WORD-PROBLEM INSTRUCTION FOR ENGLISH LEARNERS: A CULTURALLY AND LINGUISTICALLY RESPONSIVE APPROACH

A Dissertation

Presented to the Faculty of the Curry School of Education

University of Virginia

In Partial Fulfillment

Of the Requirements for the Degree

Doctor of Philosophy

By

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March 2015

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ABSTRACT

Word problems are prevalent on high-stakes achievement tests, and success on word problems has implications for promotion and graduation. English Learners (ELs) continue to perform significantly below their native English-speaking peers on mathematics achievement tests, which may be attributed to the linguistic complexity of mathematics instruction and standardized assessments. Little is known about the instructional needs and performance of ELs at-risk for mathematics difficulty (MD). A multiple-methods design (i.e., qualitative methods and single-subject multiple baseline methods) is used to investigate word-problem instruction for ELs in a culturally and linguistically diverse public elementary school. Specifically, I studied one teacher's mathematics instruction for ELs over several months and empirically tested the efficacy of a word-problem intervention for ELs with MD (N=9) that combines culturally and linguistically responsive practices and schema instruction (CLR-SI). The study is unique in that it combines research on effective instruction for ELs and students with MD. CLR-SI has not been investigated for either ELs or students with MD. In addition, I studied teacher perceptions and beliefs regarding mathematics instruction for diverse learners. Findings from this study have implications for teachers, administrators, and researchers of ELs with MD.

Keywords: English Learners, culturally responsive teaching, mathematics difficulty, word-problem instruction, schema instruction

DEDICATION

This dissertation is dedicated to the many students, teachers, and educational leaders I've worked with over the past decade. You are the reason I entered into the teaching profession and the reason I remain in education. Working with you was a privilege that forever changed what is important to me, as well as what I hope to see possible in this world. Each of you continues to fuel my passion for educational equity for historically underserved populations. This dissertation is inspired by and dedicated to you.

ACKNOWLEDGMENTS

I would like to thank my dissertation committee (Drs. Michael Kennedy, Amanda Kibler, and John Lloyd), and especially my committee co-chairs (Drs. Stanley Trent and Sarah Powell). Each of you provided invaluable feedback from inception to completion of this project. A special thank you to Dr. Frank and Linda Butts and the Curry Foundation, as well as Ms. Doris Buffett and the Center on Children, Families, and the Law who generously provided funding for the mathematics instruction in this project. I would also like to thank the school administration, teachers, and students who shared their space, time, and perspectives throughout this project. This school challenges notions of what "at-risk" means, and the quality of education students receive from your staff exceeds what can be measured by standardized assessments. This dissertation could not have happened without the dedication and hard work of my incredible research assistants (Addie Boyd-Pratt, Julia Colopy, Irene Cunningham, and Stephen Giannotti) who spent hours tutoring students and scoring word problems. A special thank you to Julia who assisted in both data collection and provided childcare as I analyzed the results. I would also like to thank both Cara Rose Williams and Nancy Trifiletti for providing highquality childcare and being flexible with my ever-changing data collection schedule. In addition, I would like to thank my dissertation writing group members (Hilary Dack and Christine Hardigree) for their resourcefulness and encouragement over the past year. Finally, I would like to thank my husband, David Driver, and sweet daughter Annabelle. Your encouragement, love, patience, and support made this work possible.

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CHAPTER I: INTRODUCTION

Early success in mathematics is important because it is a predictor of later achievement in school (Morgan, Farkas, & Wu, 2011), and mathematics success can lead to increased college and career opportunities. Achievement in mathematics is often measured by high-stakes standardized tests such as the National Assessment of Educational Progress (NAEP; National Governing Board, 2009) that rely heavily on word problems. A *word problem* is a mathematics calculation presented using words and sentences. To solve word problems, students use text, most often presented in English, to identify missing information, make a plan to solve the problem, and perform one or more calculations to get the solution (Powell, 2011). The language and multi-step processes inherent in word problems can pose particular difficulties for English Learners (ELs; Martiniello, 2008).

Mathematics Achievement and ELs

The term ELs refers to a wide range of students with varying linguistic, cultural, and educational backgrounds. By federal definition, ELs' native language is a language other than English, and their level of English proficiency may impede academic achievement in classrooms where the language of instruction is English (Linquanti & Cook, 2013). For the present study, I focus on the mathematics achievement of EL students who also identify with historically underserved racial and ethnic groups. Language is embedded within mathematics instruction, and ELs lag behind native-English speakers in performance on standardized mathematics achievement measures involving word problems at the elementary and secondary level (Abedi & Lord, 2001). Data from the most recent NAEP fourth-grade mathematics assessment reveals that ELs continue to perform significantly below their native English peers (NAEP, 2013).

Martiniello's (2008) analysis of word-problem think-alouds revealed that although ELs may understand the mathematical content (i.e., probability), ELs often struggle to understand the text providing information. Difficulty with text is not unique to ELs; however, ELs may struggle with the process of learning academic content in a second language. Unfamiliarity with English vocabulary, relevant background knowledge, and linguistic syntax structures in word problems contribute to incorrect problem solving (Martiniello, 2008).

Elementary mathematics can also pose challenges for ELs due to the linguistic complexity of instruction. In mathematics classrooms, students are expected to not only understand and solve problems but also explain their problem solving process in written and verbal form (Moschkovich, 1999). Many mathematical terms are new to learners (e.g., coefficient, hypotenuse) and others may be familiar sounding (e.g., sum, value, product) but have specific and complex mathematical definitions (Freeman & Crawford, 2008). The latter can be just as unfamiliar for ELs. Syntactic and semantic features of mathematical discourse such as, "the same as," "take away," and "how many go into" can also be confusing for students (McLeman, 2012). In addition, the symbols associated with mathematics (e.g., +, -, ×, ÷, =) can be very challenging for all student populations to understand (Freeman & Crawford, 2008; Gilmore, McCarthy, & Spelke, 2007; Lipton & Spelke, 2005; Powell & Fuchs, 2010).

Shaftel, Belton-Kocher, Glasnapp, and Poggio (2006) analyzed language

characteristics (i.e., multiple meaning words, complex syntax) for mathematics items on a Midwestern standardized state assessment administered at 4th, 7th, and 10th grades. Results for students with disabilities and ELs were analyzed within the overall student sample taking the state assessment. Assessment items including multiple-meaning or unclear words had a statistically significant effect at fourth grade and items with comparative terms had a statistically significant impact at seventh grade. Shaftel et al. did not find a disproportionate impact on students with disabilities or ELs compared to the overall sample. Shaftel et al.'s findings reflect the difficulty that both contentspecific terminology and language structures can pose for students, including ELs, in mathematics.

Similarly, Reardon and Galindo (2007) analyzed data from a nationally representative sample of kindergarten through fifth-grade student data from the Early Childhood Longitudinal Study–Kindergarten Cohort (ECLS-K). Approximately 9,000 students from the kindergarten class of 1998–1999 were included in the sample. Approximately 2,000 of the students in the sample identified as Hispanic or Latino. Early mathematics skills (i.e., counting, recognizing numerals, solving simple addition problems) proficiency rates varied significantly in relation to student language use. Latino students who were proficient in oral English use at the start of kindergarten demonstrated higher mathematics proficiency throughout elementary school than Latino students with limited English at the start of kindergarten. Latino students who lived in homes where English was the primary, or sole, language used demonstrated higher mathematics proficiency than Latino students living in homes where Spanish was the primary, or sole, language used.

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Reardon and Galindo (2007) also analyzed mathematics proficiency and skill by race/ethnicity. Latino (e.g., ELs and native-English speaking) and African American students demonstrated lower mathematics proficiency rates compared to Caucasian students at all grade levels (Reardon & Galindo, 2007). Latino students were found to enter kindergarten with mathematics skills significantly lower than native-born, non-Latino, Caucasian students. However, Reardon and Galindo found that this proficiency gap narrowed as students progressed through elementary school. Latino students' mathematics proficiency was similar to native-born African American students at the start of kindergarten, but at fifth grade, Latino students demonstrated higher proficiency rates than their African American peers.

It is important to note that although the gaps appear to narrow in these findings, the mathematics proficiency levels of Latino students remain below their Caucasian peers. In addition, Reardon and Galindo (2007) found considerable variation in proficiency among Latino population subgroups. Recent immigrants and groups with lower socioeconomic resources demonstrated the lowest levels of mathematics proficiency throughout elementary school. The reason some Latino EL groups made rapid gains while others did not is unclear, but it may be related to processes of increased English acquisition and effective instructional practices for ELs.

Mathematics Difficulty

A mathematics disability is considered under IDEA (2004) as a type of specific learning disability (SLD) and is often referred to as dyscalculia in scientific literature (e.g., Butterworth, 2010; Mussolin, Mejias, & Noël, 2010). Approximately 3 to 6% of school-age students struggle with a diagnosed mathematics disability (Shalev, Auerbach, Manor, & Gross-Tsur, 2000), but an even greater number of students demonstrate low mathematics performance without an official disability diagnosis. Of those students identified with a mathematics disability before fifth grade, 95% continue to struggle with mathematics at the high school level (Shalev, Manor, & Gross-Tsur, 2005). Both students with a diagnosed mathematics disability and students who display low mathematics performance are referred to in the literature as students with mathematics difficulty (MD; Vukovic, 2012).

Students with persistent MD (i.e., continued difficulty across grade levels) are more likely than students without MD to have deficits in several mathematical areas. Specifically, elementary students with MD may have difficulty counting (Geary, Hamson, & Hoard, 2000), understanding and comparing numbers (De Smedt & Gilmore, 2011), performing basic arithmetic facts (Jordan & Montani, 1997), solving computation problems and equations (Chong & Siegel, 2008), and understanding word problems (Reikerås, 2009). These difficulties can lead to poor understanding of essential concepts necessary for problem solving in middle and high school grades, including college-entry courses such as algebra. Students with continued difficulty across grade levels in arithmetic facts, computation, and solving word problems are considered to have persistent MD (Vukovic & Siegel, 2010).

Students with MD can also experience difficulty in other academic areas, such as reading and language (Andersson, 2010; Jordan, Hanich, & Kaplan, 2003). The language involved in word problems can pose challenges to students with MD. Jordan, Levine, and Huttenlocher (1995) found that language ability, as measured by the Test of Language Development, influenced mathematics performance on verbal calculations and

story problems. Students in the sample came from a variety of ethnic backgrounds and socio-economic status; however, students participating in bilingual or English as Second Language programs were excluded from the study. This exclusion has implications for how ELs are classified with MD. Bryant, Bryant, and Hammill's (2000) study on the characteristics of students with MD found professionals frequently rated students with MD as having difficulty with solving word problems.

Students who demonstrate persistent MD are often referred for special education identification. Culturally and linguistically diverse (CLD) students, including ELs, can be disproportionally represented in special education (Artiles & Trent, 1994; Sullivan, 2011; Trent, 2010). Longitudinal data analyses indicate ELs are often underrepresented in special education at kindergarten and first grade, then overrepresented in late elementary and secondary grades (Artiles, Rueda, Salazar, & Higareda, 2005; Samson & Lesaux, 2009). Klingner, Artiles, and Barletta's (2006) review of existing literature on ELs and identification practices for LD reveals the need for further research on the complexities between second language acquisition and demonstrated achievement in the second language instruction.

Students who demonstrate academic difficulty in either their native language or English are at an increased risk for special education identification (Artiles et al., 2005). Samson and Lesaux (2009) found that language minority status, teacher ratings of language and literacy skills, and reading proficiency level can all be significant predictors of placement in special education. These predictors can prove particularly complex for ELs, as they are asked to demonstrate proficiency in a second language. Identification of ELs with learning disabilities can be difficult and variable, because decisions are based on student response to instruction and assessment in a nonnative language (Richards-Tutor et al., 2013). It is difficult to determine whether students who are experiencing difficulties have had sufficient and appropriate opportunities to learn and to demonstrate their learning (Lesaux, 2006). This difficulty has implications for whether ELs are accurately identified with a disability.

There has been an increase in the availability of interventions and assessments available in Spanish (e.g., Vaughn et al., 2006); however, these are not widespread and can be difficult for educators to implement if they do not share their students' native language. These interventions and assessments also do not address the needs of ELs who speak a language other than Spanish. The intersection of language, assessment, and instruction presents the risk of misidentifying ELs for special education during the decision-making process (McMaster, Kung, Han, & Cao, 2008).

Little is known about specific characteristics of ELs with MD. Traditional classifications of MD focus on mathematics performance, which may or may not be appropriate for ELs. Understanding the degree to which linguistic challenges influence mathematics performance can inform decisions regarding MD identification. There is a need for research on instructional techniques to promote mathematical problem solving for ELs, particularly for ELs with MD. This work is important because little is known about effective strategies to improve word-problem solving for ELs, despite the prevalence of word problems on high-stakes tests of achievement and the emphasis on problem-solving in the national Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and the National Council of Teachers of Mathematics content standards (NCTM, 2000).

Culturally and Linguistically Responsive Mathematics

In the present study, I take a critical stance towards improving outcomes for ELs who also identify with historically underserved racial and ethnic minorities. Culturally responsive pedagogy is an approach to promote academic achievement for CLD students, and represents a variety of age, gender, geographic, class, and privilege in classroom instruction (Ladson-Billings, 1995b). Culturally responsive pedagogy is also referred to in the literature as culturally responsive instruction or culturally responsive teaching (CRT). A primary purpose of CRT is to achieve equitable educational outcomes for student populations who have been historically marginalized, thereby working towards larger goals of social justice (Hernandez, Morales, & Shroyer, 2013). A critical aspect of CRT includes knowing and incorporating student identities, therefore, this instructional approach lends itself for teachers working with a range of learner characteristics in their classrooms.

Pursuing instructional approaches that promote equitable outcomes is especially critical in mathematics, where CLD students perform consistently lower on standardized achievement measures than their Caucasian, native-English speaking peers (NAEP, 2013). Additionally, African American and Latino students are often underrepresented in science, technology, engineering, and mathematics (STEM) college majors (Engberg & Wolniak, 2013). Culturally and linguistically responsive mathematics may hold promise for improving the academic and social outcomes for CLD students, including ELs who identify with historically underserved racial and ethnic minorities.

Culturally responsive mathematics instruction is defined in the literature as pedagogical knowledge, teacher beliefs, and instructional practices that promote

mathematical thinking, value student funds of knowledge, and incorporate issues of power and social justice in mathematics education (Aguirre & del Rosario Zavala, 2013). Funds of knowledge refer to experiences and understandings students bring into the classroom from their home community (e.g., measuring ingredients while cooking, economic practices of a local business), which can be leveraged in instruction (Moll, Amanti, Neff, & Gonzalez, 1992). In order to address complex issues of justice and equality in instruction, Turner and Strawhun (2007) state that the problem-solving contexts teachers present must be authentic to students' lived experiences, and that students should find the problem worth solving (e.g., using a mathematics measurement unit to analyze issues of school overcrowding). On a large scale, mathematics education can be used as a tool to analyze relationships of power and privilege through social and economic structures.

Specifically for ELs, research has focused on linguistically responsive teaching (LRT). Teachers implementing LRT incorporate linguistic supports (e.g., native language, grammatical supports, vocabulary development; Echevarria et al., 2006; Goldenberg, 2013) for their EL students because of the additional challenges of learning academic content in a second. Cultural and linguistic diversity are seen as assets to classroom learning experiences (Lucas, de Oliveira, & Villegas, 2014). Linguistically responsive mathematics instruction should be informed by ELs' prior experiences with mathematics content, language experiences and proficiencies, and educational histories (Moschkovich, 2013). Moschkovich's (2013) recommendations for equitable mathematics instruction for ELs include a focus on conceptual understanding and reasoning, strategic support for ELs' participation in mathematical discussions as they

learn English by drawing on available resources (i.e., objects, drawings, graphs, and gestures), and the value of native language and home experiences in instruction. Teachers can make mathematics instruction comprehendible for ELs by using familiar content and contexts, developing English vocabulary, using native language to support content understanding, and promoting collaborative discourse (Taube & Jasper, 2009).

Purpose of the Present Study

In the present study, I view culturally and linguistically responsive instruction as complementary approaches that are interrelated. Within CRT and LRT, teachers should consider the unique learning characteristics of their students including native language, English-language proficiency, race and ethnicity, home and community culture, and past educational experiences. Culturally and linguistically responsive mathematics instruction, therefore, should incorporate linguistic supports and effective strategies for the ELs to make content accessible and promote academic achievement, while also incorporating aspects of students' culture and experiences into mathematics content. In addition to facilitating mathematics achievement, culturally and linguistically responsive mathematics instruction may also help develop ELs' English language proficiency. Similarly, teachers should view diverse student experiences, perspectives, and languages as resources in their classroom. Word-problem instruction presents a unique opportunity to study culturally and linguistically responsive mathematics instruction, because of the role of context as well as linguistic complexities inherent in problems.

In the present study, I used a multiple-methods approach to study the role of culturally and linguistically responsive word-problem instruction. Using a single-subject, multiple baseline design, I investigated the efficacy of a culturally and linguistically responsive schema intervention (CLR-SI) for elementary ELs with MD. The CLR-SI intervention was designed using CRT, LRT, and schema-based word problem instruction for students with MD. Culturally and linguistically responsive approaches were integrated into an evidence-based instructional model that uses schemas (i.e., problem types) to teach students how to solve word problems. In addition to understanding the efficacy of this intervention, I also explored how a third-grade teacher, identified by her school as an effective teacher for EL populations, provided mathematics instruction. The majority of student participants in the intervention received instruction for CLD learners. To do so, I used interpretivist methods of observation, interviews, and document analysis. Both quantitative and qualitative methods were used to make inferences regarding effective word-problem instruction for ELs with MD. The following research questions (RQ) were investigated:

- (RQ1) What beliefs guide a third-grade teacher's implementation of culturally and linguistically responsive mathematics instruction?
- (RQ2) How does a third-grade teacher instruct CLD students to solve word problems?
- (RQ3) Do ELs with or at risk for MD perform better on solving mathematical word problems after receiving CLR-SI?

The study is unique in that it integrates research on culturally and linguistically responsive instruction and an evidenced-based practice for students with MD (e.g., word-problem schema instruction). In the present study, I am interested in studying mathematics instruction for ELs with MD. Low mathematics performance in EL

populations may be caused by a variety of factors (e.g., limited English language proficiency, a lack of time and instructional opportunities, prior educational history, computational difficulty, or conceptual problem solving difficulty), and there is limited research focused on instructional mathematics approaches for ELs with MD. Culturally and linguistically responsive schema word-problem instruction has not been investigated for either ELs or students with MD. Findings from this study have implications for teachers, administrators, and researchers of ELs with MD.

CHAPTER II: LITERATURE REVIEW

In this literature review, I first describe my theoretical framework for this research. Then, I discuss culturally and linguistically responsive mathematics instruction for ELs. Next, I provide a comprehensive overview of existing research on ELs and word-problem solving. Following this, I present literature on schema-based wordproblem instruction. I conclude with rationale for the present study and research questions.

Theoretical Framework: Culturally and Linguistically Responsive Instruction

The theoretical framework driving this research is culturally and linguistically responsive instruction. This framework includes theoretical underpinnings of CRT and LRT. In this section, I discuss CRT and LRT, and then describe how the two overlap. **CRT**

CRT is grounded in the idea that all students should experience academic success, develop and/or maintain competence and confidence in their cultural identity, and develop a critical consciousness that challenges the status quo (Ladson-Billings, 1995a). CRT includes institutional (e.g., administration policies and values; allocation of resources), personal (e.g., teachers' cognitive and emotional processes), and instructional (e.g., materials, strategies, and learning activities) dimensions (Taylor, 2010). CRT also involves incorporating students' culture and prior experiences to empower them academically and socially and is grounded in ideals of social justice, educational equity, and a dedication to facilitating educational experiences (Green, 2007). Central to CRT is

an understanding of the role of culture in the lives of both teachers and their students (Shealey et al., 2011). Generally speaking, CRT embraces cultural and ethnic diversity as a positive and valuable resource in a classroom (Gay, 2010). See Figure 1 for a visual of CRT. The three dimensions of CRT (e.g., institutional, instructional, and personal) are interconnected. There are three key tenets of the instructional dimension of CRT: (a) teacher knowledge and beliefs; (b) curriculum; and (c) instruction. Each of these tenets contributes to the interactions between teachers and their students.

Figure 1 Culturally Responsive Teaching Framework

	 Inco exp soci Eml posi (Ga Idea ded (Green) 	Culturally Responsive Teaching neorporates students' culture and prior xperiences to empower them academically and ocially (Green, 2007). Imbraces cultural and ethnic diversity as a ositive and valuable resource in a classroom Gay, 2010). deals of social justice, educational equity, and a edication to facilitating educational experiences Green, 2007).		
Institutional	1	Instructional	Person	al Dimension
e.g., administrat	tion	e.g., materials.	and emot	tional processes
policies and values;		strategies, and learning	(Tay	ylor, 2010)
allocation of resources		activities		
(Taylor, 2010))	(Taylor, 2010)		
 Teacher Knowledge an Beliefs Understands students' cultural values, tradition communication patterns, learning styles. Authentically cares abou students. Understands the role of t own cultural identities, beliefs, and assumptions (Gay, 2002, 2010; Seidl & Pug 2009) 	nd s, , and it their s. gach,	 Curriculum A diversity of cultural representations are included in curricular materials (Gay, 2002) Curriculum can be modified through four approaches: contributions, additive, transformation, and social action. (Banks, 2002; Gay, 2002; Seidl & Pugach, 2009) 	 Instruction hands- demonand manong Connection lives a explicion Classred welco environ feel retore (Gay, 200) 	Instruction ction includes visuals, on experiments, istrations, modeling, eaningful discussions g teachers and students ections to students' ire purposeful and it. room climate is a ming and safe onment where students espected.

Observable Teacher Actions

- Modify or create culturally representative curricula that include transformative or social action approaches.
- Make curricular connections with students' personal experiences and cultural heritage (including language) in classroom instruction.
- Use multiple resources in instruction (e.g., objects, drawings, graphs, and gestures) and outside of the classroom including home languages and experiences.
- Visit the homes of their students to learn from their families
- Reflect on the role of their own cultural identities, beliefs, assumptions, and experiences on their teaching practices and interactions with students and families.

(Banks, 2008; Gay, 2002, 2010; Klingner et al., 2012; Moll et al., 1992; Seidl & Pugach, 2009)

Teacher knowledge and beliefs. The first tenet of CRT relates to teacher knowledge and beliefs. With CRT, teachers should understand the cultural characteristics and contributions for a variety of student groups (Seidl & Pugach, 2009). This includes knowledge of cultural values, traditions, communication, learning styles, contributions, and relational patterns (i.e., appropriate interactions between children and adults in various settings, communal living, cooperative problem solving, gender role; Gay, 2002). Knowledge of cross-cultural communication patterns (i.e., linguistic structures, discourse features, vocabulary, intonation and delivery, body language, and speaking role relations) is also included under this tenet of CRT. Understanding the communication styles of different ethnic groups, including how these styles reflect cultural values and shape learning behaviors, is important for teachers to encourage the development and sharing of diverse learners' ideas (Gay, 2002). This sharing of ideas can include both communications inside the classroom with students as well as communication with students' families outside of school.

A critical part of this tenet is also how teacher beliefs about race, class, culture, ethnicity, and experience affect instructional behaviors (Gay, 2010). Effective teachers acknowledge the role of their own cultural identities, beliefs, assumptions, and experiences on their teaching practices and interactions with students and families (Shealey et al, 2011). This awareness can include implicit bias or prejudice (Lai, Hoffman, & Nosek, 2013), stereotype threat (i.e., individuals feeling at risk of confirming negative stereotypes about racial, ethic, or social groups they identify with; Steele, 2011), and teacher efficacy (Protheroe, 2008). Teacher knowledge and beliefs can directly influence curriculum, the second tenet of CRT.

Curriculum. The second tenet of CRT relates to curriculum. In addition to prioritizing knowledge about ethnic and cultural diversity, teachers need to develop the skills necessary to design culturally responsive curriculum and instructional strategies and modify existing curricular resources to be more culturally responsive (Gay, 2002). At a basic, superficial, level teachers can incorporate cultural elements (i.e., food, holidays) and themes into curricular units. Isolated incorporation of cultural elements; however, is not sufficient to truly transform curricula for CLD students towards more equitable outcomes (Banks, 2008). CRT includes purposefully representing a variety of age, gender, geographic, class, and privilege within and across ethnic groups in pictures and activities throughout classroom learning. Meaningful curricular modification could include encouraging students to analyze learning objectives from differing perspectives, including their own, and encouraging students to make their own decisions on social issues and take action to help solve them either through class or individual projects (Banks, 2008). Teachers can modify curricular materials to better reflect the experiences of their students and use curriculum as a tool to engage CLD learners in content.

Instruction. Although curriculum is a key component of CRT, the pedagogical knowledge and skills teachers demonstrate in action are just as critical. The third, and most observable, tenet of CRT is instruction. Instruction refers to the presentation of content, learning activities, and classroom environment. In CRT, teachers make explicit connections between instructional content and students cultural heritage, experiences, pop culture, etc. (Klingner & Edwards, 2006; McIntyre, 2010). Teachers should view student culture and identity, including native language, as resources and seek to incorporate them into classroom instruction (Klingner & Edwards, 2006; Klingner et al.,

2012; Moschkovich, 2013). With CRT, teachers can draw on multiple resources inside the classroom (i.e., objects, drawings, graphs, and gestures) and outside of the classroom (i.e., native languages and experiences) to make classroom content engaging and accessible for all students.

Instruction also includes classroom climate and behavior management. Creating a classroom environment that is safe and supportive for a diverse group of students is highly emphasized in CRT. This includes going beyond "best practices" of behavior management and classroom organization and explicitly using students own experiences and perspectives as a form of cultural scaffolding to make academic content relevant and meaningful in students lives (Gay, 2002). According to Gay (2002), for teachers to effectively create a safe learning communities they need to authentically care about students' individual backgrounds and value their unique experiences (Gay, 2002). Caring for students is less tangible than other aspects of the instructional dimension of CRT; however, there are observable actions teachers can take to foster strong relationships with students. For example, teachers can visit the homes of their students to learn from their families and incorporate this knowledge into the classroom (Moll et al., 1992).

LRT

In LRT, culture and language are interconnected. LRT is often discussed in the literature in terms of effective instruction for ELs. There are many approaches to effective instruction for EL students. Goldenberg (2013) states that many general instruction strategies are also effective for ELs including: (a) clear goals and objectives; (b) appropriate and challenging material; (c) well-designed instruction and classroom routines; (d) clear and explicit instructions; (d) purposeful modeling of skills and

procedures; (e) student engagement and participation; (f) instructional feedback; (g) opportunities to apply and transfer learning to new situations; (h) practice and review; (i) structured interactions with peers; (j) frequent assessment; and (k) established classroom routines and behavior norms. In addition to generally effective practices, Goldenberg (2013) also states that EL students need additional instructional supports to be successful. How teachers and schools provide these additional supports can vary from full bilingual programming to modifications within English-only instruction (de Jong, 2010). An example of supports and modifications in English-only instruction is the use of sheltered instruction. Sheltered instruction is an approach that makes grade level content accessible to ELs by incorporating strategies that facilitate the second-language acquisition process (Echevarria, Short, & Powers, 2006). For the purpose of the present study, I conceptualize LRT more broadly as a set of principles to guide instruction for ELs.

LRT is an approach that emphasizes teachers' understanding of sociolinguistic consciousness (i.e., the relationship between language and sociocultural and sociopolitical factors), valuing linguistic diversity, advocating for the learning needs and experiences of ELs, learning about students' language experiences and proficiencies, identifying language demands in classroom instruction (i.e., complexity of text), providing instructional scaffolds (i.e., activating prior knowledge, using multi-modal materials, etc.), and applying key principles of second language learning in instruction (Lucas et al., 2014). Lucas et al. (2014) describe key instructional principles of second language proficiency; (b) access to comprehensible input just beyond students' current level of

understanding; (c) the role of social interaction; (d) transfer between first and second language learning; and (e) anxiety associated with instruction in a second language. Each of these principles is briefly described.

Conversational and academic language. The differences between conversational and academic language can be addressed in several ways. The concept of academic language, and its role in EL instruction, is debated in the literature. Generally speaking, academic language is defined as the vocabulary, grammatical structures, and linguistic functions that students will engage with and must master to be successful in content areas (Cummins, 2000). Traditional views of academic language focus on separating content-specific terminology (e.g., trapezoid) that may be considered decontextualized and cognitively demanding, from less demanding conversational terminology (e.g., shape); however, the role of context is highly debated (Bunch, 2006). Other scholars believe that academic language encompasses the context it is used in, both formally and informally, to develop meaning (Schleppegrell, 2004).

Comprehensible input. Comprehensible input (i.e., language which learners process for meaning and which contains something to be learned; Ortega, 2009) is considered to be at an instructional level slightly above the students' current level of proficiency and understanding (Rodrigo, Krashen, & Gribbons, 2004). Opportunities for comprehensible output (i.e., communication in a second language) should be considered for both the academic content and the English language used to communicate this content (Ortega, 2009). In addition to comprehensible input, comprehensible output is also critical for students to increase their awareness of their language proficiency, reflect on linguistic structure and form, and practice negotiating meaning (Lucas, Villegas, &

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Freedson-Gonzalez, 2008).

Social interaction. A key mechanism in second-language learning is the role of peer discourse, particularly with students who are linguistically and academically knowledgeable in the second language (Ortega, 2009). Social interactions among peers can be informal or structured learning opportunities, and these interactions allow ELs to practice both language and content. Teachers can carefully orchestrate student discussions around academic content "so that students have an opportunity to learn, develop, and practice the language of disciplines, while constructing new understandings about content" (McIntyre, 2010, p. 66). Social interaction moves beyond comprehensible input and output, by allowing for further engagement and negotiation of concepts with their peers (Lucas et al., 2008).

Transfer between languages. A concept critical to LRT is the understanding that proficiency in native language is a resource for proficiency in a second language (Lucas et al., 2008). Students can transfer knowledge between their two languages, both in terms of linguistic structure and academic context (Ortega, 2009). Teachers should seek out information about students' prior educational experiences, including language and literacy backgrounds to inform instruction for ELs (Lucas et al., 2014).

Anxiety. The process of learning academic content in a second language can cause anxiety for ELs (Lucas et al., 2014). ELs perceptions of self in relation to their classroom peers can have implications on their motivation and investment in learning (Norton & McKinney, 2011). Pappamihiel's (2002) analysis of EL anxiety found that EL anxiety is not necessarily related to achievement in mainstream classrooms. Anxiety can cause ELs to withdraw from social interaction, which can negatively affect their language development (Pappamihiel, 2002). Teachers should be mindful of the pressures of performing in a second language and create a classroom environment that is warm, welcoming, and safe for all students may encourage ELs to take academic and social risks (Lucas et al. 2008). Teachers should also provide classroom instructions that are clear and explicit for students to follow (Gersten, Baker, Haager, & Graves, 2005; Lucas et al., 2014).

LRT and CRT Overlap

There is a high degree of overlap between the guiding principles of CRT and LRT. In the present study, I focus on eight principles to guide inferences regarding culturally and linguistically responsive mathematics instruction. These principles, and the purposes each serves in CRT and LRT, are outlined in Figure 2. The eight principles include: (a) viewing student culture and identity, including native language, as resources and seeking to incorporate them into classroom instruction; (b) incorporating student ideas and experiences and providing relevant instructional examples to participants' daily lives and cultural heritage; (c) involving families of students; (d) creating a classroom environment that is safe and supportive for a diverse group of students; (e) facilitating oral discussions with students and encouraging peer interactions and discourse; (f) using multiple resources in instruction (e.g., graphic organizers manipulatives, objects, drawings, graphs, and gestures) to help students organize information and clarify concepts; (g) reflecting on the role of teachers' own cultural identities, beliefs, assumptions, and experiences on their teaching practices and interactions with student and families; and (h) incorporating purposeful language supports and scaffolds (e.g., vocabulary development, sentence frames etc.) into lessons.
Figure 2 Culturally and Linguistically Responsive Overlap

Instructional Aspect	СРТ	IDT
Vigwing student culture and		LKI Stratagiaally using native lenguage
identity, including native language, as resources and seeking to incorporate them into classroom instruction	historically underserved students through changing the curriculum and instruction experiences students engage in (Gay 2002; Ladson-Billings, 1995b).	and experiences to help students understand content (Gersten et al., 2005; Klingner & Edwards, 2006; Klingner et al., 2012; Moschkovich, 2013).
Incorporating student ideas and experiences and providing relevant instructional examples to participants' daily lives and cultural heritage	Engaging students in the content by making the material meaningful and relevant. Students are actively contributing to the classroom content, which in turn validates their experiences as legitimate school content (Banks, 2008; Gay, 2010; Klingner & Edwards, 2006; Ladson-Billings, 1995b).	Connecting concepts to students lives to help them develop an understanding of content in authentic and relevant situations (Echevarria et al., 2006; Gersten et al., 2005; McIntyre, 2010).
Involving families of students	Viewing students' families as a resource to learn from and visiting the homes of their students to learn from their families (Moll et al., 1992).	Connecting with students' families inside and out of school to learn from their perspectives and experiences, as well as how to support advancement in both native and secondary language (McIntyre, 2010).
Creating a classroom environment that is safe and supportive for a diverse group of students	Facilitating a classroom culture where students feel respected and valued as individuals. Teachers solicit student' ideas and incorporate into class content (Gay, 2002).	Creating well-established classroom routines and behavior norms (Goldenberg, 2013; Lucas et al., 2008).
Facilitating oral discussions with students and encouraging peer interactions and discourse	Encouraging a positive classroom climate and soliciting students' ideas and perspectives on issues that truly matter to students (Turner & Strawhun, 2007).	Providing structured, focused interactions with other students to develop English language and content understanding (Goldenberg, 2013; Lucas et al., 2008).
Using multiple resources in instruction (e.g., graphic organizers manipulatives, objects, drawings, graphs, and gestures) to help students organize information and clarify concepts	Using manipulatives and cultural artifacts (i.e., objects from their home and community) to engage students in the content, and to represent content across for various cultural backgrounds (Banks, 2008; Shumate et al., 2012).	Modeling, using gestures, and examples. Planning hands-on activities for students to use manipulatives/objects to develop language and content understanding through authentic contexts. Making concepts and the relationships among concepts visually clear as a language scaffold (Barone, 2010; Echevarria et al., 2006; Moschkovich, 2013).
Reflecting on the role of teachers' own cultural identities, beliefs, assumptions, and experiences on their teaching practices and interactions with student	Acknowledging the role of teacher identity and background including implicit bias, stereotype threat, and teacher efficacy (Shealey et al., 2011).	
Incorporating purposeful language supports and scaffolds (e.g., vocabulary development, sentence frames etc.) into lessons		Supports are used to facilitate English language development and understanding of content (Goldenberg, 2013; Lucas et al., 2008; 2014).

Culturally and Linguistically Responsive Mathematics Instruction for ELs

For the purpose of the present study, I focus on culturally and linguistically responsive mathematics instruction for ELs. Culturally and linguistically responsive mathematics instruction for ELs can include: (a) connecting mathematics to real-life experiences; (b) using mathematics as a tool to create a learning community; (c) purposefully using questions to facilitate student language development and understanding of mathematics concepts; and (d) explicitly teaching vocabulary in context and with multi-modal activities (Moschkovich, 2013; Torres-Velasquez & Lobo, 2005). There is limited empirical evidence on the effects of culturally and linguistically responsive mathematics instruction for ELs. In this section, I summarize existing research on culturally and linguistically responsive mathematics instruction for ELs.

Quantitative Studies

Shumate, Campbell-Whatley, and Lo (2012) used a multiple-treatment reversal design to study the effectiveness of culturally responsive mathematics instruction for five Latino ELs with learning disabilities in an eighth-grade resource classroom. The dependent variable was the gain in the number of correct responses on the posttest compared to the pretest using a daily quiz of simple one-step mathematical calculations. Questions included pre-algebra, algebra, and geometry. Word-problems were not included as a dependent measure. Social validity data was also collected.

The resource teacher provided instruction in each phase of the intervention. In the baseline condition (e.g., phase A), the teacher used traditional methods of lecture, textbook examples, individual questioning, and practice drills. In the intervention phase (e.g., phase B), culturally responsive mathematics instruction was defined as: explicitly

stating measureable lesson objectives, explaining how students would meet these objectives, providing guided notes, facilitating oral discussions in small groups and with partners, using graphic organizers and tools (e.g., highlighters), and providing relevant instructional examples to participants' daily experience, pop culture, and their cultural heritage. Language supports for ELs included facilitation of oral discussion and using graphic organizers to represent content. After students did not respond to the intervention phase, culturally responsive instruction was modified. This third condition, modified culturally responsive instruction (e.g., phase C), continued using the strategies from phase B but also incorporated manipulatives and game activities. The game activities used vocabulary and language to build the background knowledge relevant to the new material. Instruction in the three conditions was provided in English.

Findings from Shumate et al. (2012) indicate there was a functional relationship between the participants' mathematics performance and the modified culturally responsive instruction (e.g., phase C). In addition, participants demonstrated higher levels of interest and engagement in culturally responsive conditions compared to the traditional condition. Shumate et al.'s findings hold promise for infusing culturally and linguistically responsive strategies into mathematics instruction for Latino ELs with MD. However, it is unclear the extent to which aspects of culturally and linguistically responsive instruction impacted student performance and which are just generally effective mathematics practice.

Freeman and Crawford (2008) designed Help with English Language Proficiency (HELP) Math, which is a web-based supplemental curriculum focused on mathematics vocabulary and academic concepts for Latino middle school ELs. HELP Math is based on the SIOP model and applies instructional practices for ELs. These instructional practices include: targeting vocabulary development, providing native language vocabulary support, and building on student background knowledge in an online environment. Initial findings from HELP Math field tests across multiple states indicate promising results for ELs with higher levels of English proficiency, but findings appear less effective for new language learners (Freeman & Crawford, 2008). Later models of the program were revised to build in more language supports (i.e., summaries of content in native language).

Crawford (2013) later conducted a randomized control trial to measure achievement differences between middle school ELs who used HELP Math and students using other computer-based mathematics programs. All students in the sample (N = 396) demonstrated statistically significant gains from pre- to posttest; however, there were not statistically significant differences between the two groups. ELs with higher Englishlanguage proficiency who participated in the comparison condition performed significantly better than ELs in the HELP Math condition. This finding is surprising, and further research is needed to understand which language supports affect the performance of ELs with differing language proficiencies and cultural backgrounds.

Qualitative Studies

Moschkovich's (1999) study of mathematical discourse patterns in a bilingual classroom highlights how one teacher developed academic language in a mathematics lesson involving technical words such as trapezoid and parallelogram. This teacher did not focus on grammatical or word meaning errors, but instead, provided opportunities for students to develop understanding by focusing on the mathematical content of student contributions, asking students for clarification, revoicing student statements, affirming and building on student responses, and revoking student statements (Moschkovich, 1999).

Similarly, Moschkovich (2008) studied mathematics discourse in an eighth-grade bilingual classroom through analysis of videotaped whole-class discussions, small group discussions, and problem solving discussions between pairs of students. The researcher found that the teacher's role in facilitating mathematical discourse allowed students to engage with multiple meanings of mathematical language (i.e., slope, axis) to construct a conceptual understanding of graphing. Utilizing language as a resource to make connections and facilitate discussion can help develop mathematical concepts. This in turn may lead to greater understanding for ELs.

Aguirre and del Rosario Zavala (2013) studied culturally responsive mathematics instruction in the context of teacher professional development. The researchers used a qualitative research design to analyze the development and impact of culturally responsive mathematics instruction on teacher pedagogical decisions, visions, and mathematics instruction over time. Data was collected for six beginning teachers who taught mathematics in schools with high CLD student populations. Throughout the data collection and analysis process, the researchers developed a culturally responsive mathematics lesson analysis tool. Two themes emerged from analyses. First, the researcher found that the lesson analysis tool enabled teachers to reflect and critique their lessons in terms of mathematical thinking, language, culture, and social justice. Second, teachers negotiated multiple interpretations of how to incorporate cultural and community funds of knowledge in mathematics instruction. Specifically regarding ELs, Aguirre and del Rosario Zavala (2013) described how a teacher in the sample self-reflected on the degree to which she incorporated her ELs linguistic funds of knowledge (e.g., native language) into her classroom. In considering student native language as an instructional resource, the teacher discussed ways she could label mathematical concepts in multiple languages and solicit student input by inquiring how they would explain concepts (i.e., halves) in their native language. This selfreflection ultimately led the teacher to acknowledge her need for further training on effective EL strategies in addition to incorporating student native language and ideas into the classroom (Aguirre & del Rosario Zavala, 2013). Further research is needed to understand the effects of such practices in the context of mathematics instruction for ELs.

Mixed-method Study

As previously mentioned, a well-validated approach to teaching content area subjects, such as mathematics, to ELs is sheltered instruction (Echevarria, Short, & Powers, 2006). In sheltered instruction, teachers use the general academic curriculum (e.g., mathematics, social studies, science, English language arts) and modify instruction to make the content accessible while purposefully promoting English language development. Sheltered instruction techniques include slow and clear speech enunciation, visuals and demonstrations, instructional scaffolds, purposeful vocabulary development, peer interactions, incorporation of student experiences, adapting and supplementing curricular materials (Echevarria et al., 2006).

Echevarria et al. (2006) designed and field-tested the Sheltered Instruction Observation Protocol (SIOP) for teaching ELs in content area subjects. This study did not focus exclusively on mathematics instruction; however, mathematics was one of the core content areas in which teachers provided sheltered instruction to ELs. The SIOP tool measured the extent to which teachers incorporate language and content objectives, use supplementary materials, the meaningfulness of activities, make meaningful connections with students' background experiences, develop academic vocabulary, adjust their speech, model academic tasks, use multimodal techniques to enhance comprehension, provide explicit instruction on strategies to help students access and retain information, scaffold instruction, promote higher order thinking skills, encourage elaborated speech, and to group students appropriately, extend language and content learning, present a lesson that meets planned objectives, assess student learning, provide feedback, and review lesson concepts at the conclusion of the lesson.

Teachers of 346 ELs enrolled in public middle schools across the United States participated in the study. Teachers participated in an initial three-day training on the SIOP model and several professional development workshops throughout the school year. The researchers videotaped instruction in core academic content, including mathematics, at three separate time points in the school year (i.e., fall, winter, and spring). Videotapes were rated on the SIOP protocol and feedback was provided to participating teachers on an ongoing basis. These ratings were analyzed with ratings of the comparison teachers who did not receive any SIOP training or ongoing feedback. Qualitative data included written feedback from observations, electronic discussion, teacher evaluations and reflections, and researcher notes. Echevarria et al. (2006) found that teachers who received feedback on their performance and on SIOP protocol incorporated more elements of sheltered instruction. Students in the SIOP teacher classrooms made significantly better gains on language outcome measures then students in comparison teacher classrooms. Mathematics outcomes were not included in the study.

Summary of Literature

The existing literature on culturally and linguistically responsive mathematics instruction indicates promise for improving mathematics achievement for ELs. The present study will extend the existing literature on culturally and linguistically responsive mathematics for ELs in several important ways: focus on elementary students to investigate the effects of culturally responsive instruction on younger students, strategically incorporate language supports, and increase the complexity of mathematics instruction by focusing on word-problems. In addition, the present study will focus specifically on improving mathematics outcomes for ELs with MD.

Word-Problem Solving and ELs

Of the existing research on culturally and linguistically responsive mathematics instruction for ELs, few focus on word-problem solving instruction. In this section, I provide summaries of existing research on elementary ELs and word-problem solving. First, I discuss quantitative studies. These studies typically focused on testing the effectiveness of an instructional strategy or assessment. Then, I describe qualitative studies. These studies focused heavily on the processes and interactions students engaged in to make meaning of word problems. Finally, I present a mixed-methods study.

Quantitative Studies

Orosco, Swanson, O'Connor, and Lussier (2013) assessed the effectiveness of a mathematics comprehension strategy, Dynamic Strategic Math (DSM), on the word

problem-solving performance of ELs. DSM was defined in this study "as the researcher systematically modifying the vocabulary to the individual student's understanding level of the word problems and then providing strategy instruction with probes that assessed students' ability to solve problems" (Orosco et al., 2013, p. 97). Orosco et al. employed a multiple baseline design across 6 second-grade Latino ELs at risk for mathematics difficulty and disability. Students were selected based on the following criteria: low reading and mathematics achievement scores, teacher recommendations, Spanish as a native language, and early to intermediate English proficiency on the *California English Language Development Test*. Although the students' socio-economic status was not indicated, it is mentioned that the school had a high poverty rate.

Orosco et al. (2013) created their dependent measure, DSM Assessment Probe (DSMAP), which was designed to reveal differing levels of word-problem skill, with and without assistance, through a series of five levels. A bilingual trained classroom teacher and the first author of the study provided the intervention on a one-to-one basis for 17 sessions over a 5-week period as a supplementary intervention to the 50 minutes of general education mathematics students received per day. The intervention consisted of 20 to 25 minutes instructional sessions including: (a) pre-teaching mathematics concepts; (b) comprehension strategy instruction that connected to mathematics concepts; and (c) hints or scaffolds. Linguistic modifications, or scaffolds, included minimizing sentence length, rephrasing mathematics problems, and removing irrelevant language or sentences. Native language use was not provided as an instructional scaffold. At the conclusion of each session, students were individually administered four word problems based on their problem-solving level without help.

Compared to the baseline phase, introduction of the DSM intervention increased the level of word-problem solving for all participants. Follow-up probes three weeks later indicated that students were able to maintain a higher level of word-problem solving as a result of the DSM intervention. Social validity interviews indicated students and their teachers felt positively about DSM but recommended the incorporation of more visual aids and manipulative tools.

There are several strengths noted in Orosco et al.'s (2013) study in terms of linguistically responsive instruction. First, the researchers provide detailed participant selection criteria, including how they determined language proficiency levels. They also took an individualized approach to matching support to students' linguistic and mathematical needs. In addition, this was the only reviewed study that tested the effectiveness of an instructional intervention. Limitations include the small sample size and degree of individualization. Although the high level of individualization was effective for students, it is unclear how these findings might transfer to a classroom setting. Implications of this study include further research on explicit word problem solving instruction for ELs, including the use of visual aids and manipulative tools.

Cuellar, De La Colina, and Cmajdalka, (2005) investigated the performance of ELs on three-digit subtraction word problems with regrouping. The participants included 74 Latino students identified as ELs in four bilingual fourth-grade classrooms in two Texas school districts. Disability status was not provided for any of the participants. Students were assessed individually on two pre-selected subtraction word problems involving regrouping on the Mathematical Problem Solving Process Assessment (MPSPA). The five problem-solving process skills that were measured include: explaining the problem, estimating the final answer, representing the problem, solving the representation, and explaining the final answer. A linguistically responsive aspect of the study was that students were given the choice of solving the mathematics problems in either English or Spanish. Wilk's lambda stepwise method was used to analyze the assessment data.

Cuellar et al.'s (2005) results indicate that the process skills significantly related to arriving at a correct answer were representing the problem and solving the representation. Estimating the final answer, understanding the problem, and explaining the final answer were not significantly related to arriving at the correct final answer on multi-digit subtraction problems. Strengths of this study include the relatively large sample size and inclusion of native language in assessment. A weakness is that students language choice was not taken into account in the data analysis of process selection and application. Students were able to select their preferred language to solve word problems; however, this preference was not analyzed. Further study is needed to determine how each process might be used in instruction to help ELs solve world problems, particularly with using visual and object representation. In addition, how students' selection of language influenced, or mediated, the processes to find the correct answer should be explored.

Also related to word problems, Ambrose and Molina (2010) compared the performance of a group of Latino first graders on word problems presented in both Spanish and in English. They were also interested in whether success rates differed on various problem types in each language. The conceptual frameworks driving this study were Cognitively Guided Instruction (CGI) problem types (i.e., Part-Part-Whole; JoinChange-Unknown) as well as Cummins (2000) distinction of academic language (i.e., abstract and decontextualized) from social everyday language (i.e., concrete and context embedded). CGI is based on the theory that underlying structures of word problems influence the strategies can students use to solve a variety of problem types (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993; Carpenter, Hiebert, & Moser, 1981). For examples of CGI, refer to Figure 3 on page 45.

Participants in the Ambrose and Molina (2010) study included 16 six- to sevenyear old Latino ELs from two schools within the same school district. In this school district, 80% of the students in the district received free or reduced lunch and all mathematics instruction was in English. All students in the sample had been identified as Spanish speakers with either early intermediate or intermediate level English proficiency (i.e., could generate simple sentences in English and had limited English vocabularies) using the state language development test. Participants' receptive vocabulary ranged from 3.5 to 7.25 years age equivalent in English and 2.92 to 6.5 years age equivalent in Spanish as measured by the *Peabody Picture Vocabulary Test (PPVT*. It was not mentioned if any students in the sample were identified with disabilities. In this descriptive study, all students were interviewed in English and Spanish using a dynamic assessment model. Interviews lasted 20 to 30 minutes and were conducted first in Spanish then English. Word-problem tasks were parallel in both languages; meaning problems were similar in CGI type, magnitude, and language complexity. Participants' problem-solving strategies were coded in four categories (e.g., direct modeling, counting, derived facts, and fact strategies).

Ambrose and Molina's (2010) results indicated students' performance was slightly higher in English than in Spanish but lower than monolingual students from other comparison studies. Eight students performed better in English, seven students performed higher or slightly higher in Spanish, and one student performed similarly in both languages. Results were not presented for specific student language proficiency levels. When analyzing performance on types of problems, researchers found about 70% of the sample were successful solving both versions of the Separate-Result-Unknown problem and the Part-Part-Whole unknown problem in both languages. Students encountered some difficulties in the Join-Change-Unknown and the most difficulty in the Compare-Difference-Unknown problems in both languages.

Strengths of this study (Ambrose & Molina, 2010) include specifying participants' language proficiency in both Spanish and English on the *PPVT*, as well as analyzing academic performance in relation to language proficiency. In addition, the researchers provided resources, such as manipulative blocks, for students for students to select and use while problem solving. One key limitation of the study is that native language was assessed first for all students and all problems, thus is it unclear if an order effect is present. Comparing results to monolingual populations in past studies is informative; however, more information is needed on past participants and study conditions to determine if direct comparisons can be made. In addition, there are mixed results on the feasibility and effectiveness of assessing students in their native language when instruction and content are provided in English (Townsend & Collins, 2008). In this study, all mathematics instruction in the students' classroom was English. Ambrose and Molina (2010) found that while some ELs seemed excited about solving word

problems in Spanish, during the task many students seemed to prefer working in English. Further research is needed to understand student perspectives on solving word problems in primary and secondary languages. Future studies should investigate the effectiveness of explicitly teaching CGI problem-types for ELs in both English and their native language, as well as how problem type influences problem-solving process.

Qualitative Studies

Barwell (2003) was interested in how one EL student made sense of mathematics classroom interactions, while working on a word-problem task with a monolingual peer. This discourse analysis used an interaction theoretical framework to collect and analyze qualitative data. An interaction framework takes into account the individual perspectives, diversity, and cultural experiences of both participants by focusing on what they each do and say.

Participants from Barwell's (2003) study included one 9-year old EL who immigrated to the United Kingdom from Hong Kong and one native-English speaking student. The student from Hong Kong's native language was Cantonese, but she also spoke some Mandarin. The monolingual peer in the interaction was a native English speaker. The two students were removed from the classroom to write and solve their written word problems. Participant conversations were recorded and transcribed for analysis.

Barwell refers to his analysis of word-problem solving as patterns of attention. Three patterns of attention emerged from Barwell's (2003) analysis of the transcripts: (a) attention to narrative experience or forms of explanation (e.g., discussion on what present the student should buy her mom in the word problem); (b) the genre of word problems

(e.g., content and grammatical set up of the word problem); and (c) mathematical structure of their problems (e.g., addition, multiplication, division, etc.). Barwell found that allowing students to write their own word problems provided students the opportunity to negotiate meaning with each other despite language barriers.

Strengths of the study include Barwell's (2003) detailed analysis of how the EL student adjusts her language use based on the modeling and informal corrections provided by the monolingual student. A limitation is that the interactions were observed outside of the natural instructional setting. It is unclear how the students' discourse would be different in their mathematics class. The participants were selected for analysis after Barwell observed the two students occasionally working together in class. It is unclear how these patterns of interaction would change if the EL student worked with a monolingual peer she was less comfortable with. Barwell's patterns of attention have implications for several future studies. Genre and structure of the word problem are related to approaches such as CGI or schema instruction, where students are explicitly taught word problem structures. Attention to narrative experience has implications for how teachers can leverage and use students' personal and cultural experiences to relate to mathematical content.

Barwell (2005) expanded his analysis of attention in word-problem interactions (2003) to a larger participant pool. In this study, he reports results from a three-year study of the participation of elementary ELs in an arithmetic word-problem task. He again uses a social interaction theoretical framework to frame his discourse analysis of data taken from a larger ethnographic study.

Data in Barwell's (2005) study was collected longitudinally in fifth-grade classes

taught by the same teacher. Ten EL students (9- to 10-years old) participated in the study. Their language proficiency was described as "becoming familiar with English" and "conversationally proficient" (Barwell, 2005, p. 334). Barwell does not specify if any of the ELs in this sample were diagnosed with a disability. Data collection included ethnographic observations of mathematics lessons over several months. Barwell also collected copies of students' work, information about students' attainment, interviews with teachers, classroom observation notes, and video-recordings of mathematics lessons. Primary data consisted of 10 to 25 minute recordings taken of students working together to write and solve basic arithmetic word problems, such as addition or division, which were transcribed for analysis. For these tasks students were grouped in a combination of ways including: two EL students sharing a home language, two EL students from different language backgrounds, one EL student with one monolingual student, and two monolingual students. Although not required by Barwell, English was the only language used during the word-problem tasks.

Barwell (2005) systematically coded full transcripts for sequences of interactions which exemplified the three patterns of attention found in his earlier work: (a) attention to narrative experience or forms of explanation; (b) the genre or type of word problem; and (c) mathematical structure of their problems (Barwell, 2003). In the analysis, a fourth pattern of attention emerged: attention to written form (e.g., punctuation, spelling, verb tense, etc.). The transcript analysis indicated that students' attention to narrative experience was used to connect word problems and their own experience, as well and negotiate meaning with each other. Students used language but the focus was not on the correctness of language use but the mathematical concepts. Despite the still relatively small sample size, a key strength of this study is Barwell's (2005) expansion to a broader participant pool to extrapolate his findings from his 2003 analysis. A weakness is the limited information on the linguistic and cultural characteristics of the student population in the sample. Students are writing and solving their own word problems in all of the transcripts analyzed. Patterns of attention may differ when students are asked to solve word problems with unfamiliar contexts. Although Barwell's (2005) analysis includes attention to problem structures and narrative, he does not report if students were exposed to or solved a variety of word problem types. Future studies should continue to explore how interactions with peers, both monolingual and students who share similar language backgrounds, influence ELs understanding and performance on word problems. Investigating how problem type and structure influence this performance may also provide valuable instructional information. Finally, a key implication for both researchers and practitioners is the importance of context and relevance of word problems for ELs.

Bautista, Mulligan, and Mitchelmore (2009) explored the problem-solving strategies Filipino children demonstrate when solving addition and subtraction word problems written in either English or Filipino. The researchers did not specify a guiding theoretical framework; however, they discussed the importance of reading and language in word problems in their literature review. Participants included seven young Filipino children (grades 1 through 4) in a public school in a low-income area of Manila. The native language of students was Filipino, but academic instruction was provided in English.

Bautista et al. (2009) conducted task-based interviews in a community center

where they asked students to read and solve addition and subtraction word problems in English or Filipino. Students were first presented word problems in English and, if the student demonstrated difficulty, the same problem was then presented in Filipino. Students read the problems themselves so reading strategies could be analyzed. Findings indicate the language the word problems were presented in led to differences in students' solutions. Students were better able to comprehend and solve problems in Filipino, and there was a higher occurrence of errors and non-attempts in English problems. Bautista et al. also found that students rarely used reading strategies, such as decoding, and rarely used drawings or objects to represent word problems. Participants had difficulty solving compare-difference problems in both languages.

Strengths of Bautista et al.'s (2009) study include both a wide range of elementary grade levels and providing manipulatives for students to use when solving word problems. A limitation of the study is that the level of student proficiency in each language is unclear, as is the order in which problems were presented in each language. Future research should explore the use of manipulative tools to facilitate language comprehension, reading strategies in word problem instruction, and mathematics instruction using native language.

Mixed-method Study

Turner and Celedón-Pattichis (2011) explored mathematics problem solving for ELs in three kindergarten classrooms. The researchers used two theoretical perspectives to guide their research: opportunities to learn (e.g., time and quality of instruction) and CGI problem solving. Mixed methods were used to examine teaching practices that engaged Latina/o students in problem solving and determine student performance. Three

kindergarten teachers and their students in three low-income schools participated in the study. In one classroom, all students were ELs. In the other two classrooms, about half of the students were ELs and half were native English speakers. Classroom instruction ranged from a bilingual model, a 50/50 dual-language model, and an English-only model. All teachers had recently attended a two-week professional development seminar on CGI.

Turner and Celedón-Pattichis (2011) observed and videotaped five problemsolving lessons for each teacher. All recordings were transcribed and coded for teacher and student actions (i.e., native language use, connecting to personal experiences, questioning). Analytic induction was used to identify patterns related to instructional practices that emerged. Each pattern was triangulated across multiple data sources (i.e., researcher field notes, lesson transcripts, teacher interviews) for each of the three teacher's data from the three teachers. Seven students from each classroom (N = 21 total) who represented a range of achievement levels were also selected for problem-solving assessment interviews. Disability status was not provided for any of the students. A preand posttest of counting tasks and simple word problems was administered orally in each student's dominant language. Testing administrators were instructed to read a problem as many times as needed, and students had access to manipulative tools (i.e., cubes, counters, etc.). Researchers used Carpenter et al.'s (1993) coding scheme, which included (a) the strategy used (i.e., direct modeling, counting, recalled fact); (b) whether the strategy was valid or invalid; and (c) whether the answer was correct.

An analysis of variance (ANOVA) was used to test for differences in class mean scores across the three teachers. The difference in overall posttest means was significant, indicating that the students in the bilingual classroom performed significantly better on

the posttest. A post-hoc analysis by assessment item revealed significant differences on four of the more challenging problem types: multiplication, measurement division, partitive division, and compare. Qualitative analysis indicated that students in this teacher's class received mathematics instruction in their native language. Students in the bilingual class also had more opportunities to learn since they participated in more problem-solving lessons per week (e.g., three times a week) than the other two classrooms (e.g., one to two times a week). Their results indicate that ELs benefited from frequent opportunities to solve linguistically and contextually rich problems (i.e., drawing upon home and community-based knowledge and experiences as resources).

The main strength of Turner and Celedón-Pattichis' (2011) study was that the researchers described aspects of instruction they considered to be culturally and linguistically responsive. The researchers incorporated both a qualitative and quantitative measure to investigate teacher and student behaviors across three different classrooms. A key limitation was that the pre- and posttest sample sizes were unequal, making comparisons difficult to infer. Further studies should explore the effect of opportunities to learn on word problem solving ability, increased practice with challenging problem types, and instruction in students' native language.

Summary of Literature

Several themes persisted across studies of ELs and word problems. Native language was included to a degree in all of the seven studies. Attending to native language, and leveraging it as a resource, provides opportunities for meaningful instruction and assessment (Gersten et al., 2005; Klingner et al., 2012; Moschkovich, 2013). In addition, taking into account students' personal experiences and culture was also found to be a positive technique across numerous studies. Making connections to students' lives, as well as viewing culture and identity as resources, is important to engage ELs in instruction (Klingner & Edwards, 2006; McIntyre, 2010; Moschkovich, 2013). Finally, including multiple representations in lessons (i.e., visuals, graphic organizers, and manipulatives) was highlighted as a resource to help ELs conceptualize and solve word problems (Ambrose & Molina, 2010; Bautista et al., 2009; Turner & Celedón-Pattichis, 2011). These findings align to the curriculum and instructional tenets of CRT.

There are several critical gaps in the existing literature on ELs word-problem solving. Of the seven studies reviewed, only Orosco et al. (2013) included ELs with MD. Student proficiency in both mathematics and language was unclear in the majority of studies reviewed. Much of the existing research assumes a deficit model that lumps all ELs into one group, regardless of language and mathematics proficiency (Gutierrez & Orellana, 2006). Student populations should be well defined and distinguish language status from academic difficulties, as much as possible. In addition, Orosco et al. (2013) was the only study to test the effectiveness of an instructional strategy to improve ELs word-problem solving. There is an alarming absence of empirical evidence to guide teachers' word-problem instruction for ELs, particularly those who need additional support in mathematics.

Investigating the effectiveness of interventions that incorporate native language and cultural experiences into instruction is essential to understand effective teaching practices to improve EL word problem solving. Turner and Celedón-Pattichis' (2011) described additional benefits for students in the classroom with rich language and cultural

supports; however, none of the articles reviewed explicitly tested the efficacy of a culturally and linguistically responsive approach to word-problem solving. Likewise, although CGI was observed in several classrooms none of the studies investigated the efficacy of this approach with ELs. Several studies mentioned either the effectiveness of, or the need for, visual and tactile representations. Word-problem instruction that incorporates mathematical tools, such as manipulatives, should also be empirically tested for EL populations. The actions of teachers were analyzed (e.g., Turner & Celedón-Pattichis, 2011); however, none of the studies focused on the teachers' underlying beliefs or the school context in which they provided instruction. Finally, further research is needed to specifically investigate word problem solving for ELs with MD. One evidence-based approach for students with MD is the use of schemas, or problem types, to help students conceptualize and solve word problems. This approach is promising, yet has not been investigated specifically for ELs.

Schema Instruction: A Promising Approach

Zheng, Flynn, and Swanson's (2012) meta-analysis of word-problem interventions for students with MD indicates that studies yielding larger effect sizes include similar instructional components: advance organizers, skill modeling, explicit practice, task difficulty control, elaboration, task reduction, questioning, and providing strategy cues. Many of these approaches are a part of the evidence-based approach, schema instruction (SI), which is often used for teaching students with MD how to solve word problems and incorporates many of these components into one instructional program (Jitendra & Star, 2011). SI can be referred to in the literature as CGI (Ambrose & Molina, 2010), schema-based instruction (SBI; Jitendra et al., 2013), and schemabroadening instruction (Fuchs et al., 2008). SI combines explicit strategy instruction and meta-cognition (i.e., a person's awareness of strategies, organizing information, planning solution attempts, executing plans, and checking results; Goldberg & Bush, 2003) to improve word problem solving. See Figure 3 for types of SI instruction.

Figure 3 Schema Instruction Comparison Chart

Name	Culturally Responsive Schema Instruction (CLR-SI)	Cognitively Guided Instruction (CGI)	Schema-based Instruction (SBI)	Schema- broadening Instruction	Other
Research Base	Proposed Study	Carpenter et al., 1981; 1993	Griffin & Jitendra, 2009; Jitendra et al., 1996; 1998; 2007; 2013	Fuchs et al.,2006; 2008; 2009	Jordan & Hanich (2000)
Additive Problem Types	Change	Joining/ Separating	Change	Change	Change
	Combine	Part-Part- Whole	Group; Combine	Total	Combine
	Compare	Comparison	Compare	Difference	Compare
	N/A (Other)	Equalizing	N/A	N/A	Equalize

Principles of SI

With explicit SI, "students are taught to flexibly apply a small repertoire of strategies that reflect the processes most frequently evidenced by skilled students" (Van Luit & Naglieri, 1999, p. 99). This approach is beneficial for students with learning difficulties (Archer & Hughes, 2011; Kroesbergen, Van Luit, & Maas, 2004). In SI, a

schema is used as a framework in which students are taught to identify additive problem types (e.g., Group, Change, Compare; Jitendra & Star, 2011). SI can also be used with multiplicative problem types (Jitendra et al., 2009; Jitendra & Star, 2011), but I focus on additive problem types in this section, as additive problems are the focus of the present study and the curriculum of the students in the study. SI is directly related to CGI, as discussed in Turner and Celedón-Pattichis' (2011) and Ambrose and Molina's (2010) research on EL word-problem solving. In SI, understanding the structure of a word problem is seen as critical to successful problem solving (Kalyuga, 2006). SI is a particularly promising approach for ELs with MD because this evidence-based instruction includes both explicit strategy principles of MD research and grammatical structures to help students identify and solve word problems.

A variation of SI, referred to as schema-broadening instruction, builds off traditional models of SI by incorporating a transfer component. This "helps students recognize a novel problem (with unfamiliar problem features such as different format, additional question, irrelevant information, unfamiliar vocabulary, or information presented in charts, graphs, or pictures) as belonging to the schema for which they know a problem-solution strategy" (Powell, 2011, p. 103). Schema-broadening instruction has also shown promising results for students with MD.

SI Problem Types

There are three types of additive problem types: Group, Change, and Compare. A Group problem asks students to total two or more amounts and is also referred to as a Total or Combine problem. For example: *Isabel has two apples and four bananas. How many pieces of fruit does she have?* The second type of problem is a Change problem,

where students are given an initial amount, a change occurs, and students are asked to find the resulting amount. Change problems can increase (e.g., *Isabel has apples. Her sister gave her three more apples. How many apples does Isabel have now?*) or decrease (e.g., *Isabel has seven apples. She gave her sister two apples. How many apples does Isabel have now?*). A Compare, or Difference, problem prompts students to use the relationship between two quantities to find the unknown. For example: *Isabel has four apples. Her sister has five more apples than Isabel. How many apples does Isabel sister have?*

Once students are taught to identify a problem type in SI, they are instructed on specific strategies (i.e., diagram, equation, or plan) to solve each type (Powell, 2011). Students are presented problems in varying formats and provided strategy instruction to solve each variation. For example, in a Change problem students can solve three variations: (a) the initial amount is unknown; (b) the change amount is unknown; and (c) the end amount is unknown.

Prior SI Research for Students with MD

Multiple studies have demonstrated the efficacy of SI for students with MD (Fuchs et al., 2008, 2009; Jitendra et al., 1998; Powell & Fuchs, 2010). Powell's (2011) review on SI indicated effect sizes favoring experimental conditions ranging from ES = 0.28 to ES = 6.84. Although ELs have been included in past schema-based experiments, there has not been a study to date investigating the effectiveness of this approach for this specific student population. In this section, I provide a review of SI research for elementary students with MD.

Jitendra et al. (1998) provided small-group tutoring for 34 elementary students $(2^{nd} \text{ through 5}^{th} \text{ grade})$ with MD for 45 minutes a day for 4 weeks. SI was provided for all three additive problem types (e.g., Group, Change, Compare). Effect sizes on posttest and maintenance measures comparing the SI group with the control group ranged from moderate to large (ES = 0.57 to ES = 0.81). In addition, students were able to transfer skills to novel problems from curricula not used in the treatment. However, EL status was not indicated for participants, which makes it difficult to determine the efficacy of the SI approach for EL populations.

Fuchs et al. (2008) provided schema-broadening tutoring to third-grade students (N = 35) identified as having mathematics and reading difficulties. Students were randomly assigned to either the schema-instruction or control condition. ELs comprised 10.5% of the control condition and 6.3% of the treatment condition. In this study, SI included a transfer component, such as an extra step, novel questions, or relevant information presented in charts, graphs, or pictures. Students received individual tutoring for 20 to 30 min, 3 times a week for 12 weeks from trained research assistants. Students in the schema condition outperformed students in the control group who received regular classroom mathematics instruction without tutoring. Students in the schema condition improved on word-problem solving measures with moderate to large effects (ES = 0.69 to ES = 1.80) at immediate posttest. Results were not analyzed by EL status, which makes generalizations difficult.

In another schema-broadening study, Fuchs et al. (2009) stratified and randomly assigned 133 third-grade students with MD to three conditions: computational fluency, word-problem solving, and no-tutoring control. ELs comprised 16% of the total sample.

Students in the tutoring conditions received instruction in 20 to 30 minute sessions, three sessions per week, over the course of 16 weeks. Students in the schema-broadening condition demonstrated significantly higher posttest scores (ES = 0.28 to ES = 0.83) than students in the other two conditions on several measures. Results were not reported by EL status.

Powell and Fuchs (2010) randomly assigned third-grade students (N = 90) with MD to three conditions: SI, SI combined with equal sign instruction, and a no-tutoring control. Students in the schema-broadening and combined conditions received tutoring in 25 to 30 minute sessions, 3 times per week, for 5 weeks on total problem types. Students who received the combined tutoring were better able to solve variations of the total problem type than the other two conditions (ES = 0.22 and ES = 0.63). Similar to previous research on SI for students with MD, results are not reported for specific EL subgroups.

Most recently, Jitendra et al. (2013) extended SI research for elementary students with MD in several ways. First, SI instruction was compared to standards-based instruction instead of the typical "business as usual" condition. In addition, instruction addressed more complex two-step problems compared to previous studies, which focused on one-step word problems. Paraprofessionals were trained on the intervention conditions and provided the tutoring to students. The researchers randomly assigned 136 students with MD across 12 elementary schools to either the SI or standards-based condition. Students received 30 minutes of tutoring in addition to typical classroom instruction 5 days a week for 12 weeks. Tutoring in the SI condition explicitly taught all three additive problem types.

Although 46% of the sample identified as EL, results were not analyzed separately because of the high level of overlap between ethnicity and EL status. As a result, ethnicity (Caucasian vs. non-Caucasian) was used to analyze results. There were different word-problem solving results for students with MD based on pretest scores. Students with higher pretest scores in the SI condition performed better on the posttest and maintenance measure of word-problem solving than the standards-based condition (ES = 0.82 and ES = 1.16). There were not statistically significant differences between the Caucasian and non-Caucasian groups. However, students with lower pretest scores who received instruction in the standards-based condition performed better than students in the SI condition. The authors hypothesize that these differential effects may be because SI instruction did not include a focus on computational strategies, many students with MD who had not previously mastered basic skills did not benefit as much from the higher-level problem-solving strategy instruction. In previous studies, students with MD had either already mastered basic computation (e.g., Jitendra et al., 1998) or addition and subtraction strategies were built into the word problem instruction (e.g., Fuchs et al., 2008, 2009; Powell & Fuchs, 2010).

Several studies have studied the effects of SI for elementary students who were not classified with MD by researchers. In these studies, there are consistent positive effects favoring SI conditions (e.g., Fuchs et al., 2003, 2004, 2006, 2008, 2009, 2010; Griffin & Jitendra, 2009; Jitendra & Hoff, 1996; Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007; Jitendra, Griffin, Haria, Leh, Adams, & Kaduvettoor, 2007). Despite the inclusion of ELs in these samples, only Jitendra et al. (2007) provided additional analysis for ELs. However, Jitendra et al. (2007) reports this analysis for a combined group of ELs, students with disabilities, and Title 1 students. Thus, the efficacy of SI for EL populations remains unclear.

In sum, SI is a promising approach to improve word-problem solving for students with MD. SI methods incorporate explicit strategy instruction and metacognitive approaches, which can help students navigate the linguistic complexity of word problems. Although ELs have been included in student samples in previous SI research, effects of this approach for ELs of varying ethnicity and language proficiency are unknown. Likewise, CRT or LRT have not been documented in existing SI research. Thus, the evidence base for SI warrants further investigation to determine if this approach is beneficial for ELs with MD.

Present Study: Research Questions

The present study built off of the research presented for CRT and SI to explore word-problem instruction for ELs with MD using a mixed-methods approach. The quantitative portion of this study tested the efficacy of SI instruction on Change problems. Specifically, I used SI as an evidence-based framework and incorporate culturally responsive approaches to design a word-problem solving intervention. The intervention focused on Change problems because students with varying mathematics ability have difficulty with this problem type (Willis & Fuson, 1988), including ELs (Ambrose & Molina, 2010). In Powell, Fuchs, Fuchs, Cirino, and Fletcher's (2009) comparison of word-problem types for students with MD-only and students with MD comorbid with reading difficulties, students with MD-only performed better on Change problems compared to Total and Difference problems. For students with mathematics and reading difficulties, Change problems were more difficult when compared to the other two types (Powell et al., 2009).

The specific elements of culturally and linguistically responsive instruction (see Figure 2) that will be incorporated into the intervention are: (a) explicitly stating measureable lesson objectives; (b) facilitating oral discussions with students; (c) allowing use of native language; (d) using manipulatives (i.e., colored bears) to help illustrate and compute each problem; (e) incorporating student ideas and experiences; and (f) providing relevant instructional examples to participants' daily lives, pop culture, and cultural heritage. Other elements are equally important (i.e., home visits to learn from students families); however, they are not appropriate for the proposed research study due to time constraints and the use of research assistants to provide mathematics tutoring.

The qualitative portion of this study provided additional insight by exploring if, and how, teachers of ELs with MD provide instruction on word-problem solving. Using a variety of methods, the study sought to answer the following research questions (RQ):

- (RQ1) What beliefs guide a third-grade teacher's implementation of culturally and linguistically responsive mathematics instruction?
- (RQ2) How does a third-grade teacher instruct CLD students to solve word problems?
- (RQ3) Do ELs with or at risk for MD perform better on solving mathematical word problems after receiving CLR-SI?

CHAPTER III: METHODS

For the quantitative component of this mixed-methods study, I used a singlesubject multiple-baseline design across student participants to determine the efficacy of the word-problem intervention. For the qualitative approach, I used an interpretive case study design to make sense of data collected including in-depth observations, interviews, and field notes (Guba & Lincoln, 1994). In the following sections, I describe the setting, participants, and both qualitative and quantitative methods.

Setting

The study occurred at a culturally and linguistically diverse public elementary school (i.e., pre-kindergarten through fifth grade) in the mid-Atlantic region of the United States. Pseudonyms are used to describe the school and all student and teacher participants. Douglas Elementary School (DES) serves a high percentage of diverse learners, including refugee and EL populations. At the time of the study, there were 606 students enrolled at DES. The student population at DES was 53% male and 47% female. Racial composition included 33.1% Black, 22.2% Hispanic, 24.5% Caucasian, and 20.2% other. Student demographics also included 9.9% students with disabilities, 3.7% students identified as gifted or talented, and 76.6% receiving free or reduced lunch. In addition to the 9.9% students identified with disabilities, 28.2% of the student population was in Tier 2 or Tier 3 of DES' Response to Intervention (RTI) framework. RTI was the process DES used to provide intensive intervention for students who demonstrated persistent

academic struggle. Students whom the school considered "at risk" for having a disability first engaged in Tier 2 and/or Tier 3 intervention before the identification process began.

Of the student population at DES, 37.5% students were receiving English as Second or Other Language (ESOL) services. There were 30 different languages spoken by students at DES. In terms of EL language representation at DES, 46% spoke Spanish as their native language, 10% Arabic, 8% Mandarin, 6% Nepali, 3% Dari, 3% Russian, 3% Swahili, and 21% spoke another language not listed. Twenty-three different languages comprised the 21% "other" category.

Participants

Participants in the study included school administration, teachers, and students in third-grade classrooms at DES. Upon receiving approval from the university Institutional Review Board (IRB), along with the local school district, informed consent was obtained from the classroom teachers and school administrators participating in the study. The parents of students in the teacher's classroom received opt out letters in both English and Spanish. The third-grade team confirmed that translations in English and Spanish were sufficient for their student populations. Detailed participant information is presented within the qualitative and quantitative methods sections.

Qualitative Methods

Participants

For the case study, one third-grade teacher was the unit of analysis. The case study focused on the mathematics instruction of Ms. Jay at DES. Ms. Jay is a general education teacher who has taught for seven years. For five of those years, Ms. Jay taught third-grade at DES. Each year she had a high percentage of ELs in her class. Ms. Jay

graduated with a Master's of Teaching from a traditional teacher preparation program. Prior to the start of her teaching, Ms. Jay had engaged in limited training or professional development on teaching ELs and teaching mathematics. While teaching at DES Ms. Jay collaborated regularly with the school English as Second or Other Language (ESOL) teachers and sought out information, often through practitioner books, on effective EL instruction.

Ms. Jay's third-grade classroom was purposefully selected for two reasons. First, I was interested in studying third-grade classroom instruction because this is the first year which students are assessed on the state standardized assessment. The state assessment relies heavily on word-problems to measure student understanding, therefore I was interested in studying if and how word-problem instruction occurred during third-grade mathematics instruction. The second reason I selected Ms. Jay for study was that multiple school staff members (e.g., ESOL teachers, general education teachers, school administration) praised her ability to provide effective instruction for ELs. This recommendation fit my first research question, if and how a teacher provides culturally and linguistically responsive mathematics for CLD students.

Instruction and interactions were observed between Ms. Jay and all the students in her classroom. The majority of observations occurred in Ms. Jay's classroom. Additional interviews occurred throughout the semester with Ms. Jay's student teacher, ESOL teachers, a RTI Interventionist, and the assistant principal. These additional interviews were scheduled as a result of the reflexive approach to data collection and analysis, whenever additional perspectives and context were needed to triangulate data sources and make inferences. The racial and ethnic student demographics mirrored the larger population of DES. There were 18 third-grade students in Ms. Jay's class. Of these students, five were Latino, four were African American, four were Caucasian, three were Asian, and two students identified as biracial. Although none of Ms. Jay's students began the year identified with a disability, at least one student was in the third, most intensive, tier of the RTI framework used to identify students for special education. Ms. Jay speculated this student would likely be diagnosed with a learning disability by the end of third grade. Seven of her students were receiving Tier 2 or 3 interventions within the RTI framework. Ten of the students in Ms. Jay's classroom were meeting all third-grade benchmarks, and several of these students were pulled weekly for gifted services.

Nine of Ms. Jay's 18 students were ELs, and English proficiency was measured at DES by the *World-class Instructional Design and Assessment (WIDA)*. The *WIDA* is an English language assessment that determines student proficiency in reading, writing, listening, and speaking, and is used across a number of states in kindergarten through 12th grade (*WIDA*, 2014), including at DES. Students are scored on their linguistic complexity, language forms and conventions, and vocabulary use. Although helpful for providing a picture of second-language acquisition, the *WIDA* only provides proficiency indicators for English and not native language. The *WIDA* designates five levels of English language proficiency, with five being the highest level and one being the lowest level of English proficiency. Students who are recent immigrants often start at a level one and are considered newcomers. Six of the nine ELs in Ms. Jay's class began the school year at a level three or below. Native languages spoken in Ms. Jay's primary language

is English, but she also had a limited knowledge of conversational Spanish. Several of Ms. Jay's ELs were considered in the on-grade level or gifted group; however, the students demonstrating the lowest academic performance were her Latino and African ELs. Native languages in this group included Spanish and Arabic.

Data Sources

Multiple sources of evidence are essential to triangulate data and understand the phenomenon of study (Erickson, 1986). Over the course of four months (September 2014 to December 2014), data collection consisted of classroom and teaching observations, interviews, and document analysis with classroom teachers. Each data source included a different perspective on how Ms. Jay provided and understood mathematics instruction for CLD students. I approached data collection analysis with a theoretical framework of culturally and linguistically responsive instruction as outlined in Chapter II.

Observations. Over 1,000 minutes of Ms. Jay's mathematics instruction were observed over the course of four months. Observations occurred two to three times a week, for approximately 50 minutes each session. Observations covered all parts of mathematics instruction (i.e., introduction to new content, guided practice including group work, independent seat work, and lesson closing).

Throughout each observation, I focused on how Ms. Jay approached wordproblem solving, as well as if and how students' cultural and linguistic resources were incorporated into classroom instruction. I used a *Classroom Observation Protocol* (see Appendix A) as a resource for observational note taking. I also looked for the presence or absence of word-problem instruction, including examples of the types of problems students solve.

In addition to Ms. Jay's mathematics instruction, I observed two third-grade level team meetings. These meetings occurred mid-way through the data collection period (e.g., November). The purpose of these meetings was to analyze mathematics data and identify a group of students across classrooms that would benefit from a Tier 2 intervention provided by a school-based interventionist. Attending these meetings allowed me to observe the staff dynamics and collaboration, as well as how teachers discussed the intersection of language and ability of their students in determining a Tier 2 intervention group. Field notes were written for each observation and include a description of setting, participants, interactions, dialogue, and researcher inferences.

Interviews. Interviews were conducted with Ms. Jay at the beginning and end of the data collection period (i.e., September and December). Interview data confirmed or disconfirmed data collected during the observations. The primary interview protocol was developed based on existing literature of culturally and linguistically responsive instruction for ELs (e.g., Echevarria et al., 2006; Gersten et al., 2005; Klingner et al., 2012; McIntyre, 2010; Moschkovich, 2013). Interviews were digitally recorded and transcribed for analysis. Each interview lasted approximately 1 hour. As previously mentioned, I also conducted additional interviews with Ms. Jay's student teacher, the ESOL teachers who support third-grade, the RTI interventionist who provides Tier 2 instruction to third grade, and the assistant principal. These additional participants provided supplemental instruction or contributed to teacher professional development. Including these additional perspectives contributed to the study by providing a holistic,
contextual picture of mathematics instruction for CLD students at DES. Protocols for any additional interviews were modified from the initial protocol to fit the intended participant and time frame (i.e., end of semester). See Appendix B for the initial interview protocol.

Documents. Relevant documents (i.e., curriculum, lesson plans, student work samples) were summarized and analyzed for analysis using an adapted version of Miles and Huberman's (1994) guidelines (e.g., date, setting, significance and summary of document). See Appendix C for the document analysis protocol. Documents included student artifacts (e.g., student work samples) and teacher artifacts (e.g., *WIDA* Can-Do English proficiency objectives).

Qualitative Data Analysis

To analyze the data, I used analytic induction throughout the entire data collection processes. Analytic induction includes examining assumptions about the phenomenon (i.e., mathematics instruction), seeking to understand what actually happened, finding the structure and organization of meanings in the field, relating findings to the larger structure that surrounds it, establishing validity by constructing a plausible and coherent account, and establishing the evidence of this account (Erickson, 1986). Data collected through fieldwork was compiled electronically. Observation field notes, recorded interviews, and document summaries were read and reread. Inferences were included in each observation to generate themes and codes, and make meaning from the data (Erickson, 1986).

Analytic memos were written periodically (i.e., every few weeks) throughout the data collection process to facilitate analysis of larger themes and drive future

methodological decisions. The analytic memo template used in this study is included in Appendix D. For example, the decision to interview the RTI intervention teacher came as a result of ongoing analysis. In an interview with Ms. Jay, she spoke of the additional supports her students received through the RTI framework at DES. I then observed the discourse and decision-making that occurred during the grade-level meeting, and I learned that the purpose of the new Tier 2 mathematics group was to focus on problem solving in the context of word problems. These data highlighted the need to include the RTI interventionist, who would be providing the Tier 2 instruction perspective on mathematics instruction for CLD students.

Field notes and interview transcriptions were systematically coded until themes emerged from the data. In qualitative analysis, a code assigns symbolic meaning to descriptive and inferential data (Miles, Huberman, & Saldaña, 2014). Codes emerged through reflective analysis of field notes, interview transcripts, document analyses, and analytic memos (see Figure 4). I used Nvivo© Qualitative Software for electronic data storage and for coding purposes. The coding process had two main phases. First, I approached the data using a method of process coding to represent observable and conceptual action (Miles et al., 2014). From this first round of coding, a code representing teacher beliefs seemed to encompass many of the observation and conceptual actions. I then focused my analysis within this code to infer the guiding principles that seemed to drive Ms. Jay's mathematics instruction.

Figure 4 Codes and Sub-codes Used in Analysis

	Nvivo© Codes							
٠	Student-Teacher Relationship							
•	Teacher Prep/Experience							
•	Word Problem Instruction							
	• Problem Type Representation							
•	Teacher Beliefs about CLD Students							
	 High Teacher Efficacy 							
	 Knowing Student Progress 							
	 Encouraging Student Voice & Discourse 							
	 Linguistic Supports 							
	Vocabulary							
	 Beliefs about Math Instruction 							
	Mathematics Instruction Context							
	 School Context 							
	Student Population							
	Staff Servicing and Collaboration							

Trustworthiness

Representing participants' voice and meaning making in their specific context is a key indicator of trustworthiness in qualitative research. Trustworthiness is judged by the importance of the topic, plausibility, credibility, and relevance of the account. As a result, an in-depth amount of time was spent at the site, rich and detailed descriptions were provided in context, and multiple data collection methods were employed to triangulate findings (Erickson, 1986).

To ensure credibility of results, I engaged in over 1,000 minutes of fieldwork at the school site over the course of four months. This fieldwork time allowed me to observe patterns and related interactions of mathematics instruction. I used member checking to ensure my analysis accurately reflected the experiences of the teachers and students I observed (Brantlinger et al., 2005). For instructional observations, I tried to remain an unobtrusive observer. As the teacher and students became familiar with my presence; however, I became more active in the classroom as appropriate (e.g., a student sitting near me asks if he solved a problem correctly).

Throughout this time period, relevant documents (e.g., student work samples, classroom curriculum, professional development documents) were analyzed as supporting documentation. These multiple sources of data (i.e., observations, interviews, and documents) were triangulated through ongoing reflexive analysis to develop an understanding of the word problem instruction in EL classrooms. It is important to note that none of the participants had direct authority over this research, and that I did not have direct authority over any of the participants. The principal at the school was aware and supportive of the project.

Quantitative Methods

Participants

For the intervention, all third-grade ELs across participating teacher classrooms were screened for MD to determine eligibility for word-problem tutoring. ELs were administered a word-problem assessment (e.g., *Pennies Test*; Jordan & Hanich, 2000), and students who performed below the 25th percentile were considered to be at risk for MD. Nine ELs across three classrooms were identified as at-risk for MD and therefore eligible for the tutoring component of the study.

Each of the third-grade teachers were provided a list of eligible students to confirm if: (a) the proposed students would benefit from a word problem intervention; and (b) they were comfortable with each student being pulled out of the classroom during their scheduled morning meeting time. All teachers supported the inclusion of their

students. Ms. Jay had six of the eligible students in her classroom. I solicited additional feedback from this teacher on possible tutoring pairings of students that she felt would be most beneficial for their instruction. Students were assigned to one of the four tutoring groups. Groups 1, 3, and 4 had two students in each group. Group 2 consisted of three students. Student groups remained stable throughout the study, and there was 100% completion for all students who began the tutoring.

Surprisingly, reading level was unavailable for the majority of students. All students are assessed on district-mandated reading benchmarks multiple times a year; however, the interventionist team held assessment data. Teachers could only provide basic indicators for reading (e.g., Guided Reading Level F) on their demographic questionnaire. Available demographic information is provided for all students who participated in the word-problem tutoring in Table 1.

Name	Age (years)	Gender	Race/ ethnicity	Native Language	English Proficiency <i>WIDA</i> Level (Max Score 5)	Identified with LD	MD Screening (Max Score 14)
Juan	8	М	Latino	Spanish	3	No	1
Ana	9	F	Latino	Spanish	3	No	6
Maria	9	F	Latino	Spanish	3	No	5
Sophia	8	F	Latino	Spanish	4	No	4
Rosa	8	F	Latino	Spanish	3.8	No	4
Eli	8	М	Latino	Spanish	3	No	3
Isabel	9	F	Latino	Spanish	1	No	6
Aaron	8	М	Latino	Spanish	3.6	No	6
Gabriel	8	М	African American	Arabic and Nuba*	3.5	No	6

Table 1Word-problem Tutoring Participant Demographics

*Gabriel's teacher expressed some confusion over which language, including English, was his primary language. DES classified Gabriel as an EL and he was identified as eligible for tutoring based on his *Pennies Test* score, so was included in the study.

Measures

Several measures were administered to the nine students who participated in the tutoring component of the project. These measures were used to assess students' mathematics understanding and word-problem solving abilities, as well as the social validity of the intervention. Measures were administered in small-group and individual settings. Two measures were used to determine word-problem solving performance: *Pennies Test* and *Story Problems*. Additional pre- and posttest included measures of

computational (i.e., addition and subtraction) fluency. A measure of social validity was also included for tutoring participants.

Word-problem solving. With the *Pennies Test* (Jordan & Hanich, 2000), students are asked to solve 14 word problems. All problems are read aloud so that problem-solving performance is not confounded with reading ability. Students were given a written copy of the test to solve problems (see Appendix E). Word problems range from simple to complex and cover the following problem types: Total/Combine, Change, Difference/Compare, and Equalize (e.g., "Alex has 8 pennies. Kris has 6 pennies. What could Alex do to have as many pennies as Kris?"; Jordan & Hanich, 2000, p. 571). The maximum score is 14. The *Pennies Test* was administered to all third-grade ELs in participating teachers' classroom to screen for students eligible for tutoring (i.e., students scoring at the 25th percentile or less). The *Pennies Test* was again administered as a posttest measure for students who participated in the word-problem intervention to determine if word-problem solving changed after engaging in the tutoring project.

Due to the length of the measure and tutoring time constraints, *Pennies Test* was not an appropriate measure to use as the primary dependent variable in the multiplebaseline design. *Story Problems* (Driver & Powell, 2014) is a researcher created measure that was used as the primary dependent measure in the multiple-baseline design. *Story Problems* is a bank of word-problems featuring Change problem types. At the beginning of each tutoring lesson, research assistants (RAs) providing the tutoring gave students 3 minutes to answer as many word problems as possible. RAs read problems aloud on request. Student performance on *Story Problems* was used in the multiple-baseline design to determine if a change in performance resulted from the intervention. A 15-point rubric was created to score each word problem. RAs scored word problems on the rubric at the conclusion of each daily lesson and submitted the results over a secure online form. See Appendix F for the *Story Problems* measure and Appendix G for the rubric.

Computational fluency. Computational fluency measures were included to determine if there was a relationship between word-problem, addition, and subtraction skills. With *Addition Fluency* (Fuchs et al., 2003), students have one minute to answer 25 addition facts with sums to 12. All problems are presented vertically, and students answer as many facts as they can during the one-minute time period. The principal investigator or an RA read directions aloud prior to starting the timer then allowed students to work independently until the end of one minute. The maximum score is 25. *Addition Fluency* was administered as a pre-and posttest measure for students who participated in the word-problem intervention. See Appendix H for the *Addition Fluency* measure.

A measure of subtraction was also included, as students can confuse addition and subtraction operations. With *Subtraction Fluency* (Fuchs et al., 2003), students have one minute to answer 25 vertically presented subtraction facts with minuends to 12. After listening to directions, the students worked independently until the timer beeps. The maximum score is 25. *Subtraction Fluency* was administered as a pre-and posttest measure for students who participated in the word-problem intervention. See Appendix I for the *Subtraction Fluency* measure.

Social validity. Social validity measures if participants think the goals of the intervention are relevant to everyday life, if the intervention procedures are relevant for individuals and the larger community, and if the intervention outcomes make a difference

in the everyday lives of participants (Kazdin, 2011). At the conclusion of the study, students responded to a questionnaire soliciting feedback on the intervention. See Appendix J for the social validity questionnaire for students.

Research Design

A single-subject, multiple-baseline design was used to determine the efficacy of the proposed intervention: culturally and linguistically responsive schema instruction (CLR-SI). According to Kazdin (2011), in a "multiple-baseline design, inferences are based on examining performance across several different baselines" (p.144). In the present study, a multiple-baseline design was implemented across student participants to determine if CLR-SI affects word-problem solving performance.

In a multiple-baseline design, a baseline is first established for participants before the introduction of the independent variable. In the present study, there were two phases for each student: before the intervention is introduced (e.g., Basic Strategy condition) and after it is introduced (e.g., CLR-SI condition). A multiple-baseline design is appropriate for determining the efficacy of CLR-SI because it "demonstrates the effect of an intervention by showing that behavior changes when and only when the intervention is applied" (Kazdin, 2011, p. 145). This single-subject design is characterized by the "staggered introduction of the independent variable at different points in time" (Horner et al., 2005, p.168). In a multiple-baseline design, each subject (e.g., baseline) serves as its own control to evaluate when a change in the outcome behaviors occurs (Kazdin, 2011).

The What Works Clearinghouse Appendix E (2014) and Horner et al. (2005) provide guidelines for conducting evidence-based single-subject research. These guidelines include: (a) within-subject analysis; (b) systematic and repeated measurement of the dependent variable (DV); (c) systematic and repeated manipulation of the independent variable (IV; e.g., the intervention); (d) a minimum of at least three data points demonstrating the intervention's effect in each phase for each subject; (e) the DV is operationally described and has social significance; (f) fidelity of implementation of the IV and inter-scorer agreement for the DV is reported for at least 20% of data points in each condition (e.g., before and after the intervention is introduced) for each subject; (g) detailed descriptions are provided for participants, setting, and research procedures (Horner et al., 2005; What Works Clearinghouse Appendix E, 2014). The present study design met all of the abovementioned quality indicators.

The nine eligible students were assigned to one of four tutoring groups to assess treatment procedures. The outcome behavior in this study was student word-problem performance. Four RAs provided tutoring to groups of ELs with MD. RAs were all undergraduate students at a mid-Atlantic university. Three of the RAs were female and one was male. One RA was enrolled in a combined Bachelor's and Master's of teaching program. He hoped to pursue a secondary social studies teaching job. Two RAs were pursuing speech and communication disorder majors. One RA was a Spanish major in the college of liberal arts and sciences. She planned to pursue a teaching position in a private school setting after graduation.

Tutoring conditions. Tutoring lasted 10 weeks over the course of a semester, beginning in late September and concluding in mid-December. Tutoring sessions were conducted 3 times per week for 20 to 25 minutes a session. The principal investigator arranged tutoring sessions based on the classroom teacher's schedule. Throughout each tutoring session, students had the opportunity to earn puzzle pieces for following

directions, working hard, and completing each of the activities during sessions to promote positive behavior. Students kept track of the number of puzzle pieces earned during each session, and colored this number on a twelve-piece printed puzzle was is shaped like a book (see Appendix K). This book puzzle was designed to positively reinforce the idea that students are finishing the story, similar to when they solve word problems. When a student colored an entire puzzle, they chose a small prize from a bag of prizes.

There were two phases of word-problem instruction. Only basic addition and subtraction facts (with addends 0-9 and sums to 18) were used in each lesson. RAs provided explicit and scaffolded instruction during every lesson in both conditions. The first phase was Basic Strategy Instruction, where students practiced word-problems through strategy instruction and techniques they knew from their classroom. The second phase consisted of the word-problem intervention, CLR-SI. All RAs began by providing Basic Strategy Instruction, and only changed to the CLR-SI intervention when instructed by the principal investigator.

Basic strategy instruction. During each tutoring session in the first phase, three activities occurred. The first activity was a flash card warm-up. Each flash card displayed either two numbers or two sets of pictures. The students took turns saying the total amount of numbers or pictures on the card. The numbers on a number flash card correspond to the pictures on a picture flash card. See Appendix L for an example. The RA shows cards one at a time for 1 min. If the student answered correctly, the tutor placed the card in a correct pile. If the student answered incorrectly, the tutor asked the student to count to find the correct answer. After the student remediated an incorrect answer, the tutor placed the card in a correct pile. At the end of 1 min, the tutor and

student counted the number of flash cards in the correct pile. Student graphed their individual or collective group flash card score on a graph daily (Appendix L).

After the flash card warm up, the students demonstrated what they have retained from prior lessons on a word-problem review. RAs gave students 3 minutes to answer as many problems as possible on the *Story Problems* measure. Students completed the word-problem review every session, including the first day of tutoring, to establish baseline and performance growth across the two tutoring phases. The next part of each tutoring lesson was the actual word-problem instruction. All word problems were presented in English, which is consistent with their typical classroom instruction, and read aloud by the RAs. In the Basic Strategy Instruction phase, this consisted of general strategy instruction where students were taught to understand the problem, devise a plan, carry out the plan, look back and check (Pólya, 1945). Strategy instruction is commonly used in elementary classrooms and is considered effective for students with MD (Jitendra & Star, 2011). Specifically, RAs used the RISE strategy where students were taught to read, illustrate, solve, and explain the problem (see Figure 5).

Figure 5 Baseline RISE Strategy Poster



Students solved three to five word-problems each tutoring session. RAs followed Archer and Hughes' (2011) explicit instruction I Do-We Do-You Do model. Explicit instruction is beneficial for students with LD, including students with or at risk for MD, in previous research (e.g., Gersten et al., 2009; Kroesbergen & Van Luit, 2003; Swanson, & Hoskyn, 2001; Vaughn, Gersten, & Chard, 2000). RAs taught students to use the Counting Up strategy (Figure 6) to use as a resource when solving basic addition and subtraction facts.

Figure 6 Counting Up Strategy Poster (Powell, Driver, & Julian, in press) **Counting Up**

Say the	Count	Your
bigger	up the	answer is
number.	smaller	the last
	number.	number
		you say.

Providing a strategy for basic computation is important for participants with low pretest scores as identified by Jitendra et al.'s (2013) study. In this phase, RAs modeled the first problem using the RISE strategy, provide guided practice to the student on one to three problems, and allowed the student to solve the last problem independently. After the independent problem, students could count and color their puzzle pieces before the RA escorted them back to class. See Appendix M for a complete example of a basic strategy instruction tutoring script.

CLR-SI. Similar to the Basic Strategy phase, each CLR-SI lesson consisted of three activities: flash card warm-up, review, and instruction. The key difference in this condition was the word-problem instruction students received. Once RAs began the intervention phase, they started implementing CLR-SI. The CLR-SI condition included strategic support for ELs' participation in mathematics instruction as they learn English

by drawing on available resources (i.e., objects, drawings, graphs, and gestures), allowing the use of native language, and incorporating student experiences from outside of school (see Figure 2).

Although SI covers a variety of problem types, the present study focused on Change problem types. Ambrose and Molina's (2010) comparison of Latino first graders' performance on word problems revealed students had the most difficulty with Change and Difference/Compare problems. Change problems can be solved using addition or subtraction, and Difference/Compare problems typically use subtraction. For these reasons, as well as time-constraints, RAs only provided CLR-SI instruction on Change word-problem types.

CLR-SI follows traditional SI methods (e.g., Fuchs et al., 2008; Jitendra et al., 2013), where RAs instruct students to identify the problem type, identify the missing information, identify the known information, and to set up the appropriate equation. Key differences between CLR-SI and the Basic Strategy Instruction phase include the incorporation of culturally and linguistically responsive elements and schema instruction using Change-increase and Change-decrease problem types. Students used an expanded version of the RISE strategy to solve word problems. See Figure 7 for an example of the expanded RISE strategy. RAs again followed Archer and Hughes (2011) explicit instruction model that has demonstrated benefit to students with MD.

Figure 7 CLR-SI RISE Strategy Poster



Culturally and linguistically responsive pedagogy was incorporated into traditional SI in several ways. Specifically, CLR-SI included the following elements of culturally and linguistically responsive mathematics instruction: (a) explicitly stating measureable lesson objectives; (b) facilitating oral discussions with students; (c) allowing use of native language; (d) using graphic organizers and manipulatives (i.e., colored motors) to help illustrate and compute each problem; (e) explicitly using students' own ideas and experiences; and (f) providing relevant instructional examples to participants' daily lives, pop culture, and cultural heritage (see Figure 2).

Each CLR-SI lesson concluded with a word-problem that solicited information from students' personal lives. Students were asked to provide information relating to their experiences, interests, and pop culture (e.g., Do you have any brothers or sisters?; What is your favorite TV show?; What did you do last weekend?). This information was used to create word problems with student input, for the student to then solve. Students were allowed and encouraged to use native language if they desired. RAs affirmed and encouraged native language use if students found it helpful. Students used native language more often in groups where all students spoke Spanish. See Appendix N for a complete example of a CLR-SI tutoring script.

Data collection. Permanent products (e.g., assessment measures, student work samples) were collected at multiple times throughout the study. The principal investigator administered the initial screening measures. RAs and the principal investigator administered small-group and individual measures throughout both tutoring phases. Four RAs were hired to provide tutoring to students and assist with data collection and scoring. RAs and the principal investigator provided tutoring and scored word problems for analysis.

Student performance on the *Story Problems* measure and time constraints drove decisions of when to implement the intervention phase for each group. Data were collected and reported daily for correct rubric elements on the *Story Problems* measures during the initial phase (e.g., Basic Strategy Instruction) to determine the predictability of the data before the intervention was introduced. A minimum of three data points was used to determine predictability in each phase as outlined in the What Works Clearinghouse Appendix E (2014) guidelines for multiple-baseline research. Predictability was determined by visually analyzing the slope of data points for a linear pattern. Groups began the CLR-SI intervention on a rolling basis, every few weeks, to ensure each group had time in both phases within the ten weeks of tutoring.

Procedural fidelity. Fidelity of implementation was assessed across each phase of the intervention. RAs participated in a 2-hour training to become familiar with and

practice instruction in the two tutoring phases. I met with RAs at the school site on a regular basis and communicated biweekly through email. Two members of the research team (e.g., RAs and principal investigators) independently scored 100% of student responses for *Story Problems* and all pre- and posttest measures (e.g., *Pennies Test, Addition Fluency*, and *Subtraction Fluency*). The discrepancies between the two databases were compared and rectified by the principal investigator to reflect the student's original response.

All lessons were scripted to ensure RAs covered material in a similar manner. RAs were not required to read scripts verbatim, but were required to use scripts designated for each of the two phases. Scripts contained various prompts for RAs to facilitate and encourage peer discussion about word-problem solving. RAs became familiar with each lesson and delivered the lesson by following the framework, concepts, and vocabulary of the script. RAs delivered the lesson using only the word-problems, materials, and examples provided in the script. All RAs followed the same sequence of lessons within each condition.

To evaluate fidelity of implementation of the two tutoring phases (i.e., baseline and intervention), RAs digitally audio recorded all sessions. Of the recorded sessions, 20% were randomly sampled from each phase to ensure comparable representation of tutoring phase, tutors, and sessions. Two RAs listened to 20% of tutoring sessions in each phase, for each group. RAs did not assess fidelity of their own tutoring sessions. RAs used a checklist to look for the presence or absence of different lesson components (i.e., 15 components in Basic Strategy Instruction phase; 16-19 components in CLR-SI phase) to determine whether instruction was implemented as intended. Checklist components included introducing the warm-up activities, lesson-specific word problems, appropriate instruction provided for each phase of the tutoring project (e.g., use of manipulatives in CLR-SI phase) reinforcing key concepts through feedback (see Appendices O and P). This was to ensure each lesson addressed all of the Basic Strategy and CLR-SI instructional components and lasted within the designated time limit (e.g., 20-25 min). After listening to 20% of the intervention sessions from each group, procedural fidelity was found to be 99% in the Basic Strategy Instruction phase and 94% in the CLR-SI phase.

Data analysis. Student performance on *Story Problems* was analyzed for each student, with a clear delineation of when CLR-SI was introduced, and graphed separately using computer software. RAs entered *Story Problems* rubric data after each tutoring session into a secure electronic database. Data was then manually transferred to an Excel spreadsheet to track progress in each phase. Excel was used to create graphs and calculate descriptive statistics (e.g., mean, median, range, slope, etc.) for *Story Problems* performance.

Data for each student and group were visually analyzed for changes in: (a) level (e.g., mean score for data within a phase; (b) trend (e.g., slope of the best fitting straight line for data within a phase); (c) variability (e.g., range of data in best fitting line); (d) immediacy of effect (e.g., change in level between the last three data points in one phase and the first three in the subsequent phase); (e) the consistency of data patterns within each phase; and (f) the degree of overlap between all phases (What Works Clearinghouse Appendix E, 2014). I calculated degree of overlap using the nonoverlap of all pairs model (NAP; Parker & Vannest, 2009), which indicates the proportion of data from one phase that overlaps with data from the subsequent phase for each participant. See Figures 9 through 14 for the multiple-baseline graphs of *Story Problem* performance. In addition to the multiple baseline analysis, statistical software was used to descriptively analyze for changes in performance on pre- and posttest measures.

Researcher as Instrument Statement

As a former special education teacher in low-income schools, the majority of my experience has been with diverse students who have been identified as having a disability. However, I have limited experience teaching mathematics to ELs. As an undergraduate sociology major, I did not originally plan to pursue a career in education. My senior year of college; however, the hurricanes of 2005 hit the Gulf coast, and I decided to spend a year working with the Americorps National Civilian Community Corps (NCCC) on disaster relief and reconstruction. One project, working with the Recovery School District to reopen Louisiana public schools, significantly influenced my career trajectory. The social injustices (i.e., critical teacher shortage, insufficient resources) that I observed school-aged children face in the aftermath of Hurricane Katrina compelled me to enter the education sector and get into a classroom as quickly as possible. Through Teach for America, I taught students with learning disabilities at a middle school program in Louisiana. My students had all been retained in school at least twice and experienced low academic performance.

During my time in the classroom, as well as my experience mentoring novice teachers, I witnessed how the inequity students experienced fell along stark lines of race, socio-economic status, and disability. All of this brought me to the Curry School of Education at UVA, where I have had the opportunity to develop and refine my interests

through doctoral coursework and a variety of research experiences. I have designed and engaged in research experiments that include randomized control trials, large-scale assessment analyses, systematic literature reviews, and qualitative case studies. Specifically, I have investigated how services are provided to ELs within a RTI framework, empirically tested innovative teacher preparation methods to improve acquisition of evidence-based practices, and tested the effectiveness of mathematics interventions for elementary students with or at risk for LD.

Thus, I bring in my own biases and beliefs of instructional and identification practices into this research study. Specifically, I believe students can be misdiagnosed as having a learning disability when other cultural and linguistic factors are at play. I also believe students from ethnically diverse backgrounds, who live in low-income communities, are at increased risk of being disproportionately represented in special education. To protect against my personal bias, I constantly checked my own assumptions and attempted to not project my own biased interpretations as the interpretations of my participants. I sought to carefully describe my participants' interactions and attempted to capture the meaning they ascribe to their actions.

CHAPTER IV: RESULTS

In this chapter, I first provide findings from the qualitative component, which provides additional context for the results of the quantitative component of this study. Ms. Jay taught six of the nine students who participated in the word-problem tutoring project. After describing emerging themes, I will present the quantitative results of the tutoring project.

Qualitative Results

The findings presented from the case study of Ms. Jay's classroom emerged from an extensive period of time spent in the field. Within each data source (e.g., interviews, observations, and documents), I sought out multiple perspectives (e.g., observations of classroom teacher and grade-level team meetings, interviews with various staff members, documents including student work and teacher instructional materials) to make meaning from the data. Analysis was an ongoing process throughout data collection, and themes emerged and shifted through the coding process in response to two research questions: (RQ 1) What beliefs guide a third-grade teacher's implementation of culturally and linguistically responsive mathematics instruction?; and (RQ 2) How does a third-grade teacher instruct CLD students to solve word problems?

There was a large degree of overlap between how Ms. Jay approached mathematics instruction for CLD students and word-problem solving. Therefore, I present findings for both research questions simultaneously, as this is how instruction was observed. Word-problem instruction was infused into the larger mathematics curriculum throughout the semester. There are two elements of culturally and linguistically responsive mathematics instruction to consider: the seen and unseen. There were several examples of CRT and LRT that I observed in classroom instruction: linguistic supports (e.g., sentence frames), vocabulary instruction, allowing student use of native language, facilitating and encouraging peer discourse, nonverbal representation of content (e.g., manipulatives, visuals), incorporating student identity and experiences into instructional examples, and warm welcoming relationships between Ms. Jay and her students. The abovementioned seen aspects were influenced by Ms. Jay's knowledge and belief system (i.e., the unseen).

The unseen aspects of CRT and LRT relate to Ms. Jay's knowledge and beliefs about her students, instruction, and school community. Several beliefs, or rules, emerged throughout the semester through analysis of observation field notes and interview transcription. Each rule stems from Ms. Jay's belief in her ability to teach all students, often referred to in the literature as high teacher efficacy (Protheroe, 2008). Ms. Jay attributed to her confidence in her own teaching ability to experience and to her belief that success was possible for all students. She demonstrated a high level of efficacy as an individual and as a contributing staff member at DES. Collective efficacy refers to the belief that faculty efforts will promote positive outcomes for students (Goddard, Hoy, & Hoy, 2000). Each of Ms. Jay's guiding beliefs, or rules, stemmed from this sense of high individual and collective efficacy. Five rules appeared to govern Ms. Jay's mathematics instruction for culturally and linguistically diverse students: (a) understanding student progress; (b) valuing student voice and discourse; (c) emphasizing vocabulary and nonverbal representation; (d) solving problems in context; (e) connecting with the community; and (f) operating as a "crew". In this section, I present data to support Ms. Jay's sense of efficacy and subsequent rules. My perception of how these five rules work together to foster Ms. Jay's teacher efficacy is displayed in Figure 8.

Figure 8

Ms. Jay's Individual and Collective Teacher Efficacy in Instruction for CLD Learners



High Individual and Collective Teacher Efficacy

Across multiple interactions, I witnessed Ms. Jay express confidence in her ability to persevere through challenges and overcome instructional obstacles. DES is considered on the state's watch list for struggling schools; however, Ms. Jay remained passionate about ensuring all of the students in her classroom made significant academic growth. In our initial interview, Ms. Jay described her entry into the teaching profession:

And it was challenging, but I'm the kind of person that when something is hard I have to figure it out. I have to be able to do it and do it well. That's the perfectionist side of me. And I worked really hard. I kept volunteering throughout college to make it happen. I will not let this beat me. And I was like this is it [teaching]. This is me right here. So after I started taking the first semester classes. I applied and knew that if I didn't want to do it I could get myself out of it but once I started learning the theory behind it...it was just done. It was just me.

Ms. Jay viewed teaching as a part of her own identity. She took ownership of her students and their progress. As the grade-level chair, she expressed confidence in her ability to lead not only her students to outcomes but also students in other classrooms. In one CLT meeting observation, the third-grade teachers were grouping students who would need Tier 2 or Tier 3 mathematics supports. One teacher expressed concern for the "lowest" student in her classroom, explaining how the child did not fit into any instructional groups. Ms. Jay asked a few questions to help this teacher problem solve, then offered to provide intensive small-group instruction to this student along with her own students during mathematics block.

While other teachers seemed overwhelmed by the range of student needs in their classroom, Ms. Jay approached the diverse range of learners (e.g., gifted, ELs, students being evaluated for learning and behavior disabilities) in her classroom strategically. She employed a mix of heterogeneous and homogenous instructional groupings, recognizing

that at times she wanted her students to benefit from the academic, social, cultural, and linguistic breadth of her classroom while at other times she needed to provide targeted instruction to students working on similar skills. Ms. Jay appeared to keep each student in mind and value their identities as she planned learning activities and student groups.

One student population for which her focus on students was extremely evident was her ELs. Ms. Jay incorporated linguistic scaffolds (e.g., sentence frames, vocabulary instruction, gestures and body movements to represent language) into her mathematics instruction on a regular basis. She also created systems in her classroom that encouraged students to self-assess when they understood mathematical concepts and when they needed additional support. One example was the teacher-table, in which students could fluidly move across the classroom to ask questions and receive feedback while problem solving. The teacher table was open to any student who needed it; however, the students I observed consistently seeking support were the ELs who qualified for the tutoring component of this study (i.e., ELs with the lowest mathematics performance).

Ms. Jay explained that she wanted students to ultimately own their learning through this process; however, to ensure all students received the necessary feedback she also quietly encouraged individual students to come to the teacher-table as needed. By the time I began observations about one month into the school year, Ms. Jay said she rarely needed to encourage students to come to the table. Some students would sit with her for the entire practice period; others would come for assistance on just one problem then return to their seat. Ms. Jay was keenly aware of which students thought they needed more help than they actually did, and focused on praising their effort and problem-solving to build their confidence. She provided instructional scaffolds such as modeling, questioning, and nonverbal representations for students who struggled with the lesson and sought to gradually increase their independence. Ms. Jay's commitment to creating tailored learning opportunities for all students moved these instructional practices beyond good teaching. She connected excellent instruction to providing CLD students with equitable education.

When speaking of the ELs in her classroom, Ms. Jay emphasized the importance of having a comprehensive knowledge of where each student was in terms of their mathematics and linguistic understanding. In addition, Ms. Jay believed in having a clear vision for her students academically, emotionally and socially. She operated with the end in mind for her students, which encompassed both the state standards of learning assessment as well as her own standards for being "fourth-grade ready". She continually sought to improve her practice, as evidenced by her new approach to teaching wordproblem solving:

Well and again it's our freshman year with it. You know I am always willing to try something new if I feel like it is going to benefit them. And I feel like this is not really different from how I would teach it [word-problem solving], its just again being more transparent with them. This is what it is, this is what we are doing, this is why we do it. For me again its all based on the standards. [Instruction] is based on the things they are going to need to be able to accomplish by the end of this. I can use that to determine some flexible math groups. If I notice that I've got a kid that completely understands the skill, doesn't recognize it in a...it in any kind of context, and doesn't know the vocabulary, if I can find similar students I can utilize my time to hit exactly what they are lacking. I'm not going to drill and kill the multiplication, because maybe they've got the skill already. Let's just recognize it in the context, let's talk about it, let's do word problems.

Over the course of the semester, I witnessed consistent alignment between what

Ms. Jay planned to accomplish, what actually happened in classroom instruction, and her

reflection and response to student learning. This alignment was particularly evident in

her approach for developing vocabulary and her focus on problem solving in context, two rules which will be further discussed in this section. Ms. Jay's confidence and belief in her ability to lead her students to more than a year's worth of academic growth was supported, and challenged, by her perceptions of student progress.

Understanding student progress. In Gibson and Dembo's (1984) study of teacher efficacy, they found that teachers with high efficacy spent significantly more time monitoring and checking on student practice than teachers with low efficacy. Ms. Jay often mentioned student progress in our interviews, conversations with her student teacher, and collaboration with her grade-level team. Ms. Jay consistently used informal and formal assessments to ascertain her students' progress. She often mentioned that she could not tell me exactly what the students would be learning by the end of the week, because while she had a plan it would be adjusted based on how students responded on Monday, Tuesday, etc. In December, I asked how she felt about student progress in relation to the end-of-year state assessment. She considered the student populations in her class and replied:

They [students receiving gifted services] could do it tomorrow. Umm I have a very well defined on grade level group students. They are meeting my expectations, learning the curriculum, showing mastery of it at the end of all their post-assessments. Those are students I am not concerned about because by the end of the year I believe that they will be able to show what they know. Well...they were also the students who came ready for third grade math. They had accomplished and been proficient in second grade math. So I have my very neat group of above, my well-defined group of on, and my "below" kids that I would equate to end of year [1st], beginning of second grade math understanding and ability. And some of that comes from considering when they have come to the country. And they are not...if you think about how a student acclimates into a classroom that doesn't speak their native language. There is a very long grace period. That acclimation. And they lose a lot during this time. What they might be able to get conceptually in math is trumped by discomfort, overstimulation, and honestly just some students haven't had a lot of formal schooling. And that's not to say every student who comes here from another country hasn't had that.

But that seems to be the case for a lot of our students, especially the ones coming as refugees. So when you combine all those factors its not a surprise to me that all of my kids I am concerned about when it comes to end of year benchmarks and where they are...are also all ESOL students (e.g., ELs). It's not because they can't learn the math, it's because they just need more time.

Ms. Jay worked to adjust her instruction as inevitable time constraints occurred

(e.g., sharing instructional time with her student teacher, field trips, school-wide

meetings, and technological difficulties), while still pushing her class towards her vision.

The tension between her academic end-of-year standards, students' learning needs, and

the amount of instructional time available was evident in our end of semester interview:

Time is not on my side. Umm I wish I could be very optimistic and say that they [ELs with the lowest mathematics achievement] are going to be third grade on grade level by the end of the year [pauses]...I know better than that...from experience. I'm always going to push them as fast as I feel like they can go. However, with that in mind, I also can't go any faster then they are willing to go. So a lot of it is going to be responding to them and really looking at what they need and what they are able to accomplish. And what they are successful at. My goal is always growth, its not meeting arbitrary benchmarks. I have tools that I am using to progress monitor them. I'm going to be able to show, or maybe not show, what their growth looks like at the end of the year. I think I need to be satisfied with what I consider a grade level or more of growth. Again with the number assessments, or the number sense assessments that as a team we are working on to kind of meld the developmental continuum for the various strands of number sense, computation, place value, and all those pieces. I will be able to demonstrate and document that. It's always my goal, if I'm considering myself a Tier 2 [teacher], our goal is always a year and a half worth of growth...because eventually they will catch up. However, I don't feel confident enough to give myself that much credit. [She says this sheepishly and we both laugh]. So I will deliver them developmentally appropriate curriculum, and I will keep pushing them forward, and I will hope for the best.

While Ms. Jay expressed some doubt at her ability to lead students to grade level

standards, it stemmed from her emphasis on providing developmentally appropriate

learning for students who began the year with critical deficits in mathematics

computation, concepts, and problem-solving. Through observations, interviews, and

analysis of instructional documents and student work samples I witnessed her work

relentlessly to monitor student learning and tailor grade-level instruction to individual needs. Ms. Jay's perception of student progress was focused at the unit and daily level. One excerpt of observation field notes shows how Ms. Jay approached the understanding of one EL student while her class worked on the district-mandated computer software (e.g., ST Math).

Ms. Jay calls Isabel over to her to look at her ST Math. [She appears to check her progress, provide encouragement, and provide assistance]. "If we look at the key", Ms. Jay models how she would add three numbers together. Isabel thanks Ms. Jay and they smile at each other. Ms. Jay next calls Maria over. [She is looking at her computer and seems to be using some data to call students over with their computer]. "Oh I see how this could be tricky". "How are you figuring out where 17 is?" Maria responds quietly. Ms. Jay, "Great strategy. I can see that you know what to do but sometimes it's hard to get the graph where you want it. Can you work here at the table a bit so I can help you out?" [Ms. Jay appears to be teasing out if the difficulty is in the content or the formatting of the computer software]. Maria agrees and cheers quietly when she gets the next problem correct. Ms. Jay affirms her effort and continues to ask questions about *how* she solves each problem.

Ms. Jay often spoke of knowing "where" her students were, referring to both their mathematical understanding and the progress each student was making in the third-grade curriculum. She seemed to perceive their progress as key to informing her instructional decisions and teacher actions. In addition to this focus on her students' academic learning, she also sought to understand who her students were as social and emotional individuals.

Valuing student voice and discourse. In order to know her students, Ms. Jay seemed to place value on listening to and learning from her students' perspectives. She also appeared to value students listening to and engaging with their peers to create meaning in mathematics lessons. Partner and small group work were regular structures in her mathematics lessons. In addition to simply grouping students, Ms. Jay would give

instructions to encourage dialogue between group members. In one lesson observed, she instructed students to use arrays (e.g., visuals) to represent a multiplication sentence. "Your table needs to agree on a multiplication sentence." Students were grouped heterogeneously, in terms of mathematics ability and language, and I observed each group debating what their multiplication sentence should be, why, and how to represent it visually. Ms. Jay's purposeful use of student dialogue aligned with the grade-level ESOL teacher's description of effective mathematics instruction for ELs:

I would say ideal would be Tier 1 – effective Tier 1 teaching. I would say every student should be listening to math, reading math, doing math, talking about it, sharing your whole process orally, listening to other people share, saying back what you notice about what they said –so that, all of the sudden, you're repeating other peoples' processes – and then writing about it. And I think that all four of those components have to be involved. You have to listen, you have to speak, you have to read, and you have to write around a content topic, always.

Ms. Jay often incorporated speaking and writing into her mathematics instruction, and sought to connect her mathematics lessons with broader, more meaningful, contexts. One example of this incorporation came from an assignment in the graphing unit. The third-grade students were simultaneously preparing to lead an upcoming school-wide morning meeting in the gym. Ms. Jay and her student teacher designed a performance task that involved students meeting with the assistant principal for a needs assessment (e.g., what information would be helpful to plan this month's morning meeting?), generating questions and surveying the other third-grade classrooms based on the assistant principal's request (e.g., what songs should we play at school-wide morning meeting?), tallying and graphing the results (e.g., based on our research, we should play...). Later that month at the school-wide morning meeting, I observed the assistant principal praise Ms. Jay's students for their hard work. She explained to the student body

that Ms. Jay's class did extensive research to select the songs that DES wanted to hear. Ms. Jay's students were rewarded with a resounding cheer from the hundreds of students in the gym, as well as the lively school-wide dance party that ensued. This assignment was an example of how Ms. Jay saw and used the school as a form of culture in her instruction. The role of mathematics in context continued to emerge as a key rule influencing Ms. Jay's mathematics instruction, which will be further described in this section.

I also observed several instances where students were allowed to use their native language with peers. While Ms. Jay's students spoke a range of languages (e.g., English, Spanish, Nepali, Arabic), several students' native language was Spanish. Ms. Jay described her Spanish language proficiency as, "a basic working knowledge of conversational Spanish" and explained that she was more comfortable relying on an interpreter when meeting with families. All observed instruction was provided in English; however, Ms. Jay seemed comfortable allowing students to discuss mathematics concepts and skills in their native language. This excerpt of observation field notes describes how three ELs approached an assignment (e.g., identify and describe an alternate fire escape route for Ms. Jay's class) in the measurement unit:

Sophia, Maria, and Ana are sitting on the floor working on their papers. They begin having a conversation in Spanish about their route. They talk heatedly and seem to be trying to clarify their individual opinions. Ms. Jay looks at them but does not interject. She allows them to continue debating. She begins helping Isabel off to the side (this debate lasts for several minutes). At one point Maria and Sophia both sigh and resume writing on their independent clipboards. They continue whispering in Spanish but seem to have reached some agreement. Ms. Jay looks at Maria inquisitively, but does not say anything. Maria smiles and says, "It would be difficult to go through the field by fourth grade".

When asked about this lesson in a follow-up interview Ms. Jay explained:

I believe that a student like Maria who is using her Spanish to then explain to Ana what the math concept is, Ana is now in a place where she can connect it to what she already knows in her language and be more receptive to it when we are talking in English because she has activated her background knowledge. And Maria has then solidified what she knows by having to teach it, and explain it to someone, and code switch. I feel like it's a missed opportunity if we are not allowing students to do that. It's also not the reality of things. You know...we don't want students to feel that their English education means that they shouldn't be speaking their native languages. They will be much stronger off for it in the future if they have strong linguistics in both languages.

Valuing student voice and discourse also extended into Ms. Jay's vision for her word problem unit. She viewed her students' ability to make meaning through interaction with content as critical to building a lasting understanding. When discussing her plans for the word problems unit, she described how she wanted students to be able articulate their processes and share their findings with peers. Ms. Jay sought to have students build a shared understanding of word problems, and allow students practice creating and explaining problems for peers to solve. She wanted her students to own their learning, and to demonstrate this ownership through discourse. To reach this goal, she was committed to incorporating language supports into her mathematics instruction to make content accessible. How, and why, Ms. Jay planned and adjusted her lessons revealed what she believed to be effective instructional strategies for ELs.

Emphasizing vocabulary and nonverbal representation. When probed on effective instruction for ELs, Ms. Jay consistently referred to the need for vocabulary instruction and multiple representations of content. She planned lessons with an emphasis on mathematics language, and was thoughtful of concepts that students might struggle to understand. Both ESOL teachers and the RTI intervention teacher I interviewed echoed the importance of emphasizing mathematics vocabulary in instruction for ELs. The grade-level ESOL teacher explained: So if you don't learn that vocabulary, you're not fully understanding the explanations of the teacher, anyways. And then, especially, with word problems. A lot of it, I think, is the key mathematical vocabulary that you have to pre-teach, but I think another part of it is – if you think about words being either brick words or mortar words. That academic vocabulary is a brick word. It has a really defined meaning, and it's being attached to another brick word by these little words that we have. They're not – we don't pay attention to them, when we speak, if you're a native speaker...And so not just teaching the explicit content vocabulary, but really teaching the nuances of those mortar words that hold the bigger words together. And the structures of the sentences so that they can understand what's being asked of them.

In several conversations with Ms. Jay, she described how she approached

vocabulary instruction:

The only reason that I go above and beyond [when asked about vocabulary instruction] is because of my ESOL students. Just because the...a lot of times...when they have to unfortunately show this on a state mandated test...you know I don't want them to not be able to show what they know because I never explicitly talked about what an equation actually consists of. It's always not assuming. And making sure that is built in in some capacity in a routine where its not this huge thing...it's just we are focusing on the key math vocabulary today. It's the same thing with any other routine you would throw in math like number sense, mental math, those pieces and components. It's just got to be another component...

...I think about them [mathematics vocabulary words] a little like how I would teach background knowledge to students in reading. I've preselected the words. So what I'd love to be able to do is present the words to them out of context first. To build some background knowledge. Lets talk about it. Have you heard it before? Have you heard it in a certain subject? Have you seen it in a book? Just to kinda get a sense of where they are with their understanding. And then be able to point it out in afterwards in context. Oh gosh you remember we talked about this, lets connect it to what we are doing right now [changes voice/tone to mimic how she would actually say this to students during a lesson]. So its kind of the same process as what I would do in reading. And again I don't know that I've settled on the best course of action. I'm still kind of playing with it.

Ms. Jay and her student teacher consistently incorporated vocabulary activities

into mathematics lessons. A popular game with students was "Flashlight," where

vocabulary terms written on index cards were scattered on the floor. Students stood in a

circle surrounding the cards, holding flashlights to shine on key terms as the student

teacher read each definition. After the first mathematics unit test, the student teacher shared that their students had not done as well as hoped. The assessment measured vocabulary knowledge, computational skills, and students' ability to solve problems in context. The student teacher shared that the area of the assessment students performed most poorly was the vocabulary. Shortly after this unit test, Ms. Jay introduced a new math structure she referred to as KUD centers. These centers were modeled after the Know-Understand-Do (KUD) format that Ms. Jay learned to lesson plan from in her teacher preparation program:

So KUD is a pretty typical way for teachers to take the standards and weed it down to the most important core components that you would need to use in your lesson plans. Umm so what do students need to know about this problem, what will they need to understand, and what will they need to do that is skill-based. So I took that idea. Our school has done a lot of work around this is the last few years because of our status and test scores on the SOLs. [I wanted] just to be a little more transparent with the kids. So if I really want them to be able to see the word, and I keep using the word sum, but if my goal is to have them be able to tell me what sum is, be able to recognize it in a word problem, then let me tell them. Maybe stop...maybe for me I wanted to stop assuming they were getting it as we were moving through. For me again its all based on the standards. It's based on the things they are going to need to be able to accomplish by the end of this [year]. I can use that to determine some flexible math groups.

In addition to an emphasis on vocabulary, I frequently observed students engaging with mathematical concepts through language, visuals, and body movement. When reviewing place value, Ms. Jay had students leap to represent the hundreds place, jump to represent the tens place, and hop to represent the ones. Students excitedly practiced representing place value (e.g., show me 28, 541, etc.) by jumping in place before practicing writing numbers in expanded form (e.g., 20 + 8, 500 + 40 + 1, etc.). I also observed students acting out prepositions (e.g., before, after, in, etc.) in preparation to write about a measurement assignment. Ms. Jay also incorporated sound and rhythm into

instruction. I observed students using classroom instruments (e.g., triangles, egg shakers, tambourines) and hand clapping games to represent the pattern of skip counting, a prerequisite skill for multiplication. Manipulatives were incorporated into lessons on multiple occasions for students to concretely and visually engage with new content before moving to abstract symbols and numerals (e.g., using arrays to represent multiplication before solving multiplication sentences). This excerpt from an observation of a multiplication lesson is an example of how Ms. Jay used language supports to build mathematical understanding:

"We are going to talk about multiplication but we are going to start with the vocabulary, the lingo. Does anyone remember the flip-flop" [holds up two fingers and rotates hand]? Students excitedly call out "communicative property"...

...Ms. Jay says, "When we think about multiplication lets think about counting groups. How many are in three groups of four?" She lays manipulatives and groups numbers out on the carpet. She writes on the white board easel: $3 \times 4 =$ ______ Beneath this multiplication sentence she writes: 3 groups of 4, 3 times 4, etc. "We are going to practice reading multiplication sentences so we can say them all the same way". Students spend a few minutes reading multiplication sentences with a partner, using language of "times" and "groups"...

...Ms. Jay models the clapping pattern they used in morning meeting to greet each other. 1 times 10 equals 10. 2 times 10 equals 20. She stops and asks students, "What is two groups of 10 equal to?" [Students practiced skip counting by 10s as a warm-up earlier in the lesson and several hands go up immediately]. A student answers correctly. Sophia [EL] says "Oh now I get the times". Students begin practicing the clapping pattern with a partner as they recite the tens-times table.

This observation occurred on one of the first days of the multiplication unit.

Third grade is the first year students are formally taught multiplication; however, because

of the focus on the vocabulary associated with this skill and the multiple representations

(i.e., visual arrays, sound patterns, etc.) students were able to pick up this new concept

very quickly. I also observed an emphasis on nonverbal representation of mathematics

content in the word problem unit. Two of the word-problem strategies taught were to

"Visualize it!" and "Act it out!". Ms. Jay encouraged students to model how word problems might play out in real life (e.g., Mr. Music Teacher had 5 drums in the music room. A famous singer came to visit the school and decided to give them 4 more drums. How many drums are in the music room now?). All of the scenarios provided in the word-problem unit consisted of familiar, school-based contexts. This is representative of another key rule guiding Ms. Jay's instruction, the importance of using the school culture to facilitate her students solving problems in context.

Solving problems in context. Ms. Jay placed a large emphasis on solving problems in context. For her, this meant both word problems and connecting mathematics to real-life context (e.g., teaching measurement units by measuring common classroom objects including rulers, glue sticks, large dice with numbers, highlighter; fire escape proposal; morning meeting music survey). Word problems were written by the third-grade team and typically included students' names, teachers' names, the school mascot, and familiar school-based situations. When asked about her process for writing word problems, Ms. Jay explained:

If you are not choosing a situation that is relevant...I mean its hard to hit a situation that would be relevant to everybody...but you know if its talking about being at the beach and I've got five students who have never been to the shore. Things like that that automatically set them up for not understanding at a literacy level means they can't quite get there as a mathematician at all. So I find [context] is very [important] for ESOL students. They have a much tougher time. It's not all of them but typically.

I was also able to interview the intervention teacher, Jane, who was pulling a small group of students from each third-grade classroom to provide intensive intervention for students considered to be Tier 2 in DES' RTI framework. The focus of this small group instruction was to promote computational flexibility and problem solving in the
context of word problems. Jane's approach to word-problem solving was similar to Ms.

Jay's response:

Even if you're thinking about some of the background knowledge in word problems or vocabulary words and word...like unnecessarily verbose word problems. And...but it goes back again that I feel like some of that same thing can be true for English speakers also. It's not a word problem they're going to connect to. Or even in reading, I started pulling a book that was how this girl was real bummed that she had to go to this work camp with her family for the summer. And I was like, "How can you read that with these students who," it's like there's nothing for them to hook onto with that kind of experience. Just things that seem...we talk about how word problems are authentic and should feel like real life and you want that to feel authentic in real life. In our population, children who are children from poverty who...if you want them to see themselves reflected in the story, you have to take into account their experiences wherever they're from. Although it's not bad to expose them to other things...

In a separate interview, the grade-level ESOL teacher echoed and expanded upon

Ms. Jay and Jane's explanations:

[Describing an example she observed where the context was relevant to ELs] The biblio-burro. The library donkey. And so, instead of saying Mary went to the grocery store and bought 47 watermelons, then gave two to her friends at some place that nobody would ever go to...it's just this totally ludicrous, non-real word problems that are made-up, that are also totally not-attached to the culture that the kids live. I think it's also...I don't want to say catered to kids' cultures, because we often superficially pull what we think is somebody's culture and we really don't have much clue about what their culture is. Or we accidentally then reduce their culture, which is this big, lived experience to something overly simplified. Keeping them [word problems] varied and different and sometimes it's the very typical whitewashed word problem that you might see on an SOL test, but it might be...so that everybody is being asked to think about an experience that they might not have lived. On all sides. The white kid trying to think about, "oh gosh, what do you mean a biblio-burro? What's that? Oh, wow. I didn't know that happened, but I can still do the math behind it".

I found it essential to seek multiple staff members' viewpoints on mathematics

instruction because of the strong emphasis on school-based collaboration at DES.

Through our many interactions, Ms. Jay referred to a village mentality and attributed her

instructional approaches to her school context. Numerous times she demonstrated her commitment to a collective responsibility at DES by saying, "We are a? crew".

Operating as a "crew". Each of the staff members I spoke to (i.e., Ms. Jay, the assistant principal, ESOL teachers, interventionist) attributed the school's vision, instructional practices, and school staffing as a response to their student population. The assistant principal described the school culture as warm and caring, with a primary focus on closing achievement and opportunity gaps for their CLD students. In the teachers lounge there is a large bulletin board decorated with a ship. On each of the ship's seven oars there is a guiding principle written. One of the principles read: *Knowing the students we teach individually, culturally, and developmentally is as important as knowing the content we teach.* I asked both Ms. Jay and the assistant principal, in separate interviews, why this particular principle mattered to DES and how the staff has responded to this approach. Both spoke convincingly of a shared vision at DES with a focus on helping students succeed academically, socially, and emotionally.

I later learned from Ms. Jay that the ship represented the guiding principles of the Responsive Classroom (RC) approach, which DES has implemented for the last several years. RC focuses on teaching the whole child: academically, socially, and emotionally (Rimm-Kaufman, Fan, Chiu, & You, 2007). Ms. Jay continuously returned to the RC model when I inquired about the welcoming environment she created in her class, as well as the strong relationships she held with her students. Each day I observed Ms. Jay demonstrate care and compassion for her students' home lives and sought to know who they were as individual members of her class. When describing the impact RC has had

on DES, Ms. Jay highlighted how teachers' mindsets towards the school shifted after the

implementation of the program:

I think part of it came from years ago when our last administrator had gotten here. We were already a school that was failing...we had a bad reputation. I mean realtors were telling people that you don't want that house because that is in DES's district. We were struggling in all different ways. And if you have a culture where all the teachers are always stressed and the students are not achieving what the state is saying they need to achieve, you can see a lot of failure associated with the school. When that is not the case. These are children. We are part of a community. And I think Responsive Classroom fits what our needs were at the time. And we were all craving that. We needed to remember, this is a ship. We are crew. We are in this together it takes a village. And it really changed a lot of perspectives. A lot of us were saying oh we didn't make our AYP [annual yearly progress]. We didn't make our AYP. We aren't going to make it again.

But now you can see there are shifts in what we want for students for their education. It's not just about being able to regurgitate this on a multiplication test. It's about giving them experiences that connect to the real world. And giving them tools to be competent citizens in our country. We are learning I think more about what we would want for a well-rounded student versus maybe what the state is determining us to have to...the benchmarks we need to meet to make us feel successful...Now we have community outreach. Parents that are choosing where they live because they want to stay here. People in the community say they hear wonderful things about us. They want to hear what we are doing here. I feel like we have reimaged ourselves a little bit. And we did it all with these students at the forefront.

Through several interactions, it became clear that the majority of teachers worked

at the school because they wanted to provide high-quality instruction for historically underserved populations, specifically for CLD students and students living in poverty. There was a strong emphasis on parental outreach at DES. In one passing conversation with the intervention teacher, I learned they were expecting over 90 parents at the family literacy night happening later that evening. DES appears to make a concentrated effort to connect home and school life for their students. At the beginning of the year, classroom teachers "Step Out" into the community and visit each of their students homes before the first day of school. This is an initiative that began at DES and has since been replicated

by other schools in the district. The staff at DES also takes great pride in their semiannual student-led conferences, which are designed to empower students and their families. One goal of these conferences is to shift the educational discourse, and power, from being held exclusively by the teacher to the student and their support system. When asked about why she works at DES, Ms. Jay began speaking passionately about the school:

Once I got to know the school and their vision for the students...this school is so different from all the other schools in the county and its kind of set apart and kind of has some liberty in how we approach the needs here. We have a HUGE population of students living in poverty. We have a HUGE population of students that have very little English. A HUGE amount of kids coming from non-literate families. Once I knew this demographic, once I got to know the kids and the needs I felt like I could do more here than I could if I went to a school like I student-taught at.

Numerous times Ms. Jay mentioned how what she was doing was possible because of the school where she worked. She attributed the initiatives, support, and autonomy to find what "worked" for her diverse student population to working at DES. Ms. Jay appeared to feel empowered by her school context. Her leadership in the classroom and with her fellow teachers was clearly valued. She demonstrated high teacher efficacy in instruction while continuing to improve her practice through staff collaboration. When Ms. Jay spoke of what she was trying to accomplish with her students, she often used "we" when speaking broadly of the educational inequity facing students at DES. Ms. Jay views her efforts within a broader school context that shares her vision and mission for her students. Her teaching identity and the context of DES appears to be reciprocal in nature. Each of the abovementioned rules guiding her instructional practice is shaped by the context of DES; likewise, Ms. Jay actively

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contributes to and pushes the progress that DES is making. Her beliefs on mathematics instruction and her students are inseparable from the context in which she teaches.

Quantitative Results

In this section, I present results for the tutoring component of the study which addresses (RQ3): Do ELs with or at risk for MD perform better on solving mathematical word problems after receiving CLR-SI? First, I share results from the single-subject, multiple-baseline tutoring. Following the *Story Problems* results, I present the results from the pre- and posttest analysis on the *Pennies Test* measure and social validity responses.

Story Problems

Tutoring results on the *Story Problems* are represented by group (Figure 9) and individually (Figure 10). The scores presented for each student represent the number of correct rubric elements answered during each tutoring session. The scores presented for each tutoring group represent the average of correct rubric elements answered by students each tutoring session. I analyzed data for each student and group for changes in: (a) level (e.g., mean score for data within a phase; (b) trend (e.g., slope of the best fitting straight line for data within a phase); (c) variability (e.g., range of data in best fitting line); (d) immediacy of effect (e.g., change in level between the last three data points in one phase and the first three in the subsequent phase); (e) the consistency of data patterns within each phase; and (f) the degree of overlap between all phases (What Works Clearinghouse Appendix E, 2014). These descriptive statistics are presented in Table 2 and 3. I calculated degree of overlap using the nonoverlap of all pairs model (NAP; Parker &

Vannest, 2009), which indicates the proportion of data from one phase that overlaps with

data from the subsequent phase for each participant.

Table 2

Group Story Problems Descriptive Statistics

	Basic Strategy Instruction			CLR-SI		
Group	Mean (SD)	Median	Slope	Mean (SD)	Median	Slope
		(Range)	Coefficient		(Range)	Coefficient
Group	21.12 (10.37)	18.5	+2.55	20.13	16	+0.02
1		(28)		(11.51)	(49)	
Group	24.63 (13.86)	22	+6.08	37.42	34	+0.61
2		(56)		(19.12)	(85)	
Group	10.92 (5.94)	9	+0.06	9.5	8.5	+0.45
3		(26)		(5.86)	(27)	
Group	12.03 (7.82)	9	+0.35	32.13	27.5	+3.98
4		(30)		(17.31)	(60)	

Figure 9 Story Problems Group Data



	Basic Strategy Instruction			CLR-SI			
Student	Mean (SD)	Median	Slope	Mean (SD)	Median	Slope	
		(Range)	Coefficient		(Range)	Coefficient	
Gabriel	12.50 (2.89)	12.5 (5)	+0.00	20.14	15 (49)	+0.64	
				(12.06)			
Aaron	29.75 (6.65)	29.5	+5.10	20.13	20 (46)	-0.60	
		(16)		(11.22)			
Eli	20.29 (11.00)	20 (33)	+2.12	30.87	30 (49)	+0.91	
				(12.97)			
Isabel	21.00 (6.86)	22 (18)	+2.20	25.38 (7.97)	26.5 (26)	+0.75	
Rosa	31.57 (18.29)	36 (46)	+4.93	54.53 (19.24)	52 (68)	+0.28	
Juan	11.62 (6.98)	9 (25)	+0.09	10.00 (6.52)	8.5 (25)	+0.22	
Ana	10.23 (4.87)	10 (16)	+0.10	9.00 (5.36)	8.5 (19)	+0.63	
Maria	15.06 (7.49)	14.5 (23)	+0.39	25.00 (9.47)	27.5 (31)	+0.98	
Sophia	8.40 (6.76)	7 (24)	+0.61	39.25 (20.89)	37.5 (55)	+6.98	

Table 3Individual Story Problems Descriptive Statistics

Figure 10 Story Problems Individual Data



Group one. In the first phase (i.e., Basic Strategy Instruction), Aaron had a mean *Story Problems* score of 29.75, and Gabriel had a mean score of 12.50. The mean for Group One in the first phase was 21.12 and the trend line (i.e., slope) showed a gradual, increasing trend. According to visual analysis, neither Aaron nor Gabriel demonstrated clear improvement when the CLR-SI intervention was introduced (Figure 11). Aaron and Gabriel's mean CLR-SI phase score was 20.13. Aaron's trend line was rapidly increasing in the Basic Strategy phase. After CLR-SI was introduced; however, Aaron's trend began gradually decreasing. Aaron's final data point was lower than any of his scores in the first phase. Gabriel showed a slight, gradual increase in performance over the course of the CLR-SI intervention. The slope of Gabriel's trend line increased from +0.00 in Basic Strategy to +0.64 in the CLR-SI phase. The average for this group during the CLR-SI phase was 20.13.

When comparing nonoverlap of all pairs (NAP; Parker & Vannest, 2009) between the basic strategy instruction phase and the CLR-SI intervention phase for Aaron, 19% of points were found to be nonoverlapping. For Gabriel, 70% of data points were nonoverlapping when comparing basic strategy to intervention. The NAP average for Group Three was 44.7%.





Group two. In the first phase (i.e., Basic Strategy Instruction), Rosa had a mean *Story Problems* score of 31.57 and showed a rapidly increasing trend. Isabel had a mean score of 21.00 and showed a gradual increasing trend. Eli had a mean score of 20.29 and showed a rapidly increasing trend. The mean for Group Two in the first phase was 24.63, and the group showed a rapidly increasing trend in the first phase. According to visual analysis, none of the students in Group Two demonstrated immediate improvement when

the CLR-SI intervention was introduced (Figure 12). The average trend line during intervention was at a higher level than in the first phase, but there was not a clear change after the introduction of the intervention. The mean score for Group Two in the CLR-SI phase was 37.42. Rosa and Eli's trend lines continued to increase in the CLR-SI phase. Rosa's mean score in the CLR-SI phase was 54.53, and Eli's mean score was 30.87. Isabel's trend line was flat in the CLR-SI phase with a mean of 25.38.

When comparing NAP between the basic strategy instruction phase and the CLR-SI intervention phase for Rosa, 79% of points were found to be nonoverlapping (Parker & Vannest, 2009). For Isabel, 73% of points were nonoverlapping. For Eli, 83% of data points were nonoverlapping when comparing basic strategy to intervention. The NAP average for Group Two was 78.4%.

Figure 12 Group Two Results



Group three. In the first phase (i.e., Basic Strategy Instruction), Ana had a mean *Story Problems* score of 10.23 and showed a flat trend. Juan had a mean score of 11.62 and also showed a flat trend. The mean for Group Three in the first phase was 10.92, and showed the group trend line was flat. According to visual analysis, neither Ana nor Juan demonstrated improvement when the CLR-SI intervention was introduced (Figure 13). Their trend lines during intervention remained at flat levels. The mean score for Group Three in the CLR-SI phase was 9.50. Ana's mean score in the intervention phase was 8.50 and Juan's was 8.50.

When comparing NAP between the basic strategy instruction phase and the CLR-SI intervention phase for Ana, 43% of points were found to be nonoverlapping (Parker & Vannest, 2009). For Juan, 44% of data points were nonoverlapping when comparing basic strategy to intervention. The NAP average for Group Three was 43.5%.

Figure 13 Group Three Results



Group four. In the first phase (i.e., Basic Strategy Instruction), Sophia had a mean *Story Problems* score of 8.40 and showed a flat trend. Maria had a mean score of 15.06 and showed a flat trend. The mean for Group Four in the first phase was 12.03, and the group showed a flat trend. According to visual analysis, Sophia demonstrated immediate improvement when the CLR-SI intervention was introduced (Figure 14). Her trend lines during intervention rapidly increased in slope and level. Sophia's mean score

in the CLR-SI intervention phase was 39.25. Sophia's slope increased from +0.61 in the Basic Strategy to +6.98 in the CLR-SI phase. Maria did not demonstrate a clear improvement after the introduction of the intervention. Her trend line showed a gradual increase. Maria's mean score in the CLR-SI phase was 25.00. The mean for Group Four in the CLR-SI intervention phase was 32.13. The group average demonstrated an immediate change when the intervention was introduced and a rapidly increasing trend.





Of all nine participants, Sophia was the only student to indicate a clear change of performance after the introduction of the intervention. When comparing NAP between the basic strategy instruction phase and the CLR-SI intervention phase for Sophia, 95% of points were found to be nonoverlapping (Parker & Vannest, 2009). For Maria, 79% of data points were nonoverlapping when comparing basic strategy to intervention. The NAP average for Group Four was 87.1%.

Pennies Test

A repeated-measures analysis of covariance (ANCOVA) was conducted to determine differences between students at the start and completion of the tutoring project on the *Pennies Test*. English language proficiency, addition computational fluency, and subtraction computational fluency were included as covariates in the analysis. To control for English language proficiency, students' WIDA scores were used as a covariate. To control for computational fluency, student pretest scores on Addition Fluency and Subtraction Fluency were also used as covariates in the full model. There was a significant effect of *Pennies* posttest performance, F(1, 5) = 19.069, p < .01. The effect size, calculated using partial eta-squared, was .79. English language proficiency (e.g., WIDA level) was not significantly related to *Pennies* performance, F(1, 5) = 2.173, p =.20. There was a significant interaction with students' pretest score on *Addition Fluency* with *Pennies* performance, F(1, 5) = 10.383, p = .02. The effect size, calculated using partial eta-squared, was .68. Student pretest *Subtraction Fluency* scores were not significantly related to *Pennies* performance, F(1, 5) = 2.794, p = .16

Name	English Proficiency <i>WIDA</i> Level (Max Score 5)	Addition Fluency (Max Score 25)		Subtraction Fluency (Max Score 25)		Pennies Test (Max Score 14)	
		Pre	Post	Pre	Post	Pre	Post
Juan	3	2	3	2	1	1	10
Ana	3	11	8	2	3	6	10
Maria	3	12	7	2	5	5	11
Sophia	4	4	8	4	5	4	10
Rosa	3.8	18	15	7	8	4	9
Eli	3	6	8	2	7	3	11
Isabel	1	16	19	4	7	6	11
Aaron	3.6	15	17	5	5	6	10
Gabriel	3.5	12	13	1	2	6	8

Table 4Pre- and Posttest Pennies Test Performance

Social Validity

Students responded to a five-question social validity questionnaire at the completion of posttest. Five statements were presented and students were prompted to circle a smiley face if they agreed with the statement, a straight face if they were neutral or weren't sure how they felt, or a sad face if they did not agree with the statement (see Appendix J). For the first statement, "This tutoring was helpful," nine students (100%) agreed with this statement. For the second statement, "I liked coming to tutoring," seven students (78%) agreed with this statement. Two students (22%) felt neutral about this

statement. The next statement was, "I learned how to solve story problems in tutoring," and nine students (100%) agreed with this statement. The following statement was "I can solve story problems correctly in class," and six students (67%) agreed with this statement. Three students (33%) felt neutral about this statement. For the final statement, "I would like to continue learning how to solve different types of story problems," nine students (100%) agreed. Students were able to write in additional comments at the bottom of their questionnaire. Overall, student comments were positive and expressed satisfaction with the tutoring. Examples of student comments were: "It was really fun", "I want to continue", "good job", "I had so much fun doing tutoring", and "I love to do tutoring."

CHAPTER V: DISCUSSION

The results from this multiple-method study hold several implications for wordproblem instruction for ELs with MD. In this chapter, I discuss the findings from both the quantitative and qualitative components of the study. Findings are discussed in terms of student performance and teacher practice. I then present the methodological limitations and describe how each could be improved. To conclude, I reflect on the educational implications for instruction and future research.

Student Performance

Student performance varied greatly between the *Story Problem* and *Pennies Test* measure. Overall, students did not demonstrate a clear effect after the introduction of the intervention on the *Story Problems*. However, on the *Pennies Test* students demonstrated significant gains from pre- to posttest. The mean student score changed from approximately the 11th percentile at pretest to the 48th percentile at posttest. The *Pennies Test* has been well validated in the research (i.e., Fuchs et al., 2008; Jordan & Hanich, 2000), but was not appropriate for use as the primary dependent measure in the multiple-baseline design due to tutoring time restraints. *Story Problems* was researcher-created, and largely based on the *Pennies Test* structure. The *Story Problems* measure was more appropriate for the limited assessment time within each tutoring session; however this measure has not been validated with a large student population. Additional analysis of existing student responses may inform future intervention measures for word-

problem solving. For example, analyzing changes in specific rubric rows (e.g., Does the student write an equation to represent the word problem?) or the ways in which students approach word problems (e.g., drawing pictures, making tally marks, etc.) can provide valuable information for how to measure changes in performance. Identifying appropriate measures to demonstrate word-problem solving proficiency given instructional time constraints is essential for future intervention work.

The only student to show a clear change in performance on the Story Problems measure after the introduction of the intervention (e.g., Sophia) had unique student characteristics. She was the only one of nine students selected for the third-grade Tier 2 intervention group. This may suggest that ELs with higher conceptual understanding and computational fluency respond better to CLR-SI. Further research with larger samples is necessary to explore this hypothesis. In addition, Sophia had the highest WIDA level, a four, compared to the other students in the sample. Having advanced English language proficiency may have benefited the enhanced understanding once the CLR-SI intervention began. In contrast, the students in Group Three (e.g., Ana and Juan) demonstrated consistently low performance in both phases of the tutoring project. Juan was currently receiving the most intensive intervention in Tier 3 at DES, and Ms. Jay felt he would likely be diagnosed with a learning disability by the end of third grade. According to Ms. Jay, Ana had experienced limited schooling in her native country before moving to the United States. Further investigation is needed to understand how students with varying profiles (i.e., language proficiency, mathematics achievement, years of schooling in native country) respond to schema instruction.

Teacher Practice

Ms. Jay's primary approach to word-problem instruction focused on presenting problems in contexts her students could relate to. This was evident through the instructional examples given that included students' names and situations that might commonly occur at DES. Throughout the course of the semester it became clear that students were exposed to limited word-problem representation and variation. For example, the most common word problem observed was a Change problem with the end unknown. The tutoring component of the study also focused on Change problems. However, students in both tutoring phases were exposed to Change problems with varying unknowns. Specifically, students were asked to solve addition and subtraction problems with the start, change, and end amounts unknown. Repeated exposure to all possible problem variations may improve students problem-solving abilities over time, and warrants further investigation. Further analysis of existing data can include a closer look at the frequency and quality of word problem instruction in Ms. Jay's class. Specifically, the time Ms. Jay spent providing direct instruction on word-problem solving and time students spent working on word problems can be calculated in relation to the total mathematics time observed. In addition, a frequency count on the type and variations of word-problems can be conducted to determine the extent to which Ms. Jay's students were exposed various word-problem representations.

There were several examples of culturally and linguistically responsive instruction observed in Ms. Jay's classroom: linguistic scaffolds (e.g., sentence frames), vocabulary instruction, allowing student use of native language, facilitating and encouraging peer discourse, nonverbal representation of content (e.g., manipulatives,

visuals), incorporating student identity and experiences into instructional examples, and warm welcoming relationships between the Ms. Jay and her students. Considering that the majority of students in the intervention were in Ms. Jay's class, their prior exposure to CRT and LRT may have influenced student responsiveness to the CLR-SI. Ms. Jay embraced her students' unique identities, including their cultural and ethnic diversity, as a positive and valuable resource in a classroom (Gay, 2010). I observed her frequently making curricular connections with students' personal experiences and using multiple resources in instruction (e.g., objects, drawings, graphs, and gestures) to facilitate content understanding (Gay, 2002, 2010; Klingner et al., 2012). Ms. Jay also incorporated sheltered instruction techniques regularly, including slow and clear speech enunciation, visuals and demonstrations, scaffolded instruction (e.g., her word-problem unit), purposeful vocabulary development, encouraging peer interactions and discourse, and incorporation of student experiences (Echevarria et al., 2006). She assumed personal responsibility of the third-grade mathematics curricular materials by adapting and supplementing to meet her students' diverse learning needs.

Native language was not explicitly incorporated in the intervention; however, Ms. Jay's Spanish-speaking students often spoke to each other in their native language during transitions and occasionally during content instruction. Peer exchanges between ELs also occurred during tutoring sessions. Each tutoring session was audio recorded for fidelity purposes, therefore it is possible to transcribe and translate EL use of native language during tutoring sessions. Further analysis of when (e.g., transitions, guided practice), on what (e.g., counting warm-up flash cards, using manipulatives), and how ELs use native

language to construct meaning (e.g., self-talk, peer discourse) holds implications for mathematics instruction and should be explored.

Ms. Jay's belief in her ability stemmed from a high individual and collective teacher efficacy (Protheroe, 2008). Individual, or personal, efficacy includes a teacher's belief that they have the "skills and ability to bring about student learning" for all students (Gibson & Dembo, 1984). Teacher efficacy has a demonstrated correlation with positive student outcomes (Tschannen-Moren, Hoy, & Hoy, 1998); however, further research is needed to understand specific attributes that contribute to student learning. Collective efficacy expands this notion to the belief in a community's (i.e., school staff) ability to improve student outcomes (Protheroe, 2008). Much of Ms. Jay's individual efficacy was interrelated with a larger collective efficacy evident at DES. Further analysis of the specific contextual factors that influenced Ms. Jay's belief system and those of her colleagues (e.g., teachers and administrators) may lead to greater understanding of contextual factors that promote achievement of CLD students, including ELs. Further research is necessary to determine how teacher preparation and mentoring of in-service teachers can contribute towards shaping beliefs that drive culturally and linguistically responsive teacher actions.

Limitations

There are several limitations that should be noted in this study. First, inferences are made with a relatively small sample size in both the qualitative and quantitative portions of the study. Future studies should study culturally and linguistically responsive word-problem instruction in settings with larger populations of ELs with MD. Time, in terms of tutoring instruction and measuring student understanding, was another limitation of the study. In addition, the primary dependent measure used to assess student performance is researcher-created. Further field-testing and standardization of wordproblem solving measures is critical to improving inferences researchers are able to make regarding student performance.

The intervention also has several inherent limitations. While culturally and linguistically responsive elements are incorporated into the intervention, others were purposefully left out (i.e., home-school relationship). Additional elements of CRT could be infused into a word-problem intervention if delivered by students' teachers, because a more permanent relationship would be established. Finding RAs who are also fluent in participants' language would allow for explanation of content and facilitation of discussion in native language. There was also a degree of overlap between the Basic Strategy and CLR-SI. Students were similar provided instructional scaffolds (i.e., flash card warm-up to improve computational fluency, RISE strategy, etc.) in both phases. Students also solved word problems in all variations of Change problem types (i.e., addition problems with the starting amount unknown; subtraction problems with the change amount unknown; etc.). This variation was not consistent with classroom observations of mathematics instruction. Increased exposure and practice on wordproblems in both phases may have influenced student performance. The Basic Strategy phase may have contributed to the overall growth students demonstrated on the Pennies Test.

Due to the limited time available for tutoring over the course of the semester, only one problem type was explicitly taught in CLR-SI. Expanding the intervention to include multiple problem types (e.g., Total, Difference) would improve the inferences I was able to make regarding the efficacy of CLR-SI for ELs with or at risk for MD. Future researchers should investigate the efficacy of including all problem types in the intervention.

There were also several methodological limitations in the study. Decisions in each phase were made primarily by visually analyzing group data. In hindsight, analyzing the results both individually and by group would have provided a more rigorous context in which to make methodological decisions throughout the course of the study. Similarly, most of the descriptive statistics (e.g., slope, median, etc.) presented in Tables 2 and 3 were calculated after the study concluded. Calculating these statistics to use in decision-making regarding phase changes would have been beneficial. Despite the above-mentioned limitations, the proposed study provides insight to the challenges and opportunities found in word-problem instruction for ELs with or at risk for MD.

Educational Implications and Conclusions

There are several implications of the present study for word-problem instruction, culturally and linguistically responsive mathematics instruction, teacher preparation, and the role of school context.

Word-problem Instruction

It is relatively unclear if CLR-SI is an effective approach for word-problem instruction for ELs. The present study highlighted issues with measurement and methodological challenges associated with time. Word-problem solving is a complex process, therefore students should be given ample time to engage in instruction. The majority of word-problem intervention studies for students with MD measure change in performance over the course of weeks (i.e., Fuchs et al., 2008; 2009; Jitendra et al., 2013; Powell & Fuchs, 2010). Future intervention studies should consider the extensive time needed to assess changes in word-problem solving performance and select research designs that will allow for maximum instruction with limited assessment (i.e., randomized-control trials). Problem type and structural representation was very limited in the classroom observations and problems I observed students working on. To date, word-problem representation has not been systematically investigated in elementary curricula and assessment. Increasing the variation may help students' word-problem solving abilities, and should be explored in future studies.

The role of vocabulary in word-problem instruction also warrants further investigation. Ms. Jay prioritized vocabulary development in mathematics instruction, especially for her ELs. She emphasized the importance of vocabulary throughout every interview, and this emphasis was mirrored in her classroom instruction. Students were assessed on mathematics vocabulary in addition to basic computational skills and their ability to apply skills in context. This was purposeful, to allow Ms. Jay to tease out when mathematics vocabulary interfered with students academic performance. When her EL students performed poorly on the vocabulary section of the first unit assessment, Ms. Jay incorporated a vocabulary center into students' weekly mathematics routine. She frequently incorporated games, visuals, and body gestures to facilitate student understanding of vocabulary terms. The role of vocabulary instruction in word-problem solving was also emphasized in interviews with the ESOL teachers and RTI Intervention teacher.

Much of the research on ELs and vocabulary instruction has focused on content areas of reading, science, and social studies (Nagy et al., 2012). Research on

mathematics vocabulary is often conducted in the later grades (i.e., middle and high school; Capraro & Joffrion, 2006). Several studies have examined vocabulary development in mathematics, but do so primarily for upper elementary and secondary students with content specific words (i.e. parallel, variable; Harmon, Hedrick, & Wood, 2005). Little is known about vocabulary instruction in mathematics for ELs with or at risk for MD. Further research is needed to understand the role of vocabulary instruction in word-problem contexts.

Culturally and Linguistically Responsive Mathematics Instruction

Ms. Jay's implementation of culturally and linguistically responsive mathematics instruction was not necessarily specific to mathematics as a content area. Many of her practices (e.g., frequently making curricular connections with students' personal experiences, purposeful vocabulary development, encouraging peer interactions) can be implemented across content areas. Through analysis of Ms. Jay's instruction and interviews, her emphasis on problem solving in context emerged as a culturally responsive practice unique to mathematics. Further research in additional settings is needed to further our understanding of culturally and linguistically responsive practices unique to mathematics instruction, as well as core practices applicable for all content areas.

The results from the word-problem intervention confirm the need to better understand the variability of student characteristics within EL populations. For example, Isabel had the lowest English language proficiency but demonstrated consistently higher performance on *Story Problems* than a student such as Ana. According to Ms. Jay's reflections, Isabel had received multiple years of schooling in her native country. Ms.

Jay was confident that as Isabel's English proficiency increased she would have minimal academic difficulties. Ana has attended school in the United States longer and has a higher English proficiency than Isabel; however, Ms. Jay expressed concern over the limited information provided regarding Ana's schooling in her native country. Ms. Jay's understanding of Ana's transition to the United States was that it was complicated and has involved her moving between numerous family members since attending DES. Her concern for Ana's word-problem solving had less to do with ability and more to do with the fact that Ana was still catching up on the school experience. Juan, in contrast, has attended DES since pre-kindergarten. Ms. Jay, the ESOL teachers, and the RTI interventionists were all concerned with his limited academic achievement. Juan's English proficiency indicator was comparable to Ana's; however; he is classified at the most intensive tier of the RTI framework and will likely be identified with a learning disability before the end of third-grade.

Academic performance, native and secondary language proficiency, the number of year's ELs have lived in the United States, and their academic experiences prior to arriving are a sample of the factors that should be considered in student mathematics achievement, including word-problem solving. Students with different educational strengths and needs may respond differentially to tailored instruction. For example, Isabel may have responded better to an intervention that included more linguistic supports and less mathematical concept supports. In contrast, Juan may have responded better to an intervention focused on computational fluency and basic mathematical concepts before moving on to more complex tasks such as word-problem solving. Varying student characteristics, including mathematics proficiency levels, have

implications for how students are identified with MD and for providing aligned intervention. Further qualitative research in this area may help develop future inventories to guide teachers' instructional decision-making for ELs.

A key emphasis of culturally responsive instruction is for teachers to incorporate students' culture and prior experiences to empower them academically and socially towards a goal of social justice and educational equity (Green, 2007). Ms. Jay often incorporated students' names and school-based experiences into mathematics instruction. Ms. Jay conceptualized and incorporated the school culture at DES into mathematics examples, particularly the word problems students were asked to solve. Ms. Jay's incorporation of students' cultural heritage and community experiences was limited. She appeared to have deep relationships with her students, but her focus on context occurred primarily through presenting familiar school-based situations (e.g., bringing drums to the music teacher at DES). Further implementation of culturally responsive instruction could include encouraging students to make their own decisions on social issues that are personally relevant to their community or native country, and take action to help solve them either through class or individual projects (Banks, 2008). In mathematics, this might look similar to the project Ms. Jay's students engaged in (e.g., survey of music for morning meeting, proposal for an alternative fire escape route), but would push further by empowering students as contributors to larger goals of equity and justice.

Teacher Preparation

In order shift the balance of privilege for historically underserved student populations through high-quality mathematics instruction; teachers must be prepared to teach CLD learners. In addition to providing pre-service teachers with an understanding

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of culturally responsive instructional strategies, preparation programs should also emphasize sociocultural consciousness (i.e., understanding ones belief system is influenced by race, ethnicity, and socio-economic status), inclusive and affirming attitudes towards CLD learners, focus on developing students who are critical thinkers and problem solvers, and an understanding of their role as teachers to be agents of change (Kea, Campbell-Whatley, & Richards, 2006). Teacher preparation should include both culturally and linguistically responsive elements to prepare educators to work with ELs (Lucas et al., 2014). Further research is needed to validate the role of culturally and linguistically responsive teacher preparation in improving outcomes for CLD students, including ELs with MD.

School Context

An increased focus on culturally responsive practices in general and special education teacher training and professional development is critical to ensuring CLD students receive quality instructional support (Gay, 2002; 2010; Shealey et al., 2011). However, few studies have focused on the adoption of culturally responsive pedagogy in general and special education teacher preparation programs (Trent, Kea, & Oh, 2008). Hernandez et al. (2013) developed a model for culturally responsive science and mathematics instruction, and piloted this model with pre-service teachers. Data collected throughout the pilot study was used to verify Hernandez et al.'s culturally responsive model. The resulting model included content integration, knowledge construction, prejudice reduction, social justice, and academic development. Further study is needed to determine which programmatic aspects provide meaningful learning experiences that help shape the knowledge, beliefs, and actions of pre-service teachers preparing to lead

CLD students with MD to academic gains. Continued research is needed to further define culturally and linguistically responsive mathematics instruction and to understand how this instructional approach affects math achievement for ELs.

Once teachers leave their initial preparation programs, the context of their school can influence their classroom effectiveness (Kraft & Papay, 2014). Despite this influence, there is limited research on the role of school context on instruction for CLD students, including ELs. Ms. Jay consistently connected her instructional beliefs to working at DES. The frequent collaboration and high collective efficacy evident at the school warrants further investigation. Future analysis of existing data collected may shed additional understanding on the contextual factors at DES that shaped teachers' belief system and interactions with students.

Further research is needed to determine the features and characteristics of educators in schools where personal and collective efficacy are high. This is particularly important to understand at schools effectively serving CLD learners. These characteristics should be incorporated into teacher education and educational leadership programs.

In summary, standardized mathematics items rely heavily on word problems to assess student knowledge and skill. Although there is evidence of discrepancies in mathematics performance between ELs and their native-English speaking peers, there is limited research on effective culturally and linguistically responsive instruction to improve word-problem solving for ELs. The present study contributed to the literature through multiple-method study on mathematics instruction for ELs with MD, with a focus on word-problem solving. Further research at the student, teacher, and school level

is needed to better understand culturally and linguistically responsive word-problem

instruction for ELs with MD

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APPENDICES

Appendix A.

Observation Protocol (Kibler, Deutsch, Futch, & Molloy, 2014)

The following is the Observation Protocol to be used for data collection.

Date: Times observed: Teacher, subject/ period observed: Focal students observed:

Summary of observation: Write 1-3 sentences summarizing the observations here.

Detailed observations: Include the running record of the observation here. Pay close attention to students' language use. Note code-switching, social talk, academic dialogue, and written communication. Note the instructional activity, student behavior, dialogue, and interactions. Note the time periodically. Analytical memos are noted with brackets.

Map of space: Include a map of the spaces observed here or attached to the notes. Include names and locations of students with times when possible.)

Pertinent artifacts: Make a note if student work documents are attached.

Your roles in the field today: *Make a note of what tasks you performed as a researcher* (*taking notes, recording students, etc.*), a classroom helper (making copies), a tutor, etc. *Make a note of any ways in which these roles privileged certain insider or outsider information collected*.

Post-observation reflections: *Reflect on methodological issues, items to be clarified, and analytical commentaries.*

Appendix B.

Initial Interview Protocol (Adapted from Kibler et al., 2014)

Hi [teacher name]. It is nice to meet you/see you again. Thank you for taking the time to talk to me today so I can learn a little bit more about your classroom. And thank you in advance for allowing me to observe your mathematics instruction over the next few months. I'd like to ask a few questions to learn about your classroom.

Before we start there are a few logistics to cover. First, this will not take longer than one hour. Second, everything you share will be stored under a pseudonym and will not be linked to your identity in any of my notes. Third, I'd like to audio-record our conversation so that I can go back and transcribe the interview for my notes. Is this all right with you? Do you have any questions for me before we get started?

General Information

- What grade and subject do you teach?
- How many years have you been a teacher?
 - What grades and subjects have you taught?
 - How many of those years have you taught at [school name]?
- Can you describe your initial teacher preparation program?
- Why did you decide to pursue a teaching career?

Culturally Responsive Teaching

- Can you describe the student demographics in your classroom?
 What racial and ethnic groups are represented?
 - What percentage of students in your classroom are ELs?
 - What languages do your students speak?
 - Are you fluent in any of these languages?
- How does your student population influence your instruction?
 - Can you describe your thought process as you plan instruction for ELs?

Language & Mathematical Concept Development

- How many years have you worked with the ESOL cluster at [school name]?
- What does ESOL instruction look like for your classroom/grade level?
- How would you describe the mathematics instruction in your classroom?
- How is native language taken into account in classroom instruction?
 - Specifically for mathematics instruction?
- What training and professional development have you received on teaching ELs?
 - Have you received any professional development on teaching mathematics to ELs? If so, please describe.

Word Problem Instruction

- When do students first encounter mathematics word problems?
- What difficulties do students have when solving word problems?

- Do you notice more difficulty for any student sub groups?
 - Students with disabilities?
 - ELs?
 - Others?
- Are there certain types of word problems that students seem to have more difficulty with?
- Can you describe the steps your students take when solving word problems?
- How often are students exposed to word problems?
- How often are students assessed on word problems?
- How do you teach students to solve word problems?

Closing

- Is there any other information about your classroom or instruction that I should know before I start observations?
- What time is mathematics instruction?
- Is there a weekly schedule or anything I should be aware of? For example, assessments are always on Friday, centers on Wednesdays, etc.

Thank you again for your time. I will likely have some follow up questions as I start to observe your classroom. We can wait to schedule the final formal interview until closer to the end of the semester. I look forward to working with you on this project!

Appendix C.

Document Analysis Protocol (Adapted from Miles & Huberman,1994)

Title or description of document:

Author:

Date given:

Obtained from (specific contact):

Significance or importance of document:

Summary (including specific quotes):

Appendix D.

Analytic Memo Template (Adapted from Kibler et al., 2014)

AUTHOR: MEMO DATE: FOR DATES: SETTING/FRAME: ACTORS:

QUALITATIVE RESEARCH QUESTIONS:

I. Summary of what I did these weeks:

II. Brief reflections from this week's

- A. Fieldnotes/ observations:
- B. Interviews:
- C. Recordings/ transcriptions:
- D. Student artifacts:

III. Assessing the week's data holistically

A. Are there emerging patterns or themes across the data?

B. Lingering questions?

IV. Comparing this week's data from previous week's data A. Similarities?

B. Novelties?

V. Literature

- A. Any good connections or ideas spring from the literature this week?
- *B.* Does the data bring to mind any particular literature to seek out?

VI. Methodological notes/ reflections from the week:

Appendix E.

Pennies Test (Jordan & Hanich, 2000)

- 1. Alex has 8 pennies. Kris has 6 pennies. How many pennies does Alex need to give away to have as many as Kris?
- 2. Sue had 5 pennies. Then Mike gave her 2 more pennies. How many pennies does Sue have now?
- 3. Chelsea has 6 pennies. Max has 4 pennies. How many pennies does Max have less than Chelsea?
- 4. Nina had 9 pennies. Then she gave 3 pennies to Anthony. How many pennies does Nina have now?
- 5. Janet has 3 pennies. Andy has 5 more pennies than Janet. How many pennies does Andy have?
- 6. Carol had 4 pennies. Then Nick gave her some more pennies. Now Carol has 6 pennies. How many pennies did Nick give her?
- 7. Claire has 4 pennies. Ben has 9 pennies. How many more pennies does Claire need to have as many as Ben?



DRIVER DISSERTATION

- 8. Jen had 7 pennies. Then she gave some pennies to Joe. Now Jen has 2 pennies. How many pennies did she give to Joe?
- 9. Emily has 3 pennies. John has 6 pennies. How many pennies do they have altogether?
- 10. Maria and Kevin have 8 pennies together. Maria has 3 pennies. How many pennies does Kevin have?
- 11. Ashley has 7 pennies. Jason has 4 pennies less than Ashley. How many pennies does Jason have?
- _____ 12. Dennis has 7 pennies. Molly has 5 pennies. How many pennies does Dennis have more than Molly?
- 13. Karen had some pennies. Then Matt gave her 4 more pennies. Now Karen has 6 pennies. How many pennies did she have to start with?
- 14. Lisa had some pennies. Then she gave 3 pennies to Bill. Now Lisa has 5 pennies. How many pennies did Lisa have to start with?

Appendix F.

Story Problems Packet 1

(Driver & Powell, 2014)

Solve each problem for the unknown amount. Show your work. Circle your final answer.

- 1. AJ had some crayons. Then, his teacher gave him 3 more crayons. Now AJ has 4 crayons. How many crayons did he have to start with?
- 2. Jaylen had 8 cookies, and then she gave a few to her friend. Now Jaylen has 5 cookies. How many cookies did she give to her friend?
- 3. Mila had 4 dresses and then she bought a few more. Now, Mila has 6 dresses. How many dresses did Mila buy?
- 4. Will started with 7 pencils. Then, he gave 5 pencils to his friend. How many pencils does Will have now?
- 5. Jacob ate 5 cookies, and then he ate 4 more. How many cookies did Jacob eat?
- 6. Matt had some toy cars. He lost 1 of the cars. Now he has 5 toy cars. How many toy cars did Matt have before losing some?

Appendix G.

Story Problems Scoring Rubric (Powell & Driver, 2014)

Story Problem Rubric			
Student Name:			
RA Name:			
Tutoring Day/Condition:			
Problem #:			
Rubric			
Product	Yes = 5; No = 0		
Is the word-problem numerical answer correct?			
Is the word-problem label answer correct?			
Process	Yes = 1; No = 0		
Does the student identify the part of the problem that is unknown			
(i.e., start, change, or end)?			
Does the student write an equation to represent the word problem?			
Is the equation set-up correctly (i.e., X represents unknown; start,			
change, and end amounts correct)?			
Does the student show their work while solving the problem?			
Does the student use correct calculations while solving the problem?			
TOTAL POINTS			

Appendix H.

Name: _				
Add.				
4	5	7	6	2
<u>+ 5</u>	<u>+ 1</u>	<u>+ 5</u>	<u>+ 4</u>	<u>+ 4</u>
2	8	0	3	3
<u>+ 8</u>	<u>+ 4</u>	<u>+ 7</u>	<u>+ 7</u>	<u>+ 4</u>
5	8	5	3	9
<u>+ 3</u>	<u>+ 1</u>	<u>+ 6</u>	<u>+ 6</u>	<u>+ 3</u>
2	8	6	7	2
<u>+ 5</u>	<u>+ 3</u>	<u>+ 6</u>	<u>+ 4</u>	<u>+ 9</u>
3	5	6	7	2
<u>+ 5</u>	<u>+ 4</u>	<u>+ 2</u>	+ 2	+ 3

Appendix I.

Subtraction Fluency (Fuchs et al., 2003)

Name: _				
Subtract. 12 <u>- 5</u>	7 <u>- 4</u>	7 - 0	10 <u>- 2</u>	12 <u>- 8</u>
11	8	11	9	12
<u>- 3</u>	<u>- 6</u>	<u>- 5</u>	<u>- 6</u>	- 9
6	11	10	7	10
<u>- 4</u>	<u>- 7</u>	<u>- 8</u>	<u>- 5</u>	<u>- 3</u>
11	6	12	10	12
<u>- 4</u>	<u>- 1</u>	<u>- 5</u>	<u>- 7</u>	<u>- 4</u>
5	9	5	8	11
- 3	- 4	- 0	- 5	<u>- 8</u>

Appendix J.

Student Social Validity Measure

Directions: Please circle how you feel about each of the following statements.

Statement	Rating		
This tutoring was helpful.			
I liked coming to tutoring.	() ()		
I learned how to solve story problems in tutoring.	÷)		
I can solve story problems correctly in class.) :	•••	
I would like to continue learning how to solve different types of story problems.	():		
Comme	ents		

Appendix K.

Sample Tutoring Materials: Book Puzzle Incentive



Appendix L.

Sample Tutoring Materials: Flash Card Warm-up and Graph



Appendix M.

Sample Basic Strategy Tutoring Script

Basic Strategy Tutoring Day 6

Materials

- Digital recorder
- Puzzle pieces
- Puzzle Sheet
- Picture Flash Cards
- Timer
- Flash Card Graph
- Colored pencils
- RISE Poster
- Student Story Problems Packet
- Baseline *Student Sheet* Day 6
- Highlighter

Turn on recorder. Say group name, tutoring condition, and session number.

PICTURE FLASH CARDS (2 min)

It's time for our flash cards.

Show Picture Flash Cards.

On these cards, there are pictures. Your job is to tell me how many objects or pictures are on each card. You can figure out the answer by counting the objects, doing the math with your fingers, or figuring it out in your brain. Sometimes you might just know the answer, so tell it to me as fast as you can!

Now, you'll have 1 minute to do as many cards as you can. If you answer a card incorrectly, I'll ask you to try again until you answer correctly. Any questions?

(Questions.)

Okay. You have 1 minute. What number?

Hold up flash cards one at a time. If correct, place card in pile on desk. If incorrect, say, "Count again." Place card in pile when corrected. Stop! That was 1 minute. Let's count to see how many cards you answered correctly. Count with me. 1, 2, 3, ...

Great! You answered ___ cards correctly! ___ is your flash card score for today. ___ was your score last time. Is your score today bigger or smaller than yesterday's score?

You always want to make your score get bigger! Let's graph ____ score on this *Flash Card Graph*.

(Student colors on Flash Card Graph).

ි්් You worked hard on the flash cards, so you earn one puzzle piece!

PREVIOUS LESSON REVIEW (4 min)

Nice work. Now let's practice what we learned last time we were together.

Present student packet of Story Problems. Put a star by the problem students should start on (after the highlight mark from the previous day).

You have 3 minutes to answer as many of these word problems as you can. Let me know if you want me to read any of the problems aloud to you. Do not skip problems; just try your best. Show your work and circle your answer. You may add or subtract to solve each problem. Most important, try your best! When the timer goes off, put your pencil down and look up at me. Ready?

(Set timer for 3 minutes. Allow students to work. They may use whatever strategies they would like but cannot use the motor manipulatives).

(After the timer goes off) Great work!

Make a highlighted mark after the last problem the student attempted. This will indicate where to start the Story Problems the next tutoring session. Collect student Story Problems for scoring on the rubric and file in student packet.

ြို့ပြား Vou worked hard on our Story Problems, so you earn one puzzle piece!

LESSON (7-9 min)

Present Basic Strategy Student Sheet Day 6.

Today we're going to keep practicing story problems. Let's review our RISE poster. This poster can help us solve story problems (take out Baseline RISE poster).

First, we read the whole problem. Then we illustrate the problem. You can illustrate by drawing pictures or just writing the equation.

Next, we solve the problem. All of the problems we solve will be either addition or subtraction. The last thing we do is to explain the problem. We do this by saying the answer, including what we were solving for, and by checking our work.

Show Problem A.

Let's RISE through a problem.

A. "James started with 2 strawberries. Then his brother gave him a few more. Now, James has 7 strawberries. How many strawberries did his brother give James?"

Go ahead and read the problem to yourself.

(Student works)

Next, you can illustrate the problem.

(Student works)

Then, you solve the problem. You can use addition and subtraction strategies that you know from class to solve the problem.

(Student works)

Last, explain the problem

(Student explains)

ි්් You are working hard and following directions. You've earned a puzzle piece!

Show Problem B.

Let's try another problem. Show me how you RISE through the problem.

B. "John had 6 books. Then he gave his teacher 4 of the books. How many books does he have now?"

(Student works on the problem. Prompt student to use the RISE poster as needed).

പ്പ് Excellent work!

(If time, have student work on Problem C and D. If 5 min or less remaining, move on to the final problem. Prompt student, as needed, using RISE poster)

C. "Kayla had 9 pins. She used some pins in art class. Now Kayla has 3 pins. How many pins did she use in art class?"

D. "Jen started with some shoes. Her mother gave her a 4 more. Now, Jen has 7 shoes. How many shoes did Jen start with?"

FINAL QUESTION (3 min)

We have one last question. Are you ready to solve it?

(Student responds)

E. "Mike had some crayons. His friend gave him 5 more crayons. Now Mike has 8 crayons. How many crayons did Mike start with?"

Go ahead and RISE through this problem.

(Allow student to work, providing support as needed).

်င္သာ You worked very hard on this problem. You earn a puzzle piece!

WRAP-UP (2 min)

Now, it's time to see how many puzzle pieces you earned today. Remember, solving word problems is just like finishing a story. You find the missing information to finish the problem! Go ahead and, count your puzzle pieces.

(Counts.)

How many pieces did you earn today for working hard and following directions?

Great! You get to color in ___ pieces on this puzzle.

Present Puzzle Sheet. Give colored pencil. (Student Colors.)

Remember, when you fill up the book puzzle, you get to pick a prize from the prize box! Nice work for today.
Appendix N.

Sample CLR-SI Tutoring Script

CLR-SI Tutoring Day 6

(Change increase – change unknown)

Materials

- Digital recorder
- Puzzle pieces
- Puzzle Sheet
- Picture Flash Cards
- Timer
- Flash Card Graph
- Colored pencils
- CLR-SI RISE Poster
- CLR-SI Problem-solving mat
- CLR-SI Student Sheet Day 6
- Student Story Problem Packets
- Motor manipulatives
- Highlighter

Turn on recorder. Say group name, tutoring condition, and session number.

FLASH CARDS (2 min)

It's time for our flash cards.

Show Flash Cards.

On these cards, there are pictures or numbers. Your job is to tell me how many are on each card. You can figure out the answer by counting the objects, doing the math with your fingers, or figuring it out in your brain. Sometimes you might just know the answer, so tell it to me as fast as you can!

Now, you'll have 1 minute to do as many cards as you can. If you answer a card incorrectly, I'll ask you to try again until you answer correctly. Any questions?

(Questions.)

Okay. You have 1 minute. What number?

Hold up flash cards one at a time. If correct, place card in pile on desk. If incorrect, say, "Count again." Place card in pile when corrected.

Stop! That was 1 minute. Let's count to see how many cards you answered correctly. Count with me. 1, 2, 3, ...

Great! You answered ___ cards correctly! ___ is your flash card score for today. ___ was your score last time. Is your score today bigger or smaller than yesterday's score?

You always want to make your score get bigger! Let's graph ____ score on this *Flash Card Graph*.

(Student colors on Flash Card Graph).

ි්්රි You worked hard on the flash cards, so you earn one puzzle piece!

PREVIOUS LESSON REVIEW (4 min)

Nice work. Now let's practice what we learned last time we were together.

Present student packet of Story Problems. Put a star by the problem students should start on (after the highlight mark from the previous day).

You have 3 minutes to answer as many of these word problems as you can. Let me know if you want me to read any of the problems aloud to you. Do not skip problems; just try your best. Show your work and circle your answer. You may add or subtract to solve each problem. Most important, try your best! When the timer goes off, put your pencil down and look up at me. Ready?

(Set timer for 3 minutes. Allow students to work. They may use whatever strategies they would like but cannot use the motor manipulatives).

(After the timer goes off) Great work!

Make a highlighted mark after the last problem the student attempted. This will indicate where to start the Story Problems the next tutoring session. Collect student Story Problems for scoring on the rubric and file in student packet.

്പ് You worked hard on our Story Problems, so you earn one puzzle piece!

LESSON (7-9 min)

Present CLR-SI Student Sheet Day 6.

Today we're going to keep learning about specific types of story problems. There are three different types of story problems.

What type of story problem did we learn about yesterday?

(Student responds)

That's right, Change problems. Today we are going to continue learning to solve change problems.

Change problems start with an amount of something. Then something happens to increase (hold up one hand – make hand move up) or decrease (hold up one hand – make hand move down) the amount you started out with. You end with a new amount.

When something increases, it get's bigger (make hand move up). When something decreases, it gets smaller (make hand move down).

Look at our problem-solving mat for Change story problems (show CLR-SI problemsolving mat).

Point to the start box). We start with one amount. Then a change happens. If something increases (move hand up), we will add. If something decreases (move hand down), we will subtract. The final number is the end (point to end box).

We can use our RISE poster to solve Change problems (show CLR-SI RISE poster).

Let's look at an example:

Present Problem A.

A. "[Insert student's name] started with 1 motor. Then I gave her/him a few more. Now, [Insert student's name] has 3 motors. How many motors did I give [Insert student's name]?"

Watch how I solve problem using our RISE poster and our Change mat.

First, I read the problem. What are we solving for?

(Student responds, "motors").

That's right, motors. Then I ask myself, what type of problem is this? It is a Change problem. I know this, because [insert student's name] started with an amount, something changes, and [insert student's name] ends with a different amount.

Next, I illustrate the problem. I will write the equation (write S C = E).

I know [insert student's name] started with 1 motor (put 1 motor in the Start box). How many motors does [insert student's name] end with?

(Student responds: 3).

That's right, 3 motors (put 3 motors in the End box). So I ask myself, what part of the equation is unknown? In this problem, it's the Change. I will circle the C on my equation.

Will the amount of motors increase or decrease from the start to end? Did the amount of motors get bigger or smaller? Let's think about the problem. If I give [insert student's name] more motors, will the amount they have get bigger or smaller?

(Students responds, "bigger/increase". Model with motors if needed).

Yes, bigger. 3 is bigger than 1. So, I will write a plus sign because the amount of motors increased, got bigger, after the change (write S + C = E).

Now I need to solve this problem. I know this is an addition problem because the change increases. So I write 1 + X = 3 (write equation). I need to solve for X, the unknown. How many motors do I need to add to 1 to equal 3?

Let's use the motors to count together. We start with 1 (point to start box). Now let's count on. Each time we count I will add a motor to the Start box until we reach 3. (With student – add each motor in Start box as you count). 2...3. We reached 3! How many motors did we add?

(Student responds: 2)

That's right, 2. Now let's explain the problem. What were we solving for?

(Student responds: motors)

So I gave [insert student's name] 2 motors. Last thing we do is check our work using our Counting Up Poster. Hmm, can we use our counting up strategy when X is before the equal sign?

(Student responds).

You're right, we need to rewrite the equation before we can solve.

DRIVER DISSERTATION

(Guide students through rewriting and solving for X using the appropriate Counting Up Poster).

Is 1 + 2 the same as 3?

(Student checks work using appropriate Counting Up Poster. Counts motors if needed).

Yes it is.

്പ് You are working hard and following directions. You've earned a puzzle piece!

Present Problem B.

Let's try another problem. This time I would like you to solve the problem using our RISE poster and Change Mat.

B. "Maria had 3 chips. Then her brother gave her a few more. Now, Maria has 4 chips. How many chips did her brother give her?"

First, we read the problem. What are we solving for?

(Student responds, "chips").

That's right, chips. Then we ask, what type of problem is this?

(Student responds, "Change").

How do you know?

(Student responds)

That's right, it is a Change problem. We know this, because Maria started with an amount, something changes, and Maria ends with a different amount.

Next, we illustrate the problem. Let's write the equation (write S C = E).

How many chips does Maria start with?

(Student responds: 3)

Let's put 3 motors in the Start box. Even though the problem says chips, we can use motors to solve the problem.

(Student counts 3 motors in Start box).

How many chips does Maria end with? Put them in the end box.

(Student responds: 4 chips; puts in end box).

That's right, 4 chips.

What part of the equation is unknown? Is it the start, change, or end?

(Student responds: Change).

In this problem, it's the Change. Circle the C on the equation.

Did the amount of chips increase or decrease from the start to end? Did the amount of chips get bigger or smaller?

(Students responds, "bigger/increase")

Yes, bigger. 4 is bigger than 3.

Should we write a plus sign or minus sign? Are we going to add or subtract?

(Student responds: add/plus sign)

That's right, we write a plus sign because the amount of chips increased, got bigger, after the change (write S + C = E).

Now we need to solve this problem. We know this is an addition problem because the change increases.

Can you write the equation?

(Student writes 3 + X = 4; provide support as necessary). We need to solve for X, the unknown. How many chips do we need to add to 3 to equal 4?

Let's use the motors to count together. We start with 3 (point to Start box). Now let's count on. Each time we count I will put a motor in the start box until we reach 4. (With student – add each motor in Start box as you count). 4. We reached 4! How many motors did we add?

(Student responds: 1)

That's right, 1. Now let's explain the problem. What were we solving for?

DRIVER DISSERTATION

(Student responds: chips)

So how many chips did Maria's brother give her?

(Student responds: 1 chip).

Last thing we do is check our work using our Counting Up Poster. Can we use our counting up strategy when X is before the equal sign?

(Student responds).

You're right, we need to rewrite the equation before we can solve.

(Guide students through rewriting and solving for X using the appropriate Counting Up Poster).

Is 3 + 1 the same as 4?

(Student checks work. Counts motors if needed).

Yes it is.

ີ້ມີ Excellent work! Now, it's your turn to solve some problems.

(If time, have student work on Problem C and D. If 5 min or less remaining, move on to the final problem. Prompt student, as needed, using RISE poster, Change Mat, and Counting Up Posters)

C. "A monkey had 4 bananas. Then the monkey found a few more. Now, the monkey has 6 bananas. How many bananas did the monkey find?"

D. "Jose had 5 pencils. His teacher gave him a few more. Now, Jose has 7 pencils. How many pencils did his teacher give him?"

FINAL QUESTION (3 min)

For our last question, you are going to help me create a story problem. What is your favorite TV show?

(Student responds)

Who is a character on this show? (If unsure, prompt student to tell you about the show). What does [insert TV character's name] like to do?

(Student responds)

O.K., let's make up a Change story problem together.

E. "[Insert TV character's name] **started with 4** [insert relevant objects]. **Then, his/her friend gave** [insert TV character's name]'s a few more. Now he/she has 8 [insert relevant objects]. **How many** [insert relevant objects] **did** [insert TV character's name]'s **friend give him/her?**"

Show me how to solve this problem using our RISE poster and our Change mat. (Allow student to work, providing support as needed).

So, how many [insert relevant objects] did [insert TV character's name]'s friend give him/her?"

(Student responds).

That's right, 4 [insert relevant objects]. Does this sound like something that could happen on [insert TV show]?

Last thing we do is check our work using our Counting Up Poster.

(Student checks work).

Is 4 + 4 the same as 8?

Yes it is.

်သိုYou worked very hard on this problem. You earn a puzzle piece!

WRAP-UP (2 min)

Now, it's time to see how many puzzle pieces you earned today. Remember, solving word problems is just like finishing a story. You find the missing information to finish the problem! Go ahead and, count your puzzle pieces.

(Counts.)

How many pieces did you earn today for working hard and following directions?

__.

Great! You get to color in ___ pieces on this puzzle.

Present Puzzle Sheet. Give colored pencil.

(Student Colors.)

Remember, when you fill up the book puzzle, you get to pick a prize from the prize box! Nice work for today!

Appendix O.

Tutoring Fidelity Checklist: Basic Strategy Instruction

Lesson Component	Check if
	Present
RA presents cards and asks how many all together? [Note: RA does not have to ask this every time as students get used to the procedure].	
RA gives one minute to answer cards, if incorrect say, "Try that again."	
RA counts with the student the number correct and students graph score.	
RA gives students 3 min to answer as many word problems as they can.	
RA tells students to let them know if they want the RA to read any problems out loud.	
RA tells student to not skip problems and try their best.	
RA models RISE strategy on a word problem.	
RA prompts students with RISE strategy.	
Students practice on a word problem word problems	
Students practice on a second word problem.	
RA DOES NOT use any materials from CR-SI (includes change mat, manipulatives, counting up posters, X, terminology of "start, change, and end").	
RA awards puzzle pieces throughout lesson.	
Student counts number of puzzle pieces they earned.	
Student colors in number of puzzle pieces earned.	
RA finishes lesson between 15 min and 25 minutes.	

Appendix P.

Tutoring Fidelity Checklist: CLR-SI

Lesson Component	Check if
	Present
RA presents cards and asks how many all together? [Note: RA does not	
have to ask this every time as students get used to the procedure].	
RA gives one minute to answer cards, if incorrect say, "Try that again."	
RA counts with the student the number correct and students graph	
score.	
RA gives students 3 min to answer as many word problems as they can.	
RA tells students to let them know if they want the RA to read any	
problems out loud.	
RA tells student to not skip problems and try their best.	
RA explains/reviews Change problem structure: "Change problems start	
with an amount of something. Then something happens to increase or	
decrease the amount you started out with. You end with a new amount".	
RA models RISE CLR-SI or Counting Up strategy on a word problem.	
RA prompts students with strategy.	
Students practice on a word problem word problems	
Students practice on a second word problem.	
RA illustrates problem with manipulatives.	
RA prompts students to share personal information to create a word	
problem.	
RA guides students through solving the problem, providing strategy	
prompts and feedback as needed.	
RA awards puzzle pieces throughout lesson.	
Student counts number of puzzle pieces they earned.	
Student colors in number of puzzle pieces earned.	
RA finishes lesson between 15 min and 25 minutes.	