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AN INVESTIGATION OF THE RATE OF FLOW OF VARIOUS GASES THROUGH CAPILLARY TUBES.

BY

JOSEPH R. BRANHAM.

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A C K N O W L E D G E M E N T

I wish to express my gratitude to Dr. Graham Edgar who proposed this investigation and who by his encouragement and helpful cooperation when difficulties were encountered made possible what little success has attended my efforts.

To Dr. Robert N. Pease who directed the mathematical side of the research and who helped interpret many of the results, I am greatly obligated.

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C O N T E N T S

RATE OF FLOW OF DIFFERENT GASES THROUGH CAPILLARY TUBES

When a gas is flowing through a tube there is a certain resistance to this flow that must be overcome by a pressure difference between the ends of the tube. For a given rate of flow,a long tube requires a greater pressure difference than a short one,and a tube of small diameter,a greater pressure differene than a tube of large diameter. The rate of flow is therefore dependent upon the length of the tube,its diameter,and the pressure difference that exists between its ends. For any tube,the dimensions of which are fixed,the rate of flow and the pressure difference should be functions of one another.

Advantage is taken of this fact in the construction of flow-meters. The gas is made to flow through a tube of small diameter; the pressure difference of its ends is measured,and from this measured difference the rate of flow is deduced.

There are four types of flow-meters in general use. They are the resistance tube flow-meter,the orifice flow-meter,the Venturi flow-meter and the Pitot tube flow-meter. Only the first type is dealt with in this article.

This type consists,essentially,of a capillary tube set into the main gas line and a device whereby the pressure difference between the ends of this tube can be measured.

The usual form of the resistance tube flow-meter is shewn in figure (1).

RATE OF FLOW OF DIFFERENT GASES THROUGH THE SAME TUBE

When a gas is flowing through a tube there is a certain resistance to this flow that must be overcome by a pressure difference between the ends of the tube. For a given rate of flow, a long tube requires a greater pressure difference than a short one, and a tube of small diameter, a greater pressure difference than a tube of large diameter. The rate of flow is therefore dependent upon the length of the tube, its diameter, and the pressure difference that exists between its ends. For any tube, the dimensions of which are fixed, the rate of flow and the pressure difference should be functions of one another.

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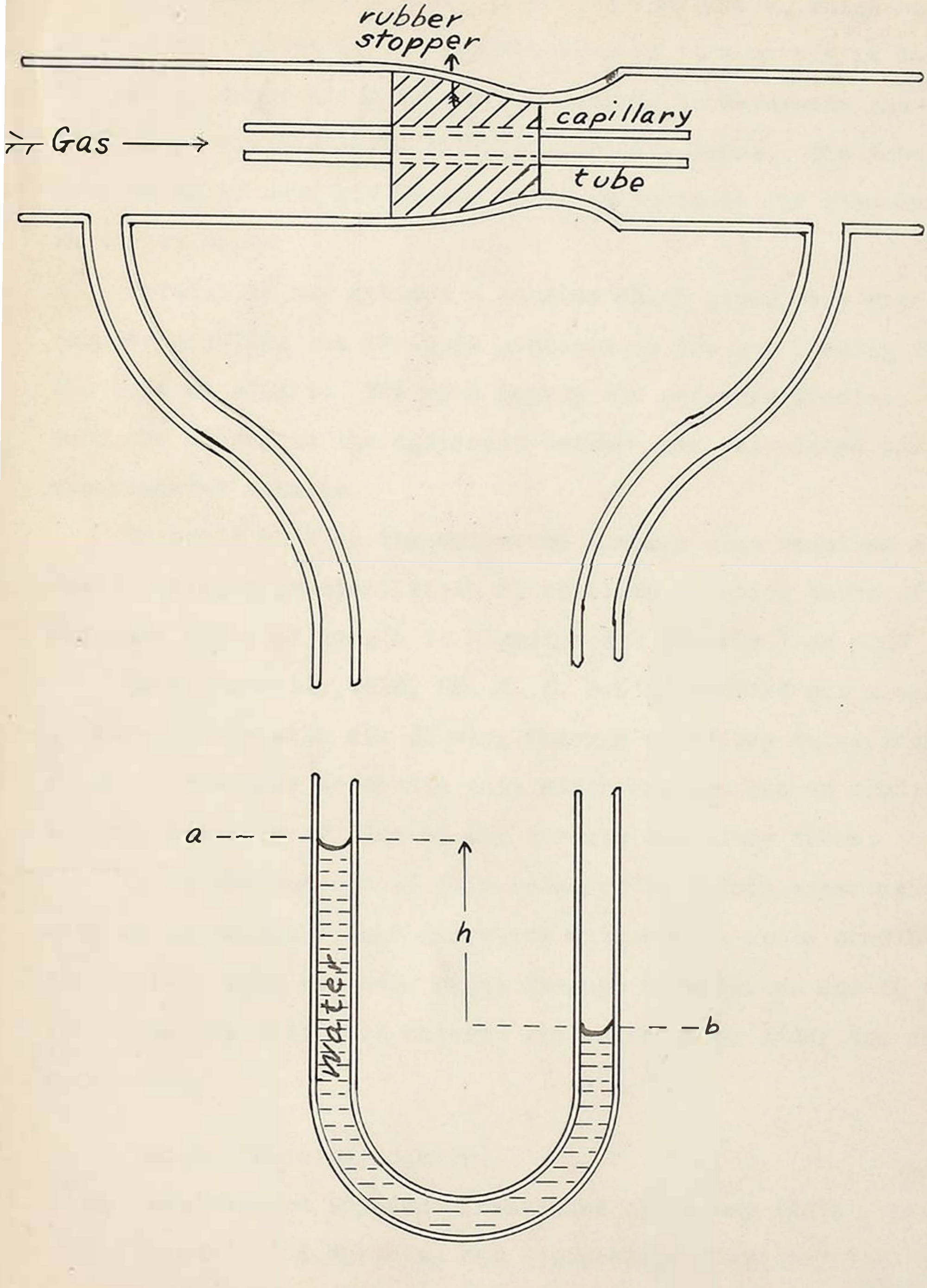
Figure 1



PURPOSE

On account of the lack of data, in literature, which could be used as a guide in the construction of flow-meters it was decided to carry out a set of experiments to determine the rate of flow of various gases through capillary tubes. The tubes were to be of such dimensions as to be suitable for flow-meter construction.

Poiseuille[1] has derived a formula which gives very exact results provided the pressure gradient on the gas flowing through the tube be slight. The more nearly the pressure gradient approaches zero, the closer is the agreement between the calculated and the experimental results.

Graham's[2] work on transpiration numbers also required a small pressure gradient, which he obtained by using tubes, of whichthe ratio of length to diameter was greater than 4000 to 1.

Quite recently, 1918, Dr. A. F. Benton[3] carried out a series of experiments with air flowing through capillary tubes, from which he was able to obtain data which enables one to predict closely the rate of flow of air through capillary tubes.

It is the purpose of this research to obtain experimental data on gases, other than air, which will enable us to predict the rate of flow of these gases through flow-meters and to find the relations that hold between different gases under the same conditions.

(1) Ganot's Physics, page 136

(2) Chemical and Physical Researches of Thomas Graham, page 108

(3) Journal of Industrial and Engineering Chemistry, Vol. 2, page 623 (1919

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a. Ganot's Physics, p. 150

b. Chemical and Physical Researches of Thomas Graham. 1876 [illegible]

c. Journal of Industrial and Engineering Chemistry 1919 [illegible]

THEORETICAL DISCUSSION

Poiseuille's Law(1) is obtained upon the supposition that there are no eddy or cross currents in the gas flowing through the tube. Under this condition of streamline flow the resistance tending to stop the flow is a measure of the gaseous viscosity alone.

Under conditions existing, where the rate of flow of gases is measured by flow-meters, there is necessarily a sudden change in the forward movement of the gas at both influx and efflux end of the capillary tube. This is bound to create turbulent regions at both ends of the tube; and if the rate is very high, or the tube very short, this turbulency may not die out but may exist throughout the intire length of the tube.

We have then three possible types of flow;(1) streamline throughout the tube; (2) streamline in the middle part of the tube with turbulency at the ends and (3), turbulency throughout the entire length of the tube.

For the first type of flow Poiseuille's law will hold but this ideal type of flow is seldom encountered in actual flow-meter measurements. For the second type of flow,we have two resistance factors, viscosity and the resistance due to the sudden change of momentum of the gas molecules, which may possibly be a function of their mass. The viscous resistance may or may not operate in the turbulent regions.

In the third type, where the entire tube is subject to turbulent flow, we may have either a single resistance arising from the change in momentum of the gas molecules,or a combinations of this resistance plus a viscous resistance.

(1) Rate $= \frac{\pi h d^4}{128 \eta l}$

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(1) Bate, ibid.

NOTATION

H = difference of pressure across flow-meter (cm. H_2O)

d = diameter of tube in mms.

l = length of tube in mms.

r = rate of flow corrected to 760 mms. pressure 25° C

η = coef. of viscosity 30° C (gms/cm.sec)

ρ = density of gas (gms per cc.)

v = $\frac{\eta}{\rho}$ kinematic viscosity (cm. /sec)

DESCRIPTION OF APPARATUS AND METHODS.

The apparatus used was set up in the following order; first, gas in containers under pressure; second, calcium chloride drying tubes; third, flow-meter; fourth, wet gas meter discharging into the atmosphere. Between the gas tanks and the calcium chloride tubes was placed a vertical glass tube about 3 meters high by 5 cms. diameter which had two openings in its stoppered lower end, by means of which water could be allowed to flow in or could be drawn off. In this way the water level in this tube could be changed at will. Into this tube led a smaller tube, branching by means of a T tube, from the main gas line. This small tube terminated in an orifice about a millimeter in diameter. It reached nearly to the bottom of the larger vertical tube containing the water so that by varying the height of the water above the lower end of the small tube we could allow any excess gas, above a definite pressure, to escape into the atmosphere. By this means we could maintain, at the influx end of the flow-meter, any chosen pressure regardless of change in pressure of the gas in the containers.

We first attempted to make all determinations at constant temperature of 25° C. by means of a thermostat and a long coil of glass tubing. It was found, however, that this was of little value because of the fact that the gas underwent a rapid change of temperature when it came into contact with the walls of the flow-meter. We therefore made all determinations at room temperature, reading the temperature of the gas at the efflux end of the flow-meter by means of a thermometer placed in a branch tube. From these readings we could correct the viscosity constant whenever necessary, using Sutherland's formula.(1) The wet gas meter was equipped with a thermometer so that we could correct the observed volumes by means of the gas laws to a standard condition, namely 760 mms. pressure, 25° C.

In actual practice the viscosity constants were assumed not to vary greatly from their value at 20° C. and in the data collected no correction has been made for this slight variation over the small range of temperature encountered.

The dimensions of the capillary tubes were determined by filling them with mercury and weighing the mercury contained in a measured length. The tubes were first tested for uniformity of bore by means of a shorter column of mercury whose length was measured in a different regions of the tube. It was found that there was some variation in bore of all the tubes used. This probably leads to an error in some cases. The average diameter was taken as correct in all calculations. The tubes used were portions of longer tubes filed and broken at right angles to their axis and the ends were not rounded by heating.

(1) (Phil. Mag. 31, 1893.) or Smithsonian Physical Tables page 165.

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The dimensions of the capillary tubes were determined by filling them with mercury and weighing the mercury contained in a measured length. The tubes were first tested for uniformity of bore by means of a similar column of mercury whose length was measured in a different section of the tube. It was found that there was some variation in bore of all the tubes used. This probably leads to an error in some cases. The average diameter has been taken as correct in all calculations. The other tubes were portions of longer tubes [illegible] their ends and the ends were then ground off by hand.

(1) Phil. Mag. (5) 36, 507 (1893).

Preliminary work, with relatively short tubes, gave rise to data which when plotted, pressure against rate, did not give curves which were regular for their entire length. These curves would be regular when the pressure and rate were below a certain limit. Beyond this limit there was apparently some factor which changed the character of the flow.

On account of this fact we decided that we must, first of all, determine the point where the curves became irregular. This point of irregularity or inflection we will call the "break".

Examination of the data showed that the breaks were more pronounced and hence more readily located when the length of the capillary tube was great when compared to its diameter. We therefore chose three long tubes of different diameters and collected data from the behavior of gases flowing through them.

The gases used were hydrogen, carbon dioxide, ethylene, and air. These four gases have the following physical properties.

gas	mol.wt.	viscosity constant 0° C.	Sutherland constant	viscosity constant 20° C.
H_2	2.016	845×10^{-7}	75.6	888×10^{-7}
CO_2	44.00	1388×10^{-7}	249.	1486×10^{-7}
C_2H_4	28.04	990×10^{-7}	249.	1060×10^{-7}
Air		1721×10^{-7}	112.	1819×10^{-7}

These gases were chosen because their volumes could be measured in a wet gas meter and because in them were found all possible combinations of high and low density, and high and low viscosities.

According to Benton, who uses a formula derived by Osborne Reynolds; r_c ; the critical rate, the point on the pressure-rate diagram where the plotted curve undergoes a sudden change of

direction is equal to ten dv. (10 dv.). That is to say, that the point of change is a function of the diameter or linear velocity and the viscosity and the density of the gas. Substi- tuting the values obtained from our data we get results which seem to be in fair agreement with this formula. This is shown as follows.

AIR

Tube		lit. min. calc	lit. min. found	
O	$r_c = \frac{15.53 \quad .0001819}{.001185}$	2.385	2.52	$r_c = 1.62\,d$
N	$r_c = \frac{17.374 \quad .0001819}{.001185}$	2.665	2.72	$r_c = 1.57\,d$
P	$r \quad \frac{7.702 \quad .0001819}{.001185}$	1.18	Diameter too small to get required rate within pressure limit	

CO_2

Tube		lit. min calc.	lit. min found	
O	$r_c = \frac{15.53 \quad .0001486}{.0018}$	1.282	1.28	$r_c = .823\,d$
N	$r_c = \frac{17.274 \quad .0001486}{.0018}$	1.432	1.37	$r_c = .789\,d$
P	$r_c = \frac{7.702 \quad .0001486}{.0018}$	.636	.625	$r_c = .812\,d$

C_2H_4

Tube		lit. min calc.	lit. min found	
O	$r_c = \frac{15.53 \quad .000106}{.001147}$	1.435	1.35	$r_c = .869\,d$
N	$r_c = \frac{17.374 \quad .000106}{.001147}$	1.508	1.41	$r_c = .811\,d$
P	$r_c = \frac{7.702 \quad .000106}{.001147}$	.713	.667	$r_c = .865d$

H_2

Tube		lit. min. calc.	found
O	$r_c = \frac{15.53 \quad .0000888}{.0000825}$	16.7	All breaks calculated for hydrogen are beyond the rate limit of the wet meter used. We cannot get any data with this meter for rates greater than 6 or 7 liters per minute.
N	$r_c = \frac{17.374 \quad .0000888}{.0000825}$	18.7	
P	$r_c = \frac{7.702 \quad .0000888}{.0000825}$	8.3	

DATA on CRITICAL RATES.

Tube	FLOW METER pressure cm. H_2O	FLOW METER temp. °C.	GAS METER temp. °C.	GAS METER cu. ft.	TIME sec.	RATE LITERS per min. 760 mm. 25° C.
		AIR				
N 900 AIR	83.5	22.8	---	.1	49.6	3.3396
	68.7	22.8	---	.1	55.1	3.060
	57.5	22.7	---	.1	58.5	2.880
	50.1	22.8	---	.1	60.4	2.808
	40.6	22.8	---	.1	64.7	2.604
	30.5	22.7	---	.1	79.4	2.124
	23.5	22.7	---	.1	101.2	1.662
	17.4	22.7	---	.1	134.2	1.257
	7.7	22.6	---	.1	291.4	.579
O 900 AIR	85.3	22.9	---	.1	61.6	2.736
	75.0	23.0	---	.1	63.0	2.676
	68.3	23.0	---	.1	65.4	2.580
	49.6	23.0	---	.1	80.0	2.106
	35.2	23.9	---	.1	106.0	1.590
	24.4	22.9	---	.1	149.2	1.128
	18.0	22.8	---	.1	199.8	.824
	12.1	22.8	---	.1	290.0	.582
	81.2	22.7	---	.1	63.6	2.652
N 800 AIR	167.0	22.4	18	.1	31.9	5.350
	143.7	22.8	18	.1	34.9	4.99
	121.7	23.0	18	.1	38.4	4.54
	105.9	23.1	18	.1	41.6	4.14
	91.0	23.1	18	.1	45.3	3.84
	74.5	23.1	19	.1	50.5	3.44
	60.6	22.9	19	.1	55.4	3.14
	49.1	22.7	19	.1	59.4	2.92

DATA on CRITICAL RATES (continued)

Tube	FLOW METER pressure cm. H_2O	FLOW METER temp. °C	GAS METER temp. °C	GAS METER cu. ft.	TIME sec.	RATE, liters per minute 760 mm. 25° C
			AIR (continued)			
N 800 AIR	38.0	22.3	19.	.1	65.3	2.66
	29.2	22.1	19.	.1	77.6	2.24
	25.8	22.1	19.	.1	86.3	2.01
	19.0	22.0	19.	.1	112.8	1.54
	15.2	21.7	20.	.1	137.6	1.26
	8.0	22.0	20.	.1	247.8	.698
O 800 AIR	179.0	23.4	23.	.1	40.2	4.18
	153.0	23.5	23.	.1	43.9	3.84
	135.0	23.6	23.	.1	47.2	3.57
	114.4	23.6	23.	.1	51.6	3.26
	95.3	23.7	23.	.1	56.6	2.98
	73.9	23.7	23.	.1	61.4	2.75
	59.3	23.7	23.	.1	65.7	2.57
	47.2	23.7	23.	.1	75.1	2.24
	37.4	23.7	23.	.1	92.2	1.83
	31.3	23.7	23.	.1	107.5	1.57
	24.8	23.7	23.	.1	132.8	1.27
	16.2	23.7	23.	.1	196.0	.859
	5.4	23.7	23.	.1	576.0	.293

DATA on CRITICAL RATES (continued)

Tube	FLOW METER pressure cm H_2O	METER temp. °C	GAS METER temp. °C	cu. ft.	TIME sec.	RATE, liters per minute 760 mm. 25° C
			AIR (continued)			
P	192.3	23.7	23	.1	185.6	.878
500	159.2	23.6	23	.1	211.1	.800
AIR	133.0	23.8	23	.1	249.0	.687
	90.7	23.8	23	.1	360.	.469
	63.2	23.8	23	.1	504.	.338
	36.6	23.8	23	.1	872.8	.194
	20.7	24.1	23	.1	1527.2	.111
P	187.2	23.5	23	.1	164.5	1.022
400	151.6	23.5	23	.1	192.0	.875
	128.7	23.5	23	.1	221.3	.761
	86.8	23.6	23	.1	314.3	.535
	64.4	23.5	23	.1	418.	.402
	35.5	23.5	23	.1	748.	.225
	19.7	24.0	23	.1	1346.	.125

DATA on CRITICAL RATES (continued)

Tube	FLOW METER pressure cm H_2O	FLOW METER temp. °C	GAS METER temp. °C	GAS METER cu. ft.	TIME sec.	RATE, liters per minute 760 mm 25° C
			CO_2			
N	84.5	23.1	---	.1	59.1	2.850
900	74.7	23.1	---	.1	63.1	2.670
CO_2	66.9	23.2	---	.1	67.5	2.496
	56.1	23.2	---	.1	74.2	2.271
	45.3	23.1	---	.1	84.8	1.9884
	36.8	23.2	---	.1	93.2	1.8072
	27.8	23.1	---	.1	105.5	1.5966
	18.1	23.1	---	.1	121.9	1.3812
	13.7	23.1	---	.1	145.1	1.1598
	9.3	23.1	---	.1	207.5	.8112
	7.0	22.5	---	.1	268.0	.630
			CO_2			
O	88.0	22.8	---	.1	77.5	2.178
900	77.4	22.8	---	.1	84.0	2.101
CO_2	69 15	22.9	---	.1	89.2	1.890
	61.8	23.0	---	.1	95.0	1.788
	52.88	23.0	---	.1	102.8	1.644
	45.10	23.0	---	.1	111.0	1.518
	38.26	23.0	---	.1	118.0	1.428
	33.55	23.0	---	.1	121.3	1.392
	28.2	23.0	---	.1	127.0	1.326
	24.5	23.0	---	.1	135.3	1.2450
	19.5	23.0	---	.1	162.2	1.0398
	15.33	23.0	---	.1	200.2	.8430
	10.58	23.0	---	.1	286.2	.5898
	5.76	23.0	----	.1	497.0	.3396
			CO_2			
N	3.8	22.2	21	.1	421.4	0.402
800	8.8	22.3	21	.1	198.8	0.852
CO_2	13.1	24.0	21	.1	141.8	1.192
	15.0	23.6	21	.1	127.6	1.325
	26.6	24.3	21	.1	101.7	1.663
	43.4	24.7	21	.1	80.4	2.106
	63.4	25.0	22	.1	65.1	2.59
	84.7	25.2	22	.1	54.9	3.073
	112.5	25.4	22	.1	46.6	3.618
	142.9	25.6	22	.1	40.4	4.17
	172.3	25.7	22	.1	36.3	4.645

DATA on CRITICAL RATES(continued)

Tube	FLOW METER pressure cm H_2O	FLOW METER temp.°C	GAS METER temp.°C	GAS METER cu. ft.	TIME sec.	RATE, liters per minute 760 mm. 25°C
			CO_2			
0	178,0	25.9	22	.1	47.8	3.525
200	142.9	26.0	22	.1	54.4	3.1
CO_2	102.0	26.1	22	.1	66.4	2.54
	65.5	26.1	22	.1	85.3	1.975
	44.4	26.2	22	.1	104.6	1.61
	29.1	26.3	22	.1	120.0	1.404
	21.7	26.5	23	.1	135.2	1.243
	15.8	26.4	23	.1	179.8	o.935
	10.8	26.5	23	.1	250.8	0.76
	0.5	26.2	23	.1	404.4	0.416
			CO_2			
P	197.8	25.1	25	.1	227.	.740
500	148.3	24.9	25	.1	259.	.649
CO_2	115.6	24.4	25	.1	288.	.583
	72.5	24.2	25	.1	389.	.432
	36.8	23.9	25	.1	721.	.233
	52.1	24.0	25	.1	525.	.320
	32.7	23.8	25	.1	1166.	.144
	90.7	23.0	25	.1	334.8	.501
	170.6	23.1	25	.1	239.4	.700

DATA on CRITICAL RATES (continued)

Tube	FLOW METER pressure cm H_2O	FLOW METER temp. °C	GAS METER temp. °C	GAS METER cu. ft.	TIME sec.	RATE, liters per minute 760 mm 25°C
			C_2H_4			
N	79.5	23.5		.1	47.6	3.516
900	47.0	23.5		.1	64.0	2.616
C_2H_4	34.7	23.5		.1	76.0	2.202
	26.6	23.4		.1	86.2	1.944
	16.8	23.4		.1	105.0	1.596
	6.28	24.2		.1	203.0	.8250
	9.6	24.2		.1	140.0	1.197
	3.17	24.2		.1	388.0	.4332
	4.08	24.2		.1	302.0	1.5556
	14.7	24.2		.1	106.8	1.5714
	23.5	24.2		.1	92.0	1.842
	10.95	24.3		.1	127.6	1.314
			C_2H_4			
N	145.5	21.8	22	.1	31.6	5.37
800	131.3	22.0	22	.1	33.8	5.04
C_2H_4	115.0	22.2	22	.1	36.4	4.67
	93.8	22.3	22	.1	40.95	4.16
	79.45	22.4	22	.1	45.0	3.79
	61.8	22.3	22	.1	51.85	3.28
	44.8	22.3	22	.1	62.6	2.72
	34.2	22.2	22	.1	72.5	2.35
	26.45	22.2	22.5	.1	83.7	2.03
	17.35	22.2	22.5	.1	101.34	1.68
	11.45	22.2	23	.1	118.0	1.44
	7.0	22.2	23	.1	168.5	1.01
	3.9	22.1	23	.1	282.7	0.605
			C_2H_4			
O	5.1	22.7	21	.1	347.1	0.492
800	7.2	22.8	21	.1	246.5	0.692
C_2H_4	9.75	23.0	21	.1	193.9	0.88
	13.8	23.0	21	.1	141.4	1.21
	19.0	23.1	21	.1	120.0	1.42
	48.0	23.1	21	.1	81.2	2.1
	65.1	23.1	21	.1	68.9	2.48
	94.1	23.2	21	.1	55.5	3.08
	118.1	23.3	21	.1	48.6	3.505
	144.1	23.4	21	.1	43.2	3.95
	166.2	23.4	21	.1	39.7	4.29

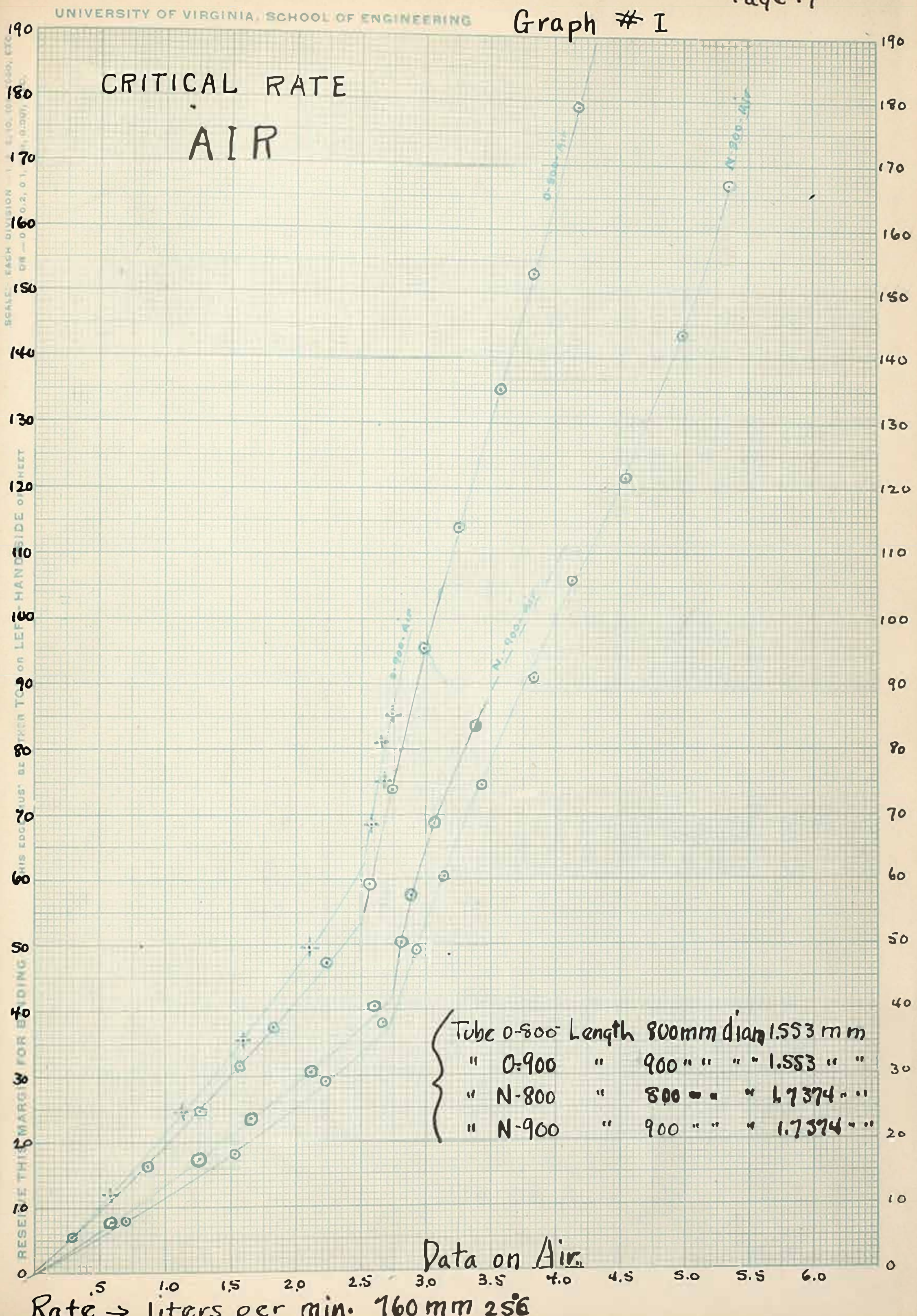
DATA on CRITICAL RATES (continued)

Tube	FLOW METER pressure cm H_2O	FLOW METER temp. °C	GAS METER temp. °C	GAS METER cu. ft.	TIME sec.	RATE, liters per minute 760 mm 25° C
			C_2H_4 (continued)			
P 500 C_2H_4	22.6	24.7	25	.1	858.4	.1955
	51.5	24.6	25	.1	423.4	.396
	63.0	24.7	25	.1	365.4	.459
	81.6	24.8	25	.1	306.	.548
	111.	24.9	25	.1	245.8	.672
	148.	25.0	25	.1	217.0	.772
	178.6	25.0	25	.1	194.2	.863
P 400 C_2H_4	188.1	24.4	24 C_2H_4	.1	169.4	.989
	161.7	24.7	24	.1	184.6	.909
	138.4	24.9	24	.1	203.0	.826
	113.6	25.0	24	.1	223.4	.750
	82.6	25.2	25	.1	252.0	.665
	60.4	25.2	25	.1	297.	.565
	42.7	25.4	25	.1	389.6	.430
	20.0	25.4	25	.1	769.8	.218
			HYDROGEN			
N 800 H_2	30.25	22.5	21	.1	32.4	5.33
	21.0	22.4	21	.1	45.85	3.77
	14.4	22.4	21	.1	65.8	2.625
	8.3	22.4	21	.1	114.2	1.51
O 800 H_2	67.2	21.2	20 H_2	.1	23.2	7.43
	55.4	21.2	20	.1	27.75	6.22
	41.7	21.3	20	.1	36.55	4.72
	29.4	21.3	20	.1	51.15	3.37
	18.5	21.3	20	.1	81.10	2.132
	8.9	21.4	20	.1	166.1	1.035
	51.0	21.7	20	.1	29.9	5.78
P 400 H_2	148.5	23.2	21 H_2	.1	82.8	2.08
	109.3	23.0	21	.1	113.0	1.514
	80.0	22.9	21	.1	155.5	1.106
	52.7	23.4	22	.1	235.2	0.728

DATA on CRITICAL RATES (continued)

Tube	FLOW METER pressure cm H_2O	FLOW METER temp. °C	GAS METER temp. °C	GAS METER cu. ft.	TIME sec.	RATE, liters per minute 760 mm 25° C
			HYDROGEN			
P	158.0	18.5	16	.1	27.6	6.25
100	138.8	18.3	16	.1	30.4	5.68
H_2	124.9	18.7	16	.1	33.15	5.21
	104.5	18.2	16	.1	37.83	4.56
	87.1	18.2	16	.1	43.85	3.936
	60.7	18.1	16	.1	59.55	2.893
	36.7	18.1	16	.1	92.7	1.861
	20.5	18.1	16	.1	157.3	1.095
	146.4	17.9	17	.1	28.9	5.94
	105.2	18.4	17	.1	37.6	4.57
	114.0	18.8	17	.1	35.20	4.87

Graph # I



Rate → liters per min. 160 mm 256

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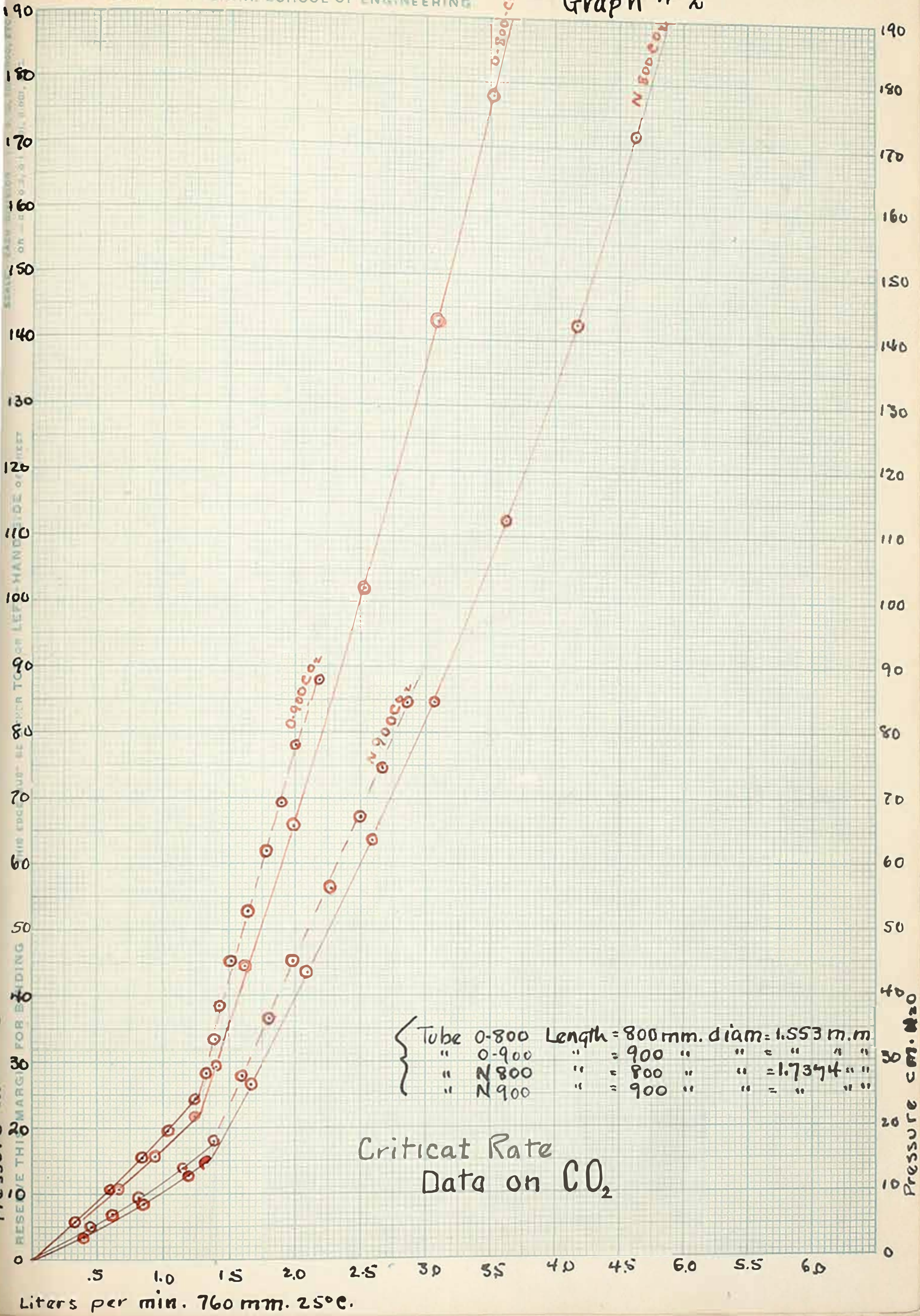
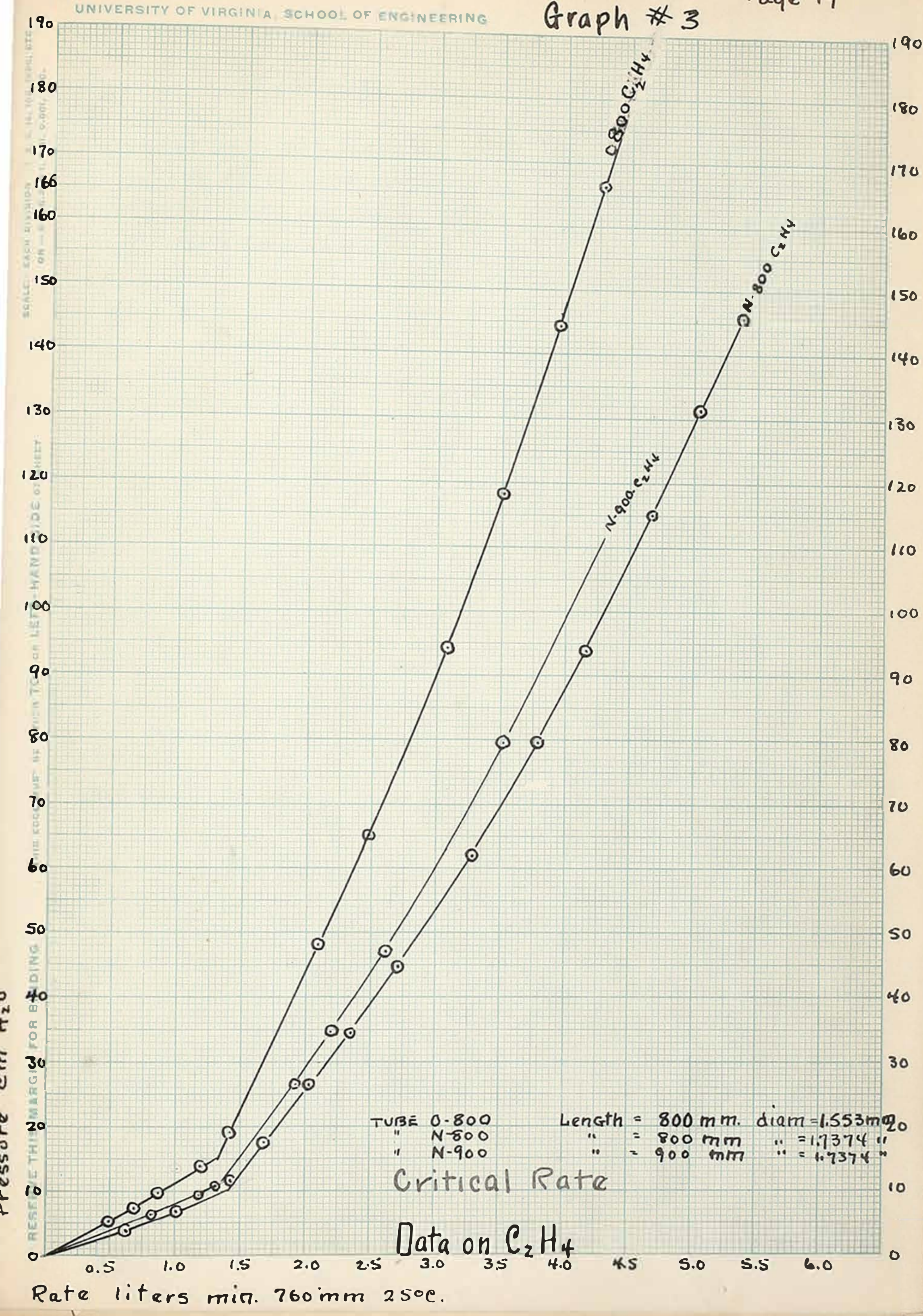
Graph # 2



Graph # 3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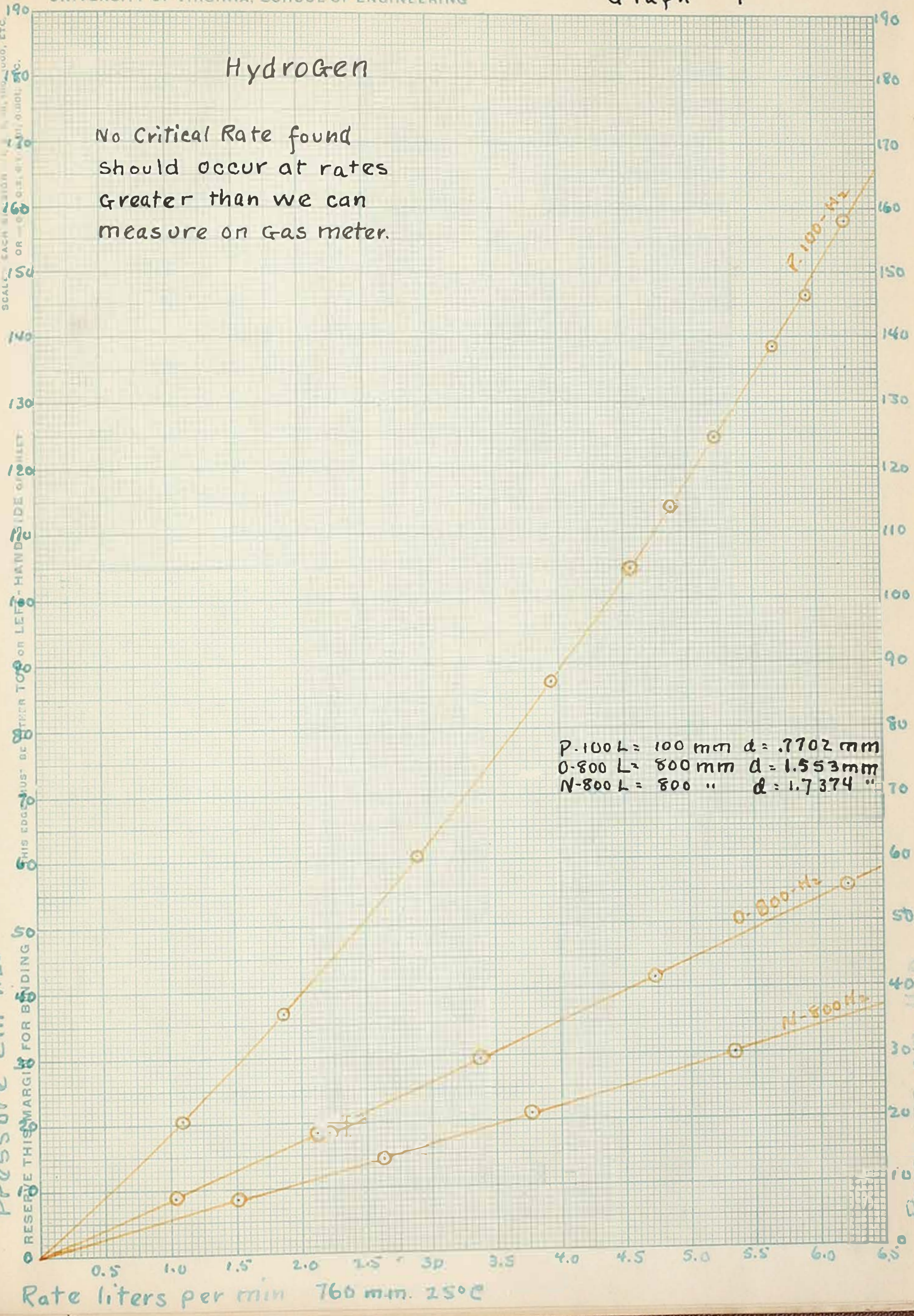
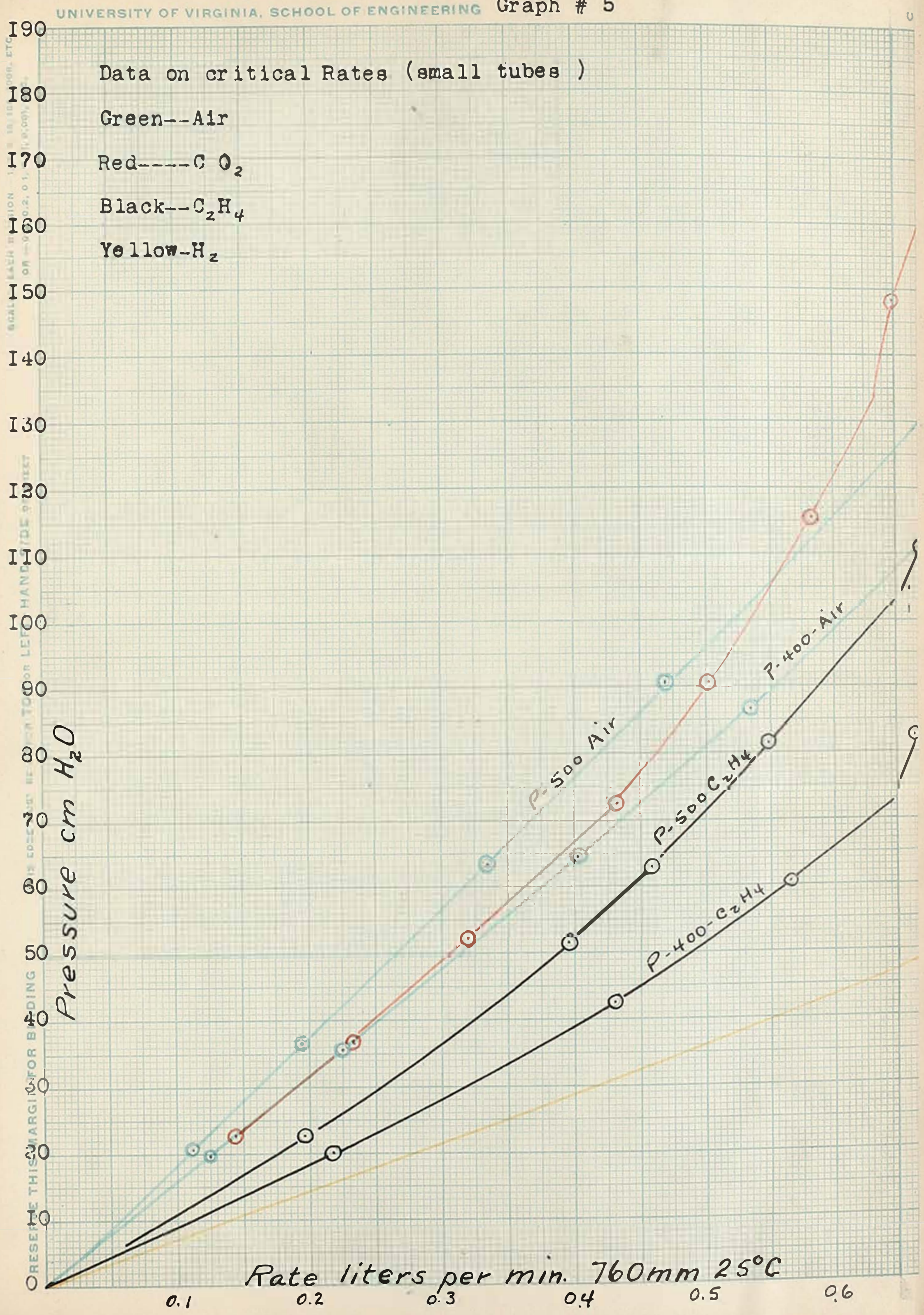
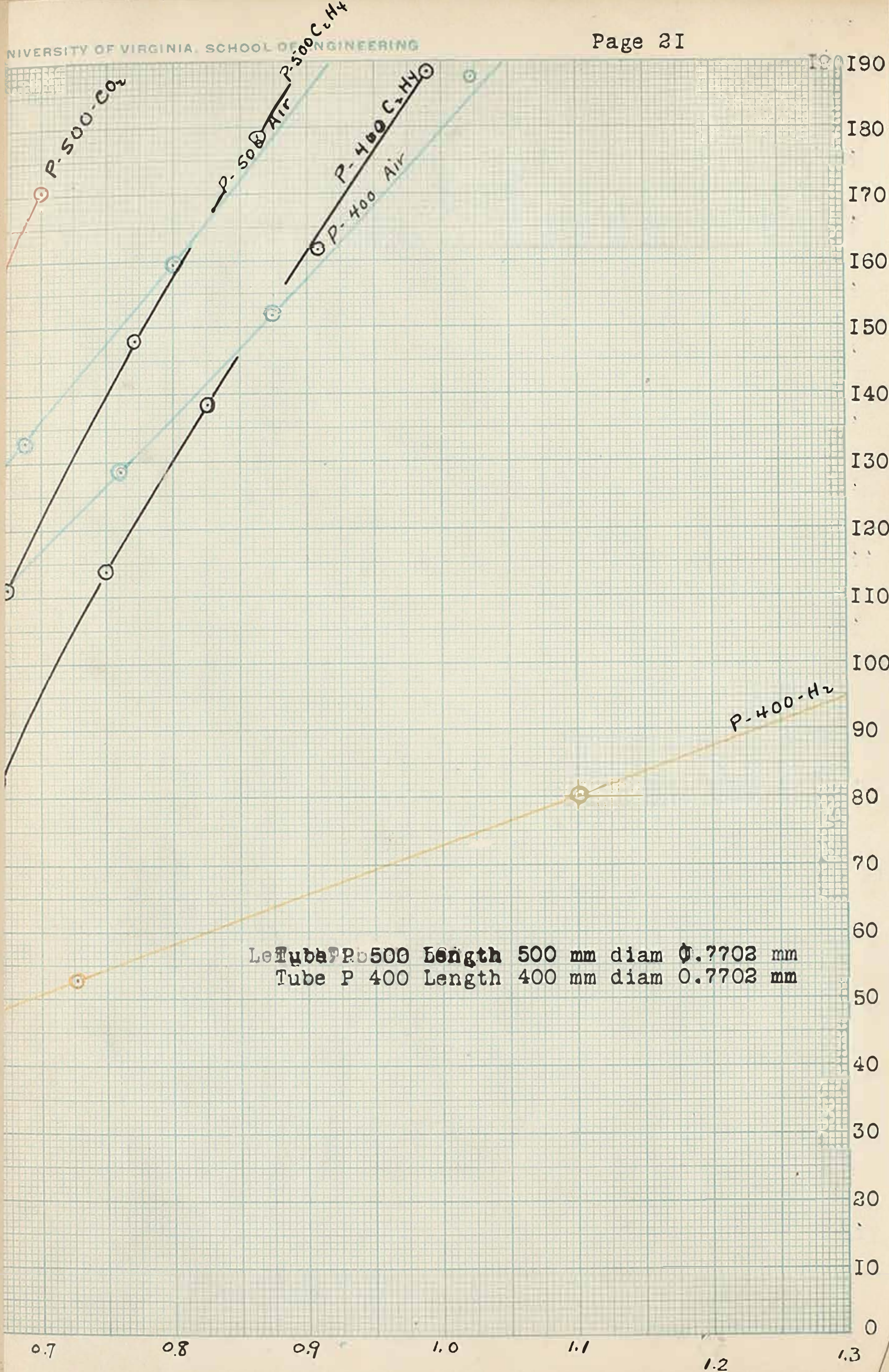
O800 C_2H_4
N-800 C_2H_4
N-900 C_2H_4
TUBE O-800 Length = 800 mm. diam = 1.553 mm
" N-800 " = 800 mm " = 1.7374 "
" N-900 " = 900 mm " = 1.7374 "
Critical Rate
Data on C_2H_4
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 166 170 180 190
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0
Rate liters min. 760 mm 25°C.
Pressure cm H_2O


Hydrogen
No Critical Rate found
should occur at rates
Greater than we can
measure on Gas meter.
P.100 L = 100 mm d = .7702 mm
0-800 L = 800 mm d = 1.553 mm
N-800 L = 800 " d = 1.7374 "
P.100-H2
O-800-H2
N-800-H2
Pressure cm H2O
Rate liters per min 760 mm. 25°C
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0

Rate →→→ liters per minute 760 mm 25°C

Graph # 5




UNIVERSITY OF VIRGINIA. SCHOOL OF ENGINEERING
P-500-CO_2
P-500 Air
P-500 C_2H_4
P-400 C_2H_4
P-400 Air
P-400-H_2
Tube P 500 Length 500 mm diam 0.7702 mm
Tube P 400 Length 400 mm diam 0.7702 mm
0.7
0.8
0.9
1.0
1.1
1.2
1.3
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190

END EFFECT

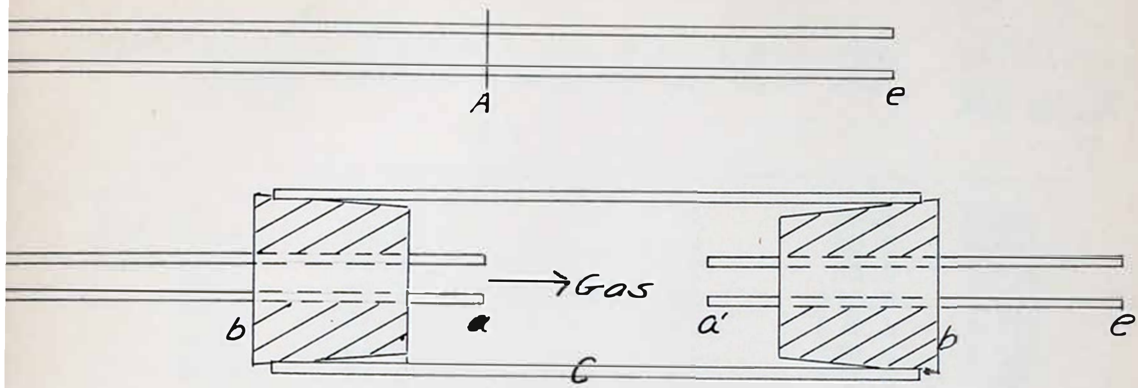
Having thus established the region where the break occurs, with a fair degree of accuracy, we decided to try to investigate the magnitude of end effect.

Poiseuille's law if plotted, rate against pressure, gives a straight line which is tangent to all of our curves at zero rate and zero pressure. Above this rate and pressure there is a deviation which increases with increase of pressure and rate. The rate of all gases is less than that calculated by Poiseuille's law. We will define "end effect" as the difference in pressure for a given rate, between that pressure actually found and the pressure which would correspond to this given rate if calculated by Poiseuille's law.

The value of the end effect can be found by combining with the pressure-rate diagram of a gas the plotted values calculated from Poiseuille's law and measuring the difference between the pressures which correspond to the same rate of flow.

Another angle of attack on this problem is to introduce into a tube an additional end effect and to measure the pressure difference at the same rates of flow between this tube and the original tube.

The method of introducing an added effect is best shown by the diagram on the following page.



Compound tube resulting from broken tube

The tube was broken at right angles to its axis at point A. The broken ends were inserted into gas tight stoppers (b) which were in turn inserted into a short tube (c) whose diameter was very great compared to that of the capillary. A large diameter in this tube allows us to neglect the slight resistance arising in it.

The gas could then flow through the tube as before, the only difference being that it encountered an additional end effect due to the extra ends. A pressure rate diagram from this tube was then compared with a pressure-rate diagram of this same tube before it was broken and the magnitude of the increase in end effect measured from the two diagrams.

The data on these two methods of calculating "end effects" is given on the following pages.

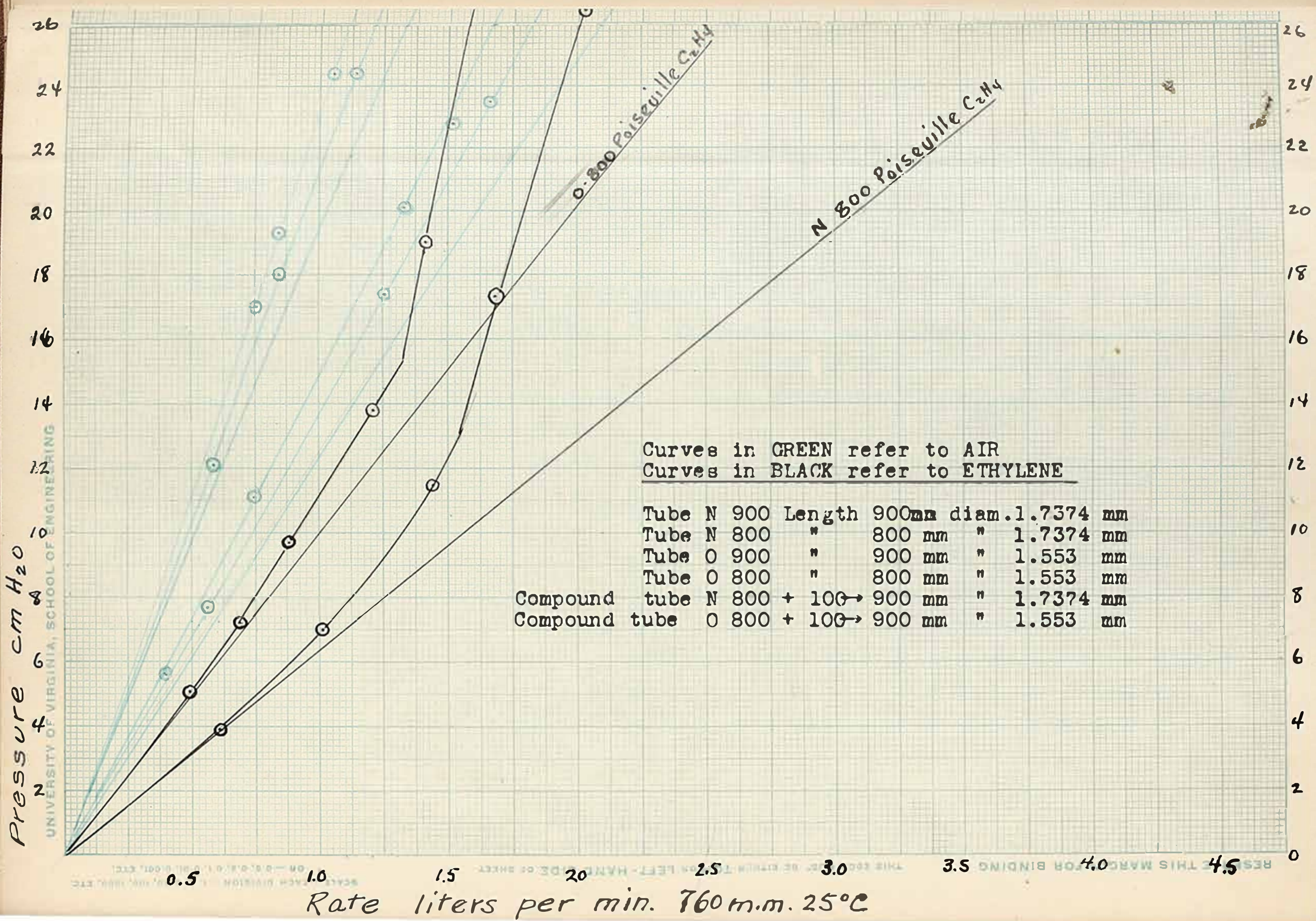
The method of describing tubes is as follows;
the diameter is indicated by a letter and the length in mm, by the figures following the letter If in the above illustration d-A was 800 mm. and A-c was 100mm the unbroken tube would be designated by a letter followed by 900; the compound tube would be the same letter followed by 800+ 100.

DATA on END EFFECT.

Tube	FLOW METER pressure cm. H_2O	FLOW METER temp. °C	GAS METER temp. °C	GAS METER cu. ft.	TIME sec.	RATE, liters per minute 760 mm 25° C
			A I R			
0	67.7	23.1		.1	70.4	2.383
800	56.4	23.1		.1	80.0	2.094
+100	43.7	23.1		.1	98.0	1.712
	36.1	23.1		.1	113.2	1.481
	17.0	23.1		.1	220.5	0.761
	24.4	23.5		.1	159.4	1.050
	19.3	23.5		.1	198.0	0.846
			Air			
N	72.9	22.4		.1	54.4	3.120
800	63.0	20.9		.1	59.0	2.880
+100	52.0	22.6		.1	61.6	2.760
	42.6	22.6		.1	68.8	2.472
	33.6	22.6		.1	82.0	2.070
	22.8	22.6		.1	111.8	1.519
	20.1	22.4		.1	127.6	1.330
	11.1	22.5		.1	228.0	0.746
	5.6	22.5		.1	411.0	0.415

DATA on END EFFECT (continued)

Tube	FLOW METER pressure cm. H o	FLOW METER temp.°C	GAS METER temp.°C	GAS METER cu. ft.	TIME sec.	RATE, liters per minute 760 mm 25°C
N			CO_2			
800+100						
	162.5	23.8		.1	42.4	3.975
	177.5	23.8		.1	39.2	4.299
	137.3	23.8		.1	46.2	3.648
	105.6	23.8		.1	54.9	3.072
	124.0	23.8		.1	50.0	3.372
	150.7	23.8		.1	44.5	3.792
	10.1	22.1		.1	215.6	0.787
	13.3	22.1		.1	169.0	0.994
	16.3	22.1		.1	145.3	1.154
	20.2	22.2		.1	126.1	1.330
	25.6	22.2		.1	114.5	1.47
	33.8	22.2		.1	101.3	1.66
	45.2	25.0		.1	86.4	1.945
	57.7	25.1		.1	76.6	2.193
	71.8	25.1		.1	67.3	2.500
	83.6	25.1		.1	62.3	2.700
	41.5	25.1		.1	58.4	1.880
O			CO_2			
800+100						
	180.1	23.8		.1	53.6	3.1410
	151.6	23.8		.1	60.4	2.790
	134.0	23.8		.1	64.9	2.600
	108.8	23.8		.1	73.0	2.310
	15.7	22.8		.1	217.6	0.780
	5.9	22.9		.1	520.0	0.3258
	26.0	23.0		.1	145.0	1.167
	36.0	23.0		.1	123.0	1.374
	48.9	23.0		.1	111.6	1.516
	59.7	23.0		.1	98.4	1.719
	70.7	23.0		.1	89.6	1.876
	83.6	22.9		.1	83.8	2.019
	87.7	22.9		.1	82.0	2.063
	29.4	22.9		.1	133.6	1.266
	20.8	22.8		.1	170.4	0.992
	11.4	22.8		.1	288.0	0.5865
	7.4	22.8		.1	420.	0.4020
N						
600+200			H_2			
	34.1	23.4	23	.1	29.7	5.725
H_2	24.1	23.4	23	.1	41.6	4.08
	17.4	23.8	23	.1	56.2	1.025
	10.0	24.1	23	.1	95.3	1.785


Curves in GREEN refer to AIR
Curves in BLACK refer to ETHYLENE
Tube N 900 Length 900mm diam. 1.7374 mm
Tube N 800 " 800 mm " 1.7374 mm
Tube 0 900 " 900 mm " 1.553 mm
Tube 0 800 " 800 mm " 1.553 mm
Compound tube N 800 + 100→ 900 mm " 1.7374 mm
Compound tube 0 800 + 100→ 900 mm " 1.553 mm
O 800 Poiseuille C_2H_4
N 800 Poiseuille C_2H_4
Pressure cm H_2O
Rate liters per min. 760 m.m. 25°C
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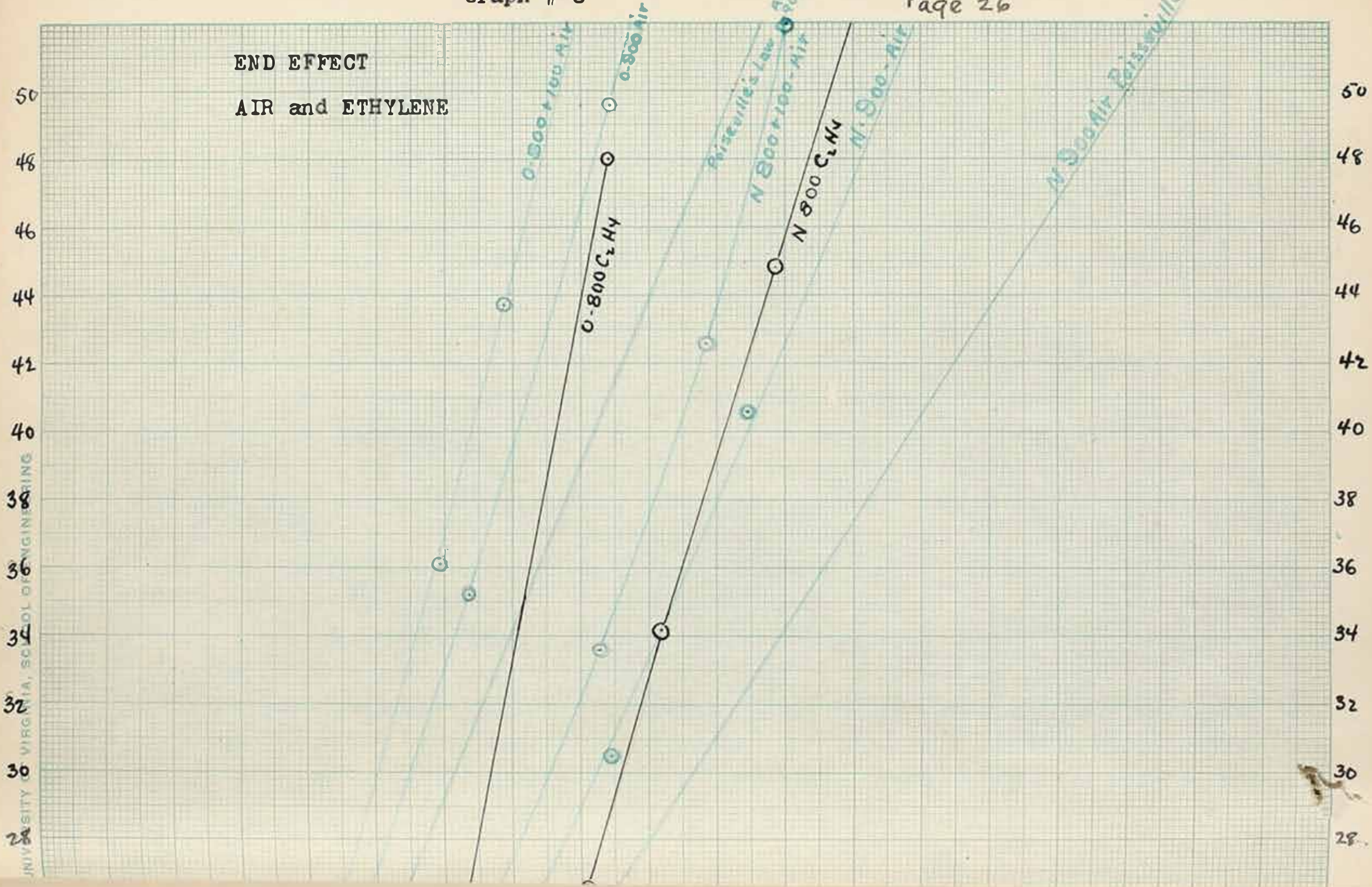
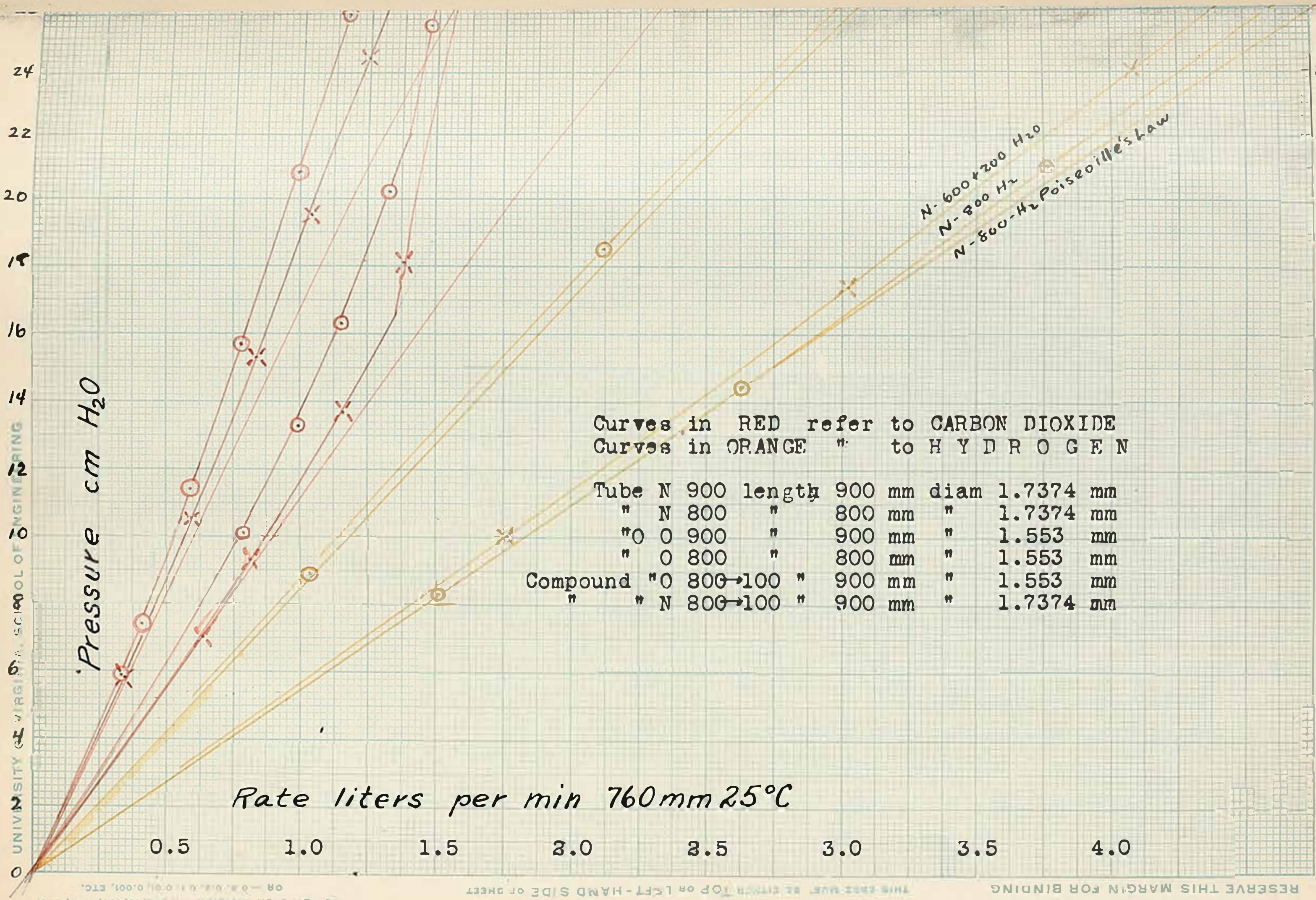
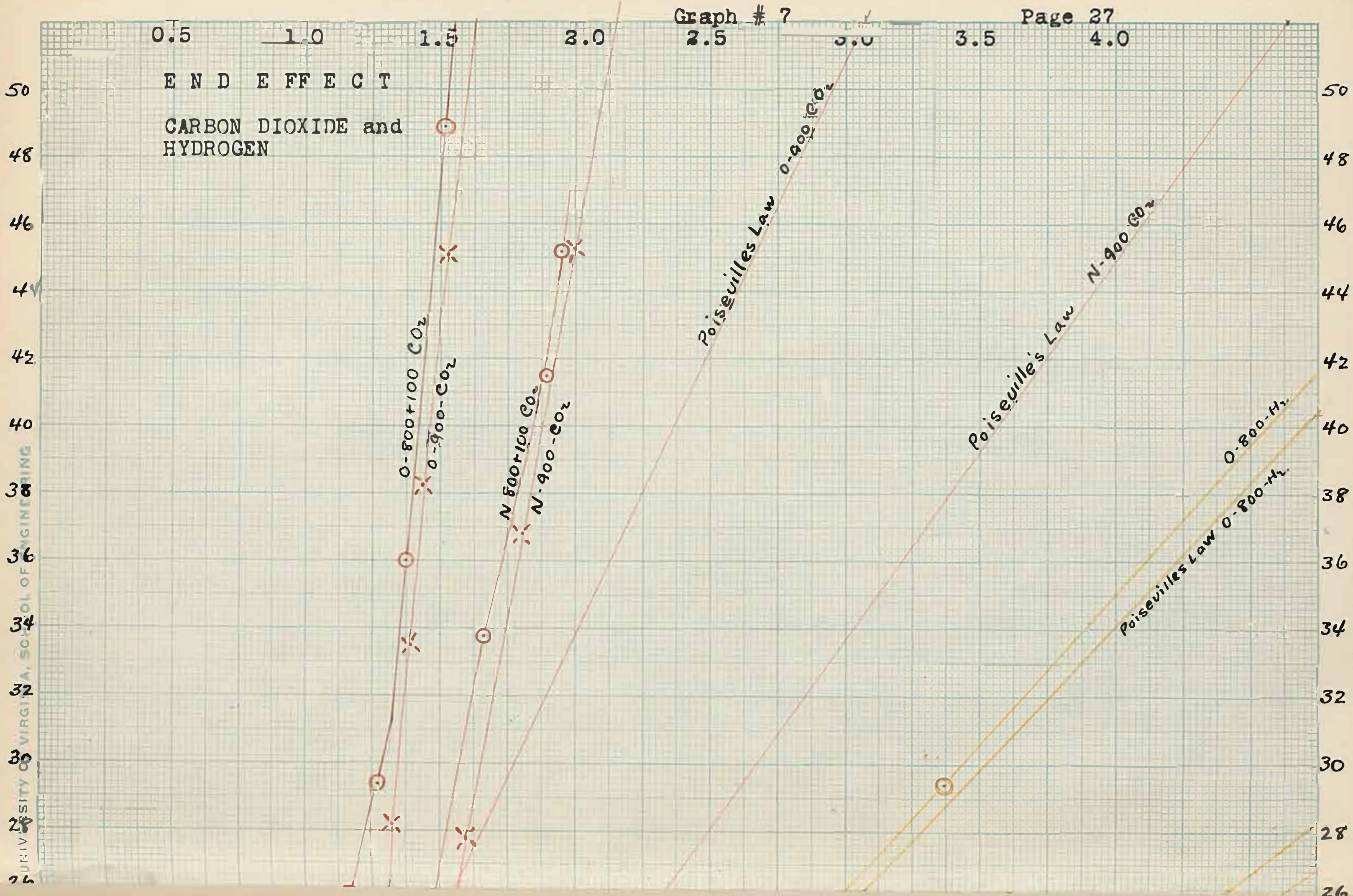
26
24
22
20
18
16
14
12
10
8
6
4
2
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5

Graph # 6


END EFFECT
AIR and ETHYLENE
0-800+100 Air
0-800 Air
0-800 C2 H4
Poiseuille's Law Air 800
N 800+100-Air
N 800 C2 H4
N 800-Air
N 800 Air Poisseuille
50
48
46
44
42
40
38
36
34
32
30
28
UNIVERSITY OF VIRGINIA, SCHOOL OF ENGINEERING



Pressure cm H_2O
Rate liters per min 760mm 25°C
N-600+200 H_2O
N-800 H_2
N-800-H_2 Poiseoille's Law
Curves in RED refer to CARBON DIOXIDE
Curves in ORANGE " to H Y D R O G E N
Tube N 900 length 900 mm diam 1.7374 mm
" N 800 " 800 mm " 1.7374 mm
" O 900 " 900 mm " 1.553 mm
" O 800 " 800 mm " 1.553 mm
Compound " O 800→100 " 900 mm " 1.553 mm
" " N 800→100 " 900 mm " 1.7374 mm
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0
2
4
6
8
10
12
14
16
18
20
22
24
RESERVE THIS MARGIN FOR BINDING


0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
END EFFECT
CARBON DIOXIDE and
HYDROGEN
50
48
46
44
42
40
38
36
34
32
30
28
26
0-800+100 CO2
0-900-CO2
N 800+100 CO2
N-900-CO2
Poiseuilles Law 0-900 CO2
Poiseuille's Law N-900 CO2
0-800-H2
Poiseuilles Law 0-800-H2

The curves plotted from the data on end effect indicate that errors in measurement, which are unimportant in so far as they affect the general shape of the curve itself, become serious obstacles when the same data is to be used in the calculation of end effect. These errors lead to results which differ among themselves when we attempt to derive a quantitative measure of end effect from the data. In order to come to any definite conclusion on the exact magnitude of end effect, the methods of measuring time, volume and pressure, particularly low pressures must be greatly refined.

Certain more or less qualitative conclusions may be drawn from the data that has been collected. Strictly speaking, these conclusions are applicable only to long tubes.

(1) For the gases used the end effect at a given rate increases as the diameter decreases, indicating that end effect may be a function of linear velocity.

(2) There are also indications that end effect is a function of the density of the gas. In all cases the end effect for hydrogen was very much less than the other gases, and in the great majority of cases the end effect of carbon dioxide was greater than that of air, which in turn, was greater than that of ethylene.

(3) The rate of change of end effect with change of rate of flow above the critical rate seems to be more nearly constant than below the critical rate. This gives rise to curves in this range that are so flattened as to approach straight lines in shape.

(4) No conclusions were drawn as to the relative magnitude of end effect for the various gases above the critical rate on account of insufficient data.

(5) The formula for the critical rate, $r_c = 10\ dv$, was in one sense confirmed. This formula indicates that the critical rate is independent of the length of the tube.

In plotting our data on compound tubes it was found that the critical rates came at the same points, within the limit of error, that they occurred with the same tube before breaking and compounding. If the critical rate were dependent upon the length of the tube, it would seem that we would have found breaks occuring in regions other than those calculated on the basis that the tube was a continous one whose length was equal to the sum of the lengths of the two segments. Some effect due to the shorter segment would surely have been noticed.

(6) It is of interest to note that below the critical rate, the rate of flow of the gases used was in the inverse order of their viscosities. Above the critical rate howiver the order is the inverse of their densities. This is illustrated graph # 8.

Further work with these gases should prove a fruitful field of research provided that methods of taking measurements can be very greatly refined.

(4) [illegible] was made [illegible] of the [illegible] magnitude of the effect of the partial pressure upon the original rate on account of insufficient data.

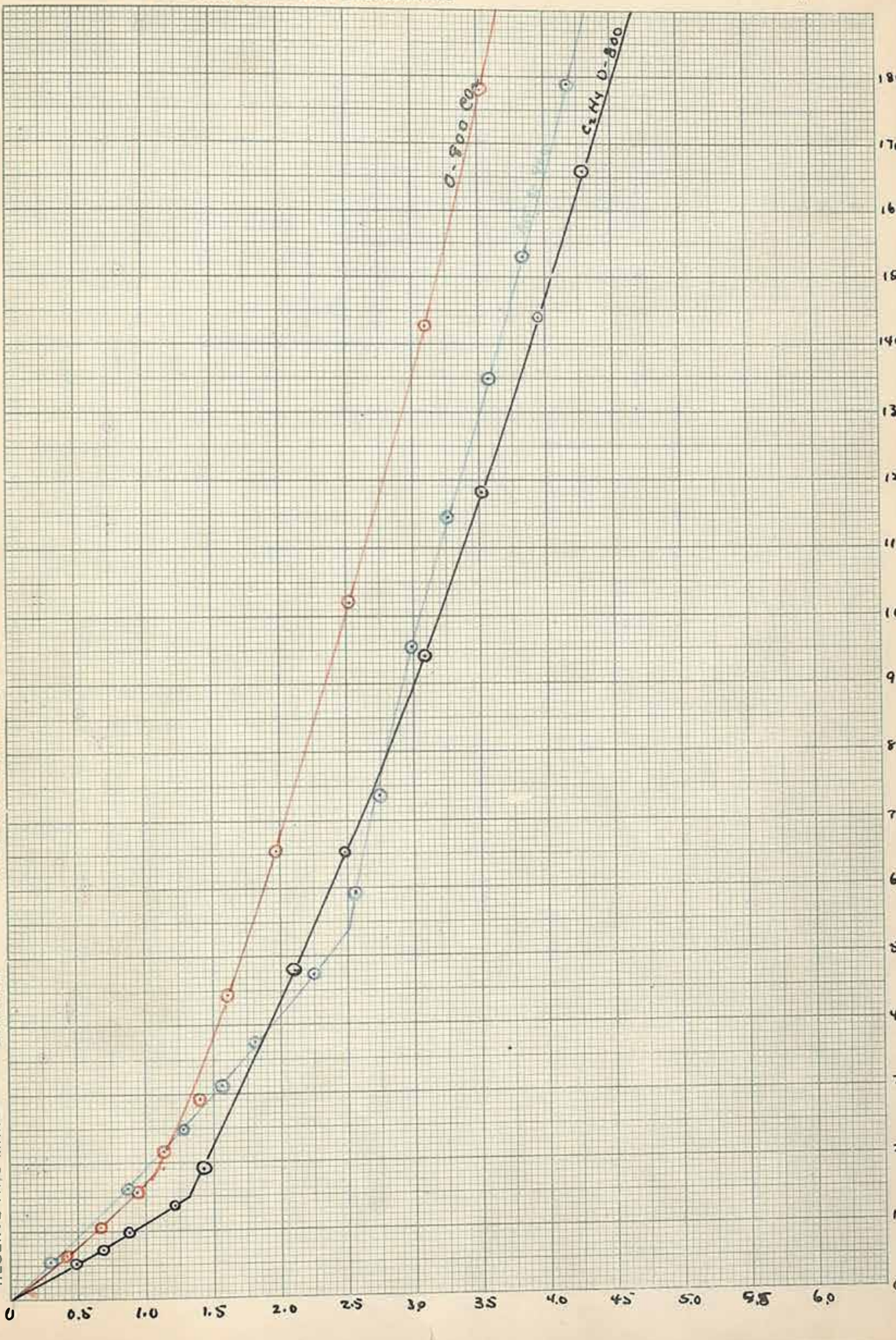
(5) The [illegible] for the critical rate, [illegible], was in the [illegible] sufficient. This [illegible] indicates that the critical rate is independent of the length of the tube.

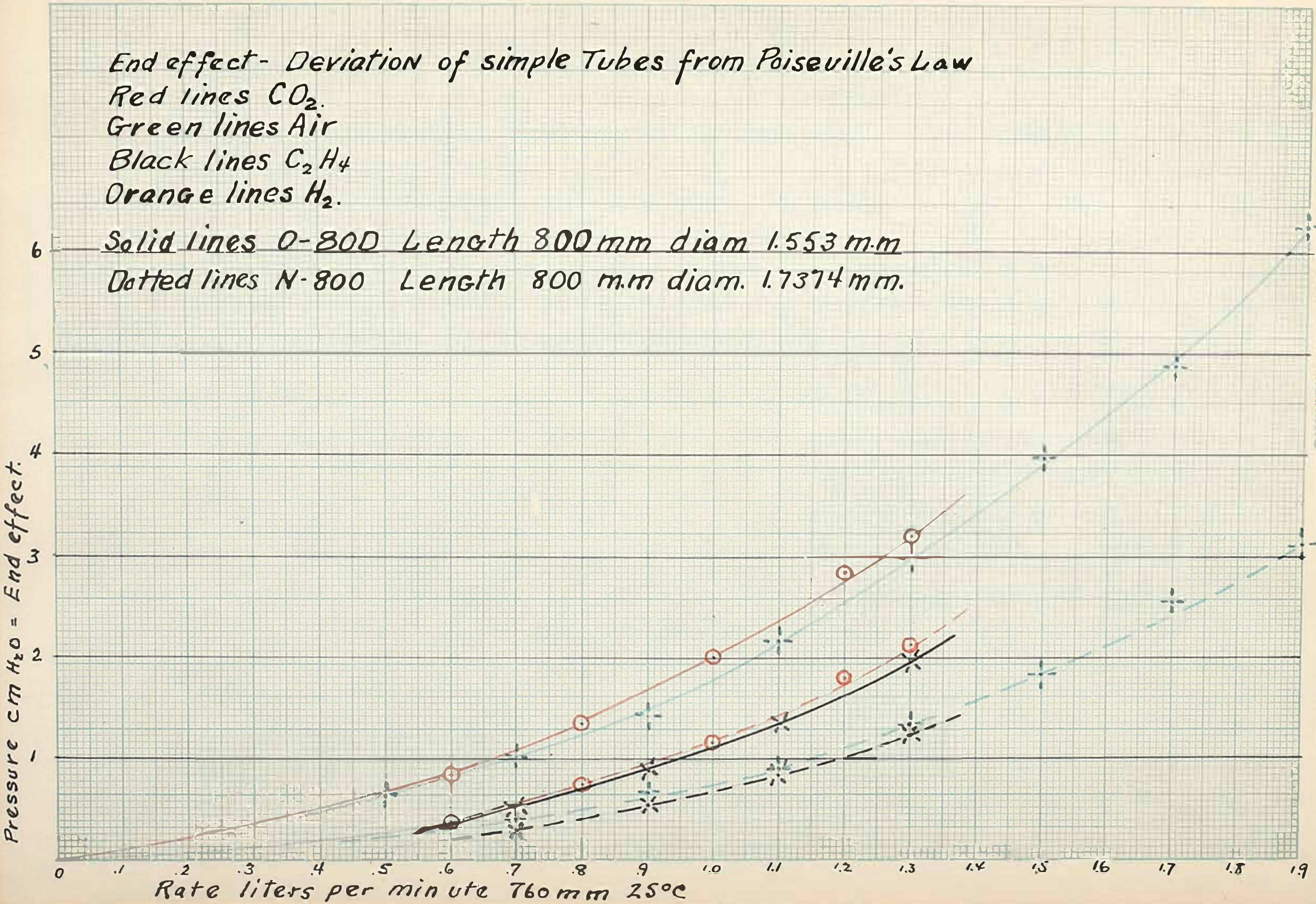
In plotting the [illegible] tubes it was found that the critical [illegible] on [illegible], within the limit of error, that the [illegible] obtained [illegible] tube before [illegible] and [illegible]. If the critical rate were dependent upon the length of the tube, it would seem that we should have found greater [illegible] than those [illegible] to the [illegible] that the [illegible] whose length was equal to the sum of the lengths of the two segments. Some effect due to the [illegible] would surely have been noticed.

(6) It is of interest to note that below the critical rate, the rate of flow of the gases [illegible] in the inverse order of their viscosities. Above the critical rate [illegible] the order is the reverse of their viscosities. This is illustrated [illegible] graph [illegible].

Further work with these gases should prove fruitful field of research provided that methods of taking measurements can be very greatly refined.

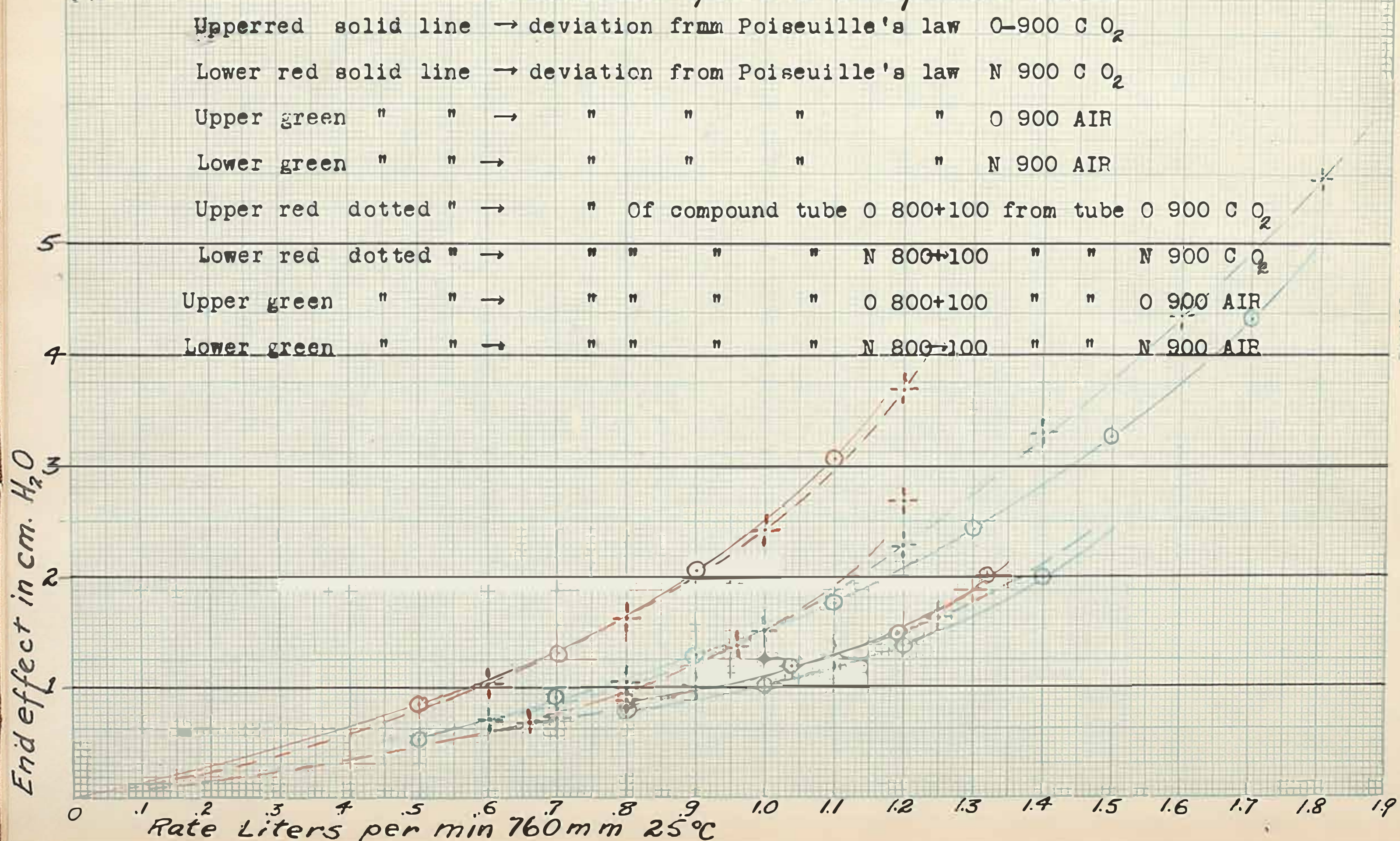
Graph # 8


0-800 CO_2
C_2H_4 0-800
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0


End effect - Deviation of simple Tubes from Poiseville's Law
Red lines CO_2.
Green lines Air
Black lines C_2H_4
Orange lines H_2.
Solid lines O-80D Length 800 mm diam 1.553 m.m
Dotted lines N-800 Length 800 m.m diam. 1.7374 mm.
Pressure cm H_2O = End effect.
1
2
3
4
5
6
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
Rate liters per minute 760 mm 25°C

Graph # 9

Curves showing agreement between Endeffect
(1) Plotted from deviation from Poiseuille's Law
(2) " " " between simple and compound tubes.



Graph # 10

Page 32

SUMMARY

A set of experiments has been carried out on the rate of flow of air, carbon dioxide, ethylene and hydrogen, flowing through long capillary tubes.

The results of the experiments have been analized graphically by means of pressure-rate graphs. These graphs show that there is a critical rate for these gases and that at this critical rate their pressure-rate curve shows a decided change of slope. (This change in slope for hydrogen was not found because it lay outside of the range of our measurements) A formula giving the location of the break in the curve, proposed by Osborne Reynolds, has been found to give results that were in agreement with the experimental data.

An additional set of experiments was carried out to determine the magnitude of end effect. This was done by two different methods and the results obtained from the two methods were checked against each other.

The results of both sets of experiments show that the rate of flow a gas in a tube cannot be deduced by simple viscosity relations from the rate of flow of some other gas in the same tube, unless both the rate and pressure of the gases very nearly approach zero.