THE RATE OF FLOW OF AIR AND OF CARBON DIOXIDE THROUGH A POROUS PLUG

Lois Corinne Ketcham Carvile

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THROUGH A POROUS PLUG

Lois Corinne Ketcham Carwile, B. S.

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THE RATE OF FLOW OF AIR AND OF CARBON DIOXIDE THROUGH A POROUS PLUG

INTRODUCTION

The object of this experiment is to study the rate of flow of air through a porous plug as affected by (a) pressure difference between the ends of the plug up to 110 centimeters of water; (b) absolute pressure from 75 to 110 centimeters of mercury; (c) temperature from 0 to 80 degrees Centigrade, that is 273 to 353 degrees absolute; and to observe any differences and similarities in the behavior of carbon dioxide.

Graham seems to have been the first to work with the flow of gases through porous plugs. He was concerned (a) with the rate of diffusion of two different gases separated by a porous partition, which he found to be inversely proportional to the square roots of the densities of the two gases[1]; and (b) with the time of passage through a small orifice into a vacuum of equal volumes of different gases, which he found to vary as the square

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1. Winkelmann, "Handbuch der Physik", Book I, Volume II, "Allgemeine Physik", page 1432.

roots of their densities[2]. This is known as Graham's law[3], for equal temperatures and pressures[4,5].

Bunsen observed the velocity of effusion of a gas through plaster of Paris under a pressure difference. He found this velocity proportional to the pressure difference within the limits observed, 72 millimeters of mercury. Finally Hansemann established Bunsen's law up to 690 mm. of mercury for plaster of Paris[6].

The present paper deals with the flow of a single gas, into an atmosphere of the same gas. Bunsen's law is verified up to 81 mm. of mercury pressure difference. The effect of temperature upon the rate of flow does not seem to have been studied, as far as available references go. The well-known Joule-Thompson porous plug experiment deals with the ratio of the temperature drop to the pressure drop when a gas is forced through a porous plug, for the purpose of estimating the deviation of actual gases from the ideal gas[7]. This is quite another problem.

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2. Graham, "Elements of Chemistry", Vol. I, p. 81. 3. Edser, "General Physics for Students", p. 536. 4. Chwolson, "Lehrbuch der Physik", Vol. I, pp. 517-518. 5. Jamin, "Cours de Physique", Tome I, pp. 98*-99*. 6. Winkelmann, op. cit., pp.1432-1434. 7. Goodenough, "Principles of Thermodynamics", p. 171.

GENERAL THEORY

One might expect the flow to be somewhere between viscous flow as of a continuous fluid, and diffusion of discrete molecules.

It is known that viscous flow is found in the case of capillary tubes, where by Poiseuille's formula[8] the volume rate of flow r_v is proportional to the pressure gradient, and inversely proportional to the coefficient of viscosity η. If we assume that for a tortuous tube such as a channel through a porous plug the effect of viscosity may be expressed by a similar law, we have

$$r_v = \frac{A_o (p_2 - p_1)}{\eta}, \qquad (1)$$

where A_o is a constant of proportionality, and $(p_2 - p_1)$ is the pressure difference between the ends of the plug. From the Kinetic Theory of Gases the coefficient of viscosity is independent of the density, according to Maxwell[9], but varies according to the n^{th} power of the absolute temperature[10], where n is a constant for a

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8. Kaye and Laby, "Physical and Chemical Constants", p. 30 (1911). 9. Jeans, "The Dynamical Theory of Gases", p. 277. 10. " " " " " " , p. 283.

given gas.

$$\eta \propto T^n = \eta_\circ \left(\frac{T}{273.1} \right)^n . \qquad (2)$$

Therefore $$r_v = \frac{A_1 (p_2 - p_1)}{T^n} . \qquad (3)$$

Converting this into mass rate r_m , sinoe

$$p\, r_v \propto r_m\, T \text{ , or } r_m = r_v\, p/T ,$$

multiply equation (1) by p/T , and we have

$$r_m = \frac{p}{T} r_v = \frac{A_0\, p\, (p_2 - p_1)}{\eta\, T}$$

$$= \frac{A\, p\, (p_2 - p_1)}{T^{n+1}} . \qquad (4)$$

Also from equation (4), at a given pressure, pressure difference, and absolute temperature,

$$\frac{\eta \text{ for gas 1}}{\eta \text{ for gas 2}} = \frac{A \text{ for gas 2}}{A \text{ for gas 1}} . \qquad (5)$$

It has been shown by Sir George Stokes that the rate of fall of a spherioal drop is inversely as the viscosity of the air through which it falls[11]. Therefore

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11. Starling, "Electrioity and Magnetism", p. 483 (1924).

the above assumption concerning a viscous flow term for a tortuous channel seems a suitable working hypothesis.

In addition, there is the flow dependent upon molecular velocity. The mass rate of flow into a vacuum is proportional to the molecular density and also to the molecular velocity[12]. $r_m \propto \nu c$. But the molecular velocity $c \propto \sqrt{T/m}$ if m is the molecular weight[13] Also[14] $\nu \propto p/T$.

Therefore $$r_m \propto \sqrt{\frac{T}{m}} \propto p\sqrt{\frac{1}{mT}} . \qquad (6)$$

For a flow not into a vacuum, the effective pressure is the difference of the two pressures. Hence

$$r_m \propto \frac{(p_2 - p_1)}{\sqrt{mT}} = \frac{B\,(p_2 - p_1)}{\sqrt{T}} . \qquad (7)$$

Therefore $$\frac{B \text{ for gas } 1}{B \text{ for gas } 2} = \frac{m \text{ for gas } 2}{m \text{ for gas } 1} . \qquad (8)$$

Combining equations (4) and (7), we have tentatively

$$r_m = \frac{A\,p\,(p_2 - p_1)}{T^{n+1}} + \frac{B\,(p_2 - p_1)}{\sqrt{T}} . \qquad (9)$$

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12. Jeans, "The Dynamical Theory of Gases", p.120.
13. " " " " " " , p. 118.
14. " " " " " " , p. 116.

APPARATUS

A cylindrical porous plug was used, closed at one end, about 17 cm. long and 2.3 cm. outside diameter, 1.7 cm. inside diameter, therefore the thickness was about 3 mm. Most of the work was done with a Pasteur filter, but a more open plaster of Paris plug was also tried which gave about three times as rapid flow. The plug fitted into a metal case (Fig. 1) surrounded by a coil through which the gas passed, reaching the temperature of an electrically heated bath before traversing the plug. A motor-driven pump maintained the gas at any desired pressure difference, the constancy of which was increased by using a battery of Edison storage cells as the source of power. The gas entered through a drying tube of calcium chloride one meter long. Connecting tubes of glass, rubber and brass were used. Regulating valves were employed freely, and throttling valves were used between the plug and the outlet when the absolute pressure was to be increased. The pressure difference between the ends of the plug was read by a water manometer, and the absolute pressures above atmospheric were read by a mercury manometer open to the air, added to the barometric height. The volume of gas passing

APPARATUS

A cylindrical porous plug was used, closed at one end, about 10 cm. long and 2.5 cm. outside diameter, 1.7 cm. inside diameter, the walls thickness was about 4 mm. Most of the work was done with a Jena filter, but a more open plug of Jena glass was also tried which gave about three times as rapid flow. The plug fitted into a metal case (Fig. 1) surrounded by a coil through which the gas passed, acquiring the temperature of an electrically heated bath before traversing the plug. A motor-driven pump maintained the gas at any desired pressure difference, the constancy of which was increased by using a battery of [illegible] as the source of power. The gas entered through a drying tube of calcium chloride one meter long. Connecting tubes of glass, rubber and brass were used, regulating valves were employed freely, and throttling valves were used between the plug and the outlet when the absolute pressure was to be increased. The pressure difference between the ends of the plug was read by a water manometer, and the absolute pressure above atmospheric were read by a mercury manometer open to the air, added to the barometric height. The volume of gas passing

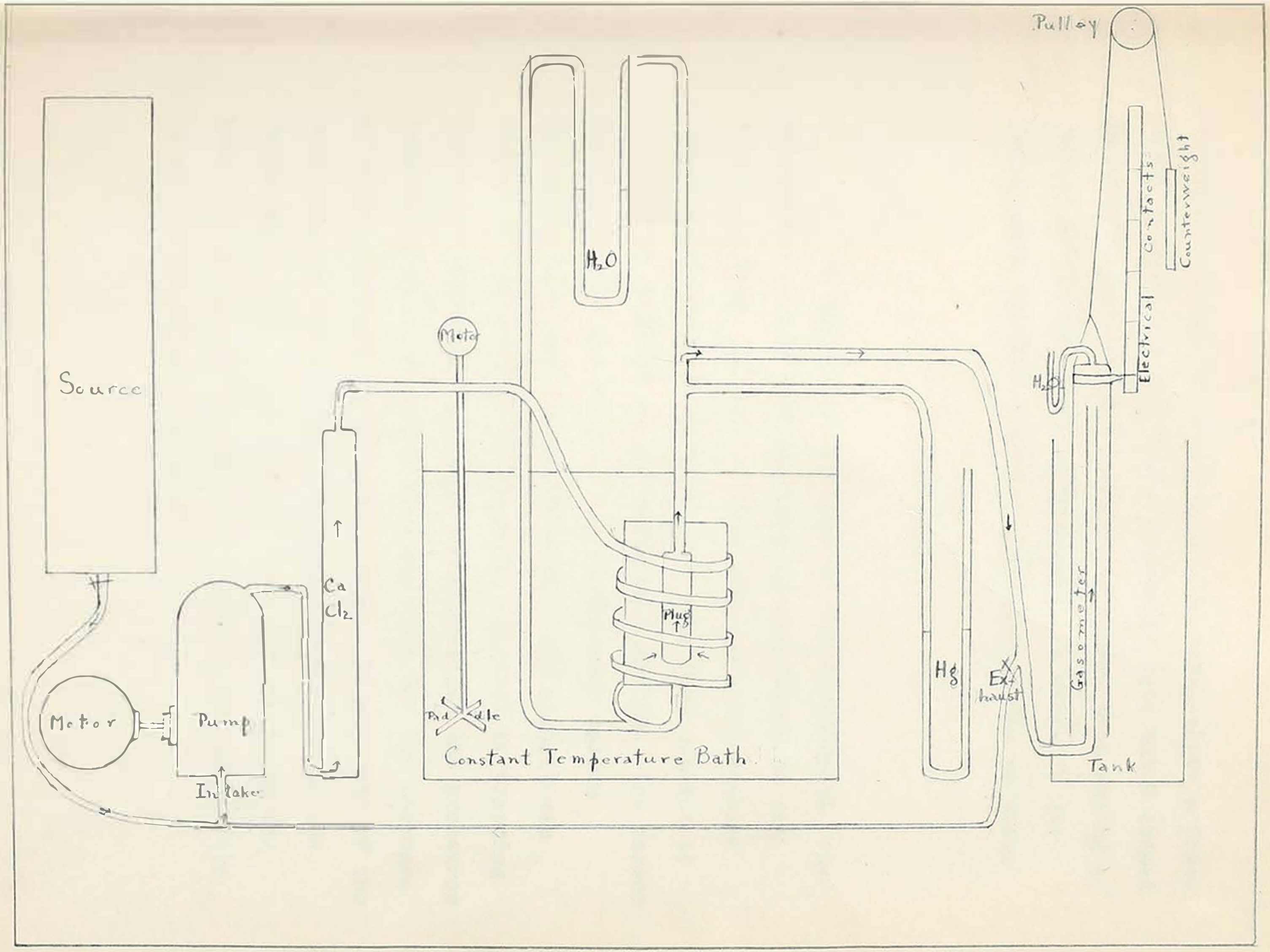

Fig. 1 APPARATUS

through the plug was measured by the rise, above a fixed water level, of a calibrated gasometer tube which opened an electrical circuit at certain volumes, this making a record of the time on a chronograph. A clock in the chronograph circuit recorded intervals of two seconds.

PRECISION

By adjusting the rate of the chronograph, one centimeter of the tape was made to correspond to one second of time, so that an easy interpolation enabled the time of rise of the gasometer tube from contact to contact to be known to one-tenth of a second. The carbon dioxide in the gas circuit was tested over sodium hydroxide and found to be 96% pure. The circuit was tested for leaks with liquid soap, and also by trapping gas there under pressure and noting whether the pressures remained fixed. No correction was made for the presence of water vapor in the gasometer tube. An estimate of the precision of the several measurements is given in the following table; it is evident that the errors in the time, barometer and temperature readings are negligible in comparison with the others.

TABLE I

QUANTITY	INSTRUMENT	MEAN RANGE	MAXIMUM DEVIATION	PERCENTAGE DEVIATION
Volume	Gasometer	500 cc.	5 cc.	1.0
Time	Chronograph	50 sec.	.1 sec.	.20
Pressure Difference	Water Manometer	50 cm.	.4 cm.	.80
Absolute Pressure	Mercury Manometer	20 cm.	.2 cm.	1.0
"	Barometer	29 in.	.01 in.	.034
Temperature	Thermometer	40 deg.	.1 deg.	.25

DATA AND DISCUSSION

Direct measurements gave the time of flow of 200, 400, 600, 800 cc. of gas at barometric pressure and gasometer temperature. This quantity (cc./sec.) is therefore proportional to the mass flow, by Boyle's law. A plot of the volume as a function of the time is a straight line passing very nearly through the origin (Fig. 2), and its slope is the rate of flow. Using pressure differences every 10 cm. from 20 to 110 cm. of water at a given temperature, one sees that the rate of flow increases with the pressure difference. A plot

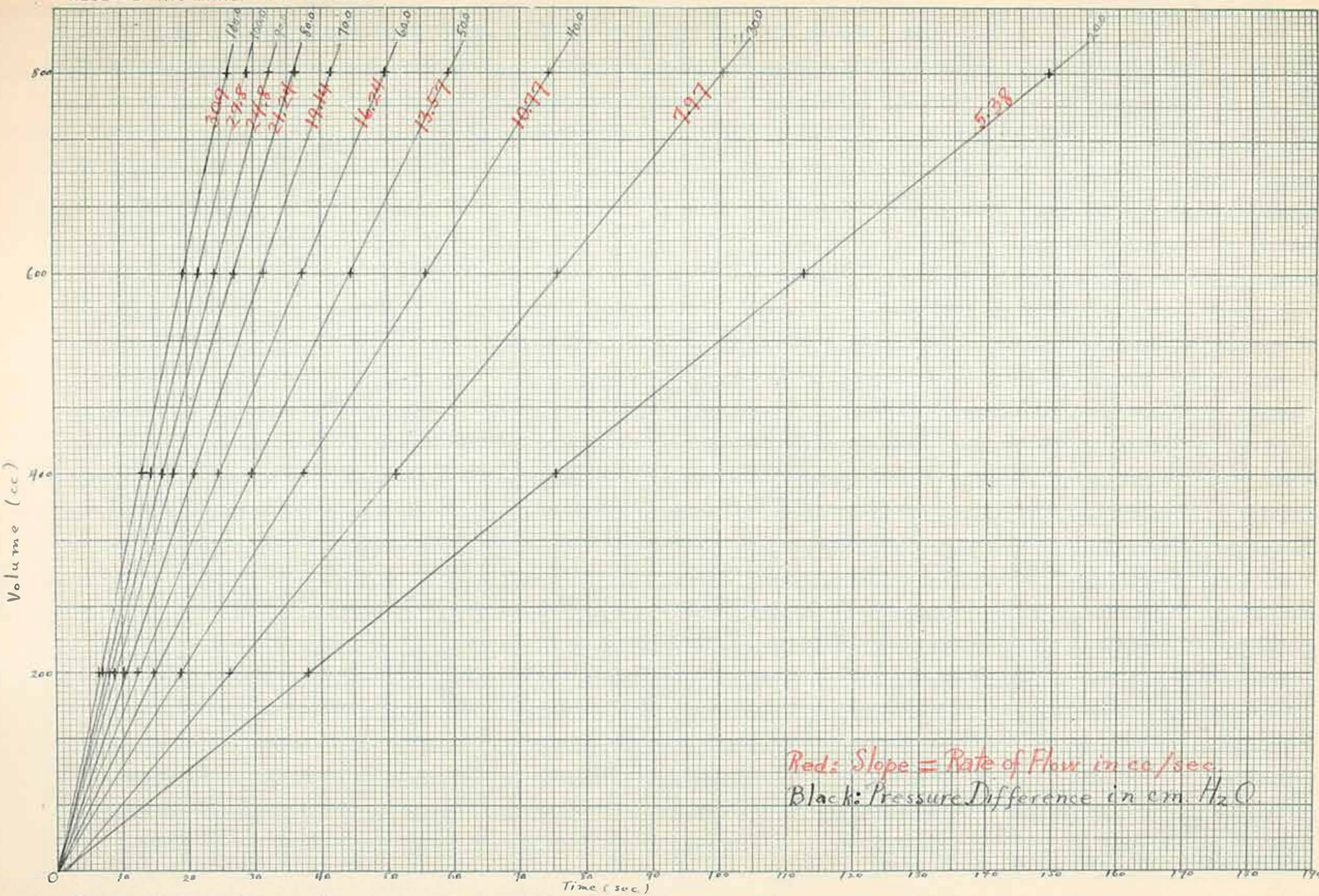

Fig. 2. Rate of Flow of Air through Pasteur Filter at 0.0°C, and Barometer 74.4 cm. of Mercury.

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SCALE: EACH DIVISION — 1, 2, 5, 10, 100, 1000, ETC.
OR — 0.5, 0.2, 0.1, 0.01, 0.001, ETC.

THIS EDGE MUST BE EITHER TOP OR LEFT-HAND SIDE OF SHEET

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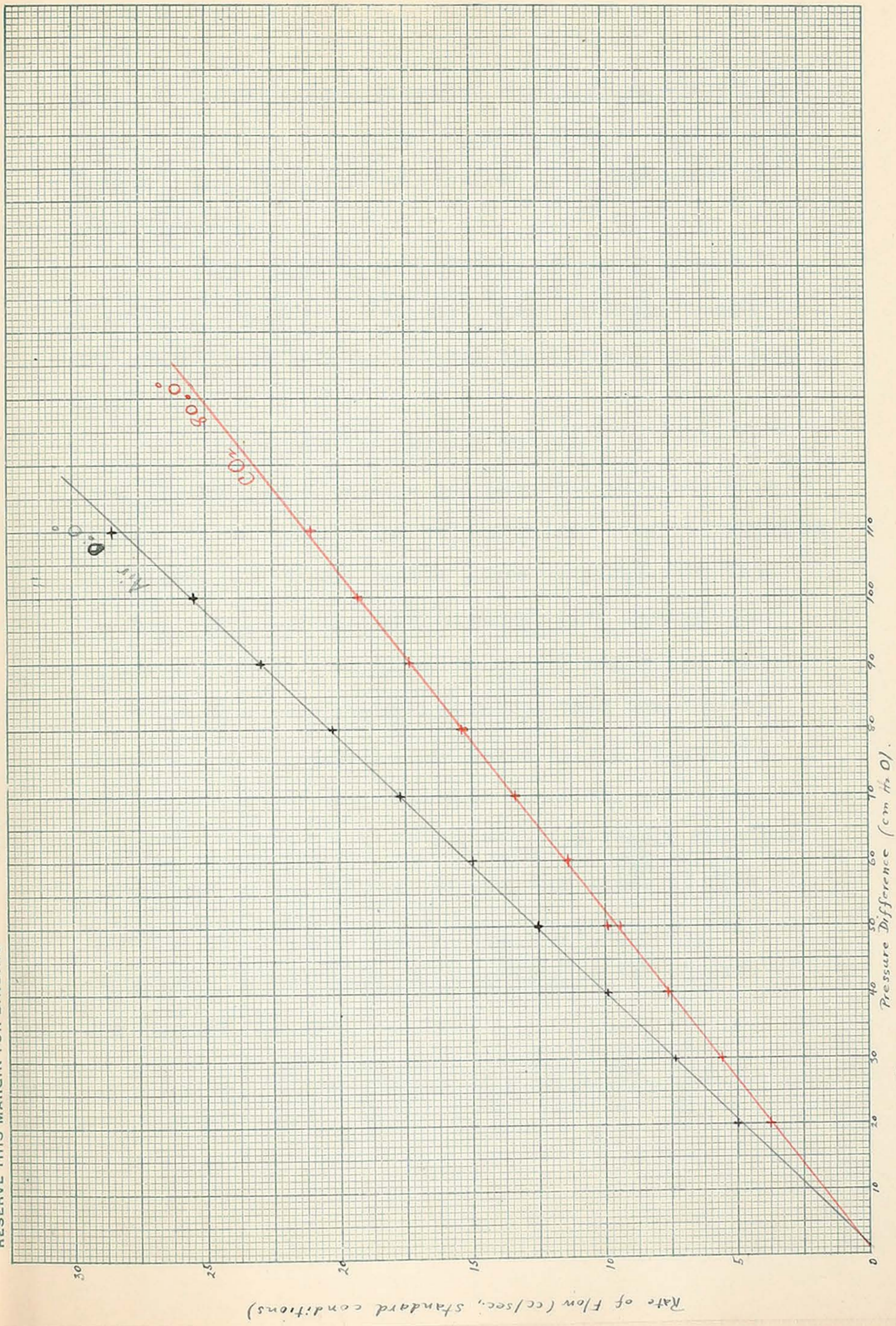

Fig. 3.

(Fig. 3) shows again a straight line through the origin. Both for air and for carbon dioxide from 0 to 80 degrees Bunsen's law is therefore verified up to one-ninth the mean pressure:

The velocity of effusion of a gas through plaster of Paris or a Pasteur filter is proportional to the pressure difference between the extremities of the plug.

A computation formula was used in order to find the rate of flow more quickly than by means of a graph. In the graph, the best straight line representing the points was drawn and its slope measured. By this formula, instead, the best straight line through the origin was used. The first point was neglected, since it was always more subject to error than the others, some time being required for the observer to get the gasometer rise steady, and also since the quantities themselves are smaller, which would in itself make the percentage error larger. Calling the times of flow of 400, 600 and 800 cc. t_2, t_3, t_4, respectively, and weighting them as 1, 2, 4, according to the squares of the lengths, we have

$$\frac{1}{7}\left(\frac{t_2}{400} + \frac{2t_3}{600} + \frac{4t_4}{800}\right) = \frac{1}{100}\left(\frac{t_2}{28} + \frac{t_3}{21} + \frac{t_4}{14}\right).$$

Therefore the mass rate $= 100 / \left(\frac{t_2}{28} + \frac{t_3}{21} + \frac{t_4}{14}\right)$. (10)

This formula gives results agreeing with the graphical easily within 1%, which is as close as the pressures could be maintained. (See Table I)

The rate of flow was reduced to standard conditions, zero degrees and 76 cm. of mercury. It is equal to the mass rate of flow if multiplied by the density. The absolute pressure was taken as the mercury manometer reading plus the barometer reading plus half the water manometer reading, all reduced to centimeters of mercury.

The relation between the rate of flow and the absolute temperature is shown in Fig. 4 to be a straight line for air for the range of temperatures, 273 to 353 degrees. The rate decreased as the temperature increased. For carbon dioxide, which is less perfect a gas than air, a decided curvature is noted. In Table II are given some temperature coefficients of the mass rate of flow.

A graph (Fig. 5) of the rate of flow as a function of the absolute pressure for air shows a linear relation. Such was found to be the case for carbon dioxide at a given temperature, the pressure difference being constant (Table III). This agrees with the

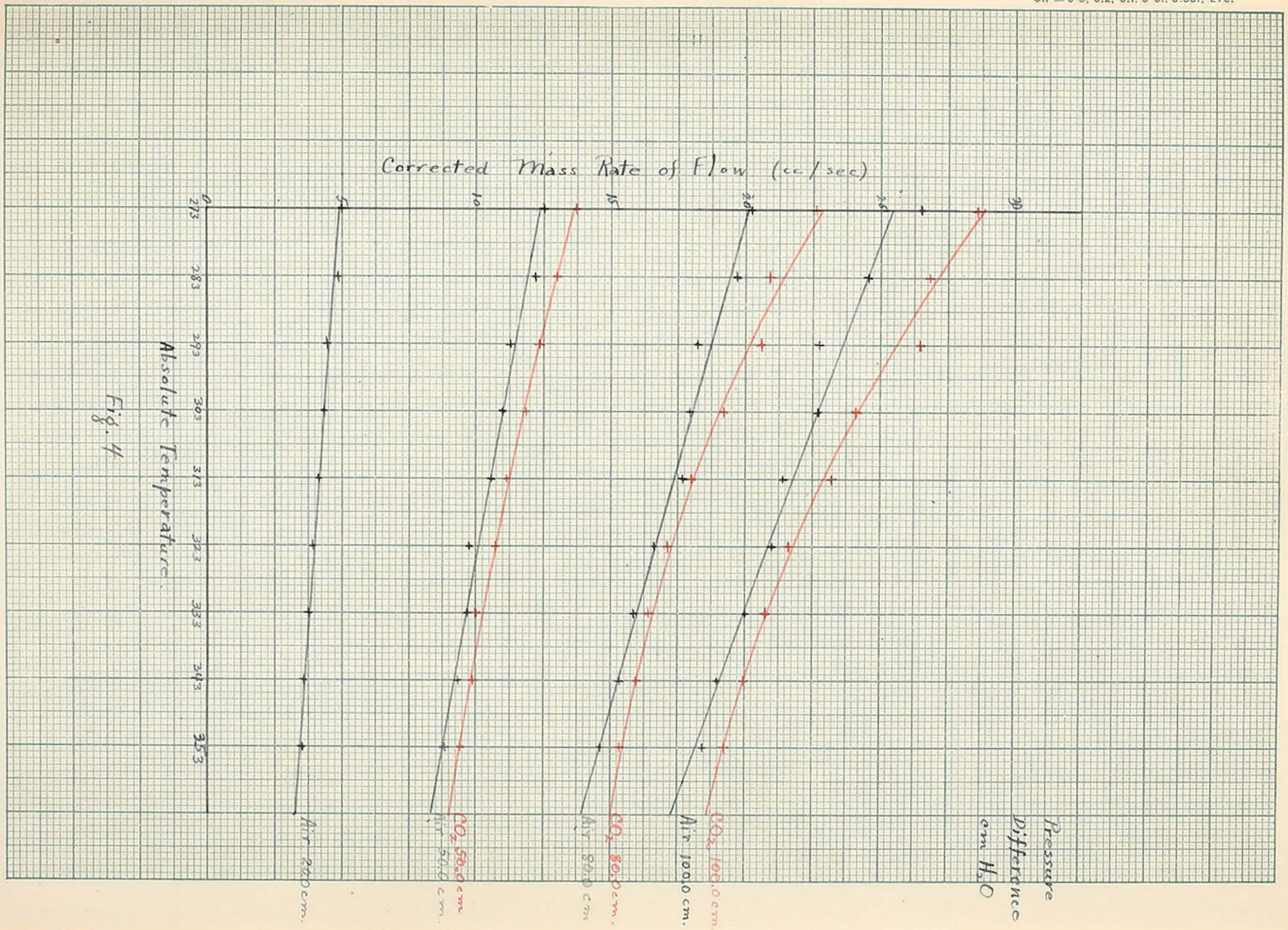
Corrected Mass Rate of Flow (cc/sec)
0
5
10
15
20
25
30
273
283
293
303
313
323
333
343
353
Absolute Temperature
Air 20.0 cm.
Air 50.0 cm.
CO_2 50.0 cm
Air 80.0 cm
CO_2 80.0 cm.
Air 100.0 cm.
CO_2 100.0 cm.
Pressure Difference cm H_2O

Fig. 4

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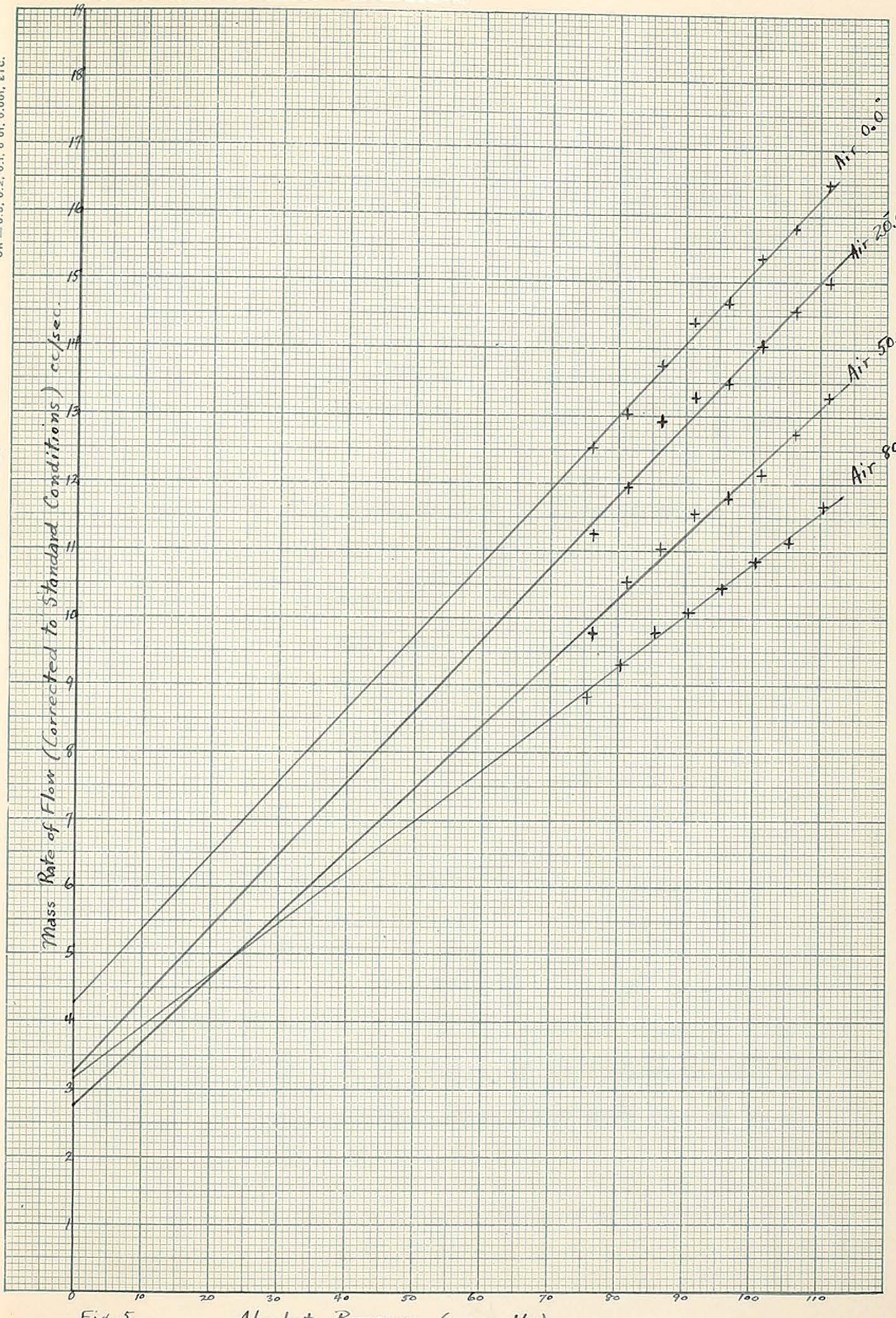

Fig 5.

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TABLE II (a) CO_2

Absolute Temperature	Pressure Difference 50.0 cm. H_2O	80.0 cm. H_2O	100.0 cm. H_2O
273	- .070	- .14	- .19
293	- .051	- .11	- .15
313	- .050	- .081	- .11
333	- .042	- .066	- .086
353	- .042	- .042	- .069

(b) Mean Temperature Coefficients for Air

Pressure Difference	Temperature Coefficient
20.0 cm.	- .020
50.0 "	- .046
80.0 "	- .069
100.0 "	- .090

* * * * * * * * * *

tentative equation (9), which is of the form

$$r_m = \frac{A'p}{T^{n+1}} + \frac{B'}{\sqrt{T}} . \qquad (11)$$

The method of Least Squares was used in order to find the actual slope and intercept of typical rate-of-flow absolute-pressure graphs. Equations were set up from

TABLE III

(See Fig. 5)

Air

Absolute Temperature	Slope	Intercept	$T^{\frac{1}{2}}$	Intercept times $T^{\frac{1}{2}}$
273	.1095	4.29	16.52	71.0
293	.1072	3.25	17.12	55.6
323	.0929	2.74	17.97	49.2
353	.0764	3.15	18.89	59.4
			Mean	60 ± 3.

CO_2

273	.1342	3.53	16.52	58.4
323	.1038	2.79	17.97	50.2
353	.0884	2.73	18.89	51.6
			Mean	53 ± 2.

the data, of the type

$$\begin{cases} \sum y - \sum a - b\sum x = 0 \\ \sum xy - a\sum x - b\sum x^2 = 0 , \end{cases}$$

and solved for a and b , the mean intercept and slope, respectively. On account of the shortness of the pressure interval only two figures are good, however. (Table III)

Differentiating equation (11) partially with respect to p , holding T constant, we have

$$\left(\frac{\partial r}{\partial p}\right)_T = \frac{A'}{T^{n+1}} .$$

Therefore slope x $T^{n+1} = A'$, or if $n + 1 = n'$,

$$\log \text{slope} + n' \log T = c .$$

The three such equations for carbon dioxide are

$$.1342 \times (273)^{n'} = c$$

$$.1038 \times (323)^{n'} = c$$

$$.0884 \times (353)^{n'} = c .$$

Solving, though these equations are not all independent, we get three values $n' = 1.53$, 1.81 , 1.63 . They are weighted according to the squares of the temperature intervals, and $n' = 1.63$, or $n = .63$. Similarly

for air, $n = .52$. But Jeans gives $n = .98$ for CO_2, and .75 for air[15]. Therefore the value here obtained is only about two-thirds of what he gives, in each case. Kaye and Laby give an approximate law (p. 31) due to Maxwell, $\eta \propto T^{\frac{1}{2}}$, which is apparently as good for our purpose as the more elaborate one of Jeans.

Multiplying equation (11) by $T^{\frac{1}{2}}$, we get

$$r_m T^{\frac{1}{2}} = \frac{A'p}{T^{n} \; \frac{1}{2}} + B' ,$$

hence B' should be constant (Fig. 6) for the family of lines obtained for one gas for arbitrary values of the parameter T. Table III gives

$$B_{Air} = 60 \pm 3 ,$$

$$B_{CO_2} = 53 \pm 2 .$$

Therefore $B_{Air} / B_{CO_2} = 1.13$; but

$$\frac{m \text{ for } CO_2}{m \text{ for air}} = \sqrt{\frac{44}{29}} = 1.23 .$$

This is a fair check of equation (8).

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15. Jeans, op. cit., p. 284.

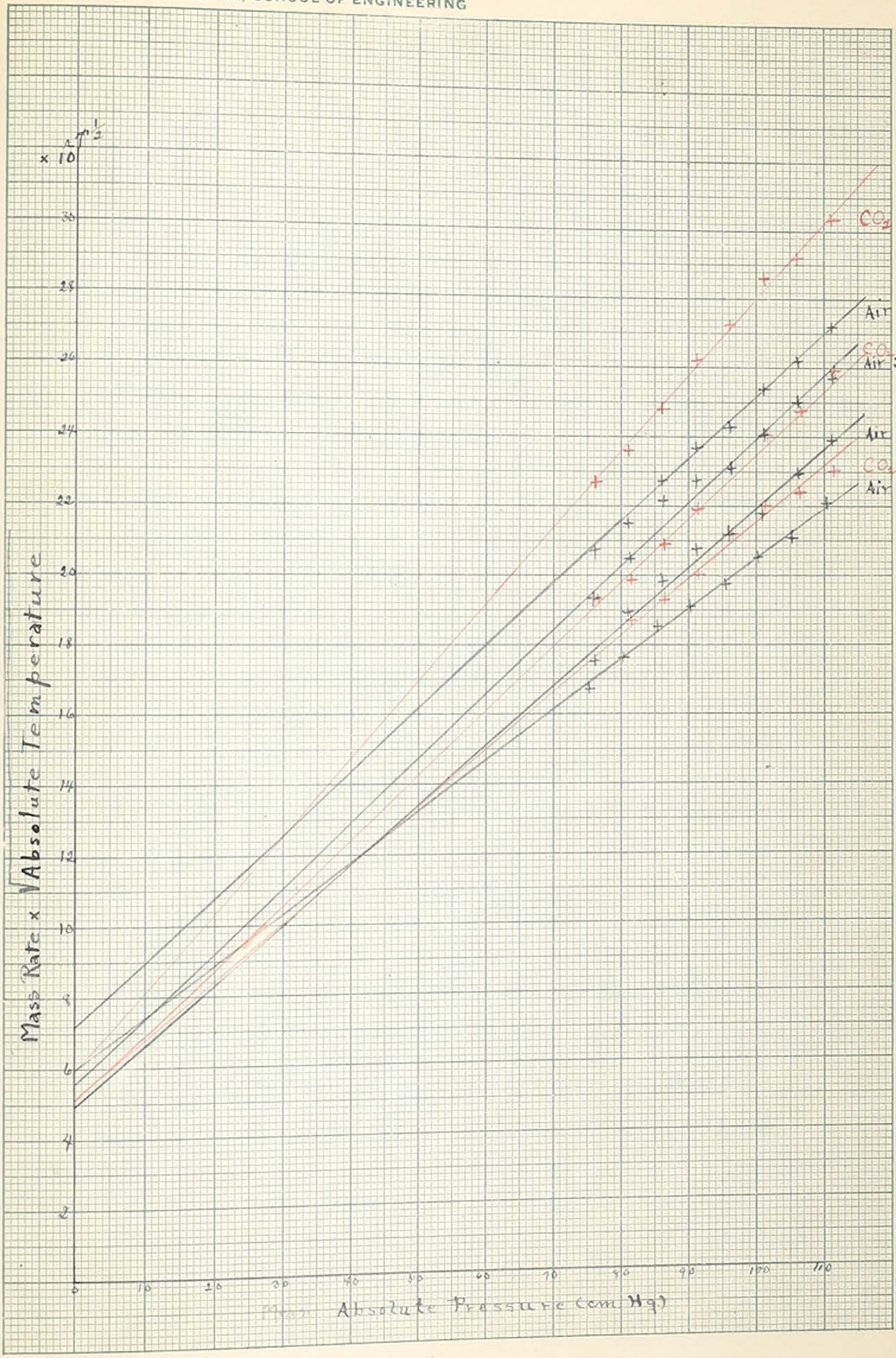
$\times 10^{-1}$
Mass Rate × √Absolute Temperature
Absolute Pressure (cm Hg)
CO_2 0.0°
Air 0.0°
CO_2 50.0°
Air 20.0°
Air 50.0°
CO_2 80.0°
Air 80.0°
0
10
20
30
40
50
60
70
80
90
100
110
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30

Fig. 6

Again, assume $n = \frac{1}{2}$. Multiply equation (11) by $T^{3/2}$, then

$$r_m T^{3/2} = A'p + B'T ,$$

whence A' should be constant for a family of lines for arbitrary values of T. A' was found to be 590 for CO_2 and 499 for air. Therefore

$$\frac{A' \text{ for } CO_2}{A' \text{ for air}} = 1.18 .$$

But

$$\frac{\eta \text{ for air}}{\eta \text{ for } CO_2} = \frac{171 \times 10^{-6}}{139 \times 10^{-6}} = 1.23 .$$

These two results agree within 2½%, and check equation (5).

CONCLUSIONS

It appears that the theory given fits the results as well as any other theory. But to establish it would require the use of a wider range of absolute pressures and temperatures than was attempted here.