How People Borrow

Properties and Interactions of Financial Instruments

by

Mrithyunjayan Nilayamgode

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
Economics

at the
University of Virginia
2024

Date of Final Oral Exam: 4/16/2024
The dissertation is approved by the following members of the Final Oral Committee:

Ana Fostel, Professor, University of Virginia
Anton Korinek, Professor, University of Virginia
Eric Young, Professor, University of Virginia
Zachary Bethune, Associate Professor, Rice University

Ana Fostel Ana Fostel

Anton Korinek Anton Korinek

Eric Young Eric Young

Zachary Bethune

How People Borrow

Properties and Interactions of Financial Instruments

Mrithyunjayan Nilayamgode

Abstract

On the whole, this thesis studies the different ways people can borrow, such as unsecured debt contracts and various types of secured debt contracts, and aims to explore how they interact with each other, and what implications this has for the pricing of the underlying assets in the economy, and for economic equity.

The first chapter builds a general equilibrium model where both secured and unsecured debt contracts are available for trade and analyzes this model to prove the existence and determine the nature of equilibria. I define a coexistence equilibrium in this economy as an equilibrium that involves active trade in both secured and unsecured debt, and study the conditions sufficient to guarantee its existence. This paper combines endogenous leverage with the anonymity of perfectly competitive markets to present a scenario where coexistence arises endogenously. I connect this behavior to the existence of endowment inequalities, and illustrate how this inequality affects agents' portfolio decisions between the two types of debt. Finally, by comparing equilibria across financial structures where only one or both kinds of contracts are available, I also demonstrate the asset pricing and redistributive implications of these results.

The second chapter studies the effect of collateral-based financial innovation in a general equilibrium model with incomplete markets and provide precise predictions about asset price effects and spillovers*. We define financial innovation is the use of new kinds of collateral or new kinds of promises backed by existing collateral. Whereas leverage and tranching have positive effects on the price of the underlying asset, credit-default swaps (CDS) has negative price effects. On the other hand, leverage has positive price spillovers on other markets, but tranching and CDS have negative price spillovers. Our results underscore a new mechanism (collateral-based financial innovation) that can explain asset price spillovers without relying on traditional (fire-sale/contagion) channels.

*This chapter is a working paper joint with Ana Fostel and Tyler Wake.

Acknowledgements

I would like to thank my dissertation committee chair, Ana Fostel, for her advice, mentorship, and guidance. She has been a greater mentor and co-author than a graduate student could ever hope for. I would also like to thank my committee members, Anton Korinek, Eric Young, and Zachary Bethune. I have benefited greatly from their feedback, advice, and assistance. I would also like to thank the faculty members of the Economics department at the University of Virginia, especially the members of the Macroeconomics group, for their valuable feedback and guidance as both educators and scholars. I would also be remiss in not taking this opportunity to thank Sheetal Sekhri, Sisir Debnath, and any others who helped me start on and guided me along the path that has led me here today.

This dissertation was made possible by the financial support of the Bankard Fund for Political Economy at the University of Virginia, and I thank them for their generous financial support.

I would also like to thank the participants of the American Economic Association (2024), the Economics Research Colloquium at the University of Virginia (2023), the Midwest Macro (2023, 2022), and the Southern Economic Association (2023, 2022) conferences for useful feedback on the papers contained herein.

I would like to thank my parents, Haridasan and Hemalatha Nilayamgode, for their lifelong support and encouragement. They instilled in me a love for learning that has driven me to this throughout my life and education. I would like to thank my sister, Medha Gayathri Nilayamgode, for her support and encouragement, and for always believing in me. Our shared interests in books and creativity have always been a key part of pushing me towards continuing to read, learn, and write throughout my life. I would like to thank my friends Revathy Suryanarayana and Tyler Wake, as well as my other graduate student friends at UVA's Department of Economics, for being my source of company and comfort throughout graduate school.

Contents

Chapter 1

Introduction

"Promises make debt, and debt makes promises."

–Dutch proverb

In the recent decades, the financial landscape has undergone significant transformations, characterized by increased complexity in financial instruments and their implications for markets and economies. This thesis delves into the intricacies of secured and unsecured debt, leveraging theoretical models to explore the interactions between and the implications of these financial instruments in a general equilibrium setting. The central inquiry of this thesis is to understand how different forms of debt coexist and influence economic variables and agent behaviors within financial markets. This investigation is contextually relevant, given the pervasive impact of debt instruments on financial stability, asset pricing, and economic inequalities.

The first chapter, inspired by a blend of theoretical insights and empirical observations, constructs a general equilibrium model where both secured and unsecured debt instruments are available. This model is not just a theoretical construct but a reflection of real-world financial markets where such instruments coexist and interact dynamically. Through this model, we explore the conditions under which these debts coexist, the mechanisms through which they impact market behavior, and their broader economic implications.

This chapter brings together two strands of literature, one on secured debt and another on unsecured debt, which is a contribution in its own right, given these strands of literature already deal with relatively complex models. Furthermore, since most credit markets in the real world have a mixture of secured and unsecured debt, this question comes with real-world significance. Third, the two kinds of debt markets are likely to have spillover effects on each other, implying that there is value in modeling both forms of debt together rather than in isolation. Finally, this paper fits into an agenda that studies the effect of financial innovation on asset markets. Once developed further, this model may help in answering questions such as how policy aimed at one debt market might spill over into the other.

The second chapter shifts focus slightly to the the effect of financial innovation within the secured debt market on the price of assets used to back the secured debt. We define collateral-based financial innovation is the use of new kinds of collateral or new kinds of promises backed by existing collateral. This chapter presents a general equilibrium incomplete markets model of two periods, two states, and three financial asset, and computes equilibrium prices without leverage, with leverage on a single risky asset, tranching of the same risky asset, and with tranching and CDS on the same risky asset. For each step, we show an innovation has both a direct price effect and spillovers. To be specific, leveraging and tranching boost the price of the underlying risky asset, whereas credit default swaps (CDS) lower it. Furthermore, the introduction of leverage, tranching, and credit default swaps using a risky asset tend to have positive, negative, and negative spillovers, respectively, on the price of other risky assets.

This chapter provides precise predictions regarding the effect of financial innovation on asset prices, allocations, and, ultimately, household welfare, making it useful for policymakers. This paper also contributes to the literature on incomplete markets and collateral, financial innovation, and price spillovers.

This thesis builds upon a rich body of literature, extending the theoretical frameworks of Dubey et al. (2005) and Fostel and Geanakoplos (2012), among others, to analyze the market implications of different forms of debt. Combining or comparing models of different kinds of financial instruments, this work aims to provide a comprehensive view of how different enforcement mechanisms and financial structures influence economic outcomes.

Chapter 2

Collateral and Punishment: Coexistence in General Equilibrium

2.1 Introduction

Writers and thinkers have often wondered why people repay their debts. Sometimes, they do it because they have to, and sometimes because they feel guilty if they choose not to. In general, however, people cannot always be trusted to repay what they borrow; the real fear of default that this engenders makes people hesitant to lend to others. For this reason, institutions were developed to enforce repayment. These institutions are usually punitive in nature – defaulters are punished in some form or the other. The implementation of these institutions can happen either ex post or ex ante. Consider, for example, the housing mortgage market; the mortgage contract states upfront that failing to repay the loan can lead to the seizure and repossession of the house being used as collateral – the use of collateral is thus an ex ante implementation of loan enforcement. On the other hand, in the case of sovereign debts, countries rarely borrow money after signing explicit agreements about what happens in the case of default; instead, if a country defaults on its debts, its creditors might choose ex post to impose some penalties on it. However, some markets, such as consumer and corporate finance, often involve both kinds of institutions – people may take a mortgage from a bank, but also use credit cards or borrow from a loan shark, and firms may have both secured and unsecured debt on their balance sheets.

How, then, do we account for the coexistence of secured and unsecured debt, or collateral and punishment? This paper aims to build a theoretical model and construct examples to study the existence, nature, and positive and normative consequences of default in a setting where both punishments and collateral simultaneously deter it. In other words, this amounts to modeling a scenario where secured and unsecured debt coexist, and default may be partial. To do this, I model a binomial economy where two kinds of assets exist – secured and unsecured. Agents are free to use either to borrow, and the aim is to understand under what circumstances agents make the decisions we observe them making in the real world.

First, we do have ample evidence that both kinds of debt do coexist, and in rather significant quantities, in both consumer and corporate finance[1].

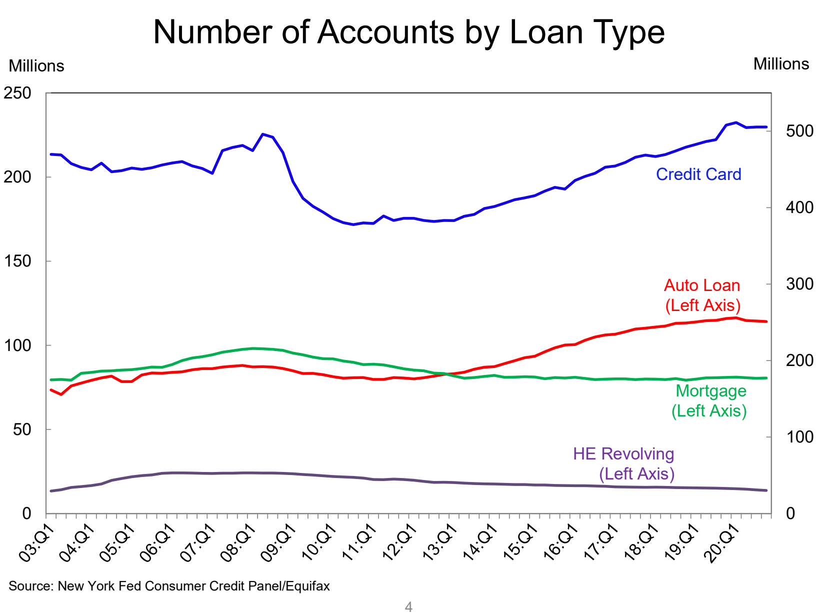


Figure 2.1: Coexistence of Debt Types

Second, there is some evidence that richer households are less likely to hold unsecured debt and more

[1]See Figure 2.1 for an illustration of this evidence using data from the New York Fed Consumer Credit panel; clearly, there are as many outstanding credit card accounts in the United States at any given point in the last few decades as all forms of secured debt combined, making unsecured debt a significant portion of consumer debt.

likely to hold secured debt[2]. Among other results, Disney et al. (2010) observe, based on data from the British Household Panel Survey, that richer households (with greater holdings of financial assets) hold less unsecured debt. At the same time, the link between firm size/revenue and portfolio choice is an area of debate, with evidence in the literature going in both directions. He (2011) cites Frank and Goyal (2008) in arguing that smaller firms take on less leverage than larger firms. On the other hand, using a supervisory data set maintained by the Federal Reserve, Chodorow-Reich et al. (2022) report that smaller firms almost always post collateral, whereas larger ones often borrow unsecured.

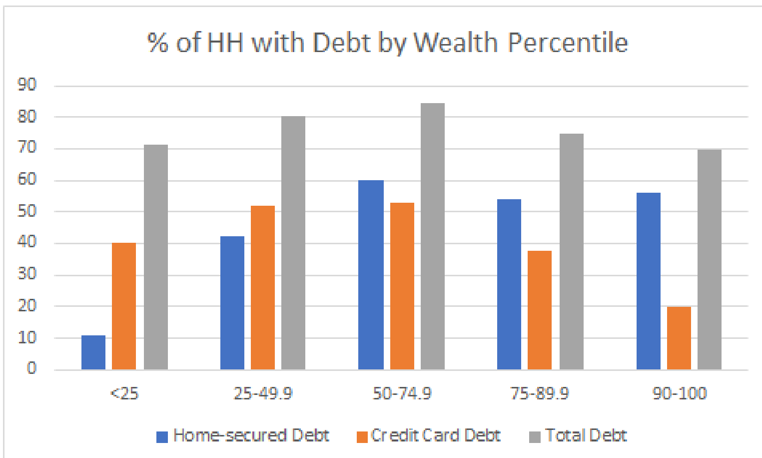


Figure 2.2: Debt Choice Over the Wealth Distribution

Third, both types of debt are known to affect asset pricing. Garriga et al. (2019); Garriga and Hedlund (2020) find credit to be an important factor in housing price dynamics. Landvoigt et al. (2015); Favilukis et al. (2017) find cheaper and easier access to credit, especially for poor households, was a major driver of the housing price boom in the 2000s. Justiniano et al. (2015) also find a close relationship between credit availability and housing prices, though they expect the relationship to be driven in the opposite direction. There is also some anecdotal evidence[3] that links student debt forgiveness and reduced borrowing requirements to the recent boom in housing prices. In the case of corporate finance, Scott (1977) is the seminal paper arguing that firm valuation can be increased by issuing secured debt. More recent work, such as Morellec (2001), finds that there is a more nuanced relationship between a firm's decision between secured and unsecured debt, and their valuation.

[2] See Figure 2.2 for a rough illustration of this, using data from the Survey of Consumer Finances.

[3] See https://www.wsj.com/articles/the-student-debt-bubble-fueled-a-housing-bubble-debt-income-obama-fannie-freddie-bd29b05c .

As such, I aim to build a perfectly competitive GE model that can endogenously explain the coexistence of secured and unsecured debt. Such a model will allow me to study both how inequality (in the form of endowment heterogeneity) affects the portfolio choice between debt types, and the effect of such coexistence on asset prices.

I model the secured part of the debt market after the literature on endogenous leverage; a financial contract in this economy consists not just of the promise it makes, but also the collateral used to back it. The necessity of collateral to secure borrowing both limits the amount that can be borrowed (and hence defaulted on) and acts as a deterrent against extreme strategic default – where the agent defaults despite the value of the collateral held by them being sufficient to repay the loan. Dubey et al. (1995); Geanakoplos (1997a); Geanakoplos and Zame (1997, 2002) were among the first papers to present the C-model (or collateral GEI model), where financial promises need to be backed by collateral requirements. Papers like Geanakoplos (2003b); Fostel and Geanakoplos (2008, 2015a) built on these ideas and developed the concepts of leverage cycles and collateral and liquidity values. These papers demonstrate how collateral requirements have profound positive and normative implications for the economy.

The unsecured part of the debt market in my model is built on another strand of the literature on default, epitomized by Dubey et al. (2005), considers another tool that can serve a similar purpose – punishment; a financial contract in this economy (also called the λ-model) consists of promises, punishments, and borrowing constraints. By using a "pangs of conscience" punishment that is increasing in the magnitude of the default, they are able to show that markets can function in an orderly fashion even in the presence of default, and that punishment and borrowing constraints can provide generic existence of equilibria in a GEI setting with default.

I seek to bring together these two strands of literature by asking similar questions while combining both methods of disciplining default. In a real-world scenario, borrowing from a bank using loans that are secured using collateral can be seen as an example of secured debt. On the other hand, borrowing with punishment-on-default can be seen as borrowing from loan sharks; they might not ask for collateral, and it might be possible to default partially, but they back their lending with threats of punishment (often physical) in case of default. The key mechanism of interest here concerns the endogenous selection of the contracts that agents trade actively and the determination of the credit surface and equilibrium leverage.

These questions cannot be studied using the two kinds of contracts separately – the selected contract and the equilibrium price (interest rate) of each type are likely to be affected by each other's existence. At the same time, we may also question whether an equilibrium of a model that includes both asset classes will necessarily involve trade in both. Is it possible that the agents choose to use either secured or unsecured credit and not the other? Under what features of the model do both asset classes "coexist" in equilibrium?

To the best of my knowledge, such a model of both kinds of debt in GE with incomplete, perfectly competitive markets does not currently exist in the literature. Existing models either focus on the case of complete markets ((Araujo and Villalba, 2022)) or rely on partial equilibrium or other frameworks ((Athreya, 2006; Donaldson et al., 2020)), making my model a novel contribution.

My analysis is relevant for a few reasons: first, it brings together two strands of literature that both deal with how default is deterred, thereby explaining the coexistence of secured and unsecured debt. In and of itself, this is a non-trivial task, given the additional complexity engendered by putting two already complex models together. Second, these questions have real-world significance: most debtor credit markets in the real world have a mixture of secured and unsecured debt. Thus, this project may be a step towards an analysis of default in such generalized settings. Third, the existence of unsecured (punishment-enforced) debt is likely to have spillover effects on the market for collateral-backed (secured) debt, and vice versa, which are likely to be crucial to explaining the kinds of questions this model can answer regarding portfolio decisions, asset pricing, or the spillover effects of government regulation in either debt market on the other. These interactions imply that there is real value in modeling both forms of debt together rather than in isolation.

To expand upon that last point, this paper fits into an agenda that explores how financial innovation affects pre-existing asset markets. In another working paper ((Fostel et al., 2023)), we explore the effect of financial innovation within the secured debt market on the price of assets used to back the secured debt. On the other hand, this paper demonstrates the effect of financial innovation introducing a new debt market (secured or unsecured) on the price of the asset used to back the secured debt contracts. This model, once developed further, can serve as a new starting point for this agenda, and may be helpful in answering other questions, such as how policy aimed at one debt market might spill over into the other. For example, one of my upcoming research goals is to to study how mortgage subsidies (a policy aimed at the secured debt market) affect agents' actions in the unsecured debt market.

This model synthesizes two complex strands of literature, each focused on one of the mechanisms that encourage debt repayment, to shed light on the coexistence of secured and unsecured debt —- a phenomenon that is not only theoretically intriguing but also empirically significant. The task of integrating these two models is far from trivial and presents its own set of challenges. However, the endeavor is well-justified given the real-world relevance of the questions at hand. Both secured and unsecured debt are pervasive and coexist in substantial proportions within consumer and corporate finance markets. Therefore, this project serves as a foundational step toward a more comprehensive analysis of default mechanisms in such generalized financial settings.

Moreover, the model offers qualitative insights into the portfolio decisions of agents across different wealth brackets. Specifically, it suggests that wealthier agents are more inclined to hold a greater proportion of secured debt, while reducing their exposure to unsecured debt. This observation has important implications for understanding financial behavior across socio-economic strata.

The rest of the paper is structured as follows. In Section 2, I set up a binomial two-period general equilibrium model, where agents have access to two goods – a numeraire consumption good and a perfectly durable non-financial asset – and menus of two kinds of debt. Secured debt is backed by the ex ante use of collateral, and each secured debt contract is characterized by the promise of repayment in units of the numeraire good, and the amount of collateral used to back that promise. Unsecured debt is backed by an ex-post scaling utility penalty in the case of default, and each unsecured debt pool is characterized by the promise of repayment in terms of the numeraire good, the penalty parameter, and the sales cap. Unsecured debt is characterized as pools because I follow Dubey et al. (2005) in modeling unsecured debt as being intermediated by pools that collect repayment from agents as a measure of retaining anonymity while allowing default to be punished. I proceed to prove the existence of equilibria in this λC-economy under standard assumptions, thereby guaranteeing that the model is internally consistent.

Next, in Section 3, I define a coexistence equilibrium in this economy as an equilibrium that involves active trade in both secured and unsecured debt. This definition allows me to identify sufficient conditions under which all equilibria of this model must feature coexistence. This relies on the ideas that secured debt offers better returns, and hence, all agents would prefer to first take out as much secured debt as they can, while in the presence of sufficient endowment heterogeneity, at least one agent would also want to take on unsecured debt as well, in order to facilitate greater access to secured debt.

Then, in Section 4, I present a simple numerical example to illustrate the features of the model, and use this example to demonstrate other features of the equilibria I am interested in. In particular, I construct an example in which richer agents hold more secured debt, and less unsecured debt, a pattern that is reflected in real world data. I use this example to present further sufficient conditions (in addition to those sufficient to guarantee coexistence) under which we obtain a coexistence equilibrium of the λC-economy that displays this property.

Finally, in Section 5, I use the same numerical example to compare the price of the non-financial asset across various economies that differ only on the basis of what financial markets are open to agents for trade. To be specific, I compare the equilibrium in the complete λC-economy to equilibria in models that are identical except that agents only have access to either secured debt (C-economy) or unsecured debt (λ-economy), but not both. This comparison tells us that, in my constructed example, moving from either of the single-debt-type economies to the λC-economy pushes up the price of the non-financial asset. This can be interpreted as an effect of financial innovation, if an indirect one in the case of moving from the C-economy to the λC-economy. This then leads me to postulate the sufficient conditions under which such pricing behavior is observed. Section 6 concludes and presents implications for future work.

2.2 Model

I use the standard binomial two-period general equilibrium model, with two time periods, $t \in \{0, 1\}$, with two states in the second time period such that the state space is $S \equiv \{0, U, D\}$, with the set of terminal states being given by $S' \equiv \{U, D\}$. Let there be one consumption good c that we treat as the numeraire, and one perfectly durable non-financial asset y[4]. Denote the price of the asset y in state s by p_s.

Let there be a continuum of agents, $h \in H$, characterized by their subjective discount factors (β^h) and probabilities (γ_s^h), utility functions (u^h), and endowments ($e^h \equiv \left(\left\{e_s^h\right\}_{s \in S}, y^h\right)$)[5] such that their

[4]In particular, I assume that the asset y directly provides utility, thereby making it a non-financial asset.

[5]Notice the inherent assumption that agents are endowed with the financial asset y only at time 0; no additional endowments of y are realized at time 1 in either state of the world.

expected utility is given by

$$U^h = u^h\left(c_0^h, y_0^h\right) + \beta^h \sum_{s\in S'} \gamma_s^h u^h\left(c_s^h, y_s^h\right).$$

We make the following standard assumptions about the utility functions and endowments of the agents:

Assumption A1. Everybody owns something in every state: $e_s^h \neq 0, \forall h \in H, s \in S$.

Assumption A2. $\forall h \in H, u^h(\cdot)$ is weakly concave in each of its arguments.

Assumption A3. $\forall h \in H, u^h(\cdot)$ is weakly monotone in each of its arguments.

Assumption A4. $\forall h \in H, u^h(\cdot)$ is continuously differentiable in each of its arguments.

The crux of our model is the existence of menus of two kinds of debt contracts - secured and unsecured. The menu of secured debt contracts is modeled in the vein of the endogenous leverage literature, as in Geanakoplos (1997a); Fostel and Geanakoplos (2008, 2015a), defined as $(j \cdot \tilde{1}, 1) \in J$, where each secured debt contract j is characterized by the promise of repayment of j units of consumption made against a collateral of one unit of the asset y. Agents choose to buy or sell as many and whichever contracts they want, taking prices as given, with the result that the choice of leverage is endogenous. Following the standard structure in the endogenous collateral literature, we know that the per-contract delivery to creditors is given by $\delta_{sj} = \min\{j, p_s\}$[6]. We define the price (amount borrowed) of the contract j by π_j.

The menu of unsecured debt contracts is modeled in the vein of the literature on punishment, as in Dubey et al. (2005), defined as $(R_i \cdot \tilde{1}, \lambda_i, Q_i) \in I$, where each unsecured debt contract or "pool" i is defined by the promise of repayment of R_i units of consumption made under a threat of utility penalty scaled by the factor λ_i and sales caps Q_i. Agents choose to buy or sell as many and whichever contracts they want, taking prices as given, with the result that the choice of pools is endogenous. Following the structure of unsecured debt contracts in Dubey et al. (2005), delivering D_i instead of R_i incurs a penalty of $\lambda_i \max\{R_i - D_i, 0\}$. Note that the penalty parameter λ_i depends only on the chosen pool, and not the person borrowing using the pool. We define π_i as the price of contract i and D_{si}^h as the repayment being made against the unsecured debt contract i in terminal state s by agent h. Repayments made against

[6] This is easily seen from the fact that no agent would repay more on a contract in any given state than what the collateral backing that contract was worth in that state.

each unsecured debt contract by agents are pooled before being repaid pro rata to creditors[7], such that the per-contract delivery to creditors can be defined as

$$\delta_{si} = R_i \frac{\sum_h D_{si}^h}{\sum_h R_i \varphi_i^h} = \frac{\sum_h D_{si}^h}{\sum_h \varphi_i^h},$$

implying that the repayment rate on unsecured debt contract i in state s is given by $K_{si} = \frac{\delta_{si}}{R_i}$.

2.2.1 Economy

Based on the above definitions of the states, goods, agents, and debt contracts, we can define the economy of our model as follows:

Definition 1. Given the state space S, the agents $h \in H$ defined by their endowments e^h and utilities u^h, and the menus of secured (J) and unsecured (I) debt contracts, and under Assumptions A1-A4, we define the economy we are working in as the λC-economy, $E_{\lambda C}$, as given by

$$E_{\lambda C} = \left(S, \left(u^h, e^h\right)_{h \in H}, J, I\right).$$

We can also further define a pair of special cases of the economy as follows:

Definition 1a. When only unsecured debt contracts are available for trade, i.e., $J = \varnothing$, the λC-economy reduces to the special case of the λ-economy,

$$E_{\lambda} = \left(S, \left(u^h, e^h\right)_{h \in H}, \varnothing, I\right).$$

Definition 1b. When only secured debt contracts are available for trade, i.e., $I = \varnothing$, the λC-economy reduces to the special case of the C-economy,

$$E_{C} = \left(S, \left(u^h, e^h\right)_{h \in H}, J, \varnothing\right).$$

[7] The concept of pooling debt here is somewhat similar to the concept of asset markets with heterogenous quality and adverse selection used in Guerrieri and Shimer (2014).

2.2.2 Budget Set

Given the prices of goods and debt contracts as well as the expected repayment rates of unsecured debt contracts, agents choose consumption and holdings of whatever debt contracts of either or both types as they want to maximize post-penalty expected utility

$$W^h = U^h - \sum_i \lambda_i \sum_s \gamma_s^h \left[\varphi_i^h R_i - D_{si}^h\right]^+$$

subject to the budget set

$$\begin{aligned} B^h(p, \pi_j, \pi_i, K_{si}) = \{ & (c, y, \theta_j, \theta_i, \varphi_j, \varphi_i, D_{sj}^h, D_{si}^h) : \\ & \left(c_0^h - e_0^h\right) + p\left(y_0^h - y^h\right) + \sum_j \pi_j \left(\theta_j^h - \varphi_j^h\right) + \pi_i \left(\theta_i^h - \varphi_i^h\right) \leq 0; \\ & \left(c_s^h - e_s^h\right) + p_s\left(y_s^h - y_0^h\right) + \sum_i D_{si}^h + \sum_j \varphi_j^h \min\{j, p_s\} \\ & \qquad - \sum_i \theta_i^h K_{si} R_i - \sum_j \theta_j^h \min\{j, p_s\} \leq 0, \forall s \in \{U, D\} \\ & \left.\sum_j \max\{0, \varphi_j^h\} \leq y_0^h \right\} \end{aligned}$$

2.2.3 Equilibrium

Having defined the environment of the model that I am working in, I will now proceed to define the solution concept I will be using.

Definition 2. A collateral-punishment (λC) equilibrium for this economy is defined as a vector comprising of prices (prices for the asset and financial contracts and expected deliveries on unsecured debt) and allocations (individual consumptions of the numeraire good and the asset, sales and purchases of both kinds of assets, and actual deliveries for both kinds of assets),

$$\left(p, (\pi_j)_j, (\pi_i, K_i)_i, \left(c^h, y^h, \left(\theta_j^h, \varphi_j^h, \left(D_{sj}^h\right)_s\right)_j, \left(\theta_i^h, \varphi_i^h, \left(D_{si}^h\right)_s\right)_i\right)_h\right),$$

such that:

1. the allocations solve the agents' maximization problems

$$\left(c^h, y^h, \left(\theta_j^h, \varphi_j^h, \left(D_{sj}^h\right)_s\right)_j, \left(\theta_i^h, \varphi_i^h, \left(D_{si}^h\right)_s\right)_i\right.$$
$$\left.\in \arg\max W^h \left(c^h, y^h, \left(\theta_j^h, \varphi_j^h, \left(D_{sj}^h\right)_s\right)_j, \left(\theta_i^h, \varphi_i^h, \left(D_{si}^h\right)_s\right)_i, p\right)\right.$$

over their budget set $B^h(p, \pi_j, \pi_i, K_i), \forall h$,

2. the market for the numeraire clears in all states

$$\sum_{h\in H}\left(c_0^h - e_0^h\right) = 0, \sum_{h\in H}\left(c_s^h - e_s^h\right) = \sum_{h\in H}(y_0^h - y_s^h)p_s, s \in S',$$

3. the markets for the collateral asset and financial contracts of both types clears at $t = 0$

$$\sum_{h\in H}\left(y_0^h - y^h\right) = 0, \sum_{h\in H}\left(\theta_i^h - \varphi_i^h\right) = \sum_{h\in H}\left(\theta_j^h - \varphi_j^h\right) = 0, \forall i, j, \text{ and}$$

4. lenders form rational expectations of the delivery from any unsecured debt contracts that are actually traded in equilibrium

$$K_{si} = \begin{cases} \frac{\sum_{h\in H} p_s D_{si}^h}{\sum_{h\in H} p_s R_i \varphi_i^h} = \frac{\delta_{si}}{R_i}, & \sum_{h\in H} p_s R_i \varphi_i^h > 0 \\ \text{arbitrary}, & \sum_{h\in H} p_s R_i \varphi_i^h = 0 \end{cases}, \forall i.$$

This general notion of an equilibrium is problematic in this context, as discussed in Dubey et al. (2005); the fact that expectations of delivery are arbitrary for unsecured debt contracts that are not actively traded in equilibrium leads to the possibility that some contracts may be go untraded simply due to what they call "whimsical pessimism" - a situation where agents arbitrarily assume that a particular contract will always be defaulted upon, resulting in it not being traded, which in turn allows the original arbitrary assumption. In order to avoid such arbitrary exclusions of certain contracts, we need to refine our equilibrium concept to account for off-equilibrium behavior. We do so by following the procedure of ϵ-boosting as described in Dubey et al. (2005).

An ϵ-boosted economy is defined as a perturbation of the economy described above where we introduce an infinitesimal agent who borrows an infinitesimal amount ϵ using each unsecured debt contract available on the menu, and always fully repays any debt he/she takes out.

Definition 2a. A collateral-punishment equilibrium of an ϵ-boosted economy is known as an ϵ-boosted collateral-punishment ($\epsilon\lambda C$) equilibrium.

Definition 3. A λC-equilibrium $\mathcal{E}_{\lambda C}$ is called a refined equilibrium if there exists a sequence of ϵ-boosted collateral-punishment ($\epsilon\lambda C$) equilibria $\mathcal{E}(\epsilon)$ s.t. $\lim_{\epsilon\to 0}\mathcal{E}(\epsilon)=\mathcal{E}_{\lambda C}$.

Being the limit of a sequence of ϵ-boosted collateral-punishment ($\epsilon\lambda C$) equilibria which do not feature "whimsical pessimism", we cn be assured that refined equilibria are also free of this problem, and can therefore be considered the core solution concept of this model.

2.2.4 Existence

Proposition 1. Consider the λC-economy $E_{\lambda C}$; then, a refined equilibrium exists.

Proof. See Appendix A.1. □

The core of this proof depends on using a fixed point theorem on a mapping from a non-empty, compact, and convex space to ensure the existence of a fixed point that can serve as an equilibrium. However, given the large number of secured and unsecured debt contracts that agents have access to, their ability to default, as well as severe market incompleteness, it is quite easy to be in a position where the conditions necessary for the use of a fixed point theorem may not apply. However, under the fairly standard assumptions described earlier, I am able to combine the methods that Fostel and Geanakoplos (2015a) and Dubey et al. (2005) use in the case of the standard collateral and punishment models respectively to restore these conditions, and adapt them to work in the case where both kinds of debt contracts coexist. This allows me to prove that a refined equilibrium exists in this economy.

2.3 Coexistence

Coexistence is defined as the existence of trade in at least one contract of each type, i.e. $\exists i \in I, j \in J$ such that $\sum_{h\in H}\theta_j^h > 0$ and $\sum_{h\in H}\theta_i^h > 0$. A refined equilibrium that satisfies the property of coexistence is called a refined coexistence equilibrium. Represent the property of coexistence by ω, and the set of all such refined equilibria by $\mathcal{E}(\omega)$. The primary question we ask in this section is under what conditions this set is non-empty, i.e., $\mathcal{E}(\omega) \neq \phi$.

Definition 4. A refined competitive equilibrium is a refined coexistence equilibrium if there is trade in at least one contract of each type, i.e. $\exists i \in I, j \in J, h, h' \in H$ such that $\theta_j^h > 0$ and $\theta_i^{h'} > 0$.

In order to prove the coexistence of both kinds of debt, we consider a λC-economy as described in previous sections with a few additional assumptions:

Assumption C1. Agents can be divided into two groups, i.e. $H \equiv H^L \bigcup H^B$, such that:

1. The utility function of agents is such that agents of type B always like the asset more than agents of type L,
$$\forall h \in H^L, h' \in H^B, \, u_Y^h\Big|_{Y^h=0} < u_Y^{h'}\Big|_{Y^{h'}=\sum\limits_{h \in H} y^h}.$$
2. Furthermore, assume that agents of type L are risk-neutral.

Assumption C2. Agents' endowments are such that:

1. At least one agent of Type L is endowed with some of the asset Y at time 0, i.e.,
$$\exists h \in H^L \text{ s.t. } y^h \neq 0.$$
2. All agents of type B need to borrow; the poorest agent is unable to afford the down payment, i.e.,
$$\min_{h' \in H^B} e_0^{h'} = 0 \text{ and } \max_{h' \in H^B} e_0^{h'} < \bar{e} \text{ for some finite } \bar{e}$$
3. Endowments in the bad state are bounded away from zero, i.e., $\exists \epsilon > 0$ s.t.$\forall h \in H^B$,
$$e_{Dc}^h > \epsilon > yp_D + c_D^h$$

We begin first by proving an intermediate result.

Lemma 1. Consider a λC-economy $E_{\lambda C}$ satisfying Assumptions C1-C2. Then, given any competitive equilibrium where secured debt contracts are being traded actively, no secured debt contract that is being actively traded can offer 100% LTV.

Proof. In a secured-debt-only equilibrium, the MU of using cash to buy the asset Y and consumption c at time 0 must be equal:

$$\frac{U_y^h(c_0^h, y_0^h) + \sum_{s \in S'} \mu_s^h \left(p_s - \delta_s(j)\right)}{p_0 - \pi_j} = \frac{U_c^h(c_0^h, y_0^h)}{1}.$$

Since $U_y^h(c_0^h, y_0^h) \neq 0$ for a non-financial asset, $p_0 \neq \pi_j \implies LTV \neq 100\%$. □

Proposition 2. Consider a λC-economy $E_{\lambda C}$ satisfying Assumptions C1-C2. Then, any refined competitive equilibrium is a refined coexistence equilibrium.

Proof. See Appendix A.2. □

While relegating the minutiae of this proof to the appendix, I will provide here a sketch of the steps taken to prove this proposition. First, we know that an equilibrium exists, as per Proposition 1. Consider any such equilibrium. We also know, from Assumptions C1 and C2.1, that this equilibrium must feature trade in goods. We further know from Assumption C2.2 that this equilibrium must feature trade in financial contracts. Given this information, the next question is whether the equilibrium could feature trade in only one kind of debt contract.

Assume that the equilibrium features trade only in secured debt contracts. Since, per Lemma 1, no secured debt contract that is being traded actively can offer 100% LTV, any purchase of the asset y must require a down payment. However, by Assumption C2.2, the poorest agent is unable to afford the down payment by using only their endowment. Hence, this agent will unilaterally deviate to using unsecured debt contracts in order to afford the down payment necessary to access secured debt contracts. This provides a contradiction.

Now assume that the equilibrium features trade only in unsecured debt contracts. I prove that, given Assumptions C2.2 and C2.3, there is a feasible and profitable deviation for some agent, taking prices as given. This comes down to the idea that, for the same amount borrowed (or promised), secured debt offers better returns, and would be the preferred choice for any agent who can afford it. In other words, it is wasteful to not leverage collateral that you have access to. This provides a contradiction.

Since any given equilibrium cannot feature trade in only one kind of debt contract, it must feature coexistence.

2.4 Debt Portfolio Composition

Next, I consider a simple numerical example to illustrate the results we have discussed so far. I will then use this example to further study some additional results.

2.4.1 Numerical Example

Consider a special case of the λC-economy, under Assumptions C1-C2, with some additional structure imposed to facilitate further analysis. Assume that there are two groups of agents, $H \equiv H^L \bigcup H^B$, such that their state utility functions are given by

$$u^L = y^L + c^L$$
$$u^B(h) = \sqrt{y^B(h)} + \alpha\sqrt{c^B(h)},$$

which clearly meet the requirements of Assumptions A2-A4 and C1. We also abstract away, for the sake of simplicity, from considering discounting across time, since that is not a particular focus of this analysis; we assume that $\beta = 1$ for all agents. Assume further that the endowments of the two groups of agents are given by

$$e^L = ((20, 1), (20, 0), (20, 0))$$
$$e^B(h) = ((8h, 0), (20, 0), (3, 0)), \forall h \in (0, 1),$$

which is in line with the requirements of assumptions A1 and C2. Further assume that agents have access to both secured and unsecured debt markets, with the menu of available debt contracts/pools being given by

$$I = \left\{ (d_i.\tilde{1}, \lambda_i, \infty; \lambda_i \in \left[\frac{1}{2}, 10\right] \right\}$$
$$J = \{(j.\tilde{1}, 1)_{j \in \mathbb{R}_+}\}.$$

I also assume that $\epsilon = 0.1$, i.e., that there is a 10% chance of ending up in the bad state of the world, D, at time 1.

Some of these additional assumptions have been chosen such that we obtain an equilibrium where, as we will see, the only secured debt contract being actively traded is one that promises $j^* = p^U$, and the only unsecured debt pools being used are ones with $\lambda^* = \frac{1}{2}$. Furthermore, all traded secured debt contracts deliver fully in the good state U but are defaulted on in the bad state, D. On the other hand, all traded unsecured debt pools deliver fully in the good state U, and exhibit partial default in the bad state D.

Any secured debt contract that promises $j \leq p^D$ will never be defaulted on if it is issued; this means that such debt contracts will always be charged a riskless rate of interest, $R_j = 1$. Then, no borrower will ever sell a contract with $j < p^D$, since the opportunity cost of collateral in doing so is forgoing the ability to borrow using $j = p^D$ at the same rate of interest. Any secured debt contract that promises $j > p^U$ will always be defaulted on if it is issued; this means that no borrower will ever sell a contract with $j > p^U$. Restricting our focus to $j \in [p^D, p^U]$, agents face a trade-off between a higher ability to borrow and the higher interest rates they will have to face in order to do so. Given that all agents are collateral-constrained (and hence borrowing-constrained) at time 0, and that they expect sufficiently higher endowments in the future (both of which conditions hold under the example I have constructed), they will choose to pay the higher interest rate to borrow as much as they can, thereby using only the contract $j^* = p^U$.

In the case of unsecured debt pools, the actual repayment in the bad state is monotonically increasing in the choice of λ; a pool with a lower penalty parameter does not present as large an incentive for repayment, and agents choose to consume more of their endowment and repay less of their promise in state D, resulting in a higher interest rate and a lower amount borrowed. However, the parameters in this example are chosen such that the benefits of choosing a higher penalty parameter in equilibrium, both in terms of the direct benefit of a lower interest rate and the indirect benefit of a relaxed time-0 budget constraint, are lower than the incurred cost of the increased penalty faced when they default in state D.

The range of λ that agents can choose from for unsecured debt pools are chosen such that we observe the type of default behavior mentioned above. The lower limit on the penalty parameter is tailored to incentivize the agents' behavior; it is not so high as to induce agents to spend all their endowment trying to repay what they can of the debt, but not so low as to encourage complete default in state D or any kind of default in state U. At the same time, the upper limit on λ is such that agents do not benefit so much from choosing higher penalties as to deviate away from choosing the lowest penalty available, as described above.

In such an equilibrium, agents choose to borrow using unsecured debt until the additional cost tomorrow of borrowing an extra unit of consumption today equals the marginal benefit today of that extra unit of consumption, or the marginal benefit of using that unit of consumption as down payment to leverage an additional amount of housing, i.e.,

$$\frac{\alpha}{2\sqrt{c_0^h}} = \frac{1}{2(p_0 - \pi_j)\sqrt{y_0^h}} = \frac{\alpha}{2\sqrt{c_U^h}} R_i^h(1-\epsilon) + R_i^h \underline{\lambda} \epsilon$$

Note that leveraging the house provides benefits both in terms of the utility flow it provides today as well as the consumption it can be sold for tomorrow; however, the leverage contract also means that while agents only pay a down payment (an not full price) for the house today, they also lose a claim to (at least) part of the value of the house tomorrow, depending on the secured debt contract that they choose. In this equilibrium, where the debt contract chosen is $j^* = p^U$, agents lose the value of the entire house tomorrow, and are effectively using the down payment to pay for the utility flow received today.

Agents also further choose food and housing allocations to spend their constrained budget in an optimal way, both in terms of intra-temporal trade-offs between food and housing, and inter-temporal trade-offs between the states 0, U, and D. This results in the following set of equations that pin down the equilibrium.

$$\frac{\alpha}{2\sqrt{c_D^h}} = \lambda$$

$$y_D^h = 1$$

$$\frac{\alpha}{2\sqrt{c_D^h}} = \frac{1}{2p_D\sqrt{y_D^h}}$$

$$r_D^h + c_D^h + p_D = e_D^h$$

$$\int_0^1 y_U^h dh = 1$$

$$p_U y_U^h + c_U^h + d_U^h = e_U$$

$$\frac{\alpha}{2\sqrt{c_U^h}} = \frac{1}{2p_U\sqrt{y_U^h}}$$

$$\pi_i^h = d_U^h(1-\epsilon) + r_D^h\epsilon$$

$$R_i^h = \frac{d_U^h}{\pi_i^h}$$

$$\int_0^1 y_0^h dh = 1$$

$$(p_0 - \pi_j)y_0^h + c_0^h = 8h + \pi_i^h$$

$$\frac{\alpha}{2\sqrt{c_0^h}} = \frac{1}{2(p_0-\pi_j)\sqrt{y_0^h}}$$

$$\pi_j = p_U(1-\epsilon) + p_D\epsilon$$

$$\frac{\alpha}{2\sqrt{c_0^h}} = \frac{\alpha}{2\sqrt{c_U^h}} R_i^h(1-\epsilon) + R_i^h\underline{\lambda}\epsilon$$

2.4.2 Portfolio Composition

In equilibrium, all agents of type B borrow using both secured debt contracts and unsecured debt pools. Moreover, richer (in terms of time-0 endowments) agents hold more secured and less unsecured debt (Figure 2.3). Since all agents borrow using the same secured debt contracts, this translates into richer agents purchasing (and hence leveraging) a larger stock of housing at time 0. Conditional on an internal solution to agent optimization (which has to be the case based on the assumptions of this model), this translates into a larger consumption of food at time 0. Finally, based on agent optimization and the agents' budget constraints, this also implies that richer agents take on less unsecured debt. Agents with different initial (time-0) endowments borrow different amounts using different unsecured debt pools; although all pools in use have the same $\lambda = \frac{1}{2}$, they promise different amounts of consumption at time 1, and hence cost (allow agents to borrow) different amounts today. Furthermore, we see that richer agents are borrowing (less) at better terms, i.e., they face lower interest rates.

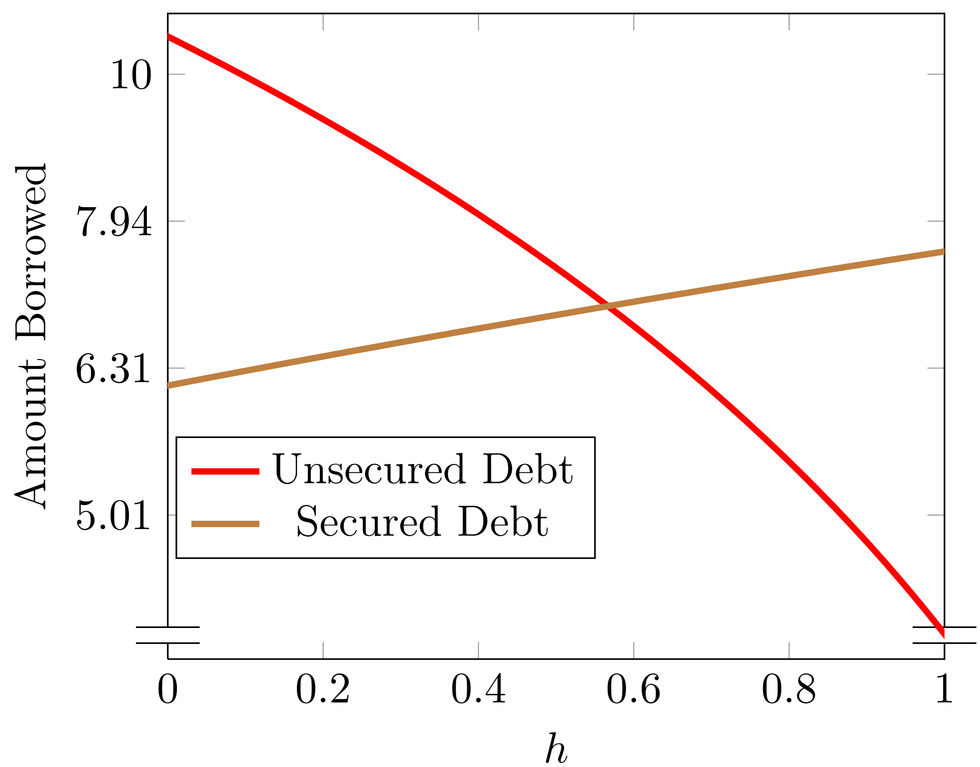


Figure 2.3: Portfolio Choice Over the Endowment Distribution

Now, let us consider under what sufficient conditions this result holds more generally. The starting point is to consider what conditions imposed in this example led to the regime described above. Two primary requirements come up in this context. First, we require the range of penalty parameters to be such that a regime of full repayment in state U and partial and interior repayment in state D is supported. Second, the state utility function for borrowers must be such that intra-temporal optimization has an interior solution in food and housing. This can be boiled down to the following additional assumptions.

Assumption P1. The utility penalty parameter for all available unsecured debt contracts is bounded both above and below, i.e., $\lambda_i \in \left[\underline{\lambda}, \bar{\lambda}\right], \forall i \in I$, s.t. $\underline{\lambda}$ is not so low as to induce strategic default in state U and $\bar{\lambda}$ is not so high as to induce repayment being the only expenditure in the bad state if such a contract is chosen.

Assumption P2. The state utility is given by $u(c, y) = v(y) + w(c)$, where $w(\cdot)$ is more concave than $v(\cdot)$.

Assumption P3. Agents are heterogeneous only in terms of time-0 endowments.

Under these assumptions, I can now make a statement regarding agents' portfolio choice.

Proposition 3. Consider a λC-economy $E_{\lambda C}$ satisfying Assumptions C1-C2 and P1-P3. Then,

- secured debt, $|\pi_j y_j^h|$, is increasing in h, and
- unsecured debt, $|\pi_i^h|$, is decreasing in h.

Proof. See Appendix A.3. □

Secured debt provides better returns and is the preferred choice when feasible. As I argued in the sketch of the coexistence proof, it is wasteful to not leverage collateral that you have access to. As agents get richer, they substitute away from unsecured debt and into secured debt, as they become capable of purchasing and leveraging larger stocks of housing. Internal optimization then implies that these richer agents must be consuming more food, both today and tomorrow, thereby implying that they must be borrowing (and hence repaying) less using unsecured debt pools.

2.5 Asset Pricing

Consider once again the numerical example from earlier. I will first present two variants of this economy, and compare equilibria across the three economies. The first variant is one where $J = \varnothing$ but $I = \left\{(d_i.\tilde{1}, \lambda_i; \lambda_i \in \left[\frac{1}{2}, 10\right]\right\}$ as in the original λC-economy; agents have access to the same menu of unsecured debt pools, but have no access to secured debt contracts. Since agents in this economy have access to unsecured debt, but not secured debt, I call this economy a λ-economy. The second variant is one where

$I = \varnothing$ but $J = \{(j.\tilde{1}, 1)_{j \in \mathbb{R}_+}\}$ as in the original λC-economy; agents have access to the same menu of secured debt contracts, but have no access to unsecured debt pools. Since agents in this economy have access to secured debt, but not unsecured debt, I call this economy a C-economy. Both these economies are identical to the λC-economy in all other parameters except the debt markets they have access to.

In the λ-economy, the only unsecured debt pools being used are ones with $\lambda^* = \frac{1}{2}$. Furthermore, all traded unsecured debt pools deliver fully in the good state U, and exhibit partial default in the bad state D. In the C-economy, the only secured debt contract being actively traded is one that promises $j^* = p^U$, and it delivers fully in the good state U but is defaulted on in the bad state, D.

In the λ-equilibrium, agents once again choose to borrow using unsecured debt until the additional cost tomorrow of borrowing an extra unit of consumption today equals the marginal benefit today of that extra unit of consumption, i.e.,

$$\frac{\alpha}{2\sqrt{c_0^h}} = \frac{\alpha}{2\sqrt{c_U^h}} R_i^h(1-\epsilon) + R_i^h \underline{\lambda}\epsilon$$

Agents also further choose food and housing allocations to spend their constrained budget in an optimal way, both in terms of intra-temporal trade-offs between food and housing, and inter-temporal trade-offs between the states 0, U, and D. This results in the following set of equations that pin down the equilibrium:

$$\frac{\alpha}{2\sqrt{c_D^h}} = \underline{\lambda}$$

$$\int_0^1 y_D^h = 1$$

$$\frac{\alpha}{2\sqrt{c_D^h}} = \frac{1}{2p_D\sqrt{y_D^h}}$$

$$r_D^h + c_D^h + p_D y_D^h = e_D^h$$

$$\int_0^1 y_U^h dh = 1$$

$$p_U y_U^h + c_U^h + d_U^h = e_U + p_U y_0^h$$

$$\frac{\alpha}{2\sqrt{c_U^h}} = \frac{1}{2p_U\sqrt{y_U^h}}$$

$$\pi_i^h = d_U^h(1-\epsilon) + r_D^h\epsilon$$

$$R_i^h = \frac{d_U^h}{\pi_i^h}$$

$$\int_0^1 y_0^h dh = 1$$

$$p_0 y_0^h + c_0^h = 8h + \pi_i^h$$

$$\frac{\alpha}{2\sqrt{c_0^h}} = \frac{1}{2p_0\sqrt{y_0^h}} + \frac{(1-\epsilon)p_U\frac{\alpha}{2\sqrt{c_U^h}} + \epsilon p_D\frac{\alpha}{2\sqrt{c_D^h}}}{p_0}$$

$$\frac{\alpha}{2\sqrt{c_0^h}} = \frac{\alpha}{2\sqrt{c_U^h}}R_i^h(1-\epsilon) + R_i^h\underline{\lambda}\epsilon$$

In a C-equilibrium, agents are restricted in terms of borrowing using secured debt by the stock of collateral owned by them. Agents also further choose food and housing allocations to spend their constrained budget in an optimal way, both in terms of intra-temporal trade-offs between food and housing, and inter-temporal trade-offs between the states 0, U, and D. This results in the following set of equations that pin down the equilibrium.

$$
\begin{aligned}
y_D^h &= 1 \\
\frac{\alpha}{2\sqrt{c_D^h}} &= \frac{1}{2p_D\sqrt{y_D^h}} \\
c_D^h + p_D &= e_D^h \\
\int_0^1 y_U^h dh &= 1 \\
p_U y_U^h + c_U^h &= e_U \\
\frac{\alpha}{2\sqrt{c_U^h}} &= \frac{1}{2p_U\sqrt{y_U^h}} \int_0^1 y_0^h dh \qquad = 1 \\
(p_0 - \pi_j)y_0^h + c_0^h &= 8h \\
\frac{\alpha}{2\sqrt{c_0^h}} &= \frac{1}{2(p_0 - \pi_j)\sqrt{y_0^h}} \\
\pi_j &= p_U(1-\epsilon) + p_D\epsilon
\end{aligned}
$$

Solving for both these equilibria as well, and comparing the solutions to the λC-equilibrium, we observe that the price of the house at time 0 is highest in the λC-equilibrium ($p_0^{\lambda C} = 4.25$), and lowest in the C-equilibrium ($p_0^C = 4.03$), with the price in the λ-equilibrium ($p_0^{\lambda} = 4.12$) being somewhere in the middle. In other words, whether we start in a world with only unsecured or only secured debt, as we head into a world where agents have access to both, this financial innovation pushes asset prices up. This holds true even in a case where the financial innovation is not directly linked to the secured debt market, as in the case when we compare the C-economy to the λC-economy.

The next question is to consider under what sufficient conditions this result holds more generally. Once again, we start from the set of assumptions that hold together the regime in the λC-equilibrium. Then, we observe that the equilibria of the λ- and λC-economies are sensitive to the choice of $\underline{\lambda}$, whereas the equilibrium of the C-economy is not. In particular, for small $\underline{\lambda}$, the increased competition from easy access to unsecured debt pushes housing prices up in the λ- and λC-economies, whereas this effect is much more muted under high $\underline{\lambda}$. This leads us to the consensus that our pricing result would only hold for intermediate values of the penalty parameter.

Proposition 4. Consider the economies $E_\lambda, E_C, E_{\lambda C}$ satisfying Assumptions C1-C2 and P1-P3. Then, $\exists \lambda_1, \lambda_2 > 0$ s.t., $\forall \underline{\lambda} \in [\lambda_1, \lambda_2]$,

$$p_0^C \leq p_0^\lambda \leq p_0^{\lambda C}.$$

Proof. See Appendix A.3. □

2.6 Conclusions and Future Work

In this paper, I build a perfectly competitive general equilibrium model that can endogenously explain the coexistence of secured and unsecured debt, where the secured debt market is modeled after the literature on endogenous leverage ((Dubey et al., 1995; Geanakoplos, 1997a; Geanakoplos and Zame, 1997, 2002)) and the unsecured debt market is based on Dubey et al. (2005). I use this model to demonstrate how inequality affects the portfolio choice between debt types, and the effect of coexistence on asset prices. As far as I am aware, a model featuring and endogenous choice between both kinds of debt in GE with incomplete, perfectly competitive markets is a contribution in and of itself.

I begin the paper by setting up a binomial two-period general equilibrium model, with agents having access to two goods - a numeraire consumption good and a perfectly durable non-financial asset - and menus of two kinds of debt, namely, secured debt backed by the use of the non-financial asset as collateral, and unsecured debt backed by a threat of punishment. I then proceed to show that this λC-economy is internally consistent by showing the existence of equilibria in this model under standard assumptions.

Next, I define a coexistence equilibrium in this economy as an equilibrium that involves active trade in both secured and unsecured debt. Using this definition, I proceed to identify sufficient conditions under which all equilibria of this model must feature coexistence. This relies on the ideas that secured debt offers better returns, and hence, all agents would prefer to first take out as much secured debt as they can, while in the presence of sufficient endowment heterogeneity, at least one agent would also want to take on unsecured debt as well.

Then, I present a simple numerical example to illustrate the features of the model, and use this example to demonstrate other features of the equilibria I am interested in. In particular, I construct an example in which richer agents hold more secured debt, and less unsecured debt, a pattern that is reflected in real world data. I use this example to present further sufficient conditions (in addition to those sufficient to guarantee coexistence) under which we obtain a coexistence equilibrium of the λC-economy that displays this property.

Finally, I use the same numerical example to compare the price of the non-financial asset across various economies that differ only on the basis of what financial markets are open to agents for trade. To be specific, I compare the equilibrium in the complete λC-economy to equilibria in models that are identical except that agents only have access to either secured debt (C-economy) or unsecured debt (λ-economy), but not both. This comparison tells us that, in my constructed example, moving from either of the single-debt-type economies to the λC-economy pushes up the price of the non-financial asset. This can be interpreted as an effect of financial innovation, if an indirect one in the case of moving from the C-economy to the λC-economy.

My model brings together two strands of literature that both deal with how debt repayment is encouraged, and helps explain the coexistence of secured and unsecured debt. This task is non-trivial in and of itself, given that I am combining two already complex models. In addition, these questions have some significance in the real world: secured and unsecured debt coexist in significant proportions in consumer and corporate finance markets. Thus, this project may be a step towards an analysis of default in such generalized settings. The models also qualitatively explains the portfolio decisions of agents across the wealth distribution, whereby richer agents tend to hold more secured debt and less unsecured debt. Last but not least, the model also speaks to the potential spillover effects of financial innovation on asset prices.

To expand upon that last point, this paper fits into an agenda that explores how financial innovation affects pre-existing asset markets. In another working paper ((Fostel et al., 2023)), we explore the effect of financial innovation within the secured debt market on the price of assets used to back the secured debt. On the other hand, this paper demonstrates the effect of financial innovation introducing a new debt market (secured or unsecured) on the price of the asset used to back the secured debt contracts. This model, once developed further, can serve as a new starting point for this agenda, and may be helpful in answering other questions.

For example, the numerical example considered in this paper suggests that financial innovation of this kind (the introduction of unsecured debt markets to a world where previously only secured debt contracts were available for trade) has a clear redistributive effect on agents' welfare. In a secured-debt-only world, agents were limited in their ability to leverage the asset by their initial wealth; the ability to borrow using unsecured debt relaxes their budget constraint, and allows agents - especially the poor - to borrow, and therefore consume, more. While this increased demand also leads to higher prices, and therefore reduces utility across the board for borrowers, the positive effect of relaxing the budget constraint is strong enough for the poorer agents that the net result of the financial innovation is to make the poor better off at the expense of the rich.

On another note, the model would also serve to understand the effect of government policies that restrict or facilitate either kind of borrowing. For example, government policies that subsidize mortgage borrowing may have unanticipated side effects that may push agents (who now value mortgages more highly) towards taking on more (and more expensive) unsecured debt. On the other hand, recent government policies that allow more gradual repayments of unsecured student loans may be interpreted as a reduction in the lower threshold of the penalty parameter, and thus, a comparative statics analysis using this model may shed some light on the spillover effects of such policy changes on goods and secured debt markets. For example, a quantitative extension of this model, once developed, could be useful in explaining the observed correlation between the rollout of these policies and the recent spike in housing prices.

Chapter 3

Collateral Expansion: Pricing Spillovers from Financial Innovation

3.1 Introduction

Financial innovation is the adoption of new technologies, new delivery methods, and new types of financial promises. It can both expand the size of market by bringing in new customers and deepen the scope of the market by expanding what customers can exchange. Understanding the second type of financial innovation is complicated because new products have ramifications on the price of existing financial products. Tracing the price effects of financial innovation in general equilibrium is especially difficult in the presence of incomplete markets. In this paper, we demonstrate the ways collateral-based financial innovation on a single asset can have pricing spillovers even to uncorrelated assets held by different people.

Collateral-based financial innovation is the use of new kinds of collateral or new kinds of promises backed by existing collateral. This form of financial innovation has had a profound influence on the U.S. economy in the past forty years, and the scope of these innovations is growing. In 2022, asset-backed security issuance peaked at an all-time high of \$5.2 T, 35% higher than the previous peak of \$3.8 T in 2003.[1] More assets, and in greater volume, are being securitized every year, and the trend is global.

[1] SIFMA.

Japan, Korea, Great Britain, France, and many G20 countries report rising amounts of ABS issuance.[2]

This paper presents a general equilibrium incomplete markets model of two periods, two states, and three financial assets. Risk-neutral households vary only in their beliefs of the state in the terminal period and align their beliefs with their purchases of the financial assets in the initial period. That choice is also influenced by the financial promises they can make using the underlying financial assets. We compute equilibrium prices without leverage, with leverage on a single risky asset, tranching of the same risky asset, then with tranching and CDS on the same risky asset.

For each step, we show an innovation has both a direct price effect and spillovers. First, leverage creates both a positive direct effect on the price of the underlying risky asset and positive spillovers to the other risky asset. The positive spillover over is generated through a wealth effect, so that agents who do not want to purchase the leveraged asset nevertheless can spend more on the un-levered asset. Second, moving from leverage to tranching generates a positive direct price effect and a negative spillover. Tranching generates negative spillovers through a substitution effect. Selling off derivative assets creates competitors to the other risky asset, lowering demand. Finally, CDS have negative direct effects and negative spillovers. CDS have a negative spillover because they lower prices relative to the tranche economy (negative price effect) and change the composition of demand for the risky asset (negative substitution effect). We demonstrate these results rigorously in proofs and explain them through intuitive examples.

The precise predictions are useful for policymakers, as they provide a road-map of how a financial innovation (or set of financial innovations) will affect prices in financial markets, allocations, and, ultimately, household welfare. We discuss welfare in a setting where agents disagree over beliefs, and we show that an agent's ex-ante welfare does not always increase from a singular financial innovation. However, if enough financial innovations were undertaken such that the Arrow-Debreu equilibrium is implemented, then no agent will be made worse off under their own beliefs relative to financial autarky, when no financial promises are traded. The difficulty is that some agents will have lower ex-ante welfare in the Arrow-Debreu equilibrium relative to some economies with even just a single financial innovation.

This paper contributes to the literature on incomplete markets and collateral, financial innovation, and price spillovers. It builds on and extends Fostel and Geanakoplos (2012), which considered the effects of financial innovations in a two-period, two-state, two-asset environment. Their finding of the direct price

[2] ASIFMA for Asian countries. AFME for European countries.

effects from leverage, tranching, and credit-default swaps (CDS) hold in this more-general setting, which is not obvious. The addition of the third asset in this paper created the scope for asset price spillovers.

Financial innovation has been a topic of great interest to economics and finance researchers for decades. Merton (1992) and Tufano (2003) trace the path of financial innovation in the United States in the late twentieth century. They provide reasons why financial innovation advanced so far so quickly, namely the high interest rates that motivated methods of expanding banks' fixed-income lending. This paper documents a mechanism by which financial innovations may have influenced the cross-section of asset prices in ways not previously understood. For example, we show how introducing leverage on a single risky asset boosts all asset prices in the economy, which is both non-obvious and insightful to historical patterns of asset prices.

The paper also add to the literature on price spillovers. One such mechanism is fire-sales. Shleifer and Vishny (2011) explain how defaults on collateralized loans can force cascading asset sales that cause negative price spillovers, especially when agents anticipate deflationary price movements ((Diamond and Rajan, 2011)). Our model shows how collateral-based financial innovations can create negative price spillovers without default or fire sales. Contagion between interconnected financial partners is another mechanism to generate spillovers. Large networks can be liable to liquidity-shock-induced negative price spillovers on long-term securities ((Allen and Gale, 2000); (Elliott et al., 2014)). Our model can generate negative price spillovers without explicit links/networks in a static, not dynamic, setting. Indeed, agents in our paper do not overlap in their holdings at all, but they are affected through general equilibrium price movements.

The paper proceeds as follows. In Section 3.2, the model of incomplete markets is defined, and attention is spent explaining the budget set of the agents. In Section 3.3, we discuss two benchmark economies that will provide insights into financial innovation: the ideal Arrow-Debreu economy with contingent claims and a financial autarky economy, where agents can buy underlying assets but not financial promises backed by the assets. In Section 3.4, we discuss leverage, where one asset is allowed to be leveraged while the other asset is unaffected. We compare the allocation and prices to the financial autarky equilibrium. In Section 3.5, we compare the equilibrium allocation and prices when the same asset which was leveraged in Section 3.4 is instead allowed to be tranched. In Section 3.6, we discuss credit-default swaps and an economy where agents are allowed to tranche the cash-flows of the riskless asset. We compare the equilibrium allocation and prices to the Tranche economy. In Section 3.7, we look across the five economies for a brief discussion of welfare. The paper concludes in Section 3.8.

3.2 The Model

We work with an extension of a tractable class of general equilibrium models with incomplete markets and collateral, called "C-Models," which were introduced by Fostel and Geanakoplos (2012).[3]

3.2.1 Time and Assets

The model is a two-period general equilibrium model. Uncertainty is represented by a tree $S = \{0, U, D\}$ with a root $s = 0$ at time 0 and two terminal states of nature $S_T = \{U, D\}$ at time 1. Let $L_0 = \{Y, Z, X\}$, $L_U = \{c_U\}$, $L_D = \{c_D\}$ be the set of commodities in states $0, U$ and D. Denote by $L_T = \cup_{s \in S_T} L_s$ the set of commodities in terminal states. Let $F_s(Y, Z, X) = d_s^Y Y + d_s^Z Z + d_s^X X$, $s \in S_T$ be an inter-period production function describing how any vector of commodities at state $s = 0$ gets transformed into a vector of commodities in each state $s \in S_T$. In our framework, time-0 commodities $k \in L_0$ can be interpreted as physical assets, or Lucas trees, that produce dividends d_s^k of the only consumption good in terminal states. We assume that $0 < d_s^k \leq 1$, for $k \in L_0$ and states $s \in S_T$. Physical assets Y and Z are risky and physical asset X is riskless so (without loss of generality) $d_U^X = d_D^X = 1$. We take the price of the riskless asset and the consumption good to be the numeraire in each state, and denote the price of Y and Z at time 0 by p_Y and p_Z respectively. Figure 3.1 shows the uncertainty and asset structure.

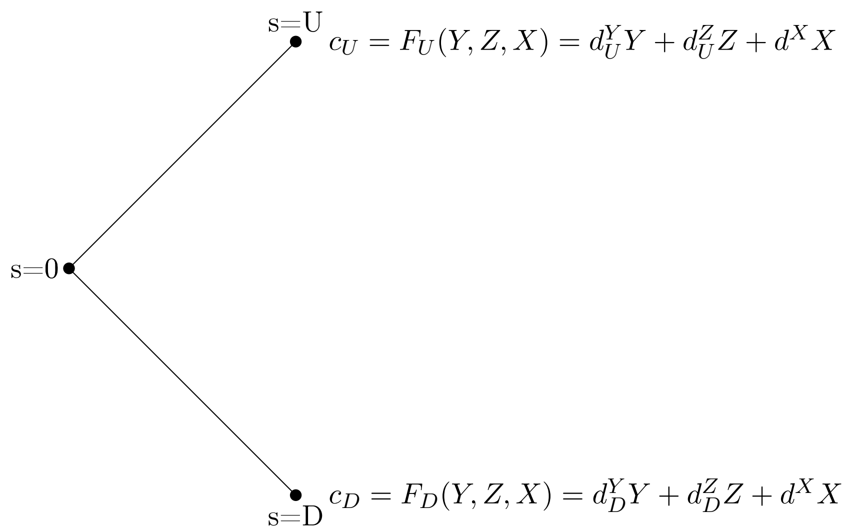
3.2.2 Agents

There is a continuum of agents that are uniformly distributed in the interval $H = [0, 1]$. Each agent $h \in H$ is characterized by a von-Neumann-Morgenstern expected utility,

$$U^h(Y, Z, X, c_U, c_D) = \gamma_U^h c_U + \gamma_D^h c_D, \tag{3.1}$$

[3] For an early treatment see Geanakoplos (2003a)

Figure 3.1: Uncertainty and Asset Structure



each agent $h \in [0, 1]$ is risk-neutral, does not discount the future, and consumes only at time 1.[4] We assume that the subjective probabilities $(\gamma_U^h, \gamma_D^h) = (\gamma(h), 1 - \gamma(h))$ are strictly increasing and continuous in h.

All agents have identical initial endowments of commodities, i.e. $e^h = e = (e_Y, e_Z, e_X, e_{c_U}, e_{c_D})$. Hence, the only source of heterogeneity among the agents is in the subjective probabilities $\gamma(h)$.

3.2.3 Collateral and Financial Contracts

We assume there is no trust and that agents can default in their promises. Hence, we assume that the only way to enforce repayment is through collateral: lenders will seize the collateral agreed by contract if borrowers do not honor their promises. Following the literature on General Equilibrium with incomplete markets and collateral starting with Geanakoplos (1997b), we define a financial contract not only by its promises (like in standard finance) but also by the collateral backing the promises. More precisely, a financial contract j is defined as a pair

$$j = j(k) = ((j_U, j_D), 1_k) \tag{3.1}$$

[4]In the terminology of Fostel and Geanakoplos (2015b) the assets are financial since they do not provide utility to agents.

consisting of a promise (j_U, j_D) of repayment in units of the consumption good at each future state, and the collateral of one unit of the physical asset 1_k, with $k \in L_0$. Define the set of total financial contracts by $J = \bigcup_{k \in L_0} J^k$, where J^k is the set of financial contracts that use one unit of physical asset $k \in L_0$ as collateral.

In a world with default, the contract's promise does not need to coincide with its actual delivery. We assume that all financial contracts are non-recourse, hence the delivery of a financial contract $j = j(k)$ in state $s \in S_T$ will be the minimum between the value of the collateral and the promise in each state:

$$\delta_s(j(k)) = \min\{j_s, d_s^k\} \tag{3.2}$$

Let π_j be the price of contract $j \in J$. Let $\varphi_j < 0$ (> 0) be the sale (purchase) of of the contract j. The sale of a financial contract corresponds to borrowing the sale price, π_j, and the purchase is tantamount to lending the same price in return for the promise. Whereas a sale of a financial contract requires the ownership of the corresponding collateral, the sale does not.

3.2.4 Budget Set

Given the prices prices $(p_Y, p_Z, (\pi_j)_{j \in J})$ of physical assets and financial contracts, each agent $h \in H$ chooses physical asset holdings x, y, z, and financial contract trades, φ_j, at time 0 and consumption c_U, c_D in terminal states to maximize utility (1) subject to the budget set defined by

$$\begin{aligned} B^h(p_Y, p_Z, (\pi_j)_{j \in J}) = \Big\{ &(x, y, z, \varphi, c_U, c_D) \in \mathbb{R}^3_+ \times \mathbb{R}^J \times \mathbb{R}^2_+ : \\ &x + p_Y y + p_Z z + \sum_{j \in J} \varphi_j \pi_j \leq e_X + p_Y e_Y + p_Z e_Z, \\ &\sum_{j \in J^k} \max(0, -\varphi_j) \leq k; k \in L_0 \\ &c_s = e_{c_s} + F_s(Y, Z, X) + \sum_{j \in J} \varphi_j \delta_s(j); s \in S_T \Big\}. \end{aligned}$$

First, at time 0, any expenditures on physical assets, net of endowments, must be financed by net borrowing using financial contracts. Note that agents cannot short physical assets or consumption goods.[5] Second, the collateral constraint requires that agents who borrow must hold the collateral at time 0.[6] Finally, terminal consumption must be financed by dividends coming from asset holdings and net financial contracts deliveries.

3.2.5 Equilibrium

A Collateral Equilibrium in this economy consists of asset and contract prices, asset holdings, contract trades and consumption decisions by all the agents $((p_Y, p_Z, (\pi_j)_{j\in J}), (x^h, y^h, z^h, \varphi^h, c_U^h, c_D^h)_{h\in H}) \in (R_+^2 \times R_+^J) \times (R_+^{L_0} \times R^J \times R_+^{L_T})^H$, such that

1. $\int_0^1 k^h dh = e_k, k \in L_0$
2. $\int_0^1 \varphi_j^h dh = 0 \ \forall j \in J$
3. $(x^h, y^h, z^h, \varphi^h, c_U^h, c_D^h) \in B^h(p_Y, p_Z, (\pi_j)_{j\in J}), \forall h.$
4. $(x, y, z, \varphi, c_U, c_D) \in B^h(p_Y, p_Z, (\pi_j)_{j\in J}) \Rightarrow U^h(x, y, z, \varphi, c_U, c_D) \leq U^h(x^h, y^h, z^h, \varphi^h, c_U^h, c_D^h), \forall h.$

In equilibrium, all markets clear and agents optimize their utilities subject to their budget sets. Geanakoplos and Zame (2014) show that collateral equilibrium always exists under standard assumptions.

3.3 Two Benchmarks and Financial Innovation

We introduce two extreme benchmarks: Arrow-Debreu and Financial Autarky. We follow the analysis by providing a collateral-based financial innovation definition that we use our analysis.

Throughout the paper we will provide intuition for our theoretical results using a numerical example characterized by the following parameterization. Physical assets' deliveries, initial endowments and subjective beliefs are given by $((d_U^X, d_D^X), (d_U^Y, d_D^Y), (d_U^Z, d_D^Z)) = ((1, 1), (1, 0.2), (0.2, 1))$, $e = (e_Y, e_Z, e_X, e_{c_u}, e_{c_d}) = (1, 1, 1, 0, 0)$ and $\gamma_U^h = \gamma(h) = 1 - (1 - h)^2$ respectively. In this example the risky assets Y and Z are negatively correlated, but our theoretical results do not depend on this feature.[7]

[5] Asset short sales would also require posting of collateral, hence they are modeled by trade on financial contracts. In section 6 consider a Credit Default Swap, which is very similar to shorting.

[6] Notice that if an agent borrow, $\varphi_j < 0$ and hence the max is given by the number of contracts sold, each of which is collateralized by one unit of physical asset. However, if an agent is lending, the collateral constraint is automatically satisfied given that $\varphi_j > 0$ the agents cannot short physical assets.

[7] See Appendix B.

3.3.1 Arrow-Debreu Economy

Our first benchmark is the case of the Arrow-Debreu economy where markets are complete. Consider the C-model described in Section 3.2 but assume complete trust (so agents do not need to post collateral) and the existence of a complete set of Arrow securities. In this case, agents will directly trade in Arrow securities and asset prices will be determined by the state prices π_U and π_D. We call this economy the "AD Economy."

Equilibrium for the parameters values discussed before is easy to characterize because of the linear utilities, the continuity of utility in h and the connectedness of the set of agents H. As Figure 3.2a shows, in equilibrium there is a marginal buyer h^{AD} who is indifferent between consumption in the U and D states. All optimistic agents $h > h^{AD}$ consume only in state U, whereas all pessimistic agents $h < h^{AD}$ consume only in state D. Normalizing state prices so that $\pi_U + \pi_D = 1$, equilibrium is described by a system of two equations in two unknowns (π_U and h^{AD}):

$$(1+1+0.2) = (1-h^{AD})\left(\frac{\pi_U(1+1+0.2)+(1-\pi_U)(1+1+0.2)}{\pi_U}\right)$$

$$\frac{\gamma_U^{h^{AD}}}{\pi_U} = \frac{1-\gamma_U^{h^{AD}}}{1-\pi_U}$$

The first equation describes the market clearing of the consumption good in the U state. The second equation states that the marginal buyer is indifferent between consumption U and D consumption.

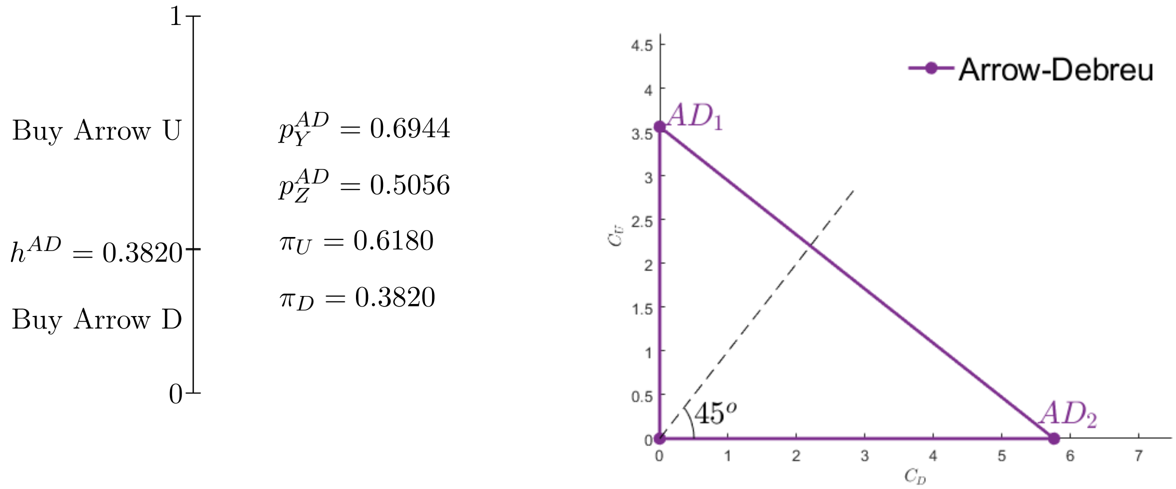
State prices are given by $\pi_U = .6180$ and $\pi_D = .3820$, implying asset prices of $p_Y = .6944, p_Z = .5056$ and $p_X = 1$. The marginal buyer is given by $h^{AD} = .3820$. Figure 3.2b shows the budget set faced by agents. All optimistic agents above the marginal buyer consume on the left upper corner at the point $AD1$, whereas the pessimistic agents below the marginal buyer consume on the right down corner at point $AD2$. State prices determine the slope of the budget line. As will be discussed in Section 7, the AD equilibrium is the first-best allocation.

3.3.2 Financial Autarky

Now we turn to the other extreme benchmark example. Consider the same C-economy as in Section 3.2, but this time assume that $J = \emptyset$ so no asset can be used as collateral to back promises and there is no

Figure 3.2: Arrow-Debreu Economy

(a) Regime (b) Budget Set



trust. Recognizing this agents will not trade any financial promise. This economy corresponds to financial autarky since only commodities and physical assets can be traded. The financial autarky economy will be called the "FA Economy."

Equilibrium for the same parameter values is also easy to characterize. As Figure 3.3a shows, in equilibrium there are two marginal buyers h_1^{FA} and h_2^{FA}. Optimistic agents $h > h_1^{FA}$ buy only asset Y, moderate agents $h_1^{FA} > h > h_2^{FA}$ buy asset X, and all pessimistic agents $h < h_2^{FA}$ buy asset Z. The marginal buyers h_1^{FA} and h_2^{FA} are indifferent between buying asset Y and X and asset X and Z respectively.

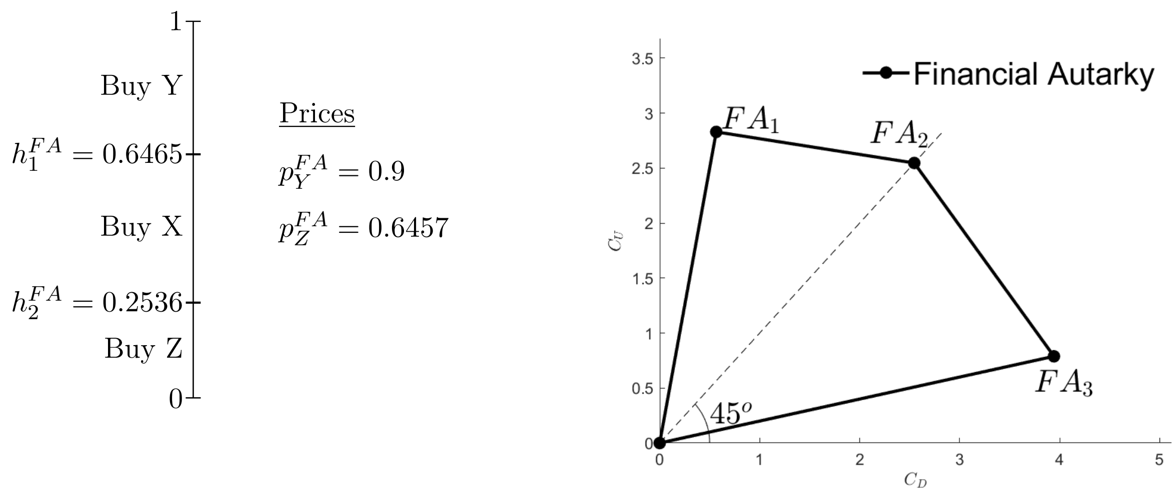
Normalizing the price of the riskless asset X, $p_X = 1$, equilibrium is described by a system of four equations in four unknowns ($h_1^{FA}, h_2^{FA}, p_Y^{FA}$ and $, p_Z^{FA}$):

$$\begin{aligned}
p_Y^{FA} &= (1 - h_1^{FA})(1 + p_Y^{FA} + p_Z^{FA}) \\
1 &= (h_1^{FA} - h_2^{FA})(1 + p_Y^{FA} + p_Z^{FA}) \\
p_Y^{FA} &= \gamma^{h_1^{FA}} + 0.2(1 - \gamma^{h_1^{FA}}) \\
p_Z^{FA} &= 0.2\gamma^{h_2^{FA}} + (1 - \gamma^{h_2^{FA}})
\end{aligned}$$

The first two equations are the market clearing conditions for the markets of Y and Z. The third equation states the the marginal buyer is indifferent between Y and X. The fourth equation states that the marginal buyer is indifferent between X and Z.

Figure 3.3: Financial Autarky Economy

(a) Regime (b) Budget Set



Asset prices are given by $p_Y = .9$ and $p_Z = .6457$. The marginal buyers are $h_1^{FA} = .6465$ and $h_2^{FA} = .2536$. Figure 3.3b shows the budget set faced by agents. The budget set is very different from the AD budget set where market are complete and there are (unique) state prices. In the FA-Economy, we can see that there are consumption bundles that were feasible in the AD economy, that are not anymore. This is because in this economy, despite the fact that there are three physical assets and two states, markets are incomplete due to the short sale constraints. All optimist agents above the marginal buyer consume on the left upper corner at the point $FA1$, the moderate consume at point $FA2$ and pessimistic agents consume at point $FA3$. Moreover, given the short sale constraint there are no state prices that price *all* the assets. The state prices given by the slope between $FA1$ and $FA2$ determines the price of asset Y, and the state prices given by the slope between $FA2$ and $FA3$ determine the price of asset Z.

3.3.3 Financial Innovation

With these two benchmarks in mind, we now embark in trying to understand intermediate cases of financial market structures. We follow Fostel and Geanakoplos (2015b) and define financial innovation as the use of new physical assets as collateral or the use of existing collateral to back new financial promises.

Starting from the Financial Autarky economy, we will consider three financial innovations in the market of the risky asset Y: Leverage, Tranching and Credit Default Swaps. These financial innovations are characterized by different sets J.

First, we consider the Leverage Economy, L-Economy, where agents can use the risky physical asset Y as collateral to back a non-contingent promise (collateralized debt). The financial innovation is the use of a new asset Y as collateral compared to the autarky economy. Second, we consider the Tranching Economy, T-Economy, where agents can use a risky asset Y as collateral to back state-contingent promises. When we go from the leverage to the tranching economy the financial innovation consists of the use of existing collateral Y to back new type of promises. Finally, we consider the Credit Default Swap economy, C-Economy, where agents can use the riskless asset X as collateral to issue a credit default swap on the risky asset Y. In this last step, the financial innovation is the use of a new asset X can as collateral compared to the Tranching economy.

In each step we will study how the financial innovation affects, not only the price in the market where it takes place (asset class Y), but also the price in asset classes not affected directly by the financial innovation, (Z).[8] Throughout this process we will always assume that the asset Z cannot be used as collateral, hence all the financial innovations affect the Y market only. The appendix shows numerical simulations showing other possibilities where financial innovation affects also the market Z. Our theoretical results are robust to other possible combinations, and it is easy to see that the logic of the proofs applies in those cases as well.[9]

3.4 Leverage

Consider the C-model described in Section 3.2. Starting from the financial autarky economy we introduce the first financial innovation: leverage. Agents can issue non-contingent promises using the asset Y as collateral. In this case $J = \bigcup_{k \in L_0} J^k$, where $J^Z = J^X = \emptyset$ and $J^Y = \{(j, j), (0, 1, 0)\}_{j \in \mathbb{R}^+}$. We call this economy the Leverage economy, L-economy.

Each debt contract j promises j units of consumption good at t=1, and is collateralized by one unit

[8] Throughout the paper we will normalize the prize of the riskless asset to be $p_X = 1$.
[9] See discussion in Appendix C.

of the asset Y. Hence, by buying Y and selling any contract j (thus borrowing π_j), agents can leverage their purchases of Y. We assume that X and Z cannot be used as collateral.

Our first result below shows that the introduction of leverage to a financial autarky economy raises the price of all risky assets, creating *positive* price spillovers across different asset classes.

Proposition 5. Consider the FA-economy and the L-economy. Then, (1) The price of asset Y is higher in the L-economy than in the FA-economy, $p_Y^{FA} < p_Y^L$, and (2) the price of asset Z is higher in the L-economy than in the FA-economy, $p_Z^{FA} < p_Z^L$.

Proof. See Appendix B. □

To illustrate the mechanism at play in the result we will use the same numerical example for the parameterization introduced above.

Leverage is endogenous in the model. All contracts are priced in equilibrium, but since collateral is scarce, a limited set may be actively traded. Fostel and Geanakoplos (2015b) show that in C-models the only contract actively traded is the max-min contract $j^* = \min\{d_s^Y\}$, ruling out default in equilibrium, with an associated price $\pi_j = j^*$ (the risk-free interest rate is zero). In our example the max-min contract $j^* = d_D^Y = .2$. Note that when agents leverage a unit of the risky asset Y by selling the max-min contract $j^* = d_D^Y = .2$, they receive $d_U^Y - d_D^Y = 1 - .2 = .8$ in state U and $d_D^Y - d_D^Y = 0$ in state D. Thus, leveraged agents are effectively buying an Arrow U security. Leverage effectively split the asset Y cash flows of $(1, .2)$ into an Arrow U security $(.8, 0)$ (held by the borrower) and a riskless bond $(.2, .2)$ (held by the lender).

As Figure 3.4 shows, in equilibrium there are two marginal buyers h_1^L and h_2^L. Optimistic agents above $h > h_1^L$ leverage asset Y: they buy Y and use it as collateral to borrow $d_D^Y = .2$ by selling debt contract promising $j = d_D^Y = .2$. Moderate agents $h_1^L > h > h_2^L$ buy asset X and lend to optimist agents buying the riskless bond $j^* = d_D^Y = .2$. Finally, all pessimistic agents $h < h_2^L$ buy asset Z. The marginal buyer h_1^L is indifferent between leveraging asset Y and a riskless investment on X and j^* and h_2^L is indifferent between the safe investment and and Z.

Normalizing the price of the riskless asset X, $p_X = 1$, equilibrium is described by a system of four equations in four unknowns (h_1^L, h_2^L, p_Y^L and p_Z^L):

$$p_Y^L = (1 - h_1^L)(1 + p_Z^L + p_Y^L) + 0.2$$

$$p_Z^L = h_2^L(1 + p_Z^L + p_Y^L)$$

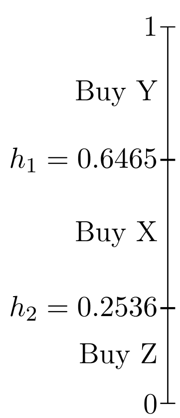
$$p_Y^L = 1\gamma^{h_1^L} + 0.2(1 - \gamma^{h_1^L})$$

$$p_Z^L = 0.2\gamma^{h_2^L} + 1(1 - \gamma^{h_2^L})$$

The first and second equations are the market clear conditions for the markets of the risky assets Y and Z. The third equation states the the marginal buyer is indifferent between leveraging Y and the safe investment in X and riskless bonds. The fourth equation states that the marginal buyers is indifferent between the safe position and the risky asset Z.

Figure 3.4: Financial Autarky v/s Leverage

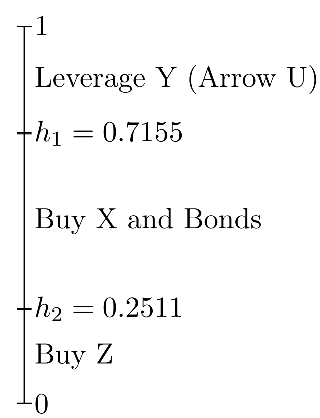
(a) Regime: Financial Autarky



(b) Prices

Financial Autarky		Leverage
0.9	p_Y	0.9352
0.6457	p_Z	0.6487
1	p_X	1

(c) Regime: Leverage



As shown in Figure 3.4, asset prices in the L-economy are given by $p_Y^L = .9352$ and $p_Z^L = .6487$, both higher than in the FA-economy. Moreover the marginal buyers are $h_1^L = .7155$ and $h_2^L = .2511$. Both marginal buyers are shifted to the extremes. Since the marginal buyer pricing asset Y is more optimistic, the price of Y increases from FA to L. For the same reason, since the marginal buyer pricing asset Z is more pessimistic, the price of Z also increases from FA to L.

Leverage creates a wealth effect that explain the increase in both prices.

First, the price of Y increases due to the big wealth effect caused by the possibility of borrowing. It

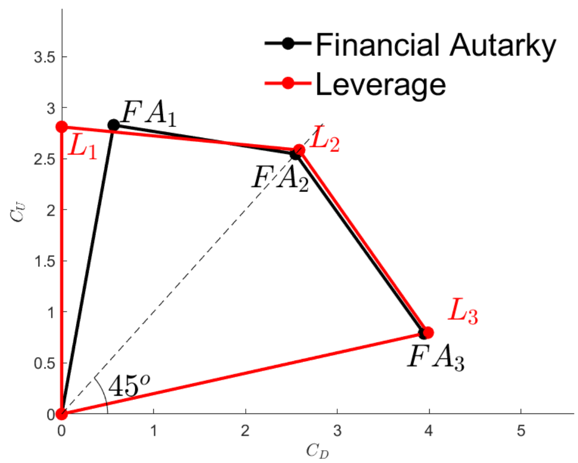


Figure 3.5: Financial Autarky v/s Leverage - Budget Sets

takes fewer agents to buy the same supply of assets and the marginal buyer becomes more optimistic. In line with Fostel and Geanakoplos (2012) the increase in price can be interpreted as collateral value (zero in the FA-economy). Assets fetch a premium when can be used as collateral to borrow.

Second, a wealth effect is what primarily explains the positive spillovers to the Z market. The increase in the price of Y makes all agents richer (including those buying Z). Note that leverage also increases the supply of riskless assets, which induces substitution from the risky asset to bonds, leaving only the most optimistic (or pessimistic) agents to price the risky assets.

Finally, Figure 3.5 presents the budget sets for both FA and L economies. The financial innovation through leverage affects the budget set faced by agents. There are consumption bundles that are not available in the FA-economy that are available in the L-economy: for example consumption only in the U state. All optimist agents above the marginal buyer consume on the left upper corner at the point $L1$, the moderate consume at point $L2$ and pessimistic agents consume at point $L3$.

Moreover, the effect of leverage on asset prices can be seen in the budget sets too. As in the FA-economy there are no state prices that price all the assets. The state prices given by the slope between $L1$ and $L2$ determines the price of asset Y. The slope of this segment is flatter than the one in the FA-economy between points $FA1$ and $FA2$, (since the marginal buyer is someone more optimistic) explaining the increase in the price of Y. Analogously, the state prices given by the slope between $L2$ and $L3$ determines the price of asset Z. The slope of this segment is steeper than the one in the FA-economy between points

$FA2$ and $FA3$ (since the marginal buyer is someone more pessimistic) explaining also the increase in the price of Z.

3.5 Tranching

Consider the C-model described in Section 3.2. Starting from the L-economy, we introduce the second financial innovation: Tranching. Agents can issue contingent promises using the asset Y as collateral. In this case $J = \bigcup_{k \in L_0} J^k$, where $J^Z = J^X = \emptyset$ and $J^Y = \{j_T : j_T = ((0, d_D^Y), (0, 1, 0))\}$. We keep assuming that that X and Z cannot be used as collateral. The set of contracts that use Y as collateral consists of the single contingent contract denoted j_T promising payoffs $(0, d_D^Y)$ collateralized by one unit of Y. We refer to this contract as a "down tranche" since it pays only in state D. Considering this single promise is without loss of generality. Fostel and Geanakoplos (2015b) show that we can always assume in this type of binomial model with financial assets that j_T is the only contract actively traded in equilibrium (in particular, no agent would find optimal to trade non-contingent contracts as in the L-economy even if they were available). We call this economy the Tranching economy, T-economy.

Our second result below shows that the introduction of tranching to a leverage economy raises the price of Y but creates *negative* price spillovers on asset class Z.

Proposition 6. Consider the L-economy and the T-economy. Then, (1) The price of asset Y is higher in the T-economy than in the L-economy, $p_Y^L < p_Y^T$, and (2) the price of asset Z is lower in the T-economy than in the L-economy, $p_Z^L > p_Z^T$.

Proof. See Appendix B. □

As we did before, to illustrate the mechanism at play in the result we will use the same numerical example for the parameterization introduced above.

In our example the down tranche contract j_T promises $(0, d_D^Y) = (0, .2)$. Notice that when buying Y and using it as collateral to issue the down tranche, an agent is effectively buying an Arrow U that pays $(d_U^Y, 0) = (1, 0)$. The buyer of the tranche is buying an Arrow D security that pays $(0, d_D) = (0, .2)$. The down tranche allows agents to completely tranche the asset payoffs of Y into Arrow securities.

As Figure 3.6 shows, in equilibrium there are three marginal buyers h_1^T, h_2^T and h_3^T. Optimistic agents $h > h_1^T$ buy asset Y and sell the down tranche j_T, effectively holding an Arrow U. Moderate agents $h_1^T > h > h_2^T$ buy asset X. More pessimistic agents $h_2^T > h > h_3^T$ hold asset Z. Finally, the most pessimistic agents $h < h_3^T$ buy the down trance j_T, effectively buying an Arrow D.

Normalizing the price of the riskless asset X, $p_X = 1$, equilibrium is described by a system of six equations in six unknowns ($h_1^T, h_2^T, h_3^T, p_Y^T, p_Z^T$ and π_D^T):

$$
\begin{aligned}
p_Y^T &= (1 - h_1^T)(1 + p_Y^T + p_Z^T) + \pi_D^T \\
p_Z^T &= (h_2^T - h_3^T)(1 + p_Y^T + p_Z^T) \\
\pi_D^T &= h_3^T(1 + p_Y^T + p_Z^T) \\
p_Y^T - \pi_D^T &= 1\gamma^{h_1^T} \\
p_Z^T &= 0.2\gamma^{h_2^T} + 1(1 - \gamma^{h_2^T}) \\
\frac{0.2(1 - \gamma^{h_3^T})}{\pi_D^T} &= \frac{0.2\gamma^{h_3^T} + 1(1 - \gamma^{h_3^T})}{p_Z^T}
\end{aligned}
$$

The first, second and third equations are the market clearing conditions for the markets of Y and Z and the down tranche j_T respectively. The fourth equation states that the marginal buyer h_1^T is indifferent between tranching Y and holding X. The fifth equation states that the marginal buyer h_2^T is indifferent between X and Z. The last equation states that the marginal buyer h_3^T is indifferent between Z and j_T.

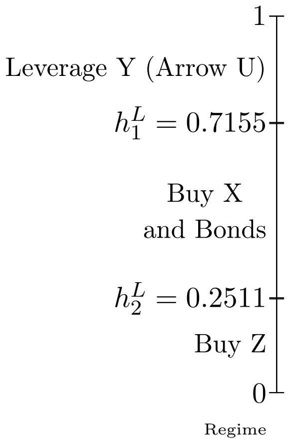
As shown in Figure 3.6, prices in the T-economy are given by $p_Y^T = 1.0062, p_Z^T = .6143$ and $\pi_D^T = .1205$. Whereas the price of Y is higher than in the L-economy, the price of Z is lower. In the T-economy the three marginal buyers are $h_1^T = .6620$, $h_2^T = .2804$ and $h_3^T = .0460$.

First, the increase in price in Y, in line with Fostel and Geanakoplos (2012), is due to the fact that in this economy the price of Y is determined by *all* the marginal buyers. In fact, from the last three equations it is easy to see that $p_Y^T = \gamma^{h_1^T} + 0.2 * f(h_2^T, h_3^T)$, where $f(h_2^T, h_3^T)$ is a decreasing function in both arguments.[10] Hence, there is the possibilities of bubbles: the price of asset Y is higher than 1, more than any agent think is worth, given that the asset payoffs are multiplied but functions that are not summing equal to one anymore. As before, the increase in price is due to a larger collateral value of asset

[10] See Appendix B.

Figure 3.6: Leverage v/s Tranche

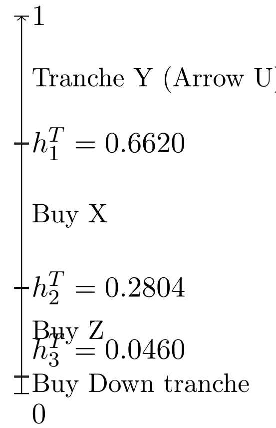
(a) Regime: Leverage



(b) Prices

Leverage		Tranching
0.9352	p_Y	1.0062
0.6487	p_Z	0.6143
1	p_X	1
-	π_T	0.1205

(c) Regime: Tranche



Y compared to the L-economy. In the T-economy the asset cash flows can be split into Arrow securities, which given the high agent heterogeneity raises its collateral value.

Second, a substitution effect is what primarily explains the negative spillovers to the Z market. Tranching of asset Y creates a new security, the down tranche, that is more valuable to pessimistic agents who were previously consuming asset Z. The new marginal buyer pricing asset Z is more optimistic reducing its price. Note that tranching, unlike leverage, reduces the supply of safe assets, inducing also substitution from safe to risky assets.

Finally, notice that in this example assets Y and Z are negatively correlated, and hence the substitution effect coming from the introduction of a down-tranche is very clear. However, as we show in Appendix B, the negative spillover result coming from the substitution effect does not rely on this feature.

Figure 3.7 presents the budget sets for both L and T economies. The financial innovation through tranching affects the budget set faced by agents. We can clearly see that there are consumption bundles that were not available in the L-economy that are available in the T-economy, for example consumption only in the D state. All optimist agents above the marginal buyer consume on the left upper corner at the point $T1$, the moderate consume at point $T2$, the next pessimistic agents consume at point $T3$, and the most pessimistic agents consume at point $T4$.

The effect of tranching on asset prices can be seen in the budget sets. As in the previous economies there are no state prices that price all the assets. The state prices given by the slope between $T1$ and

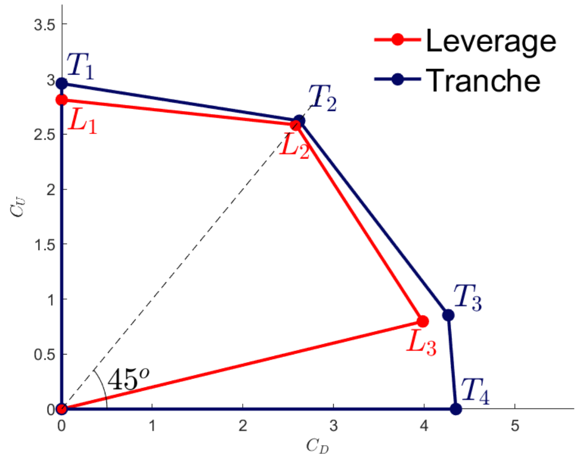


Figure 3.7: Leverage v/s Tranche - Budget Sets

$T2$, $T2$ and $T3$ and $T3$ and $T4$ all determines the price of asset Y. Whereas the slope of the segment between $T1$ and $T2$ is steeper than in $L1$ and $L2$ (reducing the valuation of d_U^Y), the slopes between $T2$ and $T3$ and $T3$ and $T4$ are both flatter than the one implied by $1 - h_1^L$ between $L1$ and $L2$ (increasing the valuation of d_D^Y). The second effect dominates explaining the increase in Y. Analogously, the state prices given by the slope between $T2$ and $T3$ determines the price of asset Z. The slope of this segment is flatter than the one in the L-economy between points $LA2$ and $LA3$ (since the marginal buyer is someone more optimist) explaining also the decrease in the price of Z.

3.6 Credit-Default Swaps

Consider the C-model described in Section 3.2. Starting from the T-economy we introduce our last financial innovation: Credit Default Swaps on Y.

A Credit Default Swap (CDS) on the asset Y is a contract that promises to pay 0 at $s = U$ when Y pays d_U^Y and promises $1 - d_D^Y$ at $s = D$ when Y pays only d_D^Y. A CDS is thus an insurance policy for Y. A seller of a CDS must post collateral, typically in the form of money. Following Fostel and Geanakoplos (2012) we can model CDS into our economy by taking J^X to consist of one contract called j_C promising $(0, 1)$ backed by one unit of asset X.

CDS can be covered or naked. A covered CDS is one in which a buyer of a CDS is obliged to hold the underlying security Y. In this case, the joint position is equivalent to a risk-free asset that pays $(1,1)$. Hence, the introduction of a secured CDS in our T-economy would not change the equilibrium. On the other hand a naked CDS is one in which the buyer of the CDS is not obliged to hold the underlying security. In what follows we consider only naked CDS.

We keep assuming that Y can be tranched as in the T-economy and that Z cannot be used as collateral. Hence we have that $J = \bigcup_{k \in L_0} J^k$, where $J^Z = \emptyset$, $J^Y = \{j_T : j_T = ((0, d_D^Y), (0,1,0))\}$ and $J^X = \{j_C : j_C = ((0,1),(1,0,0))\}$. We call this economy the CDS economy, C-economy.

Our third result below shows that the introduction of CDS on Y to a tranching economy reduces the prices of Y and creates *negative* price spillovers on asset class Z. Hence the introduction of a CDS on Y reduces the price of all risky assets.

Proposition 7. Consider the T-economy and the C-economy. Then, (1) The price of asset Y is lower in the C-economy than in the T-economy, $p_Y^T > p_Y^C$, and (2) the price of asset Z is lower in the C-economy than in the T-economy, $p_Z^T > p_Z^C$.

Proof. See Appendix B. □

As we did before, to illustrate the mechanism at play in the result we will use the same numerical example for the parameterization introduced above.

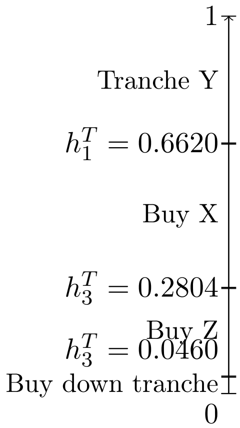
In our example the CDS contract j_C promises $(0,1)$. Notice that when buying X and using it as collateral to issue a CDS on Y, an agent is effectively buying an Arrow U that pays $(1,0)$. The buyer of the CDS is buying an Arrow D security that pays $(0,1)$. Hence writing a CDS on Y using X as collateral allows agents to completely tranche the asset payoffs of X into Arrow securities.

As Figure 3.8 shows, in equilibrium there are two marginal buyers h_1^C and h_2^C . Optimistic agents above $h > h_1^C$ buy assets Y and X and sell the down tranche j_T and the CDS j_C respectively, effectively holding an Arrow U. Moderate agents $h_1^C > h > h_2^C$ buy asset Z. Finally, the most pessimistic agents $h < h_2^T$ buy the down trance j_T and the CDS j_C from the optimistic buyers, effectively buying an Arrow D.

Normalizing the price of the riskless asset X, $p_X = 1$, equilibrium is described by a system of six

Figure 3.8: Tranche v/s CDS

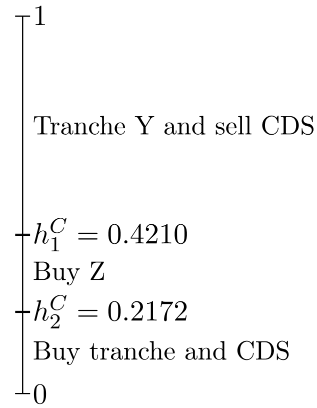
(a) Regime: Leverage



(b) Prices

Tranching		Tranche Y & Sell CDS
1.0062	p_Y	0.6922
0.6143	p_Z	0.4333
1	p_X	1
0.1205	π_T	0.0769
-	π_C	0.3847

(c) Regime: Tranche



equations in six unknowns ($h_1^C, h_2^C, p_Y^C, p_Z^C, \pi_T^C$ and π_C^C):

$$
\begin{aligned}
1 + p_Y^C &= (1 - h_1^C)(1 + p_Y^C + p_Z^C) + \pi_C^C + \pi_D^C \\
p_Z^C &= (h_1^C - h_2^C)(1 + p_Y^C + p_Z^C) \\
\frac{1}{p_Y^C - \pi_T^C}\gamma^{h_1^C} &= \frac{0.2\gamma^{h_1^C} + 1(1 - \gamma^{h_1^C})}{p_Z^C} \\
\frac{0.2}{\pi_T^C}(1 - \gamma^{h_2^C}) &= \frac{0.2\gamma^{h_2^C} + 1(1 - \gamma^{h_2^C})}{p_Z^C} \\
\frac{0.2}{\pi_T^C} &= \frac{1}{\pi_C^C} \\
\frac{1}{p_Y^C - \pi_T^C} &= \frac{1}{1 - \pi_C^C}
\end{aligned}
$$

The first two equations are the market clear conditions for the markets of the Arrow U created by tranching asset Y and writing a CDS on Y using X as collateral, and the market of Z. The third equation states that the marginal buyer h_1^C is indifferent between and Arrow U and asset Z. The fourth equation states that the marginal buyer h_2^C is indifferent between Z and an Arrow D. The last two equations are non-arbitrage conditions for the market of Arrow U and D, agents have to be indifferent between creating these securities via tranching Y or issuing a CDS using X.

As shown in Figure 3.8, prices in the C-economy are given by $p_Y^C = 0.6922, p_Z^C = .4333$, $\pi_D^C = .0769$ and $\pi_C^C = .3847$, a huge drop for asset both prices with respect to the T-economy. In the C-economy the

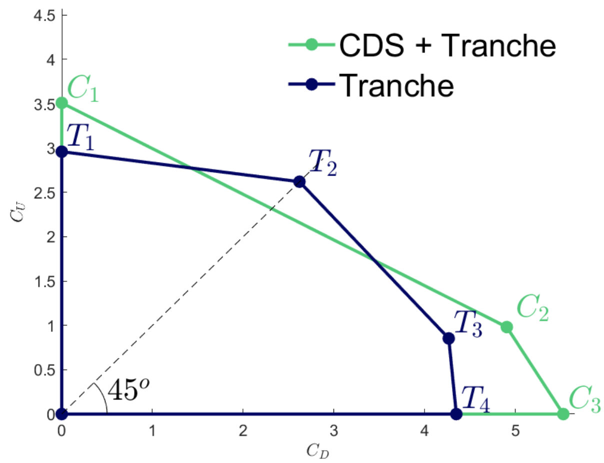


Figure 3.9: Leverage v/s Tranche - Budget Sets

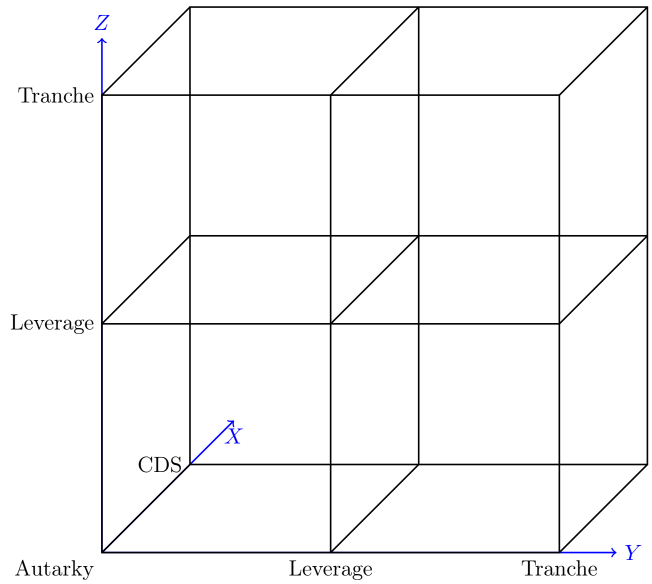
two marginal buyers are $h_1^C = .4210$ and $h_2^C = .2172$. A substitution effect is what primarily explains not only the decrease in price of asset Y but also the negative spillovers to the Z market.

The introduction of CDS creates a supply shift and large substitution effects. First, in the T-economy only Y could be used to create Arrow securities increasing its collateral value and price. In the CDS economy, asset X can do the same through the CDS market, this creates a substitution away from Y. Second, the introduction of CDS adds also more Arrow D that compete with the Z asset. Both effects explain the decrease in all risky asset prices.

Figure 3.9 presents the budget sets for both T and C economies. The financial innovation through CDS affects the budget set faced by agents. We can clearly see that there is a supply shift, increasing the supply of Arrow securities and away from safe assets. All optimist agents above the marginal buyer consume on the left upper corner at the point $C1$ (U consumption), the moderate consume at point $C2$ (buying the Z asset), and the most pessimistic agents consume at point $C3$ (D consumption).

The effect of CDS on asset prices can be seen in the budget sets. The slope of the segment connecting $C1$ and $C2$ is steeper than the segment connecting $T1$ and $T2$, i.e. a more pessimist agent is pricing the Arrow U security, this pushes the price of asset Y down. Moreover, the slope of the segment connecting $C2$ and $C3$ is flatter than the segment connecting $T2$ and $T3$, i.e. a more optimistic agent is pricing the Arrow D security, this pushes the price of asset Z down.

Figure 3.10: Generalizability of Results



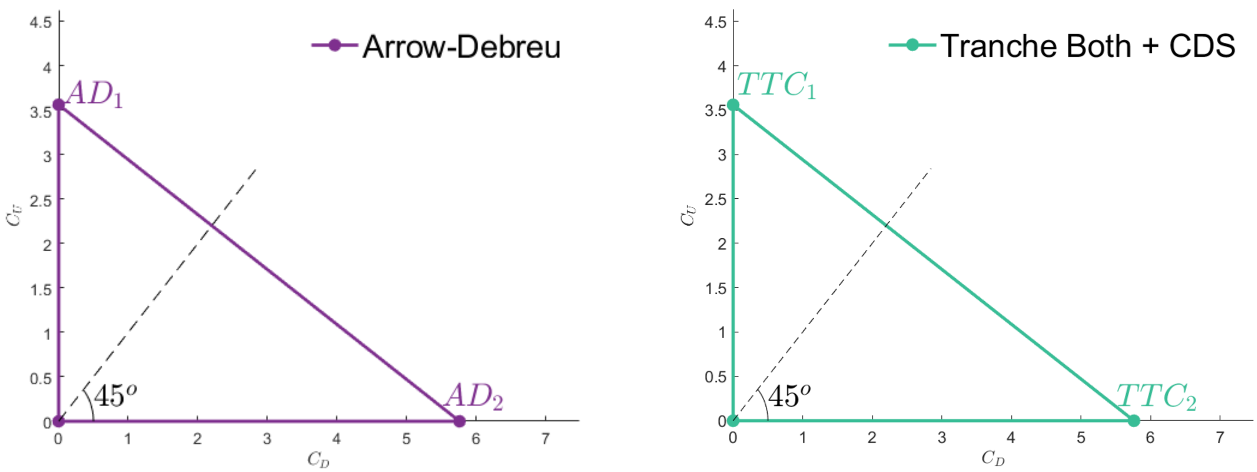
3.7 Financial Innovation, Implementing Arrow-Debreu, and Welfare

Thus far, we have explored the ways collateral-based financial innovation on a single asset can have pricing spillovers on other assets held by different people. In particular, we compute equilibrium asset prices for all assets under financial autarky, leverage on a single risky asset, tranching of the same risky asset, and tranching and CDS on the same risky asset. We consider the changes to asset prices along each of these steps, and show that leveraging an asset increases the price of all assets relative to autarky, tranching an asset increases its price and lowers the price of all other assets relative to leverage, and implementing a CDS on an asset lowers the price of all assets relative to tranching.

The next question, then, addresses the generalizability of these results. In fact, the propositions can be shown, with a little work, to apply so long as the financial innovation we consider lies along one of the *edges* outlined in the following Figure 3.10, i.e., so long as we do not *skip* innovations or apply multiple innovations at the same time. For example, while our Proposition 5 considers moving from leveraging Y to tranching it with no contracts written on Z, the direction of the spillovers remain positive even if Z were leveraged (or tranched). So long as the ex-ante financial structure is held constant, the exact financial contracts available to Z are irrelevant to the spillovers from leverage, tranching, or CDS on Y. In the figure, that would mean moving no more than one node at a time along any dimension. In Section 3.4,

Figure 3.11: Welfare Comparisons across Economies

(a) Arrow-Debreu (b) Tranche Both + CDS



we defined and briefly outlined the solution to the Arrow-Debreu benchmark version of this economy, where the set of Arrow securities $\{a_U, a_D\} \subseteq J$. It is fairly straightforward to show that when financial innovation reaches its greatest extent — both Y and Z to be tranched, and X used in a CDS — then all cash-flows are completely contingent. The financial assets can then be understood as bundles of Arrow Securities, namely:

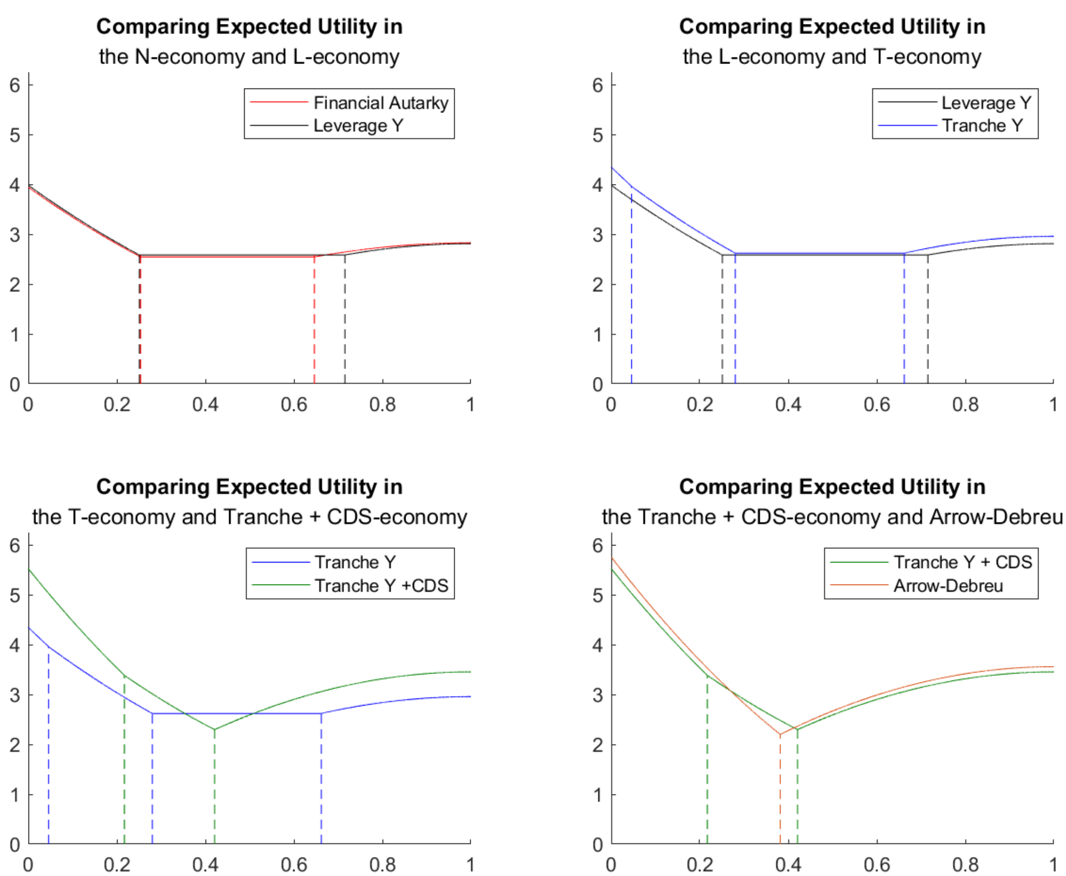
CDS on X:	$X \equiv a_u + a_D$
Tranche Y:	$Y \equiv a_u + 0.2a_D$
Tranche Z:	$Z \equiv 0.2a_u + a_D$

Therefore, the equilibrium allocation of the economy with this financial structure will be equivalent to the Arrow-Debreu equilibrium (conditional on agents not having any endowments of future goods). We illustrate this using the respective budget sets in Figure 3.11.

Thus far, we have focused on how financial integration affects flows, prices, and volatility, but the implications for agents' utility (let alone for an aggregate measure of welfare) are more subtle given the heterogeneity in the model. To evaluate the effects of financial integration on welfare, we calculate utility according to agents' subjective beliefs.

We put together the framework of this paper as a process of going from financial autarky to the above-mentioned case that replicates an Arrow-Debreu equilibrium through the process of financial innovation, going in steps from autarky to leveraging Y, to tranching Y, to the CDS economy, to the Arrow-Debreu equivalent. This allows us to do things; first, this informs our choice of benchmark comparisons for the propositions presented in this paper; second, it allows us to do an agent-by-agent welfare comparison at every step along this process (see Figure 3.12).

Figure 3.12: Welfare Comparisons across Economies



The first financial innovation allowed the leveraging of asset Y. The worst-off agents are still those who purchase the riskless asset, but they benefit from the lower price of bonds. They spend less on consumption in the state they think is less likely. It is interesting to note that agents who purchase Y are uniformally worse off, as a group, than they were in the financial autarky because the price of Y has gone up.

The second financial innovation allowed the tranching of asset Y. Welfare gains were substantial but evenly distributed. The benefits were concentrated for the pessimistic agents who purchase the down tranche (welfare gains of 8%, on average) and the agents who sell them the down tranche (5%). If we accept agents' reported expected utility, the financial innovation of tranching represented a Pareto improvement.

The third financial innovation allowed CDS against tranched asset Y. The changes are presented graphically in the bottom-left panel of Figure 3.12 below. The worst-off agent is the bottom kink in the image (h=.4208). The curvature of the belief function means he thinks the Up state will occur with 2/3 chance; however, he is unwilling to purchase asset Y at its equilibrium price and ends up purchasing asset Z, which does not align well with his beliefs. The measure of agents who are made worse off from the financial innovation is small compared with the agents who gain. Moreover, their expected utility gains are substantial, averaging 17.4%, relative to the population of those made worse-off, who suffer an average loss of 6.1%.

If all cashflows from X, Y, and Z could be made state-contingent, and agents were not endowed with future goods, the equilibrium would replicate the Arrow-Debreu allocations. The change from the CDS-economy to Arrow-Debreu would not be a Pareto improvement since a measure of agents would be worse off in their own estimation. This group of agents were holding asset Z and would be forced to purchase the relatively more-expensive Arrow Down security. That is, they would suffer more from the increase in the price of the asset they wish to purchase than they would from the wealth effect created by higher asset prices induced by the financial innovations on Z.

3.8 Conclusion

This paper proposed a mechanism by which collateral-based financial innovation can produce cross-market pricing spillovers. In a simple incomplete-markets, general equilibrium model with two states, two periods, and three financial assets where households vary in their belief over the terminal period's state, we altered the financial contracts available to households and showed how asset prices changed. The particular innovations we considered were leverage, which had positive price spillovers, and tranching and credit-default swaps, which had negative price spillovers.

The price spillovers operated through traditional income and substitution effects. We were able to demonstrate, intuitively through numerical examples and formally through proofs, how these effects combined to produce the spillovers. Leverage created positive spillovers through wealth effects that pushed up all asset prices. Tranching created negative spillovers through substitution effects, as households who were previously purchasing another financial asset switched to purchase the derivative tranche. Credit default swaps created negative spillovers through both income and substitution effects.

The contribution of the paper is its to make precise predictions regarding how financial innovation influences the asset prices in incomplete markets. That we can provide insights to cross-section asset price movements without the use of fire sales, contagion, or other mechanisms allows the study of price movements where those mechanisms are not operative. We also generalize results from previous theoretical papers ((Fostel and Geanakoplos, 2012)), showing the direct effect of financial innovations holds in a setting with multiple risky assets.

There are other avenues for further work in this research area. The paper's precise predictions motivate a direct test of the theory in a lab setting where the financial structure of the economy can be manipulated. Another idea to expand the theory of collateral and incomplete markets would be to increase the number of terminal states in the model beyond two. This would admit the potential for default and the strength of cross-price effects could be strengthened (or attenuated) when asset prices include a default risk premium. The results would also more easily map into real-world applications of the theory.

Advances in theory also open up new avenues of empirical inquiry and study. A potential application is to look across countries with different financial regimes. For example, one might compare residential real estate prices in Japan and the United States during the the late 19980's/early 1990's in the United States, when collateralized mortgage obligations (CMOs) were introduced in the latter. Might the ability to tranche U.S. mortgages have negatively influenced real-estate prices in Japan, where tranching was not possible until the early 2000's? In the decade proceeding the adoption of CMO in the United States (1971 to 1982), both countries had about the same average price growth in residential real estate: 9.65%

for Japan and 9.76% for the United States. In the years from 1983 to 2002 when only the United States used this financial instrument, the average price growth in Japan was 1.25% while it was 4.93% in the United States.[11] It is an empirical question whether financial innovation may have induced substitution out of Japanese property market and into the U.S, but it is just one example empirical question which is motivated by our results.

These ideas merit more investigation.

[11] Residential Real Estate Prices, OECD. Retrieved April 8, 2024.

Chapter 4

Conclusion

Building upon a rich body of literature, this thesis explores the market implications of different forms of debt, and aims to understand how different enforcement mechanisms and financial structures influence economic outcomes.

The findings underscore the significance of understanding the dual role of collateral and punishment in financial markets. Not only do these mechanisms ensure compliance with financial obligations, but they also have profound implications for asset pricing, financial stability, and economic disparities. The insights into the redistribution effects and asset price adjustments brought about by these financial instruments contribute to our broader understanding of financial economics.

Furthermore, this research opens several avenues for future exploration. The models and hypotheses tested provide a foundation for empirical studies that could investigate the real-world applicability of the theoretical outcomes observed. Additionally, the interaction between market regulations and these financial instruments offers a rich field of study to ascertain the optimal regulatory frameworks that can enhance market stability and efficiency.

In conclusion, the thesis reinforces the importance of understanding how financial instruments shape economic landscapes. It calls for a big-picture view of innovation in financial products and their regulation to understand how financial innovation affects not just the assets and markets it is applied to, but also other assets and markets that may exist in the economy.

Appendix A

Proofs - Chapter 2

A.1 Existence

Proposition 1. Consider the λC-economy $E_{\lambda C}$; then, a refined equilibrium exists.

Proof. **Setup:** We start by assuming that penalties are finite, $\lambda \in \mathbb{R}^I_+$. Fix a perturbation for the unsecured debt market, $(\epsilon_i)_{i \in I} >> 0$. This corresponds to a trembling-hand agent who buys and sells (for a net zero position) an amount ϵ_i of each unsecured debt i, and always fully delivers on his/her promises; this enables a refinement of the equilibrium concept to exclude cases where some unsecured debt pools go untraded simply because of undue pessimism regarding their repayment rates ((Dubey et al., 1995)). Fix a perturbation for the secured debt market, $\rho > 0$; this corresponds to bounding the promises made by all secured debt contracts from below, to exclude the case of some securities having zero deliveries. Fix a small lower bound, $b > 0$, to bound prices. To bound asset positions, fix an upper bound M s.t. more than M of any good is strictly better than twice of all endowments in the economy, i.e., $||(c, y)||_\infty > M \implies u^h(c, y) > u^h\left(2\sum_{h' \in H} e^{h'}_{c0}, 2\sum_{h' \in H} e^{h'}_{y0}\right), \forall h \in H$. Such an M exists w.l.o.g. under the assumptions made regarding utility functions in A1-4.

Price Simplex: Given the above setup, define the price simplex as

$$\Delta_b = \Bigg\{ (p, (\pi_j)_{j\in J}, (\pi_i)_{i\in I}) \in \mathbb{R}_+^{S\times 2} \times \mathbb{R}_+^J \times \mathbb{R}_+^I : p_{sc} + p_{sy} = 1, \forall s \in S; p_{sc}, p_{sy} \geq b, \forall s \in S;$$

$$\pi_i \in \left[0, \frac{1}{b}\right], \forall i \in I; \pi_j \in [0,2], \forall j \in J \Bigg\}. \quad \text{(A.1)}$$

Note that the prices are normalized differently here than in the model described in the main body of the paper (where the normalization used is $p_{sc} = 1, \forall s \in S$), but since equilibria in this model are not independent of the chosen price level normalization, we can continue with the price simplex Δ_b to prove existence, and this existence result will continue to hold when the normalization described in the main text is chosen instead.

Truncated Choice Space: Next, I bound the space of positions of commodities, assets, and deliveries for agent h, $\square^h$, defined as

$$\square^h = \Bigg\{ (c, y, \theta_j, \theta_i, \varphi_j, \varphi_i, D_j, D_i) \in \mathbb{R}_+^S \times \mathbb{R}_+^S \times \mathbb{R}_+^J \times \mathbb{R}_+^I \times \mathbb{R}_+^J \times \mathbb{R}_+^I \times \mathbb{R}_+^{S\times J} \times \mathbb{R}_+^{S\times I} :$$

$$||(c,y)||_\infty \leq M; \varphi_i^h \leq Q_i; \theta_i^h \leq 2 \sum_{h'\in H} Q_i;$$

$$||D||_\infty \leq ||Q||_\infty ||R||_\infty; \varphi_j^h \leq \sum_{h'\in H} e_y^h 0; \theta_j^h \leq \sum_{h'\in H} e_y^h 0 \Bigg\}. \quad \text{(A.2)}$$

Then, the truncated choice space for the entire economy can be defined as the Cartesian product of the indiviudal truncated choice spaces of the agents,

$$\square^H \equiv \prod_{h\in H} \square^H.$$

Space of Potential Equilibria: Then, given $S^* \equiv \{U, D\}$, a potential equilibrium can be denoted as

$$\eta \equiv (p, (\pi_j)_{j\in J}, (\pi_i)_{i\in I}, K, (c^h, y^h, \theta_j^h, \theta_i^h, \varphi_j^h, \varphi_i^h, D_j^h, D_i^h)_{h\in H}) \in \Delta_b \times [0,1]^{S^*\times I} \times \square^H \equiv \Omega_b. \quad \text{(A.3)}$$

Expected Delivery Map: Consider the mapping $\bar{K}_b : \Omega_b \to [0,1]^{S^*\times I}$ that denotes the expected delivery rates of unsecured debt contracts in this economy with the trembling-hand agent,

$$\bar{K}_{bsi} = \begin{cases} \frac{p_s R_i \epsilon_i + \sum_{h\in H} p_s D^h_{si}}{p_s R_i \epsilon_i + \sum_{h\in H} p_s R_i \varphi^h_i}, & R_i \neq 0 \\ 1, & R_i = 0 \end{cases}. \tag{A.4}$$

This map is clearly continuous by construction of the trenbling hand.

Maximizing Value of Aggregate Excess Demand: Consider the correspondence of prices $\psi^0_b : \Omega_b \Rightarrow \Delta_b$ that maximizes the value of aggregate excess demand,

$$\psi^0_b(\eta) = \arg \max_{(p,(\pi_j)_j,(\pi_i)_i)\in\Delta_b} \Bigg\{ p_{0c} \sum_{h\in H} \left(c^h_0 - e^h_0\right) + p_{0y} \sum_{h\in H} \left(y^h_0 - y^h\right) + \sum_j \pi_j \sum_{h\in H} \left(\theta^h_j - \varphi^h_j\right) + \pi_i \sum_{h\in H} \left(\theta^h_i - \varphi^h_i\right) + \sum_{s\in S^*} \left[p_{sc} \sum_{h\in H} \left(c^h_s - e^h_s\right) + p_{sy} \sum_{h\in H} \left(y^h_s - y^h_0\right) - \sum_i (1 - \bar{K}_{bsi}(\eta)) R_i \epsilon_i \right] \Bigg\}. \tag{A.5}$$

Clearly, this correspondence is non-empty and convex-valued, and upper hemi-continuous (u.h.c.).

Optimal Choice Correspondence: Now, for each agent, define by $\psi^h_b : \Omega_b \Rightarrow \square^h$ the correspondence that defines the optimal choice over the truncated budgeted set $B^h \bigcap \square^h$,

$$\psi^0_b(\eta) = \arg \max_{(c,y,\theta_j,\theta_i,\varphi_j,\varphi_i,D_j,D_i)} [w^h(c, y, \theta_j, \theta_i, \varphi_j, \varphi_i, D_j, D_i, p) : (c, y, \theta_j, \theta_i, \varphi_j, \varphi_i, D_j, D_i) \in B^h \bigcap \square^h]. \tag{A.6}$$

Note that this correspondence is non-empty-valued and convex-valued by the continuity and concavity of post-penalty utility. It is straightforward to show that the truncated budget set is continuous, in addition to being compact-valued since $B^h, \square^h$ are both compact-valued. Further note that the post-penalty expected utility function is continuous by assumption. Given the above, we are looking at the argmax of a continuous function over a continuous, compact-valued correspondence, and Berge's Maximum Principle applies, implying that the correspondence ψ^h_b is u.h.c.

Equilibrium Correspondence: Define the equilibium correspondence $\psi_b : \Omega_b \Rightarrow \Omega_b$ as the product of these three correspondences,

$$\psi^b(\eta) = \psi^0_b(\eta) \times \left\{\bar{K}_b(\eta)\right\} \times \prod_{h\in H} \psi^h_b(\eta).$$

Kakutani's FPT: Since $\psi^b(\eta)$ is a u.h.c. correspondence with non-empty, convex values on a convex, compact subset of $\mathbb{R}^n$, by Kakutani's Fixed Point Theorem, this correspondence has a fixed point η^b.

Aggregate Excess Demand: Now, using a standard price player argument, we can show that there cannot be positive aggregate excess demand for any debt contract of either type. We can calculate the negative aggregate excess demand in some unsecured debt pools, and in the commodities, and show that they are functions of the arbitrary bound on prices, b, and go to 0 as $b \to 0$. For small enough b, the bounds are small enough that consumption is bounded by twice of everything in economy; but M of either commodity would be better, if it were feasible. This bounds commodity prices and unsecured debt prices as $b \to 0$.

Convergence: All variables of interest in the equilibrium object are bounded for small b, ρ. We can then take convergent subsequences as $b, \rho \to 0$ and find a limit point $\bar{E}$ which features (by taking limits on results above) non-positive aggregate excess demand, artificial bounds that do not bind, and with $\bar{E}$ being an optimum over the actual budget set. We can conclude that aggregate excess supply is not possible since the price player would be making negative profits. This gives us an equilibrium in the presence of the given trembling-hand agent. Further taking convergent subsequences as we let the influence of this agent disappear ($\epsilon \to 0$), we get a limit point that is a refined equilibrium for the economy. □

A.2 Coexistence

Proposition 2. Consider a λC-economy that satisfies Assumptions 2-3. In such an economy, a coexistence equilibrium exists.

Proof. We have already proved the existence of a refined equilibrium in a more general version of this economy. Furthermore, under assumptions C1 and C2.1, this equilibrium must involve trade in both the goods and financial markets; hence, if we prove that the above assumptions are sufficient conditions for the non-existence of refined equilibria where either only secured debt is being used or only unsecured debt is being used, we will have proved that a refined coexistence equilibrium exists, i.e. $\mathcal{E}(\omega) \neq \phi$.

We will begin by proving that any such equilibrium cannot be one where only secured debt is being used. To do this, we need only show that the secured debt contract that is being used cannot offer $100\% LTV$, which I do in Lemma 1. In the case of a non-financial asset, even though asset deliveries in future states of the world are pinned down exactly by their prices in those states such that the second term in the numerator on the LHS is zero, the first term $U_y^h(c_0^h, y_0^h)$ is necessarily non-zero since agents derive utility from the asset Y. Hence, the denominator, which is the difference between the price of the asset at time 0 and the amount that can be borrowed against it (i.e. the down payment), must be positive, and hence, we cannot reach $100\% LTV$. If this is the case, the poorest agent $h' \in H^B$ s.t. $e_0^{h'} = 0$ is unable to access secured debt, and will choose to use unsecured debt to finance their purchase of the house (or at least the down payment), since that is a better option than being excluded from the housing market. Note that this argument relies on Assumption C2.2.

Finally, we will prove that an equilibrium where everyone uses only unsecured debt cannot be sustained. Consider an economy in an unsecured-debt-only equilibrium. Consider the same agent $h' \in H$ s.t. $e_0^{h'} = 0$, who exists by Assumption C2.2. Let (c_0, y_0) denote the time-0 consumption allocations of this agent in this equilibrium, and let i_0 denote the amount agent h' borrows using unsecured debt in this equilibrium. Now, consider a unilateral deviation for this agent, from these allocations to a position where they also take on secured debt using the contract $j_0 = p_D$. Consider a deviation where the new consumption allocations are denoted by $\hat{c}_0, \hat{y}_0 = y_0$ such that

$$\frac{U_y^{h'}(\hat{c}_0, y_0) + \sum_{s \in S'} \mu_s^{h'} \left(p_s - \delta_s(p_D)\right)}{p_0 - \pi_j} = \frac{U_c^{h'}(\hat{c}_0, y_0)}{1}. \tag{A.1}$$

Then, under the assumption that the bad state is not too bad (Assumption C2.3), we can show that $\hat{c}_0 < c_0$ (this boils down to requiring that $U_c^{h'}(c_0, y_0) > \sum_{s \in S'} \mu_s^{h'}$); the agent reduces their consumption of food, but maintains their consumption of housing at the same level as in the unsecured-only equilibrium. Let $\hat{c}_0 = c_0 - \nu$; then, given that this agent is using secured debt to borrow $p_D y_0$, they can feasibly achieve this allocation by reducing their unsecured debt borrowing to $\hat{i}_0 = i_0 - p_D y_0 - \nu$.

Given the feasibility of this deviation, it remains to show that it is profitable. But this is directly seen by observing that reducing time-0 consumption implies a higher marginal utility of consumption at time

0.

$$\frac{U_y^{h'}(\hat{c}_0, y_0) + \sum_{s \in S'} \mu_s^{h'}\,(p_s - \delta_s(j))}{p_0 - \pi^j} = \frac{U_c^{h'}(\hat{c}_0, y_0)}{1} \tag{A.2}$$

$$> \frac{U_c^{h'}(c_0, y_0)}{1} \tag{A.3}$$

$$= \frac{U_y^{h'}(c_0, y_0) + \sum_{s \in S'} \mu_s^{h'} p_s}{p_0}, \tag{A.4}$$

where the inequality in (8) follows from the fact that $\hat{c}_0 < c_0$. This violates agent optimization in an unsecured-debt-only equilibrium; thus, we have built a feasible and profitable deviation for $h' \in H$, contradicting the existence of an unsecured-debt-only equilibrium.

Since an equilibrium exists in this economy, and the equilibrium can neither be secured-debt-only nor unsecured-debt-only, the equilibrium must feature coexistence. This proves that any equilibrium of this economy must be a coexistence equilibrium. □

A.3 Portfolio Choice

Proposition 3. Consider a λC-economy $E_{\lambda C}$ satisfying Assumptions C1-C2 and P1-P3. Then,

- secured debt, $|\pi_j y_j^h|$, is increasing in h, and
- unsecured debt, $|\pi_i^h|$, is decreasing in h.

Proof. The equilibrium in the λC-economy can be characterized by the following equations.

$$w'(c_D^h) = \underline{\lambda}$$

$$y_D^h = 1$$

$$w'(c_D^h) = \frac{v'(y_D^h)}{p_D} = \frac{v'(1)}{p_D}$$

$$d_D^h + c_D^h + p_D = e_D^h$$

$$\int_0^1 y_U^h dh = 1$$

$$p_U y_U^h + c_U^h + d_U^h = e_U^h$$

$$w'(c_U^h) = \frac{v'(y_U^h)}{p_U}$$

$$\pi_i^h = d_U^h(1-\epsilon) + d_D^h \epsilon$$

$$R_i^h = \frac{d_U^h}{\pi_i^h}$$

$$\int_0^1 y_0^h dh = 1$$

$$(p_0 - \pi_j) y_0^h + c_0^h = f(h) + \pi_i^h$$

$$w'(c_0^h) = \frac{v'(y_0^h)}{p_0 - \pi_j}$$

$$\pi_j = p_U(1-\epsilon) + p_D \epsilon$$

$$w'(c_0^h) = w'(c_U^h) R_i^h (1-\epsilon) + R_i^h \underline{\lambda} \epsilon$$

Suppose, for the sake of contradiction, that π_i^h is increasing in h; since d_D^h is constant across agents, this implies that d_U^h is decreasing in h. Since c_U^h and y_U^h must either both be increasing or decreasing in h, this implies that they must both be decreasing in h. Furthermore, it is easy to show that R_i^h must be increasing in h; combining this with the earlier result about c_U^h, we can show that c_0^h, and hence y_0^h, must be decreasing in h. Then, the time-0 budget constraint would imply that π_i^h is decreasing in h, which is a contradiction.

Suppose instead that π_i^h is not monotonic in h. Then, consider an arbitrary agent h^*; there must exist a pair of agents h', h'' s.t. $h^* \in (h', h'')$ and either $\pi_i^h < \pi_i^{h'} = \pi_i^{h''}$ or $\pi_i^h > \pi_i^{h'} = \pi_i^{h''}$. Assuming monotonicity of the utility function in its arguments, it is fairly straightforward to see that either of these strict inequalities would violate agent optimization.

Then, π_i^h is decreasing in h; since d_D^h is constant across agents, this implies that d_U^h is increasing in h. Since c_U^h and y_U^h must either both be increasing or decreasing in h, this implies that they must both be increasing in h. Furthermore, it is easy to show that R_i^h must be decreasing in h; combining this with the earlier result about c_U^h, we can show that c_0^h, and hence y_0^h, must be increasing in h.

Thus, richer agents borrow less using unsecured debt and more using secured debt. $\square$

A.4 Asset Pricing

Proposition 4. Consider the economies $E_\lambda, E_C, E_{\lambda C}$ satisfying Assumptions C1-C2 and P1-P3. Then, $\exists \lambda_1, \lambda_2 > 0$ s.t., $\forall \underline{\lambda} \in [\lambda_1, \lambda_2]$,

$$p_0^C \leq p_0^\lambda \leq p_0^{\lambda C}.$$

Proof. λ-economy:

$$
\begin{aligned}
w'(c_D^h) &= \underline{\lambda} \\
\int_0^1 y_D^h &= 1 \\
w'(c_D^h) &= \frac{v'(y_D^h)}{p_D} \\
d_D^h + c_D^h + p_D y_D^h &= e_D^h \\
\int_0^1 y_U^h dh &= 1 \\
p_U y_U^h + c_U^h + d_U^h &= e_U + p_U y_0^h \\
w'(c_U^h) &= \frac{v'(y_U^h)}{p_U} \\
\pi_i^h &= d_U^h(1-\epsilon) + d_D^h \epsilon \\
R_i^h &= \frac{d_U^h}{\pi_i^h} \\
\int_0^1 y_0^h dh &= 1 \\
p_0 y_0^h + c_0^h &= 8h + \pi_i^h \\
w'(c_0^h) &= \frac{v'(y_0^h)}{p_0} + \frac{(1-\epsilon)p_U w'(c_U^h) + \epsilon p_D w'(c_D^h)}{p_0} \\
w'(c_0^h) &= w'(c_U^h) R_i^h (1-\epsilon) + R_i^h \underline{\lambda} \epsilon
\end{aligned}
$$

C-economy:

$$y_D^h = 1$$
$$w'(c_D^h) = \frac{v'(y_D^h)}{p_D}$$
$$c_D^h + p_D = e_D^h$$
$$\int_0^1 y_U^h dh = 1$$
$$p_U y_U^h + c_U^h = e_U$$
$$w'(c_U^h) = \frac{v'(y_U^h)}{p_U}$$
$$\int_0^1 y_0^h dh = 1$$
$$(p_0 - \pi_j) y_0^h + c_0^h = 8h$$
$$w'(c_0^h) = \frac{v'(y_0^h)}{p_0 - \pi_j}$$
$$\pi_j = p_U(1-\epsilon) + p_D\epsilon$$

Compare the prices:

$$\begin{aligned}
\lambda C: \quad & p_0 = & \frac{v'(y_U^h)}{w'(c_U^h)}(1-\epsilon) + \frac{v'(y_D^h)}{w'(c_D^h)}\epsilon \quad & + \frac{v'(y_0^h)}{w'(c_0^h)} \\
\lambda: \quad & p_0 = & \frac{v'(y_U^h)}{w'(c_0^h)}(1-\epsilon) + \frac{v'(y_D^h)}{w'(c_0^h)}\epsilon \quad & + \frac{v'(y_0^h)}{w'(c_0^h)} \\
C: \quad & p_0 = & \frac{v'(y_U^h)}{w'(c_U^h)}(1-\epsilon) + \frac{v'(y_D^h)}{w'(c_D^h)}\epsilon \quad & + \frac{v'(y_0^h)}{w'(c_0^h)}
\end{aligned}$$

Clearly, compared to the λC equilibrium, the C equilibrium has the same y_D and a lower c_D, depending on the set of available λ, which translates to a lower p_D. Note that this arises from the fact that the same time-0 endowment must now also cover the partial repayment of unsecured debt, while still ensuring that the borrowers still consume all the asset y in the economy.

For the same reason, every agent must be consuming less food in state U in the λC equilibrium as compared to the C equilibrium (lower c_U). It can then be argued that this leads to a lower p_U, and combined with a lower p_D, to a reduced leverage (i.e., lower π_j).

Considering the time-0 budget constraint, the question, then, is whether this reduced leverage or the increased (or newly available) unsecured borrowing has a greater effect on the tightness of the constraint. However, comparing the expressions for the two types of borrowing, we can argue that this comparison comes down to comparing the value of the pledged collateral ($p_s y_0$) and the rationally expected repayment of the unsecured promise (d_s). Furthermore, by comparing the state-s budget constraints, we see that the introduction of d_s is only partially offset by the change in prices; the rest of the change is absorbed by the change in state-s consumption (c_s). This implies that the introduction of the unsecured debt pools had a greater effect, relaxing the time-0 budget constraint, thereby increasing the value of time-0 consumption, and in turn, pushing up p_0.

Now, let us compare the λC equilibrium to the λ equilibrium. First, it can be shown that the allocations in states U and D are the same in both equilibria. This further implies that unsecured borrowing (π_i) is much greater in the λ equilibrium (as one might intuitively expect), and hence that unsecured interest rates are greater, and hence, time-0 consumption of food (c_0) lower in it than in the λC equilibrium. Integrating time-0 asset holdings (y_0^h) out of both time-0 budget constraints and comparing, we can argue that the reduction in unsecured borrowing and increased time-0 consumption of food in the λC equilibrium only partially offsets the increased liquidity available through leverage; thus, the remaining liquidity is absorbed by an increase in the time-0 asset price, p_0. $\square$

Appendix B

Proofs - Chapter 3

B.1 Leverage

Proposition 5. Consider the C-model; suppose w.l.o.g. that Y can be leveraged, but Z and X cannot. Then (1) the price of the leveraged asset Y is greater in the leverage economy than in the no-leverage economy, $p_Y^N < p_Y^L$, and (2) the price of the non-leveraged asset is weakly greater in the leveraged economy than in the no-leverage economy, $p_Z^N \leq p_Z^L$.

Proof. In our economy, $d_U^Y = d_D^Z > d_D^Y = d_U^Z$. For reference, the equilibrium equations are:

$$p_Y^N = (1 - h_1^N)(1 + p_Z^N + p_Y^N) \tag{B.1}$$

$$p_Z^N = h_2^N(1 + p_Z^N + p_Y^N) \tag{B.2}$$

$$p_Y^N = d_U^Y \gamma^{h_1^N} + d_D^Y(1 - \gamma^{h_1^N}) \tag{B.3}$$

$$p_Z^N = d_U^Y \gamma^{h_2^N} + d_D^Y(1 - \gamma^{h_2}) \tag{B.4}$$

$$p_Y^L = (1 - h_1^L)(1 + p_Z^L + p_Y^L) + d_D^Y \tag{B.5}$$

$$p_Z^L = h_2^L(1 + p_Z^L + p_Y^L) \tag{B.6}$$

$$p_Y^L = d_U^Y \gamma^{h_1^L} + d_D^Y(1 - \gamma^{h_1^L}) \tag{B.7}$$

$$p_Z^L = d_U^Z \gamma^{h_2^L} + d_D^Z(1 - \gamma^{h_2^L}) \tag{B.8}$$

The proof proceeds by cases over the inequalities. First, for the sake of contradiction assume that (i)

$p_Y^N \geq p_Y^L$ and (ii) $p_Z^N > p_Z^L$. By (i), equations 3 and 7 imply:

$$p_Y^L = d_U^Y \gamma^{h_1^L} + d_D^Y(1 - \gamma^{h_1^L}) \leq d_U^Y \gamma^{h_1^N} + d_D^Y(1 - \gamma^{h_1^N}) = p_Y^N.$$

By the monotonicity and continuity of the belief function γ^h and $d_U^Y > d_D^Y$, this means that $h_1^L \leq h_1^N$.

Similarly, by (ii), equations 4 and 8 are such that:

$$p_Z^L = d_U^Z \gamma^{h_2^L} + d_D^Z(1 - \gamma^{h_2^L}) < d_U^Z \gamma^{h_2^N} + d_D^Z(1 - \gamma^{h_2^N}) = p_Z^N.$$

By the monotonicity and continuity of the belief function γ^h and $d_D^Z > d_U^Z$, this equation implies $h_2^L > h_2^N$.

Then, manipulating equations 1 and 2:

$$1 + \frac{1}{P_Z^N} = \frac{h_1^N}{h_2^N} \qquad\qquad 1 + \frac{1}{P_Z^L} + \frac{d_D^Y}{P_Z^L} = \frac{h_1^L}{h_2^L}.$$

Now, the LHS of the first equation is less than the LHS of the second equation by assumption (i) alone. Therefore,

$$\frac{h_1^N}{h_2^N} < \frac{h_1^L}{h_2^L}.$$

But this contradicts the intermediate results $h_1^L \leq h_1^N$ and $h_2^L > h_2^N$.

Second, for the sake of contradiction assume instead that (i) $p_Y^N < p_Y^L$ and (ii) $p_Z^N > p_Z^L$. By similar arguments as above, equations 3 and 7 imply that $h_1^L > h_1^N$, and equations 4 and 8 imply that $h_2^L > h_2^N$. The contradiction arises from the market clearing condition over asset Z. Solving equation 6 for P_Z^L yields

$$p_Z^L = \frac{h_2^L}{1 - h_2^L}(1 + p_Y^L).$$

Similarly, equation 2 solved for P_Z^N is

$$p_Z^N = \frac{h_2^N}{1 - h_2^N}(1 + p_Y^N).$$

The base assumption (ii) requires that

$$p_Z^L = \frac{h_2^L}{1-h_2^L}(1+p_Y^L) < \frac{h_2^N}{1-h_2^N}(1+p_Y^N) = p_Z^N.$$

Comparing terms, $(1+p_Y^L) > (1+p_Y^N)$ is true by assumption while the intermediate conclusion $h_2^L > h_2^N$ implies that $\frac{h_2^L}{1-h_2^L} > \frac{h_2^N}{1-h_2^N}$. Therefore, the left-hand side of the equation is greater than the right-hand, producing the contradiction. Economically, the market for Z does not clear.

Third and finally, for the sake of contradiction assume instead that (i) $p_Y^L \leq p_Y^N$ and (ii) $p_Z^N \leq p_Z^L$. By similar arguments as above, equations 3 and 7 imply that $h_1^L \leq h_1^N$, and equations 4 and 8 imply that $h_2^L \leq h_2^N$. The contradiction arises from the market clearing conditions over asset Y. Solving equation 5 for P_Y^L yields

$$P_Y^L = \frac{1-h_1^L}{h_1^L}(1+p_Z^L) + \frac{d_D^Y}{h_1^L}.$$

Similarly, equation 1 solved for P_Y^N is

$$P_Y^N = \frac{1-h_1^N}{h_1^N}(1+p_Z^N).$$

The base assumption (i) requires that

$$P_Y^L = \frac{1-h_1^L}{h_1^L}(1+p_Z^L) + \frac{d_D^Y}{h_1^L} \leq \frac{1-h_1^N}{h_1^N}(1+p_Z^N) = P_Y^N.$$

Comparing terms, the intermediate finding $h_1^L \leq h_1^N$ implies that $\frac{1-h_1^L}{h_1^L} \geq \frac{1-h_1^N}{h_1^N}$ while $(1+p_Z^L) \geq (1+p_Z^N)$ is true by assumption. Because the payment d_D^Y/h_1^L is strictly positive, the left-hand side of the inequality is in fact strictly greater than the right-hand, producing the contradiction. Economically, the market Y does not clear. □

B.2 Tranche

Proposition 6. Consider the C-model; suppose w.l.o.g. that Y can be tranched, but Z and X cannot. Then (1) the price of the tranched asset Y is greater in the tranche economy than in the leverage economy,

$p_Y^L < p_Y^T$, and (2) the price of the non-tranched asset is less in the tranche economy than in the leverage economy, $p_Z^L > p_Z^T$.

Proof. In our economy, $d_U^Y = d_D^Z > d_D^Y = d_U^Z$. For reference, the equilibrium equations are as follows:

$$p_Y^L = (1 - h_1^L)(1 + p_Z^L + p_Y^L) + d_D^Y \tag{B.1}$$

$$p_Z^L = h_2^L(1 + p_Z^L + p_Y^L) \tag{B.2}$$

$$p_Y^L = d_U^Y \gamma^{h_1^L} + d_D^Y(1 - \gamma^{h_1^L}) \tag{B.3}$$

$$p_Z^L = d_U^Z \gamma^{h_2^L} + d_D^Z(1 - \gamma^{h_2^L}) \tag{B.4}$$

$$p_Y^T = (1 - h_1^T)(1 + p_Y^T + p_Z^T) + \pi_D^T \tag{B.5}$$

$$p_Z^T = (h_2^T - h_3^T)(1 + p_Y^T + p_Z^T) \tag{B.6}$$

$$\pi_D^T = h_3^T(1 + p_Y^T + p_Z^T) \tag{B.7}$$

$$p_Y^T - \pi_D^T = d_U^Y \gamma^{h_1^T} \tag{B.8}$$

$$p_Z^T = d_U^Z \gamma^{h_2^T} + d_D^Z(1 - \gamma^{h_2^T}) \tag{B.9}$$

$$\frac{d_D^Y(1 - \gamma^{h_3^T})}{\pi_D^T} = \frac{d_U^Z \gamma^{h_3^T} + d_D^Z(1 - \gamma^{h_3^T})}{p_Z^T} \tag{B.10}$$

Suppose for the sake of contradiction that $p_Z^L \leq p_Z^T$. Using equations (20) and (25) produces the following equation:

$$\begin{aligned}
p_Z^T - p_Z^L &= d_U^Z \gamma^{h_2^T} + d_D^Z(1 - \gamma^{h_2^T}) - d_U^Z \gamma^{h_2^L} - d_D^Z(1 - \gamma^{h_2^L}) \\
&= d_D^Z\big((1 - \gamma^{h_2^T}) - (1 - \gamma^{h_2^L})\big) + d_U^Z(\gamma^{h_2^T} - \gamma^{h_2^L}) \\
&= (d_D^Z - d_U^Z)(\gamma^{h_2^L} - \gamma^{h_2^T})
\end{aligned}$$

For the LHS to be positive, given the fact that $d_D^Z > d_U^Z$, $h_2^T \leq h_2^L$.

Substituting equation (23) into (22) and then subtracting equation (18) from that yields

$$
\begin{aligned}
& p_Z^T - p_Z^L = h_2^T(1 + p_Z^T + p_Y^T) - h_2^L(1 + p_Z^L + p_Y^L) - \pi_D^T, \\
\iff\ & p_Z^T - p_Z^L = h_2^T(1 + p_Z^T + p_Y^T) - h_2^L(1 + p_Z^L - p_Z^T + p_Z^T + p_Y^L - p_Y^T + p_Y^T) - \pi_D^T \\
\iff\ & p_Z^T - p_Z^L = (h_2^T - h_2^L)(1 + p_Z^T + p_Y^T) - h_2^L((p_Z^L - p_Z^T) + (p_Y^L - p_Y^T)) - \pi_D^T, \\
\iff\ & (1 - h_2^L)(p_Z^T - p_Z^L) = (h_2^T - h_2^L)(1 + p_Z^T + p_Y^T) + h_2^L(p_Y^T - p_Y^L) - \pi_D^T.
\end{aligned}
$$

Based on the finding $h_2^T < h_2^L$, the first term on the RHS is negative while $(-\pi_D^T)$ is also negative. Therefore, if the term $h_2^L(p_Y^T - p_Y^L)$ is also weakly negative, then it means that the entire RHS is negative, which would create a contradiction (the LHS is positive). Therefore, $p_Z^L \leq p_Z^T$ implies $p_Y^L < p_Y^T$, and the wealth of all agents has strictly increased, i.e.$(1 + p_Y^T + p_Z^T) > (1 + p_Z^L + p_Y^L)$. Since the number of bonds has decreased and the wealth of all agents has increased (while bond prices are fixed at 1), it must be the case that the measure of agents purchasing bonds has decreased. Mathematically,

$$
\begin{aligned}
& (1 + d_D^Y) - 1 = (h_1^L - h_2^L)(1 + p_Z^L + p_Y^L) - (h_1^T - h_2^T)(1 + p_Y^T + p_Z^T) > 0 \\
\iff\ & (h_1^L - h_2^L) > (h_1^T - h_2^T), \\
& h_2^T \leq h_2^L \implies h_1^L > h_1^T.
\end{aligned}
$$

Returning to the market clearing conditions, equations (17) and (21) can be expressed as $1 + p_Y^L = (1 - h_2^L)(1 + p_Z^L + p_Y^L)$ and $1 + p_Y^T - \pi_D^T = (1 - h_2^T)(1 + p_Y^T + p_Z^T)$. Taking the difference yields the following quantity

$$
p_Y^T - \pi_D^T - p_Y^L = (1 - h_2^T)(1 + p_Y^T + p_Z^T) - (1 - h_2^L)(1 + p_Z^L + p_Y^L).
$$

The finding $h_2^T < h_2^L$ implies $(1 - h_2^L) < (1 - h_2^T)$. Therefore, given wealth has increased, i.e. $(1 + p_Y^T + p_Z^T) > (1 + p_Z^L + p_Y^L)$, the RHS of the quantity is positive. However, by equations (19) and (24), the LHS of the quantity is $p_Y^T - \pi_D^T - p_Y^L = d_U^Y(\gamma^{h_1^T} - \gamma^{h_1^L}) - d_D^Y(1 - \gamma^{h_1^L})$. This is negative because $h_1^L > h_1^T$ implies $\gamma^{h_1^L} > \gamma^{h_1^T}$, producing the contradiction. Hence, $p_Z^T < p_Z^L$.

Suppose instead that $p_Y^T \leq p_Y^L$. By the previous result $p_Z^T < p_Z^L$, equations (4) and (9) imply $h_2^T > h_2^L$. By agent optimization $\pi_D^T > d_D^Y(1 - \gamma^{h_1^T})$, otherwise agents purchasing Y would not sell the down tranch.

Therefore, $p_Y^T > d_U^Y \gamma^{h_1^T} + d_D^Y(1-\gamma^{h_1^T})$. For the maintained assumption to hold

$$\begin{aligned} 0 < p_Y^L - p_Y^T <& d_U^Y \gamma^{h_1^L} + d_D^Y(1-\gamma^{h_1^L}) - d_U^Y \gamma^{h_1^T} + d_D^Y(1-\gamma^{h_1^T}), \\ =& (d_U^Y - d_D^Y)(\gamma^{h_1^L} - \gamma^{h_1^T}), \end{aligned}$$

which implies $h_1^L > h_1^T$.

Equation (8) can be expressed as $p_Y^T = d_U^Y \gamma^{h_1^T} + \pi_D^T$. Define $\Omega = \frac{p_Z^T}{d_U^Z \gamma^{h_3^T} + d_D^Z(1-\gamma^{h_3^T})}$. By the fact that $d_D^Z > d_U^Z$ and $h_3^T < h_2^T$, therefore $\Omega < 1$. Substituing Ω into equation (10), and then substituting equation (10) into equation (8) yields the pricing equation for Y in the T-economy,

$$p_Y^T = d_U^Y \gamma^{h_1^T} + d_D^Y(1-\gamma^{h_3^T})\Omega.$$

Taking the difference between p_Y^T and p_Y^L using the above formulation yields

$$p_Y^L - p_Y^T = (\gamma^{h_1^L} - \gamma^{h_1^T})d_U^Y - [(1-\gamma^{h_3^T})\Omega - (1-\gamma^{h_1^T}) + (1-\gamma^{h_1^T}) - (1-\gamma^{h_1^L})]d_D^Y.$$

By the above argument, we know that $\gamma^{h_1^T} < \gamma^{h_1^L}$. Therefore, to get our desired contradiction, it would suffice to prove $(\gamma^{h_1^L} - \gamma^{h_1^T})d_U^Y < [(1-\gamma^{h_3^T})\Omega - (1-\gamma^{h_1^T})]d_D^Y$. Then, the sufficient conditions is equivalent to the following sequence of inequalities:

$$(\gamma^{h_1^L} - \gamma^{h_1^T})d_U^Y \overbrace{<}^{(1)} \gamma'^{h_1^T}(h_1^L - h_1^T)d_U^Y \overbrace{<}^{(2)} \gamma'^{h_1^T}(h_1^T - h_2^T)d_D^Y \overbrace{<}^{(3)} [(1-\gamma^{h_2^T}) - (1-\gamma^{h_1^T})]d_D^Y \overbrace{<}^{(4)} [(1-\gamma^{h_3^T})\Omega - (1-\gamma^{h_1^T})]d_D^Y.$$

We proceed by arguing each inequality in turn. The first inequality in the sequence holds by the concavity of γ. By the definition of a concave function

$$(\gamma^{h_1^L} - \gamma^{h_1^T}) < \gamma'^{h_1^T}(h_1^L - h_1^T).$$

The second inequality is constructed using equations (17) and (21). The difference $(h_1^L - h_1^T)d_U^Y$ is

$$\left(\frac{1+p_Z^L+d_D^Y}{1+p_Y^L+p_Z^L} - \frac{1+p_Z^T+\pi_D^T}{1+p_Y^T+p_Z^T}\right)d_U^Y.$$

Using the equations (17), (22), and (23), the second difference $(h_1^T - h_2^T)d_D^Y$

$$\left(\frac{1+p_Z^T+\pi_D^T}{1+p_Y^T+p_Z^T} - \frac{p_Z^T+\pi_D^T}{1+p_Y^T+p_Z^T}\right)d_D^Y$$

Substituting these terms into the desired inequality yields

$$\left(\frac{1+p_Z^L+d_D^Y}{1+p_Y^L+p_Z^L} - \frac{1+p_Z^T+\pi_D^T}{1+p_Y^T+p_Z^T}\right)d_U^Y < \left(\frac{1+p_Z^T+\pi_D^T}{1+p_Y^T+p_Z^T} - \frac{p_Z^T+\pi_D^T}{1+p_Y^T+p_Z^T}\right)d_D^Y.$$

Moving terms to either side shows

$$\underbrace{\frac{(1+d_D^Y)d_U^Y}{1+p_Y^L+p_Z^L}}_{a} + \underbrace{\frac{p_Z^L d_U^Y}{1+p_Y^L+p_Z^L} - \frac{(p_Z^T+\pi_D^T)d_D^Y}{1+p_Y^T+p_Z^T}}_{b} < \underbrace{\frac{d_U^Y+d_D^Y}{1+p_Y^T+p_Z^T}}_{c} + \underbrace{\left(\frac{p_Z^T+\pi_D^T}{1+p_Y^T+p_Z^T}\right)(d_U^Y-d_D^Y)}_{d}.$$

First, compare terms (a) and (c). By the maintained assumption, wealth has decreased, i.e. $1+p_Y^L+p_Z^L > 1+p_Y^T+p_Z^T$, and the denominator of (c) is smaller than (a). The numerator of (c) is greater than the numerator of (a), which can be seen by expanding terms, $(1+d_D^Y)d_U^Y < (d_U^Y+d_D^Y) \iff d_U^Y < 1$. Therefore, (c)>(a). Next, compare terms (b) and (d). Because $h_2^L < h_2^T$ it follows from equations (2), (6), and (7) that

$$\frac{p_Z^L d_U^Y}{1+p_Y^L+p_Z^L} - \frac{(p_Z^T+\pi_D^T)d_D^Y}{1+p_Y^T+p_Z^T} < \frac{p_Z^L}{1+p_Y^L+p_Z^L}(d_U^Y-d_D^Y).$$

Of course, by the same logic, $h_2^L(d_U^Y-d_D^Y) < h_2^T(d_U^Y-d_D^Y)$, and (b)<(d). Therefore, we conclude $\gamma'^{h_1^T}(h_1^L-h_2^T)d_U^Y < \gamma'^{h_1^T}(h_1^T-h_2^T)d_D^Y$. The third inequality holds by concavity. It follows from $(\gamma^{h_2^T}-\gamma^{h_1^T}) < \gamma'^{h_1^T}(h_2^T-h_1^T)$ that $\gamma'^{h_1^T}(h_1^T-h_2^T) < (1-\gamma^{h_2^T})-(1-\gamma^{h_1^T})$. Finally, the fourth inequality holds because

$(1-\gamma^{h_3^T})\Omega > (1-\gamma^{h_2^T})$. Substituting out the definition of Ω will show the result:

$$
\begin{aligned}
1-\gamma^{h_2^T} &< (1-\gamma^{h_3^T})\Omega, \\
\iff (1-\gamma^{h_2^T})\big(d_U^Z\gamma^{h_3^T} + d_D^Z(1-\gamma^{h_3^T})\big) &< (1-\gamma^{h_3^T})\big(d_U^Z\gamma^{h_2^T} + d_D^Z(1-\gamma^{h_2^T})\big), \\
= d_U^Z\frac{\gamma^{h_3^T}}{1-\gamma^{h_3^T}} + d_D^Z &< d_U^Z\frac{\gamma^{h_2^T}}{1-\gamma^{h_2^T}} + d_D^Z, \\
= \frac{\gamma^{h_3^T}}{1-\gamma^{h_3^T}} &< \frac{\gamma^{h_2^T}}{1-\gamma^{h_2^T}}, \\
\iff \gamma^{h_3^T} &< \gamma^{h_2^T},
\end{aligned}
$$

which is true by assumptions over the regime. We have demonstrated the sequence of inequalities and produced a contradiction. We conclude $p_Y^T > p_Y^L$. □

B.3 CDS

Proposition 7. Consider the C-model; suppose w.l.o.g. that Y can be tranched, but Z cannot. Further assume that X can be used to write a CDS on Y. Then (1) the price of the tranched asset Y is greater in the tranche economy than in the CDS economy, $p_Y^C < p_Y^T$, and (2) the price of the non-tranched asset is greater in the tranche economy than in the CDS economy, $p_Z^C < p_Z^T$.

Proof. In our economy, $d_U^Y = d_D^Z > d_D^Y = d_U^Z$. For reference, the equilibrium equations are

$$p_Y^T = (1 - h_1^T)(1 + p_Y^T + p_Z^T) + \pi^T \tag{B.1}$$

$$1 = (h_1^T - h_2^T)(1 + p_Y^T + p_Z^T) \tag{B.2}$$

$$p_Z^T = (h_2^T - h_3^T)(1 + p_Y^T + p_Z^T) \tag{B.3}$$

$$\pi^T = h_3^T(1 + p_Y^T + p_Z^T) \tag{B.4}$$

$$\frac{d_U^Y}{p_Y^T - \pi^T}\gamma^{h_1^T} = 1 \tag{B.5}$$

$$1 = \frac{d_U^Z \gamma^{h_2^T} + d_D^Z(1 - \gamma^{h_2^T})}{p_Z^T} \tag{B.6}$$

$$\frac{d_D^Y}{\pi^T}(1 - \gamma^{h_3^T}) = \frac{d_U^Z \gamma^{h_3^T} + d_D^Z(1 - \gamma^{h_3^T})}{p_Z^T} \tag{B.7}$$

$$1 + p_Y^C = (1 - h_1^C)(1 + p_Y^C + p_Z^C) + \varphi^C + \pi_D^C \tag{B.8}$$

$$p_Z^C = (h_1^C - h_2^C)(1 + p_Y^C + p_Z^C) \tag{B.9}$$

$$\varphi^C + \pi_D^C = h_2^C(1 + p_Y^C + p_Z^C) \tag{B.10}$$

$$\frac{d_U^Y}{p_Y^C - \pi_D^C}\gamma^{h_1^C} = \frac{d_U^Z \gamma^{h_1^C} + d_D^Z(1 - \gamma^{h_1^C})}{p_Z^C} \tag{B.11}$$

$$\frac{d_D^Y}{\pi_D^C}(1 - \gamma^{h_2^C}) = \frac{d_U^Z \gamma^{h_2^C} + d_D^Z(1 - \gamma^{h_2^C})}{p_Z^C} \tag{B.12}$$

$$\frac{d_D^Y}{\pi_D^C} = \frac{1}{\varphi^C} \tag{B.13}$$

$$\frac{d_U^Y}{p_Y^C - \pi_D^C} = \frac{1}{1 - \varphi^C}(a) \tag{B.14}$$

$$\tag{B.15}$$

Suppose for the sake of contradiction that $p_Z^C \geq p_Z^T$. Given that agents between (h_2^C, h_1^C) are purchasing Z in the CDS-economy and $d_D^Z > d_U^Z$, the worst-off agent holding Z is the most optimistic agent h_1^C. Because the bond is always available to purchase, yet no agent chooses to do so, that agent's portfolio returns must be at least 1, i.e.

$$1 \leq \frac{d_U^Z \gamma^{h_1^C} + d_D^Z(1 - \gamma^{h_1^C})}{p_Z^C}.$$

By the indifference equation (6) for h_2^T,

$$
\begin{aligned}
1 = \frac{d_U^Z \gamma^{h_2^T} + d_D^Z(1-\gamma^{h_2^T})}{p_Z^T} &\leq \frac{d_U^Z \gamma^{h_1^C} + d_D^Z(1-\gamma^{h_1^C})}{p_Z^C}, \\
\iff (1-\gamma^{h_2^T}) &\leq (1-\gamma^{h_1^C}), \\
\iff h_1^C &\leq h_2^T,
\end{aligned}
$$

where the second equality holds by $d_D^Z > d_U^Z$ and the maintained assumption and the third equality holds by monotonicity of beliefs. The immediate implication is that $h_1^C < h_1^T$.

Comparing the indifference equation (5) for h_1^T to indifference equation (11) for h_1^C shows

$$
\begin{aligned}
\frac{d_U^Y}{p_Y^T - \pi^T}\gamma^{h_1^T} = 1 \leq \frac{d_U^Z \gamma^{h_1^C} + d_D^Z(1-\gamma^{h_1^C})}{p_Z^C} &= \frac{d_U^Y}{p_Y^C - \pi^C}\gamma^{h_1^C}, \\
\iff \frac{d_U^Y}{p_Y^T - \pi^T}\gamma^{h_1^T} &\leq \frac{d_U^Y}{p_Y^C - \pi^C}\gamma^{h_1^C}, \gamma^{h_1^T} > \gamma^{h_1^C} \\
\implies p_Y^C - \pi^C &< p_Y^T - \pi^T.
\end{aligned}
$$

The individual pricing up-state consumption in the CDS-economy is less optimistic about the deliveries than the agent pricing that consumption in the T-economy; therefore, the agent must be paying less (on net) to purchase asset Y.

Next, solving the indifference equations (7) and (12) in terms of the ratio $\frac{p_Z}{\pi}$ will yield:

$$
\frac{p_Z^T}{\pi^T} = [\frac{d_U^Z}{d_D^Y}\frac{\gamma^{h_3^T}}{1-\gamma^{h_3^T}} + \frac{d_D^Z}{d_D^Y}], \quad \text{and} \quad \frac{p_Z^C}{\pi^C} = [\frac{d_U^Z}{d_D^Y}\frac{\gamma^{h_2^C}}{1-\gamma^{h_2^C}} + \frac{d_D^Z}{d_D^Y}].
$$

Whether the ratio increased or decreased, wealth must have increased, i.e. $(1 + p_Y^C + p_z^C) > (1 + p_Y^T + p_z^T)$.

Suppose the ratio has increased, i.e. $\frac{p_Z^C}{\pi^C} > \frac{p_Z^T}{\pi^T}$, then it must be that $h_3^T < h_2^C$ by monotonicity. Given the previous finding $h_1^C \leq h_2^T$, it follows $h_1^C + h_3^T < h_2^T + h_2^C$, implying $h_1^C - h_2^C < h_2^T - h_3^T$. Substituting equations (3) and (9) for the relevant quantities shows

$$h_1^C - h_2^C < h_2^T - h_3^T,$$
$$\iff \frac{p_Z^C}{(1 + p_Y^C + p_Z^C)} < \frac{p_Z^T}{(1 + p_Y^T + p_Z^T)}.$$

Wealth must increase, i.e. $(1 + p_Y^C + p_z^C) > (1 + p_Y^T + p_z^T)$, for the strict inequality to hold.

On the other hand, suppose the ratio has decreased, i.e. $\frac{p_Z^C}{\pi^C} \leq \frac{p_Z^T}{\pi^T}$. Then it must be that $h_2^C \leq h_3^T$ and the price of the tranche has weakly increased $\pi^C \geq \pi^T$. (That is the only way for the overall ratio to have decreased.) The market clearing conditions for the down tranche are equations (4) and (10), taking their ratio yields

$$\frac{\pi^T}{\varphi^C + \pi^C} = \frac{h_3^T(1 + p_Y^T + p_Z^T)}{h_2^C(1 + p_Y^C + p_Z^C)}, \text{and by equation (13)}$$
$$\varphi^C = \frac{\pi^C}{d_D^Y} \implies \frac{\pi^T}{\pi^C} = \frac{h_3^T}{h_2^C}\frac{(1 + p_Y^T + p_Z^T)}{(1 + p_Y^C + p_Z^C)}(1 + \frac{1}{d_D^Y}).$$

Because the fraction on the LHS is less than 1, it must be that the fraction on the RHS is less than 1. Given the finding $h_2^C < h_3^T$, it means that wealth has increased, i.e. $(1 + p_Y^C + p_z^C) > (1 + p_Y^T + p_z^T)$, for the equality to hold.

We proceed using the finding wealth has increased.Returning to the market clearing equations for Y, adding equations (1) and (2) yields $1 + p_Y^T - \pi^T = (1 - h_2^T)(1 + p_Y^T + p_Z^T)$. Dividing this equation by the market clearing equation (8) yields the ratio

$$\frac{1 + p_Y^T - \pi^T}{1 - \varphi^C + p_Y^C - \pi^C} = \frac{(1 - h_2^T)}{(1 - h_1^C)}\frac{(1 + p_Y^T + p_Z^T)}{(1 + p_Y^C + p_Z^C)}.$$

We have previously found that $p_Y^C - \pi^C < p_Y^T - \pi^T$, and clearly, $1 - \varphi^C < 1$. Therefore, $1 - \varphi^C + p_Y^C - \pi^C < 1 + p_Y^T - \pi^T$, and the LHS of the ratio is greater than 1. This produces a contradiction. The RHS is less than 1. We have found that wealth has increased, so $\frac{(1+p_Y^T+p_Z^T)}{(1+p_Y^C+p_Z^C)} < 1$, and $h_1^C < h_2^T$ implies $\frac{(1-h_2^T)}{(1-h_1^C)} < 1$. Hence, $p_Z^C < p_Z^T$.

Suppose for the sake of contradiction that $p_Y^T \leq p_Y^C$. We have already established that $p_Z^C < p_Z^T$.

Solving the market clearing equations (3) and (9) in terms of p_Z and taking the ratio yields

$$\frac{p_Z^C}{p_Z^T} = \frac{\frac{(h_1^C - h_2^C)}{1-(h_1^C - h_2^C)}}{\frac{(h_2^T - h_3^T)}{1-(h_2^T - h_3^T)}} \frac{1+p_Y^C}{1+p_Y^T}.$$

By our previous finding, the LHS is less than 1; however, the maintained assumption implies $\frac{1+p_Y^C}{1+p_Y^T} > 1$. Therefore, it must be the case that

$$\frac{(h_1^C - h_2^C)}{1-(h_1^C - h_2^C)} < \frac{(h_2^T - h_3^T)}{1-(h_2^T - h_3^T)}, \iff (h_1^C - h_2^C) \qquad\qquad < (h_2^T - h_3^T).$$

Next, we show that $h_1^C < h_1^T$. Suppose not. Then, $h_1^C \geq h_1^T > h_2^T$, and to satisfy the above inequality, it must be that $h_2^C > h_3^T$. As before, solving the indifference equations (7) and (12) in terms of the ratio $\frac{p_Z}{\pi}$ will yield:

$$\frac{p_Z^T}{\pi^T} = [\frac{d_U^Z}{d_D^Y}\frac{\gamma^{h_3^T}}{1-\gamma^{h_3^T}} + \frac{d_D^Z}{d_D^Y}], \quad \text{and} \quad \frac{p_Z^C}{\pi^C} = [\frac{d_U^Z}{d_D^Y}\frac{\gamma^{h_2^C}}{1-\gamma^{h_2^C}} + \frac{d_D^Z}{d_D^Y}].$$

If $h_2^C > h_3^T$, then $\frac{p_Z^C}{\pi^C} > \frac{p_Z^T}{\pi^T}$. Given the result $p_Z^C < p_Z^T$, it follows that the price of the tranche has decreased, i.e. $\pi^C < \pi^T$, and by the maintained assumption, $p_Y^T + \pi^C < p_Y^C + \pi^T$, which is equivalent to $p_Y^T - \pi^T < p_Y^C - \pi^C$. Given $h_1^C \geq h_1^T$, it follows $1 - h_1^C \leq 1 - h_1^T$, and by equations (1) and (8)

$$\frac{1 - \varphi^C + p_Y^C - \pi^C}{(1 + p_Y^C + p_Z^C)} = (1 - h_1^C) \leq (1 - h_1^T) = \frac{p_Y^T - \pi^T}{(1 + p_Y^T + p_Z^T)}.$$

Given the intermediate finding $p_Y^T - \pi^T < p_Y^C - \pi^C$, the only way this can happen is if the agents' wealth has increased, i.e. $(1 + p_Y^C + p_Z^C) > (1 + p_Y^T + p_Z^T)$. Returning to the market clearing conditions, it follows that

$$\frac{h_1^C}{h_1^T} = \frac{p_Z^C + \varphi^C + \pi^C}{1 + p_Z^T + \pi^T}\frac{(1 + p_Y^T + p_Z^T)}{(1 + p_Y^C + p_Z^C)}.$$

The previous finding $p_Z^C < p_Z^T$ and the intermediate finding $\pi^C < \pi^T$ together imply $p_Z^C + \varphi^C + \pi^C < 1 + p_Z^T + \pi^T$. Similarly, wealth has increased, so $\frac{(1+p_Y^T+p_Z^T)}{(1+p_Y^C+p_Z^C)} < 1$. Therefore, the RHS is less than 1. The LHS is greater than 1 by the maintained assumption, which produces the contradiction. We proceed using $h_1^C < h_1^T$.

All agents in CDS have returns of at least one, so by the indifference condition,

$$\frac{d_U^Y}{p_Y^T - \pi^T}\gamma^{h_1^T} = 1 \leq \frac{d_U^Y}{p_Y^C - \pi^C}\gamma^{h_1^C},$$
$$\gamma^{h_1^T} > \gamma^{h_1^C} \implies p_Y^C - \pi^C < p_Y^T - \pi^T.$$

Therefore, the down-payment for the upstate consumption has decreased. By the maintained assumption $p_Y^C > p_Y^T$, it must be that $\pi^C > \pi^T$.

We next demonstrate that $h_1^C > h_2^T$. Suppose not, then $h_1^C \leq h_2^T$, which implies $1 - h_1^C \geq 1 - h_2^T$. By the market clearing conditions,

$$\frac{1 + p_Y^T - \pi^T}{1 - \varphi^C + p_Y^C - \pi_D^C} = \frac{(1 - h_2^T)}{(1 - h_1^C)}\frac{(1 + p_Y^T + p_Z^T)}{(1 + p_Y^C + p_Z^C)}$$

Given the intermediate finding $p_Y^C - \pi^C < p_Y^T - \pi^T$, the LHS is greater than 1. However, by the maintained assumption, $\frac{1-h_2^T}{1-h_1^C} \leq 1$, and for the RHS to be greater than 1, wealth must have decreased, i.e. $(1 + p_Y^T + p_Z^T) > (1 + p_Y^C + p_Z^C)$.

The market clearing conditions for the Down-state tranche are equations (4) and (10), and taking their ratio yields

$$\frac{\pi^T}{\varphi^C + \pi^C} = \frac{h_3^T(1 + p_Y^T + p_Z^T)}{h_2^C(1 + p_Y^C + p_Z^C)}, \text{ and by equation (13)}$$
$$\varphi^C = \frac{\pi^C}{d_D^Y} \implies \frac{\pi^T}{\pi^C} = \frac{h_3^T}{h_2^C}\frac{(1 + p_Y^T + p_Z^T)}{(1 + p_Y^C + p_Z^C)}(1 + \frac{1}{d_D^Y}).$$

Because the LHS is less than 1, the RHS must be less than 1. Given a decrease in wealth, i.e. $(1+p_Y^T+p_Z^T) > (1 + p_Y^C + p_Z^C)$, this can only happen if $h_3^T < h_2^C$. As before, solving the indifference equations (7) and (12) in terms of the ratio $\frac{p_Z}{\pi}$ will yield:

$$\frac{p_Z^T}{\pi^T} = [\frac{d_U^Z}{d_D^Y}\frac{\gamma^{h_3^T}}{1 - \gamma^{h_3^T}} + \frac{d_D^Z}{d_D^Y}], \quad \text{and} \quad \frac{p_Z^C}{\pi^C} = [\frac{d_U^Z}{d_D^Y}\frac{\gamma^{h_2^C}}{1 - \gamma^{h_2^C}} + \frac{d_D^Z}{d_D^Y}].$$

Given the finding $h_3^T < h_2^C$, it follows that $\frac{p_Z^T}{\pi^T} < \frac{p_Z^C}{\pi^C}$.This is a contradiction. Our previous findings were that $p_Z^C < p_Z^T$ and $\pi^C > \pi^T$. That is, the supposedly greater fraction has a larger denominator and a smaller numerator. We proceed using $h_1^C > h_2^T$.

From the market clearing equations (3) and (9), we found that the market for Z must shrink given an increase in p_Y, i.e. $(h_1^C - h_2^C) < (h_2^T - h_3^T)$. The immediate implication of $h_1^C > h_2^T$ is $h_3^T < h_2^C$, or the market will not shrink. By the same logic as above, this produces a contradiction. The finding $h_3^T < h_2^C$ implies that $\frac{p_Z^T}{\pi^T} < \frac{p_Z^C}{\pi^C}$, which cannot hold under the maintained assumptions. Hence, $p_Y^C < p_Y^T$. $\square$

Appendix C

Numerical Examples - Chapter 3

We will consider a numerical example where the risky assets Y and Z are negatively correlated, with the following payoffs:

$$Y : (d_Y^U, d_Y^D) = (1, 0.2)$$
$$X : (d_X^U, d_X^D) = (1, 1)$$
$$Z : (d_Z^U, d_Z^D) = (0.2, 1)$$

This environment results in equilibrium outcomes as in the table below, depending on the financial structure of the economy.

C.1 Equilibrium Equations

Leverage Z Only

$$p_Y = d_Y^U \gamma^{h_1} + d_Y^D (1 - \gamma^{h_1})$$
$$p_Z = d_Z^U \gamma^{h_2} + d_Z^D (1 - \gamma^{h_2})$$
$$p_Y = (1 - h_1)(1 + p_Z + p_Y)$$
$$p_Z = h_2(1 + p_Z + p_Y) + d_Z^U$$

	Arrow-Debreu	Financial Autarky	Leverage Y Only	Leverage Z Only	Leverage Y and Z
h_1	0.3820	0.6465	0.7155	0.6548	0.7221
h_2	-	0.2536	0.2511	0.1969	0.1951
q_U	0.6180	-	-	-	-
q_D	0.3820	-	-	-	-
p_Y	0.6944	0.9	0.9352	0.9047	0.9382
p_Z	0.5056	0.6457	0.6487	0.7160	0.7183

	Tranche Y Only	Tranche Y Leverage Z	Tranche Z Only	Tranche Z Leverage Y	Tranche Both
h_1	0.6620	0.6693	0.9340	0.6693	0.6200
h_2	0.2804	0.2237	0.6038	0.2237	0.2499
h_3	0.0460	-	0.2262	-	-
p_Y	1.0062	1.0112	0.8744	0.9125	0.9681
p_Z	0.6143	0.6821	0.7736	0.7807	0.7338
π_Y	0.1205	0.1205	-	-	0.1125
π_Z	-	-	0.1747	0.1781	0.1711

	Tranche Y + CDS	Tranche Y + CDS, Leverage Z	Tranche and CDS Both
h_1	0.4210	0.4327	0.3820
h_2	0.2172	0.3500	-
p_Y	0.6922	0.7325	0.6944
p_Z	0.4333	0.5207	0.5056
π_Y	0.0769	0.0798	0.0764
π_Z	-	-	0.1236
p_{CDSY}	0.3847	0.3735	0.3820
p_{CDSZ}	-	-	0.6180

Leverage Y and Z

Equations:

$$p_Y = d_Y^U \gamma^{h_1} + d_Y^D (1 - \gamma^{h_1})$$

$$p_Z = d_Z^U \gamma^{h_2} + d_Z^D (1 - \gamma^{h_2})$$

$$1 = d_o^U \gamma^{h_2} + d_o^D (1 - \gamma^{h_2})$$

$$p_Y = (1 - h_1)(1 + p_Z + p_Y) + d_Y^D$$

$$p_Z = h_2(1 + p_Z + p_Y) + d_Z^U$$

Tranche Y, Leverage Z

$$\begin{aligned}
p_Y &= (1-h_1)(1+p_Y+p_Z)+p_{\mathrm{TA}} \\
p_{\mathrm{TA}}+p_Z &= h_2(1+p_Y+p_Z)+d_Z^U \\
1 &= \frac{d_Y^U\gamma^{h_1}}{p_Y-p_{\mathrm{TA}}} \\
1 &= \frac{d_Z^U\gamma^{h_2}+d_Z^D(1-\gamma^{h_2})}{p_Z} \\
\frac{d_Y^D(1-\gamma^{h_2})}{p_{\mathrm{TA}}} &= \frac{d_Z^U\gamma^{h_2}+d_Z^D(1-\gamma^{h_2})}{p_Z}
\end{aligned}$$

Tranche Z

$$\begin{aligned}
p_{\mathrm{TP}} &= (1-h_1)(1+p_Y+p_Z) \\
p_Y &= (h_1-h_2)(1+p_Y+p_Z) \\
p_Z &= h_3(1+p_Y+p_Z)+\pi_Z \\
1 &= \frac{d_Z^D(1-\gamma^{h_3})}{p_Z-p_{\mathrm{TP}}} \\
1 &= \frac{d_Y^U\gamma^{h_2}+d_Y^D(1-\gamma^{h_2})}{p_Y} \\
\frac{d_Z^U\gamma^{h_1}}{p_{\mathrm{TP}}} &= \frac{d_Y^U\gamma^{h_1}+d_Y^D(1-\gamma^{h_1})}{p_Y}
\end{aligned}$$

Tranche Z, Leverage Y

$$\begin{aligned}
p_Z &= h_2(1+p_Y+p_Z)+p_{\mathrm{TP}} \\
p_{\mathrm{TP}}+p_Y &= (1-h_1)(1+p_Y+p_Z)+d_Y^D \\
1 &= \frac{d_Z^D(1-\gamma^{h_2})}{p_Z-p_{\mathrm{TP}}} \\
1 &= \frac{d_Y^U\gamma^{h_1}+d_Y^D(1-\gamma^{h_1})}{p_Y} \\
\frac{d_Z^U\gamma^{h_1}}{p_{\mathrm{TP}}} &= \frac{d_Y^U\gamma^{h_1}+d_Y^D(1-\gamma^{h_1})}{p_Y}
\end{aligned}$$

Tranche Both

$$
\begin{aligned}
p_{\text{TP}} + p_Y &= (1-h_1)(1+p_Z+p_Y) + \pi_Y \\
p_{\text{TA}} + p_Z &= h_2(1+p_Z+p_Y) + \pi_Z \\
1 &= \frac{d_Z^U \gamma^{h_1}}{\pi_Z} \\
1 &= \frac{d_Y^D (1-\gamma^{h_2})}{\pi_Y} \\
\frac{d_Y^U}{p_Y - \pi_Y} &= \frac{d_Z^U}{\pi_Z} \\
\frac{d_Z^D}{p_Z - \pi_Z} &= \frac{d_Y^D}{\pi_Y}
\end{aligned}
$$

Tranche Y + CDS, Leverage Z

$$
\begin{aligned}
p_Z &= (h_1 - h_2)(1+p_Z+p_Y) \\
p_Y - p_{\text{TA}} + \alpha - \alpha p_{\text{CDSA}} + (1-\alpha)p_{CDSZ} &= (1-h_1)(1+p_Z+p_Y) \\
\alpha &\in (0,1) \text{ indeterminate} \\
p_Z(1-\gamma^{h_2}) &= (d_Z^U \gamma^{h_2} + d_Z^D(1-\gamma^{h_2}))p_{CDSY} \\
p_Z \gamma^{h_1} &= (d_Z^U \gamma^{h_1} + d_Z^D(1-\gamma^{h_1}))p_{CDSZ}
\end{aligned}
$$

$$
\begin{aligned}
p_{CDSY} &= 1 - p_{CDSZ} = 5\pi_Y \\
1 - p_{CDSY} &= p_{CDSZ} = p_Y - \pi_Y
\end{aligned}
$$

Tranche + CDS Both

$$p_{\mathrm{TP}} + p_Y - p_{\mathrm{TA}} + \alpha(1 - p_{\mathrm{CDSA}}) + (1-\alpha)p_{CDSZ} = (1-h_1)(1 + p_Z + p_Y)$$

$$\alpha \in (0,1) \text{ indeterminate}$$

$$\gamma_1^h \pi_Y = (1-\gamma_1^h)\pi_Z$$

$$p_Y - \pi_Y = 5\pi_Z = 1 - p_{CDSY} = p_{CDSZ}$$

$$p_Z - \pi_Z = 5\pi_Y = 1 - p_{CDSZ} = p_{CDSY}$$

Bibliography

Allen, F. and D. Gale (2000). Financial contagion. *Journal of Political Economy 108*(1), 1–33.

Araujo, A. and J. M. Villalba (2022). Equilibrium efficiency with secured and unsecured assets. *Economic Theory 73*(4), 1025–1049.

Athreya, K. (2006). Fresh start or head start? uniform bankruptcy exemptions and welfare. *Journal of Economic Dynamics and Control 30*(11), 2051–2079.

Chodorow-Reich, G., O. Darmouni, S. Luck, and M. Plosser (2022). Bank liquidity provision across the firm size distribution. *Journal of Financial Economics 144*(3), 908–932.

Diamond, D. W. and R. G. Rajan (2011). Fear of fire sales, illiquidity seeking, and credit freezes. *The Quarterly Journal of Economics 126*(2), 557–591.

Disney, R., S. Bridges, and J. Gathergood (2010). House price shocks and household indebtedness in the united kingdom. *Economica 77*(307), 472–496.

Donaldson, J. R., D. Gromb, and G. Piacentino (2020). The paradox of pledgeability. *Journal of Financial Economics 137*(3), 591–605.

Dubey, P., J. Geanakoplos, and M. Shubik (2005). Default and punishment in general equilibrium 1. *econometrica 73*(1), 1–37.

Dubey, P., J. Geanakoplos, and W. R. Zame (1995). Collateral, default and market crashes. *Cowles Foundation Dis-cussion Paper*.

Elliott, M., B. Golub, and M. O. Jackson (2014). Financial networks and contagion. *American Economic Review 104*(10), 3115–53.

Favilukis, J., S. C. Ludvigson, and S. Van Nieuwerburgh (2017). The macroeconomic effects of housing wealth, housing finance, and limited risk sharing in general equilibrium. *Journal of Political Economy 125*(1), 140–223.

Fostel, A. and J. Geanakoplos (2008). Leverage cycles and the anxious economy. *American Economic Review 98*(4), 1211–44.

Fostel, A. and J. Geanakoplos (2012). Tranching, cds, and asset prices: How financial innovation can cause bubbles and crashes. *American Economic Journal: Macroeconomics 4*(1), 190–225.

Fostel, A. and J. Geanakoplos (2015a). Leverage and default in binomial economies: a complete characterization. *Econometrica 83*(6), 2191–2229.

Fostel, A. and J. Geanakoplos (2015b). Leverage and default in binomial economies: A complete characterization. *Econometrica 83*(6), 2191–2229.

Fostel, A., M. Nilayamgode, and T. Wake (2023). Collateral expansion: Pricing spillovers from financial innovation. *Working Paper*.

Frank, M. Z. and V. K. Goyal (2008). Trade-off and pecking order theories of debt. *Handbook of empirical corporate finance*, 135–202.

Garriga, C. and A. Hedlund (2020). Mortgage debt, consumption, and illiquid housing markets in the great recession. *American Economic Review 110*(6), 1603–1634.

Garriga, C., R. Manuelli, and A. Peralta-Alva (2019). A macroeconomic model of price swings in the housing market. *American Economic Review 109*(6), 2036–2072.

Geanakoplos, J. (1997a). Promises, promises. *The economy as an evolving complex system II 27*, 285.

Geanakoplos, J. (1997b). "promises, promises". pp. 285–320.

Geanakoplos, J. (2003a). "liquidity, default, and crashes". pp. 170.

Geanakoplos, J. (2003b). Liquidity, default, and crashes endogenous contracts in general. In *Advances in economics and econometrics: theory and applications: eighth World Congress*, Volume 170.

Geanakoplos, J. and W. R. Zame (1997). Collateralized security markets. *Unpublished working papers 2009*.

Geanakoplos, J. and W. R. Zame (2002). Collateral and the enforcement of intertemporal contracts. Technical report, Yale University working paper.

Geanakoplos, J. and W. R. Zame (2014). Collateral equilibrium, i: a basic framework. *Economic Theory 56*, 443–492.

Guerrieri, V. and R. Shimer (2014). Dynamic adverse selection: A theory of illiquidity, fire sales, and flight to quality. *American Economic Review 104*(7), 1875–1908.

He, Z. (2011). A model of dynamic compensation and capital structure. *Journal of Financial Economics 100*(2), 351–366.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2015). Household leveraging and deleveraging. *Review of Economic Dynamics 18*(1), 3–20.

Landvoigt, T., M. Piazzesi, and M. Schneider (2015). The housing market (s) of san diego. *American Economic Review 105*(4), 1371–1407.

Merton, R. C. (1992). Financial innovation and economic performance. *Journal of Applied Coporate Finance 4*(4), 12–22.

Morellec, E. (2001). Asset liquidity, capital structure, and secured debt. *Journal of financial economics 61*(2), 173–206.

Scott, J. H. (1977). Bankruptcy, secured debt, and optimal capital structure. *The journal of finance 32*(1), 1–19.

Shleifer, A. and R. Vishny (2011). Fire sales in finance and macroeconomics. *Journal of Economic Perspectives 25*(1), 29–48.

Tufano, P. (2003). Financial innovation. *Handbook of the Econommics of Finance* (1), 307–335.