Optimizing Electoral Boundaries:
A Network Flow Approach to Political Districting

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Optimizing Electoral Boundaries

A Network Flow Approach to Political Districting

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While many computational approaches to redistricting focus on optimizing for and evaluating population equality, compactness, and contiguity, few take into account criteria such as minority representation. Thus In this paper, I describe a model for computational redistricting that uses network flow to assign geographic units to district centers while optimizing for population equality and compactness. I use the model to generate example maps for Virginia's Congressional and General Assembly districts and evaluate the produced districts on the basis of population equality and minority representation, comparing them against the maps drawn by the Supreme Court of Virginia in the 2021 redistricting cycle. My results show that this approach is capable of generating compact and contiguous districts with population standard deviations of 0.0571% at the Congressional level, 0.3489% at the state Senate level, and 0.9006% at the state House level. The generated plans also show an increase in the number of minority opportunity districts compared to the current maps. This work demonstrates the potential for computational methods to be used in a neutral and transparent manner to draw political districts that meet a variety of criteria. Future work could expand the model to include additional criteria such as the preservation of Communities of Interest and political competitiveness.

Additional Key Words and Phrases: redistricting, political geography, network flow, optimization, clustering

1 INTRODUCTION

Every ten years, in response to data reported in the United States census, electoral district boundaries are redrawn to account for population changes in a process known as redistricting. Because of the high-stakes nature of the single-member districts which compose our electoral system, redistricting is a fiercely political battleground with a history of manipulation [9]. Historically, redistricting has been carried out by state legislatures, where the controlling party both commissions and approves the new district maps [2]. This has led to a practice known as *gerrymandering*, which is the distortion of district boundaries for political gain [9]. Thus with legislators in control of the process, it is said that, "In an election, the voters choose their politicians; but in redistricting, the politicians choose their voters" [21].

Beginning in the computer revolution of the 1960s, there have been extensive research efforts to depoliticize redistricting by automating the process. Computational approaches seek to solve redistricting as an *optimization problem*, where the goal is to minimize or maximize some objective function while satisfying a set of constraints [9]. However, selecting the *best possible* map from the set of legal maps is an *NP-Hard* problem [15], thus computational approaches must rely on *heuristic optimization* to find plans that are *good enough* [9].

Redistricting involves the clustering of geographic census units called *blocks* in such a way that satisfies federal and sate requirements, and the redistricting criteria that a plan must abide by varies from state to state. In Virginia, which has a redistricting commission tasked with drawing its maps, plans must satisfy the following requirements:

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- *Population equality*, which is defined to be a deviation from the ideal size of no more than ±*one person* for Congressional districts and a deviation of no more than ±5% for General Assembly districts.
- *Contiguity*, which requires districts to be composed of contiguous territory, such that no district is connected only by bodies of water.
- *Compactness*, which requires that districts are drawn "employing one or more standard numerical measures of individual and average district compactness, both statewide and district by district."
- *The Voting Rights Act of 1965* (VRA) and the *Equal Protection Clause of the 14th Amendment*, which require that districts are not drawn with race as the predominant factor, and that districts "shall provide, where practicable, opportunities for racial and ethnic communities to elect candidates of their choice."
- Preservation of *Communities of Interest* (COI), which are defined to be a "neighborhood or any geographically defined group of people living in an area who share similar social, cultural, and economic interests."
- *Political Neutrality*, such that "districts shall not, when considered on a statewide basis, unduly favor or disfavor any political party" [6].

Most of the literature on computational approaches focuses on optimizing for the general criteria of population equality, contiguity, and compactness, but evaluating an algorithm's compliance with the VRA is rare [3]. Thus in this paper, I describe a heuristic optimization model that uses a *capacitated clustering* approach leveraging network flow. I use the model to generate example maps for Virginia's Congressional and General Assembly districts, and evaluate the produced districts using the criteria of population equality and minority representation. I then compare my results to the maps drawn by the Supreme Court of Virginia for the 2021 redistricting cycle in Grofman and Trende [13]. My results show that this approach is beneficial in that it can generate district plans with low population deviations and strong minority representation opportunities.

2 RELATED WORK

The first computational approach to redistricting was published in 1963 by Weaver and Hess [24], which recognizes redistricting to be analogous to the *warehouse location-allocation* problem found in operations research. In the location-allocation problem, the objective is to identify the number, location, and size of the warehouses that will most efficiently serve a set of customers with goods [7]. Weaver and Hess formulate the problem such that districts are the warehouses and population units are the customers. In order to minimize the assignment cost, a compactness measure is proposed based on the physics principle of *moment of inertia*, which is the sum of squared distances from each unit to its axis of rotation, as this measure is smallest when the units are concentrated at the center [24].

Due to the intractability of the problem, [24] uses Cooper's iterative location-allocation heuristic [7] where for each iteration, the new district centers are first located after which each population unit is allocated to a district. This procedure continues until there is no change in the location of the district centers. The allocation of each unit to a district is done using a subroutine that uses linear programming to solve what is known as the *transportation problem*, which seeks to minimize the distribution cost of transporting M goods to N locations [5], where the cost is defined to be the *moment of inertia* [24]. However, the transportation problem is solved in such a way that units can be split between districts, and requires a procedure to recombine these split units. Furthermore, the algorithm makes no guarantees of contiguity in the model and requires manually rejecting plans that are discontiguous. Despite these limitations, [24] is considered to be a seminal paper in computational redistricting and there have been many approaches that have built upon this foundation.

Rather than solving the allocation problem using a linear programming approach, George et al. [11] models the problem as one using minimum-cost network flow. In this approach, a flow network is created where each geographic unit is represented as a node with its supply equal to the population of the unit. These nodes are each connected to a set of nodes representing the district centers with demand set to the sum of the population allocated to the specific district center. All district nodes connect to a *super sink* node S with its demand set to the sum of the population of all geographic units.

The model implements two arc cost functions that the flow network seeks to optimize. The first is the population flow from the ith geographic unit to the jth district, denoted $f_{ij}(u_{ij})$, while the second is the population flow from the jth district to the *super sink* node S, denoted $g_j(v_j)$. Various formulations for these two cost functions are presented, including setting the cost of $f_{ij}(u_{ij})$ equal to the population-weighted distance $d(i, j) \cdot u_{ij}$, similar to [24], and the cost of $g_j(v_j) = 0$.

[11] is modeled such that district capacities can be within some population threshold, and implements this by setting upper and lower bounds to the arc flow, demonstrating an ability to create districts within a population deviation of ±5%. However, while this population deviation is sufficient to satisfy Virginia's General Assembly threshold, it does not meet the more strict Congressional requirements found in many states. Nevertheless, modeling the allocation problem as one using network flow is beneficial due to its performance and flexibility, with network structure and arc cost functions easily augmented to further constrain a model.

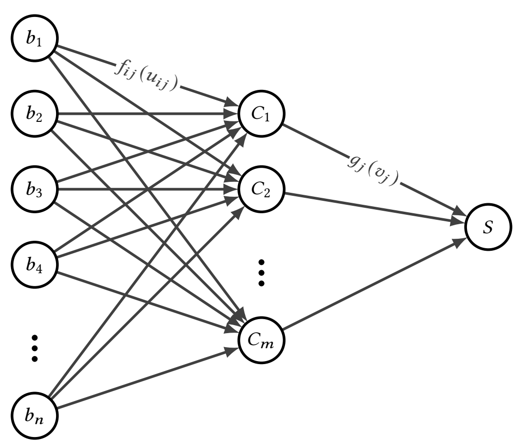

Fig. 1. Example Minimum-Cost Flow Network in [11]

The work in this paper modifies the approach in [11] using a network inspired by Bradley et al. [4], which describes a *constrained k-means* algorithm that sets capacity limits to clusters. The model in [4] is similar to that of [11] with three main differences. First, [4] is focused on cluster sizes where the cardinality of the cluster is equal to the total number of units assigned to the cluster. Thus the supply of each ith node is 1. Second, the arc cost function from the ith unit to the jth center is defined to be the euclidean distance $d(i, j)$. Finally, to satisfy capacity constraints, the set of center nodes have their demand set to the *minimum* cluster size and the artificial node has demand equal to the difference of the total supply and the sum of the minimum capacity for each cluster center.

3 METHODS

This section describes the methodology employed to generate Virginia's Congressional and General Assembly districts. In 3.1, I detail the network flow model used to assign geographic units to district centers, along with the general capacitated clustering approach. In 3.2, I describe the data required for the experiments, as well as key preprocessing steps that were taken. Finally, in 3.3, I describe the evaluation criteria used to assess the generated districts.

3.1 Model

In order to extend [4] to political districting, the supply of each ith node is set to the total population of the geographic unit instead of 1. Because some geographic units have 0 population, the algorithm also adds an artificial supply of 1 to any units without any population to guarantee appropriate flow. Further, a minimum capacity constraint is not a sufficient criteria for population equality, as excess supply can end up allocated to a single cluster. Thus this model constrains the district capacities using *lower-bound* (L_ϵ) and upper-bound (U_ϵ) thresholds.

The capacity constraints are implemented in two steps. Let N be the number of geographic units and M be the number of district centers. First, a set of M intermediary *transshipment* nodes is introduced, which have a demand of zero and to which every geographic unit connects. Each of the M transshipment nodes then connects to its respective district node with an *arc flow capacity* equal to the maximum population deviation threshold. Finally, the set of district center nodes have a *demand* equal to the minimum population deviation. Similar to [4], excess supply is handled by an *artificial node* with demand $k \cdot L_\epsilon - \sum_{i=1}^{n} s_i$, where $s_i \in S$ is the population of the ith block.

As in [11], the arc cost of each ith block to jth transshipment node is the population-weighted euclidean distance $d(i, j) \cdot s_i$, and the arc cost from each *transshipment node* to district center, and district center to *artifical node*, is 0.

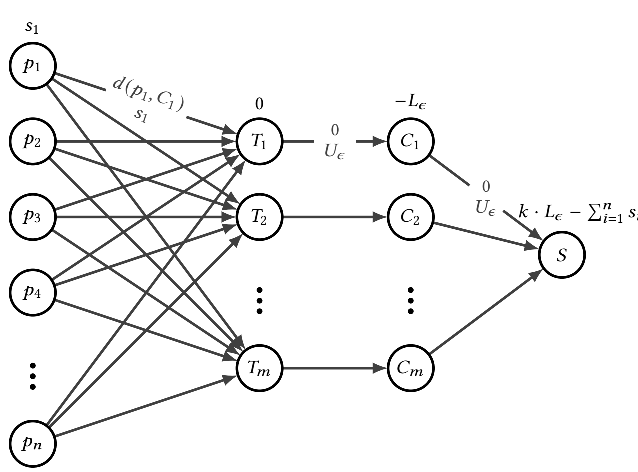

Fig. 2. Minimum-Cost Flow Network Formulation

There are two limitations to this formulation. The first is that, similar to [11, 24], the supply from each ith geographic unit can be assigned to multiple district centers. However, this occurs when a district is close to equidistant between two centers. To overcome this, each district is assigned to the $\arg\max u_{ij}$, where u_{ij} is the flow from the ith unit to the jth center. However, minimum flow constraints can be added to ensure that each unit is assigned to a single district center.

The second limitation is that this formulation does not explicitly model for contiguity, instead optimizing for *compactness* with contiguity considered to be a byproduct. Duque et al. [10] shows that network flow is one of three valid ways to model contiguity in a mathematical program, and Shirabe [22] uses network flow with explicit contiguity constraints in the context of districting, thus making it possible to augment the network model to add contiguity as a formal constraint.

For the purposes of this paper, the issue of contiguity is addressed through an *a posteriori* approach similar to that of Mulvey and Beck [20] and Liao and Guo [17]. Unlike [17, 20] which consider each pairwise swap of units to cluster centers, graph theory is leveraged to identify components which are disconnected. Let $G = (V, E)$ be a graph representing the geographic units with $V = \{x \in X\}$ and $E = \{(u, v) \mid u, v \in X \text{ and } u \sim v\}$. Let G_j be the subgraph for the geographic units of district j and C_j be the set of connected components of G_j. The switch of each of the geographic units in the set $C_j \setminus C_{j_{\max}}$ from district j to each district $d \in D, d \neq j$ is considered. If this creates a connected component, the switch is made.

Procedure Capacitated Clustering for Political Districting

```
Data: X = {(x_i, y_i, p_i)}, i = 1...n
Result: L = {j_i}, j = 1...k, i = 1...n
begin
    I ⟵ ∞
    L ⟵ ∅
    G ⟵ Graph(X)
    for m = 1 to MajorIterations do
        Z ⟵ RandomSelection(X, k)
        Δ ⟵ ∞
        while Δ > δ do
            L' ⟵ MinCostFlowAssignment(X, Z, ε)
            Z' ⟵ Centroids(X, L')
            I' ⟵ Inertia(X, Z', L')
            if I' < I then
                I ⟵ I'
                L ⟵ L'
            end
            Δ ⟵ FrobeniusNorm(Z, Z')
            Z ⟵ Z'
        end
        for j = 1 to k do
            C_j ⟵ ConnectedComponents(G_j)
            if |C_j| > 1 then
                Switch(C_j, Z)
            end
        end
    end
end
```

The general procedure in Capacitated Clustering for Political Districting follows that of the iterative location-allocation heuristic [4, 7, 11, 19, 24]. Let there be M *major iterations*, which seek to minimize the *moment of inertia*. For each major iteration, k random centers are chosen. The algorithm then iteratively alternates between solving the minimum-cost flow allocation subroutine and determining the new district centers. This procedure is repeated until convergence, or when the Frobenius norm of current and previous district centers are within some threshold δ. Once a solution is found, the contiguity check and correction is performed, after which the next major iteration begins.

3.2 Data

Three datasets from the United States Census Bureau were required for these experiments. Demographic data is from the 2020 Redistricting Data (PL 94-171) dataset, with two tables used: *P1* for *Total* and *Black* populations, and *P2* to define the *Non-Hispanic White* population. Geographic data is from the TIGER/Line Shapefiles from 2023. While it is common for redistricting algorithms to use block-level data, which is the smallest geographic subdivision, I instead opted for the *block group* granularity, which is a superset of the *block* granularity and better preserves political boundaries such as counties and cities.

There were two modifications needed to prepare the geographic data for use with the model. First, the data was projected to *EPSG:3968* such that planar coordinates could be used with the euclidean distance metric. Second, Virginia's Eastern Shore is a challenge for automated redistricting, as it is connected to mainland Virginia by a single bridge. The population of the Eastern Shore is not large enough such that it can be drawn as a single district, thus this region was geographically constrained to the mainland block group to which the bridge connects such that it would not violate the contiguity requirements of [6]. Given *a priori* knowledge of a geography, similar approaches could be taken to constrain other regions, or to ensure the preservation of political subdivisions and communities of interest.

3.3 Evaluation Criteria

The districts generated using this approach are evaluated on the basis of *population deviation* and *minority representation*. For minority representation, I consider four different district composition criteria. The first are *Black Majority* districts, which are districts which have a Black population in excess of 50%. The second are *Minority-Majority* districts, where *Minority* is defined to be the district population excluding *Non-Hispanic White*. Additionally, as Lublin et al. [18] argue, due to an increase in *politically polarized voting*, recent electoral data suggests that minorities now have a higher probability of representation in districts without a majority. [18] finds this "sweet spot" to be in the 40-50% range. Thus I also consider both *Black Opportunity* and *Minority Opportunity* districts, which are defined to be those with a *Black* or *Minority* population $\geq$ 40%.

Finally, I compare the generated districts using this criteria to Virginia's district maps from the 2021 redistricting cycle as drawn by the state Supreme Court in [13].

4 RESULTS

Experimental results are summarized below. For annotated maps of the generated districts, as well as detailed population and minority composition data, please see the appendices.

At the Congressional level, Virginia's strict population deviation requirement is difficult to meet when using a *block group* granularity. The maps drawn in [13] have a population deviation of 0%, with 10 of the 11 districts meeting the ideal population target and the last with a deviation of one person from the ideal. However, Levin and Friedler [16] find that few algorithmic approaches achieve Congressional population deviations within 0.5%. Further, DeFord and Duchin [8] note that Virginia is one of 13 states with a Congressional deviation requirement of $\pm$*one person*, arguing that overly strict population deviation requirements are often an excuse for poor redistricting practices.

Thus despite not meeting Virginia's strict Congressional requirements, this approach is able to draw districts at this level within a population deviation $< 0.1\%$ which is well within a reasonable target of $< 0.5\%$.

Table 1. Congressional District Population Deviations

Ideal Population	**Std Deviation**	**Min Deviation**	**Max Deviation**
784,672	0.0571	0.0033	0.0966

The generated Congressional maps demonstrate strong opportunities for minority representation. In [13] there are 2 *Minority-Majority* and 6 *Minority Opportunity* districts, compared to the generated map which has 4 *Minority-Majority* and 8 *Minority Opportunity* districts. However, [13] has 2 *Black opportunity* districts compared to 1 in the generated map, while neither achieves a *Black Majority* district.

Table 2. Congressional District Minority Representation

Black Majority	**Black Opportunity**	**Minority-Majority**	**Minority Opportunity**
0	1	4	8

At the state Senate level, the algorithm performs well in regards to population deviation, with a maximum deviation of 0.7285%. This is well within the alloted threshold in [6], and also significantly lower than the maximum deviation of 2.40% in [13].

Table 3. State Senate District Population Deviations

Ideal Population	**Std Deviation**	**Min Deviation**	**Max Deviation**
215,784	0.3489	0.0093	-0.7285

The minority representation in the generated state Senate maps is comparable to that of the maps in [13], and generates districts that favor *Opportunity* versus *Majority* compositions. In [13], there are 2 *Black Majority*, 15 *Minority-Majority*, 2 *Black Majority*, and 6 *Black Opportunity* districts.

Table 4. State Senate District Minority Representation

Black Majority	**Black Opportunity**	**Minority-Majority**	**Minority Opportunity**
1	6	13	22

The population deviation for state House district maps is significantly more challenging at the *block group* granularity due to the fact that there are 100 districts and block groups contain more population per unit. However, the algorithm still generated districts with a maximum population deviation of 2.4376%, less than the maximum deviation of 2.50% in [13].

The algorithm's ability to create strong opportunities for minority representation that reflects the composition of the state is best at the House level. The 8 *Black Majority* and 14 *Black Opportunity* districts surpass that of the 5 and 13, respectively, in [13]. Furthermore, 37 *Minority-Majority* and 51 *Minority Opportunity* districts were created compared to 31 and 49, respectively, in [13].

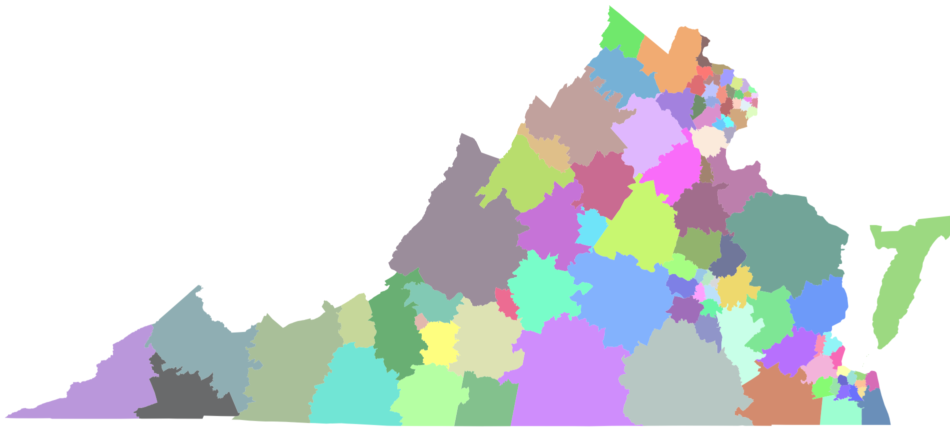

Fig. 3. Generated State House District Map

Table 5. State House District Population Deviations

Ideal Population	Std Deviation	Min Deviation	Max Deviation
86,313	0.9006	0.0104	−2.4376

Table 6. State House District Minority Representation

Black Majority	Black Opportunity	Minority-Majority	Minority Opportunity
8	14	37	51

In summary, this model is able to generate districts that come close to meeting Virginia's strict population deviation requirements at the Congressional level, and well within the requirements at the state Senate and House levels. The model also generates districts that provide strong opportunities for minority representation at all levels, with the state House districts in particular providing significantly more opportunities for minority representation than the current maps.

5 CONCLUSION

In this paper, I have described an approach for generating political district maps based on the *capacitated clustering problem*, which uses network flow to minimize the population-weighted distance from each geographic unit to its district center and optimizes for population equality and compactness. This procedure was used to generate maps for Virginia's Congressional and General Assembly districts, and was evaluated against the districts drawn by the Supreme Court of Virginia for the 2021 redistricting cycle. My results show that this approach is capable of generating districts that satisfy the requirements of contiguity and compactness, and produces districts with a higher number of *Opportunity* districts than the maps drawn by the court. While Virginia's strict Congressional population deviation requirement was not met, this approach generated Congressional districts within a 0.1% population deviation and General Assembly districts with lower population deviations than the current maps.

6 FUTURE WORK

Future work should strive to further constrain the model to consider the preservation of political subdivisions, such as counties and cities, as well as COI. Network-based approaches can be augmented such that the objective function is concave [11], and there are existing modifications to the *k-means* algorithm that constrain units in a *must-link* manner [23]. However, given *a priori* knowledge of these political subdivisions, it is possible to constrain the geographic units as demonstrated in this paper with Virginia's Eastern Shore. Additional criteria such as *political competitiveness* can also be incorporated or evaluated post-hoc.

While there have been decades of research into computational methods for redistricting, redistricting is a complex and nuanced problem and algorithmic approaches might not necessarily consider the local communities affected in the creation of districts [1, 9]. Because of this, maps continue to be drawn by humans in a way that lacks transparency and reproducibility. However, with a growing number states establishing redistricting commissions that seek to remove the power of process from the legislative body [12], algorithmic approaches may still have a place. Weaver and Hess [24] saw computers as a way to overcome legislative deadlock in the map drawing process, and, George et al. [11] was used alongside a redistricting commission to iteratively create and modify districts based on commission feedback. Thus there need not be an *all or nothing* approach to the use of computers in the map making process. With Virginia being a particular case where commission deadlock resulted in its maps being drawn by the courts [14], algorithms may provide an opportunity to offer neutrality in an inherently partisan process.

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A GENERATED CONGRESSIONAL DISTRICTS

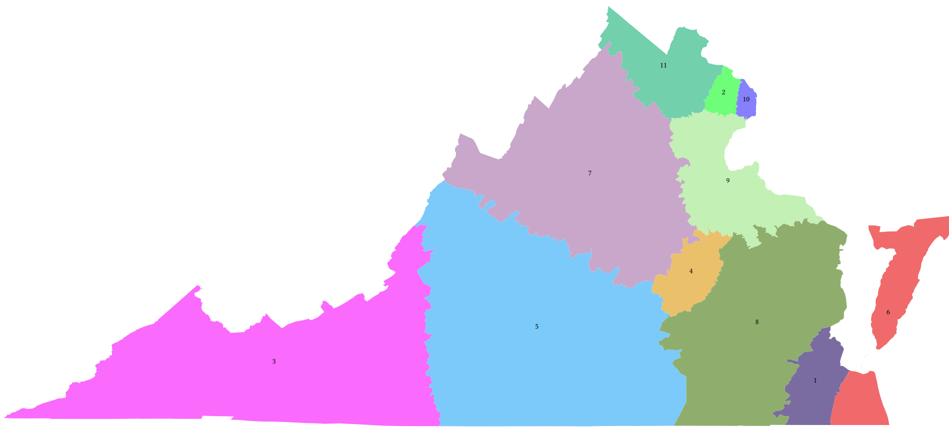

Fig. 4. Generated Congressional District Map

Table 7. Generated Congressional District Population & Minority Composition Data

District	Population	Deviation	Deviation %	Minority %	Black %
1	784,853	181	0.0231	54.9556	42.519
2	785,180	508	0.0647	50.6089	8.5522
3	784,336	−336	−0.0428	15.6249	8.0168
4	785,117	445	0.0567	41.6129	24.9056
5	783,952	−720	−0.0918	30.3295	23.1296
6	784,698	26	0.0033	45.23	28.0104
7	784,254	−418	−0.0533	23.25	10.2256
8	784,134	−538	−0.0686	45.902	34.386
9	785,430	758	0.0966	52.6631	24.6816
10	784,873	201	0.0256	51.0753	15.2035
11	784,566	−106	−0.0135	44.0509	9.7566

B GENERATED STATE SENATE DISTRICTS

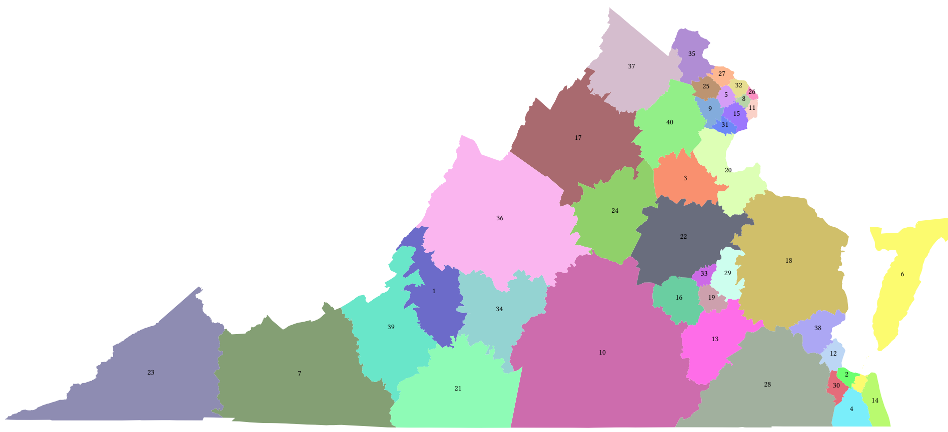

Fig. 5. Generated State Senate District Map

Table 8. Generated State Senate District Population & Minority Composition Data

District	Population	Deviation	Deviation %	Minority %	Black %
1	215,908	124	0.0575	28.2681	18.0364
2	217,261	1,477	0.6845	58.4877	42.0066
3	214,899	−885	−0.4101	38.0732	20.1336
4	215,495	−289	−0.1339	44.2971	26.0735
5	215,470	−314	−0.1455	47.369	8.2132
6	215,443	−341	−0.158	42.1188	25.0442
7	215,964	180	0.0834	9.3983	4.0127
8	215,685	−99	−0.0459	65.713	20.5791
9	215,412	−372	−0.1724	60.926	12.9807
10	216,543	759	0.3517	39.854	34.6869
11	214,872	−912	−0.4226	47.2458	17.577
12	216,133	349	0.1617	61.4844	50.5064
13	215,586	−198	−0.0918	52.4974	40.9498
14	215,804	20	0.0093	36.6745	18.1317
15	215,954	170	0.0788	52.9858	14.9912
16	215,749	−35	−0.0162	26.3301	13.5069
17	214,212	−1,572	−0.7285	20.285	5.5977
18	217,082	1,298	0.6015	29.5193	20.3914
19	216,248	464	0.215	67.7477	45.7803
20	216,313	529	0.2452	51.1255	27.5259
21	215,459	−325	−0.1506	33.8682	26.9736
22	216,793	1,009	0.4676	31.0213	14.4221
23	216,215	431	0.1997	7.3316	3.3212

Continued. . .

District	Population	Deviation	Deviation %	Minority %	Black %
24	216,786	1,002	0.4644	30.2349	14.4539
25	217,105	1,321	0.6122	58.9222	9.3628
26	217,182	1,398	0.6479	44.7371	11.2776
27	215,308	−476	−0.2206	51.5452	9.407
28	214,787	−997	−0.462	47.6193	39.8744
29	214,350	−1,434	−0.6646	56.4147	47.3702
30	215,023	−761	−0.3527	59.7327	49.7314
31	215,943	159	0.0737	69.736	27.347
32	215,115	−669	−0.31	43.301	4.8774
33	214,641	−1,143	−0.5297	35.3502	17.8382
34	215,384	−400	−0.1854	27.723	19.6398
35	216,381	597	0.2767	40.7397	8.7609
36	216,107	323	0.1497	15.8602	8.0775
37	215,677	−107	−0.0496	21.5285	6.4926
38	216,030	246	0.114	46.8518	30.3319
39	215,036	−748	−0.3466	18.1202	5.9302
40	216,038	254	0.1177	34.7101	12.0118

C GENERATED STATE HOUSE DISTRICTS

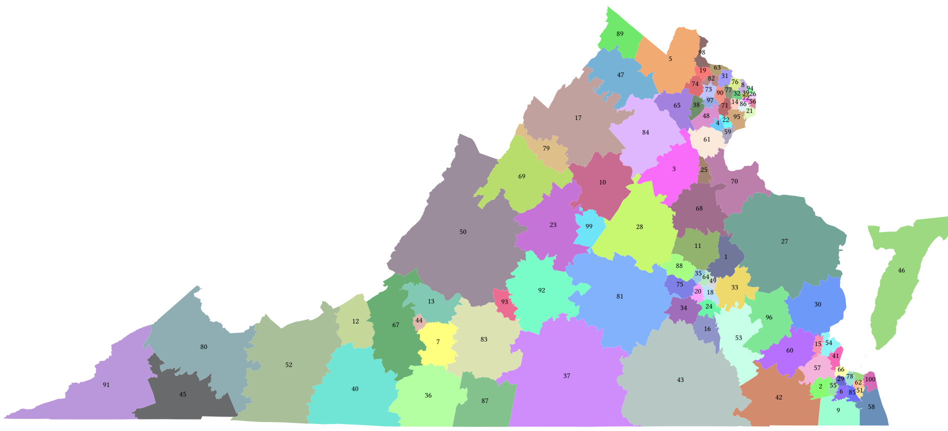

Fig. 6. Generate State House District Map

Table 9. Generated State House District Population & Minority Composition Data

District	Population	Deviation	Deviation %	Minority %	Black %
1	85,618	−695	−0.8052	28.3363	18.9446
2	85,572	−741	−0.8585	53.8856	43.0223
3	86,303	−10	−0.0116	28.8437	13.8442
4	84,389	−1,924	−2.2291	67.1474	26.033
5	85,081	−1,232	−1.4274	20.4335	4.9976
6	88,192	1,879	2.177	62.0442	49.2834
7	86,009	−304	−0.3522	18.2725	9.5746
8	86,322	9	0.0104	32.3255	4.701
9	85,239	−1,074	−1.2443	29.463	16.0044
10	87,009	696	0.8064	25.0871	11.4057
11	86,749	436	0.5051	34.9237	16.0002
12	86,550	237	0.2746	22.5523	7.294
13	86,943	630	0.7299	23.8179	11.7376
14	86,488	175	0.2028	55.1949	8.289
15	87,049	736	0.8527	50.9357	33.4628
16	86,695	382	0.4426	60.0796	51.4274
17	86,911	598	0.6928	11.9191	2.9858
18	86,084	−229	−0.2653	75.898	52.7078
19	86,418	105	0.1217	51.87	9.3349
20	87,073	760	0.8805	56.5192	36.4407
21	87,627	1,314	1.5224	58.4295	21.3907
22	86,858	545	0.6314	69.7276	23.7756
23	86,991	678	0.7855	17.1788	8.033

Continued. . .

District	Population	Deviation	Deviation %	Minority %	Black %
24	86,531	218	0.2526	54.785	32.8611
25	85,168	−1,145	−1.3266	43.6749	23.1049
26	86,185	−128	−0.1483	49.9948	15.1384
27	86,793	480	0.5561	32.9162	26.0885
28	86,323	10	0.0116	24.4489	16.7893
29	86,971	658	0.7623	61.87	51.5114
30	86,580	267	0.3093	25.2298	14.635
31	86,113	−200	−0.2317	37.7934	8.9139
32	87,062	749	0.8678	60.7912	8.2906
33	85,629	−684	−0.7925	57.832	50.6779
34	85,602	−711	−0.8237	28.2587	15.6118
35	86,543	230	0.2665	41.0143	17.4087
36	86,592	279	0.3232	32.7455	24.4711
37	86,021	−292	−0.3383	38.8347	34.7683
38	86,657	344	0.3985	57.8003	16.1672
39	87,222	909	1.0531	61.8422	15.1751
40	85,910	−403	−0.4669	10.6914	3.8133
41	85,314	−999	−1.1574	61.6394	51.6926
42	86,278	−35	−0.0406	48.9835	42.9206
43	85,516	−797	−0.9234	49.1452	43.8386
44	86,211	−102	−0.1182	39.1574	28.5868
45	85,611	−702	−0.8133	7.9125	3.1456
46	85,793	−520	−0.6025	35.7232	21.3304
47	87,694	1,381	1.6	18.0571	5.9115
48	84,209	−2,104	−2.4376	50.1799	13.1839
49	87,355	1,042	1.2072	68.8925	57.7574
50	86,253	−60	−0.0695	12.0633	6.2595
51	87,507	1,194	1.3833	54.9693	29.2308
52	86,148	−165	−0.1912	9.1238	4.5271
53	87,199	886	1.0265	51.84	38.6312
54	86,778	465	0.5387	44.5758	29.9661
55	85,935	−378	−0.4379	63.7109	54.9055
56	84,366	−1,947	−2.2557	35.5665	13.3454
57	87,011	698	0.8087	62.1875	53.5415
58	86,223	−90	−0.1043	32.2083	13.7597
59	87,138	825	0.9558	76.6187	41.6087
60	85,084	−1,229	−1.4239	55.3665	39.5539
61	86,734	421	0.4878	52.8063	25.8042
62	85,447	−866	−1.0033	44.1291	25.0494
63	87,000	687	0.7959	53.323	8.9218
64	85,750	−563	−0.6523	34.7219	23.8274
65	86,900	587	0.6801	33.1438	10.6743
66	85,775	−538	−0.6233	52.0047	32.1422
67	85,920	−393	−0.4553	14.361	4.4751
68	86,740	427	0.4947	37.7496	23.077

Continued...

District	Population	Deviation	Deviation %	Minority %	Black %
69	86,721	408	0.4727	14.9618	7.3131
70	86,573	260	0.3012	36.2191	19.9947
71	86,424	111	0.1286	43.5006	8.751
72	87,049	736	0.8527	65.2563	29.9038
73	87,248	935	1.0833	55.7732	7.1749
74	85,285	−1,028	−1.191	59.0948	9.9244
75	86,821	508	0.5886	22.2423	9.781
76	84,910	−1,403	−1.6255	45.4611	5.2726
77	86,939	626	0.7253	46.7339	5.9766
78	86,289	−24	−0.0278	61.8584	44.0242
79	85,543	−770	−0.8921	31.763	7.5985
80	87,123	810	0.9384	5.172	2.0087
81	86,035	−278	−0.3221	34.2012	27.6852
82	85,640	−673	−0.7797	64.8015	11.5367
83	86,349	36	0.0417	18.5179	12.8432
84	86,334	21	0.0243	30.4909	12.8733
85	85,234	−1,079	−1.2501	49.3277	28.3267
86	84,619	−1,694	−1.9626	57.818	19.7249
87	86,496	183	0.212	43.1812	36.9416
88	85,924	−389	−0.4507	32.6381	9.7458
89	85,756	−557	−0.6453	27.1643	7.968
90	86,595	282	0.3267	49.4567	8.9104
91	86,433	120	0.139	8.1867	4.1223
92	86,630	317	0.3673	33.8462	27.4201
93	85,710	−603	−0.6986	27.8252	17.1135
94	86,392	79	0.0915	35.6665	7.1314
95	86,981	668	0.7739	55.8306	21.415
96	86,670	357	0.4136	30.9357	19.1312
97	87,233	920	1.0659	64.6143	10.4353
98	86,995	682	0.7901	45.4095	10.4558
99	84,944	−1,369	−1.5861	36.4876	15.1323
100	87,537	1,224	1.4181	28.5388	14.218