p_h(u_H - c) + (1 - p_h)(u_L - c)] \\ &\geq x[p_l u_H + (1 - p_l)(u_H - \psi)] + (1 - x)[p_l u_H + (1 - p_l)u_L], \end{aligned}$$

which can be simplified as

$$u_H - u_L \geq \frac{c - x(p_h - p_l)\psi}{(1 - x)(p_h - p_l)}. \tag{1.8}$$

It must be binding in the optimal contract, and consequently the incentive compatibility constraint on reporting choice (1.7) is automatically satisfied. Suppose that the incentive constraint on effort decision (1.8) is not binding. Then a small reduction in u_H and an increase in u_L that just keep the participation constraint (2.2) satisfied will still satisfy the incentive constraint (2.1). This change will reduce the objective function value—therefore, it causes a contradiction. Hence, the incentive compatibility constraint on effort decision (1.8) is binding under the optimal contract. □

In Figure 2.3, the equation (1.6) is depicted by the line AB. As the principal designs the contract to control for effort choice and reporting behavior, the wedge between utilities assigned to high and low reports crucially depends on the cost of misstating earnings, the cost of putting in effort, and the likelihood of being able to manipulate earnings. Relevant comparative statics are illustrated later in this section.

Lemma 3 *If $\psi \geq c/(p_h - p_l)$, the optimal contract implements truthful reporting.*

Proof: As described in the previous section, there are two possible reporting strategies the principal can implement: One strategy is reporting truthfully, that is, $\{R(H) = H, R(L) = L\}$, and the other choice is to report honestly if high earnings are realized and overstate earnings when low earnings are realized, that is, $\{R(H) = H, R(L) = H\}$.

If $\psi \geq c/(p_h - p_l)$ and the principal implements truthful reporting, the incentive compatibility constraint on effort decision (1.5) is binding, and hence the truthful-reporting constraint (1.4) is automatically satisfied. Suppose that the incentive compatibility constraint on effort decision (1.5) is not binding. Then a small reduction in u_H and an increase in u_L that just keep the participation constraint (2.2) satisfied will still satisfy the incentive constraint (1.5). This change will reduce the objective function value—therefore, it causes a contradiction.

We can then solve for u_H and u_L as follows:

$$\begin{aligned} u_H &= \bar{U} + c + \frac{c(1-p_l)}{p_h - p_l}. \\ u_L &= \bar{U} + c - \frac{cp_l}{p_h - p_l}. \end{aligned}$$

If $\psi \geq c/(p_h - p_l)$ and the principal implements the alternative strategy, $\{R(H) = H, R(L) = H\}$. As shown in Lemma 4, the incentive compatibility constraint on effort decision (1.8) must be binding in the optimal contract, and then the incentive compatibility constraint on reporting choice (1.7) is automatically satisfied. We can solve for u_H and u_L as follows

$$\begin{aligned} u_H &= \bar{U} + c + (1 - p_h)\psi. \\ u_L &= \bar{U} + c - p_h\psi. \end{aligned}$$

Compared to the case with the alternative reporting strategy, implementing truthful-reporting strategy requires a lower u_H and a higher u_L, and hence makes the utility promises more equalized. Given the convex cost of providing utilities, it incurs a lower cost to induce effort with truth-telling strategy. Truthful reporting is the optimal recommended strategy in this case. □

If the cost of misstating earnings is large compared to the cost of exerting effort, it is relatively easy to motivate truthful earnings reports, and honest reporting becomes implementable. When truth-telling strategy is feasible to implement, it is always in the principal's best interest to achieve truthful reporting. The reason is that although manipulation is personally costly to the manager, since the principal must design a compensation contract that meets the manager's participation constraint, the cost of manipulation undertaken by the manager is ultimately borne by the principal.

Proposition 1.1 $u_H > u_L$ *always holds.*

Proof: Straightforward from Lemma 4 and Lemma 3. □

The property implies that similar to the standard contracting problem, the reports of high earnings are associated with a larger compensation in order to motivate the preferable effort.

When does earnings management occur? The following proposition establishes the necessary and sufficient condition for earnings management to emerge under the optimal contract.

Proposition 1.2 *$\psi < c/(p_h - p_l)$ is the necessary and sufficient condition for earnings management to occur under the optimal contract.*

Proof: Straightforward from Lemma 1 and Lemma 3. □

Here, the optimal contract is fully characterized, and the condition for earnings management to take place is derived explicitly. We now consider an economy with a large number of shareholders and managers engaging in such a contracting relationship. There are several implications of the model that are worth noting, stated as follows.

Proposition 1.3 *Suppose that $\psi < c/(p_h - p_l)$ holds. Then $u_H - u_L$ is decreasing in ψ.*

Proof: From Lemma 4,

$$u_H - u_L = \frac{c - x(p_h - p_l)\psi}{(1-x)(p_h - p_l)}, \tag{1.9}$$

and the right-hand-side of (1.9) is decreasing in ψ. □

Suppose that, possibly due to a more stringent accounting rule or corporate governance policy, misstating earnings becomes more costly to the manager. Then, if the low outcome realizes and the manager has an opportunity to misreport, he will over-report but this overstatement of earnings is more costly. The manager has more incentive to avoid the unfavorably high manipulation cost, and this works as an additional incentive for the manager to work hard. Thus, the principal does not have to provide as much monetary incentive (that is, $u_H - u_L$) to satisfy the incentive compatibility constraint (2.1).

Proposition 1.4 *Suppose that $\psi < c/(p_h - p_l)$ holds. Then $u_H - u_L$ is increasing in x.*

Proof: As in Proposition 1.3, (1.9) holds. (1.9) can be rewritten as follows:

$$u_H - u_L = \frac{c - (p_h - p_l)\psi}{(1-x)(p_h - p_l)} + \psi,$$

and the right-hand-side is increasing in x. □

In Figure 2.3, the line AB shifts to $A'B$ as x increases. Suppose that x becomes greater and so the manager is more likely to be able to manipulate earnings. The manager hence enjoys a higher chance of having discretion to overstate earnings in case low earnings are realized, which leads to less incentive for effort under any given compensation schemes. A larger reward for high earnings reports is thus required to incentivize effort, and executive compensation consequently involves more performance pay.[16]

Compared to an economy in the absence of opportunities to manage earnings ($x = 0$), in an economy where it is possible ($x > 0$) and not too costly to manipulate earnings ($\psi < c/(p_h - p_l)$), it is optimal for the shareholders to provide stronger monetary incentives to executives through the compensation packages they offer.

[16]Our result is different from Peng and Roell (2008) in that their equilibrium compensation contract increases the sensitivity of pay to the stock market value only as a result of an reduction in the elasticity of prices to the manager's reports. Unlike theirs, the results in this paper do not require restrictions on the functional forms; especially, we do not impose restrictions on the form of optimal contracts in either binary or continuous case.

When x, the index for the prevalence of earnings management holding corporate productivity constant, rises, the model predicts an executive compensation structure that is more responsive to performance.[17]

Proposition 1.5 *Suppose that each industry is populated by a large number of firms operated by managers with constant disutility from exerting effort but with different levels of manipulation cost ψ. Earnings management is more pervasive in the industry where the cost of exerting effort c is larger.*

Proof: Suppose that in each industry there are different firms, each of which is run by a manager with a different level of manipulation cost ψ_i. Then $\psi^* = c/(p_h - p_l)$ represents the threshold level of manipulation cost below which the manager manipulates earnings, and hence the percentage of managers with $\psi_i < \psi^*$ is the percentage of managers potentially engaging in earnings management in the industry. When spending effort is more costly to the manager, or to put it in another way, it is more difficult to monitor effort in one industry, the threshold ψ^* to achieve truthful reports rises, resulting in a larger fraction of managers with a manipulation cost below the threshold level and therefore a higher prevalence of earnings management in the industry. □

Notice that when it is more difficult to monitor productive effort, it also becomes

[17]It is worth pointing out that the principal in the model compensates the manager with utility promises contingent on earnings reports, and a larger utility differential does not necessarily translate into a larger wage difference. In this model, incentive compatibility constraint on effort choice (2.1) determines utility differential between high and low reports, and participation constraint (2.2) pins down the exact levels of promised utilities

$$\begin{aligned} u_H &= \bar{U} + \frac{c(1-p_l)}{(p_h - p_l)}. \\ u_L &= \bar{U} + \frac{c(1-p_l)}{(p_h - p_l)} - \frac{c - x(p_h - p_l)\psi}{(1-x)(p_h - p_l)}. \end{aligned}$$

Since u_H is independent of ψ or x, the change of $(u_H - u_L)$ is solely due to the change of u_L. It is straightforward to map utility wedge into wage differential in this case.

more difficult to induce truthful reports. Any change in disutility from putting forth effort not only affects the manager's effort decision, but also has a considerable impact on the equilibrium financial reporting strategy. The optimal contract is designed to balance the incentives for productive effort against the incentives to manipulate earnings, and the substitutability of earnings management for productive effort in improving the reported earnings and hence the compensation aggravates the incentive problems. The agency cost is magnified in this environment compared to the standard moral hazard principal-agent model.

Proposition 1.6 *Suppose that the economy is populated by a large number of firms operated by managers that share the same manipulation cost but differ in the cost of exerting effort c. When the cost of manipulation ψ increases, earnings management will become less pervasive in the economy.*

Proof: In an economy where the cost of spending effort c is firm or industry specific, the threshold level of c below which truthful reports are produced is $\psi(p_h - p_l)$. Only the managers with a level of disutility incurred from putting effort less than the threshold level report truthfully. If ψ grows, the percentage of honest managers increases accordingly, and earnings management becomes less prevalent in the economy. □

The aforementioned predictions derived from the model are consistent with empirical evidence, which will be discussed in section 5.

1.4 Extension to continuous earnings

In this section, we extend the two-earnings-level specification to the case with a continuum of earnings. Analogous to the binary model elaborated above, a risk-neutral principal hires a risk-averse, work-averse manager for one period. Expending high effort incurs a utility cost to the manager, that is, $a(h) = c$, while low effort involves no cost: $a(l) = 0$. The manager's unobserved effort decision and an exogenous state realization together determine the firm's earnings, which are privately observed by the manager. The conditional distributions of earnings given high and low effort follow normal distributions: $f(y|e = h) \sim N(\mu_h, \sigma_h)$, $f(y|e = l) \sim N(\mu_l, \sigma_l)$, where $\mu_h > \mu_l$. After the manager exerts effort, he privately learns whether he has an opportunity to inflate earnings in his favor. With probability x, he has discretion to overstate earnings by a certain amount, that is, a constant, a, and there is a utility cost involved in such earnings manipulation $\phi(R(y)-y)$. In particular, $\phi(a) = \psi > 0$. With probability $(1 - x)$, the applicable accounting rules are so hard-and-fast that the manager has no option but truthfully present earnings. The manager's outside option is $\bar{U}$.

We extend the formulation to the case with continuous earnings by characterizing the optimal wage function contingent on the earnings reports. The principal's objective is to minimize the expected wage payment of inducing effort, taking into consideration that the manager has an incentive to exaggerate the firm's earnings. The manager is risk averse with respect to income, and thereby his utility function $u(w)$ is strictly concave with respect to the wage w. The optimal wage function $w(y)$ solves the following problem:

$$\begin{aligned} \min_{w(\cdot)} \quad & E[w(y)|h] \\ & = xE[w(R(y))|h] + (1-x)E[w(y)|h] \end{aligned}$$

subject to

$$h = \arg\max_{e\in\{l,h\}} xE\{u[w(R(y))]-\phi(R(y)-y)-a(e)\}+(1-x)E\{u[w(y)]-a(e)\}, \quad y_e \sim N(\mu_e, \sigma_e). \tag{1.10}$$

$$E[u|h] = xE\{u[w(R(y))] - \phi(R(y) - y) - a(e)|h\} + (1 - x)E\{u[w(y)] - a(e)|h\} \geq \bar{U}. \tag{1.11}$$

The objective function is the expected wage payment for the principal to motivate effort. The first term is the expected payment to implement effort when the manager has an opportunity to artificially inflate earnings, and the second term is the wage if he does not have such an opportunity. The principal designs an optimal compensation contract that satisfies the incentive constraint on the effort decision (2.13) and the participation constraint (2.14). In addition to these constraints, when the manager has an opportunity to exaggerate earnings, the "recommended reporting strategy" has to be the in the best interest of the manager. This incentive constraint on the reporting strategy in the continuous case is:

$$R(y) = \arg\max_{r\in\{y,y+a\}} u[w(r)] - \phi(r - y) \qquad \forall y \sim N(\mu_h, \sigma_h). \tag{1.12}$$

More specifically,

$$R(y) = y \qquad \text{if } u[w(y + a)] - u[w(y)] < \psi \tag{1.13}$$

$$R(y) = y + a \qquad \text{if } u[w(y + a)] - u[w(y)] \geq \psi \tag{1.14}$$

The optimal wage schedule is computed using Simulated Annealing algorithm with Gauss Hermit quadrature. The Schumaker approximation is utilized to preserve the shape of wage functions in interpolation and extrapolation. For the point of illustration, we report the results from a numerical example in which the parameterization is specified as follows: $u(w) = log(w)$, $y_h \sim N(7, 1)$, $y_l \sim N(2, 1)$, $c = 1$, $a = 5$, $\bar{U} = 2$. It is worth noting that the comparative features of the model are robust with respect to functional forms and parameter values.

In Figure 1.4, we compare the optimal wage schedule for the case without earnings management opportunities ($x = 0$), and that for the case where the opportunity to manipulate realizes with certainty and manipulation is costless ($x = 1$ and $\psi = 0$). Changing from the former case to the latter one, the distribution of reports shifts to the right by a in a parallel manner. The optimal wage function correctly discounts the report by the amount of overstatement. Therefore, a parallel rightward shift of wage schedules by a maintains the same distribution of wages and suffices to provide incentives for effort at the least cost. Because the likelihood that the realized output is drawn from the earnings distribution conditional on high effort as opposed to low effort decreases with the report, and decreases at a faster rate as the report falls due to the properties of normal distributions, the optimal contract is strictly increasing and concave.[18] Given the concavity of wages, a parallel shift of the wage function leads to a steeper slope at any particular level of reports, and thus provides sufficient incentives for effort even when it is easier to manage earnings.

As long as $x < 1$, the distribution of reports has a second mode at $\mu_e + a$, $e \in \{l, h\}$. As x increases, there is a greater concentration on the second mode of the reports' distributions. If the opportunity for earnings management becomes more likely, the manager will enjoy higher payments due to the increased density at the second mode. Given the concavity of the wage function, the manager would benefit more from increased chances of manipulating earnings if he exerted low effort. As shown in Figure 1.4, in response to an increase in x, in addition to a parallel rightward shift by $a\Delta x$, the wage schedule will shift downwards to absorb the slackness in the

[18]The discontinuities in the wage function are caused by the discretization of the state space.

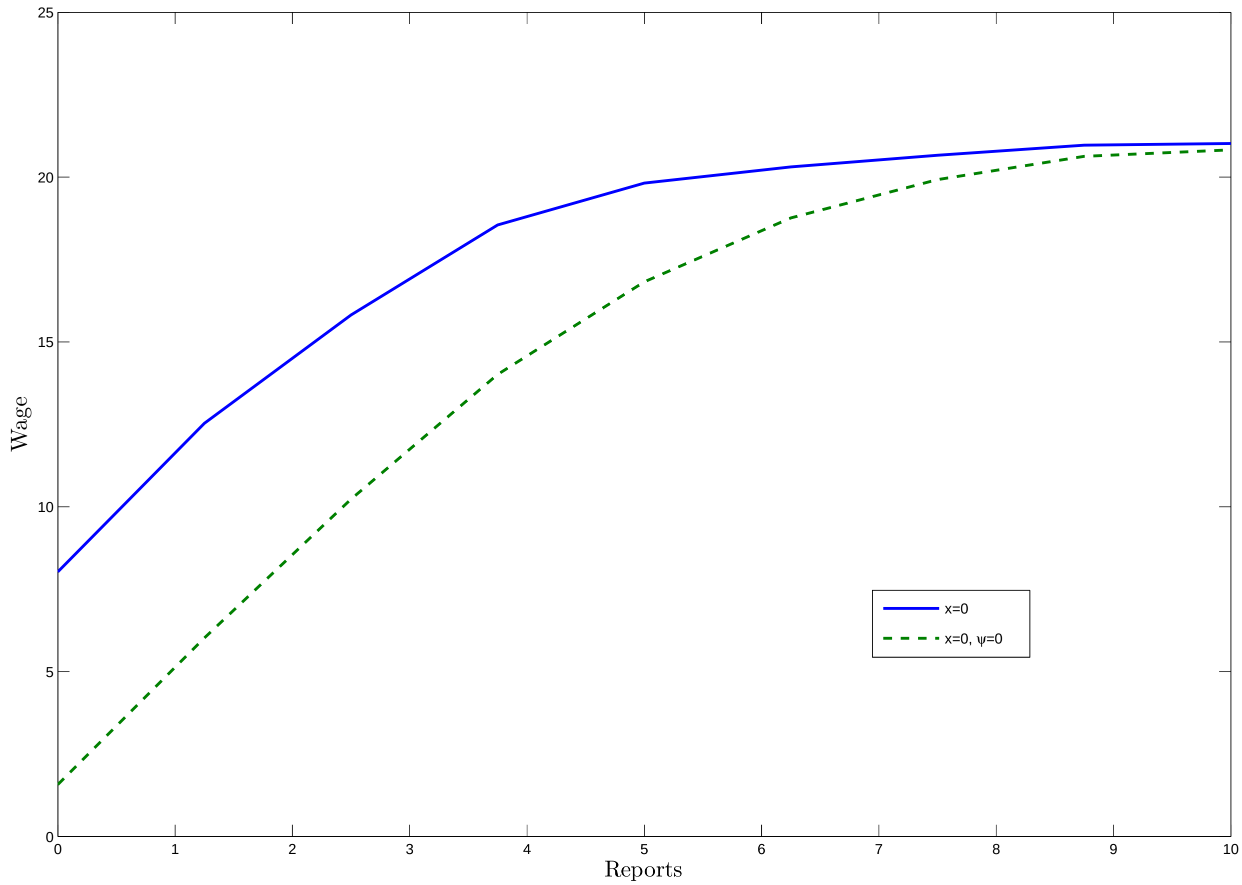

Figure 1.3: Wage function with and without opportunities for costless manipulation

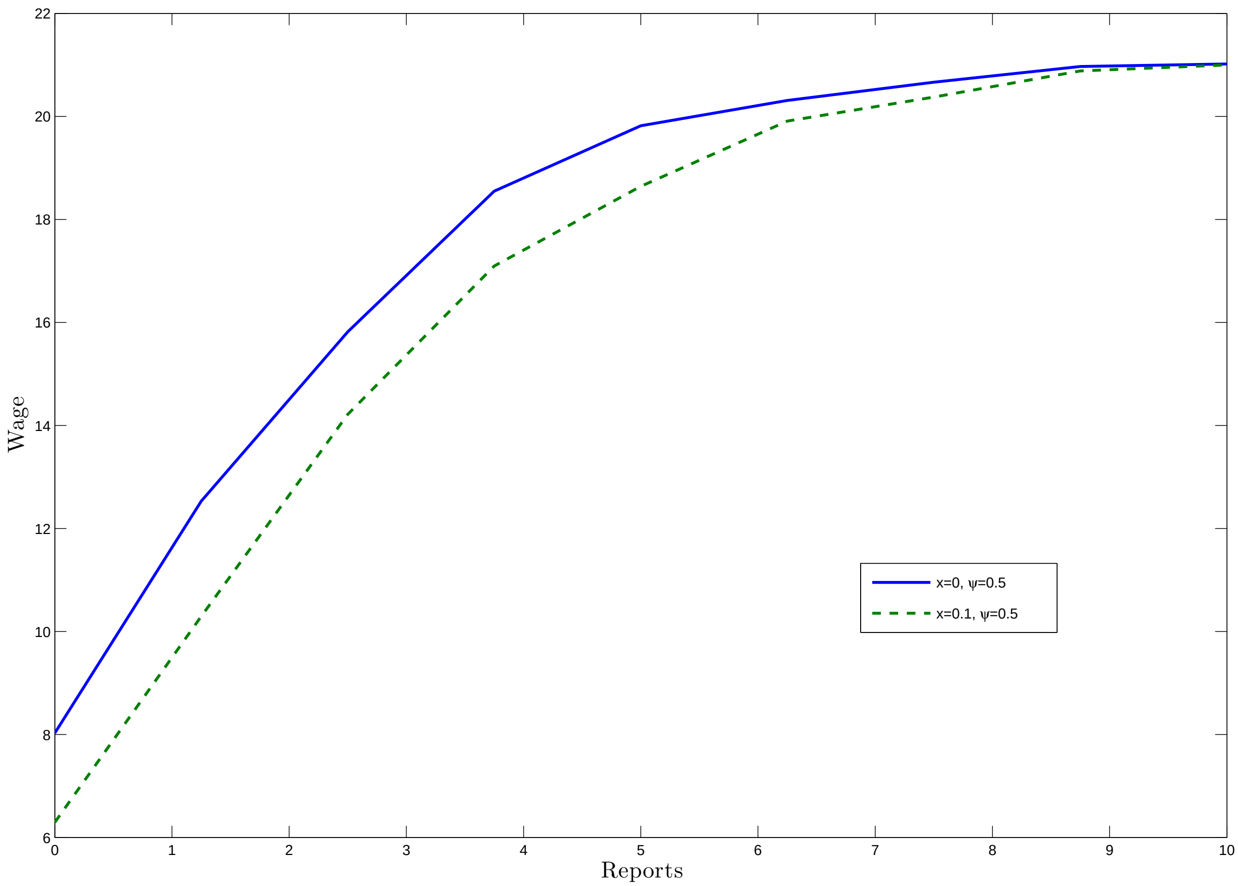

Figure 1.4: Wage function with different x

participation constraint, and tilt upwards to provide enough incentives to exert effort. The enhanced pay-performance sensitivities are in accordance with the binary model results (See Proposition 1.4).

Now we consider the impact of the manipulation cost on the compensation contract. All else equal, if the cost of manipulation ψ is extremely large, the optimal wage function coincides with that in the baseline case without earnings management. Since the condition (2.16) holds over the entire domain of earnings, truthful reporting is always sustained. If the cost associated with overstating earnings is rather small, the condition for earnings management to occur (2.17) is satisfied regardless of actual earnings levels, and the manager overstates earnings whenever he has the chance. The optimal wage schedule in this case discounts the reports equally over the entire domain and shifts downward parallelly by $x * a$. We focus on those values of manipulation cost that lie between the extremes. Over this range, the reporting decision varies with actual earnings, and both truthful reporting and earnings manipulation emerge as equilibrium reporting strategies under the optimal contract. Given a concave utility function, the wage function translates into a set of concave utility promises. As the actual earnings expand, the manager faces a decreasing utility gain and a constant utility cost of overstating earnings. As a consequence, there exists a threshold level of earnings, above which the manager does not find it worthwhile to manipulate earnings and truth-telling strategy is thus maintained. Below this threshold, the manager achieves personal gains from manipulation, and inflates earnings whenever he has a chance.[19]

Figure 5 illustrates how the structure of compensation contract varies with levels of manipulation cost. Everything else equal, a decrease in the cost of earnings management results in an increase in the manager's expected utility under any given compensation schemes. If earnings management becomes less costly to the manager, given compensation schemes, the manager has less incentive to work because of the increased expected wage and hence a lower marginal utility. In order to provide sufficient monetary incentive for effort, the compensation becomes more responsive to earnings reports, leading the range over which the condition (2.17) is satisfied to enlarge. Consequently, earnings management is more pervasive. More specifically, as overstating earnings becomes less costly, the optimal compensation scheme shifts downwards since there is additional slackness in participation constraint, and tilts

[19] In this numerical example, there exists a parameterization such that the threshold level equals the conditional mean of earnings given high effort μ_h. That is the case when we set $x = 0.06$ and $\psi = 1.06$.

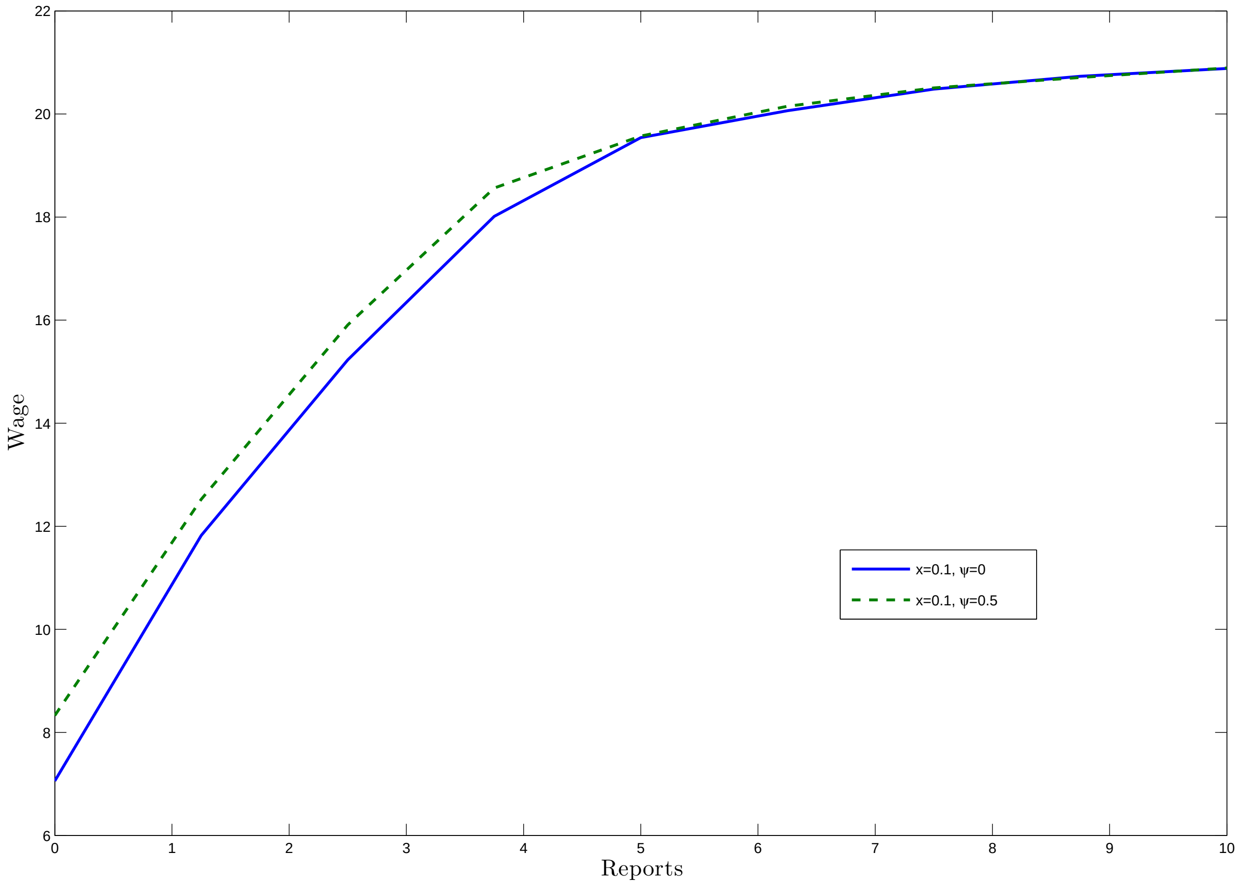

Figure 1.5: Wage function with different ψ

up to motivate effort. The changes of pay-performance sensitivities and earnings management prevalence in response to changes of manipulation cost echo the binary model (See Proposition 1.3 and 1.6).

Figure 6 compares compensation contracts with different levels of a. If the report distribution merely shifted to the right by $x\Delta a$ in response to a change of a, then a parallel shift of wage function would be optimal. However, an increase in the amount of overstatement causes the second mode (the mode created by the possibility of overstatement) of the report distribution to shift outwards by Δa. Others being equal, an increment of the second mode improves the manager's compensation, in

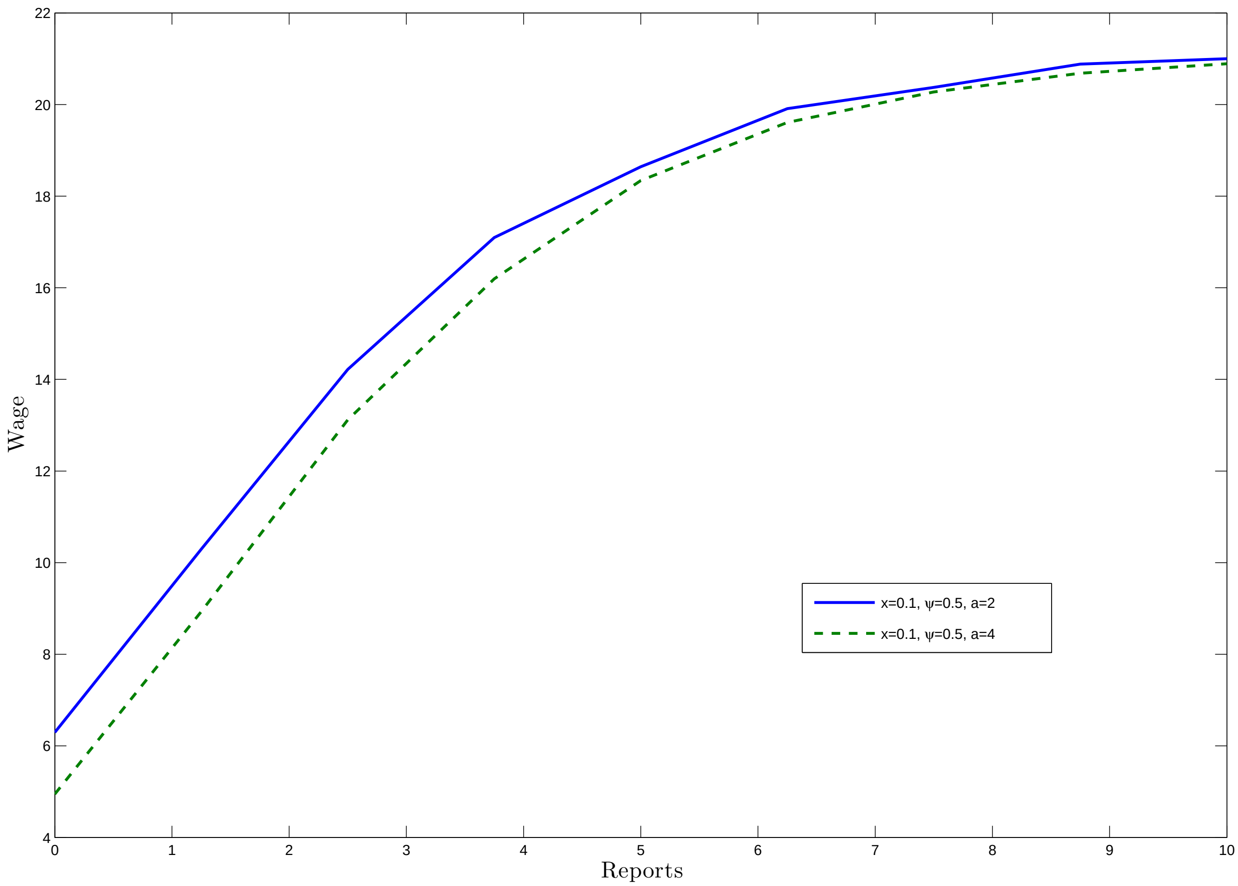

Figure 1.6: Wage function with different a

favor of low effort relative to high effort due to the concavity of contracts. In addition to a parallel shift of wage schedules by $x\Delta a$, the optimal contract has a lower wage and a steeper slope at any given level of reports to extract wage surplus and provide sufficient incentives for effort.

1.5 Empirical implications

The model results illustrated in this paper are useful in classifying the endogeneity of earnings management in executive compensation design and thus help understand some stylized facts on earnings management and executive compensation. In particular, the model provides an explanation for the observed association between earnings management and compensation schemes. The results also have implications for the impact of corporate governance changes on the structure of executive compensation and the prevalence of earnings management.

1.5.1 Positive link between earnings management practice and executive compensation structure

The model has a clear empirical implication described by Proposition 4: a positive relationship between the likelihood of earnings management and the responsiveness of executive compensation to earnings reports. We expect to see that executive pay is more sensitive to performance as earnings management becomes more pervasive. Both time-series and cross-sectional studies provide evidence in support of this prediction.

Time-series trends in earnings management and executive compensation

As indicated by Figure 7 from Cohen et al. (2004), the research documenting the trend in earnings management over time indicates that firms' management of earnings grew steadily from the late 1980s until the passage of the Sarbanes-Oxley Act (Brown 2001; Bartov et al. 2002; Lopez and Rees 2001; Cohen et al. 2004). This literature also provides evidence that managerial propensity to avoid negative earnings surprises has increased significantly over time (Brown 2001; Bartov et al. 2002; Matsumoto 2002). In this strand of the literature, one of the most common metrics used to detect earnings management is the magnitude of discretionary accruals, which proxies for the discretion used by managers to achieve their financial reporting goals.[20] A com-

[20]To identify discretionary accruals, many studies begin with total accruals, measured as the difference between reported net income and cash flows from operations. Total accruals are then

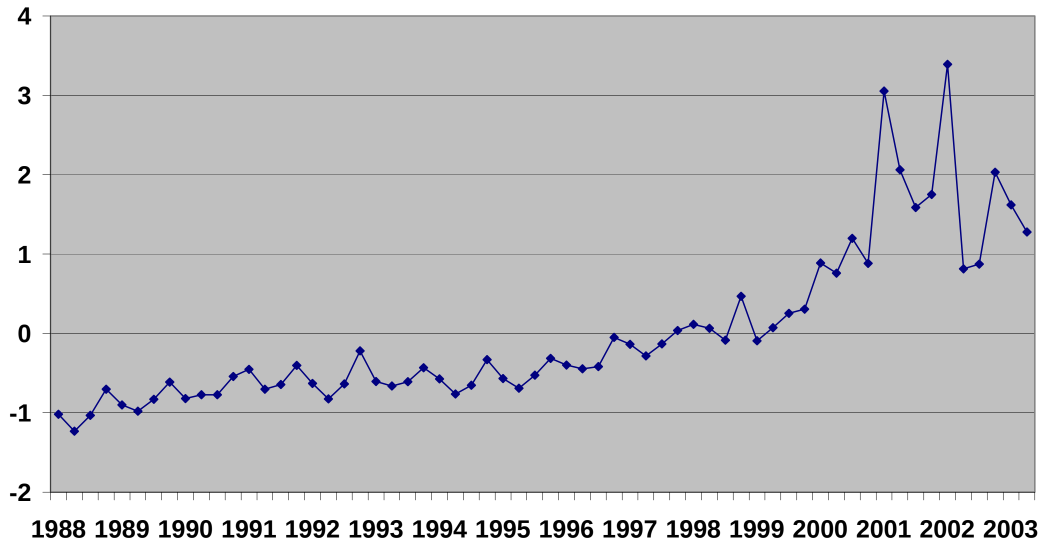

Figure 1.7: Trends in Earnings Management 1987-2003 (Source: Cohen et al. 2005)

monly used approach is Jones (1991) model of accruals, which estimates discretionary accruals as the residual from a regression of total accruals on lagged firm size, the change in firm sales, and gross property plant and equipment scaled by total firm assets.

Meanwhile, one of the striking developments in the American economy since the 1990s has been the enormous increase in performance-based executive compensation(See Figure 8 from Cohen et al. (2005)) . Hall and Liebman (1998) show that the median exposure of CEO wealth to firm value tripled during the 1990s, when stock options replaced base salaries as the single largest component of compensation. Option grants in manufacturing firms swelled from 27% to 36% of total compensation, more than doubling in dollar terms (Muphy 1999; Bergstresser and Philippon 2006). The comovement between the two time series supports the model implication that

regressed on variables that are proxies for normal accruals, such as revenues to allow for typical working capital needs, and gross fixed assets to allow for normal depreciation. Discretionary accruals are the unexplained residual of total accruals.

there is a predictable link between the pervasiveness of earnings management and the use of performance-based compensation.

Cross-sectional studies on earnings management and executive compensation

This model prediction also fits well with the cross-sectional pattern of earnings management and executive compensation found in the data. Gao and Shrieves (2002) utilize ExecuComp and CRSP database, covering 1,200 firms over the period 1992 to 2000. They show that earnings management intensity, as measured by the absolute value of discretionary current accruals scaled by asset size, is related to the managerial compensation contract. They find that the amounts of stock options and bonuses, and the incentive intensity of stock options, are positively related to earnings management intensity, whereas salaries are negatively related.

Bergstresser and Philippon (2006) use the COMPUSTAT and the ExecuComp datasets from the 1990s to assess whether the level of earnings management estimated using Jones (1991) model is related to the use of stock-based CEO compensation. They find evidence that companies with higher levels of earnings management are the companies with more "incentivized" CEOs — those whose overall compensation is more sensitive to company share prices.

It is worth noting that in the literature on manipulation, Goldman and Slezak (2006) produce a reduced pay-performance sensitivity due to precise discounting of overstated performance, and they relate their result to the low sensitivity of pay to firm value reported by Jensen and Murphy (1990) and Murphy (1999). Contrast to theirs, shareholders in our framework cannot perfectly infer the true performance, and therefore a more high powered contract is needed to incentivize effort. We further cite direct empirical evidence on the *relationship* between earnings management and executive pay in both time series and cross section. On a somewhat related note, there is a widely-held view that incentive pay is the *cause* of earnings management. In this paper, we present a model that draws a layered portrait of the underlying causality: Additional incentive is necessary in executive pay in the presence of potential to manipulate, in spite that it leads to manipulation as a side effect. Our approach enhances our understanding of determinants of optimal pay-performance sensitivities and complements the existing view in an interesting and plausible way.

1.5.2 Influence of changes in corporate governance on compensation structure and earnings management

The Sarbanes-Oxley Act of 2002 (SOX) was a regulatory reaction to a number of high-profile corporate accounting scandals, including those affecting Enron, Tyco, and WorldCom. SOX aims to improve the reliability of corporate disclosures by modifying the governance, the reporting, and the disclosure rules for public companies. One of the requirements introduced by SOX is that CEOs and CFOs must certify the veracity of their financial statements (Section 302). SOX also penalizes fraud by requiring the return of incentive-based compensation and profits from stock sales in the event of earnings restatements (Section 304).

The parameter of policy interest is the manipulation cost managers incur when cooking the books, which reflects corporate governance environment determined by external regulatory authorities. The policy implications derived by Proposition 1.3 and Proposition 6 predict that when corporate governance policy becomes more stringent, as is the case with the passage of SOX, executive compensation will be less responsive to performance, and earnings management will be less widespread in the

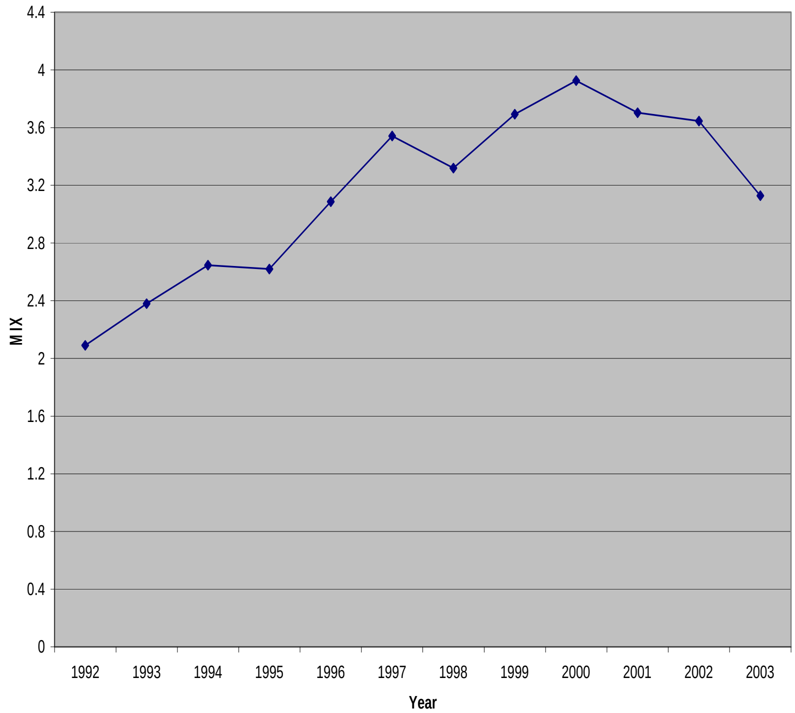

Figure 1.8: Compensation Mix Over Time, 1992-2003
The incentive compensation mix is defined as the ratio of the Black-Scholes value of option grants plus bonus compensation to the salary of CEO (Source: Cohen et al. 2005)

economy.

Cohen et al. (2005) investigate whether there has been a significant change in the structure of compensation subsequent to the passage of SOX (the post-SOX period). They use COMPUSTAT and ExecuComp databases for the period of 1992-2003, and their evidence indicates that the ratio of incentive compensation to fixed salary declined significantly in the post-SOX period, suggesting an increase in base salary as compared to incentive compensation (See Figure 8 from Cohen et al (2005)). Narayanan and Seyhun (2005) also show that the average grant size has steadily declined since the passage of SOX, and the total number of options granted shows a downward trend in the post-SOX period. In addition, Bayer and Burhop (2008) study a major shift in legal rules of corporate governance in Germany and document a significant decrease in pay-performance sensitivities of executive pay after the reform. We interpret these results as evidence in line with the model prediction that more stringent corporate governance leads to a reduction in the proportion of performance-based compensation.

The empirical studies on earnings management practices in response to SOX provide evidence compatible with our results as well. Cohen et al. (2004) use a modified cross-sectional Jones model to examine the level of earnings management surrounding the passage of SOX. They focus on earnings management and the informativeness of accounting disclosures across two main time periods: the pre-SOX period extending from Q1 1987 through Q2 2002, and the post-SOX period extending from Q3 2002 through Q4 2003. They also find that earnings management reversed abruptly after the passage of SOX, and the result is robust to industry performance (See Figure 7). Narayanan and Seyhun (2005) analyze managerial influencing of grant date stock prices based on a database of over 569,000 option grants reported to the SEC by insiders, and their results suggest that managerial influencing of stock prices is still prevalent after SOX, but it has diminished in magnitude.

1.6 Conclusion

In this paper, we model the shareholders-manager relationship to study the implications of earnings management for executive compensation and analyze the responses of compensation schemes to corporate governance reforms. We show that equilibrium pay-performance sensitivities increase with manipulation of corporate performance, consistent with empirical evidence in both time series and cross section. In addition, we find that public policies that impose additional manipulation cost on executives lead to less incentive pay. The time-series variations in the structure of executive compensation after legal reforms of corporate governance follow the pattern predicted by the model.

We apply the model to illustrate the incentive structure inherent in the optimal contract that controls for productive effort and managerial reporting. In this model of managerial reporting under moral hazard, productive effort and earnings management are substitutes to the manager in the sense that they both improve reported performance and thus the compensation. Earnings management undermines incentives to put forth effort, and therefore exacerbates the agency problem. Using a tension between inducing managerial effort and motivating truthful reporting, we explicitly derive the conditions under which eliciting the truth is not feasible, and earnings management is thereby sustainable as equilibrium behavior.

The crucial feature we focus on here is that the principal does not observe actual earnings when he has to compensate the manager. It would provide useful insights to extend the model to a dynamic setting where the reconciliation of discrepancies between reported earnings and true earnings can be explicitly addressed.

Finally, it should be noted that the model developed here may be useful for studying the issue of how incentives and pay vary across firms. It it natural to think that management manipulation of financial records incurs a larger personal cost in the firms with more effective internal corporate governance. One way to further test the model implications is to check indicators of strong governance, such as the proportion of independent directors, efficiency of internal audit, and frequency of BOD meetings, and explore whether they are associated with less incentive pay in mangerial compensation contracts. Application of this model to these issues would be interesting avenue for future research.

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Chapter 2

Asset Returns with Earnings Management

This chapter investigates stock return dynamics in an environment where executives have an incentive to maximize their compensation by artificially inflating earnings. A principal-agent model with financial reporting and managerial effort is embedded in a Lucas asset-pricing model with periodic revelations of the firm's underlying profitability. The return process generated from the model is consistent with a range of financial anomalies observed in the return data: volatility clustering, asymmetric volatility, and increased idiosyncratic volatility. The calibration results further indicate that earnings management by individual firms does not only deliver the observed features in their own stocks, but can also be strong enough to generate market-wide patterns.

2.1 Introduction

[1]Executives' desire to use financial reports, especially bottom-line earnings, to pursue their own financial interests gives rise to the phenomenon of *earnings management*, which is defined as intentional manipulation of reported earnings by knowingly choosing accounting methods and estimates that do not accurately reflect the firm's underlying fundamentals. The accounting irregularities at Enron and WorldCom that precipitated the stock market downturn of 2002 and the corporate scandals that triggered the financial meltdown in 2008, notably Freddie Mac and AIG,[2] indicate that such behavior can engender significant economic consequences, especially in the financial markets. This paper explicitly examines the asset pricing implications of earnings management.

This intentional manipulation of financial information must be reflected in the pricing of stocks, since it affects the inference of the investors who value the stock

[1]I sincerely thank the seminar participants at Boston University, Carnegie Mellon University, Cornell Univerity, Darden School of Business and Department of Economics of the University of Virginia, INSEAD, Peking University, the Federal Reserve Board of Governors, the Federal Reserve Bank of Cleveland, the Federal Reserve Bank of Richmond, the 2008 Midwest Theory Conference, the Robert H. Smith School of Business at the University of Maryland, the University of Chicago Booth School of Business, the Wharton School of Business of the University of Pennsylvania, the World Bank, and Tsinghua University. While I cannot name everyone, I must single out some for particular thanks. I am greatly indebted to Stanley Baiman, Philip Berger, David Bowman, Mark Carey, Pingyang Gao, Jonathan Glover, Richard A. Lambert, Nathan Larson, Pierre Liang, Yohei Okawa, Michael G. Palumbo, Nagpurnanand R. Prabhala, Ponpoje Porapakkarm, Haresh Sapra, Catherine Schrand, Abbie Smith, Xuan Tam, and Robert E. Verrecchia for their helpful comments.

[2]Morgan Stanley determined the accounting tactics, while legal, enabled Freddie Mac, and to a lesser extent Fannie Mae, to overstate the value of their reserves. Both companies also pushed inevitable losses into future by sharply curtailing their repurchase of soured mortgages out of the securitizations they have guaranteed. "Fannie Mae and Freddie Mac were 'playing games with their accounting' to meet reserve requirements, prompting the government to seize control of the companies," U.S. Senator Richard Shelby said (Bloomberg [September 9, 2008]). In the case of AIG, PricewaterhouseCoopers prompted an announcement about the material accounting weaknesses related to the valuation of AIG's derivatives holdings. Prosecutors insisted that five former executives from the American International Group deliberately mounted a fraud to manipulate its financial statements, after a string of AIG scandals early this decade."Accounting flaws at American International Group significantly understated the insurance giant's losses on complex financial instruments linked to mortgages and corporate debt," AIG Said in an official public statement (The New York Times [February 12, 2008]).

of a firm. Empirical studies (e.g., Turner et al. [2001], Wu [2002], and Palmrose et al. [2004]) suggest that distorted information flow can cause adverse capital market reactions. In these studies, on average, stock returns fall by about 10% on the days around earnings restatement announcements. Figure 2.1, reproduced from Wu [2002], documents how stock returns react to restatements.[3] However, due to the lack of theoretical guidance and difficulty of detecting earnings management with accuracy, comparatively little is known about the potential systematic impact of earnings management on stocks.

The objective of the present study is to analyze the implications of earnings management for dynamic return patterns. In particular, this paper shows that earnings management is a possible explanation for a number of stylized financial facts, namely, volatility clustering, asymmetric volatility, and increased idiosyncratic volatility. These results underscore why earnings management is of central importance in pricing financial assets, in understanding the risk implied by empirical financial anomalies, and in contemplating the ongoing debate on regulations of financial markets and executive compensation.

I conduct this exercise within a Lucas asset-pricing model that is standard in all aspects, except that the investors hire a manager to operate the firm and report the firm's earnings. In particular, a principal-agent model with financial reporting and productive effort is embedded in a simple variant of the Lucas asset-pricing model. As in the standard Lucas asset-pricing model, a group of investors is the representative owner of an apple orchard. The infinitely-lived trees (firms) produce perishable apples (earnings) each period, and the harvest varies from period to period depending on the weather. In contrast with the standard Lucas model, the process of apple production

[3] I thank Min Wu for providing Figure 3 of Wu [2002], which is reproduced as Figure 1 in the current paper.

Figure 2.1: Cumulative abnormal returns around restatements: day (-125,+125)
Source: Wu [2002]

is not entirely exogenous in the current paper. The manager exerts an unobserved effort that affects the production, and possibly has discretion over the quantity of apples reported to the investors. The reported earnings are paid to the investors as dividends. The investors engage in a (single-period) contractual relationship with a newly hired manager in every period and pay the manager a fraction of the reported earnings as compensation. The key feature I focus on here is the manager's ability to manipulate earnings reports. Earnings management occurs in the model when the reported apple harvest (earnings) differs from the true amount.[4]

There are periodic investigations concerning the underlying true earnings of the firm. In the final period of each auditing cycle, the uncertainty about true earnings is resolved, and the investors bear monetary costs in the event that earnings management is detected.[5] The investors are assumed to be risk-neutral; thus the price of the firm in each period is given by the discounted expected future dividends net of the labor wage and the financial loss associated with earnings management.

The return sequences generated from the model mimic a set of stylized facts in stock return data. First and foremost, the model returns exhibit volatility clustering. Because earnings management patterns vary with underlying true performance, certain levels of earnings lead to higher frequency of restatements than others,[6] creating

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[4]The modeling technique presented here bears some similarities with Shorish and Spear [2005]. The similarities and differences between their paper and this paper will be discussed later in this section.

[5]This analysis does not make a distinction between earnings management and fraud. While the accounting choices that explicitly violate Generally Accepted Accounting Principles (GAAP) clearly constitute both earnings management and fraud, according to the SEC, systematic choices made within the boundries of GAAP can constitute earnings management as long as they are used to obscure the true performance of a firm and will lead to adverse consequences for the firm in the same way as fraud.

Following this notion, there is no economic difference between fraud and earnings management in the model: in both cases the reported number is different from the true amount, and such behavior hurts the firm's future prospects.

[6]The notion of "restatements" in the paper does not necessarily imply actual restatement announcements but rather the broadly-defined adverse consequences of earnings management. The periodic revelations of true earnings in the model hence capture the negative consequences of earn-

larger swings in the return sequence. Return volatility becomes state-dependent in the model. As the state (that is, actual earnings) exhibits persistence over time, return volatility is time-varying and persistent. In addition to the direct impact due to possible *future* manipulation, an indirect effect reflecting suspicion of *previous* misreporting amplifies the persistence in volatility. The possibility of earnings management creates a range of reports that are associated with belief revision and intense suspicion of manipulation. The anticipation of restatements increases uncertainty and hence volatility. The volatility persists as reported earnings persist. Although the conditional heteroskedasticity observed in many financial markets has led to ARCH and GARCH models that are intensively used in analyzing stock returns, the underlying microeconomic motives are still not well understood. This paper presents the persistence in earnings management behavior as a likely source of the persistence in stock return volatility.

The model data capture another stylized fact in the finance literature: asymmetric volatility in stock returns. The mechanism is also twofold. First, earnings management goes hand-in-hand with weak economic performance, due to stronger financial incentives to inflate earnings when the performance is weaker. Because current low earnings lead to more frequent future earnings manipulation and resultant drastic consequences, low returns lead to high volatility in subsequent returns. Second, earnings reports within certain range are viewed as symptomatic of intentional misstatement. The inference of earnings management reduces the current price and increases the uncertainty over subsequent outcomes, thereby intensifying asymmetric volatility.[7] The existing literature on asymmetric volatility falls into two categories:

ings management that periodically show up in returns and can be understood as reflecting the reversing nature of earnings management.

[7]Following Shin [2003], Rogers et al. [2007] empirically document that strategic disclosure, defined as the reporting of good news and the withholding of bad news, provides an explanation for asymmetric return volatility. They find that asymmetric volatility is more pronounced in the return

the leverage effect proposed by Black and Scholes [1973], Merton [1974], and Black [1976] and the volatility feedback effect put forward by French et al. [1987] and Campbell and Hentschel [1992]. However, Christie [1982] and Schwert [1989] find that the leverage effect is too small to account for the asymmetry in volatility, and Campbell and Hentschel [1992] find that the volatility feedback effect normally has little impact on returns. This paper shows that a mechanism exists for earnings management to generate the observed asymmetric behavior in stock returns. The calibration results further suggest that this channel can be quantitatively important.

Last but not least important, as earnings management becomes more likely in the model, asset returns exhibit greater volatility. The dramatic consequences of earnings management generate active fluctuations in the return sequence and thus intensify return volatility. This work adds to a growing literature that studies individual stock return volatility. Campbell et al. [2001] document that the level of average stock return volatility increased considerably from 1962 to 1997 in the United States. Furthermore, most of this increase is attributable to idiosyncratic stock return volatility as opposed to the volatility of the stock market indices. Rajgopal and Venkatachalam [2008] explore whether deteriorating financial reporting quality, as measured by earnings quality and dispersion in analyst forecasts of future earnings, can plausibly explain the increase in idiosyncratic volatility over the past four decades. Their results from cross-sectional and time-series regressions indicate a strong association between idiosyncratic return volatility and financial reporting quality. The current model replicates the positive relationship between the likelihood of earnings manage-

series of individual firms that are more likely to disclose strategically as measured by their litigation risk incentives. Patterns in return volatility in market indices are also consistent with strategic disclosure as an explanation. As earnings management represents strategic decisions in mandatory reporting, different from strategic disclosure with verifiable reports, I do not present their findings as direct empirical evidence for this model. However, their paper suggests that financial reporting decisions can matter in generating the observed patterns in stock returns, in line with the prediction of the current model.

ment and the volatility of individual returns, and thus contributes to the theoretical explanations of the data.

In this paper, the contracting system in a principal-agent model with financial reporting and moral hazard is first examined as a point of departure. This principal-agent model is developed and analyzed in greater detail in Sun [2008]. The purpose of this step is to provide the underlying economic motive for earnings management in the model, to understand how motives to induce managerial effort and to motivate truthful reports differentially affect the optimal contract, and to identify how earnings management decisions vary with actual economic performance. This principal-agent model lays out a micro-foundation for asset pricing in that it generates a set of earnings reports that may or may not be systematically biased. This model of managerial reporting under moral hazard is built on Dye [1988]. The message space is limited to a single-dimensional signal while the privately informed agent receives two dimensions of private information; therefore the Revelation Principle is not applicable.[8]

In order to highlight the role that earnings management plays in price formulation, the principal-agent model with reporting choices is embedded into an otherwise standard Lucas asset-pricing model. In particular, by switching on and off the measure for earnings management in the model, I maintain the focus on earnings management and make the comparison with the standard asset-pricing model transparent. This

[8] A recent paper by Crocker and Slemrod [2007] considers an alternative environment where the Revelation Principle can be applied. In solving the model, they assume a monotonically increasing reporting function; actual earnings can therefore be recovered by inverting the reporting function. In their setting, the principal knows the exact amount of actual earnings as a function of the report, while in the current model the principal faces uncertainty over whether earnings management occurs. The manager possesses a second dimension of private information in this model, and hence the reporting function is no longer invertible. As a model that constructs an explanation for earnings management, the current contract work can be viewed as complementary to theirs. As a microeconomic foundation for the investigation into asset pricing with earnings management, their model would generate prices that are fully revealing in the equilibrium; whereas the investors in this model try to infer the true outcomes through Bayesian learning, but cannot perfectly see through earnings management.

modeling approach is related to Shorish and Spear [2005], where the owner of the firm hires a manager to maximize the firm's value, and there is asymmetric information about the manager's effort level between the owner and the manager. Along this line of agency-based asset pricing, Gorton and He [2006] show that when compensation depends on the firm's market performance, stock prices are set to induce the optimal effort level. In contrast with these papers, the current paper focuses on earnings management incentive in the contractual relationship and price formulation by assuming additional asymmetric information regarding output realizations.

This analysis also relates to the literature on asset pricing under asymmetric information, such as Detemple [1986], Wang [1993], and Cecchetti et al. [2000]. In particular, Wang [1993] presents a dynamic asset-pricing model in which the investors can be either informed or uninformed: the informed investors know the future dividend growth rate, while the uninformed investors do not. He finds that the existence of uninformed investors can lead to risk premia much higher than those under symmetric and perfect information. Distinguished from previous studies that examine the impact of information asymmetry and heterogeneous beliefs among investors, the study reported in this paper analyzes information asymmetry between corporate executives and outside investors as a whole.

There have not been many theoretical studies that examine the economic impact of earnings management. Fischer and Verrecchia [2000] is an early and notable exception. They show that more bias in the report reduces the correlation between share price and reported earnings, and they also study how the cost to the manager of biasing the report and the market's uncertainty about the manager's objective affect the slope and the intercept term in a regression of market price on earnings reports. Subsequently, Guttman et al. [2006] use a signaling model similar to Fischer and Verrecchia [2000] to explain the discontinuity observed in the distribution of reports. While these papers do not model the contractual relationship between shareholders and the manager, Kwon and Yeo [2008] consider a single-period model where the principal takes into account how compensation affects productive effort and market expectations when designing the optimal contract. In their paper, a rational market can simply recalibrate or discount the reported performance when the manager overstates earnings, and correctly guess the true performance. They show that such rational market discounting leads to less productive effort by the manager and less performance pay by the principal. In contrast with the studies presented in these papers, the current study considers stock returns under earnings management in a dynamic setting, with a central focus on the return properties beyond the first moment. This study further provides a quantitative evaluation of the model.

Existing studies have analyzed earnings management behavior and stylized financial facts in isolation, and a systematic investigation into the link between earnings management and financial anomalies has not yet been undertaken. By incorporating earnings management into an otherwise standard asset-pricing model, this paper presents a mechanism through which financial misrepresentation may lead to a set of stylized financial facts. This paper suggests that there may be a unifying cause for these empirical regularities in the financial markets. In addition, the calibration results indicate that earnings management can be quantitatively important in explaining dynamic return patterns. This quantitative analysis further suggests that earnings management by individual firms may not only generate patterns in their own stock returns, but also be powerful enough to drive the observed effects in stock market indices.

The remainder of this paper proceeds as follows. Section 2 lays out the setup of the model. Section 3 discusses the general results and presents the properties of simulated returns from the model. As one step toward calibration, Section 4 extends the model to continuous earnings. Section 5 presents a quantitative evaluation of the model. Section 6 checks the robustness of model dynamics by adopting an alternative calibration strategy and incorporating stochastic investigation. Section 7 contains concluding remarks.

2.2 Model

The core of this paper is based on a Lucas asset-pricing model in which the investors hire a manager to operate the firm and report the firm's earnings. The investors design a contract that controls the manager's effort decision and reporting choice. In every period, the principal (investors) offers a newly hired manager a single-period contract. Earnings y are stochastic and take two possible values, $y \in \{l, h\}$, where $l < h$. The firm's production is associated with a simple Markov process:

$$\Pr(y_{t+1} = j | y_t = i) = \pi_{ij}, \qquad \forall i \in \{l, h\}, \quad \forall j \in \{l, h\}$$

The manager makes earnings announcements, and the reported earnings $R(y)$ are then paid out as dividends to the investors.[9] The underlying true earnings are periodically revealed.[10] For the purpose of illustration as well as tractability, it is assumed that after every two periods the uncertainty about the underlying earnings in the past two periods is resolved, and the investors bear financial losses if earnings management

[9] This analysis does not explicitly model how the manager finances the discrepancy in the reports. In reality, the manager can obtain funds from the firm's suppliers by talking them into some sham transactions or borrow money from banks. Although without the active help from suppliers and banks, companies could not have deceived investors and analysts alike, a recent Supreme Court ruling shields third parties, including suppliers and banks, from being held responsible for knowingly participating in financial data manipulation. The source of funds is therefore chosen to be left outside the model for simplification without causing any modeling inconsistencies.

[10] The model results do not hinge on the particular time structure of information disclosure. In Section 6.2, I examine a version of the model where investigation is conducted on a stochastic basis in each period, and the results are robust to this change in the assumption.

is detected. The investors know the auditing periodicity. The price of the firm in each period is given by discounted expected future dividends net of the executive compensation and monetary costs of earnings management.

One interpretation of the model is that the manager finances the discrepancy in the report from a market outside the economy, and the firm's owner (the investors) must repay a large amount of money at the time earnings management is detected (this is a part of the monetary loss that the investors have to bear during investigations). Because the current manager is replaced in the next period, the significant repayment burden imposed on the investors does not directly affect the manager's incentive.[11] Another, and much broader, interpretation of the model is that the manager may engage in activities that boost current earnings at the expense of future (long-term) benefits. In particular, the manager may follow myopic strategies and take economically suboptimal actions to inflate current earnings, such as forsaking profitable investment and postponing R&D and capital spending plans.[12] This interpretation corresponds to a more general notion of earnings management this model captures, which is an overstatement of current earnings that has negative consequences for the firm's future prospects.

2.2.1 Optimal contract

The contractual environment follows Sun [2008]. A risk-neutral principal (investors) hires a risk-averse agent (manager) for one period. Figure 2.2 details the timeline of

[11] The model let the financial cost of earnings management almost entirely fall on the investors by dismissing the manager before the investigation, which reflects a general view that investors suffer most from earnings manipulation (please see footnote 20 for more details) whereas executives tend to absorb personal gains. While a dynamic contract with auditing technology can be an interesting extension of the model, the current version suffices in delivering overstatement in the equilibrium so as to derive its asset pricing implication.

[12] Grahama et al. [2005] document that 78% of executives in the survey admit to sacrificing long-term value to maintain predictability in earnings.

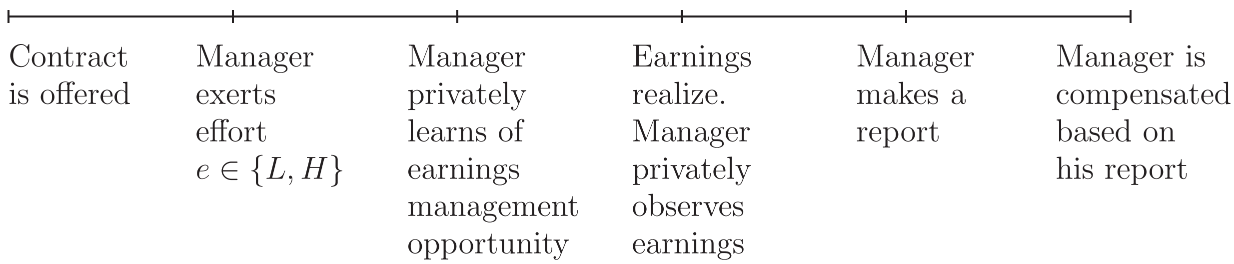

Figure 2.2: Timeline of contracting within each period

the contracting arrangement between the principal and the manager. In the beginning of each period, the manager accepts the take-it-or-leave-it contract offered by the principal for one period. Earnings are stochastic and influenced by the manager's effort. The unobserved effort level of the manager, e, can take two values, low (L) and high (H). The manager incurs disuility from exerting effort, denoted by the cost function $a(e)$. In particular, high effort is associated with a cost of $a(H) = c$, and low effort involves no cost: $a(L) = 0$. Earnings take two possible values, represented by $y \in \{l, h\}$, where $l < h$. Let p_e be the probability that earnings are h when the effort is e, with $p_H > p_L$. After exerting effort, the manager privately learns whether he has the opportunity to manage earnings. With probability x, the manager has discretion over how much earnings to report.[13] With probability $(1-x)$, the manager is prohibited from manipulating earnings. Then the manager privately observes the

[13] Here, whether the manager has the opportunity of managing earnings is assumed to be a random event, and the outcome is the manager's private information. Generally Accepted Accounting Principles (GAAP) provide guidelines on how to record and summarize each type of economic transaction, and hence define the accounting latitude available to senior management in financial reporting. In practice, certain economic activities, those where there is no hard-and-fast rule for which accounting method to use, lead to more discretion than others. In any particular period, economic transactions of this type may or may not take place. By virtue of being closer to the operations process, only the manager knows the extent of these activities and hence the degree of reporting latitude available.

In modelling language, the stochastic opportunity to manage earnings adds an additional noise in financial reports that investors cannot perfectly filter out. Due to the additional uncertainty, the investors need to make inferences as to whether earnings management occurs, and earnings management is not fully unravelled in the equilibrium. Alternatively, an environment where the opportunity to manipulate reports realizes with certainty while the size of manipulation is stochastic would be essentially identical to the current model.

earnings, and makes an earnings announcement.

If the manager produces an inaccurate report, the manager incurs a personal cost, denoted by $\phi(\cdot)$. ϕ is a function of the discrepancy between true earnings and reported earnings. When the manager reports honestly, he incurs no cost: $\phi(0) = 0$.[14] When the manager overstates earnings, there is a positive cost $\phi(h - l) = \psi > 0$. Earnings management occurs in the model when the reported earnings differ from true earnings. More specifically, earnings management emerges in this environment if the manager announces that high earnings (h) have been achieved when the actual realization of earnings is low (l).

As the contract must be designed based on mutually observed variables, the manager's compensation can be based only on the earnings report. As long as the manager's reported earnings fall in the set $\{l, h\}$, the principal cannot directly detect whether the manager has misstated earnings.[15] It is also assumed that the manager is essential to the operation of the firm, so the contract must be such that the manager (weakly) prefers to work for the principal regardless of whether the manager gains the opportunity to manage earnings.

To distinguish from high and low actual earnings, high and low reported earnings are denoted by $\tilde{h}$ and by $\tilde{l}$. The contract between the risk-neutral principal and the risk-averse agent includes a set of wages contingent on the reports, which can be

[14] There are two frictions in the model that restrain earnings management: earnings management opportunity that realizes with probability x and the cost involved in misstating earnings ϕ. This model can be also considered with only one friction: the cost of manipulation with a simple stochastic structure. The manipulation cost now in the model follows a binary distribution with two possible realizations ∞ and ψ.

[15] Following Dye [1988], the model presented in this section places restrictions on the manager's ability to communicate the truth. In addition to the unobserved effort level, the manager observes two dimensions of information, the value of actual earnings and the realization of misstatement opportunity. However, the manager is permitted to communicate only a one-dimensional signal, which is an earnings announcement. Communication is restricted in that the manager cannot fully communicate the full dimensionality of his information, and hence the Revelation Principle is not applicable.

alternatively characterized as a set of contingent utilities. The manager's utility level corresponding to compensation level w_i, $i \in \{\tilde{l}, \tilde{h}\}$, is denoted as $U(w_i) = u_i$, where $U(\cdot)$ is a strictly increasing and strictly concave utility function. Let $U^{-1}(\cdot) = V(\cdot)$. Then $V(u_i)$ is the cost to the principal of providing the agent with utility u_i. Because $U(\cdot)$ is a strictly increasing and strictly concave function, $V(\cdot)$ is a strictly increasing and strictly convex function.

In this environment, the contract must not only induce effort but also control for the manager's reporting incentive. This study assumes that the difference in the earnings is large enough that the principal always wants to implement high effort. The objective of the manager is to maximize utility by choosing a level of effort and a reporting strategy represented by $R(y)$, subject to the contract offered. When the manager has no discretion, we denote the report by $\bar{R}(h)$. By assumption, $\bar{R}(h) = \tilde{h}, \bar{R}(l) = \tilde{l}$. The manager's utility is of the form $U_m(e, R(y)) = xE[u_{R(y)} - \phi(R(y) - y) - a(e)] + (1-x)E[u_{\bar{R}(y)} - a(e)]$. The first term is the manager's expected utility if the manager has sufficient discretion over reporting. The second term is the manager's expected utility if the manager has to truthfully report. The principal chooses the utility values u_i, $i \in \{\tilde{l}, \tilde{h}\}$, and recommended reporting choices $R(y)$ for each realization of earnings that minimize the expected cost of inducing effort.[16]

Formally, the optimal contract solves

$$\min_{u_{\tilde{h}}, u_{\tilde{l}}, R(h), R(l)} \quad E[V(u)|H]$$
$$= x[p_H V(u_{R(h)}) + (1 - p_H)V(u_{R(l)})] + (1 - x)[p_H V(u_{\tilde{h}}) + (1 - p_H)V(u_{\tilde{l}})]$$

[16]As in the standard principal-agent model, the principal is the residual claimant, and hence entitled to receive the firm's earnings. The one-step departure from the standard model here is that the principal in this model does not observe the true earnings when the principal has to compensate the manager.

subject to

$$H = \arg\max_{e \in \{L,H\}} xE[u_{R(y)} - \phi(R(y) - y) - a(e)] + (1-x)E[u_{\bar{R}(y)} - a(e)], \qquad \forall y \in \{l, h\}. \tag{2.1}$$

$$E[u|H] = xE[u_{R(y)} - \phi(R(y) - y) - a(e)|H] + (1-x)E[u_{\bar{R}(y)} - a(e)|H] \geq \bar{U}. \tag{2.2}$$

The objective function is the expected cost for the principal to motivate high effort. The first term is the cost of implementing high effort when the manager has an opportunity to manage earnings, and the second term is the cost if the manager does not have the opportunity. The first constraint is the incentive constraint for the manager's effort choice — here, it is assumed that the principal wants to induce high effort. The second is the participation constraint, where $\bar{U}$ is the manager's outside option. In addition to these constraints, when the manager has an opportunity to misstate earnings, the principal faces another constraint. As the reporting decision has been necessarily delegated to the manager, the "recommended reporting strategy" has to be voluntarily followed by the manager:

$$R(y) = \arg\max_{r \in \{\tilde{l}, \tilde{h}\}} u_r - \phi(r - y) \qquad \forall y \in \{l, h\}. \tag{2.3}$$

The optimal contract includes a set of utility promises $\{u_{\tilde{h}}, u_{\tilde{l}}\}$ and the recommended action $\{e^*, R(y)\}$. Following the convention, it is assumed that the principal wants to induce high effort, so $e^* = H$. Figure 2.3 summarizes the main results. The optimal contract is described as the curve ABC, which depicts how the wedge between promised utilities assigned to reports of high and low earnings varies with different values of manipulation cost ψ. The shaded area below the 45° line shows the combination of the compensation differential and manipulation cost that induces truthful reporting. Below I restate the relevant results shown in Sun [2008].

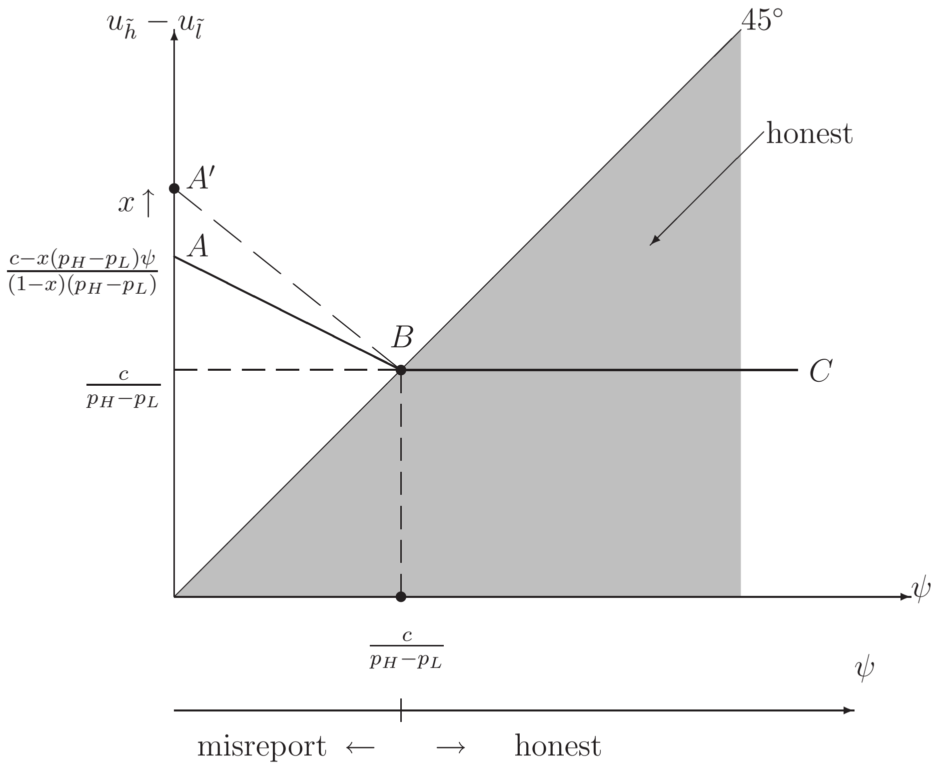

Figure 2.3: Main results

Proposition 2.1 *$\psi < c/(p_H - p_L)$ is the necessary and sufficient condition for earnings management to occur under the optimal contract.*

Lemma 4 *If $\psi < c/(p_H - p_L)$ holds, the optimal contract satisfies*

$$u_{\tilde{h}} = \bar{U} + \frac{c(1-p_L)}{(p_H - p_L)}, \tag{2.4}$$

$$u_{\tilde{l}} = \bar{U} + \frac{c(1-p_L)}{(p_H - p_L)} - \frac{c - x(p_H - p_L)\psi}{(1-x)(p_H - p_L)}. \tag{2.5}$$

The contract model yields a number of empirical implications of how earnings management affects executive compensation, which are detailed in Sun [2008]. For example, the contract shifts to $A'BC$ when x increases, producing a positive relationship between the likelihood of earnings management[17] and the sensitivity of compensation to performance that is present in both time series and cross section. In the current paper, this principal-agent model derives the manager's motive to manage earnings and also serves as a micro-foundation for asset pricing. Given a sequence of true earnings, the contract model generates a set of reports that may or may not be systematically biased. Because the realization of manipulation opportunity is stochastic, the investors are not able to make perfect inferences as to whether a report has been manipulated. As a micro-foundation for asset pricing, the central features this contract model boils down to are (1) the investors' inability to see through earnings management and (2) a focus on the upward manipulation of earnings.[18] The analysis below assumes that

[17] x is a measure for the likelihood of earnings management for individual firms in cross section studies and an indicator for the prevalence of earnings management in an economy as a whole in time series analysis.

[18] This paper has a central focus on misreporting on upside. The reason is that overstatement of earnings is more widespread than understatement in the data and more problematic in general. The average amount of restated earnings is hugely negative, and over 75% of restating firms restated their earnings downwards, indicating a strong drive to appear more productive than they actually are. Burgstahler and Dichev [1997] also estimate that 8-12 percent of the firms with small pre-

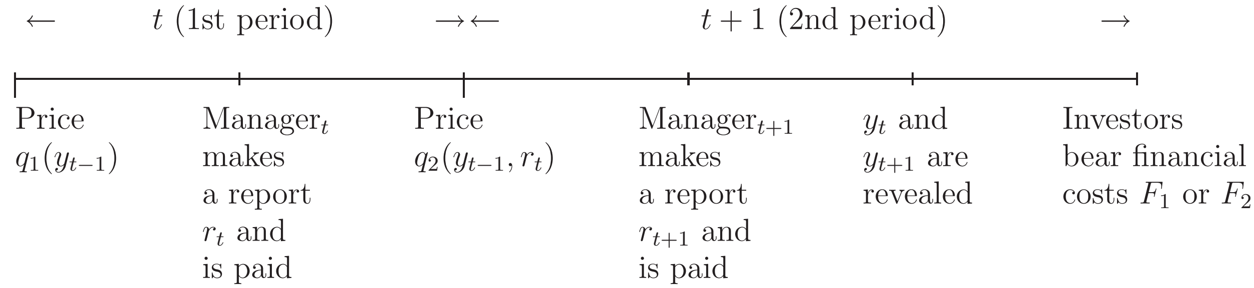

Figure 2.4: Model timeline

ψ is sufficiently low relative to $c/(p_H - p_L)$ so that the manager always overstate earnings when the earnings are low and the earnings management opportunity arises.

2.2.2 Asset prices

Now, this contract model is embedded into a dynamic model of asset pricing. It is assumed that the earnings process is persistent: the true earnings at time t, y_t, depend on y_{t-1} in addition to the manager's current effort. In particular, under the high effort by the manager (which is always the case in the equilibrium I consider), I assume that the true earnings follow a Markov process with transition probability $\pi_{yy'}$, where y is the earnings at time $t-1$ and y' is the earnings at time t. The asset price is determined as the present value of dividends, which are reported earnings net of the compensation and financial losses associated with earnings management. Figure 2.4 chronicles the timeline of the model. It describes the timing of the events in two consecutive periods t and $t+1$, and this two-period auditing cycle repeats over time. Because the model is stationary, all the relevant past information is summarized in the previously revealed earnings and current reported earnings.

managed earnings decreases manipulate earnings to achieve earnings increases, and 30-44 percent of the firms with small pre-managed losses manage earnings to create positive earnings. As long as the asymmetry between overstatement and understatement remains, in other words, the magnitude, frequency and consequences of overstatement are not exactly identical to those of understatement, the model results and intuition hold.

In the first period of the two-period auditing cycle (hereafter, period 1), the price of the firm $q_1(y_{t-1})$ is determined based on the revelation of the previous period's earnings y_{t-1}. Having the manager's reporting incentive in mind, the investors form their expectations about future dividend income based on the revelation of the firm's previous earnings y_{t-1}. In the second period of each cycle (hereafter, period 2), given the earnings report in the first period r_t and the true outcome in the ending period of the last cycle y_{t-1}, the firm is priced as $q_2(y_{t-1}, r_t)$. After the manager reports the earnings and pays them out entirely to the investors, the investigation takes place. When the investigation is conducted, the true realization of earnings in each period of the cycle is revealed, and the investors bear financial costs associated with any misstatement of earnings that occurs during the cycle. If the report is inflated in one of the two periods, the investors incur an amount of financial losses F_1. If earnings management occurs in both periods, the investors must pay an amount of monetary costs F_2, where $F_2 \geq 2F_1$.

I assume that the investors have linear utility and maximize the sum of the expected dividends. Then the value of the firm can be formulated as follows. In the beginning of an auditing cycle, given the revelation of the true outcome in the end of the last cycle y_{t-1}, the price of the firm $q_1(y_{t-1})$ is given by the expected sum of the net dividends and asset price in the next period (the time subscript is dropped when the timing is clear):

$$\begin{aligned} q_1(h) =& \pi_{hh}[d_{\tilde{h}} + \beta q_2(h, \tilde{h})] + \pi_{hl} x [d_{\tilde{h}} + \beta q_2(h, \tilde{h})] \\ &+ \pi_{hl}(1-x)[d_{\tilde{l}} + \beta q_2(h, \tilde{l})], \end{aligned} \tag{2.6}$$

and

$$\begin{aligned} q_1(l) =& \pi_{lh}[d_{\tilde{h}} + \beta q_2(l, \tilde{h})] + \pi_{ll} x [d_{\tilde{h}} + \beta q_2(l, \tilde{h})] \\ &+ \pi_{ll}(1-x)[d_{\tilde{l}} + \beta q_2(l, \tilde{l})], \end{aligned} \tag{2.7}$$

where d_r is the net dividend income and β is the investors' discount factor. The net dividend income equals the reported earnings less the compensation, that is, $d_r = r - w(r)$, where $r \in \{\tilde{l}, \tilde{h}\}$.

Regardless of the revelation of y_{t-1} in period 1, the investors may encounter three possible states in period 2. The first term in (2.6) and (2.7) is the expected net dividend income if the manager sends an honest report of high earnings in the next period. The second term in (2.6) and (2.7) represents the case in which the actual realization of earnings is low, but the manager makes an overstatement of earnings. The third term in the prices is the case in which the manager truthfully reports low earnings.

Given the first-period report r_t and the previously revealed outcome y_{t-1}, the investors update their belief about the true state in period 1. If the first-period report is low, it is for certain an honest report. If the report sent by the manager is high, it may be an overstated report that leads to immediate penalties. The posterior belief of the first-period report being truthful is derived following Bayes' Rule. If the previously revealed outcome is high, the conditional probability of $y_t = h$, denoted by γ_1, is

$$\begin{aligned}
\gamma_1 &= \Pr(y_t = h | r_t = \tilde{h}, y_{t-1} = h) \\
&= \frac{\Pr(y_t = h, r_t = \tilde{h} | y_{t-1} = h)}{\Pr(r_t = \tilde{h} | y_{t-1} = h)} \\
&= \frac{\Pr(r_t = \tilde{h} | y_t = h, y_{t-1} = h) \Pr(y_t = h | y_{t-1} = h)}{\Pr(r_t = \tilde{h} | y_{t-1} = h)} \\
&= \frac{\pi_{hh}}{\pi_{hh} + \pi_{hl} x},
\end{aligned}$$

If the previously revealed outcome is low, the conditional probability of $y_t = h$, denoted by γ_2, is

$$\begin{aligned}
\gamma_2 &= \Pr(y_t = h | r_t = \tilde{h}, y_{t-1} = l) \\
&= \frac{\Pr(y_t = h, r_t = \tilde{h} | y_{t-1} = l)}{\Pr(r_t = \tilde{h} | y_{t-1} = l)} \\
&= \frac{\Pr(r_t = \tilde{h} | y_t = h, y_{t-1} = l) \Pr(y_t = h | y_{t-1} = l)}{\Pr(r_t = \tilde{h} | y_{t-1} = l)} \\
&= \frac{\pi_{lh}}{\pi_{lh} + \pi_{ll} x}.
\end{aligned}$$

The price of the firm $q_2(y_{t-1}, r_t)$ is determined using these posterior probabilities. There are two cases. First, if period 1's report is low, the investors know that the realization of earnings is low.

$$\begin{aligned}
&q_2(l, \tilde{l}) = q_2(h, \tilde{l}) = \\
&\qquad \pi_{lh} \left[d_{\tilde{h}} + \beta q_1(h)\right] + \pi_{ll} x \left[d_{\tilde{h}} - F_1 + \beta q_1(l)\right] + \pi_{ll}(1 - x)\left[d_{\tilde{h}} + \beta q_1(l)\right]. \qquad (2.8)
\end{aligned}$$

Because actual earnings follow a Markov process, the most recent realization of earnings is the only useful information for predicting future earnings. The price in response to a low report (which implies a realization of low earnings) is thus independent of the previous revelation of earnings, equal the expected payoff over three possible states in the next period. The first term in (2.8) is the expected net dividend income if the manager sends an honest report of high earnings in the current period. The second term in (2.8) represents the case in which the manager makes an overstatement of earnings that leads to immediate financial losses. The third term in prices is associated with the situation in which the manager truthfully reports low earnings.

If the report just sent by the manager in period 1 is high, the report may or may not be truthful. Prices are determined as follows:

$$q_2(h, \tilde{h}) = \tag{2.9}$$

$$\gamma_1 \left\{ \pi_{hh} \left[d_{\tilde{h}} + \beta q_1 \left(h \right) \right] + \pi_{hl} x \left[d_{\tilde{h}} - F_1 + \beta q_1 \left(l \right) \right] + \pi_{hl} \left(1 - x \right) \left[d_{\tilde{l}} + \beta q_1 \left(l \right) \right] \right\}$$
$$+ \left(1 - \gamma_1 \right) \left\{ \pi_{lh} \left[d_{\tilde{h}} - F_1 + \beta q_1 \left(h \right) \right] + \pi_{ll} x \left[d_{\tilde{h}} - F_2 + \beta q_1 \left(l \right) \right] + \pi_{ll} \left(1 - x \right) \left[d_{\tilde{l}} - F_1 + \beta q_1 \left(l \right) \right] \right\},$$

$$q_2(l, \tilde{h}) = \tag{2.10}$$

$$\gamma_2 \left\{ \pi_{hh} \left[d_{\tilde{h}} + \beta q_1 \left(h \right) \right] + \pi_{hl} x \left[d_{\tilde{h}} - F_1 + \beta q_1 \left(l \right) \right] + \pi_{hl} \left(1 - x \right) \left[d_{\tilde{l}} + \beta q_1 \left(l \right) \right] \right\}$$
$$+ \left(1 - \gamma_2 \right) \left\{ \pi_{lh} \left[d_{\tilde{h}} - F_1 + \beta q_1 \left(h \right) \right] + \pi_{ll} x \left[d_{\tilde{h}} - F_2 + \beta q_1 \left(l \right) \right] + \pi_{ll} \left(1 - x \right) \left[d_{\tilde{l}} - F_1 + \beta q_1 \left(l \right) \right] \right\}.$$

The first term in (2.9) and (2.10) corresponds to the case where the first-period report is honest. In this case, there are three possible situations in the next period. In particular, if the realization of the second-period earnings is low and the manager has an opportunity to inflate earnings, the manager will report high. An amount of monetary penalties F_1 will be charged and thus subtracted in the pricing equation. The second term in (2.9) and (2.10) represents the case in which the first-period report is false. There are again three possible states in the second period. The investors suffer from an amount of financial losses F_1 if the manager truthfully presents earnings in period 2 and an amount F_2 if the manager manipulates earnings in period 2.

The manager's overstatement of earnings enables the investors to enjoy a higher level of current period consumption than they would in the absence of earnings management; however, this practice also exposes the investors to the loss from earnings restatement risk, that is, the subsequent financial cost after the periodic investigations. The net dividends in period 1 equal the reported earnings net of the compensation, that is, $d_r = y - w(r)$, where $r \in \{\tilde{l}, \tilde{h}\}$.[19] If $\beta F_1 > (d_{\tilde{h}} - d_{\tilde{l}})$, the cost of financial misreporting overwhelms the benefit. Everything else constant, all the prices decrease as x rises. I restrict my attention to this case throughout this analysis.[20]

2.2.3 Comparative statics

The price differential between $q_1(h)$ and $q_1(l)$ measures how sensitive the firm's price $q_1(y_{t-1})$ is in response to the investigation results y_{t-1}. How does $q_1(h) - q_1(l)$ change as the opportunity of earnings management, x, changes? To examine this, let us first ignore that the wage of the manager actually changes with x. It can be shown that

[19] If $\beta F_1 = (d_{\tilde{h}} - d_{\tilde{l}})$, the cost of earnings management is offset by its benefit exactly, and it is straightforward to determine that prices become independent of x. If $\beta F_1 < (d_{\tilde{h}} - d_{\tilde{l}})$, the benefit of earnings management overwhelms its cost, then earnings management is not only beneficial to the manager, but also to the investors. The prices increase with the frequency of earnings management.

[20] This is a better description of reality than the other two cases, as indicated in the calibration exercise. The Securities and Exchange Commission has collected over $10 billion penalties in fraud cases since 2002, and the amount of settlement fines has been growing over time. In addition, as a typical yet somewhat extreme example, the meltdown of Enron caused over 4,500 employees to lose their jobs and pension funds worth over $1 billion. The stock's value plummeted from $90 to below 50 cents, wiping out $60 billion of shareholders' assets. The loss of confidence in corporate financial reporting could also hurt business and investment opportunities. Furthermore, the reduced availability and higher cost of capital may as well cause firms to postpone capital spending plans and accelerate layoffs. Although the production inefficiency due to earnings restatements, including a declaration of bankruptcy and the lack of investment caused by reputation damage, is not specifically modeled in this framework, it is implicitly included in the monetary losses F_1 and F_2 that are incurred during the periodic investigations.

as long as the firm's stochastic production process is persistent, that is, $\pi_{hh} > \pi_{lh}$, the price becomes more responsive to investigation results as x increases. Under the condition that $\beta F_1 > (d_{\tilde{h}} - d_{\tilde{l}})$, both $q_1(h)$ and $q_1(l)$ fall as x escalates. However, $q_1(l)$ diminishes faster than $q_1(h)$, because a low previous output implies that future outputs tend to be low as well, imposing greater exposure to earnings restatement risk.

The analysis above does not consider that wages and thus net dividend income change with x. However, the same qualitative result holds even if the change in the compensation is taken into account. The optimal contract in this environment is characterized by (2.4) and (2.5). It can be seen that the compensation for the report of high earnings is independent of x, and the compensation for low earnings reports decreases as x expands. Therefore, as x becomes greater, the net dividend income from a report of high earnings, that is, $d_{\tilde{h}} = \tilde{h} - w(\tilde{h})$, remains the same, whereas the net dividend from a low earnings report, $d_{\tilde{l}} = \tilde{l} - w(\tilde{l})$, increases, resulting in a smaller dividend differential between high and low reports. Assuming that the monetary costs F_1 and F_2 do not vary with x, as the financial gain from earnings management, represented by $d_{\tilde{h}} - d_{\tilde{l}}$, diminishes, earnings management becomes more financially costly to the investors. The prices thus drop more as x rises. The change in the compensation schedule in response to the change of x internalizes the financial gain from earnings management, and it reinforces the amplification of the price differential and hence the price volatility.

Keeping the revelation of previous earnings constant, the price wedge in response to different reports in the ending period of one cycle, as measured by $q_2(h, \tilde{h}) - q_2(h, \tilde{l})$, does not necessarily have a monotonic relationship with x. To see this in a relatively straightforward manner, let us first ignore the effect of x on the manager's wages. $q_2(h, \tilde{h})$ is decreasing in x because of two forces that reinforce each other. First, as x

rises, it is more likely to have false reports in future. These falsified reports lead to the investors' financial losses. Second, it is also more likely that the previous report r_t is a false report, resulting in penalties waiting to be paid. Because $\tilde{l}$ in $q_2(h, \tilde{l})$ is surely an honest report, the second force is absent. However, we do not necessarily obtain a smaller gap between $q_2(h, \tilde{h})$ and $q_2(h, \tilde{l})$ as x increases. Because of the high persistence in the earnings process, the first force works stronger for $q_2(h, \tilde{l})$ than for $q_2(h, \tilde{h})$. The impact of changes in x on the price volatility remains ambiguous in this case.

There are additional effects to consider if we take into account the impact of x on compensation schedule. Recall that the compensation structure in this environment exhibits the property that as earnings management becomes more likely, the compensation wedge is magnified, leading to a smaller dividend differential. As earnings management becomes more costly to the investors, prices decline more when x increases. This response of the wage payment to changes in x strengthens the first mechanism that is at work for both $q_2(h, \tilde{h})$ and $q_2(h, \tilde{l})$ without affecting the other mechanism that works only for $q_2(h, \tilde{h})$. Although the net effect of x on the price volatility could spin either way in the second period of one cycle, incorporating the change in the compensation scheme generates higher price volatility than otherwise.

From this point forward in this paper, I will ignore the wage values in the price calculation, so as not to complicate the mechanism and conflate with the main argument. The channel that earnings management influences returns through wages should be quantitatively weak, because executive compensation, although sizable and growing, does not constitute a substantial fraction of firms' earnings.[21]

The asset return is calculated as the sum of the current period price and dividends

[21]CEOs of public companies earn a mean of \$600,000 this decade, which is about 0.5% of the firms' average earnings.

divided by the previous period price and then subtracted by one. The return volatility in the model is measured as an equally weighted average of the return volatility in each period of one auditing cycle. In the revelation stage, it is straightforward to show that volatility rises with x. When earnings management opportunities become more likely, more frequent earnings restatements generate greater fluctuations in the returns and thus higher volatility. Earnings management may dampen return volatility in the reporting periods, because there is less variation in the reports. In addition, the prices in response to high reports are discounted to reflect possible earnings management, leading the price range to shrink. However, earnings management amplifies the movement of returns in the revelation stage significantly. As long as F_1 is not too small, the amplification effect of earnings restatement risk in the revelation stage is dominant, and hence average return volatility increases with x.

Analogously, in order to compare the conditional volatility difference in response to earnings revelations, I use the difference between the equally weighted average of the return volatility in one cycle following a revelation of high earnings and that following a revelation of low earnings. If $x = 0$, it is straightforward to show that the difference is zero. With a positive value of x, earnings restatement risk increases the volatility difference in the revelation stage, because low earnings generate financial incentives for the manager to overstate earnings while high earnings do not. As long as F_1 is large enough, the asymmetry in return volatility is present when earnings management is possible.

2.3 Results

In this section, I solve the model numerically and present the results from model simulations. Table 2.1 shows the parameter values in the numerical example. The

Parameter	Value
h	50
l	0
π_{hh}	0.8
π_{ll}	0.8
β	0.95
F_1	$1.2(h-l)/\beta$
F_2	$2F_1$

Table 2.1: Parameter values in the numerical example with binary earnings

primary purpose in this section is to illustrate that earnings management can generate a number of stylized financial facts. The quantitative results will be presented in Section 5.[22]

2.3.1 Volatility clustering and asymmetric volatility

For the illustrative purpose, I use $x = 0$ and $x = 0.1$ as an example to demonstrate the impact of earnings management throughout this section. The simulated return sequence from the model captures the stylized facts of conditional volatility:

[22]It is worth noting that the asset pricing model is consistent with the contract model in the sense that it is optimal for the investors to implement high effort when designing executive compensation, although earnings management leads to monetary penalties imposed on the investors. Recall that in the contract model with two-earnings-level specification, the principal always wants to induce high effort. In the following analysis, wage values are assumed to be negligibly small relative to firms' earnings. In a standard principal-agent model without earnings management, high effort is desirable as long as high earnings are different enough from low earnings. With the possibility of earnings management and revelations, it is still beneficial for the principal to induce high effort if the value of high effort outweighs the possible monetary loss associated with earnings management. That is,

$$[p_H h + (1-p_H)l] - [p_L h + (1-p_L)l] > xF_1 \tag{2.11}$$

And recall that for earnings management to exert influence on stock returns, the discounted monetary penalties associated with earnings management must be different from the amount of overstatement, and this analysis focuses on the case that earnings management is costly to the investors. That is,

$$\beta F_1 > h - l \tag{2.12}$$

The numerical example used here satisfies both (2.11) and (2.12). The assumption that high effort is desirable for the principal remains valid, after taking into account the negative consequence of earnings management.

first, conditional volatility exhibits persistence; second, stock returns are negatively correlated with the volatility of subsequent returns.

The EGARCH (1,1) model of the return series is estimated using Maximum Likelihood method with 10,000 artificially generated observations. The EGARCH (1,1) model used is $\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$, where E is the expectation operator, ϵ_t is the innovation, and σ_t is the conditional variance of the innovation. The G term captures volatility clustering (that is, persistence of volatility). A positive value of the A term in the equation implies that a deviation of the standardized innovation from its expected value causes the variance to be larger than otherwise. The L coefficient allows this effect to be asymmetric.[23]

Table 2.2 presents the results. The upper panel presents the case without earnings management, that is, $x = 0$. In this case, there is no GARCH or ARCH effect present in the simulated return data. As x becomes positive, return volatility becomes serially correlated. Before estimation, the Lagrange Multiplier (LM) test is applied to the return data, and the LM test strongly rejects the i.i.d. residual hypothesis at the 95% confidence level. The coefficients of the EGARCH (1,1) model are all statistically significant beyond the 95% confidence level. In addition, the conditional variance process is strongly persistent (with G coefficient = 0.60). The negative value of the coefficient L shows evidence of asymmetry in the model return behavior — negative surprises increase volatility more than positive surprises.

The persistence and asymmetry in the conditional volatility of stock returns in the model are generated by earnings management incentive together with a persistent earnings process. When true earnings are revealed to be low, the persistence in the

[23] If $L = 0$, then a positive surprise ($\epsilon_{t-1} > 0$) has the same effect on volatility as a negative surprise of the same magnitude. If $-1 < L < 0$, a positive surprise increases volatility less than a negative surprise. If $L < -1$, a positive surprise actually reduces volatility while a negative surprise increases volatility. For further reference, see Hamilton [1994, p. 668].

x=0	Coefficient	Std.Error	t-statistic
K	-5.0000	0.4153	-12.0387
G	-0.0001	0.6829	0.0001
A	0.0000	0.0087	0.0000
L	0.0009	0.0092	0.1049

x=0.1	Coefficient	Std.Error	t-statistic
K	-1.8621	0.3136	-5.9380
G	0.5999	0.0663	9.0545
A	0.0407	0.0058	6.9856
L	-0.1125	0.0278	-4.0553

Table 2.2: EGARCH(1,1) estimation results (binary earnings)
Variance equation:
$\log\sigma_t^2 = K + G\log\sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$

earnings-generating process implies that earnings tend to stay low for a while, so earnings management is likely to occur in the current and future periods. A higher frequency of occurrence of earnings management increases future return volatility. If the previous earnings are revealed to be high, the current and future earnings are likely to remain high. Overstatement of earnings has little chance of occurring; thereby future returns are relatively stable in this case. As a result, the volatility of the return series is persistent, and returns are negatively correlated with the subsequent volatility.

Note that the core intuition does not hinge upon the two-period time structure of information disclosure. The mechanism that drives EGARCH property stays in effect when the model is extended to incorporate additional periods and stochastic investigation in Section 6.2. As long as restatements generate returns movements and there is a persistent component to earnings management because of the persistence in underlying profitabilities, the substance of the model dynamics remains.

x	Standard Deviation
0	0.0954
0.1	0.1015
0.2	0.1086

Table 2.3: Volatility of the model returns (binary earnings)

2.3.2 Return volatility

Table 2.3 presents the volatility of the simulated returns. Monetary losses that incur during revelations generate large swings in the return sequence and hence produce volatility. When earnings management and earnings restatements occur more frequently, returns become more volatile. Campbell et al. [2001] document that idiosyncratic stock return volatility increased considerably from 1962 to 1997 in the United States. Rajgopal and Venkatachalam [2008] report a strong association between idiosyncratic return volatility and financial reporting quality, as measured by both earnings quality and forecast dispersion, in both cross-sectional and time-series regressions. In line with the empirical findings, as x increases in the model, implying that the informativeness of earnings reports becomes weakened, the returns exhibit greater volatility.

2.4 Extension to continuous earnings

In this section,the model is extended to the case with a continuum of earnings. This model is used for the quantitative analysis in the next section. In the continuous case, I assume that earnings follow an AR(1) process: $y' = \rho y + k + \epsilon$, where $\rho < 1$, k is a constant, and ϵ is a white noise process with zero mean and standard deviation σ.

2.4.1 Optimal contract

Analogous to the binary model elaborated above, a risk-neutral principal (investors) hires a risk-averse agent (manager) for one period. Expending high effort incurs a utility cost, that is, c, to the manager, whereas low effort involves no cost. The manager's effort decision and an exogenous state realization together determine the firm's economic earnings, which are privately observed by the manager. The conditional distributions of earnings given high and low effort follow normal distributions: $f(y|e = H) \sim N(\mu_H, \sigma_H)$ and $f(y|e = L) \sim N(\mu_L, \sigma_L)$, where $\mu_H > \mu_L$. After exerting effort, the manager privately learns whether an opportunity is available to inflate earnings in the manager's favor. With probability x, the manager has discretion to overstate earnings by a constant amount a, and a utility cost $\phi(R(y) - y)$ is involved in such earnings manipulation. In particular, $\phi(a) = \psi > 0$. With probability $(1-x)$, the applicable accounting rules are so hard-and-fast that the manager has no option but truthfully present earnings. The manager's outside option is $\bar{U}$.

The model is extended to the case with continuous earnings by characterizing the optimal wage function contingent on the earnings reports. The principal's objective is to minimize the expected wage payment that implements effort, taking into consideration that the manager has an incentive to exaggerate the firm's earnings. The manager is risk averse with respect to income, and thereby his utility function $U(w)$ is strictly concave with respect to the wage w. The optimal wage function solves the following problem:

$$\min_{w(\cdot)} xE\left[w\left(R(y)\right)|H\right] + (1-x)E\left[w(y)|H\right]$$

subject to

$$H = \arg\max_{e\in\{L,H\}} xE\Big\{U\left[w\left(R(y)\right)\right] - \phi\left(R(y) - y\right) - a(e)\Big\} + (1-x)E\Big\{U\left[w(y)\right] - a(e)\Big\}, \quad y_e \sim N(\mu_e, \sigma_e). \tag{2.13}$$

$$E[u|H] = xE\Big\{U\left[w\left(R(y)\right)\right] - \phi\left(R(y) - y\right) - a(e)|H\Big\} + (1-x)E\Big\{U\left[w(y)\right] - a(e)|H\Big\} \geq \bar{U}. \tag{2.14}$$

The objective function is the expected wage payment for the principal to motivate effort. The first term is the expected payment to implement effort when the manager has an opportunity to artificially inflate earnings, and the second term is the wage if the manager does not have such an opportunity. The principal designs a compensation contract that satisfies the incentive constraint on the effort decision (2.13) and the participation constraint (2.14). In addition to these constraints, when the manager has an opportunity to exaggerate earnings, the "recommended reporting strategy" has to be in the manager's best interest. This incentive constraint on the reporting strategy in the continuous case is:

$$R(y) = \arg\max_{r\in\{y,y+a\}} U\left[w(r)\right] - \phi(r-y) \qquad \forall y \sim N(\mu_H, \sigma_H). \tag{2.15}$$

More specifically,

$$R(y) = y \qquad \text{if } U\left[w(y+a)\right] - U\left[w(y)\right] < \psi, \tag{2.16}$$

$$R(y) = y + a \qquad \text{if } U\left[w(y+a)\right] - U\left[w(y)\right] \geq \psi. \tag{2.17}$$

The optimal wage schedule is numerically computed in Sun [2008], utilizing Simulated Annealing algorithm with Gauss Hermite quadrature. The Schumaker approximation is used to preserve the shape of wage functions in interpolation and extrapolation. In the numerical implementation, it is always the case that under the optimal contract, there exists a threshold level of earnings y^*, above which the manager does not find it worthwhile to manipulate earnings and truth-telling strategy is thus maintained. Below this threshold, the manager achieves personal gains from manipulation and inflates earnings whenever possible. Thereafter, this paper focuses on this threshold-style of reporting behavior.

The intuition behind the existence of the threshold earnings that separates truthful reporting and earnings management is as follows. Given that the manager is risk averse, a wage function that is not too convex translates into a set of concave utility promises. As actual earnings expand, the manager faces a decreasing utility gain but a constant utility cost from overstating earnings. As a consequence, earnings management occurs when the realized earnings are relatively low, and a truthful reporting strategy is sustained if actual earnings are high.

2.4.2 Asset prices

The pricing formulation is extended to the continuous case as follows.[24] Based on the revelation of previous earnings, the price in period 1 is determined as the expected sum of the dividends and price in the next period:

$$\begin{aligned} q_1(y) = &\Pr[y' \geq y^*|y]E\left[(\rho y + k + \epsilon) + \beta q_2(y, \rho y + k + \epsilon)|y' \geq \ y^*\right] \\ &+ \Pr[y' < y^*|y]xE\left[(\rho y + k + \epsilon + a) + \beta q_2(y, \rho y + k + \epsilon + a)|y' < y^*\right] \\ &+ \Pr[y' < y^*|y](1-x)E\left[(\rho y + k + \epsilon) + \beta q_2(y, \rho y + k + \epsilon)|y' < y^*\right]. \quad (2.18) \end{aligned}$$

The first term in the pricing function represents the case when the actual earnings in the next period exceed the threshold level of earnings that elicits the truth, and

[24] Again, the labor wage is assumed to be negligibly small compared with the firm's earnings, therefore compensation does not affect net dividends or asset prices.

therefore the manager reports honestly. The second term in (2.18) is the case when the next period's actual earnings fall below the threshold earnings, and the manager has an opportunity to manage earnings. The manager in this case overstates earnings. In particular, the next period's report is $r = \rho y + k + \epsilon + a$. The third term in (2.18) represents the situation in which the next period's earnings are below the threshold earnings, but the manager does not have the earnings management opportunity. In this case, the manager has to truthfully represent the earnings.

The price in period 2 is a function of the previously revealed earnings and the earnings report in period 1.

$$q_2(y, r) = p\Omega + (1-p)\tilde{\Omega},$$

where

$$\begin{aligned} \Omega \equiv & \Pr[y'' \geq y^* | y' = r] E\left[(\rho r + k + \epsilon) + \beta q_1(\rho r + k + \epsilon) | y'' \geq y^*\right] \\ & + \Pr[y'' < y^* | y' = r] x E\left[(\rho r + k + \epsilon + a) - F_1 + \beta q_1(\rho r + k + \epsilon) | y'' < y^*\right] \\ & + \Pr[y'' < y^* | y' = r]\,(1-x)\,E\left[(\rho r + k + \epsilon) + \beta q_1(\rho r + k + \epsilon) | y'' < y^*\right], \end{aligned}$$

and

$$\begin{aligned} \tilde{\Omega} \equiv & \Pr[y'' \geq y^* | y' = r - a] E\left[(\rho(r-a) + k + \epsilon) - F_1 + \beta q_1\left(\rho(r-a) + k + \epsilon\right) | y'' \geq \ y^*\right] \\ & + \Pr[y'' < y^* | y' = r - a] x E\left[(\rho(r-a) + k + \epsilon + a) - F_2 + \beta q_1\left(\rho(r-a) + k + \epsilon\right) | y'' < y^*\right] \\ & + \Pr[y'' < y^* | y' = r - a](1-x) E\left[(\rho(r-a) + k + \epsilon) - F_1 + \beta q_1\left(\rho(r-a) + k + \epsilon\right) | y'' < y^*\right]. \end{aligned}$$

Here, Ω is the expected present value of the dividends when the first-period report is truthful, and $\tilde{\Omega}$ corresponds to the case where the first-period report is false. Similar to the pricing function in period 1, the first term in Ω and $\tilde{\Omega}$ represents the case when the second-period earnings are higher than the threshold earnings, and the reported earnings are truthful. In $\tilde{\Omega}$, F_1 is subtracted because investors must bear monetary penalties for the earnings management practice in period 1 of this auditing cycle. The second term in Ω and $\tilde{\Omega}$ represents the case when the actual earnings in period 2 are lower than the threshold earnings, and the manager has discretion to inflate earnings by a. In this case, the investors pay F_1 for the overstatement if the first-period report is honest (as in Ω) and F_2 if the first-period report is also falsified (as in $\tilde{\Omega}$). The third term is the case when the manager does not have sufficient discretion over reporting in period 2 and has to truthfully report the earnings that fall below the threshold earnings. In $\tilde{\Omega}$, the deduction of F_1 is due to the earnings overstatement by the manager in period 1.

The posterior belief of having an accurate report in period 1, that is, $p = \Pr[y' = r|y]$, is derived following Bayes' Rule,

$$p = \begin{cases} 1 & \text{if } r \in [y^* + a, \infty), \\ \dfrac{f(r-k-\rho y)}{f(r-k-\rho y) + xf(r-a-k-\rho y)} & \text{if } r \in (y^*, y^* + a), \\ \dfrac{(1-x)f(r-k-\rho y)}{(1-x)f(r-k-\rho y) + xf(r-a-k-\rho y)} & \text{if } r \in (-\infty, y^*]. \end{cases} \tag{2.19}$$

Note that the compensation contract endogenously determines the threshold level y^* that elicits the truth. As actual earnings follow an AR(1) process, the implied conditional distributions of earnings given effort change over time, leading to changes of compensation contracts and hence threshold levels. In the simulation of prices and returns, the endogeneity of y^* requires calculations of the optimal contract for each possible earnings distribution implied by previous earnings. Sun [2008] specifies the parameterization of the principal-agent model such that the threshold level equals the conditional mean of actual earnings given high effort. The following proposition states the conditions under which the wage schedule shifts in a parallel manner when the earnings distribution moves. More specifically, the optimal contract and the underlying earnings distribution move together in the same direction by an equal amount. Therefore, the threshold level is always equal to the mean of earnings given high effort, even when the mean level itself varies over time.[25]

Proposition 2.2 *Suppose that the values of the parameters* $(a, \psi, c, \bar{U}, \sigma_H, \sigma_L)$ *are fixed, and* $f(y|e = H)$ *and* $f(y|e = L)$ *shift in a parallel manner by* δ*, keeping* $(\mu_H - \mu_L)$ *fixed. Then a parallel shift of the wage function* $w(r)$ *by* δ *is a solution to the principal's problem, and therefore the threshold level* y^* *will shift by* δ *as well.*

Proof: See Appendix.

Below, I restrict the attention to the parameterization specified in Sun [2008] and the conditions stated above. In the first period of each auditing cycle, the investors have perfect knowledge of the value of y^* given the revelation of previous earnings. In the second period, they form an expectation of actual earnings in period 1 based on the report in period 1 and the previously revealed earnings. The investors use this expectation to infer the current distribution of earnings for both compensation design purposes and firm valuation purposes.

[25] A possible alternative interpretation of the existence of threshold level outside the model is that executives strive to beat the consensus earnings forecast by financial analysts, and the best forecast is the conditional mean of earnings given the previous earnings reports.

The threshold level y^* can be derived as follows:

$$y^* = \begin{cases} \rho y + k & \text{in period 1,} \\ \rho\left[pr + (1-p)(r-a)\right] + k & \text{in period 2.} \end{cases}$$

For the baseline case without earnings management ($x = 0$), reported earnings are always truthful, and the pricing function can be derived analytically. In this case, there is no difference between the reporting period (that is, period 1 of each auditing cycle) and the revelation period (that is, period 2 of each auditing cycle). The pricing equations in each period thus coincide with each other, equal to the sum of discounted expected future earnings.

$$\begin{aligned} q(y) &= E\Big\{(\rho y + k + \epsilon) + \beta\left[\rho(\rho y + k + \epsilon) + k + \epsilon\right] + \beta^2\{\rho\left[\rho(\rho y + k + \epsilon) + k + \epsilon\right] + k + \epsilon\} + \cdots\Big\} \\ &= \lim_{n\to\infty} \frac{\rho\left[1 - (\beta\rho)^n\right]}{(1-\beta\rho)} y + \lim_{n\to\infty} \sum_n \frac{\beta^{n-1} k}{(1-\rho)} - \lim_{n\to\infty} \sum_n \frac{\beta^{n-1}\rho^n k}{(1-\rho)} \\ &= \frac{\rho y}{(1-\beta\rho)} + \frac{k}{(1-\beta)(1-\beta\rho)}. \end{aligned} \tag{2.20}$$

Since actual earnings follow $y' = \rho y + k + \epsilon$, we can lag and substitute (2.20) into the earnings process to yield

$$q(y') = \rho q(y) + \frac{(1-\rho)k}{(1-\beta)(1-\beta\rho)} + \frac{\rho}{(1-\beta\rho)}(k + \epsilon).$$

The price follows an AR(1) process with the same autoregressive parameter as the earnings process but with different mean and variance.

The system of integral equations that characterizes the asset prices with earnings management does not yield an analytical solution. Instead, the prices are computed using Monte Carlo integration. Here, a numerical example is presented to illustrate

Parameter	Value
ρ	0.77
k	0.23
a	2.1
β	0.98
F_1	31.8
F_2	$2F_1$

Table 2.4: Parameter values in the numerical example with continuous earnings

how earnings management affects asset prices. Table 2.4 shows the parameter values specified in the price computation. With a couple of exceptions, most of the parameter values are taken from the calibration implemented in the next section. For the purpose of illustration, I enlarge the value of x and F_1, compared with the value calibrated in the next section, to demonstrate the impact of earnings management on price dynamics.

Figure 2.5 shows how period 1's price varies with revealed previous earnings and how period 2's price varies with reported earnings, keeping previously revealed earnings fixed. The dotted line and the light line that overlap with each other represent the price of period 1 (as a function of y) and that of period 2 (as a function of r) in the baseline case. The dashed line is period 1's price (as a function of y) with earnings management, and the dark line is period 2's price (as a function of r) for a given level of previous earnings y. Compared to the baseline case, a positive value of x makes the prices in both periods lower for a given level of previous earnings and earnings report. The price is discounted to reflect future monetary losses because of a possibly manipulated report in the current period. The shift of prices is parallel (except for some deviation in period 2), because the possibility of having a false report in the current period is independent of y under the current assumptions.

With earnings management opportunity, the price of period 1 and that of period 2

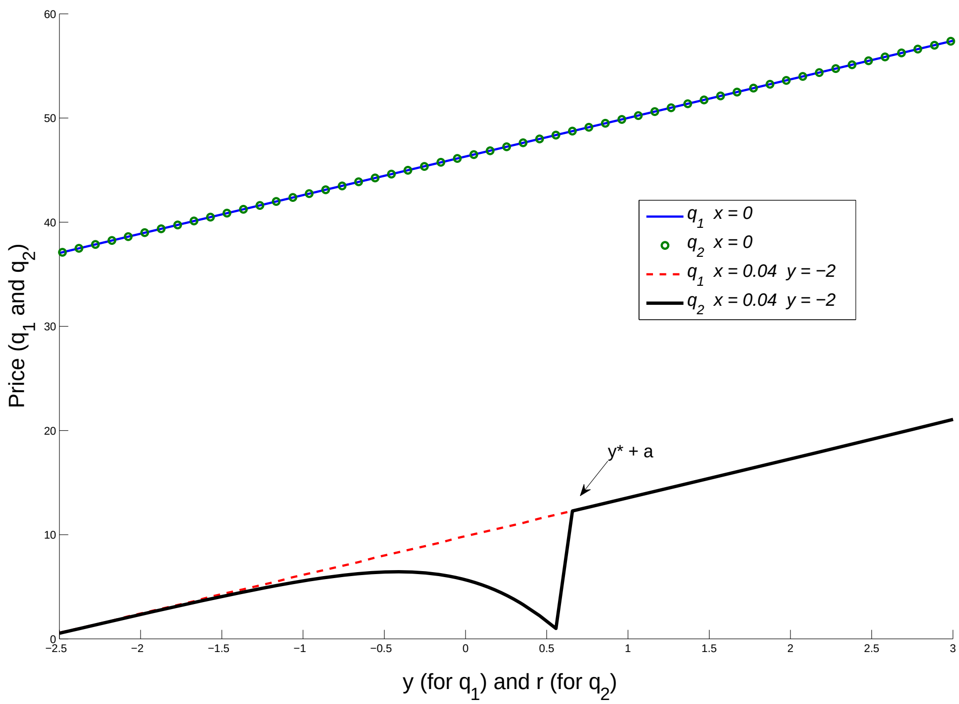

Figure 2.5: Pricing function with continuous earnings

differ only to reflect the additional information coming from the comparison between y and r. In period 2, the comparison between y and r reveals some information about the possibility that r is a false report, as shown in (2.19). Note that y^* is the conditional mean of the true earnings, which is a function of y. If r is very small, it is unlikely that the report has been inflated. If r is very large, it cannot be a manipulated report because there is no incentive to manage earnings when true earnings are greater than y^*. In particular, if $r > y^* + a$, the investors can infer (with probability 1) that r is a truthful report. In the medium range of r, the probability is large that r is a false report.

In the particular case with normal distributions of earnings, the following result holds.

Lemma 5 *if $r \in (-\infty, y^*]$ or $r \in (y^*, y^* + a)$, p is strictly decreasing in y.*

Proof: See Appendix.

In Figure 2.6, period 2's price is plotted as a function of reported earnings for different levels of previously revealed earnings y. The dark, light, and dashed line represent a relatively low, medium, and high level of previous earnings respectively. If the previously revealed earnings are higher, the threshold level that induces truthful reporting is thus higher. The sharp drop-off of prices occurs at a higher level of reports.

2.5 Quantitative results

In this section I describe how I calibrate the model. Because this model describes individual stock returns, the calibration strategy is to simulate realizations of productivity shocks and earnings management opportunities for a large number of individual

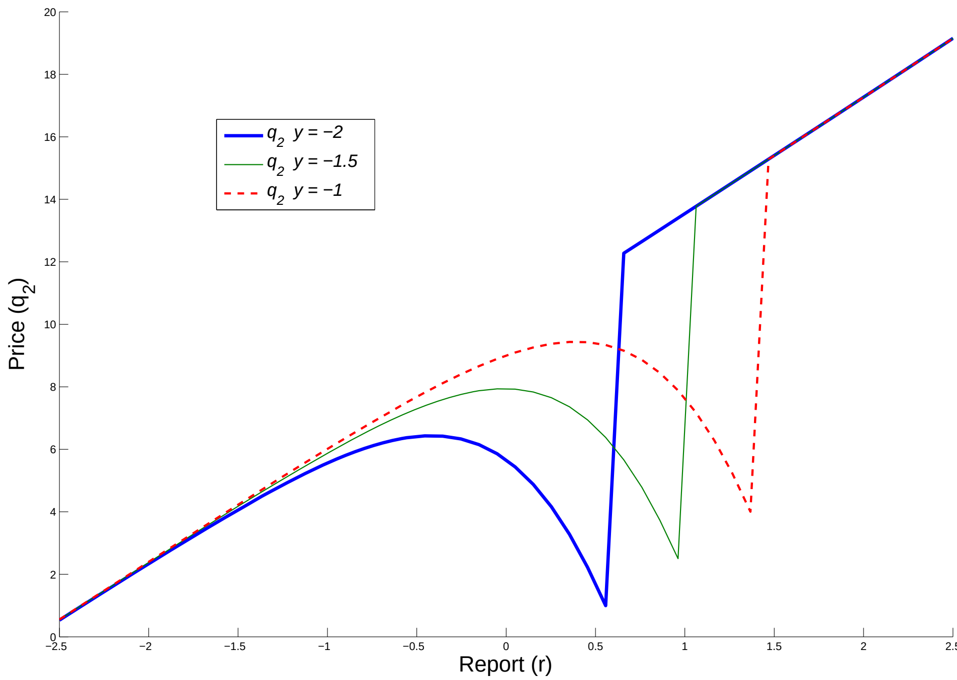

Figure 2.6: Pricing function in period 2

firms, gather the return sequences together, and then set the parameter values so as to match the aggregate targets.

To capture fluctuations in stock market indices, the calibrated model incorporates aggregate uncertainty: an aggregate productivity shock. The production process that individual firms follow is thus specified as $y' = \rho y + \epsilon_a + \epsilon_i$, where $\epsilon_a \sim N(0, \sigma_a^2)$ and $\epsilon_i \sim N(0, \sigma_i^2)$. Here, ϵ_a and ϵ_i represent aggregate productivity shock and idiosyncratic productivity shock respectively, and they are independent. Aggregate productivity shock is assumed to be observable to both managers and investors. In doing so, I maintain the focus on the asymmetric information between managers and investors regarding idiosyncratic performance, without causing additional inference problems.[26]

In the rest of this section, I first calibrate the model using Compustat industrial quarterly data after restatement corrections as actual earnings process, and investigate the statistical properties of returns generated from the model.[27] This case represents the benchmark calibration. Second, counterfactual experiments are conducted by considering different levels of earnings management prevalence to assess the impact of earnings management in the financial markets.

[26] If aggregate productivity shock is unobservable to investors, earnings reports from all the firms in the economy convey information regarding the aggregate state of the economy. In pricing individual firms, investors should utilize earnings reports from all the firms to filter out the aggregate shock and then make inferences about individual outcomes. As earnings management is considered as a phenomenon arising from asymmetric information about idiosyncratic performance, the possible information asymmetry regarding aggregate economy is beyond the scope of this paper.

[27] Since earnings management may be more prevalent than earnings restatements, there can be a potential discrepancy between restated earnings and true earnings, which does not hurt the validity of the results. If we interpret the model literally, the model in this paper examines the impact of earnings management behavior that leads to SEC enforcement actions or earnings restatements. Thus, earnings management practice that goes unnoticed over the firm's entire life cycle is outside the scope of the model. When following the general notion of earnings management, that is, a misstatement of earnings that leads to adverse consequences for the firm, I acknowledge the downward bias in the estimated prevalence of earnings management, a correction of which would rather improve the model fit to the data. I will also consider matching the moments of unrestated earnings later in the next section to check the robustness of model properties.

Parameter	Description	Value
β	Discount factor	0.98
ρ	Autoregressive parameter	0.77
k	Constant drift	0.23
σ_a	Std.Dev of aggregate productivity shock	0.07
σ_i	Std.Dev of idiosyncratic productivity shock	0.11
x	Earnings management prevalence	0.04
a	Amount of overstatement	0.07
F_1	Monetary loss for one restatement	1.06
F_2	Monetary loss for two restatements	2.12

Table 2.5: Benchmark parameterization

2.5.1 Benchmark calibration

Table 2.5 contains the benchmark parameter values. The period length is set to be half a year. The annual periodicity of restatements is thus in accordance with the empirical finding that the average number of restated fiscal quarters is about four (Wu [2002]).[28] The discount factor β is chosen to be 0.98 so that the implied semiannual real interest rate is 2 percent.

The autoregressive parameter ρ, the constant drift k, and standard deviations of productivity shocks σ_a and σ_i are calibrated using Compustat data. I include all available observations in the quarterly industrial Compustat database from Q1 1971 to Q4 2006 to study public firms' earnings. Compustat quarterly files provide data on a restated basis. When a company reports for a new quarter and at the same time reports different data than originally reported for the corresponding quarter of the prior year, the data for the corresponding quarter of the prior year is changed and said to be restated.[29] In this benchmark calibration, the net income process from Compustat is taken as actual earnings process.

[28] Wu [2002] analyzes 932 earnings restatements from Jan 1997 through Dec 2001. The restated period varies from one quarter to eight years, with an average of 4.2 quarters in the sample.

[29] These restatements can be due to mergers, acquisitions, discontinued operations, and accounting changes.

	Mean	Std.Dev	Autocorr	Std.Dev of avg. earnings
Scaled earnings	0.06	0.21	0.77	0.12

Table 2.6: Moments of semiannual scaled earnings (1971 - 2006)

In the results reported here, I use the sum of net income over both quarters (Compustat quarterly data item #69) to study firms' earnings. The results are also computed using earnings before extraordinary items (Compustat quarterly data item #8), and the results are generally consistent for these two alternative measures of earnings. The earnings data are drawn from a broad spectrum of firm sizes and are therefore scaled following the approach in the literature. The earnings variable is scaled by beginning-of-the-period market value of common equity, computed as the close price in the end of the previous period multiplied by the number of common shares outstanding (i.e., [one-period-lagged Compustat quarterly data item #14] $\times$ [Compustat quarterly data item #61]). Following the convention, I also winsorize the data at 1 percent extreme values from each tail to reduce the impact of outliers and data errors.

The descriptive statistics of semiannual earnings in the sample are presented in Table 2.6. I normalize the steady-state level of actual earnings to be one, that is, $\bar{y} = \dfrac{k}{1-\rho} = 1$. The value of ρ is chosen to match with the average autocorrelation of firms' earnings, which is the third entry in Table 2.6. This gives $\rho = 0.77$ and $k = 1 - \rho = 0.23$. The standard deviation of aggregate productivity shock σ_a is set to be 0.07 to match with the time variation of average earnings across firms, shown in the fourth column in Table 2.6. As aggregate productivity shock and idiosyncratic productivity shock are independent of one another, given the variance of aggregate productivity shock, the standard deviation of idiosyncratic productivity shock is calculated to be $\sigma_i = 0.11$.

The parameter x is calibrated to be 0.04, yielding an overall restatement rate 2 percent. This feature is in line with the average frequency of restatement announcements among publicly traded companies over the period of Jan 1997 to Sep 2005 (GAO [2002] and GAO [2006]).[30] Wu [2002] documents that the average amount of restated earnings in her sample is −$9.8 million, while the average number of restated quarters is 4.2.[31] As the model is calibrated on a semiannual basis, I choose the amount of overstatement to be half of $9.8 million in each period, that is, $4.9 million. After scaled by average market value of listed companies and then normalized by average scaled earnings, a is 0.07.

To measure the monetary loss that the investors incur in the event of earnings restatements in the model, the current paper focuses on the average immediate market-adjusted loss in market capitalization of restating companies, that is, $75.5 million for each restatement announcement (GAO [2002] and GAO [2006]).[32] I choose the three-trading-day window to focus on regarding the market response to the exclusion of other factors. This measure provides a lower bound for the financial losses the investors suffer from restatements, and the associated result serves as a lower bound for evaluating the importance of earnings management in the financial markets. The scaled and normalized measure for the financial loss associated with each restatement

[30] To identify and collect financial statements, GAO [2002, 2006] use Lexis-Nexis, an online periodical database, to conduct an intensive keyword search using variations of the word "restate." They include only announced restatements that were being made to correct previous material misstatements of financial results, while exclude announcements involving stock splits, changes in accounting principles, and other financial statement restatements that were not made to correct mistakes in the application of accounting standards.

[31] Wu [2002] analyzes 932 earnings restatements from Jan 1997 through Dec 2001. The raw restated earnings magnitude runs from $1.1 billion downward to $470 million upward.

[32] To determine the immediate impact on stock prices, GAO [2002] analyzes 689 earnings restatements that were announced from January 1997 to March 2002. GAO [2006] examines 1061 restatement announcements from July 2002 to September 2005. For each of these cases, they examine the company's stock price on the trading days before, of, and after the announcement date to assess the immediate impact and calculate the change in market capitalization. I take an average of the immediate market-adjusted loss in market capitalization in the two samples.

	Standard Deviation
Model	0.0714
Data	0.1063

Table 2.7: Comparison of data volatility (benchmark calibration)

is $F_1 = 1.06$. F_2 is then set to be 2.12.

2.5.2 Results

I report the simulation results on the parsimoniously parameterized model using the benchmark calibration for 500 firms and compare the statistical properties with S&P 500 index returns data. To get compound semiannual returns, I obtain S&P 500 quarterly returns from CRSP quarterly files from Jan 1931 to Dec 2007.[33]

Table 2.7 shows that relative to S&P 500 Index data, the volatility of the model-generated data is moderately lower. Table 2.8 compares EGARCH estimation results from the model returns and S&P 500 Index returns. The coefficients of the EGARCH (1,1) model are all statistically significant beyond the 95% confidence level. Consistent with the data, the conditional variance process is strongly persistent, although the magnitude of G coefficient is not as much as the data show. Since the coefficient L has a negative value, the model displays asymmetric volatility.

The intuition for the EGARCH effect in the binary example with two levels of earnings can be extended to the current model with a continuum of earnings. The general unifying story is that earnings management goes hand-in-hand with weak performance, because the financial incentive to artificially inflate earnings is strong when the earnings realization is poor. Relatively low earnings lead to more frequent future restatements and associated sharp drops of returns than high earnings, generating

[33] I consider a longer-period sample for stock returns than company earnings, excluding the 1929 stock market crash. The longer time span is chosen due to the semiannual frequency of the model.

Model data	Coefficient	Std.Error	t-statistic
K	-5.0000	2.5890	-1.9312
G	0.5260	0.2454	2.1436
A	0.0529	0.0235	2.2474
L	-0.0234	0.0139	-1.6784

S&P 500 data	Coefficient	Std.Error	t-statistic
K	-1.1029	0.4893	-2.2540
G	0.7365	0.1173	6.2789
A	0.3057	0.1542	1.9830
L	-0.2557	0.1127	-2.2703

Table 2.8: Comparison of EGARCH(1,1) estimation results (benchmark calibration)
Variance equation:
$\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$.

greater movements in the return data. The return volatility becomes state-dependent, and the state (actual earnings) is persistent. Return volatility is thus persistent and asymmetric. In addition to this direct impact, an indirect effect due to *suspicion* of earnings management amplifies the persistence and asymmetry in return volatility. As shown in Figure 2.6, the possibility of earnings management creates a region of reports at the lower end that cause active learning and intensive suspicion of misstatement. Investors lower the price in anticipation of restatements. The uncertainty regarding the firm's fundamental value and subsequent outcomes is increased in this case, and some of the earnings reports under suspicion indeed lead to restatements and market fluctuations. Because the reported numbers tend to persist, the volatility also persists and exhibits asymmetry.

Although the model is consistent with volatility clustering and asymmetric volatility in the data, the magnitude is somewhat smaller. The A coefficient and L coefficient in S&P 500 Index returns are an order of magnitude greater than can be reproduced in the model. In light of the difficulty of measuring monetary costs of earnings management, the discrepancy is not as large as it appears. For example, GAO [2002] and GAO [2006] show that restatement announcements have a negative effect on stock prices beyond their immediate impact. They find persistent market capitalization declines for restating companies. After controlling for the movement in the overall market, they report an average of $79.3 million loss in market value from 20 trading days before through 20 trading days after a restatement announcement (the intermediate impact) and an average of $136.1 million loss in market value from 60 trading days before through 60 trading days after the announcement (the longer-term impact). In addition, the use of market capitalization loss as a proxy for monetary loss that the investors incur precludes other potentially important factors.[34] The effects of such errors would be to bias the financial loss downwards, a correction of which would result in the model moving closer to the data. Measurement errors in the frequency of earnings management would have a similar effect on dynamic return patterns. Another plausible explanation for the discrepancy between model prediction and observational data is the simplicity of the model. Thus, although the overall fit of the model is good, it is not surprising, given the level of abstraction, that there are elements of the fine structure of returns the model is not designed to capture.

2.5.3 Counterfactual experiment

GAO [2002] and GAO [2006] document a significant upward trend in the number of restatements over time. To gain insight on policy-related issues, it is of interest to examine how the magnitude of financial anomalies varies with the extent of earnings

[34]For example, the loss of confidence in corporate financial reporting could also hurt business and investment opportunities. Furthermore, the reduced availability and higher cost of capital may as well cause firms to postpone capital spending plans and accelerate layoffs. How to accurately measure the efficiency loss associated with earnings management is a question that warrants further research.

x=0	Coefficient	Std.Error	t-statistic
K	-4.9882	0.6962	-7.1651
G	0.0028	0.7555	0.0037
A	0.0116	0.0279	0.4133
L	0.0009	0.0007	1.2500

x=0.04	Coefficient	Std.Error	t-statistic
K	-5.0000	2.5890	-1.9312
G	0.5260	0.2454	2.1436
A	0.0529	0.0235	2.2474
L	-0.0234	0.0139	-1.6784

x=0.1	Coefficient	Std.Error	t-statistic
K	-2.9453	1.6996	-1.7330
G	0.6786	0.1855	3.6589
A	0.0353	0.0203	1.7393
L	-0.0255	0.0129	-1.9729

Table 2.9: EGARCH(1,1) estimation results with different levels of x
Variance equation:
$\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$.

x	Standard Deviation
0	0.0424
0.04	0.0714
0.1	0.1044

Table 2.10: Volatility of the model returns with different level of x

management. Here, I consider the economies with different prevalence of earnings management. Specifically, I consider various values of x to assess the importance of earnings management. In these economies with different values of x, the other parameters are chosen to match the same aggregate targets as in the benchmark calibration.

Table 2.9 presents the results. The extreme case of $x = 0$ in this model, shown in the first panel, corresponds to the standard Lucas asset-pricing model. In this case, earnings management does not exist. The estimated EGARCH coefficients are substantially reduced and insignificant. Neither long-memory persistence nor asymmetric behavior is present in the model data.

As x is increased to 0.04 as in the calibrated model, the EGARCH estimation results on the simulated return data demonstrate the presence of strong persistence and asymmetry in volatility. When $x = 0.1$, G and L coefficients become larger in magnitude and more significant. There are strong indications that incorporating earnings management intensifies both persistence and asymmetry in return volatility.

Table 2.10 contains the standard deviation of returns in the simulated data. Consistent with the empirical studies mentioned in Section 1 and Section 3.2, as x increases (implying that the informativeness of earnings reports becomes weakened), the returns exhibit greater volatility. Monetary losses incurred upon revelations generate large swings in the return sequence and hence raise volatility.

Models such as the one considered in this paper can be used to predict the consequences of a particular corporate governance rule on financial reporting. The comparison of asset returns dynamics with different prevalence of earnings management underlines the importance of earnings management in pricing financial assets and understanding the empirical anomalies; therefore this analysis adds to the current debate about advantages of strict implementation of corporate governance policies, such as the Sarbanes-Oxley Act.

2.6 Robustness check

In this section, robustness checks of the baseline model are conducted, both in terms of quantitative evaluations and model specifications. First, following an alternative calibration strategy, I recalibrate the model to Compustat Unrestated data and study the return patterns. Second, I consider a setting in which investigations are conducted stochastically, so as to make sure that model dynamics are *not* driven by the deterministic auditing cycles.

2.6.1 Alternative calibration

Of particular interest is the sensitivity of the quantitative results to the specification of restated data as true earnings. An alternative to the benchmark calibration strategy is to take unrestated data and match them with the reported earnings generated from the model. In contrast to the conventional Compustat quarterly dataset that contains restated statements, Compustat Unrestated dataset covers the initial 10Q filing for a quarter that may be subject to SEC filings and earnings restatements in subsequent quarters. Here, I recalibrate the model using the Compustat Unrestated dataset.

The Compustat Unrestated dataset starts in 1987 for U.S. companies, covering a shorter time span than the Compustat restated dataset. Table 2.11 presents the

	Mean	Std.Dev	Autocorr	Std.Dev of avg.	Avg. of Std.Dev
Scaled reports	0.10	0.22	0.82	0.03	0.15

Table 2.11: Moments of semiannual scaled reports (1987 - 2006)

Parameter	Description	Value
ρ	Autoregressive parameter	0.82
k	Constant drift	0.18
σ_a	Std.Dev of aggregate productivity shock	0.02
σ_i	Std.Dev of idiosyncratic productivity shock	0.08
β	Discount factor	0.98
x	Earnings management prevalence	0.04
a	Amount of overstatement	0.03
F_1	Monetary loss for one restatement	0.49
F_2	Monetary loss for two restatements	0.98

Table 2.12: Alternative parameterization

moments of semiannual reported earnings scaled by beginning-of-the-period market value. Here, ρ, σ_a, and σ_i are calibrated to match the average autocorrelation of firms' earnings, time variation of average reports across firms, and average time variation of reports within firms, shown in the third, fourth, and fifth entry respectively in Table 2.11. This gives $\rho = 0.82, \sigma_a = 0.02$, and $\sigma_i = 0.08$. As the steady-state report is normalized to 1, k is then set to be 0.18.

The rest of the parameters are chosen to match the same targets as in the benchmark calibration, and that gives $\beta = 0.98, x = 0.04, a = 0.03, F_1 = 0.49$, and $F_2 = 0.98$, as presented in Table 2.12. Some values are different from the benchmark calibration because of the normalization of reported earnings to unity, compared with the normalization of restated earnings to unity.

Table 2.13 contains measures of EGARCH effect for the model returns and S&P 500 Index returns. The results are similar to those with the benchmark parameterization, except that the G coefficient somewhat overshoots. The stronger persistence

Model data	Coefficient	Std.Error	t-statistic
K	-2.7503	1.4893	-1.8467
G	0.8049	0.1056	7.6225
A	0.0339	0.0166	2.0492
L	-0.0231	0.0106	-2.1911

S&P 500 data	Coefficient	Std.Error	t-statistic
K	-1.1029	0.4893	-2.2540
G	0.7365	0.1173	6.2789
A	0.3057	0.1542	1.9830
L	-0.2557	0.1127	-2.2703

Table 2.13: Comparison of EGARCH(1,1) estimation results (alternative calibration)
Variance equation:
$\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$.

	Standard Deviation
Model	0.0300
Data	0.1063

Table 2.14: Comparison of data volatility (alternative calibration)

in volatility than in the benchmark calibration is attributable to the higher calibrated persistence in firms' earnings. This result confirms that most of the volatility clustering in the model has to come from the persistent component in earnings management, which directly stems from the persistent component in earnings. This element of the model is crucial in making it consistent with the observed heteroskedasticity. The finding that EGARCH effect is quite similar for different calibration strategies suggests that, even though the parameters may differ across economies, the nature of return dynamics can still be quite similar.

Table 2.14 compares the volatility of the model and the data. Compared with the benchmark parameterization, the model volatility is reduced. The reason is that the value of monetary loss associated with earnings management is calibrated to be lower (in particular, less than half in size), leading to a more moderate reaction of asset returns to restatements. A smaller fluctuation of the returns during restatements produces lower volatility.

2.6.2 Stochastic investigation

In the baseline model, investigations are conducted deterministically every two periods. To examine how this assumption affects the results, here I consider a setting where investigations take place stochastically, which can be more generally understood as an environment where the value-destroying effect of earnings management may or may not show up in returns every period. As in Section 2 and Section 3, there are two levels of earnings: $y \in \{l, h\}$. Actual earnings follow a Markov process

$$\Pr(y_{t+1} = j | y_t = i) = \pi_{ij}, \qquad \forall i \in \{l, h\}, \quad \forall j \in \{l, h\}.$$

The investigation regarding financial reporting is now assumed to be stochastic and occurs with probability λ every period. When an investigation takes place, all the previous earnings since the most recent investigation are revealed. The financial statements in the corresponding periods when earnings management occurs have to be restated, and the investors bear monetary losses. More specifically, the amount of financial losses upon restatements is a strictly increasing function of the number of periods in which the manager manipulates earnings. The timeline of the model events in each period is described in Figure 2.7.

Note that the derivation of the posterior probability of having a false report at each point in time requires utilizing the entire history of reports since the most recent investigation up to the current report. In particular, when the manager makes an earnings announcement every period, the investors not only infer the current realization and predict future earnings, but also revise their belief about each previous

Investigation takes place with probability λ and monetary costs incur	Manager exerts effort $e \in \{L, H\}$	Manager privately learns of earnings management opportunity	Earnings realize. Manager privately observes earnings	Manager makes a report and is paid	Asset price realizes and dividends are paid

Figure 2.7: Model timeline with stochastic investigation

report in history.

Fortunately, in this setting all the relevant information in the reporting history can be summarized with a small set of state variables. In what follows, the problem is reduced to a variational problem in which history dependence can be summarized and asset price can be characterized by the following five state variables (See Appendix B for detailed examples of what each state variable represents.).[35]

- γ: the conditional probability (with the information from the current report) that the current true earnings are high;
- Z: the expected number of periods involving earnings management since the last investigation until the most recent low report ($Z = 0$ if there is no low report since the last investigation until the previous period);
- N: the number of consecutive high reports until the previous period since the last low report or the last investigation, whichever is more recent;
- r: the current earnings report, $r \in \{\tilde{l}, \tilde{h}\}$;
- $\bar{y}$: the true earnings before the series of consecutive N high reports starts.

Given the reporting incentive in this binary setting, the current true earnings are revealed under two circumstances. The first situation is when the investigation

[35] I gratefully thank Toshihiko Mukoyama for his help in solving this problem.

regarding financial reporting takes place. In this case, the entire history of earnings realizations is revealed. The second is when the manager sends a low report. If the reported earnings are low, although the credibility of financial statements in prior periods remains ambiguous, the current earnings are low with certainty. In the following, I derive the pricing functions that describe a stationary solution to the problem using these state variables. The stock price at time t is denoted by $q_t = P(\gamma_t, Z_t, N_t, r_t, \bar{y}_t)$.

Let the monetary cost of earnings management be a linear function of the number of restating periods upon investigation. Specifically, the loss $F = \kappa n$, where κ is a constant and n is the number of periods involving earnings management since the most recent investigation. As the investors update their beliefs in the standard Bayesian fashion, γ' evolves following Bayes' Rule:

$$\gamma' = \begin{cases} \dfrac{\gamma\pi_{hh} + (1-\gamma)\pi_{lh}}{\gamma\pi_{hh} + (1-\gamma)\pi_{lh} + \gamma(1-\pi_{hh})x + (1-\gamma)(1-\pi_{lh})x}, & r = \tilde{h} \text{ at } t+1, \\ 0, & r = \tilde{l} \text{ at } t+1. \end{cases}$$

First, the price in response to a high report, $P(\gamma, Z, N, \tilde{h}, \bar{y})$, is derived.[36]

$$P(\gamma, Z, N, \tilde{h}, \bar{y}) = \tilde{h} + \beta\left[(1-\lambda)W_n^{\tilde{h}} + \lambda W_i^{\tilde{h}}\right]. \tag{2.21}$$

Here, β is the discount factor. $W_n^{\tilde{h}}$ represents the expected price if the investigation does not occur in the beginning of the next period, and $W_i^{\tilde{h}}$ represents the expected price if an investigation occurs. Both prices are conditional on a current high report.

If the investigation does not take place in the beginning of the next period, the

[36] Again, the impact of wage values in price calculations is not considered in the current analysis.

expected price is

$$W_n^{\tilde{h}} = \mu P(\gamma', Z, N+1, \tilde{h}, \bar{y}) + (1-\mu) P(0, Z, N+1, \tilde{l}, \bar{y}). \tag{2.22}$$

The first term in (2.22) is the expected price if the next report is high. The second term is the expected price when the report in the next period is low. Note that a low report is always truthful, and thus γ is updated to 0. μ denotes the conditional probability that the manager makes a high report in the next period:

$$\mu = \gamma \pi_{hh} + \gamma(1-\pi_{hh})x + (1-\gamma)\pi_{lh} + (1-\gamma)(1-\pi_{lh})x$$

If the investigation takes place in the next period, the expected price is

$$\begin{aligned} W_i^{\tilde{h}} =& -\kappa[Z + f(N+1; \bar{y}))] \\ &+ \gamma \left[\xi_1 P\left(\frac{\pi_{hh}}{\xi_1}, 0, 0, \tilde{h}, h \right) + (1-\xi_1) P(0, 0, 0, \tilde{l}, h) \right] \\ &+ (1-\gamma) \left[\xi_2 P\left(\frac{\pi_{lh}}{\xi_2}, 0, 0, \tilde{h}, l \right) + (1-\xi_2) P(0, 0, 0, \tilde{l}, l) \right]. \end{aligned} \tag{2.23}$$

where ξ_1 represents the conditional probability of having a high report in the next period, given the current true earnings are high. ξ_2 is the probability of having a high report conditional on that the current true earnings are low.

$$\xi_1 = \pi_{hh} + (1-\pi_{hh})x$$

$$\xi_2 = \pi_{lh} + (1-\pi_{lh})x$$

The first term in (2.23) is the expected amount of financial penalties for earnings management. $f(N+1; \bar{y})$ denotes the expected number of falsified reports among the $(N+1)$ consecutive reports of high earnings since the last low report or the last investigation, whichever is more recent. The function $f(N+1;\bar{y})$ is calculated from the model fundamentals in a recursive manner, and the method is illustrated in Appendix C. The number of the expected restating periods is thus the sum of $f(N+1;\bar{y})$ and the expected number of periods involving earnings management from the last investigation through the most recent low report, Z. Recall that γ is the conditional probability that the current high report is truthful. The second term in (2.23) thus represents the expected price if the current high report is truthful. The third term is the case in which the current earnings are low and have been overstated.

Now let us consider the asset price if the current report is low.

$$P(0,Z,N,\tilde{l},\bar{y}) = \tilde{l} + \beta\left[(1-\lambda)W_n^{\tilde{l}} + \lambda W_i^{\tilde{l}}\right]. \tag{2.24}$$

where $W_n^{\tilde{l}}$ and $W_i^{\tilde{l}}$ represent the expected price if the investigation does not occur in the next period and the expected price if the investigation occurs, respectively, conditional on a current low report.

If the investigation does not take place in the next period, the expected price is

$$W_n^{\tilde{l}} = \xi P\left(\frac{\pi_{lh}}{\xi}, Z, 0, \tilde{h}, l\right) + (1-\xi)P(0,Z,0,\tilde{l},l)$$

where ξ denotes the conditional probability that the manager makes a high report in the next period:

$$\xi = \pi_{lh} + (1-\pi_{lh})x$$

Parameter	Value
h	20
l	10
π_{hh}	0.8
π_{ll}	0.8
β	0.98
κ	15
λ	0.5

Table 2.15: Parameter values in the numerical example with stochastic investigation

If the investigation takes place in the next period:

$$\begin{aligned} W_i^{\tilde{l}} = & -\kappa[Z + f(N;\bar{y})] \\ & + \xi P\left(\frac{\pi_{lh}}{\xi}, 0, 0, \tilde{h}, l\right) \\ & + (1-\xi)P\left(0, 0, 0, \tilde{l}, l\right) \end{aligned} \tag{2.25}$$

The first term in (2.25) is the expected monetary cost of earnings management, which is a linear function of the expected number of restating periods. The second term is the expected price if the realization of actual earnings is high in the next period, and the third term corresponds to the case in which the realization is low. Thus, from (2.21) and (2.24), the price in each period can be solved recursively.

pricing functions are computed numerically. Table 2.15 contains the parameter values. The Figure 2.8 displays $f(N, \bar{y})$, the shape of which may vary with parameterizations. Figure 2.9 and Figure 2.10 show how the price associated with a high report changes with γ and N. As the monetary cost of earnings management is a linear function of the number of restated financial statements, the price in response to a high report is linearly increasing in γ and linearly decreasing in Z. As shown in Figure 2.11, the price in response to a low report is also linearly decreasing in Z,

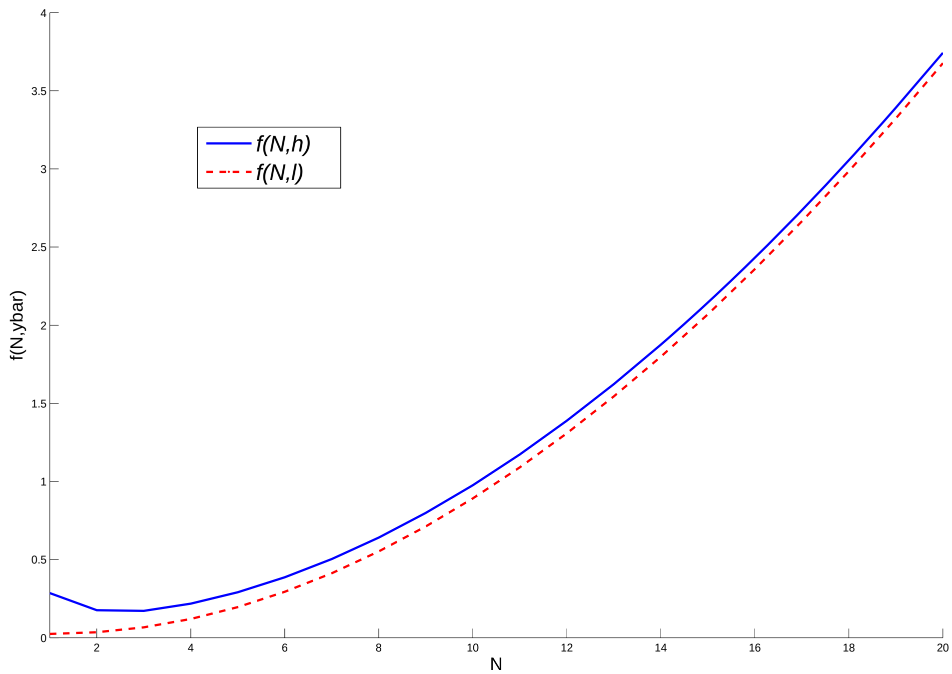

Figure 2.8: The expected number of inflated reports among N consecutive high reports $f(N, \bar{y})$

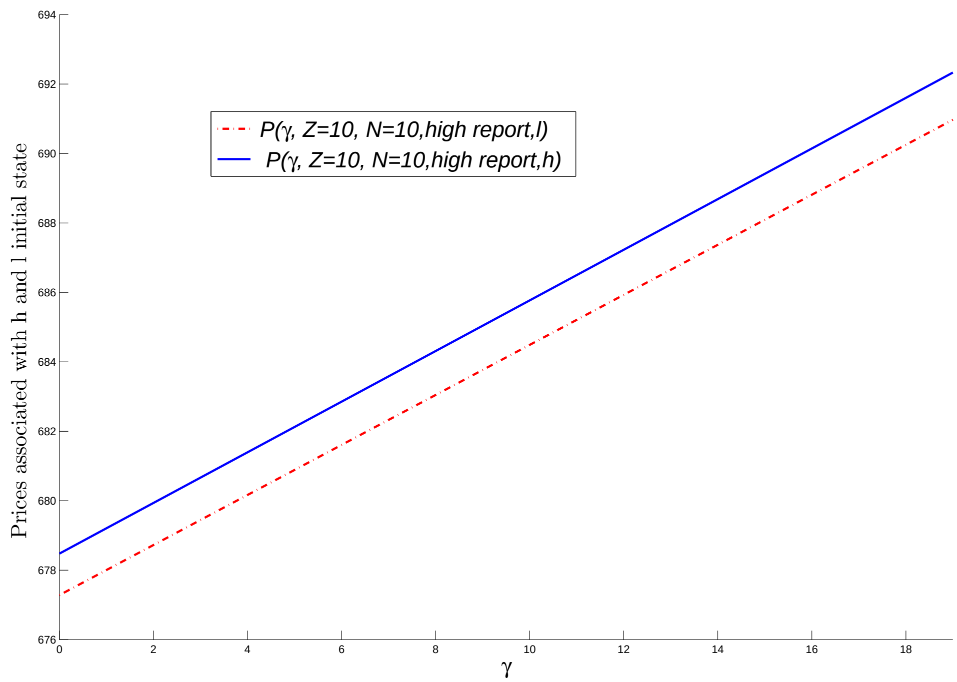

Figure 2.9: Price for a high report as a function of γ

with γ updated to 0.

The model is simulated for 10,000 periods. In order to illustrate the influence of earnings management incentive on dynamic return patterns, I compare the model returns with $x = 0$ and those with $x = 0.1$. Table 2.16 presents the EGARCH estimation results on the model returns. In a model without earnings management ($x = 0$), there is no persistence in return volatility (shown in the upper panel). As earnings management becomes possible, the coefficients of the EGARCH model are all statistically significant. Persistence and asymmetry are present in the model return volatility. In addition, Table 2.17 shows that the model returns become more volatile as x increases. The same set of results and intuition from the model with deterministic monitoring carry through.

This model of stochastic investigation assumes a constant exogenous probability

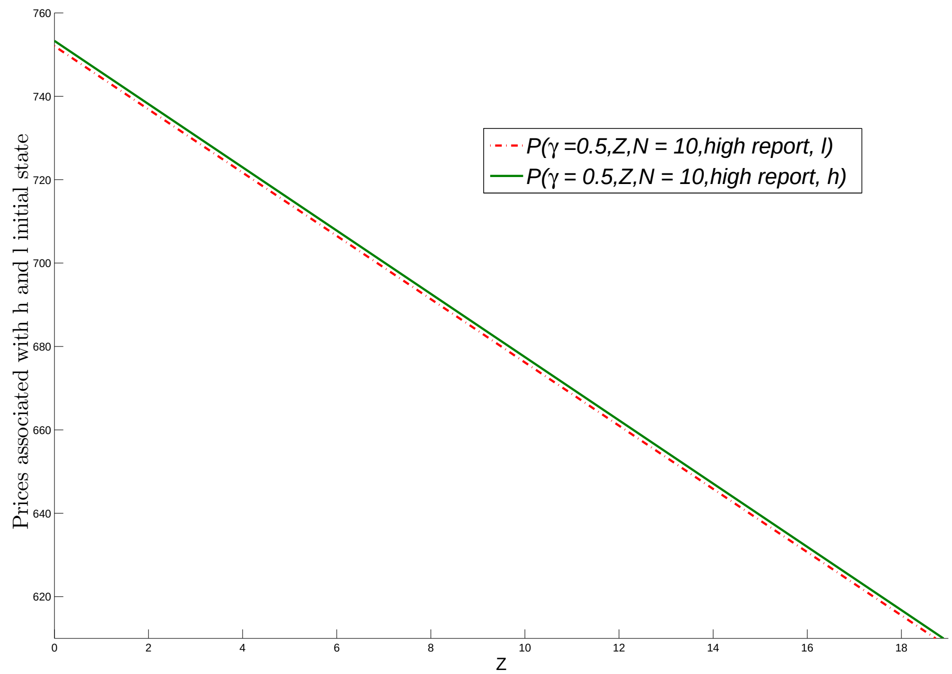

Figure 2.10: Price for a high report as a function of Z

x=0	Coefficient	Std.Error	t-statistic
K	-5.0000	12.8300	-0.3897
G	0.0576	0.0880	0.6552
A	0.0033	0.0119	0.2838
L	0.0041	0.0066	0.6195

x=0.1	Coefficient	Std.Error	t-statistic
K	-2.0291	0.2979	-6.8092
G	0.7441	0.0376	19.7951
A	0.1068	0.0207	5.1616
L	-0.0841	0.0197	-4.2789

Table 2.16: EGARCH(1,1) estimation results (stochastic investigation)
Variance equation:
$$\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$$

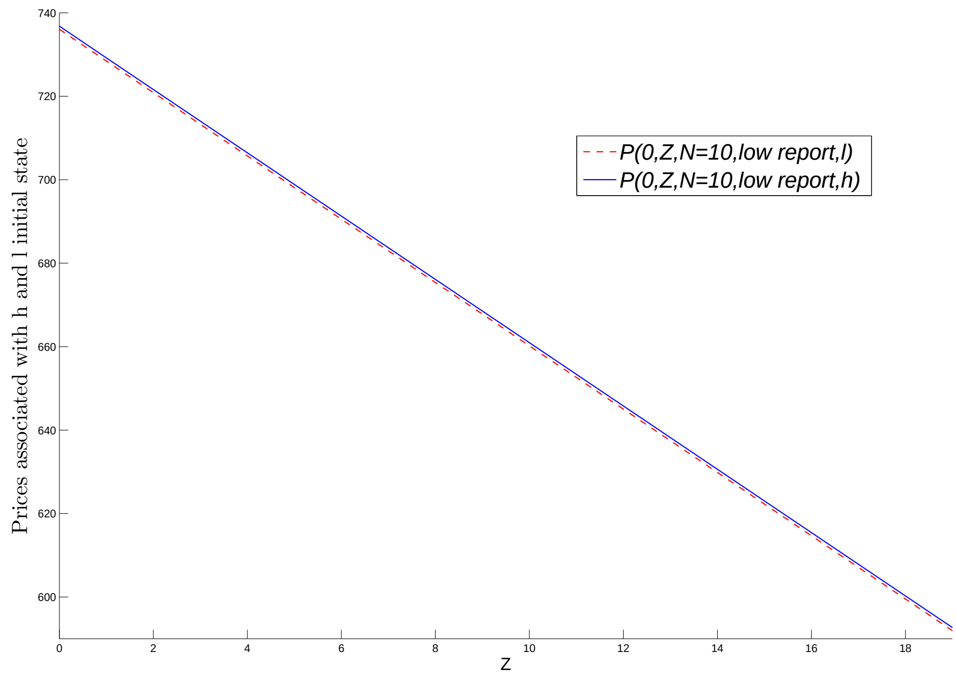

Figure 2.11: Price for a low report as a function of Z

x	Standard Deviation
0	0.0134
0.1	0.0193
0.2	0.0201

Table 2.17: Volatility of the model returns (stochastic investigation)

of monitoring in every period. However, it is natural to argue that monitoring would occur with a higher probability in bad times, since there tends to be little interest in investigating when the market is booming. Accounting fraud does come in waves, and is detected more intensively during market collapses. As monitoring occurs more often when the aggregate state of the economy is bad and earnings management is prevalent, the asymmetric behavior in stock returns tends to be more pronounced. An monitoring probability that varies with aggregate economic prospects would strengthen the mechanism illustrated in this paper and intensify these observed features of asset returns.

2.7 Conclusion

This paper examines dynamic asset return patterns in an economy in which information about the underlying profitability is obscured. An important ingredient in the current formulation of the asset-pricing problem is that executives intentionally manipulate financial information in their own best interests. Executives possess two dimensions of private information: realizations of actual earnings and realizations of earnings management opportunities. Because different combinations of these two can generate identical earnings reports, there is no strict monotonicity of the report with respect to the actual earnings, and hence the reporting function is not invertible. Although the investors are fully rational and they learn in a standard Bayesian fashion, they cannot perfectly filter out the manipulation component in the reports. Asset prices are therefore not fully-revealing in the equilibrium. Earnings management causes a pricing distortion — honest firms are undervalued, while firms that manipulate their accounting numbers are overpriced.

This study shows that an asset-pricing model with earnings management delivers the observed features of asset return data: volatility clustering, asymmetric volatility, and increased individual volatility. To the best of my knowledge, existing models cannot replicate all of these features in a representative-agent framework. A contribution of the paper is to show that incorporating creative accounting in an otherwise standard asset-pricing model can mimic a number of stylized financial facts, and earnings management may play a crucial role in price formulations in the financial markets. This research also advances the understanding of the implications of endogenously-determined earnings management behavior for the dynamic patterns of asset returns.

The quantitative analysis indicates that, in addition to producing aforementioned patterns in individual stocks, earnings management may also generate aggregate consequences in stock market indices. Importantly, these effects are symptoms of inefficiency and risk, and they are likely to be more pronounced during episodes of weak economic performance when the financial incentive to inflate earnings is particularly strong. The mechanism illustrated in this paper also presents a likely source of non-fundamental volatility and financial risk.

The current model of shareholders-manager behavior in a financial market is a simplified one. In particular, the current analysis does not explicitly model how the manager finances the discrepancy in the reports. As elaborated in section 2, leaving the source of funds outside the model is for the purpose of simplification without causing a modeling inconsistency. This formulation can also be viewed as a simple way of illustrating the idea that the manager can divert resources from profitable investment to current payouts. Formulating this idea explicitly requires a production economy with investment, and I take the current framework as the first step towards the ultimate goal. A full understanding of the welfare consequences and macroeconomic implications is a task for future research.

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2.8 Appendix

Appendix A: Proofs

Proof of Proposition 2.2:

After the parallel shift of $f(y|e = H)$ and $f(y|e = L)$ by δ, the conditional distribution of actual earnings given effort is denoted by $g(y|e = i) = f(y - \delta|e = i)$, $\forall i \in \{L, H\}$. The principal has a utility function given by $V(y - w)$.

The Lagrangian for the principal's problem in this case is

$$\mathcal{L} = \int_{\underline{y}+\delta}^{\bar{y}+\delta} \Big\{ V\left[y - w(r(y))\right] g(y|e = H) + \lambda \left[u(w)g(y|e = H) - \bar{U}\right] \\ + \mu \left[u(w)g(y|e = H) - u(w)g(y|e = L) - c\right] \Big\} dy$$

The reporting function $r(y)$ is given by

$$r(y) = \begin{cases} y + a & \text{if } u\left[w(y + a)\right] - u\left[w(y)\right] > \psi, \text{ and earnings management opportunity realizes} \\ y & \text{otherwise.} \end{cases}$$

Differentiating with respect to $w(r)$ inside the integral sign, we obtain the first-order condition. Assuming that it is optimal to elicit high effort, an optimal incentive compensation scheme $w(r)$ satisfies

$$\frac{V'[y - w(r)]}{u'[w(r)]} = \lambda + \mu \left[1 - \frac{g\left(y(r)|e = L\right)}{g\left(y(r)|e = H\right)}\right], \tag{2.26}$$

Assume that the principal is risk-neutral, and the manager's utility function takes the logarithm form given by $u(w) = \log(w)$. (2.26) simplifies to

$$\begin{aligned} w(r) &= \lambda + \mu \left[1 - \frac{g\left(y(r)|e=L\right)}{g\left(y(r)|e=H\right)} \right] \\ &= \lambda + \mu \left[1 - \frac{f\left(y(r)-\delta|e=L\right)}{f\left(y(r)-\delta|e=H\right)} \right] \end{aligned} \tag{2.27}$$

The solutions also satisfy the complementary slackness conditions

$$\lambda \int_{\underline{y}+\delta}^{\bar{y}+\delta} \left\{ u(w) g(y|e=H) - \bar{U} \right\} dy = 0,$$

$$\mu \int_{\underline{y}+\delta}^{\bar{y}+\delta} \left\{ u(w) g(y|e=H) - u(w) g(y|e=L) - c \right\} dy = 0.$$

which can be rewritten as

$$\lambda \int_{\underline{y}+\delta}^{\bar{y}+\delta} \left\{ u(w) f(y-\delta|e=H) - \bar{U} \right\} dy = 0, \tag{2.28}$$

$$\mu \int_{\underline{y}+\delta}^{\bar{y}+\delta} \left\{ u(w) f(y-\delta|e=H) - u(w) f(y-\delta|e=L) - c \right\} dy = 0. \tag{2.29}$$

The following inequalities should also be satisfied

$$\lambda \geq 0, \quad \mu \geq 0. \tag{2.30}$$

Let $w^*(r)$ be the solution to the principal's problem before the parallel shift of $f(y|e=H)$ and $f(y|e=L)$. λ^* and μ^* are the corresponding Lagrangian multipliers. Then $w^*(r)$, λ^*, and μ^* satisfy the first-order condition

$$w^*(r) = \lambda^* + \mu^* \left[1 - \frac{f\left(y(r)|e=L\right)}{f\left(y(r)|e=H\right)} \right]$$

together with the complementary slackness conditions

$$\lambda^* \int_{\underline{y}}^{\bar{y}} \left\{ u(w) f(y|e = H) - \bar{U} \right\} dy = 0,$$

$$\mu^* \int_{\underline{y}}^{\bar{y}} \{ u(w) f(y|e = H) - u(w) f(y|e = L) - c \} \, dy = 0.$$

and the inequalities

$$\lambda^* \geq 0, \quad \mu^* \geq 0.$$

It follows that

$$w^*(r - \delta) = \lambda^* + \mu^* \left[1 - \frac{f\left(y(r) - \delta | e = L\right)}{f\left(y(r) - \delta | e = H\right)} \right]$$

$$\lambda^* \int_{\underline{y}+\delta}^{\bar{y}+\delta} \left\{ u\left[w^*(r - \delta)\right] f(y - \delta | e = H) - \bar{U} \right\} dy = 0,$$

$$\mu^* \int_{\underline{y}+\delta}^{\bar{y}+\delta} \{ u\left[w^*(r - \delta)\right] f(y - \delta | e = H) - u\left[w^*(r - \delta)\right] f(y - \delta | e = L) - c \} \, dy = 0.$$

$$\lambda^* \geq 0, \quad \mu^* \geq 0.$$

It is straightforward to determine that $w(r) = w^*(r - \delta)$, $\lambda = \lambda^*$, and $\mu = \mu^*$ satisfy (2.27), (2.28), (2.29), and (2.30). The reporting choice $r(y)$ remains unchanged in this case. Therefore, a parallel shift of the wage function by δ solves the principal's problem. □

The idea underlying this analysis is that given a parallel shift of conditional distributions of output, a parallel shift of the wage payment schedule by the same amount provides the same incentive to the manager and same marginal value of effort to the risk-neutral principal. First, because the distribution of wage payment remains unchanged after parallel shifts of the wage function and output distribution by an identical amount, the manager does not have an incentive to deviate from the recommended effort and reporting choice.

Second, the risk-neutral principal designs the compensation based on the monetary value of high effort relative to low effort, which is the difference in the residuals. The residual is the expected earnings net of compensation payment, conditional on high and low effort. The monetary value of effort can be denoted by $\Big[$(expected earnings given high effort - expected payment given high effort) $-$ (expected earnings given high effort $-$ expected payment given low effort)$\Big]$. It can be rewritten as $\Big[$(expected earnings given high effort $-$ expected earnings given low effort)$-$(expected payment given high effort $-$ expected payment given low effort)$\Big]$. As long as $(\mu_H - \mu_L)$ and the wage distribution remain constant, the principal does not have any incentives to change the shape of incentive schemes.

A parallel shift of the wage schedule by an equal amount as the shift of output distributions provides the manager with the same incentive and the principal with the same value, and therefore it is an optimal contract in this case.

Proof of Lemma 4:

If $r \in (y^*, y^* + a)$,

$$\begin{aligned}
p &= \Pr[y' = r|y] \\
&= \frac{f(r-k-\rho y)}{f(r-k-\rho y) + x f(r-a-k-\rho y)} \\
&= \frac{1}{1 + x\left[\dfrac{f(r-a-k-\rho y)}{f(r-k-\rho y)}\right]} \\
&= \frac{1}{1 + x\exp\left[\dfrac{1}{2\sigma}(r-k-\rho y)^2 - \dfrac{1}{2\sigma}(r-a-k-\rho y)^2\right]} \\
&= \frac{1}{1 + x\exp\left[\dfrac{a}{2\sigma}(2r-2k-2\rho y-a)\right]}
\end{aligned}$$

Using the same property of normal distributions, it is straightforward to check that p is decreasing in r when $r < y^*$.

$$p = \frac{1}{1 + (1-x)\exp\left[\dfrac{a}{2\sigma}(2r-2k-2\rho y-a)\right]}.$$

□

Appendix B: Examples of state variables in the model with stochastic investigation

As the monetary losses upon investigation depend on the number of restated financial statements, the expected number of periods in which the manager inflates earnings since the most recent realization up to now is necessary in characterizing the prices. If there are N consecutive high reports and no low reports after the most recent investigation, a function of $f(N;\bar{y})$ determines the expected number of periods involving earnings management until the last period. If there is any low report after the last investigation, the sum of Z and $f(N;\bar{y})$ summarizes the history. In addition, γ and r incorporate the information regarding the current true state conveyed by the current report.

To be clear on what each variable represents, a set of clarifying examples are provided in the following. Now let today be $t = 10$ and let the last investigation happen at the beginning of $t = 5$. Suppose that the true state of $t = 4$ is revealed to be y_4.

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}, \tilde{h}\}$, then at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, and 7; $N = 1$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 8 is known to be low (recall that all the low reports are honest reports).

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}\}$, then at $t = 10$, $Z = 0$ (there is not any low report after the last investigation until the previous period); $N = 5$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = y_4$, because it is the known true state before the consecutive high reports.

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}\}$, then at $t = 10$, $Z = 0$ (there is not any low report after the last investigation until the previous period); $N = 5$; and $r = \tilde{l}$. $\bar{y} = y_4$, because it is the known true state before the consecutive high reports. Note that $\gamma = 0$ at $t = 10$, because the current low report is an honest one.

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}, \tilde{l}, \tilde{h}\}$, then at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, and 8; $N = 0$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 9 is known to be low (all the low reports are honest reports). Note that in the case of $N = 0$, $\bar{y}$ is set to be y_{t-1} ($N = 0$ occurs only when the report at $(t-1)$ is low or the investigation happens at the beginning of t).

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}\}$, then at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, 7, and 8; $N = 0$; and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 9 is known to be low (Again, all the low reports are honest reports).

Let today be $t = 5$ and let the investigation happen at the beginning of $t = 5$.

- If $r_5 = \tilde{h}$, then $Z = 0$, $N = 0$, $r = \tilde{h}$, and $\bar{y} = y_4$.
- If $r_5 = \tilde{l}$, then $Z = 0$, $N = 0$, $r = \tilde{l}$, and $\bar{y} = y_4$.

Appendix C: Calculation of $f(N;\bar{y})$ in the model with stochastic investigation

Let the information set $\mathcal{R}_N^{\bar{y}} \equiv \{\bar{y}, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\}$. y_n represents the true earnings in period n, $\forall n \in \{1, 2, \cdots, N\}$. Thus $f(N;\bar{y})$ can be written as

$$\begin{aligned} f(N;\bar{y}) = & \Pr[y_1 = l|\mathcal{R}_N^{\bar{y}}] + \Pr[y_2 = l|\mathcal{R}_N^{\bar{y}}] + \cdots \\ & + \Pr[y_n = l|\mathcal{R}_N^{\bar{y}}] + \cdots + \Pr[y_N = l|\mathcal{R}_N^{\bar{y}}] \end{aligned}$$

The problem of deriving $f(N;\bar{y})$ in a recursive way is transformed into an equivalent problem, that is, to recursively derive

$$\Pr[y_n = l|\mathcal{R}_N^{\bar{y}}] = 1 - \Pr[y_n = h|\mathcal{R}_N^{\bar{y}}], \qquad \forall n \in \{1, 2, \cdots, N\}.$$

Note that

$$\mathcal{R}_N^h \equiv \{h, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\}$$
$$\mathcal{R}_N^l \equiv \{l, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\}$$

The proof includes two steps. In step 1, $\Pr[y_1 = h|\mathcal{R}_1^l]$ and $\Pr[y_1 = h|\mathcal{R}_1^h]$ are calculated. In step 2, I show that $\Pr[y_n = h|\mathcal{R}_{N+1}^l]$ and $\Pr[y_n = h|\mathcal{R}_{N+1}^h]$, $\forall n \in \{1, 2, \cdots, N+1\}$, can be calculated using $\Pr[y_n = h|\mathcal{R}_N^l]$ and $\Pr[y_n = h|\mathcal{R}_N^h]$, $\forall n \in \{1, 2, \cdots, N\}$.

As the first step, $\Pr[y_1 = h|\mathcal{R}_1^l]$ and $\Pr[y_1 = h|\mathcal{R}_1^h]$ are derived as follows.

$$\begin{aligned}
\Pr[y_1 = h|\mathcal{R}_1^l] &= \Pr[y_1 = h|\bar{y} = l, r_1 = \tilde{h}] \\
&= \frac{\Pr[y_1 = h, r_1 = \tilde{h}|\bar{y} = l]}{\Pr[r_1 = \tilde{h}|\bar{y} = l]} \\
&= \frac{\Pr[r_1 = \tilde{h}|y_1 = h, \bar{y} = l] \times \Pr[y_1 = h|\bar{y} = l]}{\Pr[r_1 = h|y_1 = h, \bar{y} = l] \times \Pr[y_1 = h|\bar{y} = l] + \Pr[r_1 = h|y_1 = l, \bar{y} = l] \times \Pr[y_1 = l|\bar{y}} \\
&= \frac{\pi_{lh}}{\pi_{lh} + (1 - \pi_{lh})x}, \\
\Pr[y_1 = h|\mathcal{R}_1^h] &= \Pr[y_1 = h|\bar{y} = h, r_1 = \tilde{h}] \\
&= \frac{\Pr[y_1 = h, r_1 = \tilde{h}|\bar{y} = h]}{\Pr[r_1 = \tilde{h}|\bar{y} = h]} \\
&= \frac{\Pr[r_1 = \tilde{h}|y_1 = h, \bar{y} = h] \times \Pr[y_1 = h|\bar{y} = h]}{\Pr[r_1 = h|y_1 = h, \bar{y} = h] \times \Pr[y_1 = h|\bar{y} = h] + \Pr[r_1 = h|y_1 = l, \bar{y} = h] \times \Pr[y_1 = l|} \\
&= \frac{\pi_{hh}}{\pi_{hh} + (1 - \pi_{hh})x}.
\end{aligned}$$

In step 2, I first show that $\Pr[y_n = h|\mathcal{R}_{N+1}^l]$ can be calculated if $\Pr[y_n = h|\mathcal{R}_N^l]$ is known. For $n \in \{1, 2, \cdots, N+1\}$,

$$\Pr[y_n = h | \mathcal{R}_N^l, r_{N+1} = \tilde{h}] = \frac{\Pr[y_n = h, r_{N+1} = \tilde{h} | \mathcal{R}_N^l]}{\Pr[r_{N+1} = \tilde{h} | \mathcal{R}_N^l]}. \tag{2.31}$$

The denominator in (2.31), $\Pr[r_{N+1} = \tilde{h} | \mathcal{R}_N^l]$, is derived as the following.

$$\begin{aligned}
\Pr[r_{N+1} = \tilde{h} | \mathcal{R}_N^l] =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = h | \mathcal{R}_N^l] + \Pr[r_{N+1} = \tilde{h}, y_{N+1} = l | \mathcal{R}_N^l] \\
=& \Pr[r_{N+1} = \tilde{h} | y_{N+1} = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = h | \mathcal{R}_N^l] \\
& + \Pr[r_{N+1} = \tilde{h} | y_{N+1} = l, \mathcal{R}_N^l] \times \Pr[y_{N+1} = l | \mathcal{R}_N^l] \\
=& \Pr[y_{N+1} = h | \mathcal{R}_N^l] + x \left[1 - \Pr[y_{N+1} = h | \mathcal{R}_N^l]\right],
\end{aligned}$$

where

$$\begin{aligned}
\Pr[y_{N+1} = h | \mathcal{R}_N^l] =& \Pr[y_{N+1} = h, y_N = h | \mathcal{R}_N^l] + \Pr[y_{N+1} = h, y_N = l | \mathcal{R}_N^l] \\
=& \Pr[y_{N+1} = h | y_N = h, \mathcal{R}_N^l] \times \Pr[y_N = h | \mathcal{R}_N^l] \\
& + \Pr[y_{N+1} = h | y_N = l, \mathcal{R}_N^l] \times \Pr[y_N = l | \mathcal{R}_N^l] \\
=& \pi_{hh} \Pr[y_N = h | \mathcal{R}_N^l] + \pi_{lh} \left[1 - \Pr[y_N = h | \mathcal{R}_N^l]\right].
\end{aligned} \tag{2.32}$$

As $\Pr[y_N = h | \mathcal{R}_N^l]$ is known from the supposition, this can be calculated. The denominator is obtained

$$\begin{aligned}
\Pr[r_{N+1} = \tilde{h} | \mathcal{R}_N^l] =& \pi_{hh} \Pr[y_N = h | \mathcal{R}_N^l] + \pi_{lh} \left[1 - \Pr[y_N = h | \mathcal{R}_N^l]\right] \\
& + x\{1 - \pi_{hh} \Pr[y_N = h | \mathcal{R}_N^l] - \pi_{lh} \left[1 - \Pr[y_N = h | \mathcal{R}_N^l]\right]\}.
\end{aligned} \tag{2.33}$$

Now let us consider the numerator in (2.31). For $n = N+1$, $\Pr[y_{N+1} = h, r_{N+1} =$

$\tilde{h}|\mathcal{R}_N^l]$ can be rewritten as

$$\begin{aligned}\Pr[y_{N+1} = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^l] &= \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = h|\mathcal{R}_N^l] \\ &= \Pr[y_{N+1} = h|\mathcal{R}_N^l],\end{aligned}$$

where $\Pr[y_{N+1} = h|\mathcal{R}_N^l]$ is derived in (2.32).

For $n \in \{1, 2, \cdots, N\}$, the numerator $\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^l]$ can be rewritten as

$$\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^l] = \Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}_N^l] \times \Pr[y_n = h|\mathcal{R}_N^l].$$

Here, $\Pr[y_n = h|\mathcal{R}_N^l]$ is known from the supposition. Now we only need to check if $\Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}_N^l]$ can be calculated. I rewrite

$$\Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}_N^l] = \Theta + \Lambda,$$

where

$$\begin{aligned}\Theta =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\ =& \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, y_n = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\ =& 1 \times \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\ =& \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l], \end{aligned} \tag{2.34}$$

$$\begin{aligned}\Lambda =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = l | y_n = h, \mathcal{R}_N^l] \\ =& \Pr[r_{N+1} = \tilde{h} | y_{N+1} = l, y_n = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = l | y_n = h, \mathcal{R}_N^l] \\ =& x \left[1 - \Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l]\right] \\ =& x[1 - \Theta]. \end{aligned} \tag{2.35}$$

If $n = N$, it is straightforward to determine that

$$\Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l] = \pi_{hh}.$$

Now let us consider $\Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l]$ if $n < N$. Because actual earnings y follow a Markov process, all the past information is fully summarized in the most recent realization, and the prior realizations are informationally irrelevant. Thus,

$$\begin{aligned}\Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l] &= \Pr[y_{N+1} = h | y_n = h, \bar{y} = l, r_1 = \tilde{h}, \cdots, r_N = \tilde{h}], \\ &= \Pr[y_{N+1} = h | \bar{y} = h, r_{n+1} = \tilde{h}, \cdots, r_N = \tilde{h}]\end{aligned}$$

and

$$\Pr[y_{N+1} = h | \bar{y} = h, r_{n+1} = \tilde{h}, \cdots, r_N = \tilde{h}] = \Pr[y_{N-n+1} | \bar{y} = h, r_1 = \tilde{h}, \cdots, r_{N-n} = \tilde{h}].$$

Recall that $\mathcal{R}_{N-n}^h \equiv \{\bar{y} = h, r_1 = \tilde{h}, \cdots, r_{N-n} = \tilde{h}\}$. Therefore,

$$\Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l] = \begin{cases} \Pr[y_{N-n+1} = h | \mathcal{R}_{N-n}^h] & \text{if } n < N, \\ \pi_{hh} & \text{if } n = N. \end{cases} \tag{2.36}$$

and

$$\begin{aligned}\Pr[y_{N-n+1} = h|\mathcal{R}^h_{N-n}] =& \Pr[y_{N-n+1} = h, y_{N-n} = h|\mathcal{R}^h_{N-n}] + \Pr[y_{N-n+1} = h, y_{N-n} = l|\mathcal{R}^h_{N-n}]\\ =& \Pr[y_{N-n+1} = h|y_{N-n} = h, \mathcal{R}^h_{N-n}] \times \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}]\\ &+ \Pr[y_{N-n+1} = h|y_{N-n} = l, \mathcal{R}^h_{N-n}] \times \Pr[y_{N-n} = l|\mathcal{R}^h_{N-n}]\\ =& \pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}] + \pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}]\right],\end{aligned}$$

where $\Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}]$ is known from the supposition, since $N - n < N$. Therefore, Θ and Λ can be both calculated. Hence, the numerator in (2.31) can be derived following this procedure. The numerator is obtained

$$\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N] = \begin{cases} \pi_{hh} \Pr[y_N = h|\mathcal{R}^l_N] + \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}^l_N]\right] & \text{if } n = N+1, \\ \Pr[y_N = h|\mathcal{R}^l_N] \left[\pi_{hh} + x(1-\pi_{hh})\right] & \text{if } n = N, \\ \Pr[y_n = h|\mathcal{R}^l_N]\Big\{\pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}] + & \text{if } n < N. \\ \pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}]\right] & \\ +x\{1 - \pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}] - \pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}]\right]\}\Big\} & \end{cases} \tag{2.37}$$

Now combining the expressions (2.33) and (2.37), it has been shown that $\Pr[y_n = h|\mathcal{R}^l_N, r_{N+1} = \tilde{h}]$ can be calculated using $\Pr[y_n = h|\mathcal{R}^l_N, r_N = \tilde{h}]$. The same procedure can be repeated for $\Pr[y_n = h|\mathcal{R}_N^h, r_{N+1} = \tilde{h}]$ as follows.

$$\Pr[y_n = h|\mathcal{R}_N^h, r_{N+1} = \tilde{h}] = \frac{\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^h]}{\Pr[r_{N+1} = \tilde{h}|\mathcal{R}_N^h]}.$$

where the denominator is

$$\begin{aligned}\Pr[r_{N+1} = \tilde{h}|\mathcal{R}_N^h] =& \pi_{hh} \Pr[y_N = h|\mathcal{R}_N^h] + \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}_N^h]\right] \\ &+ x\{1 - \pi_{hh} \Pr[y_N = h|\mathcal{R}_N^h] - \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}_N^h]\right]\}.\end{aligned}$$

and the numerator is

$$\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^h] =$$
$$\begin{cases} \pi_{hh} \Pr[y_N = h|\mathcal{R}_N^h] + \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}_N^h]\right] & \text{if } n = N+1, \\ \\ \Pr[y_N = h|\mathcal{R}_N^h] \left[\pi_{hh} + x(1 - \pi_{hh})\right] & \text{if } n = N, \\ \\ \Pr[y_n = h|\mathcal{R}_N^h] \Big\{ \pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h] + & \text{if } n < N. \\ \pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h]\right] \\ +x\{1 - \pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h] - \pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h]\right]\} \Big\} \end{cases}$$