

Concrete Canoe Capstone - Final Report

Paxton Gunn, Nicholas Lacy, Ella Maulfair, Yajaira Membreno Lara, Dervon Parchment,
Mason Pearce

Introduction

Project Problem Statement

Concrete is one of the most widely used construction materials in the world but also utilizes some of the most environmentally damaging processes during its life cycle. The Concrete Canoe Capstone Design Team has been tasked by the Concrete Canoe Competition Committee (C4) to develop a design and construction process for the creation of a concrete canoe. The design must be lightweight, durable, environmentally sustainable, and able to produce 100 canoes using the same mold and methods. The team plans on adhering to design standards that are outlined in the Request for Proposal (RFP) that was sent out by C4. As the team aims to meet the standards of the committee, we are aware of the greater sustainability implications of our project and the work that can come out of it.

The team has also been challenged with not only meeting the requirements of C4, but additionally, meeting them in the form of a 3D-printed concrete canoe. The team will be pioneering a new form of concrete canoe construction and will take on extensive research, development, design and construction activities to meet this goal. This research will support the UVA Concrete Canoe team in creating a canoe that will allow them to be better positioned to win the Regional Competition and represent UVA at the 2026 National Competitions, which are likely to be held at the University.

Design Objectives

The Concrete Canoe Capstone team will be leading advanced research and development in 3D-printed concrete (3DPC) to develop the first ever process for designing and building a 3D-printed post-tensioned concrete canoe. The team will also aid future UVA concrete canoe teams by creating detailed documents on how to replicate the process of 3D-printing a concrete canoe. The team will consist of 3 sub-teams: Mix Design, Hull Design and Construction. Each team will lead different parts of the design process.

The Mix Design team is tasked with researching, developing and testing a concrete mix design that adheres to the standards of the RFP while also being printable. They will make multiple mix tables; perform compression, flexural beam, and dog bone testing; and mix the concrete that is to be printed by the machine. Through this extensive research and mix testing, the Mix Design team will accumulate valuable knowledge on what lightweight aggregates are printable, and how they affect the strength of the concrete mix.

The Hull Design team will design and analyze the shape of the canoe hull. The team will formulate curve equations, model the canoe in AutoCAD, and perform structural and buoyancy calculations. They are also tasked with learning and developing the Geometric Code (G-Code) that is to be inputted into the 3D-printer if future teams were to use it for their canoes. While the 3D-printer may not be used this semester, the research and design done by the Hull Design team will help the UVA Concrete Canoe team design and model a full-scale 3D-printed concrete canoe, as well as developing a G-Code that is successful in printing the curves of the canoe.

The Construction team will design the canoe printing process and the post-tensioning system that will be used to attach the 3D-printed pieces of the canoe. Due to size limitations of the 3D-printer, the canoe cannot be printed all in one piece. This presents a challenge of how to successfully attach the pieces, so the canoe remains strong and hydrodynamic. This will include performing post-tensioning calculations and spacing calculations. At the end of the process, the Construction team will oversee the creation of the final prototype. The developments made by the Construction team will be used to create a repeatable process for connecting the canoe curves that can be scaled up to create a full-size 3D-printed post-tensioned concrete canoe.

At the end of the year, the Concrete Canoe Capstone Design team will produce a scale model of a 3D-printed post-tensioned concrete canoe prototype that will be used to move the UVA Concrete Canoe team into a new era of 3D printed concrete canoes.

Background

Concrete production currently contributes to about 8% of global CO₂ emissions. 3D-printing concrete significantly enhances sustainability in construction by reducing waste, improving material efficiency, and enabling eco-friendly design. Unlike traditional methods that usually rely on formwork, which is often discarded after a single use, 3D-printing deposits concrete precisely where it's needed, layer by layer, eliminating excess and minimizing material waste. This process also supports the use of sustainable concrete mixes that incorporate supplementary cementitious materials like fly ash or slag, as well as recycled aggregates, which collectively lower the cement content and reduce the carbon footprint of construction. Additionally, 3D-printing reduces the energy demand of construction by streamlining the building process, cutting down on the use of heavy machinery and shortening build times. It also enables the creation of optimized structures with hollow geometries that use less material without compromising strength. These performance-based designs would allow engineers to build more with less, increasing overall efficiency. 3D-printing offers the potential for localized, on-demand production, reducing the need to transport heavy materials and components over long distances. This makes it especially promising for resource-constrained or disaster-affected regions, where sustainable, scalable solutions are urgently needed.

The Concrete Canoe Competition, hosted by the American Society of Civil Engineers (ASCE), challenges university teams to design, build, and race a concrete canoe, testing their ability to balance strength, buoyancy, and weight reduction. While traditional construction

methods for these canoes rely on molds and hand-layering, this project aims to introduce 3D-printing into the competition for the first time, setting a precedent for future sustainable designs.

Design

The project scope has shifted due to malfunctioning in the 3D concrete printer, and the team will be designing a mold, tensioning system, and mix design to be used for casting a concrete canoe. Sections of the canoe will be hand-cast to imitate what the 3D printed sections would look like. This is supposed to be a sustainable design creating a reusable mold and a mix with sustainable aggregates. Hull design curves, models, and calculations can be found in Appendix D. Mix design and strength results can be found in Appendix D. Tensioning calculations and materials can be found in Appendix D.

Hull

Due to the shift in project scope, our plan went from creating a hull design to be 3D-printed by a concrete printer to creating a hull design to be used in casting sections of a concrete canoe replicating the 3D-printing process. Our curve design can be used for both hand-molding and 3D printing, although it is better when hand paved to ensure consistent depth.

The final curves are raised to the fifth power to allow for a more flat-bottomed design, and easier tensioning and gluing process. The final design's hull curves can be found in Appendix D which includes the interior and exterior curves needed for the injection molds and female molds. The middle 30 inches of the canoe were created using injection molds while either end of the canoe was created using female molds. There are ten exterior curves and ten interior curves with the largest curve representing the middle 30 inches of the canoe. The remaining nine curves represent the final nine inches of the canoe on either end. The canoe is designed to be 48 inches long, 10 inches wide, and 6 inches high. The final design allows for a freeboard of 4.13 inches as seen by the calculations for this can be found in Appendix D. To accommodate the swap to hand-casting, we designed a Styrofoam mold made using hot knives and saws. This mold was duplicated to create the entire centerpiece, then wrapped in several layers of duct tape. This was an important step as it allowed us to re-use the molds for the future.

As we shifted our scope from iteration 1 to the final design, our use of G-code and 3D printing evolved as well. Initially, we modeled our print using Prusa3D, which gave us a digital model to edit as well as the ability to print and test. However, once it was converted to G-code we realized that the printer would struggle greatly to print the design. In iteration 2, we used our digital model to play with curves, running prints and tests to give us a general idea of how our project would come out. The G-code we developed was primarily used to troubleshoot the printer, as we were unsure how to fix it due to the lack of communication from the maker. Finally, in the final design, we had the digital models for show, not pursuing anymore tests with them as we had moved into our practical design creation.

Mix

Due to the shift in project scope, our plan has gone from making a 3D-printable mix to one that is optimal for traditional molding. This mix will have the same focuses as a traditional concrete canoe team mix, that is, being as light and as strong as possible. Furthermore, our mix will have sustainability as a key focus of its design philosophy. The primary reason for this is that we believe we are designing for the future, and the future of concrete must be greener if it is an industry that wishes to continue to be profitable in an ever more environmentally conscious world.

Due to the large change in our mix's goals, much of our data and research from last semester is no longer of use to us, this is not, however, a terrible setback. Due to us switching to a traditional concrete canoe construction we can pull from the data of our own concrete canoe team to inform our mix design decisions. From our work with the traditional concrete canoe team at UVA we have learned some very important lessons that will inform our design to a great degree.

The largest takeaway from the concrete canoe teams mixes is the type of aggregate we will be using. On paper, poraver seems to be optimal for a concrete canoe, however in actuality it is often nearly impossible to work with, mostly due to the difficulty in cleaning equipment that encounters it. We also know that ASCE tends to ban poraver in concrete canoe competitions which means any work done with poraver by the capstone team now has the potential to be useless to future teams if poraver is used.

In future mixes we will be using lightweight aggregates that have been proven to be nearly as effective as poraver but also demonstrate greater strengths and increased workability. These include things such as puffed shale, pumice, and other lightweight materials. Some research will be done to determine an aggregate best for our needs i.e. Lightweight, cheap, and strong.

The two final mix designs, Grout Mix and Base Mix (Appendix D), represent iterations of a sustainable concrete mix optimized for traditional mold-based casting rather than 3D-printing, following a major shift in the capstone project's scope. Both mixes use Portland Limestone Cement (PLC), blast slag, and water in identical proportions, supported by pumice as the lightweight aggregate. This substitution reflects lessons learned from the general concrete canoe team, particularly the decision to avoid poraver due to its poor strength, cleaning difficulty, and potential disqualification under ASCE competition rules. The key difference between the final mixes is the inclusion of a high-range water reducer (HRWR) in the Grout Mix. HRWR improves workability without increasing water content, leading to a stronger and more durable final product. This provides an advantage especially critical for achieving high strength-to-weight ratios. While both mixes prioritize sustainability through supplementary cementitious materials (SCMs) like slag and natural aggregates like pumice, the Grout Mix's inclusion of HRWR makes it more suitable where flowability and placement ease are vital for acting as a glue for the molds. These carefully tailored formulations not only align with

competition goals of strength and lightness but also embody the broader mission of innovating greener construction materials for a more sustainable future.

Construction

Despite the change in our project scope going from 3D-printing concrete sections to sectional molding, the post-tensioning calculations have remained consistent. Our objective still depends on the integration of a post-tensioning system to unify the molded sections. The calculations we performed allowed us to determine the total number of threaded rods our canoe will require, as well as their spacing for effective post-tensioning.

The first iteration of the design consisted of 3D-printing the sections of the canoe, and utilizing a post-tensioning system, as well as a concrete mortar to join the sections and seal any gaps that could potentially leak. During this iteration, we performed two sets of post-tensioning calculations (can be found in Appendix D: Construction): one set for the 4' scale model, and another set for a full-size concrete canoe. These calculations were meant to provide a guideline for future teams in post-tensioning their canoe. Unfortunately, the team faced issues with 3D-printer and had to pivot to hand-casting the sections of the canoe.

The Construction team tailored the second iteration of the design to fit the constraints faced by the malfunctioning of the 3D-printer. Injection and female molds were designed and built for the middle and ends of the canoe using the interior and exterior curves designed by the Hull Design team. A spacing guide was used to ensure that the plastic tubing was placed evenly throughout each curve. After tubing was in place, concrete was poured into each mold and cured under a plastic tent. After curing, the team intended to connect the sections by running threaded rod through the tubes and using hex nuts on each end to tension the rods. Unfortunately, the team discovered that most of the tubes had been clogged with concrete paste and unclogging them was too risky. To attach the pieces, the team choose to use a concrete mortar between each of the sections.

Design Constraints

The team plans on adhering to design standards that are outlined in the Request for Proposal (RFP) that was sent out by C4. Along with the requirements from the RFP, the team will be using ASTM standards for concrete practices. Some of the technical considerations for the project include material properties such as strength and density of concrete and structural design such as hull shape and buoyancy. Regulatory constraints for the canoe include the use of safety gear when constructing the canoe and material restrictions. Some logistical constraints the team is dealing with are molding and formwork, curing time for concrete, and team coordination as there are three sub teams that must work together to produce the canoe.

The team must adhere to ASTM standards for concrete practices. Additionally, safety standards for handling concrete materials and constructing the canoe must be observed according to competition regulations and general industry safety protocols.

Additionally, the team found that the functionality of the 3D-printer would be a major constraint on the project. While trying to use the printer, the team found that the quality of the prints was extremely poor due to the age and lack of maintenance on the printer. This caused the team to have to pivot to a different construction method that would mimic the properties of the cross-section as if they were 3D-printed.

Recommendations for the Future

Throughout the course of this capstone project, the capstone team has gone through many iterations of trial and error and have learned many lessons along the way. To help future UVA concrete canoe teams, the capstone team has compiled a list of things that we would have done differently and recommendations for the future.

Improving the Process of 3D-Printing a Concrete Canoe

Hull Design:

The hull shape for a 3D-printed canoe should be made from flat-bottomed curves for ease of printing. This will produce a sturdier, more bulky canoe. The team should run through many iterations of equations to find the one that is most efficient yet can still be easily printed. If the prototype is being molded and tensioned in the future, the team could get away with a more streamlined shape to produce a more efficient canoe in the water. A flat-bottomed canoe is created from higher power equations while a more streamlined canoe is created from lower power equations. The type of curve can also depend on the type of mix used, so coordination with the mix team is necessary.

When beginning to develop the 3D model, use a pre-made canoe from one of several online sites. This creates a “save-state” which makes backtracking much easier if you decide to change the curves or hull shape. G-code is difficult, especially with the current state of the environmental lab printer. The biggest recommendation we can make is to budget \$20 for a 3-month subscription to a CAD to G-code website (average cost). When trying to convert the design, the team had to go through 3 different websites to take advantage of free trials, which made trouble shooting the code next to impossible. By focusing on one website, it will allow tests to have a much more central point of interest. Finally, when running the G-code always begin with a cube test. This is simply using G-code of a two-layered cube to test out the axis design. This will allow you to troubleshoot any issues on any axis before trying to run the actual design, which will help save not only concrete mix but prevent unnecessary wear and tear on the machine.

Mix Design:

When beginning concrete mix development, the aggregate type should be finalized before any major prototyping or mix experimentation is done. Switching aggregates late in the process

— as we experienced moving from pumice to Poraver — can drastically change the behavior of the mix and set progress back significantly.

All team members should also be properly trained in both mix design and correct concrete mixing procedures. Having only one or two people familiar with the process creates major risks if any issues arise. Redundancy in knowledge is critical for a smooth operation.

When refining a mix, teams should invest time into researching admixtures. Admixtures are powerful tools that can fine-tune a good mix into a great one if used correctly. Understanding what each admixture does and how it interacts with your specific mix is essential to achieving the desired properties.

Lastly, it's important to design mixes that are not just high-performing but also *useful* for the project goals. It's easy to get caught up in maximizing strengths like compressive strength or density, but a mix that excels on paper might be completely impractical in actual application if it doesn't suit the project's needs.

Construction:

A challenge we faced was ensuring consistent tube spacing throughout the canoe. Although our initial spacing guide provided a uniform placement on one side, there were several factors that contributed to the inconsistencies along the full length of the canoe. With the tubing having acquired bending, along with tube shifting during the concrete pour, made it difficult to maintain a straight, through and through alignment. Moving forward, and maintaining the tubing method, it would be beneficial to have the tubing straightened out beforehand while also developing a more rigid spacing system to better secure the tubes placement throughout the pouring.

For improvement in the tensioning system, material selection should be improved upon to reduce the amount of space it occupies within the hull, increase the amount of concrete around the tensioning elements, and enhance the overall strength of the canoe structure. An alternative material to that of our threaded rod could be steel wire. It would contribute as well furthermore flexibility, facilitating the installation throughout the curved sections.

Another future consideration is the possibility of utilizing the 3D-printer available at the A-School. Should it be used, an alternative method or mold would need to be developed to accurately place the tubing within the printed structure to ensure proper alignment and tensioning without the risk of tube shifting either during the printing or curing process. Integrating tubing into a printed canoe could require a new material strategy but could result in greater precision for an overall cleaner final product.

Topics of Focus for Future Capstone Teams

While researching and testing methods for 3D-printing a concrete canoe, the team encountered topics of research that would be of interest to the main canoe team to continue pursuing. Introducing tensioning to the canoe is a valuable topic as it allows for an increase in tensile strength and reduction in hull thickness. The second being researching and testing different lightweight aggregates beyond poraver for the concrete mix design.

The team developed a design for a post-tensioning system as the main way of connecting the 3D-printed sections of the canoe. In addition to designing the system for our prototype size of 4.00', the team also designed a system for a full-size canoe. Post-tensioning is a valuable topic of research for the team and should be researched further. However, it is not very practical when applied to traditional concrete canoe construction methods. In addition to continuing post-tensioning research, the team also recommends that pre-tensioning becomes a new topic of research. Pre-tensioning the concrete canoe is a popular method of providing tensile support to the canoes. The cables are positioned over the mold and tensioned prior to concrete placement. After the concrete has cured, the cables are released. Using tensioned cables as a form of tensile support allows for a reduction in hull thickness which decreases the overall weight of the canoe, making it better equipped to perform well in the races as well as the aesthetics competition.

The mix design team developed a concrete mix for our prototype with the main aggregate being pumice stone. Pumice is a great aggregate for concrete canoe construction as it is lightweight and strong. In the past, the UVA Concrete Canoe Team's main lightweight aggregate has been poraver which are small beads of expanded glass. While extremely lightweight, poraver is also very weak, and is not always permitted by the RFP. To have a solid foundation for future teams, research into other lightweight aggregates that are lightweight without compromising strength would be recommended.

Conclusion

The Concrete Canoe Capstone team was tasked with designing and fabricating a concrete canoe that is entirely 3D-printed. The canoe must adhere to the standards set in the Request for Proposal that is sent out each year by C4. The team split into three sub-teams that each took on different design and fabrication tasks.

The process of using a 3D concrete printer begins with developing a digital design of the canoe in software such as Solid Works or AutoCAD, which is then converted into G-code to guide the printer's movements layer by layer. Equally important is formulating a printable concrete mix. An easily replicable mix that maintains enough flowability to extrude smoothly while setting quickly enough to support successive layers provided with its own challenges. At UVA, the current 3D concrete printer faces several technical challenges. It requires servicing and new spare parts before it can be reliably used, presenting a logistical hurdle for ongoing projects. Printing a cross-section of a concrete canoe introduces its own unique complexities, such as managing overhangs and ensuring geometric precision, which differ significantly from the traditional male or female mold methods used by the broader concrete canoe team. These

challenges highlight both the potential and current limitations of additive manufacturing in structural concrete applications. Despite these challenges the team was able to pivot from its original plans to 3D print the canoe while also providing the groundwork for future teams to continue working towards that end goal.

Incorporating 3D concrete printing into the concrete canoe competition could have several impactful implications. First, it offers the potential to streamline and innovate the construction process by reducing the need for large, time-consuming molds and allowing for more complex, precise geometries that would be difficult to achieve with traditional methods. This could open new possibilities in design optimization for hydrodynamics and weight distribution. Additionally, experimenting with printable mixes could lead to material innovations that improve performance while being more sustainable or cost-effective. However, the current limitations, such as printer reliability and the technical learning curve, must be addressed before this technology can be fully integrated. If resolved, 3D-printing could significantly improve the efficiency, customization, and experimental freedom of the concrete canoe building process, giving teams a competitive edge and pushing the boundaries of what's possible in student-led engineering.

Appendices

Link to Capstone Folder: [Concrete Canoe Capstone Files 2025](#)

Appendix A: Schedule

[Project Schedule](#)

The Gantt Chart linked above shows our pivot from a long-term schedule to a much faster paced deliverable schedule. The goal of this is to keep our team on schedule despite the recent pivot of our final design process. As school became a bigger time crunch at the end of the semester, we focused on creating the practical and final mold. The 3D printing was put on the side until we could confirm our main mold would work.

Appendix B: Design evolution

Due to complications with the 3D concrete printer, the design of the canoe has evolved from being fully-3D printed to semi-3D printed to not 3D printed at all. The last evolution was designed in a way that would mimic the properties of a 3D printed canoe.

The first iteration of the canoe was designed to be fully 3D printed, and the curves were developed in a way that they would be easy to print and bind together with a post-tensioning system. The holes for the post-tensioning rods would be incorporated in the wet concrete after each segment was printed. Then, each segment would be glued together to create a fully constructed post-tensioned canoe.

For the second iteration of the canoe, the middle 30 inches of the canoe would be developed by the same curve. We planned to 3D-print this section and hand-cast the end pieces of the canoe which are measured to be 9 inches each. As the curves got steeper towards the end of the canoe, we thought it would be best to hand-mold these sections since the printer is not able to precisely print the curves in a manner where the tensioning system would hold them together. The three large sections would then be glued together, and the tensioning rods would be inserted.

After multiple tests with the 3D printer, we found that that it would not print curves that would develop a functioning canoe. We found that it would be best to hand-cast the middle section and the end sections similar to how they would be 3D-printed. Once cast, they could easily be set up for a tensioning system by the construction team to develop a full-length prototype. We are still casting the canoe in three separate sections including the middle 30-inch section and the two 9-inch sections at the ends. These will be glued together and include pilot holes for the post-tensioning system.

Appendix C: Engineering Standards

Concrete Canoe Rulebook: C4 Request For Proposal (RFP)

The RFP provides a detailed outline of standards for a concrete canoe. Some standards include the amount of cementitious material allowed in a mix design. The RFP also provides instructions on different structural calculations that must be done. Due to the nature of our capstone, the team has chosen which standards from the RFP that we can apply to the canoe, like mix design restrictions and structural calculations.

Compression **ASTM C39/C39M-21**

The standard compression test consists of slowly increasing the compressive force at a standard rate on a standard size specimen. We will use the standard compression test on a variety of cube specimens in order to determine the compressive strength of our mix. From this we will also be able to extrapolate the approximate tensile strength of the mix.

Slump **ASTM C143/C143M-12**

Even though we are not using an actual 3d printer to construct our canoe we must still adhere to the requirements of a 3d printer so future teams are able to learn from our research and testing. To ensure that we create a mix that future teams can utilize we must have a very low slump value. A low slump in value results in stiffer concrete, 3D printers require stiff concrete to extrude properly and make even layers which can maintain their shape as successive layers are placed on top.

Accelerated curing **ASTM C684-99**

Due to our limited time frame, we cannot rely on standard curing for our testing. A normal run of concrete testing takes 28 days with tests occurring at 3, 7, and 28 days, or at 7, 14 and 28 days. We have elected to use accelerated curing to obtain relevant compressive strengths on an accelerated time frame.

Making and Curing Concrete Test Specimens in the Laboratory **ASTM C192-24**

The proper preparation of samples is key to consistent reliable data in the lab. For concrete the creation of these specimens is only half the battle. To get proper results each specimen must be rodded (or vibrated) usually in two lifts for standard cylinders (there are edge cases but we did not deal with these) after initial fabrication specimens are left to cure in the mold for 24 hours, then they are demolded and left in moist cure until testing.

Appendix D: Technical deliverables

Hull Design:

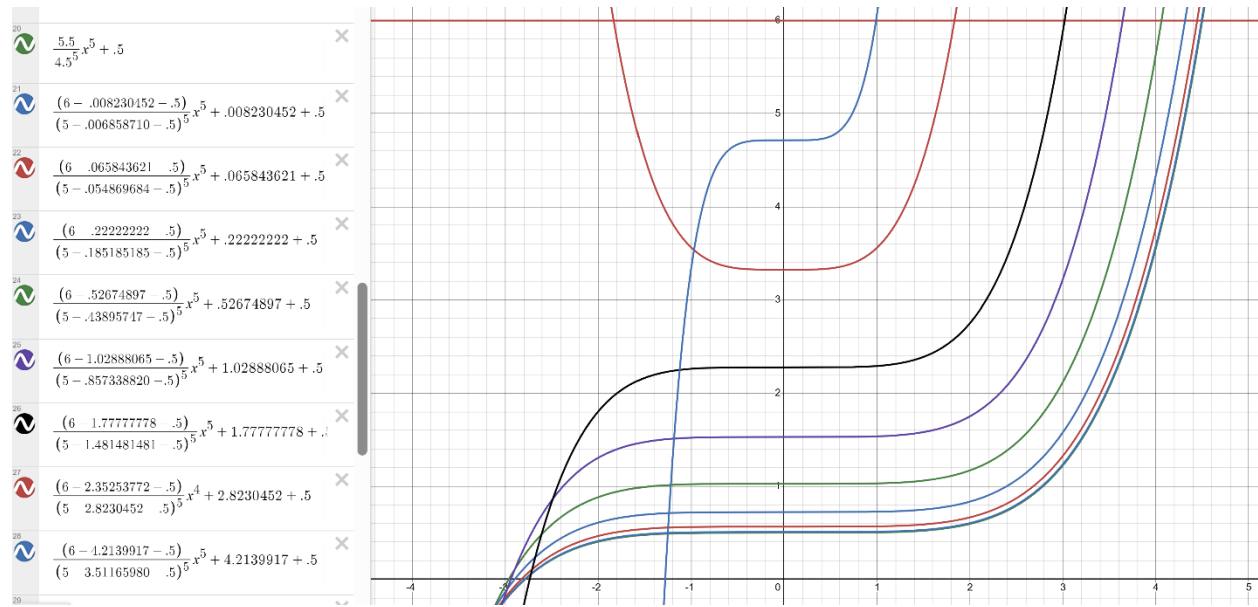


Figure 1: Hull Curve Equations (Interior)

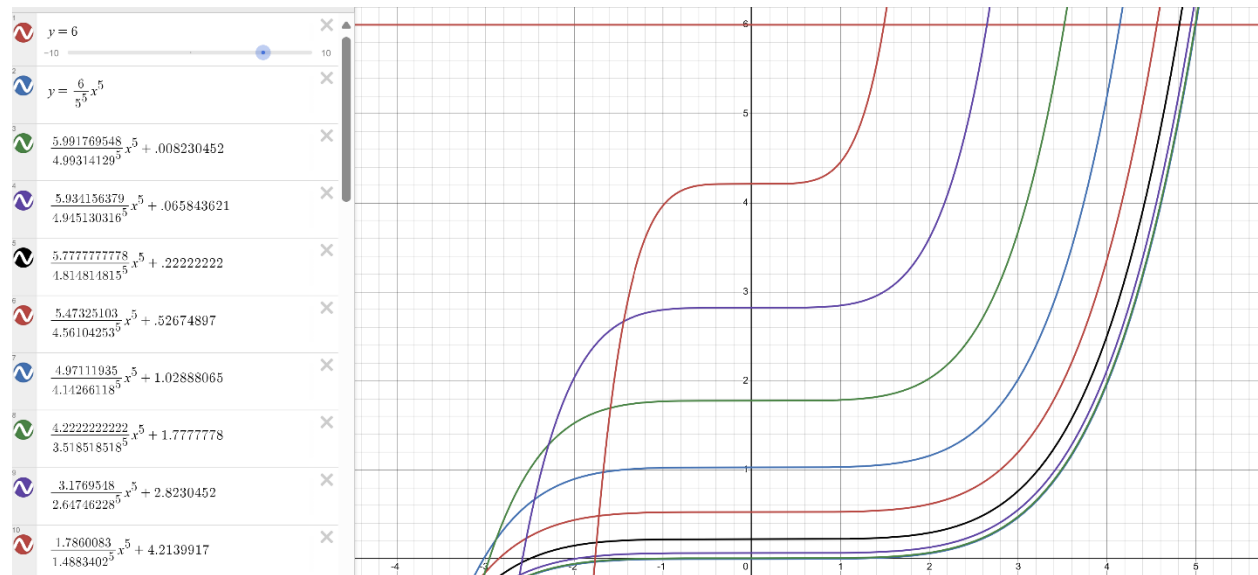


Figure 2: Hull Curve Equations (Exterior)

Figure 3: Hull Volume, Weight, and Freeboard Calculations

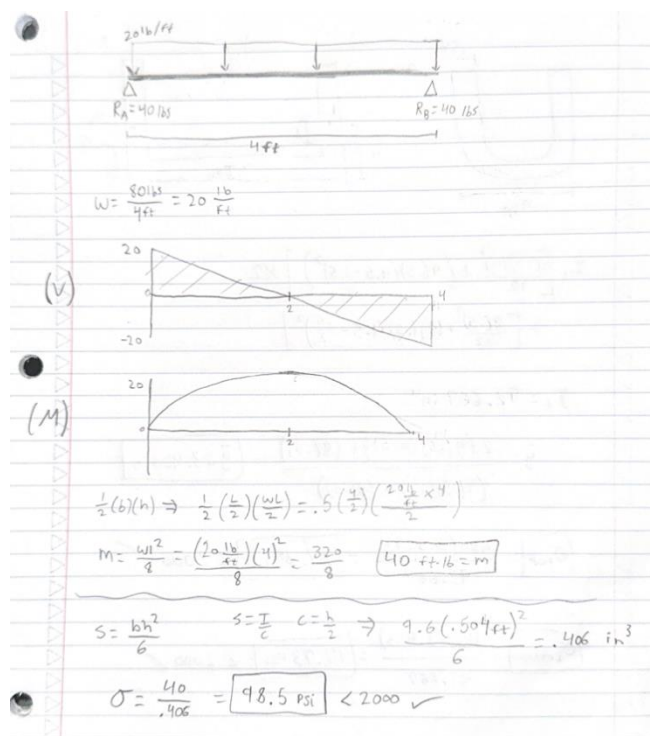


Figure 4: Shear and Moment Diagrams

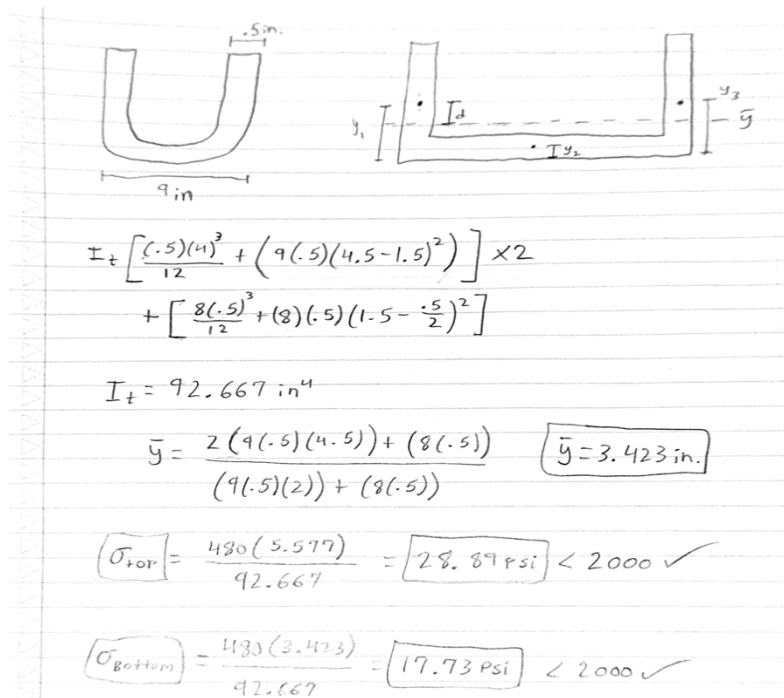


Figure 5: Stress Calculations

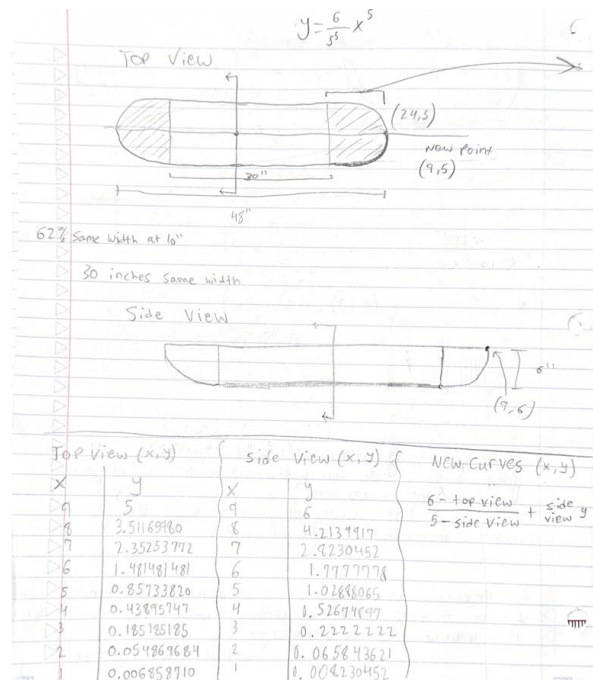


Figure 6: Hull Curve Calculations

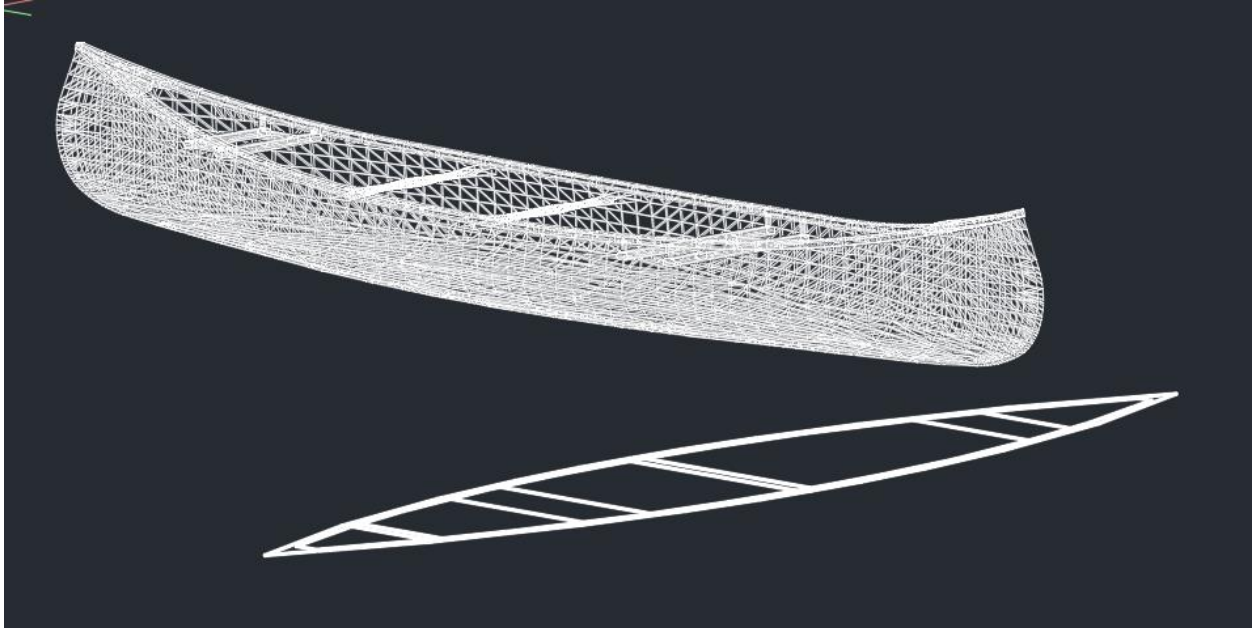


Figure 7: Hull CAD Model – Iteration 1

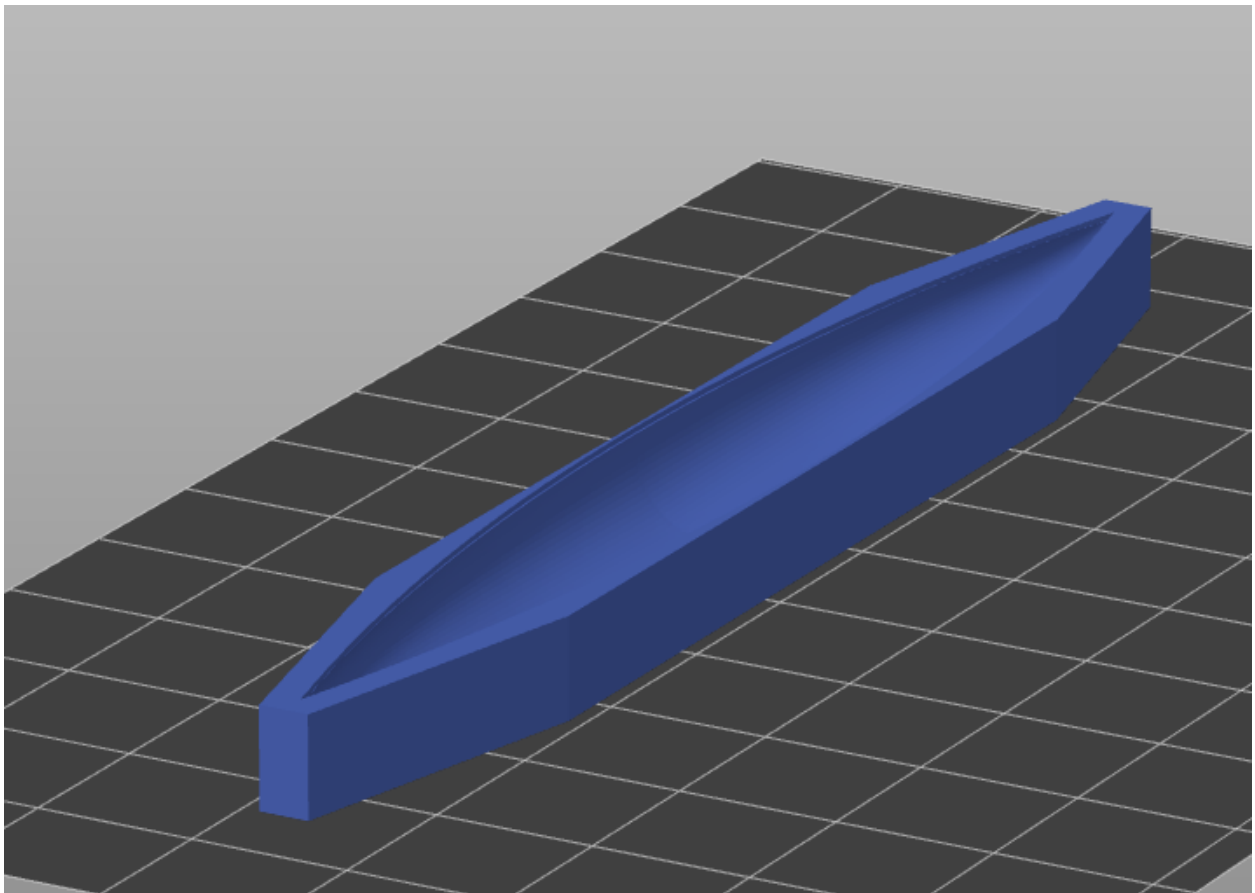


Figure 8: PrusaSlicer Model

Gcode Link:

<https://docs.google.com/document/d/1JFAGmZTkKJybd8QF03mccrwC1n9PNTiNSY6x78ChltM/edit?usp=sharing>

Mix Design:

Base Mix		
	Parts by weigh	Amount (Grams)
Cement (PLC)	1	666.33
Blast slag (BS)	1	666.33
Water	0.8	533.06
Pumice	6	3997.99
HRWR	0	0
Total		5863.71

Table 1: Final Mix Design

Grout Mix		
	Parts by weigh	Amount (Grams)

Cement (PLC)	1	666.33
Blast slag (BS)	1	666.33
Water	0.8	533.06
Pumice	6	3997.99
HRWR	0.0035	20.6
Total		5884.31

Table 2: Final Grout Mix Design

3- and 14-Day Compressive Strengths

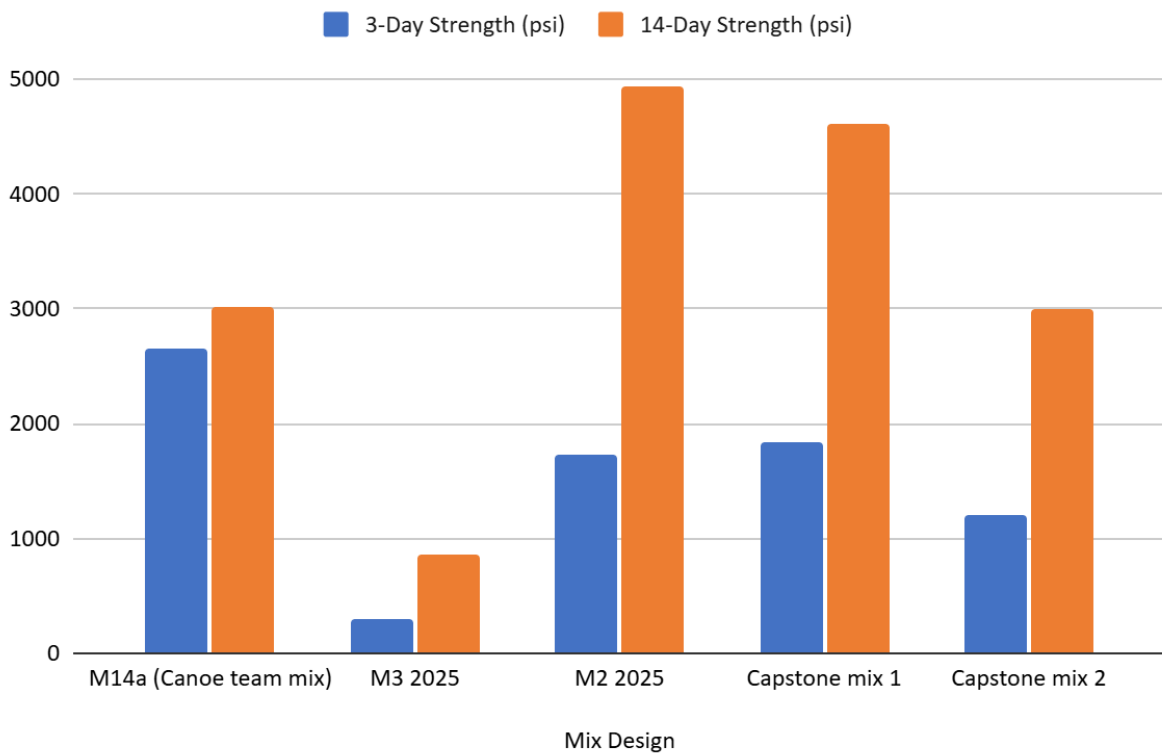


Figure 1: 3- and 14-day Strength Comparison of Possible Mix Options

Construction:

Tensioning System Materials List:

M3 (3mm diameter) threaded steel rod

M4 plastic tubing

M3 washers and hex nuts

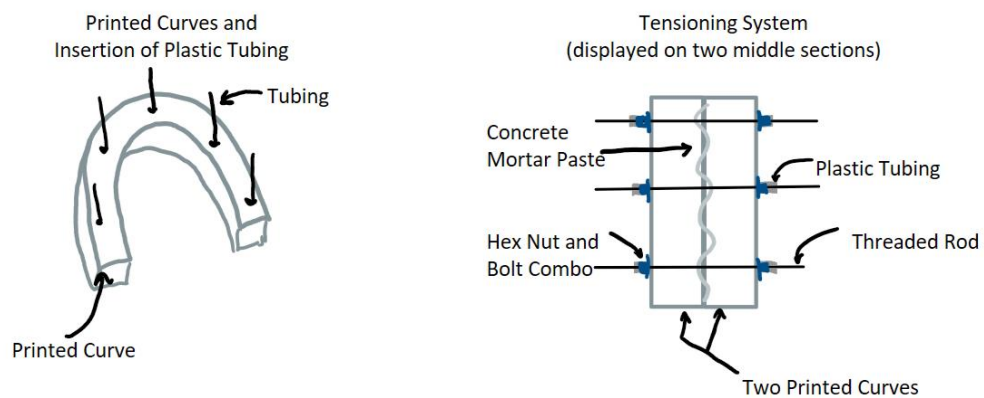


Figure 1: Tensioning System Diagrams and Materials List

Calculations for a scale model (4') canoe:

M3 (3mm OD)

Proof Load (PL): 249.918 lb

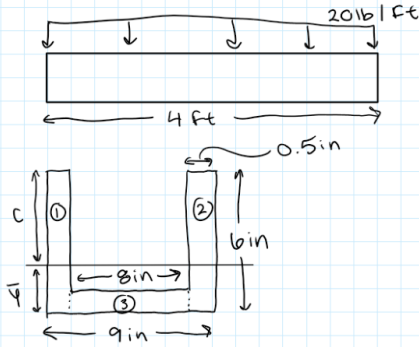
SF of 2 \rightarrow PL: 124.959 lb

$F'_c = 2000$ psi

$F'_t \approx F'_r = 7.5 \lambda \sqrt{F'_c} = 7.5(0.75) \sqrt{2000}$

$F'_t = 251.56$ psi

SF of 5 $\rightarrow F'_t = 50.31$ psi



$$M_{\max} = \frac{w l^2}{8} = \frac{20(4)^2}{8} = 40 \text{ lb} \cdot \text{ft}$$

$$= 480 \text{ lb} \cdot \text{in}$$

$$\bar{y} = \frac{\sum yA}{\sum A} = \frac{2(3)(6)(0.5) + 0.25(8)(0.5)}{2(6)(0.5) + 8(0.5)}$$

$$\bar{y} = 1.9 \text{ in} \quad c = 4.1 \text{ in}$$

$$I_{\text{total}} = \sum [I + Ad^2] \quad I = \frac{bh^3}{12}$$

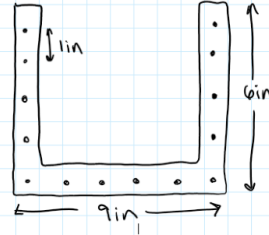
$$I_{1,2} = \frac{0.5(6)^3}{12} = 9 \text{ in}^4 \quad I_3 = \frac{8(0.5)^3}{12} = 0.083 \text{ in}^4$$

$$I = 2[9 + 0.5(6)(3 - 1.9)^2] + [0.083 + 8(0.5)(1.9 - 0.25)^2]$$

$$I = 36.233 \text{ in}^4$$

$$\sigma_{\text{bend}} = \frac{Mc}{I} = \frac{480(4.1)}{36.233} = 54.32 \text{ psi}$$

$$\text{Area in tension} = 2(4.1)(0.5) = 4.1 \text{ in}^2$$



Calculations for a full size (20') canoe:

M8 (8mm OD) threaded rod

Proof Load (PL): 1843 lb

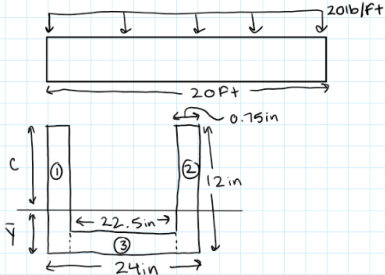
SF of 2 \rightarrow PL: 921.5 lb

$F'_c = 2000$ psi

$F'_t \approx F'_r = 7.5 \lambda \sqrt{F'_c} = 7.5(0.75) \sqrt{2000}$

$F'_t \approx 251.56$ psi

SF of 5 $\rightarrow F'_t = 50.31$ psi



$$M_{\max} = \frac{w l^2}{8} = \frac{20(20)^2}{8} = 1000 \text{ lb} \cdot \text{ft}$$

$$M_{\max} = 12000 \text{ lb} \cdot \text{in}$$

$$\bar{y} = \frac{\sum yA}{\sum A} = \frac{2(6)(12)(0.75) + (0.375)(22.5)(0.75)}{2(12)(0.75) + 22.5(0.75)}$$

$$\bar{y} = 3.28 \text{ in} \quad c = 12 - 3.28 = 8.72 \text{ in}$$

$$I_{1,2} = \frac{0.75(12)^3}{12} = 108 \text{ in}^4 \quad I_3 = \frac{22.5(0.75)^3}{12} = 0.79 \text{ in}^4$$

$$I = 2[108 + 0.75(12)(6 - 3.28)^2] + [0.79 + 22.5(0.75)(3.28 - 0.375)^2]$$

$$I = 492.37 \text{ in}^4$$

$$\sigma_{\text{bend}} = \frac{Mc}{I} = \frac{12000(8.72)}{492.37} = 212.52 \text{ psi}$$

$$\text{Area in tension} = 0.75(8.72)(2) = 13.08$$

$$212.52 - 50.31 = 162.21 \text{ psi}$$

$$162.21(13.08) = 2121.69 \text{ lb}$$

$$\frac{2121.69}{928.5} = 2.29 = 3 \text{ bars}$$

* 4 bars for symmetry *

* 1 bar every 3" for connecting pieces

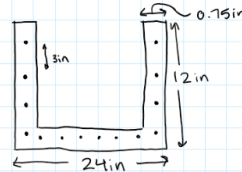


Figure 2: Tensioning Calculations for Scale and Full-Size Model



Figure 3: Test Day Pictures



Figure 4: Injection and Female Molds

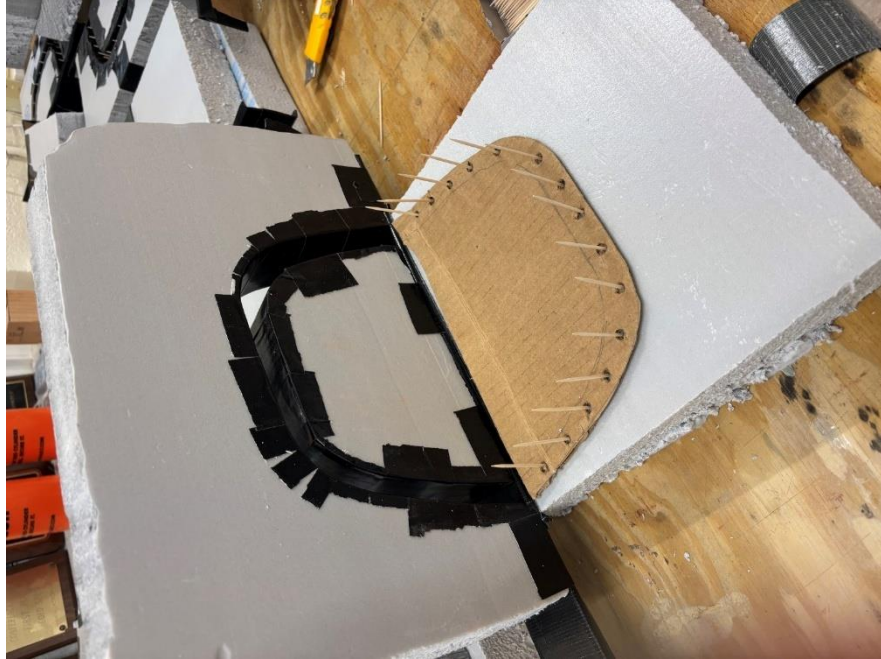


Figure 5: Tube Spacing

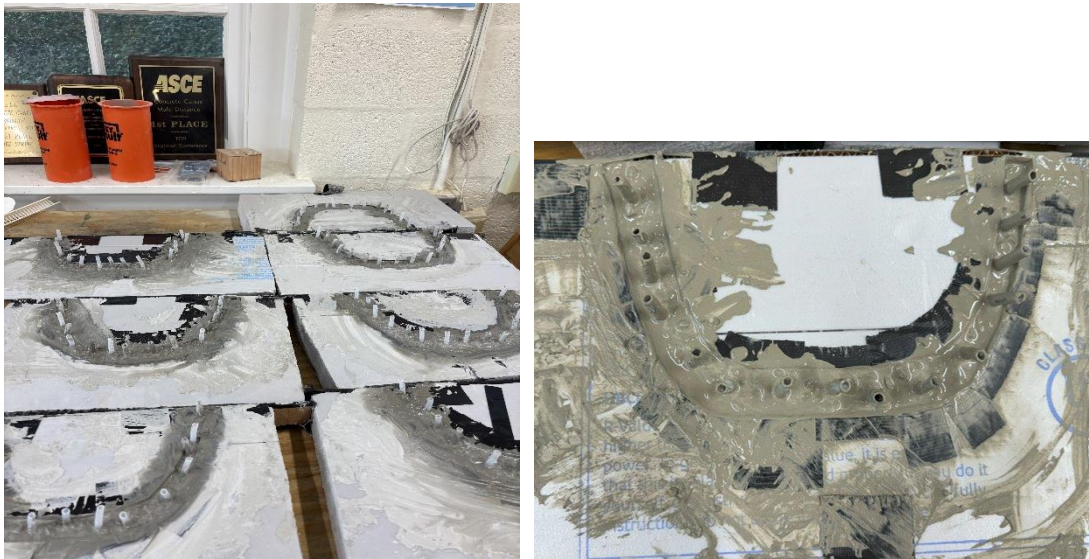


Figure 6: Concrete Placed in Molds



Figure 7: Final Prototype