Risk and Uncertainty in Labor Markets

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Abstract

In this dissertation I analyze the effects of various uncertainties on aggregate economy, especially on unemployment. For that reason I build a general equilibrium model with heterogeneous firms with Diamond-Mortensen-Pissarides style search-and-matching in the labor market. Firms can hire more than one worker and hiring decision is partially irreversible due to linear hiring costs. First, I analyze the effect of an increase in time-varying idiosyncratic volatility, which is a mean-preserving increase in the standard deviation of firm-level productivity. I show that the model that contains idiosyncratic volatility shock in addition to aggregate productivity shock explains 60% of the variation in unemployment and 54% of the variation in vacancies, while also performing well in consumption and investment dynamics. In that model if idiosyncratic volatility rises, unemployment rate increases. However, workers become more productive due to the reallocation of resources from low productive to high productive firms. Thus, even though there are fewer people working, total wage bill and consumption is larger. Output and capital also increase with volatility because the increase in capital demand of large and highly productive firms dominates the reduction in capital demand of small and less productive firms when volatility goes up. The irreversible search costs in the labor market by itself are not large enough to induce large and counteracting option value effects of volatility.

Secondly, I solve the model with low and high ambiguity aversion levels to understand the role of model uncertainty. I show that since low-volatility states are the ones with low utility, ambiguity-averse households distort the conditional expectations by putting more weight on low-volatility states when higher ambiguity aversion is considered. In addition, the distortion creates a correlation between aggregate pro-

ductivity and idiosyncratic volatility shocks, which are independent in the benchmark distribution. However, the additional effect of this distortion and the correlation it induces on the dynamics of aggregate variables is negligible.

Lastly, I add disasters in terms of capital depreciation into the baseline model and analyze effects of an increase in probability of a disaster on the economy. When probability of a disaster increases without an actual realization of disaster, the rate of return on capital net of depreciation becomes riskier and lower on average, thus household reduces investment today. Rental rate of capital goes up, capital and output decline over time. On the separation margin, since marginal benefit of a worker to a firm depends negatively on the rental rate of capital, separation thresholds increase for all firms, expanding the separation region, thus increasing the total separations. The rise in number of unemployed lowers the labor market tightness. The effect on the hiring margin is non-trivial. Lower labor market tightness makes hiring less costly for all firms, however higher disaster probability also reduces the marginal benefit of a worker to a firm. Responses of hiring threshold at various productivity levels show that the reduction in marginal benefit outweighs the reduction in marginal cost at lower productivity levels, and the opposite result holds at higher productivity levels. On aggregate matches are increasing because the number of hires made by highly productive firms is greater than the number of hires less productive firms decided not to make after the increase in disaster risk.

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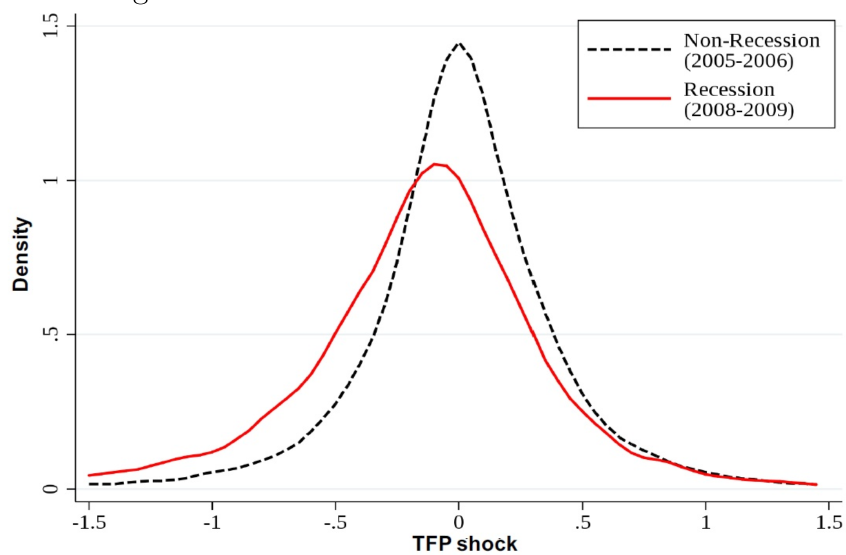
Chapter 1

Risk and Uncertainty in Labor Markets

1.1 Introduction

The effect of uncertainty on the real economy has been a popular research topic since the Great Recession, since various measures of volatility, both at macro and micro-level, increased during the recession along with the unemployment rate. One measure of micro-level volatility is the cross-sectional variance of innovations to establishment-level TFP constructed from the U.S. Census of Manufacturers data. According to Bloom et al (2016) this measure increased by 76% during the Great Recession. Figure 1.1 compares the distribution of establishment-level TFP before and after the Great Recession. The distribution after the recession has a lower mean and a higher dispersion. During the same period, the unemployment rate increased from 5% in 2008 to 10% in 2009. This is not an isolated incident, Figure 1.2 plots the establishment-level TFP dispersion series and unemployment rate since 1970s and unemployment rate is positively correlated with the establishment-level TFP dispersion with one-year lag. Motivated by those facts, I aim to understand the contribution of uncertainty, especially volatility, on unemployment by analyzing the employment decisions of multi-worker firms in this work.

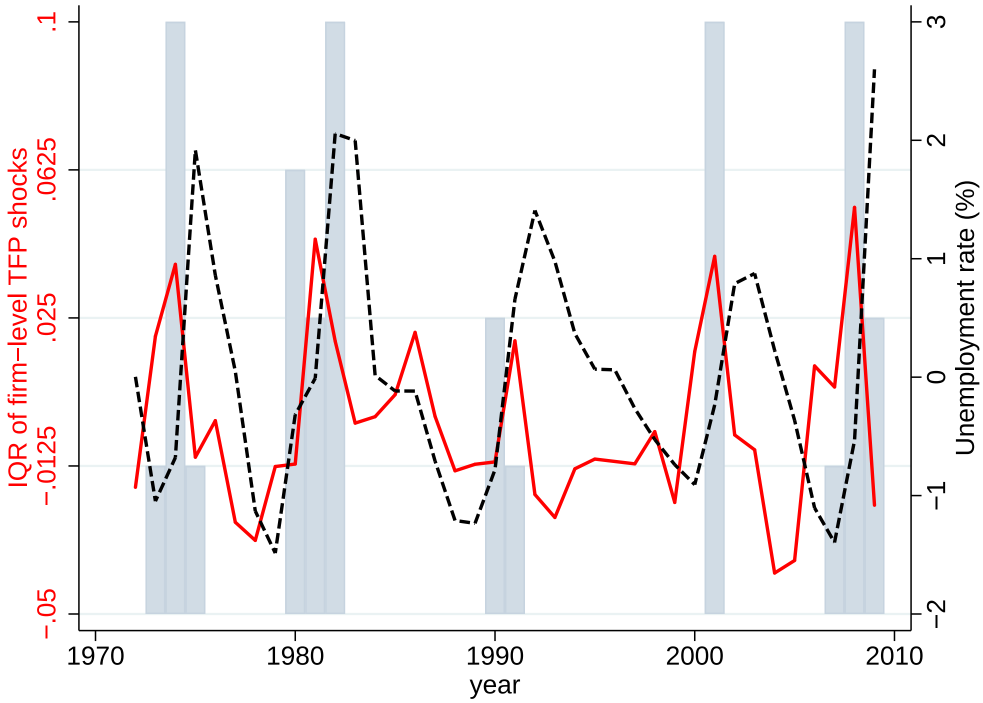
Figure 1.1: Distribution of establishment-level TFP



Note: Bloom et al. (2016) Figure 1

The model contains firms that are heterogeneous in productivity and size. Firms hire by posting costly vacancies in a frictional labor market with Diamond-Mortensen-Pissarides style search-and-matching. The linear hiring cost gives rise to productivity thresholds for separation and hiring. Thus there exists option value of inaction at both hiring and separation margins, and a range of productivity over which firms do not adjust labor. They also rent capital from the ambiguity-averse households, which pools wages of employed members, rental income from capital and profit from firms to provide equal consumption to its members and to invest in capital. Firms are subject to persistent aggregate and idiosyncratic productivity shocks. In addition, volatility

Figure 1.2: Unemployment rate and micro-level volatility



Note: Bloom et al. (2016) micro-level volatility measure and annual unemployment rate, both series are detrended with Hodrick-Prescott filter with parameter 100.

of the idiosyncratic productivity shock is time-varying and common to all firms.

Idiosyncratic volatility affects the economy through multiple channels. First, the option value of inaction for firms increases at both hiring and separation margins. At the separation margin, option value of inaction increases since the firm knows that even though today's productivity is low, due to high volatility tomorrow's productivity might be much higher and it might be costlier to re-hire a worker once separated today. Similarly, at the hiring margin, the firm may not choose to hire despite a good productivity shock today because in a volatile environment tomorrows productivity might be much lower and the hires of today may need to be fired, and maybe rehired later if conditions improve. These option-value effects shift both thresholds outward and result in an expansion of the inaction zone. As both separations and matches are expected to decline, the net result of *option value effect* on the unemployment is unclear.

Second, while the option value effect is due to the expectations about future volatility, there is also a *realized volatility effect* occurs due to the actual realization of the volatility. If volatility goes up today, firms become more dispersed both in productivity and size, and they hit the adjustment thresholds more often. As firms become more active due to volatility, the resources are reallocated towards more productive firms from less productive ones.

Third, there are also *Oi-Hartman-Abel effects* of volatility. In the frictionless case, at given prices, factor demands are convex functions of productivity. Thus, if there is a mean-preserving spread of productivity, expected factor demands are larger. This positive effect is present in this model since labor demand is convex outside the inaction zone, and capital demand is convex but less so inside the inaction zone. With convex capital demand, firms that experience larger positive shocks demand more capital thus aggregate capital demand goes up.

Lastly, there are *general equilibrium effects* operating mainly through labor market tightness. Due to Oi-Hartman-Abel effects, value of firms increases with volatility. Firms demand more capital and labor. The initial surge in vacancies causes labor market tightness to rise. When tightness increases the vacancies become harder to fill and thus their expected cost increases, shifting the hiring threshold to the right. In addition, a higher labor market tightness makes finding a job easier for the workers and improve their outside options. The increase in value of unemployment causes separations to go up. These effects combined will result in both separations and matches rising, but since separations increase more, unemployment increases.

On the household side, the volatility shock is welfare-improving due to wages and capital increasing with volatility. Wages are determined by Stole-Zwiebel bargaining as in Elsby and Michaels (2013). In a multi-worker firm, firm bargains with the last worker over the marginal surplus and all the other workers of the firm earn this bargained wage. The bargained wage depends on the marginal productivity of the marginal worker, which is convex in idiosyncratic productivity. For the values of productivity inside the inaction zone, wage becomes even more sensitive to the changes in productivity. If volatility rises, number of unemployed agents increases, however the employed ones end up working in more productive firms and earning more. Then the total wage bill of the representative household goes up, leading to an increase in consumption.

The simulated business cycle moments show that incorporating idiosyncratic volatility shocks in a model with search-and-matching and capital improves the fit for a set of business cycle moments. The model can account for 60% of the variation in unemployment and 54% of the variation in vacancies. It also performs well in explaining consumption and investment dynamics. The idiosyncratic volatility shock improves the amplification in labor market variables through its effect on the firm distribution, which is a slow-moving object. Moreover, since the firm bargains with the marginal worker over the marginal surplus and all the other workers of the firm earn this bargained wage, it is possible to generate a large average surplus and a small marginal surplus. This feature of the model brings cyclicality of labor market tightness closer to the data as Elsby and Michaels (2013) show.

This work is related to the growing literature on the effects of uncertainty shocks. Bloom (2009) shows in a model with non-convex adjustment costs in capital and labor adjustments, higher volatility increases the option value of waiting and leads to an initial freeze in hiring and investment decisions. In the presence of labor attrition and capital depreciation, the freeze leads to an immediate decline in capital, labor and output. Bloom et al. (2016) extends Bloom (2009) to general equilibrium setting and find significant negative effects of volatility shocks. In my model separations happen endogenously and capital depreciates at the aggregate level. Thus, when a volatility shock hits firms are able to keep their sizes the same. Moreover, in my model labor adjustment costs are due to search-and-matching frictions and calibrated to match the labor market flows. I show that when the search costs are calibrated to match the labor market flows, they do not create irreversibilities in employment decision as strong as in Bloom (2009) and Bloom et al. (2016).

Schaal (2017) also analyzes the effect of volatility on unemployment; he uses a different labor market structure, but reaches the same conclusion that search costs are not large enough to create strong option value effects. Both in my model and in Schaal (2017) the weak option value effect arising from search frictions are not enough to dominate the other positive effects of volatility. In Schaal (2017), free-entry condition creates a channel through which firms are affected by the Oi-Hartman-Abel effects. In my model, there is no entry, but the flexibility on capital dimension depresses the option value effect and also makes the Oi-Hartman-Abel effects stronger.

This work is also related to the literature on ambiguity aversion. I model the ambiguity aversion of the agents with Hansen and Sargent (2008) multiplier-preferences. In this class of preferences, conditional expectations over future exogenous states are distorted towards the low-utility states to guard against possible model misspecification. The idiosyncratic volatility is one of the exogenous states of the economy, thus magnitude and direction of the distortion depend on the level of volatility and how it affects household utility. Tallarini (2000) finds that considering higher risk aversion in risk-sensitive preferences improves the performance of the standard RBC model in matching asset pricing data, while not changing the results for aggregate quantities. [1] In a related study, Bidder and Smith (2012) analyze the effects of an increase in aggregate volatility and whether it changes with ambiguity-aversion in a representative agent model. The model is armed with various assumptions to deliver joint declines in output, capital and consumption (to address co-movement issues in RBC models) in response to a shock to the volatility of aggregate productivity. Aggregate volatility shock in this model delivers small third-order effects. They find that since higher volatility yields lower consumption, the ambiguity-averse agents assign higher probabilities to high-volatility states. However, they also find that this distortion does not have a significant effect on the response of aggregate quantities, echoing Tallarini (2000) result. In this work I consider higher ambiguity in a model in which an idiosyncratic volatility shock produces larger first-order effects, due to the existence of option value.

I show that due to volatility shocks positive effect on capital and consumption, low-volatility states are the ones with low utility, thus the ambiguity averse household puts more weight on low-volatility states when taking expectations. Since firms are owned by the ambiguity-averse households, they use the households' stochastic discount factor when discounting future values. This way the households' concern about low-utility states is transferred to the firms and basically firms are told by the households to put more weight to the low-utility states of the world. I show that concern for model uncertainty creates a correlation between aggregate productivity and idiosyncratic volatility in the worst-case distribution. Even though the probabil-

[1] The risk aversion parameter in the risk-sensitive preferences corresponds to the ambiguity aversion parameter in multiplier-preferences. In the former it represents the aversion towards quantifiable risk, while in the latter it shows the aversion to model uncertainty.

ity distortion directly affects the stochastic discount factor and the stochastic steady state levels of capital, output and rental rate of capital, its effect on the aggregate dynamics is negligible.

Lastly, I also solve a version of the model with Epstein-Zin preferences and disasters in terms of higher capital depreciation rate as in Gourio (2012). The disaster risk is used in macro-finance literature within representative agent frameworks to explain asset pricing facts such as equity premium puzzle, however its effects on heterogeneous firms in a general equilibrium setting has not been studied yet, to the best of my knowledge. In this version of the model when probability of a disaster increases, household invests less since now their investment is subject to a higher depreciation rate on average. At the same time, unemployment goes up, capital and output goes down. Even though consumption goes up on impact because of setting intertemporal elasticity of substitution greater than one following Gourio (2012), this version of the model is able to create a mild recession in response to an increase in disaster risk, in contrast to the first model with productivity risk. The movement of productivity thresholds for hiring and separation, and the movement of firm distribution are creating non-trivial labor market dynamics in the model with disaster risk. Moreover, due to the presence of labor adjustment costs and slow-moving firm distribution, adjustments after the disaster risk shock are not instantaneous and responses demonstrate more propagation compared to Gourio (2012) results.

1.2 Model with volatility shock

1.2.1 Household's Problem

There is a representative household in the economy with L members. The number of employed members is determined with the random search-and-matching in the labor market. The household owns the firms, and also rents capital to the firms. Given the rental rate of capital r, the total wage bill W, and the aggregate states capital K, aggregate productivity A, idiosyncratic volatility σ_z, the household chooses how much to consume and invest. The total wage bill is the sum of the wages of the employed, the unemployment benefits of the unemployed, and the profits from the firm net of the lump-sum tax. The government is collecting taxes only to finance the unemployment benefits, $T = (L - \sum n(n_{-1}, z, K, A, \sigma_z, \Gamma)\Gamma(n_{-1}, z))b$, where $\Gamma(n_{-1}, z))$ is the distribution of firms over the number of workers they had last period n_{-1} and the idiosyncratic productivity z. In the equilibrium the total wage bill is given as:

$$W(K, A, \sigma_z, \Gamma) = \sum w(n, z, K, A, \sigma_z, \Gamma)n(n_{-1}, z, K, A, \sigma_z, \Gamma)\Gamma(n_{-1}, z)$$

$$+ \sum \Pi(n, z, K, A, \sigma_z, \Gamma)\Gamma(n_{-1}, z)$$

The household takes the total wage bill W as given when solving its problem thus household's decisions do not depend on the distribution of its employed members over the firms $\Gamma(n_{-1}, z)$ directly. Moreover, due to perfect insurance assumption, the household provides equal consumption to all its members.

The representative household is ambiguity-averse, which means that it does not know the true distribution of the shocks driving the economy, has a benchmark distribution in mind, however entertains a set of possible distributions around this benchmark distribution. Since it has aversion to having ambiguity over possible distributions, it chooses to act on the worst-case one to guard itself from possible adverse consequences of model uncertainty. This leads to acting on a distorted version of the benchmark distribution. I assume that the household has multiplier-preferences as in Hansen and Sargent (2008). With that class of preferences the ambiguity-aversion parameter is $\varphi > 0$ and the Bellman equation takes a special form with log-exponential continuation value as shown in Section A.3. If $\varphi \to \infty$, the preferences collapse into the expected utility preferences, and the smaller the φ the higher the ambiguity-aversion is. This representation is the same as in Tallarini (2000) risk-sensitive preferences, however in Tallarini (2000) the parameter φ shows the aversion to quantifiable risks.

The problem of the household is:

$$V^H(K, A, \sigma_z, \Gamma) = \max_{C,K'} \Big\{ \log(C) - \beta\varphi \log E\Big[\exp(\frac{-V^H(K', A', \sigma_z', \Gamma')}{\varphi})\Big]\Big\}$$

subject to the budget constraint

$$C + K' = (1 - \delta + r(K, A, \sigma_z, \Gamma))K + W(K, A, \sigma_z, \Gamma)$$

and the law of motion for the firm distribution $\Gamma' = H(K, A, \sigma_z, \Gamma)$.

This problem will give us the decision rules $C(K, A, \sigma_z, \Gamma)$, $K'(K, A, \sigma_z, \Gamma)$, the household's value function $V^H(K, A, \sigma_z, \Gamma)$ and the distortion to the probabilities as:

$$m(K', A', \sigma_z', \Gamma') = \frac{\exp(-V^H(K', A', \sigma_z', \Gamma')/\varphi)}{E\Big[\exp(-V^H(K', A', \sigma_z', \Gamma')/\varphi)\Big]}$$

With the expected utility the distortion would be 1 at every state. With the ambiguity aversion, the expected value of this distortion is 1. It is larger (smaller) than 1 at the states for which $V^H(K', A', \sigma_z', \Gamma')$ is low (high). The magnitude and the direction of the distortion are dependent on the ambiguity-aversion parameter φ, the future values of exogenous states A', σ_z', and it is conditional on the current aggregate states K, A, σ_z and Γ through $K'(K, A, \sigma_z, \Gamma)$.

1.2.2 Firm's problem

There is a constant measure 1 of firms in the economy. There is no entry or exit, however the firms can increase or decrease the number of workers they have. Firms differ with respect to the number of their workers they had last period n_{-1}, and their idiosyncratic productivity z. The firm decides on how many workers to employ this period n, how many vacancies v it needs to post to get n workers, if hiring, and how much capital k to use. If the firm decides to hire, it has to post vacancies with cost of κ, but firing workers is costless. For a hiring firm, the total vacancy posting cost is κv. Then at a given rental rate of capital r and labor market tightness θ, the firm with n_{-1} workers and productivity z solves the following problem:

$$V^F(n_{-1}, z, K, A, \sigma_z, \Gamma)$$

$$= \max_{n,v,k} \Big\{ Azk^{\alpha_k} n^{\alpha_n} - r(K, A, \sigma_z, \Gamma)k - w(n, z, K, A, \sigma_z, \Gamma)n - \kappa v$$

$$+ \beta C(K, A, \sigma_z, \Gamma) E_{A'|A} E_{\sigma_z'|\sigma_z} \Big[\frac{m(K', A', \sigma_z', \Gamma')}{C(K', A', \sigma_z', \Gamma')} E_{z'|z} \Big[V^F(n, z', K', A', \sigma_z', \Gamma') \Big] \Big] \Big\}$$

subject to the law of motion for $\Gamma(n_{-1}, z)$ (4), and to the law of motion for employment:

$$\triangle n \mathbf{1}^+ = q(\theta)v$$

where $\triangle n$ is the change in number of workers, $q(\theta)$ is the job-filling probability and $\mathbf{1}^+$ is an indicator function that is 1 if firm is hiring and 0 otherwise. The optimal choice of capital satisfies $r = \alpha_k Azk^{\alpha_k - 1} n^{\alpha_n}$. If I substitute the optimal capital and vacancy choices into the problem above, the firm's problem becomes:

$$V^F(n_{-1}, z, K, A, \sigma_z, \Gamma)$$

$$= \max_{n} \Big\{ \Big(\frac{1-\alpha_k}{\alpha_k} \Big) \Big(\frac{\alpha_k Azn^{\alpha_n}}{r(K, A, \sigma_z, \Gamma)^{\alpha_k}} \Big)^{\frac{1}{1-\alpha_k}} - w(n, z, K, A, \sigma_z, \Gamma)n - \frac{\kappa}{q(\theta)} \triangle n \mathbf{1}^+$$

$$+ \beta C(K, A, \sigma_z, \Gamma) E_{A'|A} E_{\sigma_z'|\sigma_z} \Big[\frac{m(K', A', \sigma_z', \Gamma')}{C(K', A', \sigma_z', \Gamma')} E_{z'|z} \Big[V^F(n, z', K', A', \sigma_z', \Gamma') \Big] \Big] \Big\}$$

There is a kink in the value function at $\triangle n = 0$. It is similar to the kinked labor and capital adjustment costs used by Bloom et at. (2016) however here the labor adjustment cost, $\frac{\kappa}{q(\theta)}$, is endogenously determined since the job-filling probability q depends on the labor market tightness $\theta(K, A, \sigma_z, \Gamma)$. This problem gives us productivity thresholds for hiring and separation decisions for a firm of size n_{-1}. In fact, it is possible to calculate number of workers that would be optimal at each level of productivity under separation (costless adjustment) and hiring (costly adjustment) cases, and to derive the optimal labor demand for any size firm as a combination of these two decision rules.

The hiring thresholds can be calculated by assuming that not only the positive adjustments but all adjustments incur the hiring cost. Let $n_h(z)$ denote the optimal number of workers to employ with productivity z under costly adjustment case. The following is the first-order condition with respect to employment with adjustment costs, where $\pi(n, z)$ is the profit of a firm excluding the adjustment cost and aggregate shocks are ignored for ease of notation.

$$\frac{\partial \pi(n_h, z)}{\partial n_h} + \beta E_{z'|z}\left[{V_{n_h}}^F(n, z')\right] = \frac{\kappa}{q(\theta)}$$

Similarly, the firing thresholds can be calculated by assuming that all adjustments are costless. Let $n_f(z)$ denote the optimal number of workers to have with productivity z under costless adjustment case.

$$\frac{\partial \pi(n_f, z)}{\partial n_f} + \beta E_{z'|z}\left[{V_{n_f}}^F(n, z')\right] = 0$$

Then the optimal labor demand for a firm of size n_{-1} is of the form:

$$n(n_{-1}, z) = \begin{cases} n_h(z) & if \quad z > n_h^{-1}(n_{-1}) \\ n_{-1} & if \quad z \in [n_f^{-1}(n_{-1}), n_h^{-1}(n_{-1})] \\ n_f(z) & if \quad z < n_f^{-1}(n_{-1}) \end{cases}$$

This problem will give us the employment decision rule $n(n_{-1}, z)$. Figure 1.3 shows the labor demand function of a firm that has 25 workers, as an example. It is optimal for this firm to reduce its workforce to a level that satisfies the first-order condition for firing if it draws a productivity lower than $1.03 = n_f^{-1}(25)$. Similarly, if this firm draws a productivity higher than $1.15 = n_h^{-1}(25)$ then it is optimal to expand until the condition for hiring threshold is satisfied. For the intermediate values of z, it is optimal to keep the number of workers the same.

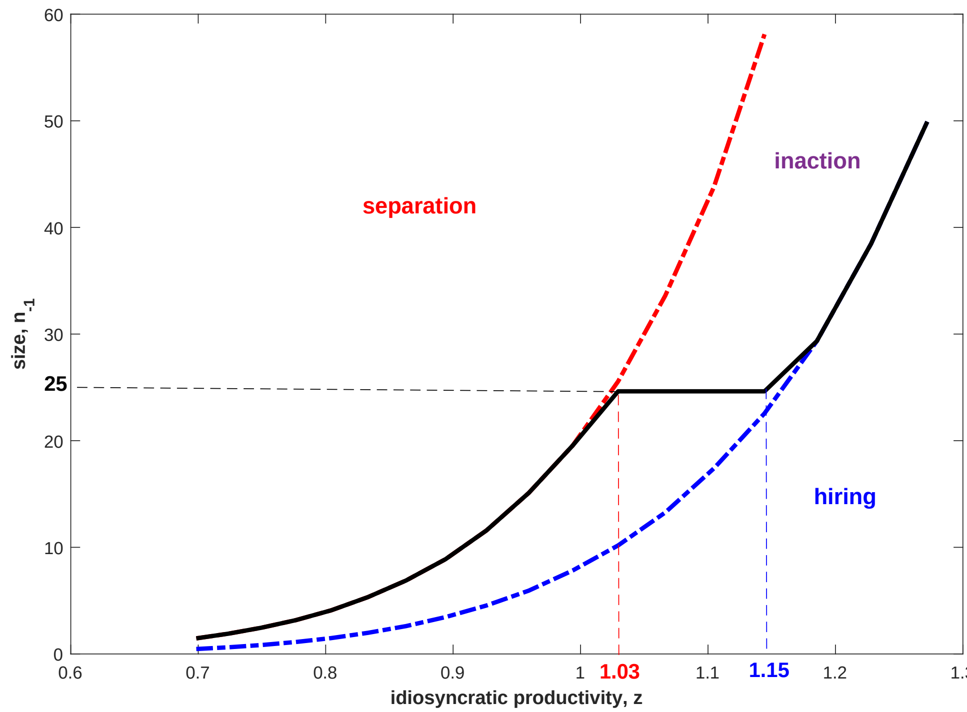
After obtaining the employment decision rule, capital decision rule is calculated as a function of it:

$$k(n_{-1}, z, K, A, \sigma_z, \Gamma) = \Big(\frac{\alpha_k A z n(n_{-1}, z, K, A, \sigma_z, \Gamma)^{\alpha_n}}{r(K, A, \sigma_z, \Gamma)}\Big)^{\frac{1}{1-\alpha_k}}$$

1.2.3 Wage setting

Whether there is constant or diminishing marginal returns of labor in production matters for the wage bargaining. In the classical search-and-matching model with linear production function labor has a constant marginal product. This assumption provides tractability by making the rents that firm and workers bargain over the same for each worker within a firm. Then it becomes possible for the firm to bargain with each of its workers independently. If we assume that firms have diminishing

Figure 1.3: Labor demand of a firm



marginal product of labor instead, these rents will depend on the number of workers in each firm. On top of that due to the decreasing returns to scale in production, working with N one firm-one worker matches is not equivalent to working with one large firm with N workers. It breaks the independence of the rents of each individual employment relationship within a firm. Now the firm and workers should bargain over the marginal surplus. The bargaining solution of Stole and Zwiebel (1996) is suitable for this case.

Workers' bargaining power is $\eta \in (0, 1)$. If workers had no bargaining power the firm would pay the unemployment benefit b to make workers indifferent between employment and unemployment and workers would decide to work. If workers had all the bargaining power then they would get all the benefits of hiring the marginal worker for the firm. The benefits are the marginal product of labor for this worker adjusted for the lower wages due to decreasing returns to scale and the savings made on vacancy costs in the future by hiring today. The wage that results from Stole and Zwiebel (1996) bargaining is a weighted average of these two limiting cases and given by [2]

$$w(n,z,K,A,\sigma_z,\Gamma) = \eta\Bigg\{\frac{\alpha_n(1-\alpha_k)}{1-\alpha_k-\eta(1-\alpha_k-\alpha_n)}\Big(\frac{\alpha_k}{r(K,A,\sigma_z,\Gamma)}\Big)^{\frac{\alpha_k}{1-\alpha_k}}(Az)^{\frac{1}{1-\alpha_k}}n^{\frac{\alpha_k+\alpha_n-1}{1-\alpha_k}}$$

$$+\beta\kappa C(K,A,\sigma_z,\Gamma)E_{A'|A}E_{\sigma_z'|\sigma_z}\Big[\frac{m(K',A',\sigma_z',\Gamma')}{C'(K',A',\sigma_z',\Gamma')}\theta'(K',A',\sigma_z',\Gamma')\Big]\Bigg\}+(1-\eta)b$$

1.2.4 Recursive competitive equilibrium

A recursive competitive equilibrium in this economy is a set of quantity functions $\{C, K', n, k\}$, price functions $\{r, \theta, W\}$, wage function w, and value functions $\{V^H, V^F\}$. V^H and $\{C, K'\}$ are the value and policy functions solving (2) while V^F and $\{n, k\}$ are the value and policy functions solving (7) and wage function w solves the bargaining problem between a firm and marginal worker. The price functions $\{r, \theta, W\}$ clear

(i) the capital market

$$K = \sum\sum\Big(\frac{\alpha_k Azn(n_{-1},z,K,A,\sigma_z,\Gamma)^{\alpha_n}}{r}\Big)^{\frac{1}{1-\alpha_k}}\Gamma(n_{-1},z),$$

[2] Derivation of the wage function is in Section A.4.

(ii) the labor market

$$\frac{1}{(1+\theta^{-g})^{1/g}} = \frac{\sum\sum\Big(n(n_{-1},z,K,A,\sigma_z,\Gamma)-n_{-1}\Big)_+\Gamma(n_{-1},z)}{\Big(L-\sum\sum n_{-1}\Gamma(n_{-1},z)\Big)}$$

and (iii) the goods market

$$W = \sum\sum w(n,z,K,A,\sigma_z,\Gamma)n(n_{-1},z,K,A,\sigma_z,\Gamma)\Gamma(n_{-1},z)$$

$$+\sum\sum \Pi(n,z,K,A,\sigma_z,\Gamma)\Gamma(n_{-1},z).$$

Lastly, the evolution of the firm distribution $\Gamma(n_{-1},z)$ is consistent, which means that $H(K,A,\sigma_z,\Gamma)$ is generated by the appropriate integration of $n(n_{-1},z,K,A,\sigma_z,\Gamma)$ and $k(n_{-1},z,K,A,\sigma_z,\Gamma)$, along with the exogenous stochastic evolution of A,z,σ_z and the endogenous evolution of K.

1.2.5 Stationary equilibrium

In the stationary equilibrium the aggregate productivity A is 1 and the idiosyncratic volatility is at its mean $\bar{\sigma_z}$. Household's problem gives

$$r^* = \frac{1}{\beta} - 1 + \delta$$

At r^*, each firm solves the following problem at given labor market tightness θ

and firm distribution $\Gamma(n_{-1}, z)$

$$V(n_{-1}, z, K; \theta, \Gamma) = \max_{n} \Bigg\{ \Bigg[\Big(\frac{\alpha_k}{r^*}\Big)^{\frac{\alpha_k}{1-\alpha_k}} z^{\frac{1}{1-\alpha_k}} n^{\frac{\alpha_n}{1-\alpha_k}} - wn - \frac{\kappa}{q(\theta)} \triangle n \mathbf{1}^{+} \Bigg]$$

$$+ \beta E_{z'|z} \Big[V(n, z'; \theta, \Gamma) \Big] \Bigg\}$$

where

$$w(n, z; \theta) = \eta \Bigg[\frac{\alpha_n (1-\alpha_k)}{1-\alpha_k - \eta(1-\alpha_k-\alpha_n)} \Big(\frac{\alpha_k}{r^*}\Big)^{\frac{\alpha_k}{1-\alpha_k}} z^{\frac{1}{1-\alpha_k}} n^{\frac{\alpha_k+\alpha_n-1}{1-\alpha_k}}$$

$$+ \beta\kappa\theta \Bigg] + (1-\eta) b$$

The steady state of the economy can be characterized by two relationships as in Elsby and Michaels (2013). First one is the *job creation condition*, which is a generalization of the labor demand condition as a function of labor market tightness. The unemployment implied by the job creation is given as

$$U_{JC}(\theta) = \Big(L - \sum \sum n(n_{-1}, z; \theta) \Gamma(n_{-1}, z) \Big)$$

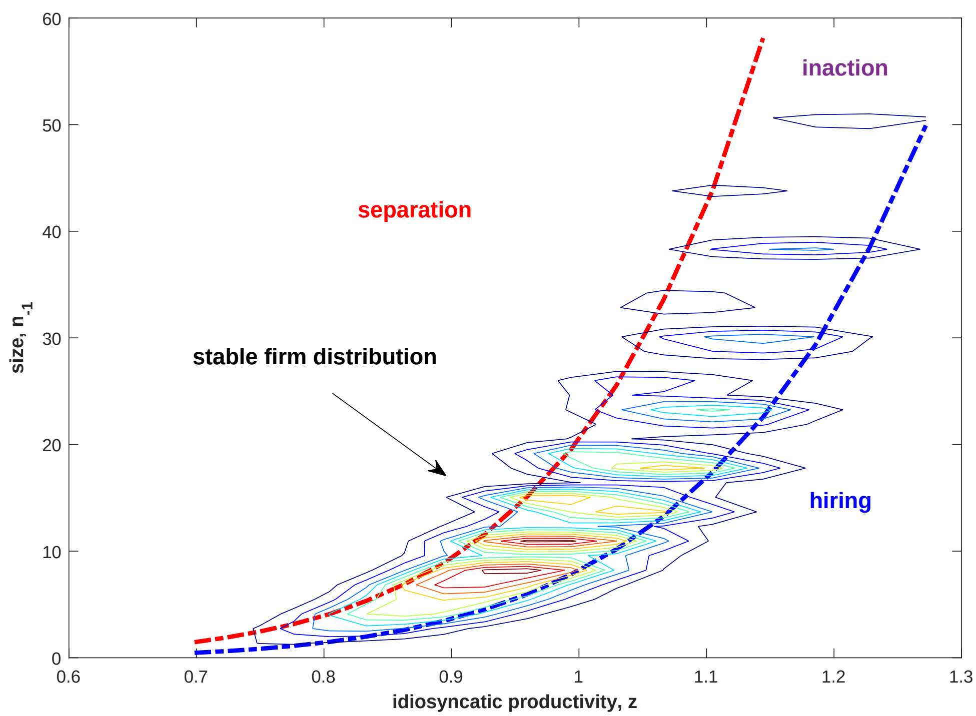
Second one is the *Beveridge curve relation*. It is obtained from the evolution of unemployment over time through inflows by separations and outflows by matches as $\Delta U = S(\theta) - f(\theta)U$. In the stationary equilibrium the aggregate employment, thus unemployment is constant, and the unemployment implied by the Beveridge curve relation is given as

$$U_{BC}(\theta) = S(\theta)/f(\theta)$$

The steady-state level of labor market tightness θ^* and the stable firm distribution $\Gamma^*(n_{-1}, z)$ solve these two conditions together, and ensure that total number of separations $S(\theta^*)$ is equal to the total number of matches $f(\theta^*)\Big(L - \sum\sum n(n_{-1}, z; \theta^*)\Gamma(n_{-1}, z)\Big)$. In other words, θ^* clears the search-and-match market and $\Gamma^*(n_{-1}, z)$ is the invariant distribution under the operator T that is generated by the aggregation of individual firm's optimal decisions, i.e. $T(\Gamma^*) = \Gamma^*$.

Figure 1.4 shows the contour map of the resulting stable firm distribution $\Gamma^*(n_{-1}, z)$. It has mass around both adjustment bands, meaning that in the stationary equilibrium some firms are always hitting the adjustment bands as their idiosyncratic productivity evolves, however in the absence of any aggregate shocks the aggregate employment stays the same since at θ^* separations and matches are equal.

Figure 1.4: Firm distribution at the stationary equilibrium



1.3 Solution method

I solve the model with value function iteration. The solution of this model is challenging for three reasons. First, since it is a heterogeneous firm model with aggregate shocks, it requires keeping track of the firm distribution over time. Second, with a constant firm mass with no entry or exit, there is no free entry condition. Instead, the labor market tightness must be solved as an additional equilibrium object. Third, it is stated by Krusell and Smith (2006) and Young (2010) that in the standard Krusell-Smith algorithm when the agents respond to the forecasted prices instead of the actual prices, markets do not clear. To avoid that problem, I use the algorithm proposed in Young (2010) and make sure that the agents are responding to the actual prices both today and in the future, instead of the forecasted prices. To achieve that, I embedded a simulation and projection step in between each value function iteration step. The details of the solution algorithm are given in Section A.5.

To simulate the model with aggregate shocks and construct the approximate pricing functions $r(K, A, \sigma_z)$, $W(K, A, \sigma_z)$, and $\theta(K, A, \sigma_z)$, I need to calculate the prices that clear the capital, labor and good markets at each period. Given a time series of aggregate states, A, σ_z, I find the prices that solve the market clearing conditions (11), (12) and (13). Along the way the cross sectional distribution $\Gamma_t(n_{-1}, z, K, A, \sigma_z)$ is updated by the histogram approach described in Young (2010). Then I project the resulting time series for r, W, θ on the aggregate states to obtain $r(K, A, \sigma_z), W(K, A, \sigma_z)$, and $\theta(K, A, \sigma_z)$, which are be used to update the value function in the next iteration. The coefficients of price functions are given in Table 1.1. In principle, prices are also functions of the firm distribution $\Gamma(n_{-1}, z)$ at each period, however I do not include a moment of this distribution in the regressions. The reason is that the firm distribution at time t is known by the firm, since it is generated by updating the firm distribution at time $t-1$ according to the employment choices at time $t-1$. Similarly the capital stock at time t is also determined at time $t-1$ as a function of the aggregate states at time $t-1$. Thus, the explanatory variable K_t contains the same information that is needed to determine Γ_t and it is not necessary to include another variable related to the firm distribution in the price regressions.

Table 1.1: Price functions - Expected utility

	constant	$\log(K/K_{ss})$	$\log(A)$	$\log(\sigma_z/\bar{\sigma^z})$	R^2
$\log(r/r_{ss})$	-0.00026	-0.66918	1.08917	0.06102	
	(0.00003)	(0.00128)	(0.00175)	(0.00037)	0.99
$\log(W/W_{ss})$	0.00008	0.33044	1.07891	0.04492	
	(0.00003)	(0.00151)	(0.00207)	(0.00044)	0.99
$\log(\theta/\theta_{ss})$	0.00789	1.69794	4.95413	0.51271	
	(0.00078)	(0.03784)	(0.05197)	(0.01098)	0.98
$\log(C/C_{ss})$	0.01265	0.76069	0.15233	0.00816	
	(0.00004)	(0.00178)	(0.00245)	(0.00052)	0.99

Note: Regressions are run for the expected utility case ($\varphi = 20000$) for 500 periods. Standard errors are in parentheses.

1.4 Calibration

1.4.1 Functional forms and exogenous processes

I assume a production function with decreasing returns to have a distribution of firms with different sizes.

$$y = Azk^{\alpha_k} n^{\alpha_n}, \qquad \alpha_k + \alpha_n < 1$$

I choose the following matching function as in Den Haan et al.(2000) to make sure that the job-finding and job-filling probabilities are always less than 1.

$$M(U,V) = \frac{UV}{(U^g + V^g)^{\frac{1}{g}}}$$

Then the job-finding and job-filling probabilities are respectively given as follows:

$$f = \frac{1}{(1+\theta^{-g})^{\frac{1}{g}}}, \qquad q = \frac{1}{(1+\theta^{g})^{\frac{1}{g}}}$$

where $\theta = V/U$ is the labor market tightness.

The idiosyncratic and aggregate productivity shocks follow the AR(1) processes respectively:

$$\log(z_t) = \rho_z \log(z_{t-1}) + \sigma_t^z \sqrt{(1-\rho_z^2)}\epsilon_t^z, \quad \epsilon_t^z \sim N(0,1)$$

$$\log(A_t) = \rho_A \log(A_{t-1}) + \sigma^A \sqrt{(1-\rho_A^2)}\epsilon_t^A, \quad \epsilon_t^A \sim N(0,1)$$

where the time-varying volatility of the idiosyncratic shock has a mean $\bar{\sigma_z}$ and has the following process:

$$\sigma_t^z = (1-\rho_{\sigma^z})\bar{\sigma}_z + \rho_{\sigma^z}\sigma_{t-1}^z + \sigma_{\sigma^z}\sqrt{(1-\rho_{\sigma^z}^2)}\epsilon_t^{\sigma^z}, \quad \epsilon_t^{\sigma^z} \sim N(0,1)$$

I use Rouwenhorst (1995) method to discretize the processes. For the idiosyncratic productivity process I first discretize the volatility process σ_t^z into three grid points for low, medium and high volatility states. Next I use Rouwenhorst method three times to get different grids for $\log(z_t)$ at each volatility state.

1.4.2 Calibration strategy

Table 1.2 summarizes the parameter calibration and Table 1.3 lists the moments that result from the calibration. The time period in the model is one quarter. I set the capital depreciation rate to 2.5% to match annual depreciation rate of 10%. The quarterly rental rate of capital is set to 3.1% resulting in a discount factor β of 0.994. The production function parameter for capital α_k is set to 0.31 to ensure the capital-output ratio of 10 and thus the investment-output ratio of 0.25.

The production function parameter for labor α_n that is set to 0.57 leads to an labor income-output ratio of 0.60, which is close to its data counterpart of 0.64. I set the matching function parameter g to 1.60 as in Schaal (2017), who estimates the job-finding function $f = \dfrac{1}{(1+\theta^{-g})^{\frac{1}{g}}}$ using the job-finding rate series constructed by Shimer (2007) and a measure of labor market tightness constructed with the vacancy series from Conference Board's Help Wanted Index and JOLTS, and the unemployment rate series from the BLS.

The unemployment benefit b, the vacancy posting cost κ and the size of the labor force L are calibrated together to match the separation rate $S/N = 0.03$, unemployment rate $(L-N)/L = 0.045$ and the average firm-size of $N = 17$. The unemployment benefit controls the separations, while the vacancy posting cost affects the matches. In the stationary equilibrium the separations are equal to the matches, thus the ratio of b and κ determines the labor market tightness to be 0.73, which is close to its data counterpart of 0.72. The levels of b and κ determine the average firm size. I target the vacancy posting cost κ to be 33% of the average wage in the economy as in Schaal (2017). The resulting value for the unemployment benefit is 84% of the average wage, thus the calibration is close to Hagedorn and Manovskii (2008) case. In addition, the unemployment benefit corresponds to 50% of the average output per person, which is lower than 63% in Schaal (2017) and 71% in Hall and Milgrom (2008).

The persistence of aggregate and idiosyncratic productivity shocks are both set to 0.95 and consistent with Khan and Thomas (2008). The standard deviation of the aggregate productivity is set to 0.012 to match the standard deviation of output in the data which is 0.017. The mean idiosyncratic volatility $\bar{\sigma^z}$ is set to 0.12, and it is close to the mean of the uncertainty measure Gilchrist et al. (2014) obtained from the profits of Compustat firms, which is 0.15. The standard deviation of idiosyncratic volatility is also set to 5% of its mean value as in Gilchrist et al. (2014). The persistence of the volatility shock is backed out from simulating the uncertainty process in Bloom et al. (2016).

Table 1.2: Calibration

Parameter	Meaning	Value
β	Discount factor	0.994
δ	Depreciation rate	0.025
α_k	$y = Azk^{\alpha_k}n^{\alpha_n}$	0.31
α_n	$y = Azk^{\alpha_k}n^{\alpha_n}$	0.57
b	Value of leisure	0.928
η	Worker's bargaining power	0.40
g	Matching function parameter	1.60
κ	Vacancy posting cost	0.36
ρ_z	Persistence of idios. prod.	0.95
$\bar{\sigma_z}$	Mean vol. of idios. prod.	0.12
ρ_{σ_z}	Persistence of vol. of idios. prod.	0.976
σ_{σ_z}	Std. dev. of vol. of idios. prod.	0.006
ρ_A	Persistence of agg. prod.	0.95
$\bar{\sigma_A}$	Mean vol. of agg. prod.	0.012

1.5 Results

1.5.1 Business cycle statistics

The simulated business cycle moments show that incorporating idiosyncratic volatility shocks in a model with search-and-matching and capital improves the fit for a set of business cycle moments. The model can explain 60% of the variation in unemployment and 54% of the variation in vacancies, which more in line with the data than Shimer (2005) and comparable to Schaal (2017) results. It also performs well in consumption and investment dynamics, which is missing in Schaal (2017).

The idiosyncratic volatility shock improves the amplification in labor market variables through its effect on the firm distribution, which is a slow-moving object. Moreover, since the firm bargains with the marginal worker over the marginal surplus and all the other workers of the firm earn this bargained wage, it is possible to generate a

Table 1.3: Moments

Moment	Meaning	Model	Data
F	Firm mass	1	Normalization
L	Labor force	17.53	u = 4.5%
N	Avg. firm size	16.74	17
W/Y	Labor income share	0.60	0.64
K/Y	Capital output ratio	10	$I/Y = 0.25$
Π/Y	Profit income ratio	0.08	0.10
S/N	Separation rate	0.03	0.04
θ	Labor market tightness	0.73	0.72
κ/W	Vacancy cost/Avg. wage	0.33	Schaal (2017)
$(\kappa/q)/W$	Expected hiring cost/Avg. wage	0.44	
b/W	Unemp. benefit/Avg. wage	0.84	
$b/(Y/N)$	Unemp. benefit/Labor productivity	0.50	

large average surplus and a small marginal surplus. This feature of the model brings cyclicality of labor market tightness closer to the data as Elsby and Michaels (2013) show.

The correlations of hirings and separations with the output are especially affected by the inclusion of the volatility shock, comparing last two rows of Column 5 and Column 7 in the top panel of Table 1.4. First, I explain, by referring to impulse-responses, how including volatility shock improves the correlation of hirings and output, turning the large negative correlation to a small but positive one. As Figure 1.10 in Section 1.5.4 shows, if a negative aggregate productivity shock hits fewer vacancies are posted initially and labor market tightness decreases. Then with a lower labor market tightness, the hiring cost $\kappa/q(\theta)$ is lower and matches recover quickly, even surpass their pre-shock level, thus creating a negative correlation between hirings and output. I also find that a positive volatility shock increases output and hirings at the same time as Figure 1.7 in Section 1.5.3 demonstrates. When both shocks are considered together in the model, I find that the positive correlation of hirings and output created by the volatility shock dominates the negative correlation created by the aggregate productivity shock, and the result is a mild positive correlation.

Similarly, the inclusion of volatility shock reduces the large negative correlation of separations and output to a small but still negative one. With only negative aggregate productivity shock, separations increase since many workers are now not productive enough to work. But at the same time, due to low labor market tightness, job finding rate is lower and there are less incentives to separate for the workers. The former effect dominates and the result is a large negative correlation between separations and output. However, as shown in Figure 1.7, a positive volatility increases both separations and output due to the strong positive effect of job finding rate on separations. Thus, when considered together the positive correlation induced by volatility shock reduces the negative correlation of separations and output, and brings it closer to the data.

1.5.2 Effects of higher idiosyncratic volatility

Option value effect

The option value of inaction increases at both margins with higher volatility. At the separation margin, the option value of inaction increases since the firm knows that even though today's productivity is low, due to high volatility the productivity might be higher tomorrow and it might be costlier to re-hire a worker once separated today. Similarly, at the hiring margin, the firm may not choose to hire despite a good productivity shock today because in a volatile environment tomorrows productivity might be much lower and the hires of today may not be worthy of their costs, thus

Table 1.4: Business cycle statistics

	Data		Model (only A)		Model (A and σ_z)	
	$\sigma(x)$	$\rho(x,Y)$	$\sigma(x)$	$\rho(x,Y)$	$\sigma(x)$	$\rho(x,Y)$
Y	0.017	1	0.015	1	0.018	1
I	0.070	0.90	0.035	0.851	0.040	0.852
C	0.013	0.90	0.012	0.834	0.014	0.847
U	0.121	-0.86	0.050	-0.958	0.073	-0.650
V	0.138	0.70	0.024	0.926	0.074	0.539
Hirings	0.058	0.68	0.014	-0.605	0.059	0.056
Separations	0.042	0.70	0.015	-0.745	0.060	-0.016
	Schaal (2017)		Bloom et al. (2016)		Shimer (2005)	
	$\sigma(x)$	$\rho(x,Y)$	$\sigma(x)$	$\rho(x,Y)$	$\sigma(x)$	$\rho(x,Y)$
Y	0.017	1	0.020	1	0.017	1
I			0.012	0.90		
C			0.009	0.50		
U	0.089	-0.722			0.007	-0.982
V	0.053	0.267			0.021	0.993
Hirings	0.049	0.202			0.003	0.448
Separations						
Layoffs	0.086	-0.600			0.001	0.931
Quits	0.071	0.648				

Note: Time series are at quarterly frequency and presented in log-deviation from an HP trend with parameter 1600. Statistics for Schaal (2017) and Shimer (2005) are from Schaal (2017) Table 3 and 4. Statistics for Bloom et al. (2016) are from Bloom et al. (2016) Table 7. See Section A.2 for data sources.

leading to separations and searching again. These option-value effects are expected to shift both thresholds outward and result in an expansion in the inaction zone, in the absence of other effects. As both separations and matches are expected to decline, the net result of option value effect on the unemployment is not clear. The strength of the option value effect depends on the size of the hiring cost and the mass of firms close to the adjustment thresholds.

Realized volatility effect

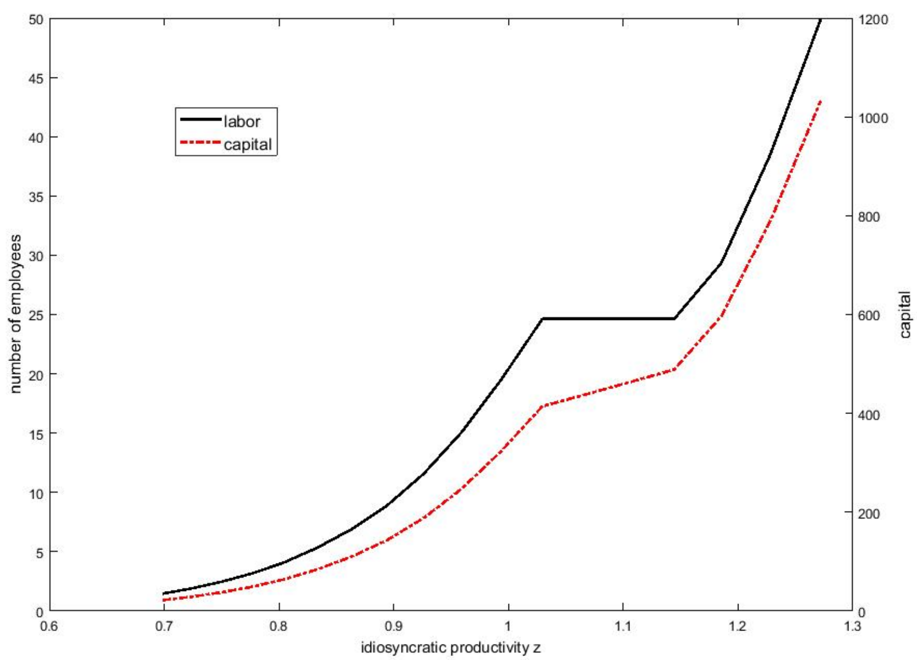
The realized volatility effect occurs due to the actual realization of the volatility.

If volatility increases today, dispersion across firm increases and firms may hit the adjustment thresholds more often. As firms become more active due to high volatility, the resources are reallocated towards more productive firms.

Oi-Hartman-Abel effect

Aside from the option value effect, there is also Oi-Hartman-Abel effects of volatility. In the frictionless case, at given prices, the factor demands are convex functions of the productivity. Thus, if there is a mean-preserving spread of productivity, i.e. productivity becomes more volatile, the factor demands are expected to increase. This positive effect is present in this model since labor demand is convex outside the inaction zone, and capital demand is convex but less so inside the inaction zone as shown in Figure 1.5.

Figure 1.5: Labor and capital demand of a firm



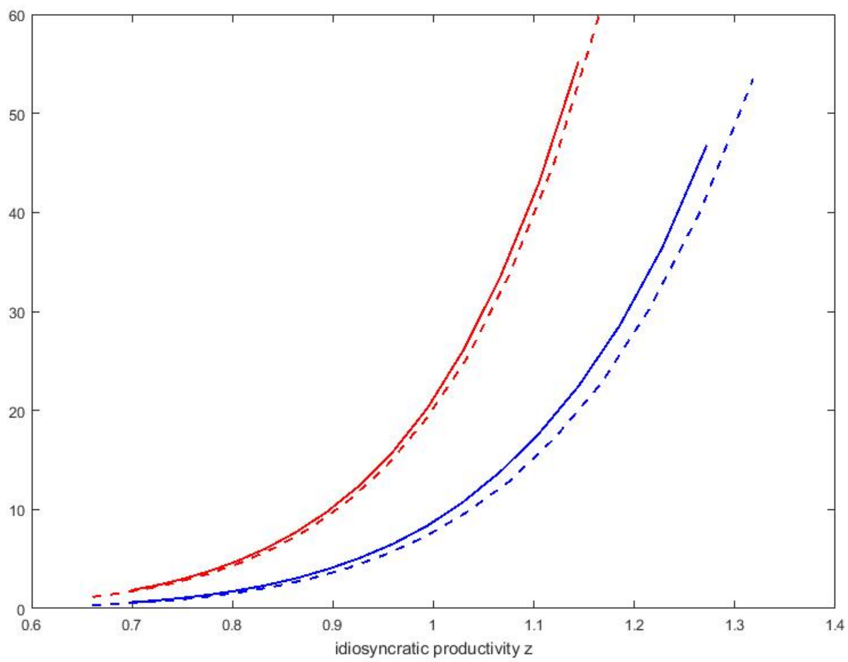
General equilibrium effects

In the general equilibrium, labor market tightness clears the labor market as the rental rate of capital clears the capital market. The general equilibrium effect of changes in the labor market tightness are particularly important since it affects the matching decision through its effect on the hiring costs and separation decision through its effect on job-finding probability.

Adjustment bands at low and high volatility

Figure 1.6 displays the adjustment bands for low and high volatility states. With higher volatility both adjustment bands move to the right, due to combination of all the effects listed above. The shift of hiring band to the right is mostly due to option value effect since in a highly volatile environment firms now require higher productivity to incur irreversible hiring costs. The shift of separation band to the right is mainly due to the general equilibrium effect of higher labor market tightness. In a tight labor market finding a job is easier for workers, thus there is more incentive to separate. Even though the option value effect is expected to shift the separation band to the left and reduce the separations, in this case the general equilibrium effect dominates the option value effect and the bands shifts to right. Higher labor market also affects the hiring band through its effect on the hiring costs. With a higher labor market tightness, job-filling probability of a firm declines and the expected hiring costs go up, contributing to the rightward shift of the hiring band.

Figure 1.6: Adjustment bands at low and high volatility



Note: Solid lines: Low volatility, Dashed lines: High Volatility,
Red: Separation bands, Blue: Hiring bands

1.5.3 Impulse-response analysis of a volatility shock

Figure 5 shows the responses of various variables to a one-standard deviation positive shock to the volatility. If volatility increases, expected value of a match increases due to Oi-Hartman-Abel effect, leading firms to post more vacancies and hire more. The initial surge in vacancies causes labor market tightness to rise. When tightness increases, vacancies become harder to fill and thus their expected cost increases, shifting the hiring threshold to the right. In addition, a higher labor market tightness makes finding a job easier for the workers and improve their outside options. The increase in the value of unemployment causes separations to go up. These effects combined will result in both separations and matches increasing, but since separations increase more, unemployment goes up. In addition, with convex capital demand, the firms that experience larger positive shocks demand more capital thus aggregate capital demand goes up. Since at the beginning of the period the capital supply is pre-determined, increase in capital demand immediately results in an increase in the rental rate of capital.

In addition, as Figure 1.8 shows, the number of inactive firms i.e. the mass of firms inside the inaction zone, declines when the shock hits, showing that the realized volatility effect is stronger than the option value effect and firms become more active in response to higher volatility. Consistent with the shifts int he adjustment bands, the mass of firms separating goes up as the mass of firms hiring goes down. The reallocation effect is also evident with the fact that matches are increasing as the mass of firms hiring is decreasing. Thus it must be the case that with the positive volatility shock, there are firms with very favorable productivity draws, which are not many in numbers but are hiring more workers per firm and increasing the overall productivity of the workforce.

I decompose the total responses to a one-standard deviation positive shock to the volatility and display results in Figure 1.9. The dashed line shows the response if the shock affects the expectations only, meaning that the actual volatility is not changing with the shock. Compared to the total response, the dashed line shuts down the realized volatility effect. I call this case the constant realized volatility case. The constant realized volatility case includes all the other remaining effects; option value, Oi-Hartman-Abel and general equilibrium. The realized volatility effect causes significant reallocation in the economy from less productive firms towards more productive firms. The effect of reallocation on output, consumption and capital is large and pos-

Figure 1.7: Responses to a 1 std.dev. positive σ_z shock

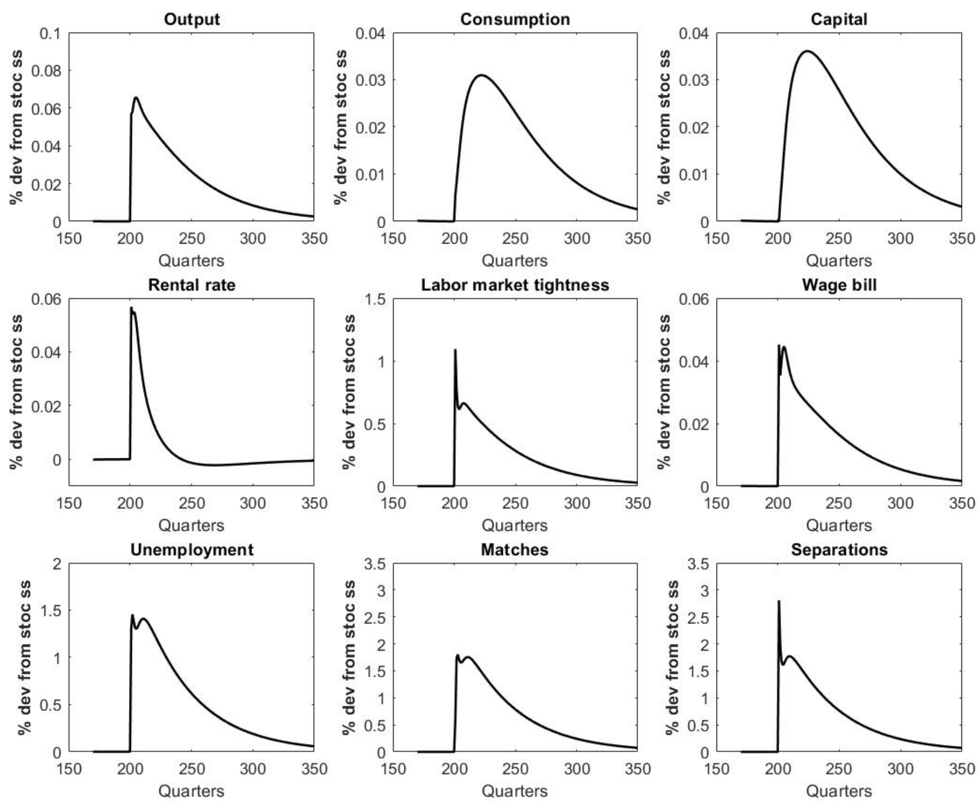
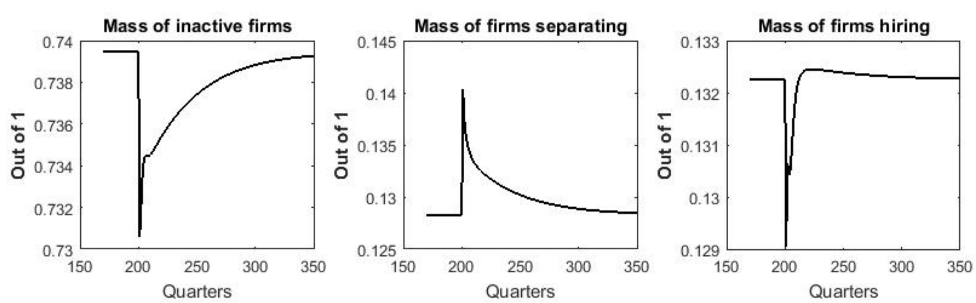
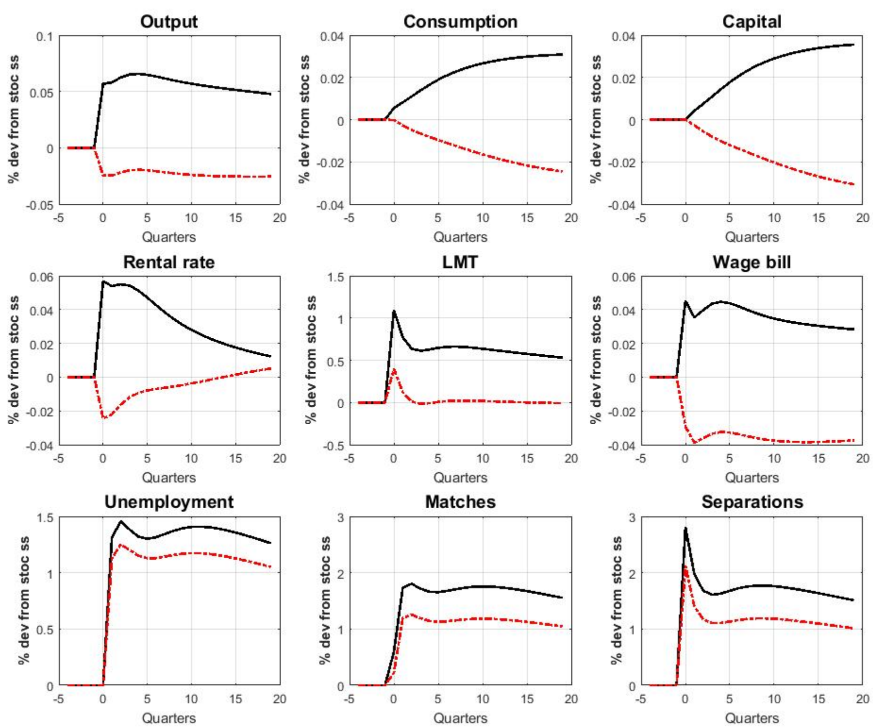


Figure 1.8: Responses to a 1 std.dev. positive σ_z shock - Firm mass



itive since without realized volatility they all would be decreasing. Moreover, Figure 1.9 also shows that the rise in unemployment is mostly due to combination of the other remaining effects, not due to reallocation. Even in the absence of realized volatility, it possible to see the effect of convexity in the increasing matches and the general equilibrium effect of higher labor market tightness in the increasing separations. In addition, the capital demand is now decreasing since without an actual increase in volatility, less number of workers are employed by not the most productive firms and producing less in total. At the period shock hits capital supply is pre-determined, thus lower capital demand results in a lower rental rate of capital.

Figure 1.9: Decomposition of response to σ_z shock



Note: Black solid line: actual response, Red dashed line: if expectations about volatility changes only (shuts down realized volatility effect)

1.5.4 Effects of lower aggregate productivity

Figure 1.10 shows the response of aggregate variables to a decline in aggregate productivity. As expected aggregate output, consumption and capital all decline and unemployment increases. Lower aggregate productivity increases separations for all firms, while it decreases matches on impact. With fewer vacancies posted and more workers separated, labor market tightness is lower after the shock. Matches recover quickly because lower labor market tightness increases job-filling probability $q(\theta)$, which in turn incentives more matches due to lower cost of $\kappa/q(\theta)$. However the general equilibrium effects are affecting firms with different productivity levels differently. Figure 1.11 plots how adjustment bands for firing and hiring change when aggregate productivity decreases. One thing to notice is that the adjustment band for hiring decreases especially for low productive firms on impact, but then it increases for all productivity levels as the general equilibrium effects, such as the changes in the labor market tightness, in rental rate of capital and firm distribution, take place. By looking at these changes in the hiring thresholds I deduct that the initial decline on matches is due to low productive firms reducing hiring, while the following increase in matches is mainly due to the high productive firms that benefit from the lower rental rate of capital and lower labor market tightness.

Figure 1.10: Responses to a 1 std.dev. negative A shock

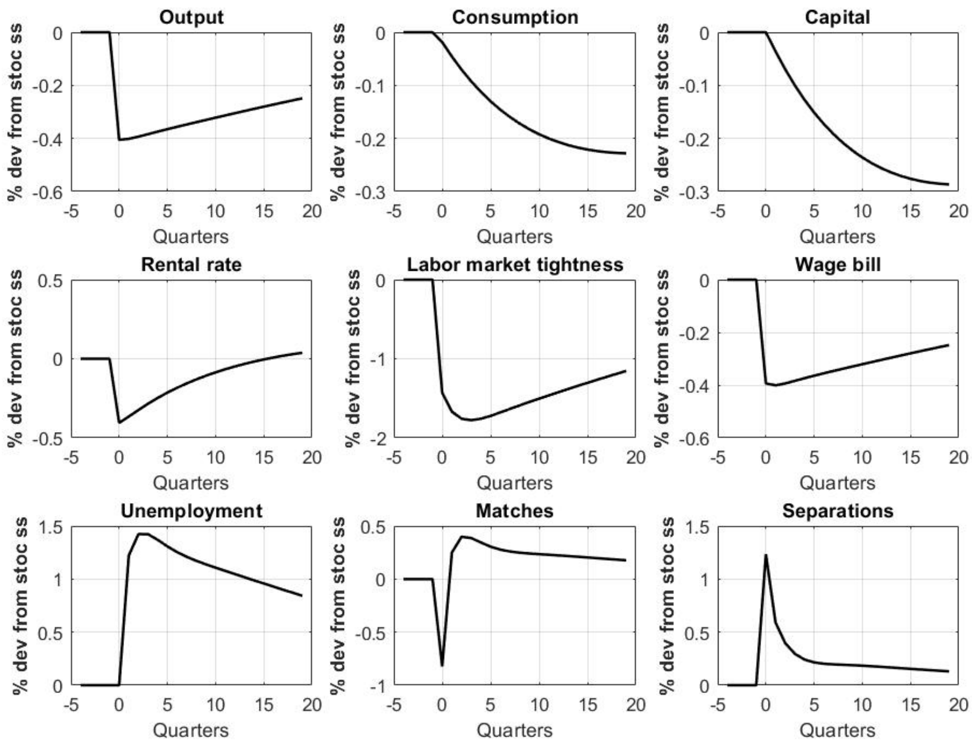
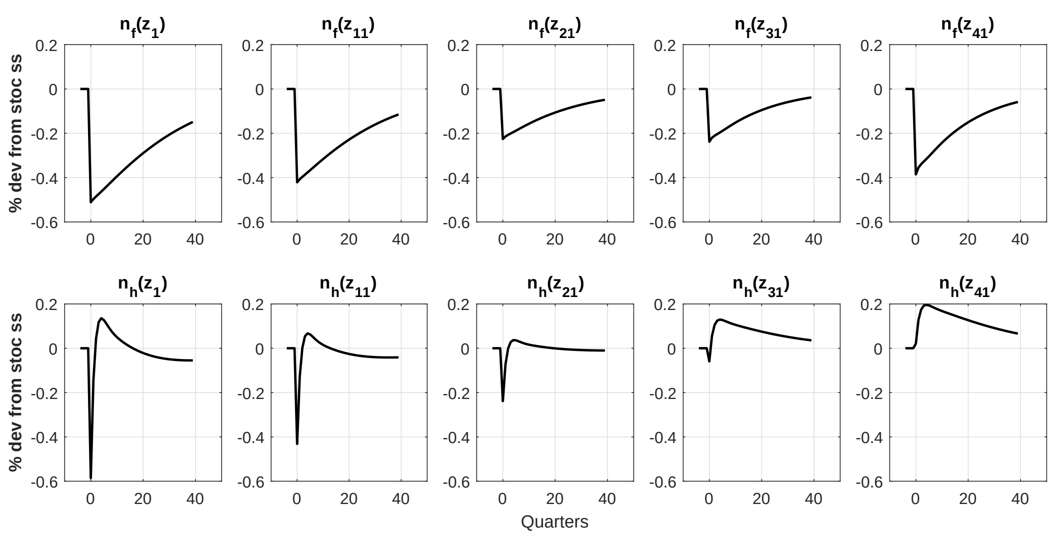


Figure 1.11: Responses of adjustment bands to a 1 std.dev. positive A shock



Note: Upper (lower) panel is the responses of the firing (hiring) thresholds at various idiosyncratic productivity levels.

1.6 Higher ambiguity aversion

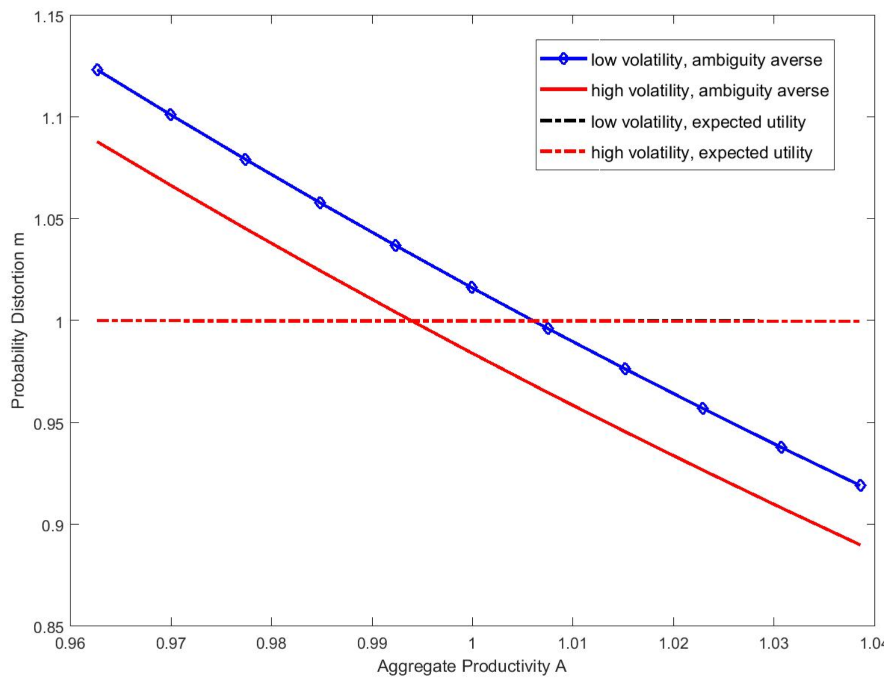
1.6.1 Probability distortion

Figure 1.12 displays the probability distortion for each aggregate productivity and idiosyncratic volatility combination that can be experienced next period, conditional on $A = 1$, $\sigma_z = \bar{\sigma_z}$ and steady state level of capital today. I set ambiguity aversion parameter φ to 20000 for low ambiguity (or expected utility) case and set it to 10 for the high ambiguity case. Hansen and Sargent (2008) derive the equivalence between Epstein-Zin preferences with intertemporal elasticity of substitution equal to 1 and multiplier preferences and show that the ambiguity aversion parameter can be written as $\varphi = -1/(1-\beta)(1-\varphi_{EZ})$, where β is the discount factor and φ_{EZ} is the risk aversion parameter in Epstein-Zin preferences. This conversion formula implies that the high ambiguity aversion case corresponds to a risk aversion of 17.6, as the low ambiguity aversion case corresponds to a risk aversion of 1 i.e. standard expected utility specification.

With high ambiguity aversion, the probability distortion is higher for low aggregate productivity levels and low idiosyncratic volatility levels. With low ambiguity aversion (or expected utility preferences), the probability distortion is 1 for every future state.

Table 1.5 tabulates the same conditional probability distortion for each level of A and σ_z to demonstrate the correlation created at the worst-case distribution. As the highest A and highest volatility combination is distorted to have less weight (0.890) compared to the baseline of 1, the lowest A and lowest volatility combination is distorted to have disproportionately more weight (1.123).

Figure 1.12: Probability distortion



1.6.2 Volatility shock with higher ambiguity aversion

Figure 1.13 shows the impulse-response analysis for 1 standard deviation idiosyncratic volatility shock, at two different ambiguity aversion levels. The blue ones are for an economy with $\varphi = 20000$, which is equivalent to the household having expected utility preferences, while the red dotted lines are for the economy with $\varphi = 10$, meaning that household is ambiguity averse. At the stochastic steady state, the economy with higher ambiguity aversion has higher capital, lower rental rate of capital and slightly higher output. The responses as the percentage deviations from the corresponding stochastic steady states are the same at both levels of ambiguity aversion.

Considering higher ambiguity aversion does not lead to any change in aggregate

Table 1.5: Probability distortion

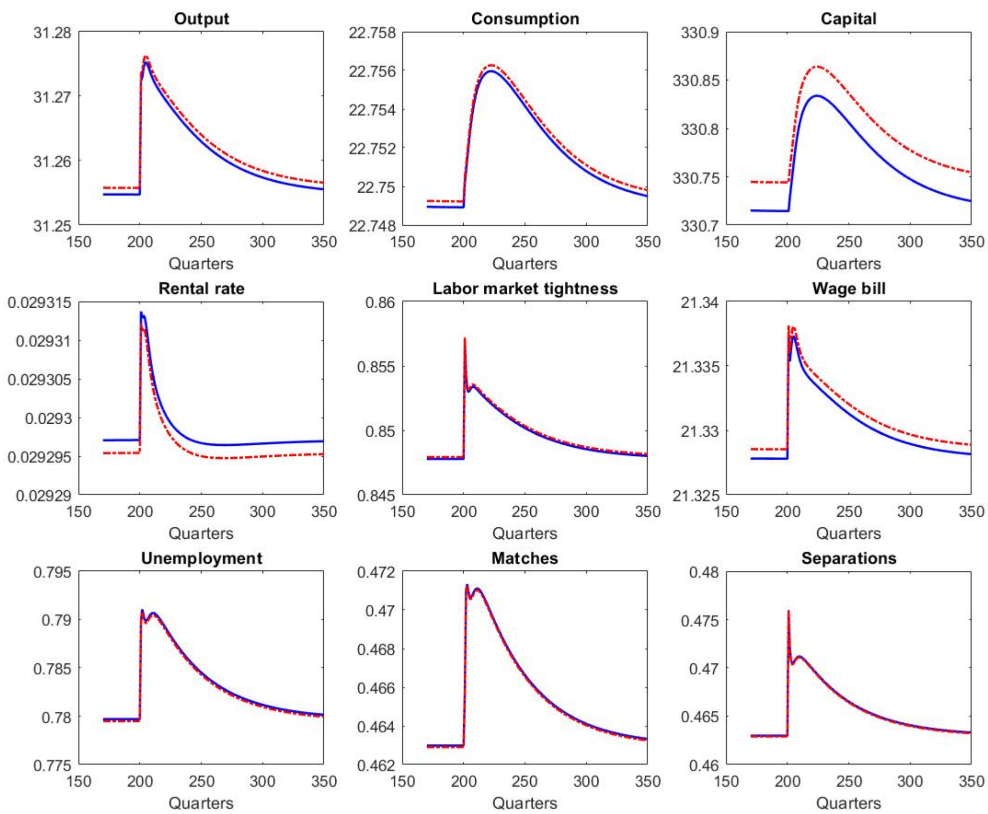
	σ_{z1}	σ_{z2}	σ_{z3}
A_1	**1.123**	1.105	1.088
A_2	1.101	1.083	1.067
A_3	1.079	1.062	1.045
A_4	1.058	1.041	1.025
A_5	1.037	1.020	1.004
A_6	1.016	1.000	0.984
A_7	0.996	0.980	0.965
A_8	0.976	0.961	0.946
A_9	0.957	0.941	0.927
A_{10}	0.938	0.923	0.908
A_{11}	0.919	0.904	**0.890**

dynamics but it increases the market price of risk $\sigma(SDF)/E[SDF]$, since it increases the volatility of the stochastic discount factor. With $\varphi = 20000$ the average market price of risk over 500 periods is 0.00087, while it is 0.0022 with $\varphi = 10$.

1.7 Model with disaster risk

In this section I explain a version of the model with disasters in terms of higher depreciation rate for capital, similar to Gourio (2012). Disaster risk version is interesting and relevant because it is a shock that can lead to a decline in investment. In previous sections results of an increase in idiosyncratic volatility show how important dynamics of capital demand is in whether increase in risk leads to a recession or not. In this section by considering an increase in disaster risk, I am considering a case in which an increase in risk is reducing the capital supply, and even though there may be some increase in capital demand, it is possible to observe a recession in response to a rise in risk.

Figure 1.13: Responses to a 1 std.dev. positive σ_z shock - Higher ambiguity



Note: High ambiguity aversion (dashed red line), low ambiguity aversion (solid blue line)

1.7.1 Household's problem

Given the distribution of its employed members over the firms $\Gamma(n_{-1}, z)$, the rental rate of capital r, total wage bill W, and aggregate states capital K, disaster probability p, and disaster state x, the household chooses how much to consume and invest.

The problem of the household in presence of disaster risk is:

$$V^H(K,p,x,\Gamma) = \max_{C,I} \Bigg\{ (1-\beta) C^{1-\gamma}$$

$$+ \beta E_{p'|p} \Big[(1-p') V^H(K',p',0,\Gamma')^{1-\phi} + p' V^H(K',p',1,\Gamma')^{1-\phi} \Big]^{\frac{1-\gamma}{1-\phi}} \Bigg\}^{\frac{1}{1-\gamma}}$$

subject to

$$C + I = r(K,p,x,\Gamma)K + W(K,p,x,\Gamma)$$

$$K' = ((1-\delta)K + I)(1 - x b_k)$$

where x is 1 if there is a disaster, and 0 if there is no disaster and b_k is the size of the disaster in terms of extra capital depreciation.

The disaster probability follows a persistence AR(1) process around the mean $\bar{p}$:

$$p_t = (1-\rho_p)\bar{p} + \rho_p p_{t-1} + \sigma^p \sqrt{(1-\rho_p^2)} \epsilon_t^p, \qquad \epsilon_t^p \sim N(0,1)$$

This problem will give us: the decision rules $C(K,p,x,\Gamma)$, $K'(K,p,x,\Gamma)$ and the value function $V^H(K,p,x,\Gamma)$.

The stochastic discount factor is given as:

$$SDF = \beta \left(\frac{C(K,p,x,\Gamma)}{C(K',p',x',\Gamma')} \right)^{\gamma} \left(\frac{V^H(K',p',x',\Gamma')}{E_{p'|p} \Big[(1-p') V^H(K',p',0,\Gamma')^{1-\phi} + p' V^H(K',p',1,\Gamma')^{1-\phi} \Big]^{\frac{1}{1-\phi}}} \right)^{\gamma-\phi}$$

1.7.2 Firm's problem

For given rental rate of capital r, labor market tightness θ and wage bill W, a firm with n_{-1} workers and z productivity solves the following problem:

$$V^F(n_{-1}, z, K,p, x, \Gamma) = \max_{n,v,k} \Bigg\{ zk^{\alpha_k} n^{\alpha_n} - \kappa v - wn - rk$$

$$+ \beta C(K,p,x,\Gamma)^\gamma E_{p'|p}\Big[(1-p')\frac{m(K',p',0,\Gamma')}{C(K',p',0,\Gamma')^\gamma} E_{z'|z}\Big[V^F(n,z',K',p',0,\Gamma')\Big]$$

$$+ p'\frac{m(K',p',1,\Gamma')}{C(K',p',1,\Gamma')^\gamma} E_{z'|z}\Big[V^F(n,z',K',p',1,\Gamma')\Big]\Big] \Bigg\}$$

where the distortion to the probabilities is now

$$m(K',p',x',\Gamma') = \left(\frac{V^H(K',p',x',\Gamma')}{E_{p'|p}\Big[(1-p')V^H(K',p',0,\Gamma')^{1-\phi} + p'V^H(K',p',1,\Gamma')^{1-\phi}\Big]^{\frac{1}{1-\phi}}} \right)^{\gamma-\phi}$$

Idiosyncratic productivity follows an AR(1) process with a constant standard deviation $\bar{\sigma^z}$:

$$\log(z_t) = \rho_z \log(z_{t-1}) + \sigma^z \sqrt{(1-\rho_z^2)}\epsilon_t^z, \quad \epsilon_t^z \sim N(0,1)$$

If we substitute the constraint $\triangle n \mathbf{1}^+ = qv$ and optimal capital level that satisfies

$r = \alpha_k z k^{\alpha_k - 1} n^{\alpha_n}$ into the problem above, the firm's problem becomes:

$$V^F(n_{-1}, z, K, p, x, \Gamma) = \max_n \Bigg\{ \Big(\frac{\alpha_k z n^{\alpha_n}}{r}\Big)^{\frac{1}{1-\alpha_k}} (\frac{1-\alpha_k}{\alpha_k}) - wn - \frac{\kappa}{q(\theta)} \triangle n \mathbf{1}^+$$

$$+ \beta C(K, p, x, \Gamma)^\gamma E_{p'|p} \Big[(1-p') \frac{m(K', p', 0, \Gamma')}{C(K', p', 0, \Gamma)^\gamma} E_{z'|z} \Big[V^F(n, z', K', p', 0, \Gamma')\Big]$$

$$+ p' \frac{m(K', p', 1, \Gamma')}{C(K', p', 1, \Gamma')^\gamma} E_{z'|z} \Big[V^F(n, z', K', p', 1, \Gamma')\Big]\Big]\Bigg\}$$

Solution to this problem will give the decision rule $n(n_{-1}, z, K, p, x, \Gamma)$ and from that we can get the capital demand of a firm as

$$k(n_{-1}, z, K, p, x, \Gamma) = \Big(\frac{\alpha_k z n(n_{-1}, z, K, p, x, \Gamma)^{\alpha_n}}{r}\Big)^{\frac{1}{1-\alpha_k}}$$

1.7.3 Wage Setting

Similar to the version of the model with volatility shocks, the wage that results from Stole and Zwiebel (1996) bargaining is:

$$w(n, z, K, p, x, \Gamma) = \eta \Bigg\{ \frac{\alpha_n(1-\alpha_k)}{1 - \alpha_k - \eta(1 - \alpha_k - \alpha_n)} \Big(\frac{\alpha_k}{r(K, p, x, \Gamma)}\Big)^{\frac{\alpha_k}{1-\alpha_k}} z^{\frac{1}{1-\alpha_k}} n^{\frac{\alpha_k + \alpha_n - 1}{1-\alpha_k}}$$

$$+ \beta C(K, p, x, \Gamma)^\gamma E_{p'|p} E_{z'|z} \Big[(1-p') \frac{m(K', p', 0, \Gamma')}{C(K', p', 0, \Gamma')^\gamma} \theta(K', p', 0, \Gamma')$$

$$+ p' \frac{m(K', p', 1, \Gamma')}{C(K', p', 1, \Gamma')^\gamma} \theta(K', p', 1, \Gamma')\Big]\Bigg\} + (1-\eta) b$$

1.7.4 Solution

This version is solved the same way the model with volatility shock is solved. The detailed algorithm is in Section A.7.

Calibration

The additional parameters for the disaster risk model are given in Table ????. The remaining parameters are same as in volatility model, except for the aggregate productivity and idiosyncratic volatility shock models since these shocks are absent in this version. Intertemporal elasticity of substitution (IES) is 1.2 and risk aversion parameter ϕ is 3. In this theoretical exercise I calibrate capital depreciation disasters as small and not-so-rare occurrences. When disaster occurs non-depreciated capital from last period $(1-\delta)K$ and new investment I depreciate at an additional 4.3%, which is one-tenth of the disaster size Barro (2006) considers. The probability of disaster is set to 2% per quarter on average, which is higher than the 0.72% Gourio (2012) considers. I assume the disaster probability has a persistence of 0.96.

Table 1.6: Additional parameters - Disaster risk

Parameter	Meaning	Value
ϕ	Risk aversion	3
$1/\gamma$	Intertemporal elasticity of substitution	1.2
b_k	Size of the disaster	0.043
$\bar{p}$	Mean of disaster prob.	0.02
ρ_p	Persistence of disaster. prob.	0.96
σ_p	Std. dev. of disaster prob.	0.013

Market Clearing

To simulate the model with aggregate shocks and construct the approximate pricing functions $r(K,p,x)$, $WB(K,p,x)$, and $\theta(K,p,x)$, I need to calculate the prices that clear the capital, labor and good markets each period. Given a realization of aggregate states, p, x, and the current firm distribution $\Gamma(n_{-1}, z)$, r, WB, and θ solve the following three equations:

(i) the capital market

$$K = \sum\sum\left(\frac{\alpha_k z n(n_{-1}, z, K, p, x, \Gamma)^{\alpha_n}}{r}\right)^{\frac{1}{1-\alpha_k}}\Gamma(n_{-1}, z),$$

(ii) the labor market

$$\frac{1}{(1+\theta^{-g})^{1/g}} = \frac{\sum\sum\Big(n(n_{-1}, z, K, p, x, \Gamma) - n_{-1}\Big)_{+}\Gamma(n_{-1}, z)}{\Big(L - \sum\sum n_{-1}\Gamma(n_{-1}, z)\Big)}$$

and (iii) the goods market

$$W = \sum\sum w(n, z, K, p, x, \Gamma)n(n_{-1}, z, K, p, x, \Gamma)\Gamma(n_{-1}, z)$$

$$+\sum\sum\Pi(n, z, K, p, x, \Gamma)\Gamma(n_{-1}, z).$$

I solve these equations at every period for a given time series of aggregate state variables. While solving the model, I used Rouwenhorst (1995) method to discretize the AR(1) process for the disaster probability. At each period in simulation first I simulate a series of disaster probabilities p_t then I draw a value for $\hat{x}_t$ from a Uniform(0,1) distribution and I check whether $\hat{x}$ is greater than p_t. If $\hat{x} > p_t$ I set the disaster realization x_t to 1, otherwise to 0. Along the way the cross sectional distribution $\Gamma_t(n_{-1}, z)$ is be updated by the histogram approach described in Young (2010). Then I project the resulting time series for r, WB, θ on the aggregate states to obtain $r(K,p,x), WB(K,p,x)$, and $\theta(K,p,x)$, which are be used to update the value function in the next iteration.

Table 1.7: Price functions - Disaster risk

	constant	$\log(K/K_{ss})$	$\log(p/\bar{p})$	x	R^2
$\log(r/r_{ss})$	-0.00016	-0.65988	0.00095	0.00071	
	(0.00001)	(0.00034)	(0.00025)	(0.00003)	0.99
$\log(WB/WB_{ss})$	-0.00018	0.33796	0.00091	0.00037	
	(0.00001)	(0.00042)	(0.00032)	(0.00003)	0.99
$\log(\theta/\theta_{ss})$	-0.00901	1.63409	0.01964	0.02838	
	(0.00018)	(0.01181)	(0.00880)	(0.00084)	0.99
$\log(C/C_{ss})$	-0.00100	0.64686	0.22313	0.02218	
	(0.00001)	(0.00010)	(0.00007)	(0.00001)	0.99

Note: Regressions are run for 1500 periods. Standard errors are in parentheses.

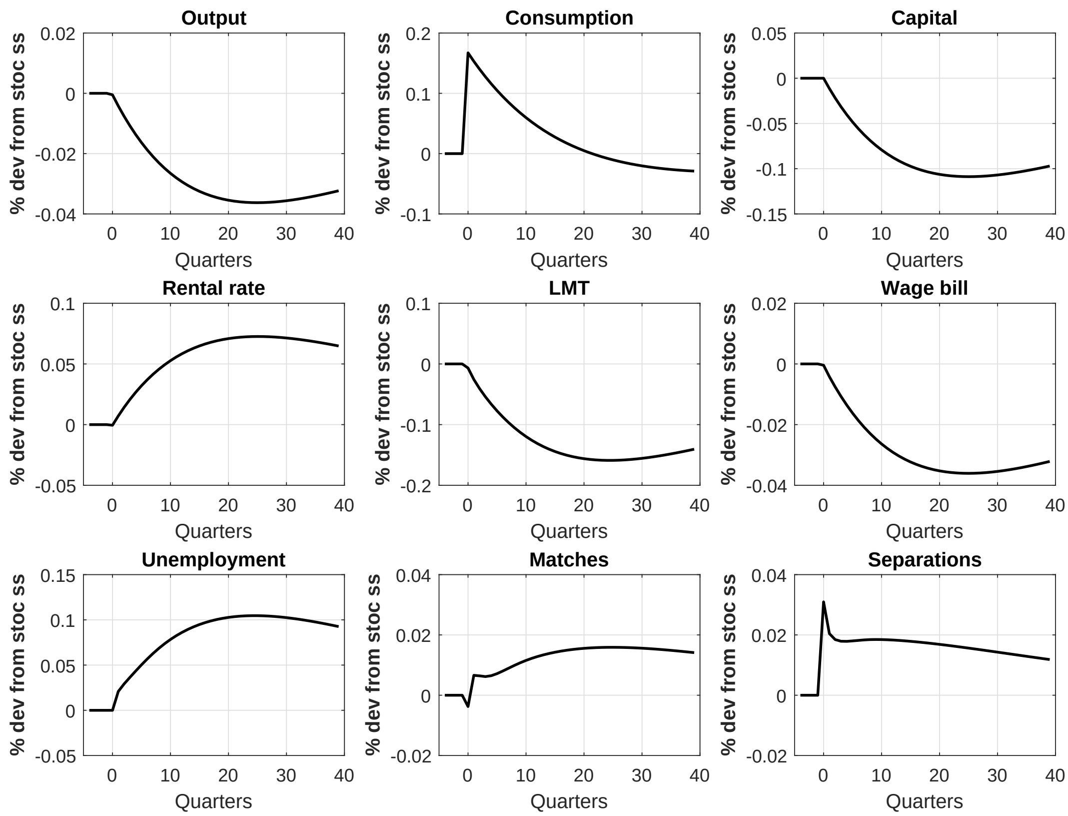
1.7.5 Results

In response to an increase in disaster risk, household decreases investment. Capital and output decline as rental rate of capital increases. Consumption increases on impact but then declines as in Gourio (2012). The movement of the adjustment thresholds for hiring and firing and the movement of firm distribution are creating non-trivial labor market dynamics in the model with disaster risk. Unemployment increases in response to higher disaster risk, since separations increase more than the matches in total. I observe that in response to an increase in disaster risk, productivity thresholds for firing decrease for all firm sizes, while the productivity thresholds for hiring decrease for the small firms and increase even more for the large firms. The parallel shift to the right in the firing threshold is causing the separation region to expand for firms of all sizes but the counterclockwise twist in hiring threshold is decreasing the hiring region for small and low productive firms while increasing it for large and highly productive firms. At the aggregate the increase in matches of large firms dominates the lack of hiring in small firms and total number of matches increases. As a result since separations are across the board for all sizes and increase more than matches, unemployment rate increases. The twist in the hiring threshold and the expanding matching region for highly productive firms points to the fact that marginal benefit of a worker to a small firm declines more in face of higher disaster risk compared to that of a large firm. In addition, labor adjustment costs and slow-moving firm distribution create more propagation compared to Gourio (2012) results.

1.8 Conclusion

In this paper, I build a general equilibrium model with heterogeneous firms to analyze the effects of uncertainty on aggregate economy, in particular on unemployment. There is Diamond-Mortensen-Pissarides style search-and-matching in the labor market, firms can hire more than one worker and hiring decision is partially irreversible due to linear hiring costs. First, I considered the effects of an increase in time-varying idiosyncratic volatility. The model gives rise to multiple effects of volatility, such as option value, realized volatility, Oi-Hartman-Abel and general equilibrium effects, and each of them has different implications for aggregate variables. I found that the irreversible adjustment costs created by search frictions in the labor market do not create strong option value effects to cause an economic downturn when volatility increases. In addition, there are strong realized volatility effects that causes the

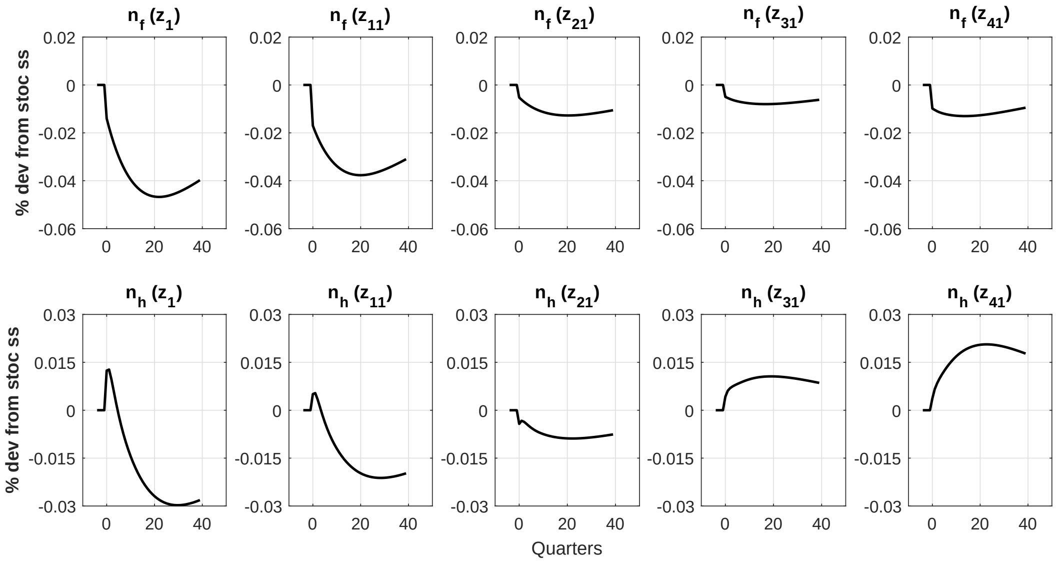
Figure 1.14: Responses to a 1 std.dev. positive p shock



firms to become more active in face of high volatility. As firms hit the adjustment thresholds more often, reallocation of resources from less productive firms to more productive ones increases. As a result, even though in total there are less people working in the economy, on average the workers become more productive. Moreover, the out-of-steady-state dynamics of the model also shows a comparable performance to Schaal (2017) in explaining labor market volatilities, while also performing well in explaining consumption and investment dynamics. The inclusion of volatility shock brings many business cycle moments in line with data.

I also considered a possible role of aversion to model uncertainty, in form of am-

Figure 1.15: Responses of adjustment bands to a 1 std.dev. positive p shock



Note: Upper (lower) panel is the responses of the firing (hiring) thresholds at various idiosyncratic productivity levels.

biguity aversion, in presence of idiosyncratic volatility shocks. With ambiguity aversion agents act on a worst-case distribution, which is distorted towards low utility states. I show that since low-volatility states are the ones with low utility, ambiguity-averse household distorts the conditional expectations by putting more weight on low-volatility states when taking expectations. In addition, the distortion creates a correlation, which is absent in the benchmark distribution, between low aggregate productivity and low idiosyncratic volatility states since the worst-case scenario is facing unfavorable realizations of both shocks at the same time. However, the additional effect of this distortion on the dynamics of aggregate variables is negligible.

Finally, I solved a version of the model with disasters in the form of additional capital depreciation. If probability of a disaster increases, investment goes down. At the same time, unemployment goes up, capital and output goes down. The difference of this version from the model with volatility shock is that in this version of the model an increase in risk creates a recession. The rise in unemployment is due to a larger increase in separations compared to that in matches. Total separations increase, because separations from firms of all sizes and productivities increase. Total matches also increase however it is mainly due to highly productive firms, because marginal value of a worker declines less for more productive and large firms compared to less productive and small firms. Moreover, due to the presence of labor adjustment costs and slow-moving firm distribution, adjustments after the disaster risk shock are not instantaneous and responses show more propagation compared to Gourio (2012).

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Appendix

A.1 Calculation of establishment-level TFP

Bloom et al. (2016) use the annual U.S. Census of Manufacturers panel data for 15,673 establishments with 25+ years of data, from 1972 to 2011. They calculate the total factor productivity (TFP) of establishment i at year t, $\log(\hat{z}_{i,t})$, by using Foster, Haltiwanger, and Krizan (2000) approach. TFP shock ($e_{i,t}$) is the residual from estimation of the following AR(1):

$$\log(\hat{z}_{i,t}) = \rho \log(\hat{z}_{i,t-1}) + \mu_i + \lambda_t + e_{i,t}$$

where μ_i is the establishment-level fixed effect and λ_t is the year fixed effect.

The micro-level volatility measure I plot in Figure 1 is the interquartile range (IQR) of $e_{i,t}$, which is made available online by the authors at `https://people.stanford.edu/nbloom/sites/default/files/census_data.zip`.

A.2 Data description

- Output: the quarterly GDP in 2005 dollars from 1972Q1 to 2009Q4 from the NIPA tables constructed by the Bureau of Economic Analysis.

- Unemployment: the seasonally adjusted monthly unemployment rate constructed by the BLS from the Current Population Survey over the period January 1972-December 2009 (for people aged 16 and over). The series are averaged over quarters.

- Total civilian labor force: for people aged at least 16 from the BLS over the period January 1972-December 2009. The series are averaged over quarters.

- Vacancy: the quarterly average of the monthly vacancy measure from the Job Openings and Labor Turnover Survey. Since the measure is available only since 2001, the Conference Boards Help Wanted Index is used to complete the measure from 1972Q1 to 2000Q4.

- Hirings, layoffs and total separations: the quarterly sums of the JOLTS measures from January 2001 to December 2009. The series are normalized by total labor force.

A.3 Derivation of household's Bellman equation

The agents have a benchmark model in mind, but the parameters are estimated with some standard error. Thus, there is a set of alternative models that can be realized. The ambiguity averse agents want to guard themselves against possible models that will result in lower utility. The following two-step optimization problem enables agents to make decisions that are robust to model uncertainty. First, they minimize utility by choosing the probability distortion $m(K', A', \sigma_z')$ that would result in the worst-case scenario and then maximize utility by choosing other variables (C, K') using the conditional probabilities distorted by $m(K', A', \sigma_z')$.

$$V^H(K, A, \sigma_z) = \max_{C,K'} \min_{m(K',A',\sigma_z')>0} \bigg\{ \log(C) + \beta E\Big[m(K', A', \sigma_z')V^H(K', A', \sigma_z') + \varphi m(K', A', \sigma_z') \log(m(K', A', \sigma_z'))\Big]\bigg\}$$

$$\text{st. } E[m(K', A', \sigma_z')] = 1$$

$$C + K' = (1 - \delta + r(K, A, \sigma_z))K + W(K, A, \sigma_z).$$

φ is the penalty parameter on $m(K', A', \sigma_z') \log(m(K', A', \sigma_z'))$, which is the conditional relative entropy ie. measure of how far the worst-case model is from the benchmark model. For small $\varphi > 0$ it makes sense to care about the model uncertainty and ambiguity aversion is high. For large $\varphi \to \infty$ it does not make sense to worry about the model uncertainty and the preferences are equivalent to the expected utility case.

The solution to the inner minimization problem gives the following probability distortion:

$$m(K', A', \sigma_z') = \frac{\exp(-V^H(K', A', \sigma_z')/\varphi)}{E[\exp(-V^H(K', A', \sigma_z')/\varphi)]}$$

Substituting this probability distortion into the problem above gives us the log-exponential Bellman equation for the outer maximization problem:

$$V^H(K, A, \sigma_z) = \max_{C,K'} \left\{ \log(C) - \beta\varphi \log E\Big[\exp(\frac{-V^H(K', A', \sigma_z')}{\varphi})\Big] \right\}$$

subject to the budget constraint

$$C + K' = (1 - \delta + r(K, A, \sigma_z))K + W(K, A, \sigma_z)$$

A.4 Wage bargaining

The derivation of the wage as a result of a Nash bargaining between a firm and a marginal worker follows Elsby and Michaels (2013). Let the exogenous aggregate states to be denoted as $\Omega = \{A, \sigma_z\}$. For the ease of notation throughout the derivation I surpass the dependence of functions on aggregate state variables (K, Ω), except when taking conditional expectations and also use $\hat{m}$ as a shorthand for the probability distortion $m(K', \Omega')$. The firm problem is then given as:

$$V^F(n_{-1}, z, K, \Omega) = \max_n \Bigg\{ y(n, z) - rk(n, z) - w(n, z)n(n_{-1}, z) - \frac{\kappa}{q(\theta}\triangle n\mathbf{1}^+$$

$$+ \beta E_{\Omega'|\Omega}\Big[\hat{m}\frac{C}{C'}E_{z'|z}\Big[V^F(n, z', K', \Omega')\Big]\Big] \Bigg\}$$

Since $r = \alpha_k y/k$, in the current period's profit I replace $y - rk$ with $rk(1 - \alpha_k)/\alpha_k$. The optimal decision for employment can be divided into two parts. For hiring firms i.e. the firms with productivity draws that are higher than the hiring threshold for their size $n_h^{-1}(n_{-1})$:

$$\frac{(1-\alpha_k)}{\alpha_k}\frac{\partial k(n,z)}{\partial n}r - w(n,z) - \frac{\partial w(n,z)}{\partial n}n(n_{-1},z)$$

$$-\frac{\kappa}{q(\theta)} + \beta E_{\Omega'|\Omega}\Big[\hat{m}\frac{C}{C'}E_{z'|z}\Big[{V_n}^F(n,z',\Omega')\Big]\Big] = 0$$

For firing firms i.e. the firms with productivity draws that are lower than the firing threshold for their size $n_f^{-1}(n_{-1})$:

$$\frac{(1-\alpha_k)}{\alpha_k}\frac{\partial k(n,z)}{\partial n}r - w(n,z) - \frac{\partial w(n,z)}{\partial n}n(n_{-1},z)$$

$$+\beta E_{\Omega'|\Omega}\Big[\hat{m}\frac{C}{C'}E_{z'|z}\Big[{V_n}^F(n,z',\Omega')\Big]\Big] = 0$$

Then the marginal value of a worker in a firm that wants to be size n, and has productivity z is denoted as $J(n,z)$. If a firm needs to hire to reach the size n, then the marginal value of a worker in that firm is given as:

$$J(n,z) = \frac{(1-\alpha_k)}{\alpha_k}\frac{\partial k(n,z)}{\partial n}r - w(n,z) - \frac{\partial w(n,z)}{\partial n}n$$

$$+\beta E_{\Omega'|\Omega}\Big[\hat{m}\frac{C}{C'}E_{z'|z}\Big[{V_n}^F(n,z',\Omega')\Big]\Big] = \frac{\kappa}{q(\theta)}$$

If a firm needs to reduce its number of workers to n and draws productivity z, then the marginal value of the n^{th} worker in that firm is given as:

$$J(n,z) = \frac{(1-\alpha_k)}{\alpha_k}\frac{\partial k(n,z)}{\partial n}r - w(n,z) - \frac{\partial w(n,z)}{\partial n}n$$

$$+\beta E_{\Omega'|\Omega}\Big[\hat{m}\frac{C}{C'}E_{z'|z}\Big[{V_n}^F(n,z',\Omega')\Big]\Big] = 0$$

The effect of today's employment decision on the next period's value function can be decomposed into three parts depending on the next period's productivity z' as:

$$E_{z'|z}\Big[{V_n}^F(n,z',\Omega')\Big] = \underbrace{E_{z'>n_h^{-1}(n)}\Big[\frac{\kappa}{q(\theta')}\Big]}_{\text{hiring}} + \underbrace{E_{n_f^{-1}(n)<z'<n_h^{-1}(n)}\Big[J(n,z')\Big]}_{\text{inaction}} + \underbrace{E_{z'<n_f^{-1}(n)}\Big[0\Big]}_{\text{firing}}$$

With this decomposition $J(n,z)$ can be rewritten as:

$$J(n,z) = \frac{(1-\alpha_k)}{\alpha_k}\frac{\partial k(n,z)}{\partial n}r - w(n,z) - \frac{\partial w(n,z)}{\partial n}n$$

$$+ \beta E_{\Omega'|\Omega}\left[\hat{m}\frac{C}{C'}\left(E_{z'>n_h^{-1}(n)}\Big[\frac{\kappa}{q(\theta')}\Big] + E_{n_f^{-1}(n)<z'<n_h^{-1}(n)}\Big[J(n,z')\Big] + 0\right)\right]$$

On the household side, the marginal value of being n^{th} worker in a firm of size n and productivity z is

$$W(n,z) = w(n,z) + \beta E_{\Omega'|\Omega}\Big[\hat{m}\frac{C}{C'}\Big(E_{z'<n_f^{-1}(n)}\Big[W(n',z')(1-s) + U's\Big]$$

$$+ E_{n_f^{-1}(n)<z'<n_h^{-1}(n)}\Big[W(n',z')\Big] + E_{z'>n_h^{-1}(n)}\Big[W(n',z')\Big]\Big)\Big]$$

Similarly the value of being unemployed is:

$$U = b + \beta E_{\Omega'|\Omega}\Big[\hat{m}\frac{C}{C'}\left((1-f(\theta'))U' + f(\theta')E_n\Big[E_{z'>n_f^{-1}(n)}\Big[W(n',z')\Big]\Big]\right)\Big]$$

According to Stole and Zwiebel (1996), with large firms the bargained wage is the same as the outcome of Nash bargaining over the marginal surplus, so $(1-$

$\eta)(W(n,z)-U)=\eta J(n,z)$ holds. For hiring firms:

$$W(n',z')-U'=\frac{\eta}{(1-\eta)}J(n',z')=\frac{\eta}{(1-\eta)}\frac{\kappa}{q(\theta')}$$

For inactive firms:

$$W(n',z')-U'=\frac{\eta}{(1-\eta)}J(n,z')$$

For firing firms:

$$W(n',z')-U'=\frac{\eta}{(1-\eta)}J(n',z')=0 \;\Rightarrow\; W(n',z')=U'$$

Then the wage bargaining rule $W(n,z)-U=\dfrac{\eta}{(1-\eta)}J(n,z)$ can be expanded as

$$w(n,z)-b+\beta E_{\Omega'|\Omega}\Big[\hat{m}\frac{C}{C'}\Big(E_{z'>n_h^{-1}(n)}\Big[\frac{\eta}{(1-\eta)}\frac{\kappa}{q(\theta')}\Big]$$

$$+E_{n_f^{-1}(n)<z'<n_h^{-1}(n)}\Big[\frac{\eta}{(1-\eta)}J(n,z')\Big]-\frac{\eta}{(1-\eta)}\frac{f(\theta')}{q(\theta')}\kappa\Big)\Big]$$

$$=\frac{\eta}{(1-\eta)}\Big[\frac{\partial(y-rk)}{\partial n}-w(n,z)-\frac{\partial w(n,z)}{\partial n}n$$

$$+\beta E_{\Omega'|\Omega}\Big[\hat{m}\frac{C}{C'}\Big(E_{z'>n_h^{-1}(n)}\Big[\frac{\kappa}{q(\theta')}\Big]+E_{n_f^{-1}(n)<z'<n_h^{-1}(n)}\Big[J(n,z')\Big]\Big)\Big]\Big]$$

With a few more steps the equation above reduces to the following differential equation:

$$w(n,z) = \eta\Big[\frac{(1-\alpha_k)}{\alpha_k}\frac{\partial k(n,z)}{\partial n}r - w(n,z) - \frac{\partial w(n,z)}{\partial n}n$$

$$+ \beta E_{\Omega'|\Omega}\Big[\hat{m}\frac{C}{C'}\frac{f(\theta')}{q(\theta')}\kappa\Big]\Big] + (1-\eta)b$$

The solution to the differential equation (after substituting $k(n,z)$ in as a function of n explicitly) is the following wage function:

$$w(n,z) = \eta\Big[\frac{\alpha_n(1-\alpha_k)}{1-\alpha_k-\eta(1-\alpha_k-\alpha_n)}\Big(\frac{\alpha_k}{r}\Big)^{\frac{\alpha_k}{1-\alpha_k}}(Az)^{\frac{1}{1-\alpha_k}}n^{\frac{\alpha_k+\alpha_n-1}{1-\alpha_k}}$$

$$+ \beta E_{\Omega'|\Omega}\Big[\hat{m}\frac{C}{C'}\theta'\kappa\Big]\Big] + (1-\eta)b$$

A.5 Solution algorithm for the model with volatility

- Guess $V_s^H(K, A, \sigma_z)$ and $V_s^F(n_{-1}, z, K, A, \sigma_z)$ for the first iteration. Guess also the forecast functions for the future labor market tightness $\hat{\theta}(K', A', \sigma_z')$ and the future consumption $\hat{C}(K', A', \sigma_z')$. These forecast functions will be used only in the first iteration. In the following iterations I will use the projection functions I get from the previous iterations.

- Simulate series of A and σ_z for T periods.

- In the first period, start with the steady state level of capital, K_{ss} and steady state firm distribution $\Gamma_{ss}(n_{-1}, z)$.

- At every period, the Levenberg-Marquardt routine finds the prices $P = (r, wb, \theta)$ at which the following problems are solved:

  - Household and firm problems are solved at the simulated values of A and σ_z in the following order:

    Household's problem:

$$\max_{K'} \left\{ \log((1-\delta+r)K+wb-K')-\beta\varphi \log E_{A'|A} E_{\sigma_z'|\sigma_z} \left[\exp(\frac{-V_s^H(K', A', \sigma_z')}{\varphi}) \right] \right\}$$

    Get K', $C = (1 - \delta + r)K + wb - K'$, and $m_s(K', A', \sigma_z')$.

    Firm's problem:

$$\max_{n}\Big\{\Big(\frac{\alpha_k Azn^{\alpha_n}}{r^{\alpha_k}}\Big)^{\frac{1}{1-\alpha_k}}\Big(\frac{1-\alpha_k}{\alpha_k}\Big)-w_s(n,z,K,A,\sigma_z)n-\frac{\kappa}{q(\theta)}\triangle n\mathbf{1}^{+}$$

$$+\beta CE_{A'|A}E_{\sigma_z'|\sigma_z}\Big[\frac{m_s(K',A',\sigma_z')}{\hat{C}(K',A',\sigma_z')}E_{z'|z}\Big[V_s^F(n,z',K',A',\sigma_z')\Big]\Big]\Big\}$$

where the wage at this iteration is given as:

$$w_s(n,z,K,A,\sigma_z)=\eta\Big\{\frac{\alpha_n(1-\alpha_k)}{1-\alpha_k-\eta(1-\alpha_k-\alpha_n)}\Big(\frac{\alpha_k}{r}\Big)^{\frac{\alpha_k}{1-\alpha_k}}(Az)^{\frac{1}{1-\alpha_k}}n^{\frac{\alpha_k+\alpha_n-1}{1-\alpha_k}}$$

$$+\beta\kappa CE_{A'|A}E_{\sigma_z'|\sigma_z}\Big[\frac{m_s(K',A',\sigma_z')}{\hat{C}(K',A',\sigma_z')}\hat{\theta}(K',A',\sigma_z')\Big]\Big\}+(1-\eta)b$$

Get $n(n_{-1},z,K,A,\sigma_z)$.

– Calculate the aggregate quantities for the market clearing with the current firm distribution $\Gamma(n_{-1},z)$. Solve for the (r,wb,θ) that satisfy the following market clearing conditions

$$K=\sum\sum\Big(\frac{\alpha_k Azn(n_{-1},z,K,A,\sigma_z)^{\alpha_n}}{r}\Big)^{\frac{1}{1-\alpha_k}}\Gamma(n_{-1},z)$$

$$\frac{1}{(1+\theta^{-g})^{1/g}}=\frac{\sum\sum\Big(n(n_{-1},z,K,A,\sigma_z)-n_{-1}\Big)_{+}\Gamma(n_{-1},z)}{\Big(L-\sum\sum n_{-1}\Gamma(n_{-1},z)\Big)}$$

$$wb = \sum \sum w(n, z, K, A, \sigma_z) n(n_{-1}, z, K, A, \sigma_z) \Gamma(n_{-1}, z)$$

$$+ \sum \sum \Pi(n, z, K, A, \sigma_z) \Gamma(n_{-1}, z)$$

  - Update the firm distribution $\Gamma(n_{-1}, z)$ with the non-stochastic simulation method of Young (2010) for the next period.

- Regress the time series of (r, wb, θ), on the time series of (K, A, σ_z) to get projection functions $r(K, A, \sigma_z)$, $wb(K, A, \sigma_z)$, and $\theta(K, A, \sigma_z)$.

- Update the value functions to $V^H_{s+1}(K, A, \sigma_z)$ and $V^F_{s+1}(n_{-1}, z, K, A, \sigma_z)$ by solving the household and firm problems again at every point of the aggregate state-space (K, A, σ_z), by using the prices given by the projection functions.

- Repeat until the value functions converge.

A.6 Price functions for the high ambiguity aversion case

Table A.1: Price functions - Ambiguity Averse

	constant	$\log(K/K_{ss})$	$\log(A)$	$\log(\sigma_z/\bar{\sigma^z})$	R^2
$\log(r/r_{ss})$	-0.00028	-0.66993	1.08944	0.06102	
	(0.00003)	(0.00122)	(0.00195)	(0.00042)	0.99
$\log(W/W_{ss})$	0.00006	0.32927	1.08002	0.04496	
	(0.00003)	(0.00154)	(0.00222)	(0.00045)	0.99
$\log(\theta/\theta_{ss})$	0.00865	1.72727	4.94166	0.51345	
	(0.00076)	(0.03733)	(0.05531)	(0.01082)	0.98
$\log(C/C_{ss})$	0.01575	0.76461	0.14944	0.01026	
	(0.00005)	(0.00232)	(0.00317)	(0.00051)	0.99

Note: Regressions are run for the ambiguity averse case ($\varphi = 10$) for 500 periods. Standard errors are in parentheses.

A.7 Solution algorithm for the model with disasters

- Guess $V_s^H(K,p,x)$ and $V_s^F(n_{-1},z,K,p,x)$ for the first iteration. Guess also the forecast functions for the future labor market tightness $\hat{\theta}(K',p',x')$ and the future consumption $\hat{C}(K',p',x')$. These forecast functions will be used only in the first iteration. In the other iterations I will use the projection functions I get from the previous iterations.

- Simulate series of p and x for T periods.

- In the first period, start with the steady state level of capital, K_{ss} and steady state firm distribution $\Gamma_{ss}(n_{-1},z)$.

- At every period, the Levenberg-Marquardt routine finds the prices $P=(r,wb,\theta)$ at which the following problems are solved:

  - Household and firm problems are solved at the simulated values of p and x in the following order:

    Household's problem:

$$\max_{C,I}\left\{(1-\beta)C^{1-\gamma}+\beta E_{p'|p}\left[(1-p')V^H(K',p',0)^{1-\phi}+p'V^H(K',p',1)^{1-\phi}\right]^{\frac{1-\gamma}{1-\phi}}\right\}^{\frac{1}{1-\gamma}}$$

    s.t.

$$C+I=rK+W$$

$$K' = ((1-\delta)K + I)(1 - xb_k)$$

Get K', $C = (1-\delta+r)K + wb - K'/(1-xb_k)$, and $m_s(K', A', \sigma_z')$.

Firm's problem:

$$\max_n \Bigg\{ \Big(\frac{\alpha_k z n^{\alpha_n}}{r}\Big)^{\frac{1}{1-\alpha_k}} (\frac{1-\alpha_k}{\alpha_k}) - wn - \frac{\kappa}{q(\theta)} \triangle n \mathbf{1}^+$$

$$+ \beta C(K,p,x)^\gamma E_{p'|p} E_{z'|z} \Big[(1-p') \frac{m(K',p',0)}{C(K',p',0)^\gamma} V^F(n,z',K',p',0)$$

$$+ p' \frac{m(K',p',1)}{C(K',p',1)^\gamma} V^F(n,z',K',p',1)\Big]\Bigg\}$$

where the wage at this iteration is given as:

$$w(n,z,K,p,x) = \eta \Bigg\{ \frac{\alpha_n(1-\alpha_k)}{1-\alpha_k - \eta(1-\alpha_k-\alpha_n)} \Big(\frac{\alpha_k}{r}\Big)^{\frac{\alpha_k}{1-\alpha_k}} z^{\frac{1}{1-\alpha_k}} n^{\frac{\alpha_k+\alpha_n-1}{1-\alpha_k}}$$

$$+ \beta C(K,p,x)^\gamma E_{p'|p} E_{z'|z} \Big[(1-p') \frac{m(K',p',0)}{C(K',p',0)^\gamma} \theta(K',p',0)$$

$$+ p' \frac{m(K',p',1)}{C(K',p',1)^\gamma} \theta(K',p',1)\Big]\Bigg\} + (1-\eta)b$$

Get $n(n_{-1}, z, K, p, x)$.

– Calculate the aggregate quantities for the market clearing with the current firm distribution $\Gamma(n_{-1}, z, K, p, x)$. Solve for the (r, wb, θ) that satisfy the following market clearing conditions

$$K = \sum\sum\left(\frac{\alpha_k Azn(n_{-1}, z, K, p, x)^{\alpha_n}}{r}\right)^{\frac{1}{1-\alpha_k}}\Gamma(n_{-1}, z)$$

$$\frac{1}{(1+\theta^{-g})^{1/g}} = \frac{\sum\sum\Big(n(n_{-1}, z, K, p, x) - n_{-1}\Big)_{+}\Gamma(n_{-1}, z)}{\Big(L - \sum\sum n_{-1}\Gamma(n_{-1}, z)\Big)}$$

$$wb = \sum\sum w(n, z, K, p, x)n(n_{-1}, z, K, p, x)\Gamma(n_{-1}, z)$$

$$+\sum\sum \Pi(n, z, K, p, x)\Gamma(n_{-1}, z)$$

  - Update the firm distribution $\Gamma(n_{-1}, z)$ with the non-stochastic simulation method of Young (2010) for the next period.

- Regress the time series of (r, wb, θ), on the time series of (K, p, x) to get projection functions $r(K, p, x)$, $wb(K, p, x)$, and $\theta(K, p, x)$.

- Update the value functions to $V^H_{s+1}(K, p, x)$ and $V^F_{s+1}(n_{-1}, z, K, p, x)$ by solving the household and firm problems again at every point of the aggregate state-space (K, p, x), by using the prices given by the projection functions.

- Repeat until the value functions converge.