Macro Labor Dynamics: Aggregation and Insurance

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Chapters

Misspecification or Misallocation?
Productivity Shocks in the Labor Wedge

I show that for almost all sectors output growth is highly positively correlated with labor productivity growth; 97% of sector's have a correlation higher than 0.6 and 73% of sectors have a correlation over 0.8. These high correlations are consistent with productivity shocks driving variations in sectoral output. At the aggregate level, however, the correlation between output growth and labor productivity growth is only 0.35. I estimate a three-sector DSGE model with sector specific labor and capital alongside an analogous aggregate model. I find that in the aggregate model the labor wedge plays a larger role in explaining variations in output growth than do the sector-specific labor wedges in the three-sector model. In the aggregate model the labor wedge accounts for 24% of the variation in output growth from 2010-2019, in the three-sector model the labor wedges together account for 13%. This overestimation of the importance of the labor wedge at the aggregate level is the result of the aggregate model ignoring assuming a common wage, and therefore a common wedge, for all labor. The aggregate labor wedge should not be interpreted as proof of an intratemporal inefficiency.

Unemployment Insurance and COVID-19
with Jaeki Jang[1]
University of Virginia

This paper examines the welfare implications of the unemployment insurance (UI) expansion policy under COVID-19. We build an equilibrium search and matching model with an incomplete market structure. We find that the UI policy has little to no effect on unemployment. Moreover, we find that the expansion harmed households by an average of $2,400. Much of the benefits are paid to wealthy households choosing not to work during the pandemic due to health risk and large costs are imposed by the accumulation of government debt.

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Misspecification or Misallocation?

Productivity Shocks in the Labor Wedge

1 Introduction

Labor productivity growth, measured as relative changes in real output per hour worked, tends to decrease during recessions and increase during expansions. Standard macroeconomic models feature a diminishing marginal product of labor. Increasing the labor input increases output but marginal output per hour worked, and therefore average output per hour worked, falls. This implies a negative correlation between output growth and labor productivity growth.[1] To produce procyclical labor productivity, as observed in the data, business cycle modelers have used exogenous productivity shocks and endogenous labor hording.

Real Business Cycle (RBC) models which only have productivity shocks produce nearly perfect correlation between labor productivity and real output. A labor hording model with perfect labor hording, that is, a model where firms never change their labor input would also produce a perfect positive correlation. In the U.S., the correlation between output growth and labor productivity growth from 2001-2019 was 0.4. To match this feature of the data, modern business cycle models use labor hording or productivity shocks in combination with other sources of exogenous variation. In the case of productivity shocks, the degree to which output fluctuations are estimated to be the result of fluctuations in productivity depends on the co-movement of labor and output, and is thus tied to the degree of procyclicality of labor productivity.

[1] Nearly a century ago, before quality data was available, this was a theoretical prediction by J.M. Clark (1928).

I document that the correlation between output growth and labor productivity growth is much higher at the sector level than in the aggregate. As laid out below, most sectors have a correlation between their own productivity growth and own output growth in excess of 0.8 and only one sector has a lower correlation than the aggregate economy. The low procyclicality of labor productivity is, in part, the result of aggregation and does not reflect the high correlation between output and labor productivity present at the sector level. The high correlation between output growth and labor productivity growth at the sector level suggests a large role for productivity shocks in explaining variations in sector level output.

Charlie, Kehoe, and McGraten's (2007) (CKM) work on business cycle accounting demonstrates several isomorphisms between a standard neoclassical baseline model with time varying "wedges" in equilibrium conditions and more sophisticated models. The efficiency, or productivity, wedge bridges the gap between output implied by the production function given inputs and the actual output observed. The labor wedge measures the amount by which the marginal rate of substitution (MRS) between consumption and leisure varies from the marginal product of labor (MPL), as in a decentralized competitive equilibrium with no inefficiencies these should each be set equal to the wage. Finally the investment wedge measures the difference between the intertemporal marginal rate of substitution in consumption and the rate of return on capital. Importantly, these wedges are not merely measurement errors outside of the model; they are observed by agents in the model, and agents make decisions based on expectations of future wedges. They find that for US data the wedge which accounts for the most variation in aggregate output, particularly during recessionary periods, is the labor wedge. The efficiency wedge plays a smaller role and the investment wedge is nearly irrelevant. CKM's intent is to guide further research into models that better explain the nature of the wedges.

Following CKM, other researchers have focused on the labor and productivity wedges. It is worth noting that while the investment wedge has little explanatory power, that does not mean that financial frictions are unimportant. As demonstrated by CKM, input-financing frictions which lead to inefficient allocations of resources across sectors would show up in the efficiency wedge rather than the investment wedge. Karabarbounis (2014) confirms the finding that the labor wedge plays an important role in determining aggregate volatility, particularly in recessions, and demonstrates that most of the variation in the labor wedge is caused by a gap between the household MRS and the wage as opposed to the firm's MPL and the wage. He recommends more study of models which better explain the household side of the labor wedge.

More recently, Bigio and La'O (2020), building on Barquee and Farhi (2019), construct a static, non-parametric, firm-level model with no government spending and endogenous labor supply. They allow for productivity shocks and two forms of distortions: sector-level markups and taxes or financial frictions. They derive envelope conditions, along the lines of Hulton's Theorem, to show that starting in an economy with no distortions, adding distortions has no first-order effects on the efficiency wedge but does have first order effects, equal to that sector's share of GDP, on the labor wedge. They find that in the Cobb-Douglas special case "sectoral productivity shocks have zero effects on the labor wedge"

A necessary condition for an efficient allocation in neoclassical representative agent macroeconomic models with endogenous labor supply is that the representative firm's marginal product of labor (MPL) should equal the representative household's marginal rate of substitution (MRS) between consumption and leisure. As such, the labor wedge is commonly interpreted as proof of allocative inefficiency. Sala, Söderström, and Trigari (2010) state that "The labor wedge is...a measure of inefficiency in the allocation of labor." Río and Lores (2021) state that the "labor wedge must primarily reflect distortions in the labor supply". I argue that the labor wedge is, in part, a result of model misspecification. A multi-sector model with productivity shocks and no distortions is capable of producing an aggregate labor wedge.

To study the role sector-level productivity shocks play in variations in aggregate output I construct two models. The first is an aggregate neoclassical model with one representative firm with one production technology and one representative household. I allow for two sources of exogenous variation: productivity shocks and a labor wedge. I then estimate the model on quarterly data on output and labor growth from 2006 to 2019. I find that the labor wedge accounts for 24% of the variation in output growth after the Great Recession. Next, I estimate a three sector general equilibrium model of the US economy with productivity shocks and labor wedges for each sector. I find that the labor wedges together account for 13% of the variation in aggregate output.

Finally I conduct two experiments estimating the aggregate model on data simulated from the three-sector model. First I estimate the aggregate model on simulated data from the three sector model. I find that the aggregate model attributes a similar share of variation in changes in output to the labor wedge as the true data generating process. Next, I simulate data from the aggregate model without any sectoral labor wedges. When I estimate the aggregate model on the simulated data I still find that the aggregate labor wedge explains 5.2% of the variation in output. Critically, even though there was no intratemporal inefficiency in data generating process one was still estimated to exist in the aggregate model as a result of model missspecification.

In the next section I will describe my data and present the differences in labor productivity correlations at the two levels of aggregation. Sector 3 will present the two models. Section 4 will discuss the calibration and estimation of the model. Section 5 presents the simulation exercises. Section 6 concludes.

2 Data

My data on quarterly real sector level value added output comes from the Bureau of Economic analysis and quarterly sector level hours is constructed from the Bureau of Labor Statistics monthly data. I have data on 60 private subsectors (three-digit NAICS) in the US from 2006Q2 to 2019Q1, a total of 52 quarters. Correlations discussed below are all in terms of growth rates for which I have 51 observations for each variable. The correlation between aggregate output and aggregate labor productivity, henceforth the 'aggregate' correlation, is .3492. 97% of subsectors have a correlation higher than 0.6 and 73% of subsectors have a correlation over 0.8. The within correlations are much larger than the aggregate on average. Only one sector, administrative services, has a lower within correlation. This is not a case of the average being weighed down.

Table 1 shows the the aggregate correlation as well as its 95% confidence interval. For comparison, the simple average of the sectors within correlations are shown. I also give the average of the within sectors weighted by total output over the period. The average values are far larger than the aggregate correlation, far larger even than the upper bound of the confidence interval. The confidence intervals for the averages are based on the confidence interval of a correlation from the same size sample and the the point estimate of the average.

In my sector-level model, consumption and investment are produced using different bundles of inputs so I use annual input-output tables from the BEA to get expenditure shares for each sector from 2006-2019. To make the the model tractable, I aggregate the 60 sectors into 3 sectors. First, if at least 1% of a sector's output is used for investment in the IO tables, not counting inventory changes, I consider it an investment sector. Next sectors whose output is used primarily as inputs are upstream while those producing mostly consumption goods I designate downstream. Lastly, I do not consider the financial sector because (1) it doesn't fit neatly as either upstream or downstream and (2) my sectoral model is not designed to capture peculiarities of the financial services sector, particularly as the Great Recession is in my data set. This leaves the following sector choices:

- Investment
  - Mining; Construction; Manufacturing; Information; Professional and Business Services
- Upstream
  - Utilities; Wholesale Trade; Transportation and Warehousing
- Downstream
  - Retail trade; Education, Health care, and social assistance; Arts, entertainment, recreation, accommodation, and food services; Other services except government

These sectors together account for 80% of GDP. All references to aggregate output throughout this paper refer to the sum of real value added output of these included sectors.

3 Models

3.1 Aggregate Model

The aggregate model will be a traditional stationary RBC model where the social planner maximizes the present value of expected utility:

$$\max_{C_t, L_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \phi_t \log(\bar{L} - L_t) \right) \right] \tag{1}$$

Where β is the discount factor, C_t is consumption at time t, σ is the inter-temporal elasticity of substitution. ϕ_t is the weight on the relative utility from leisure. $\bar{L}$ is the labor endowment and L_t is labor at time t. Output is produced with capital K_t and labor L_t according to a Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{2}$$

Where A_t is total factor productivity which evolves according to an AR(1) process:

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t} \tag{3}$$

Relative utility from leisure also evolves according to an AR(1) process:

$$\log \phi_t = \rho_\phi \log \phi_{t-1} + (1 - \rho_\phi) \log \bar{\phi} + \epsilon_{L,t} \tag{4}$$

$\epsilon_{A,t}$ and $\epsilon_{L,t}$ are independent and normally distributed with mean zero and variances σ_A^2 and σ_L^2 respectively. ϕ_t andA_t are the labor wedge and efficiency wedges.I choose to model the labor wedge as a time varying relative utility from leisure. This wedge should be interpreted as a measure of how much household intratemporal preferences would have to be shifting in order to fit the data. Capital evolves according to:

$$K_{t+1} = (1-\delta)K_t + Inv_t \tag{5}$$

And finally, each period output must clear:

$$Y_t = C_t + Inv_t \tag{6}$$

In equilibrium the MRS and the MPL are set equal by the wage:

$$(1-\alpha)\frac{Y_t}{L_t} = \phi_t \frac{C_t^\sigma}{\bar{L} - L_t} \tag{7}$$

The labor wedge is the ratio between ϕ_t and $\bar{\phi}$ and the productivity wedge is the ratio between A_t and $\bar{A} = 1$.

3.2 Three Sector Model

I construct a stationary general equilibrium three sector model with intermediate inputs, and sector specific labor and capital. The social planner will maximize the expected present value of households' utility which they derive from leisure for each labor type and a CES-consumption bundle.

$$\max_{C_{i,t}, L_{i,t}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \sum_{i=I,U,D} \phi_i \log(\bar{L}_i - L_{i,t}) \right) \right] \tag{8}$$

$\phi_{i,t}$ is a sector specific weight on the relative utility from leisure from working in that sector, $\bar{L}_i$ is the sector specific labor endowment, and $L_{i,t}$ is labor in sector i at time t.

The three sectors are investment I, upstream U, and downstream D. The consumption bundle is:

$$C_t = (\gamma_I C^s_{I,t} + \gamma_U C^s_{U,t} + \gamma_D C^s_{D,t})^{\frac{1}{s}} \tag{9}$$

Where γ_i and $\frac{s}{s-1}$ are the consumption weights and elasticity of substitution respectively and $C_{i,t}$ is consumption of good i at time t. The household has different members with different professions who cannot switch sectors. As a result the household cares about where its members are employed in addition to their total level of employment. This specification of sector-specific labor is important to prevent negative co-movement of labor across sectors. With a single type of labor and no frictions the MRS and therefor the MPL would be equal across sectors. If one sector experienced a positive productivity shock, the household would reallocate labor away from less productive sectors to the more productive sector. Moreover, as documented by Christiano and Fitzgerald (1998) and others, even if a positive productivity shock is common across sectors it will cause an increase in the value of the investment good. Thus, even if both sectors receive equal productivity shocks the marginal revenue product of the investment sector will rise by more. Sector specific labor prevents massive reallocation of labor among sectors at business cycle frequencies, but still allows for total labor to vary over the business cycle. The law of motion for each sector's capital stock is:

$$K_{i,t+1} = (1-\delta)K_{i,t} + Inv_{i,t} \tag{10}$$

Where δ is the depreciation rate of capital and $Inv_{i,t}$ is sector specific capital investment at time t. Output from each sector is produced using Cobb-Douglas production functions.

$$Y_{i,t} = A_{i,t}K_{i,t}^{\alpha_{i,1}}L_{i,t}^{\alpha_{i,2}}M_{i,I,t}^{\alpha_{t,3}}M_{t,U,t}^{\alpha_{i,4}}M_{i,D,t}^{\alpha_{i,5}} \tag{11}$$

Where $Y_{i,t}$ is output, $A_{i,t}$ is productivity,$K_{i,t}$ capital, $L_{i,t}$ is labor, and $M_{i,j,t}$ is materials from sector j used in sector i at time t. I assume each sector's output is constant returns to scale, that is, $\sum_{j=1}^{5} \alpha_{i,j} = 1, \forall i$. Output is used only for consumption and material inputs for each other sector. The exception to this is the investment sector which produces the investment good in addition to consumption and intermediate inputs.

$$C_{I,t} + M_{I,I,t} + M_{U,I,t} + M_{D,I,t} + Inv_t = Y_{I,t} \tag{12}$$

Following Huffman and Wynne (1999) I include intratemporal capital adjustment costs. The investment good is allocated to sectors according to:

$$Inv_t = (\xi_I Inv_{I,t}^{\chi} + \xi_U K_{U,t}^{\chi} + \xi_D K_{D,t}^{\chi})^{\frac{1}{\chi}} \tag{13}$$

Where $\chi > 1$ to produce a production possibilities frontier for the investment goods that is concave to the origin. This is done to prevent large swings in investment from one sector to another due to small changes in the relative marginal products of capital across sectors. These intratemporal adjustment costs capture the fact that the process to produce capital used in one sector, such as heavy mining equipment, cannot be freely altered to produce capital for another sector, such as cash registers. Sector productivity evolves according to an AR(1) process

$$\log A_{i,t} = \rho_A \log A_{i,t-1} + \epsilon_{A,i,t} \tag{14}$$

Relative utility from leisure is subject to shocks:

$$\log \phi_{i,t} = (1-\rho_\phi) \log \bar{\phi}_i + \rho_\phi \log \phi_{i,t-1} + \epsilon_{L,i,t} \tag{15}$$

Where $\epsilon_{A,i,t}$ and $\epsilon_{L,i,t}$ are drawn from a zero mean multivariate normal distributions with variance-covariance matrices Σ_A and Σ_L. I assume that, as in the aggregate model, the labor wedges are independent of the productivity wedges.

In equilibrium the marginal revenue product of labor (MRPL) and the MRS for each sector are set equal to the sector specific wage, and therefor one another, substituting equilibrium prices yields:

$$\frac{\gamma_i C_t^{(1-s)(1+\sigma)}}{C_{i,t}^{1-s}} \frac{\alpha_{i,2} Y_{i,t}}{L_{i,t}} = \phi_{i,t} \frac{C_t^\sigma}{(\bar{L}_i - L_{i,t})} \tag{16}$$

The labor wedge is the ratio between ϕ_t and $\bar{\phi}$ and the productivity wedge is the ratio between A_t and $\bar{A} = 1$.

4 Calibration and Estimation

4.1 Aggregate Model

The aggregate model has six non-shock parameters. I normalize $\bar{L} = 1$, set $\beta = 0.99$ because the model is quarterly, and take $\sigma = 2$ as is common in the literature. I set α to match the value-added-weighted average capital share of expenditures from the IO tables. Finally, $\bar{\phi}$ and δ are set so that in the steady state the household employs one third of their labor endowment and the investment to GDP ratio is 12%. I solve and estimate the model using a linear approximation around the steady state. The aggregate model is estimated using two time series: relative changes in total real output and hours worked at the quarterly level from the sectors discussed in Section 2, the data ranges from the second quarter of 2006 through 2019. While quarterly data on GDP and hours worked is available for much longer periods of time I am limited by the availability of data on quarterly sector-level real output. I estimate with a Monte-Carlo Markov-Chain sampling method with 50,000 draws from a Metropolis-Hastings sampling algorithm. Table 2 shows the parameters and their estimated values as well as the 90% confidence interval for the posterior distribution. To test for convergence I perform the Geweke (1992) diagnostic to test for equality of means early versus later in the Markov Chain, post burn in. The null hypothesis is that the means are equal. As shown in Table 3, the p-values for each variable are greater than 0.05 so I fail to reject the null hypothesis that the series has converged. Figure 1 shows the trace plot for the posterior density of the aggregate model. Figure 2 shows priors and posteriors for the estimated parameters.

4.2 Three Sector Model

The three-sector model has thirty-two non-shock parameters. I set $\beta = .99$ and $\sigma = 2$ as in the aggregate model. I normalize the total labor endowment of the household to one: $1 = \bar{L_I} + \bar{L_U} + \bar{L_D}$ and set the relative labor endowments to match the relative total hours worked in each sector from the BLS data over the entire time period. I use the BEA IO tables to set the Cobb-Douglas share parameters $\alpha_{i,j}$ to match the value-added weighted average expenditure shares for each input and sector. The IO tables also provide the share of each sectors' output used as consumption and so I set γ_i equal to each sectors' share of total consumption. As with the aggregate model I solve and estimate the model using a linear approximation around the steady state. The 3-sector model is estimated using six time series: relative changes in real output and hours worked at the quarterly level from the sectors discussed in Section 2, with the same window of time as before. I estimate with a Monte-Carlo Markov-Chain sampling method with 100,000 draws from a Metropolis-Hastings sampling algorithm. Table 4 shows the parameters and their estimated values as well as the 90% confidence interval for the posterior distribution for the aggregate model.

To test for convergence I again perform the Geweke diagnostic. As shown in Table 5, the p-values for each variable are greater than 0.05 with higher tapering so I fail to reject the null hypothesis that the series has converged. The changing p-values across levels of tapering are a result of serial correlation in the Markov Chain. Figure 3 shows the trace plot for the posterior density of the 3-sector model. Figures 4-6 show priors and posteriors for the estimated parameters.

5 Results

5.1 Shock Decomposition

I estimate the wedges necessary to produce the observed data for both models and decompose the effects of those wedges changes in output. Because I do not observe initial values for the state variables in my model, they must be estimated. The estimated initial state has an effect on the observable variables is large for the year of observations, but becomes less important for later observations. My models are ill-equipped to handle the Great Recession and, as mentioned earlier, I do not use data from the financial services sector. As such I treat the first three years of data as a sort of "burn in" to insulate the effects of the wedges on aggregate output growth from the effect of the estimated initial state.

Figure 1 shows the effects of the labor and productivity wedges on changes in output in the aggregate model. The solid line represents the observed changes in output from 2010 to 2019 and the bars show what the changes in output would have been, had the shock to the other wedge been zero. I calculate the share of variation in output attributable to the labor wedge as:

$$\frac{\sum_t |SDL_t|}{\sum_t [|SDL_t| + |SDP_t|]} \tag{17}$$

Where SDL_t and SDP_t are the decomposed effects of shocks to the wedges plotted on Figure 7 and the denominator gives the total height of the bars in Figure 7. I find that labor wedge accounts for 23.94% of the variation in changes in output.

Figure 9 shows the combined effects of the labor and productivity wedges on changes in output in the three-sector model. The bars now show the net effect of the estimated wedges of each type. Figure 9 shows the effect of each wedge on output growth. The solid line in each is again is again the data. To calculate the share of variation in output attributable to the labor wedge I add up the shares of variation attributable to each labor wedge:

$$\frac{\sum_{i=U,I,D} \sum_t |SDL_{t,i}|}{\sum_t \left[\sum_{i=U,I,D} |SDL_{t,i}| + \sum_{i=U,I,D} |SDP_{t,i}|\right]} \tag{18}$$

Table 4 shows the effect of each wedge on changes in aggregate output. Overall, the labor wedges account for 13.30% of the variation in changes in output.

Figure 10 provides a comparison of the importance and direction of the decomposed effects of the labor wedges on GDP across the models. The bars are the share of variation in output growth attributable to labor wedges calculated according to Equation (17) and Equation (18) without taking the absolute value or summing over time in the numerator. The sign of the bars give the direction of the effect while the height indicates the relative strength of the effect compared to the effect of productivity wedges. Notice that signs match is all but two quarters, indicating that when the aggregate labor wedge is estimated to have a positive effect on output growth the net effect of the sectoral labor wedges tend also to be estimated to have a positive effect on output growth.

5.2 Estimation on Simulated Data

I conduct two exercises where I estimate the aggregate model on data simulated by the three-sector model. First, I simulate data from the three-sector model on changes in aggregate output and labor using the estimated parameters. I simulate 1000 periods to avoid a small sample concern. Next, I estimate the aggregate model on this data using the same methodology as before. Because I chose the data generating process I know that sectoral labor wedges account for 18.98% of the variation in changes in output for the simulated data. I find that the aggregate model estimates the aggregate labor wedge to account for 17.7% of the variation in changes in output. Priors and posteriors for the aggregate parameters are included in Figure 11 and this estimation passes the Geweke test. The difference between the share of variation attributed to the labor wedges is small but not zero. The aggregate model overestimates the importance of labor wedges in determining output growth.

Next, I remove the sectoral labor wedges so that $\phi_{i,t} = \bar{\phi}_i$ and simulate data from the three-sector model, holding the other parameters at their estimated values. I simulate 30 samples of 51 periods of aggregate output and labor growth to match the sample size of my data. The initial 3-sector model state is drawn from the ergodic distribution. Under this setup, all of the variation in changes in aggregate output are due to productivity shocks. Figure 14 shows the share of variation in changes in output estimated to be caused by the aggregate labor wedge for each of the 30 samples, using Equation (17). Moreover, this is not the case of a restrictive prior on the variance of labor wedge shocks as shown in Figure 12, which gives priors and posteriors for σ_ϕ in one of the samples. The prior put more weight on much smaller shocks to the relative disutility from working than the posterior. Figure 14 shows the distribution of posterior means for σ_ϕ across the 30 samples. An aggregate labor wedge is consistently measured even though for each sector the MRS equals the MRPL. In this case the aggregate labor wedge is not an indication of factor misallocation, but of model misspecificaiton.

Why is a labor wedge measured in the aggregate model when none exist at the sector level? The first, and most important, issue is that the three-sector model has sector-specific capital and labor. In Bigio and La'O's static model there is no capital and one labor input used for all sectors. This means there is one MRS for the household and a common MRPL for each firm. Bigio and La'O are able to find an aggregate production function in the Cobb-Douglass case based on the single factor of labor and an efficiency wedge which depends on sector-level productivity and the input-output structure of the model. Put simply, in their model each unit of labor is equally valuable so expressing total output and the disutility from working as a function of total labor, without regard for where it is employed, is possible.

In my three-sector model, there is no such object as aggregate labor. While one could add up each sector's labor, as is done in the data and as I do for the simulation exercises, this does not respect the differences in productivity (let alone network location) of the types of labor. The same problem is true for the capital input. A transformation to effective units of labor and capital *might* allow for the derivation of an aggregate production function. However, such a modification would require the new aggregate model to be estimated on a transformation of labor data at the sector level. My argument remains; the aggregate model, estimated using a single input of total hours worked, can overestimate the role of the labor wedge in determining changes in output.

5.3 Analytic Result

Consider the following data generating process: output is produced according to a function of several types of labor as well other inputs, either other factors of production or shocks.

$$Y = F(L_1, ..., L_n, ...) \tag{19}$$

A representative household receives per-period utility from consumption and leisure:

$$U(C) + V(L_1, ..., L_n) \tag{20}$$

Where the labor inputs and consumption are time varying but with subscripts suppressed. Let $F()$ and $V()$, with respect to all L_i, be continuously differentiable, increasing, and concave. Let $U()$ be continuously differentiable, increasing, and concave. Suppose there are no labor wedges and that each type of labor is efficiently employed. The MPL for each type of labor should equal its MRS:

$$\frac{\partial F}{\partial L_i} = -\frac{\frac{\partial V}{\partial L_i}}{\frac{dU}{dC}} \tag{21}$$

Finally suppose we can solve the model to find L_i as a function of aggregate variables, including aggregate labor $L = \sum L_i$.

$$L_i = g_i(L, ...) \tag{22}$$

Plugging $L_i = g_i(L, ...)$ into the production and utility functions gives an aggregate model:

$$Y = F(g_1(L, ...), ..., g_n(L, ...), ...) \tag{23}$$

$$U(C_t) + V(g_1(L, ...)), ..., V(g_n(L, ...)) \tag{24}$$

We can calculate MPL and MRS:

$$MPL = \sum \frac{\partial F}{\partial L_i} \frac{\partial g_i}{\partial L} \tag{25}$$

$$MRS = -\frac{\sum \frac{dV}{dL_i} \frac{\partial g_i}{\partial L}}{\frac{dU}{dC}} \tag{26}$$

These are equal because equation (21) holds for each labor type so each term in the MPL sum has a corresponding equal term in the MRS sum. Thus, it is possible to have multiple labor types and guarantee no spurious aggregate labor wedge. This requires three important things:

- Equilibrium labor can be expressed as a function of aggregate variables for each type

- We can find those functions
- We know the disaggregated model to begin with

If $L_i = g_i(L, ...)$ exists, finding it would generally require knowledge of the actual data generating process. In reality, we don't know the data generating process; all models are incorrect. Some are more useful than others, certainly. If in the real world there are different types of labor, then even if each type of labor is efficiently employed a labor wedge can exist in a model purely due to misspecification. Of course, inefficiently employed labor types can also produce an aggregate labor wedge.[2] Next I present an example of a "near miss" in model specification to illustrate how misspecification can produce a labor wedge in aggregate models.

5.3.1 Misspecification Example

Consider an economy perfectly described by a simple RBC model with log-log preferences and Cobb-Douglas production with capital and two kinds of labor. Output is given by:

$$Y_t = A_t K_t^{1-\alpha_1-\alpha_2} L_{1,t}^{\alpha_1} L_{2,t}^{\alpha_2} \tag{27}$$

There are two labor clearing equations:

$$\frac{\alpha_i Y_t}{L_{i,t}} = \frac{\phi_i C_t}{\bar{L}_i - L_{i,t}} \tag{28}$$

[2]A misspecified model might not even find a labor wedge with inefficiently employed labor types, given a very contrived underlying data generating process.

Similar to the three-sector model above $\bar{L}_i$ is endowment of labor type i and ϕ_i is the relative utility from leisure for labor type i. Let $1 = \bar{L}_1 + \bar{L}_2$. In this simpler setup however, ϕ_i will be constant for each sector. Variation in output will only come from shocks to A_t. By construction this efficient model has no labor wedges. Rearranging (20) yields expressions for each type of labor as functions of parameters and the ratio of consumption to output:

$$L_{i,t} = \frac{\alpha_i \bar{L}_i}{\alpha_i + \phi_i \frac{C_t}{Y_t}} \tag{29}$$

Now suppose a naive economist set out to model this economy but failed to recognize the two types of labor. If he chooses a Cobb-Douglas production function then he will calibrate the share parameter on labor to be $\alpha_1 + \alpha_2$ as that gives the total share of income to the types of labor, which we can assume he observes. Output in his model will be:

$$Y_t = \tilde{A}_t K_t^{1-\alpha_1-\alpha_2} L_t^{\alpha_1+\alpha_2} \tag{30}$$

Where $\tilde{A}_t$ is now different from the true A_t. If he assumes a log-log utility function for the household and derives the labor market clearing condition he will find:

$$\frac{(\alpha_1 + \alpha_2) Y_t}{L_t} = \frac{\tilde{\phi} C}{1 - L_t} \tag{31}$$

Where $L_t = L_{1,t} + L_{2,t}$ which again, is observable to our hypothetical economist who normalizes the total supply of labor to 1. $\tilde{\phi}$ is the object of interest in this exercise. It can be solved for as a function of total labor, the consumption-output ratio and parameters:

$$\tilde{\phi} = \frac{(\alpha_1 + \alpha_2)(1 - L_t)}{L_t \frac{C_t}{Y_t}} \tag{32}$$

Substituting in $L_t = L_{1,t} + L_{2,t}$ yields:

$$\tilde{\phi} = \frac{(\alpha_1 + \alpha_2)(\phi_2(\alpha_1 - \bar{L_1}(\alpha_1\phi_2 - \alpha_2\phi_1) + \phi_1\frac{C_t}{Y_t})}{\alpha_1\alpha_2 + (\alpha_2\phi_1 + \bar{L_1}(\alpha_1\phi_2 - \alpha_2\phi_1))\frac{C_t}{Y_t}} \tag{33}$$

If $\alpha_1\phi_2 = \alpha_2\phi_1$ then $\tilde{\phi} = \phi_1 + \phi_2$ and each type of labor will employ the same share of its endowment. Otherwise, $\tilde{\phi}$ must vary with the consumption to GDP ratio and our hypothetical economist will find a labor wedge solely due to misunderstanding the bifurcated nature of labor in this economy and misspecifying the model.

5.4 The Labor Productivity Puzzle

Prior to 1983 the correlation between output growth and labor productivity growth was higher, from 1960-1983 it was 0.7. The decline in the correlation has been christened the "Labor Productivity Puzzle" by some and solutions to the puzzle within the RBC framework have focused on unobserved additional factors of production. Gali and Gambetti (2009), Gali and van Rens (2010), and Nucci and Mariana (2009) argue that variations in effort or effective labor can produce the low correlation. McGratten and Prescott (2012) propose large variations in unobservable intellectual capital production to solve the puzzle. The other common approach is to include demand shocks in the form of government expenditure shocks. A government demand shock can increase output without directly increasing labor productivity and thus lower their correlation as demonstrated by Fairise and Langot (1994).

As mentioned in Section 2 the within correlations are much larger on average than the aggregate correlation. This counter-intuitive product of aggregation is a result of three aggregation inequalities:

- Aggregate labor productivity not the sum of sectors' labor productivity.

- The growth rate of a sum is not the sum of its components' growth rates.

- Unlike the covariance, the correlation is not a linear operator; the sum of correlations in not equal to the correlation of sums.

Some algebra will illuminate these difficulties. The correlation between aggregate labor productivity and aggregate output can be written as

$$corr(\eta, \gamma) = corr(\sum_i \eta_i \lambda_i \theta_i + \theta_i \Delta\lambda_i + \eta_i \Delta\lambda_i, \sum_i \gamma_i \upsilon_i) \tag{34}$$

where η and γ are the growth rates of aggregate labor productivity and output respectively. η_i and γ_i are the sector level growth rates. λ_i is the ratio of sector labor to aggregate labor with $\Delta\lambda_i$ denoting its change. θ_i is the ratio of sector labor productivity to aggregate labor productivity. υ_i is the ratio of sector output to aggregate output. Equation 34 shows the first two inequalities.

An example will make the economic implication of the third inequality clear: let x and y be standard normal variables with covariance σ_{xy} and consider:

$$corr(x + y, f(x) + g(y)) \tag{35}$$

which is analogous to a two sector economy where output growth is a deterministic function of labor productivity growth for each sector and the weighting terms discussed above are all constant across sectors and time. The correlation here is dependent on the derivatives of f and g. If we assume f and g are linear with positive coefficients on x and

y of a and b we can solve for the correlation:

$$\frac{(a+b)\sqrt{1+\sigma_{xy}}}{\sqrt{2(a^2+2ab\sigma_{xy}+b^2)}} \tag{36}$$

which achieves its maximum value of one when $a = b$ or when $\sigma_{xy} = 1$. Each sector has a constant linear relationship between labor productivity growth and output growth, but so as long as the sectors' relationships are different the aggregate correlation will be less than one.

6 Conclusion

The labor wedge should not be interpreted solely as an inefficiency. While the household's MRS should equal the firm's MRPL in an efficient allocation, if an aggregate model does not adequately match the data generating process then it can have the wrong MRPL and MRS. In that case, a gap between the MPL and MRS is a sign of misspecification, not missallocation. Productivity shocks are capable of generating an aggregate labor wedge if labor is composed of distinct types of labor. Using my 3-sector model I find that the role labor wedges play in determining variations in output growth is considerably less than an aggregate model would imply.

A larger model, with more sectors, might plausibly find an even smaller role for labor wedges. However, if wedges are not assumed to be independent the number of parameters to be estimated grows exponentially. Since a sector-specific labor wedge causes an inefficient allocation it may be the case that variations in productivity are caused by variations in allocative efficiency, as is well understood in the literature. Finally, both labor wedges and efficiency wedges could result from common sources of variation or distortions.

Because the aggregate labor wedge can result from productivity shocks if there is efficiently employed differentiated labor it should be viewed as proof of an inefficient allocation.

7 Tables

Table 1: LP growth- VA growth Correlation

Aggregate Correlation	.3492	[.0898,.5641]
Unweighted Average Corr.	.8546	[.7611,.9133]
Output Weighted Avg. Corr.	.8430	[.7430,.9061]

Table 2: Aggregate Model Parameter Estimates

Description	Symbol	Estimate	90% CI
Efficiency Wedge Persistence	ρ_A	0.9769	[0.9596,0.9956]
Efficiency Wedge Variance	σ_A	0.0072	[0.0060,0.0084]
Labor Wedge Persistence	ρ_ϕ	0.9918	[.9814,.9990]
Labor Wedge Variance	σ_ϕ	0.0103	[0.0087,0.0120]

Table 3: Aggregate Model Geweke Test for Equality of Means p-values

Symbol	Posterior Mean	No Tapering	4% Tapering	8% Tapering	15% Tapering
ρ_A	0.9769	0.081	0.700	0.706	0.707
σ_A	0.0072	0.000	0.180	0.196	0.232
ρ_ϕ	0.9918	0.000	0.527	0.518	0.511
σ_ϕ	0.0103	0.173	0.803	0.807	0.798

Table 4: Three-Sector Model Parameter Estimates

Description	Symbol	Estimate	90% CI
Consumption Bundle Elasticity of Substitution	$\frac{1}{1-s}$	1.2097	[1.1944,1.2275]
Investment Bundle Elasticity of Substitution	$\frac{1}{1-\chi}$	-10.4822	[-769.2308,-4.9505]
Efficiency Wedge Persistence	ρ_{A_I}	0.9334	[0.8493,0.9990]
	ρ_{A_U}	0.8519	[0.7597,0.9442]
	ρ_{A_D}	0.9194	[0.8352,.9988]
Efficiency Wedge Standard Deviations	$\sigma^2_{A_I}$	0.0074	[0.0058,0.0089]
	$\sigma^2_{A_U}$	0.0898	[0.0764,0.1022]
	$\sigma^2_{A_D}$	0.0381	[0.0308,0.0455]
Efficiency Wedge Correlations	$\frac{\sigma_{A_{IU}}}{\sigma_{A_I}\sigma_{A_U}}$	0.2689	[0.1282,0.4233]
	$\frac{\sigma_{A_{ID}}}{\sigma_{A_I}\sigma_{A_D}}$	0.5045	[0.3573,0.6313]
	$\frac{\sigma_{A_{UD}}}{\sigma_{A_U}\sigma_{A_D}}$	-0.4192	[-0.5485,-0.2961]
Labor Wedge Persistence	ρ_{ϕ_I}	0.9371	[0.8844,0.9988]
	ρ_{ϕ_U}	0.8593	[0.7444,0.9860]
	ρ_{ϕ_D}	0.9787	[0.9482,0.9990]
Labor Wedge Standard Deviations	$\sigma^2_{\phi_I}$	0.0458	[0.0396,0.0519]
	$\sigma^2_{\phi_U}$	0.0162	[0.0136,0.0185]
	$\sigma^2_{\phi_D}$	0.0114	[0.0096,0.0132]
Labor Wedge Correlations	$\frac{\sigma_{\phi_{IU}}}{\sigma_{\phi_I}\sigma_{\phi_U}}$	-0.0118	[-0.1702,0.1514]
	$\frac{\sigma_{\phi_{ID}}}{\sigma_{\phi_I}\sigma_{\phi_D}}$	0.1000	[-0.0688,0.2726]
	$\frac{\sigma_{\phi_{UD}}}{\sigma_{\phi_U}\sigma_{\phi_D}}$	0.0495	[-0.0922,0.1864]

Table 5: Three-Sector Model Geweke Test for Equality of Means p-values

Description	Symbol	Posterior Mean	No Tapering	4% Tapering	8% Tapering	15% Tapering
Consumption Bundle Elasticity of Substitution	$\frac{1}{1-s}$	1.2097	0.000	0.179	0.234	0.278
Investment Bundle Elasticity of Substitution	$\frac{1}{1-\chi}$	-10.4822	0.000	0.041	0.067	0.073
Efficiency Wedge Persistence	ρ_{A_I}	0.9334	0.000	0.549	0.618	0.628
	ρ_{A_U}	0.8519	0.000	0.495	0.505	0.474
	ρ_{A_D}	0.9194	0.000	0.076	0.076	0.075
Efficiency Wedge Standard Deviations	$\sigma^2_{A_I}$	0.0074	0.000	0.453	0.496	0.540
	$\sigma^2_{A_U}$	0.0898	0.000	0.023	0.044	0.053
	$\sigma^2_{A_D}$	0.0381	0.000	0.104	0.152	0.165
Efficiency Wedge Correlations	$\frac{\sigma_{A_{IU}}}{\sigma_{A_I}\sigma_{A_U}}$	0.2689	0.000	0.061	0.103	0.152
	$\frac{\sigma_{A_{ID}}}{\sigma_{A_I}\sigma_{A_D}}$	0.5045	0.000	0.619	0.644	0.667
	$\frac{\sigma_{A_{UD}}}{\sigma_{A_U}\sigma_{A_D}}$	-0.4192	0.000	0.115	0.127	0.106
Labor Wedge Persistence	ρ_{ϕ_I}	0.9371	0.000	0.530	0.564	0.545
	ρ_{ϕ_U}	0.8593	0.000	0.118	0.152	0.144
	ρ_{ϕ_D}	0.9787	0.000	0.179	0.234	0.476
Labor Wedge Standard Deviations	$\sigma^2_{\phi_I}$	0.0458	0.000	0.011	0.035	0.057
	$\sigma^2_{\phi_U}$	0.0162	0.000	0.040	0.032	0.062
	$\sigma^2_{\phi_D}$	0.0114	0.000	0.610	0.611	0.583
Labor Wedge Correlations	$\frac{\sigma_{\phi_{IU}}}{\sigma_{\phi_I}\sigma_{\phi_U}}$	-0.0118	0.000	0.208	0.258	0.253
	$\frac{\sigma_{\phi_{ID}}}{\sigma_{\phi_I}\sigma_{\phi_D}}$	0.1000	0.000	0.049	0.091	0.115
	$\frac{\sigma_{\phi_{UD}}}{\sigma_{\phi_U}\sigma_{\phi_D}}$	0.0495	0.564	0.970	0.970	0.964

Table 6: Three-Sector Model Shock Decomposition

Wedge	Contribution (%)
Investment Productivity	26.83
Upstream Productivity	34.18
Downstream Productivity	24.12
Investment Labor	8.12
Upstream Labor	2.21
Downstream Labor	2.73
Initial State Estimate	1.8

8 Figures

Figure 1: Aggregate Model: Posterior Trace Plot

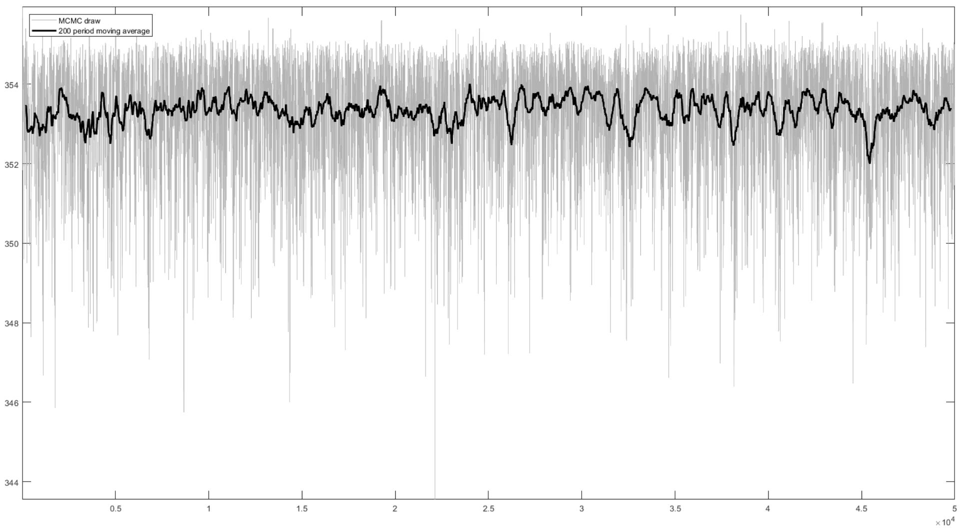


Figure 2: Aggregate Model: Priors and Posteriors

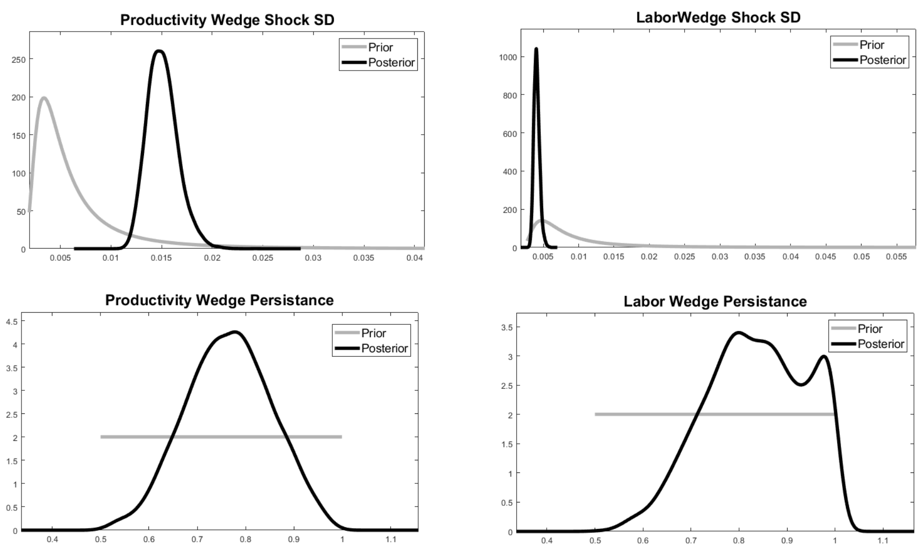


Figure 3: 3-Sector Model: Posterior Trace Plot

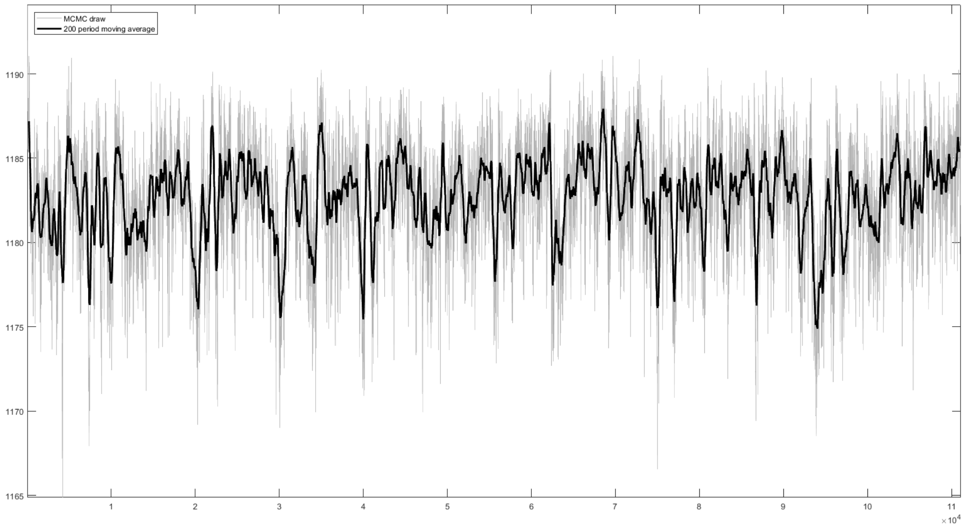


Figure 4: 3-Sector Model: Priors and Posteriors 1

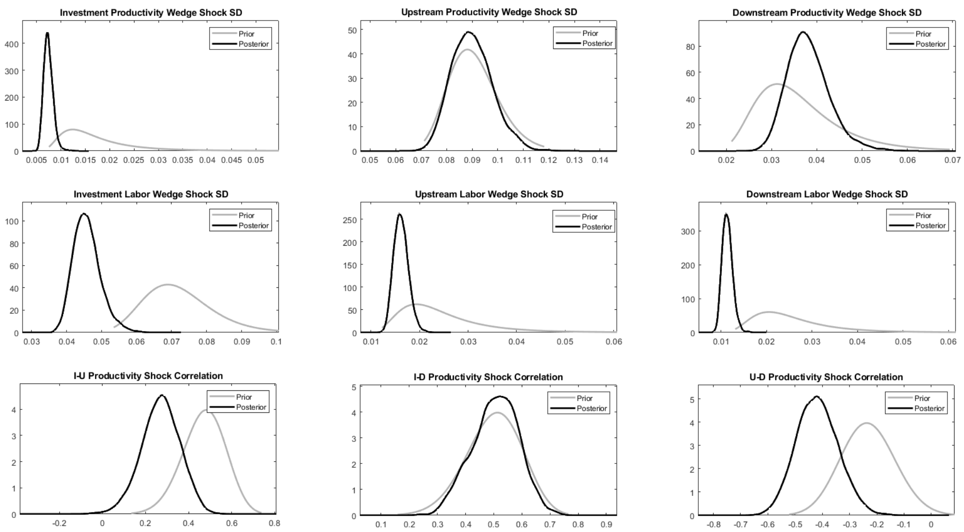


Figure 5: 3-Sector Model: Priors and Posteriors 2

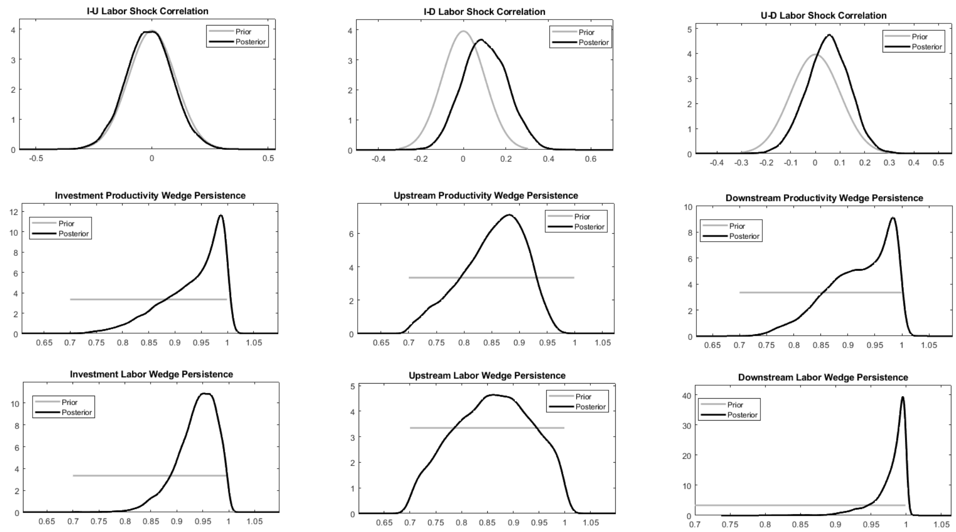


Figure 6: 3-Sector Model: Priors and Posteriors 3

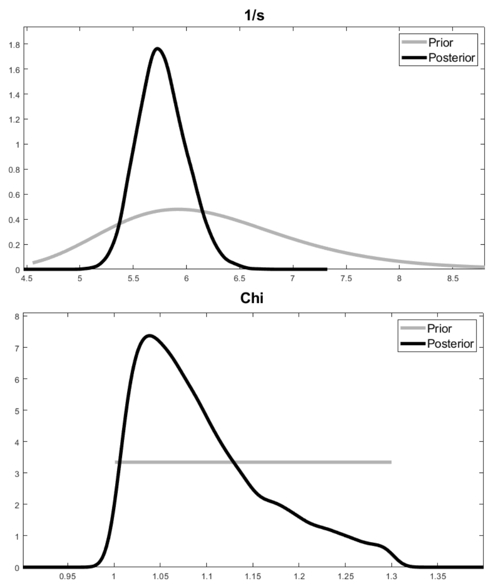
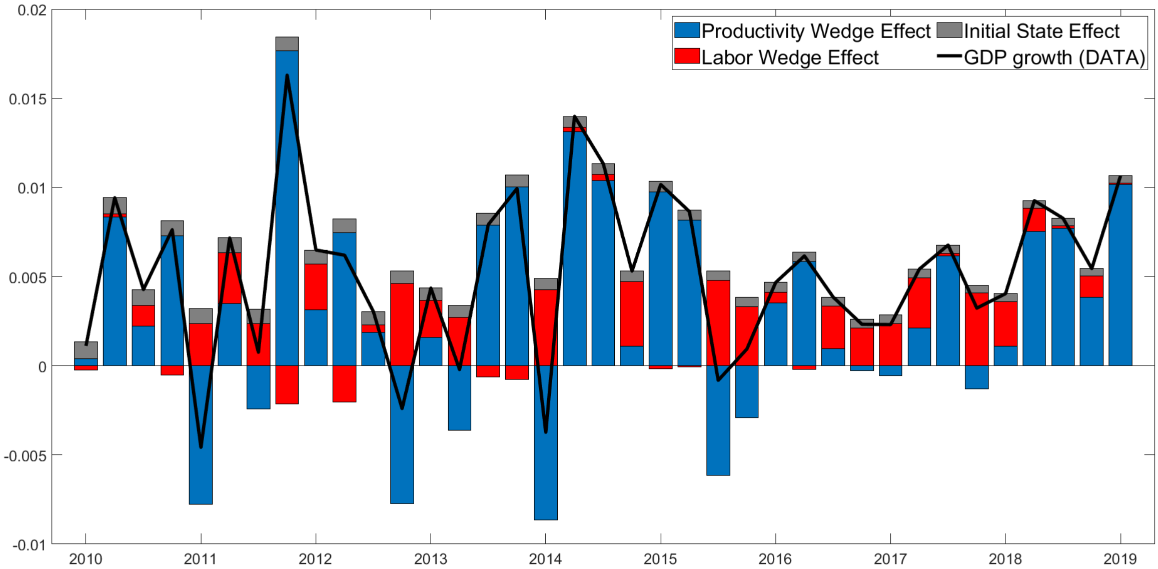


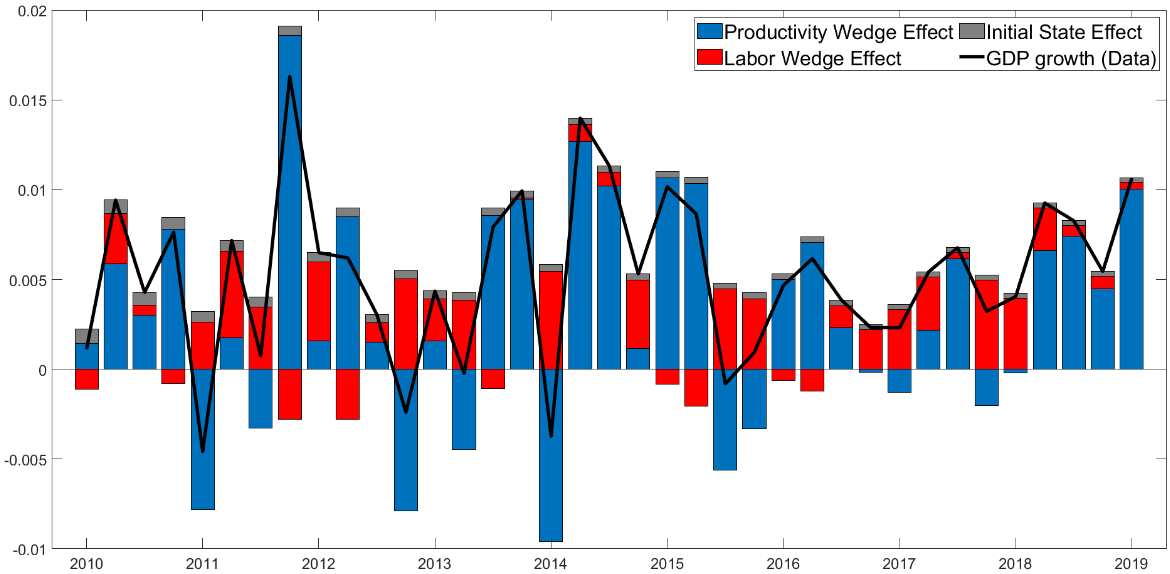
Figure 7: Aggregate Model: Effect of Labor and Productivity Wedges on GDP



(a) *The effect of the productivity wedge is shown in blue and the effect of the labor wedge is shown in red.*

(b) *Growth data is de-meaned for the estimation, but the data set includes the Great Recession, so this portion of the data has a positive mean.*

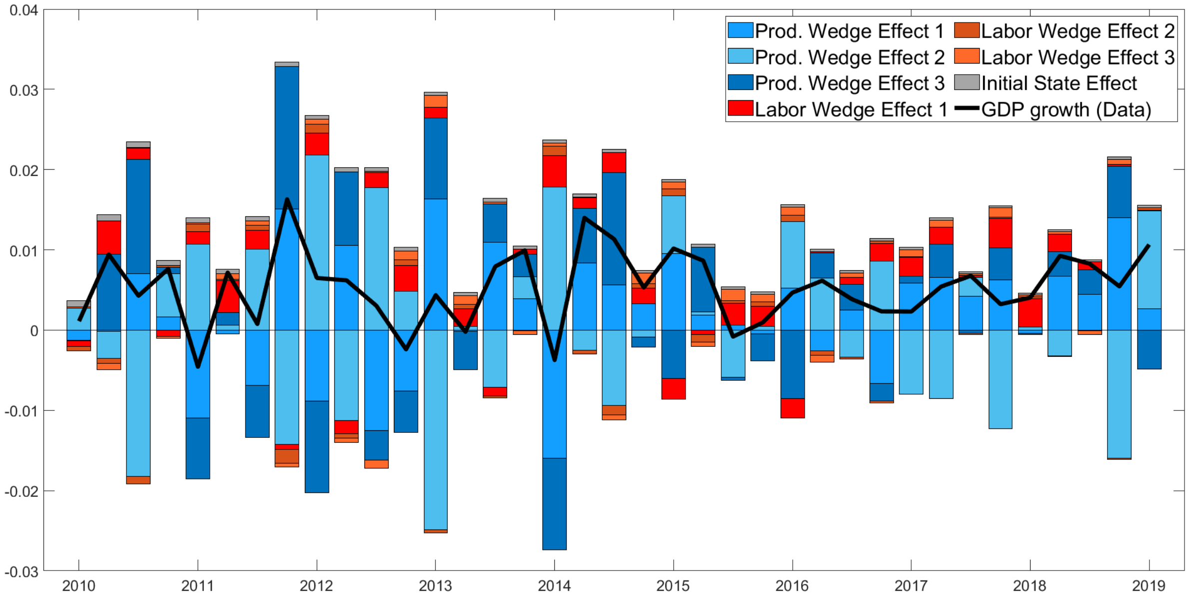
Figure 8: 3-Sector Model: Net Effect of Labor and Productivity Wedges on GDP



(a) *The net effect of the productivity wedges are shown in blue and the net effect of the labor wedges are shown in red.*

(b) *Growth data is de-meaned for the estimation, but the data set includes the Great Recession, so this portion of the data has a positive mean.*

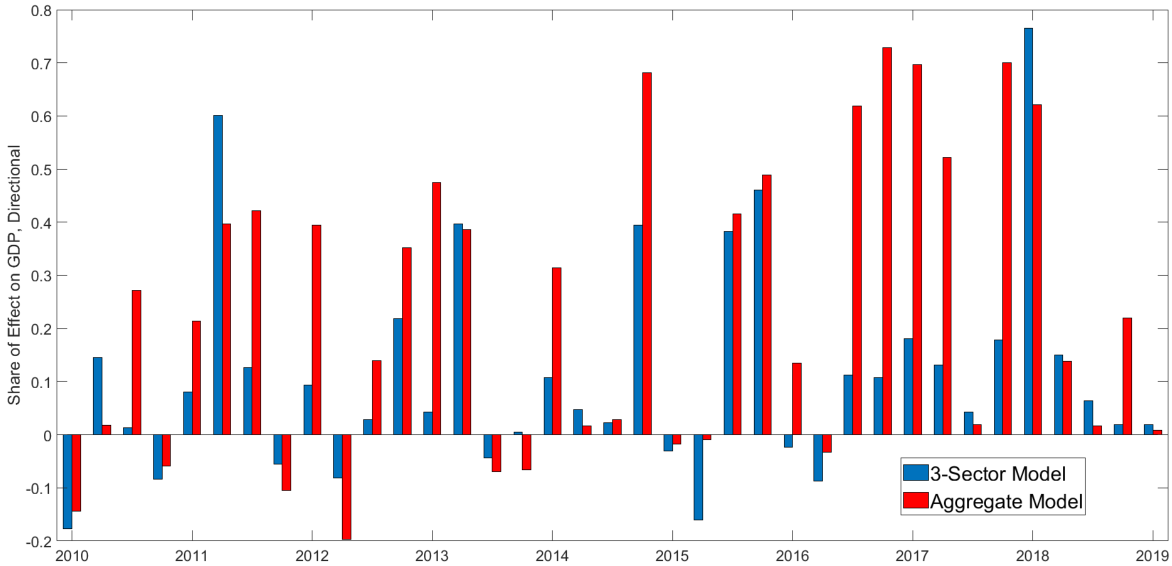
Figure 9: 3-Sector Model: Effect of Labor and Productivity Wedges on GDP



(a) *The effect of each productivity wedge are shown in blue tones and the effect of each labor wedge are shown in red tones.*

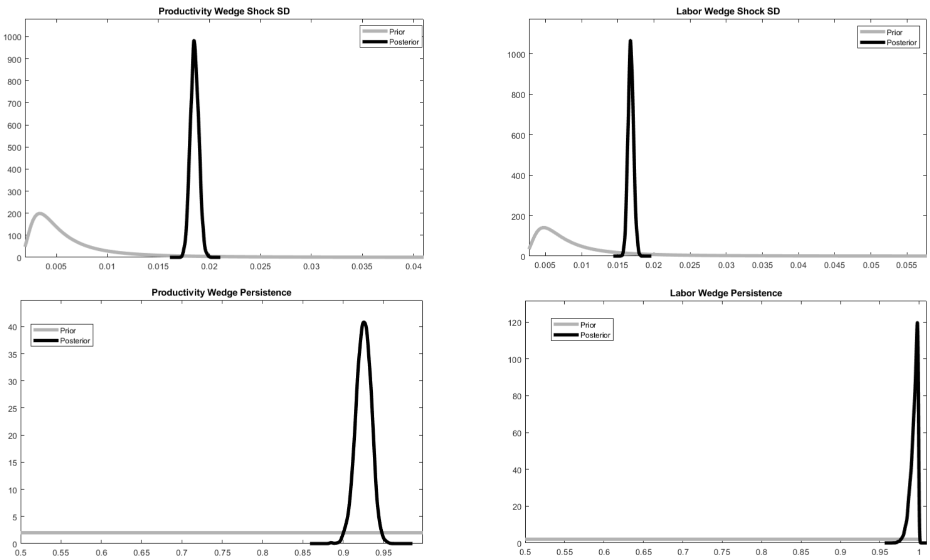
(b) *Growth data is de-meaned for the estimation, but the data set includes the Great Recession, so this portion of the data has a positive mean.*

Figure 10: Both Models: Effect of Labor Wedges on GDP



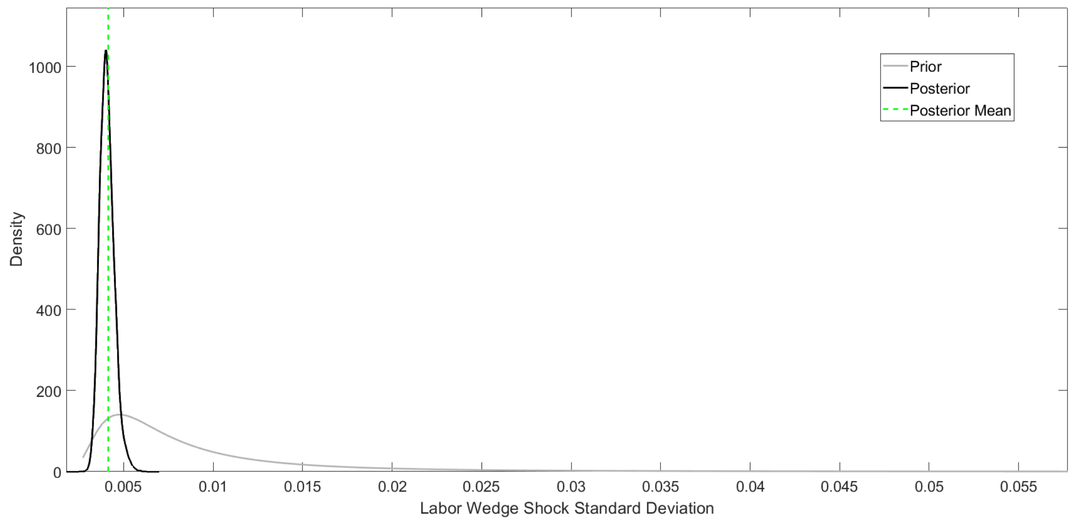
(a) *The net effect of 3-sector labor wedges are shown in blue and the effect of the aggregate labor wedge is shown in red. Each is expressed as a share of total effects on GDP growth from their model and the direction of the effects are given by the sign.*

Figure 11: Prior and Posterior Distributions for Simulation Exercise 1



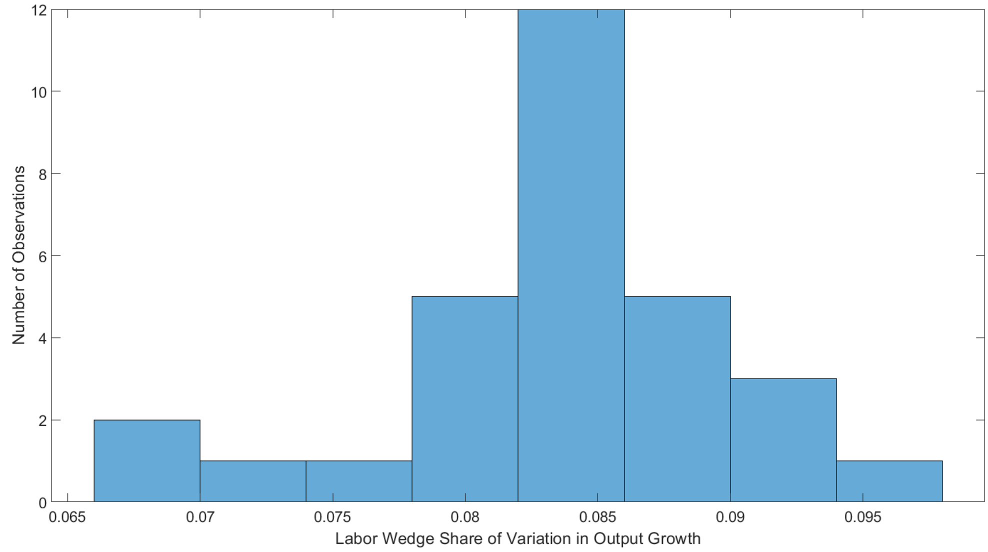
(a) *These are the distributions for the simulation exercise based on data generated by the 3-sector model with both productivity and labor wedges.*

Figure 12: Prior and Posterior Distributions for Simulation Exercise 2



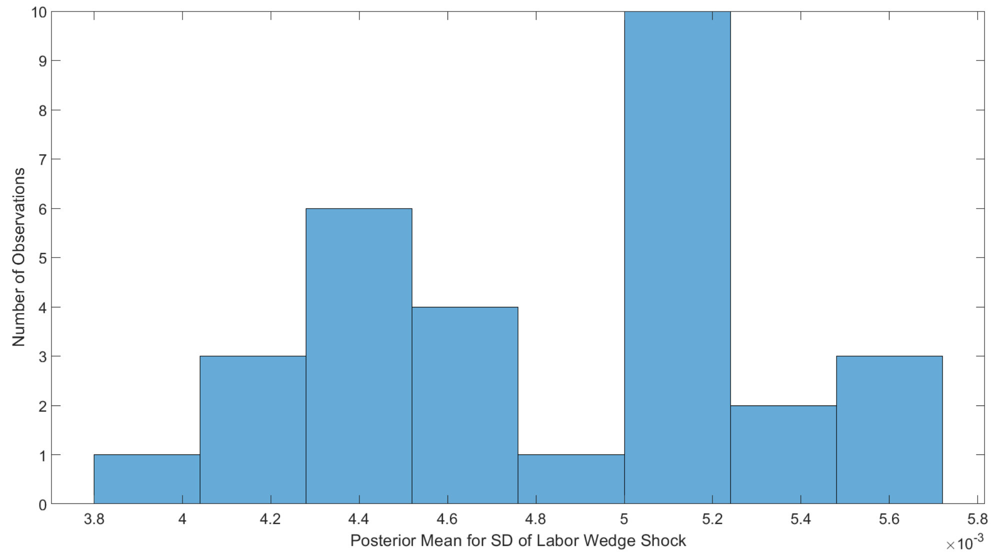
(a) *These are the distributions for the simulation exercise based on data with no labor wedges at the sector level. Notice that the prior allowed for much smaller aggregate labor wedges than were estimated.*

Figure 13: Simulation Effect of Aggregate Labor Wedge on GDP



(a) *This histogram shows the portion of variation in changes in GDP estimated to be caused by the aggregate labor wedge. The data was simulated in the three sector model with no labor wedges.*

Figure 14: Distributions of Estimates for σ_ϕ



(a) *This histogram shows the estimated values for the SD of shocks to the aggregate labor wedge. The data was simulated in the three sector model with no labor wedges.*

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Unemployment Insurance and Covid-19

with Jaeki Jang

1 Introduction

The unemployment rate spiked from 3.8% in the first quarter of 2020 to 13.0% in the second quarter with the onset of the COVID-19 pandemic in the US. Around 20 million jobs were lost in April 2020. In response to the unprecedented shock to unemployment, congress passed the CARES Act at the end of March 2020, which included the Federal Pandemic Unemployment Compensation (FPUC) and Pandemic Unemployment Assistance (PUA). FPUC provides an additional $600 per week and PUA relaxes eligibility guidelines for unemployment benefits. Households will respond to the expansion of the unemployment benefits differently depending on their characteristics such as education levels and wealth. In this paper, we study the effects of expanded unemployment insurance (UI) on labor market outcomes both in the aggregate and by groups of people. Furthermore, we examine the welfare consequences of UI during the pandemic and the process of recovery.

We build a Diamond-Mortensen-Pissarides (DMP) equilibrium search and matching model which incorporates an incomplete asset market structure and COVID shocks. Agents are heterogeneous in terms of ability and wealth and face two types of shocks: employment and COVID shocks. The former are idiosyncratic and exogenously separate workers from work, and the latter can lead to endogenous separations even after a match between a potential worker and a vacant firm is formed because there is no wage that makes both parties better off. The COVID shocks enter into the model by lowering the aggregate productivity and raising disutility from working. An increase in disutility captures the risks of being infected. Therefore, the shocks affect both the supply and the demand side of labor markets.

One of the key innovations in our model is that agents hold assets to insure against their risk of being unemployed. To our knowledge, there is no paper in the literature on

UI during the pandemic with an equilibrium model that allows households to hold assets. We need assets in order to better analyze the effects of UI policy as labor supply decisions heavily depend on wealth. Also, we can focus on moral hazard issues of households in the presence of generous UI policy. Furthermore, in order to run a gigantic program like the CARES Act, which amounts to $2.2 trillion dollars, government needs to issue bonds to finance the program, which requires some agents to hold assets.

Households are different in terms of their innate abilities which affect the productivity of firms as well as labor income. Responses to COVID shocks and the value of compensation should be different for people with different abilities. Our model can also capture the interesting trade-offs on wages that the COVID shocks generate. A falling productivity lowers wages, but a rise in disutility from working raises wages for those still working in order for firms to hire workers.

We find that expanding UI had little to no effect on the unemployment rate, which is consistent with the literature (Altonji et al. 2020, Bartik et al. 2022, Boar and Mongey 2020, Finamor and Scott 2021). However, we find that the UI expansion harmed households by $2,400 on average for two reasons:

1. Many rich households choose to become unemployed due to the increased health risk of working. Most of the UI payments go to these rich households which have low marginal utilities of consumption.

2. The debt the government is forced to take on to finance the UI expansion crowds out capital, lowering wages and dividends over time and must eventually be repaid by future taxes.

This paper contributes to the growing literature on labor markets and pandemic (Atkeson 2020, Alvarez, Argente and Lippi 2021, Cortes and Forsythe 2020, Forsythe et al. 2020, Glover et al. 2020, Gupta, Simon and Wing 2020, Krueger, Uhlig and Xie 2020). More specifically, we contribute to the literature on UI and pandemic by allowing households to hold assets and government to finance its debt. There are several papers that have structural models but do not allow individuals to save (Fang, Nie and Xie 2020, Birinci et al. 2021, Mitman and Rabinovich 2021, Marinescu, Skandalis and Zhao 2021). Ganong et al. 2021 examine spending responses and job search efforts of individuals with different assets. However, they only focus on households and lack decisions of firms and government.

In Section 2, we describe the model and explain how it works. We analyze the model and present corresponding results in Section 3. We conclude in Section 4.

2 Model

2.1 Households

Agents are endowed with unchanging idiosyncratic ability z are either employed or unemployed. If unemployed, agents either receive unemployment benefits $h(z)$. If employed, agents receive wage w which is match specific and renegotiated each period. There are three assets: capital k, equity holdings x, and the government bond b. Agents hold equity with a share price p in all firms simultaneously and earn dividends d. Capital depreciates at rate δ and is rented at rate r. The face value of a government bond (with one period maturity) that sells for b is qb, where q is the price of bond. Assets have equal return so we can define:

$$a \equiv (1 + r - \delta)k + (p + d)x + qb$$

where the return on asset is defined as m and agents' positions on the assets are undetermined but their total holdings of a is known.

Agents choose their asset holdings next period a' to maximize their present value of utility from consumption, c. An employed worker solves the following problem:

$$W(a,z) = \max_{c,a'} u(c) - \rho + \beta[\Theta(1-\sigma)W(a',z) + \Theta\sigma U(a',z) + (1-\Theta)U(a',z)] \tag{1}$$

$$\text{subject to} \qquad c + a' = ma + (1-\tau)w$$

where ρ is the disutility a worker receives from working, σ is probability of exogenous separation from the firm, β is the discount factor, and Θ is an indicator function, detailed later, equal to zero if the match is endogenously separated. The match-specific wage is determined each period through Nash Bargaining. Lastly, τ is a tax rate on labor earnings. An employed household would stay employed if both endogenous and exogeneous separation do not occur. Otherwise, she becomes unemployed. Let the asset decision rule for employed workers be $a'_e = \varphi_e(a,z)$.

Unemployed agents receive income h, which indicates home production and social benefits from the government. The value function for an unemployed agent is:

$$U(a,z) = \max_{c,a'} u(c) + \beta[\lambda_w \Theta W(a',z) + (1-\lambda_w)U(a',z) + \lambda_w(1-\Theta)U(a',z)] \tag{2}$$

$$\text{subject to} \qquad c + a' = ma + h$$

where λ_w is the probability that an unemployed worker is matched to a job.

An unemployed agent would work if she is matched to a vacant firm and the match is profitable. Otherwise, she receives income h and stays unemployed in the next period. Let the asset decision rule for unemployed households be $a'_u = \varphi_u(a,z)$.

2.2 Firms

Each firm employs a single worker and chooses the level of capital to rent from households at rate r. An agent at a job produces $zAF(k)$ where z is an idiosyncratic productivity. A

is a deterministic aggregate productivity level, and $F(k)$ is a concave function of capital per filled job. A firm with a worker pays any flow profits in dividends to the shareholders. The value function for a firm with a filled position is:

$$J(a,z) = \max_k zAF(k) - rk - w + \frac{1}{m}[V + \Theta(1-\sigma)J(a',z) - V] \tag{3}$$

where V is a value of a vacant firm, which satisfies:

$$V = -\xi + \frac{1}{m}[V + \lambda_f \int \Theta J(\varphi(x), z)\frac{f_u(a,z)}{u} dadz] \tag{4}$$

where ξ is the cost of posting a vacancy and λ_f is the probability that a vacant firm is filled with a worker. $f_u(a,z)$ is the population of unemployed workers with asset holdings a and ability z, and u is the total unemployed workers. We assume free entry for new vacant firms so the value of a vacancy in equilibrium is equal to zero. The flow value of firm equity is the sum of matched firm profits minus the search costs of vacant firms.

2.3 Matching and wage negotiation

Unemployed workers will be matched with vacant firms according to a matching function: $M(u,v)$,where v represents the total measure of vacancies. We denote the probability that a vacant job is matched with a worker as $\lambda_f = M(u,v)/v = M(u/v,1) = M(1\backslash\theta,1)$ where $\theta \equiv v/u$, which indicates labor market tightness. A match between an unemployed worker and a vacant job will be completed if the following two conditions satisfy:

$$W(a,z) \geq U(a,z) \tag{5}$$

$$J(a,z) \geq V \tag{6}$$

The above two conditions guarantee that the both parties receive surplus from the match. $\Theta(a,z)$ is an indicator function with a value of 1 if there exists a wage that satisfies both conditions. If either condition does not satisfy, then the value of W becomes the same as U and $J(a,z) = 0$. An existing match will have the wage that solves:

$$\max_{w}\{W(x) - U(x)\}^{\kappa}\{J(x) - V\}^{1-\kappa} \tag{7}$$

where κ represents the bargaining power of the worker.

2.4 Government

The government issues one-period bonds b, taxes income τ, and pays social benefits in the form of unemployment compensation h. The government's budget constraint is:

$$\tau \int w(a,z)\frac{f_e(a,z)}{e} dadz + B' = hu + qB \tag{8}$$

where B is the total amount of government bonds and $f_e(a,z)$ is a distribution for workers with asset a and ability z. In steady state the government will be restricted to set $B = B' = 0$ and will do so by adjusting labor tax τ. During the pandemic and a transition phase after, however, the government will be allowed to increase its debt level.

3 Results

3.1 Steady State Calibration

We first solve the steady state of our model to serve as the bookends of the pandemic, and post-pandemic transition periods. We set utility and production functions:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \tag{9}$$

$$F(k) = k^{\alpha} \tag{10}$$

With $\gamma = 2$ and $\alpha = 0.3$. We set $\beta = 0.995$ and $\delta = 0.005$ since each time period is six weeks. Following Shimer (2005), we set the exogenous separation rate $\sigma = 0.05$ and choose the matching function:

$$M(u, v) = \chi u^{\eta} v^{1-\eta}$$

with parameters $\chi = 0.6$ and $\eta = 0.72$. We also set the parameter governing Nash Bargaining, $\kappa = 0.72$, using Hosios efficiency condition. Departing from Shimer and KMS we set the cost of posting a vacancy $\xi = 1$ in order to produce a labor market tightness consistent with pre-pandemic levels. In the steady state, unemployment benefits are equal to the 30% of the wage paid to workers of the same ability and asset holdings. In order to pin down the wage of a worker with no assets we set a minimum level of benefits which, in equilibrium, is equal to 30% of their wage. We assume that in the steady-state there is no disutility from working.

To solve the steady state we define value function analogues which take their future value from the actual value functions and are computed at a given wage:

$$\tilde{W}(a, z, w) = \max_{c,a'} u(c) - \rho + \beta[\Theta_e(1-\sigma)W(a', z) + \Theta_e \sigma U(a', z) + (1-\Theta_e)U(a', z)] \quad (11)$$
$$\text{subject to} \qquad c + a' = ma + (1-\tau)w$$

$$\tilde{U}(a, z, w) = \max_{c,a'} u(c) + \beta[\lambda_w \Theta_u W(a', z) + (1-\lambda_w)U(a', z) + \lambda_w(1-\Theta)U(a', z)] \quad (12)$$
$$\text{subject to} \qquad c + a' = ma + h$$

$$\tilde{J}(a, z, w) = \max_{k} zAF(k) - rk - w + \frac{1}{m}[V + \Theta(1-\sigma)J(\tilde{\varphi}_e(a, z, w), z) - V] \quad (13)$$

and then use Nash Bargaining to find the wage that maximizes:

$$(\tilde{W}(a,z,w) - \tilde{U}(a,z,w))^{\kappa}\tilde{J}(a,z,w))^{1-\kappa} \tag{14}$$

Using the fact that $V = 0$ in all states due to free entry. Once $w(a,z)$ is calculated we then calculate the value functions $W(a,z), U(a,z)$ and $J(a,z)$ and policy rules $\varphi_e(a,z)$ and $\varphi_u(a,z)$. We use this process to recalculate the wage at each iteration of finding the value functions. The steady state is then found as the pair of labor market tightness θ and capital K that sets the value of an entrant from equation (3) equal to zero and sets total asset holdings of households at the invariant distribution equal to the value of firm equity plus capital.

3.2 Transition

The transition starts with a population of firms and employed and unemployed workers with asset holdings determined by the steady state invariant distribution. The future value for value functions in teh final transition period are the steady-state value functions. The transition has five phases:

1. Pandemic phase
    - Aggregate productivity (A) falls
    - Disutility from working (ρ) rises
2. UI expansion phase
    - Occurs during the pandemic phase
3. Post-Pandemic
    - Aggregate productivity (A) instantly returns to its SS value
    - Disutility from working (ρ) instantly returns to zero

- Government debt is rolled over

4. Taxation

- Government debt is paid off with a new tax (τ_2) on firms and vacancies

5. Terminal Phase

- A short phase to connect the taxation phase to the future steady-state values

The third phase is much longer than the others and has the purpose of lessening the importance to first-period welfare of the eventual tax policy. The optimal policy for repaying debts accrued during the pandemic is unclear, and in reality the debt is simply being rolled over. To use the zero-debt steady state value functions as an endpoint the debt must be paid off, so in phase 4 a flat tax is imposed on firms both vacant and full. Phase 5 allows the unemployment rate and asset distribution to recover towards the steady state.

We set the pandemic to last about two years, beginning February 2020 and ending February 2022. Phase 2 begins in the second period and the UI benefits expansion lasts 4 months. We do not include in our analysis the later, smaller, UI expansions that were not part of the CARES Act. The government runs up debt for ten years[1] before paying it off over the course of two years. Phase 5 is two years.

To solve their problems, agents need to know next-period's value functions as well as the interest rate and matching probabilities, which can be calculated from labor market tightness. During phase 4 agents also need to know the amount of the tax. By assuming paths over time for i, θ, and τ_2 agents can negotiate wages and solve for their asset and employment decisions working backwards. The model is then simulated forward using asset and endogenous separation policy rules. The paths of i, θ, and τ_2 are updated to (1) clear the asset market (2) set the value of a vacant firm to zero, and (3) pay off the debt according

[1] We are experimenting with lengthening this phase.

to the rule:

$$B(t+1) = B(t)\frac{n-1}{n} \tag{15}$$

during phase 4 only, where n is the number of periods left to pay off the debt. In the last period of phase 4 the debt must be fully paid off.

3.3 Calibration

To calibrate the size of the shocks to productivity and disutility from working we use two data points: the 10 percentage-point increase in unemployment from January 2020 to March 2020 and the 7% increase in median real weekly earnings from Q4 2020 to Q2 2020 over the trend from the previous 2 years. We set the size of the UI benefit expansion to make total UI benefits equal to 100% of the steady-state median wage, following reporting on the motivation for the $600 per week expansion.

Figure 1 shows paths over time for the unemployment rate, labor market tightness, government debt, and the interest rate for the first 50 periods simulated by the model.

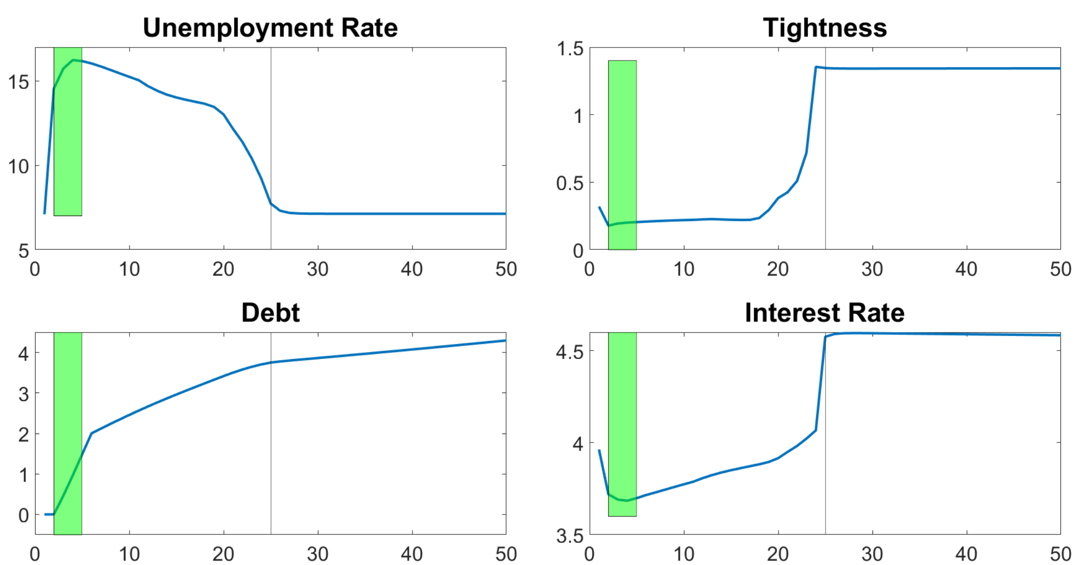

Figure 1: *Paths for unemployment, tightness, debt, and the interest rate over the first 50 periods, a little over 4 years, from the start of the pandemic. The vertical line marks the end of the pandemic and the green bar the duration of expanded benefits.*

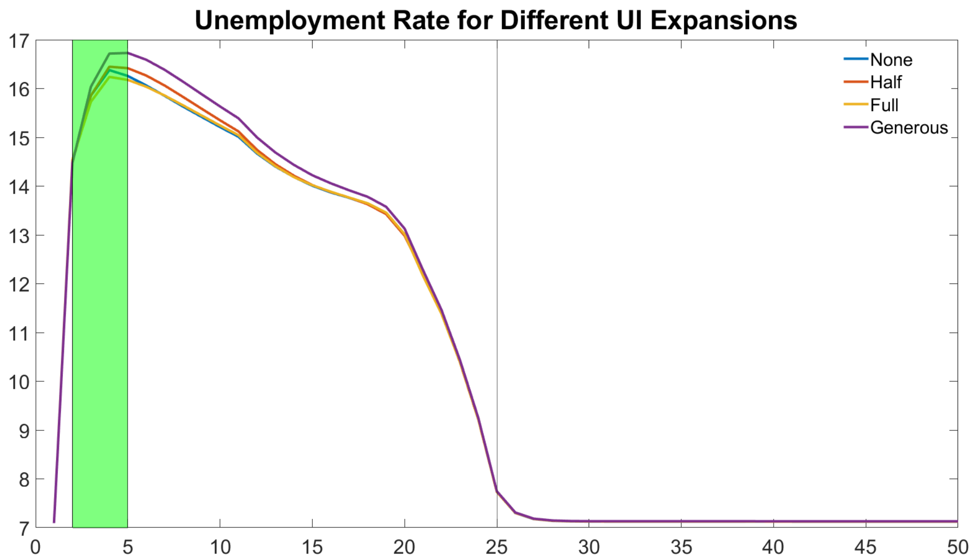

Figure 2: *Paths for unemployment across the four different levels of UI expansion*

3.4 Counterfactual UI Levels

We simulate three alternative levels of UI expansion to compare welfare and simulated variables over different policy options.

1. No expansion: 30% Median UI
2. Half expansion: 65% Median UI
3. Full expansion: 100% Median UI
4. Generous expansion: 135% Median UI

As shown in Figure 2 there is very little difference in the unemployment rate across different levels of unemployment benefits. The most generous benefits, equivalent to an additional $900 per week cause an additional 0.3 percentage point increase in the unemployment rate for two periods, but this difference disappears before the pandemic ends.

Figure 3 shows the path of average wages for low ability workers across the counterfactuals for the first 150 periods and Figure 4 shows the same for high-ability workers. While benefits are being paid, the difference across counterfactuals is compositional.

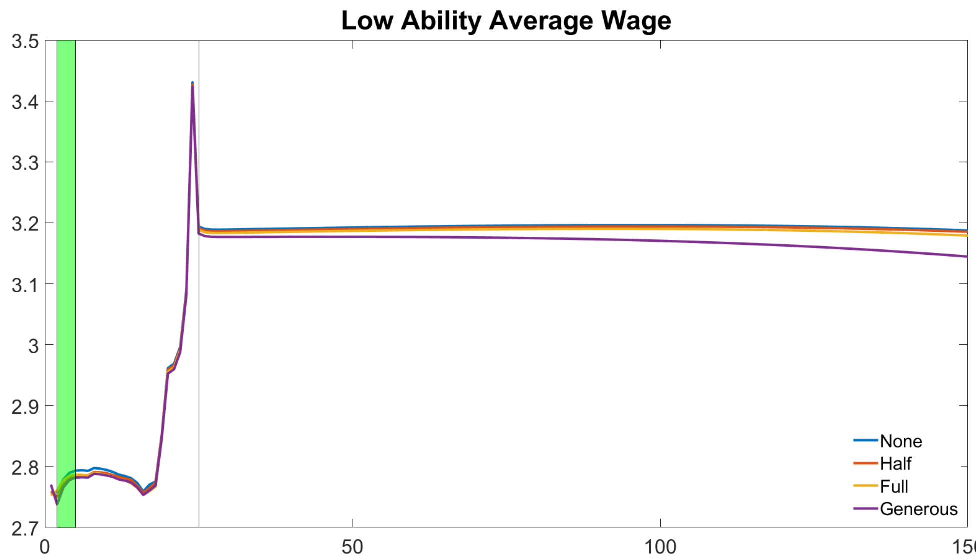

Figure 3: *Paths for average wages of low ability workers across the four different levels of UI expansion*

Endogenous separation occurs from high asset individuals who are normally high wage earners. The slight differences in the unemployment rate across counterfactuals causes the slight difference in average wages. For these graphs, we include a longer time horizon to show how the higher debt level is depressing wages by crowding out capital. The most generous expansion causes a particularly large reduction in wages.

Intuitively, debt grows faster with higher benefits being paid. Even without any UI expansion the pandemic causes a deficit by increasing unemployment past the steady-state level the labor income tax is designed to fund. Post pandemic debt grows exponentially until it is forced down by policy.

Capital falls quickly during the pandemic as shown by 6 because output falls and households want to dissave. Post-pandemic capital rises, but is crowded out over time by government debt. In the most generous UI expansion, this crowding out is so severe that capital never grows post pandemic.

Next we calculate value function the of different agents, averaged over the initial asset distribution, across the counterfactuals. We find that all UI expansions reduce welfare for

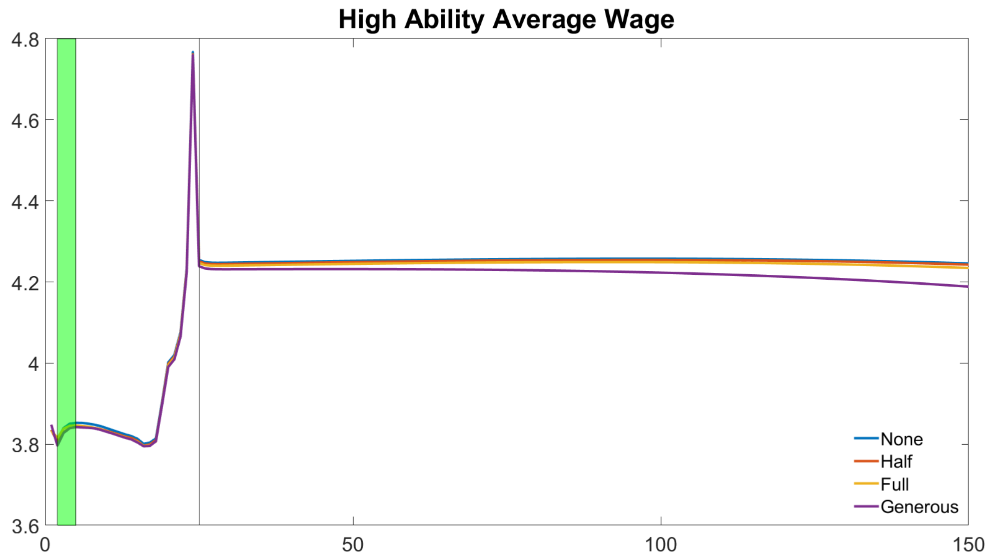

Figure 4: *Paths for average wages of low ability workers across the four different levels of UI expansion*

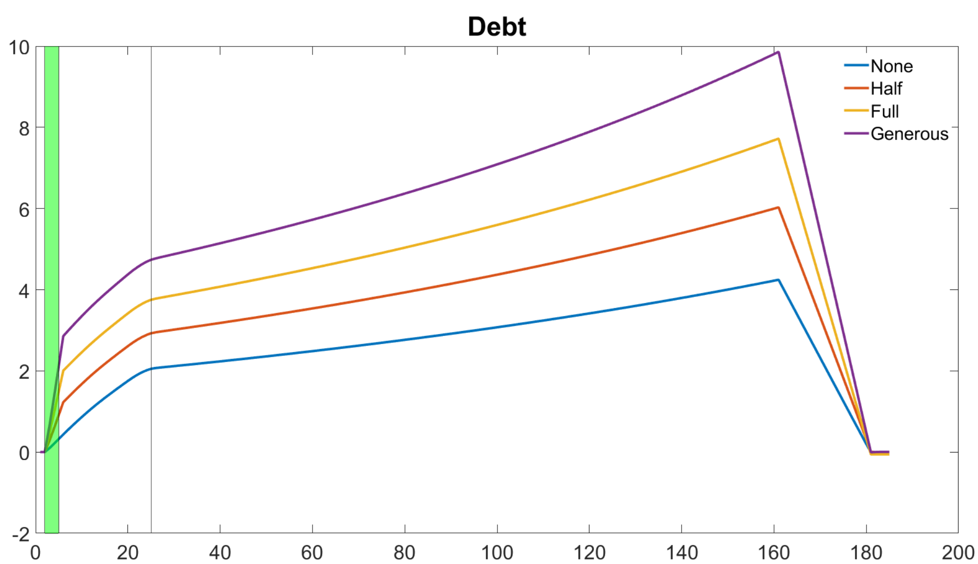

Figure 5: *Paths for government debt across the four different levels of UI expansion*

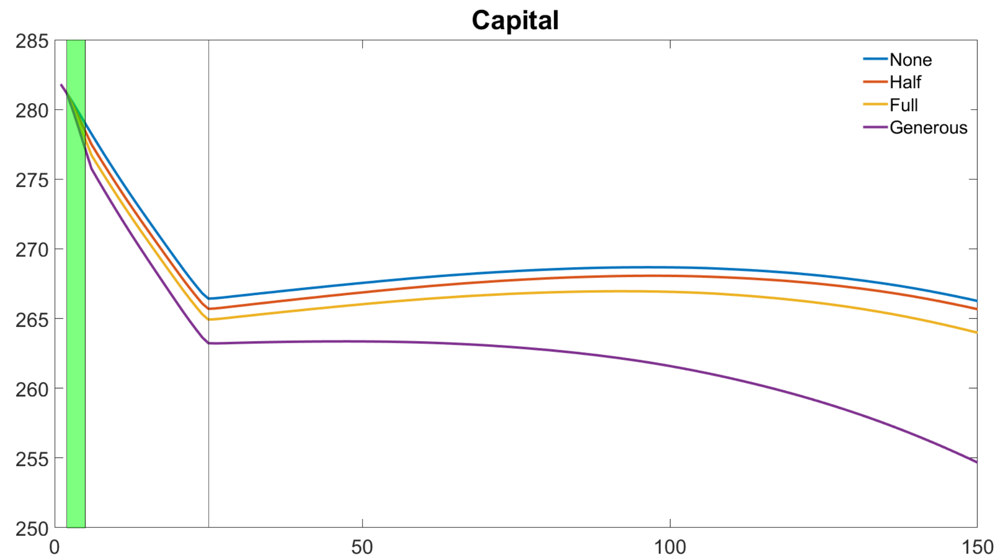

Figure 6: *Paths for capital across the four different levels of UI expansion*

employed and unemployed individuals across ability types, with larger expansions being worse. We calculate equivalent variations from the counterfactual with no UI expansion and find that the $600 per week UI expansion policy was equivalent to a $2,411.90 reduction in household wealth to the average household. Table 1 shows the equivalent variation for the different counterfactuals across different agent types.

Table 1: Equivalent Variation of UI Expansion

Type	$300	$600	$900
High Ability Employed	-$600.40	-$2667.30	-$5,250.80
Low Ability Employed	-$440.10	-$2,167.80	-$6,843.70
High Ability Unemployed	-$641.20	-$2,740.70	-$5,431.10
Low Ability Unemployed	-$423.40	-$1,964.10	-$6,154.00

Equivalent loss of SS assets to implementing UI expansion, relative to counterfactual with no expansion.

It may seem counter intuitive to the reader that UI expansion would not even help low-ability unemployed workers on average. There are several factors at play:

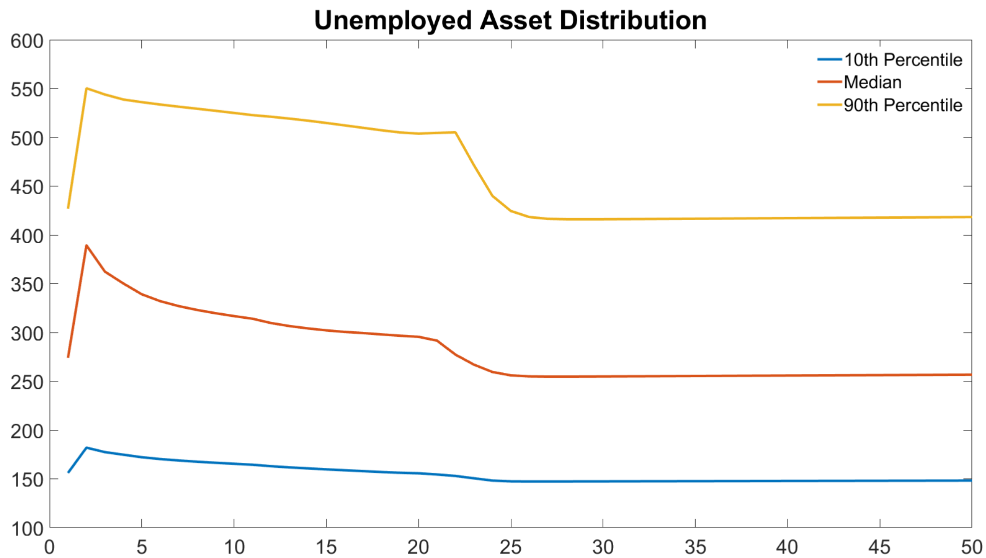

Figure 7: *Paths for top and bottom deciles and median asset holdings of unemployed workers.*

- Agents are infinitely lived and only ever briefly unemployed, so unemployed households care greatly about the future value of being employed.
- Endogenous separation is occurring in high asset households, so most of the newly unemployed households in the pandemic are high ability, high wealth households choosing not to work.
- Most low ability households have sufficient assets to weather a temporary unemployment spell with the basic 30% UI benefits.

The second point is the key to understanding why even from a utilitarian standpoint the UI expansion has a negative effect. High wealth households quit their jobs to avoid paying the disutility from working. These households have the lowest marginal utility from consumption, yet they are allocated additional resources from the expansion at the expanse of crowding out and eventual taxes faced by firms. Low ability households are particularly harmed by high levels of government debt because they face lower wages due to capital being crowded out and hold less asset so they do not gain from the slight increase in interest rates.

Figures 7 and 8 show the top and bottom deciles as well as the median asset holdings

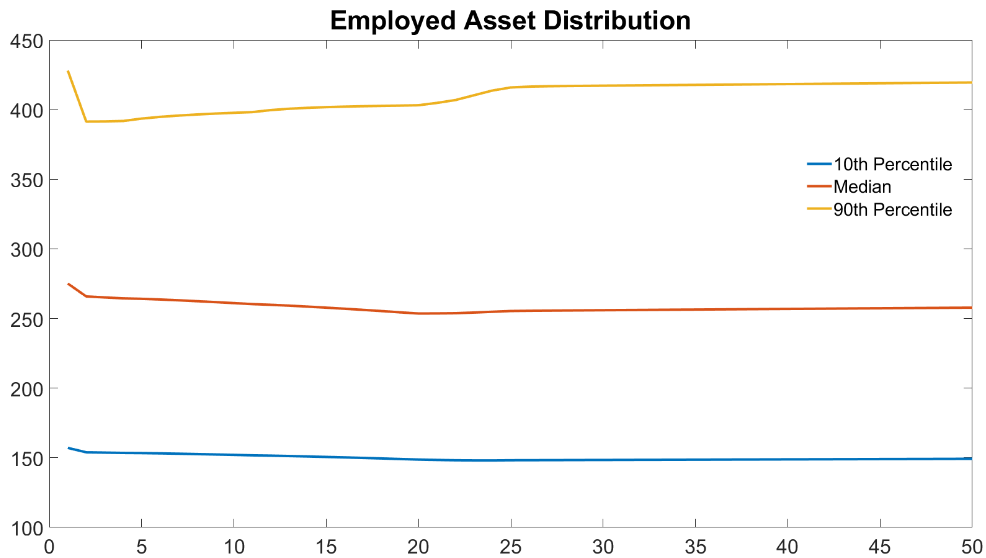

Figure 8: *Paths for top and bottom deciles and median asset holdings of employed workers.*

of unemployed and employed workers over time. Notice the surge of high wealth individuals becoming unemployed during the pandemic. Pre-pandemic the median asset holding of employed and unemployed workers is almost equal, during the pandemic the unemployed are much richer as a group than the employed. Over time, unemployed workers first dissave and then choose to accept employment when matched.

4 Conclusion

Expanding unemployment benefits in our model inefficiently transfers wealth to mostly rich households because they are the ones who choose to become unemployed in response to the increased health risk from working. These high wealth households do not even benefit on average because over time the additional government debt crowds out capital, reducing wages and dividends, and must eventually be repaid with high taxes on firms. Even low ability agents who start the pandemic unemployed (before endogenous separation occurs) stand to lose on average because of the large costs of financing the program.

One limitation of our model is that we do not have endogenous job destruction for the lowest wage workers. Future work is needed to study the interplay between incomplete assets and mechanisms already in the literature, such as sector specific productivity shocks or costly technologies which reduce or eliminate the health risk of working.

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