Autonomous Vehicle Tracking and Collision Avoidance Using Adaptive Control Algorithms

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Abstract

With the rapid economic growth, global car ownership increasing leads more researchers to focus their work on making cars more intelligent and safer. Autonomous vehicle control (AVC) problems have drawn increasing research in the last decades because of their potential to reduce vehicle accidents and commuting time. Vehicle traffic control at street intersections is one of the most important problems for its special and complex situations in AVC. Adaptive control, which is known for its power to deal with system uncertainties, is a useful method to solve AVC problems without the knowledge of the system parameters.

This thesis studies the control problems of a vehicle passing an intersection: the designed controller should make the controlled vehicle pass the intersection quickly and avoid any collision. In other words, the control design needs to create a proper reference trajectory satisfying certain traffic requirements. Then, it requires to make the vehicle's trajectory track arbitrary vehicle reference trajectory as quickly as possible. In this research, the state-space model of the vehicle dynamics, containing several uncertain parameters, is established. The adaptive control method is adopted to deal with the systems parameter uncertainties in such vehicle control problems. For this study, two adaptive control designs are developed to solve the problem: a baseline adaptive control design and an enhanced adaptive control design. Unlike the classic PI controller which can only make the vehicle track constant velocity trajectories, both two adaptive control designs can achieve asymptotic tracking of arbitrary vehicle velocity trajectories. The enhanced adaptive design can even further improve the system tracking performance. Analysis and simulation results have demonstrated the effectiveness of the proposed adaptive control systems.

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Chapter 1

Introduction

With unrivaled economic growth, more and more vehicles running on the road, which leads to a considerable amount of car accidents and quite low traffic efficiency. According to [19] and [20], only 8.7% of households in America do not own a vehicle, and 36560 people died in vehicle accidents in 2018 only. Thus, autonomous driving technology becomes a research hot-spot to solve such problems. In the last decades, with the unparalleled improvements in communication, positioning, and sensing areas, so many aided driving imaginations can come true. As a multi-disciplinary research area, autonomous driving techniques could make great progress based on advanced communication, sensing, and positioning technologies. More intelligent autonomous driving systems can be developed to improve the performance of the technology to reduce car accidents and improve commuting efficiency.

In the autonomous driving research area, the control problems always attract much attention from researchers. Adaptive control methods are used to deal with the uncertainties of the vehicle system. In this thesis, we thoroughly study two different adaptive designs for vehicle control to make the vehicle track arbitrary vehicle velocity reference trajectories for the specific intersection passing scenario. Moreover, the differences between adaptive designs and nominal designs are analyzed comprehensively.

1.1 Literature Review

In this section, vehicle control problems studies are discussed, including three parts: autonomous vehicle control, intersection vehicle control, and adaptive control. Some of the important researches can be the basis of our study in this area, whose ideas and results are reported as follows.

Autonomous vehicle control. [1], [3], and [5] offer several uncertainties for system solutions, including a fuzzy system, auto tuned PID controller, and a data driven method. In [1], Wenchang Li's team proposed a hierarchical structure for Electric Vehicles' adaptive car-following control systems. The system contained an upper control layer to obtain the ideal acceleration with a sliding mode control and a lower layer to ensure passenger comfort. For the uncertain parameters of the system, their optimization strategy was using a fuzzy system technique. Through their test results, the adaptive fuzzy sliding mode control method has better system's robustness in contrast to the sliding mode control method. In [3], Xingchen Wu led a group to come up with an idea to apply an intelligent algorithm to tune the parameters of PID controller with improved chaotic ant swarm for vehicle control systems to improve the comfort of driving, fuel economy, and distance for safety. In their results, the system could improve the efficiency of traffic and ride comfort for passengers, however, the improvement of fuel saving was not enough. In [5], a data based method was introduced to deal with the uncertainties of the autonomous vehicle system by Pin Wang's group. They applied the Principle Components Analysis technique and Time Delay Neural Network technique to their steering error compensation scheme, which was combined with a feedforward controller for the path tracking task. The better system performance of this scheme was proved by their tested maximum path error result.

In [2], İkbal Eski's group did some research on speed control of heavy duty vehicles. They applied some different types of controllers to heavy duty vehicles including, PID controller, model-based neural network controller, and adaptive neural network-based fuzzy inference control systems. Then, they also proposed a robust adaptive neural network-based fuzzy inference control system. According to the comparison results of those controllers by them, the tested control strategies cannot track the changing desired speed in the heavy duty vehicle system or cannot track it fast enough.

In [4], Yugong Luo's team considered applying the nonlinear model predictive method to a novel adaptive cruise control system for hybrid electric vehicles. They also introduced a position-based nonlinear longitudinal inter-vehicle dynamics model. According to the results of their experiments, the novel adaptive cruise control system can not only finish the tracking task but also improve the fuel-saving ability and driving comfort for passengers.

In [6], Libiao Jiang's group considered using some algorithms to realize vehicles' path tracking based on a vehicle kinematics model. In their work, HD-Maps, A-star, and model predictive control algorithms were applied to construct out-parking maps, generate the desired path and finish the path tracking task.

Intersection vehicle control. Based on connected vehicle techniques in [21-24], scholars put much effort to reduce collisions and improve traffic efficiency in the intersection areas. In [8, 25, 26], researchers utilized model predictive control methods to control the vehicles in the intersection. [25, 27] proposed global coordination based algorithms to solve the intersection vehicle control problems. In [30], Yuheng Zhang's team developed a vehicle-to-infrastructure (V2I) communication based centralized global optimization robust intersection control algorithm. Besides, [28-29, 31] proposed distributed control and optimization plans for intersections, which do not need global coordination information. [32] introduced an intersection network management algorithm and utilized distributed linear controllers to fulfill the vehicle control. [29, 33-34] present several ideas based on the intersection partition or slot division direction. In [29], Yougang Bian's team divide intersection and surrounding areas into several parts according to different vehicles' task in those areas. Similarly, [34] also demonstrated an intersection partition algorithm based on vehicle-to-vehicle (V2V) communication. [37-41] also demonstrated several intersection control and optimization plans to eliminate the collision possibility and get the utmost out of the intersections. In [37], Joyoung Lee's group developed a vehicle intersection algorithm by eliminating the vehicles' trajectory overlaps for scenarios constituted by fully automated vehicles completely. Moreover, [38] also considered using vehicle real-time state estimation to overcome the uncertainties in the measurements. Different from the above ideas, [35-36] present two methods to fulfill the requirements through intersection signal lights control. In [42], Akhil M's team presented a collision avoidance technique combining vehicle detectors and smartphones to reduce the cost of the collision-free process.

Adaptive control. Adaptive control approaches are also widely applied in vehicle control areas. In [9], Milan Stork led a team to work on designing an algorithm to solve vehicle path following problem based on an adaptive control method. They used model reference adaptive control to estimate the system parameters and control the vehicle to track the trajectory. However, the desired trajectory generation algorithm is still needed. Similarly, in [10], an output adaptive controller was proposed to realize the trajectory tracking for skid steered autonomous vehicles, by Ruben Fuentes-Alvarez's group. This proves the adaptive control method's powerful ability to deal with the uncertainties of the system. Also, in [11], the adaptive control algorithm was used to control the connected vehicle team, by Xu Jin and his partners. Moreover, they used the adaptive method to overcome the sensor and actuator attacking effects.

1.2 Research Problems and Thesis Outline

Research Problems. The deep research of vehicle positioning and connected vehicles in [14,21-23,46-47] offers new directions to solve the AVC problems with the techniques of high accuracy vehicle positions by Global Positioning System (GPS) or Global Navigation Satellite System (GNSS) based on positioning techniques. While considering the AVC problem, it is inevitable to develop an intersection vehicle control system. According to [43], the basic requirements of automated vehicles (AV) include not only AVs ensure passenger safety but also AVs own the ability to run in AV and manually driven vehicle mixed situations. Also, vehicle systems in the real world have serious uncertainties, because of the influence from the fickle road conditions and vehicle internal structures and loads. This causes that we cannot get the precise system parameters in solving an intersection vehicle control problem. Moreover, to accommodate the fast pace of modern life, the desired control algorithm should make the vehicle pass through the intersection as fast as possible.

In this thesis, we consider a typical AVC problem in an intersection scene. As shown in Figure 1.1, there is a four-way intersection with only one lane in each direction. In this problem, we assume all vehicles are connected vehicles, which means every vehicle can share its information synchronously through the V2V communication techniques and vehicle positioning techniques. And all vehicles must not exceed the speed limit. For this problem, our task is to design a scheme to control a vehicle pass the intersection as fast as possible and to avoid collisions with other vehicles in the intersection by managing its speed. At the same time, this scheme should also fulfill the AVC problem requirements. Specifically, the controlled vehicle can work in the AV and manually driven vehicle mixed situations safely. In summary, we will finish the following two tasks in following this thesis:

Task I: Find a proper desired vehicle velocity trajectory for the controlled vehicle to satisfy the intersection passing task based on the other vehicle's position and velocity information.

Task II: Design a model reference adaptive control system for the controlled vehicle to track the desired vehicle velocity trajectory asymptotically, without the knowledge of vehicle system parameters.

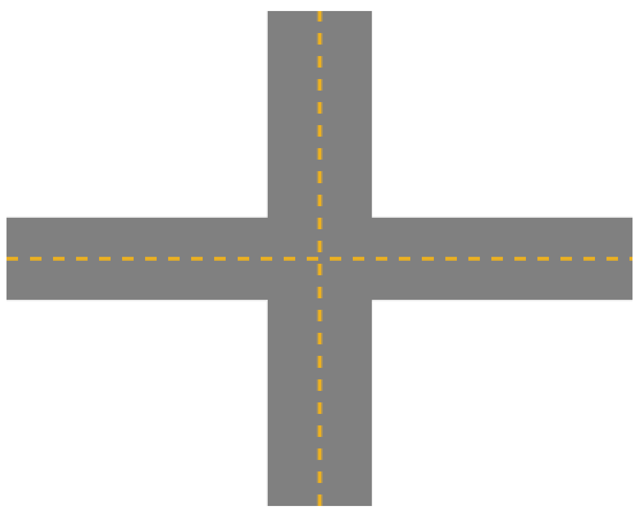


Figure 1.1: Intersection schematic diagram.

Thesis Outline. This thesis proposes two adaptive control systems to make the controlled vehicle pass the intersection safely and fast, which does not need the knowledge of the parameters of the vehicle system, including the mass, angle of road slope, and friction coefficient. Those two adaptive control schemes including a baseline design and an enhanced design.

The contents of this thesis are organized as follows.

In Chapter 2, we introduce the related research background. An introduction of adaptive control techniques is presented. At the same time, the motivation of the research and the detailed research problem statements are also showed in this chapter.

In Chapter 3, we design a baseline adaptive control structure to generate the desired velocity trajectory and solve the velocity trajectory tracking problem. About the velocity trajectory tracking problem, two different control systems are discussed here, a nominal control system for a known system and an adaptive control system for a system with uncertainties. The stability analysis for these two systems is derived to show the effectiveness of such designs. The control systems are simulated, and their performance is analyzed and discussed thoroughly. The evaluation of comparison between nominal and adaptive control systems is offered.

In Chapter 4, an enhanced control structure is considered to improve the trajectory tracking performance. Similar to Chapter 3, a nominal control system and an adaptive control system are discussed in this chapter too. The stability analysis and simulation of the enhanced designs are also demonstrated. A comparison between those two designs is provided.

Chapter 5 offers the conclusions of this study and possible future work along the dimension of the research of this thesis.

Chapter 2

Research Background

Before starting the discussion of the adaptive control of the vehicle systems, it is necessary to present some basic background about the control problem. In this chapter, some basic adaptive control knowledge is introduced at first. Then, the research motivation and the research problems are expatiated.

2.1 Adaptive Control Techniques

Traditional feedback control techniques can work well when the parameters are known. In practice, systems usually contain uncertainties in the operation process. In industrial processes, electrical devices, and other automatic systems, external noise, disturbances, and internal system parameter variations can lead to unexpected system uncertainties which traditional feedback control techniques cannot deal with satisfactorily.

Therefore, adaptive control techniques are developed to finish difficult uncertain system control tasks. The basic idea of adaptive control is to employ a parameter adaptation scheme to estimate those unknown parameters and replace the unknown parameters in the feedback controller with their estimates [12]. There are two adaptive controller design methods. The first method is called Direct Adaptive Control, which can be applied when we can use either the system state variables or the output variables. In this method, the controller parameters are directly updated online. The second method is called Indirect Adaptive Control, which can be applied without the use of the system state variables. This method updates the controller parameters through the design equations using the estimates of the system parameters. Both adaptive control designs can update the controller parameters to achieve the control objective for an unknown system with possible disturbances. We will discuss the direct adaptive control thoughts in detail. According to [13], there are two adaptive feedback control designs: state feedback adaptive control and output feedback adaptive control. The important contents excerpted from [13] are as followed.

State Feedback Adaptive Control for Output Tracking

Consider a system in the state space form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2.1}$$

for some unknown constant matrices $A \in R^{n\times n}$, $B \in R^{n\times 1}$ and $C \in R^{1\times n}$, with $n > 0$, where $x(t) \in R^{n\times 1}$ is the state vector, $u \in R$ is the control input and $y(t) \in R$ is the system output. The input-output description of this system is

$$y(s) = C(sI - A)^{-1}Bu(s) = \frac{Z(s)}{P(s)}u(s), \tag{2.2}$$

where

$$Z(s) = z_m s^m + \cdots + z_1 s + z_0 \tag{2.3}$$

with $z_m \neq 0$, and $P(s)$ is a monic polynomial of degree n. Our goal is to design an adaptive state feedback model reference controller for generating the system input $u(t)$ which ensures closed-loop signal boundedness and asymptotic tracking of an independent signal $y_m(t)$ by the system output $y(t)$, without the knowledge of the system parameters, where

$$y_m(t) = W_m(s)[r](t),\ W_m(s) = \frac{1}{P_m(s)}, \tag{2.4}$$

with $P_m(s)$ being the desired closed-loop characteristic polynomial of degree $n - m$.

The model reference adaptive controller structure is

$$u(t) = K(t)x(t) + k_r(t)r(t), \tag{2.5}$$

where $r(t)$ is an external input signal, $K(t) = (k_1(t), k_2(t), ..., k_n(t)) \in R^{1\times n}$ and $k_r(t) \in R$ are the adaptive estimates of the unknown parameters $k_r^* \in R$ and $K^*(t) = (k_1^*(t), k_2^*(t), ..., k_n^*(t)) \in R^{1\times n}$, which can be calculated when the system parameters are known. The goal of adaptive control is updating the estimates of K and k_r to achieve the system's tracking task without the knowledge of K^*, k_r^* the which are defined from system parameters to satisfy

$$\det(sI - A - BK^*) = P_m(s)Z(s)\frac{1}{z_m},\ k_r^* = \frac{1}{z_m}. \tag{2.6}$$

With the adaptive controller (2.5), the system (2.1) becomes

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B(K(t)x(t) + k_r(t)r(t)) \\ &= (A + BK^*)x(t) + Bk_r^* r(t) + B((K(t) - K^*)x(t) + (k_r(t) - k^*)r(t)) \\ y(t) &= Cx(t) \end{aligned} \tag{2.7}$$

From (2.4) it follows that

$$C(sI - A - BK^*)^{-1}Bk_r^* = \frac{Z(s)k_r^*}{\det(sI - A - BK^*)} = \frac{1}{P_m(s)} = W_m(s). \qquad (2.8)$$

According to (2.2), (2.5) and (2.6), the closed-loop tracking error equation is

$$e(t) = y(t) - y_m(t) = \rho^* W_m(s)[(K - K^*)x + (k_r - k_r^*)r](t) + Ce^{(A+BK^*)t}x(0), \quad (2.9)$$

where $\rho = z_m$, and $Ce^{(A+BK^*)t}x(0)$ converges to zero exponentially fast as $A + BK^*$ is stable. We need the following assumptions to design an adaptive state feedback model reference controller:

(A1) $Z(s)$ is a stable polynomial;

(A2) the degree m if $Z(s)$ is known; and

(A3) the sign of z_m is known.

To derive an estimation error equation, we introduce

$$\theta(t) = (K(t), k_r(t))^T,\ \theta^* = (K^*, k_r^*)^T, \qquad (2.10)$$

$$\omega(t) = (x^T(t), r(t))^T, \qquad (2.11)$$

$$\zeta(t) = W_m(s)[\omega](t), \qquad (2.12)$$

$$\xi(t) = \theta^T(t)\zeta(t) - W_m(s)[\theta^T\omega](t), \qquad (2.13)$$

and define the estimation error

$$\epsilon(t) = e(t) + \rho(t)\xi(t), \qquad (2.14)$$

where $\rho(t)$ is an estimate of $\rho^* = z_m$. Then, substituting (2.8) - (2.12) in (2.13), we have

$$\epsilon(t) = \rho^*(\theta(t) - \theta^*)^T \zeta(t) + (\rho(t) - \rho^*)\xi(t) + Ce^{(A+BK^*)t}x(0). \tag{2.15}$$

Ignoring the effect of the exponentially decaying term $Ce^{(A+BK^*)t}x(0)$, we have the estimation error expression

$$\epsilon(t) = \rho^*(\theta(t) - \theta^*)^T \zeta(t) + (\rho(t) - \rho^*)\xi(t). \tag{2.16}$$

This error equation suggests the following adaptive laws:

$$\dot{\theta}(t) = -\frac{\Gamma \text{sign}[z_m]\zeta(t)\epsilon(t)}{1 + \zeta^T(t)\zeta(t) + \xi^2(t)}, \tag{2.17}$$

$$\dot{\rho}(t) = -\frac{\gamma \text{sign} z_m \xi(t)\epsilon(t)}{1 + \zeta^T(t)\zeta(t) + \xi^2(t)}, \tag{2.18}$$

where sign$[z_m]$ is the sign of the parameter z_m, $\Gamma = \Gamma^T > 0$ and $\gamma > 0$ are adaptation gains. Theorem 2.1 ensures the system properties.

Theorem 2.1 *All signals in the closed-loop system with the plant (2.1) or (2.2), the reference model (2.4), and the controller (2.5) updated by the adaptive law (2.17)-(2.18) are bounded, and the tracking error* $e(t) = y(t) - y_m(t)$ *satisfies*

$$\lim_{t\to\infty}(y(t) - y_m(t)) = 0 \tag{2.19}$$

$$\int_0^\infty (y(t) - y_m(t))^2 dt < \infty \tag{2.20}$$

Output Feedback Adaptive Control for Output Tracking

To make the system (2.1), which can be expressed as

$$P(s)[y](t) = k_p Z(s)[u](t), \tag{2.21}$$

track the reference model characterized by

$$P_m(s)[y_m](t) = r(t), \tag{2.22}$$

where $P_m(s)$ is a monic stable polynomial and $r(t)$ is bounded and piecewise continuous external input signal. We need to make following assumptions to deign a adaptive controller to meet the control objective:

(A1) $Z(s)$ is a Hurwitz polynomial;

(A2) an upper bound for the unknown degree of $P(s)$ is known;

(A3) the sign of k_p is known; and

(A4) the degree of $P_m(s)$ is n^* equaling to the relative degree of $k_p \frac{Z(s)}{P(s)}$.

The model reference control structure is

$$u(t) = \theta_1^{*T}\omega_1(t) + \theta_2^{*T}\omega_2(t) + \theta_{20}^* y(t) + \theta_3^* r(t), \tag{2.23}$$

where

$$\begin{aligned} \omega_1(t) &= \frac{a(s)}{\Lambda(s)}[u](t) \\ \omega_2(t) &= \frac{a(s)}{\Lambda(s)}[y](t) \\ a(s) &= (1, s, \cdots, s^{n-2})^T, \end{aligned} \tag{2.24}$$

$\theta_1^* \in R^{n-1}$, $\theta_2^* \in R^{n-1}$, $\theta_{20}^* \in R$ and $\theta_3^* \in R$ can be calculated when the system transfer function $G(s) = C(sI - A)^{-1}B$ is known, and $\Lambda(s)$ is any monic Hurwitz polynomial of degree of $n - 1$.

Theorem 2.2 *When θ_1^*, θ_2^*, θ_{20}^*, θ_3^* satisfies*

$$\theta_1^{*T} a(s)P(s) + (\theta_2^{*T} a(s) + \theta_{20}^{*\,T}\Lambda(s))k_p Z(s) = \Lambda(s)(P(s) - k_p\theta_3^* Z(s)P_m(s)), \quad (2.25)$$

the controller (2.23) ensures that all signals in the closed-loop system are bounded and

$$y(t) - y_m(t) = \epsilon_0(t) \quad (2.26)$$

for some initial condition-related exponentially decaying $\epsilon_0(t)$.

To make the closed-loop control system output $y(t)$ track the desired system output $y_m(t)$ without the knowledge of the parameters of transfer function $G(s)$, the adaptive control structure is introduced:

$$u(t) = \theta_1(t)^T\omega_1(t) + \theta_2(t)^T\omega_2(t) + \theta_{20}(t)y(t) + \theta_3(t)r(t), \quad (2.27)$$

where $\theta_1(t) \in R^{n-1}$, $\theta_2(t) \in R^{n-1}$, $\theta_{20}(t) \in R$ and $\theta_3(t) \in R$ are the estimates from an adaptive law based on the tracking error $y(t) - y_m(t)$ of the ideal controller parameters θ_1^*, θ_2^*, θ_{20}^* and θ_3^* where $y_m(t)$ is the reference signal of the system output $y(t)$.

Adaptive control system for $n^* \geq 1$. About the adaptive law this case is almost the same as state feedback adaptive control for output tracking case except for $\rho^* = k_p$. The estimation error is defined as

$$\epsilon(t) = e(t) + \rho(t)\xi(t), \tag{2.28}$$

where

$$\xi(t) = \theta^T(t)\zeta(t) - \frac{1}{P_m(s)}[\theta^T\omega](t) \tag{2.29}$$

$$\zeta(t) = \frac{1}{P_m(s)}[\omega](t). \tag{2.30}$$

The adaptive update law for $\theta(t)$ and $\rho(t)$ is

$$\dot{\theta}(t) = -\frac{\Gamma \text{sign}[z_m]\zeta(t)\epsilon(t)}{1 + \zeta^T(t)\zeta(t) + \xi^2(t)} \tag{2.31}$$

$$\dot{\rho}(t) = -\frac{\gamma \text{sign} z_m \xi(t)\epsilon(t)}{1 + \zeta^T(t)\zeta(t) + \xi^2(t)} \tag{2.32}$$

$\Gamma = \Gamma^T > 0$ and $\gamma > 0$ are adaption gains.

Theorem 2.3 summarizes the adaptive control system properties.

Theorem 2.3 *All signals in the closed-loop system with plant (2.21), reference model (2.22), controller (2.27), adaptive scheme (2.28) - (2.32) are bounded, and the tracking error* $e(t) = y(t) - y_m(t)$ *belongs to* L^2 *and reduces to zero asymptotically with time.*

This method can be used for the systems whose relative degree between the unknown parameters $P(s)$ and $Z(s)$ greater or equal to 1 [13, 44].

Adaptive control system for $n^* = 1$. For the system (2.21), we also want to make this system track the reference model (2.22) with adaptive controller structure

(2.23). For this model reference control problem, we define

$$\theta^* = (\theta_1^{*T}, \theta_2^{*T}, \theta_{20}^*, \theta_3^*)^T \in R^{2\bar{n}}, \tag{2.33}$$

$$\theta(t) = (\theta_1^T(t), \theta_2^T(t), \theta_{20}(t), \theta_3(t))^T \in R^{2\bar{n}}, \tag{2.34}$$

$$\omega(t) = (\omega_1^T(t), \omega_2^T(t), y(t), r(t))^T \in R^{2\bar{n}}, \tag{2.35}$$

$$\widetilde{\theta}(t) = \theta(t) - \theta^*, \quad e(t) = y(t) - y_m(t). \tag{2.36}$$

Here, we only consider a special situation which the relative degree of the system transfer function $G_0 = k_p \frac{Z(s)}{P(s)}$ is 1. The adaptive law for $\theta(t)$ is

$$\dot{\widetilde{\theta}}(t) = \dot{\theta}(t) = -\text{sign}[k_p]\Gamma\omega(t)e(t), \tag{2.37}$$

where $\Gamma = \Gamma^T > 0$, $\theta(0)$ can be chosen arbitrarily. For this system, we have following theorem.

Theorem 2.4 *The adaptive controller (2.27) with the adaptive law(2.37), applied to the system (2.21) with relative degree 1, guarantees that all closed-loop signals are bounded and the tracking error $e(t)$ goes zero as t goes to infinity.*

This theorem implies this adaptive scheme can achieve the control objective without the system parameter knowledge. In this thesis, we will employ the Adaptive control system for $n^* = 1$. More detailed information can be found in [13].

2.2 Autonomous Vehicle System Dynamic Model

Basic vehicle dynamic equation. According to Newton's laws of motion, the basic moving vehicle dynamic equation is

$$m\ddot{x}(t) = u(t) - b\dot{x}(t) - mg\sin\delta, \tag{2.38}$$

where m is the mass of the car, b is the friction coefficient, g is the gravity acceleration, δ is the road slope angle, $x(t)$ is the position of the vehicle and $u(t)$ is the engine force. In this problem, the values of m, b and δ are uncertain to the controller generating $u(t)$.

System parameterization. Define the tracking error signal

$$e(t) = x(t) - x_d(t), \tag{2.39}$$

and, we introduce an auxiliary error signal

$$z(t) = \dot{e}(t) + \lambda e(t) \tag{2.40}$$

for a chosen positive design parameter $\lambda > 0$. Then, we can express the controlled vehicle equation (2.33): $m\ddot{x}(t) = u(t) - b\dot{x}(t) - mg\sin\delta$, as

$$\begin{aligned} m\dot{z}(t) &= m\ddot{x}(t) - m(\ddot{x}_d(t) - \lambda\dot{e}(t)) \\ &= u(t) - b\dot{x}(t) - mg\sin\delta - m(\ddot{x}_d(t) - \lambda\dot{e}(t)). \end{aligned} \tag{2.41}$$

Introducing the unknown parameter vector

$$\theta^* = [b, m\sin\delta, m]^T \tag{2.42}$$

and the regressor vector

$$\omega(t) = [\dot{x}, g, \ddot{x}_d - \lambda\dot{e}(t)]^T, \tag{2.43}$$

we derive the parameterization:

$$\theta^*\omega(t) = b\dot{x}(t) - mg\sin\delta - m(\ddot{x}_d(t) - \lambda\dot{e}(t)). \tag{2.44}$$

Then, equation (2.36) can be expressed as

$$m\dot{z}(t) = u(t) - \theta^* w(t). \tag{2.45}$$

2.3 Research Motivations

Since unprecedented precise vehicle position and connected vehicle technology growth[14]-[17], the development of autonomous vehicles is rapid, and autonomous vehicles start to appear in people's view even in their daily life: California starts accepting fully driverless testing applications, National Electric Vehicle Sweden aims to develop autonomous vehicles which do not need safety drivers [17], Bayerische Motoren Werke, abbreviated to BMW, plans to develop a fully autonomous and all-electric car before 2025 [45].

In the autonomous vehicles field, control is one of the most important areas. PI controllers are applied widely in such autonomous vehicles field. PI controllers are proved that they can ensure tracking of constant vehicle velocity trajectories stated in [18]. In this thesis, we consider developing a collision-free control system to make vehicles pass a signal-free intersection as fast as possible. It is necessary to develop a tracking control design to track arbitrary vehicle velocity trajectories for this system.

The technical objective of this thesis is to design effective control systems which can ensure desired asymptotic tracking of arbitrary vehicle velocity trajectories, not just constant vehicle velocity trajectories. A PI controller can ensure tracking of constant vehicle velocity trajectories but not arbitrary vehicle velocity trajectories. Besides, designing a stable PI control system requires a certain knowledge of system parameters. However, the desired velocity trajectories for the problem of this thesis are not changeless, and it is difficult to obtain the precise system parameters in practice.

Thus, it is necessary to develop adaptive tracking control systems, which can achieve asymptotic tracking of arbitrary vehicle velocity trajectories in the presence of system parameter uncertainties. A more powerful enhanced adaptive control design can further improve the system tracking performance.

2.4 Technical Issues

In Chapter 1, we stated a typical AVC problem: how to design a safe and high-efficiency vehicle control system in the signal-free intersection area situation according to the vehicles' position and speed information. We will present this problem in a more concrete form. To begin with, it is indispensable to construct a coordination system for the intersection area. As shown in Figure 2.2, we set the west-southern corner of the overlap part of the horizontal and vertical roads is $(0, 0)$ and the width of each road is ten meters. We assume there are two vehicles are in the intersection adjacent region driving within the speed limit. We need to control one of these vehicles to make both of them pass the intersection area safely and within the minimum time.

Because both two vehicles are connected vehicles, we introduce two variables $x(t)$ and $x_o(t)$ to express the front position information of the controlled vehicle and the other vehicle for this problem. Correspondingly, $\dot{x}(t)$ express the controlled vehicle speed.

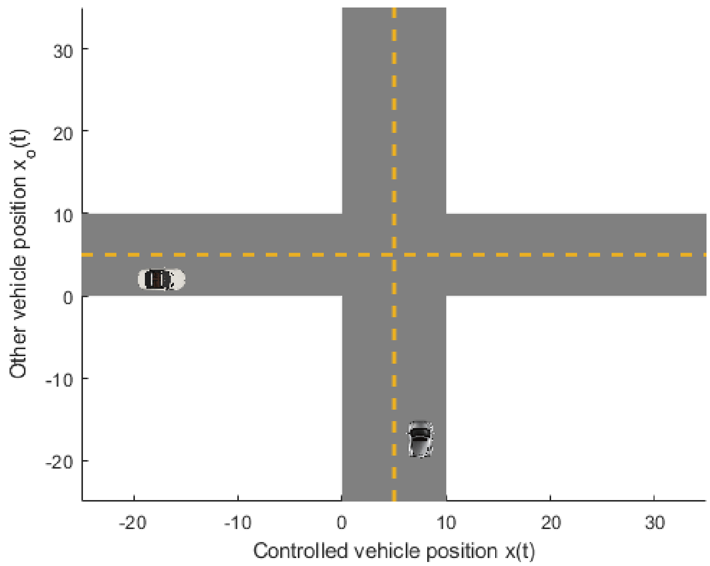


Figure 2.1: Intersection coordinates: controlled vehicle x, other vehicle x_o.

More specifically, our tasks to solve this research problem become

- Design the desired velocity trajectory $x_d(t)$ for $x(t)$, which can make both of these two vehicles pass the intersection area as fast as possible and avoid any collisions.

- Develop an model reference adaptive controller to make $x(t)$ to track $x_d(t)$ asymptotically, without using the knowledge of vehicle system parameters m, b and δ in equation (2.38).

Chapter 3

Adaptive Vehicle Tracking Control: Baseline Design

In this chapter, an adaptive tracking control system is introduced. It contains a desired velocity trajectory generation scheme for the controlled vehicle to pass the intersection as quickly as possible and to avoid colliding with the other vehicles in such an intersection. Different from the methods in the literature, this system can be applied in the AVs and manually driven vehicles mixed situations. Then, an adaptive tracking controller to make the vehicle's actual velocity trajectory to track the desired velocity trajectory asymptotically, overcoming the uncertainties and unintended disturbance of the vehicle's dynamic system. After the discussion of this baseline control system, we will modify the control signal to gain a higher performance in the next chapter.

3.1 Tracking Control System Structure

To solve the intersection vehicle control problem described at the end of Chapter 2, we form this tracking control system into an adaptive control problem.

Control objective. There are two parts of the control objective: the desired velocity trajectory generation task and the trajectory tracking task. Based on the position and the speed information of other vehicles in the intersection, the system should generate the desired velocity trajectory, which can make the controlled car pass the intersection fast and avoid other vehicles. The adaptive control system should make the controlled vehicle system with uncertainties track the desired trajectory as quickly as possible.

Desired trajectory generation. Firstly, the system needs to generate a proper desired trajectory for the controlled vehicle to pass the intersection as fast as possible and avoid any collision. To express the specific logic of the desired velocity trajectory generation, we need to discuss two possible situations: two vehicles' driving directions are the same initially, and two vehicles' driving directions are different initially.

(1) **Two vehicles' driving directions are same initially.** This is a situation that is relatively easy to deal with. When these vehicles have the same driving direction, they are on the same lane. The initial positions of the two vehicles in the intersection area are shown in Figure 3.1. In this problem, we take the controlled vehicle intends to move eastward.

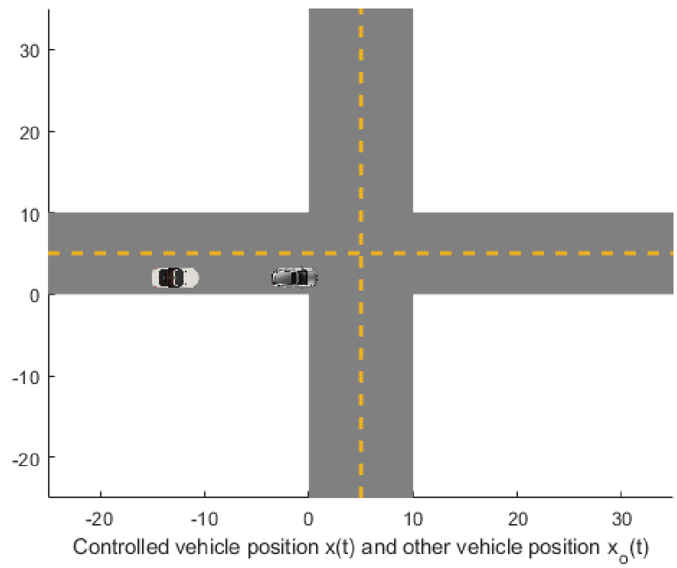


Figure 3.1: Positions of controlled vehicle $x(t)$ and other vehicle $x_o(t)$.

In this situation, if the controlled vehicle is in front of the other vehicle, we only need to make the controlled vehicle pass the intersection area as the speed limit. When the other vehicle is in front, we need to make our controlled vehicle follow the other vehicle until it leaves the controlled vehicle's lane. To express the desired trajectory into equation, we define the controlled vehicle driving direction and other vehicle driving direction $D(t) = \{e, w, n, s\}$ and $D_o(t) = \{e, w, n, s\}$, where

(i) e indicates the vehicle driving direction is eastward;

(ii) w indicates the vehicle driving direction is westward;

(iii) n indicates the vehicle driving direction is northward;

(iv) s indicates the vehicle driving direction is southward.

The desired velocity trajectory is

$$x_d(t) = \begin{cases} x_o(t) - l & 0 \le x_o(t) - x(t) \le l' \wedge D_o(t) = D(t) \\ v_{limit}(t - t_{leave}) + x_{leave} - d & \text{else} \end{cases} \tag{3.1}$$

where $x(t)$ is the position of the controlled vehicle, $x_o(t)$ is the position of the other vehicle, l is a long enough distance to avoid these two vehicles colliding, v_{limit} is the speed limit of this intersection area, t_{leave} is the time that the other vehicle changes its driving direction and leaves the intersection(the initial value of t_{leave} is 0), x_{leave} is the position of the controlled vehicle when the other vehicle changes its driving direction and leaves the intersection (the initial value of x_{leave} is $x(0)$), d is the necessary distance delay to avoid the vehicle over the speed limit, l' is used to judge whether the distance between these two vehicles is close enough to adjust the controlled vehicle speed, $D(t)$ is the driving direction of the controlled vehicle, and $D_o(t)$ is the driving direction of the other vehicle.

In Equation(3.1), $0 \le x_o(t) - x(t) \le l'$ means the other vehicle is in the front of the controlled vehicle, and the distance between these two vehicles is too close; $D_o(t) = D(t)$ means these two vehicles' driving direction are the same. When all these two restrictions are satisfied, we make the controlled vehicle follow the other vehicle. For all other situations, we make the controlled vehicle drive as the speed limit.

(2) **Two vehicles' driving directions are different initially.** Discussing the vehicles' behaviors in the intersection area is necessary before designing such an appropriate desired velocity trajectory under this situation. It is easy to find that every car has three different possible strategies in such an intersection area: go straight, left turn, and right turn. If we know vehicles' strategies in the

intersection, we can find the accurate overlap point of those vehicles' driving paths. Then, we only need to make the controlled vehicle pass such a point at the moment which is different from the other vehicle. However, we cannot know the exact strategies of other vehicles unless all vehicles are fully automated, because all the ways to estimate the vehicles' strategies may lead to collisions since these ways' failure rate is not zero. As stated in Chapter 2, we need to make sure the desired trajectory can work in not only the situations comprising all AVs, but also the scenes containing both AVs and manually driven vehicles. Under this situation, to make our system safe enough, we need to define two important zones in the intersection area: danger zone and action zone.

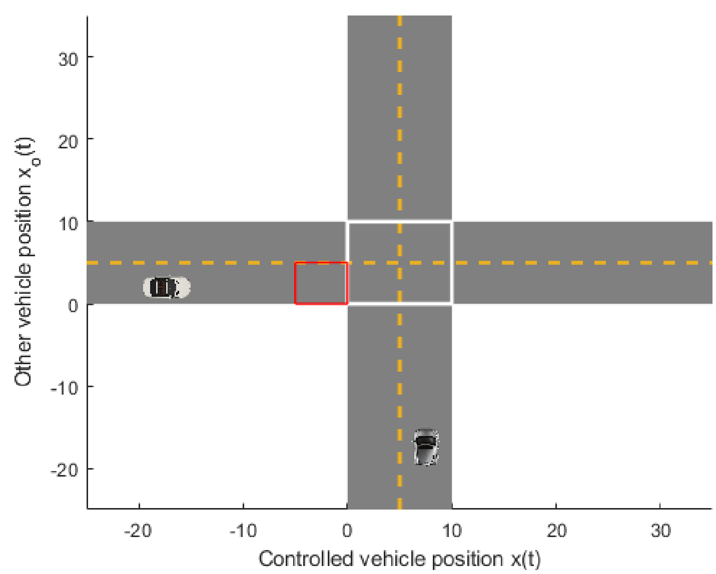


Figure 3.2: Intersection segmentation with the controlled vehicle position $x(t)$ and other vehicle position $x_o(t)$.

As shown in Figure 3.2, the rectangle area framed by the white line is the danger zone because it covers all the places the collision may happen. The rectangle area framed by the red line is called the action zone, which means the controlled vehicle should adjust its movement in these areas. To eradicate all possible collisions, we stipulate only one vehicle can enter the danger zone at the same moment. There are two possible actions the controlled car can select when it reaches the entrance of the danger zone, which are stopping at the entrance or go through it as fast as possible. The specific logic to make that selection is that we judge whether the collision may happen according to whether the controlled vehicle's tail can clear from the danger zone before the other vehicle reaches the entrance of the danger zone when the controlled vehicle reaches the action zone. If the controlled vehicle's tail can clear from the danger zone earlier, and there are no vehicles in the danger zone at that moment, we would make the controlled vehicle pass the danger zone without stopping. Otherwise, the controlled vehicle should drive to the danger zone entrance as the speed limit and wait at the entrance until the other vehicle passing the intersection area completely. Then we will make the controlled vehicle pass through the danger zone as the speed limit if the other vehicle does not change its driving direction to the same as our controlled vehicle after it passes the intersection. If the other vehicle changes to the same driving direction as our controlled vehicle, we will make our controlled vehicle follow it. In summary, there are three possible desired trajectories corresponding to three situations:

- When the controlled vehicle is inside of the action zone, and it detects a collision may happen, it should stop at the entrance of the intersection;
- When the other vehicle change to the same lane as the controlled vehicle's and the distance between these two vehicles is close enough, the controlled vehicle will follow the other vehicle;
- For all other cases, the controlled vehicle will drive with the speed limit.

Thus, the desired desired velocity trajectory $x_d(t)$ is:

$$x_d(t) = \begin{cases} 0 & \begin{cases} D_o(t) = n \\ \wedge \frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)}{v_{limit}}| \\ \wedge x_{action} < x(t) < 0 \\ \wedge x_o(t) < 10 + l_{vo} \end{cases} \\ 0 & \begin{cases} D_o(t) = \{s, w\} \\ \wedge \frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)-10}{v_{limit}}| \\ \wedge x_{action} < x(t) < 0 \\ \wedge x_o(t) > 0 - l_{vo} \end{cases} \\ x_o(t) - l & \begin{cases} D_o(t) = D(t) \\ \wedge x_o(t) \geq 10 + l_{vo} \geq x(t) \\ \wedge 0 \leq x_o(t) - x(t) \leq l' \end{cases} \\ v_{limit}(t - t_0) + x' - d & \text{else,} \end{cases} \tag{3.2}$$

where l is a long enough distance to avoid these two vehicles colliding, d_{limit} is the necessary distance delay that the controlled vehicle accelerates to v_{limit}, t_0 is the moment when the latest time that $x_d(t) = 0$'s and $x_d(t) = x_o(t) - l$'s restricted condition dissatisfying (the initial value of t_0 is 0), x' is the position of the controlled vehicle at the moment when the latest time that $x_d(t) = 0$'s and $x_d(t) = x_o(t) - l$'s restricted condition dissatisfying (the initial value of x' is $x(0)$), d is the necessary distance delay to avoid the vehicle over the speed limit,

x_{action} is the start location of the action zone, l_v is the length of the controlled vehicle, l_{vo} is the length of the other vehicle, and l' is used to judge whether the distance between these two vehicles is close enough to adjust the controlled vehicle speed.

In the restrictions of $x_d(t) = 0$ in Equation (3.2), $D_o(t) = n$ means the other vehicle's driving direction is northward, and $D_o(t) = \{s, w\}$ means the other vehicle's driving direction is southward or westward; $\frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)}{v_{limit}}|$ or $\frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)-10}{v_{limit}}|$ indicates that the time that the controlled vehicle clearing from the danger zone is longer than the minimum time that the other vehicle reaching the danger zone entrance, which means it is possible to cause a collision because we cannot ensure the controlled vehicle's tail clears from the danger zone before the other vehicle enters the danger zone; $x_{action} < x(t) < 0$ means the controlled vehicle is in the action zone, whose length should be long enough to make the controlled vehicle stop from the speed limit to zero; $x_o(t) < 10 + l_{vo}$ and $x_o(t) > 0 - l_{vo}$ means the other vehicle has not passed the danger zone. When these four restrictions are satisfied at the same time, the controlled vehicle needs to stay at the entrance of the danger zone until the other vehicle passes the danger zone completely to avoid colliding.

In the restrictions for $x_d(t) = x_o(t) - l$ in Equation (3.2), $D_o(t) = D(t)$ indicates the driving direction of these two vehicles are the same, the other vehicle may change to the controlled vehicle's lane; $x_o(t) \geq 10$ and $x_o(t) \leq 0 - l_{vo}$ means the other vehicle has passed the danger zone completely; $0 \leq x_o(t) - x(t) \leq l'$ means the other vehicle is in the front of the controlled vehicle, and the distance between these two vehicles is too close. When these three restrictions are satisfied, we will make the controlled vehicle follow the other vehicle.

In all other situations, we will make the controlled vehicle drive as the speed limit to ensure efficiency.

Trajectory tracking preparation. In Chapter 2, we introduced the basic vehicle system as equation (2.38) and system parameterization (2.39) - (2.45). We introduced the tracking error $e(t)$, the auxiliary error signal $z(t)$, unknown parameter vector θ^*, and the regressor vector $\omega(t)$:

$$e(t) = x(t) - x_d(t), \tag{3.3}$$

$$z(t) = \dot{e}(t) + \lambda e(t), \tag{3.4}$$

$$\theta^* = [b, m\,\sin\delta, m]^T, \tag{3.5}$$

$$\omega(t) = [\dot{x}, g, \ddot{x}_d - \lambda\dot{e}(t)]^T, \tag{3.6}$$

where m is the mass of the car, b is the friction coefficient, g is the gravity acceleration, δ is the road slope angle, $x(t)$ is the position of the vehicle, $x_d(t)$ is the desired velocity trajectory and λ is the chosen positive design parameter $\lambda > 0$. Then we formulated the parameterized system, which can be expressed as

$$\theta^*\omega(t) = b\dot{x}(t) - mg\,\sin\delta - m(\ddot{x}_d(t) - \lambda\dot{e}(t)), \tag{3.7}$$

$$m\dot{z}(t) = u(t) - \theta^* w(t), \tag{3.8}$$

where $u(t)$ is the engine force or brake force acting on the vehicle.

3.2 Nominal Control System

Before develop the adaptive tracking control system, we need to consider the nominal control system which is created based on the knowledge of the system.

For the parameterized system (3.7)-(3.8), we design a linear non-adaptive controller generating $u(t)$ with the knowledge of θ^* for this system

$$u(t) = \theta^{*T}\omega(t) - k_1 z(t), \tag{3.9}$$

where $k_1 > 0$ is a design parameter.

Theorem 3.1 *The control law $u(t) = \theta^{*T}\omega(t) - k_1 z(t)$ applied to the vehicle system (2.38), ensures that the position tracking error $e(t) = x(t) - x_d(t)$ and velocity tracking error $\dot{e}(t) = \dot{x}(t) - \dot{x}_d(t)$ both converge to zero exponentially.*

Proof: The combination of (3.8) and (3.9) is

$$\begin{aligned} m\dot{z}(t) &= -k_1 z(t) \\ \dot{z}(t) &= -\frac{k_1}{m} z(t). \end{aligned} \tag{3.10}$$

After solving this differential equation, we can get

$$z(t) = z(t_0)e^{\int_{t_0}^{t}(-\frac{k_1}{m})d\sigma}. \tag{3.11}$$

When $t_0 = 0$,

$$z(t) = z(0)e^{-\frac{k_1}{m}t}. \tag{3.12}$$

Thus, $\lim_{t\to\infty} z(t) = 0$ exponentially. For $\dot{e}(t) + \lambda e(t) = z(t)$, this is a first-order linear differential equation. According to the general solution for first order linear differential equation, the tracking error $e(t)$ of this this system can be expressed as

$$e(t) = e^{-\lambda t}(e(0) + z(0)(\frac{1}{\lambda - \frac{k_1}{m}}(e^{\lambda t - \frac{k_1 t}{m}} - 1))), \tag{3.13}$$

Therefore, $\lim_{t\to\infty} e(t) = 0$ is obvious, which means the tracking error of this system can converge to zero exponentially. Then, $\dot{e}(t)$ can also converge to 0 exponentially, since $z(t) = \dot{e}(t) + \lambda e(t)$. In other words, this control design can make the vehicle track the desired velocity trajectory exponentially.

3.3 Adaptive Control System

In many practical conditions, we cannot know the exact parameters θ^*. We propose a linear adaptive controller generating $u(t)$ for this situation:

$$u(t) = \theta^T(t)\omega(t) - k_1 z(t), \tag{3.14}$$

where $k_1 > 0$ is a design parameter, and $\theta(t)$ is generated from the adaptive law

$$\dot{\theta}(t) = -\Gamma\omega(t)z(t), \tag{3.15}$$

for a chosen symmetric and positive definite matrix $\Gamma \in R^{3\times 3} : \Gamma = \Gamma^T > 0$, as the adaptation gain matrix. In this control deign, $\theta(t)$ is the adaptive estimate of θ^*, which may not converge to θ^* but ensuring $\lim_{t\to\infty} e(t) = 0$.

This adaptive control system has the following desired properties, which are stated in the following theorem.

Theorem 3.2 *The adaptive control system consisting (3.14) and (3.15), applied to the vehicle system (2.38), ensures that all system signals are bounded and the position tracking error $e(t) = x(t) - x_d(t)$ and velocity tracking error $\dot{e}(t) = \dot{x}(t) - \dot{x}_d(t)$ both converge to zero asymptotically.*

Proof: Choose positive definite function:

$$V(z, \widetilde{\theta}) = \frac{m}{2} z^2(t) + \frac{1}{2} \widetilde{\theta}^T \Gamma^{-1} \widetilde{\theta} \tag{3.16}$$

with $\widetilde{\theta} = \theta(t) - \theta^*$. Then

$$\begin{aligned} \dot{V} &= mz(t)\dot{z}(t) + \Gamma^{-1}\widetilde{\theta}^T \dot{\widetilde{\theta}} \\ &= z(t)(u(t) - \theta^*\omega(t)) - \widetilde{\theta}\omega(t)z(t) \\ &= z(t)(\theta^T(t)\omega(t) - k_1 z(t) - \theta^*\omega(t)) - \widetilde{\theta}^T(t)\omega(t)z(t) \\ &= z(t)(\widetilde{\theta}^T(t) - k_1 z(t)) - \widetilde{\theta}^T(t)\omega(t)z(t) \\ &= -k_1 z^2(t) \leq 0. \end{aligned} \tag{3.17}$$

Thus,

$$z(t) \in L^\infty,\ \widetilde{\theta}(t) \in L^\infty,\ z(t) \in L^2. \tag{3.18}$$

Because $z(t) = \dot{e}(t) + \lambda e(t)$,

$$e(t) \in L^\infty,\ e(t) \in L^2,\ \dot{e}(t) \in L^\infty. \tag{3.19}$$

Hence, the theorem's result holds, which means the tracking error and the speed tracking error converge to zero asymptotically. In other words, this adaptive control design can make the vehicle to track the desired velocity trajectory without using the knowledge of the system parameters θ^*.

3.4 Simulation Studies

In this section, we construct a simulation system to test the performance of our adaptive vehicle tracking control system based on several different cases. In this simulation study, we consider controlling a vehicle in a specific intersection scenario without a signal light.

Simulation System. For the intersection as shown in Figure 3.3, the intersection is the rectangle with vertexes $(0,0), (0,10), (10,0), (10,10)$ and each road's width is 10 meters.

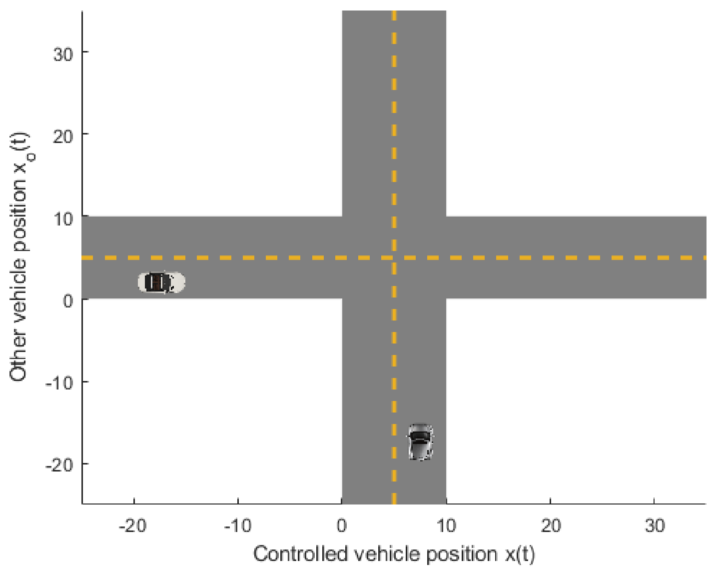


Figure 3.3: Intersection coordinates: controlled vehicle x, other vehicle x_o.

The dynamics equation of the vehicle is given in equation (2.38) which is $m\ddot{x}(t) = u(t) - b\dot{x}(t) - mg\sin\delta$. In the begin of chapter 3, we finish system parameterization. We have the parameter vector as:

$$\begin{aligned} \theta^* &= [\theta_1^*, \theta_2^*, \theta_3^*]^T \\ &= [b, m\sin\delta, m]^T, \end{aligned} \tag{3.20}$$

and the regressor vector $\omega(t)$ as

$$\begin{aligned}\omega(t) &= [\omega_1(t), \omega_2(t), \omega_3(t)]^T \\ &= [\dot{x}, g, \ddot{x}_d - \lambda \dot{e}(t)]^T.\end{aligned} \tag{3.21}$$

Next, we will demonstrate simulation the results of nominal control system $u(t) = \theta^{*T}(t)\omega(t) - k_1 z(t)$ and the adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t)$ in four different the other vehicle's trajectories $x_o(t)$.

Simulation conditions. In this problem, we always set the drive direction of the controlled vehicle is eastward $D(t) = e$. We consider four different situations to test our system:

- In Case I, we set the other vehicle will always stay in the lane at the speed of $\dot{x}_o = 5$ meters per second from $x_o(0) = 1$ to eastward. Thus, the other vehicle's velocity trajectory is $x_o(t) = 5t + 1$, $D_o(t) = e$ for this case. The initial position of controlled vehicle is $x(0) = -10.5$, and its initial speed is $\dot{x}(0) = 6.7$ meters per second.

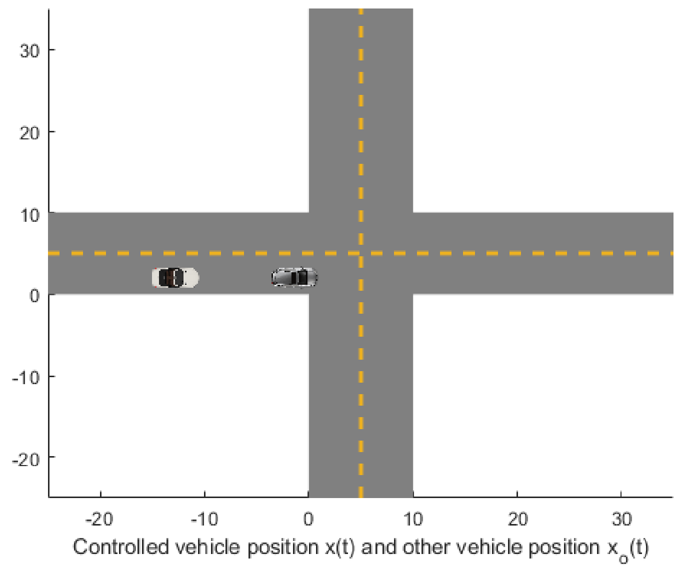


Figure 3.4: The initial driving directions and positions of two vehicles in Case I.

- In Case II, the other vehicle's velocity trajectory is $x_o(t) = 5t - 7.5$ means it moves forward at a speed of $\dot{x}_o = 5$ meters per second from $x_o(0) = -7.5$ to the eastward initially. However, it will change its lane and driving direction to $x_o(t) = n$ when it leaves the danger zone. The initial position of controlled vehicle is $x(0) = -19$, and its initial speed is $\dot{x}(0) = 6.7$ meters per second.

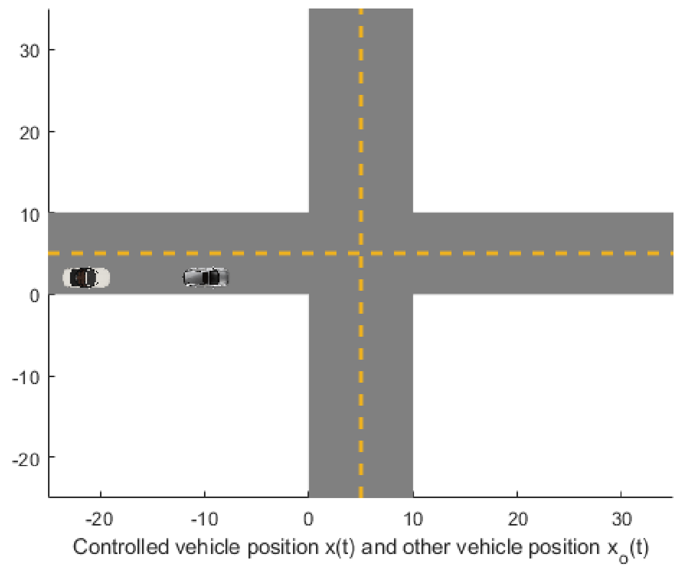


Figure 3.5: The initial driving directions and positions of two vehicles in Case II.

- In Case III, the other car's trajectory is $x_o = 6.7t - 10$, $D_o(t) = n$, which means the vehicle moves forward at a speed of $\dot{x}_o = 6.7$ meters per second from $x_o(0) = -10$ to the northward. The initial position of controlled vehicle is $x(0) = -5$, and its initial speed is $\dot{x}(0) = 6.7$ meters per second. In this case, collisions may happen. Thus, our controller needs to modify the desired trajectory.

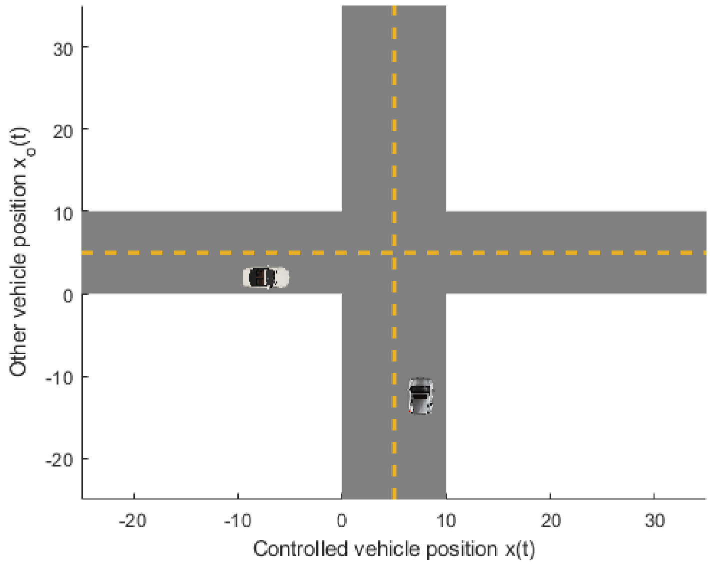


Figure 3.6: The initial driving directions and positions of two vehicles in Case III.

- In Case IV, the other car's trajectory is $x_o = -6.7t + 32$, $D_o(t) = s$, which means the vehicle moves forward at a speed of -6.7 meters per second from $x = 32$ to southward. The initial position of controlled vehicle is $x(0) = -5$, and its initial speed is $\dot{x}(0) = 5$ meters per second.

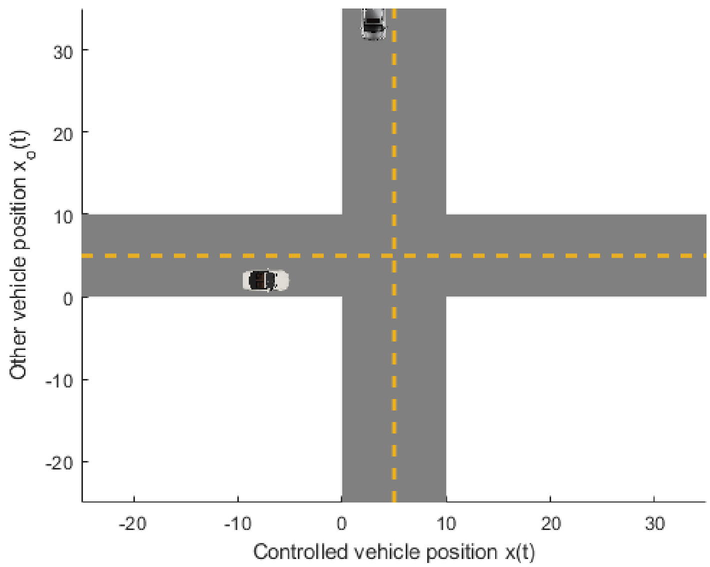


Figure 3.7: The initial driving directions and positions of two vehicles in Case IV.

According to the above, the desired trajectory for Case I and Case II is

$$x_d(t) = \begin{cases} x_o(t) - l & 0 \leq x_o(t) - x(t) \leq l' \wedge D_o(t) = D(t) \\ v_{limit}(t - t_{leave}) + x_{leave} - d & \text{else} \end{cases} \tag{3.22}$$

where $x(t)$ is the position of the controlled vehicle, $x_o(t)$ is the position of the other vehicle, l is a long enough distance to avoid these two vehicles colliding, v_{limit} is the speed limit of this intersection area, t_{leave} is the time that the other vehicle changes its driving direction and leaves the intersection(the initial value of t_{leave} is 0), x_{leave} is the position of the controlled vehicle when the other vehicle changes its driving direction and leaves the intersection (the initial value of x_{leave} is $x(0)$), d is the necessary distance delay to avoid the vehicle over the speed limit, l' is used to judge whether the distance between these two vehicles is close enough to adjust the controlled vehicle speed, $D(t)$ is the driving direction of the controlled vehicle, and $D_o(t)$ is the driving direction of the other vehicle.

In Equation(3.22), $0 \leq x_o(t) - x(t) \leq l'$ means the other vehicle is in the front of the controlled vehicle, and the distance between these two vehicles is too close; $D_o(t) = D(t)$ means these two vehicles' driving direction are the same. When all two restrictions are satisfied, we make the controlled vehicle follow the other vehicle. For all other situations, we make the controlled vehicle drive as the speed limit.

For Case III-IV, the desired velocity trajectory is

$$x_d(t) = \begin{cases} 0 & \begin{cases} D_o(t) = n \\ \wedge \frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)}{v_{limit}}| \\ \wedge x_{action} < x(t) < 0 \\ \wedge x_o(t) < 10 + l_{vo} \end{cases} \\ 0 & \begin{cases} D_o(t) = \{s, w\} \\ \wedge \frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)-10}{v_{limit}}| \\ \wedge x_{action} < x(t) < 0 \\ \wedge x_o(t) > 0 - l_{vo} \end{cases} \\ x_o(t) - l & \begin{cases} D_o(t) = D(t) \\ \wedge x_o(t) \geq 10 + l_{vo} \geq x(t) \\ \wedge 0 \leq x_o(t) - x(t) \leq l' \end{cases} \\ v_{limit}(t - t_0) + x' - d & \text{else,} \end{cases} \tag{3.23}$$

where l is a long enough distance to avoid these two vehicles colliding, d_{limit} is the necessary distance delay that the controlled vehicle accelerates to v_{limit}, t_0 is the moment when the latest time that $x_d(t) = 0$'s and $x_d(t) = x_o(t) - l$'s restricted condition dissatisfying (the initial value of t_0 is 0), x' is the position of the controlled vehicle at the moment when the latest time that $x_d(t) = 0$'s and $x_d(t) = x_o(t) - l$'s restricted condition dissatisfying (the initial value of x' is $x(0)$), d is the necessary distance delay to avoid the vehicle over the speed limit, x_{action} is the start location of the action zone, l_v is the length of the controlled vehicle, l_{vo} is the length of the other vehicle, and l' is used to judge whether the distance between these two vehicles is close enough to adjust the controlled vehicle speed.

In the restrictions of $x_d(t) = 0$ in Equation (3.23), $D_o(t) = n$ means the other vehicle's driving direction is northward, and $D_o(t) = \{s, w\}$ means the other vehicle's driving direction is southward or westward; $\frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)}{v_{limit}}|$ or $\frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)-10}{v_{limit}}|$ indicates that the time that the controlled vehicle clearing from the danger zone is longer than the minimum time that the other vehicle reaching the danger zone entrance, which means it is possible to cause a collision because we cannot ensure the controlled vehicle's tail clears from the danger zone before the other vehicle enter. the danger zone; $x_{action} < x(t) < 0$ means the controlled vehicle is in the action zone, whose length should be long enough to make the controlled vehicle stop from the speed limit to zero; $x_o(t) < 10 + l_{vo}$ and $x_o(t) > 0 - l_{vo}$ means the other vehicle has not passed the danger zone. When these four restrictions are satisfied at the same time, the controlled vehicle needs to stay at the entrance of the danger zone until the other vehicle passes the danger zone completely to avoid colliding.

In the restrictions for $x_d(t) = x_o(t) - l$ in Equation (3.23), $D_o(t) = D(t)$ indicates the driving direction of these two vehicles are the same, the other vehicle may change to the controlled vehicle's lane; $x_o(t) \geq 10$ and $x_o(t) \leq 0 - l_{vo}$ means the other vehicle has passed the danger zone completely; $0 \leq x_o(t) - x(t) \leq l'$ means the other vehicle is in the front of the controlled vehicle, and the distance between these two vehicles is too close. When these three restrictions are satisfied, we will make the controlled vehicle follow the other vehicle.

In all other situations, we will make the controlled vehicle drive as the speed limit to ensure efficiency.

For parameters in the desired trajectory choosing, we set the v_{limit} = 15 miles per hour = 6.7 meters per second to be the speed limit, l = 10 meters to be the following distance to avoid the collision, the distance delay d_{limit} is the maximum value of $\{0.9|v_{limit} - \dot{x}(0)|, 1.5\}$, the distance delay d is the maximum value of $\{0.9|\dot{x}_d(t_0) - \dot{x}(t_0)|, 1.5\}$ to avoid the vehicle over the speed limit, the start location of action zone $x_{action} = -5$, the length of vehicles $l_v = l_{vo} = 4.7$ meters [49], and $l' = 11.5$ meters is used to judge whether the distance between these two vehicles is close enough to adjust the controlled vehicle speed.

For the system parameters, we set $m = 1800$kg according to [49], keeping two significant figures, $g = 9.8\text{m/s}^2$, $b = 0.10$, $sin\delta = 0.10$. Thus,

$$\theta^* = [0.1, 180, 1800]^T. \tag{3.24}$$

3.4.1 Nominal Control System Evaluation

In this subsection, we tested the nominal control system performance in those four cases, which includes the generated different desired trajectories and the corresponding tracking error convergence. The simulation results demonstrated that our control system can achieve the control objective in all four cases.

In the stated four situations, we use the nominal control system

$$u(t) = \theta^{*T}\omega(t) - k_1 z(t), \tag{3.25}$$

to finish the tracking control task, when we assuming know the exact system parameters. About the design parameters, we choose $\lambda = 2$, $k_1 = 2700$.

Control input signal limitation. In this problem, we set the controlled vehicle can accelerate to 60 miles per hour from 0 in 3.1 seconds [49], and it can decelerate to 0 from 60 miles per hour within 133 feet [50]. When we substitute the vehicle mass $m = 1800$ kilograms to the uniform acceleration motion formula, the reasonable control input signal range should be about $[-16000, 16000]$ without considering the driving resistance and keeping two significant figures.

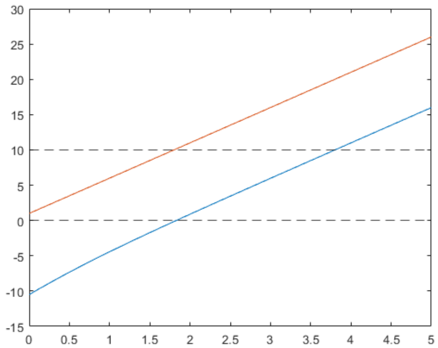
System response. Figure 3.8 shows the system response of Case I. From Figure 3.8(a), there are no contact points between these two vehicles' velocity trajectories and the distance between these two vehicles is always larger than 10 meters, which indicates two vehicles have not collided with each other. From Figure 3.8(b), the system tracking error $e(t) = x(t) - x_d(t)$ converge to zero within 4 seconds. From Figure 3.8(c), the controlled vehicle speed $\dot{x}(t)$ converge to $\dot{x}_o(t)$ within 4 second, and the high speed in this process is less than the speed limit $v_{limit} = 6.7$ meters per seconds. Figure 3.8(d) presents the control input $u(t) \in [-16000, 16000]$.

Figure 3.9 shows the system response of Case II. From Figure 3.9(a), the controlled vehicle follows the other vehicle at first and accelerates to the speed limit v_{limit} once the other vehicle leaves the lane. When $t \leq 4.4$ seconds, the distance between two vehicles is always larger than 10 meters, which means the collision does not happen when two vehicles driving direction is the same. Although there may be a contact point between these two vehicles' velocity trajectories after $t = 9$ second, the collision does not happen, because the other vehicle leaves this lane after it passes the danger zone at $t = 4.4$ second. Also, there are two phases of the controlled vehicle: following the other vehicle and driving as the speed limit to ensure efficiency. From Figure

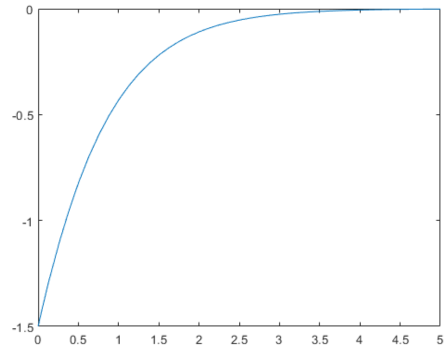
3.9(b), tracking error $e(t) = x(t) - x_d(t)$ of the system can converge to zero within 4 seconds in both two phases. According to Figure 3.8(c), the speed $\dot{x}(t)$ always converge to the desired speed $\dot{x}_d(t)$ within 4 seconds in both two phases. We also do not find the vehicle velocity $\dot{x}(t)$ exceeds the speed limit in this case. Figure 3.9(d) presents the control input signal $u(t) \in [-16000, 16000]$.

The system response of Case III is presented in Figure 3.10. In Figure 3.10(a), we can see the controlled vehicle velocity trajectory has two phases: it decelerates from $\dot{x}(0) = 6.7$ meters per second to zero before it reaches $x = 0$, after the other vehicle passes the danger zone, it continues driving as the speed of 6.7 meters per second. The contact point of two vehicles' trajectories is before $x = 0$, and the two trajectory parts whose vertical coordinates belong to $x = [0, 10]$ are at different times. These proofs mean two vehicles do not appear in the danger zone at the same time. Thus, the collision does not happen. According to Figure 3.10(b), for two different statuses, the tracking error $e(t) = x(t) - x_d(t)$ always can converge to zero within 4 seconds. In Figure 3.10(c), the controlled vehicle speed $\dot{x}(t)$ can converge to the desired speed $\dot{x}_o(t)$ in all two statuses. In each phase, the overshoot of speed does not appear. The control input signal $u(t) \in [-16000, 16000]$ is shown in Figure 3.10(d).

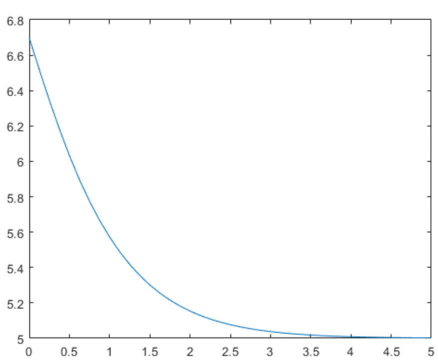
Figure 3.11 shows the system response of Case IV. In Figure 3.11(a), we can find the controlled vehicle drives as the speed limit $x_{limit} = 6.7$ meters per second all the time. The contact point of the two vehicles' trajectories is out of the zone between $x = 0$ and $x = 10$, and the two trajectory parts whose vertical coordinates belong to $x = [0, 10]$ are at different times. These proofs indicate the two vehicles do not collide in this case. The tracking error $e(t) = x(t) - x_d(t)$ in Figure 3.11(b) converges to zero within 4 seconds. According to Figure 3.10(c), there is no speed overshoot. Figure 3.9(d) shows the control input signal $u(t) \in [-16000, 16000]$.



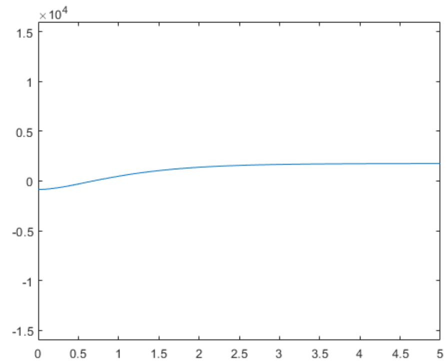
(a) Vehicle position $x(t)$(blue) and $x_o(t)$(red) in m



(b) Tracking error $e(t)$ in m

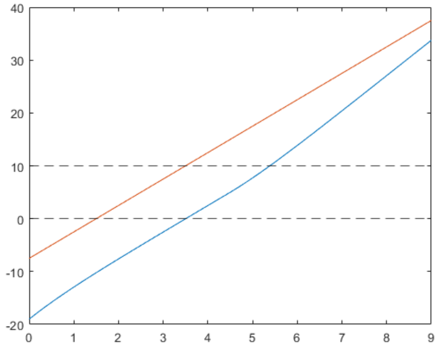


(c) Vehicle speed $\dot{x}(t)$ in m/s

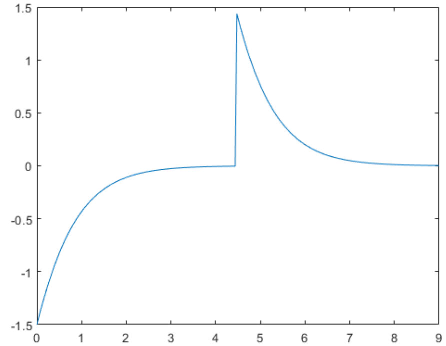


(d) Control input signal $u(t)$ in N

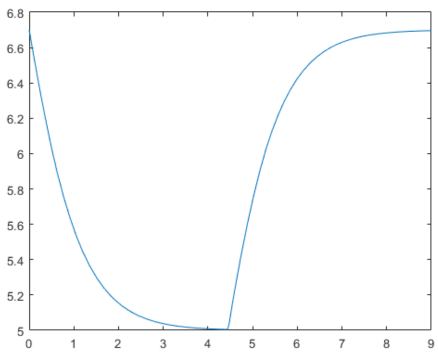
Figure 3.8: System response for the baseline nominal control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t)$ (Case I).



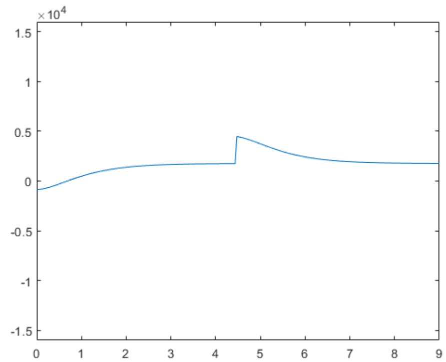
(a) Vehicle position $x(t)$(blue) and $x_o(t)$(red) in m



(b) Tracking error $e(t)$ in m

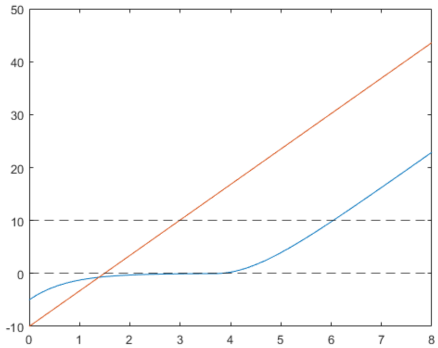


(c) Vehicle speed $\dot{x}(t)$ in m/s

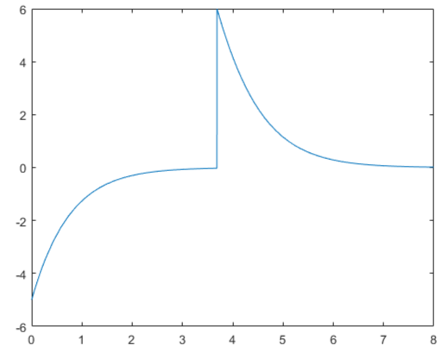


(d) Control input signal $u(t)$ in N

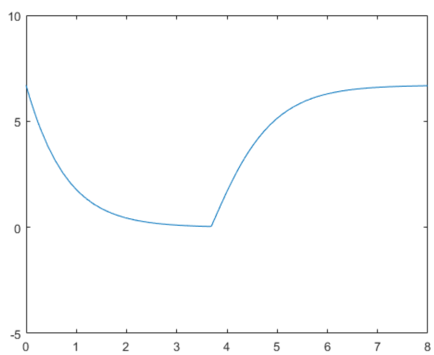
Figure 3.9: System response for the baseline nominal control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t)$ (Case II).



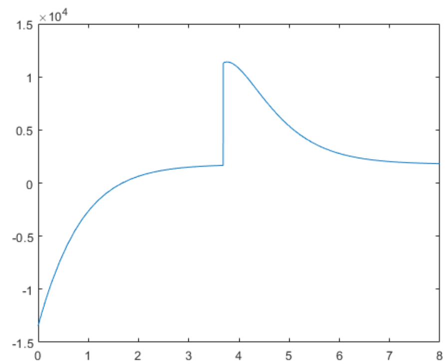
(a) Vehicle position $x(t)$(blue) and $x_o(t)$(red) in m



(b) Tracking error $e(t)$ in m

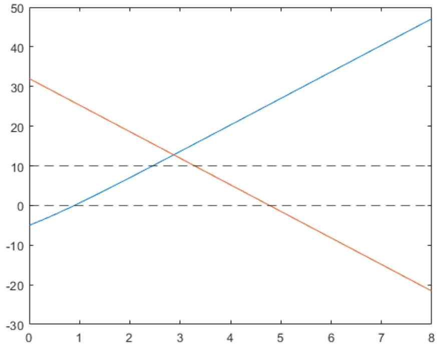


(c) Vehicle speed $\dot{x}(t)$ in m/s

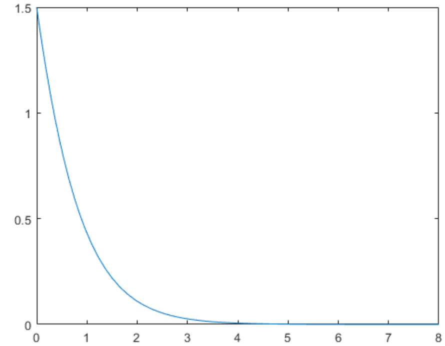


(d) Control input signal $u(t)$ in N

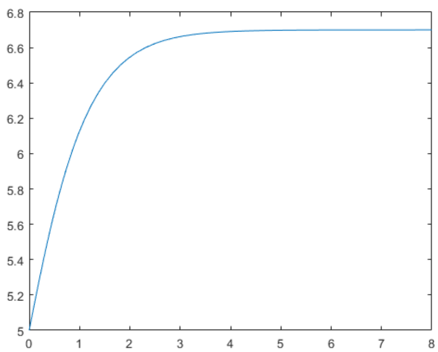
Figure 3.10: System response for the baseline nominal control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t)$ (Case III).



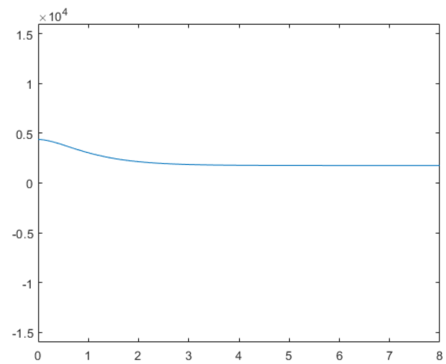
(a) Vehicle position $x(t)$(blue) and $x_o(t)$(red) in m



(b) Tracking error $e(t)$ in m



(c) Vehicle speed $\dot{x}(t)$ in m/s



(d) Control input signal $u(t)$ in N

Figure 3.11: System response for the baseline nominal control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t)$ (Case IV).

Summary. According to the system response, in all four cases, all possible collisions are avoided. The tracking errors $e(t) = x(t) - x_d(t)$ in all cases converge to zero and the vehicle speed $\dot{x}(t)$ converges to the desired speed $\dot{x}_d(t)$. For each case, the overshoot of speed does not appear, which indicates that our vehicle's speed does not exceed the speed limit in four cases. In all four cases, the control input signal $u(t)$ does not exceed the reasonable range.

According to the above, the nominal scheme can meet our requirements that make the vehicle pass the intersection fast, collision-free, and avoid breaking the speed and the control input limit.

3.4.2 Adaptive Control System Evaluation

In this subsection, we tested the adaptive control system's ability to deal with the system uncertainties in four different cases.

The system equations, parameterization, and system parameters are the same as the nominal controller applied system. Differently, we will apply the input control signal as:

$$u(t) = \theta^T(t)\omega(t) - k_1 z(t), \tag{3.26}$$

where $\lambda = 2$, $k_1 = 2700$ and the adaptive law

$$\dot{\theta}(t) = -\Gamma\omega(t)z(t), \tag{3.27}$$

where $\Gamma = 0.1I_3$. The initial value of parameter estimator is $\theta(0) = [0.098, 117.6, 1176]^T$.

Control input signal limitation. As stated in Subsection 3.4.1, we set the controlled vehicle can accelerate to 60 miles per hour from 0 in 3.1 seconds [49], and it can decelerate to 0 from 60 miles per hour within 133 feet [50]. When we substitute the vehicle mass $m = 1800$ kilograms to the uniform acceleration motion formula, the reasonable control input signal range should be about $[-16000, 16000]$ without considering the driving resistance and keeping two significant figures.

System response. Figures 3.12-3.13 show the system response of Case I. From Figure 3.12(a), the distance between these two vehicles is always larger than 10 meters, which indicates two vehicles have not collided with each other. From Figure 3.12(b), the system tracking error $e(t) = x(t) - x_d(t)$ converge to zero within 4 seconds. From

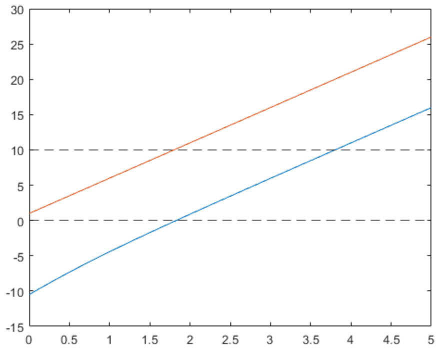
Figure 3.12(c), the controlled vehicle speed $\dot{x}(t)$ converge to $\dot{x}_o(t)$ within 4 second, and the high speed in this process is less than the speed limit $v_{limit} = 6.7$ meters per seconds. Figure 3.12(d) presents the control input $u(t) \in [-16000, 16000]$. Through Figure 3.13, the parameter errors $\theta_1(t) - \theta_1^*$ and $\theta_2(t) - \theta_2^*$ are not bounded, but $\theta_3(t) - \theta_3^*$ is bounded.

Figures 3.14-3.15 show the system response of Case II. From Figure 3.14(a), the controlled vehicle follows the other vehicle at first and accelerates to the speed limit v_{limit} once the other vehicle leaves the lane. When $t \leq 4.4$ seconds, the distance between two vehicles is always larger than 10 meters, which means the collision does not happen when two vehicles driving direction is the same. Although there may be a contact point between these two vehicles' velocity trajectories after $t = 9$ second, the collision does not happen, because the other vehicle leaves this lane after it passes the danger zone at $t = 4.4$ second. Also, there are two phases of the controlled vehicle: following the other vehicle and driving as the speed limit to ensure efficiency. From Figure 3.14(b), tracking error $e(t) = x(t) - x_d(t)$ of the system can converge to zero within 4 seconds in both two phases. According to Figure 3.14(c), the speed $\dot{x}(t)$ always converges to the desired speed $\dot{x}_d(t)$ within 4 seconds in both two phases. We also do not find the vehicle velocity $\dot{x}(t)$ exceeds the speed limit in this case. Figure 3.14(d) presents the control input signal $u(t) \in [-16000, 16000]$. According to Figure 3.15, the parameter errors $\theta_1(t) - \theta_1^*$ and $\theta_2(t) - \theta_2^*$ are not bounded in both two phases, but $\theta_3(t) - \theta_3^*$ is bounded in both two phases.

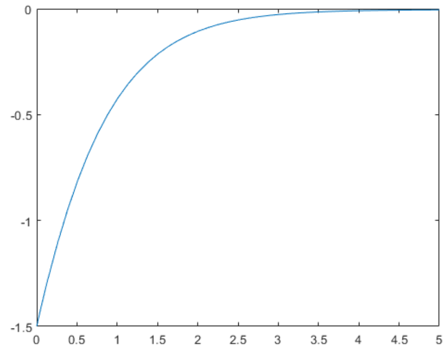
The system response of Case III is presented in Figures 3.16-3.17. In Figure 3.16(a), we can see the controlled vehicle velocity trajectory has two phases: it decelerates from $\dot{x}(0) = 6.7$ meters to zero per second before it reaches $x = 0$, after the other vehicle passes the danger zone, it continues driving as the speed of 6.7 meters per second. The contact point of two vehicles' trajectories is before $x = 0$, and the two trajectory parts whose vertical coordinates belong to $x = [0, 10]$ are at different times. These proofs mean two vehicles do not appear in the danger zone at the same time. Thus, the collision does not happen. According to Figure 3.16(b), for two different statuses, the tracking error $e(t) = x(t) - x_d(t)$ always can converge to zero within 4 seconds. In Figure 3.16(c), the controlled vehicle speed $\dot{x}(t)$ can converge to the desired speed $\dot{x}_o(t)$ in all two statuses. In each phase, the overshoot of speed does not appear. The control input signal $u(t) \in [-16000, 16000]$ is shown in Figure 3.16(d). From Figure 3.17, the parameter errors $\theta_1(t) - \theta_1^*$ and $\theta_2(t) - \theta_2^*$ are not bounded in both two phases, but $\theta_3(t) - \theta_3^*$ is bounded in both two phases.

Figures 3.18-3.19 show the system response of Case IV. In Figure 3.18(a), we can find the controlled vehicle drives as the speed limit $x_{limit} = 6.7$ meters per second all the time. The contact point of the two vehicles' trajectories is out of the zone between $x = 0$ and $x = 10$ and the two trajectory parts whose vertical coordinates belong to $x = [0, 10]$ are at different times. These proofs mean the two vehicles do not collide in this case. The tracking error $e(t) = x(t) - x_d(t)$ in Figure 3.18(b) converges to zero within 4 seconds. According to Figure 3.18(c), there is no speed overshoot. Figure 3.18(d) shows the control input signal $u(t) \in [-16000, 16000]$. According to Figure 3.19, the parameter errors $\theta_1(t) - \theta_1^*$ and $\theta_2(t) - \theta_2^*$ are not bounded, but $\theta_3(t) - \theta_3^*$ is bounded.

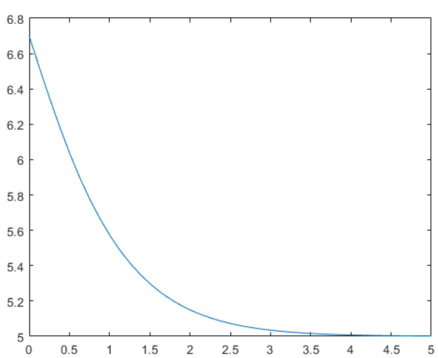
Figures 3.20-3.23 present the system tracking error $e(t) = x(t) - x_d(t)$ difference and the vehicle speed difference between the baseline nominal control system and the baseline adaptive control system in four cases. We found in all the cases, the tracking error $e(t) = x(t) - x_d(t)$ waveform of these two control systems are almost overlapped.



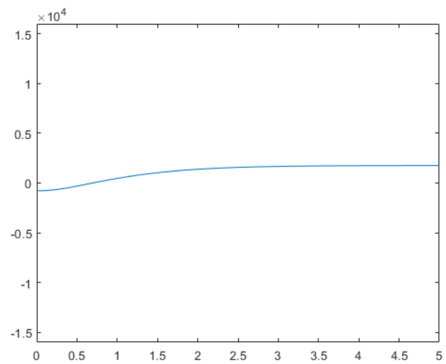
(a) Vehicle positions $x(t)$(blue) and $x_o(t)$(red) in m



(b) Tracking error $e(t)$ in m

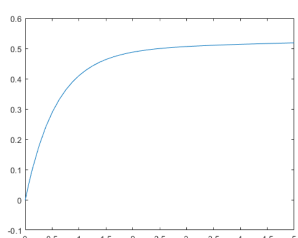


(c) Vehicle speed $\dot{x}(t)$ in m/s

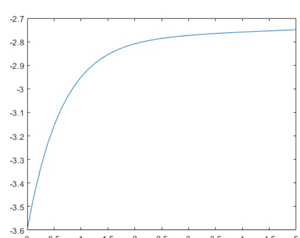


(d) Control input $u(t)$ in N

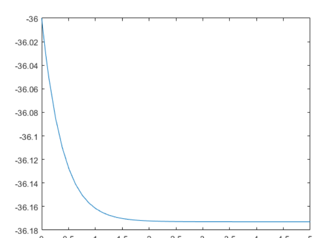
Figure 3.12: System response for the baseline adaptive control system $u(t) = \theta(t)^T\omega(t) - k_1 z(t)$ (Case I).



(a) Parameter error $\theta_1(t) - \theta_1^*$ in kg/s

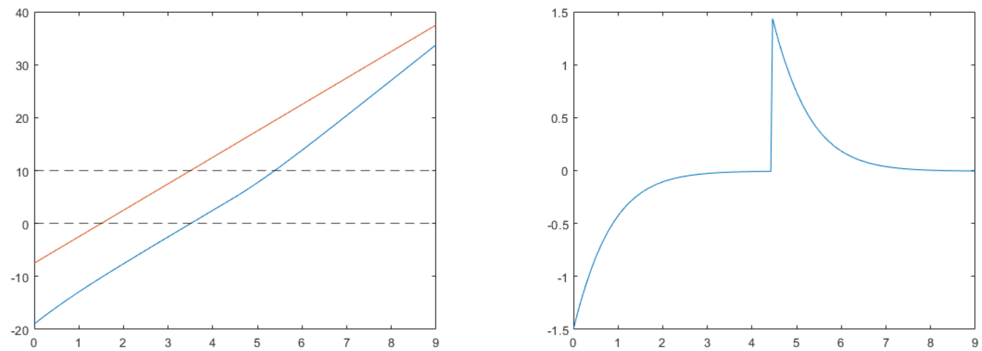


(b) Parameter error $\theta_2(t) - \theta_2^*$ in kg



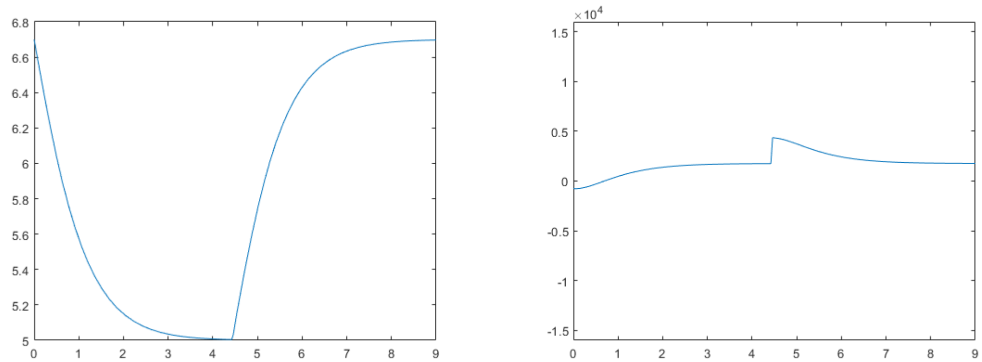
(c) Parameter error $\theta_3(t) - \theta_3^*$ in kg

Figure 3.13: Parameters errors of $\theta(t)$ for the baseline adaptive control system $u(t) = \theta(t)^T\omega(t) - k_1 z(t)$ (Case I).



(a) Vehicle positions $x(t)$(blue) and $x_o(t)$(red) in m

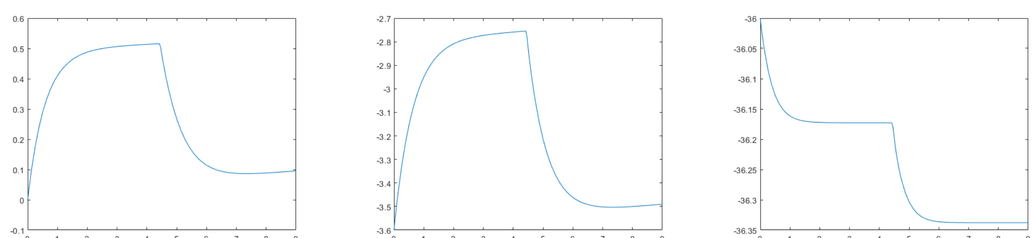
(b) Tracking error $e(t)$ in m



(c) Vehicle speed $\dot{x}(t)$ in m/s

(d) Control input $u(t)$ in N

Figure 3.14: System response for the baseline adaptive control system $u(t) = \theta(t)^T\omega(t) - k_1 z(t)$ (Case II).

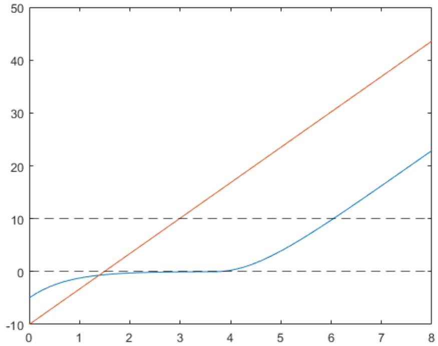


(a) Parameter error $\theta_1(t) - \theta_1^*$ in kg/s

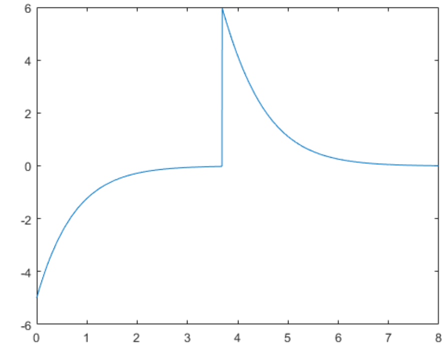
(b) Parameter error $\theta_2(t) - \theta_2^*$ in kg

(c) Parameter error $\theta_3(t) - \theta_3^*$ in kg

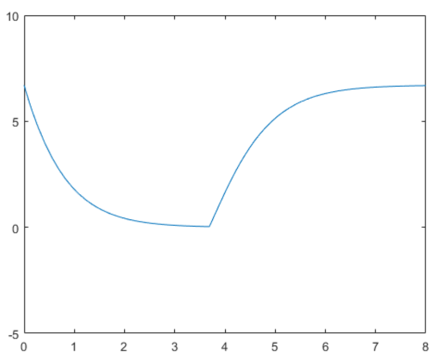
Figure 3.15: Parameters errors of $\theta(t)$ for the baseline adaptive control system $u(t) = \theta(t)^T\omega(t) - k_1 z(t)$ (Case II).



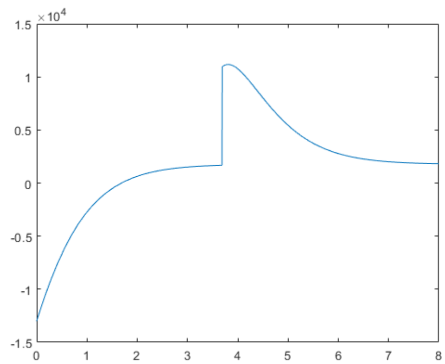
(a) Vehicle positions $x(t)$(blue) and $x_o(t)$(red) in m



(b) Tracking error $e(t)$ in m

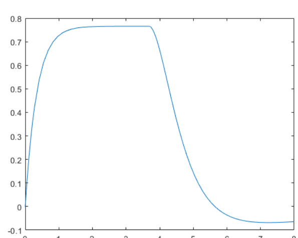


(c) Vehicle speed $\dot{x}(t)$ in m/s

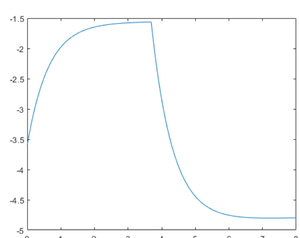


(d) Control input $u(t)$ in N

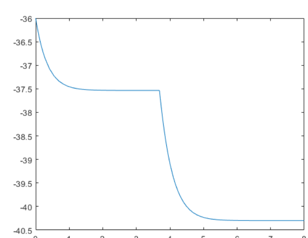
Figure 3.16: System response for the baseline adaptive control system $u(t) = \theta(t)^T\omega(t) - k_1 z(t)$ (Case III).



(a) Parameter error $\theta_1(t) - \theta_1^*$ in kg/s

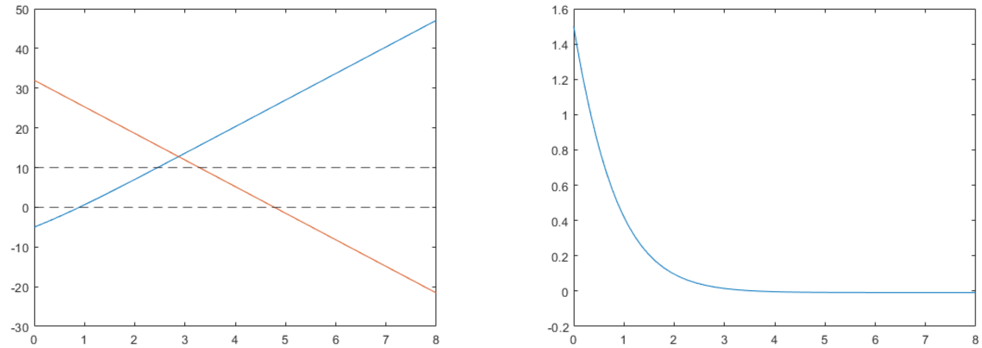


(b) Parameter error $\theta_2(t) - \theta_2^*$ in kg



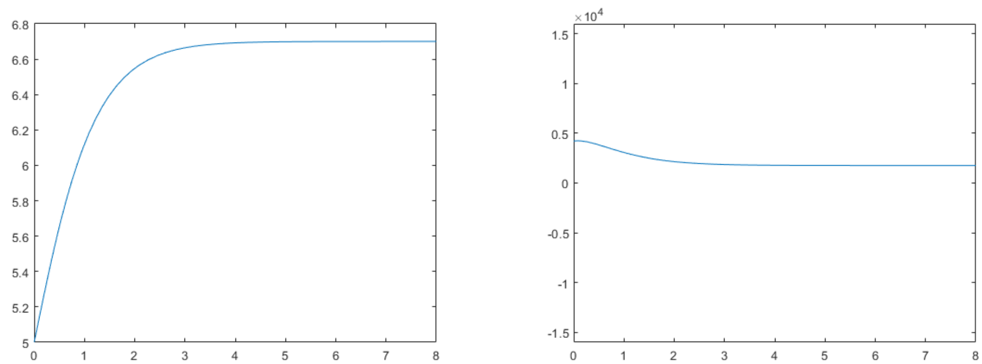
(c) Parameter error $\theta_3(t)_-\theta_3^*$ in kg

Figure 3.17: Parameters errors of $\theta(t)$ for the baseline adaptive control system $u(t) = \theta(t)^T\omega(t) - k_1 z(t)$ (Case III).



(a) Vehicle positions $x(t)$(blue) and $x_o(t)$(red) in m

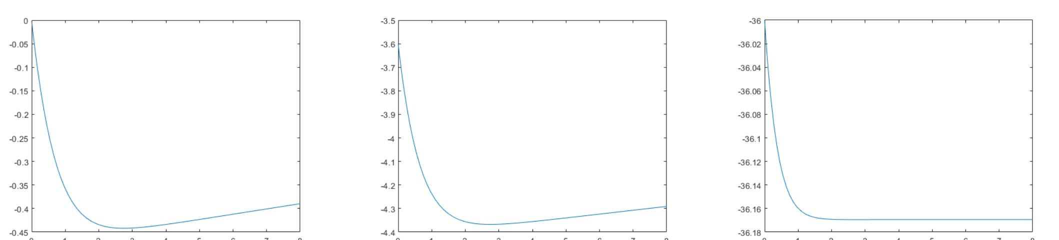
(b) Tracking error $e(t)$ in m



(c) Vehicle speed $\dot{x}(t)$ in m/s

(d) Control input $u(t)$ in N

Figure 3.18: System response for the baseline adaptive control system $u(t) = \theta(t)^T\omega(t) - k_1 z(t)$ (Case IV).

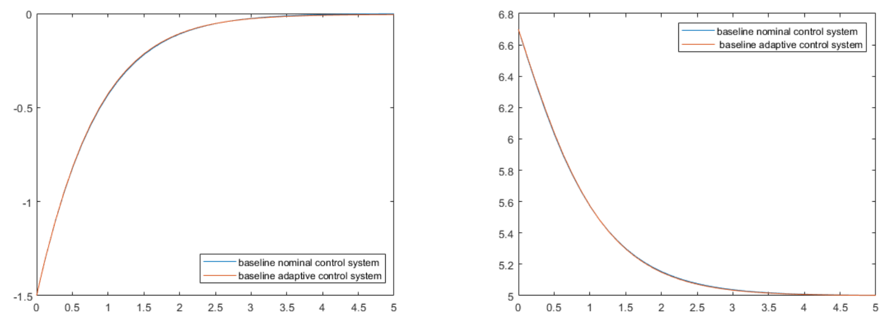


(a) Parameter error $\theta_1(t) - \theta_1^*$ in kg/s

(b) Parameter error $\theta_2(t) - \theta_2^*$ in kg

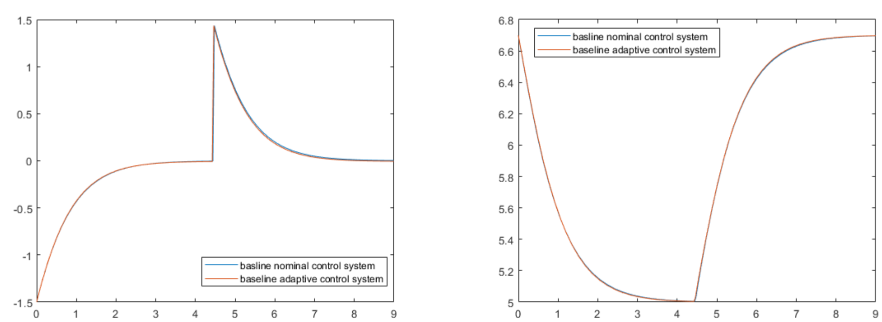
(c) Parameter error $\theta_3(t) - \theta_3^*$ in kg

Figure 3.19: Parameters errors of $\theta(t)$ for the baseline adaptive control system $u(t) = \theta(t)^T\omega(t) - k_1 z(t)$ (Case IV).



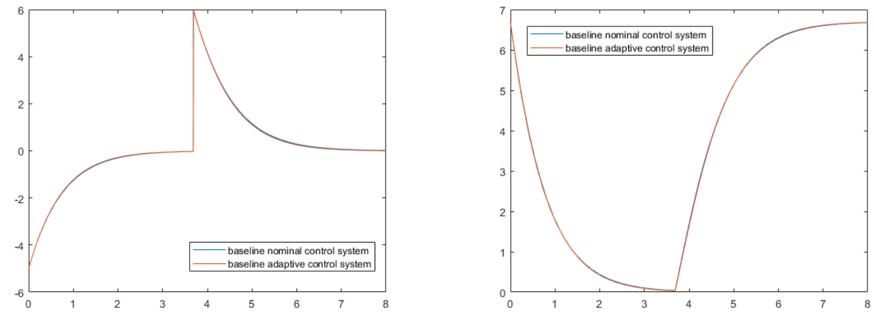
(a) The tracking error difference in m (b) The vehicle speed difference in m/s

Figure 3.20: Comparison between the nominal control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t)$ and the adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t)$ of Case I.



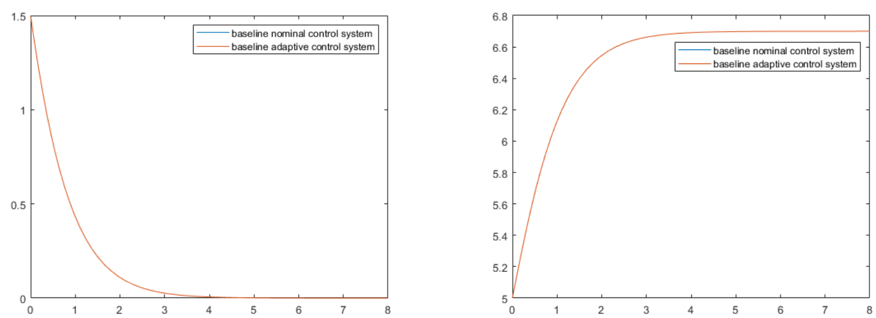
(a) The tracking error difference in m (b) The vehicle speed difference in m/s

Figure 3.21: Comparison between the nominal control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t)$ and the adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t)$ of Case II.



(a) The tracking error difference in m (b) The vehicle speed difference in m/s

Figure 3.22: Comparison between the nominal control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t)$ and the adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t)$ of Case III.



(a) The tracking error difference in m (b) The vehicle speed difference in m/s

Figure 3.23: Comparison between the nominal control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t)$ and the adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t)$ of Case IV.

Summary. According to the system response, in all four cases, all possible collisions are avoided. The tracking errors $e(t) = x(t) - x_d(t)$ in all cases converge to zero and the vehicle speed $\dot{x}(t)$ converges to the desired speed $\dot{x}_d(t)$. The tracking error and vehicle speed of the nominal control system and adaptive control system in all four cases are almost the same.

According to the above, the adaptive scheme can meet our requirements without using the knowledge of system parameters that make the vehicle pass the intersection fast, collision-free without breaking the speed and control input limit. In the simulation, we applied the initial parameter initial value is 0.98 times of the real value for example. When the initial parameter error changes, the system response will also be changed. In general, this control design can finish the control objective, which is the requirements of AVC problems in intersection areas.

The control system of the intersection vehicle system we designed in this chapter contains the nominal control system and adaptive scheme, which still can be improved. As a consequence, the enhanced control signal $u(t)$ will be introduced in Chapter 4. The parameterization and desired trajectory generation process will also be of great importance.

Chapter 4

Adaptive Vehicle Tracking Control: Enhanced Design

In this chapter, we will propose another control system to finish the control task stated in Chapter 2. Based on the baseline controller discussed in Chapter 3, we introduce a new enhanced control input signal in this chapter, which can not only solve the system parameter uncertainties even improve the system's transient performance. The improvement is necessary for traffic safety and traffic efficiency reasons. The presented simulation study will verify the system's performance.

4.1 Tracking Control System Structure

Similarly to Chapter 3, we need to solve the intersection vehicle control problem described at the end of Chapter 2. Our control task is to make the vehicle pass the intersection shown in Figure 4.1 as fast as possible and avoid the collision. Thus, we formulate such a problem into an adaptive tracking control problem.

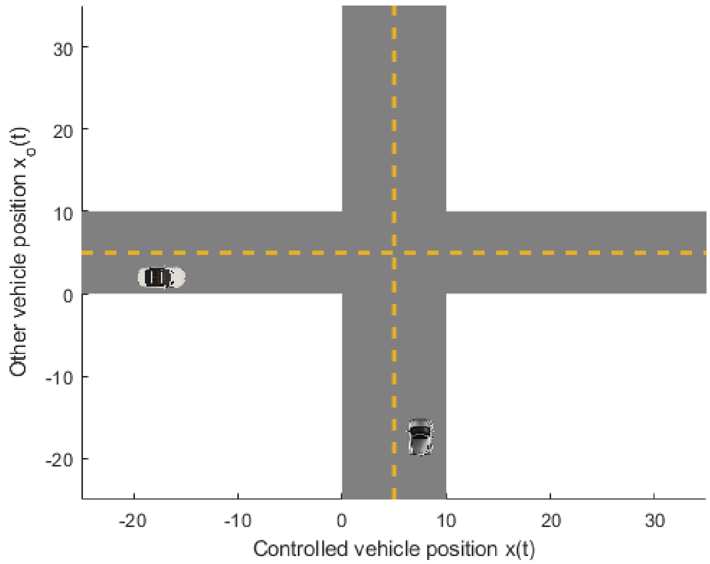


Figure 4.1: Intersection coordinates: controlled vehicle x, other vehicle x_o.

Control objective. There are two parts of the control objective: the desired trajectory generation part and the trajectory tracking part. Based on other vehicles' position information in the intersection, the system should generate the desired velocity trajectory, which can make the vehicle pass the intersection area fast and avoid collisions with other vehicles. The adaptive control system should make the controlled system with uncertainties track the trajectory as quickly as possible.

Desired trajectory generation. As stated in Chapter 3, to meet the control objective, we also stipulate two vehicles cannot enter the intersection simultaneously when their initial driving directions are not the same. Thus, we set the same desire trajectory as Chapter3 in the cases that the controlled vehicle moving eastward.

(1) **When these two vehicles' driving directions are the same initially,** we

choose

$$x_d(t) = \begin{cases} x_o(t) - l & 0 \leq x_o(t) - x(t) \leq l' \wedge D_o(t) = D(t) \\ v_{limit}(t - t_{leave}) + x_{leave} - d & \text{else} \end{cases} \tag{4.1}$$

where $x(t)$ is the position of the controlled vehicle, $x_o(t)$ is the position of the other vehicle, l is a long enough distance to avoid these two vehicles colliding, v_{limit} is the speed limit of this intersection area, t_{leave} is the time that the other vehicle changes its driving direction and leaves the intersection(the initial value of t_{leave} is 0), x_{leave} is the position of the controlled vehicle when the other vehicle changes its driving direction and leaves the intersection (the initial value of x_{leave} is $x(0)$), d is the necessary distance delay to avoid the vehicle over the speed limit, l' is used to judge whether the distance between these two vehicles is close enough to adjust the controlled vehicle speed, $D(t)$ is the driving direction of the controlled vehicle, and $D_o(t)$ is the driving direction of the other vehicle.

In Equation(4.1), $0 \leq x_o(t) - x(t) \leq l'$ means the other vehicle is in the front of the controlled vehicle, and the distance between these two vehicles is too close; $D_o(t) = D(t)$ means these two vehicles' driving direction are the same. When all two restrictions are satisfied, we make the controlled vehicle follow the other vehicle. For all other situations, we make the controlled vehicle drive as the speed limit.

(2) **When these two vehicles' driving directions are different initially,** the logic of desired trajectory generation is:

- When the controlled vehicle is inside of the action zone, and it detects a collision may happen, it should stop at the entrance of the intersection;
- When the other vehicle change to the same lane as the controlled vehicle's

and the distance between these two vehicles is close enough, the controlled vehicle will follow the other vehicle;

– For all other cases, the controlled vehicle will drive with the speed limit.

Thus, the corresponding $x_d(t)$ expression is

$$x_d(t) = \begin{cases} 0 & \begin{cases} D_o(t) = n \\ \wedge \frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)}{v_{limit}}| \\ \wedge x_{action} < x(t) < 0 \\ \wedge x_o(t) < 10 + l_{vo} \end{cases} \\ 0 & \begin{cases} D_o(t) = \{s, w\} \\ \wedge \frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)-10}{v_{limit}}| \\ \wedge x_{action} < x(t) < 0 \\ \wedge x_o(t) > 0 - l_{vo} \end{cases} \\ x_o(t) - l & \begin{cases} D_o(t) = D(t) \\ \wedge x_o(t) \geq 10 + l_{vo} \geq x(t) \\ \wedge 0 \leq x_o(t) - x(t) \leq l' \end{cases} \\ v_{limit}(t - t_0) + x' - d & \text{else,} \end{cases} \tag{4.2}$$

where l is a long enough distance to avoid these two vehicles colliding, d_{limit} is the necessary distance delay that the controlled vehicle accelerates to v_{limit}, t_0 is the moment when the latest time that $x_d(t) = 0$'s and $x_d(t) = x_o(t) - l$'s restricted condition dissatisfying (the initial value of t_0 is 0), x' is the position of the controlled vehicle at the moment when the latest time that $x_d(t) = 0$'s and $x_d(t) = x_o(t) - l$'s restricted condition dissatisfying (the initial value of x' is $x(0)$), d is the necessary distance delay to avoid the vehicle over the speed limit, x_{action} is the start location of the action zone, l_v is the length of the controlled vehicle, l_{vo} is the length of the other vehicle, and l' is used to judge whether the distance between these two vehicles is close enough to adjust the controlled vehicle speed.

In the restrictions of $x_d(t) = 0$ in Equation (4.2), $D_o(t) = n$ means the other vehicle's driving direction is northward, and $D_o(t) = \{s, w\}$ means the other vehicle's driving direction is southward or westward; $\frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)}{v_{limit}}|$ or $\frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)-10}{v_{limit}}|$ indicates that the time that the controlled vehicle clearing from the danger zone is longer than the minimum time that the other vehicle reaching the danger zone entrance, which means it is possible to cause a collision because we cannot ensure the controlled vehicle's tail clears from the danger zone before the other vehicle enters the danger zone; $x_{action} < x(t) < 0$ means the controlled vehicle is in the action zone, whose length should be long enough to make the controlled vehicle stop from the speed limit to zero; $x_o(t) < 10 + l_{vo}$ and $x_o(t) > 0 - l_{vo}$ means the other vehicle has not passed the danger zone. When these four restrictions are satisfied at the same time, the controlled vehicle needs to stay at the entrance of the danger zone until the other vehicle passes the danger zone completely to avoid colliding.

In the restrictions for $x_d(t) = x_o(t) - l$ in Equation (4.2), $D_o(t) = D(t)$ indicates the driving direction of these two vehicles are the same, the other vehicle may change to the controlled vehicle's lane; $x_o(t) \geq 10$ and $x_o(t) \leq 0 - l_{vo}$ means the other vehicle has passed the danger zone completely; $0 \leq x_o(t) - x(t) \leq$

l' means the other vehicle is in the front of the controlled vehicle, and the distance between these two vehicles is too close. When these three restrictions are satisfied, we will make the controlled vehicle follow the other vehicle.

In all other situations, we will make the controlled vehicle drive as the speed limit to ensure efficiency.

Trajectory tracking preparation. According to Chapter 2, we formulated the vehicle movement model $m\ddot{x}(t) = u(t) - b\dot{x} - mgsin\delta$ into the parameterized model to design the adaptive cntrol scheme:

$$\begin{aligned} \theta^*\omega(t) &= b\dot{x}(t) - mgsin\delta - m(\ddot{x}_d(t) - \lambda\dot{e}(t)) \\ m\dot{z}(t) &= u(t) - \theta^*\omega(t), \end{aligned} \tag{4.3}$$

by defining the tracking error expression

$$e(t) = x(t) - x_d(t), \tag{4.4}$$

the auxiliary signal

$$z(t) = \dot{e}(t) + \lambda e(t), \tag{4.5}$$

the unknown parameter vector

$$\theta^* = [b, m\,\text{sin}\delta, m]^T, \tag{4.6}$$

and the regressor vector

$$\omega(t) = [\dot{x}, g, \ddot{x}_d - \lambda\dot{e}(t)]^T, \tag{4.7}$$

where m is the mass of the car, b is the friction coefficient, g is the gravity acceleration, δ is the road slope angle, $x(t)$ is the position of the vehicle, $x_d(t)$ is the desired velocity trajectory, λ is the chosen positive design parameter $\lambda > 0$ and $u(t)$ is the engine force or the brake force of the vehicle.

4.2 Nominal Control System

To improve the system tracking performance, we introduce an enhanced nonlinear control signal in this chapter based on the baseline control design

$$u(t) = \theta^{*T}\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}, \tag{4.8}$$

where $k_i > 0$, $i = 1, 2, 3$, are design parameters, sign$[z(t)]$ represents the sign of $z(t)$, $\alpha \in (0, 1)$ and $\beta \in (1, \infty)$ are also design parameters, and θ^* is the known system parameters.

Theorem 4.1 *The control signal (4.8), applied to the vehicle system(2.38), ensures that all system signals are bounded and the position tracking error $e(t)$ and velocity tracking error $\dot{e}(t)$ both converge to zero exponentially.*

Proof: We choose positive definite function

$$V(z) = \frac{m}{2} z^2(t) \tag{4.9}$$

With Equation (4.8), we express $m\dot{z}(t) = u(t) - \theta^{*T}\omega(t)$ as

$$m\dot{z}(t) = -k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}. \tag{4.10}$$

Then, we can get

$$\begin{aligned}\dot{V} &= mz(t)\dot{z}(t)\\ &= z(t)(-k_1z(t) - k_2\text{sign}[z(t)]|z(t)|^{\alpha} - k_3\text{sign}[z(t)]|z(t)|^{\beta})\\ &= -k_1z^2(t) - k_2\text{sign}[z(t)]z(t)|z(t)|^{\alpha} - k_3\text{sign}[z(t)]z(t)|z(t)|^{\beta}\\ &= -k_1z^2(t) - k_2|z(t)|^{1+\alpha} - k_3|z(t)|^{1+\beta}.\end{aligned} \tag{4.11}$$

Because $k_i > 0$, $i = 1, 2, 3$,

$$\dot{V} < 0. \tag{4.12}$$

Therefore,

$$z(t) \in L^{\infty} \tag{4.13}$$

and

$$\begin{aligned}&\int_0^{\infty}(k_1z^2(t) + k_2|z(t)|^{1+\alpha} + k_3|z(t)|^{1+\beta})dt\\ = -&\int_0^{\infty}\dot{V}dt = V(z(0) - V(z(\infty)) \leq \infty,\end{aligned} \tag{4.14}$$

which means

$$z(t) \in L^2,\ z(t) \in L^{1+\alpha},\ z(t) \in L^{1+\beta}. \tag{4.15}$$

Because $z(t) = \dot{e}(t) + \lambda e(t)$,

$$e(t) \in L^{\infty},\ e(t) \in L^2,\ \dot{e}(t) \in L^{\infty}. \tag{4.16}$$

Hence, the theorem's result holds.

With Theorem 4.1, the convergence of this nominal control design indicates this design can finish our tracking control objective which is to make the vehicle moves as the desired velocity trajectory. According to [48], when $e(t) \in L^2$, $e(t) \in L^{1+\alpha}, e(t) \in$

$L^{1+\beta}$ and $z(t) \in L^2$, $z(t) \in L^{1+\alpha}, z(t) \in L^{1+\beta}$, $e(t)$ and $z(t)$ can converge faster than when $e(t) \in L^2$, $z(t) \in L^2$ only.

4.3 Adaptive Control System

About the cases we cannot know the precise system parameters, a nonlinear enhanced control system will ensure the tracking result:

$$u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^\alpha - k_3 \text{sign}[z(t)]|z(t)|^\beta, \qquad (4.17)$$

where $k_i > 0$, $i = 1, 2, 3$, are design parameters, sign$[z(t)]$ represents the sign of $z(t)$, $\alpha \in (0, 1)$ and $\beta \in (1, \infty)$ are also design parameters. And $\theta(t) \in R^3$ is generated from the adaptive law

$$\dot{\theta}(t) = -\Gamma\omega(t)z(t), \qquad (4.18)$$

where Γ is a chosen symmetric and positive definite matrix $\Gamma \in R^{3\times3} : \Gamma = \Gamma^T$. In this enhanced control design, $\theta(t)$ is the adaptive estimate of real parameter vector θ^*, which may not converge to θ^*.

This adaptive control system has the following desired properties, which are stated in the following theorem.

Theorem 4.2 *The adaptive scheme consisting of (4.17) and (4.18), applied to the vehicle system(2.38), ensures that all system signals are bounded and the position tracking error $e(t)$ and velocity tracking error $\dot{e}(t)$ both converge to zero asymptotically.*

Proof: Choose positive definite function:

$$V(z,\widetilde{\theta}) = \frac{m}{2}z^2(t) + \frac{1}{2}\widetilde{\theta}^T\Gamma^{-1}\widetilde{\theta} \tag{4.19}$$

with $\widetilde{\theta} = \theta(t) - \theta^*$. Then

$$\begin{aligned}
\dot{V} &= mz(t)\dot{z}(t) + \Gamma^{-1}\widetilde{\theta}\dot{\widetilde{\theta}} \\
&= z(t)(u(t) - \theta^*\omega(t)) - \widetilde{\theta}\omega(t)z(t) \\
&= z(t)(\theta^T(t)\omega(t) - k_1 z(t) - k_2\mathrm{sign}[z(t)]|z(t)|^\alpha - k_3\mathrm{sign}[z(t)]|z(t)|^\beta - \theta^*\omega(t)) \\
&- \widetilde{\theta}\omega(t)z(t) \\
&= z(t)(-k_1 z(t) - k_2\mathrm{sign}[z(t)]|z(t)|^\alpha - k_3\mathrm{sign}[z(t)]|z(t)|^\beta) \\
&= -k_1 z^2(t) - k_2\mathrm{sign}[z(t)]z(t)|z(t)|^\alpha - k_3\mathrm{sign}[z(t)]z(t)|z(t)|^\beta \\
&= -k_1 z^2(t) - k_2|z(t)|^{1+\alpha} - k_3|z(t)|^{1+\beta}.
\end{aligned} \tag{4.20}$$

Because $k_i > 0$, $i = 1, 2, 3$,

$$\dot{V} < 0. \tag{4.21}$$

Therefore,

$$z(t) \in L^\infty,\ \widetilde{\theta}(t) \in L^\infty, \tag{4.22}$$

and

$$\begin{aligned}
&\int_0^\infty (k_1 z^2(t) + k_2|z(t)|^{1+\alpha} + k_3|z(t)|^{1+\beta})dt \\
&= -\int_0^\infty \dot{V}dt = V(z(0) - V(z(\infty)) \leq \infty,
\end{aligned} \tag{4.23}$$

which means

$$z(t) \in L^2,\ z(t) \in L^{1+\alpha},\ z(t) \in L^{1+\beta} \tag{4.24}$$

Because $z(t) = \dot{e}(t) + \lambda e(t)$,

$$e(t) \in L^{\infty},\ e(t) \in L^2,\ \dot{e}(t) \in L^{\infty}. \tag{4.25}$$

Hence, the theorem's result holds, which means the tracking error and the speed tracking error converge to zero asymptotically. In other words, this adaptive control design can make the vehicle to track the desired velocity trajectory although missing the system parameters. According to [48], when $e(t) \in L^2$, $e(t) \in L^{1+\alpha}, e(t) \in L^{1+\beta}$ and $z(t) \in L^2$, $z(t) \in L^{1+\alpha}, z(t) \in L^{1+\beta}$, $e(t)$ and $z(t)$ can converge faster than when $e(t) \in L^2$, $z(t) \in L^2$ only.

4.4 Simulation Studies

This section will present and discuss the simulation results of the enhanced control system for the vehicle control problem in the intersection area. Similar to the last chapter discussed, we consider four different cases testing the enhanced control system:

- In Case I, we set the other vehicle will always stay in the lane at the speed of $\dot{x}_o = 5$ meters per second from $x_o(0) = 1$ to eastward. Thus, the other vehicle's velocity trajectory is $x_o(t) = 5t + 1$, $D_o(t) = e$ for this case. The initial position of controlled vehicle is $x(0) = -10.5$, and its initial speed is $\dot{x}(0) = 6.7$ meters per second.

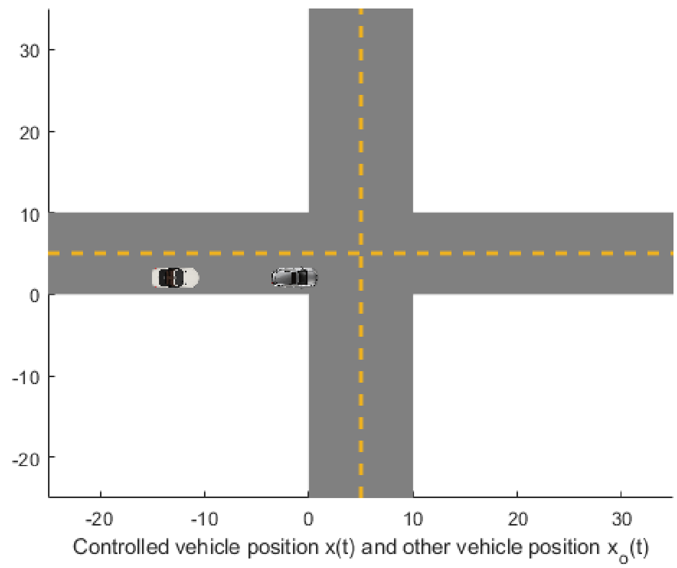


Figure 4.2: The initial driving directions and positions of two vehicles in Case I.

- In Case II, the other vehicle's velocity trajectory is $x_o(t) = 5t - 7.5$ means it moves forward at a speed of $\dot{x}_o = 5$ meters per second from $x_o(0) = -7.5$ to the eastward initially. However, it will change its lane and driving direction to $D_o(t) = n$ when it leaves the danger zone. The initial position of controlled vehicle is $x(0) = -19$, and its initial speed is $\dot{x}(0) = 6.7$ meters per second.

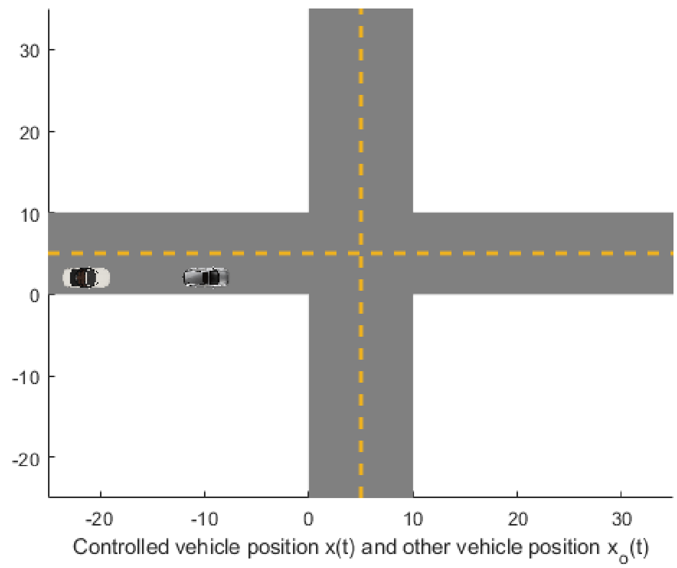


Figure 4.3: The initial driving directions and positions of two vehicles in Case II.

- In Case III, the other car's trajectory is $x_o = 6.7t - 10$, $D_o(t) = n$, which means the vehicle moves forward at a speed of $\dot{x}_o = 6.7$ meters per second from $x_o(0) = -10$ to the northward. The initial position of controlled vehicle is $x(0) = -5$, and its initial speed is $\dot{x}(0) = 6.7$ meters per second. In this case, collisions may happen. Thus, our controller needs to modify the desired trajectory.

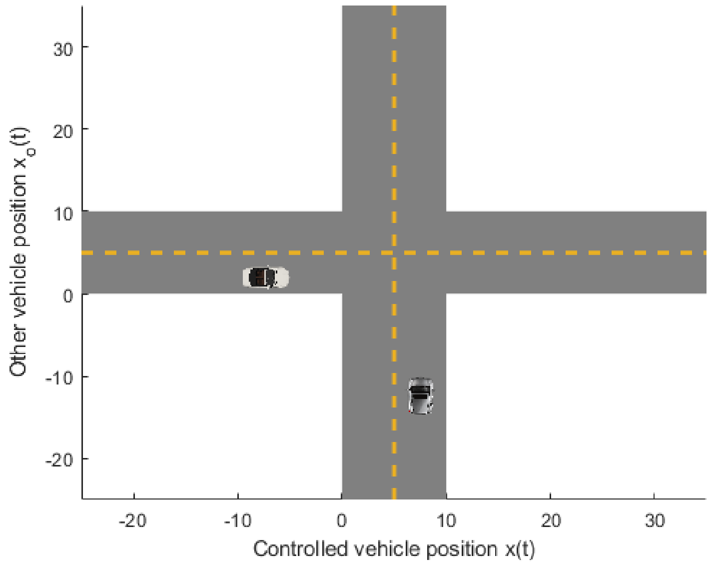


Figure 4.4: The initial driving directions and positions of two vehicles in Case III.

- In Case IV, the other car's trajectory is $x_o = -6.7t + 32$, $D_o(t) = s$, which means the vehicle moves forward at a speed of -6.7 meters per second from $x = 32$ to southward. The initial position of controlled vehicle is $x(0) = -5$, and its initial speed is $\dot{x}(0) = 5$ meters per second.

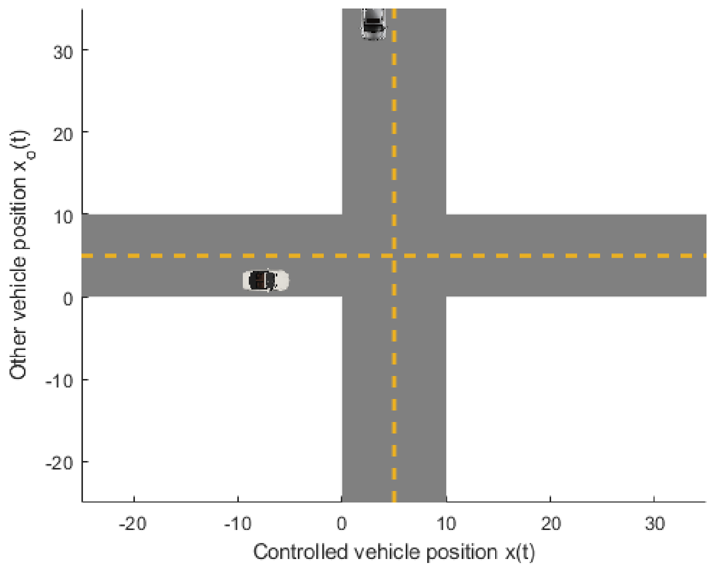


Figure 4.5: The initial driving directions and positions of two vehicles in Case IV.

According to the above, the desired trajectory for Case I and Case II is

$$x_d(t) = \begin{cases} x_o(t) - l & 0 \leq x_o(t) - x(t) \leq l' \wedge D_o(t) = D(t) \\ v_{limit}(t - t_{leave}) + x_{leave} - d & \text{else} \end{cases} \tag{4.26}$$

where $x(t)$ is the position of the controlled vehicle, $x_o(t)$ is the position of the other vehicle, l is a long enough distance to avoid these two vehicles colliding, v_{limit} is the speed limit of this intersection area, t_{leave} is the time that the other vehicle changes its driving direction and leaves the intersection(the initial value of t_{leave} is 0), x_{leave} is the position of the controlled vehicle when the other vehicle changes its driving direction and leaves the intersection (the initial value of x_{leave} is $x(0)$), d is the necessary distance delay to avoid the vehicle over the speed limit, l' is used to judge whether the distance between these two vehicles is close enough to adjust the controlled vehicle speed, $D(t)$ is the driving direction of the controlled vehicle, and $D_o(t)$ is the driving direction of the other vehicle.

In Equation(4.26), $0 \leq x_o(t) - x(t) \leq l'$ means the other vehicle is in the front of the controlled vehicle, and the distance between these two vehicles is too close; $D_o(t) = D(t)$ means these two vehicles' driving direction are the same. When all two restrictions are satisfied, we make the controlled vehicle follow the other vehicle. For all other situations, we make the controlled vehicle drive as the speed limit.

For Case III-IV, the desired trajectory $x_d(t)$ is

$$x_d(t) = \begin{cases} 0 & \begin{cases} D_o(t) = n \\ \wedge \frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)}{v_{limit}}| \\ \wedge x_{action} < x(t) < 0 \\ \wedge x_o(t) < 10 + l_{vo} \end{cases} \\ 0 & \begin{cases} D_o(t) = \{s, w\} \\ \wedge \frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)-10}{v_{limit}}| \\ \wedge x_{action} < x(t) < 0 \\ \wedge x_o(t) > 0 - l_{vo} \end{cases} \\ x_o(t) - l & \begin{cases} D_o(t) = D(t) \\ \wedge x_o(t) \geq 10 + l_{vo} \geq x(t) \\ \wedge 0 \leq x_o(t) - x(t) \leq l' \end{cases} \\ v_{limit}(t - t_0) + x' - d & \text{else,} \end{cases} \tag{4.27}$$

where l is a long enough distance to avoid these two vehicles colliding, d_{limit} is the necessary distance delay that the controlled vehicle accelerates to v_{limit}, t_0 is the moment when the latest time that $x_d(t) = 0$'s and $x_d(t) = x_o(t) - l$'s restricted condition dissatisfying (the initial value of t_0 is 0), x' is the position of the controlled vehicle at the moment when the latest time that $x_d(t) = 0$'s and $x_d(t) = x_o(t) - l$'s restricted condition dissatisfying (the initial value of x' is $x(0)$), d is the necessary distance delay to avoid the vehicle over the speed limit, x_{action} is the start location of the action zone, l_v is the length of the controlled vehicle, l_{vo} is the length of the other vehicle, and l' is used to judge whether the distance between these two vehicles is close enough to adjust the controlled vehicle speed.

In the restrictions of $x_d(t) = 0$ in Equation (4.27), $D_o(t) = n$ means the other vehicle's driving direction is northward, and $D_o(t) = \{s, w\}$ means the other vehicle's driving direction is southward or westward; $\frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)}{v_{limit}}|$ or $\frac{|x(t)|+10+l_v+d_{limit}}{v_{limit}} \geq |\frac{x_o(t)-10}{v_{limit}}|$ indicates that the time that the controlled vehicle clearing from the danger zone is longer than the minimum time that the other vehicle reaching the danger zone entrance, which means it is possible to cause a collision because we cannot ensure the controlled vehicle's tail clears from the danger zone before the other vehicle enters the danger zone; $x_{action} < x(t) < 0$ means the controlled vehicle is in the action zone, whose length should be long enough to make the controlled vehicle stop from the speed limit to zero; $x_o(t) < 10 + l_{vo}$ and $x_o(t) > 0 - l_{vo}$ means the other vehicle has not passed the danger zone. When these four restrictions are satisfied at the same time, the controlled vehicle needs to stay at the entrance of the danger zone until the other vehicle passes the danger zone completely to avoid colliding.

In the restrictions for $x_d(t) = x_o(t) - l$ in Equation (4.27), $D_o(t) = D(t)$ indicates the driving direction of these two vehicles are the same, the other vehicle may change to the controlled vehicle's lane; $x_o(t) \geq 10$ and $x_o(t) \leq 0 - l_{vo}$ means the other vehicle has passed the danger zone completely; $0 \leq x_o(t) - x(t) \leq l'$ means the other vehicle is in the front of the controlled vehicle, and the distance between these two vehicles is too close. When these three restrictions are satisfied, we will make the controlled vehicle follow the other vehicle.

In all other situations, we will make the controlled vehicle drive as the speed limit to ensure efficiency.

For parameters in the desired trajectory choosing, we set the $v_{limit} = 15$ miles per hour $= 6.7$ meters per second to be the speed limit, $l = 10$ meters to be the following distance to avoid the collision, the distance delay d_{limit} is the maximum value of $\{0.9|v_{limit} - \dot{x}(0)|, 1.5\}$, the distance delay d is the maximum value of $\{0.9|\dot{x}_d(t_0) - \dot{x}(t_0)|, 1.5\}$ to avoid the vehicle over the speed limit, the start location of action zone $x_{action} = -5$, the length of vehicles $l_v = l_{vo} = 4.7$ meters [49], and $l' = 11.5$ meters is used to judge whether the distance between these two vehicles is close enough to adjust the controlled vehicle speed.

For the system parameters, we set $m = 1800$kg according to [49] and keeping two significant figures, and $g = 9.8\text{m/s}^2$, $b = 0.10$, $sin\delta = 0.10$. Thus,

$$\theta^* = [0.1, 180, 1800]^T. \tag{4.28}$$

4.4.1 Nominal Control System Evaluation

Simulation conditions. The simulation system is the same as last chapter, the controller design parameters are $\lambda = 2$, $k_1 = 2700$, $k_2 = 500$, $k_3 = 900$, $\alpha = 0.5$ and $\beta = 1.05$ for the control input signal

$$u(t) = \theta^{*T}\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}. \tag{4.29}$$

Control input signal limitation. In this problem, we set the controlled vehicle can accelerate to 60 miles per hour from 0 in 3.1 seconds [49], and it can decelerate to

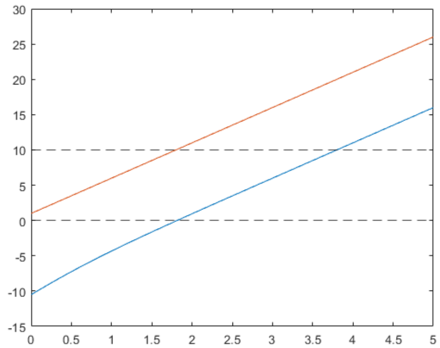
0 from 60 miles per hour within 133 feet [50]. When we substitute the vehicle mass $m = 1800$ kilograms to the uniform acceleration motion formula, the reasonable control input signal range should be about $[-16000, 16000]$ without considering driving resistance and keeping two significant figures.

System response. Figure 4.6 shows the system response of Case I. From Figure 4.6(a), the distance between these two vehicles is always larger than 10 meters, which indicates two vehicles have not collided with each other in this case. From Figure 4.6(b), the system tracking error $e(t) = x(t) - x_d(t)$ converge to zero within 3.5 seconds. From Figure 4.6(c), the controlled vehicle speed $\dot{x}(t)$ converge to $\dot{x}_o(t)$ within 3.5 second, and the high speed in this process is less than the speed limit $v_{limit} = 6.7$ meters per seconds. Figure 4.6(d) presents the control input $u(t) \in [-16000, 16000]$.

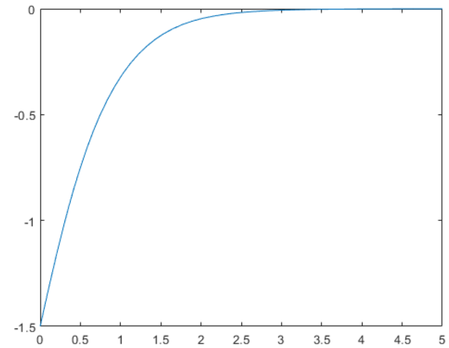
Figure 4.7 shows the system response of Case II. From Figure 4.7(a), the controlled vehicle follows the other vehicle at first and accelerates to the speed limit v_{limit} once the other vehicle leaves the lane. When $t \leq 4.4$ seconds, the distance between two vehicles is always larger than 10 meters, which means the collision does not happen when two vehicles driving direction is the same. Although there may be a contact point between these two vehicles' velocity trajectories after $t = 9$ second, the collision does not happen, because the other vehicle leaves this lane after it passes the danger zone at $t = 4.4$ second. Also, there are two phases of the controlled vehicle: following the other vehicle and driving as the speed limit to ensure efficiency. From Figure 4.7(b), tracking error $e(t) = x(t) - x_d(t)$ of the system can converge to zero within 3.5 seconds in both two phases. According to Figure 4.7(c), the speed $\dot{x}(t)$ always converges to the desired speed $\dot{x}_d(t)$ within 3.5 seconds in both two phases. We also do not find the vehicle velocity $\dot{x}(t)$ exceeds the speed limit in this case. Figure 4.7(d) presents the control input signal $u(t) \in [-16000, 16000]$.

The system response of Case III is presented in Figure 4.8. In Figure 4.8(a), we can see the controlled vehicle velocity trajectory has two phases: it decelerates from $\dot{x}(0) = 6.7$ meters per second before it reaches $x = 0$, after the other vehicle passes the danger zone, it continues driving as the speed of 6.7 meters per second. The contact point of two vehicles' trajectories is before $x = 0$, and the two trajectory parts whose vertical coordinates belong to $x = [0, 10]$ are at different times. These facts mean two vehicles do not appear in the danger zone at the same time. Thus, the collision does not happen. According to Figure 4.8(b), for two different statuses, the tracking error $e(t) = x(t) - x_d(t)$ always can converge to zero within 3.5 seconds. In Figure 4.8(c), the controlled vehicle speed $\dot{x}(t)$ can converge to the desired speed $\dot{x}_o(t)$ in all two statuses. In each phase, the overshoot of speed does not appear. The control input signal $u(t) \in [-16000, 16000]$ is shown in Figure 4.8(d).

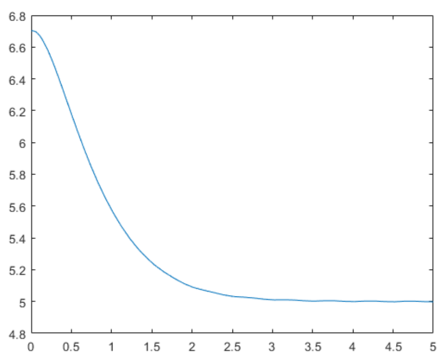
Figure 4.9 shows the system response of Case IV. In Figure 4.9(a), we can find the controlled vehicle drives as the speed limit $x_{limit} = 6.7$ meters per second all the time. The contact point of the two vehicles' trajectories is out of the zone between $x = 0$ and $x = 10$, and the two trajectory parts whose vertical coordinates belong to $x = [0, 10]$ are at different times. these facts prove the two vehicles do not collide in this case. The tracking error $e(t) = x(t) - x_d(t)$ in Figure 4.9(b) converges to zero within 3.5 seconds. According to Figure 4.9(c), there is no speed overshoot. Figure 4.9(d) shows the control input signal $u(t) \in [-16000, 16000]$.



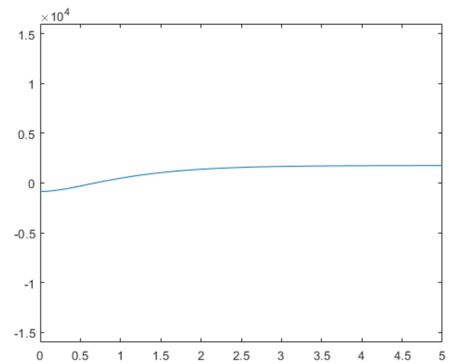
(a) Vehicle position $x(t)$(blue) and $x_o(t)$(red) in m



(b) Tracking error $e(t)$ in m

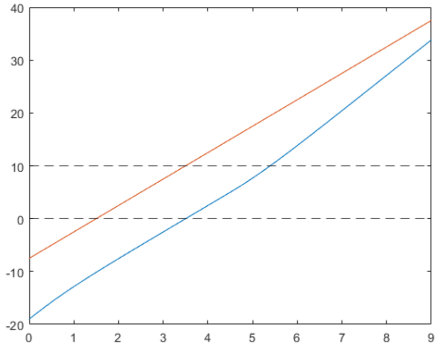


(c) Vehicle speed $\dot{x}(t)$ in m/s

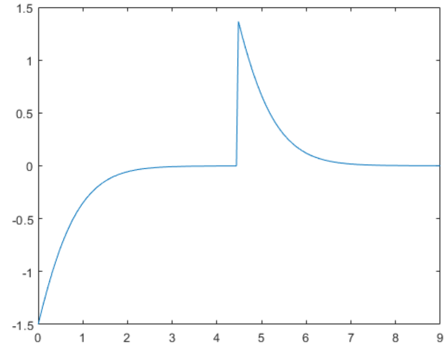


(d) Control input signal $u(t)$ in N

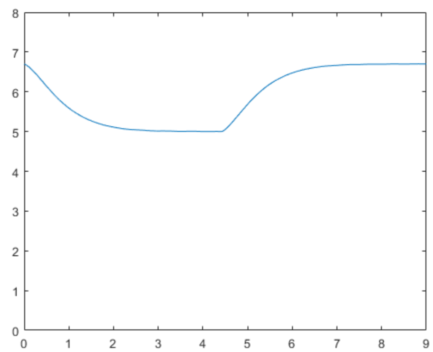
Figure 4.6: System response for the enhanced nominal control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}$ (Case I).



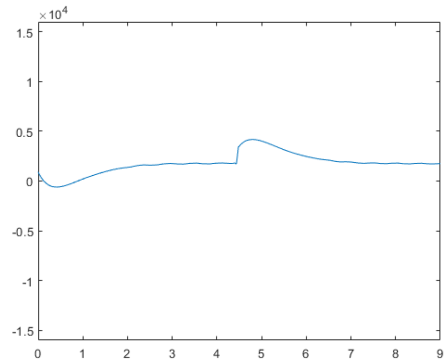
(a) Vehicle position $x(t)$(blue) and $x_o(t)$(red) in m



(b) Tracking error $e(t)$ in m

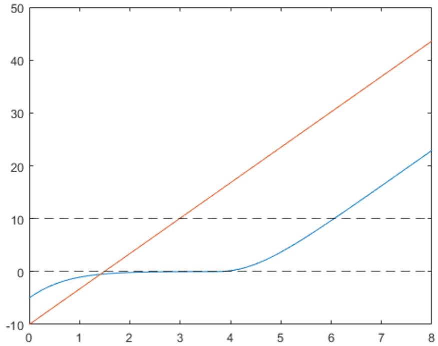


(c) Vehicle speed $\dot{x}(t)$ in m/s

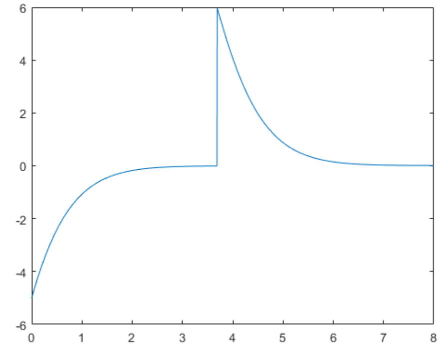


(d) Control input signal $u(t)$ in N

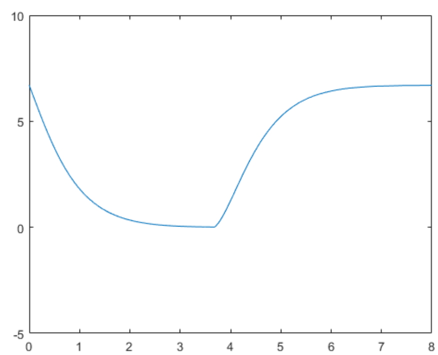
Figure 4.7: System response for the enhanced nominal control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}$ (Case II).



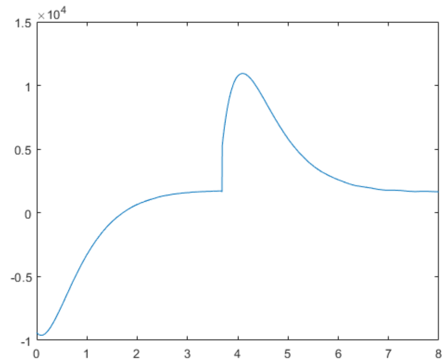
(a) Vehicle position $x(t)$(blue) and $x_o(t)$(red) in m



(b) Tracking error $e(t)$ in m

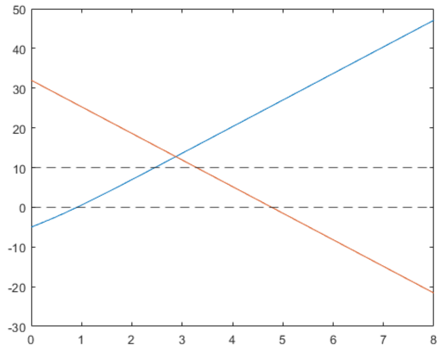


(c) Vehicle speed $\dot{x}(t)$ in m/s

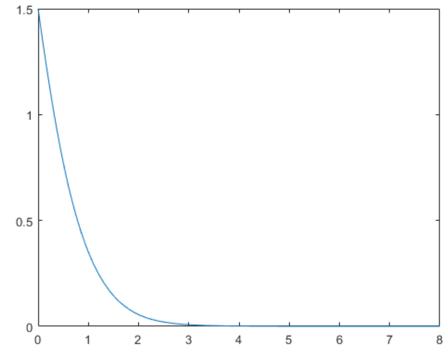


(d) Control input signal $u(t)$ in N

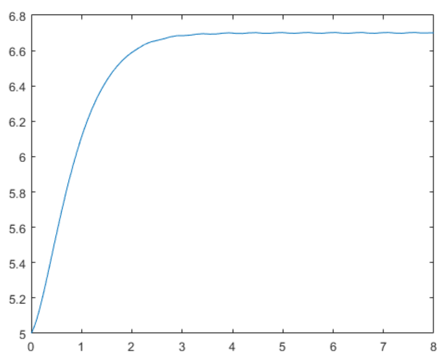
Figure 4.8: System response for the enhanced nominal control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}$ (Case III).



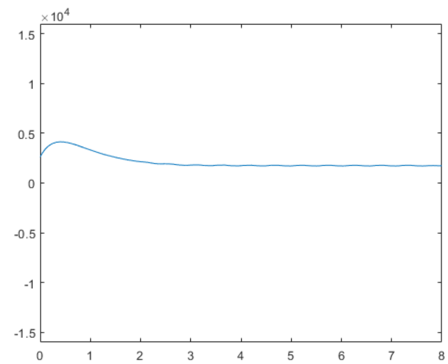
(a) Vehicle position $x(t)$(blue) and $x_o(t)$(red) in m



(b) Tracking error $e(t)$ in m



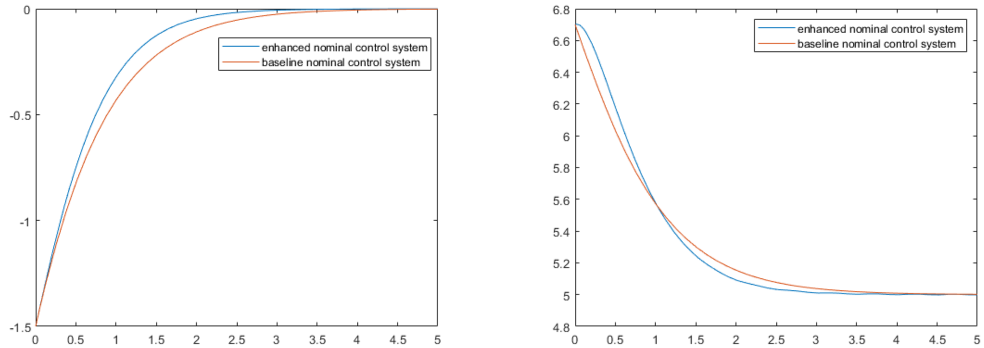
(c) Vehicle speed $\dot{x}(t)$ in m/s



(d) Control input signal $u(t)$ in N

Figure 4.9: System response for the enhanced nominal control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}$ (Case IV).

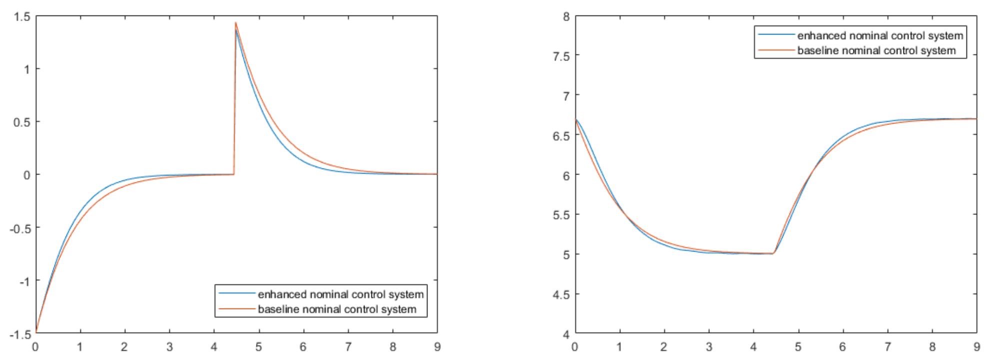
Comparsion of two designs. In this part, we will present the system performance comparison of the baseline control design and enhanced control design.



(a) The tracking error difference in m

(b) The vehicle speed difference in m/s

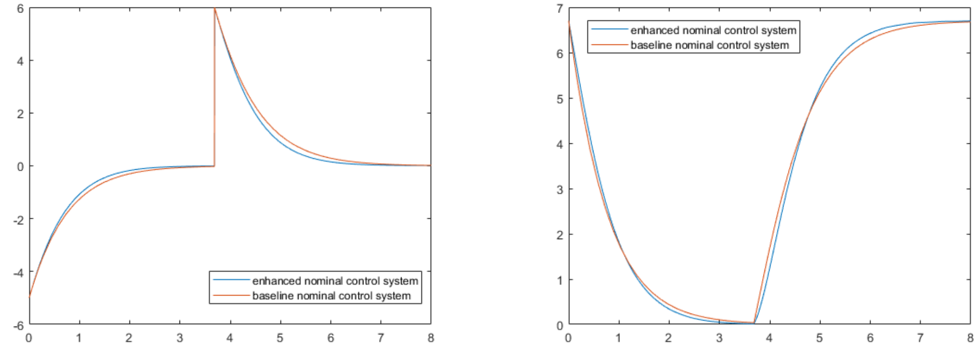
Figure 4.10: Comparison between the nominal baseline control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t)$ and the nominal enhanced control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}$ of Case I.



(a) The tracking error difference in m

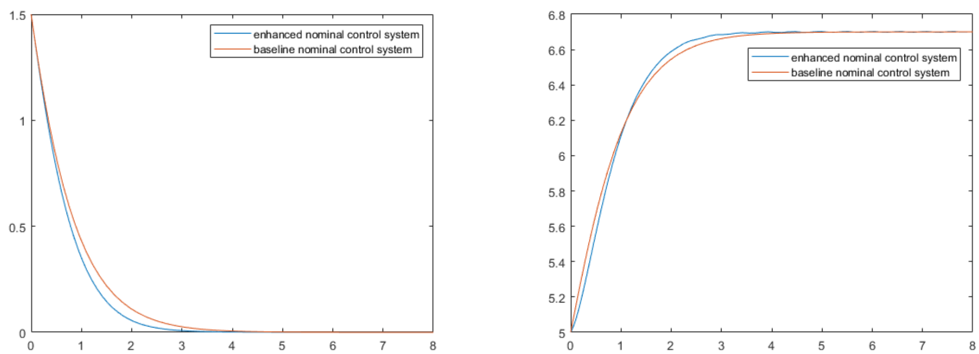
(b) The vehicle speed difference in m/s

Figure 4.11: Comparison between the nominal baseline control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t)$ and the nominal enhanced control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}$ of Case II.



(a) The tracking error difference in m (b) The vehicle speed difference in m/s

Figure 4.12: Comparison between the nominal baseline control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t)$ and the nominal enhanced control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}$ of Case III.



(a) The tracking error difference in m (b) The vehicle speed difference in m/s

Figure 4.13: Comparison between the nominal baseline control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t)$ and the nominal enhanced control system $u(t) = \theta^{*T}\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}$ of Case IV.

Summary. From the simulation results shown in Figures 4.6-4.9, we found that the proposed nominal control systems can solve the control problem of intersection vehicle control tasks effectively. In every case, the tracking errors converge less than 3.5 seconds, and the vehicle speed converges to the desired velocity within 3.5 seconds. In Figures 4.6-4.9(a), we can see that there are no contact points of these vehicles' trajectories in the danger zone. Also, in Figures 4.6-4.9(c), there is not any speed overshoot, which means the speed exceeding the speed limit will not happen. In Figures 4.6-4.9(d), all the control input signals $u(t)$ are in the reasonable range $[-16000, 16000]$.

According to Figures 4.10-4.13, we find that both the tracking error waveform and the vehicle speed waveform of the enhanced control system are better than the baseline design in all four cases. The tracking error and the vehicle speed can always converge to the desired value earlier in the enhanced control system, which brings more confidence to deal with the emergency happening in the intersection area.

In summary, the nominal enhanced control design has better system performance than the nominal baseline control design.

4.4.2 Adaptive Control System Evaluation

In this subsection, we consider using the adaptive parameter estimation method to deal with the system uncertainties.

Simulation conditions. We will apply the control input:

$$u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^\alpha - k_3 \text{sign}[z(t)]|z(t)|^\beta, \tag{4.30}$$

where $\lambda = 2$, $k_1 = 2700$, $k_2 = 500$, $k_3 = 900$, $\alpha = 0.5$ and $\beta = 1.05$. The adaptive law is

$$\dot{\theta}(t) = -\Gamma\omega(t)z(t), \tag{4.31}$$

where $\Gamma = 0.1I_3$. The initial value of parameter estimator is $\theta(0) = [0.098, 117.6, 1176]^T$. The vehicle simulation system is the same as in the last chapter.

Control input signal limitation. In this problem, we set the controlled vehicle can accelerate to 60 miles per hour from 0 in 3.1 seconds [49], and it can decelerate to 0 from 60 miles per hour within 133 feet [50]. When we substitute the vehicle mass $m = 1800$ kilograms to the uniform acceleration motion formula, the reasonable control input signal range should be about $[-16000, 16000]$ without considering driving resistance and keeping two significant figures.

System response. Figures 4.14-4.15 show the system response of Case I. From Figure 4.14(a), the distance between these two vehicles is always larger than 10 meters, which indicates two vehicles have not collided with each other. From Figure 4.14(b), the system tracking error $e(t) = x(t) - x_d(t)$ converge to zero within 3.5 seconds. From Figure 4.14(c), the controlled vehicle speed $\dot{x}(t)$ converge to $\dot{x}_o(t)$ within 3.5 second, and the highest speed in this process is less than the speed limit $v_{limit} = 6.7$ meters per seconds. Figure 4.14(d) presents the control input $u(t) \in [-16000, 16000]$. Through Figure 4.15, the parameter errors $\theta_1(t) - \theta_1^*$ and $\theta_2(t) - \theta_2^*$ are not bounded, but $\theta_3(t) - \theta_3^*$ is bounded.

Figures 4.16-4.17 show the system response of Case II. From Figure 4.16(a), the controlled vehicle follows the other vehicle at first and accelerates to the speed limit v_{limit} once the other vehicle leaves the lane. When $t \leq 4.4$ seconds, the distance between two vehicles is always larger than 10 meters, which means the collision does not happen when two vehicles driving direction is the same. Although there may be a contact point between these two vehicles' velocity trajectories after $t = 9$ second, the collision does not happen, because the other vehicle leaves this lane after it passes the danger zone at $t = 4.4$ second. Also, there are two phases of the controlled vehicle: following the other vehicle and driving as the speed limit to ensure efficiency. From Figure 4.16(b), tracking error $e(t) = x(t) - x_d(t)$ of the system can converge to zero within 3.5 seconds in both two phases. According to Figure 4.16(c), the speed $\dot{x}(t)$

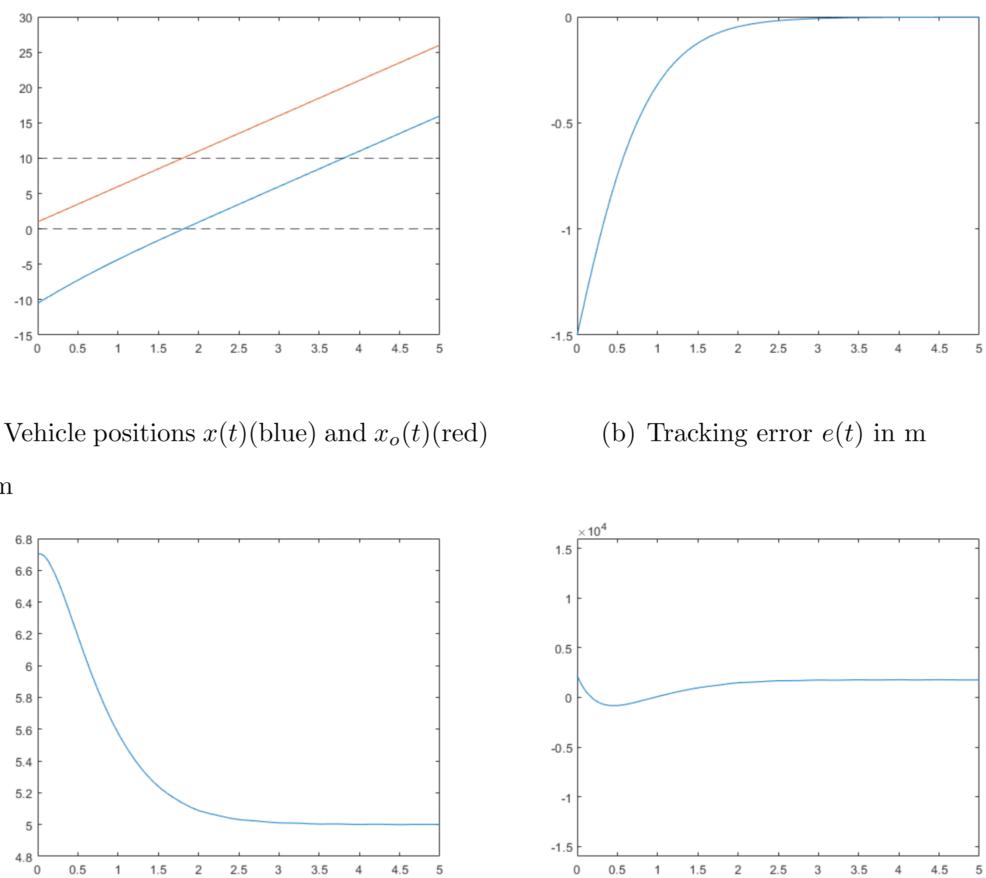
always converges to the desired speed $\dot{x}_d(t)$ within 3.5 seconds in both two phases. We also do not find the vehicle velocity $\dot{x}(t)$ exceeds the speed limit in this case. Figure 4.16(d) presents the control input signal $u(t) \in [-16000, 16000]$. According to Figure 4.17, the parameter errors $\theta_1(t) - \theta_1^*$ and $\theta_2(t) - \theta_2^*$ are not bounded in both two phases, but $\theta_3(t) - \theta_3^*$ is bounded in both two phases.

The system response of Case III is presented in Figures 4.18-4.19. In Figure 4.18(a), we can see the controlled vehicle velocity trajectory has two phases: it decelerates from $\dot{x}(0) = 6.7$ meters per second before it reaches $x = 0$, after the other vehicle passes the danger zone, it continues driving as the speed of 6.7 meters per second. The contact point of two vehicles' trajectories is before $x = 0$, and the two trajectory parts whose vertical coordinates belong to $x = [0, 10]$ are at different times. This means two vehicles do not appear in the danger zone at the same time. Thus, the collision does not happen. According to Figure 4.18(b), for two different statuses, the tracking error $e(t) = x(t) - x_d(t)$ always can converge to zero within 3.5 seconds. In Figure 4.18(c), the controlled vehicle speed $\dot{x}(t)$ can converge to the desired speed $\dot{x}_o(t)$ in all two statuses. In each phase, the overshoot of speed does not appear. The control input signal $u(t) \in [-16000, 16000]$ is shown in Figure 4.18(d). From Figure 4.19, the parameter errors $\theta_1(t) - \theta_1^*$ and $\theta_2(t) - \theta_2^*$ are not bounded in both two phases, but $\theta_3(t) - \theta_3^*$ is bounded in both two phases.

Figures 4.20-4.21 show the system response of Case IV. In Figure 4.20(a), we can find the controlled vehicle drives as the speed limit $x_{limit} = 6.7$ meters per second all the time. The contact point of the two vehicles' trajectories is out of the zone between $x = 0$ and $x = 10$, and the two trajectory parts whose vertical coordinates belong to $x = [0, 10]$ are at different times. This proves the two vehicles do not collide in this case. The tracking error $e(t) = x(t) - x_d(t)$ in Figure 4.20(b) converges to zero within 3.5 seconds. According to Figure 4.20(c), there is no speed overshoot. Figure

4.20(d) shows the control input signal $u(t) \in [-16000, 16000]$. According to Figure 4.21, the parameter errors $\theta_1(t) - \theta_1^*$ and $\theta_2(t) - \theta_2^*$ are not bounded, but $\theta_3(t) - \theta_3^*$ is bounded.

Figures 4.22-4.25 present the system tracking error $e(t) = x(t) - x_d(t)$ difference and the vehicle speed difference between the baseline nominal control system and the baseline adaptive control system in four cases. We found in all the cases, the tracking error $e(t) = x(t) - x_d(t)$ waveform of these two control systems are almost overlapped.



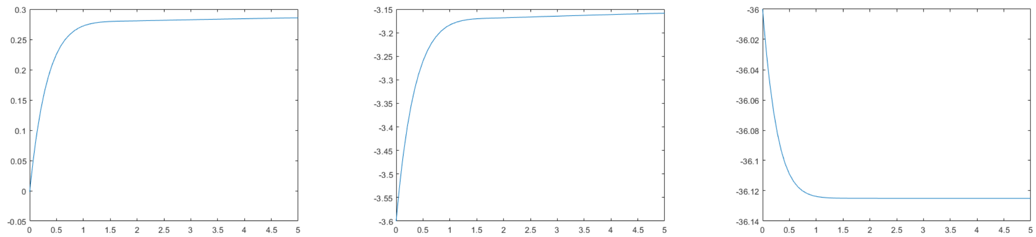
(a) Vehicle positions $x(t)$(blue) and $x_o(t)$(red) in m

(b) Tracking error $e(t)$ in m

(c) Vehicle speed $\dot{x}(t)$ in m/s

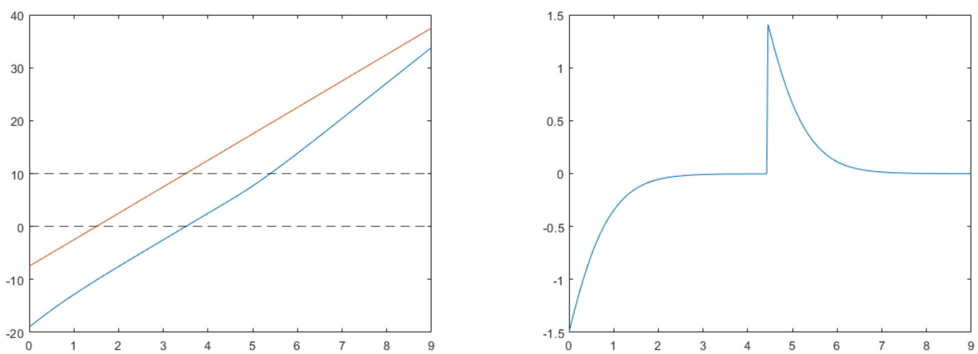
(d) Control input $u(t)$ in N

Figure 4.14: System response for the enhanced adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}$ (Case I).

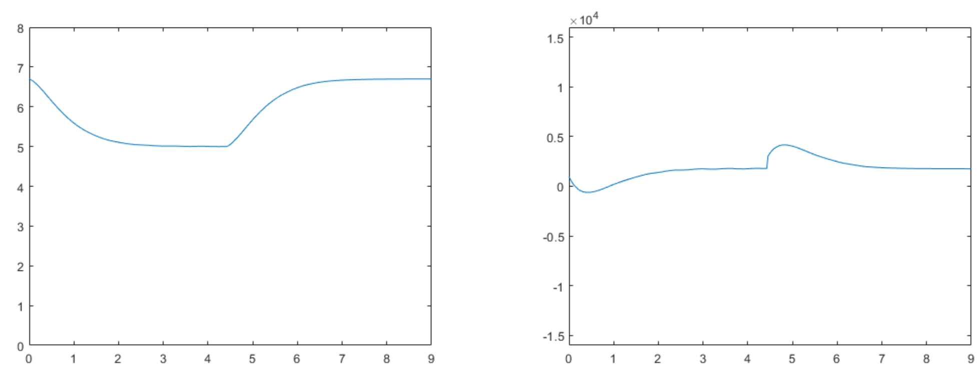


(a) Parameter error $\theta_1(t) - \theta_1^*$ in kg/s (b) Parameter error $\theta_2(t) - \theta_2^*$ in kg (c) Parameter error $\theta_3(t) - \theta_3^*$ in kg

Figure 4.15: Parameters errors of $\theta(t)$ for the enhanced adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^\alpha - k_3 \text{sign}[z(t)]|z(t)|^\beta$ (Case I).

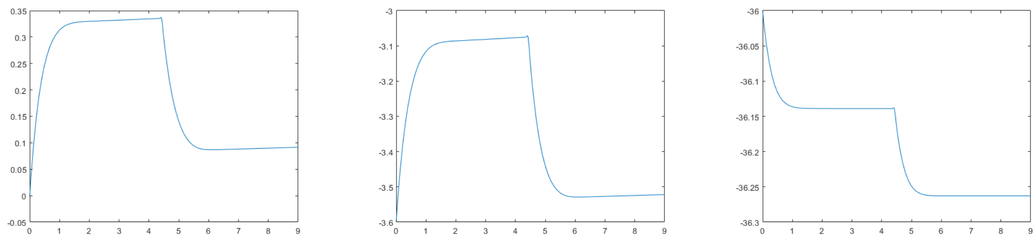


(a) Vehicle positions $x(t)$(blue) and $x_o(t)$(red) in m (b) Tracking error $e(t)$ in m



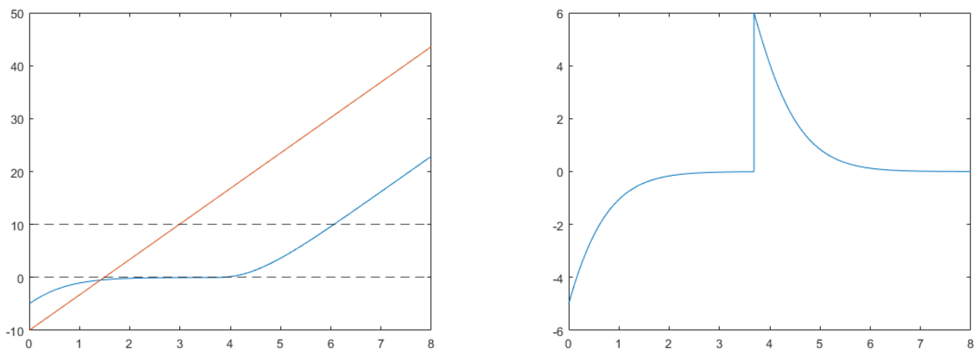
(c) Vehicle speed $\dot{x}(t)$ in m/s (d) Control input $u(t)$ in N

Figure 4.16: System response for the enhanced adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^\alpha - k_3 \text{sign}[z(t)]|z(t)|^\beta$ (Case II).

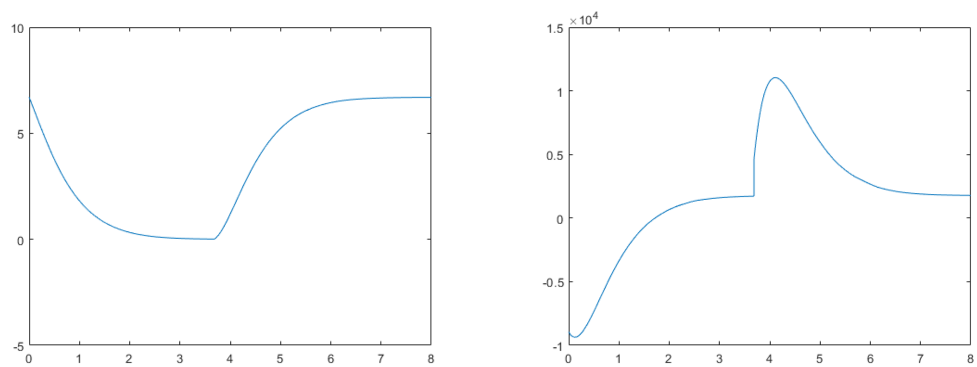


(a) Parameter error $\theta_1(t) - \theta_1^*$ in kg/s (b) Parameter error $\theta_2(t) - \theta_2^*$ in kg (c) Parameter error $\theta_3(t) - \theta_3^*$ in kg

Figure 4.17: Parameters errors of $\theta(t)$ for the enhanced adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^\alpha - k_3 \text{sign}[z(t)]|z(t)|^\beta$ (Case II).

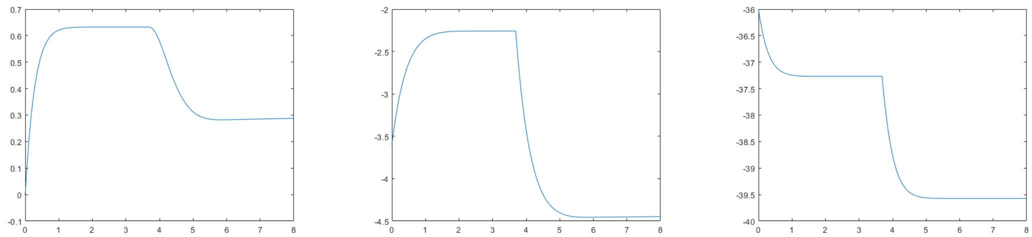


(a) Vehicle positions $x(t)$(blue) and $x_o(t)$(red) in m (b) Tracking error $e(t)$ in m



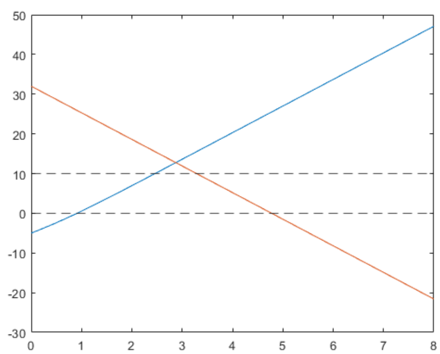
(c) Vehicle speed $\dot{x}(t)$ in m/s (d) Control input $u(t)$ in N

Figure 4.18: System response for the enhanced adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^\alpha - k_3 \text{sign}[z(t)]|z(t)|^\beta$ (Case III).

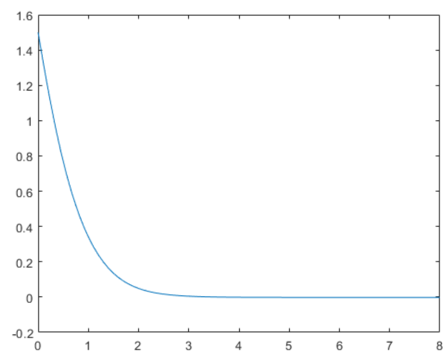


(a) Parameter error $\theta_1(t) - \theta_1^*$ in kg/s (b) Parameter error $\theta_2(t) - \theta_2^*$ in kg (c) Parameter error $\theta_3(t)_-\theta_3^*$ in kg

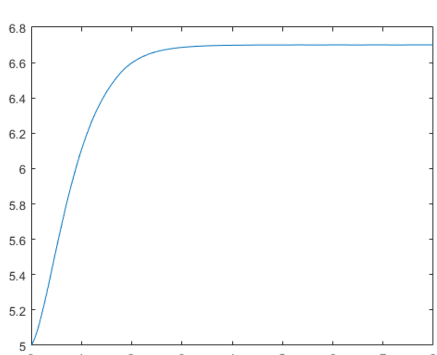
Figure 4.19: Parameters errors of $\theta(t)$ for the enhanced adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2\text{sign}[z(t)]|z(t)|^\alpha - k_3\text{sign}[z(t)]|z(t)|^\beta$ (Case III).



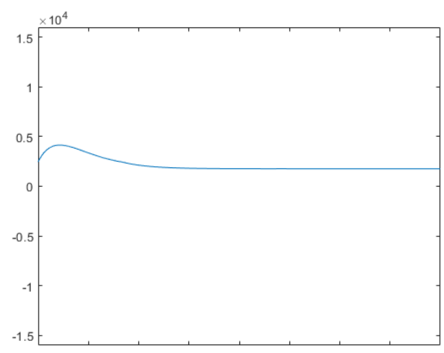
(a) Vehicle positions $x(t)$(blue) and $x_o(t)$(red) in m



(b) Tracking error $e(t)$ in m

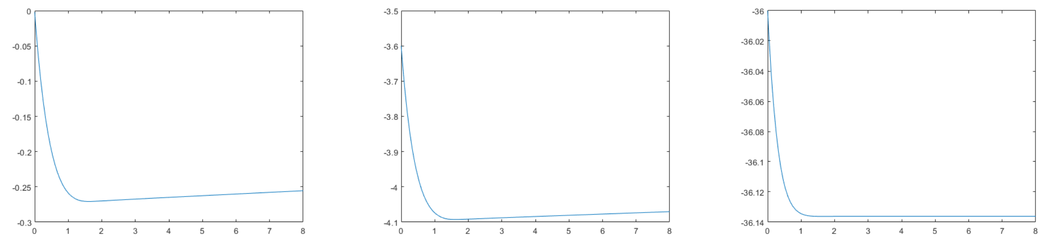


(c) Vehicle speed $\dot{x}(t)$ in m/s



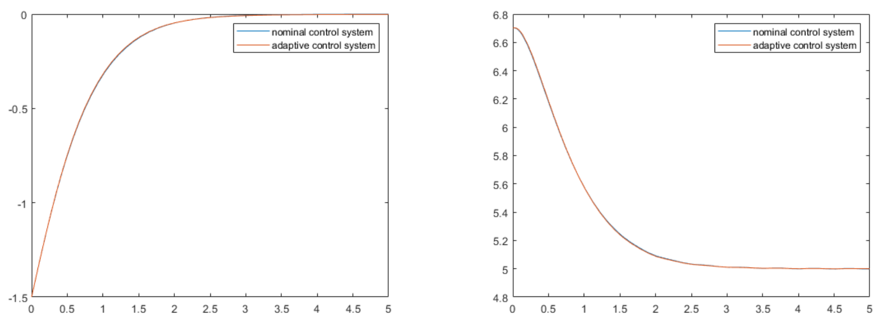
(d) Control input $u(t)$ in N

Figure 4.20: System response for the enhanced adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2\text{sign}[z(t)]|z(t)|^\alpha - k_3\text{sign}[z(t)]|z(t)|^\beta$ (Case IV).



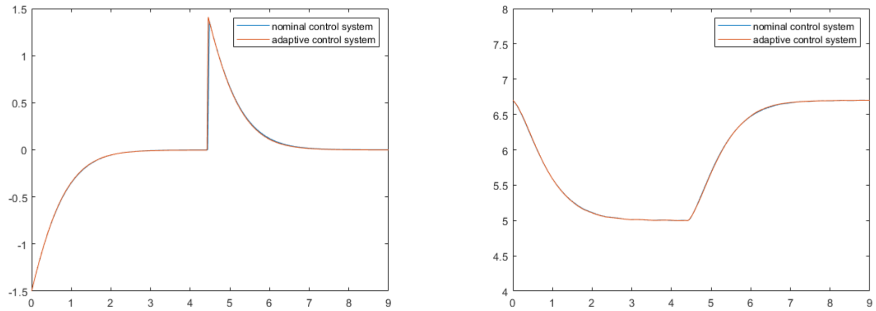
(a) Parameter error $\theta_1(t) - \theta_1^*$ in kg/s (b) Parameter error $\theta_2(t) - \theta_2^*$ in kg (c) Parameter error $\theta_3(t)_{-}\theta_3^*$ in kg

Figure 4.21: Parameters errors of $\theta(t)$ for the enhanced adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2\text{sign}[z(t)]|z(t)|^\alpha - k_3\text{sign}[z(t)]|z(t)|^\beta$ (Case IV).



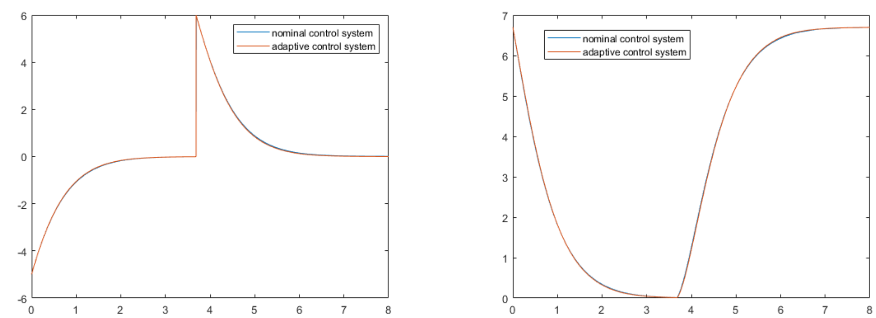
(a) The tracking error difference in m (b) The vehicle speed difference in m/s

Figure 4.22: Comparison between the nominal control $u(t) = \theta^{*T}\omega(t) - k_1 z(t) - k_2\text{sign}[z(t)]|z(t)|^\alpha - k_3\text{sign}[z(t)]|z(t)|^\beta$ and the adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2\text{sign}[z(t)]|z(t)|^\alpha - k_3\text{sign}[z(t)]|z(t)|^\beta$ of Case I.



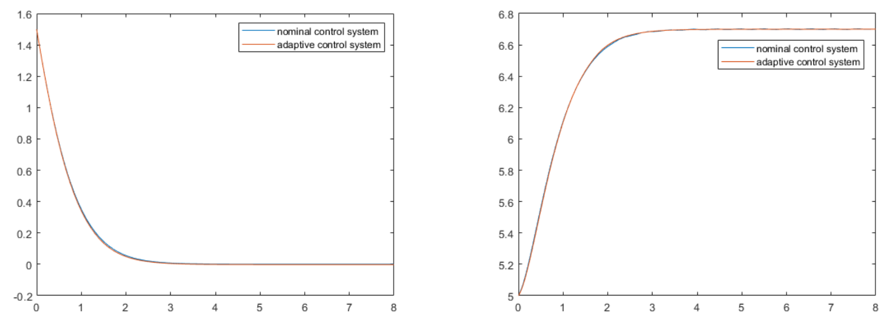
(a) The tracking error difference in m (b) The vehicle speed difference in m/s

Figure 4.23: Comparison between the nominal control $u(t) = \theta^{*T}\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}$ and the adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}$ of Case II.



(a) The tracking error difference in m (b) The vehicle speed difference in m/s

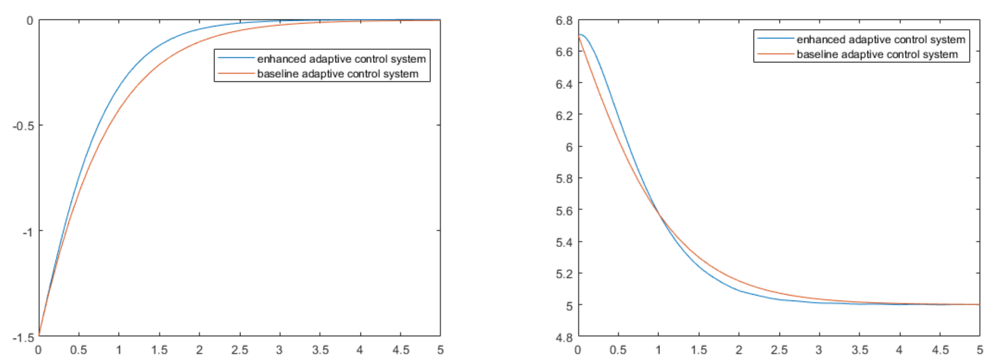
Figure 4.24: Comparison between the nominal control $u(t) = \theta^{*T}\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}$ and the adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2 \text{sign}[z(t)]|z(t)|^{\alpha} - k_3 \text{sign}[z(t)]|z(t)|^{\beta}$ of Case III.



(a) The tracking error difference in m (b) The vehicle speed difference in m/s

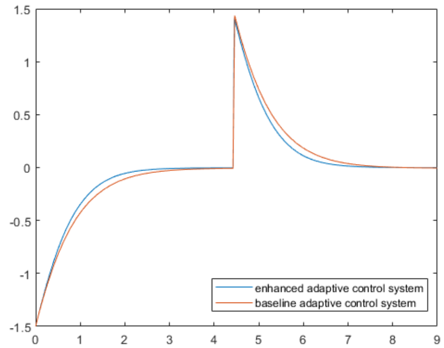
Figure 4.25: Comparison between the nominal control $u(t) = \theta^{*T}\omega(t) - k_1 z(t) - k_2\text{sign}[z(t)]|z(t)|^{\alpha} - k_3\text{sign}[z(t)]|z(t)|^{\beta}$ and the adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2\text{sign}[z(t)]|z(t)|^{\alpha} - k_3\text{sign}[z(t)]|z(t)|^{\beta}$ of Case IV.

Comparison of two designs. The comparisons between the enhanced adaptive control system and the baseline adaptive control system in four cases are presented in Figures 4.25-4.29.

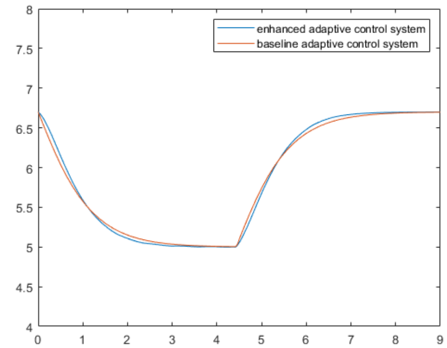


(a) The tracking error difference in m (b) The vehicle speed difference in m/s

Figure 4.26: Comparison between the baseline adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t)$ and the enhanced adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2\text{sign}[z(t)]|z(t)|^{\alpha} - k_3\text{sign}[z(t)]|z(t)|^{\beta}$ of Case I.

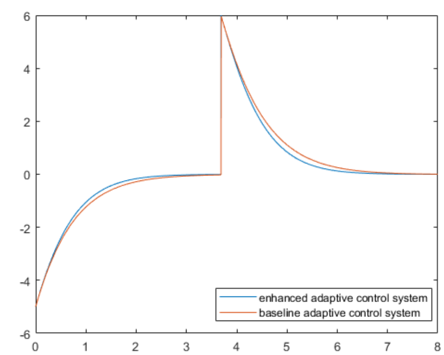


(a) The tracking error difference in m

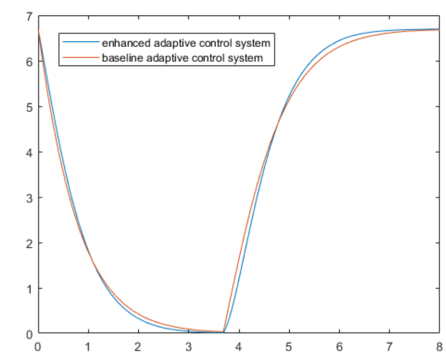


(b) The vehicle speed difference in m/s

Figure 4.27: Comparison between the baseline adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t)$ and the enhanced adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2\text{sign}[z(t)]|z(t)|^\alpha - k_3\text{sign}[z(t)]|z(t)|^\beta$ of Case II.

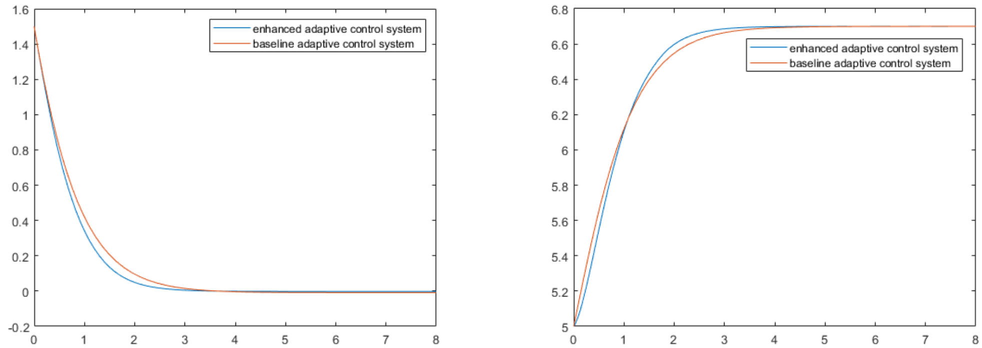


(a) The tracking error difference in m



(b) The vehicle speed difference in m/s

Figure 4.28: Comparison between the baseline adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t)$ and the enhanced adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2\text{sign}[z(t)]|z(t)|^\alpha - k_3\text{sign}[z(t)]|z(t)|^\beta$ of Case III.



(a) The tracking error difference in m (b) The vehicle speed difference in m/s

Figure 4.29: Comparison between the baseline adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t)$ and the enhanced adaptive control system $u(t) = \theta^T(t)\omega(t) - k_1 z(t) - k_2\text{sign}[z(t)]|z(t)|^\alpha - k_3\text{sign}[z(t)]|z(t)|^\beta$ of Case IV.

Summary. From the simulation results shown in Figures 4.22 - 4.25, we found that the proposed enhanced adaptive control system performances are the same as the enhanced nominal control system performances, which indicates the enhanced adaptive control system can solve intersection vehicle control problems with the system parameter uncertainties. In every case, the tracking error $e(t) = x(t) - x_d(t)$ can converge to zero in less than 3.5 seconds, and the speed $\dot{x}$ can reach the desired speed within 3.5 seconds, which means the actual trajectory can track the desired trajectory within 3.5 seconds. In Figures 4.14(a), 4.16(a), 4.18(a), and 4.20(a), collisions do not happen. In the simulation, we applied the initial parameter initial value is 0.98 times of the real value for example. When the initial parameter error changes, the system response will also be changed. According to the response, all system responses prove that the adaptive controller can achieve the control objective for the intersection AVC problem without using the knowledge of the system parameters.

According to Figures 4.26-4.29, we find that both the system response results in the enhanced control system are better than in the baseline design in all four cases.

The tracking error $e(t) = x(t) - x_d(t)$ and the vehicle speed $\dot{x}(t)$ can always converge to the desired value earlier in the enhanced control system. Thus, the enhanced design can also improve the system performance in the adaptive control system.

In sum, both the nominal and adaptive control systems can satisfy the control objective. With this enhanced adaptive control design, the vehicle can reach the desired velocity trajectory in a shorter time with less tracking error $e(t) = x(t) - x_d(t)$.

Chapter 5

Conclusions and Future Work

5.1 Conclusions

The two adaptive control systems proposed in this thesis are verified their ability to achieve our AVC in intersections control objective to make the autonomous vehicle pass the intersection as fast as possible without collisions. The work shown in this thesis is a complete study of a general control methodology in such a specific situation, which contains desired speed trajectory, adaptive laws, control input signals, and stability analysis for both of those two control design schemes.

Chapter 2 briefly summarized past research about vehicle control and intersection vehicle control. The limitation of those research indicates that our study is necessary for the AVC problem. Also, the imperative vehicle system parameterization study is presented.

The baseline control algorithm, including desired speed trajectory generation, adaptive laws, and control system stability verification, is stated in Chapter 3. In Chapter 4, we modified the control input signal of the system structure proposed in Chapter 3 to improve the system performance. The stability of the enhanced control system is also presented in Chapter 4.

The desired trajectory generation logistic is that our system will eradicate the possibility that two vehicles appear in the danger zone simultaneously. Then, our nominal or adaptive controller finishes the desired trajectory tracking task with or without using the system parameter knowledge.

Simulation studies on a vehicle of four different possible situations are discussed in Chapter 3 and Chapter 4. The results can prove the system properties from theoretical deduce. Also, through the comparison of the performances of the nominal control system and adaptive control system, we find that the adaptive control design can achieve our control objective without system parameter knowledge because the responses of the adaptive control system and nominal control system are almost the same. The simulation results in Chapter 4 demonstrate the distinct performance improvement by the proposed enhanced adaptive design algorithm.

Consequently, the combination of our desired trajectory generation and adaptive trajectory tracking control system can solve the AVC problem stated in Chapter 2. The controlled scheme can make the controlled vehicle pass a signal-free intersection as fast as possible without any collision.

According to the simulation results, the proposed automated driving system adjusts the controlled vehicle's action to avoid possible collisions, which indicates at most only one vehicle needs to stop at the intersection area to avoid collisions, and the other vehicle can always drive at its speed. However, in human driving situations, both vehicles need to slow down or even stop at this intersection to avoid collisions because the drivers may not know the positions of other vehicles. Thus, our automated system can improve traffic efficiency. Nonetheless, to eliminate the possibility of collision, we always assume the other vehicle driving as the speed limit to judge whether a collision may happen. This restriction is too strict because the time that the other vehicle reaching the danger zone does not merely depend on its distance from the danger zone entrance but also its speed. The controlled vehicle could have enough time to pass the intersection before the other vehicle if the other vehicle's speed is slow enough. However, if the other vehicle drives at a slow enough speed initially, our system lets the controlled vehicle enter the danger zone. While the controlled vehicle is passing the danger zone, the other vehicle may accelerate to the speed limit suddenly, and if the other vehicle's initial position is close enough to the danger zone, the other vehicle may enter the danger zone before the controlled vehicle clears from the danger zone to cause a collision. Therefore, we have to use such strict restrictions to ensure driving safety. If we can find an accurate scheme to estimate the other vehicle's velocity trajectory, we can release this restriction.

5.2 Future Research Topics

In this thesis, we proposed two control systems to achieve autonomous vehicle tracking control and collision avoidance with adaptive control algorithms for intersection areas. However, some parts of this research are not mature enough, and several challenges still exist which are required to solve. We will introduce several extension research in this research area.

More complex situations consideration. This research only takes into account only two vehicles in such an intersection, and there is only one lane in each direction. To meet the AVC problem's requirements of improving road efficiency and safety, we also need to consider lane change situations to increase the efficiency of road use. Moreover, there are more than two vehicles in several busy intersections. To make this study more practical, we will study more complicated situations in the future.

The comfort of passengers. Although the control system can finish the tracking task in a short time, the comfort of passengers is also necessary to consider for the AVC problems. We also need to control the brake and acceleration torque to ensure passenger comfort.

System disturbance rejection. The desired speed trajectory generation proposed in this thesis needs precise position information, which is almost impossible in real engineering applications. Therefore, it is imperative to create a new desired trajectory generation scheme with the inaccuracy of position information.

More efficient desired velocity trajectory. The desired velocity trajectory generation plays a very important role in this AVC problem. However, in this research, to ensure safety, we have to stipulate only one vehicle can enter the intersection when two vehicles are not in the same lane initially. At this time, machine learning techniques are wildly used in many fields, if we can apply machine learning techniques to design an algorithm with a one hundred percent accuracy rate to estimate and predict other vehicles‘ actions in this intersection–go straight, turn right or turn left. Then we can find the exact potential collision area through calculation. In that case, our desired velocity trajectory only needs to ensure the vehicles do not appear in such exact potential collision area at the same time rather the whole intersection area. Without a doubt, this idea can lead to a much higher traffic efficiency.

More real vehicle movement model. As we all know, in the real world, the vehicle movement process is much more complex than the equation (2.33). The vehicle movement is restricted by many parameters, including but not limited to the maximum engine torque and engine power. We also need to consider formulating a more realistic vehicle movement model with such limitations.

Bibliography

[1] W. Li, T. Chen, J. Guo and J. Wang, "Adaptive Car-Following Control of Intelligent Electric Vehicles," *2018 IEEE 4th International Conference on Control Science and Systems Engineering (ICCSSE)*, Wuhan, China, 2018, pp. 86-89

[2] Eski, Ikbal and S. Yildirim. "Neural network-based fuzzy inference system for speed control of heavy duty vehicles with electronic throttle control system." *Neural Computing and Applications* 28 (2016): 907-916.

[3] X.Wu, G.Qin, H.Yu, S.Gao, L.Liu, and Y.Xue, "Using improved chaotic ant swarm to tune PID controller on cooperative adaptive cruise control", Optik, 127, 3445-3450.(2016)

[4] Y. Luo, T. Chen, S. Zhang and K. Li, "Intelligent Hybrid Electric Vehicle ACC With Coordinated Control of Tracking Ability, Fuel Economy, and Ride Comfort," in *IEEE Transactions on Intelligent Transportation Systems, vol. 16, no. 4*, pp. 2303-2308, Aug. 2015

[5] P. Wang, T. Shi, C. Zou, L. Xin and C. Chan, "A Data Driven Method of Feedforward Compensator Optimization for Autonomous Vehicle Control," *2019 IEEE Intelligent Vehicles Symposium (IV)*, Paris, France, 2019, pp. 2012-2017

[6] L. Jiang, W. Ai and J. Li, "Intelligent Vehicle's Control for Tracking Target Path," *2019 IEEE 4th Advanced Information Technology, Electronic and Automation Control Conference (IAEAC)*, Chengdu, China, 2019, pp. 196-200

[7] H. Yang and K. Oguchi, "Intelligent vehicle control at signal-free intersection under mixed connected environment," in *IET Intelligent Transport Systems, vol. 14*, no. 2, pp. 82-90, 2 2020

[8] X. Zhao, J. Wang, G. Yin and K. Zhang, "Cooperative Driving for Connected and Automated Vehicles at Non-signalized Intersection based on Model Predictive Control," *2019 IEEE Intelligent Transportation Systems Conference (ITSC)*, Auckland, New Zealand, 2019, pp. 2121-2126

[9] M. Stork, J. Pinker and P. Weissar, "Adaptive Control System for Autonomous Vehicle Path Following," *2019 International Conference on Applied Electronics (AE)*, Pilsen, Czech Republic, 2019, pp. 1-4

[10] R. Fuentes-Alvarez, I. Chairez, K. Adams, S. Salazar and R. Lopez, "Output Adaptive Control of a Skid Steering Autonomous Vehicle," *2019 16th International Conference on Electrical Engineering, Computing Science and Automatic Control (CCE)*, Mexico City, Mexico, 2019, pp. 1-6

[11] X. Jin, W. M. Haddad, Z. Jiang and K. G. Vamvoudakis, "Adaptive Control for Mitigating Sensor and Actuator Attacks in Connected Autonomous Vehicle Platoons," *2018 IEEE Conference on Decision and Control (CDC)*, Miami Beach, FL, 2018, pp. 2810-2815

[12] R. Qi, G. Tao, B. Jiang ,*Adaptive Control: A Tutorial Introduction. In: Fuzzy System Identification and Adaptive Control*. Communications and Control Engineering. Springer, 2019

[13] G. Tao, *Adaptive Control Design and Analysis*, John Wiley and Sons Inc., New York, NY, 2003.

[14] V. A. Cherkasova and O. I. Brikova, "Development and Modeling of an Automatic Drive Micropositioning Control System for Laser Equipment Positioning Tasks," *2019 III International Conference on Control in Technical Systems (CTS)*, St. Petersburg, Russia, 2019, pp. 118-120.

[15] K.Kim, J. Im, M. Heo and G.Jee, Precise Vehicle Position and Heading Estimation Using a Binary Road Marking Map, *Journal of Sensors*, 2019, pp. 1-18.

[16] J. Siegel, D. Erb and S. Sarma, "Algorithms and Architectures: A Case Study in When, Where and How to Connect Vehicles," in *IEEE Intelligent Transportation Systems Magazine*, vol. 10, no. 1, pp. 74-87, Spring 2018.

[17] E. Uhlemann, "Time for Autonomous Vehicles to Connect [Connected Vehicles]," in *IEEE Vehicular Technology Magazine*, vol. 13, no. 3, pp. 10-13, Sept. 2018.

[18] F.Lin, K.Wang, Y.Zhao, and S.Wang, "Integrated Avoid Collision Control of Autonomous Vehicle Based on Trajectory Re-Planning and V2V Information Interaction", *Sensors (Basel, Switzerland)*, 20(4), 1079, 2020

[19] B. Peterson. "Car Ownership Statistics". https://www.valuepenguin.com/auto-insurance/car-ownership-statistics#national (accessed Mar. 31, 2021).

[20] National Highway Traffic Safety Administration. "Automated Vehicles for Safety". https://www.nhtsa.gov/technology-innovation/automated-vehicles-safety (accessed Mar. 31, 2021).

[21] N. Lu, N. Cheng, N. Zhang, X. Shen and J. W. Mark, ”Connected Vehicles: Solutions and Challenges,” in *IEEE Internet of Things Journal*, vol. 1, no. 4, pp. 289-299, Aug. 2014, doi: 10.1109/JIOT.2014.2327587.

[22] A. Gawanmeh, M. Alwadi and A. Ghawanmeh, ”Validating connected vehicles architecture,” *2014 International Conference on Connected Vehicles and Expo (ICCVE)*, Vienna, 2014, pp. 945-946, doi: 10.1109/ICCVE.2014.7297697.

[23] C. Prehofer, K. Schorp, S. Kugele, D. Clarke and M. Duchon, ”Towards a 3-tier architecture for connected vehicles,” *2014 International Conference on Connected Vehicles and Expo (ICCVE)*, Vienna, 2014, pp. 679-680, doi: 10.1109/ICCVE.2014.7297634.

[24] ”About connected vehicles.” its.dot.gov. https://its.dot.gov/research_archives/connected_vehicle/connected_vehicle.htm(accessed Mar. 20, 2021)

[25] M. A. S. Kamal, J. Imura, T. Hayakawa, A. Ohata and K. Aihara, ”A Vehicle-Intersection Coordination Scheme for Smooth Flows of Traffic Without Using Traffic Lights,” in *IEEE Transactions on Intelligent Transportation Systems*, vol. 16, no. 3, pp. 1136-1147, June 2015, doi: 10.1109/TITS.2014.2354380.

[26] K. Kim, ”Collision free autonomous ground traffic: A model predictive control approach,” *2013 ACM/IEEE International Conference on Cyber-Physical Systems (ICCPS)*, Philadelphia, PA, 2013, pp. 51-60.

[27] Y. Guan, Y. Ren, S. E. Li, Q. Sun, L. Luo and K. Li, ”Centralized Cooperation for Connected and Automated Vehicles at Intersections by Proximal Policy Optimization,” in *IEEE Transactions on Vehicular Technology*, vol. 69, no. 11, pp. 12597-12608, Nov. 2020, doi: 10.1109/TVT.2020.3026111.

[28] B. Yang and C. Monterola, "A simple distributed algorithm for lightless intersection control based on non-linear interactions between vehicles," *2017 IEEE 20th International Conference on Intelligent Transportation Systems (ITSC)*, Yokohama, 2017, pp. 1-6, doi: 10.1109/ITSC.2017.8317587.

[29] Y. Bian, S. E. Li, W. Ren, J. Wang, K. Li and H. X. Liu, "Cooperation of Multiple Connected Vehicles at Unsignalized Intersections: Distributed Observation, Optimization, and Control," in *IEEE Transactions on Industrial Electronics*, vol. 67, no. 12, pp. 10744-10754, Dec. 2020, doi: 10.1109/TIE.2019.2960757.

[30] Y. Zhang, L. Liu, Z. Lu, L. Wang and X. Wen, "Robust Autonomous Intersection Control Approach for Connected Autonomous Vehicles," in *IEEE Access*, vol. 8, pp. 124486-124502, 2020, doi: 10.1109/ACCESS.2020.3002825.

[31] T. Yu, S. Cui and Z. Wang, "Pure Electric Intelligent Connected Vehicle Driving Strategy at Intersection Based on Energy Analysis," *2019 4th International Conference on Electromechanical Control Technology and Transportation (ICECTT)*, Guilin, China, 2019, pp. 81-84, doi: 10.1109/ICECTT.2019.00025.

[32] X. Chen, B. Xu, X. Qin, Y. Bian, M. Hu and N. Sun, "Non-Signalized Intersection Network Management With Connected and Automated Vehicles," in *IEEE Access*, vol. 8, pp. 122065-122077, 2020, doi: 10.1109/ACCESS.2020.3007226.

[33] L. Chai, B. Cai, W. ShangGuan and J. Wang, "Connected and autonomous vehicles coordinating method at intersection utilizing preassigned slots," *2017 IEEE 20th International Conference on Intelligent Transportation Systems (ITSC)*, Yokohama, 2017, pp. 1-6, doi: 10.1109/ITSC.2017.8317934.

[34] S. Aoki and R. Rajkumar, "V2V-based Synchronous Intersection Protocols for Mixed Traffic of Human-Driven and Self-Driving Vehicles," *2019 IEEE*

25th International Conference on Embedded and Real-Time Computing Systems and Applications (RTCSA), Hangzhou, China, 2019, pp. 1-11, doi: 10.1109/RTCSA.2019.8864572.

[35] Guojiang Shen, Tingfang Ma and Youxian Sun, "Application of fuzzy control theory in multi-phase traffic control of single intersection," *Proceedings of the 4th World Congress on Intelligent Control and Automation (Cat. No.02EX527)*, Shanghai, China, 2002, pp. 1017-1022 vol.2, doi: 10.1109/WCICA.2002.1020730.

[36] C. Wuthishuwong and A. Traechtler, "Coordination of multiple autonomous intersections by using local neighborhood information," *2013 International Conference on Connected Vehicles and Expo (ICCVE)*, Las Vegas, NV, 2013, pp. 48-53, doi: 10.1109/ICCVE.2013.6799768.

[37] J. Lee and B. Park, "Development and Evaluation of a Cooperative Vehicle Intersection Control Algorithm Under the Connected Vehicles Environment," in *IEEE Transactions on Intelligent Transportation Systems*, vol. 13, no. 1, pp. 81-90, March 2012, doi: 10.1109/TITS.2011.2178836.

[38] D. Zhou, Z. Ma and J. Sun, "Autonomous Vehicles' Turning Motion Planning for Conflict Areas at Mixed-Flow Intersections," in *IEEE Transactions on Intelligent Vehicles*, vol. 5, no. 2, pp. 204-216, June 2020, doi: 10.1109/TIV.2019.2955854.

[39] S. U. Park, B. K. Kim, S. C. You and J. Y. Lee, "Group Mutual Exclusion Algorithm for Intersection Traffic Control of Autonomous Vehicle," *2017 International Conference on Computational Science and Computational Intelligence (CSCI)*, Las Vegas, NV, 2017, pp. 1633-1636, doi: 10.1109/CSCI.2017.284.

[40] Y. Bichiou and H. A. Rakha, "Developing an Optimal Intersection Control System for Automated Connected Vehicles," in *IEEE Transactions on Intelli-*

gent Transportation Systems, vol. 20, no. 5, pp. 1908-1916, May 2019, doi: 10.1109/TITS.2018.2850335.

[41] S. A. Fayazi and A. Vahidi, "Mixed-Integer Linear Programming for Optimal Scheduling of Autonomous Vehicle Intersection Crossing," in *IEEE Transactions on Intelligent Vehicles*, vol. 3, no. 3, pp. 287-299, Sept. 2018, doi: 10.1109/TIV.2018.2843163.

[42] M. Akhil, N. Vasudevan, U. Ramanadhan, A. Devassy, D. Krishnaswamy and A. Ramachandran, "Collision avoidance at intersections using vehicle detectors and smartphones," *2015 International Conference on Connected Vehicles and Expo (ICCVE)*, Shenzhen, 2015, pp. 389-390, doi: 10.1109/ICCVE.2015.39.

[43] "Ensuring American leadership in automated vehicle technologies," National Science Technology Council and the Department of Transportation, United States of America, January 2020.

[44] Jiaxing Guo, Yu Liu and G. Tao, "Multivariable MRAC with state feedback for output tracking," *2009 American Control Conference*, St. Louis, MO, USA, 2009, pp. 592-597, doi: 10.1109/ACC.2009.5160489.

[45] Danielle Muoio. "These 19 companies are racing to build self-driving cars in the next 5 years". BUSINESSINSIDER.com. https://www.businessinsider.com/companies-making-driverless-cars-by-2020-2017-1 (accessed on March 30th, 2021)

[46] K. Jo, K. Chu and M. Sunwoo, "Interacting Multiple Model Filter-Based Sensor Fusion of GPS With In-Vehicle Sensors for Real-Time Vehicle Positioning," in *IEEE Transactions on Intelligent Transportation Systems*, vol. 13, no. 1, pp. 329-343, March 2012, doi: 10.1109/TITS.2011.2171033.

[47] C. -h. Park and J. -h. Han, "Performance Evaluation of GNSS and Motion Sensor Integrated Positioning Algorithm for Land Vehicle Monitoring," *2020 International Conference on Information and Communication Technology Convergence (ICTC)*, 2020, pp. 1592-1595, doi: 10.1109/ICTC49870.2020.9289475.

[48] Gang Tao, "Model reference adaptive control with $L^{1+\alpha}$ tracking", *International Journal of Control*, vol. 64, no. 5, pp. 859-870, 1996, doi: 10.1080/00207179608921661.

[49] "Model 3 | Tesla". Tesla.com. https://www.tesla.com/model3 (accessed Apr. 22, 2021)

[50] Sonam Rai and Munsif Vengattil. "Consumer Reports recommends Tesla's Model 3 after braking fix". REUTERS.com. https://www.reuters.com/article/us-tesla-model-3/consumer-reports-recommends-teslas-model-3-after-braking-fix-idUSKCN1IV29D (accessed Apr. 22, 2021)