Subject Area Knowledge of Private School Secondary Mathematics Teachers: Impact of Prior Coursework on Instructional Practice

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Abstract

Hiring secondary mathematics teachers has become a difficult task for schools across the country. Both public and private schools alike face the task of hiring a knowledgeable and qualified teacher. One small independent, private school in the Southeastern United States has confronted the issue of hiring and retaining mathematics teachers who possess the knowledge to positively impact their students' development and acquisition of algebraic reasoning skills for several years. The problem they face is understanding which courses listed on applicants' transcripts should they focus on when determining which teacher qualifies for their open teaching positions. This case study takes a look into the courses a secondary mathematics teacher at a private independent school reflects upon from their undergraduate career in order to teach their current classes. With a better understanding of their classes comes a better understanding of what coursework should be emphasized to ensure that a teacher has the background and expertise the school is looking. The purpose of this study is to better understand what courses mathematics teachers utilize while planning and teaching the lessons they covered in their high school mathematics classrooms. The findings from this study will help independent schools hire and retain qualified mathematics teachers who have the content knowledge needed to influence student learning.

Keywords: Calculus, teacher training, mathematical knowledge for teaching, specialized content knowledge

TABLE OF CONTENTS

Appendixes

CHAPTER 1: INTRODUCTION

To improve P-12 education, researchers and policymakers alike focus on the teachers and their abilities within their subject area as they are the frontlines of the school (Wayne & Youngs, 2003). As Wayne and Youngs (2003) stated, student achievement “depends substantially on the teachers they are assigned” (p. 89). Therefore, school administrators need to ensure their teachers are effective and highly prepared for the job.

The most influential, school-based factor contributing to student achievement is the teacher (Stronge & Hindman, 2003). Stronge and Hindman (2003) concluded that teachers produce a strong cumulative effect on student achievement. For example, students placed with highly effective teachers, who were knowledgeable and cared for their students, for three years in a row score significantly higher on state mathematics assessments than those who had three low-performing teachers in a row (Stronge & Hindman, 2003). Stronge and Hindman go on to say, “a student who has a high-performing teacher for just one year will remain ahead of their peers for at least the next few years of school” (p. 48). However, if a student has an ineffective teacher, “the opposite is true, and the negative influence on student achievement may not be fully remediated for up to three years” (Stronge & Hindman, 2003, p. 48). The issue for schools is how to determine if a teacher will be effective in the classroom.

To determine if a teacher can effectively influence student learning in the classroom, a definition of what makes an effective teacher must be established. The question of what defines an effective and well-qualified teacher has appeared before. As Ingersoll (2001) stated, “there is little consensus on how to define a qualified teacher” (p. 42). The lack of consensus suggests that effective teaching is an elusive concept (Stronge & Hindman, 2003). Pulling together several definitions of effective teaching, Stronge and Hindman (2003) described six domains of effective teachers: prerequisites of effective teachers, the teacher as a person, classroom management and organization, organizing for instruction, implementing instruction, and monitoring student progress and potential. Stronge and Hindman suggest when interviewing prospective teachers, school administrators and hiring teams focus on these domains to help hire teachers who have the qualities that enhance student achievement. One of the more difficult of these domains to determine for a candidate are the prerequisite skills. The prerequisite skills of effective teachers include verbal ability, certification, knowledge of pedagogy, working with special needs students, and content knowledge within a subject area (Stronge & Hindman, 2003). The concept of content knowledge within a subject area is one of the more challenging aspects to determine for a high school mathematics teacher. One question investigated in this area has centered around how many content area courses are needed to provide this knowledge (Ball et al., 2008; Ferguson & Womack, 1993; Guyton & Farokhi, 1987; Monk, 1994; Telese, 2012).

Problem Statement

A small private, independent school in North Carolina has faced the issue of hiring and retaining effective mathematics teachers for the last 7 years or more. During that time, they have hired more than 14 different mathematics teachers to fill only five teaching positions. Many of the teachers were replaced due to their inability to effectively teach their respective classes. The school often relies upon a third party, placement agency to aid in searching for potential teachers. Many of the applicants they receive have little to no teaching experience. Moreover, most applicants are new graduates with a various degrees and mathematical backgrounds, often with no education background.

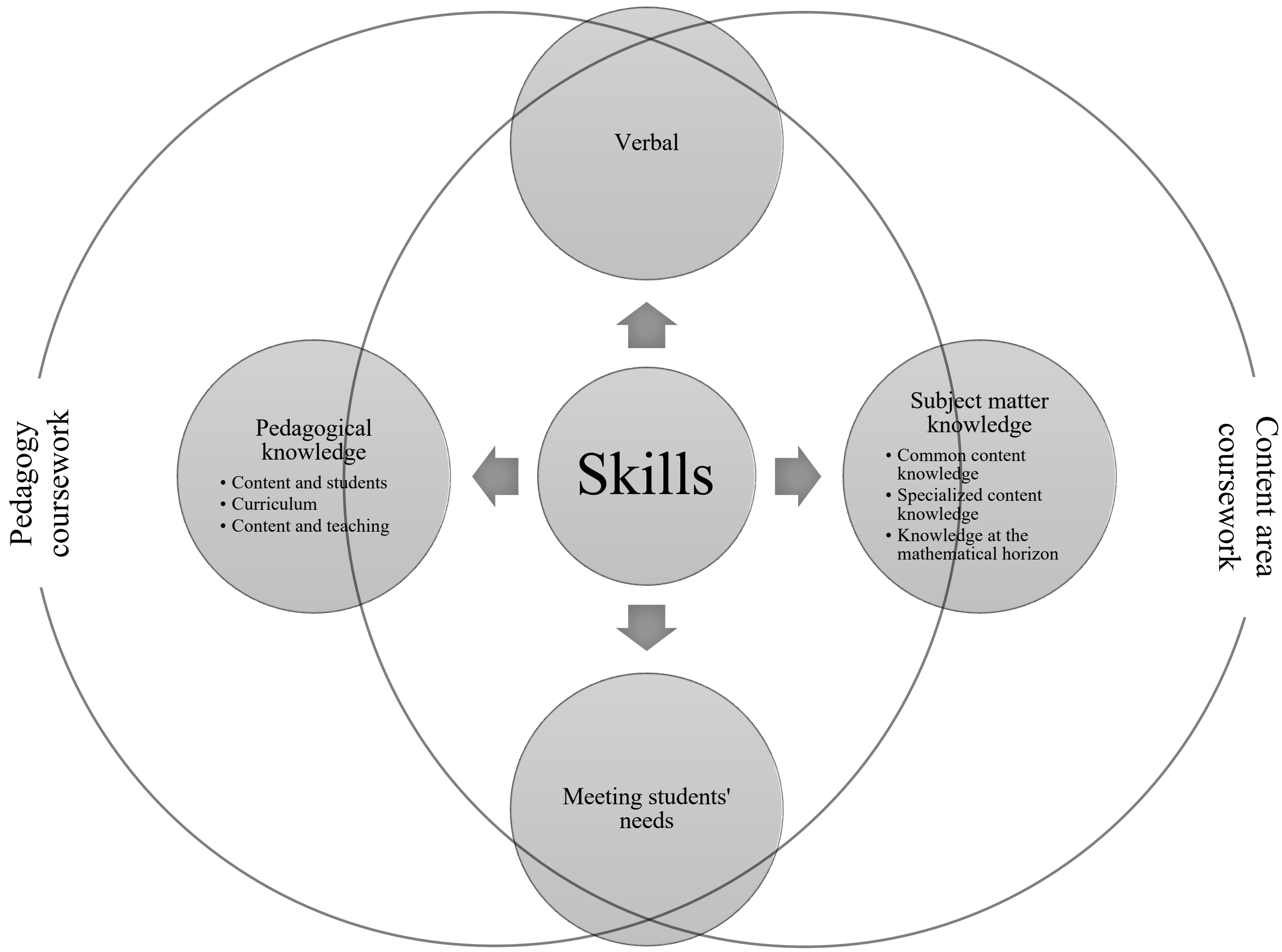
When the school receives an applicant's resume and educational background, such as transcripts, it is the job of the division head, namely the Upper School Headmaster, to determine which teachers meet the qualifications and standards set by the school to be a teacher. The headmaster often reaches out to the various department chairs to better understand the coursework they see when looking at applicants' transcripts. The Head of School asked, "which mathematics courses should we look for when reading through math teachers' transcripts?" (R. Teague, personal communication, October 25, 2021). The mathematics department chair noted the importance of pedagogy and methods courses within the transcript but also mentioned that many applicants do not have these courses (J. Hall, personal communication, October 25, 2021). During a video conference on October 25, 2021, both the department chair and Head of School mentioned that the school is willing to work with teachers to build and grow their pedagogical knowledge once they begin working at the school. The school offers payment assistance for continuing education courses and professional development opportunities (R. Teague, personal communication, October 25, 2021). For this reason, the school wishes to focus on the content area coursework taken by the teacher and how that coursework affects the learning within the secondary mathematics classroom.

Conceptual Framework

The conceptual framework for this study begins with the skills teachers need to positively affect students' learning. I intend to focus on their content knowledge, specifically the knowledge gained while taking mathematical content courses during their collegiate careers. The following concept map (see Figure 1) is intended to illustrate the framework used to guide this study.

Figure 1

Skills Necessary to Teaching and Where They are Developed



The framework begins by defining the skills needed to be an effective teacher. These skills include the teacher's verbal ability to communicate the material, knowledge of pedagogy, and ability to work with students' needs, and the presence of content knowledge (Stronge & Hindman, 2003). Of these skills, content knowledge and pedagogical training are evident within the applicant's transcripts. The teacher candidate's verbal ability and ability to meet student needs appear during other times within the interview process.

Ball et al. (2008) described mathematical content knowledge as the combination of pedagogical content knowledge (i.e., knowledge of content and students, knowledge of curriculum and knowledge of content and teaching) and subject matter knowledge (i.e., common content knowledge, specialized content knowledge, and knowledge at the mathematical horizon). The candidate's pedagogical content knowledge, as mentioned before, is something the school's administration team feels they can work with and improve over time (R. Teague, personal communication, October 25, 2021). The focus then shifts to the candidate's content knowledge.

The skills that students need in in high school consist of generalizing mathematical ideas and expressing those generalizations in formal yet age-appropriate ways (Blanton & Kaput, 2005). Many students take Algebra I during their 9th-grade year. The skills developed during this course extend beyond the Algebra I class. Skills, such as algebra, analytical geometry, functional reasoning and trigonometry, are deemed essential to completing college-level mathematics courses such as Calculus (Stewart, 2012). The question becomes which courses built the teachers' knowledge and skills. Therefore, I look to determine which content area coursework the teacher reflects upon while planning and teaching their classes. The courses deemed essential to the teachers may then shed light on the content areas that provide the teachers with the appropriate knowledge to plan and teach their lessons. With this concept map, I also acknowledge that teachers may acquire skills other than subject matter knowledge from their content area coursework, such as verbal skills and how to meet students' needs. Moreover, teachers may feel they learned pedagogical content knowledge from their content area instructors, (e.g., content and teaching).

It is reasonable to suggest that if a teacher expects the students to learn a particular set of skills, they must also possess the same set of skills (Darling-Hammond et al., 2005). Therefore, it would not be a stretch to suggest that teachers know how to do the mathematics, understand how to apply the mathematics, and understand the reasoning governing the mathematics (Blanton & Kaput, 2005, 2005; Darling-Hammond et al., 2005; Grossman et al., 2005). These skills are essential for effective teaching (Conley, 2003).

Using the problem-solving metaphor to examine teacher metacognition concerning instructional practice, the problem to be solved is how a teacher uses knowledge from previous coursework to plan and teach their lesson (Artzt & Armour-Thomas, 1998). Applying the problem-solving metaphor calls for collecting lesson plans, observing the teaching of those lessons, and conducting debriefing interviews following the lesson (Artzt & Armour-Thomas, 1998). By analyzing the interviews and observations, I look to understand the knowledge the teachers rely on from previous experiences to teach their high school mathematics classes.

Purpose of the Study

In a typical public school district, the personnel department screens the teacher applicants based upon resumes and qualifications while school-level administrators interview applicants for specific positions (Stronge & Hindman, 2003). The personnel department often uses criteria such as degree type, student teaching experience, certifications held, and experience within the classroom to screen applicants. The degree type is often the first piece of information judged, followed by certificates held (Stronge & Hindman, 2003). These two measures, albeit indirectly, imply the level of content and pedagogical knowledge possessed by the applicant. For this process, many administrators and hiring team members, often comprised of department chairs and senior teachers, rely on their impressions and professional judgment to select the best candidates for each position.

However, for some independent, private schools, the lack of a personnel department to screen applicants calls for a different approach to hiring teachers. One such school in the Southeastern United States relies on school administrators and department chairs to handle the screening process. As a teacher at two independent schools for 16 years, I served on several hiring teams for the schools, hiring teachers for all subjects and heads of school. For the final five years of my teaching career, I served as the mathematics department chair for my school. Shortly after becoming the department chair, I found myself assisting in the search for a new mathematics teacher for my department. Over my five years as department chair, I assisted in hiring ten new mathematics teachers for only five teaching positions.

My Upper School Headmaster, the equivalent of a high school principal, often relied on an external teacher placement agency to act as the gatekeepers. The agency sorted through applicants to find those who potentially fit the mission and philosophy of our school. After narrowing the selection, the agency sent the potential applicants' paperwork to the school for further consideration. Many of the teacher applicants did not have educational backgrounds but did possess strong mathematical experiences. These backgrounds ranged, however, from science (e.g., Physics) to business degrees. After looking through the applicants provided by the placement agency, the question asked by my administrators was which mathematical content courses do teacher applicants need and how many should they have. The school, attempting to stabilize the mathematics department and retain quality teachers, is looking to understand which of the mathematics courses taken by teachers positively impacts students' learning.

The school first acknowledges that many of their teacher applicants are considered novice teachers with less than years of teaching experience, many of those with no classroom experience (R. Teague, personal communication, October 25, 2021). Moreover, the school feels the amount of content knowledge is the critical aspect of the applicants' resumes. This notion is due to a belief that pedagogical knowledge can be developed, supported, and grown through support such as mentoring within the department and professional development opportunities (J. Hall, personal communication, October 25, 2021). The question posed by the administration of the school is: "What should we look for in the applicants' transcripts when hiring a new mathematics teacher?" (R. Teague, personal communication, October 25, 2021).

A recent study proposed that 9^{th}-grade mathematics teachers who took calculus and foundations of mathematics courses during college positively influenced their 9^{th}-grade students' scores on a test of algebraic reasoning (Choi & Cox, 2020). Choi and Cox (2020) also suggested the presence of a leveling-off point to the number of mathematics courses that positively affect student achievement. Choi and Cox also implied the possible presence of a third mathematics course to look for in applicants' transcripts, other than calculus and foundations of mathematics, but were unable to determine the specific course.

This study aims to identify the college-level content area coursework that teachers reflect upon when planning and teaching their high school mathematics courses. This study attempts to determine how those teachers' college mathematics experiences impact their teaching of mathematics. By gaining a deeper understanding of how high school mathematics teachers at a small independent, private school use their content knowledge, I look to contribute knowledge that will aid administrators in determining which candidates possess the potential to teach mathematics based upon their prior coursework.

A significant amount of research in this area appears focused on what coursework the teachers had before teaching and how that coursework affected student achievement scores in the classrooms (Ferguson & Womack, 1993; Guyton & Farokhi, 1987; Monk, 1994; Telese, 2012).

The results are mixed and contradictory, leading to more questions than answers (Hiebert et al., 2019). One question posed by Hiebert et al. (2019): "What kind of knowledge acquired in these courses did teachers actually use in their teaching?" (p. 24). Few studies have looked into what material from previous coursework mathematics teachers find useful in their classrooms versus what information they have never applied to teaching a lesson. The purpose of this study is to examine that gap by understanding what occurs in the high school classroom for those teachers and how they apply the knowledge and understandings gained during their pre-service course work to their new careers. The information learned during this study will aid those administrators as they determine which candidates possess the potential to positively impact their students' algebraic reasoning skills in future mathematics courses. To guide this study, I look to answer the following research question:

Research Question 1: How do teachers use their knowledge from previous coursework to plan their lessons?

Research Question 2: How do teachers use their knowledge from previous coursework to teach their lessons?

Significance of the Study

Hiring a high school mathematics teacher who can enhance student achievement requires finding a teacher who possesses the qualities Strong and Hindman (2003) mentioned. The most common 9^{th}-grade mathematics course taken by student is Algebra I (Tyson & Roksa, 2016). In this course, students learn how to think mathematically for formalizing patterns, functions, and generalizations (National Council of Teachers of Mathematics [NCTM], 2000). Researchers often refer to these skills as algebraic reasoning (Blanton & Kaput, 2005; Kaput, 2008; Smith & Thompson, 2008).

The ability to reason and think algebraically extends well beyond the algebra classroom (Blanton & Kaput, 2005; Kaput, 2008; Smith & Thompson, 2008). Students learn how to apply algebraic reasoning skills to solve several different types of problems during the Algebra I class. The skills and techniques learned in algebra classes are spread throughout a student's high school career (e.g., Chemistry, Physics and English) and beyond into their everyday lives (e.g., financial planning) (Conley, 2003; NCTM, 2000). For students to have the opportunity to be successful in higher-level mathematics courses, they must possess a strong algebra background (Kaput, 2008).

Conley (2003) suggests the application of the knowledge and skills learned in the algebra classroom extends well beyond the mathematics classroom. Conley mentions one such theme raised by teaching professionals, the habits of mind, when describing the skills needed by students entering college. Students develop these skills during high school and carry them into their college careers and beyond. These skills include critical and analytical thinking skills, problem-solving skills, and the ability to explain one's reasoning in either a verbal or written form (Conley, 2003). Researchers consider these skills essential components of algebraic reasoning (Blanton & Kaput, 2005; Kaput, 2008; NCTM, 2000; Smith & Thompson, 2008). Conley also suggests that many of these habits are more important than specific content knowledge when predicting success in college.

For students to gain content knowledge, teachers must possess the knowledge themselves. One place that teachers could have acquired this knowledge is in their college-level content area coursework. The problem lies with understanding which content area coursework taken in college supplied this knowledge to the teacher and which of those courses have the most influential association with students' understanding and learning of the algebraic reasoning skills they need to succeed in future classes.

The findings of this study have the potential to benefit the school administration of a small independent, private school and the students they serve. For a school that does not require teachers to possess a license in their given areas, the findings of this study may allow the school administrators to better understand the content area coursework taken by applicants and determine the applicants' potential to effectively teach students. Given a better understanding of a teacher's mathematical background, administrators will have the ability to make more informed decisions about who receives an invitation to visit the school. With the school observing more knowledgeable teachers, they will have the opportunity to hire strong, more prepared teachers who can improve the algebraic reasoning skills of their students. By hiring teachers who are better prepared to influence their students, the school may bring stability to a department that has experienced a high turnover rate over the last several years. The benefits of this stability will affect the overall morale of the department, the school, and the students.

The impact of this study potentially reaches beyond this specific private school. Other schools (e.g., public schools, charter schools, and other private schools) could use the results from this study to aid in the hiring of mathematics teachers who can potentially positively impact their students' learning. Ultimately, this could lead schools of education to reevaluate the mathematics education programs and determine or standardize which mathematics courses they should require of future mathematics educators.

Definition of Key Terms

For this study, I will use Blanton and Kaput's (2005) definition of algebraic reasoning as a process in which students generalize mathematical ideas, establish those generalizations through discussion and argumentation, and express the generalizations in a formal yet age-appropriate manner.

Also, for this study, I will define an independent school using the following characteristics: financial integrity, administrative integrity, and academic integrity (Virginia Association of Independent Schools [VAIS], 2020). The financial integrity of independent schools is characterized by funding that is separate from any other parent institution, and the school operates as a non-profit institution (VAIS, 2020). Independent schools are also governed by a separate Board of Trustees distinct from any parent board, a separate board responsible for setting school policy, and a Head of School who has the authority to administer free of interference (VAIS, 2020). Lastly, independent schools are characterized by academic freedom, freedom to promote diversity in teaching and curriculum approaches, and freedom from restricting teachers to a single doctrine or philosophy (VAIS, 2020).

Another aspect of independent schools that I must note is that although many schools recommend teaching certifications, teachers are not required to possess a license in North Carolina (North Carolina Association of Independent Schools, 2021; U.S. Department of Education, 2009). Some schools offer partial compensation to teachers who wish to continue their education and pursue an endorsement or license in their content area (Fork Union Military Academy, 2020; Gaston Day School, 2021).

Two essential terms needing clarification relate to the administration team of the school. The first is the Head of School. The Head of School, also called the President, is in charge of the entire school. They oversee the academic and business operations of the school. The Head of School often has the final say in the hiring process and oversees all contract negotiations. The school itself is made up of three separately functioning divisions; the Lower (i.e., elementary) School, the Middle School, and the Upper (i.e., high) School. Each division has a separate headmaster. Each headmaster oversees all academics within their division. The Upper School

Headmaster, for example, acts in a manner similar to a high school principal. When the school begins to search for teachers to fill vacancies, the headmasters are in charge of leading that search.

I also need to define the key term long-term effect. By this term, I mean the knowledge gains made by students extend beyond the one year where they learned the knowledge. This is essential for many skills within Algebra I as they are often applied in other courses.

I will define the term effective teacher as a teacher who can effectively pass on the knowledge that students need to be successful in their mathematics class and future mathematics classes.

By the term content area coursework, I look to describe the mathematics courses that the teachers took during their collegiate careers, either undergraduate or graduate.

For the terms subject matter knowledge (SMK), pedagogical content knowledge (PCK), and specialized content knowledge (SCK), I will use the definitions provided by Shulman (1986) and Ball et al. (2008). Shulman described SMK as the understanding of the structures of the subject matter. SCK is a subdomain of SMK (Ball et al., 2008). This subdomain of pure content knowledge is knowledge unique to the work of teaching (Ball et al., 2008). PCK, as described by Shulman, goes beyond subject matter knowledge to encompass the knowledge necessary for teaching.

CHAPTER 2: LITERATURE REVIEW

This literature review aims to develop a broader understanding of teacher knowledge, how teacher knowledge grows, and the impact teacher knowledge has on students' learning, all within the context of hiring a mathematics teacher. The goal of this literature review is to prepare school administrators and hiring team members to select candidates with the proper skills to teach high school mathematics, which includes teacher knowledge and skills deemed essential to high school students. This review begins with understanding the types of teachers' knowledge. I look to connect a teacher's knowledge of their subject with their students' ability to succeed in the mathematics classroom. This connection of knowledge focuses primarily on the teachers' subject matter and content knowledge. Teachers obtain most of this knowledge through content area coursework during their collegiate careers. Using content area coursework, I look to establish a connection between the amount of coursework taken by the teacher and student achievement. This connection adds to our understanding of how much content knowledge high school teachers need to affect their students' learning of mathematics. Finally, I look at the skills that students need proficiency in before they graduate high school and move forward into college. These skills include knowing how to do mathematics, applying mathematics, and mathematical reasoning.

Types of Teachers' Knowledge

Shulman (1986) defined the major categories of teacher knowledge, which include: subject matter content knowledge (SMK), general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge (PCK), knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge of educational ends. Among these categories, subject matter content knowledge and pedagogical content knowledge are deemed two of the most important aspects of teacher knowledge to assess during the interview process (Harris et al., 2010; Stronge & Hindman, 2003).

Subject Matter Knowledge

Shulman (1986) described SMK as the amount and organization of knowledge in the teacher's mind. Shulman suggested teachers not only be capable of defining the accepted truths within their subject but possess the knowledge and understanding to explain why the truths are considered necessary, worth knowing, and how they relate to other concepts and ideas learned both in and outside that a particular discipline. Shulman noted that "we expect that the subject matter content understanding of the teacher be at least equal to that of his or her lay colleague, the mere subject matter major" (p. 9). The teachers must know more than just something is so; they must also understand why it is so and what ideas and concepts govern its existence (Shulman, 1986). This statement suggests mathematics teachers need to know that a given principle works and understand how and why it works. Moreover, the teacher must possess the ability to break down the mathematical theory governing that principle and illustrate it so that students can follow along.

Shulman (1986) also suggested that teachers understand the grounds on which a belief is asserted and under what circumstances that belief can be weakened or even denied. Again, mathematics teachers should possess a deeper understanding of the principles and guidelines that govern how ideas such as theorems and postulates are applied. Teachers are expected to not only understand when and how to use a concept but also "to understand why a given topic is particularly central to a discipline whereas another may be somewhat peripheral" (Shulman, 1986, p. 9).

Focusing on subject matter knowledge specifically in mathematics, Ball et al. (2008) split Shulman's idea of SMK into three subdomains: common content knowledge (CCK), specialized content knowledge (SCK), and knowledge at the mathematical horizon. CCK refers to the common mathematical knowledge needed by teachers and nonteachers alike (Ball et al., 2008). This knowledge consists of facts such as addition facts and multiplication tables. Students often memorize these facts without necessarily understanding the more profound guiding principles that determine those facts. Knowledge at the mathematical horizon referred to how topics relate throughout mathematics included in the curriculum (Ball et al., 2008). This knowledge is part of the horizontal and vertical curriculum alignment knowledge Shulman (1986) suggested teachers possess. Horizon knowledge also plays a role in understanding why an idea is considered essential or not. This type of knowledge is useful when answering the most common question in my high school mathematics class: "When are we ever going to use this?"

Specialized content knowledge (SCK) is distinct from CCK in that SCK is mathematical knowledge typically used solely for teaching. SCK includes, but is not limited to, the ability to look for patterns in student errors and determining whether a nonstandard approach will work in a more general application (Ball et al., 2008). Again, as Shulman (1986) suggested, teachers must possess a deeper understanding of the governing principles behind the mathematical facts and principles taught during class.

Pedagogical Content Knowledge

Pedagogical content knowledge (PCK) goes beyond simply knowing the subject towards possessing the knowledge necessary for teaching (Shulman, 1986). Ball et al. (2008) combined Shulman's notions of SMK and PCK to develop measures of mathematical knowledge for teaching (MKT). The new measure of MKT consists of the SMK subdomains of CCK, SCK, and mathematical horizon, as mentioned earlier, combined with the PCK subdomains that Ball et al. developed. Ball et al. broke down PCK into three new subdomains: knowledge of content and students, knowledge of content and teaching, and knowledge of the curriculum.

Shulman (1986) suggested that PCK contains knowledge of useful forms of representations of ideas, analogies, illustrations, examples, explanations, and demonstrations used to represent and formulate the subject in a manner understandable to others. PCK also includes an understanding of what makes the learning process of a specific topic easy or difficult, along with the preconceptions and misconceptions students carry with them when they enter the classroom (Shulman, 1986). Ball et al. (2008) referred to this subsection of PCK as knowledge of content and students. Teachers do not obtain this knowledge by completing content area courses alone. Instead, teachers learn this knowledge in methods courses designed to instruct students in these particular areas of thinking and application (Ball et al., 2008; Ferguson & Womack, 1993; Guyton & Farokhi, 1987; Monk, 1994).

The domain, knowledge of content and teaching, consists of knowing about mathematics and education (Ball et al., 2008). Mathematics teachers must design their lessons beginning with broad examples that start the students thinking about a concept followed by specific examples designed to deepen the students' understanding of the content (Ball et al., 2008). To choose the appropriate examples to use in class, "teachers evaluate the instructional advantages and disadvantages of representations used to teach a specific idea" (Ball et al., 2008, p. 401).

Knowledge of curriculum consists of knowing how the curriculum aligns laterally throughout the course from unit to unit as well as vertically from one course to another (Ball et al., 2008; Shulman, 1986). This knowledge allows the teacher to build upon the topics that students have already experienced in some form in previous units and courses. Knowledge of curriculum combined with knowledge of the mathematical horizon allows the teachers to design a curriculum that flows continually and builds upon previously learned mathematical concepts.

Teacher Subject Matter Preparation and Student Achievement

Ball et al. (2008) described content knowledge as knowledge of the subject and its organizing structures. Teachers must possess more than a superficial understanding of facts and concepts that are repeated at will. They must also understand the organizing principles, systems, and rules that allow those facts to be applied under certain conditions (Ball et al., 2008; Shulman, 1986). For mathematics teachers, this idea means they need to not only know that a given procedure works but also understand why it works and under what circumstances a property or procedure cannot be used.

Wilson et al. (2002), commissioned by the U.S. Department of Education to summarize existing research concerning teacher preparation, attempted to answer the question: "What kind of subject matter preparation, and how much of it, do prospective teachers need?" (p, 191). At the time, several studies conducted relied on proxy measures to describe subject matter knowledge such as majors or amount of content area coursework (Wilson et al., 2002).

Monk (1994) conducted an analysis using the Longitudinal Study of American Youth (LSAY) data collected from 1987 to 1990. This study collected student achievement scores and teacher data, including undergraduate and graduate coursework as well as fields of study, to determine which factors influenced student achievement. Students at both sophomore and junior levels showed a positive relationship between their increased performance on achievement tests and the number of undergraduate mathematics courses their teachers took (Monk, 1994). For the 11th-grade students, an increase of one mathematics course for a teacher with five or fewer courses was associated with a significant 1.2% increase in the student's math test scores (Monk, 1994). However, the addition of coursework beyond the fifth course had little to no effect on student performance. This finding led Monk to suggest the notion of a leveling-off point to the number of content courses that positively impact student achievement. The idea of a leveling-off point implies that additional training not directly related to the subject taught does not necessarily have positive collateral effects (Monk, 1994).

In a similar study, Guyton and Farokli (1987) found that teachers' grades in content area courses did not predict student achievement. Their study indicated no evidence to suggest that an instructor with a strong background in their academic discipline teaches effectively with little to no pedagogical education. In other words, they suggest that an instructor with a high grade-point average in their field of study, such as mathematics, with little to no courses in pedagogy, does not necessarily possess the skills needed to teach effectively.

In another study, Telese (2012) pulled student and teacher data from the 2005 National Association of Education Progress (NAEP) database. The data consisted of 8^{th}-grade student achievement scores and teacher background information (Telese, 2012). This background information included the number of advanced mathematics courses the teacher took during their entire college career, both undergraduate and graduate-level (Telese, 2012). The option for responses included "none," "one or two," "three or four," and "five or more" (Telese, 2012, p. 105). The study indicated that the number of mathematics content courses taken by the 8^{th}-grade mathematics teacher was a better predictor of student achievement than the number of mathematics education courses (Telese, 2012). The association between the number of content courses taken by the teacher and student achievement was found to be statistically significant and suggests that taking more mathematics courses leads to higher student achievement levels (Telese, 2012).

At first glance, his finding appears to contradict Monk's (1994) findings and his notion of a plateau effect occurring after the fifth content course taken. However, Telese (2012) did not truly measure the number of mathematics courses beyond the fifth course. The highest measure used by Telese was "five or more" courses (p. 105). Monk's findings suggested that the number of content courses taken by the teacher significantly influenced student performance up and to the fifth mathematics course. The plateau effect presented by Monk did not come into play until after taking the fifth course. How Telese measured the number of content courses may have affected the results and disguised the existence of the leveling-off point.

To examine the effects of teachers' mathematical knowledge on student achievement Rowan et al. (1997) pulled data from the National Education Longitudinal Study of 1988 (NELS: 88). During the NELS:88, teacher knowledge was measured using a math knowledge quiz, whether the teacher majored in mathematics at either the undergraduate or graduate level, and how the teacher emphasizes higher-order thinking while teaching (Rowan et al., 1997). Both student achievement in mathematics and teacher knowledge scores were included with the NELS: 88 data (Rowan et al., 1997). After conducting a correlation analysis, the researchers concluded that the findings suggested that student achievement increased with the teacher's score on the NELS knowledge quiz (Rowan et al., 1997). Rowan et al. tested whether a teacher holding a degree in mathematics significantly affected student achievement. They found that student achievement increased based on teachers holding a degree in mathematics. However, this association was deemed statistically insignificant due to an alpha greater than .05 and an effect size of .015.

In a separate study, Ferguson and Womack (1993) attempted to connect the teachers' GPA in their major and scores on the National Teachers Exam (NTE) specialty scores to teacher performance. Teacher performance was judged using an instrument consisting of 107 Likert-response items (Ferguson & Womack, 1993). Using an ANOVA test and a stepwise regression analysis, Ferguson and Womack determined that teachers' GPA and NTE scores had a minimal effect on teaching performance.

In a recent study using data from the High School Longitudinal Study (HSLS: 09), Choi and Cox (2020) suggested that the coursework taken by 9^{th}-grade teachers had a significant positive effect on their students' algebraic reasoning achievement scores measured during their 9^{th}-grade year. Of the nine collections of courses (e.g., Calculus/Differential Equations/Analysis or Foundations/Logic/History/Philosophy) taken by teachers while in college, only two showed significant associations with improving students' scores on a test of algebraic reason, namely Calculus/Diff. Equations/Analysis and Foundations/Logic/History/Philosophy (Choi & Cox, 2020). These collections of courses fall into the strands of algebraic reasoning described by Kaput (2008). Success in these courses depends upon a strong sense of algebraic reasoning when entering the course. In turn, these courses develop a stronger sense of that reasoning that can be applied while teaching 9th-grade mathematics.

Pre-service Teacher Training to Develop SMK

Mathematics Majors or Mathematics Education Majors

Several studies examined the association between degree type, major in mathematics or mathematics education, and student achievement (Ferguson & Womack, 1993; Guyton & Farokhi, 1987; Monk, 1994; Rowan et al., 1997). These studies noted that the simple accumulation of credits by the teacher within a subject area did not increase student achievement scores. Both Monk (1994) and Telese (2012) indicate a significant association between student achievement and the number of mathematics courses taken by the teacher. However, Monk proposed a plateau effect on the number of courses taken after the fifth course. Several studies have suggested that coursework in education has a more substantial impact on student achievement than content area coursework (Ferguson & Womack, 1993; Guyton & Farokhi, 1987; Monk, 1994).

Along with his notion of a leveling-off point to the number of courses that affect student achievement, Monk (1994) also suggested that having a major in mathematics had no apparent bearing on pupil performance. This finding, combined with the idea that more than five courses have minimal effect on student performance, may suggest that the amount of coursework that a high school mathematics teacher takes during college is less significant than the actual courses taken.

Guyton and Farokli (1987) suggested that grades in education courses were better predictors of teaching success than grades in content courses. Ferguson and Womack (1993) supported the notion of coursework in education as a stronger predictor of teaching performance in comparison to GPA in major and scores achieved on the national teacher exam. Monk (1994) took this a step further, suggesting that additional coursework in pedagogy had more powerful effects on student achievement than additional content preparation. Monk also indicated that degree and teaching experience measures tended to be either negatively or unrelated to student achievement. Rowan et al. (1997) were also unable to find significant evidence that a teacher possessing a degree in mathematics increased student achievement scores. These findings may suggest that an undergraduate degree in mathematics may not be as important as the particular courses taken during a teacher's undergraduate career.

Building SCK

Several studies have provided evidence that a teacher, well-grounded in their content area, possess the skills necessary to teach effectively (Ferguson & Womack, 1993; Guyton & Farokhi, 1987; Monk, 1994). None of these studies considered the specific content courses taken by teachers and how they may affect student achievement.

Hiebert et al. (2019) focused on the particular knowledge that teachers use while teaching their classes. Hiebert et al. (2019) looked to determine the type of knowledge acquired in content courses teachers use in their teaching. To accomplish this, they focused on Ball et al.'s (2008) idea of specialized content knowledge (SCK) (Hiebert et al., 2019). SCK is strictly mathematical, and though not mixed with knowledge of students or teaching, it is essential for dealing effectively with mathematical issues that arise when teaching (Ball et al., 2008; Hiebert et al., 2019).

In this study, pre-service teaching students took courses specifically designed to explain the math content they would ultimately teach their classes (Hiebert et al., 2019). Hiebert et al. (2019) determined a relationship between the content knowledge developed during their students' teacher preparation program and the content knowledge they demonstrated zero to four years after graduation. This finding suggested that specific mathematics content studied during a teacher preparation program, even as a freshman, can be retained and applied to complete SCK for teacher tasks several years after graduation. Therefore, classes designed to build SCK may benefit teachers long after they take the course and, in turn, benefit their students.

Teachers' Impact on Student Achievement

Some previous studies have suggested that teachers lack the essential knowledge for teaching mathematics as well as the notion that teachers' intellectual resources affect student understanding (Coleman et al., 1966, Hill et al., 2005; Stigler & Hiebert, 1999, as cited in

Tchoshanov et al., 2008). Tchoshanov et al. (2008) supports these notions and showed that teacher knowledge and student achievement paralleled each other. After surveying teachers to determine their knowledge of given mathematical topics Tchoshanov et al. concluded that "student performance patterned by objective mirrors teacher performance on a teacher knowledge survey…this means if teachers have difficulty mastering a particular objective, then it impacts student achievement in the same objective" (p. 41). This finding implies that low teacher knowledge in topics such as patterns, relationships, algebraic reasoning, and measurement correlates with low student achievement in the same areas (Tchoshanov et al., 2008).

Several studies have suggested that teacher knowledge impacts student achievement (Ferguson & Womack, 1993; Guyton & Farokhi, 1987; Hiebert et al., 2019; Monk, 1994; Rowan et al., 1997; Telese, 2012). The amount of impact attributed to teachers' subject matter knowledge is still not completely understood. Many researchers used proxy measures of teacher knowledge, such as GPA in major, number of courses, and NTE scores, and found conflicting results (Ferguson & Womack, 1993; Guyton & Farokhi, 1987; Monk, 1994; Telese, 2012). Monk (1994) suggested a plateau effect on the number of content courses taken by a mathematics teacher. However, in a similar study, Telese (2012) found no evidence suggesting the presence of a leveling-off point. Meanwhile, Hiebert et al. (2019) indicated that courses specifically designed to teach the specialized content knowledge needed by the teachers had a lasting effect on their teaching. Moving forward, I intend to look at how specific college-level mathematics courses taken by high school mathematics teachers affect their students' algebraic reasoning skills. I aim to determine which courses build the algebraic reasoning skills teachers intend to pass on to their students. As Tchoshanov et al. (2008) suggested, student and teacher achievement parallel each other, and the teachers' mastery of a particular objective affects student achievement same objective.

There is a connection between teacher knowledge and student achievement in general with revealing patterns regarding specific mathematical domains, processes, and levels of cognitive demand, in particular. Tchoshanov et al.'s (2008) study showed that the areas in which teachers performed the lowest coincided with the areas in which students struggled the most. Understandably, no one person will be an expert in every field of mathematics, nor should they be. The question then becomes, how do we prepare our beginning mathematics teachers to enter a high school mathematics classroom with the feeling of preparedness needed to perform the job satisfactorily? Instead of focusing on degrees in mathematics, should we design a mathematics teacher education program that consists of specific courses that provide the pre-service teachers with the mathematical knowledge necessary to teach the skills students need? If so, what courses should we include in such a program? What skills are we looking for teachers to have when they leave these courses?

Cognitive Domains

The International Association for the Evaluation of Educational Achievement (IEA) created a project is assess achievement in mathematics and science at the fourth and eighth-grade levels called Trends in International Mathematics and Science Study (TIMSS). This study measures progress in educational achievement every four years (Mullis et al., 2005). A vital piece of the foundation of the TIMSS assessment are three cognitive domains: 1) knowing, 2) applying, and 3) reasoning. These cognitive domains describe the range of skills and sets of behaviors expected of students as they engage with the mathematics content (Mullis et al., 2005). These skills and behaviors translate to the teacher level in a straightforward manner.

Knowing Domain

The first cognitive domain, knowing, covers facts, procedures, and concepts (Mullis et al., 2005). The knowing domain is equivalent to remembering and classifying under Bloom's taxonomy (D. Lee & Huh, 2014). This domain covers behaviors such as recalling, recognizing, computing, retrieval, measuring, and classifying (D. Lee & Huh, 2014; Mullis et al., 2005). These behaviors allow individuals to make extensions beyond their existing knowledge, judge the validity of statements, and create representations for problem-solving (Mullis et al., 2005).

Under the knowing domain, Mullis et al. (2005) suggest that a person's ability to use mathematics depends upon mathematical knowledge and familiarity with concepts. This notion implies that the more relevant knowledge someone can recall and the more comprehensive the range of concepts they understand, the greater their potential to engage in a broader range of problem-solving situations and develop mathematical understandings (Mullis et al., 2005). The procedures designed within a course form a bridge between more basic knowledge and the use of mathematics for solving routine problems, including those experienced in everyday life (D. Lee & Huh, 2014; Mullis et al., 2005). An important aspect of this domain is how knowledge of concepts enables connection-making between elements that would be otherwise retained as isolated facts (Mullis et al., 2005). The ability to connect concepts is important to teachers as they prepare and teach their lessons (Darling-Hammond et al., 2005).

Applying Domain

The second cognitive domain, applying, corresponds to understanding and applying under Bloom's taxonomy (D. Lee & Huh, 2014). The domain focuses on the ability to "apply knowledge and conceptual understanding to solve problems" (Mullis et al., 2005, p. 33). This domain covers behaviors such as selecting, representing, modeling, implementing, and solving routine problems (D. Lee & Huh, 2014; Mullis et al., 2005).

Under the applying domain, individuals apply mathematical knowledge of facts, skills, procedures, or understanding of concepts to create representations and solve problems (Mullis et al., 2005). Problems may be set in real-life or quasi-real-life situations or concerned purely with mathematical questions involving numeric or algebraic expressions (Mullis et al., 2005). The problems solved in this domain, more routine in nature, provide practice in particular mathematical methods or techniques (Mullis et al., 2005). Such problems are considered the standard in mathematics classrooms. To use such examples, it stands to reason that teachers must possess knowledge of those examples beforehand. As Darling-Hammond et al. (2005) mentioned, “teachers need to do more than implement particular techniques” (p. 392) while teaching. They must also apply their knowledge and understanding to solve the problem of teaching class.

Reasoning Domain

The final domain involves the capacity for logical, systematic thinking, including “intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to solve non-routine problems” (Mullis et al., 2005, p. 37). These non-routine problems may be purely mathematical in nature or based in real-life settings. However, both situations involve applying knowledge and skills to new situations (Mullis et al., 2005). This domain corresponds to implementing an unfamiliar task under apply and analyze and evaluating in Bloom’s taxonomy (D. Lee & Huh, 2014). The reasoning domain covers behaviors such as analyzing, generalizing, synthesizing, justifying, and solving non-routine problems in real-life contexts (D. Lee & Huh, 2014; Mullis et al., 2005).

One behavior associated with the reasoning domain is working on complex problems with no immediately obvious method of solution, and the individual must decide on their procedures for solving such a problem (D. Lee & Huh, 2014). The skills of analyzing, synthesizing, and generalizing allow individuals to recognize how ideas and theories fit together and relate to one another (Darling-Hammond et al., 2005).

Algebraic Reasoning

What is Algebraic Reasoning?

A concept falling within the reasoning domain is the notion of algebraic reasoning. Kaput (2008) suggested that "algebraic reasoning is comprised of complex symbolization processes that serve purposeful generalization and reasoning with generalizations" (p. 9). Blanton and Kaput (2005) defined algebraic reasoning as "a process in which students generalize mathematical ideas from a set of particular instances, establish generalizations through the discourse of argumentation, and express them in increasingly formal and age-appropriate ways" (p. 413). The implied skill of purposeful generalization is seen within the reasoning domain mentioned by Mullis et al. (2005).

In 1991, NCTM developed the *Professional Standards for Teaching Mathematics*. In these standards, NCTM proposed five significant shifts that needed to occur in mathematics classrooms. One notion presented by NCTM suggested shifting towards the connection of mathematics, its ideas, and applications while moving away from treating mathematics as an isolated body of concepts.

Algebraic reasoning takes various forms (Blanton & Kaput, 2005; Kaput, 2008). These forms include using arithmetic as a domain for expressing and formalizing generalizations and generalizing patterns to describe functional relationships (Blanton & Kaput, 2005). These two forms lead to skills needed in higher-level mathematics such as logic and calculus (Blanton & Kaput, 2005). Blanton and Kaput (2005) also suggested that algebraic reasoning uses modeling to express and formalize generalizations and to generalize about mathematical systems abstracted from relations and computations. These two forms lead to skills utilized in higher-level mathematics, such as calculus and analysis (Blanton & Kaput, 2005).

Effects of Algebraic Reasoning

Algebraic reasoning skills are needed for the increasingly quantitative workplace, preparation for literate citizenship, the ability to make life decisions that depend on quantitative understanding such as mortgage payments and investments, and preparation for higher education and without these understandings, "citizens are handicapped" (Grossman et al., 2005, p. 213). Students must learn algebraic reasoning skills in an increasingly quantitative world and apply them in multiple situations during their school careers. It is essential for the students to retain these algebraic reasoning skills well beyond the mathematics classroom and apply them to real-world situations (Blanton & Kaput, 2005; Conley, 2003; Kaput, 2008; NCTM, 1991).

The utility of algebraic reasoning extends beyond secondary education. For example, Brown et al. (2015) noted that one out of every two students who entered a STEM field bachelor's degree program between 2003 and 2009 switched majors to a non-STEM field or dropped out of postsecondary school. Brown et al. noted that the decision to continue in STEM-field majors was usually "based on the successful completion of a gateway course, often calculus" (p. 2). For students to succeed in these so-called gateway courses, they need strong algebraic reasoning skills such as problem-solving, real-world applications, critical thinking, and creating generalizations (Brown et al., 2015; Kaput, 2008).

By learning and honing algebraic reasoning skills, students develop the confidence to continue through more rigorous mathematics education throughout high school (Kowski, 2013). Many researchers have observed that skills and abilities, such as algebraic reasoning, can be strong predictors for postsecondary educational success (Adelman, 1999, 2006; Brown et al., 2015; S. W. Lee & Mao, 2021; Perkins et al., 2004; Schneider et al., 2003). Studies have shown a link between high school mathematics courses and postsecondary academic success (Adelman, 1999; Perkins et al., 2004). Students who take advanced mathematics courses are more likely to be college-ready, receive higher scores on college admissions tests, and are more likely to complete a bachelor's degree (Adelman, 1999; Kowski, 2013; S. W. Lee & Mao, 2021; Schneider et al., 2003). Thus, having a teacher who can help students develop these skills is an important piece of the hiring puzzle for schools. It stands to reason that for students to gain these skills, their teachers must also possess the skills involved in algebraic reasoning (Grossman et al., 2005; Wayne & Youngs, 2003).

Problem Solving Metaphor

To investigate the specific mathematics courses that teachers reflect upon while planning and teaching the classes, I used Artzt and Armour-Thomas's (1998) "Problem Solving Metaphor" developed during a study to investigate teacher cognition. Using a "teaching as problem-solving" strategy to examine the metacognition, Artzt and Armour-Thomas developed a metacognitive framework allowing them to examine the range of thoughts used by a teacher during three stages of teaching: planning, interactive, and post-active (Artzt & Armour-Thomas, 1998). Using seven beginning (student) teachers and seven experienced (between seven to twenty-five years of experience) teachers, they observed the teaching of a class and conducted pre- and post-observation interviews with each participant to better understand the components of metacognition used during the various stages of teaching (Artzt & Armour-Thomas, 1998). The study was designed to indicate where teachers focused while teaching, namely, whether their knowledge, beliefs, and goals centered on their students or their own practices (Artzt & Armour-Thomas, 1998).

Over the last twenty-five years, these multiple studies demonstrate some relationships between mathematics teachers' pre-service coursework and their abilities as effective teachers. However, a gap still remains in understanding teachers' choices when using prior coursework and which coursework proves most useful while teaching. This study aims to take a modern, metacognitive approach to understand what undergraduate mathematics courses novice teachers reflect upon while teaching their classes to gain a better understanding of which courses are essential to a successful and productive teacher. To accomplish this, one must move away from the quantitative studies that have been run in the past with mixed results. To better understand the thought processes during the planning, execution, and reflection stages of teaching, I suggest using a qualitative approach focused on the teachers' application of their knowledge.

This study will attempt to assess how private school teachers use the knowledge learned during their collegiate career to plan and teach their lessons. This study will employ the problem-solving metaphor in examining teacher metacognition to understand how teachers use their skills and knowledge learned in their content area coursework to teach a mathematics lesson to their students. The study will focus on the cognitive and metacognitive behaviors of the teachers as they attempt to teach the lesson by looking at the three stages of problem-solving: (a) preparation, (b) actual problem-solving, (c) verification of the solution (Artzt & Armour-Thomas, 1998; Garofalo & Lester, 1985).

CHAPTER 3: METHODS

This study aims to provide insight as to which content area coursework in mathematics teachers use during the planning and teaching of mathematics classes. This insight will help private school administrators determine which teacher candidates can positively impact students' algebraic reasoning skills in future mathematics courses. This chapter explains the study's design, restates the research questions, and describes the setting. I will describe the participants, data collection methods, and data analysis plans. Finally, strategies to ensure trustworthiness are listed.

Research Question

The research question developed for this study came about as a result of carefully researching literature in relation to my problem of practice.

Central Question

How do teachers at a small private, independent school use their knowledge from previous coursework to plan and teach their lessons?

Study Design

I conducted a case study using an emergent approach to apply the problem-solving metaphor to examine how teachers apply their mathematical knowledge to planning and teaching their lessons (Creswell & Guetterman, 2019). This study focused on secondary mathematics courses ranging from 9^{th}-grade to 12^{th}-grade. I excluded college-level courses such as AP Calculus AB and BC. These courses require knowledge directly gained from specific collegiate coursework.

I used all four members of the mathematics department at the school who teach various courses ranging from Algebra I to Pre-Calculus. Each teacher took part in an 'interview-observation-interview' process. The first pre-observation interview provided context for the lesson observed, while the second, post-observation interview allowed for debriefing and discussion the observation. The observation consisted of taking notes on how the mathematics was taught, including but not limited to the vocabulary, notations, and procedures used during the lesson. The observation of the teacher allowed me to see how the teacher used their knowledge of mathematics while teaching their class.

Observations

This study consisted of classroom observations intended to better understand how high school mathematics teachers apply the knowledge gained during their collegiate mathematics career to instruction. The observations provided insight into how the teacher uses the information to articulate mathematical concepts in a meaningful manner. I compared the observation note to how they planned the lesson as discussed during the pre-observational interview. Using the observation protocol in Appendix A, I kept written notes on the usage of vocabulary, topics covered, level of engagement among the students, questions asked by the students and the teacher, the teacher's reaction to those questions, and the thoroughness of the answers provided.

The information collected during the observation provided insight into how well a teacher used their mathematical knowledge for teaching (MKT) by looking at how they used proper vocabulary and notations throughout the teaching of the lesson, as well as providing meaningful examples of complex ideas and methods (Ball et al., 2008). The observation also provided insight into how the teacher broke down the complex concepts learned in higher schools of mathematical thought and present them so that a young high school math student can understand and apply them. Again, this comes from reviewing how they use vocabulary and examples to explain the concepts.

Lastly, the notes gathered during the observation served as a guide for the post-observation interview. Reflecting upon the lesson covered, teachers were asked to describe why they chose to use a particular example or use specific vocabulary at given instances by answering questions such as: "What course from college did you think about while you were presenting this piece of information?" The information gathered during the observation provides an understanding of how the mathematics teacher applies their MKT during a lesson.

Interviews

Pre-observational Interviews. I interviewed the teachers twice, one before the observation and one following the observation, each lasting approximately thirty minutes. The pre-observation interview was conducted to gather information about the teacher (e.g., degree major, oral transcript of mathematics courses taken, and degrees held) and determine the learning objectives of the class ultimately observed. The information gathered during the interview will provided direction for the observation as to what to expect during the observation. This information included the specific vocabulary and examples that the teacher planned to use during the lesson.

Post-observation Interview. Using the notes taken during the observation, I revisited specific moments from the lesson during the post-observation interviews as points of discussion. By revisiting these moments, the teacher was able to reflect on their teaching and determine where they felt they learned the material, albeit definitions, notations, or procedures. The post-observation interview will occur as soon as possible upon completion of the class to ensure the best recollection of the events.

Both interviews were semi-formal in format. I used an interview protocol (see Appendix B) to ask guiding questions and build follow-up questions based on the teacher's responses.

These questions allowed the teachers to describe their mathematical understanding of the concepts.

Role of the Researcher

I hold a bachelor's degree in Pure/Theoretical Mathematics and a Master of Education in Mathematics Curriculum and Instruction. I believe this strong background in mathematical theory allows me to teach mathematics so that students understand how to do the mathematics and understand why they are doing the mathematics.

My mathematics background and teaching experience combine to give me a deeper understanding of the knowledge required to teach a vast array of mathematics courses. With a strong knowledge of the vocabulary and methods used to introduce a variety of math courses, I can evaluate the data gathered during observations and interviews. To maintain the validity of my data, however, I must keep myself as a mathematician out of the interview process. I do not want to influence the teachers to say what they think I want them to say. Therefore, I did not provide feedback on their mathematical processes. However, I did ask them to explain why they chose a particular method. I attempted to maintain the researcher aspect throughout the study. My mathematics skills and knowledge only arose during the interpretation of the data.

Study Context

The problem of practice involved in this study begins at a small, independent school in the Southeastern United States. The school itself is a PK through 12th-grade private school in the suburbs of a major city. As an independent school, they do not receive funding or support from any state agencies. The school consists of three main divisions: the lower school (i.e., pre-K to 4th-grade), the middle school (i.e., 5th- to 8th-grade), and the upper school (i.e., 9th- to 12th-grade). The head of each division leads the hiring of teachers for the school. However, the middle school headmaster does not decide alone as they work in conjunction with the upper school head to determine who is hired for each position.

Over the previous nine years, the mathematics department has experienced the most turnover among its teachers. The department consists of six teachers for 15 mathematics courses for grades five through 12. In that amount of time and for various reasons, close to 20 new teachers were hired to teach multiple levels of mathematics. The school posted all academic job offerings with a teacher placement agency that placed teachers in private schools across the Southeastern United States. The agency would send candidates' resumes, cover letters, and transcripts to the school to work through. If we hired one of their candidates, we would pay them a small fee for their service. Of all of the teachers employed, more than 70% came via a placement agency.

As a hiring team member and later as the mathematics department chair, I was often asked, "When looking at their transcripts, which classes are more important?" When looking for AP Calculus or Statistics teachers, my answer to this question was pretty straightforward; the teacher must have a strong background in Calculus or Statistics. When looking for high school mathematics teachers, answering this question was a little more complicated. This study provides more insight into which courses the administrators and hiring team members should emphasize while scrutinizing transcripts of mathematics teacher applicants for non-AP level courses.

This study examined how teachers apply the knowledge and understandings gained during their pre-service coursework to their new careers. Algebra I is the first mathematics course many students take when entering high school (Tyson & Roksa, 2016). The skills and techniques learned during this course set the foundation for all future mathematics courses. Stewart (2012) suggested that "success in calculus depends to a large extent on knowledge of the mathematics that precedes calculus: algebra, analytic geometry, functions and trigonometry" (p.xxiv). For many schools of higher education, calculus acts as a gateway course to many different majors within the STEM field (Brown et al., 2015). Learning and mastering the skills taught in high school mathematics courses that cover the material mentioned by Stewart is essential for success in college-level courses such as calculus. Therefore, it is necessary to understand which classes allowed the teachers to hone and master the skills they intend to instill in their students.

Participants

The participants in this study consist of all four teachers in the mathematics department at a small private, independent school in the Southeastern United States. After speaking to the Head of School, I received verbal permission to observe and interview all teachers within the department. Before proceeding, I sought out written consent to conduct the study from the Head of School (see Appendix E). These teachers combine to teach all high school mathematics courses. Each teacher is responsible for all sections of a particular core mathematics course (e.g., Algebra 1). The teachers have various undergraduate and graduate degrees, which may influence how they apply their knowledge.

Each of the participants has a minimum of eight years of teaching experience. Two of the participants hold degrees in applied mathematics, one with applications to statistics and the other with applications in science. Of the other two teachers, one has a degree in physics and astronomy while the other has an engineering degree. Two of the teachers have master's degrees, with one having a master's degree in finance and another with masters' degrees in engineering and mathematics education. Even though these teachers have various backgrounds, they each took the same core mathematics courses during college (i.e., Calculus I, Calculus II, Calculus III, and Differential Equations).

Data Description

The data consists of teacher demographics (i.e., highest degree level, area of the degree(s), years teaching, and courses taught), interview and observation notes, and oral transcripts that was transcribed into an Excel file. The pre-observation interview occurred before the observation to discuss the upcoming lesson, how they created the lesson, and the past experiences they pulled their knowledge from for teaching the lesson. This information guided the data collected during the observation of the class. The observation noted how the teacher followed the content described in their lesson plan, how the students appear to understand the lesson, what questions the students ask, how the teacher responds, and what vocabulary the teacher uses. The information gathered during the pre-observation interview and observation guided the direction of the post-observation interview. This interview focused on how the teacher used their knowledge to teach and adapt their lesson as they moved throughout the period. I asked questions about how well they felt the class held to the lesson they had planned and questions relating the information and examples used to previous knowledge such as content area coursework.

Demographics

Each teacher described their educational background, including where they went to college, their undergraduate and graduate degrees, what mathematics courses they took, and how many years they taught. The specific mathematics courses that each teacher taught were also collected. I also asked each teacher which of their classes they felt were the most difficult to teach. This question allowed me to compare their college-level coursework and knowledge with the courses they teach. I asked them the final question, "if you could return to college and take a class, what would it be and why?" This question was intended to help me understand what course they felt would help improve the skills and knowledge they deemed necessary for teaching their subjects.

Pre-observation Interview

I conducted a preliminary interview before the observation to gather information on the teacher's planning process and the content to be taught. I recorded the interview to allow for a more straightforward transcription of the data into Excel. Using the interview protocol (Appendix B), the teacher provided answers to questions such as "what is your background in mathematics?" and "what was the focus of your degree?". Following these questions, I asked about the particular lesson the teacher planned for the observed class. The most important of these questions dealt with where they derived specific knowledge of the topic, namely "which course or courses do [they] reflect upon while planning for that lesson?" Other questions centered around the examples the teacher used when teaching, what terminology they intended to use to explain concepts, and what were their plans for judging student understanding.

Observation

The purpose of the observation was to see how the teacher applied the concepts learned during their coursework in mathematics. Using the information gathered during the preliminary interview, I listened for specific vocabulary and how it is used. I also noted the examples used during the lesson, the students' questions, and the teachers' responses to those questions. Using the observation protocol (Appendix A), I kept written notes of moments that matched what the teacher planned and where things differed from the lesson plan. Specific questions that seemed to challenge either the students or the teacher were also noted in the observation protocol and marked to revisit during the post-observation interview.

Post-Observation Interview

The post-observation interview took place during the teachers planning period or after the school day. Meeting as soon as possible following the observation allows for clarity of thoughts and the ability to recall specific events that took place during the observation for both the observer and the teacher.

The interview consisted of questions based upon the observation and including how they used their content knowledge to answer questions posed by students during class. The interview protocol (see Appendix B) will guide the interview along with insights gained during the observation of the class lesson. After reflecting upon the observed class, I also asked if they felt the same content area courses mentioned during the pre-observation interview were influential to teaching their lesson and to explain why they made that decision.

Data Analysis

I imported interview transcripts and observation notes into a spreadsheet to allow more accessible coding and analysis. The analysis included two rounds of line-by-line coding to identify patterns and trends in the data (Bazeley, 2013). During the first coding round, a priori codes provided a general sense of trends present in the data. These codes included: the language used (e.g., appropriate vocabulary), the coursework specifically mentioned (e.g., Calculus), the appropriate notation used, and the reason for choosing a particular course. Appendix C contains the codebook used through this process. The codes (see Appendix C) were derived from the main research question: What courses do novice teachers reflect upon while planning and teaching?

These codes attempt to answer this question while focusing on the teachers' reflections while teaching.

Using these codes during the first round showed a trend of using calculus, foundations, and differential equations to provide the knowledge necessary to perform and understand various calculations. In contrast, courses in physics and finance provided the understanding of how to apply those skills. After concluding the first coding round, new codes emerged that focused on why the teachers felt each particular course was helpful during the planning and teaching of their lessons (i.e., knowledge of steps, applications, and understanding and focusing on steps or application). The emergent codes highlighted trends and patterns in the math teacher's usage of previous coursework to plan and teach.

Ethical Considerations

Ethical considerations are an essential part of any research study (Garzon, 2014). In all research process steps, one must engage in ethical practices (Creswell & Guetterman, 2019). Institutional review boards (IRBs) ensure ethical considerations in the research process are clearly defined and treated adequately. IRBs protect human subjects during research and ensure that no harm is done to those subjects (Creswell & Guetterman, 2019). Therefore, I received IRB approval (see Appendix D) before collecting data.

The data gathered in this study came from the teachers themselves. To protect their anonymity, I issued each teacher an alias so that only I know the identity belonging to each pseudonym. I stored the data in my UVA Box account to ensure no one has access to the data other than myself.

I also have a personal connection with each of these teachers. Before leaving the school to pursue my doctorate, I was their department chair. I played a role in the hiring process of many of the teachers involved in this study. I ensured that I did not allow the history between us to influence how they teach their lesson or answer the questions during the interviews. To do this, I clearly stated that this process and results will not affect their careers.

I implemented ethical assurances before data collection. The Institutional Review Board provided approval for the study (see Appendix D), and I also obtained permission to conduct the study at the selected school. Participants in this study received invitational recruitment letters prior to their participation (see Appendix F). Each participant signed informed consent letters accepting their participation in individual interviews and observations (see Appendix G). The complied data were coded, and the names were removed for the participants and the participating school. These limitations and delimitations need addressing to make these findings more relevant and transferable.

CHAPTER 4: RESULTS

This study aimed to explore how high school mathematics teachers' college mathematics experiences impact their teaching. The findings of the study will provide the administrators of a small private, independent school in the Southeastern United States points to consider when in deciding which candidates possess potential in teaching mathematics based upon their prior coursework. The research questions answered in this study were:

Research Question 1: How do teachers use their knowledge from previous coursework to plan their lessons?

Research Question 2: How do teachers use their knowledge from previous coursework to teach their lessons?

To better understand which courses are essential to a successful and productive mathematics teacher in the classroom, I collected data from the Algebra 1, Geometry, Algebra 2, and Pre-Calculus teachers. Due to the small nature of the school, one teacher is responsible for all sections of a particular course. The data gathered from the four teachers consisted of demographic data and interview and observational transcripts gathered during two semi-structured interviews and one observation of each teacher. The data were analyzed using a line-by-line coding technique using a prior codes and emergent codes (see Appendix C) to determine trends and themes within the data.

Algebra I

Effects on Planning

The interview with the Algebra I teacher, Jordan, began with inquiring about the lesson to be covered:

Me: What lesson do you have planned for class today?

Jordan: We are starting a new unit on exponents and exponential functions. For most of these students, this will be completely new information for them. I don't think a lot of them have worked with this material before, so we are going to start with the basics.

Jordan's plan for the class began with the necessary definitions of terms before working their way to the main idea and use of exponents. The questions that followed revolved around where Jordan felt most of their knowledge and understanding of the content was developed.

Me: What college-level coursework do you feel you reflected upon to plan your lesson?

Jordan: I feel like I am pulling mostly from my calculus classes and my foundations classes.

Me: What is your reasoning behind these choices?

Jordan: Calculus provided the experience to work with exponents and master the rules and definitions that guide the application of exponents. It also provided the opportunity to work with exponents and develop a sense of understanding how they work and how to manipulate them according to the rules of operations with exponents.

In Jordan's explanation, the calculus courses provided them with the knowledge and skills necessary to perform the tasks that accompany working with exponents.

The second course Jordan mentioned was coursework in foundations. Jordan noted that they took not one class in foundations but two. The first was Foundations of Logic, which focused on logical reasoning and the basics of proof writing. The second was the Foundations of Mathematics, which concentrated on proving many of the theorems taught, as they put it, "accepted as true" in other lower-level mathematics courses. An example of this was the quadratic formula used to solve second-degree polynomials. Jordan mentioned that:

> "The quadratic formula is a formula that many students learn to use early in their high school careers, but rarely do they see where it comes from. In my foundations of mathematics course, we learned where the formula comes from and how to derive it, not just use it."

Next, I asked Jordan how they felt these two courses aided in planning their lesson on exponents. Jordan responded that "the foundations courses provided the knowledge and way of thinking necessary for understanding how and why the concepts are used in several different types of situations. This is important both in and outside of the classroom." In Jordan's opinion, the foundations courses provided the skills needed to reason through the problems and topics covered within the unit and those needed to break down the material so that the students will understand.

Effects on Teaching

When Jordan began the next unit on exponents, they started with the definitions of new terms used throughout the unit. The vocabulary consisted of terms such as constant, variable, exponent, monomial, and polynomial. After the observation, I looked to determine where they developed their knowledge and understanding of such terms and found the definitions used during class for the given terms.

Me: Where did you find the definitions that you covered in class today?

Jordan: I actually pulled them from another algebra textbook that another teacher left for me when they decided to move on to a new school.

Me: Why did you use that book instead of your class text?

Jordan: That particular text had more comprehensive definitions of the terms. Like the definition that I used for monomial where they specifically describe the exponent as being a non-negative integer versus saying it's a positive whole number. I want them to see and hear the definitions in the actual mathematical terms.

Me: What about the knowledge of the topic and skills used? Where do you feel you acquired those?

Jordan: I still feel like the foundations courses that I took played a big role. They really provided the logic skills that I needed to understand the concepts and definitions that I discussed.

For this particular lesson in Algebra I, it appears that Jordan relied upon the procedural knowledge of the topic gained from calculus courses. In contrast, foundations of mathematics and logic courses appear to influence and aid their understanding of the topics covered in their lessons. The last question that I asked Jordan centered how possible improvements to their mathematical knowledge.

Me: If you could return to college and take any course to help with your teaching of Algebra I, what would it be?

Jordan: I would go back and retake calculus.

Me: Why is that?

Jordan: I want the opportunity to develop a better understanding of the concepts within calculus instead of feeling like I learned how to repeat what the professor was doing on the board. I know how to do the work in calculus, but I want to understand more of the "why do we do it this way' and 'where did it come from."

Jordan desires to understand how to do the math and show the students how to do the math. However, there is also a desire to understand math on a deeper level.

Geometry

Effects on Planning

The geometry teacher, Peyton, in the middle of a unit on the Pythagorean theorem, was focusing on the notion of Pythagorean triples. When asked what college-level coursework they reflected upon to plan their lesson, Peyton responded with two separate classes.

Me: What lesson do you have planned for your class today?

Peyton: Today, we are going to discuss Pythagorean triples, namely what they are, how they are used, and why do we use them.

Me: Which of the previous courses you took do you feel you reflect upon developing your plan for this lesson?

Peyton: I have to say there are two that I go back to when I think about how I am going to teach this. The first is calculus. I feel like those classes provided the opportunity to build the procedural knowledge necessary to perform the required calculations for the class.

Me: What do you mean by that?

Peyton: Calculus allowed me the time to become comfortable with the methods of solving that type of problem and the notations that are used. Through the sheer number of problems that we did for that course, I had the time and opportunity to practice and become accustomed to the different methods of approaching different types of problems and using the many different types of notations.

Peyton also commented that they reflected on how their Calculus III professor taught the course. They felt that their calculus professor was different from other professors because every concept covered in class used a real-world application as an example. The purpose of learning the method did not center on the simple repetition of steps to work through the problem but instead on how to use those steps to solve a specific problem. Peyton said that this professor believed that "teaching everyone how to use the method to find a usable answer was more important than being able to crunch their way through the problem." Peyton's goal was to teach similarly to their Calculus III professor by showing the students "how to do the problem while also showing them why they may have to do the problem."

The second content area that Peyton mentioned was their college-level physics courses.

Me: Why do you feel that Physics was influential to teaching Geometry?

Peyton: Physics reinforced the [math] skills and allowed me to put meaning to the work as to why are we doing this or what is the point of this. Calculus has a fair amount of geometry concepts built into it. Physics allowed me to put that to use in a practical way, for me at least.

Peyton felt that physics provided the opportunity to see the methods learned in calculus put to use in what they referred to as "practical situations." In their opinion, physics allowed them to develop a better understanding of the mathematical concepts beyond the simple repetition of steps in an algorithm.

Effects on Teaching

Peyton began class by walking through a few examples from the previous night's homework. While illustrating the problems, Peyton focused on using the correct vocabulary related to the right triangles and the proper notation associated with the Pythagorean theorem.

After observing the class, I asked Peyton what course they felt they reflected upon while teaching their lesson. Again, the response was calculus and physics. Peyton felt that calculus provided them with the opportunity to work with and develop the procedural knowledge necessary to perform the calculations. Peyton also mentioned that their experience in the calculus course allowed them to "become comfortable with the methods and notations used during the calculations."

> Me: How do you think calculus made you comfortable working with the notations and methods?
>
> Peyton: It is like doing anything else. The first time you work with something completely new, you are anxious and nervous about it. But the more you do it and see it, the easier it gets. After doing about a thousand or so problems in calculus, you just get used to it.

Peyton also credited physics with the development of their notation skills by saying that "it forces you to use proper notation to keep track of what you are doing" and "[both classes] make you show all of your work and maintain that notation throughout the work in order to keep from making simple mistakes."

As for the development of vocabulary skills, Peyton admitted that they had not taken a geometry course since high school. Peyton commented that they developed their vocabulary throughout their school career, and "it comes from my high school Geometry course and years of teaching the topic." Peyton has taught Geometry for more than 15 years. When I asked where they pulled the definitions used during the lesson, Peyton admitted they came directly from the textbook.

Peyton had a limited background in mathematics, having taken only four mathematics courses while in college (i.e., Calculus II, Calculus III, Differential Equations, and Partial Differential Equations). They felt calculus provided them the opportunity to build the skills necessary to do the math while obtaining the understanding of how, when, and why to apply that skill from their physics courses. When asked, if given the opportunity, what mathematics course they would return to college to take, Peyton responded with Statistics. The reasoning for this was that Peyton had very little knowledge of the subject and felt "to be a better, all-around teacher," they needed statistics. Peyton went on to say:

> "The last department chair understood a little bit of everything. He would sit and talk calculus with a student and be able to explain everything to them. Then another student would walk in with a statistics question, usually from the AP Biology class, and he could explain that to them as well. I have never taken a statistics course and have very limited knowledge of what is going on in that class. I want to be able to understand that information and, in turn, better help the students when they need help in that area."

Algebra II

Effects on Planning

Alex's lesson focused on solving equations using logarithms and applying that process to answer questions involving compounding interest.

Me: What lesson are you discussing in class today?

Alex: Today, we are looking at how to use and solve logarithmic and exponential functions.

Me: What courses do you feel you are reflecting on to plan your lesson for today?

Alex: I feel like a lot of it comes from my calculus classes. Calculus provided not only the time to practice working with logarithms but also helped develop a deeper understanding of the rules that govern the application of those functions.

Me: How do you feel calculus helped with that development?

Alex: I would have to say through the repetition of working through vast amounts of problems. The skills of working with these functions just developed over time. Calculus class was the first time that I really worked with logs and exponential functions. I don't think we ever really used or understood *e* when I was in high school. I really learned how to work *e* and *ln* in my calculus classes.

Me: How are you planning on teaching the class how to work with these functions?

Alex: I want to start with the steps of how to work with them, but then I want to shift into how we apply this knowledge of working with these functions. I want them to see that these functions are used in more ways than just SAT questions. They have a purpose and a usage.

Alex wanted to show the students that working with these functions was more than just an exercise in repeating particular steps multiple times. Alex wanted the students to see the application of these functions in real-world situations and understand the properties and rules that govern their use in those applications.

Effects on Teaching

Alex began with a review of applying the change of base formula to allow calculator usage to find the log of a number with a base other than 10. This process involved turning all logarithms, other than base *e*, into base-ten, allowing the students to use the *log* function built into the calculator. The lesson then moved to solving exponential equations with bases other than ten and solving logarithmic equations by applying rules for adding and subtracting logarithms. Finally, the lesson ended with using natural logs to solve problems involving compound interest. This application included comparing the differences between continuously compounded interest and interest compounded every quarter.

When asked to reflect on the lesson taught during class, Alex reiterated the usefulness of their college calculus class to provide them with the procedural knowledge necessary to perform the calculations covered in class. However, they also noted the benefit of their finance courses taken during graduate school.

Me: What courses do you feel you reflected upon while teaching?

Alex: Well, I still feel that calculus built the knowledge for understanding how to manipulate these types of functions. But, as I sit and think about it now, I think that the courses I took in finance really came into play while I was creating those examples dealing with interest. Those examples were two-fold, I wanted them to solve it, yes, but I also want them to understand what is going on when they talk about interest, interest rate, and which type of interest is best for what they are trying to do. I want them to walk out of class with some useful knowledge.

Alex had a master's degree in finance and suggested that finance courses allowed for a deeper understanding of the concept of interest. This understanding was used to create meaningful examples and explain the use of logarithms, natural logs, and exponential functions during the teaching of the lesson.

For Alex, calculus provided the background knowledge necessary for understanding the overall procedure required for the mathematics taught. In contrast, finance courses provided the understanding of when and how to apply those skills. Alex also mentioned the desire to return to college and take a teaching methods course for secondary mathematics. They feel they would benefit from this type of class because it would offer more teaching techniques to apply in their classroom.

Pre-calculus

Effects on Planning

The pre-calculus teacher, Sean, planned for a lesson on factoring polynomials and finding zeros of the polynomials. The lesson focused on finding the zeros of the polynomial by applying the Rational Root Theorem. However, instead of just defining the Rational Root Theorem, Sean planned to have the students derive the concept of the theorem on their own.

Me: Tell me a little about your lesson today

Sean: I'm planning on working with them to find the factors and roots of polynomials using the Rational Root Theorem, but I'm not giving them the theorem until the next class. I want them to work through the process and try to figure out the notion of the theorem on their own first.

Me: What courses do you feel you reflect upon to do this lesson in this manner?

Sean: I have to say that my linear algebra class played the largest role.

Me: Why is that?

Sean: I feel I learned more about how to teach from the professor. Not necessarily the mathematics used in the class but the method for teaching.

Me: How is that?

Sean: I had the same professor for both Linear Algebra I and II, and that professor demanded rigor and justification of the work. We learned the processes, but we also had to justify all of the answers that we came up with.

Sean wanted the students to do the same thing with the roots and factors of the polynomials. They went on to say that "I want them to justify their steps…it's not about the repetition of the steps but building the deeper understanding of the material." The lesson's goal was to allow the students to build upon their knowledge of roots and factors to justify the processes they were using and have a deeper understanding of the theorem they would discuss formally in a future class.

Effects on Teaching

The class observed illustrated the notion of a "productive struggle" (Lynch et al., 2018). Sean allowed the students to struggle with finding an initial root to the polynomial based upon what they knew about the polynomial factors and how those factors translated into the roots of the polynomial. With time and a small amount of direction, the students began to understand that the possible rational roots of the polynomial came from the factors of the constant divided by the factors of the leading coefficient, which is the basis of the Rational Root Theorem. The lesson was more focused on the derivation of this theorem than the steps used to test the possible roots. Students were allowed to use any method to test the possible roots other than using a calculator to graph the polynomial.

After teaching the lesson, I asked Sean where they felt they pulled the knowledge for teaching the lesson.

Me: So, thinking about the lesson today, what courses do you feel you reflected upon while teaching the class?

Sean: I feel like I thought about calculus or differential equations more than I did linear algebra.

Me: Really? Why do you think you made that change?

Sean: I feel like in those classes, we learned trial-and-error methods like u-substitution and integration by parts. You know how you try to integrate, but the direct method fails, so you try u-substitution, and if that fails, you go to by parts and so on. We keep working until we find a method that works. This process really builds a sense of tenacity in the kids. They see they may fail multiple times, but if they keep working, they can find the answer. I want them to have that tenacity to keep working.

Me: Why do you feel this tenacity is important?

Sean: It is important to understand that you may not always find the answer on the first try. Sometimes you make mistakes, and that is ok.

Sean felt that working through multiple methods beginning with an obvious method and working to the more complex processes (e.g., u-substitution, integration by parts, and trigonometric substitutions), allowed them to build the "tenacity." Building this tenacity enables the students to work through complex problems using the trial-and-error method without getting discouraged after the first non-successful method. Sean wanted their students to have the opportunity to "make connections between the types of problems and the methods used to solve them so that next time they are faced with a similar problem, they don't have to work as hard."

While teaching, I also noticed that Sean used various notations for the functions used as examples (e.g., y and f(x)).

Me: So, while you were teaching, I noticed that you were using different notations for your functions. First, it was *y* =, and next, it was *f(x)* =. Where did you develop this practice?

Sean: Well, I have to admit, when I was in high school, all my teacher ever used was *y* = to represent functions. But when I got to college, all of a sudden, there were all of these other notations, and I was like…wait a sec. It really threw me for a loop. So, I want my students to see that functions can be named in multiple ways. You know, whatever I need to fit my problem.

Sean then explained using parametric equations in calculus and how mathematicians often rewrite a single function as multiple functions of a single variable, making differentiation and integration easier.

Sean felt that they pulled knowledge from calculus and differential equations to teach their lessons and reflected on the teaching strategies learned from the linear algebra professor to share that knowledge with their students. The final question that I asked Sean was what course they would take if given the opportunity.

Me: If you were given a chance to return to school and take any class you wanted to aid in your teaching, what would that course be?

Sean: I would have to say I would take a course in game theory.

Me: Why is that?

Sean: It just kind of overlaps knowledge and skills found in discrete math, linear algebra, and probability and statistics.

Sean felt that this course would allow them to create better connections between the mathematics taught and the students' interests, making the math "more enjoyable."

Knowing, Applying, and Reasoning

Three themes emerged from the synthesis of the findings. The first theme was that calculus courses provided the teachers with the necessary knowledge to understand the content they planned and covered during their lesson. Table 1 illustrates the emergence of this theme. The emergence of this theme suggests that calculus provides the teachers with the needed experience in the topic area, allowing them to build the skills and abilities necessary to understand and demonstrate the methods taught during class.

Table 1

Calculus Leads to Knowing

Theme	Questions	Themes and emergent discoveries
Calculus leads to knowing	Which of your college-level courses do you feel you are reflecting upon to plan for this lesson?	Calculus aids with knowing Teachers felt that experience in calculus allowed them to know material better and become accustomed to the methods used.
	Which of your college-level courses do you feel you reflected upon while teaching this lesson?	"Calculus provided the experience to work with exponents and master the rules and definitions that guide the application of exponents." "Calculus provided not only the time to practice the work, but also helped develop a deeper understanding of the rules that govern the application of those functions." "Calculus allowed me to become comfortable with doing the work quickly and efficiently. We practiced doing a lot of problems in Calculus."

This theme corresponds with the knowing cognitive domain outlined by the TIMSS framework (D. Lee & Huh, 2014; Mullis et al., 2005). Calculus provides teachers with the skills necessary for knowing how to do the mathematics involved in their classrooms. These skills include recalling, recognizing, computing, retrieving, measuring, and classifying. In both the planning and teaching stages of their classes, the teachers commented that they gained experience in the computing component of this domain. These skills also fall within Blanton and Kaput's (2005) explanation of algebraic reasoning. As Blanton and Kaput described, algebraic reasoning uses arithmetic to express and formalize generalizations to explain functional relationships, another skill developed within calculus.

The second emerging theme was that calculus, physics, and finance coursework provided knowledge of how to apply the information to solve routine problems (see Table 2). These courses provided the real-world application necessary for the teachers to put meaning to the mathematical concepts and then show that meaning to the students.

Table 2

Calculus, Physics, and Finance Leads to Applying

Theme	Question	Themes and emergent discoveries
Calculus, Physics, and Finance lead to application	Which of your college-level courses do you feel you reflected upon while teaching this lesson?	Calculus, finance, and physics courses help with understanding how to apply the mathematics to solve problems Teachers felt that experience in these classes allowed them to understand how to apply the mathematics they taught. "In those [calculus] classes, we definitely learned trial and error methods…working until we find a method that works." "Finance built the understanding of how these functions are used in real world situations." "Physics allowed me to put meaning to the work as to 'why' are we doing this problem or what is the point of knowing how to do this."

This theme corresponds with the applying domain within the TIMSS framework (Lee & Huh, 2014; Mullis et al., 2005). The teachers felt that courses in calculus, physics, and finance aided in developing the skills necessary for the representation of relationships, selecting appropriate methods, creating appropriate models (e.g., equations or diagrams), and solving routine problems (Lee & Huh, 2014; Mullis et al., 2005). These courses reflected their fields of study as they learned how to apply the skills learned in their mathematics courses. These courses appear to provide context for the teachers' knowledge of mathematics and how to use that knowledge in meaningful ways.

The final theme suggested that coursework in foundations allows the teachers to develop logical, systematic thinking capacity. This logical thinking allows the teachers to analyze and generalize the concepts within mathematics and apply their knowledge to new, novel situations. Table 3 illustrates this theme.

Table 3

Foundations and Calculus Lead to Reasoning

Theme	Questions	Themes and emergent discoveries
Foundations and calculus lead to reasoning	Which of your college-level courses do you feel you are reflecting upon to plan for this lesson?	Foundations helped with the skills to reason through the mathematics Calculus helped with the skills to solve non-routine problems Courses provided the skills to analyze and generalize the processes
	Which of your college-level courses do you feel you reflected upon while teaching this lesson?	"The foundations courses provided the knowledge for understanding how and why the concepts are used in situations other than math class." "Foundations provided the skills to reason through the problem and break down the lesson in a manner that

	"... allows the class to better understand the concepts covered." "It [foundations] helps me understand the underlying 'whys' of the lesson." "[Calculus] builds tenacity…they see that even though a particular method fails in a specific situation does not mean that the problem cannot be solved."

The final theme corresponds with the reasoning domain of the TIMSS framework (D. Lee & Huh, 2014; Mullis et al., 2005). Teachers mentioned that they developed the skills necessary to analyze, generalize, and solve non-routine problems while taking courses in calculus and foundations. These skills also carry over into the realm of algebraic reasoning, where the ability to analyze different situations and form generalizations based upon those situations are critical (Blanton & Kaput, 2005).

Three out of the four teachers also mentioned a desire to return to school and take coursework to increase their mathematical knowledge, application, and reasoning skills. This notion suggests that the teachers are mindful of the abilities and understandings that may lead to success in their classrooms. With this in mind, they look to improve their skills in areas they feel they are weakest.

Summary

This chapter presented the results of this case study that assessed the content area coursework high school mathematics teachers reflect upon while planning and teaching their lessons at a small private, independent school in the Southeastern United States. I collected data from the four upper school mathematics department members via two semi-structured interviews and one observation of each class. As a result, the three following themes emerged: 1) Calculus courses played a role in the teachers' knowing how to do the mathematics involved in their classes; 2) Coursework in calculus, physics, and finance provided knowledge of how to apply the material; 3) Coursework in calculus and foundations provided the teachers with the mathematical knowledge to relate what they are teaching in mathematics to unfamiliar situations and complex contexts.

CHAPTER 5: DISCUSSION and CONCLUSIONS

The purpose of this case study was to inform the administration team of a small private, independent school what content area coursework their upper school mathematics teachers reflect upon while planning and teaching their lessons. This information derived from this study will inform the decisions made by the administration team as they work through the process of hiring a new mathematics teacher. The problem addressed in this study was the small private, independent school's attempts to hire teachers with appropriate mathematical backgrounds based upon their college transcripts. The school's administration team has a limited understanding of the mathematics backgrounds necessary to teach high school mathematics courses. This study is to provide them with more insight into what college-level mathematics courses are important to see on applicants' transcripts during the search process.

Discussion

Three themes emerged from the data analysis in response to the research questions of this study. These themes corresponded with the three cognitive domains outlined in the 2007 TIMSS framework (D. Lee & Huh, 2014; Mullis et al., 2005). From these themes emerge three implications. The first implication was the overall usefulness of calculus courses as teachers planned and taught their lessons. Every teacher mentioned how they reflected upon knowledge and skills gained in their college calculus courses. Each emergent theme contained calculus as a contributing course. Based on the teachers' reporting, I found that calculus courses influenced the knowing, applying, and reasoning aspects of content knowledge that teachers used during the planning and teaching of their respective classes.

The second implication that emerged from the themes suggests that teachers may benefit from coursework in foundations of mathematics and logic. These courses allowed for developing the reasoning skills used to analyze, generalize, and solve non-routine problems. These skills, namely analyzing and generalizing, are critical components of algebraic reasoning (Blanton & Kaput, 2005; Kaput, 2008). Reasoning allows the teachers to use their knowledge of mathematics to create generalizations towards specific ideas, such as expressions and functions (Kaput, 2008). For teachers to effectively build their students' algebraic reasoning skills, they must also possess the skills (Grossman et al., 2005; Wayne & Youngs, 2003).

The third implication suggested that the teachers benefit from a course that allows them the opportunity to apply their mathematical knowledge. This course may be outside of the area of mathematics (e.g., Physics or Finance). These courses are not courses in Applied Mathematics but courses in other fields of study where mathematics is applied. These courses allowed the teacher to see how they can apply the mathematics in an actual real-life situation similar to the non-routine problems observed within the reasoning domain. These courses may not benefit the reasoning skills needed for understanding the deeper workings of mathematics. However, they appear to affect the teachers' abilities to explain how the mathematics was used in specific situations.

The three themes that emerged aligned with the cognitive domains measured by the TIMSS framework (D. Lee & Huh, 2014; Mullis et al., 2005). The teachers appeared to have no knowledge of this framework during the data collection process. However, they did show knowledge of what skills the students need to develop within their courses. It stands to reason, if we expect the students to become proficient in these skills, the teachers must also possess the same skills.

Recommendations

The following recommendations were based upon the implications and findings of this study. Private school leaders expect their teachers to possess all the skills needed to enter a classroom, teach a meaningful lesson, and provide the students with the opportunity to develop a deep understanding of the content. A teacher must first possess a deep understanding of the concepts discussed within the course to do so. To measure the extent to which applicants have this knowledge, some school administrators turn to college transcripts when evaluating a resume.

Practical Recommendation

This study indicated that high school mathematics teachers reflect upon a range of college-level courses to plan and teach their lessons. The combination of these courses that provides the teacher with the knowledge, application, and reasoning skills needed to provide a rich explanation of the material covered in a high school classroom. Thus, when a small private, independent school in the Southeastern United States looks to hire a new mathematics teacher, the recommendation to the hiring committee is to look for mathematics coursework in calculus and foundations. The second recommendation is to look for coursework in an area that allows the teacher to apply the knowledge they gained in their mathematics courses in real-world situations. However, these recommendations were based on a small sample of teachers pulled from a single private, independent school. It is reasonable that teachers may reflect upon other courses from their collegiate career or other courses they have taught. More research must be conducted using a larger sample of teachers from multiple schools to better understand the courses reflected upon and their effects on students' learning.

Further Recommendation

Many schools of education have different mathematics requirements for students pursuing a degree in mathematics education (Appalachian State University, 2021; James

Madison University, 2021; UNC School of Education, 2021; UNC-Asheville, 2019; University of Virginia, 2021). Most schools require their students to take at least two courses in calculus; however, not all schools require coursework in foundations (Appalachian State University, 2021; James Madison University, 2021; UNC School of Education, 2021; UNC-Asheville, 2019; University of Virginia, 2021). Few schools required a science course or application course, such as finance, higher than entry-level. The findings of this, and future research in this area, may lead schools of education to rethink their mathematics education majors curriculum to include coursework in foundations and one, if not two, courses in an application setting outside of mathematics. These courses could consist of courses in the science or economics departments. More work needs to be done in this area for this recommendation to be effective. More studies need to be conducted using more teachers and schools from across the country.

Future Studies

Moving forward, more work needs to be done in this area using a larger sample size from multiple schools, both public and private, to ensure the results are generalizable. There is also a need to determine whether the mathematics courses taken by the teacher have an effect on students' mathematical abilities. I purpose a large mixed-methods study involving students and novice teachers with less than five years of teaching experience. For the teachers, I propose using the same interview-observation-interview cycle used in this study. However, I would increase the number of these cycles to ensure that teachers reflect upon various lessons and topics within their given subject. By including more cycles involving various topics, the teachers will have the opportunity to possibly reflect upon a wider variety of content area coursework. By including the students, I could measure their gains in the three cognitive domains by using a pre-test/post-test format. Using teachers with less than five years of teaching experience would possibly increase the likelihood of reflecting college-level course work instead of previous teaching experience.

Delimitations and Limitations

This study focused on the effects of content area coursework on high school mathematics teachers planning and teaching of their classes. I am aware that other factors may influence and affect these outcomes. Multiple studies suggested the importance of teachers taking courses in pedagogy and methods and the positive effects those have on student achievement (e.g., Ball et al., 2008, Ferguson and Womack, 1993, and Monk, 1994). Moreover, studies suggested that simple accumulation of credits within a subject does not imply that an individual possesses the skills to teach that subject (e.g., Monk, 1994). The purpose of this study was to better understand which content area courses could be considered the most important courses for future mathematics teachers to take during their pre-service, collegiate careers.

The focus of this study was confined to only secondary mathematics teachers. Elementary School mathematics teachers were intentionally omitted from the analysis to focus on secondary mathematics. This study was also limited to looking at private school teachers only. By looking at private school teachers, I could concentrate on the coursework they took for their degrees, which may or may not have come through a school of education.

This study contained a few limitations. The sampling size was small, sampling occurred at only one school, and there were varying amounts of teaching experience within the sample of teachers. There was a possibility of response bias due to a preexisting relationship between the participants and the researcher. This limitation was addressed with in-depth questions from the researcher and explanations from the participants. Triangulation of data allowed for the comparison of answers to questions about planning. Evidence of those answers within the teaching of the lessons and evidence of those answers within the teaching of the lessons. The triangulation was further developed during the second interview of the teachers, where they were able to reflect upon their teaching and thinking while teaching.

Conclusions

This study looked to provide the administration of a small private, independent school with the information necessary to aid in the hiring of effective mathematics teachers by understanding which content area coursework they should see in applicants' resumes. A qualitative study was performed to investigate what courses the current upper school mathematics teachers reflect upon while planning and teaching their lessons. Based on the study's findings, I recommended that the administration looks for course work in calculus, foundations, and an area outside the field of mathematics that allowed for the application of mathematics, such as physics or finance. These courses were the areas that the mathematics teachers at this school reflected the most while planning and teaching their lessons.

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Appendix A

Observation Protocol

Observation protocol

- **Research question:**
    - How do novice teachers use the knowledge from previous mathematics coursework to teach their lesson?
- **Environment:**
    - Formal, professional environment; classrooms and students will vary; courses observed will also vary
- **Prior to observation:**
    - There will be an interview prior to the observation focusing on how the teacher uses their mathematical knowledge to design an appropriate lesson
        - Read through the comments/notes from that interview to gain an idea of what they plan to use.
        - Construct a list of key concepts/ideas they plan to use.
    - Note the presence of artifacts around the classroom or lack thereof; look for posters, student work, illustrations, manipulatives, etc.
        - Are the artifacts relevant to the class that is in the room, meaning if it is an Algebra 1 class, are the artifacts useful to them or are they for another math class?
- **During the observation:**
    - Using the list created from the interview notes, what concepts have they used or not used?

        - Note how they used the concept.
        - Does their usage match the pre-observation/lesson plan they created?
        - How are the students receiving the information?
            - Is there confusion or understanding
    - Note the questions the students ask during the lesson?
    - Note the level of class engagement.
    - Note the evidence of student understanding or lack thereof.
    - Note the usage of proper vocabulary

Formal write-up format:

<table>
<tr><td colspan="2">Date:
Time:
Location:
Observed:
Subject of class:
Period of class:</td></tr>
<tr><td>Observation Notes</td><td>Comments</td></tr>
<tr><td>Record facts and details based on the criteria listed in "During Observations"</td><td>Comments or questions about observations made during class</td></tr>
<tr><td colspan="2" align="center">Reflection and Summary</td></tr>
<tr><td colspan="2"></td></tr>
</table>

Appendix B

Interview Protocol

"Hello, my name is Wesley Cox. I am looking to gather data on what course taken during your pre-service, collegiate career you reflect upon the most while you plan for a lesson, teach a lesson, and judge the effectiveness of a lesson. I want to start by saying thank you for your willingness to participate in this study. Please be aware of the confidentiality agreement and know that nothing you say here will make its way back to any administrators or supervisors. Also know that I will be recording the interview just so that I can reflect upon those notes for clarification later. Also know there are no right or wrong answers to any of these questions, please answer them to the best of your ability. "

Guiding Questions:

Pre-observation

1. First off, tell me a little bit about yourself, your background, where you went to school and what classes do you teach?
    a. Where did you go to school?
    b. What mathematics courses do you remember taking?
    c. What classes do you teach?
    d. Of those courses, which do you find to be the most challenging, content?
        i. Why?
    e. Did you study that much in college?
2. Tell me about the lesson that you are going to teach today?
    a. What resources do you intend to use to guide your lesson?
    b. What is the most important thing that you want to see that shows they understand?

i. Why?

3. How did you develop the method that you are showing the class to ____________?
4. What college-level course do you feel you reflect upon while you planned this lesson?
5. If you were given the opportunity to return to take more courses to improve your teaching of mathematics, what course(s) would that include?

Post-observation (Potential questions. The questions will vary based upon the observation)

1. It is good to talk with you again. Last time we met; I was observing your __________ class. How do you think the lesson went?
   a. There was a question that someone asked that seemed to make you think a bit, do you remember what it was about?
   b. How did you handle that?
   c. What came to mind while you were trying to explain how that worked?
2. As you were explaining __________ you mentioned ____________. What made you think of explaining it that way?
   a. Was there a flash back to a previous math course in your career that made you think of it?
   b. What was it?
3. I noticed while you were writing the example on the board, you used __________ notation. Why did you use that notation instead of something else?
   a. Where did you learn that notation?
   b. Is that something that you are going to expect them to know and understand for the next assessment?

4. I noticed while some were working that one student did his problem a little bit different than how you were showing it. How would you interpret his work?
    a. Is this something that you would accept as an answer or not?
        i. Why?
5. When we met to discuss your lesson plan you mentioned reflecting back to _________, do you still feel that way, or do you feel that you reflected back to a different course or courses?

"As always, I really appreciate your time and willingness to participate in this study. Thank you so much for all of your input."

Appendix C

Codebook

a priori codes

Code Name	Definition	Inclusionary Criteria	Exclusionary Criteria	Example
Language	When the teacher describes how the mathematical vocabulary will be used.	Includes when the teacher uses the appropriate vocabulary to describe or name parts. Also, when used to correct any misconceptions.	Does not include vocab. relating to operations such as addition and subtraction.	The roots of a polynomial are the solutions when we set the polynomial to equal zero and solve
Courses	When the teacher describes the course(s) they reflect upon to understand the concepts they need to teach a given topic.	Includes when the teacher specifically recalls a college level course they reflect back upon. (This may also be a non-mathematics course) This may also include an AP course taken during high school.	Does not include reflecting back to a previous chapter or unit covered during the teaching of a current course.	The foundations and logic courses provided the knowledge for understanding how and why the concepts are used in situations other than math class
Notation	When the teacher uses the appropriate notation or explains the use of that notation.	Includes all mathematical notations.		"When a decimal repeats, we use a bar or line over top of the piece that repeats."

Reasons	Why the teacher feels a particular class is useful	Includes ideas like deeper understanding, real world application, and experience with the work		Calculus provided the experience to work with exponents and master the rules and definitions that guide the application of exponents

Code category	**Code**	**Code definition**
Language	lang	teacher uses mathematically correct vocabulary
Courses	course - calc	reflects to "Calculus"
Courses	course - found	reflects to "Foundations" or "Logic"
Courses	course - linear	reflects to "Linear Algebra"
Courses	course - phys	reflects to "Physics"
Courses	course - diff	reflects to "Differential Equations"
Courses	course - finance	reflects to "Finance"
Notation	notation	teacher uses mathematically correct notations
Reasons	reason - exp	the class provided the opportunity to work with the material and build experience and comfort with the idea
Reasons	reason - under	The class provided the opportunity to develop deeper understandings of the concepts other than just repetition of processes or algorithms

Emergent codes

Code Name	Definition	Inclusionary Criteria	Exclusionary Criteria	Example
Knowledge	The teacher used the knowledge gained from content area coursework to explain the method and provide real-world context to the mathematics	Taught steps that can be repeated Taught with intent to develop a deeper understanding of how and when to apply the mathematics concept	The focus of the lesson as it pertains to "how to" or "why".	Step 1: Remove the coefficient Why does continuously compounded interest build faster that quarterly compounded interest?

	taught during the lesson			
Focus	The main focus of the lesson	The teacher is focused on developing students' skills with repetition of an algorithm The teacher is focused on teaching and providing understanding of how and when to apply the use of the algorithm or knowledge thereof in new situations	An example does not imply focus	Which investment option is better for doubling your money at a rate of 9%: continuous or quarterly? Why?

Code category	**Code**	**Code definition**
Knowledge	know - steps	knowledge is used to show steps for completing the math
Knowledge	know - app	knowledge is used to show application of the mathematics
Knowledge	know - under	knowledge is used to develop understanding why
Focus	focus - steps	focus of the teaching is about repeating steps and "how to" do the mathematics
Focus	focus - app	focus of the teaching is the application of the material, namely, how to use the material to make generalizations and apply the knowledge in new situations

Appendix D

IRB Approval

Institutional Review Board for Social & Behavioral Sciences
University of Virginia

Office of the Vice President for Research
Human Research Protection Program
Institutional Review Board for the Social and Behavioral Sciences

IRB-SBS Chair: Moon, Tonya
IRB-SBS Director: Blackwood, Bronwyn

Protocol Number (4854) Approval Certificate

The UVA IRB-SBS reviewed "Subject area knowledge of secondary private school teachers" and determined that the protocol met the qualifications for approval as described in 45 CFR 46.

Principal Investigator: Cox, Wesley
Faculty Sponsor: Choi, Kyong Mi
Protocol Number: 4854
Protocol Title: Subject area knowledge of secondary private school teachers **Is this research funded?** No

Review category: Exempt Review
1. Normal educational practice in educational settings

Review Type:

Modifications: No
Continuation: No
Unexpected Adverse Events: No

Approval Date: 2022-02-25

As indicated in the Principal Investigator, Faculty Sponsor, and Department Chair Assurances as part of the IRB requirements for approval, the PI has ultimate responsibility for the conduct of the study, the ethical performance of the project, the protection of the rights and welfare of human subjects, and strict adherence to any stipulations imposed by the IRB-SBS.

The PI and research team will comply with all UVA policies and procedures, as well as with all applicable Federal, State, and local laws regarding the protection of human subjects in research, including, but not limited to, the following:

1. That no participants will be recruited or data accessed under the protocol until the Investigator has received this approval certificate.
2. That no participants will be recruited or entered under the protocol until all researchers for the project including the Faculty Sponsor have

   completed their human investigation research ethics educational requirement (CITI training is required every 3 years for UVA researchers). The PI

   ensures that all personnel performing the project are qualified, appropriately trained, and will adhere to the provisions of the approved protocol.

3. That any modifications of the protocol or consent form will not be implemented without prior written approval from the IRB-SBS Chair or designee except when necessary to eliminate immediate hazards to the participants.
4. That any deviation from the protocol and/or consent form that is serious, unexpected, and related to the study or a death occurring during the study will be reported promptly to the SBS Review Board in writing.
5. That all protocol forms for continuations of this protocol will be completed and returned within the time limit stated on the renewal notification letter.
6. That all participants will be recruited and consented as stated in the protocol approved or exempted by the IRB-SBS board. If written consent is required, all participants will be consented by signing a copy of the consent form unless this requirement is waived by the board.

7. That the IRB-SBS office will be notified within 30 days of a change in the Principal Investigator for the study.
8. That the IRB-SBS office will be notified when the active study is complete.
9. The SBS Review Board reserves the right to suspend and/or terminate this study at any time if, in its opinion, (1) the risks of further research are prohibitive, or (2) the above agreement is breached.

Date this Protocol Approval Certificate was generated: 2022-02-25

Appendix E

Approval from Research Site

From: *********@********day.org

Subject: Permission to Conduct Research on GDS campus

Date: January 11, 2022 at 8:31 AM

To: wac5T@virginia.edu

Wes,

I gave you full permission to conduct research on the campus for your dissertation. We are excited to have you back and hope to learn something valuable from your work.

R. Teague

The Anderson Davis Warlick Head of School

Appendix F

Recruitment Letter

February 1, 2022

Dear <First Name> <Last Name>

I am writing to ask for your participation in a case study about how teachers use the subject matter knowledge learned during their college mathematics courses to teach their classes. The purpose of this study is to determine which college-level mathematics courses have the strongest influence on how mathematics teachers perform their day-to-day tasks of planning for class and teaching a class. This work is being done to provide insight into what courses administrators need to see when reviewing resumes of math teaching applicants. This is part of my doctoral studies and will be included in my doctoral dissertation.

Your name was provided by the Head of School as a potential participant in this survey. If you choose to participate in the study, your responses are completely confidential and will only be reported as part of group summaries.

I ask if you are willing to participate, please complete the attached informed consent form and return to me either by emailing it to wac5t@virginia.edu or giving it to me upon my arrive at the school. If you choose not to participate, simply reply to me directly stating that you wish to opt out of the study. Please know that this information will not be shared with any administrators at the school and will have no effect on your status within the school.

If you have any questions or comments about this survey, please feel free to contact me at wac5t@virginia.edu or the address below my signature.

Thank you very much for assisting me with the important study.

Sincerely,

Wesley Cox
School of Education and Human Development
University of Virginia
wac5t@virginia.edu
74 Acorn Ct. | Zion Crossroads, VA 22942

IRB protocol # 4854
To obtain more information about the study, ask questions about the research procedures, express concerns about your participation, please contact: Tonya R. Moon, Ph.D. Chair, Institutional Review Board for the Social and Behavioral Sciences. Telephone: (434) 924-5999, Email: irbsbshelp@virginia.edu, Website: www.virginia.edu/vpr/irb/sbs

Appendix G

Informed Consent

Informed Consent Agreement

Please read this consent agreement carefully before you decide to participate in the study.

Purpose of the research study: The purpose of the study is to understand which mathematics courses teachers take during their collegiate careers substantially influence their teaching of high school mathematics classes and how those courses affect their students' algebraic reasoning skills. The information gained from this study will provide direction for administrators looking to hire new teachers at a small private, independent school in North Carolina.

What you will do in the study: To understand what college-level mathematics courses mathematics teachers reflect upon while planning and teaching their classes, I plan to conduct two interviews (one prior to teaching to the class and one following the class) and an observation of the class taught. The information gathered will center on the specific content courses used to help you as the teacher plan and teach your mathematics class. The interviews and classes will not be recorded in any way. Only handwritten notes will be collected. The only information that will be collected will consist of a copy of the lesson plans for the class that is to be observed. You will be asked demographic information such as the number of years teaching each course and what college-level mathematics courses they took during their collegiate careers. No identifying information will be collected from you. During the interviews, you may feel free to skip any question you do not feel comfortable answering. All answers provided during the interviews will be considered confidential.

Time required: The study will require about 1 hour of your time. This will consist of two 30-minute interviews and 1 classroom observation.

Risks: There are no anticipated risks in this study.

Benefits: There are no direct benefits to you for participating in this research study. The study may help us understand what college-level mathematics courses administrators need to look for on applicants resumes to better determine their ability to meet the needs of their students.

Confidentiality: The data gathered will be stored in the online storage app, UVA Box. Only I will have access to the data and the identities belonging to each response. Upon completion of the capstone, I will destroy the information. I will share the results with the head of school upon completion of the study, but I will share no identifying markers with him.
The information that you give in the study will be handled confidentially. Your name and other information that could be used to identify you will not be collected or linked to the data. To aid in this each participant will be assigned a pseudonym to remove any possible identifiers within the data other than the course taught. Because of the nature of the data and the limited number of mathematics teachers, it may be possible to deduce your identity; however, there will be no attempt to do so, and your data will be reported in a way that will not identify you.

Voluntary participation: Your participation in the study is completely voluntary. Your decision to participate will have no effect your position at the school.

Right to withdraw from the study: You have the right to withdraw from the study at any time without penalty.

How to withdraw from the study:
If you want to withdraw from the study, please tell the researcher as soon as possible. There is no penalty for withdrawing or withdrawing will not affect your experience as an employee. Your answers and information will be removed from the data set upon your withdrawal.

Payment: You will receive no payment for participating in the study.

Using data beyond this study: The data from this study will not be used for any other study. The data will be stored in a secure online server for one year and then destroyed.

If you have questions about the study, contact:
Wesley Cox (Researcher/Student)
Department of Curriculum, Instruction, and Special Education
University of Virginia
PO Box 400763 Charlottesville, VA 22903
Telephone: (434) 996-4700
wac5t@virginia.edu

Kyong Mi Choi (Faculty Advisor)
Department of Curriculum, Instruction, and Special Education
University of Virginia
PO Box 400763 Charlottesville, VA 22903
Telephone: (434) 924-5909
kc9dx@virginia.edu

To obtain more information about the study, ask questions about the research procedures, express concerns about your participation, or report illness, injury or other problems, please contact:
Tonya R. Moon, Ph.D.
Chair, Institutional Review Board for the Social and Behavioral Sciences
One Morton Dr Suite 500
University of Virginia, P.O. Box 800392
Charlottesville, VA 22908-0392
Telephone: (434) 924-5999
Email: irbsbshelp@virginia.edu
Website: https://research.virginia.edu/irb-sbs
Website for Research Participants: https://research.virginia.edu/research-participants
UVA IRB-SBS # 4854

Agreement:
I agree to participate in the research study described above.

Print Name: ____ Date: _____________

Signature: ____

You will receive a copy of this form for your records.