Morgan, Herbert Rollo

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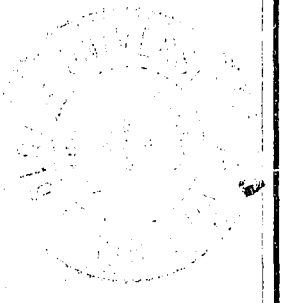
THE ORBIT OF ENCELADUS.



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THE ORBIT OF ENCELADUS.

THE ORBIT OF ENCELADUS.

By HERBERT R. MORGAN.

Having been engaged in observing the Satellites of Saturn for a number of years, I became interested in reducing the observations made at this observatory, and this paper contains some of the results. Orbits of Mimas, Rhea, and Titan have been computed, but there are more observations of these satellites to be reduced.

Enceladus, the second satellite of Saturn, was discovered in 1789 by Sir William Herschel. Its revolution around Saturn is made in thirty-three hours, at a mean distance of one hundred and fifty-seven thousand miles. Although small and quite faint, and, moreover, close to the rings, Enceladus can be followed to conjunction on fair nights with the 26-in. refractor without much difficulty.

The interest in this satellite, as pointed out by H. Struve, centers in the fact that its mean motion is nearly twice the mean motion of Dione, the fourth satellite. The perturbations arising from this relation give a libration in mean longitude, and motions of the node and perisaturnium of Enceladus.

The formulæ used in the reductions are those given by H. Struve in his work on the satellites, pp. 55–62,* and the symbols have the following meaning:

E, mean longitude, reckoned from the Vernal equinox on the ecliptic.
P, longitude of perisaturnium, reckoned from the Vernal equinox on the ecliptic.
I, N, inclination and node on Earth's equator, reckoned from the Vernal equinox on the equator.
γ, θ, inclination and node on Saturn's equator, reckoned from node of Saturn's equator on Earth's equator.
π, longitude of perisaturnium, reckoned as θ.
e, eccentricity.
n, mean daily motion.

* Publications de Pulkovo, Séries II, Vol. XI.

$\triangle$, mean distance from Saturn at Saturn's mean distance from earth, $\log(\rho) = 0.97950$.

x_i, y_i, coördinates of satellite referred to Saturn's center.

The assumed elements were also derived from the same work, with the exception of the longitudes for 1898 and 1900. In these years a preliminary solution of the equations of condition gave the longitudes both of Enceladus and of the comparison satellites. In these years the elements of the comparison satellite were taken as known, and the observations combined into one solution for Enceladus.

At Prof. Stone's suggestion the mass of Saturn, $m = \frac{1}{3500}$, was taken as a constant, and terms introduced in the equations for the correction to the micrometer screw $M = 9''.900$ of the form $(x_1 - x_2)\frac{dM}{M}$ for the equations in $x_1 - x_2$, and $(y_1 - y_2)\frac{dM}{M}$ for the equations in $y_1 - y_2$. These terms have a different sign for 1894 than for the later years, and the correction for this year was therefore made subtractive.

The orbits for 1894 are referred to the Earth's equator, and those of 1898 and 1900 to Saturn's equator as the fundamental planes.

The observations in 1894 were made by Professors Stone and Lovett, and are published in the Astronomical Journal, Vol. XV, p. 110, and Astronomische Nachrichten, Vol. 143, p. 291, respectively. Those in 1898 and 1900 were made by myself, and are published in the Astronomical Journal, Vol. XIX, p. 117, and Astronomische Nachrichten, Vol. 154, p. 91. In 1894 and 1898 four, and some times only two, measures of angle and distance were made on a night. In 1900 eight measures of each were made. The ratio of the probable errors of an equation should, therefore, be greater than $1/2 : 1$. The results show this.

I wish to express my sincere thanks to Prof. Stone for his kind encouragement and valuable suggestions in all my work at the observatory, and especially in this. Also, I wish to acknowledge my indebtedness to Mr. E. R. Jones for his assistance in the least square solution of the last two orbits. Mr. Jones' work was afterwards checked by me.

Assumed Elements.

	Enceladus.	*Tethys.*	*Dione.*	*Rhea.*	
	Epoch: May 8.0 1894. G. M. T.				
E	331°.900	66°.595	275°.059	220°.748	
P	—	—	—	—	
e	0.0	0.0	0.0	0.0	
I	6° 53′ 34″	6° 53′ 34″	6° 53′ 34″	6° 53′ 34″	Apparent equator,
N	125 34 36	125 34 36	125 34 36	125 34 36	1894.35.
n	262°.7318	190°.6980	131°.5349	79°.6901	
$\triangle$	34″.38	42″.583	54′.540	76″.170	

Saturn's equator: $N_1 = 125° 34' 36$, $I_1 = 6\ 53\ 34$, 1894.35.

	Epoch: July 8.0 1898. G. M. T.				
E	249°.557	149°.180	312°.014	190°.258	
π	—	—	—	—	
e	0.0	0.0	0.0	0.0	
γ	0.0	64′.4	4′.0	22′.0	Apparent equator of
θ	—	27°.5	222°.	293.°	Saturn, 1898.52.
n	262°.7318	190°.6980	131°.5349	79°.6901	
$\triangle$	34″.38	42″.583	54″.540	76″.170	

Saturn's equator: $N_1 = 125° 54'.4$, $I_1 = 6\ 57.0$, 1898.52.

	Epoch: July 8.0 1900, G. M. T.				
E	164°.910	38°620	213°.264	44°.058	
π	—	—	—	—	
e	0.0	0.0	0.0	0.0	
γ	0.0	64′.4	4′.0	22′.0	Apparent equator of
θ	—	242°.5	160°.	293°.	Saturn, 1900.52.
n	262°.7318	190°.6980	131°.5349	79°.6901	
$\triangle$	34″.38	42″.583	54″.540	76″.170	

Saturn's equator: $N_1 = 125° 53'.9$, $I_1 = 6\ 56.9$, 1900.52.

OBSERVATION-COMPUTATION. ENCELADUS-TETHYS.

1894.	Greenwich M. T.	x_1	x_2	Comp. x_1-x_2	Obs. x_1-x_2	$O-C$	y_1	y_2	Comp. y_1-y_2	Obs. y_1-y_2	$O-C$
		"	"	"	"	"	"	"	"	"	"
March	26.850365	+ 44.34	— 22.40	+ 66.74	+ 66.83	+ 0.09	— 1.98	— 7.43	+ 5.45	+ 4.52	— 0.93
	30.621447	+ 44.58	+ 29.78	+ 14.80	+ 14.54	— 0.26	— 1.82	— 4.35	+ 2.53	+ 2.23	— 0.30
April	2.767625	— 10.41	— 30.66	+ 20.25	+ 19.20	— 1.05	+ 9.85	— 5.73	+ 15.58	+ 16.18	+ 0.60
	14.752936	+ 42.81	+ 22.02	+ 20.79	+ 21.08	+ 0.29	— 2.77	— 5.98	+ 3.21	+ 2.61	— 0.61
	15.699209	— 42.63	+ 20.49	— 63.12	— 63.01	+ 0.11	+ 2.85	+ 7.45	+ 4.60	— 4.28	+ 0.32
	24.622638	— 13.10	— 23.15	+ 10.05	+ 9.75	— 0.30	— 9.86	— 6.98	— 2.88	— 3.46	— 0.58
	24.705636	— 24.79	— 32.50	— 7.71	+ 7.42	— 0.29	— 9.11	— 5.00	— 4.11	— 4.59	— 0.48
	25.668168	+ 27.22	+ 27.56	+ 0.35	— 0.61	— 0.26	+ 8.82	— 4.53	+ 13.35	+ 14.22	+ 0.87
	27.675630	+ 39.73	— 32.38	+ 72.11	+ 73.65	+ 1.54	+ 6.32	+ 2.97	+ 3.35	+ 4.02	+ 0.67
	29.491176	+ 32.88	+ 31.61	+ 1.27	+ 1.10	— 0.17	+ 7.90	+ 5.22	+ 2.68	+ 2.61	— 0.07
	29.541407	+ 37.88	+ 35.38	+ 2.50	+ 2.23	— 0.27	+ 6.82	+ 3.70	+ 3.12	+ 3.02	— 0.10
	29.685829	+ 45.99	+ 35.52	+ 10.47	+ 10.64	+ 0.17	+ 2.77	— 1.38	+ 4.15	+ 4.09	— 0.06
May	1.695010	+ 44.67	— 37.18	+ 81.85	+ 82.76	+ 0.91	— 1.16	— 0.28	— 0.88	— 1.01	— 0.13
	3.659388	+ 39.99	+ 35.72	+ 4.27	+ 4.15	— 0.12	— 3.55	+ 3.43	— 6.98	— 7.80	— 0.82
	3.717016	+ 34.82	+ 37.35	— 2.53	— 2.16	+ 0.37	— 5.21	+ 1.47	— 6.68	— 6.87	— 0.19
	8.589327	— 15.27	— 35.32	+ 20.05	+ 20.13	+ 0.08	+ 8.44	+ 1.26	+ 7.18	+ 8.42	+ 1.24
	8.647186	— 6.66	— 30.99	+ 24.33	+ 24.72	+ 0.39	+ 9.15	+ 3.22	+ 5.93	+ 7.24	+ 1.31
	9.673651	— 6.02	— 20.45	+ 14.43	+ 14.13	— 0.30	— 9.56	— 7.04	— 2.52	— 3.11	— 0.59
	9.753166	— 17.75	— 30.19	+ 12.44	+ 11.39	— 1.05	— 9.30	— 5.45	— 3.85	— 4.56	— 0.71
	10.682804	+ 15.74	+ 32.63	— 16.89	— 17.12	— 0.23	+ 9.37	— 2.56	+ 11.93	+ 12.50	+ 0.57
	12.568513	+ 15.49	— 35.68	+ 51.17	+ 51.36	+ 0.19	+ 9.34	— 3.26	+ 12.60	+ 13.65	+ 1.05
	15.563347	— 35.58	— 23.21	— 12.37	— 12.03	+ 0.34	— 7.04	+ 5.06	— 12.10	— 12.71	— 0.61
	21.524216	— 43.23	+ 36.67	— 79.90	— 79.88	+ 0.02	+ 1.35	+ 1.58	— 0.23	+ 0.42	+ 0.65
June	2.606790	+ 42.67	+ 13.77	+ 28.90	+ 30.04	+ 1.14	+ 4.16	+ 7.11	— 2.95	— 2.59	+ 0.36
	3.547150	— 42.44	— 36.05	— 6.39	— 6.08	+ 0.31	— 4.24	— 1.33	— 2.91	— 2.96	— 0.05
	7.571286	— 38.65	— 32.95	— 5.70	— 6.00	— 0.30	+ 2.95	— 3.95	+ 6.90	+ 8.10	+ 1.20
	11.550290	— 16.33	— 19.06	+ 2.73	+ 2.22	— 0.51	+ 7.52	— 6.62	+ 14.14	+ 14.97	+ 0.83
	23.523672	+ 42.34	+ 30.75	+ 11.59	+ 11.33	— 0.26	— 0.22	— 2.18	+ 1.96	+ 1.71	— 0.25
July	8 558821	+ 42.11	+ 32.30	+ 9.81	+ 9.35	— 0.46	+ 1.69	— 1.00	+ 2.69	+ 2.29	— 0.40

COEFFICIENTS OF THE EQUATIONS OF CONDITION FOR $x_1 - x_2$. ENCELADUS-TETHYS.

1894.		dE_1	sin $I_1 dN_1$	dI_1	dE_2	$e_2 \sin P_2$	$e_2 \cos P_2$	sin $I_2 dN_2$	dI_2	$\frac{dM}{M}$	v	Observer.
											"	
March	26	1.1461n	0.0968	0.3636n	1.4790	1.7079n	1 5922	0.7674n	0.6265	1.8250	+ 0.212	L
	30	1.1321n	0.0573	0.3524n	1.3615	1.7385n	0.1753	0.5035n	0.5464	1.1625	— 0.197	S
April	2	1.6577	0 8817n	0.8213	1.3401	1.6106n	1.5379	0.6679n	0.4698	1.2834	— 0.592	L
	14	1.2703n	0.2943	0.4746n	1.4857	1.8121n	0.6513	0.6673n	0.6563	1.3238	+ 0.410	S
	15	1.2792	0.3085n	0.4824	1.5001n	1.7370n	1.5794	0.7834	0.6386n	1.7994n	+ 0.143	L
	24	1.6500n	0.9083	0.7929n	1.4713	1.7119n	1.5712	0.7640n	0.6012	0.9891	+ 0.138	S
	24	1.5954n	0.8776	0.7294n	1.2760	1.5919n	1.4899	0.6280n	0.3 01	0.8707	— 0.008	L
	25	1.5768	0.8654n	0.7078	1.4069	1.7635n	9.4924	0.5501n	0.5838	9.7859n	+ 0.049	L
	27	1.3821	0.7306n	0.4857	1.2779n	1.6979n	9.1330n	0.3543	0.4698n	1.8672	+ 0.544	L
	29	1.5155	0.8234n	0.6364	1.3043n	1.6045n	1.5013	0.6483	0.4088n	0.0424	— 0.366	L
	29	1.4288	0.7631n	0.5379	1,0927n	1.5510n	1.3888	0.5093	0.1571n	0.3480	— 0.391	L
	29	0.7976	0.4002n	9.6852	1.0783	1.6310n	0.4185	9.9783n	0.2996	1.0271	+ 0.166	S
May	1	1.0931n	9.8602	0.3293n	0.6362n	1.5716n	0.9094	9.6284n	9.9711n	1.9178	+ 0.132	L
	3	1.3675n	0.4370	0.5597n	1.0417n	1.5458n	1.3605	0.4800	0.0880n	0.6177	— 0.356	L
	3	1.4844n	0.6164	0.6620n	0.1124n	1.5447n	1.1196	0.1476	9.1840	0.3340n	+ 0.256	L
	8	1.6385	0.8427n	0.7939	1.0705n	1.6271n	0.9404	9.9438	0.2949n	1.3039	— 0.258	S
	8	1.6591	0.8808n	0.8087	1.3141n	1.7123n	0.0046n	0.4036	0.5008n	1.3930	— 0.243	L
	9	1.6596n	0.9039	0.8008n	1.4921	1.7372n	1.5561	0.7766n	0.6175	1.1503	+ 0.107	L
	9	1.6284n	0.8949	0.7611n	1.3368	1.6216n	1.5099	0.6715n	0.4394	1.0565	— 0.757	L
	10	1.6359	0.8983n	0.7699	1.2496	1.6834n	9.7532n	0.2975n	0.4451	1.2336n	+ 0.110	L
	12	1.6359	0.8977n	0.7692	1.0067	1.5424n	1.3357	0.4623n	0.0315	1.7106	+ 0.135	L
	15	1.4598n	0.7850	0.5639n	1.4589n	1.7927n	0.0363	0 6183	0.6260n	1.0801n	— 0.365	S
	21	1.1475	9.9787n	0.3890	0.2044n	1.5386n	1.1027	0.1752	9.1664	1.9024n	+ 0.563	L
June	2	1.1270	0.5723n	0.1444	1.5233n	1.7795n	1.5044	0.7884	0.6485n	1.4777	+ 0.390	L
	3	1.1418n	0.5807	0.1640n	9.4872	1.5360n	1.0388	0.1091n	9.5318n	0.7836n	+ 0.286	S
	7	1.3382	0.3751n	0.5330	1.1482	1.5530n	1.3893	0.5468n	0.1991	0.7784n	+ 0.098	L
	11	1.6124	0.8056n	0.7640	1.4782	1.7295n	1.5216	0.7607n	0.5941	0.3472	+ 0.149	L
	23	0.9409n	8.1613	0.2202n	1.2178	1.6542n	0.1720n	0.2397n	0.4154	1.0543	— 0.265	S
July	8	0.0097	0.2088n	9.4886n	1.0270	1.5873n	0.1266	9.8457n	0.2575	0.9706	— 0.483	L

COEFFICIENTS OF THE EQUATIONS OF CONDITION FOR $y_1 - y_2$. ENCELADUS-TETHYS.

1894.		dE_1	sin $I_1 dN_1$	dI_1	dE_2	$e_2 \sin P_2$	$e_2 \cos P_2$	sin $I_2 dN_2$	dI_2	$\frac{dM}{M}$	v	Observer.
											"	
March	26	1.0180n	1.6726	0.7177n	0.6301n	0.2427n	1.0483n	1.1854	1.5162	0.6552	− 0.572	L
	30	1.0163n	1.6740	0.6571n	0.8643	1.0100n	1.0542n	1.5410n	1.2123	0.3488	+ 0.174	S
April	2	0.5546	1.3216n	1.6158	0.7941n	0.3742n	1.1303n	1.4002	1.4266	1.2093	− 0.488	L
	14	0.9924n	1.6702	0.9464n	0.7540	0.9679n	0.9470n	1.4671n	1.3893	0.4167	− 0.661	S
	15	0.9904	1.6694n	0.9631	0.5429	0.2318n	1 0027n	1.0874n	1.5357n	0.6318n	− 0.293	L
	24	0.1432	0.0678n	1.6556n	0.6005n	0.1930n	1.0205n	1.1755	1.5208	0.5390n	+ 0.028	S
	24	0.6051	1.1466n	1.6316n	0.7989n	0.4015n	1.1242n	1.4306	1.3988	0.6618n	− 0.087	L
	25	0.6604n	1.2247	1.6212	0.8206	0.9798n	1.0096n	1.5283n	1.2580	1.1528	+ 0.295	L
	27	0.8806n	1.5128	1.5015	0.8694n	0.9685n	1.0755n	1.5656	1.0311n	0.6037	+ 0.199	L
	29	0.7672n	1.3703	1.5852	0.7769	0.3526n	1.1092n	1.4080n	1.4188n	0.4161	− 0.388	L
	29	0.8501n	1.4762	1.5311	0.8959	0.5564n	1.1734n	1.4982n	1.2919n	0.4794	− 0.415	L
	29	0.9744n	1.6373	1.2131	0.8928	0.9251n	1.1218n	1.5797n	0.5218	0.6112	+ 0.033	S
May	1	0.9869n	1.6729	0.1887n	0.8968n	0.8461n	1.1500n	1.5749	0.6419	0.0047n	+ 0.347	L
	3	0.9566n	1.6578	1.1090n	0.8487	0.5795n	1.1468n	1.5047n	1.1743n	0.8919n	− 0.209	L
	3	0.9141n	1.6271	1.3193n	0.8870	0.7650n	1.1570n	1.5587n	0.9929n	0.8368n	+ 0.516	L
	8	0.6592	1.4310n	1.5746	0.8832n	0.9102n	0.9451n	1.5772	0.4466n	0.9252	+ 0.155	S
	8	0.4567	1.2822n	1.6189	0.8477n	0.9593n	1.0499n	1.5563	1.0858n	0.8596	+ 0.216	L
	9	9.4302n	0.8341	1.6493n	0.5013n	0.1990n	0.9705n	1.0548	1.5342	0.4924n	+ 0.099	L
	9	0.3504	0.7420n	1.6473n	0 7382n	0.2800n	1.0819n	1.3701	1.4423	0.6591n	− 0.190	L
	10	0.2537n	0.5193	1.6495	0.8612	0.9460n	1.0725n	1.5663n	0.9556	1.0968	− 0.172	L
	12	0.2360n	0.4735	1.6492	0.8412n	0.5782n	1.1379n	1.5047	1.2590	1.1350	+ 0.111	L
	15	0.7959	1.4236n	1.5547n	0.7540n	0.9440n	0.9438n	1.4886	1.3333n	1.1042n	− 0.432	S
	21	0.9626	1.6645n	0.4041	0 8645	0.7278n	1.1373n	1.5474n	1.0254n	9.6214	+ 0.157	L
June	2	0.9030n	1.5740	1.3717	0.1942	0 3269n	0.8732n	0.5139n	1.5437n	0.4126n	+ 0.258	L
	3	0.8996	1.5699n	1.3790n	0.8539n	0.7301n	1.1240n	1.5434	0.9789	0.4719n	− 0.209	S
	7	0.9254	1.6440n	1.0231	0.7800n	0.4243n	1 0934n	1.4441	1.3374	0.9082	+ 0.431	L
	11	0.6703	1.4492n	1.5358	0.4317n	0.1776n	0.9249n	0.9776	1.5219	1.1754	− 0.166	L
	23	0.9394n	1.6407	0.4491	0.8249	0.8996n	1.0399n	1.5418n	0.8767	0.2339	+ 0.081	S
July	8	0.9254n	1.6087	1.0702	0.8365	0.8529n	1.0708n	1.5385n	0.2400	0.3589	− 0.148	L

NORMAL EQUATIONS. ENCELADUS-TETHYS.

	dE_2	$\sin I_1 dN_1$	dI_1	$-dE_2$	$-e_2 \sin P_2$	$-e_2 \cos P_2$	$-\sin I_2 dN_2$	$-dI_2$	$\frac{dM}{M}$	n
										"
dE_1	28868	— 11690	+ 2867	— 4443	— 4649	+ 705	+ 3500	— 612	+ 108	+ 73.30
$\sin I_1 dN_1$		34166	+ 839	+ 3337	— 818	— 3051	— 13368	+ 577	— 846	— 293.24
dI_1			27525	+ 221	— 2162	— 2094	— 3129	— 4188	+ 5228	+ 453.63
$-dE_2$				13949	— 7241	+ 3495	— 7700	+ 578	+ 3915	— 212.04
$-e_2 \sin P_2$					64397	— 20848	+ 1945	— 1496	— 14737	— 57.50
$-e_2 \cos P_2$						18863	— 321	— 1208	+ 3585	— 104.27
$-\sin I_2 dN_2$							26259	+ 3834	— 969	+ 166.96
$-dI_2$								13716	+ 2380	— 68.23
$\frac{dM}{M}$									35701	+ 296.34
									$[nn]=$	21.69

THESE EQUATIONS GIVE:

Corrections.			Corrected Elements. Epoch 1894. May 8.0, G. M. T.					
	Enc.	Te		Enc.	Prob. Error.	Te.	Prob. Error.	
log dE	8.23620n	7.78831n	E	330°.914	± 0°.130	66°.243	± 0°.087	No. eq'n's. 58
log sin IdN	6.75828n	8.00758n	N	125°18′.2	± 50′.7	120°43′.1	± 43′.4	$[vv_x]$ 3′.223
log dI	7.22420n	8.17653	I	6 47.8	± 7.2	7 45.2	± 5.0	$[vv_y]$ 2.273
log $e \sin P$	7.58155n		$e \sin P$	— 0.00382	±0.00121			$[vv]$ 5.496
log $e \cos P$	7.89000n		$e \cos P$	— 0.00776	±0.00215			Prob. error of an equation ± 0″.226
log $\frac{dM}{M}$	7.84934		e	0.00865				
			P	206° 11′				
			M	9″.8300	±0″.0128			

OBSERVATION-COMPUTATION. ENCELADUS-DIONE.

1894.	Greenwich M. T.	x_1	x_2	Comp. x_1-x_2	Obs. x_1-x_2	$O-C$	y_1	y_2	Comp. y_1-y_2	Obs. y_1-y_2	$O-C$
		"	"	"	"	"	"	"	"	"	"
March	21.730775	− 24.19	− 28.16	+ 3.97	+ 4.04	+ 0.07	+ 11.79	+ 4 91	+ 6.88	+ 7.00	+ 0.12
	27.640058	+ 32.99	+ 33.81	− 0.82	− 1.07	− 0.25	+ 12.11	+ 4.61	+ 7.50	+ 7.43	− 0.07
April	2.778945	+ 50.93	− 31.75	+ 82.68	+ 83.74	+ 1.06	− 5.54	− 5.40	− 0.14	+ 0.41	+ 0.55
	15.708526	+ 23.47	+ 21.82	+ 1.65	+ 1.40	− 0.25	+ 12.58	+ 7.29	+ 5.29	+ 5.15	− 0.14
	15.768839	+ 30.84	+ 29.39	+ 1.45	+ 1.18	− 0.27	+ 11.99	+ 5.97	+ 6.02	+ 5.64	− 0.38
	24.789120	+ 39.76	− 37.20	+ 76.96	+ 76.67	− 0.29	− 8.17	− 2.29	− 5.88	− 5.06	+ 0.82
	26.688747	+ 28.13	+ 24.65	+ 3.48	+ 3.18	− 0 30	+ 11.95	+ 6.74	+ 5.21	+ 4.36	− 0.85
	27.684948	+ 21.08	− 31.54	+ 52.62	+ 53.26	+ 0.64	− 11.04	+ 3.28	− 14.32	− 14.35	− 0.03
	29.498537	+ 36.56	+ 32.28	+ 4.28	+ 4.01	− 0.27	+ 10 98	+ 5.02	+ 5.96	+ 5.85	− 0.11
	29.547426	+ 41.58	+ 35.69	+ 5.89	+ 5.74	− 0.15	+ 10.21	+ 3.52	+ 6.69	+ 6.72	+ 0.03
May	29.711061	+ 54.27	+ 33.89	+ 20.38	+ 20.46	+ 0.08	+ 6.80	− 2.28	+ 9.08	+ 9.07	− 0.01
	1.673194	− 34.79	− 37.42	+ 2.63	+ 2.28	− 0.35	+ 8.91	− 1.07	+ 9.98	+ 9.92	− 0.06
	2.565184	+ 58.70	+ 21.28	+ 37.42	+ 36.78	− 0.64	+ 3.70	− 5.72	+ 9.42	+ 10.28	+ 0.86
	9.680909	− 52.13	− 21.48	− 30.65	− 31.24	− 0.59	+ 3.95	− 6.93	+ 10.88	+ 10.70	− 0.18
	10.579911	+ 48.99	+ 37.14	+ 11.85	+ 11.47	− 0.38	+ 8.33	+ 1.02	+ 7.31	+ 7.08	− 0.23
	10.675500	+ 54.95	+ 33.20	+ 21.75	+ 21.92	+ 0.17	+ 6.18	− 2.31	+ 8.49	+ 8.76	+ 0.27
	12.546708	− 41.58	− 34.48	− 7.10	− 7.49	− 0.39	+ 7.10	− 3.93	+ 11.03	+ 11.07	+ 0.04
	12.618051	− 34.24	− 37.04	+ 2.80	+ 2.59	− 0.21	+ 8.62	− 1.61	+ 10.23	+ 10.28	+ 0.05
	15.544851	− 10.65	− 25.57	+ 14.92	+ 14.91	− 0.01	+ 11.35	+ 4.56	+ 6.79	+ 6.83	+ 0.04
	21.504725	+ 46 97	+ 36.38	+ 10.59	+ 10.55	− 0.04	+ 8.53	+ 2.22	+ 6.31	+ 6.60	+ 0.29
June	21.576587	+ 51.96	+ 35.98	+ 15.98	+ 15.91	− 0.07	+ 7.05	− 0.21	+ 7.26	+ 7.14	− 0.12
	2.636467	− 28.59	+ 18.17	− 46.76	− 46.67	+ 0.09	− 10.85	+ 6.83	− 17.68	− 16.95	+ 0.73
	3.569014	− 26.28	− 35.90	+ 9.62	+ 9.78	+ 0.16	+ 9.15	− 0.60	+ 9.75	+ 9.88	+ 0.13
	7.579827	+ 36.30	− 33.48	+ 69.78	+ 70.80	+ 1.02	− 7.37	− 3.71	− 3.66	− 2.99	+ 0.67
	11.591471	− 44.47	− 24.37	− 20.10	− 20.71	− 0.61	+ 5.30	− 6.00	+ 11.30	+ 11.51	+ 0.21
	14.535653	− 23.44	− 35.19	+ 11.75	+ 11.85	+ 0.10	+ 9.26	− 0.43	+ 9.69	+ 9.55	− 0.14
	23.537248	+ 54.42	+ 29.67	+ 24.75	+ 24.82	+ 0.07	+ 3 95	− 2.59	+ 6.54	+ 6.61	+ 0.07

COEFFICIENTS OF THE EQUATIONS OF CONDITION FOR $x_1 - x_2$. ENCELADUS-DIONE.

1894.	dE_1	dE_2	$e_2 \sin P_2$	$e_2 \cos P_2$	$\frac{dM}{M}$	τ'	Observer.
March 21	1.7342	1.3921n	1.7539n	0.3920	0.6060	+ 0.094	S
27	1.6957	1.2142n	1.6845n	1.4755	0.0298n	+ 0.025	S
April 2	1.4945n	1.3066	1.5949n	1.5208	1.9228	+ 0.670	L
15	1.7402	1.4875n	1.7240n	1.5800	0.1464	— 0.029	L
15	1.7094	1.3728n	1.6345n	1.5453	0.0736	— 0.031	L
24	1.6476n	0.7300	1.5374n	1.2463	1.8846	— 0.865	L
26	1.7204	1.4526n	1.6959n	1.5664	0.5023	— 0.091	L
27	1.7456n	1.3082n	1.7115n	9.5536n	1.7264	— 0.100	L
29	1.6712	1.2805n	1.5948n	1.4887	0.6028	— 0.063	L
29	1.6284	1.0576n	1.5470n	1.3718	0.7591	+ 0.033	L
29	1.3851	1.2041	1.6684n	9.8034	1.3109	+ 0.106	S
May 1	1.6823	9.7731n	1.5519n	1.0594	0.3578	— 0.131	L
2	0.9425	1.4880	1.8129n	0.5452	1.5656	— 0.699	L
9	1.4411	1.4823	1.7273n	1.5570	1.4947n	+ 0.066	L
10	1.5156	9.9951	1.5518n	1.0283	1.0596	— 0.279	S
10	1.3289	1 2217	1.6725n	9.3295n	1.3408	+ 0.172	L
12	1.6193	1.1357	1.5575n	1.4007	0.8746n	+ 0.028	L
12	1.6798	0.2697	1.5412n	1.1241	0.4138	+ 0.032	L
15	1.7609	1.4265n	1.7723n	9.4011n	1.1735	— 0.070	S
21	1.5368	0.6874n	1.5333n	1.2020	1.0231	+ 0.073	L
21	1.4189	0.8553	1.5847n	0.6855	1.2016	— 0.027	L
June 2	1.6956n	1.4941n	1.7451n	1.5269	1.6690n	+ 0.149	L
3	1.7058	0.5188n	1.5527n	0.8803	0.9902	+ 0.305	S
7	1.6408n	1.1061	1.5453n	1.3665	1.8500	+ 0.562	L
11	1.5416	1.4143	1.6717n	1.5176	1.3162n	— 0.030	L
14	1.7083	0.6197n	1.5511n	0.8142	1.0737	+ 0.131	S
23	1.0075	1.2646	1.6741n	0.2958n	1.3948	+ 0.006	S

COEFFICIENTS OF THE EQUATIONS OF CONDITION FOR $y_1 - y_2$. ENCELADUS-DIONE.

1894.	dE_1	$\sin I_1 dN_1$	dI_1	dE_2	$e_2 \sin P_2$	$e_2 \cos P_2$	$\sin I_2 dN_2$	dI_2	$\frac{dM}{M}$	v	Observer.
										"	
March 21	0.8427	1.5559n	1.6759	0.8509n	1.0157n	1.0358n	1.5230	1.2680n	0.8451	+ 0.312	S
27	0.7869n	1.3289	1.7274	0.8575	0.2679	1.1684n	1.4729n	1.3391n	0.8711	+ 0.002	S
April 2	1.0872n	1.7608	1.3002n	0.8139n	0.4186n	1.1416n	1.4247	1.4045	9.6118	— 0.029	L
15	0.5368n	0.9683	1.7569	0.5805	0.2166n	1.0175n	1.1388n	1.5280n	0.7120	+ 0.133	L
15	0.7113n	1.2417	1.7411	0.7521	0.2974n	1.1013n	1.3616n	1.4556n	0.7510	— 0.179	L
24	0.9908n	1.7047	1.5236n	0.8868n	0.7146n	1.1670n	1.5452	1.1231	0.7046n	+ 0 272	L
26	0.6321n	1.1373	1.7487	0.6342	0.1894n	1.0347n	1.2225n	1.5101n	0.6400	— 0.593	L
27	0.7908n	1.5494	1.6818n	0.8615n	0.9729n	1.0633n	1.5602	1.0872n	1.1569n	— 0.402	L
29	0.7905n	1.3671	1.7217	0.7892	0.3828n	1.1163n	1.4236n	1.4041n	0.7670	+ 0.072	L
29	0.8669n	1.4681	1.6961	0.8520	0.5801n	1.1507n	1.5042n	1.2759n	0.8274	+ 0.153	L
29	1.0250n	1 6692	1.5473	0.8809	0.9520n	1.0968n	1.5740n	0.8752	0.9576	— 0.118	S
May 1	0.9438	1.6725n	1.5755	0.8931n	0.7969n	1.1573n	1.5659	0.9019	0 9965	— 0 048	L
2	1.0765n	1.7411	1.3374	0.7334	0.9460n	0.9282n	1.4649n	1.3847	1.0120	+ 0.591	L
9	1.0644	1.7680n	1.1384	0.5321n	0.1837n	0.9821n	1.6982	1.5289	1.0296	— 0.193	L
10	0.9516n	1.5878	1.6298	0.8828	0.7866n	1 1471n	1.5628n	0.8976n	0.8498	— 0.207	S
10	1.0229n	1.6785	1.5216	0.8661	0.9405n	1.0810n	1.5692n	0.8948	0.9423	+ 0.156	L
12	0.9941	1.7184n	1.4678	0.8180n	0.4935n	1.1261n	1.4744	1.3260	1.0443	+ 0.041	L
12	0.9331	1.6708n	1.5682	0.8746n	0.7392n	1.1472n	1.5519	1.0271	1.0119	+ 0.085	L
15	0.6124	1.4272n	1.7152	0.7842n	0.9526n	0.9733n	1.5120	1.2791n	0.8344	+ 0.235	S
21	0.9175n	1.5542	1.6447	0.8543	0.6699n	1.1366n	1.5321n	1.1341n	0.8192	+ 0.353	L
21	0.9802n	1.6340	1.5757	0.8740	0.8480n	1.1206n	1.5698n	1.5209n	0.8540	— 0.093	L
June 2	0.5953	1.1173n	1.7330n	0.4004	0.2112n	0.9204n	0.9255n	1.5320n	1.2292n	+ 0.785	L
3	0.8451	1.6087n	1.6140	0.8597n	0.7841n	1.1190n	1.5547	0.7844	0.9947	+ 0.181	S
7	0.9411n	1.6841	1.5026n	0.7907n	0.4597n	1.0994n	1.4580	1.3151	0.4760n	+ 0.130	L
11	1.0012	1.7294n	1.3272	0.5907n	0 1231n	0.9887n	1.2009	1.4880	1.0613	+ 0.203	L
14	0.8094	1.5810n	1.6233	0 8514n	0.7856n	1.1084n	1.5492	0.7180	0.9799	— 0.085	S
23	1.0178n	1.6970	1.3756	0.8150	0.9090n	1.0237n	1.5356n	0.9814	0.8199	— 0.081	S

NORMAL EQUATIONS. ENCELADUS-DIONE.

	dE_1	$\sin I_1 dN_1$	dI_1	$-dE_2$	$-e_2 \sin P_2$	$-e_2 \cos P_2$	$-\sin I_2 dN_2$	$-dI_2$	$\frac{dM}{M}$	n
dE_1	51146	− 9231	− 729	− 5831	− 27796	+ 12369	+ 4170	+ 1043	− 5807	−281.05
$\sin I_1 dN_1$		45258	− 2175	+ 4001	− 1533	− 2919	− 19343	− 4289	− 1386	+124.20
dI_1			46546	+ 2194	− 3415	− 8753	− 9606	− 7798	+ 8568	−144.92
$-dE_2$				11214	+ 2732	− 2121	− 5304	− 1713	+ 3460	− 20.30
$-e_2 \sin P_2$					55728	− 18965	− 227	+ 176	− 15941	+ 70.52
$-e_2 \cos P_2$						17455	− 47	+ 902	+ 1561	− 85.04
$-\sin I_2 dN_2$							25071	+ 6923	− 912	+ 57.01
$-dI_2$								12660	+ 775	+ 84.81
$\frac{dM}{M}$									30338	+166.00
									[nn]=	8 70

THESE EQUATIONS GIVE:

Corrections.			Corrected Elements. Epoch 1894. May 8.0, G. M. T.					
	Enc.	Di.		Enc.	Prob. Error.	Di.	Prob. Error.	
log dE	7.76124n	7.62389n	E	331°.570	± 0°.127	274°.818	± 0°.075	No. eq'n's. 54
log sin IdN	7.22854	7.48828	N	126° 23'.1	± 58'.1	127° 2'.8	± 40'.7	$[vv_x]$ 2''.322
log dI	7.58955	7.61420n	I	7 7.0	± 7.3	6 39.4	± 4.5	$[vv_y]$ 2.152
log $e \sin P$	6.94752n		$e \sin P$	− 0.00089	± 0.00163			$[vv]$ 4.474
log $e \cos P$	7.76888n		$e \cos P$	− 0.00587	± 0.00251			
log $\frac{dM}{M}$	7.80721		e	0.00594				Prob. error of an equation ± 0''.213
			P	188°.6				
			M	9''.8365	± 0''.0158			

OBSERVATION-COMPUTATION. ENCELADUS-RHEA.

1894	Greenwich M. T.	x_1	x_2	Comp $x_1 - x_2$	Obs. $x_1 - x_2$	$O-C$	y_1	y_2	Comp. $y_1 - y_2$	Obs. $y_1 - y_2$	$O-C$
		"	"	"	"	"	"	"	"	"	"
March	23.848190	+ 30.00	+ 20.04	+ 9.96	+ 9.59	− 0.37	+ 18.45	− 6.69	+ 25.14	+ 25.84	+ 0.70
April	2.803713	+ 83.10	− 33.85	+ 116.95	+ 117.77	+ 0.82	+ 0.62	− 4.63	+ 5.25	+ 6.23	+ 0.98
	15.677762	+ 55.92	+ 17.29	+ 38.63	+ 38.84	+ 0.21	+ 15.07	+ 7.75	+ 7.32	+ 7.80	+ 0.48
	15.779441	+ 64.12	+ 30.50	+ 33.62	+ 33.95	+ 0.33	+ 13.47	+ 5.69	+ 7.78	+ 8.55	+ 0.77
	24.768656	+ 61.04	− 36.52	+ 97.56	+ 97.50	− 0.06	+ 13.88	− 3.00	+ 16.88	+ 16.28	− 0.60
	25.719707	+ 69.80	+ 20.82	+ 48.98	+ 49.66	+ 0.68	− 7.40	− 5.95	− 1.45	− 1.52	− 0.07
	27.580839	− 83.12	− 37.35	− 45.77	− 45.78	− 0.01	− 2.17	− 0.42	− 1.75	− 1.94	− 0.19
	27.666834	− 82.26	− 33.12	− 49.14	− 49.39	− 0.25	− 0.06	+ 2.67	− 2.73	− 2.20	+ 0.53
	29.520112	+ 75.93	+ 33.99	+ 41.94	+ 42.19	+ 0.25	+ 9.44	+ 4.38	+ 5.06	+ 5.13	+ 0.07
	29.567958	+ 78.00	+ 36.61	+ 41.39	+ 41.57	+ 0.18	+ 8.44	+ 2.84	+ 5.60	+ 5.75	+ 0.15
May	1.657430	− 69.20	− 37.37	− 31 83	− 32.00	− 0.17	− 11.73	− 1 63	− 10.10	− 10.00	+ 0.10
	3.669145	+ 49.86	+ 36.18	+ 13.68	+ 13.39	− 0.29	+ 15.32	+ 3.11	+ 12.21	+ 12.01	− 0.20
	9.710805	+ 29.76	− 25.43	+ 55.19	+ 55.54	+ 0.35	− 14.77	− 6.40	− 8.37	− 9.18	− 0.81
	12.591384	+ 39.26	− 36.53	+ 75.79	+ 75.48	− 0.31	+ 16.02	− 2.51	+ 18.53	+ 19.20	+ 0.67
	16.605742	− 16.08	− 30.26	+ 14.18	+ 13.68	− 0.50	+ 15.71	− 5.31	+ 21.02	+ 21.32	+ 0.30
	21.510524	+ 27.49	+ 36.49	− 9.00	− 9.30	− 0.30	+ 16.33	+ 2.03	+ 14.30	+ 14.71	+ 0.41
	21.581471	+ 34.89	+ 35 84	− 0.95	− 1.16	− 0.21	+ 15.96	− 0.38	+ 16.34	+ 17.00	+ 0.66
June	3.583135	− 24.70	− 35.61	+ 10.91	+ 11.12	+ 0.21	+ 14.20	− 0.14	+ 14.34	+ 14.78	+ 0.44
	23.554251	+ 55.88	+ 28.15	+ 27.73	+ 27.60	− 0.13	− 8.60	− 3.09	− 5.51	− 5.79	− 0.28

COEFFICIENTS OF THE EQUATIONS OF CONDITION FOR $x_1 - x_2$. ENCELADUS-RHEA.

1894.	dE_1	dE_2	$e_2 \sin P_2$	$e_2 \cos P_2$	$\frac{dM}{M}$	v	Observer.
						"	
March 23	1.8890	1.5006	1.8194n	0.8859	0.9820	+ 0.038	L
April 2	0.8789n	1.2182	1.5651n	1.4742	2.0710	+ 0. 01	L
15	1.7925	1.5249n	1.7652n	1.5721	1.5893	— 0.037	L
15	1.7284	1.3451n	1.6191n	1.5320	1.5309	+ 0.206	L
24	1.7532	0.9461	1.5394n	1.3253	1.9890	— 0.301	L
25	1.6564n	1.4951	1.8179n	0.6505	1.6960	+ 0.335	L
27	0.3182	0.5517n	1.5676n	0.9532	1.6607n	+ 0.174	S
27	1.0788	1.2466n	1.6848n	9.2900	1.6936n	— 0.200	L
29	1.5268	1.1984n	1.5696n	1.4445	1.6252	+ 0.037	L
29	1.4550	0.9037n	1.5385n	1 3029	1.6188	— 0.010	L
May 1	1.6598n	0.3248	1.5426n	1.1480	1.5051n	— 0.183	L
3	1.8204	0.9731n	1.5410n	1.3285	1.1267	— 0.173	L
9	1.8856n	1.4336	1.6841n	1.5498	1 7446	— 0.010	L
12	1.8584	0.8038	1.5359n	1.2507	1.8778	— 0.399	L
16	1.9042	1.3256	1.6166n	1.4992	1.1360	— 0.025	L
21	1.8838	0.5910n	1.5342n	1.1745	0.9683n	+ 0.012	L
21	1.8659	0.9014	1.5908n	0.6265	0.0663n	+ 0.088	L
June 3	1.8806	0.7496n	1.5673n	0.7480	1.0463	+ 0.338	S
23	1.7281n	1.3148	1.6983n	0.3448n	1.4409	— 0.465	S

COEFFICIENTS OF THE EQUATIONS OF CONDITION FOR $y_1 - y_2$. ENCELADUS-RHEA.

1894.		dE_1	$\sin I_1 dN_1$	dI_1	dE_2	$e_2 \sin P_2$	$e_2 \cos P_2$	$\sin I_2 dN_2$	dI_2	$\frac{dM}{M}$	v	Observer.
											"	
March	23	0.6805n	1.0929	1.8987	0.7331	0.9710n	0.9354n	1.4300n	1.4248	1.4123	+ 0.390	L
April	2	1.2727n	1.9234	0.9412	0.8503n	0.5187n	1.1623n	1.4703	1.3468	0.7947	+ 0.912	L
	15	1.0107n	1.5963	1.8486	0.4383	0.2904n	0.9683n	0.9349n	1.5492n	0.8923	+ 0.051	L
	15	1.0888n	1.6960	1.8074	0.7734	0.3358n	1.1134n	1.3884n	1.4376n	0.9319	+ 0.390	L
	24	1.0478n	1.6539	1.8271	0.8726n	0.6478n	1.1628n	1.5256	1.2152	1.2117	— 0.957	L
	25	1.2082n	1.9052	1.4401n	0.7300	0.9496n	0.9272n	1.4571n	1.3988	0.1810n	+ 0.150	L
	27	1.2441	1.9141n	1.2200n	0.9013n	0.8439n	1.1563n	1.5750	0.6961	0.2867n	— 0.288	S
	27	1.2473	1.9236n	0.8462n	0.8760n	0.9630n	1.0862n	1.5699	0.9696n	0.3423n	+ 0.350	L
	29	1.1712n	1.8156	1.6904	0.8207	0.4721n	1.1343n	1.4637n	1.3548n	0.7106	— 0.198	L
	29	1.1883n	1.8376	1.6497	0.8699	0.6536n	1.1584n	1.5280n	1.2002n	0.7594	— 0.161	L
May	1	1.1127	1.7439n	1.7713n	0.8876n	0.7559n	1.1595n	1.5565	1.0222	0.9998n	+ 0.226	L
	3	0.9146n	1.4885	1.8698	0.8576	0.6152n	1.1508n	1.5165	1.2365n	1.0794	— 0.584	L
	9	0.9360n	1.7028	1.8169n	0.6354n	0.1705n	1.0273n	1.2378	1.5009	0.9627n	— 0.476	L
	12	0.7488n	1.2565	1.8901	0.8597n	0.6583n	1.1452n	1.5299	1.1697	1.2832	+ 0.249	L
	16	0.7752	1.5856n	1.8563	0.7337n	0.2758n	1.0760n	1.3708	1.4372	1.3288	— 0.061	L
	21	0.4646n	1.6254	1.8980	0.8577	0.6879n	1.1372n	1.5371n	1.1048n	1.1675	+ 0.050	L
	21	0.6540n	1.1071	1.8919	0.8736	0.8567n	1.1178n	1.5705n	1.4027n	1.2305	+ 0.192	L
June	3	0.8789	1.6678n	1.8146	0.8611n	0.8143n	1.1133n	1.5596	0.5833	1.1697	+ 0.005	S
	23	1.1130n	1.8471	1.5535n	0.8000	0.9166n	1.0015n	1.5254n	1.0828	0.7627n	— 0.062	S

NORMAL EQUATIONS. ENCELADUS-RHEA.

	dE_1	$\sin I_1 dN_1$	dI_1	$-dE_2$	$-e_2 \sin P_2$	$-e_2 \cos P_2$	$-\sin I_2 dN_2$	$-dI_2$	$\frac{dM}{M}$	n
										"
dE_1	66731	− 12580	− 4629	− 5509	− 20677	+ 11873	+ 3834	+ 178	+ 11583	− 158.96
$\sin I_1 dN_1$		59909	+ 16395	+ 3890	− 1124	− 4169	− 19031	− 499	+ 2991	+ 34.96
dI_1			67791	+ 1908	− 2703	− 8061	− 8261	− 7386	+ 13650	+ 347.07
$-dE_2$				7271	− 3512	− 25	− 3911	+ 5449	+ 5449	+ 5.92
$-e_2 \sin P_2$					37453	− 12717	+ 147	− 21522	− 21522	− 52.73
$-e_2 \cos P_2$						11782	− 321	− 1182	+ 11670	− 20.85
$-\sin I_2 dn_2$							18376	+ 5525	− 867	− 12.95
$-dI_2$								8310	− 14	− 45.71
$\frac{dM}{M}$									50862	+ 237.61
									[nn] =	7.47

THESE EQUATIONS GIVE:

Corrections.			Corrected Elements. Epoch, 1894, May 4.0, G. M. T.					
	Enc.	Rh.		Enc.	Prob. Error.	Rh.	Prob. Error.	
$\log dE$	7.94810n	7.61490n	E	0°.466	± 0°.215	261°.750	± 0°.069	No. eq'n's. 38
$\log \sin I dN$	6.34902n	7.11959n	N	125° 28'.2	± 8.2'.8	124° 56'.9	± 40'.8	$[vv_r]$ 0".970
$\log dI$	7.62059n	7.49558	I	6 39 .3	± 11.9	7 4 .3	± 4.7	$[vv_s]$ 3.075
$\log e \sin P$	7.41643n		$e \sin P$	− 0.00261	± 0.02194			$[vv]$ 4.045
$\log e \cos P$	7.65503n		$e \cos P$	− 0.00452	± 0 00392			Prob. error of an equation ± 0".252
$\log \frac{dM}{M}$	7.75785		e	0.00521				
			P	210°.0				
			M	9".8433	± 0".0155			

OBSERVATION-COMPUTATION.

1898.	Greenwich M. T.	cf. Satellite	x_i	x_4	Comp. $x_i - x_4$	Obs. $x_i - x_4$	$O-C$	y_i	y_4	Comp. $y_i - y_4$	Obs. $y_i - y_4$	$O-C$
			"	"	"	"	"	"	"	"	"	"
April	21.712292	Te.	− 42.75	− 18.33	− 24.42	− 23.68	+ 0.74	+ 2.83	− 13.00	+ 15.83	+ 15.84	+ 0.01
	21.772501	Rh.	− 76.93	− 25.74	− 51.19	− 51.49	− 0.30	− 2.99	− 10.32	+ 7.33	+ 6.84	− 0.49
June	6.571989	Te.	+ 20.60	+ 14.67	+ 5.93	+ 5.50	− 0 43	+ 18.01	+ 14.40	+ 3.61	+ 3.43	− 0.18
	11.563608	Te.	− 44.18	− 35.23	− 8.95	− 8.51	+ 0.44	− 3.67	− 3.88	+ 0.21	− 0.18	− 0.39
	11.595438	Di.	− 10.73	− 36.08	+ 25 35	+ 25.43	+ 0.08	− 24.46	− 1.62	+ 22.84	− 23.36	− 0.52
	30.626110	Di.	+ 4.67	− 25.68	+ 30.35	+ 30.78	+ 0.43	− 24.61	− 10.95	− 13.66	− 13.79	− 0.13
	30.643055	Te.	− 38.91	− 27.57	− 11.34	− 10.44	+ 0.90	+ 9.86	− 10.06	+ 19.92	+ 19.41	− 0.51
July	1.636979	Te.	+ 34.69	+ 27.16	+ 7.53	+ 7.74	+ 0.21	− 12.64	− 10.25	− 2.39	− 2.11	+ 0.28
	1.655266	Rh.	+ 29.96	+ 25.08	+ 4.88	+ 5.58	+ 0.70	− 31.59	− 11.20	− 20.39	− 20.66	− 0.27
	3.544715	Rh.	− 62.26	− 35.94	− 26.32	− 26.19	+ 0.13	+ 21 08	+ 0.66	+ 20.42	+ 20.30	− 0.12
	3.560640	Di.	− 20.90	− 35.73	+ 14.83	+ 14.30	− 0.53	− 22.91	+ 1.79	− 24.70	− 24.91	− 0.21
	3.582088	Te.	+ 28.62	− 35.16	+ 63.78	+ 63.77	− 0.01	− 15.37	+ 3.30	− 18.67	− 19.27	− 0.60
	25.586908	Te.	+ 12.93	− 28.98	+ 41.91	+ 41.55	− 0.36	+ 18.68	+ 8.63	+ 10.05	+ 9.74	− 0.31
Aug.	1.518573	Te.	− 42.42	− 19.54	− 22.88	− 23.09	− 0.21	− 4.13	+ 12.49	− 16.62	− 16.33	+ 0.29
	1 541153	Di.	+ 45.04	− 16.45	+ 61.49	+ 61.19	− 0.30	+ 13.85	+ 13.30	+ 0.55	+ 0.53	− 0.02
	1.562322	Rh.	+ 75.53	− 13.39	+ 88.92	+ 87.94	− 0.98	− 6.68	+ 13.93	− 20.61	− 21.00	− 0.39
	5.512199	Te.	− 36 86	− 31.38	− 5.48	− 5.48	0.00	+ 9.98	+ 6.38	+ 3.60	+ 3.02	− 0.58
	5.550423	Di.	− 36.75	− 28.33	− 8.42	− 8.71	− 0.29	− 17.66	+ 8.65	− 26.31	− 26.26	+ 0.05
	6.552386	Te.	+ 28.00	− 16.49	+ 44.49	+ 45.01	+ 0.52	− 14.66	− 13.17	− 1.49	− 1.64	− 0.15
	8.527976	Te.	+ 17.41	+ 4.51	+ 12.90	+ 12.57	− 0.33	− 17.59	+ 14.81	− 32.40	− 32.09	+ 0.31
	15.495467	Te.	+ 29.70	+ 21.05	+ 8.65	+ 8.01	− 0.64	+ 13.47	+ 11.65	+ 1.82	+ 1.94	+ 0.12
	15.517886	Di.	+ 54 09	+ 23.70	+ 30.39	+ 30.01	− 0.38	− 0.96	+ 10.65	− 11.61	− 11.78	− 0.17
	15.537977	Rh.	+ 52.66	+ 25.86	+ 26.80	+ 26.55	− 0.25	− 23.10	+ 9.66	− 32.76	− 32.39	+ 0.37
	16.499592	Te.	− 35.04	− 28.95	− 6.09	− 6.38	− 0.29	− 10.50	+ 7.80	− 18.30	− 19.40	− 1.10
	16.530657	Di.	− 39.02	− 26.11	− 12.91	− 13.17	− 0.26	− 16.18	+ 9.50	− 25.68	− 25.51	+ 0.17
	22.481543	Te.	− 37.34	+ 32.36	− 69.70	− 68.83	+ 0.87	+ 8.59	+ 4.19	+ 4.40	+ 4.17	− 0.23
	22.498198	Di.	− 49.94	+ 33.00	− 82.94	− 82.12	+ 0.82	+ 8.36	+ 3.10	+ 5.26	+ 5.61	+ 0.35
	22.522712	Rh.	− 34.70	+ 33.60	− 68.30	− 67.39	+ 0.91	+ 28.33	+ 1.48	+ 26.85	+ 27.26	+ 0.41

COEFFICIENTS OF THE EQUATIONS OF CONDITION

1898.	FOR $x_i - x_4$. dE	$e \sin \pi$	$e \cos \pi$	$\frac{dM}{M}$	v	FOR $y_i - y_4$. dE	$e \sin \pi$	$e \cos \pi$	$\gamma \sin \theta$	$\gamma \cos \theta$	$\frac{dM}{M}$	v
					"							"
April 21	1.4728n	1.6813	1.5967	1.3878n	− 0.004	0.9044	1.2963n	0.7737	1.4629n	1.0759n	1.1995	− 0.313
21	1.3726n	1.6430	1.4899	1.7092n	+ 1.064	1.0517	1.3720n	0.8872	1.4939n	0.5493n	0.8651	+ 0.153
June 6	1.5206	1.6741	1.6887	0.7731	+ 0.153	0.8043n	1.2689n	0.6969	1.4352	1.2558	0.5575	+ 0.204
11	0.9511n	1.4447	1.4459	0.9518n	− 0.150	1.1833	1.4443n	1.1172	1.4925n	1.0034	9.3222	− 0.005
11	0.5715n	1.3557	1.4627	1.4039	− 0.052	1.1946	1.4368n	1.1858	1.4666n	1.1612	1.3587n	+ 0.232
30	1.4031n	1.6342	1.5674	1.4822	− 0.043	1.0459	1.3836n	0.7301	1.4994n	0.8865n	1.1354n	− 0.100
30	1.3661n	1.6173	1.5400	1.0546n	− 0.345	1.0767	1.4008n	0.7844	1.5062n	0.7181n	1.2993	+ 0.106
July 1	1.3743n	1.0451	1.7280	0.8768	+ 0.148	1.0703n	1.2317n	1.2824	1.0043	1.4897n	0.3784n	− 0.093
1	1.1128n	1.0969	1.7492	0.6884	− 0.293	1.0357n	1.2012n	1.2684	0.8738	1.4999n	1.3094n	+ 0.533
4	0.1805	1.2328	1.5934	1.4203n	+ 0.064	1.1917	1.4217n	1.2158	1.4341n	1.2484	1.3100	− 0.375
3	0.6164	1.1782	1.5242	1.1712	+ 0.474	1.1892	1.4096n	1.2383	1.4116n	1.2934	1.3927n	− 0.059
3	0.8821	1.1066	1.5578	1.8047	− 0.342	1.1821	1.3895n	1.2625	1.3756n	1.3442	1.2711n	+ 0.308
25	1.3001	0 9446	1.6859	1.6223	− 0.064	1.0980	1.2706n	1.2751	1.1385n	1.4562	1.0022	− 0.052
Aug. 1	1.4606	1 1754	1.7717	1.3594n	+ 0.011	0.9270	1.1390n	1.2008	0.3893n	1.4961	1.2206n	− 0.428
1	1.4877	1.2555	1.7856	1.7888	− 0.328	0.8522	1.1117n	1.1633	9.9036	1.4973	9.7404	− 0.175
1	1.5077	1.3255	1.7942	1.9490	+ 0.211	0.7630	1.0936n	1.1207	0.5846	1.4941	1.3141n	+ 0.324
5	1.1684	0.9342	1.6251	0.7388n	− 0.123	1.1329	1.3200n	1.2669	1.2565n	1.4068	0.5563	+ 0.223
5	1.3006	0.9338	1.6833	0.9253n	+ 0.112	1.0885	1.2619n	1.2671	1.1247n	1.4514	1.4202n	− 0.227
6	1.4834n	1.6324	1.6649	1.6483	− 0.130	0.8535	1.2807n	0.5578	1.4228n	1.2184n	0.1732n	− 0 040
8	1.5343	1.5758	1.7584	1.1106	− 0.032	0.2902n	1.1515n	0.7646	1.2730	1.3949	1.5105n	− 0.179
15	1.4295	1.6203	1.6011	0.9370	+ 0.410	0.9600n	1.3283n	0.5838	1.4542	1.0684	0.2601	− 0.047
15	1 3906	1.6067	1.5642	1.4828	+ 0.067	1.0115n	1.3563n	0.6429	1.4700	0.9406	1.0649n	+ 0.286
15	1.3483	1.5881	1.5317	1.4281	− 0.020	1.0493n	1.3775n	0.7102	1.4798	0.7760	1.5153n	− 0.096
16	1.2552	0.9158	1.6575	0.7846n	+ 0.125	1.0986	1.2751n	1.2629	1.1666n	1.4311	1.2625n	− 0 876
16	1.3110	0.9638	1.7007	1.1109n	+ 0.081	1.0536	1.2249n	1.2529	1.0292n	1.4594	1.4096n	− 0.328
22	0.6834	1.4172	1.4336	1.8432n	− 0.545	1.1474n	1.4216n	1.0190	1.4717	0 8390n	0.6435	+ 0.387
22	0.8543	1.3721	1.4342	1.9188n	− 0.405	1.1559n	1.4213n	1.0631	1.4626	0.9610n	0.7210	− 0.204
22	1.3319	1.2977	1.4464	1.8344n	− 0.529	1.1637n	1.4158n	1.1198	1.4442	1.0913n	1.4289	− 0.362

NORMAL EQUATIONS.

	dE	$e \sin \pi$	$e \cos \pi$	$\gamma \sin \theta$	$\gamma \cos \theta$	$\frac{dM}{M}$	$-n$
dE	17688	— 11	+ 11147	— 7047	+ 3968	+ 4152	+279.90
$e \sin P$		36400	+ 21730	+ 2994	— 4543	+ 4199	−146.55
$e \cos P$			58040	— 2348	+ 3898	+ 6436	+ 84.06
$\gamma \sin \theta$				15716	— 2446	+ 47	−143.39
$\gamma \cos \theta$					12209	— 4505	+ 73.74
$\frac{dM}{M}$						53378	+293.12
						$[nn]=$	11.65

THESE EQUATIONS GIVE:

Corrections.		Corrected Elements. Epoch 1898. July 8.0, G. M. T.				
	Enc.		Enc.	Prob. Error.		
$\log dE$	8.1392	E	250°.346	± 0°.116	No. eq'n's.	56
$\log \gamma \sin \theta$	$7.3055n$	$\gamma \sin \theta$	— 6'.95	± 6'.80	$[vv_x]$	2''.893
$\log \gamma \cos \theta$	7.2187	$\gamma \cos \theta$	+ 5.69	± 7.71	$[vv_y]$	2.514
$\log e \sin \pi$	$7.5959n$	$e \sin \pi$	— 0.00394	± 0.00142	$[vv]$	5.407
$\log e \cos \pi$	$6.6623n$	$e \cos \pi$	— 0.00046	± 0.00116		
$\log \frac{dM}{M}$	7.6926	γ	8'.98		Prob. error of an equation ± 0''.222	
		θ	309°.313			
		e	0.00397			
		π	263°.354			
		M	9''.9488	± 0''.0045		

OBSERVATION-COMPUTATION.

1900.	Greenwich M. T.	cf. Satellite	x_i	x_4	Comp. $x_i - x_4$	Obs. $x_i - x_4$	$O-C$	y_i	y_4	Comp. $y_i - y_4$	Obs. $y_i - y_4$	$O-C$
			′	″	″	″	″	″	″	″	″	″
June	21.629877	Te.	− 39.65	− 36.22	− 3.43	− 3.22	+ 0.21	− 8.81	+ 0.08	− 8.89	− 8.62	+ 0.27
July	2.612444	Di.	− 43.41	− 35.93	− 7.48	− 7.56	− 0.08	+ 16.56	− 1.86	+ 14.70	+ 14.94	+ 0.24
	2.638727	Te.	− 1.87	− 35.17	+ 33.30	+ 33.19	− 0.11	− 19.04	+ 3.76	− 22.80	− 22.86	− 0.06
	2.667038	Rh.	+ 72.83	− 33.78	+106.61	+107.38	+ 0.77	+ 14.37	+ 5.74	+ 8.63	+ 9.28	+ 0.65
	4.600534	Te.	− 12.84	+ 35.50	− 48.34	− 48.87	− 0.53	− 18.27	+ 3.02	− 21.29	− 21.21	+ 0.08
	4.641031	Di.	− 35.17	+ 36.15	− 71.32	− 71.73	− 0.41	− 20.10	+ 0.01	− 20.11	− 20.29	− 0.18
	6.559485	Te.	− 22.63	− 29.21	+ 6.58	+ 6.24	− 0.34	− 16.43	− 9.42	− 7.01	− 6.66	+ 0.35
	6.584855	Rh.	+ 24.52	− 31.48	+ 56.00	+ 56.44	+ 0.44	+ 33.43	− 7.85	+ 41.28	+ 41.76	+ 0.48
	13.542215	Di.	− 44.16	− 35.98	− 8.18	− 8.36	− 0.18	+ 16.04	− 0.27	+ 16.31	+ 16.65	+ 0.34
	13.572932	Rh.	− 46.01	− 35.70	− 10.31	− 10.20	+ 0.11	− 28.53	+ 2.00	− 30.53	− 30.61	− 0.08
	27.546892	Di.	− 7.51	− 6.48	− 1.03	− 1.18	− 0.15	+ 24.95	+ 15.62	+ 9.33	+ 9.58	+ 0.25
	27.586380	Te.	− 43.57	− 0.08	− 43.49	− 44.36	− 0.87	− 2.71	+ 15.88	− 18.59	− 18.83	− 0.24
	27.611876	Rh.	− 76.10	+ 4.07	− 80.17	− 80.92	− 0.75	− 8.72	+ 15.78	− 24.50	− 24.76	− 0.26
	30.542518	Di.	+ 25.14	+ 29.63	− 4.49	− 4.69	− 0.20	+ 22.49	+ 8.69	+ 13.80	+ 14.08	+ 0.28
	30.568073	Te.	+ 41.15	+ 31.70	+ 9.45	+ 9.54	+ 0.09	− 6.62	+ 7.09	− 13.71	− 13.78	− 0.07
	30.593397	Rh.	+ 57.70	+ 33.32	+ 24.37	+ 24.29	− 0.08	− 23.97	+ 5.40	− 29.37	− 28.89	+ 0.48
Aug.	28.489962	Te.	− 30.69	+ 33.18	− 63.87	− 64.47	− 0.60	− 12.55	− 3.46	− 9.09	− 9.07	+ 0.02
	28.515447	Di.	− 46.03	+ 32.05	− 78.08	− 78.74	− 0.66	− 12.75	− 5.18	− 7.57	− 7.49	+ 0.08
	28.542109	Rh.	− 74.18	+ 30.40	−104.58	−105.35	− 0.77	+ 6.54	− 6.90	+ 13.44	+ 13.44	+ 0.00
	31.533974	Rh.	+ 26.89	− 1.69	+ 28.58	+ 28.92	+ 0.34	− 31.45	− 15.22	− 16.23	− 16.22	+ 0.01

COEFFICIENTS OF THE EQUATIONS OF CONDITION

	FOR $x_1 - x_4$					FOR $y_1 - y_4$						
1900.	dE	$e \sin \pi$	$e \cos \pi$	$\frac{dM}{M}$	v	dE	$e \sin \pi$	$e \cos \pi$	$\gamma \sin \theta$	$\gamma \cos \theta$	$\frac{dM}{M}$	v
					"							"
June 21	9.2572	1.4898	1.2773	0.5081n	− 0.303	1.2036	1.2175n	1.4373	1.2231n	1.4453	0.9354n	− 0.085
July 2	0.6222	1.4598	1.3620	0.8787n	− 0.032	1.2019	1.1866n	1.4462	1.1444n	1.4665	1.1745	+ 0.119
2	0.9284	1.4461	1.4357	1.5210	+ 0.317	1.1925	1.1297n	1.4524	1.0139n	1.4877	1.3591n	+ 0.122
2	1.1126	1.4492	1.5045	2.0309	+ 0.005	1.1750	1.0572n	1.4510	0.7965n	1.5026	0.9676	− 0.356
4	0.8330	1.5468	1.1565	1.6890n	+ 0.027	1.1970n	1.2860n	1.3985	1.3454	1.3736n	1.3265n	− 0.095
4	9.1343	1.4862	1.2831	1.8557n	− 0.201	1.2049n	1.2348n	1.4324	1.2407	1.4365n	1.3074n	+ 0.169
6	1.3271n	1.7009	1.0635	0.7955	+ 0.267	1.1126	1.3148n	1.2620	1.4786n	1.0750	0.8236n	− 0.175
6	1.2480n	1.6635	1.0417	1.7516	− 0.125	1.1450	1.3201n	1.3037	1.4552n	1.1846	1.6208	+ 0.075
13	9.7766n	1.4846	1.2807	0.9223n	+ 0.052	1.2041	1.2462n	1.4267	1.2544n	1.4275	1.2213	+ 0.040
13	0.6523	1.4508	1.3733	1.0084n	− 0.238	1.2008	1.1936n	1.4428	1.1470n	1.4627	1.4859n	+ 0.092
27	1.5437	1.7365	1.6464	0.0704n	− 0.049	0.4617	0.8317n	1.1821	1.3567	1.3476	0.9813	− 0.076
27	1.5510	1.7709	1.5990	1.6470n	+ 0 300	8.5599	0.9457n	1.1208	1.4212	1.2505	1.2749n	+ 0.189
27	1.5480	1.7845	1.5567	1.9080n	− 0.122	0.2591n	1.0198n	1.0916	1.4512	1.1644	1.3938n	+ 0.163
30	1.2890	1.6751	1.0921	0.6716n	− 0.075	1.1220n	1.3242n	1.2607	1.4663	1 0869n	1.1486	− 0.041
30	1.2002	1.6354	1.0821	0.9795	− 0.225	1.1513n	1.3268n	1.3026	1.4415	1.1917n	1.1393n	+ 0.103
30	1.0820	1.5926	1.1056	1.3854	+ 0.091	1.1730n	1.3206n	1.3401	1.4091	1.2706n	1.4607n	− 0.558
Aug. 28	0.8867n	1.3984	1.4262	1.8094n	− 0.129	1.1736n	1.1569n	1.4213	1.0554	1.4506n	0.9577n	+ 0.046
28	1.0614n	1.3958	1.4864	1.8962n	+ 0.112	1.1586n	1.0993n	1.4223	0.9027	1.4676n	0.8742n	− 0.009
28	1.1859n	1.4100	1.5405	2.0226n	+ 0.045	1.1357n	1.0309n	1.4161	0.6388	1.4787n	1.1283	+ 0.223
31	1.5296n	1.7514	1.5737	1.4612	− 0.118	9.8797	0.9726n	1.0820	1.4086n	1.2072n	1.2101n	− 0.031

NORMAL EQUATIONS.

	dE	$e \sin \pi$	$e \cos \pi$	$\gamma \sin \theta$	$\gamma \cos \theta$	$\frac{dM}{M}$	$-n$
							"
dE	10750	+ 4674	+ 3319	− 4180	+ 5875	− 578	+ 24.65
$e \sin P$		37836	+ 12376	− 767	− 486	− 7520	+ 201.46
$e \cos P$			24667	+ 720	+ 1375	− 11066	+ 47.31
$\gamma \sin \theta$				8806	− 4222	− 2947	+ 30.71
$\gamma \cos \theta$					11418	+ 1391	− 37.85
$\frac{dM}{M}$						62773	− 478.17
						$[nn]=$	5.94

THESE EQUATIONS GIVE:

Corrections.		Corrected Elements. Epoch 1900. July 8.0, G. M. T.				
	Enc.		Enc.	Prob. Error.		
$\log dE$	7.5243	E	165°.102	± 0°.093	No. eq'n's.	40
$\log \gamma \sin \theta$	7.3073	$\gamma \sin \theta$	+ 6'.98	± 5'.60	$[vv_1]$	0".596
$\log \gamma \cos \theta$	7.4214n	$\gamma \cos \theta$	− 9.07	± 5.24	$[vv_2]$	0.692
$\log e \sin \pi$	7.6814	$e \sin \pi$	+ 0.00480	±0.00076	$[vv]$	1.288
$\log e \cos \pi$	7.6301n	$e \cos \pi$	− 0.00427	±0.00096		
$\log \frac{dM}{M}$	7.8814n	γ	11'.44		Prob. error of an equation ± 0".131	
		θ	142°.440			
		e	0.00642			
		π	131°.625	"		
		M	9".8247	±0.0055		

A check solution was made for 1894 by substituting, in the normal equations, corrections to the elements of the comparison satellites, and transferring these terms to the absolute terms, and then combining the three sets of normal equations into one set, in which the unknown quantities referred to Enceladus only. A solution of this set gave the same as the mean of the three solutions given above. The latter were weighted according to the number of observations. The following are the elements of Enceladus:

	1894.35 May 8.0, G. M. T.	1898.52 July 8.0, G. M. T.	1900.52 July 8.0, G. M. T.
	°	°	°
E	331.281	250.346	165.102
γ	1′.5	9′.0	11′.4
θ	131°.3	309°.3	142°.4
e	0.00581	0.00397	0.00642
P	202°.1	28°.9	257°.2

The mean longitude for 1889.25 + t, (April 0.0), as given by H. Struve is

$$E = 199°.330 + nt + dE. \quad (t = \text{number of days from April 0.0})$$

where:

$$dE = + 11'.24 \sin (143° + 92°.4t) + 20'.0 \sin (75° + 29°.3t)$$

(t = number of years from 1889.25.)

Hence we have, assuming this value of dE, and $n = 262°.732$:

	E °	E (reduced to 1900.52.) °	E (Struve) °	Diff. °
1894.35	331.281	164.161	164.679	— 0.518
1898.52	250.346	164.968	"	+ 0.289
1900.52	165.102	164.682	"	+ 0.003

The agreement for 1900 is very close, but the difference for 1894 may indicate that there are other terms to be considered in dE.

The eccentricity, $e = 0.0058$, is a little larger than that found before.

The longitudes of perisaturnium, P, agree with the theoretical motion given by Struve: $\Delta P = (2n_1 - n) = + 123°.4$, per year; where n_1 is the mean motion of Dione.

The inclination, γ, being a small quantity, the longitudes of the node, θ, are uncertain, but agree approximately with the theoretical motion, $\Delta\theta = -\beta n = -152°.7$, per year; β being a function of the constants e_i, n_i, and Δ_i, and of the masses, m_i.

These results, then, are only another proof of the excellence, in theory and observation, of Struve's work.

UNIVERSITY OF VIRGINIA,
December 31, 1900.