APPENDIX 3A

LPJmL Dynamic Global Vegetation Model[1]

Standard Outputs

and

Inventory of Process Equations

[1] Excerpted from Schaphof 2018 (equation sources and authorities are provided in the technical supplement to cited paper, available at GMD - LPJmL4 – a dynamic global vegetation model with managed land – Part 1: Model description (copernicus.org)).

Standard Outputs Computed by LPJmL 4

	Variable	Units
Carbon pools	Soil carbon	$gC\,m^{-1}$
	Litter carbon	$gC\,m^{-1}$
	Vegetation carbon	$gC\,m^{-1}$
	Above ground biomass	$gC\,m^{-1}$
Carbon fluxes	Monthly net primary production	$gC\,m^{-1}\,month^{-1}$
	Monthly gross primary production	$gC\,m^{-1}\,month^{-1}$
	Monthly soil respiration	$gC\,m^{-1}\,month^{-1}$
	Annual fire carbon emissions	$gC\,m^{-1}\,a^{-1}$
Water fluxes	Monthly interception	$mm\,month^{-1}$
	Monthly transpiration	$mm\,month^{-1}$
	Monthly evaporation	$mm\,month^{-1}$
	Monthly runoff	$mm\,month^{-1}$
	Monthly discharge	$hm^{-3}\,day^{-1}$
	Monthly grid cell albedo	-
	Monthly fraction of absorbed PAR	-
	Foliage projected cover	-
	Crop yields	$gC\,m^{-1}\,a^{-1}$
	Sowing dates	day of the year

Equation Table Describing the Different Processes Represented in the LPJmL4 Model

Parameter/Variable	abbreviation	unit	Equation
		Energy balance	
Photosynthetic active radiation	PAR	$\mathrm{mol\,m^{-2}\,day^{-1}}$	$\mathrm{PAR} = 0.5 \cdot c_q \cdot R_{s_{\mathrm{day}}}$
conversion factor from J to mol for solar radiation at 550 nm	c_q		$c_q = 4.6 \cdot 10^{-6}$
daily incoming solar irradiance	$R_{s_{\mathrm{day}}}$	$\mathrm{J\ m^{-2}\ day^{-1}}$	$R_{s_{\mathrm{day}}} = (c + d \cdot \mathrm{ni}) \cdot Q_0 \cdot (\sin(\mathrm{lat}) \cdot \sin(\delta) \cdot h_{1/2} + \cos(\mathrm{lat}) \cdot \cos(\delta) \cdot h_{1/2})$
potential evapotranspiration	PET	$\mathrm{mm\ day^{-1}}$	$\mathrm{PET} = \mathrm{PT} \cdot E_{\mathrm{eq}}$
equilibrium evapotranspiration	E_{eq}	$\mathrm{mm\ day^{-1}}$	$E_{\mathrm{eq}} = \frac{s}{s+\gamma} \cdot \frac{R_{n_{\mathrm{day}}}}{\lambda}$
daily surface net radiation	$R_{n_{\mathrm{day}}}$	$\mathrm{J\ m^{-2}\ day^{-1}}$	
latent heat of vaporization	λ	$\mathrm{J\ kg^{-1}}$	$\lambda = 2.495 \times 10^6 + 2380 \cdot T_{\mathrm{air}}$
slope of the saturation vapour pressure curve	s	$\mathrm{Pa\ K^{-1}}$	$s = 2.502 \times 10^6 \cdot \exp[17.269 \cdot T_{\mathrm{air}}/(237.3 + (237.3 + T_{\mathrm{air}}))]/(237.3 + T_{\mathrm{air}})^2$
psychrometric constant	γ	$\mathrm{Pa\ K^{-1}}$	$\gamma = 65.05 + 0.064 \cdot T_{\mathrm{air}}$
Priestley-Taylor coefficient	PT		
net surface radiation	R_n	$\mathrm{W\ m^{-2}}$	
incoming solar irradiance (downward) at the surface	R_s	$\mathrm{W\ m^{-2}}$	$R_s = (c + d \cdot \mathrm{ni}) \cdot Q_0 \cdot \cos(z)$ or as input
outgoing (upward positive) net long-wave radiation flux at the surface	R_l	$\mathrm{W\ m^{-2}}$	$R_l = (b + (1-b) \cdot \mathrm{ni}) \cdot (A - T_{\mathrm{air}})$ or as input
albedo	β		$\beta = \sum_{\mathrm{PFT}=1}^{n_{\mathrm{PFT}}} \beta_{\mathrm{PFT}} \cdot \mathrm{FPC}_{\mathrm{PFT}} + F_{\mathrm{bare}} \cdot (F_{\mathrm{snow}} \cdot \beta_{\mathrm{snow}} + (1 - F_{\mathrm{snow}}) \cdot \beta_{\mathrm{soil}})$
albedo bare soil	β_{soil}		
albedo snow	β_{snow}		
plant compartments specific albedo	β_{PFT}		
coverage of bare soil	F_{bare}		
coverage of snow	F_{snow}		
empirical constant	b		see Prentice et al. (1993)
empirical constant	A		see Prentice et al. (1993)
mean daily air temperature	T_{air}	°C	
net outgoing daytime long-wave flux	$R_{l_{n_{\mathrm{day}}}}$	$\mathrm{J\ m^{-2}\ day^{-1}}$	$R_{l_{\mathrm{day}}} = R_l \cdot \mathrm{daylength} \cdot 3600$
angular distance between the sun's rays and the local vertical	z		
proportion of bright sky	ni		$\mathrm{ni} = 1 - \mathrm{cloudiness}$
empirical constant	c		see Prentice et al. (1993)

Parameter/Variable	abbreviation	unit	Equation
empirical constant	d		see Prentice et al. (1993)
solar constant	Q_0	W m^{-2}	$Q_0 = Q_{00} \cdot (1 + 2 \cdot 0.01675 \cdot \cos(2 \cdot \pi \cdot i/365))$
solar zenith angle	z		$\cos(z) = \sin(\text{lat}) \cdot \sin(\delta) + \cos(\text{lat}) \cdot \cos(\delta) \cdot \cos(h)$
latitude	lat	radians	
hour angle	h		
solar declination	δ	radians	$\delta = -23.4 \cdot \pi/180 \cdot \cos(2 \cdot \pi \cdot (i + 10)/365)$
half-day length	$h_{1/2}$	angular units	$h_{1/2} = \arccos(-(\sin(\text{lat}) \cdot \sin(\delta))/(\cos(\text{lat}) \cdot \cos(\delta)))$
duration of sunshine of a single day	daylength	hours	$\text{daylength} = 24 \cdot \frac{h_{1/2}}{\pi}$
Soil temperatures	T_{soil}	°C	$\frac{\partial T_{\text{soil}}}{\partial t} = \alpha \cdot \frac{\partial^2 T_{soil}}{\partial z^2}$
thermal diffusivity	$\alpha = \lambda/c$	m^2 s^{-1}	
thermal conductivity	λ	W m^{-1} K^{-1}	
soil layer	l		
time step	t		
stability criterion	r		$r = \frac{\alpha \Delta t}{(\Delta z)^2}$
Heat capacity	c	J K^{-1} m^{-3}	$c = c_{\text{min}} \cdot m_{\text{min}} + c_{\text{water}} \cdot m_{\text{water}} + c_{\text{ice}} \cdot m_{\text{ice}}$
soil minerals	c_{min}		
soil water content	c_{water}		
soil ice content	c_{ice}		
corresponding shares of c_{min}, c_{water}, c_{ice}	m	m^3	

Plant physiology

absorbed photosynthetically active radiation	APAR	mol m^{-2} day^{-1}	$\text{APAR}_{\text{PFT}} = \text{PAR} \cdot \text{FAPAR}_{\text{PFT}} \cdot \alpha_{\text{aPFT}}$
fractional absorbed photosynthetically active radiation	$\text{FAPAR}_{\text{PFT}}$		$\text{FAPAR}_{\text{PFT}} = \text{FPC}_{\text{PFT}} \cdot \Big((\text{phen}_{\text{PFT}} - \text{F}_{\text{SnowGC}}) \cdot (1 - \beta_{\text{leaf,PFT}}) - ((1 - \text{phen}_{\text{PFT}}) \cdot c_{\text{fstem}} \cdot \beta_{\text{stem,PFT}})\Big)$
scaling factor to scale leaf-level photosynthesis in LPJmL4 to biome level	α_{aPFT}		
daily phenological status	phen_{PFT}		
fraction of snow in the green canopy	F_{SnowGC}		
foliage projective cover of the respective PFT	FPC_{PFT}		$\text{FPC}_{\text{PFT}} = \text{CA}_{\text{ind}} \cdot P \cdot \text{FPC}_{\text{ind}}$
masking of the ground by stems and branches without leaves	c_{fstem}		
gross photosynthesis rate	A_{gd}	gC m^{-2} day^{-1}	$A_{\text{gd}} = \Big(J_E + J_C - \sqrt{(J_E + J_C)^2 - 4 \cdot \theta \cdot J_E \cdot J_C}\Big)/(2 \cdot \theta) \cdot \text{daylength}$
light-limited photosynthesis rate	J_E	mol C m^{-2} $hour^{-1}$	$J_E = C_1 \cdot \frac{\text{APAR}}{\text{daylength}}$
for C_3-Photosynthesis			$C_1 = \alpha_{C_3} \cdot T_{\text{stress}} \cdot \left(\frac{p_i - \Gamma_*}{p_i + 2 \cdot \Gamma_*}\right)$

Parameter/Variable	abbreviation	unit	Equation
for C_4-Photosynthesis			$C_1 = \alpha_{C_4} \cdot T_{\text{stress}} \cdot \left(\frac{\lambda}{\lambda_{\max C4}}\right)$
internal partial pressure of CO_2	p_i	Pa	$p_i = \lambda \cdot p_a$
ambient partial pressure of CO_2	p_a	Pa	
parameter describing the ratio of the intercellular to the ambient CO_2 concentration	λ		
PFT-specific temperature inhibition function	T_{stress}		
intrinsic quantum efficiencies for CO_2 uptake in C_3 plants	α_{C_3}		
intrinsic quantum efficiencies for CO_2 uptake in C_4 plants	α_{C_4}		
CO_2 compensation point	Γ_*		$\Gamma_* = \frac{[O_2]}{2 \cdot \tau}$
specificity factor	τ		$\tau = \frac{V_c \cdot K_C}{V_m \cdot K_O}$
Michaelis-Menten constant of CO_2	K_C		
Michaelis-Menten constant of O_2	K_O		
partial pressure of O_2	O_2	Pa	
Rubisco-limited photosynthesis rate	J_C	mol C m^{-2} $hour^{-1}$	$J_C = C_2 \cdot V_m$
maximum Rubisco capacity	V_m	gC m^{-2} day^{-1}	$V_m = \frac{1}{b} \cdot \frac{C_1}{C_2}((2 \cdot \theta - 1) \cdot s - (2 \cdot \theta \cdot s - C_2) \cdot \sigma) \cdot \text{APAR}$
	σ		$\sigma = \sqrt{1 - \frac{C_2 - 2}{C_2 - \theta s}}$
	s		$s = 24/\text{daylength} \cdot b$
	C_2		$C_2 = \frac{p_i - \Gamma_*}{p_i + K_C\left(1 + \frac{[O_2]}{K_O}\right)}$
leaf respiration	R_{leaf}	gC m^{-2} day^{-1}	$R_{\text{leaf}} = V_m \cdot b$
daily net photosynthesis	A_{nd}	gC m^{-2} day^{-1}	
dark respiration	R_d	gC m^{-2} day^{-1}	$R_d = (1 - \text{daylength}/24) \cdot R_{\text{leaf}}$
daily net daytime photosynthesis	A_{dt}	gC m^{-2} day^{-1}	$A_{\text{dt}} = A_{\text{nd}} + R_d$
canopy conductance	g_c	mm s^{-1}	$g_c = g_{\min} + \frac{1.6 A_{dt}}{p_a(1-\lambda)}$
PFT-specific minimum canopy conductance	$g_{\min}$	mm s^{-1}	
daily phenology status	phen_{PFT}		$\text{phen}_{\text{PFT}} = f_{\text{cold}} \cdot f_{\text{light}} \cdot f_{\text{water}} \cdot f_{\text{heat}}$
limited by cold temperatures	f_{cold}		
relation to light	f_{light}		
relation to water availability	f_{water}		
limited by heat stress	f_{heat}		
inflection point of the respective logistic function	b_x		
slope of the respective logistic function	sl_x		
change rate parameter	τ_x		
CN ratio of above-ground tissue	$\text{CN}_{\text{sapwood}}$		
CN ratio of below-ground tissue	CN_{root}		
Temperature	T (T_{air}, T_{soil})	°C	

Parameter/Variable	abbreviation	unit	Equation
phenology	phen_{PFT}		
autotrophic respiration aboveground tissue	R_{sapwood}	$\text{gC m}^{-2}\text{ day}^{-1}$	$R_{\text{sapwood}} = P \cdot r_{\text{PFT}} \cdot k \cdot \frac{C_{\text{sapwood,ind}}}{CN_{\text{sapwood}}} \cdot g(T_{\text{air}})$
autotrophic respiration belowground tissue	R_{root}	$\text{gC m}^{-2}\text{ day}^{-1}$	$R_{\text{root}} = P \cdot r_{\text{PFT}} \cdot k \cdot \frac{C_{\text{root,ind}}}{CN_{\text{root}}} \cdot g(T_{\text{soil}}) \cdot \text{phen}_{\text{PFT}}$
respiration rate	r_{PFT}	$\text{gC gN}^{-1}\text{ day}^{-1}$	
temperature function	$g(T)$		$g(T) = \exp\left[308.56 \cdot \left(\frac{1}{56.02} - \frac{1}{(T+46.02)}\right)\right]$
leaf respiration	R_{leaf}		$R_{\text{leaf}} = V_m \cdot b$
static parameter	b		
daily net primary production	NPP	$\text{gC m}^{-2}\text{ day}^{-1}$	$\text{NPP} = 0.75 \cdot (\text{GPP} - R_{\text{leaf}} - R_{\text{sapwood}} - R_{\text{root}})$

Plant functional types (PFT)

leaf mass	$C_{\text{leaf,ind}}$	$\text{gC} \cdot \text{ind}^{-1}$	
fine root mass	$C_{\text{root,ind}}$	$\text{gC} \cdot \text{ind}^{-1}$	
sapwood mass	$C_{\text{sapwood,ind}}$	$\text{gC} \cdot \text{ind}^{-1}$	
heartwood mass	$C_{\text{heartwood,ind}}$	$\text{gC} \cdot \text{ind}^{-1}$	
average individual leaf area	LA_{ind}	$\text{m}^2 \cdot \text{ind}^{-1}$	$\text{LA}_{\text{ind}} = k_{\text{la:sa}} \cdot \text{SA}_{\text{ind}}$
ratio of leaf to sapwood area	$k_{\text{la:sa}}$		
sapwood cross-sectional area	SA_{ind}		
grass leaf biomass	C_{leaf}	gC m^{-2}	$C_{\text{leaf}} = \text{lr}_{\text{max}} \cdot \omega \cdot C_{\text{roots}}$
leaf-to-root mass ratio	lr		$\text{lr} = \text{lr}_p \cdot W_{\text{supply}} / W_{\text{demand}}$
maximum leaf-to-root mass ratio	lr_{max}		
tree height	H	m	$H = k_{\text{allom2}} \cdot D^{k_{\text{allom3}}}$
stem diameter	D	m	
crown area	CA_{ind}	$\text{m}^2 \cdot \text{ind}^{-1}$	$\text{CA}_{\text{ind}} = k_{\text{allom1}} \cdot D^{k_{\text{rp}}}$
constant wood density	WD	gC m^{-2}	$H = \frac{C_{\text{sapwood,ind}} \cdot k_{\text{la:sa}}}{\text{WD} \cdot C_{\text{leaf,ind}} \cdot \text{SLA}}$
individual leaf area index	LAI_{ind}		$\text{LAI}_{\text{ind}} = \frac{C_{\text{leaf,ind}} \cdot \text{SLA}}{\text{CA}_{\text{ind}}}$
specific leaf area	SLA	$\text{m}^2\text{ gC}^{-1}$	$\text{SLA} = \frac{2 \times 10^{-4}}{DM_C} \cdot 10^{(\beta_0 - \beta_1 \cdot \log(\alpha_{\text{leaf}})/\log(10)}$
leaf longevity	α_{leaf}	months	
parameter for SLA calculation	β_0		Kattge et al. (2011)
parameter for SLA calculation	β_1		Kattge et al. (2011)
dry matter carbon content of leaves	DM_C		Kattge et al. (2011)
foliar projective cover	FPC_{ind}		$\text{FPC}_{\text{ind}} = 1 - \exp(-k \cdot \text{LAI}_{\text{ind}})$
mean number of individuals per unit area	P	ind m^{-2}	
establishment rate	k_{est}	$\text{saplings m}^{-2}\text{ a}^{-1}$	
background mortality rate	$\text{mort}_{\text{greff}}$	$\text{ind m}^{-2}\text{ a}^{-1}$	$\text{mort}_{\text{greff}} = P \cdot \frac{k_{\text{mort1}}}{1 + k_{\text{mort2}} \cdot \text{greff}}$
yearly growth efficiency	greff		
asymptotic maximum mortality rate	k_{mort1}		

Parameter/Variable	abbreviation	unit	Equation
parameter governing the slope of the relationship between mortality and growth efficiency	k_{mort2}		
heat stress	$\text{mort}_{\text{heat}}$	ind $\text{m}^{-2}\,\text{a}^{-1}$	$\text{mort}_{\text{heat}} = P \cdot \frac{\text{gdd}_{\text{tw}}}{\text{tw}_{\text{PFT}}}$
parameter value of the heat damage function	tw_{PFT}		
temperatures above threshold (accumulated)	gdd_{tw}	°C	
Nesterov index	$\text{NI}(N_d)$		$\text{NI}(N_d) = \sum_{if\text{P}(d)\leq 3\text{mm}}^{N_d} T_{\max}(d) \cdot (T_{\max}(d) - T_{\text{dew}}(d))$
daily maximum temperature	$T_{\max}$	°C	
dew-point temperature	T_{dew}	°C	
positive temperature day	d		
probability of fire spread	P_{spread}		$P_{\text{spread}} = \begin{cases} 1 - \frac{\omega_0}{m_e}, & \omega_0 \leq m_e \\ 0, & \omega_0 > m_e \end{cases}$
litter moisture	ω_0		
moisture of extinction	m_e		
fire danger index	FDI		$\text{FDI} = \max\left\{0, 1 - \frac{1}{m_e} \cdot \exp\left(-\text{NI} \cdot \sum_{p=1}^{n} \frac{\alpha_p}{n}\right)\right\}$
slope of the probability risk function	α_p		
Human-caused ignitions	$n_{h,\text{ig}}$		$n_{h,\text{ig}} = P_D \cdot k(P_D) \cdot a(N_D)/100$
population density	P_D	ind km^{-2}	$k(P_D) = 30.0 \cdot \exp(-0.5 \cdot \sqrt{P_D})$
propensity of people to produce ignition events	$a(N_D)$	ignitions individual^{-1} d^{-1}	$a(N_D) = \frac{N_{h,\text{obs}}}{t_{\text{obs}} \cdot \text{LFS} \cdot P_D}$
average number of human-caused fires	$N_{h,\text{obs}}$		
observation years	t_{obs}		
grid cell area	A	m^2	$A_b = \min(E(n_{\text{ig}}) \cdot \text{FDI} \cdot A_f, A)$
mean fire area	a_f	ha	$\overline{a_f} = \frac{\frac{\pi}{4 \cdot L_B} \cdot D_T^2}{10000}$
independent estimates of the numbers of lightning human-caused ignition events	$n_{l,\text{ig}}$ $n_{h,\text{ig}}$		
forward rate of spread	$\text{ROS}_{f,\text{surface}}$	m min^{-1}	$\text{ROS}_{f,\text{surface}} = \frac{I_R \cdot \zeta \cdot (1+\Phi_w)}{\rho_b \cdot \epsilon \cdot Q_{\text{ig}}}$
reaction intensity	I_R	kJ m^{-2} min^{-1}	
propagating flux ratio	ζ		
multiplier that accounts for the effect of wind	Φ_w		
fuel bulk density	ρ_b	kg m^{-3}	
effective heating number	ϵ		
heat of pre-ignition	Q_{ig}	kJ kg^{-1}	
fire duration	t_{fire}	min	$t_{\text{fire}} = \frac{241}{1+240 \cdot \exp(-11.06 \cdot \text{FDI})}$
length to breadth ratio of elliptical fire	L_B		
length of major axis	D_T	m	$D_T = \text{ROS}_{f,\text{surface}} \cdot t_{\text{fire}} + \text{ROS}_{b,\text{surface}} \cdot t_{\text{fire}}$
surface as the backward rate of spread	ROS_b		
crown damage	CK		$P_m(\text{CK}) = r_{\text{CK}} \cdot \text{CK}^p$

Parameter/Variable	abbreviation	unit	Equation
resistance factor	r_{CK}	0-1	
		Crop functional types (CFT)	
phenological heat unit	PHU		$\mathrm{PHU} = -0.1081 \cdot (\mathrm{sdate} - \mathrm{keyday})^2 + 3.1633 \cdot (\mathrm{sdate} - \mathrm{keyday}) + \mathrm{PHU}_{w_{\mathrm{high}}}$
harvest indices	$\mathrm{HI}_{\mathrm{opt}}$		
heat units	HU		
heat units accumulated	$\mathrm{HU}_{\mathrm{sum}}$		$\mathrm{HU}_{\mathrm{sum}} = \sum_{t'=\mathrm{sdate}}^{t} \mathrm{HU}_{t'} \cdot v_{\mathrm{rf}} \cdot p_{\mathrm{rf}}$
phenological development stage	fPHU		$\mathrm{fPHU} = \mathrm{HU}_{\mathrm{sum}}/\mathrm{PHU}$
reduction factor for vernalization	v_{rf}		$v_{\mathrm{rf}} = (\mathrm{vdsum} - 10.0)/(\mathrm{PVD} - 10.0)$
reduction factor for photoperiod	p_{rf}		$p_{\mathrm{rf}} = (1 - p_{\mathrm{sens}}) \cdot \min(1, \max(0, (\mathrm{daylength} - p_b)/(p_s - p_b))) + p_{\mathrm{sens}}$
day of solstice	keyday		
minimum base temperature for the accumulation of heat unit	$T_{\mathrm{base}_{\mathrm{low}}}$		
20-year moving average annual temperature	atemp_{20}		
CFT-specific scaling factor	$\mathrm{pf}_{\mathrm{CFT}}$		
Vernalization requirements	PVD		$\mathrm{PVD} = \mathrm{vern}_{\mathrm{date20}} - \mathrm{sdate} - \mathrm{pPVD}_{\mathrm{CFT}}, \quad 0 \leq \mathrm{PVD} \leq 60$
CFT-specific vernalization factor	$\mathrm{pPVD}_{\mathrm{CFT}}$		
julian day of the year of sowing	sdate		
multi-annual average of the first day of the year when temperatures rise above a CFT-specific vernalization threshold	$\mathrm{vern}_{\mathrm{date20}}$		
effective number of vernalizing days	vdsum		
parametrized sensitivity to photoperiod	p_{sens}		
duration of daylight (sunrise to sunset)	daylength	hours	
base photoperiod	p_b	hours	
aturation photoperiod	p_s	hours	
maximum leaf area index	$\mathrm{LAI}_{\mathrm{max}}$		
fraction of total biomass that is allocated to the roots	f_{root}		$f_{\mathrm{root}} = \frac{0.4 - (0.3 \cdot \mathrm{fPHU}) \cdot \mathrm{wdf}}{\mathrm{wdf} + \exp(6.13 - 0.0883 \cdot \mathrm{wdf})}$
ratio between accumulated daily transpiration and accumulated daily water demand	wdf		
onset of senescence	ssn		
turning points in the phenological development	fPHU_c, fPHU_k		

Parameter/Variable	abbreviation	unit	Equation
corresponding fraction of the maximum green LAI	$\mathrm{fLAI}_{\mathrm{max}_c}$, $\mathrm{fLAI}_{\mathrm{max}_k}$		$\mathrm{fLAI}_{\mathrm{max}} = \frac{\mathrm{fPHU}}{\mathrm{fPHU}+c\cdot(\frac{c}{k})^{\frac{\mathrm{fPHU}_c-\mathrm{fPHU}}{\mathrm{fPHU}_k-\mathrm{fPHU}_c}}}$
onset of senescence as point in the phenological development	$\mathrm{fPHU}_{\mathrm{sen}}$		
daily increment	$\mathrm{LAI}_{\mathrm{inc},t}$		$\mathrm{LAI}_{\mathrm{inc},t} = (\mathrm{fLAI}_{\mathrm{max}_t} - \mathrm{fLAI}_{\mathrm{max}_{t-1}}) \cdot \mathrm{LAI}_{\mathrm{max}}$
maximum green LAI	$\mathrm{fLAI}_{\mathrm{max}}$		
LAI	LAI		$\mathrm{LAI}_t = \sum_{t'=\mathrm{sdate}}^{t} \mathrm{LAI}_{\mathrm{inc}_{t'}} \cdot \omega$
harvest index	HI		$\mathrm{HI} = \begin{cases} \mathrm{fHI}_{\mathrm{opt}} \cdot \mathrm{HI}_{\mathrm{opt}}, & \text{if } \mathrm{HI}_{\mathrm{opt}} \geq 1 \\ \mathrm{fHI}_{\mathrm{opt}} \cdot (\mathrm{HI}_{\mathrm{opt}} - 1.0) + 1.0, & \text{otherwise} \end{cases}$
	$\mathrm{fHI}_{\mathrm{opt}}$		$\mathrm{fHI}_{\mathrm{opt}} = 100 \cdot \mathrm{fPHU}/(100 \cdot \mathrm{fPHU} + \exp(11.1 - 10.0 \cdot \mathrm{fPHU}))$
storage organ	C_{so}	gC m^{-2}	$C_{\mathrm{so}} = \mathrm{HI} \cdot (C_{\mathrm{leaf}} + C_{\mathrm{so}} + C_{\mathrm{pool}})$
Excess biomass	C_{pool}	gC m^{-2}	

Soil and litter carbon pools

heterotrophic respiration	R_h	gC m^{-2} day^{-1}	$R_h = R_{h,\mathrm{litter}} + R_{h,\mathrm{fastSoil}} + R_{h,\mathrm{slowSoil}}$
carbon pool size of soil or litter per layer	C_l	gC m^{-2} $layer^{-1}$	$\frac{dC_{(l)}}{dt} = -k_{(l)} \cdot C_{(l)}$
decomposition rates for litter	k	a^{-1} $layer^{-1}$	$k_{(l,p)} = \frac{1}{\tau_{10_{(p)}}} \cdot g(T_{\mathrm{soil}}) \cdot f(\theta)$
mean residence time	τ_{10}	a	
soil volume fraction of the layer	θ		
fraction of soil organic carbon per layer	Cf_l		$\mathrm{Cf}_{(l)} = 10^{k_{\mathrm{soc}} \cdot \log_{10}(d_{(l)})}$
relative share of the layer l	$d_{(l)}$		
soil layer depth	k_{soc}	mm	
total amount of soil carbon	$C_{s_{\mathrm{total}}}$	gC	$C_{(l)} = \sum_{\mathrm{PFT}=1}^{n_{PFT}} d_{(l)}^{k_{\mathrm{soc}_{\mathrm{PFT}}}} \cdot C_{s_{\mathrm{total}}}$
mean annual decomposition rate	k_{mean}	gC a^{-1}	
mean decomposition rate for each PFT	$k_{\mathrm{mean}_{\mathrm{PFT}}}$		$k_{\mathrm{mean}_{\mathrm{PFT}}} = \sum_{l=1}^{n_{\mathrm{soil}}} (k_{\mathrm{mean}_{(l)}} \cdot \mathrm{Cf}_{(l,\mathrm{PFT})})$
annual carbon shift rates	C_{shift}	a^{-1}	$C_{\mathrm{shift}_{(l,\mathrm{PFT})}} = \frac{Cf_{(l,\mathrm{PFT})} \cdot k_{\mathrm{mean}_{(l)}}}{k_{\mathrm{mean}_{\mathrm{PFT}}}}$
infiltration rate of rain water into the soil	infil	mm	$\mathrm{infil} = \mathrm{Pr} \cdot \sqrt{1 - \frac{\mathrm{SW}_{(0)} - \mathrm{WPW}_{(0)}}{W_{\mathrm{sat}_{(0)}} - \mathrm{WPW}_{(0)}}}$

Water balance

soil water content at saturation	W_{sat}	mm	
soil water content at wilting point	W_{pwp}	mm	
total actual soil water content	SW	mm	

Parameter/Variable	abbreviation	unit	Equation
daily precipitation	Pr	mm	routed in 4 mm portion in the infiltration equation
soil water content between saturation and field capacity	FW	mm	
soil layer	l		
travel time through the soil layer	TT	hours	$\mathrm{TT}_{(l)} = \frac{\mathrm{FW}_{(l)}}{\mathrm{HC}_{(l)}}$
hydraulic conductivity	HC	$\mathrm{mm\,h^{-1}}$	$\mathrm{HC}_{(l)} = K_{s_{(l)}} \cdot \left(\frac{\mathrm{SW}_{(l)}}{W_{\mathrm{sat}(l)}}\right)^{\beta_{(l)}}$
saturated conductivity	K_s	$\mathrm{mm\,h^{-1}}$	
percolation	perc	$\mathrm{mm\,day^{-1}}$	$\mathrm{perc}_{(l)} = \mathrm{FW}_{(t,l)} \cdot \left[1 - \exp\left(\frac{-\Delta t}{\mathrm{TT}_{(l)}}\right)\right]$
Interception	I	$\mathrm{mm\,day^{-1}}$	$I = \sum_{\mathrm{PFT}=1}^{n_{\mathrm{PFT}}} I_{\mathrm{PFT}} \cdot \mathrm{LAI}_{\mathrm{PFT}} \cdot \mathrm{Pr}$
PFT-specific interception storage parameter	$\mathrm{I}_{\mathrm{PFT}}$		
PFT-specific leaf area per unit of grid cell area	$\mathrm{LAI}_{\mathrm{PFT}}$		
daily precipitation	Pr	$\mathrm{mm\,day^{-1}}$	
Soil evaporation	E_s	$\mathrm{mm\,day^{-1}}$	
vegetation cover	f_v	%	
evaporation-available soil water	w_{evap}		
plant transpiration	E_T	$\mathrm{mm\,day^{-1}}$	$E_T = \min(S, D) \cdot f_v$
daily water stress	ω		
Soil water supply	S		$S = E_{\max} \cdot w_r \cdot \mathrm{phen}_{\mathrm{PFT}}$
PFT-specific maximum water transport capacity	$E_{\max}$	$\mathrm{mm\,day^{-1}}$	
water accessible for plants	w_r		$w_r = \sum_{l=1}^{n_{\mathrm{soil}}-1} w_l \cdot \mathrm{rootdist}_l$
relative water content at field capacity	w		
fraction of roots from surface to z	rootdist		$\mathrm{rootdist} = 1 - \beta_{\mathrm{root}}^{z}$
soil depth	z	mm	
root distribution parameter	β_{root}		
fraction of water that corresponds to their foliage projected cover	S_{PFT}		$S_{\mathrm{PFT}} = S \cdot \mathrm{FPC}_{\mathrm{PFT}}$
root biomass	$\mathrm{bm}_{\mathrm{root}}$	$\mathrm{gC\,m^{-2}}$	
Atmospheric demand	D		$D = (1.0 - \mathrm{wet}) \cdot E_{\mathrm{eq}} \cdot \alpha_m / (1 + g_m/g_c)$
maximum Priestley-Taylor coefficient	α_m		
conductance scaling factor	g_m		
fraction of E_{eq} that was used to vaporize intercepted water from the canopy	wet		
homogeneous segments of length	L		
outflow of a linear reservoir cascade	Q_{out}		$Q_{\mathrm{out}}(t) = Q_{\mathrm{in}} \cdot \frac{1}{K \cdot \Gamma(n)} \left(\frac{t}{K}\right)^{n-1} \cdot \exp(-t/K)$
instantaneous inflow	Q_{in}		

Parameter/Variable	abbreviation	unit	Equation
gamma function	$\Gamma(n)$		
storage parameter	K		
linear reservoir segment of length	L	km	$K = \frac{L}{v}$
flow velocity	v	$m\ s^{-1}$	
CFT-specific irrigation threshold	it		
amount of water required in the upper 50 cm soil	NIR	mm	$\mathrm{NIR} = W_{\mathrm{fc}} - w_a - w_{\mathrm{ice}}, \quad \mathrm{NIR} \geq 0$
available soil water	w_a	mm	
frozen soil water	w_{ice}	mm	
water at field capacity	W_{fc}	mm	
conveyance efficiency	E_c		
application requirements	AR	mm	$\mathrm{AR} = W_{\mathrm{sat}} - W_{fc} - W_{\mathrm{pwp}}) \cdot d_u - w_{\mathrm{fw}}, \quad \mathrm{AR} \geq 0$
gross irrigation requirements	GIR	mm	$\mathrm{GIR} = \frac{\mathrm{NIR+AR-Store}}{E_c}$
storage buffer	Store		
water distribution uniformity scalar	d_u		
available free water	w_{fw}	mm	
annual variation coefficients for precipitation	$\mathrm{CV}_{\mathrm{prec}}$		
annual variation coefficients for temperature	$\mathrm{CV}_{\mathrm{temp}}$		
biomass after the last harvest event	$\mathrm{MC}_{\mathrm{leaf}}$	$\mathrm{gC\,m^{-2}}$	