ESSAYS ON WAGE INEQUALITY: OCCUPATIONS, AUTOMATION, AND TRADE

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ABSTRACT

My dissertation consists of three chapters that examine how occupations intersect with automation and international trade to shape wage inequality: The first two chapters examine how effects of automation on spatial wage inequality differ across occupations. The third chapter investigates how occupations matter in determining winners and losers from international trade.

In Chapter 1, I first show a novel stylized fact: Since 1980, spatial wage inequality in the US has increased for non-routine occupations, while it has decreased for routine occupations. Since non-routine occupations are at the extremes of the skill distribution, while routine occupations are in the middle, existing models that focus on skill groups cannot explain the novel stylized fact I find. Second, I document that, since 1980, the price of machines in the US has experienced a substantial decline. Finally, I provide background on place-based policies in the US, which aim to reduce spatial wage inequality.

In Chapter 2, I develop a quantitative spatial model with three forces: spatial differences in total factor productivity; the supply of machines being more elastic than that of labor; and machines substituting for routine occupations while complementing non-routine occupations. I calibrate my model to the US data in 1980 and show that automation alone explains about 30 percent of the observed changes in spatial wage

inequality. Then, I show that place-based policies entail a trade-off between wage differences across places (i.e., spatial wage inequality) and wage differences within places. By raising total factor productivity in lagging places, place-based policies reduce wage differences across places. However, while place-based policies raise wages for all occupations in treated places, wages for non-routine occupations increase more. Non-neutral effects across occupations arise from the difference in supply elasticity between machines and labor.

In Chapter 3, I study how occupations matter in determining winners and losers from international trade. A growing body of literature points out that a worker's occupation plays a crucial role in determining winners and losers from international trade: switching occupations induced by international trade is costly, as occupation-specific human capital accumulation plays a critical role in wage determination. In my dissertation, I propose an additional channel through which occupation plays a role in deciding winners and losers from trade: the comparative advantage of different skill types across occupations. In my model, workers are not perfectly mobile across occupations, as different workers have comparative advantages in different occupations. Due to this imperfect mobility, workers with a comparative advantage in the occupation whose price falls lose from trade, while those with a comparative advantage in the occupation whose price rises benefit from trade.

JEL Classification: F16, J24, J31, O33, R12

Keywords: Occupation, Wage inequality, Automation, Place-based policy, Trade, Comparative advantage

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Chapter 1

Spatial Wage Inequality: Stylized Facts

1.1 Introduction

I study how automation affects spatial wage inequality. Automation refers to the replacement of jobs previously performed by human labor with machines as machines become more productive and their prices decrease. Spatial wage inequality refers to the differences in wages across places within a country. Understanding technology is becoming increasingly important as its rate of advancement continues to accelerate, even more so with the recent advent of artificial intelligence. Spatial wage inequality is a pressing issue for policymakers who aim to promote economic development in lagging places through place-based policies.

The literature shows that wages across places have diverged for college workers while converging for non-college workers in the US since 1980 (Ganong and Shoag 2017; Giannone 2022). In my dissertation, I show a novel stylized fact: spatial wage inequality in the US has decreased for routine occupations while increasing for non-routine occupations since 1980. Routine occupations refer to occupations whose job can be accomplished by machines following explicitly specified instructions, whereas non-routine occupations involve jobs where the instructions are not sufficiently well understood to be executed by machines (Autor, Levy, and Murnane 2003). Occupations and skill groups are distinct: the former is categorized based on whether the jobs performed can be replaced by machines (routine vs. non-routine), whereas the latter is categorized based on education level. Examples of routine occupations include clerical, sales, and repetitive production occupations (i.e., middle-skilled), whereas examples of non-routine occupations encompass managers and technicians (i.e., highest-skilled), as well as protective and cleaning service occupations (i.e., lowest-skilled). The stylized fact I find is that spatial wage inequality has increased for non-routine occupations—whether highest-skilled or lowest-skilled—while spatial wage inequality has decreased for routine occupations, which are in the middle of the skill distribution. In other words, trends in spatial wage inequality since 1980 differ across occupations (i.e., routine vs. non-routine), not skill types. Therefore, existing models that feature skill groups (Ganong and Shoag 2017; Giannone 2022) cannot explain the different trends in spatial wage inequality across occupations, since non-routine occupations are at the extremes of the skill distribution, while routine occupations are in the middle of the skill distribution.

In my model, automation, driven by the fall in the price of machines, increases spatial wage inequality for non-routine occupations and decreases it for routine occupations through three forces: 1) total factor productivity (TFP) varies across places, 2) the supply of machines is more elastic than that of labor, and 3) machines substitute for routine occupations while complementing non-routine occupations. Suppose workers have idiosyncratic preferences for places, leading to an upward-sloping labor supply curve in each place for every occupation. Consequently, wages for each occupation are higher in places with higher TFP. On the other hand, suppose the supply of machines is perfectly elastic, meaning that the supply of machines is more elastic than that of labor. While places with higher TFP have a larger demand for labor and machines, machines relative to labor in equilibrium are larger in places with higher TFP since the supply of machines is more elastic. That is, more productive places are more machine-intensive. Due to the difference in machine intensity across places, as the price of machines falls, more machines are adopted in the more productive (hence, more machine-intensive) places than in less productive ones, which is consistent with the fact documented by Beaudry, Doms, and Lewis (2010). As machines are adopted disproportionately in the more productive places, and machines are substitutable for routine labor, the demand for routine labor in the more productive places decreases relative to less productive ones. Hence, the initial wage difference for routine workers across places shrinks. When the price of machines falls, the opposite pattern holds for non-routine workers, who are complemented by machines. As more machines are adopted in the more productive places than in less productive ones, the demand for non-routine labor in the more productive places increases relative to less productive ones. This widens the initial wage difference for non-routine workers across places.

I build a quantitative spatial model that incorporates the above mechanism for two purposes. The first purpose is to quantify the effect of automation on spatial wage inequality. I conduct a counterfactual analysis where the price of machines falls as it does in the data, while all other parameters of my model remain the same. According to my model, automation alone explains 36 percent of the observed decrease in spatial wage inequality for routine workers. For non-routine occupations which are at the extremes of the skill distribution, I divide them into two groups: non-routine skilled and non-routine unskilled. Automation alone explains 30 percent of the observed increase in spatial wage inequality for non-routine skilled workers and 31 percent for non-routine unskilled workers.

While the first purpose of the quantitative model is to explain past changes in spatial wage inequality, the second is to derive implications for a place-based policy from my model. Consider a place-based policy that increases TFP in initially low-TFP places, reducing the TFP gap between initially low- and high-TFP places. Examples of such place-based policies include the recent Inflation Reduction Act, the Appalachian Regional Commission, and the Tennessee Valley Authority in the US, as well as the Structural Funds in the European Union, all of which aim to boost the productivity of lagging places through infrastructure investment. As TFP increases in treated places, wages for each occupation rise there, and spatial wage inequality for each occupation falls, which is the objective of place-based policies. However, according to my model, place-based policies entail an unintended consequence: While place-based policies raise wages for all occupations in treated places, wages for non-routine workers increase more. In my model, Hicks-neutral technical progress (i.e., increasing TFP) in treated places has non-neutral effects across occupations in equilibrium due to the difference in supply elasticity between machines and labor. As TFP rises in treated places, factors of production move there. However, the supply of machines rises relative to the supply of workers in treated places because the supply of machines is more elastic than that of labor. Since machines are complementary to non-routine labor but substitute for routine labor, this raises the demand for non-routine labor relative to routine labor, resulting in higher wages for non-routine labor relative to routine labor in treated places. Hence, my model implies that place-based policies entail a trade-off between wage differences across places and wage differences within places, *even* when place-based policies feature Hicks-neutral technical progress in lagging places, due to the difference in supply elasticity between machines and labor.

Using the quantitative model, I conduct a policy experiment where I increase the

TFP of each place within the first tercile of the distribution to match the level of the 75th percentile. The results show that spatial wage inequality for each occupation decreases by about 20 percent. On the other hand, wages for non-routine skilled workers relative to routine workers in treated places increase, ranging from 2 percent (in the place whose initial TFP is equal to the first tercile) to 3.7 percent (in the place that originally has the smallest TFP). Similarly, wages for non-routine unskilled workers relative to routine workers in treated places rise, ranging from 1.1 percent to 2.2 percent.

My dissertation contributes to three strands of literature. The first pertains to spatial wage divergence and convergence. Since 1980, wages have diverged across places in the US: Berry and Glaeser (2005) show that wages in initially high-wage places have grown faster than those in initially low-wage places (i.e., β-divergence), and Gaubert et al. (2021) show that the dispersion of wages across places has since risen (i.e., σ-divergence). The literature points out that the divergence in skill-specific returns to moving to high-wage places deters low-skill migration to high-wage places, resulting in spatial wage divergence. Rising housing costs in high-wage places discourage the migration of unskilled workers to high-wage places, leading to spatial wage divergence (Ganong and Shoag 2017). Local skill-biased agglomeration, interacting with national skill-biased technical change, causes spatial wage convergence to stop only for skilled workers (Giannone 2022). My dissertation is similar to Gaubert et al. (2021) since I focus on spatial wage inequality (i.e., the dispersion of wages across places). While Gaubert et al. (2021) show that spatial wage inequality—measured as the dispersion across places of *mean wages within a place*—has risen since 1980, I show the new fact that the change in spatial wage inequality differs by occupation: Spatial wage inequality has increased for non-routine occupations, while it has decreased for routine occupations. My model highlights the importance of occupations, as which occupations technology replaces or complements is central to my mechanism. Existing models that feature skill groups (Ganong and Shoag 2017; Giannone 2022) cannot explain the differential trends in spatial wage inequality across occupations since non-routine occupations are at the extremes of the skill distribution, while routine occupations are in the middle of the skill distribution.

Second, my dissertation is related to the growing literature on spatial sorting and inequality within places. There are two major sources of spatial sorting: production and amenities. Regarding production channels, national skill-biased technical change and local skill-biased agglomeration lead to a larger skill wage premium in places with more skilled workers, causing skilled workers to migrate to these places relative to unskilled workers (Baum-Snow, Freedman, and Pavan 2018; Giannone 2022). Eckert (2019) proposes another production channel: with reduced communication costs, places with a comparative advantage in skilled tradable services increase their specialization in this sector, which leads to an increase in local demand for skilled workers and raises the local skill wage premium. In terms of the amenity channel, skilled workers value amenities more, and places with more skilled workers create more amenities, which attracts even more skilled workers and contributes to larger local welfare inequality (Diamond 2016; Shapiro 2006). My dissertation contributes to the production channels and highlights spatial sorting by occupation. In my model, differences in machine intensity across places, rather than local skill-biased agglomeration, drive spatial sorting by occupation. More productive places are more machine-intensive since the supply of machines is more elastic than that of labor: while places with higher TFP have a larger demand for labor and machines, machines relative to labor in equilibrium are larger in places with higher TFP. As machines complement non-routine workers, more productive places have a larger local wage premium for non-routine workers, attracting non-routine workers (who are at the extremes of the skill distribution) relative to routine workers (who are in the middle of the skill distribution). This pattern of spatial sorting by occupation aligns with the fact that productive places disproportionately attract both the most skilled and least skilled workers (Eeckhout, Pinheiro, and Schmidheiny 2014). This cannot be explained by the production channels proposed in previous literature that study spatial sorting by skill.

Lastly, my contribution pertains to place-based policies. The literature on place-based policies has focused on two major questions: the economic rationale for these policies and their causal effects on outcomes of interest. One rationale for place-based policies is to promote spatial wage equity (Gaubert, Kline, and Yagan 2021). Another is to address localized market failures, such as those arising from agglomeration and congestion (Fajgelbaum and Gaubert 2020), or the underprovision of productive public goods (Bartik 2020). The literature shows that place-based policies that increase productivity in lagging places through infrastructure investments reduce spatial wage inequality (Becker, Egger, and von Ehrlich 2010; Jaworski and Kitchens 2019; Kline and Moretti 2014). However, the evidence on the effectiveness of place-based policies offering business subsidies, such as enterprise zones, is mixed (Neumark and Simpson 2015). My dissertation is similar to Gaubert, Kline, and Yagan (2021) since I focus on spatial wage inequality, rather than inefficiencies stemming from localized market failures, as the rationale for place-based policies. In my model, place-based policies reduce spatial wage inequality by raising TFP in lagging places, consistent with the empirical findings of Becker, Egger, and von Ehrlich (2010), Jaworski and Kitchens (2019), and Kline and Moretti (2014).

However, my dissertation highlights an unintended consequence of place-based policies: while place-based policies raise wages for all occupations in treated places, wages for non-routine workers increase more. Thus, my contribution is to suggest that place-based policies entail a trade-off between wage differences across places and wage differences within places.

The remainder of this chapter is organized as follows. Section 1.2 describes the data and stylized facts. Section 1.3 provides background on place-based policies in the US. Section 1.4 concludes by discussing what model is needed to explain the facts on spatial wage inequality and to derive implications for place-based policies.

1.2 Data and Facts

1.2.1 Trend in Spatial Wage Inequality

The purpose of this subsection is to examine how spatial wage inequality evolved between 1980 and 2019. In particular, I investigate how spatial wage inequality evolved differently by occupation. This necessitates tasks related to measuring places, occupations, and local real wages, which are explained below. More details can be found in Appendix A.1.

I use the Decennial Census data from 1980 to 2000 and the American Community Survey data from 2010 and 2019, obtained from the Integrated Public Use Microdata Series (Ruggles et al. 2023). I define places based on the 1990 commuting zones (CZs).[1] I categorize occupations into three groups: routine, non-routine skilled, and

[1]The 1980 Census data report place of residence at the county group level, while data from the 1990 and 2000 Censuses and the American Community Survey data provide place of residence at the public use microdata area (PUMA) level, with varying definitions of PUMAs over the years. I use

non-routine unskilled, based on Acemoglu and Autor (2011). The routine occupation means that its job can be accomplished by machines following explicitly specified instructions. I define sales; office and administrative support; production, craft, and repair; and operator, fabricator, and laborer as the routine occupation. The remaining occupations are non-routine, but these non-routine occupations are heterogeneous in skill levels. I divide non-routine occupations into two groups based on skill levels. The non-routine skilled occupation comprises managers, professionals, and technicians. The non-routine unskilled occupation consists of protective services; food preparation and cleaning services; and personal care. In terms of the skill distribution, the routine occupation is in the middle, the non-routine skilled at the top, and the non-routine unskilled at the bottom of the skill distribution.[2]

To measure local real wages, I take the following steps. First, I regress log hourly wages on demographics (age, gender, race, and ethnicity) and fixed effects for commuting zone-occupation combinations for each year. I use the commuting zone j-occupation ℓ fixed effect in year t as the nominal wage for occupation ℓ in commuting zone j in year t. Second, I measure housing prices by regressing gross monthly rents on commuting zone fixed effects for each year, controlling for the year the housing was built, as well as the number of bedrooms and units.[3] I use the commuting zone j fixed

crosswalks provided by Autor and Dorn (2013) to ensure consistency in defining commuting zones over time, totaling 722 commuting zones, excluding those in Alaska and Hawaii.

[2] Polarization literature (e.g., Acemoglu and Autor (2011)) shows that non-routine occupations are at the extremes of the skill distribution, while the routine occupation is in the middle of the skill distribution. The non-routine skilled occupation that I define here is called the non-routine cognitive occupation in polarization literature, and it requires problem-solving, intuition, persuasion, and creativity. The non-routine unskilled occupation that I define here corresponds to the non-routine manual occupation in polarization literature, and it requires situational adaptability, visual and language recognition, and in-person interactions. While the occupational classification I use is based on Acemoglu and Autor (2011), Autor, Levy, and Murnane (2003) pioneered in analyzing what workers do (e.g., routine, non-routine cognitive, or non-routine manual tasks) in their jobs and measuring the task input of occupations using the Dictionary of Occupational Titles.

[3] As pointed out by Moretti (2013), rental costs better reflect the user cost of housing compared to home prices. Home prices reflect both the user cost and expectations of future appreciation, as

effect in year t as the housing price for commuting zone j in year t. Third, assuming non-housing prices are the same across commuting zones in a given year, I measure the price levels of commuting zones for each year by exponentiating non-housing and housing prices using the expenditure shares estimated by Davis and Ortalo-Magne (2011).[4] Then, I normalize local price levels so that the average across commuting zones equals 1 in 1980. Finally, the real wage for occupation ℓ in commuting zone j in year t is the nominal wage for occupation ℓ in commuting zone j in year t (as determined in the first step) divided by the price level in commuting zone j in year t (as determined in the last step).

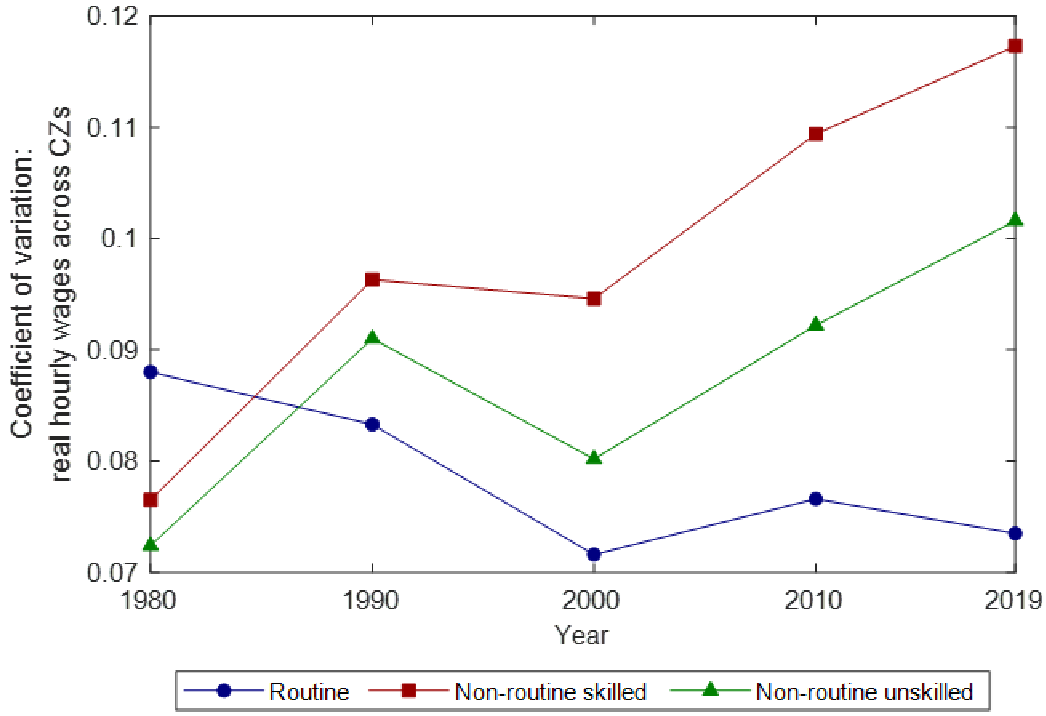
I measure spatial wage inequality for each occupation in a given year as the coefficient of variation of real wages across commuting zones for that occupation in that year. Figure 1.1 shows how spatial wage inequality evolved from 1980 to 2019 for each occupation. The blue line indicates the coefficient of variation of real wages across commuting zones for the routine occupation. It decreased by 16 percent (from 0.0880 to 0.0735) between 1980 and 2019. The red line corresponds to the non-routine skilled occupation, where the coefficient of variation increased by 53 percent (from 0.0765 to 0.1173) between 1980 and 2019. The green line corresponds to the non-routine unskilled occupation, which saw a 40 percent increase in spatial wage inequality (from 0.0724 to 0.1016) over the same period.

Figure 1.1 shows the novel stylized fact I find in my dissertation: Spatial wage inequality has increased for the non-routine occupations, while it has decreased for

houses are assets.

[4] Davis and Ortalo-Magne (2011) estimate the expenditure share on housing to be 25 percent. The assumption that non-housing prices are the same across commuting zones is based on the fact that most variation in local prices stems from housing. Additionally, data on local non-housing prices are scarce; the Bureau of Labor Statistics releases local CPI data only for 23 metropolitan statistical areas. Diamond and Moretti (2021) use non-publicly available data to measure local non-housing prices. However, the information on local non-housing prices is limited to 443 commuting zones, whereas there are 722 commuting zones in total.

Figure 1.1: Trend in Spatial Wage Inequality



Note: This figure shows how spatial wage inequality (measured as the coefficient of variation of real hourly wages across commuting zones) evolved from 1980 to 2019 for each occupation.

the routine occupation since 1980. While Gaubert et al. (2021) show that spatial wage inequality—measured as the dispersion across places of *mean wages within a place*—has risen since 1980, I show that the change in spatial wage inequality differs across occupations. The rise in spatial wage inequality shown by Gaubert et al. (2021) stems from non-routine workers. Figure 1.1 is also different from what Giannone (2022) found. Giannone (2022) shows that for skilled workers (measured as college workers), wages in high-wage places in 1980 have grown faster than in low-wage places, whereas for unskilled workers (measured as non-college workers), wages in high-wage places in 1980 have grown more slowly than in low-wage places.[5]

[5]In other words, there was β-divergence for college workers, and β-convergence for non-college workers. In terms of spatial wage inequality, measured as the dispersion of wages across places, I show that there was σ-divergence (an increase in spatial wage inequality) for non-routine workers

In contrast, I show that the trends in spatial wage inequality (measured as the dispersion of wages across places) since 1980 differ across occupations. Spatial wage inequality has increased for the two non-routine occupations, regardless of skill type. The non-routine skilled occupation is characterized by the highest skill, while the non-routine unskilled occupation is characterized by the lowest skill. On the other hand, the routine occupation, for which spatial wage inequality has decreased, lies in the middle of the skill distribution. In summary, the novel stylized fact I find is that trends in spatial wage inequality since 1980 differ across occupations (i.e., routine vs. non-routine), not skill types.

1.2.2 Falling Price of Machines

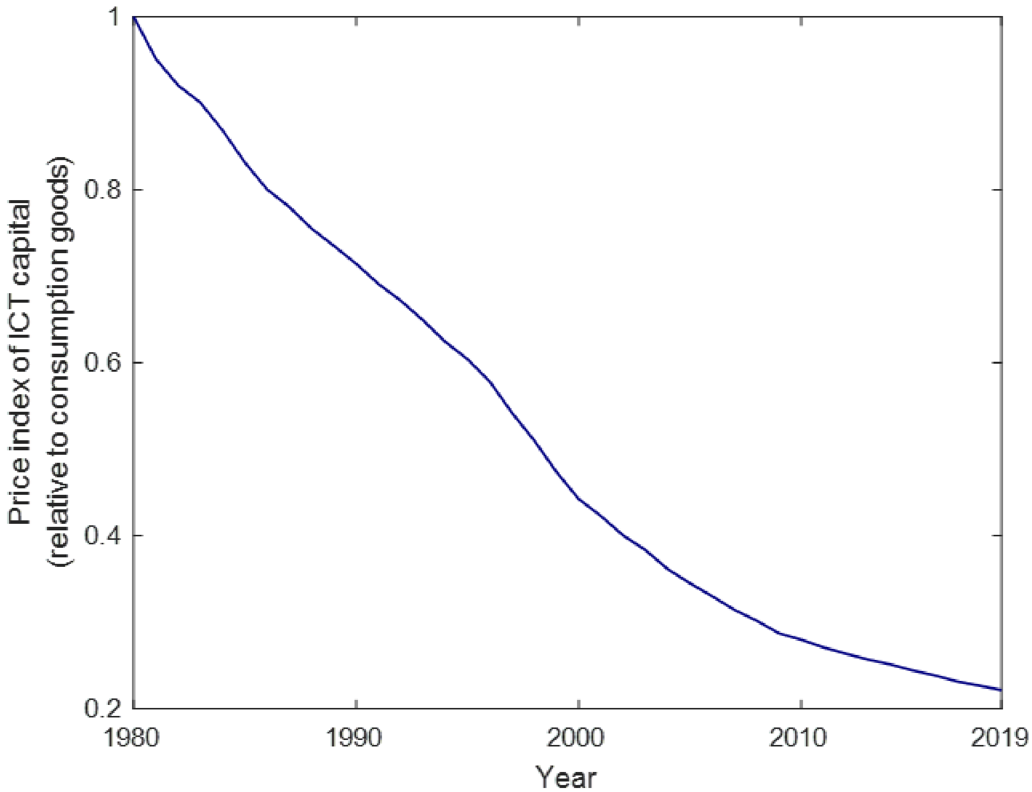
In this subsection, I show the decline in the price of machines relative to consumption goods between 1980 and 2019. I define machines as ICT (information and communication technology) capital, and I measure the price of machines using the data from the National Income and Product Accounts, available at the Bureau of Economic Analysis (BEA). More details can be found in Appendix A.2. I measure the price of consumption goods using the BEA data on non-durable goods and non-housing services. I then normalize the price of machines relative to consumption goods to 1 in 1980.

Figure 1.2 shows the trend in the price of machines relative to consumption goods between 1980 and 2019. Consistent with the literature, the price of machines relative to consumption goods has experienced a substantial decline since 1980.[6]

and σ-convergence (a decrease in spatial wage inequality) for routine workers.

[6]Many papers showed that the decline in the price of capital or equipment relative to consumption goods began to accelerate in 1980 (He and Z. Liu 2008; Karabarbounis and Neiman 2014; vom Lehn 2020). However, Eden and Gaggl (2018) showed that the price of ICT capital has declined substantially relative to consumption goods since 1980, whereas the price of non-ICT capital relative

Figure 1.2: Falling Price of Machines



Note: This figure shows the decline in the price of machines (measured as ICT capital) relative to consumption goods between 1980 and 2019. The price of machines relative to consumption goods is normalized to 1 in 1980.

1.3 Background: Place-Based Policies in the US

In many countries, governments implement place-based policies to reduce spatial wage inequality. In this subsection, I provide background on place-based policies in the US. Broadly, there are two types. The first involves providing subsidies to businesses that relocate to lagging places, typically characterized by low income, low population, and high unemployment. The most well-known example is enterprise zones. The second

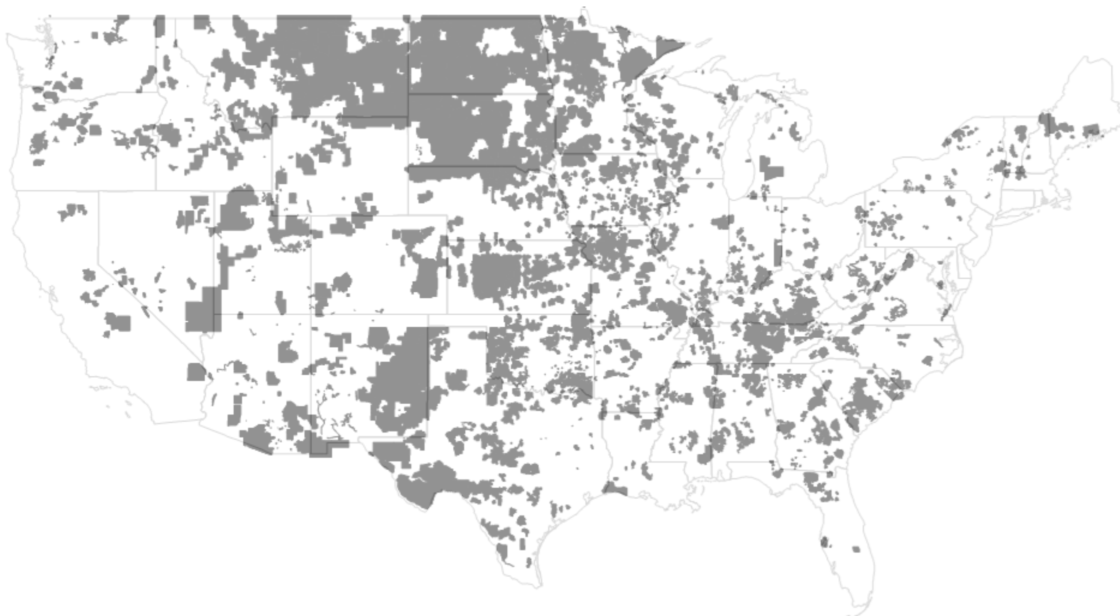
to consumption goods has remained stable.

type involves public infrastructure investments in lagging places. Historical examples include the Tennessee Valley Authority in the 1930s and the Appalachian Regional Commission in the 1960s.

The first type—providing business subsidies—has been widely used in the US. Spending on business subsidies as a percentage of business value-added tripled from 1990 to 2015 (Bartik 2020). Despite this effort, evidence on the causal effects of such policies in reducing spatial wage inequality is mixed, as surveyed by Neumark and Simpson (2015). A few policies, however, were successful in reducing spatial wage inequality—for example, the Tennessee Valley Authority and the Appalachian Regional Commission, which involved infrastructure investments in electricity and roads in lagging places (Jaworski and Kitchens 2019; Kline and Moretti 2014). Neumark and Simpson (2015) suggest that whether a policy enhances the productivity of lagging places is a key factor in its effectiveness.

Recently, the US government has been planning the largest infrastructure investment in lagging places since the Tennessee Valley Authority. The Inflation Reduction Act, signed by President Biden in 2022, includes the Empowering Rural America (ERA) initiative. The ERA allocates 9.7 billion dollars for infrastructure investments in electricity and broadband access. Figure 1.3, obtained from the Department of Agriculture, shows the places eligible for the ERA. The shaded places in the figure indicate eligible places. Notably, the covered places are often concentrated in specific regions: the central US (i.e., the West North Central and West South Central Census Divisions) and the South (i.e., the East South Central and South Atlantic Census Divisions).

Figure 1.3: Eligible Places for the Empowering Rural America



Note: Shaded places in this figure represent the eligible places covered by the Empowering Rural America, which makes infrastructure investments in electricity and broadband access in lagging places.

1.4 Conclusion

Focusing on skill types, the literature shows that spatial wage inequality has diverged for skilled workers and converged for unskilled workers since 1980. In this chapter, I present a novel stylized fact: spatial wage inequality has increased for the non-routine occupations, while it has decreased for the routine occupation since 1980. In particular, spatial wage inequality has increased for the two non-routine occupations, regardless of skill type. The non-routine skilled occupation is characterized by the highest skill, while the non-routine unskilled occupation is characterized by the lowest skill. On the other hand, the routine occupation, for which spatial wage inequality has decreased, is in the middle of the skill distribution. Accordingly, existing models that feature skill groups cannot explain the novel stylized fact I document.

Accompanying the different trends in spatial wage inequality across occupations since 1980 is the continual decrease in the price of machines since 1980. Recently, the US government has been planning the ERA—the largest place-based policy since 1930, which involves infrastructure investments in electricity and broadband access in lagging places. I need a model that serves two purposes. The first purpose is to explain the facts on spatial wage inequality: that is, whether the decrease in the price of machines leads to different trends in spatial wage inequality across occupations. The second purpose is to derive implications for a place-based policy such as the ERA: that is, whether the policy will reduce spatial wage inequality as intended. I discuss my model and conduct quantitative analyses in Chapter 2.

Chapter 2

Automation, Spatial Wage Inequality, and Place-Based Policy

2.1 Model

In my model, automation—driven by a decline in the price of machines—decreases spatial wage inequality for routine occupations and increases it for non-routine occupations through three forces: 1) total factor productivity (TFP) varies across places, 2) the supply of machines is more elastic than that of labor, and 3) machines substitute for routine occupations while complementing non-routine occupations. Suppose workers have idiosyncratic preferences for places, resulting in an upward-sloping labor supply curve in each place for every occupation. Accordingly, wages for each occupation are higher in places with higher TFP, leading to spatial wage inequality for each occupation. On the other hand, suppose the supply of machines is perfectly elastic, meaning that the supply of machines is more elastic than that of labor. While places with higher TFP have a larger demand for labor and machines, machines relative to labor in equilibrium are larger in places with higher TFP since the supply of machines is more elastic. That is, more productive places are more machine-intensive. Due to the difference in machine intensity across places, as the price of machines falls, more machines are adopted in the more productive (hence, more machine-intensive) places than in less productive ones. As machines are adopted disproportionately in the more productive places, and machines are substitutable for routine labor, the demand for routine labor in the more productive places decreases relative to less productive ones. Hence, the initial spatial wage inequality for routine occupations shrinks. On the other hand, when the price of machines falls, the opposite pattern holds for non-routine occupations, which are complemented by machines. As more machines are adopted in the more productive places than in less productive ones, the demand for non-routine labor in the more productive places increases relative to less productive ones. This widens the initial spatial wage inequality for non-routine occupations.

The three forces outlined above have implications for place-based policies. Consider a place-based policy that increases TFP in initially low-TFP places, reducing the TFP gap between initially low- and high-TFP places. As TFP increases in treated places, the marginal product of labor for all occupations rises. Hence, wages for each occupation rise there, and spatial wage inequality for each occupation falls, which is the objective of place-based policies. However, according to my model, place-based policies entail an unintended consequence: While place-based policies raise wages for all occupations in treated places, wages for non-routine occupations increase more. In my model, Hicks-neutral technical progress (i.e., increasing TFP) in treated places has non-neutral effects across occupations in equilibrium due to the difference in supply elasticity between machines and labor. As TFP rises in treated places, factors of production move there. However, the supply of machines rises relative to the supply of workers in treated places because the supply of machines is more elastic than that of labor. Since machines are complementary to non-routine labor but substitute for routine labor, this raises the demand for non-routine labor relative to routine labor, resulting in higher wages for non-routine labor relative to routine labor in treated places. Hence, my model implies that place-based policies entail a trade-off between wage differences across places and wage differences within places, even when place-based policies feature Hicks-neutral technical progress in lagging places, which results from the difference in supply elasticity between machines and labor.

In Subsection 2.1.1, I develop a simplified model to highlight the mechanism through which automation, driven by the fall in the price of machines, increases spatial wage inequality for non-routine workers while decreasing it for routine workers. Then, in Subsection 2.1.2, I develop a quantitative model to address two questions. First, if the price of machines declines as it does in the data, while all other parameters of my model remain the same, how much does automation explain the observed changes in spatial wage inequality? Second, I conduct a policy experiment to derive implications for place-based policies from my model. Based on the experiment, do place-based policies entail any unintended consequences in addition to reducing spatial wage inequality?

2.1.1 Simplified Model: Mechanism

Preferences. There are three occupations indexed by $\ell \in \{R, S, U\}$. R stands for the routine occupation, S for the non-routine skilled occupation, and U for the non-routine unskilled occupation. Within each occupation, there is a unit mass of workers. There are two places indexed by $j \in \{1, 2\}$. Workers consume a good and have idiosyncratic preferences for places. The utility function for worker ω who lives in j is $\mathcal{U}_j(\omega) = C_j \epsilon_j(\omega)$, where C denotes the consumption of the good and ϵ the preference shock. $\epsilon_j(\omega)$ is drawn independently for each (ω, j) from a Fréchet distribution with a finite variance parameter $\kappa > 1$.[1].

Labor supply. The good is freely traded across places and I choose it as the numeraire. Each worker in occupation ℓ supplies one unit of labor for wage $W_{j\ell}$ in place j. As shown in Appendix B.1, using the property of the Fréchet distribution, the labor supply of occupation ℓ in place j takes the following logit form:

$$\ell_j = \frac{W_{j\ell}^{\kappa}}{W_{1\ell}^{\kappa} + W_{2\ell}^{\kappa}},$$

where the spatial labor supply elasticity is equal to κ for each occupation. Each place faces an upward-sloping labor supply curve for each occupation, meaning higher wages must be offered to attract workers with lower tastes for that place.

Technology. In each place j, a perfectly competitive firm produces the good Y. There are four factors of production: routine labor R, machines M, non-routine skilled labor S, and non-routine unskilled labor U. The production function for Y_j consists of three nests:

$$\begin{aligned} \Omega_{jR} &= \left[\alpha_R R_j^{\frac{\sigma_R - 1}{\sigma_R}} + (1-\alpha_R) M_j^{\frac{\sigma_R - 1}{\sigma_R}}\right]^{\frac{\sigma_R}{\sigma_R - 1}} \text{ where } \sigma_R > 1, \\ \Omega_{jS} &= \left[\alpha_S S_j^{\frac{\sigma_S - 1}{\sigma_S}} + (1-\alpha_S) \Omega_{jR}^{\frac{\sigma_S - 1}{\sigma_S}}\right]^{\frac{\sigma_S}{\sigma_S - 1}} \text{ where } \sigma_S < 1, \\ Y_j &= T_j \, U_j^{\rho} \, \Omega_{jS}^{1-\rho}, \end{aligned} \tag{2.1}$$

where $(\alpha_R, \alpha_S, \rho) \in (0,1)^3$. Ω_{jR} is the routine task input in place j, consisting of routine labor and machines, such that routine labor and machines are substitutes (i.e., $\sigma_R > 1$). Ω_{jS} indicates intermediates in place j, consisting of non-routine

[1]The Fréchet distribution I use is $Pr(\epsilon_j \leq \epsilon) = exp(-\epsilon^{-\kappa})$. κ governs the dispersion of idiosyncratic preferences for places, with lower values of κ indicating greater dispersion. $\kappa > 1$ (or $\frac{1}{\kappa} < 1$) places an upper bound on the degree of dispersion of idiosyncratic preferences across places, ensuring that expected utility across places is finite.

skilled labor and the above routine task input, such that non-routine skilled labor and routine task input are complements (i.e., $\sigma_S < 1$). Y_j is a Cobb-Douglas function of non-routine unskilled labor and the intermediates described above, such that non-routine unskilled labor and intermediates are complementary.[2] Finally, T_j refers to TFP in place j. TFP is exogenous and varies across places.[3] The takeaways are that machines substitute for the routine occupation while complementing the non-routine occupations, and there are differences in TFP across places.

Expenditure shares. Machines are supplied perfectly elastically at an exogenous price, P_M, in the international market.[4] Let χ_{jR} and χ_{jS} be the marginal cost of routine task input (i.e., Ω_{jR}) and that of intermediates (i.e., Ω_{jS}) in place j, respectively:

$$\chi_{jR} \equiv \left[\alpha_R^{\sigma_R} W_{jR}^{1-\sigma_R} + (1-\alpha_R)^{\sigma_R} P_M^{1-\sigma_R}\right]^{\frac{1}{1-\sigma_R}},$$

$$\chi_{jS} \equiv \left[\alpha_S^{\sigma_S} W_{jS}^{1-\sigma_S} + (1-\alpha_S)^{\sigma_S} \chi_{jR}^{1-\sigma_S}\right]^{\frac{1}{1-\sigma_S}}.$$

[2]The nested CES production function in Equation 2.1 is widely used in the literature on skill-biased technical change and polarization in macroeconomics (Eden and Gaggl 2018; He and Z. Liu 2008; Karabarbounis and Neiman 2014; Krusell et al. 2000; Orak 2017; vom Lehn 2020). In my dissertation, I incorporate spatial elements into this type of model. The way of nesting R_j, M_j, S_j, and U_j within the production function in Equation 2.1 is based on vom Lehn (2020). I use a Cobb-Douglas production function to model the complementarity between non-routine unskilled labor and intermediates. This is similar to Eeckhout, Pinheiro, and Schmidheiny (2014), where the most skilled (corresponding to non-routine skilled labor in my model, as it is at the top of the skill distribution) and the least skilled (corresponding to non-routine unskilled labor in my model, as it is at the bottom of the skill distribution) are complementary in production.

[3]For example, exogenous differences in TFP across places can be microfounded by exogenous geographical differences in location within the context of costly inter-regional trade. Another explanation is historically determined institutions: some places, for example, have universities or better governance due to historical reasons. Finally, differences in TFP across places could also be a result of agglomeration externalities. I take these regional differences as given here.

[4]In other words, the price of machines is the same across places. This feature remains the same when I model the production of machines instead. In line with Giannone (2022), who considers *national* skill-biased technical change, suppose machines are produced by a *national* representative firm competitively using the production function $M = qI$ where M stands for machines, q for productivity, and I for investment in units of the consumption good. Due to perfect competition, the supply of machines is given by $P_M = \frac{1}{q}$, indicating that the price of machines is the same across places. Furthermore, an increase in productivity q decreases the price of machines, which, in turn, drives automation in my model.

Note that P_M is the same across places because machine supply is perfectly elastic. That is, machine supply is more elastic than labor supply, and this is the final main ingredient of my model. Now, based on cost minimization and the zero-profit condition, the expenditure shares on factors of production in place j are

$$\lambda_{jR} = \alpha_R^{\sigma_R} W_{jR}^{1-\sigma_R} \chi_{jR}^{\sigma_R-1} \cdot (1-\alpha_S)^{\sigma_S} \chi_{jR}^{1-\sigma_S} \chi_{jS}^{\sigma_S-1} \cdot (1-\rho),$$

$$\lambda_{jS} = \alpha_S^{\sigma_S} W_{jS}^{1-\sigma_S} \chi_{jS}^{\sigma_S-1} \cdot (1-\rho),$$

$$\lambda_{jU} = \rho,$$

$$\lambda_{jM} = (1-\alpha_R)^{\sigma_R} P_M^{1-\sigma_R} \chi_{jR}^{\sigma_R-1} \cdot (1-\alpha_S)^{\sigma_S} \chi_{jR}^{1-\sigma_S} \chi_{jS}^{\sigma_S-1} \cdot (1-\rho),$$

where λ_{jf} is the expenditure share on factor $f \in \{R, S, U, M\}$ in place j.

Factor demand. Having obtained the expenditure shares above, the conditional factor demands in place j are

$$W_{j\ell}\ell_j = \lambda_{j\ell} Y_j \text{ for } \ell \in \{R, S, U\}, \quad P_M M_j = \lambda_{jM} Y_j.$$

As machines are traded internationally, $\sum_j P_M M_j = \sum_j \lambda_{jM} Y_j$ holds. The zero-profit condition in place j is

$$T_j = \rho^{-\rho}(1-\rho)^{\rho-1} W_{jU}^{\rho} \chi_{jS}^{1-\rho}.$$

Spatial equilibrium. Recall that the good is the numeraire and the price of machines is exogenous. Given the parameters $\{\alpha_R, \alpha_S, \rho, \sigma_R, \sigma_S, P_M, \kappa, T_1, T_2\}$, the spatial equilibrium is $\{W_{jR}, W_{jS}, W_{jU}, R_j, S_j, U_j, M_j, Y_j, C_j\}_{j=1}^2$, such that workers and firms optimize given prices, and labor markets for each occupation clear.

Lemma 2.1 (Spatial Wage Inequality)**.** Suppose TFP is higher in place 1 than in place 2 ($T_1 > T_2$). Then, in the spatial equilibrium, wages of all occupations are higher in place 1 than in place 2 at any price of machines. That is, $W_{1\ell} > W_{2\ell}$, $\forall P_M$ for $\ell \in \{R, S, U\}$.

Proof. See Appendix B.3.

The intuition behind spatial wage inequality is straightforward. As TFP is higher in place 1, the marginal products of labor for all occupations are higher in place 1 relative to place 2. On the labor supply side, workers in each occupation have idiosyncratic preferences for places, resulting in an upward-sloping labor supply curve for place 1 relative to place 2. Thus, in the spatial equilibrium, wages are higher in place 1 than in place 2 for all occupations. The ratio $\frac{W_{1\ell}}{W_{2\ell}}$ (> 1) reflects spatial wage inequality for occupation ℓ.

Lemma 2.2 (Spatial Difference in Machine Intensity)**.** Let $T_1 > T_2$. Then, place 1 is more machine-intensive than place 2 at any price of machines. That is, the expenditure share on machines is higher in place 1 than in place 2: $\lambda_{1M} > \lambda_{2M}$, $\forall P_M$.

Proof. See Appendix B.4.

The key force behind the different machine intensity across places is that the price of machines is the same everywhere, which means that the supply of machines is more elastic than that of labor. While place 1, with higher TFP, has a larger demand for labor and machines, machines relative to labor in equilibrium are larger in place 1 since the supply of machines is more elastic. That is, place 1 is more machine-intensive, with a larger expenditure share on machines.

Proposition 2.1 (Effect of Automation on Spatial Wage Inequality)**.** Let $T_1 > T_2$. Then, automation—driven by the fall in the price of machines—decreases spatial wage inequality for the routine occupation $\left(\text{i.e., } \frac{W_{1R}}{W_{2R}} \text{ declines}\right)$ but increases spatial wage inequality for the non-routine occupations $\left(\text{i.e., } \frac{W_{1S}}{W_{2S}} \text{ and } \frac{W_{1U}}{W_{2U}} \text{ rise}\right)$.[5]

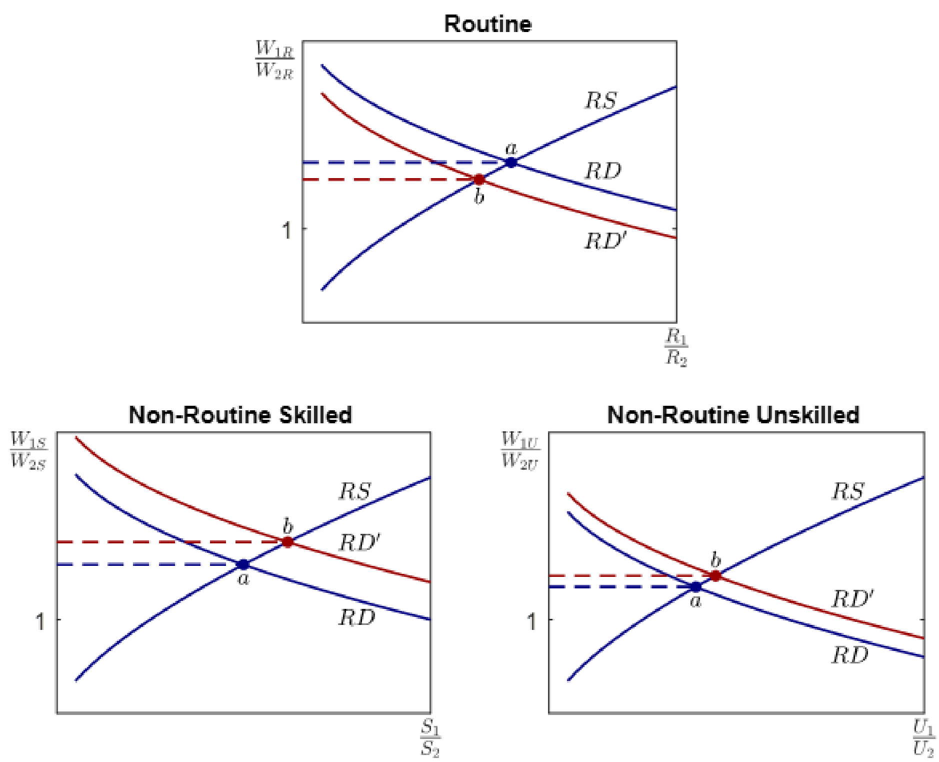
Proof. See Appendix B.5.

Figure 2.1 shows the effect of automation on spatial wage inequality as stated in Proposition 2.1. The change in the ratio $\frac{W_{1\ell}}{W_{2\ell}}$, from a to b, indicates the change in spatial wage inequality for occupation ℓ, due to the fall in the price of machines. As explained in Lemma 2.1, wages of all occupations are higher in place 1 at a because TFP is higher in place 1 $\left(\text{i.e., } \frac{W_{1\ell}}{W_{2\ell}} > 1 \text{ for any occupation } \ell\right)$. As explained in Lemma 2.2, place 1 is more machine-intensive than place 2 because TFP is higher in place 1. Due to the difference in machine intensity across places, as the price of machines falls, more machines are adopted in the more machine-intensive place (i.e., place 1) compared to place 2. As more machines are adopted in place 1 and machines substitute for routine labor, the demand for routine labor in place 1 decreases relative to place 2: the relative demand curve shifts down from RD to RD' as shown in the figure in the top panel of Figure 2.1. Hence, the initial wage difference for routine workers across places shrinks: $\frac{W_{1R}}{W_{2R}}$ becomes smaller in b compared to a. When the price of machines falls, the opposite pattern holds for non-routine workers. As more machines are adopted in place 1 than in place 2, the demand for non-routine skilled labor in place 1 relative to place 2 increases since machines complement non-routine skilled labor. Then, the demand for non-routine unskilled labor in place 1 relative to place 2 also rises as non-routine skilled labor and non-routine unskilled labor are

[5] In my model where machines are not produced, automation is driven by an exogenous decline in the price of machines. In the model where machines are produced (see footnote 4), automation is driven by an increase in the productivity of machines, which manifests as a decline in the price of machines.

complementary. That is, the relative demand curve shifts up from RD to RD' as shown in the two figures in the bottom panel of Figure 2.1. This widens the initial wage differences for the non-routine occupations across places: $\frac{W_{1S}}{W_{2S}}$ and $\frac{W_{1U}}{W_{2U}}$ become larger in b compared to a.

Figure 2.1: Effect of Automation on Spatial Wage Inequality



Note: This figure shows the effect of automation on spatial wage inequality by occupation, where automation is driven by the fall in the price of machines. As stated in Proposition 2.1, TFP is higher in place 1 than in place 2. In each panel, RD shifts to RD' as the price of machines falls. The resulting change in the ratio $\frac{W_{1\ell}}{W_{2\ell}}$, from a to b, indicates the change in spatial wage inequality for occupation $\ell \in \{R, S, U\}$ due to the decline in the price of machines.

In my model, the more productive place is more machine-intensive, and as the price of machines falls, more machines are adopted in the more machine-intensive place relative to the less machine-intensive one. This is in line with the fact documented by Beaudry, Doms, and Lewis (2010): PC intensity, defined as the ratio of personal computers to workers, increased disproportionately in the initially more PC-intensive places from 1980 to 2000. In my model, the differential adoption of machines across places—as the price of machines falls—decreases spatial wage inequality for the routine occupation, while increasing it for the non-routine unskilled occupation as well as the non-routine skilled occupation. My model highlights the importance of occupations, as which occupations technology replaces or complements is central to my mechanism. Existing models that feature skill groups (Ganong and Shoag 2017; Giannone 2022) cannot explain the differential trends in spatial wage inequality across occupations, given that non-routine occupations are at the extremes of the skill distribution, while routine occupations are in the middle of the skill distribution.

While Proposition 2.1 is aimed at understanding the effect of automation on spatial wage inequality, the purpose of Proposition 2.2 is to derive implications for place-based policies from my model. I model a place-based policy as an increase in TFP in initially low-TFP places.[6]

Proposition 2.2 (Trade-off in Place-Based Policy)**.** Let $T_1 > T_2$. Let T_2 increase to $T_2' \in (T_2, T_1]$ as a result of a place-based policy undertaken in place 2. Then,

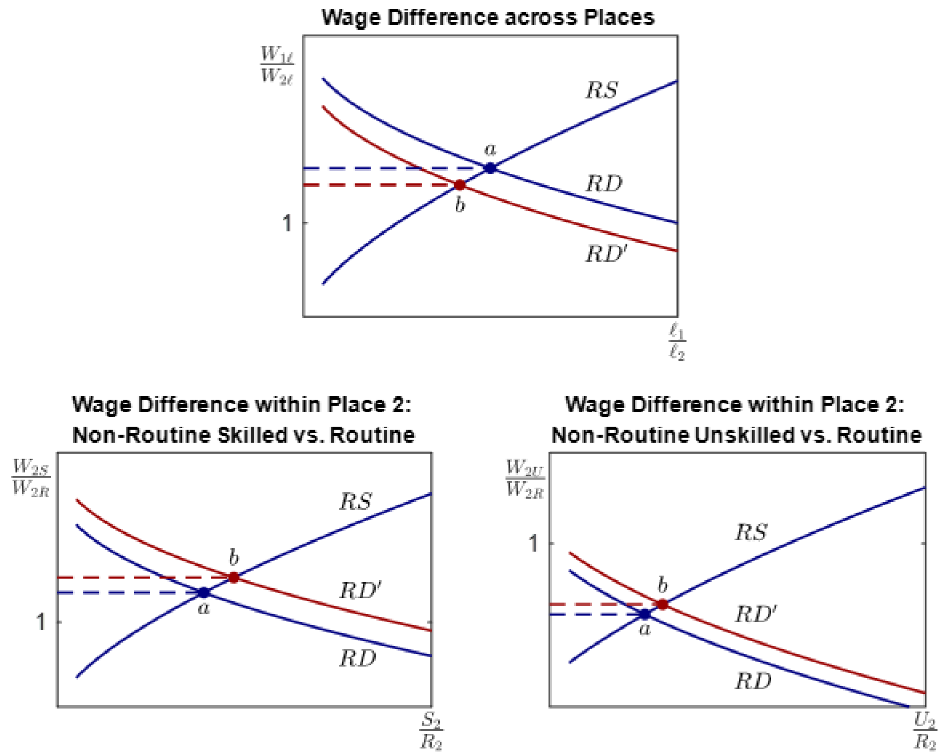
(i) Spatial wage inequality falls for each occupation: $\frac{W_{1\ell}}{W_{2\ell}}$ falls $\forall \ell \in \{R, S, U\}$.

(ii) While wages for all occupations increase in place 2, wages for the non-routine occupations rise more: $\frac{W_{2S}}{W_{2R}}$ and $\frac{W_{2U}}{W_{2R}}$ increase.

Proof. See Appendix B.6.

[6] This can be microfounded by infrastructure investments in low-TFP places, such that initially low-TFP places become more productive as a result of the infrastructure investments. Examples of place-based policies that aim to boost the productivity of lagging places through infrastructure investment include the recent ERA, the Appalachian Regional Commission, and the Tennessee Valley Authority in the US, as well as the Structural Funds in the European Union.

Figure 2.2: Trade-off in Place-Based Policy



Note: This figure shows the trade-off in a place-based policy. As stated in Proposition 2.2, TFP is higher in place 1, but the place-based policy raises TFP in place 2, thereby reducing the TFP gap between the two places. In each panel, RD shifts to RD' as a result of the place-based policy. The top panel describes part (i) of Proposition 2.2, showing that the place-based policy reduces wage differences across places for each occupation $\ell \in \{R, S, U\}$. The bottom panel describes part (ii) of Proposition 2.2, showing that the place-based policy affects wage differences within place 2: wages for the non-routine occupations relative to the routine occupation in place 2 rise.

Part (i) of Proposition 2.2 is illustrated in the top panel of Figure 2.2. Suppose TFP rises in place 2, reducing the TFP gap between the two places as a result of a place-based policy. The increase in TFP in place 2 raises the marginal product of all occupations in place 2. Consequently, the demand for each occupation in place 1 relative to place 2 decreases: the relative demand curve for each occupation $\ell \in \{R, S, U\}$ shifts down from RD to RD', as shown in the top panel of Figure 2.2. Hence, the initial wage difference across places for all occupations shrinks: $\frac{W_{1\ell}}{W_{2\ell}}$ becomes smaller at b compared to a for each occupation $\ell \in \{R, S, U\}$.

Regarding part (ii) of Proposition 2.2, it should be emphasized that Hicks-neutral technical progress (i.e., an increase in TFP) in place 2 has *non-neutral* effects across occupations in *equilibrium* due to the difference in supply elasticity between machines and labor. The increase in TFP in place 2 raises the marginal product of all factors of production in place 2 relative to place 1, causing the factors to flow from place 1 to place 2. However, the supply of machines rises relative to the supply of workers in place 2 because the supply of machines is more elastic than that of labor. Since machines are complementary to the non-routine occupations but substitute for the routine occupation, this raises the demand for the non-routine occupations relative to the routine occupation in place 2. That is, the relative demand curve shifts up from RD to RD', as shown in the two figures in the bottom panel of Figure 2.2. Consequently, wages for the non-routine occupations relative to the routine occupation in place 2 rise: $\frac{W_{2S}}{W_{2R}}$ and $\frac{W_{2U}}{W_{2R}}$ become larger in b compared to a. It is important to note that while the increase in TFP in place 2 raises wages for all occupations in place 2, wages for the non-routine occupations increase more.

In summary, Proposition 2.2 highlights that a place-based policy entails a trade-off between wage differences *across* places and wage differences *within* a place. The place-based policy implemented in place 2 raises wages for all occupations in place 2, reducing wage differences across places for each occupation. However, while wages increase for all occupations in place 2, wages for the non-routine occupations rise more, thus affecting wage differences within place 2. Part (i) of Proposition 2.2 aligns with empirical studies showing the causal effects of place-based policies on reducing spatial wage inequality by boosting the productivity of lagging places through infrastructure investment (Becker, Egger, and von Ehrlich 2010; Jaworski and Kitchens 2019; Kline and Moretti 2014). Part (ii) of Proposition 2.2 contains my contribution: while reducing wage inequality across places is an economic rationale for place-based policies (Gaubert, Kline, and Yagan 2021), my model implies that place-based policies entail an unintended consequence: affecting wage inequality within places.

2.1.2 Quantitative Model

The purpose of Subsection 2.1.1 was to highlight the mechanism based on the simplified model: how automation increases spatial wage inequality for non-routine occupations while decreasing it for routine occupations, and how place-based policies not only reduce spatial wage inequality but also entail an unintended consequence. In this subsection, I describe the model I build on from a canonical quantitative spatial model (Redding and Rossi-Hansberg 2017), where I incorporate the same mechanism as in Subsection 2.1.1 and add three additional features. First, I incorporate housing into the model. In the data, spatial wage inequality refers to differences in real wages across places, where local real wages are defined as local nominal wages adjusted for local prices, with variation in local prices arising from local housing prices. Second, I introduce local amenities into the model. Amenities are capitalized into housing prices: higher amenities in a place attract more people, driving up its housing price. While differences in TFP across places generate spatial differences in real wages, local amenities also contribute to these wage disparities.[7] Finally, I extend the number of places to any integer greater than 2, as the data includes 722 commuting zones.

Preferences. There are three occupations indexed by $\ell \in \{R, S, U\}$, where R stands for the routine occupation, S for the non-routine skilled occupation, and U for the

[7] Amenities and idiosyncratic preferences for places serve different purposes in my model, as in Moretti (2013). People do not have idiosyncratic preferences for amenities in my model. The role of amenities is to explain differences in real wages across places, as amenities are capitalized into housing prices. In contrast, the role of idiosyncratic preferences for places is to generate an upward-sloping supply curve in each place.

non-routine unskilled occupation. Within each occupation ℓ, there is a measure N_ℓ of workers such that $\sum_\ell N_\ell = 1$. There are J places indexed by $j \in \{1, ..., J\}$. Workers consume a good, housing, and amenities while having idiosyncratic preferences for places. The utility function for worker ω who lives in j is

$$\mathcal{U}_j(\omega) = B_j C_j^{\gamma} H_j^{1-\gamma} \epsilon_j(\omega) \;\text{ where }\; \gamma \in (0,1),$$

where B denotes amenities, C the good, H housing, and ϵ the preference shock. $\epsilon_j(\omega)$ is drawn independently for each (ω, j) from a Fréchet distribution with a finite variance parameter $\kappa > 1$.

Labor supply. The good is freely traded across places and I choose it as the numeraire. I denote the housing price in place j by P_{jH}. Hence, the price level in place j is given by $P_j = \gamma^{-\gamma}(1-\gamma)^{\gamma-1} P_{jH}^{1-\gamma}$. Each worker in occupation ℓ earns income $I_{j\ell}$ in place j. Using the property of the Fréchet distribution, the labor supply of occupation ℓ in place j takes the following logit form:

$$\frac{\ell_j}{N_\ell} = \frac{\left(B_j \frac{I_{j\ell}}{P_j}\right)^{\kappa}}{\sum_i \left(B_i \frac{I_{i\ell}}{P_i}\right)^{\kappa}},$$

where the spatial labor supply elasticity is equal to κ for each occupation. Each place faces an upward-sloping labor supply curve for each occupation, meaning higher real incomes must be offered to attract workers with lower tastes for that place. Expected utility across places for occupation ℓ, denoted as $\overline{V_\ell}$, is

$$\overline{V_\ell} = \delta \left(\sum_i \left(B_i \frac{I_{i\ell}}{P_i} \right)^{\kappa} \right)^{1/\kappa},$$

where $\delta = \Gamma\left(\frac{\kappa-1}{\kappa}\right)$ and $\Gamma(\cdot)$ denotes the Gamma function.

Housing supply. Place j is endowed with an exogenous supply of housing, $\overline{H}_j$. Each worker in place j owns an equal share of the housing in that place, referred to as a regional portfolio. Let ϕ_j denote the housing income of a worker in place j from this regional portfolio. Each worker in occupation ℓ supplies one unit of labor for wage $W_{j\ell}$ in place j, so that the income of a worker in occupation ℓ residing in place j is given by $I_{j\ell} = W_{j\ell} + \phi_j$. Since the housing expenditure share is $1-\gamma$, total housing income in place j constitutes a fraction $1-\gamma$ of the total income in place j, meaning total wages in place j make up a fraction γ of the total income in place j. Since the ratio of total housing income to total wages is $\frac{1-\gamma}{\gamma}$, a worker's housing income in place j, ϕ_j, is $\frac{1-\gamma}{\gamma}$ times the per-capita wage in place j: $\phi_j = \frac{1-\gamma}{\gamma}\frac{W_{jR}R_j+W_{jS}S_j+W_{jU}U_j}{R_j+S_j+U_j}$.

Technology. Technology is exactly the same as in the simplified model. From Equation 2.1, the conditional factor demands in place j are

$$W_{j\ell}\ell_j = \lambda_{j\ell}Y_j \text{ for } \ell \in \{R, S, U\}, \quad P_M M_j = \lambda_{jM} Y_j.$$

As in the simplified model, machines are supplied perfectly elastically at an exogenous price, P_M, in the international market. The zero-profit condition in each place j is given by

$$T_j = \rho^{-\rho}(1-\rho)^{\rho-1}W_{jU}^{\rho}\chi_{jS}^{1-\rho}, \quad \forall j \in \{1, ..., J\}. \tag{2.2}$$

Market clearing. Labor markets clear if

$$\frac{\left(B_j \frac{I_{j\ell}}{P_j}\right)^{\kappa}}{\sum_i \left(B_i \frac{I_{i\ell}}{P_i}\right)^{\kappa}} N_\ell = \frac{\lambda_{j\ell}}{W_{j\ell}} Y_j, \quad \forall \ell \in \{R, S, U\}, \quad \forall j \in \{1, ..., J\}, \tag{2.3}$$

with the aggregate labor market clearing for each occupation: $\sum_j \ell_j = N_\ell$, $\forall \ell \in \{R, S, U\}$. Housing markets clear if

$$P_j = \gamma^{-1} \overline{H}_j^{\gamma-1} \Big[(\lambda_{jR} + \lambda_{jS} + \lambda_{jU}) Y_j \Big]^{1-\gamma}, \quad \forall j \in \{1, ..., J\}. \tag{2.4}$$

As machines are traded internationally, $\sum_j P_M M_j = \sum_j \lambda_{jM} Y_j$ holds. Trivially, the market for the good also clears: $\sum_j C_j = \sum_j (\lambda_{jR} + \lambda_{jS} + \lambda_{jU}) Y_j$, where the good is chosen as the numeraire. Notice that Equations 2.2 to 2.4 comprise $5 \times J$ equations with $5 \times J$ unknowns: $\{W_{jR}, W_{jS}, W_{jU}, P_j, Y_j\}_{j=1}^J$.

Spatial equilibrium. Given parameters $\Big\{\alpha_R, \alpha_S, \rho, \sigma_R, \sigma_S, P_M, \kappa, \gamma, \{T_j, B_j, \overline{H}_j\}_{j=1}^J\Big\}$, the spatial equilibrium consists of allocations $\{R_j, S_j, U_j, M_j, Y_j, C_j, H_j\}_{j=1}^J$, prices $\{W_{jR}, W_{jS}, W_{jU}, P_j, \phi_j\}_{j=1}^J$, and expected utility $\{\overline{V_R}, \overline{V_S}, \overline{V_U}\}$ such that workers and firms optimize given prices and all markets clear. As in the canonical quantitative spatial model (Redding and Rossi-Hansberg 2017), a unique spatial equilibrium exists in my model because the production function exhibits constant returns to scale and two congestion forces—housing prices and idiosyncratic preferences for places—are present. In particular, the absence of agglomeration externalities helps ensure existence and uniqueness.

Mechanism. Even in the quantitative model, the forces at play remain the same as those in the simplified model. First, TFP varies across places. Second, the supply of machines is more elastic than that of labor. Third, machines substitute for the routine occupation while complementing the non-routine occupations.

2.2 Calibration

In Section 2.2, I recover the structural parameters of my quantitative spatial model described in Subsection 2.1.2. Recovering these structural parameters serves as the foundation for the two quantitative analyses conducted in Section 2.3. The first quantitative analysis is to quantify the effect of automation on spatial wage inequality by occupation, using the quantitative model. If the price of machines declines as it does in the data, while all other parameters of my model remain the same, how much does automation explain the observed changes in spatial wage inequality? The second quantitative analysis is to derive implications for place-based policies from my model. Using the quantitative model, I conduct a policy experiment to examine the effect of a place-based policy on wage differences across places and wage differences within places. To answer these two questions, recovering the structural parameters of my model is essential.

The first column in Table 2.1 lists the structural parameters of my model. The first three parameters are borrowed from the literature. First, I borrow the housing expenditure share, $1 - \gamma$, from Davis and Ortalo-Magne (2011), who estimate it to be 0.25. The second one is σ_R, the elasticity of substitution between the routine occupation (corresponding to R in the table) and machines (M). The third one is σ_S, the elasticity of substitution between the non-routine skilled occupation (S) and the routine task input (Ω_R), which includes both the routine occupation and machines. I borrow these two parameters from vom Lehn (2020) and set $\sigma_R = 1.5$ and $\sigma_S = 0.5$.[8]

[8] In vom Lehn (2020), the estimated σ_R is in the range of 1.3–1.5, and the estimated σ_S is in the range of 0.3–0.5. Krusell et al. (2000) estimate the elasticity of substitution between non-college workers and equipment to be 1.67 and that between college workers and equipment to be 0.67. Karabarbounis and Neiman (2014) estimate the elasticity of substitution between labor and capital to be 1.26.

I simultaneously recover the remaining parameters listed in Table 2.1 using the following data from 1980: $\Big\{\{W_{jR}, W_{jS}, W_{jU}, P_j, R_j, S_j, U_j\}_{j=1}^{722}, P_M\Big\}$. I denote by $W_{j\ell}$ the hourly wage for occupation $\ell \in \{R, S, U\}$ in commuting zone j. Next, P_j represents the price level in commuting zone j. I denote by ℓ_j the hours worked by occupation $\ell \in \{R, S, U\}$ in commuting zone j. Finally, P_M is the price of ICT capital relative to the price of consumption goods, which is normalized to 1 in 1980. See Section 1.2 and Appendices A.1 and A.2. for more details on the data.

Although I recover the parameters simultaneously, I explain how to recover them part by part. First, α_R and α_S are the distribution parameters for the routine occupation and the non-routine skilled occupation, respectively, in the two lower-tier CES production functions. In addition, ρ is the expenditure share parameter for the non-routine unskilled occupation from the top-tier Cobb-Douglas production function. These three share parameters, α_R, α_S, and ρ, are calibrated to match the labor income shares for the three occupations in 1980. Next, κ refers to the spatial labor supply elasticity, which I allow to vary across occupations in the calibration. Holding others constant, a higher spatial labor supply elasticity for an occupation implies smaller dispersion of real wages across places for that occupation. I calibrate κ_R, κ_S, and κ_U to match the coefficients of variation of real wages across commuting zones for the three occupations in 1980.

Finally, I am left with the location fundamental parameters $\left\{T_j, B_j, \overline{H}_j\right\}_{j=1}^{722}$, where the first vector represents TFP, the second amenities, and the last housing endowment. As my model has a unique equilibrium, I can recover the location fundamental parameters through model inversion, in which the model is inverted to recover parameters that rationalize the observed data as the model's equilibrium, based on a one-to-one mapping from the data to the parameters (Redding and Rossi-Hansberg

2017). For any given parameter vector $X = (\alpha_R, \alpha_S, \rho, \kappa_R, \kappa_S, \kappa_U)$, I retrieve $\left\{T_j, B_j, \overline{H}_j\right\}_{j=1}^{722}$ from the 1980 data and the equilibrium conditions of my model.

First, the zero-profit condition in Equation 2.2 determines T_j, conditional on the parameters α_R, α_S, and ρ:

$$T_j = \rho^{-\rho}(1-\rho)^{\rho-1} W_{jU}^{\rho} \Big(\chi_{jS}(W_{jR}, W_{jS}, P_M; \alpha_R, \alpha_S, \rho)\Big)^{1-\rho}.$$

Note that χ_{jS}, the marginal cost of intermediates Ω_{jS}, is a function of the observed data from 1980 $\big($i.e., W_{jR}, W_{jS}, and $P_M\big)$ and the parameters to be calibrated $\big($i.e., α_R, α_S, and $\rho\big)$. Second, the housing market clearing condition in Equation 2.4 pins down the housing endowment, $\overline{H}_j$. Equation 2.4 can be rewritten as:

$$W_{jR}R_j + W_{jS}S_j + W_{jU}U_j = \gamma^{\frac{1}{1-\gamma}} \overline{H}_j P_j^{\frac{1}{1-\gamma}}.$$

I recover $\overline{H}_j$ using the observed data from 1980 $\big($i.e., W_{jR}, W_{jS}, W_{jU}, R_j, S_j, U_j, and $P_j\big)$ and the value of γ set to 0.75. Third, adding the labor market clearing condition in Equation 2.3 to the above housing market clearing condition yields:

$$\sum_{\ell} W_{j\ell} \frac{\left(B_j \frac{I_{j\ell}}{P_j}\right)^{\kappa_\ell}}{\sum_i \left(B_i \frac{I_{i\ell}}{P_i}\right)^{\kappa_\ell}} N_\ell = \gamma^{\frac{1}{1-\gamma}} \overline{H}_j P_j^{\frac{1}{1-\gamma}}.$$

Note that the income in place j for occupation ℓ, $I_{j\ell}$, consists of the wage and the housing income, where the housing income in place j is a function of the observed data from 1980 $\big($i.e., W_{jR}, W_{jS}, W_{jU}, R_j, S_j, and $U_j\big)$ and γ. Additionally, N_ℓ is the fraction of workers employed in occupation ℓ in 1980. Hence, conditional on κ_R, κ_S, and κ_U, I retrieve B_j using the observed data from 1980 $\big($i.e., W_{jR}, W_{jS}, W_{jU}, R_j, S_j, U_j, and $P_j\big)$, $\overline{H}_j$ as recovered above, and the value of γ set to 0.75.

Table 2.1: Calibration Result

	Description	Source/Target	Value
Parameter			
$1-\gamma$	Housing expenditure share	Davis and Ortalo-Magne (2011)	0.25
σ_R	EOS between R and M	vom Lehn (2020)	1.5
σ_S	EOS between S and Ω_R	vom Lehn (2020)	0.5
α_R	Distribution parameter of R	$\lambda_R = 0.552$	0.926
α_S	Distribution parameter of S	$\lambda_S = 0.311$	0.188
ρ	Expenditure share parameter of U	$\lambda_U = 0.102$	0.102
κ_R	Spatial labor supply elasticity of R	$CV_R = 0.088$	2.778
κ_S	Spatial labor supply elasticity of S	$CV_S = 0.077$	2.925
κ_U	Spatial labor supply elasticity of U	$CV_U = 0.072$	2.734
Location Fundamental			
$\{T_j\}$	TFP	Model-inverted by data on wages, prices, hours	Various
$\{B_j\}$	Amenities		
$\{\overline{H}_j\}$	Housing		

Note: This table shows the structural parameters of my model. The first three parameters are borrowed from the literature, while the remaining parameters are recovered through calibration and model inversion using the data from 1980. λ_ℓ refers to the labor income share for occupation $\ell \in \{R, S, U\}$, while CV_ℓ is the coefficient of variation of real wages across commuting zones for occupation ℓ. The location fundamentals, $\{T_j, B_j, \overline{H}_j\}$, vary across places, so their values are not listed.

In summary, I perform model inversion for any $X = (\alpha_R, \alpha_S, \rho, \kappa_R, \kappa_S, \kappa_U)$ and choose X such that six model moments match the six data moments from 1980: the labor income shares and the coefficients of variation of real wages across commuting zones for the three occupations. Model-inverted parameters $\left\{T_j, B_j, \overline{H}_j\right\}_{j=1}^{722}$ are evaluated at the values of the calibrated parameters.

The middle panel of Table 2.1 shows the six data moments from 1980 and the values of the calibrated parameters. First, λ_ℓ is the labor income share for occupation $\ell \in \{R, S, U\}$ in 1980. Since I do not model non-ICT capital, I proportionally scale up the labor income share.[9] Second, CV_ℓ is the coefficient of variation of real wages

[9]In my model, machines refer to ICT capital, which has experienced a swift decline in its price

across commuting zones for occupation ℓ in 1980. Calibrated values of κ_R, κ_S, and κ_U average around 2.8, which is similar to the value estimated by Bryan and Morten (2015).

Figure 2.3 shows the location fundamentals recovered through model inversion. The first subfigure shows TFP by commuting zone in percentiles. For example, the most productive commuting zone is the one encompassing San Francisco. Other productive commuting zones include those encompassing San Jose, Los Angeles, Seattle, the East North Central Census Division, Washington DC, New York City, Newark, Houston, among others. In my model, high-TFP places are more machine-intensive, and machines are adopted disproportionately in the high-TFP places as the price of machines falls. This is in line with the fact documented by Beaudry, Doms, and Lewis (2010): the ratio of personal computers to workers was higher in the productive places compared to others in 1980, and this ratio increased disproportionately in the productive places from 1980 to 2000. In contrast, less productive commuting zones are often concentrated in specific regions, aligning with Figure 1.3, which shows the eligible places for the ERA. In particular, unproductive commuting zones below the 33rd percentile are mostly located in the central regions (i.e., the West North Central and the West South Central Census Divisions) or in the southern regions (i.e., the East South Central and the South Atlantic Census Divisions). For example, the commuting zone encompassing Marengo County in southern Alabama lies at the 30th percentile. In my policy experiment, unproductive commuting zones below the 33rd percentile are the target of place-based policies.

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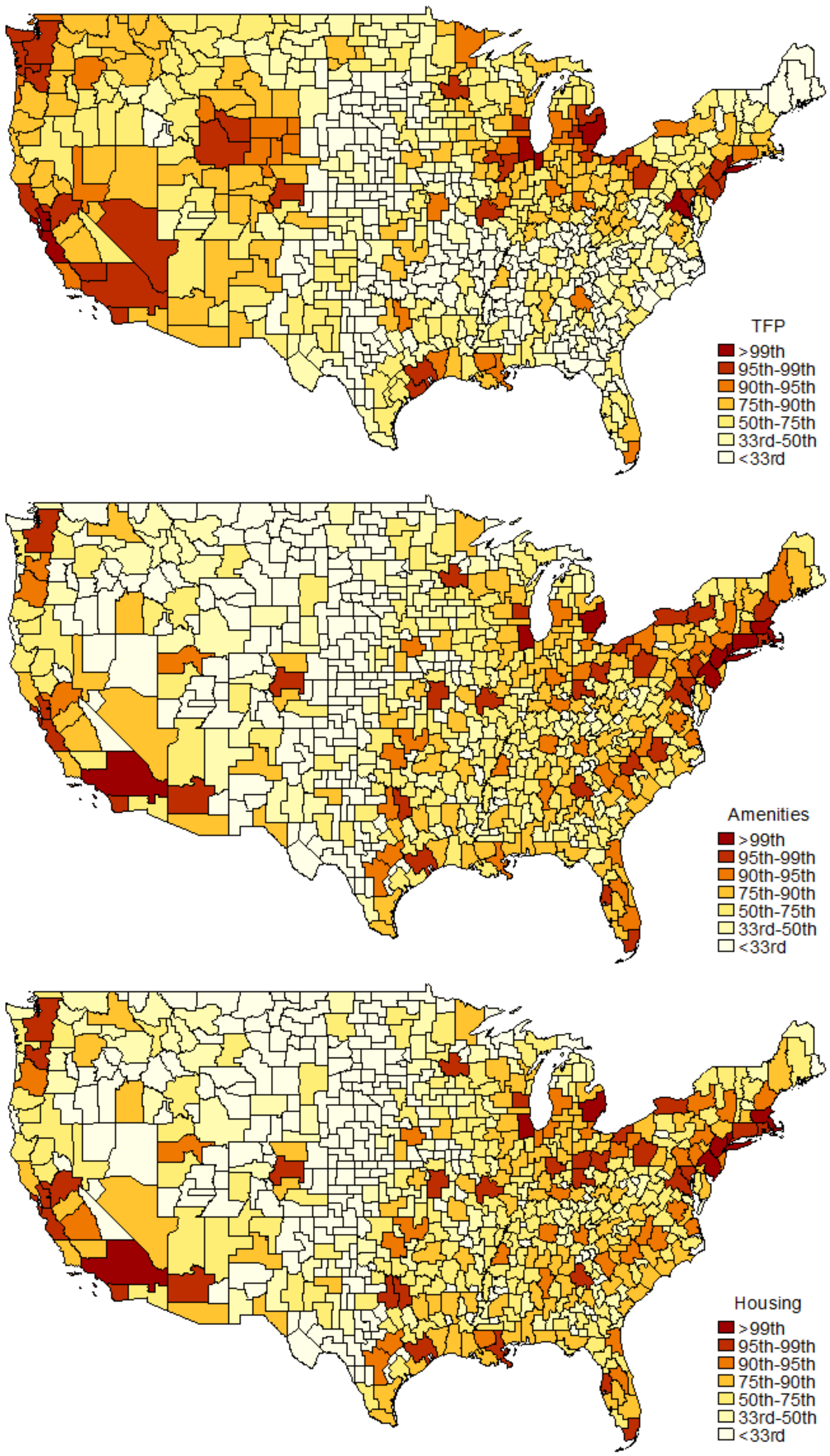
since 1980. Since I do not model non-ICT capital, I proportionally scale up the labor income share. I set the share of ICT capital in 1980 to be 2.4 percent, aligning with the value in Eden and Gaggl (2018). Hence, the labor income share in 1980, $\frac{2}{3}$ (Karabarbounis and Neiman 2014), is scaled up to 96.5 percent $\left(= \frac{\frac{2}{3}}{\frac{2}{3}+0.024}\right)$.

Figure 2.3: Recovered Location Fundamentals



Note: This figure shows location fundamentals recovered through model inversion using data from 1980. The first subfigure shows TFP, the second amenities, and the third housing endowments.

The second subfigure in Figure 2.3 shows recovered amenities by commuting zone in percentiles. Higher amenities in a place attract more people, driving up its housing prices. Amenities are relatively low in the central regions (the West North Central and West South Central Census Divisions) as well as in the Mountain Census Division. Finally, the third subfigure in Figure 2.3 shows recovered housing endowments by commuting zone in percentiles. The recovered housing endowments align with the housing data from 1980, obtained from the National Historical Geographic Information System (Ruggles et al. 2023). The correlation between the recovered housing and the data exceeds 0.99.

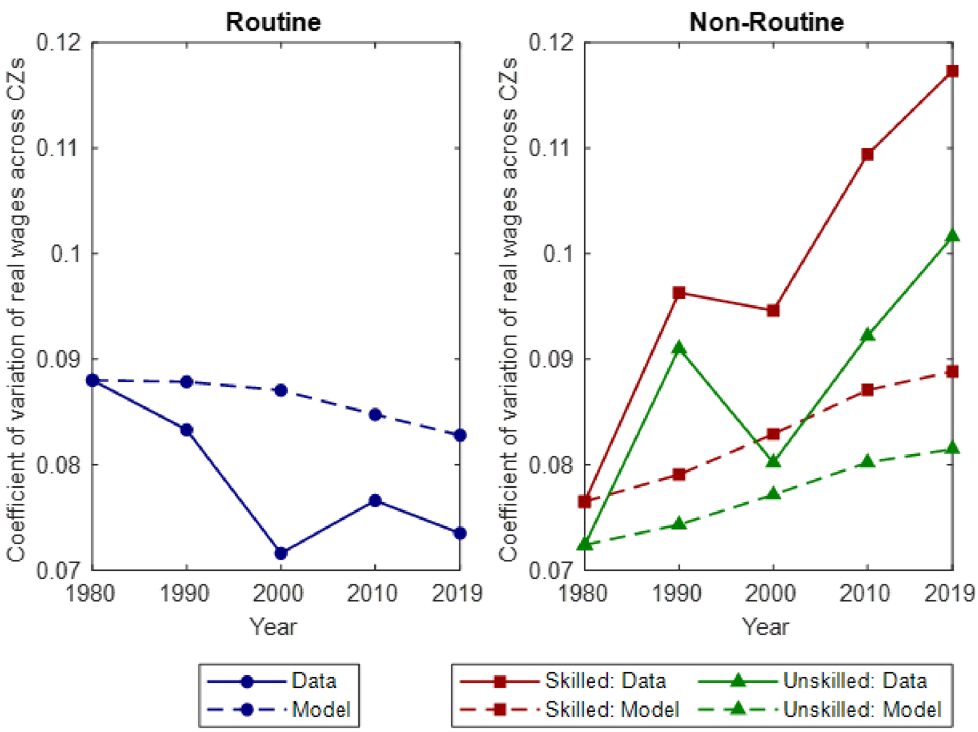
2.3 Quantitative Analysis

2.3.1 Quantifying Effect of Automation on Spatial Wage Inequality

The purpose of this subsection is to quantify the effect of automation on spatial wage inequality by occupation. Given that I recovered parameters of interest in Section 2.2, I conduct a counterfactual analysis where the price of machines falls, as it does in the data, while all other parameters in my model remain the same. Then, I examine how much automation, driven by the fall in the price of machines, explains the observed changes in spatial wage inequality by occupation in the data.

Figure 2.4 plots spatial wage inequality over time, measured as the coefficient of variation of real wages across commuting zones. The first panel shows the evolution of spatial wage inequality for the routine occupation. The solid blue line represents the data, whereas the dashed blue line represents the counterfactual in which only

Figure 2.4: Quantifying Effect of Automation on Spatial Wage Inequality by Occupation



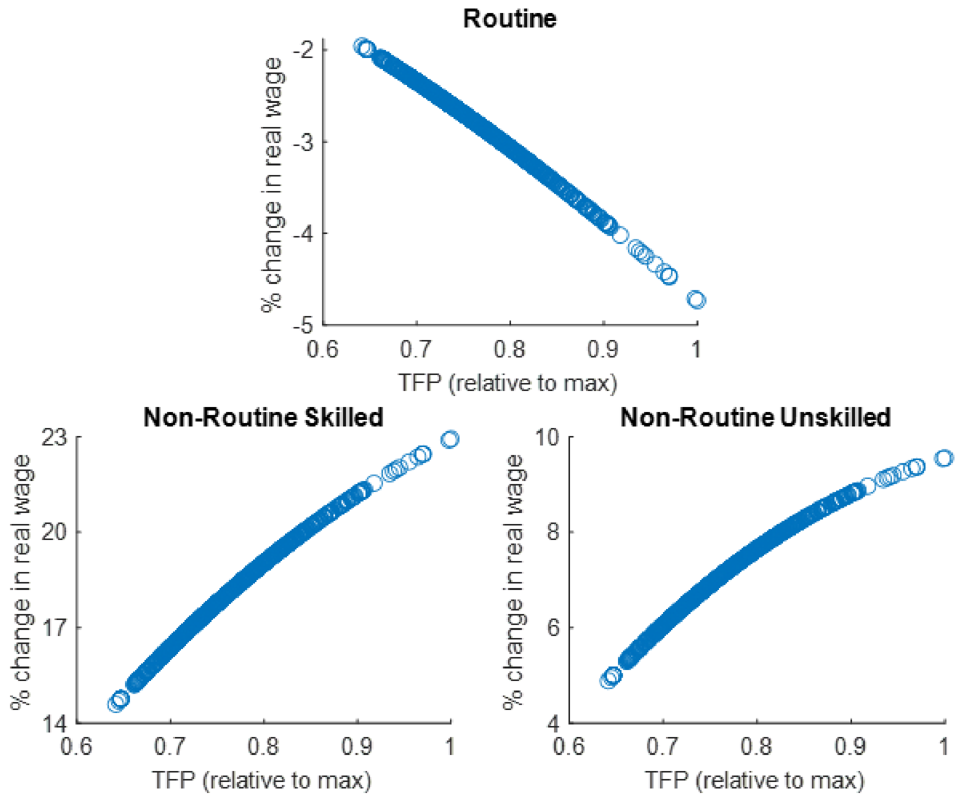
Note: In this figure, I quantify the effect of automation on spatial wage inequality by occupation. The three solid lines in both panels display spatial wage inequality over time, as measured from the data. The three dashed lines in both panels show spatial wage inequality over time based on the counterfactual analysis in which the price of machines falls, as it does in the data, while all other parameters in my model remain the same.

the price of machines falls, as it does in the data. The coefficient of variation for the routine occupation, derived from the data, decreased by 16.5 percent (from 0.0880 to 0.0735) between 1980 and 2019. According to the counterfactual analysis, it decreased by 5.9 percent (from 0.0880 to 0.0828) over the same period. My model indicates that automation alone explains 35.8 percent of the observed decrease in spatial wage inequality for the routine occupation.

The second panel in Figure 2.4 shows spatial wage inequality for the two non-routine occupations over time. The solid red line represents spatial wage inequality for the non-routine skilled occupation based on the data, whereas the dashed red line is the counterpart from the counterfactual where only the price of machines falls, as it does in the data. On the other hand, the solid green line corresponds to spatial wage inequality for the non-routine unskilled occupation based on the data, while the dashed green line comes from the same counterfactual. According to the data, the coefficient of variation for the non-routine *skilled* occupation increased by 53.3 percent (from 0.0765 to 0.1173) from 1980 to 2019. According to the counterfactual analysis, it increased by 16.1 percent (from 0.0765 to 0.0888) during the same period. My model indicates that automation alone explains 30.2 percent of the observed increase in spatial wage inequality for the non-routine skilled occupation. For the non-routine *unskilled* occupation, the coefficient of variation increased by 40.3 percent (from 0.0724 to 0.1016) in the data from 1980 to 2019, whereas it increased by 12.5 percent (from 0.0724 to 0.0815) in the counterfactual analysis during the same period. My model indicates that automation alone explains 31 percent of the observed increase in spatial wage inequality for the non-routine unskilled occupation.

Figure 2.5 shows the predicted changes in real wages from 1980 to 2019 based on the counterfactual, where only the price of machines falls as observed in the data, by commuting zone ranked by TFP. TFP is normalized to 1 for the commuting zone with the highest TFP. The top panel shows that commuting zones with higher TFP experience larger decreases in real wages for the routine occupation. The subfigures in the bottom panel show the exact opposite pattern for the two non-routine occupations. This is because machines are adopted disproportionately in places with high TFP as the price of machines falls, and machines substitute for routine labor while complementing non-routine labor. Figure 2.6 shows the predicted changes in real wages from 1980 to 2019 based on the same counterfactual by commuting zone,

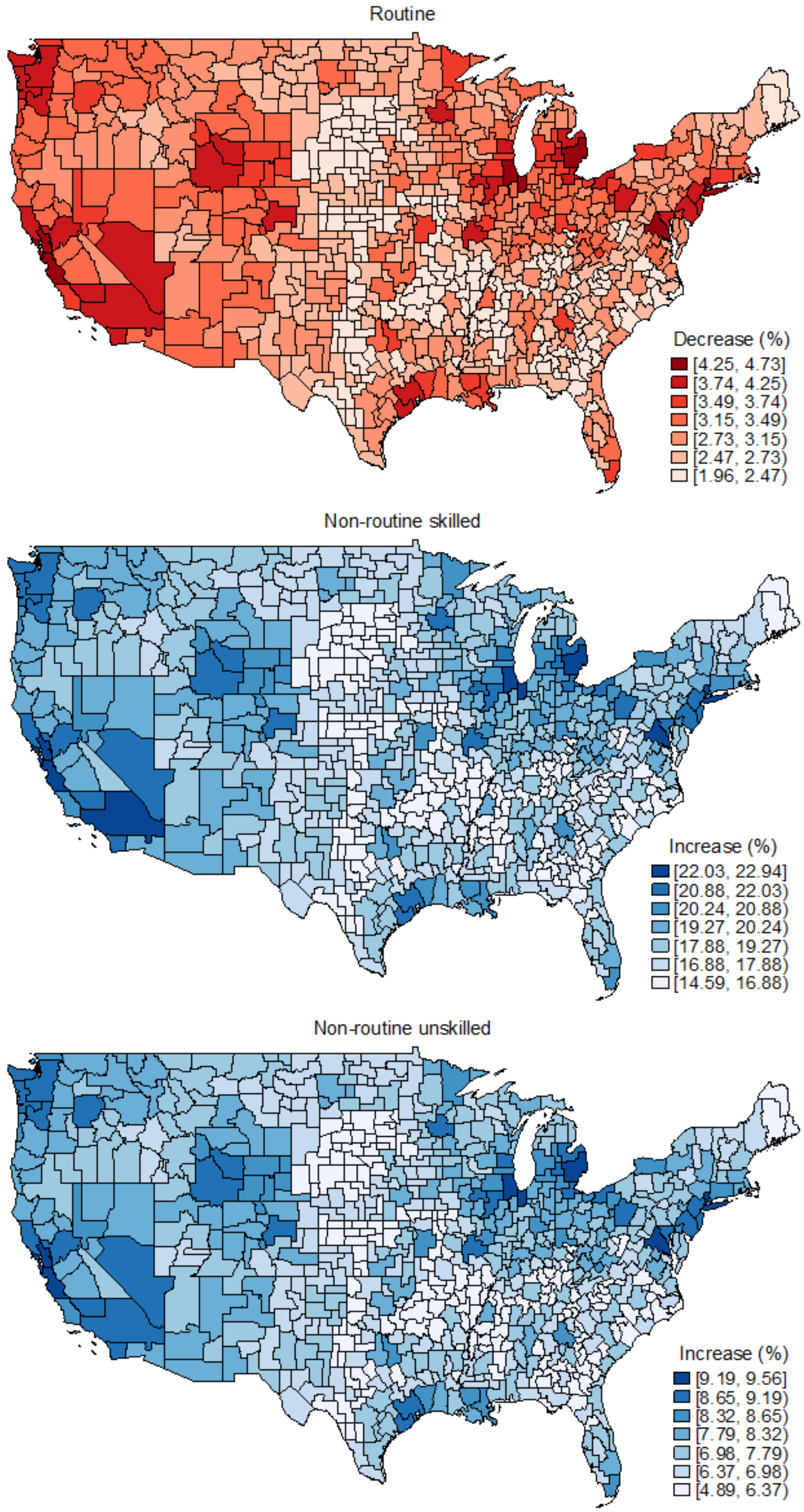
Figure 2.5: Predicted Changes in Real Wages by Commuting Zone Ranked by TFP



Note: This figure shows the predicted changes in real wages from 1980 to 2019 based on the counterfactual analysis, by commuting zone ranked by TFP. In the counterfactual analysis, the price of machines falls as it does in the data, while all other parameters of my model remain the same. TFP is normalized to 1 for the commuting zone with the highest TFP. The first subfigure corresponds to the routine occupation, the second to the non-routine skilled occupation, and the third to the non-routine unskilled occupation.

represented in a map. The first subfigure shows the predicted decreases in real wages from 1980 to 2019 by commuting zone for the routine occupation. The second and third subfigures display the predicted increases in real wages from 1980 to 2019 by commuting zone for the non-routine skilled and the non-routine unskilled occupation. The predicted changes in real wages by commuting zone, shown in Figure 2.6, generally align with the observed changes from 1980 to 2019. However, in Rust Belt areas, the predictions are less aligned with the observed changes in real wages from 1980 to 2019 for non-routine occupations than for routine occupations.

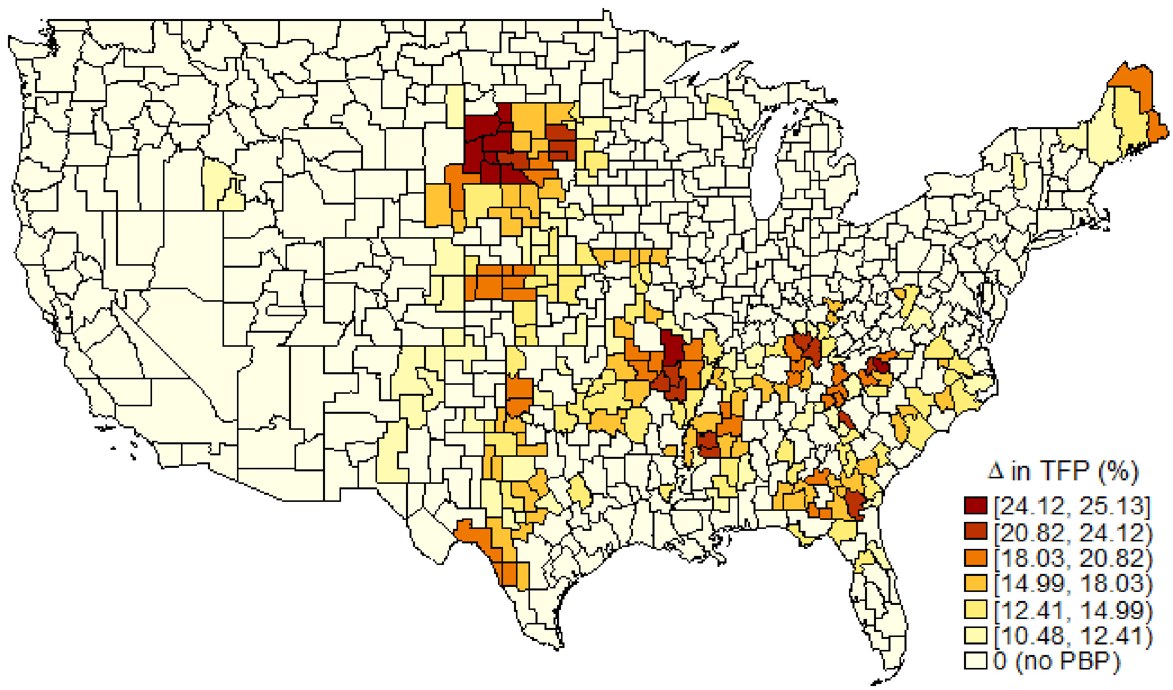
Figure 2.6: Predicted Changes in Real Wages by Commuting Zone



Note: This map shows the predicted changes in real wages from 1980 to 2019 based on the counterfactual, where the price of machines falls as it does in the data, while all other parameters of my model remain the same. The first subfigure corresponds to the routine occupation, the second to the non-routine skilled occupation, and the third to the non-routine unskilled occupation.

2.3.2 Implication for Place-Based Policy

Figure 2.7: Changes in TFP in Treated Places Due to Place-Based Policy



Note: This figure shows the changes in TFP in the policy experiment, where the TFP of each commuting zone within the first tercile of the distribution increases to match the level of the 75th percentile. The value of 0 in the legend indicates no changes in TFP for the commuting zones above the first tercile of the TFP distribution, as no place-based policies are implemented there.

In Subsection 2.3.1, I quantified the effect of automation on spatial wage inequality by occupation. In this subsection, I derive implications for a place-based policy from my model. Consider a place-based policy that increases TFP in initially low-TFP places, reducing the TFP gap between initially low- and high-TFP places. Examples of such place-based policies include the ERA, which aims to boost the productivity of lagging places through infrastructure investments in electricity and broadband access, as described in Section 1.3. The objective of this subsection is to examine whether place-based policies entail an unintended consequence in addition to reducing spatial wage inequality. Using the recovered parameters of my model, I conduct a policy experiment where TFP of each commuting zone within the first tercile of the distribution increases to match the level of the 75th percentile. Figure 2.7 shows the changes in TFP in the treated places due to the place-based policy. The targeted commuting zones within the first tercile of the TFP distribution are mostly concentrated in the central regions (i.e., the West North Central and the West South Central Census Divisions) or in the southern regions (i.e., the East South Central and the South Atlantic Census Divisions), aligning with Figure 1.3, which shows the eligible places for the ERA.

Table 2.2: Implication of Place-Based Policy on Spatial Wage Inequality

	Coefficient of variation of real wages across CZs	
	Pre-policy	Post-policy
Routine	0.0880	0.0710
Non-routine skilled	0.0765	0.0574
Non-routine unskilled	0.0724	0.0551

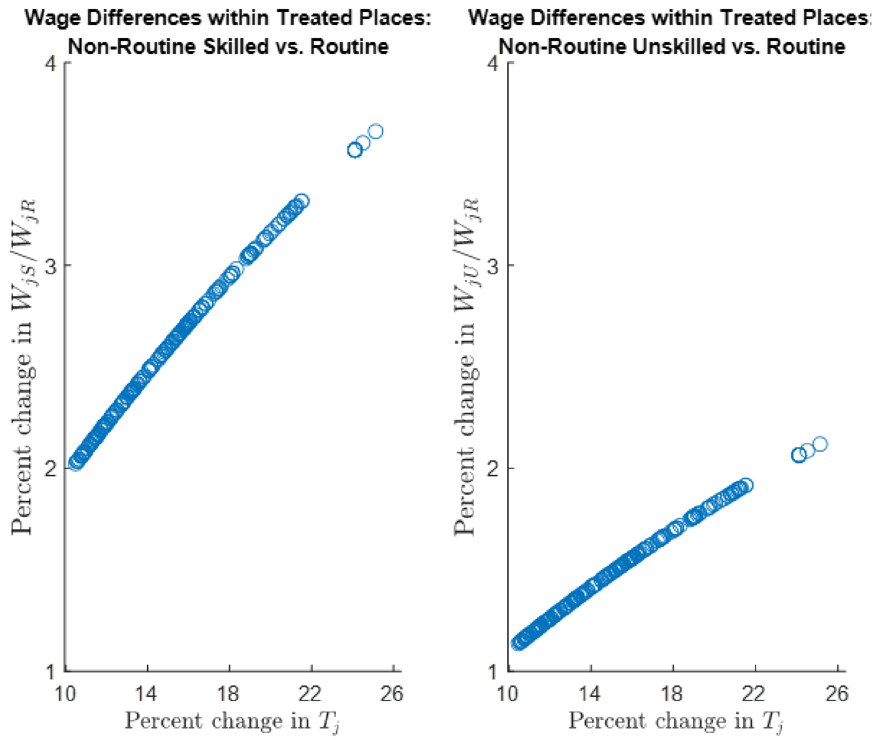
Note: This table shows the implication of place-based policies on spatial wage inequality by occupation. The second column presents spatial wage inequality as observed in the data from 1980, while the third column indicates spatial wage inequality under the policy experiment, where the TFP of each commuting zone within the first tercile of the distribution increases to match the level of the 75th percentile.

Table 2.2 shows spatial wage inequality, measured as coefficients of variation of real wages across commuting zones under two scenarios. The second column presents spatial wage inequality as observed in the data from 1980, while the third column indicates spatial wage inequality under the policy experiment. My model predicts that the place-based policy decreases spatial wage inequality for each occupation by about 20 percent. As the TFP of each commuting zone originally within the lowest third of the distribution increases, real wages in those places rise for each occupation. Consequently, spatial wage inequality falls for every occupation. The prediction from my model that place-based policies reduce spatial wage inequality by raising TFP in lagging places is consistent with the empirical findings of Becker, Egger, and von Ehrlich (2010), Jaworski and Kitchens (2019), and Kline and Moretti (2014).

Next, I examine how this policy is predicted to affect wage differences within treated places. Figure 2.8 shows predicted changes in wages for the non-routine occupations relative to the routine occupation under the policy experiment. The first panel plots changes in wages for the non-routine skilled occupation relative to the routine occupation. To illustrate, consider the commuting zone that originally has the smallest TFP. As its TFP rises to match the level of the 75th percentile in the distribution, TFP increases by 26 percent, and the wage for the non-routine skilled occupation relative to the routine occupation increases by 3.7 percent. Regarding the commuting zone whose initial TFP is equal to the first tercile of the distribution, as its TFP increases to match the level of the 75th percentile, TFP increases by 10 percent, and the wage for the non-routine skilled occupation relative to the routine occupation increases by 2 percent. The interpretation of the second panel in Figure 2.8 is exactly the same, except that it plots changes in wages for the non-routine unskilled occupation relative to the routine occupation. Wages for the non-routine unskilled occupation relative to the routine occupation rise, ranging from 1.1 percent to 2.2 percent.

Figure 2.8 shows that place-based policies raise wages for the non-routine occupations relative to the routine occupation, thus affecting wage differences within places. It also shows that the larger the increases in TFP in the treated places, the larger the increases in wages for the non-routine occupations relative to the routine occupation.

Figure 2.8: Implication of Place-Based Policy on Wage Differences within Places



Note: This figure shows the implication of place-based policies on wage differences within treated places. The first panel shows the predicted changes in wages for the non-routine skilled occupation relative to the routine occupation, while the second panel shows the predicted changes in wages for the non-routine unskilled occupation relative to the routine occupation, under the policy experiment where the TFP of each commuting zone within the first tercile of the distribution increases to match the level of the 75th percentile.

In summary, my model implies that place-based policies entail a trade-off between wage differences across places and wage differences within places. By raising TFP in lagging places, place-based policies reduce wage differences across places, as can be seen in Table 2.2. This is an intended consequence of place-based policies. However, while place-based policies raise wages for all occupations in the treated places, wages for the non-routine occupations increase more, as can be seen in Figure 2.8. This affects wage differences within places, which is an unintended consequence of place-based policies.

2.4 Conclusion

In my dissertation, I develop a quantitative spatial model in which automation has differential effects on spatial wage inequality across occupations. In my model, automation, driven by the fall in the price of machines, increases spatial wage inequality for the non-routine occupations and decreases it for the routine occupation through three forces: spatial differences in TFP; the supply of machines being more elastic than that of labor; and machines substituting for the routine occupation while complementing the non-routine occupations. According to these forces, places with high TFP are high-wage and machine-intensive places. As the price of machines falls, machines are adopted disproportionately in high-TFP places. This reduces spatial wage inequality for the routine occupation and increases it for the non-routine occupations since machines substitute for the routine occupation, while complementing the non-routine occupations. According to my quantitative spatial model, automation alone explains about 30 percent of the observed changes in spatial wage inequality. My model highlights the importance of occupations, as which occupations technology replaces or complements is central to my mechanism. Existing literature that features skill groups cannot explain the differential trends in spatial wage inequality across occupations since the non-routine occupations are at the extremes of the skill distribution, while the routine occupation is in the middle of the skill distribution.

My dissertation has implications for place-based policies, which seek to foster economic development in lagging places and hence promote spatial wage equity. My model implies that place-based policies entail a trade-off between wage differences across places and wage differences within places. By raising TFP in lagging places, place-based policies reduce wage differences across places, which is an intended consequence of place-based policies. However, my model highlights an unintended consequence of place-based policies: while place-based policies raise wages for all occupations in treated places, wages for the non-routine occupations increase more, thus affecting wage differences within places. This is because Hicks-neutral technical progress in the treated places has non-neutral effects across occupations in equilibrium due to the difference in supply elasticity between machines and labor.

The above discussions leave room for two questions for future research. First, what would be the effect of recent advancements in technology, such as artificial intelligence, on spatial wage inequality? To answer this question, we need more information on which jobs currently performed by human labor are being replaced or complemented by new technology. Second, how can we design place-based policies that reduce spatial wage inequality without affecting wage differences within places? This highlights the importance of "people-based" policies—specifically, education or training that allows workers to transition out of the routine occupation, in addition to place-based policies. If policymakers prioritize wage equity, I believe they should promote wage equity across occupations in addition to wage equity across places.

Chapter 3

Trade, Sorting across Occupations, and Wage Inequality

3.1 Introduction

A growing body of research shows that a worker's occupation plays a key role in shaping winners and losers from international trade. The force at play is that occupation-specific human capital accumulation plays a crucial role in wage determination (Kambourov and Manovskii 2009b). While international trade induces worker reallocation across industries and occupations, wage changes are largely driven by occupational transitions rather than industry transitions. Traiberman (2019) builds a structural model showing that a majority of the dispersion in wage effects from international trade is explained by occupation of employment. Reduced-form analyses support this conclusion. Ebenstein et al. (2014) find that occupational, rather than industry-level, exposure to international trade leads to wage losses as workers lose occupation-specific human capital when switching occupations. Utar (2018) finds that workers with footloose human capital experience no wage losses, as they transition smoothly across occupations in response to international trade.

In my dissertation, I study an additional channel through which occupations matter in determining winners and losers from international trade. While previous literature emphasizes occupation-specific human capital accumulation, I focus on how workers with different skill levels sort into occupations based on their comparative advantage (Gibbons et al. 2005).[1] The channel I show in my dissertation is that when international trade raises the price premium of an occupation, only workers with a comparative advantage in that occupation benefit. To illustrate, consider an economy with workers of different skill types, which are unobservable to the econometrician. High-skilled workers have a comparative advantage in one occupation, while less-skilled workers have a comparative advantage in the other. The resulting occupational wage premium reflects sorting by skill, which is unobserved to the econometrician. Suppose there are two goods, one of which is intensive in the skill-intensive occupation. When a skill-abundant country engages in trade with a skill-scarce country, trade raises the price of the skill-intensive good, which in turn raises the price of the skill-intensive occupation and lowers the price of the other occupation. As a result, more high-skilled workers choose the skill-intensive occupation—the one with the wage premium. However, less-skilled workers do not, despite the rise in the wage of the skill-intensive occupation and the decline in the wage of the less skill-intensive occupation, because they have a comparative advantage in the less skill-intensive occupation. Due to occupational sorting, workers with a comparative advantage in the skill-intensive occupation benefit, while those in the other occupation lose from international trade.

To test this mechanism, I construct a sample of US workers displaced from manufacturing industries between 1993 and 2007, using data from the Displaced

[1] Gibbons et al. (2005) show that workers sort into occupations based on skill, which is unobservable to the econometrician. They find that occupational wage differentials are largely due to unobserved worker skills, while industry wage differentials are not. The mixed results they find for industry wage differentials align with previous literature (Gibbons and Katz 1992; Krueger and Summers 1988). Since my dissertation focuses on sorting by skill, I use occupations instead of industries to illustrate the comparative advantage mechanism.

Worker Supplement. This dataset provides information on workers' pre-displacement and re-employment occupations. I measure occupational wage premia and Chinese import penetration by pre-displacement occupation, drawing from the March Current Population Survey, UN Comtrade, and the NBER-CES Manufacturing Industry Database. I document two facts. First, I find that an increase in Chinese import penetration in a worker's pre-displacement occupation (i.e., exploiting variation in Chinese import penetration across pre-displacement occupations) is associated with higher re-employment occupational wage premia only for workers with high pre-displacement occupational wage premia. That is, these workers transition into even higher-wage occupations, while workers from low-wage occupations do not experience similar upward mobility. Second, I find that an increase in Chinese import penetration in a worker's pre-displacement occupation is associated with higher wages for those who switch to higher-wage occupations and lower wages for those who do not. That is, trade-induced wage gains accrue to workers who successfully transition into high-wage occupations, while others experience wage declines.

The remainder of the paper is organized as follows. Section 3.2 describes where my dissertation stands in the literature. Section 3.3 presents the data and facts. Section 3.4 discusses my model. Section 3.5 concludes the paper.

3.2 Literature

My dissertation contributes to the literature in three ways. First, I show how international trade affects occupational sorting. A growing body of research embeds the comparative advantage of heterogeneous workers (Roy 1951) into an international context. This literature employs Roy models to impose partial mobility restrictions on workers across sectors and examines the effects of globalization, mostly in static settings. Ohnsorge and Trefler (2007) and Costinot and Vogel (2010) develop theoretical frameworks and explore their implications for international trade. Other studies construct structural models to analyze the effect of international trade on the US (Galle, Rodríguez-Clare, and Yi 2023), Brazil (Adão 2016), and multiple countries (Lee 2020). Meanwhile, R. Liu and Trefler (2019) examine the effect of offshoring on the US. A few papers extend Roy models to dynamic settings: Dix-Carneiro (2014) and Traiberman (2019) develop dynamic models in which workers accumulate industry-specific and occupation-specific human capital, respectively, within the Roy framework and the international trade context. The closest paper to mine is Traiberman (2019), as in both his paper and mine, workers sort into occupations based on skill, which is unobservable to the econometrician. However, he does not analyze how international trade affects occupational switching. My dissertation's contribution is to show that workers with a comparative advantage in skill-intensive occupations switch to higher-wage occupations in response to international trade, while other workers do not.

Second, I show that occupation plays a role in determining winners and losers from international trade through occupational sorting. The question of who benefits and who is hurt by trade has been a long-standing topic in international trade. In the canonical neoclassical Heckscher-Ohlin model, the Stolper-Samuelson theorem states that the abundant factor benefits, while the scarce factor loses, in each trading country. On the other hand, some papers suggest that the industry of employment plays an important role in deciding winners and losers from trade (Adão 2016; Artuç, Chaudhuri, and McLaren 2010; Artuç and McLaren 2015; Attanasio, Goldberg, and Pavcnik 2004; Autor et al. 2014; Lee 2020; Pierce and Schott 2016; Revenga 1992).

Meanwhile, other studies emphasize the role of the region of residence (Autor, Dorn, and Hanson 2013; Caliendo, Dvorkin, and Parro 2019; Dix-Carneiro and Kovak 2017; Galle and Lorentzen 2024; Galle, Rodríguez-Clare, and Yi 2023; Kovak 2013; Topalova 2007). Finally, the strand of literature most relevant to my dissertation shows that occupation plays a key role in determining who benefits and who is hurt by trade (Ebenstein et al. 2014; Traiberman 2019; Utar 2018). These papers focus on occupation-specific human capital accumulation, which plays a key role in wage determination; thus, occupational switching explains much of the dispersion in wage effects from trade. My contribution is to highlight an additional channel through which occupation influences the determination of winners and losers from trade: workers with a comparative advantage in skill-intensive occupations benefit by sorting into higher-wage occupations, while other workers lose.

Third, my dissertation relates to a large body of literature on occupations. One strand of research examines how different occupations involve distinct tasks (Autor, Levy, and Murnane 2003; Autor and Price 2013; Deming 2017; García-Couto 2025), finding that the automation and offshoring of middle-wage occupations—characterized by routine tasks——lead to polarization (Acemoglu and Autor 2011; Autor and Dorn 2013; Ebenstein et al. 2014; Hummels et al. 2014). Another strand emphasizes the comparative advantage of workers in different occupations, where unobserved productivities are modeled as either Fréchet or log-supermodular. The comparative advantage framework is used to study occupational choice problems (Hsieh et al. 2019; Lagakos and Waugh 2013). It is also used to understand occupational wage premia—the differences in wages among observationally equivalent workers, except for occupations. Whereas occupational wage premia capture sorting based on unobservable skills, industry wage premia do not (Gibbons and Katz 1992;

Gibbons et al. 2005; Krueger and Summers 1988). Finally, some papers show that occupation-specific human capital accumulation plays a crucial role in wage determination (Gathmann and Schönberg 2010; Kambourov and Manovskii 2009b). Kambourov and Manovskii (2009a) show that occupational mobility and wage inequality are closely linked through occupation-specific human capital. In my dissertation, I focus on occupational wage premia among these occupational features and show how certain workers switch to high-wage occupations in response to international trade, while others do not.

3.3 Data and Facts

3.3.1 Data

I use the Displaced Worker Supplement (DWS), a supplement to the Current Population Survey (CPS), conducted from 1996 to 2008 (Flood et al. 2021). The 1996–2008 DWS was completed by individuals displaced within the three years preceding each survey (i.e., from 1993 to 2007).[2] The DWS provides repeated cross-sections of displaced workers, containing information on both the year of displacement and the year of the survey. I obtain two types of information from the DWS. The first type concerns displacement: the year of displacement, the occupation and industry from which the worker was displaced, and years of tenure on the lost job. The second type pertains to the time of the survey (i.e., not the time of displacement): the survey year, re-employment status, re-employment occupation, weekly wage (if re-employed), education, and demographics. I examine

[2]DWS defines a displaced worker as someone who lost their job due to 1) a plant or company closing or relocating, 2) insufficient work, or 3) the elimination of their position or shift.

how Chinese import penetration affects occupational choice and wages using repeated cross-sections of the DWS. Hence, I construct a sample of individuals displaced from manufacturing industries between 1993 and 2007 who were re-employed at the time of the survey. My sample consists of 3,514 observations.

Next, I describe how I compute occupational wage premia. I use the March CPS surveys from 1993 to 2007, which use the 1990 Census Occupation Classification. Since the data span from 1993 to 2007, I use a balanced panel of three-digit occupation classifications from this period, obtained from Autor and Dorn (2013). I estimate coefficients on occupation fixed effects in a Mincerian wage equation.[3] These coefficients represent occupational wage premia. The occupational wage premium for an occupation is an estimate of the difference between the log wage of that occupation and that of the baseline occupation. If the wage premium for an occupation is 0.5, workers in that occupation earn 65 percent ($\approx \exp(0.5) - 1$) more than those in the baseline occupation. Gibbons et al. (2005) build a model in which workers of different skill types have comparative advantages across different occupations, and show that occupational wage premia in the data—wage differences across occupations holding observables constant—are largely driven by occupational sorting based on skills, which are unobservable to the econometrician. Following Gibbons et al. (2005), I interpret occupational wage premia as reflecting sorting by unobservable skills. I match occupational wage premia to the DWS sample by pre-displacement and re-employment occupation.

Finally, I measure Chinese import penetration for each occupation and year of displacement, following Ebenstein et al. (2014) and Traiberman (2019). This process involves two steps. In the first step, I obtain US manufacturing imports

[3]I regress log weekly wage on years of education, experience, experience squared, race, sex, state of residence fixed effects, year fixed effects, industry fixed effects, and occupation fixed effects.

and exports for the years 1993–2007 at the six-digit Harmonized System product level from UN Comtrade. The DWS sample follows the three-digit 1990 Census Industry Classification (CIC), so I use concordances provided by the World Integrated Trade Solution to match trade data to the 1990 CIC. Additionally, I retrieve US manufacturing shipments from 1993 to 2007 at the 1990 CIC level from the NBER-CES Manufacturing Industry Database. I then compute Chinese import penetration at the 1990 CIC for each year between 1993 and 2007. Let PEN_{jt} denote the Chinese import penetration of industry j, classified under the 1990 CIC, in year t. PEN_{jt} is given by:

$$PEN_{jt} = \frac{M_{jt}^{CHN}}{Y_{jt} + M_{jt} - X_{jt}}, \tag{3.1}$$

where M_{jt}^{CHN} denotes US imports from China, and the denominator represents US absorption in industry j in year t, since Y refers to US shipments, M to US imports, and X to US exports. In the second step, I compute Chinese import penetration at the occupation level for each year from 1993 to 2007. I assume that occupation-specific import penetration depends on the distribution of workers in an occupation across industries in 1992 (i.e., one year before the analysis period, 1993–2007). I use the 1992 March CPS data to obtain the number of workers employed in occupation k and industry j, denoted $N_{kj,1992}$, and the total number of workers in occupation k in 1992, denoted $N_{k,1992}$. Chinese import penetration for occupation k in year t, denoted PEN_{kt}, is given by:

$$PEN_{kt} = \sum_{j} \frac{N_{kj,1992}}{N_{k,1992}} PEN_{jt}. \tag{3.2}$$

I match the computed Chinese import penetration to the DWS sample for each occupation in each year of displacement. Once matched, I denote PEN_k as the Chinese import penetration for *pre-displacement* occupation k, with k already capturing the year of displacement.

3.3.2 Facts

Using the datasets described above, I show two facts. First, exploiting variation in Chinese import penetration across pre-displacement occupations, I find that an increase in Chinese import penetration in a worker's pre-displacement occupation is associated with higher re-employment occupational wage premia, but only for workers with high pre-displacement occupational wage premia. To show this first fact, I run the following regression:

$$WP_{ik'} = \gamma_0 + \gamma_1 PEN_k + \gamma_2 WP_{ik} + \gamma_{12} PEN_k \times WP_{ik} + Z_i \phi + \epsilon_{ikk'}, \tag{3.3}$$

where i denotes a worker, and k and k' represent the pre-displacement and re-employment occupations, respectively. Accordingly, WP_{ik} and $WP_{ik'}$ represent worker i's pre-displacement and re-employment occupational wage premia, respectively. Next, PEN_k represents Chinese import penetration in pre-displacement occupation k. The vector Z_i consists of individual characteristics: years of education, demographics (age, female dummy, white dummy, and married dummy), state of residence, years of tenure on the lost job, year of re-employment, and year of displacement. Chinese import penetration coefficients are identified through variation in Chinese import penetration across pre-displacement occupations. Finally, $\epsilon_{ikk'}$ represents an error term. I assume that errors are correlated within pre-displacement occupations but uncorrelated across them. Accordingly, I cluster standard errors by pre-displacement occupation.

However, observed Chinese import penetration in Equation 3.3 may partially reflect US import demand shocks, in addition to China's export supply shock. To isolate the China supply-driven component in US imports, I follow Autor et al. (2014) and

Table 3.1: Chinese Import Penetration and Re-employment Occupational Wage Premia

Dependent Variable: Re-employment occupational wage premia	
Chinese import penetration	-0.9684^{**}
	(0.4791)
Pre-displacement occupational wage premia	0.3874^{***}
	(0.0329)
Interaction	1.0092^{**}
	(0.4131)
R^2	0.4310
Observations	3,514

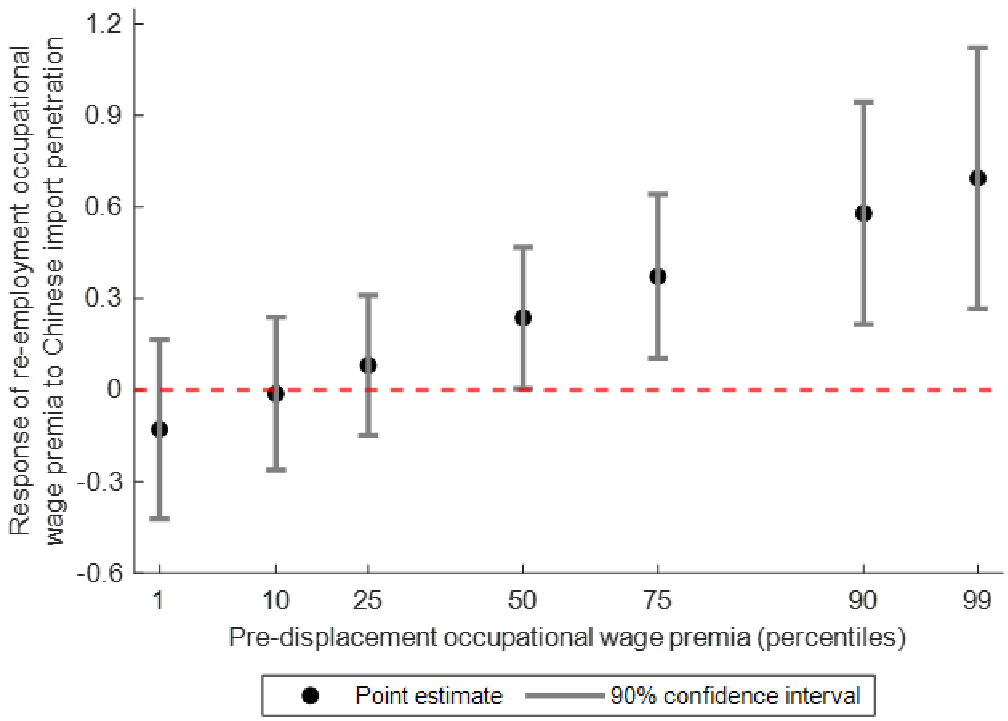
Note: The regression controls for years of education, demographics (age, female dummy, white dummy, and married dummy), state of residence, years of tenure on the lost job, year of re-employment, and year of displacement. Standard errors in parentheses are clustered by pre-displacement occupation. ** and *** denote statistical significance at the 5 percent and 1 percent levels, respectively. First-stage F-statistics for Chinese import penetration and the interaction term are 61.6116 and 107.1160, respectively.

construct an instrument as follows. In Equation 3.1, I first replace imports from China to the US (i.e., M_{jt}^{CHN}) with imports from China to non-US high-income countries for each year t, constructing a new industry-level Chinese import penetration measure.[4] Second, I plug this new measure into Equation 3.2 to obtain the occupation-level measure for each year t. Finally, I match the measure from the second step to the DWS sample for each occupation in each year of displacement. Once matched, I denote $P\tilde{E}N_k$ as an instrument for the Chinese import penetration for pre-displacement occupation k, with k capturing the year of displacement. I instrument PEN_k using $P\tilde{E}N_k$, and $PEN_k \times WP_{ik}$ using $P\tilde{E}N_k \times WP_{ik}$.

Table 3.1 presents the coefficients of interest estimated in Equation 3.3 using two-stage

[4] As outlined by Autor et al. (2014), non-US high-income countries are Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland. The rationale behind the instrument is that, while these countries are similarly exposed to China's export supply shock, import demand shocks are weakly correlated across them.

Figure 3.1: Response of Re-employment Occupational Wage Premia to Chinese Import Penetration by Pre-displacement Occupational Wage Premia



Note: This figure shows the response of re-employment occupational wage premia to Chinese import penetration evaluated by different values of pre-displacement occupational wage premia.

least squares. The response of re-employment occupational wage premia to Chinese import penetration, controlling for other variables, is given by $\hat{\gamma}_1 + \hat{\gamma}_{12} WP_{ik}$. While the coefficient for Chinese import penetration ($\hat{\gamma}_1$) is negative, the coefficient for the interaction term between Chinese import penetration and pre-displacement occupational wage premia ($\hat{\gamma}_{12}$) is positive. In particular, an increase in Chinese import penetration in a worker's pre-displacement occupation is associated with higher re-employment occupational wage premia for workers with disproportionately high pre-displacement occupational wage premia. This is shown in Figure 3.1, where I evaluate the response of re-employment occupational wage premia to Chinese import penetration based on different values of pre-displacement occupational wage premia. For example, an increase in Chinese import penetration is associated with higher re-employment occupational wage premia for workers at the 50th percentile of pre-displacement occupational wage premia, with a precisely estimated point estimate of 0.2369. For workers with even higher pre-displacement occupational wage premia, Chinese import penetration is associated with even higher re-employment occupational wage premia. Conversely, for workers with low pre-displacement occupational wage premia at the first, tenth, and twenty-fifth percentiles, point estimates are smaller (even negative at the first and tenth percentiles) and are imprecisely estimated. Table 3.1 and Figure 3.1 illustrate how workers reallocate across occupations in response to Chinese import penetration: workers from high-wage pre-displacement occupations transition into even higher-wage occupations, while workers from low-wage pre-displacement occupations do not experience similar upward mobility.

While the first fact pertains to workers' reallocation across occupations in response to Chinese import penetration, the second fact highlights its association with wages. I show that an increase in Chinese import penetration is associated with higher wages for workers who switch to high-wage occupations but with lower wages for those who do not. To show this, I first define two groups of workers: those at or above the 50th percentile in pre-displacement occupational wage premia (i.e., workers who switch to higher-wage occupations in response to Chinese import penetration) and the remaining workers. I then run the following regression for the two separate groups:

$$logW_{i,t+1} = \delta + \theta PEN_k + Z_i\zeta + u_{ik,t+1},$$

where $W_{i,t+1}$ denotes worker i's wage in the year of re-employment $t+1$. Next, PEN_k represents Chinese import penetration in pre-displacement occupation k, and

I use $\tilde{PEN}_k$ as an instrument for PEN_k, as above. The vector Z_i consists of the same individual characteristics controlled for in Equation 3.3. The coefficient for Chinese import penetration is identified through variation across pre-displacement occupations. $u_{ik,t+1}$ denotes the error term, which I cluster by pre-displacement occupation.

Table 3.2: Chinese Import Penetration and Re-employment Wage

	(1)	(2)
Dependent Variable: Re-employment wage (log)		
Chinese import penetration	0.5738	-1.3971^{**}
	(0.9938)	(0.6686)
R^2	0.1771	0.1527
Observations	1,762	1,752

Note: The sub-sample in column 1 consists of workers at or above the 50th percentile in pre-displacement occupational wage premia, while the sub-sample in column 2 consists of the remaining workers. The regression controls for years of education, demographics (age, female dummy, white dummy, and married dummy), state of residence, years of tenure on the lost job, year of re-employment, and year of displacement. Standard errors in parentheses are clustered by pre-displacement occupation. ** denotes statistical significance at the 5 percent level. First-stage F-statistics are 26.3183 in column 1 and 93.4492 in column 2, respectively.

Table 3.2 presents the estimated coefficients of interest for the two separate groups. The sub-sample in column 1 consists of workers at or above the 50th percentile in pre-displacement occupational wage premia, while the sub-sample in column 2 consists of the remaining workers. In column 1, an increase in Chinese import penetration is associated with higher re-employment wages, although the coefficient is imprecisely estimated. In contrast, in column 2, an increase in Chinese import penetration is associated with lower re-employment wages. These results suggest distributional effects of Chinese import penetration: workers who switch to higher-wage occupations win, while the remaining workers lose.

3.4 Model

So far, I presented two facts. The first fact is how Chinese import penetration is associated with labor reallocation across occupations: workers from high-wage pre-displacement occupations transition into even higher-wage occupations, while workers from low-wage pre-displacement occupations do not experience similar upward mobility. The second fact is how Chinese import penetration is associated with wages: Chinese import penetration is associated with higher wages for workers who switch to high-wage occupations but with lower wages for those who do not. In this section, I propose a model to study how trade influences occupational switching, thereby affecting winners and losers from trade. The key forces at play are differences in skill endowments between countries and the comparative advantage of different skill types across occupations.

3.4.1 Environment

Preferences. There are two countries, indexed by $n \in \{N, S\}$. In each country, there are two types of workers, H and L, where H denotes high-skilled workers and L denotes less-skilled workers. Country n is endowed with a measure $\overline{H}^n$ of high-skilled workers and a measure $\overline{L}^n$ of less-skilled workers. In my model, the only difference across countries is the endowment of high-skilled and less-skilled workers. In each country, each worker consumes goods X and Y and supplies one unit of labor inelastically. Preferences are homothetic and identical across worker types and countries, represented by the utility function

$$\mathcal{U} = Q_X^B Q_Y^{1-B} \text{ where } B \in (0,1).$$

Production of goods. In each country, perfectly competitive firms producing good $j \in \{X, Y\}$ require occupations 1 and 2. The technologies for goods X and Y are identical across countries and are given by

$$Q_X = Q_{X1}^{\alpha} Q_{X2}^{1-\alpha}, \quad Q_Y = Q_{Y1}^{\beta} Q_{Y2}^{1-\beta} \text{ where } 0 < \beta < \alpha < 1,$$

where Q_{jk} denotes the amount of occupation $k \in \{1,2\}$ required for the production of good $j \in \{X, Y\}$. In my model, $\alpha > \beta$, indicating that good X is intensive in occupation 1.

Production of occupations. In each country, perfectly competitive firms producing occupation $k \in \{1, 2\}$ require labor from workers. The technology for each occupation k is the same across countries, given by

$$Q_k = A_{Hk} H_k + A_{Lk} L_k, \quad k \in \{1, 2\},$$

where H_k and L_k indicate the number of high-skilled and less-skilled workers employed in occupation k, respectively. A_{Hk} and A_{Lk} represent the productivity of high-skilled and less-skilled workers in occupation k, respectively. While high-skilled workers have an absolute advantage in both occupations:

$$A_{H1} > A_{L1}, \quad A_{H2} > A_{L2},$$

high-skilled workers have a comparative advantage in occupation 1:

$$A_{H1}/A_{L1} > A_{H2}/A_{L2}. \tag{3.4}$$

Labor supply. In each country, each worker of skill type $i \in \{H, L\}$ is offered the value of her marginal product, $P_k A_{ik}$, by the representative firm producing occupation k, where P_k denotes the price of occupation k. The Roy mechanism in my model is that workers choose occupations based on their comparative advantage, as given in Equation 3.4. That is, each worker of skill type i chooses the occupation that maximizes her wage, W_i:

$$W_i = \max_{k \in \{1,2\}} \{P_k A_{ik}\}, \quad i \in \{H, L\}. \tag{3.5}$$

Using her wage, each worker consumes goods X and Y at prices P_X and 1, respectively (i.e., good Y is chosen as the numeraire).

3.4.2 Autarky Equilibrium

Demand for goods. In each country $n \in \{N, S\}$, the demand for goods X and Y are given by

$$P_X^n Q_X^n = B I^n, \quad Q_Y^n = (1 - B) I^n, \tag{3.6}$$

where I^n denotes the total income in country n (i.e., $I^n = W_H^n \overline{H}^n + W_L^n \overline{L}^n$).

Demand for occupations. In each country n, the cost minimization problem for competitive firms producing goods X and Y leads to the demand for occupations

$$Q_1^n = \frac{\alpha P_X^n Q_X^n + \beta Q_Y^n}{P_1^n}, \quad Q_2^n = \frac{(1-\alpha) P_X^n Q_X^n + (1-\beta) Q_Y^n}{P_2^n}. \tag{3.7}$$

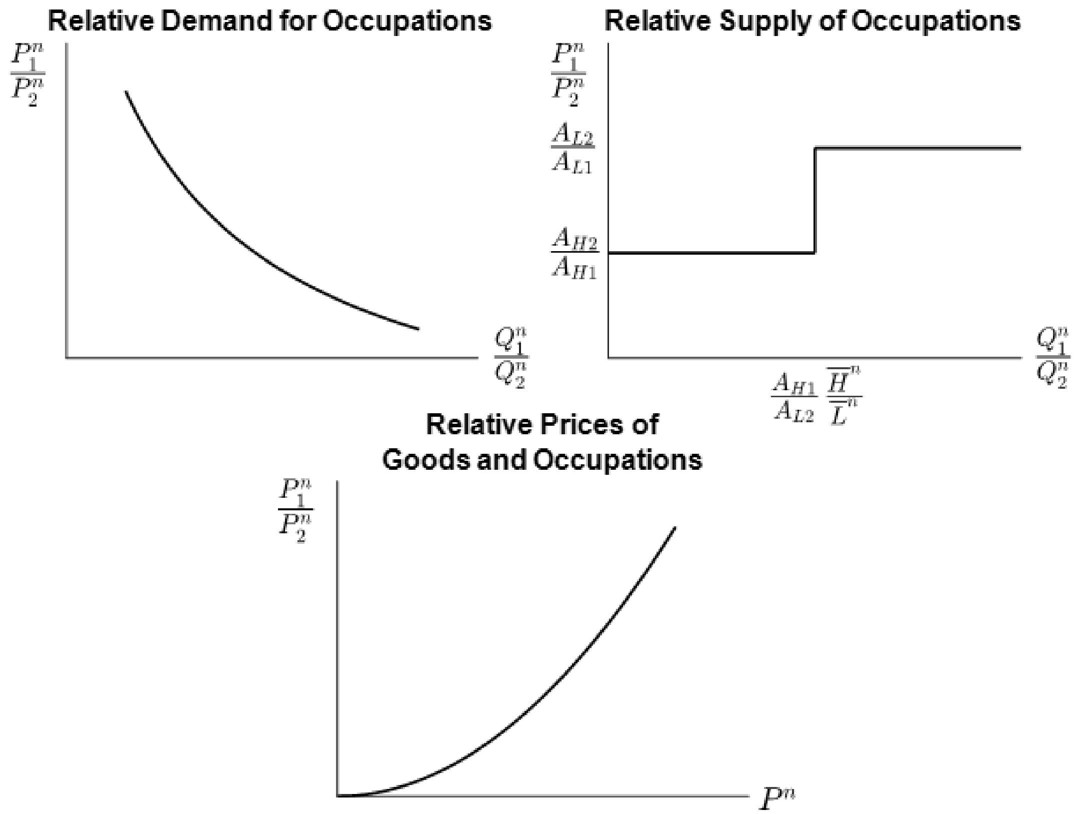
I combine Equation 3.6 and Equation 3.7 using the market clearing condition for goods to derive the demand for occupation 1 relative to occupation 2

$$\frac{Q_1^n}{Q_2^n} = \psi \Big(\frac{P_1^n}{P_2^n} \Big)^{-1} \text{ where } \psi \equiv \frac{\alpha B + \beta(1-B)}{(1-\alpha)B + (1-\beta)(1-B)}. \tag{3.8}$$

The demand for occupation 1 relative to occupation 2 is shown in the first figure in the top panel of Figure 3.2.

Supply of occupations. In each country n, the equation for comparative advantage (Equation 3.4) and the equation for labor supply (Equation 3.5) lead to the supply of occupation 1 relative to occupation 2, as shown in the second figure in the top panel of Figure 3.2. The two points shown on the vertical axis satisfy the inequality, $\frac{A_{H2}}{A_{H1}} < \frac{A_{L2}}{A_{L1}}$, which comes from the equation for comparative advantage (Equation 3.4): less-skilled workers have a comparative advantage in occupation 2. $\frac{A_{H2}}{A_{H1}}$ serves as the cutoff that determines the allocation of high-skilled workers across occupations: if $\frac{P_1^n}{P_2^n} < \frac{A_{H2}}{A_{H1}}$ (equivalently, $P_1^n A_{H1} < P_2^n A_{H2}$), high-skilled workers choose occupation 2, following the equation for labor supply (Equation 3.5). On the other hand, if $\frac{P_1^n}{P_2^n} = \frac{A_{H2}}{A_{H1}}$ (equivalently, $P_1^n A_{H1} = P_2^n A_{H2}$), high-skilled workers are indifferent between occupations. Finally, if $\frac{P_1^n}{P_2^n} > \frac{A_{H2}}{A_{H1}}$ (equivalently, $P_1^n A_{H1} > P_2^n A_{H2}$), high-skilled workers choose occupation 1. Similarly, $\frac{A_{L2}}{A_{L1}}$ is the cutoff which determines the allocation of less-skilled workers across occupations: if $\frac{P_1^n}{P_2^n} < \frac{A_{L2}}{A_{L1}}$, less-skilled workers choose occupation 2; if $\frac{P_1^n}{P_2^n} = \frac{A_{L2}}{A_{L1}}$, less-skilled workers are indifferent between

Figure 3.2: Relative Demand for and Supply of Occupations under Autarky



Note: The two figures in the top panel show the demand for and supply of occupation 1 relative to occupation 2 in country n. The figure in the bottom panel illustrates the relationship between the price of good X relative to good Y in country n (denoted P^n) and the price of occupation 1 relative to occupation 2 in country n $\left(\text{denoted } \frac{P_1^n}{P_2^n}\right)$.

occupations; if $\frac{P_1^n}{P_2^n} > \frac{A_{L2}}{A_{L1}}$, less-skilled workers choose occupation 1. Hence, if $\frac{P_1^n}{P_2^n} = \frac{A_{H2}}{A_{H1}}$, implying that $\frac{P_1^n}{P_2^n} < \frac{A_{L2}}{A_{L1}}$, high-skilled workers are indifferent between occupations, while less-skilled workers choose occupation 2. On the other hand, if $\frac{P_1^n}{P_2^n} \in \left(\frac{A_{H2}}{A_{H1}}, \frac{A_{L2}}{A_{L1}}\right)$, high-skilled workers choose occupation 1, while less-skilled workers choose occupation 2. This leads to the supply of occupation 1, $Q_1^n = A_{H1}\overline{H}^n$, and the supply of occupation 2, $Q_2^n = A_{L2}\overline{L}^n$, in country n. Finally, if $\frac{P_1^n}{P_2^n} = \frac{A_{L2}}{A_{L1}}$, implying that $\frac{P_1^n}{P_2^n} > \frac{A_{H2}}{A_{H1}}$, high-skilled workers choose occupation 1, while less-skilled workers are indifferent between occupations.

Zero-profit condition. In each country n, the zero-profit conditions for competitive firms producing goods X and Y are

$$\begin{aligned} P_X^n &= \tilde{\alpha}(P_1^n)^{\alpha}(P_2^n)^{1-\alpha} \quad \text{where} \quad \tilde{\alpha} \equiv \alpha^{-\alpha}(1-\alpha)^{\alpha-1}, \\ 1 &= \tilde{\beta}(P_1^n)^{\beta}(P_2^n)^{1-\beta} \quad \text{where} \quad \tilde{\beta} \equiv \beta^{-\beta}(1-\beta)^{\beta-1}, \end{aligned} \tag{3.9}$$

where good Y is chosen as the numeraire in each country under autarky. Combining the zero-profit conditions for country n yields

$$P^n = \frac{\tilde{\alpha}}{\tilde{\beta}}\left(\frac{P_1^n}{P_2^n}\right)^{\alpha-\beta},$$

where P^n denotes the price of good X relative to good Y in country n. Since $\alpha > \beta$, the figure in the bottom panel of Figure 3.2 shows a positive relationship between the price of occupation 1 relative to occupation 2 and the price of good X relative to good Y in country n.

Competitive equilibrium under autarky. Good Y is chosen as the numeraire in each country under autarky. Given parameters $\{\alpha, \beta, B, A_{H1}, A_{H2}, A_{L1}, A_{L2}, \overline{H}^n, \overline{L}^n\}$, the competitive equilibrium under autarky in country n consists of prices $\{P_X^n, P_1^n, P_2^n, W_H^n, W_L^n\}$ and allocations $\{Q_X^n, Q_Y^n, Q_1^n, Q_2^n, H_1^n, H_2^n, L_1^n, L_2^n\}$, such that workers and firms optimize given prices, and markets for good X and all occupations clear.

Figure 3.2 helps illustrate how the competitive equilibrium under autarky is attained. The two figures in the top panel determine the price of occupation 1 relative to occupation 2 in country n $\left(\text{i.e., } \frac{P_1^n}{P_2^n}\right)$. As $\frac{P_1^n}{P_2^n}$ is determined, the figure in the bottom panel pins down the price of good X relative to good Y in country n (i.e., P^n), which also pins down the price of good X in country n (i.e., P_X^n) since Y is the numeraire. Hence, Equation 3.9 determines the values of P_1^n and P_2^n. What plays a critical role in determining the competitive equilibrium in each country is relative skill abundance: $\frac{\overline{H}^n}{\overline{L}^n}$ shown on the horizontal axis in the second figure in the top panel. I discuss how different skill endowments across countries lead to different relative prices of occupations and, consequently, different labor allocations across countries in the following lemma.

Lemma 3.1 (Labor Allocations across Occupations under Autarky)**.** Suppose $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi \frac{A_{L2}}{A_{H2}}$ and $\frac{\overline{H}^S}{\overline{L}^S} \leq \psi \frac{A_{L1}}{A_{H1}}$. Then, under autarky,

(i) In country N, high-skilled workers are indifferent between occupations, while less-skilled workers are employed in occupation 2.

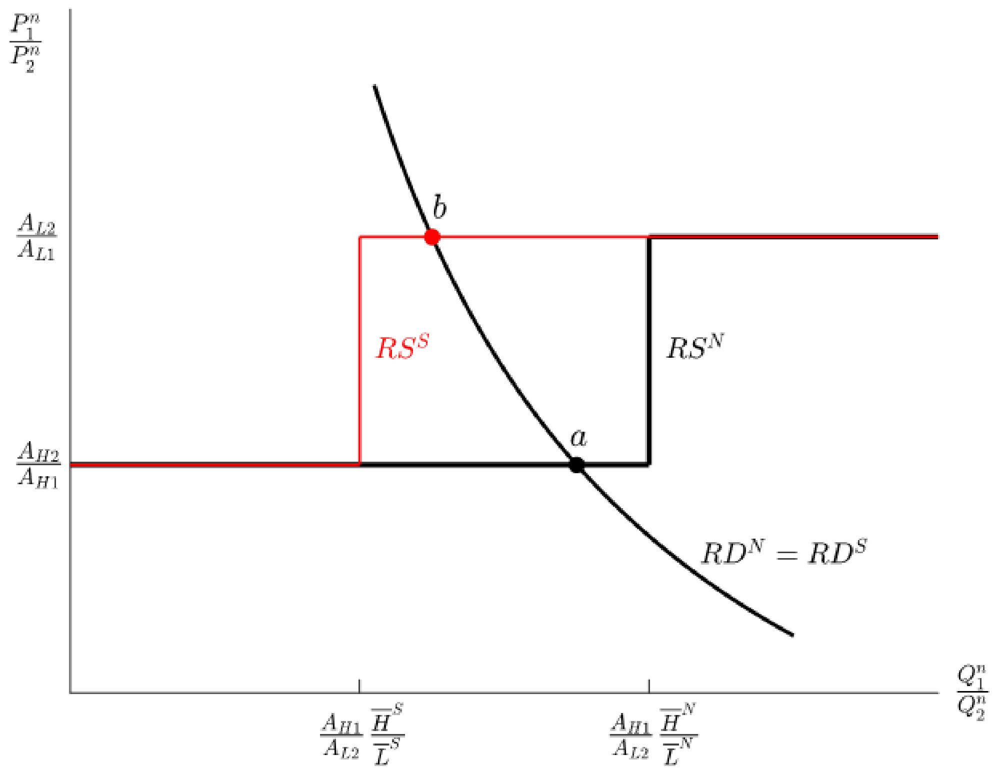
(ii) In country S, high-skilled workers are employed in occupation 1, while less-skilled workers are indifferent between occupations.

Proof. See Appendix C.1.

Intuitively, $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi \frac{A_{L2}}{A_{H2}}$ in Lemma 3.1 means that country N has a large number of high-skilled workers relative to less-skilled workers, while $\frac{\overline{H}^S}{\overline{L}^S} \leq \psi \frac{A_{L1}}{A_{H1}}$ means that country S has a large number of less-skilled workers relative to skilled workers. Note that the thresholds of relative skill abundance satisfy the inequality $\psi \frac{A_{L2}}{A_{H2}} > \psi \frac{A_{L1}}{A_{H1}}$, due to the comparative advantage equation (Equation 3.4). Since $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi \frac{A_{L2}}{A_{H2}}$ and $\frac{\overline{H}^S}{\overline{L}^S} \leq \psi \frac{A_{L1}}{A_{H1}}$, it follows that $\frac{\overline{H}^N}{\overline{L}^N} > \frac{\overline{H}^S}{\overline{L}^S}$.

Figure 3.3 shows how the cross-country difference in relative skill abundance affects the cross-country difference in the relative price of occupation under autarky. RS^N in black refers to the supply of occupation 1 relative to occupation 2 in country N, whereas RS^S in red refers to the supply of occupation 1 relative to occupation 2 in country S. RD represents the demand for occupation 1 relative to occupation 2, which is the same across countries, as shown in Equation 3.8. Skill abundance

Figure 3.3: Cross-Country Differences in Relative Skill Abundance and Occupational Prices under Autarky



Note: This figure shows the cross-country difference in the relative price of occupation under autarky, which stems from the cross-country difference in relative skill abundance: $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi \frac{A_{L2}}{A_{H2}}$ and $\frac{\overline{H}^S}{\overline{L}^S} \leq \psi \frac{A_{L1}}{A_{H1}}$. RS^N in black refers to the supply of occupation 1 relative to occupation 2 in country N, whereas RS^S in red refers to the supply of occupation 1 relative to occupation 2 in country S. RD represents the demand for occupation 1 relative to occupation 2, which is the same across countries.

in country N leads to the lower price of occupation 1 relative to occupation 2 $\left(\frac{P_1^N}{P_2^N} = \frac{A_{H2}}{A_{H1}}\right)$. Since $\frac{P_1^N}{P_2^N} = \frac{A_{H2}}{A_{H1}}$ (equivalently, $P_1^N A_{H1} = P_2^N A_{H2}$), high-skilled workers are indifferent between occupations. On the other hand, since $\frac{P_1^N}{P_2^N} < \frac{A_{L2}}{A_{L1}}$ (equivalently, $P_1^N A_{L1} < P_2^N A_{L2}$), less-skilled workers choose occupation 2. The exact opposite pattern holds for country S: skill scarcity in country S leads to the higher price of occupation 1 relative to occupation 2 $\left(\frac{P_1^S}{P_2^S} = \frac{A_{L2}}{A_{L1}}\right)$. At this relative price of occupation, high-skilled workers are employed in occupation 1, whereas less-skilled workers are indifferent between occupations.

Lemma 3.2 (Occupational Wage Premium)**.** The average wage for workers employed in occupation 1 is higher than that for those employed in occupation 2.

Proof. See Appendix C.2.

The intuition behind Lemma 3.2 is that workers sort into occupations based on skill. The average wage for occupation 1 is higher because high-skilled workers sort into occupation 1, given their comparative advantage in it. In particular, if skill is unobservable in the data, the occupational wage difference—holding observables constant—reflects sorting by skill that is unobservable to the econometrician.

3.4.3 Free-Trade Equilibrium

Throughout this section, I suppose that $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi \frac{A_{L2}}{A_{H2}}$ and $\frac{\overline{H}^S}{\overline{L}^S} \leq \psi \frac{A_{L1}}{A_{H1}}$. By Lemma 3.1, under autarky, high-skilled workers are indifferent between occupations, while less-skilled workers are employed in occupation 2 in country N. The exact opposite pattern holds for country S: high-skilled workers choose occupation 1, while less-skilled workers are indifferent between occupations. In this subsection, I describe how free trade between the two countries affects the occupational choices of workers and leads to wage inequality.

Zero-profit condition. In each country n, the zero-profit conditions for competitive firms producing goods X and Y are $P_X = \tilde{\alpha}(P_1^n)^{\alpha}(P_2^n)^{1-\alpha}$ and $1 = \tilde{\beta}(P_1^n)^{\beta}(P_2^n)^{1-\beta}$, where good Y is chosen as the numeraire.[5] In Appendix C.3, I show that under free trade, prices for each occupation are equalized across countries. Since the prices for

[5] I implicitly assume incomplete specialization, meaning that both goods X and Y are produced in both countries under free trade.

each occupation are equal across countries, the zero-profit conditions become

$$P_X = \tilde{\alpha}(P_1)^{\alpha}(P_2)^{1-\alpha}, \;\; 1 = \tilde{\beta}(P_1)^{\beta}(P_2)^{1-\beta}. \tag{3.10}$$

Combining two zero-profit conditions yields

$$P = \frac{\tilde{\alpha}}{\tilde{\beta}}\Big(\frac{P_1}{P_2}\Big)^{\alpha-\beta},$$

where P denotes the price of good X relative to good Y, which is the same across countries due to free trade. Since $\alpha > \beta$, the figure in the bottom panel of Figure 3.4 shows a positive relationship between the price of good X relative to good Y and the price of occupation 1 relative to occupation 2 under free trade.

Demand for goods. Under free trade, the demand for goods X and Y are given by

$$P_X Q_X = BI, \;\; Q_Y = (1-B)I, \tag{3.11}$$

where $Q_j = \sum_n Q_j^n$ for $j \in \{X, Y\}$, and I denotes the total income in the world economy $\Big(\text{i.e., } I = \sum_n \big(W_H \overline{H}^n + W_L \overline{L}^n\big)\Big)$. Under free trade, wages are equalized across countries for each occupation because the prices for each occupation are the same across countries.

Demand for occupations. Under free trade, the cost minimization problem for competitive firms producing goods X and Y leads to the demand for occupations:

$$Q_1 = \frac{\alpha P_X Q_X + \beta Q_Y}{P_1}, \;\; Q_2 = \frac{(1-\alpha) P_X Q_X + (1-\beta) Q_Y}{P_2}, \tag{3.12}$$

where $Q_k = \sum_n Q_k^n$ for occupation k. The demand for each occupation is summed across countries because the prices for each occupation are equal across countries. I combine Equation 3.11 and Equation 3.12 using the market clearing condition for goods to derive the demand for occupation 1 relative to occupation 2:

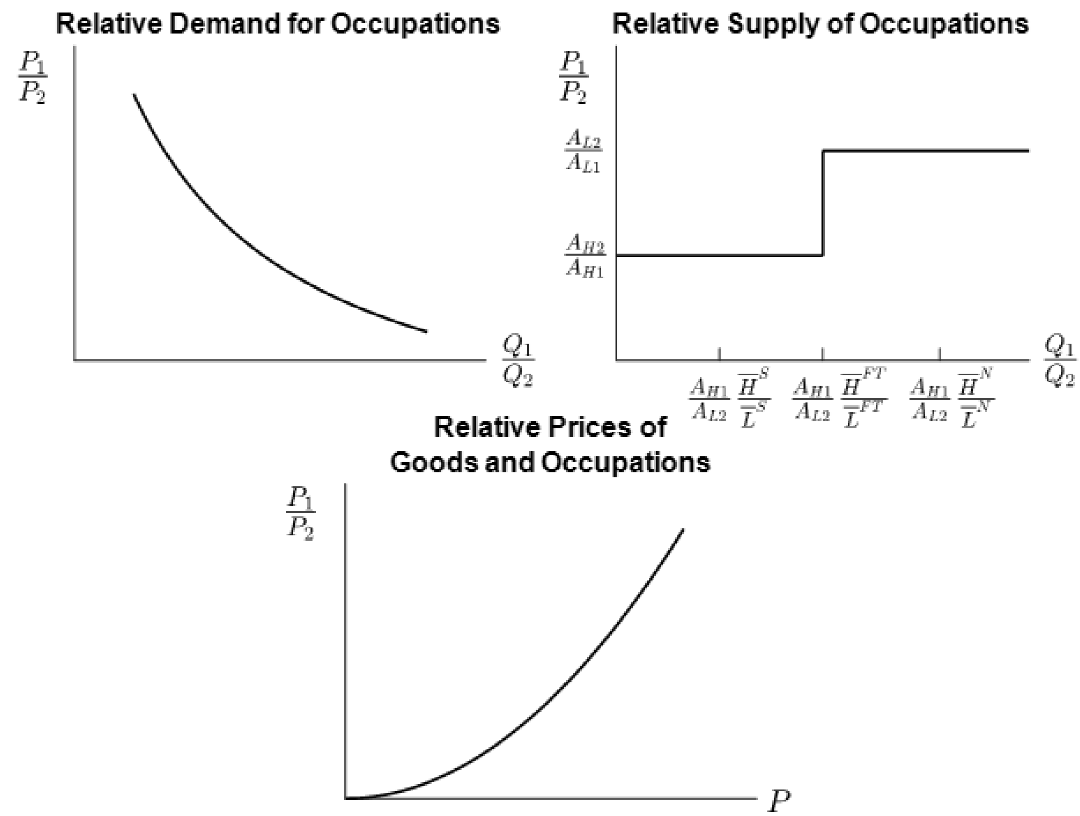
$$\frac{Q_1}{Q_2} = \psi\Big(\frac{P_1}{P_2}\Big)^{-1}.$$

The first figure in the top panel of Figure 3.4 shows the relative demand for occupation under free trade.

Supply of occupations. Under free trade, the supply of occupations is determined by the equation for comparative advantage (Equation 3.4) and the equation for labor supply (Equation 3.5), with prices equalized across countries for each occupation. The second figure in the top panel of Figure 3.4 shows the supply of occupation 1 relative to occupation 2 under free trade. On the horizontal axis, $\overline{H}^{FT}$ $\Big(\equiv \overline{H}^N + \overline{H}^S\Big)$ represents high-skilled workers in the world economy, while $\overline{L}^{FT}$ $\Big(\equiv \overline{L}^N + \overline{L}^S\Big)$ denotes less-skilled workers in the world economy. Hence, $\frac{\overline{H}^{FT}}{\overline{L}^{FT}}$ represents relative skill abundance in the world economy, which differs from the relative skill abundance in each country n, denoted as $\frac{\overline{H}^n}{\overline{L}^n}$. As shown previously, $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi\frac{A_{L2}}{A_{H2}}$ and $\frac{\overline{H}^S}{\overline{L}^S} \leq \psi\frac{A_{L1}}{A_{H1}}$, which I suppose throughout this section, imply that country N is skill-abundant $\Big($i.e., $\frac{\overline{H}^N}{\overline{L}^N} > \frac{\overline{H}^S}{\overline{L}^S}\Big)$. Hence, relative skill abundance in the world economy, $\frac{\overline{H}^{FT}}{\overline{L}^{FT}}$, falls within the range $\Big(\frac{\overline{H}^S}{\overline{L}^S}, \frac{\overline{H}^N}{\overline{L}^N}\Big)$.

The interpretation of the second figure in the top panel of Figure 3.4 is analogous to that of the counterpart in Figure 3.2. If $\frac{P_1}{P_2} = \frac{A_{H2}}{A_{H1}}$, implying that $\frac{P_1}{P_2} < \frac{A_{L2}}{A_{L1}}$, then in both countries, high-skilled workers are indifferent between occupations, while less-skilled workers choose occupation 2. On the other hand, if $\frac{P_1}{P_2} \in \big(\frac{A_{H2}}{A_{H1}}, \frac{A_{L2}}{A_{L1}}\big)$, then in both countries, high-skilled workers choose occupation 1, while less-skilled workers choose occupation 2. This leads to the supply of occupation 1, $Q_1 = A_{H1}\overline{H}^{FT}$, and the supply of occupation 2, $Q_2 = A_{L2}\overline{L}^{FT}$, in the world economy. Finally, if $\frac{P_1}{P_2} = \frac{A_{L2}}{A_{L1}}$, implying that $\frac{P_1}{P_2} > \frac{A_{H2}}{A_{H1}}$, then in both countries, high-skilled workers choose occupation 1, while less-skilled workers are indifferent between occupations.

Figure 3.4: Relative Demand for and Supply of Occupations under Free Trade



Note: The two figures in the top panel show the demand for and supply of occupation 1 relative to occupation 2 under free trade. The figure in the bottom panel illustrates the relationship between the price of good X relative to good Y under free trade (denoted P) and the price of occupation 1 relative to occupation 2 under free trade $\left(\text{denoted } \frac{P_1}{P_2}\right)$.

Competitive equilibrium under free trade. Good Y is chosen as the numeraire. Given the parameters $\{\alpha, \beta, B, A_{H1}, A_{H2}, A_{L1}, A_{L2}, \overline{H}^N, \overline{L}^N, \overline{H}^S, \overline{L}^S\}$, the competitive equilibrium under free trade consists of prices $\{P_X, P_1, P_2, W_H, W_L\}$ and allocations

$\{Q_X^n, Q_Y^n, Q_1^n, Q_2^n, H_1^n, H_2^n, L_1^n, L_2^n\}_{n\in\{N,S\}}$, such that workers and firms optimize given prices, and markets for good X and all occupations clear.

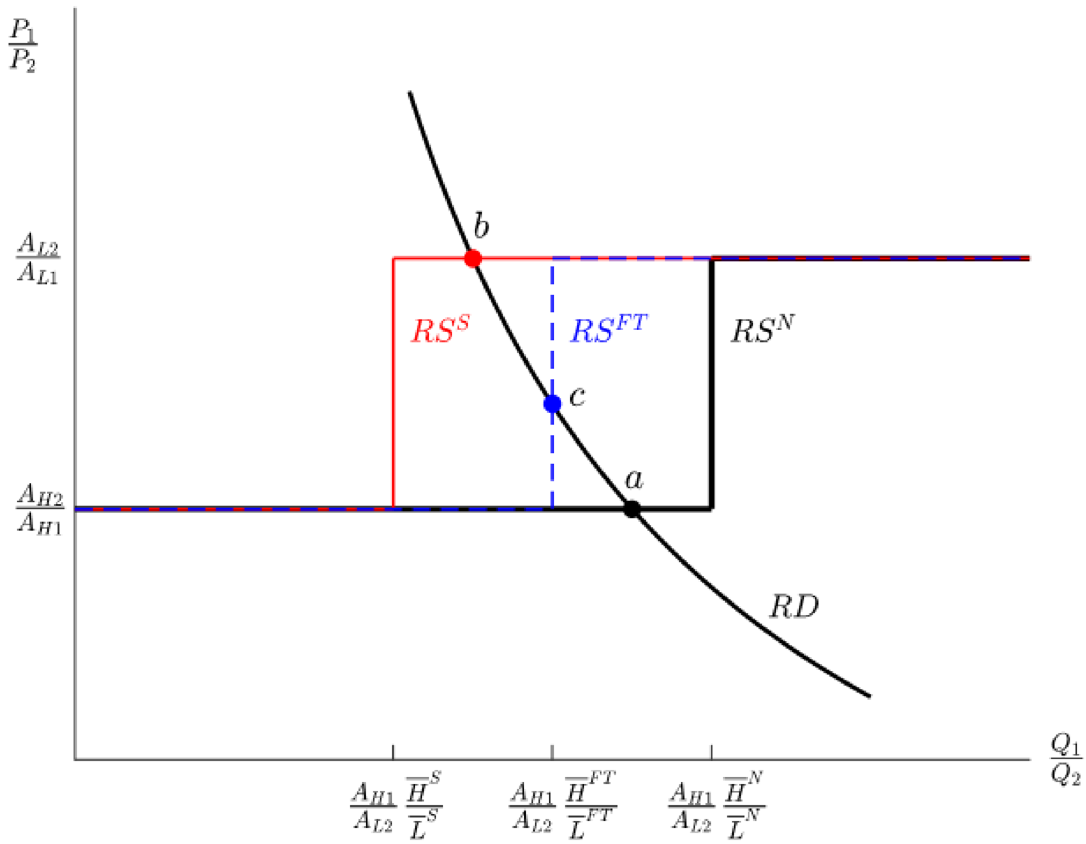
As in the autarky equilibrium, Figure 3.4 helps illustrate how the competitive equilibrium under free trade is attained. The two figures in the top panel determine the price of occupation 1 relative to occupation 2 $\left(\text{i.e., } \frac{P_1}{P_2}\right)$. As $\frac{P_1}{P_2}$ is determined, the figure in the bottom panel pins down the price of good X relative to good Y (i.e., P), which also pins down the price of good X (i.e., P_X) since Y is the numeraire. Hence, Equation 3.10 determines the values of P_1 and P_2.

Proposition 3.1 (Labor Reallocation across Occupations under Free Trade)**.** Suppose $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi\frac{A_{L2}}{A_{H2}}$, $\frac{\overline{H}^S}{\overline{L}^S} \leq \psi\frac{A_{L1}}{A_{H1}}$, and $\frac{\overline{H}^{FT}}{\overline{L}^{FT}} \in \left(\psi\frac{A_{L1}}{A_{H1}}, \psi\frac{A_{L2}}{A_{H2}}\right)$. Then,
(i) In country N, high-skilled workers who were employed in occupation 2 under autarky switch to occupation 1 under free trade, while less-skilled workers remain employed in occupation 2 under free trade.
(ii) In country S, less-skilled workers who were employed in occupation 1 under autarky switch to occupation 2 under free trade, while high-skilled workers remain employed in occupation 1 under free trade.

Proof. See Appendix C.4.

Figure 3.5 helps in understanding Proposition 3.1. The figure shows the demand for and supply of occupation 1 relative to occupation 2 where RS^{FT} in blue represents the supply of occupation 1 relative to occupation 2 under free trade. For comparison, RS^N in black and RS^S in red represent the counterpart under autarky for country N and S, respectively. RD denotes the demand for occupation 1 relative to occupation 2. Since $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi\frac{A_{L2}}{A_{H2}}$ and $\frac{\overline{H}^S}{\overline{L}^S} \leq \psi\frac{A_{L1}}{A_{H1}}$ from Proposition 3.1, in autarky, RD and RS_N intersect at a while RD and RS_S intersect at b. Now, $\frac{\overline{H}^{FT}}{\overline{L}^{FT}} \in \left(\psi\frac{A_{L1}}{A_{H1}}, \psi\frac{A_{L2}}{A_{H2}}\right)$ from Proposition 3.1 leads RS^{FT} to intersect RD at c. The intuition is that, while

Figure 3.5: Occupation Market under Free Trade vs. Autarky



Note: This figure shows the demand for and supply of occupation 1 relative to occupation 2. RS^{FT} in blue represents the supply of occupation 1 relative to occupation 2 under free trade. RS^N in black and RS^S in red represent the counterpart under autarky for country N and country S, respectively. RD denotes the demand for occupation 1 relative to occupation 2.

country N is skill-abundant and country S is skill-scarce under autarky, the relative skill abundance in the world economy lies in between. The change in the relative supply of occupations due to free trade increases the price of occupation 1 relative to occupation 2 in country N. Since $\frac{P_1}{P_2} > \frac{A_{H2}}{A_{H1}}$ (equivalently, $P_1 A_{H1} > P_2 A_{H2}$), high-skilled workers in country N choose occupation 1. In other words, workers in country N who were indifferent between occupations under autarky now switch to occupation 1 under free trade. However, despite the rise in the price of occupation 1 relative to occupation 2 in country N, less-skilled workers remain in occupation

2. This is because they have a comparative advantage in occupation 2. As a result, the inequality $\frac{A_{H2}}{A_{H1}} < \frac{A_{L2}}{A_{L1}}$ holds, as shown on the vertical axis in Figure 3.5. Since $\frac{P_1}{P_2} < \frac{A_{L2}}{A_{L1}}$ (equivalently, $P_1 A_{L1} < P_2 A_{L2}$), less-skilled workers continue to choose occupation 2 under free trade.

Part (i) of Proposition 3.1 is in line with the fact I documented above: Consider the US and China as a skill-abundant and a skill-scarce country, respectively. In response to international trade with China, US workers with high pre-displacement occupational wage premia switch to even higher-wage occupations, while workers from low-wage occupations do not experience similar upward mobility. High-paying occupations in the data correspond to occupation 1 in my model, as stated in Lemma 3.2: Since workers of different skill types sort into occupations based on their comparative advantage, wage differences across occupations, holding other observables constant, reflect sorting by skill, which is unobservable to the econometrician. In my model, high-skilled workers who were employed in occupation 2 (i.e., the low-wage occupation) under autarky switch to occupation 1 (i.e., the high-wage occupation) under free trade. On the other hand, less-skilled workers do not switch to occupation 1, as they have a comparative advantage in occupation 2.

My model highlights the role of comparative advantage across occupations, rather than industries. In my model, industries refer to goods X and Y, which are traded internationally, while occupations function as intermediate goods in the production of these goods. Workers with different skill types have a comparative advantage in different occupations. This results in an occupational wage premium that reflects occupational sorting by skill. The literature shows that workers sort into occupations, rather than industries, based on their skill (Gibbons and Katz 1992; Gibbons et al. 2005; Krueger and Summers 1988). Through the comparative advantage mechanism, my dissertation shows how international trade affects occupational sorting. While Traiberman (2019) also models comparative advantage across occupations, his paper does not analyze how international trade affects occupational sorting.

Finally, part (ii) of Proposition 3.1 addresses labor reallocation across occupations in country S under free trade. As shown in Figure 3.5, the change in the relative supply of occupations due to free trade lowers the price of occupation 1 relative to occupation 2 in country S. Hence, the exact opposite pattern holds for country S: less-skilled workers who were indifferent between occupations under autarky now switch to occupation 2 under free trade, while high-skilled workers remain employed in occupation 1. Note that, under free trade, high-skilled workers choose occupation 1, and less-skilled workers choose occupation 2 in both countries. Hence, it follows that $\frac{Q_1}{Q_2} = \frac{A_{H1}}{A_{L2}} \frac{\overline{H}^{FT}}{\overline{L}^{FT}}$, as shown on the horizontal axis of Figure 3.5.

Proposition 3.2 (Wage Inequality under Free Trade)**.** Suppose $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi \frac{A_{L2}}{A_{H2}}$, $\frac{\overline{H}^S}{\overline{L}^S} \leq \psi \frac{A_{L1}}{A_{H1}}$, and $\frac{\overline{H}^{FT}}{\overline{L}^{FT}} \in \left(\psi \frac{A_{L1}}{A_{H1}}, \psi \frac{A_{L2}}{A_{H2}}\right)$. Then,
(i) In country N, while the wage for high-skilled workers rises, the wage for less-skilled workers falls in the transition from autarky to free trade.
(ii) In country S, while the wage for less-skilled workers rises, the wage for high-skilled workers falls in the transition from autarky to free trade.

Proof. See Appendix C.5.

While Proposition 3.1 discusses labor reallocation across occupations under free trade, Proposition 3.2 focuses on the change in wages resulting from free trade. Figure 3.5 shows the change in the *relative* price of occupations under free trade, which helps in understanding labor reallocation across occupations. In Appendix C.5, however, I show the changes in the prices of each occupation in each country during the transition from autarky to free trade. The intuition behind part (i) of Proposition 3.2 is that, when the skill-abundant country N trades with the skill-scarce country S, high-skilled workers become relatively scarce and less-skilled workers relatively abundant from the perspective of country N. Hence, the price of occupation 1, in which high-skilled workers have a comparative advantage, rises, while the price of occupation 2 falls in country N. Less-skilled workers in country N choose occupation 2, despite the decrease in its price, because they have a comparative advantage in it. Therefore, their wage falls under free trade. On the other hand, the wage for high-skilled workers in country N rises due to the increase in the price of occupation 1, in which they have a comparative advantage.

Part (i) of Proposition 3.2 aligns with the fact I documented above: US workers who switch to higher-wage occupations in response to international trade with China earn higher wages, while those who do not switch to higher-wage occupations earn lower wages. Workers who switch and those who do not in the data correspond to high-skilled and less-skilled workers in my model, respectively. In my model, less-skilled workers remain in occupation 2 and experience wage loss because they have comparative advantage in the occupation whose price falls under free trade. On the other hand, high-skilled workers who were indifferent between occupations under autarky switch to occupation 1 (the one with the wage premium) under free trade. They experience wage gains from the increase in the price of occupation 1, in which they have a comparative advantage.

Finally, part (ii) of Proposition 3.2 pertains to wage inequality in country S. Compared to part (i), the exact opposite pattern holds for skill-scarce country S trading with skill-abundant country N: high-skilled workers become relatively abundant and less-skilled workers relatively scarce from the perspective of country S. With the same comparative advantage mechanism, the wage for high-skilled workers falls, while that for less-skilled workers rises in country S.

While Proposition 3.2 may seem similar to the Stolper-Samuelson theorem, which states that the abundant factor benefits and the scarce factor loses in each trading country, the underlying mechanism is different. The key forces in my model are twofold: differences in skill abundance across countries and workers' comparative advantage across occupations. In the Heckscher-Ohlin framework, factors of production are perfectly mobile across industries (or occupations, in this case). In contrast, in my model, workers are not perfectly mobile across occupations, as different workers have comparative advantages in different occupations. Due to this imperfect mobility, workers with a comparative advantage in the occupation whose price falls lose from trade, while those with comparative advantage in the occupation whose price rises benefit from trade. My dissertation emphasizes the importance of occupation in determining winners and losers from trade. The literature shows that occupation plays a key role in determining these outcomes: trade induces reallocation across occupations and occupation-specific human capital accumulation is crucial in determining wages (Ebenstein et al. 2014; Traiberman 2019; Utar 2018). In contrast, I highlight the role of comparative advantage across occupations in the distributional effects of trade.

3.5 Conclusion

Who benefits and who is hurt by international trade has been a long-standing question in the field. A growing body of literature points out that a worker's occupation plays a crucial role in determining winners and losers from international trade: switching occupations induced by international trade is costly, as occupation-specific human capital accumulation plays a critical role in wage determination. In my dissertation, I propose an additional channel through which occupation plays a role in deciding winners and losers from trade: the comparative advantage of different skill types across occupations. In my model, workers are not perfectly mobile across occupations, as different workers have comparative advantages in different occupations. Due to this imperfect mobility, workers with a comparative advantage in the occupation whose price falls lose from trade, while those with a comparative advantage in the occupation whose price rises benefit from trade.

While my dissertation shows that international trade induces wage inequality through occupational sorting by skill, future questions remain. First, what are the other possible occupational mobility constraints through which international trade affects wage inequality? My dissertation highlights occupational sorting by skill as an occupational mobility barrier, while the literature focuses on occupation-specific human capital accumulation. However, there could be other constraints that make occupational switching costly and thereby lead to wage inequality—for example, geographical barriers if certain occupations are spatially concentrated, as well as institutional barriers. Second, how much does each barrier quantitatively contribute to the wage inequality effects of international trade? Finally, on a related note, what are the most effective policies to address wage inequality? Since my dissertation highlights occupational sorting by skill, policies that facilitate skill acquisition for less-skilled workers could help mitigate wage inequality. However, understanding other possible barriers to occupational switching and the magnitude of each barrier could inform the design of the most effective policies.

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Appendices

Appendix A

Spatial Wage Inequality: Stylized Facts

A.1 Data on Local Real Wages

The sample of workers I use comprises individuals who were between age 16 and 64 and who were working in the year which precedes the survey. I drop residents of institutional group quarters, unpaid family workers, self-employed workers, and workers employed in agriculture. Decennial Census data and American Community Survey data provide weeks worked and usual number of hours per week. I measure hours worked as weeks worked multiplied by usual number of hours per week. Hourly wage is yearly wage and salary income divided by hours worked in a year. Following Autor and Dorn (2013), I multiply top-coded wage and salary income by 1.5, and I set hourly wages not to exceed this value divided by 50 weeks times 35 hours. I set hourly wages below the first percentile of the hourly wage distribution to the value of the first percentile.

I describe in detail how I measure local price levels. I define the price level in commuting zone j in year t as

$$logP_{jt} = log\big(\gamma^{-\gamma}(1-\gamma)^{\gamma-1}\big) + \gamma logP_{Nt} + (1-\gamma)logP_{jHt},$$

where $1-\gamma$ is the expenditure share on housing, P_{Nt} is the non-housing price in year t, and P_{jHt} is the housing price in commuting zone j in year t. As described in the main text, I assume non-housing prices to be the same across commuting zones in a given year. Housing price is measured as gross monthly rents, adjusted for the year the housing was built, the number of bedrooms, and the number of units. The above equation implies

$$logP_t = log\big(\gamma^{-\gamma}(1-\gamma)^{\gamma-1}\big) + \gamma logP_{Nt} + (1-\gamma)logP_{Ht},$$

where $logP_t \equiv \frac{1}{722}\sum_j logP_{jt}$, $logP_{Ht} \equiv \frac{1}{722}\sum_j logP_{jHt}$, and 722 is the number of commuting zones. With a few lines of algebra, I get

$$logP_{jt} = logP_t + (1-\gamma)\big(logP_{jHt} - logP_{Ht}\big).$$

Recall that the term on the left-hand side, P_{jt}, is the local price level I want to measure. I measure P_t as personal consumption expenditure in year t, which is available from the BEA. I use an expenditure share on housing of 25 percent (i.e., $1-\gamma = 0.25$), as estimated by Davis and Ortalo-Magne (2011). Note that P_{jHt} and P_{Ht} are observed data. Finally, I normalize local price levels so that the average of local price levels across commuting zones is 1 in 1980.

A.2 Data on ICT Capital

I define machines as ICT capital, and I measure the price of machines using the data from the National Income and Product Accounts, available at the BEA. ICT capital consists of computers and peripheral equipment; communication equipment;

medical equipment and instruments; nonmedical instruments; photocopy and related equipment; office and accounting equipment; and software, based on Eden and Gaggl (2018).

The BEA provides quality-adjusted quantity indexes for different capital types. To measure the price index of ICT capital in year t, P_{Mt}, I use the Törnqvist index:

$$log\frac{P_{Mt}}{P_{M,t-1}} = \sum_{k} log\Big(\frac{p_{kt}}{p_{k,t-1}}\frac{s_{kt}+s_{k,t-1}}{2}\Big),$$

where p_{kt} is the price of ICT capital type k in year t and s_{kt} is the expenditure share on ICT capital type k in year t. P_{Mt} is a chain-weighted index. For machines whose price has changed rapidly, a chain-weighted index is better than a fixed-weight index, which is subject to substitution bias.

Appendix B

Automation, Spatial Wage Inequality, and Place-Based Policy

B.1 Derivation of Labor Supply

Let $j \in \{1, ..., J\}$ index places, where J denotes the number of places. Each worker in occupation ℓ supplies one unit of labor for wage $W_{j\ell}$ in place j. Workers consume good using their wage and have idiosyncratic preferences. The preference shock, ϵ, is drawn independently across individuals, occupations, and places from a Fréchet distribution with a finite variance parameter $\kappa > 1$, where $Pr(\epsilon_j \leq \epsilon) = \exp(-\epsilon^{-\kappa})$. The cumulative distribution function below will be useful in the upcoming proof:

$$
\begin{aligned}
F(u) &\equiv Pr\big(max_j\{\mathcal{U}_{j\ell}\} \leq u \big) \\
&= Pr\big(max_j\{W_{j\ell}\epsilon_j\} \leq u \big) \\
&= Pr\Big(\epsilon_j \leq \frac{u}{W_{j\ell}}, \ \ \forall j \Big) \\
&= \prod_j exp\Big(-\Big(\frac{u}{W_{j\ell}}\Big)^{-\kappa} \Big) \\
&= \prod_j exp\big(- W_{j\ell}^{\kappa} u^{-\kappa} \big) \\
&= exp\Big(- \Big(\sum_j W_{j\ell}^{\kappa} \Big) u^{-\kappa} \Big).
\end{aligned}
\tag{B.1}
$$

Workers choose places to maximize their utility. Let ℓ_j denote the labor supply of occupation ℓ in place j. Within each occupation, there is a unit mass of workers. Then,

$$
\begin{aligned}
\ell_j &= Pr\big(\mathcal{U}_{j\ell} > \mathcal{U}_{i\ell}, \;\; \forall i \neq j\big) \\
&= Pr\big(W_{j\ell}\epsilon_j > W_{i\ell}\epsilon_i, \;\; \forall i \neq j\big) \\
&= Pr\Big(\epsilon_i < \frac{W_{j\ell}}{W_{i\ell}}\epsilon_j, \;\; \forall i \neq j\Big) \\
&= \int \kappa\epsilon^{-\kappa-1} exp\big(-\epsilon^{-\kappa}\big) \prod_{i\neq j} exp\Bigg(-\Big(\frac{W_{j\ell}}{W_{i\ell}}\epsilon\Big)^{-\kappa}\Bigg)\, d\epsilon \\
&= \int \kappa\epsilon^{-\kappa-1} exp\Bigg(-\sum_i \Big(\frac{W_{j\ell}}{W_{i\ell}}\epsilon\Big)^{-\kappa}\Bigg)\, d\epsilon \\
&= \int \kappa\epsilon^{-\kappa-1} exp\Bigg(-\Big(\sum_i W_{i\ell}^{\kappa}\Big)\Big(W_{j\ell}\,\epsilon\Big)^{-\kappa}\Bigg)\, d\epsilon \\
&= \frac{W_{j\ell}^{\kappa}}{\sum_i W_{i\ell}^{\kappa}} \int \kappa\epsilon^{-\kappa-1} \frac{\sum_i W_{i\ell}^{\kappa}}{W_{j\ell}^{\kappa}} exp\Bigg(-\Big(\sum_i W_{i\ell}^{\kappa}\Big)\Big(W_{j\ell}\,\epsilon\Big)^{-\kappa}\Bigg)\, d\epsilon \\
&= \frac{W_{j\ell}^{\kappa}}{\sum_i W_{i\ell}^{\kappa}} \int \kappa(W_{j\ell}\,\epsilon)^{-\kappa-1} W_{j\ell}\Big(\sum_i W_{i\ell}^{\kappa}\Big) exp\Bigg(-\Big(\sum_i W_{i\ell}^{\kappa}\Big)\Big(W_{j\ell}\,\epsilon\Big)^{-\kappa}\Bigg)\, d\epsilon \\
&= \frac{W_{j\ell}^{\kappa}}{\sum_i W_{i\ell}^{\kappa}} \int \underbrace{\kappa u^{-\kappa-1}\Big(\sum_i W_{i\ell}^{\kappa}\Big) exp\Bigg(-\Big(\sum_i W_{i\ell}^{\kappa}\Big)u^{-\kappa}\Bigg)\, du}_{=dF(u)} \\
&= \frac{W_{j\ell}^{\kappa}}{\sum_i W_{i\ell}^{\kappa}},
\end{aligned}
$$

where the underbraced term being equal to $dF(u)$ in the second-to-last equation follows from Equation B.1.

B.2 Spatial Equilibrium

The spatial equilibrium is determined by the following 7×2 equations:

$$R_j = \frac{W_{jR}^{\kappa}}{W_{1R}^{\kappa} + W_{2R}^{\kappa}} \text{ for } j \in \{1,2\}, \tag{B.2}$$

$$S_j = \frac{W_{jS}^{\kappa}}{W_{1S}^{\kappa} + W_{2S}^{\kappa}} \text{ for } j \in \{1,2\}, \tag{B.3}$$

$$U_j = \frac{W_{jU}^{\kappa}}{W_{1U}^{\kappa} + W_{2U}^{\kappa}} \text{ for } j \in \{1,2\}, \tag{B.4}$$

$$W_{jR}R_j = \lambda_{jR}Y_j \text{ for } j \in \{1,2\}, \tag{B.5}$$

$$W_{jS}S_j = \lambda_{jS}Y_j \text{ for } j \in \{1,2\}, \tag{B.6}$$

$$W_{jU}U_j = \lambda_{jU}Y_j \text{ for } j \in \{1,2\}, \tag{B.7}$$

$$T_j = \rho^{-\rho}(1-\rho)^{\rho-1}W_{jU}^{\rho}\chi_{jS}^{1-\rho} \text{ for } j \in \{1,2\}, \tag{B.8}$$

where $\chi_{jS} \equiv \left[\alpha_S^{\sigma_S}W_{jS}^{1-\sigma_S} + (1-\alpha_S)^{\sigma_S}\chi_{jR}^{1-\sigma_S}\right]^{\frac{1}{1-\sigma_S}}$ is the marginal cost of intermediates (i.e., Ω_{jS}) in place j, and $\chi_{jR} \equiv \left[\alpha_R^{\sigma_R}W_{jR}^{1-\sigma_R} + (1-\alpha_R)^{\sigma_R}P_M^{1-\sigma_R}\right]^{\frac{1}{1-\sigma_R}}$ is the marginal cost of the routine task input (i.e., Ω_{jR}) in place j. $\lambda_{j\ell}$ is the expenditure share on factor $\ell \in \{R,S,U\}$ in place j:

$$\lambda_{jR} = \alpha_R^{\sigma_R}W_{jR}^{1-\sigma_R}\chi_{jR}^{\sigma_R-1} \cdot (1-\alpha_S)^{\sigma_S}\chi_{jR}^{1-\sigma_S}\chi_{jS}^{\sigma_S-1} \cdot (1-\rho),$$

$$\lambda_{jS} = \alpha_S^{\sigma_S}W_{jS}^{1-\sigma_S}\chi_{jS}^{\sigma_S-1} \cdot (1-\rho),$$

$$\lambda_{jU} = \rho.$$

Now, let me reduce the above system of equations by substitution. From Equation B.8, we have

$$\frac{W_{1U}}{W_{2U}} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\rho}} \left(\frac{\chi_{1S}}{\chi_{2S}}\right)^{-\frac{1-\rho}{\rho}}. \tag{B.9}$$

Using Equations B.4 and B.7, we obtain

$$\frac{Y_1}{Y_2} = \left(\frac{W_{1U}}{W_{2U}}\right)^{1+\kappa}. \tag{B.10}$$

Combining Equations B.2, B.5, and B.10, yields

$$\frac{W_{1R}}{W_{2R}} = \left(\frac{\lambda_{1R}}{\lambda_{2R}}\right)^{\frac{1}{1+\kappa}} \left(\frac{Y_1}{Y_2}\right)^{\frac{1}{1+\kappa}} = \left(\frac{\lambda_{1R}}{\lambda_{2R}}\right)^{\frac{1}{1+\kappa}} \frac{W_{1U}}{W_{2U}}. \tag{B.11}$$

Similarly, combining Equations B.3, B.6, and B.10, leads to

$$\frac{W_{1S}}{W_{2S}} = \left(\frac{\lambda_{1S}}{\lambda_{2S}}\right)^{\frac{1}{1+\kappa}} \left(\frac{Y_1}{Y_2}\right)^{\frac{1}{1+\kappa}} = \left(\frac{\lambda_{1S}}{\lambda_{2S}}\right)^{\frac{1}{1+\kappa}} \frac{W_{1U}}{W_{2U}}. \tag{B.12}$$

Dividing Equation B.11 by Equation B.12, after a few lines of algebra, we have

$$\frac{W_{1S}}{W_{2S}} = \left(\frac{W_{1R}}{W_{2R}}\right)^{\frac{\kappa+\sigma_R}{\kappa+\sigma_S}} \left(\frac{\chi_{1R}}{\chi_{2R}}\right)^{\frac{\sigma_S-\sigma_R}{\kappa+\sigma_S}}. \tag{B.13}$$

Finally, substituting Equation B.9 into Equation B.11, and simplifying, yields

$$\left(\frac{\chi_{1S}}{\chi_{2S}}\right)^{1-\sigma_S+\frac{1-\rho}{\rho}(1+\kappa)} = \frac{W_{1R}}{W_{2R}}^{-\kappa-\sigma_R} \left(\frac{\chi_{1R}}{\chi_{2R}}\right)^{\sigma_R-\sigma_S} \left(\frac{T_1}{T_2}\right)^{\frac{1+\kappa}{\rho}}. \tag{B.14}$$

Equation B.13 shows that spatial wage inequality for the non-routine skilled occupation $\left(\text{i.e., } \frac{W_{1S}}{W_{2S}}\right)$ is a function of spatial wage inequality for the routine occupation $\left(\text{i.e., } \frac{W_{1R}}{W_{2R}}\right)$ since $\frac{\chi_{1R}}{\chi_{2R}}$ is a function of $\frac{W_{1R}}{W_{2R}}$. Similarly, Equation B.14 shows that $\frac{W_{1S}}{W_{2S}}$ is a function of $\frac{W_{1R}}{W_{2R}}$ since $\frac{\chi_{1S}}{\chi_{2S}}$ is a function of $\frac{W_{1S}}{W_{2S}}$ and $\frac{W_{1R}}{W_{2R}}$. Hence, Equations

B.13 and B.14 are two equations with two unknowns $\left(\text{i.e., } \frac{W_{1R}}{W_{2R}} \text{ and } \frac{W_{1S}}{W_{2S}}\right)$. Equation B.12 shows that spatial wage inequality for the non-routine unskilled occupation $\left(\text{i.e., } \frac{W_{1U}}{W_{2U}}\right)$ is a function of spatial wage inequality for the non-routine skilled occupation $\left(\text{i.e., } \frac{W_{1S}}{W_{2S}}\right)$ as $\frac{\lambda_{1S}}{\lambda_{2S}}$ is a function of $\frac{W_{1S}}{W_{2S}}$ and $\frac{W_{1R}}{W_{2R}}$, but $\frac{W_{1R}}{W_{2R}}$ is a function of $\frac{W_{1S}}{W_{2S}}$, according to Equations B.13 and B.14.

B.3 Proof of Lemma 2.1

I will prove the lemma using Equations B.12 to B.14, by contradiction. For contradiction, suppose $W_{1R} = W_{2R}$. Recall that P_M is the same across places, since the supply of machines is perfectly elastic. So, χ_{jR}, which is the marginal cost of the routine task input (i.e., Ω_{jR}) is equalized across places. That is, $\chi_{1R} = \chi_{2R}$. Then, by Equation B.13, the wage for non-routine skilled workers is also equalized across places. That is, $W_{1S} = W_{2S}$. So far, we have $\chi_{1R} = \chi_{2R}$ and $W_{1S} = W_{2S}$. Then, χ_{jS}, which is the marginal cost of intermediates (i.e., Ω_{jS}) is also equalized across places. That is, $\chi_{1S} = \chi_{2S}$. By Equation B.14, this contradicts the assumption that $T_1 > T_2$. Hence, $W_{1R} = W_{2R}$ is wrong. Similarly, $W_{1R} < W_{2R}$ also leads to a contradiction. Therefore, it must be the case that $W_{1R} > W_{2R}$.

Now, since $W_{1R} > W_{2R}$ and P_M is the same across places as the supply of machines is perfectly elastic, the marginal cost of the routine task input is also higher in place 1. That is, $\chi_{1R} > \chi_{2R}$. Intuitively, it also holds that $\frac{W_{1R}/\chi_{1R}}{W_{2R}/\chi_{2R}} > 1$. Recall that χ_{jR} is the marginal cost of the routine task input (i.e., Ω_{jR}) in place j. Since the wage for routine workers is higher in place 1, while P_M is the same across places, the fraction spent on routine workers out of the marginal cost of the routine task input $\left(\text{i.e., } \frac{W_{jR}}{\chi_{jR}}\right)$ is higher in $j = 1$ than in $j = 2$. That was the intuition behind $\frac{W_{1R}/\chi_{1R}}{W_{2R}/\chi_{2R}} > 1$. Now, proceeding with the mathematical proof, since $W_{1R} > W_{2R}$ and $\sigma_R > 1$, it follows

$$\alpha_R^{\sigma_R}(1-\alpha_R)^{\sigma_R}{W_{1R}}^{1-\sigma_R}P_M^{1-\sigma_R} < \alpha_R^{\sigma_R}(1-\alpha_R)^{\sigma_R}{W_{2R}}^{1-\sigma_R}P_M^{1-\sigma_R}.$$

Adding $\alpha_R^{2\sigma_R}W_{1R}^{1-\sigma_R}W_{2R}^{1-\sigma_R}$ to both sides, we obtain

$$\begin{aligned}&\alpha_R^{2\sigma_R}W_{1R}^{1-\sigma_R}W_{2R}^{1-\sigma_R} + \alpha_R^{\sigma_R}(1-\alpha_R)^{\sigma_R}{W_{1R}}^{1-\sigma_R}P_M^{1-\sigma_R}\\ &< \alpha_R^{2\sigma_R}W_{1R}^{1-\sigma_R}W_{2R}^{1-\sigma_R} + \alpha_R^{\sigma_R}(1-\alpha_R)^{\sigma_R}{W_{2R}}^{1-\sigma_R}P_M^{1-\sigma_R}.\end{aligned}$$

With a few lines of algebra, we have

$$1 > \frac{\alpha_R^{\sigma_R}W_{1R}^{1-\sigma_R}}{\alpha_R^{\sigma_R}W_{1R}^{1-\sigma_R} + (1-\alpha_R)^{\sigma_R}P_M^{1-\sigma_R}}\frac{\alpha_R^{\sigma_R}W_{2R}^{1-\sigma_R} + (1-\alpha_R)^{\sigma_R}P_M^{1-\sigma_R}}{\alpha_R^{\sigma_R}W_{2R}^{1-\sigma_R}} = \left(\frac{W_{1R}/\chi_{1R}}{W_{2R}/\chi_{2R}}\right)^{1-\sigma_R}.$$

Since $\sigma_R > 1$, it follows

$$\frac{W_{1R}/\chi_{1R}}{W_{2R}/\chi_{2R}} > 1. \tag{B.15}$$

Now, by Equation B.13, we obtain

$$\frac{W_{1S}}{W_{2S}} = \left(\frac{W_{1R}}{W_{2R}}\right)^{\frac{\kappa+\sigma_R}{\kappa+\sigma_S}}\left(\frac{\chi_{1R}}{\chi_{2R}}\right)^{\frac{\sigma_S-\sigma_R}{\kappa+\sigma_S}} = \left(\frac{W_{1R}/\chi_{1R}}{W_{2R}/\chi_{2R}}\right)^{\frac{\sigma_R}{\kappa+\sigma_S}}\left(\frac{W_{1R}}{W_{2R}}\right)^{\frac{\kappa}{\kappa+\sigma_S}}\left(\frac{\chi_{1R}}{\chi_{2R}}\right)^{\frac{\sigma_S}{\kappa+\sigma_S}} > 1,$$

since $\frac{W_{1R}/\chi_{1R}}{W_{2R}/\chi_{2R}} > 1$, $\frac{W_{1R}}{W_{2R}} > 1$, and $\frac{\chi_{1R}}{\chi_{2R}} > 1$. Hence, it follows that $W_{1S} > W_{2S}$.

Finally, as $\chi_{1R} > \chi_{2R}$ and $W_{1S} > W_{2S}$, the marginal cost of intermediates is also higher in place 1. That is, $\chi_{1S} > \chi_{2S}$. With a few lines of algebra, Equation B.12 becomes

$$\frac{W_{1U}}{W_{2U}} = \left(\frac{W_{1S}}{W_{2S}}\right)^{\frac{\kappa+\sigma_S}{1+\kappa}}\left(\frac{\chi_{1S}}{\chi_{2S}}\right)^{\frac{1-\sigma_S}{1+\kappa}}.$$

Since $\frac{W_{1S}}{W_{2S}} > 1$, $\frac{\chi_{1S}}{\chi_{2S}} > 1$, and $\sigma_S < 1$, it follows that $W_{1U} > W_{2U}$. Observe that the whole proof holds, whatever the level of price of machine is. That is, if $T_1 > T_2$, then $W_{1\ell} > W_{2\ell}$ for $\ell \in \{R, S, U\}$, $\forall P_M$.

B.4 Proof of Lemma 2.2

I will prove if $T_1 > T_2$, then $\lambda_{1R} < \lambda_{2R}$, $\lambda_{1S} > \lambda_{2S}$, and $\lambda_{1M} > \lambda_{2M}$, $\forall P_M$. Note that $\lambda_{jU} = \rho$, $\forall j$ and $\lambda_{jR} + \lambda_{jS} + \lambda_{jU} + \lambda_{jM} = 1$, $\forall j$. So, it suffices to show that if $T_1 > T_2$, then $\lambda_{1R} < \lambda_{2R}$ and $\lambda_{1S} > \lambda_{2S}$. First, let me show that $\lambda_{1R} < \lambda_{2R}$. The expenditure share on routine labor in place 1 relative to place 2, $\frac{\lambda_{1R}}{\lambda_{2R}}$, can be simplified to

$$\frac{\lambda_{1R}}{\lambda_{2R}} = \Big(\frac{W_{1R}/\chi_{1R}}{W_{2R}/\chi_{2R}}\Big)^{1-\sigma_R}\Big(\frac{\chi_{1R}/\chi_{1S}}{\chi_{2R}/\chi_{2S}}\Big)^{1-\sigma_S}.$$

From Equation B.15 (which holds under the assumption that $T_1 > T_2$) and the assumption that $\sigma_R > 1$, it follows

$$\Big(\frac{W_{1R}/\chi_{1R}}{W_{2R}/\chi_{2R}}\Big)^{1-\sigma_R} < 1. \tag{B.16}$$

Next, I will show that $\big(\frac{\chi_{1R}/\chi_{1S}}{\chi_{2R}/\chi_{2S}}\big)^{1-\sigma_S} < 1$. The intuition comes from Lemma 2.1 which states that non-routine skilled labor is more expensive in the place with higher TFP. $\big(\frac{\chi_{1R}/\chi_{1S}}{\chi_{2R}/\chi_{2S}}\big)^{1-\sigma_S} < 1$ can be rewritten as $\big(\frac{\chi_{1S}/\chi_{1R}}{\chi_{2S}/\chi_{2R}}\big)^{1-\sigma_S} > 1$. Since $\sigma_S < 1$, it follows that $\frac{\chi_{1S}}{\chi_{1R}} > \frac{\chi_{2S}}{\chi_{2R}}$. That is, the marginal cost of intermediates relative to that of the routine task input is higher in place 1, which has a higher wage for non-routine skilled labor. That was the intuition behind $\big(\frac{\chi_{1R}/\chi_{1S}}{\chi_{2R}/\chi_{2S}}\big)^{1-\sigma_S} < 1$. Now, proceeding with the mathematical proof, dividing both sides of Equation B.13 by $\frac{\chi_{1R}}{\chi_{2R}}$, we obtain

$$\frac{W_{1S}/W_{2S}}{\chi_{1R}/\chi_{2R}} = \Big(\frac{W_{1R}/\chi_{1R}}{W_{2R}/\chi_{2R}}\Big)^{\frac{\kappa+\sigma_R}{\kappa+\sigma_S}} > 1,$$

where the inequality follows from Equation B.15. Since $\frac{W_{1S}/W_{2S}}{\chi_{1R}/\chi_{2R}} > 1$ and $\sigma_S < 1$, it follows that $\big(\frac{W_{1S}}{W_{2S}}\big)^{1-\sigma_S} > \big(\frac{\chi_{1R}}{\chi_{2R}}\big)^{1-\sigma_S}$. In other words, $W_{1S}^{1-\sigma_S}\chi_{2R}^{1-\sigma_S} > W_{2S}^{1-\sigma_S}\chi_{1R}^{1-\sigma_S}$. Multiplying both sides by $\alpha_S^{\sigma_S}(1-\alpha_S)^{\sigma_S}$ and then adding $(1-\alpha_S)^{2\sigma_S}\chi_{1R}^{1-\sigma_S}\chi_{2R}^{1-\sigma_S}$ to both sides, we have

$$\begin{aligned}&\alpha_S^{\sigma_S}(1-\alpha_S)^{\sigma_S}W_{1S}^{1-\sigma_S}\chi_{2R}^{1-\sigma_S} + (1-\alpha_S)^{2\sigma_S}{\chi_{1R}}^{1-\sigma_S}{\chi_{2R}}^{1-\sigma_S}\\ > &\alpha_S^{\sigma_S}(1-\alpha_S)^{\sigma_S}W_{2S}^{1-\sigma_S}\chi_{1R}^{1-\sigma_S} + (1-\alpha_S)^{2\sigma_S}{\chi_{1R}}^{1-\sigma_S}{\chi_{2R}}^{1-\sigma_S}.\end{aligned}$$

A few steps of algebra yield

$$1 > \frac{(1-\alpha_S)^{\sigma_S}\chi_{1R}^{1-\sigma_S}}{(1-\alpha_S)^{\sigma_S}\chi_{2R}^{1-\sigma_R}}\frac{\alpha_S^{\sigma_S}W_{2S}^{1-\sigma_S} + (1-\alpha_S)^{\sigma_S}\chi_{2R}^{1-\sigma_S}}{\alpha_S^{\sigma_S}W_{1S}^{1-\sigma_S} + (1-\alpha_S)^{\sigma_S}\chi_{1R}^{1-\sigma_S}} = \Big(\frac{\chi_{1R}/\chi_{1S}}{\chi_{2R}/\chi_{2S}}\Big)^{1-\sigma_S}.$$

That is,

$$\Big(\frac{\chi_{1R}/\chi_{1S}}{\chi_{2R}/\chi_{2S}}\Big)^{1-\sigma_S} < 1. \tag{B.17}$$

From Equations B.16 and B.17, we have

$$\frac{\lambda_{1R}}{\lambda_{2R}} = \Big(\frac{W_{1R}/\chi_{1R}}{W_{2R}/\chi_{2R}}\Big)^{1-\sigma_R}\Big(\frac{\chi_{1R}/\chi_{1S}}{\chi_{2R}/\chi_{2S}}\Big)^{1-\sigma_S} < 1.$$

In other words, $\lambda_{1R} < \lambda_{2R}$.

Second, I will show that $\lambda_{1S} > \lambda_{2S}$. The expenditure share on non-routine skilled labor in place 1 relative to place 2, $\frac{\lambda_{1S}}{\lambda_{2S}}$, can be simplified to

$$\frac{\lambda_{1S}}{\lambda_{2S}} = \Big(\frac{W_{1S}/\chi_{1S}}{W_{2S}/\chi_{2S}}\Big)^{1-\sigma_S}.$$

Note that the marginal cost of intermediates in place j, χ_{jS}, consists of the wage for non-routine skilled labor, W_{jS}, and the marginal cost of the routine task input, χ_{jR}. Hence, Equation B.17 implies

$$\Big(\frac{W_{1S}/\chi_{1S}}{W_{2S}/\chi_{2S}}\Big)^{1-\sigma_S} > 1.$$

Therefore, we obtain

$$\frac{\lambda_{1S}}{\lambda_{2S}} = \Big(\frac{W_{1S}/\chi_{1S}}{W_{2S}/\chi_{2S}}\Big)^{1-\sigma_S} > 1.$$

In other words, $\lambda_{1S} > \lambda_{2S}$.

So far, I showed that if $T_1 > T_2$, then $\lambda_{1R} < \lambda_{2R}$ and $\lambda_{1S} > \lambda_{2S}$. So, it follows that $\lambda_{1M} > \lambda_{2M}$ since $\lambda_{jU} = \rho$, $\forall j$ and $\lambda_{jR} + \lambda_{jS} + \lambda_{jU} + \lambda_{jM} = 1$, $\forall j$. Note that this whole proof holds, whatever the level of price of machine is.

B.5 Proof of Proposition 2.1

First, I will show that as the price of machines falls, spatial wage inequality for non-routine skilled workers increases. From Lemma 2.1, the wage for non-routine skilled workers is higher in place 1 (i.e., the place with higher TFP) at any price of machines. Now, using Equation B.6, I take the derivative of the demand for non-routine skilled labor in place 1 relative to place 2 $\left(\text{i.e., } \frac{S_1}{S_2}\right)$ with respect to the price of machines, P_M, to get

$$\frac{\partial}{\partial P_M}\left(\frac{S_1}{S_2}\right) = -\frac{1-\sigma_S}{1-\rho}(\lambda_{1M} - \lambda_{2M})\frac{S_1/S_2}{P_M}.$$

Observe that $\frac{\partial}{\partial P_M}\left(\frac{S_1}{S_2}\right) < 0$ since $\sigma_S < 1$, $\rho < 1$, and $\lambda_{1M} > \lambda_{2M}$, as shown in Lemma 2.2. Since $\frac{\partial}{\partial P_M}\left(\frac{S_1}{S_2}\right) < 0$, the demand for non-routine skilled labor in place 1 relative to place 2 goes up as the price of machines declines, which increases spatial wage inequality for non-routine skilled labor $\left(\text{i.e., } \frac{W_{1S}}{W_{2S}}\right)$.

Second, I will prove that spatial wage inequality falls for routine workers as the price of machines falls. From Equation B.17 (which holds under the assumption that $T_1 > T_2$), we have

$$\left(\frac{\chi_{1S}}{\chi_{1R}}\right)^{1-\sigma_S} > \left(\frac{\chi_{2S}}{\chi_{2R}}\right)^{1-\sigma_S}.$$

With a few steps of algebra, we obtain

$$(1-\alpha_S)^{-\sigma_S}\left(\frac{\chi_{1S}}{\chi_{1R}}\right)^{1-\sigma_S} - 1 > (1-\alpha_S)^{-\sigma_S}\left(\frac{\chi_{2S}}{\chi_{2R}}\right)^{1-\sigma_S} - 1.$$

Then, since $\lambda_{1M} > \lambda_{2M}$, I multiply the left-hand side of the above inequality by λ_{1M}

and the right-hand side by λ_{2M} to get

$$\left[(1-\alpha_S)^{-\sigma_S}\left(\frac{\chi_{1S}}{\chi_{1R}}\right)^{1-\sigma_S}-1\right]\lambda_{1M} > \left[(1-\alpha_S)^{-\sigma_S}\left(\frac{\chi_{2S}}{\chi_{2R}}\right)^{1-\sigma_S}-1\right]\lambda_{2M}.$$

Then, rearrange the inequality above to obtain

$$(1-\alpha_S)^{-\sigma_S}\left[\left(\frac{\chi_{1S}}{\chi_{1R}}\right)^{1-\sigma_S}\lambda_{1M}-\left(\frac{\chi_{2S}}{\chi_{2R}}\right)^{1-\sigma_S}\lambda_{2M}\right] > \lambda_{1M}-\lambda_{2M}.$$

Multiply the inequality by $\sigma_R - \sigma_S$ (> 0). Then, since $\sigma_R > 1$, we have

$$\begin{aligned}(\sigma_R-\sigma_S)(1-\alpha_S)^{-\sigma_S}&\left[\left(\frac{\chi_{1S}}{\chi_{1R}}\right)^{1-\sigma_S}\lambda_{1M}-\left(\frac{\chi_{2S}}{\chi_{2R}}\right)^{1-\sigma_S}\lambda_{2M}\right]\\ &> (\sigma_R-\sigma_S)(\lambda_{1M}-\lambda_{2M})\\ &> (1-\sigma_S)(\lambda_{1M}-\lambda_{2M}),\end{aligned} \tag{B.18}$$

which will be useful for the upcoming proof.

Going back to the actual proof, recall that the wage for routine workers is higher in place 1 at any price of machines, as stated in Lemma 2.1. Now, I use Equation B.5 to take the derivative of the demand for routine labor in place 1 relative to place 2 (i.e., $\frac{R_1}{R_2}$) with respect to P_M to get

$$\frac{\partial}{\partial P_M}\left(\frac{R_1}{R_2}\right) = \frac{1}{1-\rho}\frac{R_1/R_2}{P_M} \times$$
$$\left[(\sigma_R-\sigma_S)(1-\alpha_S)^{-\sigma_S}\left\{\left(\frac{\chi_{1S}}{\chi_{1R}}\right)^{1-\sigma_S}\lambda_{1M}-\left(\frac{\chi_{2S}}{\chi_{2R}}\right)^{1-\sigma_S}\lambda_{2M}\right\}-(1-\sigma_S)(\lambda_{1M}-\lambda_{2M})\right].$$

Notice that $\frac{\partial}{\partial P_M}\left(\frac{R_1}{R_2}\right) > 0$ by Equation B.18. Since $\frac{\partial}{\partial P_M}\left(\frac{R_1}{R_2}\right) > 0$, the demand for routine labor in place 1 relative to place 2 falls as the price of machines declines, which decreases spatial wage inequality for routine workers (i.e., $\frac{W_{1R}}{W_{2R}}$).

Finally, according to Lemma 2.1, the wage for non-routine unskilled workers is higher in place 1 at any price of machines. Use Equation B.9 and take the derivative of $\frac{W_{1U}}{W_{2U}}$ with respect to P_M to get

$$\frac{\partial}{\partial P_M}\Big(\frac{W_{1U}}{W_{2U}}\Big) = -\frac{1}{\rho}(\lambda_{1M} - \lambda_{2M})\frac{W_{1U}/W_{2U}}{P_M}.$$

Since $\lambda_{1M} > \lambda_{2M}$, it follows that $\frac{\partial}{\partial P_M}\big(\frac{W_{1U}}{W_{2U}}\big) < 0$. In other words, spatial wage inequality for non-routine unskilled labor $\big($i.e., $\frac{W_{1U}}{W_{2U}}\big)$ goes up as the price of machines falls.

B.6 Proof of Proposition 2.2

I begin with the first part of Proposition 2.2. According to Lemma 2.1, spatial wage inequality for each occupation arises from the difference in TFP across places. As the difference in TFP across places shrinks, spatial wage inequality for each occupation falls. In the limit where $T_2' = T_1$, there is no spatial wage inequality at all for any occupation.

Moving on to the second part of Proposition 2.2, the ratio of the marginal product of non-routine skilled labor to that of routine labor in place 2 is given by

$$\frac{W_{2S}}{W_{2R}} = \frac{\alpha_S}{\alpha_R(1-\alpha_S)}\Big(\frac{R_2}{S_2}\Big)^{\frac{1}{\sigma_S}}\left[\alpha_R + (1-\alpha_R)\Big(\frac{M_2}{R_2}\Big)^{\frac{\sigma_R-1}{\sigma_R}}\right]^{\frac{\sigma_R}{\sigma_R-1}\left(\frac{1}{\sigma_S}-\frac{1}{\sigma_R}\right)},$$

which does not depend on T_2 due to Hicks neutrality. The key term on the right-hand side is $\frac{M_2}{R_2}$. Now, on the supply side, as T_2 goes up, $\frac{M_2}{R_2}$ rises since the supply of machines is more elastic than that of labor. Note that $\sigma_R > \sigma_S$, which implies that the exponent, $\frac{1}{\sigma_S} - \frac{1}{\sigma_R} > 0$. Hence, the equilibrium relative wage, $\frac{W_{2S}}{W_{2R}}$, goes up as TFP rises in place 2.

Next, the ratio of the marginal product of non-routine unskilled labor to that of routine labor in place 2 is given by

$$\frac{W_{2U}}{W_{2R}} = \frac{\rho}{\alpha_R(1-\alpha_S)(1-\rho)} \frac{R_2}{U_2} \Bigg\{ \alpha_S \left(\frac{S_2}{R_2}\right)^{\frac{\sigma_S-1}{\sigma_S}} \left[\alpha_R + (1-\alpha_R)\left(\frac{M_2}{R_2}\right)^{\frac{\sigma_R-1}{\sigma_R}}\right]^{\frac{\sigma_R}{\sigma_R-1}\left(\frac{1}{\sigma_S}-\frac{1}{\sigma_R}\right)} + (1-\alpha_S)\left[\alpha_R + (1-\alpha_R)\left(\frac{M_2}{R_2}\right)^{\frac{\sigma_R-1}{\sigma_R}}\right]^{\frac{\sigma_R}{\sigma_R-1}\left(1-\frac{1}{\sigma_R}\right)} \Bigg\},$$

where the idea is the same as before: Hicks-neutral technical progress has non-neutral effects across occupations in equilibrium since the supply of machines is more elastic than that of labor. The term $\frac{M_2}{R_2}$ appears twice on the right-hand side. On the supply side, as T_2 goes up, $\frac{M_2}{R_2}$ rises since the supply of machines is more elastic than that of labor. The exponent on the first term, $\frac{1}{\sigma_S} - \frac{1}{\sigma_R} > 0$, since $\sigma_R > \sigma_S$, and the exponent on the second term, $1 - \frac{1}{\sigma_R} > 0$ since $\sigma_R > 1$. Hence, the equilibrium relative wage, $\frac{W_{2U}}{W_{2R}}$ goes up as TFP rises in place 2.

Appendix C

Trade, Sorting across Occupations, and Wage Inequality

C.1 Proof of Lemma 3.1

First, I derive the supply of occupation 1 relative to occupation 2 in country n under autarky:

$$\frac{Q_1^n}{Q_2^n} = \begin{cases} 0, & \text{if } \frac{P_1^n}{P_2^n} < \frac{A_{H2}}{A_{H1}} \\ \left[0, \frac{A_{H1}}{A_{L2}} \frac{\overline{H}^n}{\overline{L}^n}\right], & \text{if } \frac{P_1^n}{P_2^n} = \frac{A_{H2}}{A_{H1}} \\ \frac{A_{H1}}{A_{L2}} \frac{\overline{H}^n}{\overline{L}^n}, & \text{if } \frac{P_1^n}{P_2^n} \in \left(\frac{A_{H2}}{A_{H1}}, \frac{A_{L2}}{A_{L1}}\right) \\ \left[\frac{A_{H1}}{A_{L2}} \frac{\overline{H}^n}{\overline{L}^n}, \infty\right), & \text{if } \frac{P_1^n}{P_2^n} = \frac{A_{L2}}{A_{L1}} \\ \infty, & \text{if } \frac{P_1^n}{P_2^n} > \frac{A_{L2}}{A_{L1}}, \end{cases} \tag{C.1}$$

which is consistent with the second figure in the top panel of Figure 3.2.

Suppose $\frac{P_1^n}{P_2^n} < \frac{A_{H2}}{A_{H1}}$. Equivalently, $P_1^n A_{H1} < P_2^n A_{H2}$ holds, meaning that high-skilled workers choose occupation 2. On the other hand, $\frac{A_{H2}}{A_{H1}} < \frac{A_{L2}}{A_{L1}}$ holds because less-skilled workers have a comparative advantage in occupation 2. Since $\frac{A_{H2}}{A_{H1}} < \frac{A_{L2}}{A_{L1}}$, $\frac{P_1^n}{P_2^n} < \frac{A_{H2}}{A_{H1}}$ implies that $\frac{P_1^n}{P_2^n} < \frac{A_{L2}}{A_{L1}}$, which is equivalent to $P_1^n A_{L1} < P_2^n A_{L2}$, meaning that less-skilled workers choose occupation 2. Since no workers choose occupation 1, $\frac{Q_1^n}{Q_2^n} = 0$.

Next, suppose $\frac{P_1^n}{P_2^n} = \frac{A_{H2}}{A_{H1}}$. Equivalently, $P_1^n A_{H1} = P_2^n A_{H2}$ holds, meaning that high-skilled workers are indifferent between occupations. Since less-skilled workers have a comparative advantage in occupation 2, $\frac{A_{H2}}{A_{H1}} < \frac{A_{L2}}{A_{L1}}$ holds. This leads to $\frac{P_1^n}{P_2^n} = \frac{A_{H2}}{A_{H1}} < \frac{A_{L2}}{A_{L1}}$, implying that $P_1^n A_{L1} < P_2^n A_{L2}$. Hence, less-skilled workers choose occupation 2. Since high-skilled workers are indifferent between occupations, consider an extreme case where all high-skilled choose occupation 2. Then, $\frac{Q_1^n}{Q_2^n} = 0$. Consider another extreme case where all high-skilled workers choose occupation 1. Then, $\frac{Q_1^n}{Q_2^n} = \frac{A_{H1}}{A_{L2}} \frac{\overline{H}^n}{\overline{L}^n}$ holds as all high-skilled workers choose occupation 1 and all less-skilled workers choose occupation 2. Therefore, $\frac{Q_1^n}{Q_2^n} \in \left[0, \frac{A_{H1}}{A_{L2}} \frac{\overline{H}^n}{\overline{L}^n}\right]$.

Then, suppose $\frac{P_1^n}{P_2^n} \in \left(\frac{A_{H2}}{A_{H1}}, \frac{A_{L2}}{A_{L1}}\right)$. Equivalently, $P_1^n A_{H1} > P_2^n A_{H2}$ holds, while $P_1^n A_{L1} < P_2^n A_{L2}$ holds. This means that high-skilled workers now choose occupation 1 and less-skilled workers occupation 2. Hence, $\frac{Q_1^n}{Q_2^n} = \frac{A_{H1}}{A_{L2}} \frac{\overline{H}^n}{\overline{L}^n}$.

Next, suppose $\frac{P_1^n}{P_2^n} = \frac{A_{L2}}{A_{L1}}$. Since less-skilled workers have a comparative advantage in occupation 2, $\frac{A_{H2}}{A_{H1}} < \frac{A_{L2}}{A_{L1}}$ holds. As a result, $\frac{P_1^n}{P_2^n} = \frac{A_{L2}}{A_{L1}} > \frac{A_{H2}}{A_{H1}}$, which means that $P_1^n A_{L1} = P_2^n A_{L2}$ and $P_1^n A_{H1} > P_2^n A_{H2}$. Hence, high-skilled workers choose occupation 1 and less-skilled workers are indifferent between occupations. Since less-skilled workers are indifferent between occupations, consider an extreme case where all less-skilled workers choose occupation 2. Then, $\frac{Q_1^n}{Q_2^n} = \frac{A_{H1}}{A_{L2}} \frac{\overline{H}^n}{\overline{L}^n}$. Consider another extreme case where all less-skilled workers choose occupation 1. Then, $\frac{Q_1^n}{Q_2^n} = \infty$. Hence, $\frac{Q_1^n}{Q_2^n} \in \left[\frac{A_{H1}}{A_{L2}} \frac{\overline{H}^n}{\overline{L}^n}, \infty\right)$.

Finally, suppose $\frac{P_1^n}{P_2^n} > \frac{A_{L2}}{A_{L1}}$. Since less-skilled workers have a comparative advantage in occupation 2, $\frac{A_{H2}}{A_{H1}} < \frac{A_{L2}}{A_{L1}}$ holds. Consequently, we have $\frac{P_1^n}{P_2^n} > \frac{A_{L2}}{A_{L1}} > \frac{A_{H2}}{A_{H1}}$, which means that $P_1^n A_{H1} > P_2^n A_{H2}$ and $P_1^n A_{L1} > P_2^n A_{L2}$. Hence, all workers choose occupation 1, leading to $\frac{Q_1^n}{Q_2^n} = \infty$.

As derived in the main text, the demand for occupation 1 relative to occupation 2 in country n under autarky is given by

$$\frac{Q_1^n}{Q_2^n} = \psi\Big(\frac{P_1^n}{P_2^n}\Big)^{-1}.$$

Now, I will prove the first part of the lemma using proof by contradiction. Suppose $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi\frac{A_{L2}}{A_{H2}}$. First, in equilibrium, it cannot be the case that $\frac{Q_1^N}{Q_2^N} = 0$ or $\frac{Q_1^N}{Q_2^N} = \infty$. Workers consume goods X and Y, and to ensure market clearing for goods, both occupations must be supplied. Next, for contradiction, suppose $\frac{P_1^N}{P_2^N} \in \big(\frac{A_{H2}}{A_{H1}}, \frac{A_{L2}}{A_{L1}}\big)$. Then, $\frac{Q_1^N}{Q_2^N} = \frac{A_{H1}}{A_{L2}}\frac{\overline{H}^N}{\overline{L}^N}$ holds. By the market clearing condition for occupations, we have $\psi\big(\frac{P_1^N}{P_2^N}\big)^{-1} = \frac{A_{H1}}{A_{L2}}\frac{\overline{H}^N}{\overline{L}^N}$. Since $\frac{P_1^N}{P_2^N} \in \big(\frac{A_{H2}}{A_{H1}}, \frac{A_{L2}}{A_{L1}}\big)$, it follows that $\frac{\overline{H}^N}{\overline{L}^N} \in \big(\psi\frac{A_{L1}}{A_{H1}}, \psi\frac{A_{L2}}{A_{H2}}\big)$, which contradicts the assumption that $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi\frac{A_{L2}}{A_{H2}}$. Then, suppose $\frac{P_1^N}{P_2^N} = \frac{A_{L2}}{A_{L1}}$ for contradiction. Similarly, the market clearing condition for occupations leads to $\frac{\overline{H}^N}{\overline{L}^N} \leq \psi\frac{A_{L1}}{A_{H1}}$, which is a contradiction. Finally, suppose $\frac{P_1^N}{P_2^N} = \frac{A_{H2}}{A_{H1}}$. The market clearing condition for occupations yields $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi\frac{A_{L2}}{A_{H2}}$, which is consistent with the assumption. Therefore, in country N, where $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi\frac{A_{L2}}{A_{H2}}$, we have $\frac{P_1^N}{P_2^N} = \frac{A_{H2}}{A_{H1}}$. At this relative price of occupation, high-skilled workers are indifferent between occupations, while less-skilled workers choose occupation 2.

The proof of the second part of the lemma is analogous to the proof above. Suppose $\frac{\overline{H}^S}{\overline{L}^S} \leq \psi\frac{A_{L1}}{A_{H1}}$. Using proof by contradiction and the market clearing condition for occupations, we obtain $\frac{P_1^S}{P_2^S} = \frac{A_{L2}}{A_{L1}}$. At this relative price of occupation in country S, high-skilled workers are employed in occupation 1, while less-skilled workers are indifferent between occupations.

C.2 Proof of Lemma 3.2

Lemma 3.1 shows cross-country differences in labor allocation across occupations under autarky. In that lemma, cross-country differences in relative skill abundance play a crucial role $\left(\text{i.e., } \frac{\overline{H}^N}{\overline{L}^N} \geq \psi \frac{A_{L2}}{A_{H2}} \text{ and } \frac{\overline{H}^S}{\overline{L}^S} \leq \psi \frac{A_{L1}}{A_{H1}}\right)$. On the other hand, Lemma 3.2, which pertains to the occupational wage premium, holds regardless of cross-country differences in relative skill abundance. I will show that the average wage for occupation 1 is higher than that for occupation 2, regardless of a country's relative skill abundance.

Equation C.1 contains five possible cases for the price of occupation 1 relative to occupation 2 in country n, $\frac{P_1^n}{P_2^n}$. However, in equilibrium, $\frac{P_1^n}{P_2^n}$ must satisfy one of the following three cases: either $\frac{P_1^n}{P_2^n} = \frac{A_{H2}}{A_{H1}}$, $\frac{P_1^n}{P_2^n} \in \left(\frac{A_{H2}}{A_{H1}}, \frac{A_{L2}}{A_{L1}}\right)$, or $\frac{P_1^n}{P_2^n} = \frac{A_{L2}}{A_{L1}}$. This is because it is not possible for $\frac{Q_1^n}{Q_2^n} = 0$ or $\frac{Q_1^n}{Q_2^n} = \infty$ to hold in equilibrium. Workers consume goods X and Y, and to ensure market clearing for goods, both occupations must be supplied.[1] Observe that in any of the three cases, $\frac{P_1^n}{P_2^n} \geq \frac{A_{H2}}{A_{H1}}$ holds. In addition, high-skilled workers have an absolute advantage in both occupations. So, $A_{H2} > A_{L2}$ holds. Hence, I obtain $\frac{P_1^n}{P_2^n} \geq \frac{A_{H2}}{A_{H1}} > \frac{A_{L2}}{A_{H1}}$, which implies

$$P_1^n A_{H1} > P_2^n A_{L2}, \tag{C.2}$$

which will be useful for the upcoming proof.

Now, let $\overline{W}_k^n$ denote the average wage for occupation k in country n. H_k^n and L_k^n represent the number of high-skilled and less-skilled workers employed in occupation k in country n, respectively. First, suppose $\frac{P_1^n}{P_2^n} = \frac{A_{H2}}{A_{H1}}$. In Appendix C.1, I showed

[1] This argument holds for the free-trade equilibrium, too. This is because I implicitly assume incomplete specialization, meaning that both goods X and Y are produced in both countries under free trade.

that if $\frac{P_1^n}{P_2^n} = \frac{A_{H2}}{A_{H1}}$, high-skilled workers are indifferent between occupations, while less-skilled workers choose occupation 2. That is, $H_1^n + H_2^n = \overline{H}^n$ and $L_2^n = \overline{L}^n$. Then,

$$\begin{aligned}\overline{W}_1^n - \overline{W}_2^n &= W_H^n - \left(\frac{H_2^n}{H_2^n + \overline{L}^n} W_H^n + \frac{\overline{L}^n}{H_2^n + \overline{L}^n} W_L^n\right) \\ &= \frac{\overline{L}^n}{H_2^n + \overline{L}^n}(W_H^n - W_L^n) \\ &= \frac{\overline{L}^n}{H_2^n + \overline{L}^n}(P_1^n A_{H1} - P_2^n A_{L2}) > 0,\end{aligned}$$

where the third equality follows from $W_H^n = P_1^n A_{H1}$ (which also equals $P_2^n A_{H2}$) and $W_L^n = P_2^n A_{L2}$, and the inequality at the end follows from Equation C.2.

Second, suppose $\frac{P_1^n}{P_2^n} \in \left(\frac{A_{H2}}{A_{H1}}, \frac{A_{L2}}{A_{L1}}\right)$. In Appendix C.1, I showed that if $\frac{P_1^n}{P_2^n} \in \left(\frac{A_{H2}}{A_{H1}}, \frac{A_{L2}}{A_{L1}}\right)$, then high-skilled workers choose occupation 1, while less-skilled workers choose occupation 2. That is, $H_1^n = \overline{H}^n$ and $L_2^n = \overline{L}^n$. Then,

$$\overline{W}_1^n - \overline{W}_2^n = W_H^n - W_L^n = P_1^n A_{H1} - P_2^n A_{L2} > 0,$$

where the third equality follows from $W_H^n = P_1^n A_{H1}$ and $W_L^n = P_2^n A_{L2}$, and the inequality at the end follows from Equation C.2.

Finally, suppose $\frac{P_1^n}{P_2^n} = \frac{A_{L2}}{A_{L1}}$. In Appendix C.1, I showed that if $\frac{P_1^n}{P_2^n} = \frac{A_{L2}}{A_{L1}}$, high-skilled workers choose occupation 1 and less-skilled workers are indifferent between occupations. That is, $H_1^n = \overline{H}^n$ and $L_1^n + L_2^n = \overline{L}^n$. Then,

$$\begin{aligned}\overline{W}_1^n - \overline{W}_2^n &= \frac{\overline{H}^n}{\overline{H}^n + L_1^n} W_H^n + \frac{L_1^n}{\overline{H}^n + L_1^n} W_L^n - W_L^n \\ &= \frac{\overline{H}^n}{\overline{H}^n + L_1^n} (W_H^n - W_L^n) \\ &= \frac{\overline{H}^n}{\overline{H}^n + L_1^n} (P_1^n A_{H1} - P_2^n A_{L2}) > 0,\end{aligned}$$

where the third equality follows from $W_H^n = P_1^n A_{H1}$ and $W_L^n = P_2^n A_{L2}$ (which also equals $P_1^n A_{L1}$), and the inequality at the end follows from Equation C.2.

C.3 Proof of Occupation Price Equalization

I assume incomplete specialization: both goods X and Y are produced in both countries under free trade. Based on this assumption, I will prove that under free trade, prices for each occupation are equalized across countries. In each country n, the zero-profit conditions for competitive firms producing goods X and Y are given by

$$P_X = \tilde{\alpha}(P_1^n)^{\alpha}(P_2^n)^{1-\alpha}, \tag{C.3}$$

and

$$1 = \tilde{\beta}(P_1^n)^{\beta}(P_2^n)^{1-\beta}, \tag{C.4}$$

respectively, where good Y is chosen as the numeraire. The equalities in Equations C.3 and C.4 arise from incomplete specialization. Since P_X is the same across countries, Equation C.3 imply

$$\left(\frac{P_1^N}{P_1^S}\right)^{\alpha} = \left(\frac{P_2^S}{P_2^N}\right)^{1-\alpha}. \tag{C.5}$$

Combining Equations C.3 and C.4 yields

$$P = \frac{\tilde{\alpha}}{\tilde{\beta}}\Big(\frac{P_1^n}{P_2^n}\Big)^{\alpha-\beta},$$

where P denotes the price of good X relative to good Y. Since P is the same across countries, we obtain

$$\frac{P_1^N}{P_1^S} = \frac{P_2^N}{P_2^S}. \tag{C.6}$$

Substituting Equation C.6 into Equation C.5 yields $P_2^N = P_2^S$. Thus, $P_1^N = P_1^S$ follows from Equation C.6.

C.4 Proof of Proposition 3.1

Before proving the proposition, I will first derive the supply of occupation 1 relative to occupation 2 under free trade:

$$\frac{Q_1}{Q_2} = \begin{cases} 0, & \text{if } \frac{P_1}{P_2} < \frac{A_{H2}}{A_{H1}} \\ \Big[0, \frac{A_{H1}}{A_{L2}}\frac{\overline{H}^{FT}}{\overline{L}^{FT}}\Big], & \text{if } \frac{P_1}{P_2} = \frac{A_{H2}}{A_{H1}} \\ \frac{A_{H1}}{A_{L2}}\frac{\overline{H}^{FT}}{\overline{L}^{FT}}, & \text{if } \frac{P_1}{P_2} \in \big(\frac{A_{H2}}{A_{H1}}, \frac{A_{L2}}{A_{L1}}\big) \\ \Big[\frac{A_{H1}}{A_{L2}}\frac{\overline{H}^{FT}}{\overline{L}^{FT}}, \infty\Big), & \text{if } \frac{P_1}{P_2} = \frac{A_{L2}}{A_{L1}} \\ \infty, & \text{if } \frac{P_1}{P_2} > \frac{A_{L2}}{A_{L1}}, \end{cases}$$

which is consistent with the second figure in the top panel of Figure 3.4.

The derivation of this equation is analogous to that of Equation C.1. First, suppose $\frac{P_1}{P_2} < \frac{A_{H2}}{A_{H1}}$. Equivalently, $P_1 A_{H1} < P_2 A_{H2}$ holds, meaning that high-skilled workers choose occupation 2 in both countries. On the other hand, $\frac{A_{H2}}{A_{H1}} < \frac{A_{L2}}{A_{L1}}$ holds because less-skilled workers have a comparative advantage in occupation 2. Since $\frac{A_{H2}}{A_{H1}} < \frac{A_{L2}}{A_{L1}}$, $\frac{P_1}{P_2} < \frac{A_{H2}}{A_{H1}}$ implies that $\frac{P_1}{P_2} < \frac{A_{L2}}{A_{L1}}$, which is equivalent to $P_1 A_{L1} < P_2 A_{L2}$, meaning that less-skilled workers choose occupation 2 in both countries. Since no workers choose occupation 1, the supply of occupation 1 relative to occupation 2 under free trade, $\frac{Q_1}{Q_2} = 0$.

Next, suppose $\frac{P_1}{P_2} = \frac{A_{H2}}{A_{H1}}$. Equivalently, $P_1 A_{H1} = P_2 A_{H2}$ holds, meaning that high-skilled workers in both countries are indifferent between occupations. Since less-skilled workers have a comparative advantage in occupation 2, $\frac{A_{H2}}{A_{H1}} < \frac{A_{L2}}{A_{L1}}$ holds. This leads to $\frac{P_1}{P_2} = \frac{A_{H2}}{A_{H1}} < \frac{A_{L2}}{A_{L1}}$, implying that $P_1 A_{L1} < P_2 A_{L2}$. Hence, less-skilled workers in both countries choose occupation 2. Since high-skilled workers in both countries are indifferent between occupations, consider an extreme case where all high-skilled in both countries choose occupation 2. Then, $\frac{Q_1}{Q_2} = 0$. Consider another extreme case where all high-skilled workers in both countries choose occupation 1. Then, $\frac{Q_1}{Q_2} = \frac{A_{H1}}{A_{L2}} \frac{\overline{H}^{FT}}{\overline{L}^{FT}}$ holds as all high-skilled workers in both countries choose occupation 1 and all less-skilled workers in both countries choose occupation 2. Therefore, $\frac{Q_1}{Q_2} \in \left[0, \frac{A_{H1}}{A_{L2}} \frac{\overline{H}^{FT}}{\overline{L}^{FT}}\right]$.

Then, suppose $\frac{P_1}{P_2} \in \left(\frac{A_{H2}}{A_{H1}}, \frac{A_{L2}}{A_{L1}}\right)$. Equivalently, $P_1 A_{H1} > P_2 A_{H2}$ holds, while $P_1 A_{L1} < P_2 A_{L2}$ holds. This means that high-skilled workers in both countries now choose occupation 1 and less-skilled workers in both countries choose occupation 2. Hence, $\frac{Q_1}{Q_2} = \frac{A_{H1}}{A_{L2}} \frac{\overline{H}^{FT}}{\overline{L}^{FT}}$.

Next, suppose $\frac{P_1}{P_2} = \frac{A_{L2}}{A_{L1}}$. Since less-skilled workers have a comparative advantage in occupation 2, $\frac{A_{H2}}{A_{H1}} < \frac{A_{L2}}{A_{L1}}$ holds. As a result, we have $\frac{P_1}{P_2} = \frac{A_{L2}}{A_{L1}} > \frac{A_{H2}}{A_{H1}}$, which means that $P_1 A_{L1} = P_2 A_{L2}$ and $P_1 A_{H1} > P_2 A_{H2}$. Hence, high-skilled workers in both countries choose occupation 1 and less-skilled workers in both countries are indifferent between occupations. Since less-skilled workers in both countries are indifferent between occupations, consider an extreme case where all less-skilled workers in both countries choose occupation 2. Then, $\frac{Q_1}{Q_2} = \frac{A_{H1}}{A_{L2}} \frac{\overline{H}^{FT}}{\overline{L}^{FT}}$. Consider another extreme case where all less-skilled workers in both countries choose occupation 1. Then, $\frac{Q_1}{Q_2} = \infty$. Hence, $\frac{Q_1}{Q_2} \in \left[\frac{A_{H1}}{A_{L2}} \frac{\overline{H}^{FT}}{\overline{L}^{FT}}, \infty\right)$.

Finally, suppose $\frac{P_1}{P_2} > \frac{A_{L2}}{A_{L1}}$. Since less-skilled workers have a comparative advantage in occupation 2, $\frac{A_{H2}}{A_{H1}} < \frac{A_{L2}}{A_{L1}}$ holds. Consequently, we have $\frac{P_1}{P_2} > \frac{A_{L2}}{A_{L1}} > \frac{A_{H2}}{A_{H1}}$, which means that $P_1 A_{H1} > P_2 A_{H2}$ and $P_1 A_{L1} > P_2 A_{L2}$. Hence, all workers in both countries choose occupation 1, leading to $\frac{Q_1}{Q_2} = \infty$.

Next, as derived in the main text, the demand for occupation 1 relative to occupation 2 under free trade is given by

$$\frac{Q_1}{Q_2} = \psi \left(\frac{P_1}{P_2} \right)^{-1}.$$

There are three assumptions in Proposition 3.1: $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi \frac{A_{L2}}{A_{H2}}$, $\frac{\overline{H}^S}{\overline{L}^S} \leq \psi \frac{A_{L1}}{A_{H1}}$, and $\frac{\overline{H}^{FT}}{\overline{L}^{FT}} \in \left(\psi \frac{A_{L1}}{A_{H1}}, \psi \frac{A_{L2}}{A_{H2}}\right)$. The first two are assumed in Lemma 3.1, which pertains to autarky. The lemma shows that, under these two assumptions, in country N under autarky, high-skilled workers are indifferent between occupations, while less-skilled workers are employed in occupation 2; in country S under autarky, high-skilled workers are employed in occupation 1, while less-skilled workers are indifferent between occupations.

The third assumption in Proposition 3.1 pertains to free trade. Using this assumption, I will prove that, under free trade, all high-skilled workers in both countries choose occupation 1, while all less-skilled workers in both countries choose occupation 2. This suffices to prove Proposition 3.1: in country N, high-skilled workers who were employed in occupation 2 under autarky switch to occupation 1 under free trade, while less-skilled workers remain employed in occupation 2 under free trade; in country S, less-skilled workers who were employed in occupation 1 under autarky switch to occupation 2 under free trade, while high-skilled workers remain employed in occupation 1 under free trade.

I will proceed with a proof by contradiction. Suppose $\frac{\overline{H}^{FT}}{\overline{L}^{FT}} \in \left(\psi\frac{A_{L1}}{A_{H1}}, \psi\frac{A_{L2}}{A_{H2}}\right)$. First, in the free-trade equilibrium, it cannot be the case that $\frac{Q_1}{Q_2} = 0$ or $\frac{Q_1}{Q_2} = \infty$. Workers consume goods X and Y, and to ensure market clearing for goods, both occupations must be supplied. Next, for contradiction, suppose $\frac{P_1}{P_2} = \frac{A_{H2}}{A_{H1}}$. Then, by the market clearing condition for occupations, we have $\frac{Q_1}{Q_2} = \psi\frac{A_{H1}}{A_{H2}} \leq \frac{A_{H1}}{A_{L2}}\frac{\overline{H}^{FT}}{\overline{L}^{FT}}$, which implies that $\frac{\overline{H}^{FT}}{\overline{L}^{FT}} \geq \psi\frac{A_{L2}}{A_{H2}}$, which contradicts the assumption that $\frac{\overline{H}^{FT}}{\overline{L}^{FT}} \in \left(\psi\frac{A_{L1}}{A_{H1}}, \psi\frac{A_{L2}}{A_{H2}}\right)$. Then, suppose $\frac{P_1}{P_2} = \frac{A_{L2}}{A_{L1}}$ for contradiction. Similarly, the market clearing condition for occupations leads to $\frac{\overline{H}^{FT}}{\overline{L}^{FT}} \leq \psi\frac{A_{L1}}{A_{H1}}$, which is a contradiction. Finally, suppose $\frac{P_1}{P_2} \in \left(\frac{A_{H2}}{A_{H1}}, \frac{A_{L2}}{A_{L1}}\right)$. Then, the market clearing condition for occupations yields $\frac{\overline{H}^{FT}}{\overline{L}^{FT}} \in \left(\psi\frac{A_{L1}}{A_{H1}}, \psi\frac{A_{L2}}{A_{H2}}\right)$, which is consistent with the assumption. Therefore, under free trade, we have $\frac{P_1}{P_2} \in \left(\frac{A_{H2}}{A_{H1}}, \frac{A_{L2}}{A_{L1}}\right)$. At this relative price of occupation, high-skilled workers in both countries choose occupation 1, while less-skilled workers in both countries choose occupation 2.

C.5 Proof of Proposition 3.2

In Appendix C.4, I proved Proposition 3.1, which shows labor reallocation across occupations in response to international trade. Here, on the other hand, I prove Proposition 3.2, which shows effects of international trade on wage inequality.

First, I derive wages for each skill type in each country under autarky. The zero-profit condition for the competitive firm producing good Y in country n is given by

$$1 = \tilde{\beta}(P_1^n)^{\beta}(P_2^n)^{1-\beta}, \tag{C.7}$$

where good Y is chosen as the numeraire.

As assumed in Proposition 3.2, suppose $\frac{\overline{H}^N}{\overline{L}^N} \geq \psi \frac{A_{L2}}{A_{H2}}$. It follows that $\frac{P_1^N}{P_2^N} = \frac{A_{H2}}{A_{H1}}$, as shown in Appendix C.1. Plugging $\frac{P_1^N}{P_2^N} = \frac{A_{H2}}{A_{H1}}$ into Equation C.7 pins down the prices of occupations in country N:

$$P_1^N = \frac{1}{\tilde{\beta}}\Big(\frac{A_{H2}}{A_{H1}}\Big)^{1-\beta}, \quad P_2^N = \frac{1}{\tilde{\beta}}\Big(\frac{A_{H1}}{A_{H2}}\Big)^{\beta}.$$

As high-skilled workers in country N are indifferent between occupations, we have $W_H^N = P_1^N A_{H1}$ (which also equals $P_2^N A_{H2}$). Since less-skilled workers in country N choose occupation 2, $W_L^N = P_2^N A_{L2}$ holds. Hence,

$$W_H^N = \frac{1}{\tilde{\beta}}\Big(\frac{A_{H2}}{A_{H1}}\Big)^{1-\beta} A_{H1}, \quad W_L^N = \frac{1}{\tilde{\beta}}\Big(\frac{A_{H1}}{A_{H2}}\Big)^{\beta} A_{L2}.$$

Next, suppose $\frac{\overline{H}^S}{\overline{L}^S} \leq \psi \frac{A_{L1}}{A_{H1}}$, as assumed in Proposition 3.2. Then, we have $\frac{P_1^S}{P_2^S} = \frac{A_{L2}}{A_{L1}}$, as shown in Appendix C.1. Similarly, plugging $\frac{P_1^S}{P_2^S} = \frac{A_{L2}}{A_{L1}}$ into Equation C.7 pins down the prices of occupations in country S:

$$P_1^S = \frac{1}{\tilde{\beta}}\Big(\frac{A_{L2}}{A_{L1}}\Big)^{1-\beta}, \quad P_2^S = \frac{1}{\tilde{\beta}}\Big(\frac{A_{L1}}{A_{L2}}\Big)^{\beta}.$$

As high-skilled workers in country S choose occupation 1, $W_H^S = P_1^S A_{H1}$ holds. Since less-skilled workers in country S are indifferent between occupations, we have

$W_L^S = P_2^S A_{L2}$ (which also equals $P_1^S A_{L1}$). Hence,

$$W_H^S = \frac{1}{\tilde{\beta}}\Big(\frac{A_{L2}}{A_{L1}}\Big)^{1-\beta} A_{H1}, \;\; W_L^S = \frac{1}{\tilde{\beta}}\Big(\frac{A_{L1}}{A_{L2}}\Big)^{\beta} A_{L2}.$$

Now, I derive wages for each skill type under free trade, which are equalized across countries. In each country under free trade, the zero-profit condition for the competitive firm producing good Y is given by

$$1 = \tilde{\beta}(P_1)^{\beta}(P_2)^{1-\beta}, \tag{C.8}$$

where good Y is chosen as the numeraire. As assumed in Proposition 3.2, suppose $\frac{\overline{H}^{FT}}{\overline{L}^{FT}} \in \big(\psi\frac{A_{L1}}{A_{H1}}, \psi\frac{A_{L2}}{A_{H2}}\big)$. Then, in both countries, high-skilled workers choose occupation 1, while less-skilled workers choose occupation 2, as shown in Appendix C.4. Hence, the supply of occupation 1 relative to occupation 2 under free trade is given by $\frac{Q_1}{Q_2} = \frac{A_{H1}}{A_{L2}}\frac{\overline{H}^{FT}}{\overline{L}^{FT}}$. Substituting this into the demand for occupation 1 relative to occupation 2, $\frac{Q_1}{Q_2} = \psi\big(\frac{P_1}{P_2}\big)^{-1}$, pins down:

$$\frac{P_1}{P_2} = \psi\frac{A_{L2}}{A_{H1}}\frac{\overline{L}^{FT}}{\overline{H}^{FT}}.$$

Plugging this relative price of occupation into Equation C.8 yields

$$P_1 = \frac{1}{\tilde{\beta}}\left(\psi\frac{A_{L2}}{A_{H1}}\frac{\overline{L}^{FT}}{\overline{H}^{FT}}\right)^{1-\beta}, \;\; P_2 = \frac{1}{\tilde{\beta}}\left(\frac{1}{\psi}\frac{A_{H1}}{A_{L2}}\frac{\overline{H}^{FT}}{\overline{L}^{FT}}\right)^{\beta}.$$

In both countries, high-skilled workers choose occupation 1, while less-skilled workers choose occupation 2. Hence, wages for each skill type in each country are given by

$$W_H = \frac{1}{\tilde{\beta}}\left(\psi\frac{A_{L2}}{A_{H1}}\frac{\overline{L}^{FT}}{\overline{H}^{FT}}\right)^{1-\beta} A_{H1}, \quad W_L = \frac{1}{\tilde{\beta}}\left(\frac{1}{\psi}\frac{A_{H1}}{A_{L2}}\frac{\overline{H}^{FT}}{\overline{L}^{FT}}\right)^{\beta} A_{L2}.$$

Then, I prove the first part of Proposition 3.2. In country N, the wage for high-skilled workers rises, while the wage for less-skilled workers falls in the transition from autarky to free trade:

$$\frac{W_H}{W_H^N} = \frac{P_1}{P_1^N} = \frac{\frac{1}{\tilde{\beta}}\left(\psi\frac{A_{L2}}{A_{H1}}\frac{\overline{L}^{FT}}{\overline{H}^{FT}}\right)^{1-\beta}}{\frac{1}{\tilde{\beta}}\left(\frac{A_{H2}}{A_{H1}}\right)^{1-\beta}} = \left(\psi\frac{A_{L2}}{A_{H2}}\frac{\overline{L}^{FT}}{\overline{H}^{FT}}\right)^{1-\beta} > 1,$$
$$\frac{W_L^N}{W_L} = \frac{P_2^N}{P_2} = \frac{\frac{1}{\tilde{\beta}}\left(\frac{A_{H1}}{A_{H2}}\right)^{\beta}}{\frac{1}{\tilde{\beta}}\left(\frac{1}{\psi}\frac{A_{H1}}{A_{L2}}\frac{\overline{H}^{FT}}{\overline{L}^{FT}}\right)^{\beta}} = \left(\psi\frac{A_{L2}}{A_{H2}}\frac{\overline{L}^{FT}}{\overline{H}^{FT}}\right)^{\beta} > 1,$$

where the inequalities follow from the assumption in Proposition 3.2: $\frac{\overline{H}^{FT}}{\overline{L}^{FT}} \in \left(\psi\frac{A_{L1}}{A_{H1}}, \psi\frac{A_{L2}}{A_{H2}}\right)$. Since $\frac{\overline{H}^{FT}}{\overline{L}^{FT}} < \psi\frac{A_{L2}}{A_{H2}}$, it follows that $\psi\frac{A_{L2}}{A_{H2}}\frac{\overline{L}^{FT}}{\overline{H}^{FT}} > 1$.

Finally, proof of the second part of Proposition 3.2 is analogous. In country S, the wage for high-skilled workers falls, while the wage for less-skilled workers rises in the transition from autarky to free trade:

$$\frac{W_H}{W_H^S} = \frac{P_1}{P_1^S} = \frac{\frac{1}{\tilde{\beta}}\left(\psi\frac{A_{L2}}{A_{H1}}\frac{\overline{L}^{FT}}{\overline{H}^{FT}}\right)^{1-\beta}}{\frac{1}{\tilde{\beta}}\left(\frac{A_{L2}}{A_{L1}}\right)^{1-\beta}} = \left(\psi\frac{A_{L1}}{A_{H1}}\frac{\overline{L}^{FT}}{\overline{H}^{FT}}\right)^{1-\beta} < 1,$$
$$\frac{W_L^S}{W_L} = \frac{P_2^S}{P_2} = \frac{\frac{1}{\tilde{\beta}}\left(\frac{A_{L1}}{A_{L2}}\right)^{\beta}}{\frac{1}{\tilde{\beta}}\left(\frac{1}{\psi}\frac{A_{H1}}{A_{L2}}\frac{\overline{H}^{FT}}{\overline{L}^{FT}}\right)^{\beta}} = \left(\psi\frac{A_{L1}}{A_{H1}}\frac{\overline{L}^{FT}}{\overline{H}^{FT}}\right)^{\beta} < 1,$$

where the inequalities follow from the assumption in Proposition 3.2: $\frac{\overline{H}^{FT}}{\overline{L}^{FT}} \in \left(\psi \frac{A_{L1}}{A_{H1}}, \psi \frac{A_{L2}}{A_{H2}}\right)$. Since $\frac{\overline{H}^{FT}}{\overline{L}^{FT}} > \psi \frac{A_{L1}}{A_{H1}}$, it follows that $\psi \frac{A_{L1}}{A_{H1}} \frac{\overline{L}^{FT}}{\overline{H}^{FT}} < 1$.