Financial Deregulation and the Great Moderation

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Abstract

I investigate the effects of financial deregulation on the volatility of macroeconomic variables. First, I develop a model of small business borrowing that explicitly incorporates borrowing costs. I embed the optimal small business borrowing contract model into a Dynamic Stochastic General Equilibrium model. I find that the borrowing costs are significant to the macroeconomy in my model, and that the pattern of borrowing costs changes over time. I obtain first and second order accurate parameter estimates, and conclude that the second order is significant to the model dynamics and parameter estimates.

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1 Introduction

In the 1980's, United States macroeconomic variables exhibited a substantial decrease in volatility (Stock and Watson (2003))[1]. According to McConnell and Perez-Quiros (2000), this moderation occurred abruptly in the first quarter of 1984[2]. The phenomenon is known in the literature as the "Great Moderation" (Stock and Watson (2003). Although the timing of the Great Moderation is well-documented, there is still debate over its causes.

One possible cause of the Great Moderation is improved monetary policy. Inflation volatility and macroeconomic volatility tend to move together (Blanchard and Simon (2001)), so it is possible that a decline in the variance of inflation in the 1980's led to the Great Moderation. Inflation volatility and macroeconomic volatility were both high in the 1970's and early 1980's. The 1970's was also a period of poor monetary policy performance (Romer and Romer (2002)). After 1979, the Federal Reserve committed to a policy of low inflation (Stock and Watson (2003)). However, inflation variability did not fall until 1984 (Bernanke (2004)). The lag between the Federal Reserve's policy change and the variance of inflation weakens the link between the two events, and Stock and Watson (2003) suggest that monetary policy changes had little to do with the Great Moderation.

Another possible explanation is the decreased variability of structural shocks in the mid-1980's (Stock and Watson (2003)). In their 2007 paper, Justiniano and Primiceri evaluate the explanatory ability of the variance of structural shocks by estimating a Dynamic Stochastic General Equilibrium model with time-varying volatilities of

[1] Also discussed by Sims and Zha (2006)

[2] Also Kim and Nelson (1999), Stock and Watson (2002), Chauvet and Potter (2001), Herrara and Pesavento (2005)

structural shocks. After estimating their model, Justiniano and Primiceri (2007) perform a counterfactual experiment in which the variance of the investment-specific shock is set at its pre-1984 level. The investment specific shock refers to the price associated with converting investment to capital. In this experiment, GDP and other macroeconomic variables are significantly more volatile than in the data. Justiniano and Primiceri (2007) conclude that the decreased variance of the investment-specific shock was a major contributor to the Great Moderation.

Justiniano and Primiceri's (2007) shocks are structural in the DSGE model, but do not have a clear interpretation. Without a theoretical context for exploring the investment-specific shock, we can only conclude that the volatility of the investment specific shock declined on its own. By explicitly modeling borrowing costs, I provide a framework to test the interpretation suggested by Justiniano and Primiceri (2007) that the investment shock represents borrowing costs and that financial deregulation in the 1980's caused these costs to be come more stable.

To study the effect of the mean and variability of borrowing and lending costs on macroeconomic variables, I model a lending contract between individual savers and small businesses. This model is based on an agency cost model by Carlstrom and Fuerst (1997). The terms of the contract depend on the borrowing and lending costs. After constructing the optimal contract for small business loans, I incorporate it into a Dynamic Stochastic General Equilibrium Model. The model includes a number of standard structural shocks. In one version of the model, I allow the variances of the shocks to change over time.

I first estimate the model using a first order accurate method. Although this method cannot account for agents' knowledge of stochastic volatility, it is simpler to apply when the parameter space is large. I also perform a second order accurate approximation and compare the first- and second-order results. I find that, although the second-order accurate approximation is much more computationally intensive than the first order, it provides crucial information about the parameters and the behavior of the economy. I do not, however, find significant differences in model dynamics or parameter estimates between the time-invariant and time-varying versions of the model. The model fits major macroeconomic variables, such as output, investment and consumption, by design - I use these variables to estimate the model parameters. With a few modifications to the main model, I also capture the cycles of other variables such as the interest rate and the rental rate of capital.

I also show changes in the cycles of borrowing costs starting in the 1980's. These changes coincide with a time of financial deregulation in the United States. A number of regulatory changes, enacted during the late 1970's and early 1980's, gave increased power and flexibility to lenders. For example, the Depository Institutions Deregulation and Monetary Control Act (DIDMCA), passed in 1980, removed the power of the Federal Reserve to set interest rates on savings accounts. Because a large amount of small business lending comes in the form of bank loans, the increased flexibility of lenders may have affected the entrepreneur's search costs. The DIDMCA also allowed banks to merge. Larger institutions had more leverage over borrowers, thus decreasing the monitoring cost faced by the lender. The coincidence of changes in borrowing cost cycles and financial deregulation may imply that the two are connected, although further experiments using data from different countries would be need to bolster this argument.

In Section 2, I provide a brief literature review. In Section 3, I present the dynamic stochastic general equilibrium model. The model solution equations, steady state, data, and estimation processes are discussed in Section 4. Section 5 contains the results for the benchmark model. In Section 6, I review results from alternative model specifications. Section 7 concludes.

2 Literature Review

The Great Moderation refers to a decrease in the volatility of U.S. macroeconomic variables that occurred in the mid-1980's. However, similar phenomena have occurred throughout history. The post-World War II moderation of macroeconomic variables is the first moderation to be discussed at length in the literature, and this research may be able to contribute to our understanding of the more recent moderation. DeLong and Summers (1984) find that a moderation in the variability of the rate of change of annual GNP occurred in the United States after World War II. Diebold and Rudebusch (1992) concur with this finding, noting that expansionary periods were much longer after World War II than before. This indicates a post-war decrease in the volatility of production and other macroeconomic variables. The lengthening of the business cycle after World War II is statistically significant (Diebold and Rudebusch (1992)).

The literature explores a number of explanations for the moderation that occurred after World War II. DeLong and Summers (1984) find that increased government involvement and the growing availability of private credit stabilized consumption after World War II. DeLong and Summers (1984) also investigate the avoidance of financial panics as a stabilizing force, but find that this did not play a role. Watson (1994) dismisses economic explanations for the post-war moderation, instead concluding that the stabilization was caused by improved data collection. Before the war, the NBER collected only a few economic time series. Watson (1994) concludes that pre-war volatility was the result of too few data points and that the post-war decrease in variance was due to the increased number of time series analyzed. The literature fails to find any convincing economic or structural explanations for the moderation that occurred after World War II.

More recently, the growth rate of U.S. GDP has stabilized significantly since the mid-1980's. McConnell and Perez-Quiros (2000) are the first to examine the statistical significance and timing of this more recent moderation using quarterly U.S. GDP growth from the second quarter of 1953 to the second quarter of 1999. They first regress GDP growth on a constant and linear time trend, and find that the trend coefficient is negative but statistically insignificant (McConnell and Perez-Quiros (2000)). In a regression of the square of GDP growth, however, the trend coefficient is negative and statistically significant to the 1% level (McConnell and Perez-Quiros (2000)). Finally, McConnell and Perez-Quiros (2000) test the hypothesis that a structural break occurred in the standard deviation of GDP growth. They reject the null that there is no structural break (i.e. $\sigma_1 = \sigma_2$) and identify the first quarter of 1984 as the break point. Kim and Nelson (1999), using a Bayesian approach, also find evidence that a structural break occurred in the first quarter of 1984. Although other literature finds slightly different dates [3], the first quarter of 1984 is generally accepted as the beginning date of the Great Moderation.

The moderation in economic variability is not limited to the United States. Other G7 countries, including the Italy, Germany, Japan and the U.K., also experienced

[3] Stock and Watson (2003) find that the structural break occurred around the second quarter of 1983

moderations in the 1980's or 1990's (Stock and Watson (2005)). Mills and Wang (2003) find evidence that the structural breaks in the variance of GDP growth occurred at different times in each country. The dates of the structural breaks range from Germany in 1974 to the U.K. in 1993 (Mills and Wang (2003)). It is clear, then, that although the moderation is international in nature, it did not occur in response to a phenomenon that affected all countries in the same way or at the same time.

Although the timing of the Great Moderation is well-documented both in the United States and abroad, its causes remain unclear. One possible cause is improved monetary policy and a decline in inflation volatility in the United States, which occurred at roughly the same time as the Great Moderation. According to Blanchard and Simon (2001), inflation and macroeconomic volatility often move together. The 1970's was a period of high inflation volatility and poor monetary policy performance (Romer and Romer (2002)). The Federal Reserve committed to a policy of low inflation in 1979 (Stock and Watson (2003)), and it is possible that this caused a decline in macroeconomic variability in the United States. Although Stock and Watson (2003) find some role for monetary policy in the Great Moderation, the link is weak because of the five year lag between the policy change and the volatility change (Bernanke (2004)). Ultimately, the literature suggests that changing monetary policy had little effect on macroeconomic variability (Stock and Watson (2003), Bernanke (2004)).

Others argue that the Great Moderation was caused by changes in the structure of the economy. For example, Kahn, McConnell and Perez-Quiros (2002) suggest that improved information about demand for durable goods has led to less durable goods production volatility, which in turn has decreased overall output volatility. This finding is supported by McConnell and Perez-Quiros' (2000) earlier observation that a decrease in the variance of durable goods output accounts for most of the decreased variance of total output. Although the volatility of the *production* of durable goods has declined, Kahn, McConnell and Perez-Quiros (2002) find no concurrent fall in the variance of *final sales* of durable goods. This, they argue, indicates that producers have better knowledge of demand and are less likely to over-produce or under-produce durable goods (Kahn, McConnell and Perez-Quiros (2002)). This theory depends necessarily on the lack of change in the volatility of durable goods sales. However, other literature has found convincing evidence for a structural break in sales of durable goods as well as their production[4]. These findings significantly undermine the inventory management explanation of the Great Moderation.

A more accepted theory is that the Great Moderation was caused by a decline in the volatility of shocks to the economy. Stock and Watson (2003) find that more stable shocks account for some of the decline in GDP volatility. More recently, Justiniano and Primiceri (2007) find that shocks played a significant role in the Great Moderation. Using a Dynamic Stochastic General Equilibrium model with time-varying volatilities of structural shocks, they find that a more stable investment-specific shock can account for most of the increased stability of GDP (Justiniano and Primiceri (2007)). The relevance of this shock also extends to investment, labor and consumption volatility (Justiniano and Primiceri (2007)).

The Justiniano and Primiceri (2007) model suggests that the investment-specific shock may be interpreted as a shock to borrowing costs. To investigate this theory, I incorporate borrowing costs into my model explicitly. The model that I develop is based on the optimal contract model developed by Carlstrom and Fuerst (1997). In

[4] Including Ahmed, Levin and Wilson (2002), Herrera and Pesavento (2002), Kim, Nelson and Piger (2001), Stock and Watson (2002)

this model, entrepreneurs borrow investment goods. The lender incurs a monitoring cost in case of borrowing default. The Carlstrom and Fuerst model builds on Bernanke and Gertler (1989) and Fuerst (1995). My model builds on this earlier work by introducing a search cost, incurred by all borrowers, in addition to the monitoring cost, which is incurred only in case of default.

Justiniano and Primiceri (2007) use first order methods to estimate their model. I will produce second-order accurate parameter estimates for my model. I implement a particle filter, along the lines of Fernandez-Villaverde and Rubio-Ramirez(2006) and An and Schorfheide (2007). The Fernandez-Villaverde and Rubio-Ramirez (2006) work demonstrates the use of the particle filter to estimate a nonlinear and non-normal model. In particular, linear methods calculate the likelihood function for a linear approximation of the model. In contrast, the particle filter calculates the likelihood for a second-order approximation of the model and does not require normally distributed shocks. Earlier work (Fernandez-Villaverde, Rubio-Ramirez and Santos (2006) as well as Fernandez-Villaverde and Rubio-Rameriz (2005)) demonstrates that the linearization required by first-order techniques can have a significant impact on estimation results. The errors from linearization are compounded in each period, so that they will be much more significant in larger samples. I will use a large sample of approximately 150 periods, so this is especially relevant to my work.

3 The Dynamic Stochastic General Equilibrium Model

The dynamic stochastic general equilibrium model contains a firm and two families of individuals. Each individual is a member of a family that makes investment, employment and consumption decisions for its members (as in Alexopoulos (2006)). At the beginning of each period, an employment agency assigns some individuals to work for the firm and some to be entrepreneurs. Entrepreneurs are members of one family and non-entrepreneurs are members of the other. Each family makes consumption and investment decisions for its members. The non-entrepreneurial family also chooses the amount of labor for its members. In each period, all households contribute the capital that they have earned to the family's capital stock. The family then rents its capital to the firm. This is the entrepreneurial family's sole source of income. The non-entrepreneurial family also collects the wages earned by each of its members.

The economy also has a final goods firm that produces consumption goods using labor and capital inputs. In this economy, the firm also produces capital goods. It uses its own capital in production and also rents capital from the families. If small businesses were the only producers of capital in the economy, the model might overstate the effects of deregulation that affects small businesses only. This structure also allows me to explore the effects of financial deregulation related to corporate investment activities in future work.

3.1 Model Shocks

The model contains five exogenous shocks. In the baseline version of the model, the variances of these shocks are fixed over time. In an alternate version of the model, the shock variances themselves follow an exogenous process and are also subject to shocks.

3.1.1 Model Shock Processes

The firm is subject to a technology shock, A_t, which affects its production of consumption goods. This shock is a unit root and its growth rate, $z_t = log\frac{A_t}{A_{t-1}}$ follows the exogenous process:

$$ln\ z_t = \rho_z ln z_{t-1} + \epsilon_{z,t}$$

where $\epsilon_{z,t} \sim N(0, \theta^2_{z,t})$.

The firm is also subject to a capital adjustment cost shock, ζ_t, which affects its conversion of investment to capital. This shock represents costs associated with time-to-build, and is applied to the rate of change of capital stock from the prior period to the current period. This shock takes the form:

$$ln\ \zeta_t = (1 - \rho_\zeta)\mu_\zeta + \rho_\zeta ln\zeta_{t-1} + \epsilon_{\zeta,t}$$

where $\epsilon_{\zeta,t} \sim N(0, \theta^2_{\zeta,t})$.

The stochastic lending search costs and borrower monitoring costs associated with lending between entrepreneurs and non-entrepreneurs are given by:

$$\mu_t = (1 - \rho_\mu)\mu_\mu + \rho_\mu \mu_{t-1} + \epsilon_{\mu,t}$$

$$\rho_t = (1 - \rho_\rho)\mu_\rho + \rho_\rho \rho_{t-1} + \epsilon_{\rho,t}$$

where $\epsilon_{\mu,t} \sim N(0, \theta^2_{\mu,t})$ and $\epsilon_{\rho,t} \sim N(0, \theta^2_{\rho,t})$. Note that the structure of the borrowing costs does not require them to be nonnegative.

Finally, the non-entrepreneurial family, which works for the firm, is subject to a shock to the disutility of labor:

$$ln\ \psi_t = (1 - \rho_\psi)\mu_\psi + \rho_\psi ln\ \psi_{t-1} + \epsilon_{\psi,t}$$

where $\epsilon_{\psi,t} \sim N(0, \theta^2_{\psi,t})$.

3.1.2 Time-Varying Volatility

In one version of the dynamic model, the shocks have time-varying stochastic volatility. In this formulation, the variance of the structural shock follows an autoregressive process. Using the variance of the technology shock, $\theta_{z,t}$, as an example, the variance follows the following process:

$$ln\ \theta_{z,t} = (1 - \rho_{\theta,z}) ln\ \mu_{\theta,z} + \rho_{\theta,z} ln\ \theta_{z,t-1} + \epsilon_{\theta,z,t}$$

where $\epsilon_{\theta,z,t} \sim N(0, \nu_z)$. This formulation allows me to examine changes in the structural shocks over time. However, the increased flexibility does require an additional 10 parameters to be estimated, which comes at a nontrivial computing time cost.

3.2 The Firm

The risk-neutral firm produces consumption goods using capital and labor inputs. The consumption good production process is subject to a stochastic technology shock, A_t, which grows at rate z_t. The firm obtains labor at the market-clearing wage rate, w_t. It uses some of its profits to finance capital production (similar to Poveda and Coen-Pirani (2005)). The firm's capital production process is subject to a capital adjustment cost shock, ζ_t. This shock represents time-to-build associated with capital investment. The firm uses all of its capital, given by K_t^F, in its production of consumption goods, and also rents capital from the families, in the amount K_t^H, at the market clearing rate q_t. The firm combines its capital, K_t^F, with the capital rented from the families, K_t^H, to obtain the total amount of capital available for final goods production, K_t, using the following function:

$$K_t = (K_t^H)^{\gamma}(K_t^F)^{1-\gamma}$$

In each period, the firm chooses the amount of investment in capital production, I_t^F. Then the shareholder dividend is given by:

$$D_t = f(K_t, L_t) - w_t L_t - q_t K_t^H - I_t^F$$

The firm's capital accumulation equation is given by:

$$K_{t+1}^F = (1-\delta)K_t^F + \zeta_t \left(1 - \left(\frac{I_t^F}{I_{t-1}^F}\right)^2\right) I_t^F$$

The firm's production function is:

$$f(K_t, L_t) = A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha}$$

The firm's optimization problem is to maximize shareholder dividends, subject to the constraints given in Equations 9 through 12. The problem is given by:

$$\max_{I_t, K_t^H, L_t, K_t^H} E_0 \sum_{t=0}^{\infty} \beta^t D_t$$

$$\begin{aligned} s.t. \qquad D_t &= f(K_t, L_t) - w_t L_t - q_t K_t^H - I_t \\ f(K_t, L_t) &= A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} \\ K_t &= (K_t^H)^{\gamma} (K_t^F)^{1-\gamma} \\ K_{t+1}^F &= (1-\delta) K_t^F + \zeta_t \left(1 - \left(\frac{I_t^F}{I_{t-1}^F}\right)^2\right) I_t^F \end{aligned}$$

From the first order condition for labor (L_t), the marketing clearing wage rate is:

$$w_t = (1-\alpha) A_t^{1-\alpha} \left(\frac{K_t}{L_t}\right)^{\alpha}$$

The first order condition for rental capital, K_t^H, is:

$$q_t = \frac{\alpha\gamma (A_t L_t)^{1-\alpha} K_t^{\alpha}}{K_t^H}$$

The first order condition for firm-owned capital is:

$$\beta_{t+1}^{t+1} E_0 \left[\left(\frac{\alpha(1-\gamma)(A_{t+1} L_{t+1})^{1-\alpha} K_{t+1}^{\alpha}}{K_{t+1}^F} \right) + \left(\frac{1-\delta}{\zeta_{t+1}} \right) \right] = \beta_t^t E_0 \left(\frac{1}{\zeta_t} \right)$$

The firm's problem pins down the the wage rate faced by the family when making decisions for its members. The firm also contributes to the total stock of capital in the economy.

3.3 The Small Business Borrowing Contract

Individuals may be entrepreneurs (small-business owners) or non-entrepreneurs. Small businesses may borrow from non-entrepreneurs. I construct a static model of the optimal borrowing contract between a borrower and a lender that is a modification of Carlstrom and Fuerst's (1997) agency cost model. The borrower is a capital-producing small business. The borrower invests personal wealth and borrowed funds in the business. The optimal lending contract maximizes the profit of the business, while leaving the lender indifferent.

3.3.1 Small Business Capital Production

A business produces ω units of the capital good per unit of investment. The production process, ω, is stochastic and is distributed uniformly on $[0, 2]$. The business's capital production is also affected by its ability, j, which is uniformly distributed on the interval $[0.5, 1.5]$. A j value of 0.5 means that the business has low ability and a j value of 1.5 means that the business has high ability. These distributions are chosen to allow me to find a closed form solution to the optimal contract problem. Also, the range of j allows entrepreneurial ability to be applied multiplicatively to capital production. The borrower is also subject to a cost, ρ, associated with finding financing. This search cost means that a percentage, $(1-\exp(\rho))$, of the total capital is consumed by the potential borrower in the search for a lender. A business of ability

j produces capital in the amount of:

$$j \exp(\rho)\ \omega\ i(j)$$

where j is ability, ω is the realization of the stochastic production process, ρ is the financing search cost, and $i(j)$ is the total amount of investment capital.

3.3.2 The Financial Contract

The financial contract between a business of ability j and a lender will determine the total amount to be invested, $i(j)$, the interest rate, $r(j)$, and the threshold value of $\omega(j)$ below which the borrower will default, called $\bar{\omega}$. The contract is a one-period contract, so that lending must be negotiated in every period. The total amount of investment capital used by the business is $i(j)$ and $i^e(j)$ represents its internal investment. Then $i(j) - i^e(j)$ is the amount borrowed, $i^f(j)$. The business that borrows $i^f(j)$ agrees to repay the lender $(1 + r(j))i^f(j)$ capital goods tomorrow. In addition, the business must pay a search cost, ρ, destroying a portion of the capital it produces. Because the capital production function is stochastic, the borrower may not be able to honor the terms of the contract. Default will occur when the proceeds from the capital project, $\exp(\rho)j\omega(j)i(j)$, are less than the debt obligation to the lender, $(i(j) - i^e(j))(1 + r(j))$:

$$j \exp(\rho)\omega(j)i(j) < (i(j) - i^e(j))(1 + r(j))$$

Then the threshold value of $\omega(j)$, $\bar{\omega}(j)$, below which the borrower will default, is given by:

$$\bar{\omega}(j) = \frac{(1 + r(j))(i(j) - i^e(j))}{\exp(\rho)\ j\ i(j)} \tag{1}$$

If r^k is the end-of-period price of capital, the business's expected profit can now be written in terms of $r^k, i(j), r(j), \bar{\omega}$, ρ and j:

$$r^k \left[\int_{\bar{\omega}(j)}^{2} \omega(1 + r(j))ji(j)\Phi(d\omega) - (1 - \Phi(\bar{\omega}))(1 + r(j))(i(j) - i^e(j)) \right]$$

or, equivalently:

$$r^k ji(j)\ \exp(\rho) \left[\int_{\bar{\omega}(j)}^{2} \omega\Phi(d\omega) - (1 - \Phi(\bar{\omega}(j)))\bar{\omega}(j) \right]$$

Let the function $f(\bar{\omega}(j))$ be defined as:

$$f(\bar{\omega}(j)) = \left[\int_{\bar{\omega}(j)}^{2} \omega\Phi(d\omega) - (1 - \Phi(\bar{\omega}(j)))\bar{\omega}(j) \right]$$

so that the business expected profit can be represented as:

$$r^k ji(j)\ \exp(\rho) f(\bar{\omega}(j)) = r^k ji(j)\ \exp(\rho) \left[\int_{\bar{\omega}(j)}^{2} \omega\Phi(d\omega) - (1 - \Phi(\bar{\omega}(j)))\bar{\omega}(j) \right]$$

When the borrower defaults, the lender seizes all of the proceeds of the entrepreneur's capital project. The lender also incurs costs associated with monitoring the borrower in the case of default. To monitor the project and discover the borrower's realization of ω_t, the lender destroys a percentage, $(1 - \exp\ (\mu))$, of the capital seized from the project. The lender's expected income from lending to a business with ability

h is given by:

$$r^k \left[\exp(\mu) \int_0^{\bar{\omega}(h)} \exp(\rho)\omega i(h)h\Phi(d\omega) + (1 - \Phi(\omega(\bar{h})))(1 + r(h))(i(h) - i^e(h)) \right]$$

or, equivalently:

$$r^k h \exp(\rho)\, i(h) \left[\exp(\mu) \int_0^{\bar{\omega}(h)} \omega\Phi(d\omega) + (1 - \Phi(\bar{\omega}(h)))\bar{\omega}(h) \right]$$

Let the function $g(\bar{\omega}(h))$ be defined as:

$$g(\bar{\omega}(h)) = \left[\exp(\mu) \int_0^{\bar{\omega}(h)} \omega\Phi(d\omega) + (1 - \Phi(\bar{\omega}(h)))\bar{\omega}(h) \right]$$

Then the lender income may be represented as:

$$r^k h \exp(\rho)\, i(h) g(\bar{\omega}(h)) = r^k h \exp(\rho)\, i(h) \left[\exp(\mu) \int_0^{\bar{\omega}(h)} \omega\Phi(d\omega) + (1 - \Phi(\bar{\omega}(h)))\bar{\omega}(h) \right]$$

To make the notation less cluttered, we will keep in mind that $i, \bar{\omega}, r$ and n are functions of entrepreneurial ability and remove the function notation, so that the entrepreneur's income becomes:

$$r^k ij \exp(\rho) \left[\int_{\bar{\omega}}^{2} \omega\Phi(d\omega) - (1 - \Phi(\bar{\omega})\bar{\omega} \right] = r^k ij \exp(\rho) f(\bar{\omega}, j)$$

The individual income is given by:

$$r^k ih \exp(\rho) \left[\exp(\mu) \int_0^{\bar{\omega}} \omega\Phi(d\omega) + (1 - \Phi(\bar{\omega}))\bar{\omega} \right] = qih \exp(\rho) g(\bar{\omega}, h)$$

The lender may choose not to lend. In order for the lender to loan money to the business, the expected income from lending must at least equal the amount being loaned:

$$r^k ih \exp(\rho) \left[\exp(\mu) \int_0^{\bar{\omega}} \omega \Phi(d\omega) + (1 - \Phi(\bar{\omega}))\bar{\omega} \right] = qih \exp(\rho) g(\bar{\omega}, h) \geq i - i^e \quad (2)$$

3.3.3 The Optimal Contract

The optimal contract for a business with ability j is found by solving for the $i, \bar{\omega}, r$ combination that maximizes the business's expected profit, subject to the lender participation constraint (Equation (2)), the equation that determines the threshold value of ω (Equation (1)), and the equation that describes the total amount of investment:

$$\max_{i,\bar{\omega},r} r^k ij \exp(\rho) \left[\int_{\bar{\omega}}^2 \omega \Phi(d\omega) - (1 - \Phi(\bar{\omega})\bar{\omega} \right]$$

$$s.t. \quad r^k ij \exp(\rho) \left[\exp(\mu) \textstyle\int_0^{\bar{\omega}} \omega \Phi(d\omega) + (1 - \Phi(\bar{\omega}))\bar{\omega} \right] = i - i^e$$

$$\bar{\omega} ij \exp(\rho) = (1 + r)(i - i^e)$$

$$i = i^e + i^f$$

Equivalently, the problem in terms of $f(\bar{\omega})$ and $g(\bar{\omega})$ is given by:

$$\max_{i,\bar{\omega},r} r^k ij \exp(\rho) f(\bar{\omega})$$

$$s.t. \qquad r^k ij \ \exp(\rho) g(\bar{\omega}) \geq i - i^e \tag{3}$$

$$\bar{\omega} ij \ \exp(\rho) = (1 + r)(i - i^e) \tag{4}$$

$$i = i^e + i^f \tag{5}$$

I obtain the terms of the optimal contract by solving the maximization problem. Let the optimal contract terms for an entrepreneur with ability j be given by $\bar{\omega}_t^*(j)$, $r_t^*(j)$, and $i_t^*(j)$.

The lender's participation constraint must hold with equality, so Equation (3) can be used to solve for i as a function of $\bar{\omega}$. In addition, Equation (4) can be rearranged to define r in terms of i and $\bar{\omega}$[5]. When these equations are substituted into the objective function, the problem reduces to:

$$\max_{\bar{\omega}} r^k j \ \exp(\rho) f(\bar{\omega}) \frac{i^e}{1 - r^k j \ \exp(\rho) g(\bar{\omega})}$$

The first order condition with respect to $\bar{\omega}$ is:

$$qj \ \exp(\rho) f'(\bar{\omega}) \frac{i^e}{1 - qj \ \exp(\rho) g(\bar{\omega})} + (qj \ \exp(\rho))^2 f(\bar{\omega}) g'(\bar{\omega}) \frac{i^e}{(1 - qj \ \exp(\rho) g(\bar{\omega}))^2} = 0$$

The first order condition can be [6] solved for ω^*:

$$\omega^* = \frac{-(2r^k \ \exp(r)(i + i^e) - 4i)}{(i - i^e)(2r^k \ \exp(r) - (r^k)^2 \ \exp(2r)} \tag{6}$$

[5]For details, see Appendix B

[6]For full details, see Appendix B

After finding $\bar{\omega}$, I can solve for $r(j)$ and $i(j)$ using Equation (4) and Equation (5):

$$i^* = \frac{i^e}{1 - r^k j \, \exp(\rho) g(\bar{\omega}^*)} \tag{7}$$

$$1 + r^* = \left(\frac{\bar{\omega} i^* j \, \exp(\rho)}{i^* - i^e} \right) \tag{8}$$

3.4 Families

Every individual is part of a family based on his employment assignment. An individual with entrepreneurial ability j higher than the cutoff ability $\bar{j}$ will be part of the family of entrepreneurs. Individuals with ability j less than $\bar{j}$ are members of the non-entrepreneurial family. The value of $\bar{j}$ will be chosen by the employment agency in each period such that the household at the cutoff point, where $j = \bar{j}$, would produce the same amount of capital whether an entrepreneur or not.

The families are similar to those used in Alexopoulos (2006). They make all investment and consumption decisions for their members. Each individual contributes the capital that he has accumulated via entrepreneurial activity or lending activity to the family stock of capital. The family's income comes from renting its capital stock to the production firm. The non-entrepreneurial family also receives income from wages. The entrepreneurial family does not receive wage income because it is composed of individuals working in small self-owned businesses.

The entrepreneurial household produces capital via its small business. The non-entrepreneurial family's capital is produced by lending to the small business. This is its only source of capital production. The mechanism for inter-family lending or borrowing is described in the preceding section. In each period, the family makes consumption and investment decisions for each family member. It chooses how much to invest in capital production and distributes this amount between all family members. Entrepreneurs will borrow investment goods from non-entrepreneurs, and the lending contract will be structured such that the entrepreneur must invest all of his investment goods in his enterprise. Each family's optimal investment and consumption decision will be dependent on $\bar{j}$, the cutoff ability value, which is chosen by the employment agency. Entrepreneurs also, by definition, invest all working hours in their business, so the entrepreneurial family does not choose labor hours.

3.4.1 The Family of Entrepreneurs

The family of entrepreneurs is composed of individuals with ability $j \in [\bar{j}, 1.5]$. Each entrepreneur is given an equal amount of investment, i^e, regardless of ability. In addition, the entrepreneur receives investment from the lender, $i^f(j)$ based on his ability level. Given the functions $\bar{\omega}^*(j)$, $i^{f*}(j)$ and $r^{k*}(j)^{k*}$, defining the optimal contract for ability j, an entrepreneur of ability j produces capital in the amount of:

$$\left(i^e + i^{f*}(j)\right) \; j \; \exp(\rho) \; f(\bar{\omega}^*(j), j)$$

Then the total capital production of the family composed of individuals with ability $j \in [\bar{j}, 1.5]$ is given by:

$$N \int_{\bar{j}}^{1.5} j(i^e + i^{f*}(j)) \; \exp(\rho) f(\bar{\omega}^*(j), j) dj$$

Based on the expected addition to the capital stock in each period, the family can make investment and consumption decisions. The total amount invested, $i_t^e + i^{f*}(j_t) =$

i_t, is given by equation (42) from the optimal contract section. The family's capital production is subject to the stochastic costs of borrowing: borrower search cost, μ_t, and lender monitoring cost, ρ_t. Finally, the entrepreneur works full-time. His labor amount is fixed at 1 and is not a choice variable. The entrepreneurial family's problem is described by the following:

$$\max_{c_t^e, i_t^e, K_{t+1}^e} \sum_{t=0}^{\infty} N\beta_t^t(1.5 - \bar{j}_t)\left(ln(c_t^e) - \frac{1}{1+\nu}\right)$$

$$s.t. \qquad (c_t^e + i_t^e)(1.5 - \bar{j}_t) = q_t K_t^e$$

$$K_{t+1}^e = (1-\delta)K_t^e + \exp(\rho_t) \int_{\bar{j}_t}^{1.5} i_t j f(\bar{\omega}_t^*(j), j) dj$$

$$i_t = \frac{i_t^e}{1 - r_t^k j \exp(\rho_t) g(\bar{\omega}_t^*)}$$

The law of motion of capital can be rewritten to incorporate the third constraint, so that the second constraint becomes:

$$K_{t+1}^e = (1-\delta)K_t^e + \exp(\rho_t) i_t^e \int_{\bar{j}_t}^{1.5} \frac{j f(\bar{\omega}_t^*(j), j)}{1 - E_t(q_{t+1}) \exp(\rho_t) j g(\bar{\omega}_t^*(j), j)} dj$$

Note that the integral, $\int_{\bar{j}_t}^{1.5} \frac{j f(\bar{\omega}_t^*(j), j)}{1 - E_t(q_{t+1})(1-\rho_t) j g(\bar{\omega}_t^*(j), j)} dj$ is independent of the entrepreneur's decision variables so that it is a constant in relation to the maximization problem. To simplify the equations, then, let:

$$\gamma_t^e = \int_{\bar{j}_t}^{1.5} \frac{j f(\bar{\omega}_t^*(j), j)}{1 - E_t(q_{t+1}) \exp(\rho_t) j g(\bar{\omega}_t^*(j), j)} dj$$

The solution to the entrepreneurial family's optimization problem is given by the resource constraint and the law of motion of capital, along with the Euler Equation:

$$\frac{\beta_t^t(1.5-\bar{j}_t)}{\exp(\rho_t)\gamma_t^e c_t^e} = \beta_{t+1}^{t+1} E_t\left[\frac{q_{t+1}}{c_{t+1}^e} + \frac{(1.5-\bar{j}_t)(1-\delta)}{\exp(\rho_t)\gamma_{t+1}^e c_{t+1}^e}\right]$$

The integral represented by γ_t^e cannot be solved analytically. Because I will use only first- and second-order accurate solutions to the model, I will use a second-order Taylor series to approximate the function to be integrated[7]. The approximated function can be integrated with respect to j to solve for the approximate value of γ_t^e.

3.4.2 The Family of Non-Entrepreneurs

The non-entrepreneurial family is made up of individuals with ability $j \in [0.5, \bar{j}]$. Non-entrepreneurs earn wages and also accumulate capital goods by lending to entrepreneurs. Individual wages are contributed to the family's wealth and capital earnings are contributed to the family's capital stock. A non-entrepreneur lending to an entrepreneur of ability j accumulates capital in the amount of:

$$i_t^* \; j \; g(\bar{\omega}^*(j), j)$$

The total capital production of the non-entrepreneurial family, lending to entrepreneurs of ability $j \in [\bar{j}, 1.5]$, is given by:

$$\int_{\bar{j}}^{1.5} j i_t^* g(\bar{\omega}^*(j), j) dj$$

[7] Details can be found in Appendix E

The optimal amount of investment, i_t^* is determined by Equation (7) from the solution to the optimal contract problem:

$$i_t^* = \frac{i_t^e}{1 - r_t^k exp(\rho_t) j_t g(\bar{\omega}_t^*)}$$

Further, the sum of the total entrepreneurial investment, i_t^e, and non-entrepreneurial investment, i_t^f , must be equal to total investment in the small business, i_t^*. This implies that i_t^f is fully dependent on i_t^e, so investment is not a decision variable for the non-entrepreneurial family. From the law of motion of capital, this implies that the non-entrepreneurial family's capital stock in period $t+1$ is also determined by the optimal contract problem. Therefore, the family will choose consumption and labor hours only. The non-entrepreneurial family is subject to a shock to labor disutility, ψ_t. Then the non-entrepreneurial family's problem is given by:

$$\max_{c_t^f, l_t} E_0 \left[\sum_{t=0}^{\infty} N \beta_t^t (\bar{j} - 0.5) \left(ln(c_t^f) - \psi_t \frac{l_t^{1+\nu}}{1+\nu} \right) \right]$$

$$s.t. \quad (c_t^f + i_t^f)(\bar{j}_t - 0.5) = q_t K_t^f + (\bar{j}_t - 0.5) w_t l_t$$

$$l_t \leq 1$$

The solution to the maximization problem is given by (details can be found in Appendix D):

$$\beta_t^t(\bar{j}_t - 0.5)\left(\frac{w_t}{c_t^f} - \psi_t l_t^\nu\right) = \lambda_{2t}$$
$$\lambda_{2t}(1 - l_t) = 0$$
$$c_t^f = \frac{q_t K_t^f}{(\bar{j}_t - 0.5)} + w_t - i_t^f$$

In addition, the period $t + 1$ capital stock is given by:

$$K_{t+1}^f = (1 - \delta)K_t^f + N \exp(\rho_t) \int_{\bar{j}_t}^{1.5} j(i_t)g(\bar{\omega}_t^*(j), j)dj$$

Again, from the optimal contract problem (Equation (7)), recall that i_t is given by:

$$i_t = \frac{i_t^e}{1 - r_t^k j \exp(\rho_t)g(\bar{\omega}_t^*)}$$

Inserting this expression into the Non-Entrepreneurial Family's capital accumulation equation, obtain:

$$K_{t+1}^f = (1 - \delta)K_t^f + N \exp(\rho_t)i_t^e \int_{\bar{j}_t}^{1.5} \frac{jg(\bar{\omega}_t^*(j), j)}{1 - r_t^k j \exp(\rho_t)g(\bar{\omega}_t^*}dj$$

For simplicity, let γ_t^f represent the integral:

$$\gamma_t^f = \int_{\bar{j}_t}^{1.5} \frac{jg(\bar{\omega}_t^*(j), j)}{1 - r_t^k j \exp(\rho_t)g(\bar{\omega}_t^*}dj$$

The expression γ_t^f is an integral that cannot be solved analytically. Again, I find a second order approximation for this expression, details of which can be found in

Appendix E.

3.4.3 The Employment Decision

The employment decision is made by an employment agency. The cutoff ability, $\bar{j}$ determines how many entrepreneurs there are in each period. The agency chooses $\bar{j}$ such that the expected income for an entrepreneur with ability $\bar{j}$ equals the expected income for a non-entrepreneur with the same ability. Thus, the solution the employment decision is:

$$f(\omega(\bar{j})) - g(\omega(\bar{j})) = 0$$

All households with ability $j > \bar{j}$ will be entrepreneurs and all households with ability $j < \bar{j}$ will be non-entrepreneurs. This decision-making mechanism assigns each individual to the occupation in which he should produce the most capital, based on the expected values of the shocks to which the capital production is subjected. This allocation produces the most possible capital for the economy in each period.

4 Model Solution and Data

4.1 Model Solution Equations

There are forty one model solution equations, in addition to the equations describing the evolution of the structural shock variances. There are ten state variables and thirty one control variables. The ten state variables are: z_t, μ_t, ρ_t, ψ_t, ζ_t, K_t, K_t^F, K_t^H, K_t^e, and K_t^f. The thirty one control variables are q_t, j_t, ω_t, f_t, g_t, w_t, r_t^k, I_t, I_t^F γ_t^e, γ_t^f, i_t^e, i_t^f, c_t^f, c_t^e, , l_t^f, λ_t, C_t, Y_t, I_t^H, f_t', g_t', $\Gamma_{0,t}^e$, $\Gamma_{0,t}^f$, $\Gamma_{1,t}^e$, $\Gamma_{1,t}^f$, $\Gamma_{2,t}^e$, $\Gamma_{2,t}^f$, r_t, π_t^i, and π_t^k.

The first five state variables are the model's shocks. These evolve according to the following equations:

$$
\begin{aligned}
\ln(z_{t+1}) &= \rho_z \ln(z_t) + \epsilon_{z,t} \\
\ln(\zeta_{t+1}) &= (1-\rho_\zeta)(\mu_\zeta) + \rho_\zeta \ln(\zeta_t) + \epsilon_{\zeta,t} \\
\rho_{t+1} &= (1-\rho_\rho)\mu_\rho + \rho_\rho \rho_t + \epsilon_{\rho,t} \\
\mu_{t+1} &= (1-\rho_\mu)\mu_\mu + \rho_\mu \mu_t + \epsilon_{\mu,t} \\
\ln(\psi_{t+1}) &= (1-\rho_\psi)(\mu_\psi) + \rho_\psi \ln(\psi_t) + \epsilon\psi, t
\end{aligned}
$$

The shock, z_t, represents the growth rate of the technology shock, A_t:

$$z_t = ln\left(\frac{A_t}{A_{t=1}}\right)$$

From the firm's problem, the following equation solves for wages, w_t:

$$w_t = (1-\alpha)(A_t)^{1-\alpha}\left(\frac{K_t}{l_t^f(\bar{j}-0.5)}\right)^{\alpha}$$

The following equation from the firm's problem solves for the rental rate of capital, q_t:

$$q_t = \alpha\gamma\left[\frac{A_t l_t^f(j-0.5)}{K_t}\right]^{1-\alpha}$$

The Euler Equation from the firm's problem is:

$$\beta\alpha(1-\gamma)E_t\left[\frac{(A_{t+1})^{1-\alpha}(K_{t+1})^{\alpha}(l_{t+1}^f(j_{t+1}-.5))^{1-\alpha}}{K_{t+1}^F}\right] = \frac{1}{\xi_t} - \beta E_t\left[\frac{1-\delta}{\xi_{t+1}}\right]$$

The total amount of capital used by the firm is a combination of the capital produced by individual households, K_t^H, and capital stock held by the firm, K_t^F:

$$K_t = (K_t^H)^\gamma (K_t^F)^{1-\gamma}$$

The firm's capital stock evolves according to its capital accumulation equation:

$$K_{t+1}^F = (1-\delta)K_t^F + \zeta_t \left(1 - \left(\frac{I_t^F}{I_{t-1}^F}\right)^2\right) I_t^F$$

Finally, the firm's total production, Y_t, is given by:

$$Y = A_t^{1-\alpha} \left(K_t\right)^{\alpha} \left(l_f\right)^{(1-\alpha)}$$

The entrepreneurial family's model yields the Euler Equation:

$$\frac{\beta_t^t(1.5-\bar{j}_t)}{\exp(\rho_t)\gamma_t^e c_t^e} = \beta_{t+1}^{t+1} E_t \left[\frac{q_{t+1}}{c_{t+1}^e} + \frac{(1.5-\bar{j}_t)(1-\delta)}{\exp(\rho_{t+1})\gamma_{t+1}^e c_{t+1}^e}\right]$$

The entrepreneurial family's budget constraint is:

$$N(c_t^e + i_t^e)(1.5-\bar{j}_t) = q_t K_t^e$$

Finally, the entrepreneurial family's capital stock evolves according to the equation:

$$K_{t+1}^e = (1-\delta)K_t^e + \ \exp(\rho) i_t^e \gamma_t^e$$

The non-entrepreneurial family's optimization problem produces two equations:

$$N\beta_t^t(\bar{j}_t - 0.5)\left(\frac{w_t}{c_t^f} - \psi_t l_t^\nu\right) = \lambda_t$$
$$\lambda_t(1 - l_t) = 0$$

The non-entrepreneurial family's budget constraint is:

$$c_t^f = \frac{q_t K_t^f}{N(\bar{j}_t - 0.5)} + w_t - i_t^f$$

The non-entrepreneurial family's capital stock evolves according to the following equation:

$$K_{t+1}^f = (1-\delta)K_t^f + N \exp(\rho) i_t^e \gamma_t^f$$

The employment decision yields the following equation:

$$f(\omega(\bar{j})) - g(\omega(\bar{j})) = 0$$

The optimal contract problem provides an additional three equations. The optimal cutoff value, $\bar{\omega}_t^*$, below which the borrower will default, solves the first order condition of the optimal contract problem:

$$q_t j_t \exp(\rho_t) f'(\bar{\omega}_t) \frac{i_t^e}{1 - q_t j_t \exp(\rho_t) g(\bar{\omega}_t)} + (q_t j_t \exp(\rho_t))^2 f(\bar{\omega}_t) g'(\bar{\omega}_t) \frac{i_t^e}{(1 - q_t j_t \exp(\rho_t) g(\bar{\omega}_t))^2} = 0$$

In addition, the optimal contract problem allows us to solve for i_t^f as a function of i_t^e:

$$i_t^f = i_t^e \left[\left(\frac{1}{1 - r_t^k j_t \exp(\rho_t) g(\bar{\omega}_t)}\right)\right]$$

The optimal contract problem also allows us to solve for the interest rate using the following equation:

$$r_t = log\left(\frac{\bar{\omega}_t i_t \bar{j}_t \exp(\rho_t)}{i_t - i_t^e}\right)$$

Also from the optimal contract problem, I obtain the equations for f_t, g_t, f_t', g_t' [8]:

$$\begin{aligned} f_t = f(\bar{\omega}_t, j_t) &= \frac{1}{4}\bar{\omega}_t^2 - \bar{\omega}_t + 1 \\ g_t = g(\bar{\omega}_t, j_t) &= \frac{\exp(\mu_t)}{4}\bar{\omega}_t^2 + \bar{\omega}_t - \frac{1}{2}\bar{\omega}_t^2 \\ f_t' = f'(\bar{\omega}_t, j_t) &= \frac{1}{2}\bar{\omega}_t - 1 \\ g_t' = g'(\bar{\omega}_t, j_t) &= \frac{\exp(\mu_t)}{2}\bar{\omega}_t + 1 - \bar{\omega}_t \end{aligned}$$

The eight equations for the approximated integrals γ_t^e and γ_t^f, as well as the coefficients in those approximations ($\Gamma_{0,t}^e$, $\Gamma_{0,t}^f$, $\Gamma_{1,t}^e$, $\Gamma_{1,t}^f$, $\Gamma_{2,t}^e$, and $\Gamma_{2,t}^f$) can be found in Appendix E.

Finally, there are a number of aggregation equations. The total household consumption, investment and capital are given by:

$$\begin{aligned} C &= (1.5 - j_t)c_t^e + (j - 0.5)c_t^f \\ I_t^H &= (1.5 - j_t)i_t^e + (j - 0.5)i_t^f \\ K_t^H &= K_t^e + K_t^f \end{aligned}$$

The price of capital, q_t, is given by:

$$q_t = r_t^k - 1$$

[8]Details in Appendix A

The total investment in the economy (by both the firm and the households) is:

$$I_t = I_t^F + I_t^H$$

There are two model variables that are used in analysis. These equations give the relative price of investment:

$$\pi^i = \frac{\exp(\rho)\,\exp(\gamma^e)}{(1-\delta)}$$

and the relative price of capital:

$$\pi^k = (1-\delta)$$

4.2 Steady State

An analytical steady state does not exist for the model. As a result, I solve for the steady state using non-linear solvers in either Matlab or Fortran. To obtain the starting steady state values required by Dynare for the first order parameter estimation, I use the solve function in Matlab. However, after using the provided steady state values as a starting point, Dynare computes a new steady state for each set of possible parameter values. Thus the steady state must be found just once. However, the particle filter code to find the second order parameter estimates is implemented in Fortran. This code requires the steady state to be found for each set of parameter values. The steady state equations for some variables may be found analytically. For the system of equations that cannot be solved analytically, I use the DNEQNF (non-linear solver) function in the Fortran IMSL library to solve for the steady state in the particle filter implementation.

4.3 Data

I estimate five shocks using five data series. All data is from the Federal Reserve Economic Database, unless otherwise noted. For FRED data series, I provide the series ID in parentheses. Production, Y, is given by nominal quarterly GDP (GDP) divided by population (POP) and the GDP implicit price deflator (GDPDEF). Wages, w, are given by hourly wages of non-farm workers. The series comes from the Bureau of Labor Statistics. Wages are seasonally adjusted, and are divided by the GDP deflator. Total household investment, I^H, is given by investment in consumer durables (PCDF) plus private investment (GDPI) divided by the GDP deflator and population. Consumption is calculated as consumer spending on non-durables (PCND) and services (PCESV), divided by the GDP deflator and population. Finally, labor hours are given by average weekly hours of production and nonsupervisory employees (total private), also from the Bureau of Labor Statistics database. The data starts in Q1 of 1964 and ends in Q1 of 2010. It is necessary to use significant data from both before and after the Great Moderation (1984).

I use a number of other series to further analyze the results of the DSGE model. These series are not used in parameter estimation, instead providing a view of how well the model fits series to which it is not calibrated. Tobin's Q is the market value of installed capital divided by the replacement cost of that capital. I use a series calculated by ycharts.com. I construct the relative price of investment by dividing the price of investment by the price of consumption, as in Gabler (2006). For the price of consumption, I use the chain type price index for personal consumption expenditures

(PCECTPI) from the Federal Reserve Economic Database. I use the chain type price index for Gross Private Domestic Investment (GPDICTPI) from the Federal Reserve Economic Database for the price index of investment. To approximate the interest rate faced by small business borrowers, I use the yield on Moody's BAA corporate bonds (BAA). This series is monthly, so I adjust it to a quarterly series by taking the average yield over the months of the series. I also adjust this series for inflation. Finally, I use the real U.S. return to capital from Gomme and Rupert (2008) with some slight modifications. Gomme and Rupert assume a depreciation rate of 0.0177, while I assume a higher depreciation rate of 0.1. I correct for this difference in the final series. It should be noted that there are some other implicit parameter assumptions make by Gomme and Rupert that I cannot adjust for.

I use an additional set of data to understand the implications of the static optimal contract model, outside of the context of the DSGE model. For this analysis, I use the Gomme and Rupert (2006) calculations of U.S. real return to capital discussed in the previous paragraph. I also use the bank prime rate (MPRIME), adjusted for inflation. Finally, I calculate the per-capita value of business loans outstanding by dividing the total value of business loans outstanding (BUSLOANS) by population (POP).

I apply the Band-Pass filter to all data series used for model calibration and DSGE model comparison. Filtering removes long-frequency cyclical effects and trends from the data. I use Matlab code implementing methods discussed by Christiano and Fitzgerald (1999). The Matlab code is available on Terry Fitzgerald's Cleveland Federal Reserve website. I use the unit root option, which specifies that there is a unit root in the time series. I filter out data with a period of shorter than 1.5 years or longer than 6 years. This ensures that I eliminate any short-cycle noise and any long-term trends. I use unfiltered series to analyze the static Optimal Contract model.

4.4 First Order Estimation Procedure

The first-order estimates assume policy function linearity that may not exist in a large-scale DSGE model. Because of this linearity assumption, the estimate neglects the agents' expectations of stochastic volatility. Although the first-order estimate has some limitations, it serves a few functions. First, it can be obtained with relatively little programming using Dynare, a Matlab-based solution algorithm. In addition, earlier work on the Great Moderation (such as Justiniano and Primiceri (2007)) uses first-order accurate estimates only. Therefore, comparing the differences between the first- and second-order estimates will be a useful exercise.

4.4.1 Description

The first order accurate parameter estimates are obtained via Dynare. Dynare is a free software platform that functions as an add-on to Matlab. Dynare uses the Kalman filter to estimate the likelihood. The underlying assumption of the Kalman filter is that the system is linear and that all errors are normally distributed. Because it uses the Kalman filter, Dynare accounts only for the linear component of the model when estimating parameters (although Dynare does compute the second-order solution).

After calculating the likelihood and posterior, Dynare runs one or more Metropolis-Hastings chains to characterize the posterior distributions of the parameters. Dynare has a number of benefits. It requires relatively little programming. The model solution equations must be entered into a file, along with any relevant commands for

Dynare. Dynare also produces a wide range of diagnostic information, including convergence diagnostics, prior and posterior distribution graphs, model solution approximations, and other results. A final benefit is that Dynare is significantly faster than the second order Particle Filter code that I implement. Dynare can run 250,000 Metropolis-Hastings draws in less than a day. The particle filter requires multiple weeks on significantly better hardware to execute the same number of draws. The performance will be discussed in more detail in subsequent sections.

4.4.2 Performance

Dynare is run on a personal machine with an Intel i7-2620M chip at 2.70 GHz. The system has 8.00 GB of memory and is a 64-bit Windows 7 Operating System. The processes use Matlab 7.12.0 (R2011a). I use four Metropolis-Hastings chains to estimate 13 parameters in the time-invariant version of the model. This runs for approximately 12 hours (including the likelihood calculations). I use four Metropolis-Hastings chains of 250,000 draws to estimate 17 parameters in the time-varying version of the model. This runs for approximately 24 hours. The increase in runtime is accounted for in part by a longer time required to calculate the likelihood, but it is mostly in the Metropolis-Hastings chain step.

4.5 Second Order Estimation Procedure

The second-order accurate solution will allow non-linearities in the policy function and will help me determine whether agent expectations are an important component of my model. However, accounting for non-linearities requires a significantly more complex and time-consuming estimation procedure. As a result, the linearity assumption is far more common when estimating DSGE. However, the second order can provide valuable information about agent expectations and is, thus, a critical component to understanding a DSGE model.

4.5.1 Description

I use the particle filter to obtain the second-order accurate parameter estimates for my model. The particle filter accounts for non-linearities in the policy function that first-order accurate methods do not. First, I use the Schmitt-Grohe (2004) perturbation method to approximate the second-order model solution. I then implement the particle filter, in the manner of Fernandez-Villaverde (2010) and An and Schorfheide (2007).

To describe the implementation of the particle filter, I borrow notation from Dave and DeJong (2007). Recall that the objective of the particle filter is to evaluate the likelihood function over the time $t = 1, ..., T$. The likelihood function is given by:

$$L(X^T|\mu) = \pi_{t=1}^{T} \int p(\upsilon_t(X^t, s_0))p(s_)|X^t)ds_0$$

The expression $L(X^T|\mu)$ represents the likelihood of the series of observations, X^T given a set of parameter values, μ. The expression $p(\upsilon_t(X^t, s_0))$ represents the probability of the structural shock that is implied by the data series X^t and the initial states, s_0. I integrate over s_0 so that the likelihood is independent of the initial states.

The particle filter approximates the series of integrals in the likelihood expression. To describe the particle filter, I use the term "particle" to refer to a draw from the initial distribution of state variables conditional on the data series X^t. A sequence of particles is referred to as a "swarm". The particle filter must be initialized with an initial swarm of particles. The initial swarm of state variable values is drawn from a normal distribution. The mean of the distribution is the state variable's steady state value, and the the initial variance is taken from a simulation of the model.

For each initial particle (set of state values), I can calculate this period's control variables and next period's state variables. The control variables and next period state variables are quadratic functions of the states and shocks, since I am using a second-order approximation. To evaluate the likelihood of each particle, the particle filter calculates the forecast errors of the observables. The particle filter will return a zero likelihood if the set of parameters gives an indeterminant model solution. The forecast errors are assumed to be normally distributed with a mean of zero and variances that are some fraction of the variance of the data. Once I have calculated the likelihood of each particle draw, I can obtain the unconditional likelihood of the implied shocks by calculating a weighted average of the shocks in each time period (where the weight is the likelihood of the particle). I also re-sample from the initial swarm of particles to obtain the next period swarm. Again, the weight of each particle is its calculated likelihood.

I use the GLOBAL optimization routine from Tibor Csendes[9] (Csalinear, Csendes, and Markot (2000))to maximize the likelihood function as calculated by the particle filter. After finding the set of parameter values that maximizes the likelihood, these parameter values are used to initialize the Metropolis-Hastings process. I use a Metropolis-Hastings process to draw from the posterior distribution. For a full discussion of the particle filter implementation, refer to Appendix F.

[9]The GLOBAL fortran code is available on Csendes' website, $http : //www.inf.u - szeged.hu/~csendes/linkek_en.html$

4.5.2 Performance

I run one Metropolis-Hastings chain in the particle filter implementation. Each pass through the particle filter uses 20,000 particles. The code is written in Fortran and runs on the Fir Linux Cluster supported by the University of Virginia Alliance for Computational Science and Engineering. More information about the specific configurations of the nodes in the Fir cluster can be found on the UVACSE website[10]. The job is implemented in parallel using 4 nodes with 2 processors per node and 2 parallel processes on each node, for a total of 16 parallel processes. The time-invariant model estimates 13 parameters. Using the particle filter to calculate the likelihood, the code takes approximately 110 hours to maximize the likelihood. The Metropolis-Hastings portion of the process takes approximately 133 hours to execute 25,000 Metropolis-Hastings draws. However, this is not enough draws for the chain to converge. I have run 75,000 Metropolis-Hastings draws for the time-invariant model. In total, the runtime of the entire process is around 500 hours (approximately 3 weeks). The process must be run in parts because the maximum runtime allowed by the Linux cluster is 168 hours (1 week).

The time-varying model estimates 17 parameters. The likelihood maximization step takes slightly longer than the time-invariant case, executing in approximately 160 hours. The time-varying version of the code can execute approximately 10,000 Metropolis-Hastings draws in 168 hours. The time-varying version of the model has a significantly longer runtime than the time-invariant version, so I run fewer Metropolis-Hastings draws for this model.

[10]http://www.uvacse.virginia.edu/itc-clusters/

5 Results

In this section, I present the results from the second-order accurate time-invariant dynamic model, which serves as the benchmark model for further discussion. The second order accurate parameter estimates were obtained using the particle filter code, which is discussed in detail in section 4.5 and in Appendix F. Recall that the model contains five structural shocks: z_t, the growth rate of the firm's technology shock; ρ_t, lending search costs for entrepreneurs; μ_t, borrower monitoring costs; ψ_t, a shock to the disutility of labor; and ζ_t, the firm's capital adjustment cost shock. The shocks take the following form:

$$\begin{aligned} ln\ z_t &= \rho_z ln\ z_{t-1} + \epsilon_{z,t} \\ \mu_t &= (1-\rho_\mu)\mu_\mu + \rho_\mu \mu_{t-1} + \epsilon_{\mu,t} \\ \rho_t &= (1-\rho_\rho)\mu_\rho + \rho_\rho \rho_{t-1} + \epsilon_{\rho,t} \\ ln\ \psi_t &= (1-\rho_\psi)\mu_\psi + \rho_\psi ln\ \psi_{t-1} + \epsilon_{\psi,t} \\ ln\ \zeta_t &= (1-\rho_\zeta)\mu_\zeta + \rho_\zeta ln\ \zeta_{t-1} + \epsilon_{\zeta,t} \end{aligned}$$

The stochastic elements of the shocks, $\epsilon_{z,t}$, $\epsilon_{\mu,t}$, $\epsilon_{\rho,t}$, $\epsilon_{\psi,t}$, and $\epsilon_{\zeta,t}$, are distributed normally with means of zero and variances, respectively, of $\theta^2_{z,t}$, $\theta^2_{\mu,t}$, $\theta^2_{\rho,t}$, $\theta^2_{\psi,t}$, and $\theta^2_{\zeta,t}$. In this case of the model, the variances of the shocks are fixed over time: θ^2_z, θ^2_μ, θ^2_ρ, θ^2_ψ, θ^2_ζ.

5.1 Convergence

Due to the substantial processing time of the particle filter code, I only run one Metropolis-Hastings chain for the second order accurate parameter estimation procedure. As a result, the Brooks-Gelman convergence statistics cannot be computed. Thus, I rely on the shapes of the posterior distributions and the progress of the chains themselves to assess convergence. The posterior distributions of the parameters are generally normally distributed, with the possible exception of the search shock autocorrelation parameter (ρ_ρ). This parameter is bounded between 0 and 1 and has nontrivial density throughout the allowed range. This can be seen in Figure 9. Otherwise, the posterior distributions of the parameters are reasonably distributed (also see Figures 10 and 11).

The Metropolis-Hastings chains themselves also provide insight. The chains generally do appear to vary around established means. There are a few that do seem to have difficulty settling around a mean. In Figure 12, the autocorrelation parameter of the search cost (ρ_ρ) varies widely within its allowed range. The variation of the firm's capital adjustment cost autocorrelation parameter (ρ_ζ) also appears to be a bit larger than desirable (also in Figure 12). Finally, the autocorrelation parameter of the tech shock (ρ_z) is very stable, but near its lower bound (see Figure 12). This is an artificial lower bound, imposed because autocorrelation parameters generally do not approach it. In this case, the parameter may want to go negative. In the Figure 38 through Figure 43, I provide details regarding a test run in which I expand the bounds of the autocorrelation parameter of the tech shock to -1 to 1, rather than 0 to 1. The parameter does go lower than 0. However, the change also introduces some instability to some of the other parameter chains. The only other chain of note is the chain of the steady state value of the monitoring cost (μ_μ, which can be found in Figure 13. This chain diverges from what looks like a well-established mean. However, it does return to its previous pattern at the end of the chain. All other chains, which can be seen in Figures 12, 13, and 14, look stable around an established mean. Thus, with a few minor concerns, the chains for this model appear to have converged.

5.2 Parameter Estimates

Table 2 shows selected summary statistics for the posterior distributions of the parameter estimates. The median values of the posterior distributions are generally reasonable. The median of the Inverse Frisch Labor supply, ν, is 2.7. This is within the range of 2 to 5 that is suggested by the micro data (Gali, Gertler and Lopez-Salido (2003)). The borrower monitoring cost shock, the labor disutility shock, and the firm capital adjustment cost shock are all highly persistent, as measured by the autocorrelation parameters for these shocks (all above 0.9). The technology shock is hardly correlated with the last period's shock, and tends to revert to the mean. The search cost shock is fairly persistent, with an autocorrelation parameter of 0.68.

The search cost shock is by far the most variable, with an estimated median variance of 0.0581. The monitoring cost shock and the capital adjustment cost shock are slightly less variable than the search cost. Finally, the technology shock and the labor disutility shock are the least variable.

The steady state values of the monitoring cost, μ_μ, and the search cost, μ_ρ, also provide useful information about these costs. These give the percent of small businesses earnings that are destroyed by each cost in the steady state. The parameter estimates must be converted, so that the percent of income destroyed by search costs in the steady state is given by $1 - exp\left(\frac{\mu_\rho}{10}\right)$. The percent of income destroyed by monitoring costs in the steady state is given by $1 - exp\left(\frac{\mu_\mu}{10}\right)$. Thus, the percent of capital destroyed by search costs in the steady state is 5%. The percent of capital destroyed by monitoring costs is slightly larger at 18.5%. In total, monitoring and search costs destroy about 22.5% of the small business's profit. This is similar to the 25% agency cost estimated in Carlstom and Fuerst (1997), whose optimal contract model is similar to mine but has just one borrowing cost.

5.3 Impulse Response Functions

The impulse response functions for this model, using the posterior median parameter estimates, can be seen in Figure 28 through Figure 37. These figures show both the first order IRF (in green) and the second order IRF (in blue). Figure 28 and Figure 29 show the IRFs for selected model variables responding to a technology shock (z). The first and second order responses are nearly identical, except for the response of the firm's investment (I_H), which can be seen in 29. Firm investment responds more strongly to a technology shock when the second order effects are included. The impulse responses to the search cost (ρ) can be seen Figure 30 and Figure 31. For most of the variables shown, the first and second order IRFs have the same direction of response, but a slightly different magnitude of response. However, household capital (K_H, K_e and K_f) responds negatively to a search cost shock in the first order, but positively to the search cost shock in the second order. The response of firm capital (K_F) is slightly smaller (though still positive) in the second order. This indicates that the second order effect of the search cost on capital is significant, and that the use of the second order changes the dynamics of the model. The different impulse response functions are caused by significant second-order effects on K_e and K_f, where the first and second order responses have opposite signs. For example, entrepreneurial capital, K_e, would experience a decline of 0.0258 in response to an increase of 1 in ρ in the first order (based on the appropriate element of the gx matrix). However, in the second order, there is a positive effect when the combination of ρ and household capital increase. Although there are other negative second order effects, the positive second order effect outweighs the negative first order and second order effects. The other two variables, K_H and K_f, behave similarly. Firm investment also has a very large response to a search cost shock in the second order, far outweighing its first order response.

The responses to the monitoring cost shock (Figure 32 and Figure 33) all move in the same direction for both the first and the second order. However, the second order IRFs are smaller in magnitude for all variables except the firm's investment (which has a slightly larger response in the second order). Impulse responses to a shock to the firm's capital adjustment costs (Figure 34 and Figure 35) are slightly larger in the second order, but are directionally the same between first and second order. The response of firm investment in the second order is again very large compared to the first order. Firm investment appears to be very sensitive to the second order effects of both the capital adjustment cost shock and the search cost shock. This is caused by large second order responses to a number of state variables which are also affected by the shocks).

5.4 Recovered Shocks

In order to evaluate the usefulness of the model, it is helpful to know whether it can construct a variable series that is similar to the data. The model should have a very good fit for the data series that are used in parameter estimations. However, there is no guarantee that the model will closely fit other data series not used in estimation. To evaluate this aspect of my model, I construct implied variable series. To do so, I start with an initial set of state variable values. These are calculated by taking a likelihood-weighted average of the state variable values using 20,000 particles in the particle filter. Using these state values to initialize the process, I solve for the structural shocks in each period by comparing the calculated model variable values to the data for the 5 data series that were used in estimation. This allows me to then compute the implied values for all model variable series using last period's state variable values and the implied shock values. This process is implemented in Matlab with the exception of finding the initial steady state values, which is done by the particle filter in Fortran. The result of the process is a series of implied values for each model variable. These can then be compared to the data. Before comparing the two series, I filter both series using the band-pass filter. All variables are also in log format prior to being filtered.

First, I look at four variables that are not used in the estimation procedure: interest rate, return to capital, relative price of consumption and relative price of investment. In all figures, the model series is shown in blue and the data series is shown in green. The first set of results uses the second-order accurate parameter estimates but the first order model solution (Figure 15). In the first order, the interest rate model series has significantly more variation in the early part of the sample (through approximately 1986). It levels out toward the end of the sample period, and variability is much more aligned with the data after 1986. The variability of the return to capital is significantly lower in the model than it is in the data. The data shows a distinct decline in variance in the 1980's, which does not appear to be captured fully by the model. It is interesting to note that the model is much more similar to the data at the end of the sample. It is possible, then, that the current structure of the model is more accurate for current economic conditions than for historical conditions. The price of investment is far more variable in the model than it is in the data, although its variance does decline significantly after approximately 1986. Finally, the price of capital (Tobin's Q) varies significantly in the data, but is fixed in my model. This may be an important enhancement to the model that could expand its usefulness.

The second set of results (Figure 18) uses the second-order parameter estimates as well as the second order model solution. This version exhibits excess volatility throughout the sample period for all model variable series, except for Tobin's Q (which is fixed in the model). The interest rate and the return to capital are significantly more volatile when the second order effects are considered. The variability of the price of investment may have increased slightly with the inclusion of the second order. All three variables are significantly more variable than the data in the second order.

The excess volatility appears to come from the capital adjustment cost expression that is applied to the firm. The model uses the standard time-to-build formulation:

$$\zeta_t \left(1 - \left(\frac{I_t^F}{I_{t-1}^F}\right)^2\right) \tag{9}$$

When this expression is completely removed, the variance of the recovered variables is significantly reduced (Figure 22). The interest rate variability implied by the model is very close to the data series. The return to capital is less variable than the series in the first half of the sample, but has a good fit in approximately the last half of the sample. The price of investment is still more variable than the data, but the variability of the model series is significantly reduced when the capital adjustment cost is removed.

Because the capital adjustment cost as it is constructed in my model introduces excess volatility, I test a modified version of the capital adjustment cost. Rather than use the original time-to-build formulation, I change the expression to reflect the size of investment relative to the current capital stock, as seen below:

$$\zeta_t \left(1 - \left(\frac{I_t^F}{K_t}\right)^2\right) \tag{10}$$

This change reduces the variability from the original case, but it is higher in this case than in the model with no adjustment cost (Figure 25). In this case, implied interest rate variability is still fairly close to the data variability. Return to capital is again slightly less variable than the data suggest. Price of investment is less variable than it was with the original capital adjustment cost formulation, but more variable than the case where the capital adjustment cost is removed entirely.

I calculate the magnitude squared coherence functions for the four variables that are discussed above. The coherence function quantifies the level of similarity between the model implied series and the data series over business cycle periods (1.5 years to 6 years). The figure for the original model with the time-to-build construct of the capital adjustment cost can be found in Figure 19. In the original model, all four variables have greater coherence in the shorter cycles and less coherence in the longer cycles. The price of capital appears to have a high coherence value, but this should not be given much attention since this variable is fixed in the model. Overall, the return to capital implied series has the highest coherence of the variables that do change in the model, which confirms what I observe in the graph of the two series (Figure 18). The interest rate is the next most coherent with the data, with the price of investment being the least coherent. Again, this matches the observations from the graph of the model implied series and the data series.

When I remove the capital adjustment cost completely (Figure 23), the coherence of the return to capital is less than in the original model. However, it still has the same pattern of declining as the cycle length increases. The interest rate has about the same coherence at short and long cycles, with the least coherence in the middle-length cycles of around 3.5 to 4 years. The coherence of the price of investment series is highest in the short cycle of 1.5 years and the middle-length cycle of 4.5-5 years. It is low for cycles of around 2.5 to 3 years and for cycles of around 6 years. The overall coherence levels are lower for the model without the capital adjustment cost.

For the model with the modified capital adjustment cost, the return to capital coherence (Figure 26) is similar to the return to capital coherence in the original model. It declines as cycle length increases, and peaks around 0.45 on a scale from 0 to 1. The interest rate coherence is slightly higher in the modified model than in the original model. It is highest in the short cycle lengths of 1.5 to 2 years and is smallest for a cycle length of 4.5 years. The price of investment coherence is actually slightly lower than in the original model. It is highest for shorter cycle lengths and lowest for the long cycle lengths. Overall, the coherence appears to be lowest for the model in which I have removed the capital adjustment cost (23). Although the model series variability is higher when the capital adjustment cost is included, it appears to be important to include this shock because it allows the model to capture the data cycles better.

It is also illustrative to look at the series of recovered shocks implied by the model and the data. I will focus on the borrowing cost shocks: the loan search cost shock, ρ, and the loan monitoring cost, μ. The recovered series of these shocks can be seen in Figure 21, using the second order solution[11]. The search cost (ρ) does appear to become less variable from approximately 1986 to 1996, although there are some cycles at the end of the data series with a larger amplitude. The monitoring cost, μ, has shorter cycles prior to 1986, and much longer cycles between 1986 and 2000. This could be construed as less variability - although the cycle minima and maxima are approximately the same, the cycles occur less frequently. Generally, the borrowing costs do appear to become more stable in the 1980's. However, the change seems to lag the start of the Great Moderation in the second quarter of 1984. This is probably reasonable, given that a number of regulatory changes were enacted throughout the 1980's.

The Depository Institutions Deregulation and Monetary Control Act (DIDMCA) was passed on March 31, 1980. The changes enacted by the DIDMCA included abolishing interest rate ceilings on savings accounts, lowering or removing reserve requirements for financial institutions, and allowing financial institutions to merge. These measures enabled banks to become more flexible in offering funding, and improved their ability to monitor borrowers through resource pooling and increased leverage over borrowers. The regulatory changes were enacted in phases. The repeal of interest

[11] The recovered shocks using the first order solution can be seen in Figure 17.

rate ceilings was entirely in place by 1986, and the changes in reserve requirements had been implemented fully by 1988. Thus, the decline of borrowing cost variability around 1986 is sensible if the changes were caused by financial deregulation.

As a final check of the reasonableness of the recovered shocks, I compare the standard deviation of the recovered shocks to the estimated standard deviation of the shock processes (Table 3). The standard deviations of the recovered shocks are somewhat smaller than the estimated values, but are generally similar.

5.5 Simulations

The implied series discussed in the previous section characterize the behavior of the model over a time period that it similar to the data. To evaluate the model over a longer period, I simulate the model over 10,000 periods. I calculate the moments of the simulated series as well as the data series to determine whether the long-term simulation of the model behaves similarly to the observed data. The mean of the unfiltered series, the standard deviation of the filtered series and the output correlation of the filtered series for three variables that were not used in estimation can be found in Table 4. The mean of the unfiltered simulated relative price of investment is 0.82, which is similar to the mean of the unfiltered data, which is 0.78. The rental price of capital in the data, 7%, is also very similar to the mean value in the data, 9%. The difference in this case may be caused by differences in parameter vales between my model and Gomme and Rupert's (2003) model. This is discussed in more detail in section 4.3. Finally, the model interest rate, 9.1%, is close to the interest rate in the data, 8.4%. The variability of the model is significantly higher than the data in two of the three variables of interest. The standard deviation of the filtered data series is very small (0.0084), while the standard deviation of the filtered simulated series is significantly larger (0.19430). The simulated filtered interest rate is also significantly more variable than the filtered series. The simulated standard deviation is 0.611, compared to 0.050 in the data. The rental price of capital has the correct amount of variability compared to the data - the data has a standard deviation of 0.071, while the simulation's standard deviation is 0.074. This is consistent with the results from the implied data series, in which the implied rental price of capital was similar to the data, while the relative price of investment and the interest rate were significantly more variable than the data. As discussed in the previous section, this excess volatility is caused largely by the formulation of the firm's investment adjustment cost. Finally, I review the correlation of the three variables with output. The relative price of investment is negatively correlated with output both in the data and in the simulation. However, the negative correlation is more significant in the model than in the data. The rental price of capital is positively correlated with output in both the model and the data, and the level of correlation is roughly similar. Finally, the interest rate is negatively correlated with output in both the data and the model. However the negative correlation is more significant in the model than in the data.

I check the simulated series to the data series for selected variables used in estimation, which can be seen in Table 5. The model series are slightly more variable than the data series for all three variables (production, investment and consumption). Investment and consumption are correlated positively with output in both the data and the model, although the strength of the correlation does differ.

I also look at the mean values of the unfiltered series for additional model variables that were not used in estimation. I do not characterize the series for these variables because time series data for the variables is not readily available. The mean value of the simulated series of the ability cutoff, $\bar{j}$, is 1.0411. The variable j represents entrepreneurial ability. Ability is randomly assigned to each individual in each period. Ability, j, is distributed uniformly on the support [0.5,1.5]. The cutoff ability, $\bar{j}$, is the ability above which an individual will be an entrepreneur. Thus, a $\bar{j}$ of 1.0411 implies that approximately 46 % of families will be entrepreneurs in the average period. This is significantly higher than the U.S. rate of self-employment, which is estimated to be about 10% (Hipple (2010)). There are a few possible explanations for this difference. First, some people may start a business while remaining employed by a firm. Indeed, Kirchhoff and Phillips (1989) note that small businesses often operate part-time before becoming full-time. It is also possible that the failure rate of small businesses is quite high in the first year. Kirchhoff and Phillips (1989) estimate that 40% of small business survive for 6 years. If businesses fail at an even rate over those 6 years, this would be about a 10% failure rate per year. However, Kirchhoff and Phillips (1989) use a data set maintained by the U.S. Small Business Administration, and note that business are 2 years old, on average, at the time they are contacted for survey. Thus, the paper's results cannot measure the failure rate of small businesses in the first year, which may be higher than the average of about 10%. If the failure rate is, indeed, high in the first year, that may mean that some entrepreneurs both enter and leave self-employment within the year. Such entrepreneurs may not be measurable. In addition, the employment decision in the model is made by an agency, which selects the point at which the family would be indifferent to its employment situation (self or not). However, self-selection may not be as precise, and may use different or imperfect inputs and decision criteria. Thus, the self-selection rate may not match the model's optimal selection decision.

The mean of the simulated series of the default threshold, $\bar{\omega}$, is 0.5972. The variable ω is the stochastic rate of capital production, which is randomly assigned to each entrepreneur. The production rate is uniformly distributed on [0,2]. A small business will default on its loan when its production rate falls below 0.5972. Thus, the default rate in the average period is approximately 30%. This aligns with a study by the National Federation of Independent Business, which estimates that about 30% of small businesses lose money over the life of the business (Klein (1999)). In addition, Kirchoff and Philips (1989) find that about 40% of small businesses survive for six years. This indicates that 60% of small businesses fail within 6 years of opening. However, it's reasonable to assume that the failure rate is significantly lower in any given quarterly period. In addition, the model default rate is similar to the default rate on Standard & Poor's BB rated bonds, which are the highest-rated non-investment grade bonds. This seems like a reasonable comparison, although the return on lending to small businesses is more in line with the yield to the higher-rated Baa/BBB bonds, which are lower investment-grade. Although there are some difficulties in interpreting the failure rate, the model data series seems to be roughly in line with survival rates or default rates of small businesses.

The mean search cost, 0.95, implies that 5% of small business profits are destroyed by costs associated with finding funding. The mean monitoring cost of 0.82 implies that 18% of profits are destroyed by the costs associated with monitoring borrowers who default. Combined, these costs destroy approximately 22% of small business profits. Carlstrom and Fuerst (1997) construct an optimal contract model that is similar to the one that I use. However, they have just one agency cost. They estimate the value of this cost to be 25%, which is very close to the total cost imposed by search and monitoring costs in my model. The range of values for such a cost (in total) is between 20% and 36% (Carlstrom and Fuerst (1997)). Thus, my estimate is at the smaller end of the range.

6 Alternative Model Specifications

I estimate three alternative models as comparisons to the base model. In the base model, the variances of the shocks are fixed over time. The parameter estimation procedure is second-order accurate using the particle filter to calculate the likelihood. The first alternative model is also time-invariant, but the parameter estimates are first-order accurate. I also obtain first-order estimates for a time-varying version of the model, in which the variances of the shocks may change over time. Finally, I obtain second-order accurate estimates for the time-varying version of the model. In this section, I also present an analysis of the static optimal contract model embedded in the DSGE model.

6.1 First Order Time Invariant

I obtain the first-order accurate parameter estimates using Dynare with 4 Metropolis-Hastings chains of 250,000 draws each.

6.1.1 Convergence

The Brooks-Gelman convergence diagnostics (Brooks and Gelman (1998)) are computed for the Metropolis-Hastings chains. The interval measure is computed, as well as the second- and third-order measures. The multivariate diagnostic measures converge rather early in the chain, around 100,000 draws. This can be seen in Figure 46. The diagnostic measures are also produced for each estimated parameter. For many parameters, the diagnostic measures converge almost immediately. For others, convergence occurs between 50,000 and 100,000 draws. The chains for all estimated parameters converge reasonably in 250,00 draws. For reference, see Figure 47 through Figure 51

In addition to convergence diagnostics, I check the path of one Metropolis-Hastings chain to determine whether the chain is settling on a parameter value. Nearly all of the parameter chains settle very quickly on a value. The steady state values of the two costs associated with small-business borrowing, the search cost (μ_ρ) and the monitoring cost (μ_μ) take slightly longer to settle on a value. The chain for the steady state search cost μ_ρ can be found in Figure 54 and the chain for the steady state monitoring cost μ_μ can be found in Figure 55. In both cases, the eventual median value is different from the starting value. However, changing the starting values and/or priors for these parameters does not yield better results. The posteriors of the two parameters do not appear to be bimodal.

In addition to reviewing the convergence diagnostics and the Metropolis-Hastings chains, it is also important to review the characteristics of the posterior distributions of the parameters. The posterior distributions for all estimated parameters are approximately normal, as expected. For the parameters θ_ψ, ν and ρ_ψ, the posterior distributions are nearly identical to the prior distributions. The posterior distributions for the other parameters are somewhat different from the priors, but in no case is this difference extreme. Finally, no posterior distribution has an excessively large variance from the mean. Graphs of the posterior distributions can be found in Figure 44 and Figure 45. In addition, other aspects of the posterior distribution, such as mode, median, standard deviation, 10th and 90th percentiles, can be found in Table 7.

6.1.2 Parameter Estimates

The first order time-invariant parameter estimates can be found in Table 7. The estimates of the shock persistence parameters (ρ_z, ρ_ρ, etc.) indicate a high level of shock persistence, even after the data has been de-trended. In particular, the technology shock (z), monitoring cost shock (ρ), search cost shock (μ) and firm capital adjustment cost shock (ζ) all appear to be very persistent, with ρ parameter values between 0.9651 and 0.9996. The remaining shock, labor disutility (ψ), appears to be significantly less persistent with a ρ_ψ value of 0.6667.

The steady state values of the monitoring cost, μ, and the search cost, ρ, are estimated. Due to some manipulation of these shocks within the model solution equations, the μ_μ and μ_ρ parameter estimates may not be interpreted directly. Rather, the are converted using the following equations:

$$\bar{\mu} = exp\left(\frac{\mu_{\mu}}{10}\right)$$
$$\bar{\rho} = exp\left(\frac{\mu_{\rho}}{10}\right)$$

Thus, the steady state value of the search cost, ρ, is estimated to be 1.0259. This means that, on average, the small business actually receives an extra 2.59% during the search for capital. This result is significantly different from the baseline model estimate of 5% of capital *destroyed*. The steady state value of the monitoring cost, μ, is estimated to be 0.4434. This means that approximately 56% of the small business' capital is destroyed in the process of valuing the business's assets when it defaults. This result is significantly higher than the baseline estimate of 18%. Note that it appears that the search cost is smaller in magnitude in the first order estimate (to the point of going positive), while the monitoring cost is significantly larger. This motivates the question of whether the some search costs are being pushed to the monitoring cost in the first order.

The monitoring cost shock has the highest shock variance, at an estimated 0.0876. The labor disutility shock (ψ) and the firm capital adjustment shock are significantly less variable at 0.0057 and 0.0038 variances, respectively.

6.1.3 Simulation Results

I simulate selected model variables at the posterior median. I simulate the model for 10,000 periods. I apply the band-pass filter to both the simulated series and the data series to remove any very short or long-term cycles. The data and variables are in log form, except when I note the unfiltered mean, which I have converted to non-log form for ease of interpretation.

First, I look at three variables that are not used in the parameter estimation, but for which I have data. The first variable is the price of investment. The data used is the ratio of the price index of investment to the price index of consumption, as in Gabler (2006). The rental price of capital is the rate that the firm pays to rent capital from the households in each period. The data used for this variable is the estimate of U.S. return to capital from Gomme and Rupert (2008). Finally, I compare the interest rate faced by small business borrowers to the yield on BAA Moody's Corporate bonds, which are lower investment grade. These yield approximately the same average return (8.4 %) as does lending to small businesses in my model. The data is discussed in detail in section 4.3

Selected moments of the simulated variables and data can be found in Table 8 through Table 10. The un-filtered mean of the price of investment is very close to the mean of the data. However, the standard deviation of the simulation is smaller than that of the data. Both the simulated and actual series are negatively correlated with output, although the negative correlation is a bit stronger in the simulation. The rental price of capital in the model has a simulated mean of 6 %, while the data has an average of 9 %. This may be attributable to differences in model assumptions or fixed parameter values. The standard deviation of the simulated series is very close to the standard deviation of the data. However, the data series is positively correlated with output, while the simulated series is negatively correlated with output. This difference is resolved in the second order simulation, which is discussed in section 5.5.

Finally, the mean of the simulated interest rate series is 5 %, while the mean of the data series is 8.4 %. The simulated mean is significantly higher in the second-order simulation discussed in section 5.5. In the first-order version, the standard deviation of the simulated interest rate is also significantly higher than it is in the data. Finally, the interest rate is negatively correlated with output in the data, while it is positively correlated with output in the simulation.

I also look at the standard deviations and output correlations of selected variables used in parameter estimation (refer to Figure 9). The standard deviations of the simulations of consumption, investment and production are much closer to the values in the data. However, the standard deviations of all three variables are higher in the simulation than in the data itself. This is likely caused by the capital adjustment cost to which the firm is subjected, a topic which will be discussed further in section 5.4. All variables are positively correlated with output in both the simulation and the data.

Finally, I review some additional model variables for which I have limited data. The mean simulated value of the cutoff ability, $\bar{j}$, above which a household will be an entrepreneur, is 1.1. Recall that the variable j is distributed uniformly on the interval $[0.5, 1.5]$. Thus, the steady state value $\bar{j} = 1.1$ implies that approximately 40% of households are entrepreneurs in the steady state. This is about the same self-employment rate as I see in the second-order time-invariant model, which is higher than the data suggest.

The expected default rate of small-business borrowers can be measured in my model by the variable $\bar{\omega}$, which is the capital production technology shock to which the small business is subject. This shock is distributed uniformly on $[0, 2]$, as discussed in section 3.1.1. When the realization of the shock falls below the required value, the small business will default on its loan. The simulated mean of $\bar{\omega}$ is 0.6256. This implies that approximately 30% of small businesses will default in a given period. This is the same default rate estimated for the second-order time-invariant model.

The simulated mean search cost is 0.96, which means that the small business *earns* roughly 4% of the small businesses's profit is destroyed by costs associated with finding funding. The monitoring cost is significantly higher - approximately 55% of profit is consumed in the process of monitoring defaulting businesses. This seems high, and it should be noted that the second-order simulated costs using the second order accurate parameter estimates are significantly lower (see Section 5.2).

6.2 First Order Time Varying

The second case of the model allows the shocks to vary over time. In the time-varying case, the shock variances take the following form:

$$ln\ \theta_{z,t} = (1 - \rho_{\theta,z})ln\ \mu_{\theta,z} + \rho_{\theta,z} ln\ \theta_{A,t-1} + \epsilon_{\theta,z,t}$$

$$ln\ \theta_{\mu,t} = (1 - \rho_{\theta,\mu})ln\ \mu_{\theta,\rho} + \rho_{\theta,\mu} ln\ \theta_{\mu,t-1} + \epsilon_{\theta,\mu,t}$$

$$ln\ \theta_{\rho,t} = (1 - \rho_{\theta,\rho})ln\ \mu_{\theta,\mu} + \rho_{\theta,\rho} ln\ \theta_{\rho,t-1} + \epsilon_{\theta,\rho,t}$$

$$ln\ \theta_{\psi,t} = (1 - \rho_{\theta,\psi})ln\ \mu_{\theta,\psi} + \rho_{\theta,\psi} ln\ \theta_{\psi,t-1} + \epsilon_{\theta,\psi,t}$$

In the time varying model, I estimate $\rho_{\theta,z}$, $\rho_{\theta,\mu}$, $\rho_{\theta,\rho}$, $\rho_{\theta,\psi}$, $\rho_{\theta,\zeta}$, $\mu_{\theta,z}$, $\mu_{\theta,\rho}$, $\mu_{\theta,\mu}$, $\mu_{\theta,\psi}$, and $\mu_{\theta,\zeta}$. I also estimate the variances of the shock variance processes: ν_z, ν_μ, ν_ρ, ν_ψ and ν_ζ. Finally, I estimate the steady-state values of the search cost shock, ρ_t, and the monitoring cost shock, μ_t. I fix the other parameters at their time-invariant values. This is partly due to limitations of the particle filter, discussed in Section 4.5.2. I obtain the first-order accurate parameter estimates using Dynare with four Metropolis-Hastings chains of 250,000 draws each.

6.2.1 Convergence

I compute the Brooks-Gelman diagnostics (Brooks and Gelman (1998)) for the Metropolis-Hastings chains. The interval, second-, and third-order measures are computed. The multivariate measure converges around 200,000 draws, which is slightly later than the convergence of the time-invariant process. The multivariate diagnostic can be seen in Figure 66.

A few parameters (υ_ψ, υ_ζ, υ_ρ, μ_ρ, and μ_μ) converge later in the chain. However, the other twelve estimated parameters converge fairly early in the process - generally around 50,000 draws. The convergence diagnostics for the individual parameters can be seen in Figure 67 through Figure 72.

I also check the paths of the Metropolis-Hastings chains to make sure that the chain is settling around one parameter value. In particular, I want to check for cases where the chain appears to be bimodal or the chain appears to have a trend even at the end of the chain. As we saw in the time-invariant case, all parameters seem to settle on a value relatively quickly, except for the steady state search cost (μ_ρ) and the steady state monitoring cost (μ_μ). These chains do settle at a value, but are moving until they get to about 50,000 draws. The chains for both variables can be seen in Figure 74. The chains for all variables can be seen in Figure 73 to Figure 77

As a final check, I review the posterior distributions of the estimated parameters.

All posterior distributions are approximately normally distributed. With a few exceptions, the posterior distributions are not too far from the prior distributions. There are three parameters for which the prior distribution is significantly different from the posterior distribution: μ_μ, μ_ρ, and $\mu_{\theta\zeta}$. In all cases, I experimented with changing the moments of the prior to more closely match the posterior estimate. However, these experiments yielded undesirable results. Finally, no posterior distribution has an excessive variance from the mean. Graphs of the posterior distributions for each parameter can be found in Figure 64 and Figure 65. In addition, selected moments of the posterior distributions, such as mode, median, standard deviation, 10th and 90th percentiles, can be found in Table 11.

6.2.2 Parameter Estimates

Table 11 shows the first order parameter estimates for the time-varying model. The time-varying model allows the shock variances to change over time according to an auto-regressive process. Thus, rather than estimating the variance of the shock over the entire data series (as in the time-invariant case), I estimate the parameters of the processes that define the evolution of the variances. The parameters ρ_{ez}, $\rho_{e\rho}$, $\rho_{e\mu}$, $\rho_{e\psi}$ and $\rho_{e\zeta}$ describe the level of persistence of the shock variances. In my model, the technology shock variance and the firm capital adjustment cost shock variance are most persistent, with an estimated autoregressive coefficient of $\rho_{ez} = 0.81$ and $\rho_{e\zeta} = 0.82$. The monitoring cost process shock variance is slightly less persistent with an estimated $\rho_{e\mu}$ of 0.60. Finally, the search cost variance and the labor disutility shock variance are least persistent, with estimates of $\rho_{e\rho} = 0.19$ and $\rho_{e\psi} = 0.39$, respectively. Thus, the variances of the search cost shock and the labor disutility shock are most likely to revert to their steady-state values.

As expected, the estimated steady-state values of the shock variances in the time-varying case are similar to the variances estimates in the time-invariant case. The two exceptions are the variances of the firm capital adjustment cost shock and the labor disutility shock. The steady state estimates of these variances are both significantly larger than the time-invariant estimates. The comparison can be seen in Table 12.

Finally, the estimates of the steady state values of the monitoring cost shock and the search cost shock differ slightly from the time-invariant estimates. As discussed in section 6.3.3, the parameter estimates must be converted prior to the steady state costs themselves. The search cost steady state parameter estimate, μ_ρ, is -0.0494. This means that a very small amount of capital, 0.005%, is destroyed in the process of finding capital. The monitoring cost is significantly higher. A parameter estimate of $\mu_\mu = -6.7991$ means that about 49% of capital is destroyed by the borrower monitoring process. Note that these costs are slightly different from those estimated in the time-invariant process.

6.3 First Order Comparison: Time-Invariant vs. Time-Varying

6.3.1 Parameter Estimate Comparison

Many of the parameters are estimated for the first-order time-invariant case, and then fixed in the time-varying model. The following parameters, therefore, are fixed at their time-invariant values for the time-varying model estimation: υ, ρ_z, ρ_ρ, ρ_μ, ρ_ψ, and ρ_ζ. An additional list of 10 parameters are new in the time-varying model. These are the autoregressive parameters and the shock variances of the processes describing the variance of the model's exogenous shocks. There are a remaining 7 parameters that may be compared between the two models. This list includes the steady-state value of the monitoring cost, μ_μ, and the steady-state value of the search cost, μ_ρ. In addition, we may compare the estimated values of the shock variances in the time-invariant case to the steady-state values of the shock variances in the time-varying case. The comparison between these 7 parameters can be found in Table 12.

The search cost is slightly positive in the time-invariant case and slightly negative in the time-varying case. The monitoring cost is slightly more negative in the time-varying case, translating into a larger cost associated with monitoring borrowers. The standard deviations of both parameters are the same in both models. These differences are discussed in additional detail in section 6.4.3, above. The differences between the tech shock variance, the monitoring cost variance and the search cost variance are minimal. All three of these parameters have similar median values and similar standard deviations in both models. The major differences are between the labor disutility shock variance (θ_ψ) and the firm capital adjustment cost shock variance (θ_ς). In both cases, the variance of the shocks is significantly higher in the time-varying case. The standard deviations of both estimates are also significantly higher in the time-varying case.

6.3.2 Impulse Response Function Comparison

The impulse response functions are virtually identical between the time-varying and the time-invariant case. This is expected because model itself has not changed. The only change has been to the shock variances, which are fixed in the time-invariant case but not in the time-varying case. Thus, it is expected that the model responds to a shock in the same way, regardless of the formulation of the shock variance.

6.4 Second Order Time Varying

The model is unchanged from the model discussed in the first order time-varying section. However, the parameter estimates obtained in this section are second-order accurate. They are obtained using the particle filter to calculate the likelihood.

6.4.1 Convergence

The runtime of the second-order time-varying version of the parameter estimation process precludes the possibility of running multiple chains. I only run slightly over 30,000 draws of Metropolis-Hastings due to the long runtime. As a result, some of the chains appear to need more time to converge. The steady state value of the search cost, μ_ρ, currently has a slightly bimodal distribution (Figure 86). This chain clearly requires more draws to converge. The autocorrelation parameter of the technology shock variance, ρ_{ez}, appears to be poorly identified (Figure 86). A median may emerge with more Metropolis-Hastings draws. Also in Figure 86, the autocorrelation parameter of the search cost shock variance, $\rho_{e\rho}$, has significant density at 1.00 in addition to an apparent median at a lower value. The autocorrelation parameter of the labor disutility shock variance, $\rho_{e\psi}$ (Figure 87) also has significant density at both 1.00 and a lower value. Finally, the steady state value of the labor disutility shock variance, $\mu_{e\psi}$, appears to be markedly bimodal. The chain may converge over time to a value between the two apparent modes. The posterior distributions of the variances of the shock variance process look reasonably normally distributed.

The Metropolis-Hastings chains confirm that some of the parameters have not yet settled on a median value. In particular, the autocorrelation parameters for the technology shock and the search cost shock, ρ_{ez} and $\rho_{e\rho}$ are still moving quite a bit

(Figure 90). The autocorrelation parameter for the labor disutility cost shock, $\rho_{e\psi}$, is also still moving (Figure 91). The steady state values of the variances of the labor disutility shock and the firm capital adjustment cost shock also do not appear to have converged (Figure 92). The combination of the posterior distributions and the Metropolis-Hastings chains indicate that the time-varying second order parameter estimation process needs more Metropolis-Hastings draws to converge. This is hindered by the processing time required by this version of the model.

6.4.2 Parameter Estimates

The selected moments of the posterior distributions of the estimated parameters may be found in Table 13. The median steady state value of the search cost, 1.8487, translates to 16.88% of profit destroyed by search costs. This is significantly higher than the 5% estimated by the second order time-invariant model. The median steady state value of the monitoring cost, 1.1280, implies that 10.67% of profits are destroyed by monitoring costs. Combined, the monitoring and search costs destroy 25.75% of small business earnings. The combined cost was 22% in the time-invariant case. Rather than estimating larger costs in the time-varying case, the time-varying model appears to move some costs from the monitoring cost to the search cost. The shock variance processes do not appear to be highly persistent, with the exception of the capital adjustment cost variance process. The steady state values of the shock variances appear to be of similar magnitude. The exception is the steady state value of the search cost shock variance, which appears to be about 10 times smaller than the other steady state variance estimates. However, the variability of the process appears to be captured in the variance of the search cost shock variance process, which is significantly larger than the variances estimated for the other shock processes. The firm capital adjustment cost shock variance process also appears to be more variable than the other processes.

6.5 Optimal Contract Model Results

In this section, I present an extended analysis of the static optimal contract model, which represents the lending agreement between the small business and the lender. This provides more insight into the behavior of the optimal contract independent of the DSGE model.

6.5.1 Solving for Costs

The solution to the optimal contract problem yields the optimal loan amount and interest rate. These combine to determine the cutoff value of capital production, $\bar{\omega}$, below which the borrower will default. I will use historical data for loan amounts and interest rates to solve for financing search costs, ρ, and lender monitoring costs, μ, as functions of the data.

First, I use Equation (6) and Equation (8) to solve for ρ as a function of μ. Combining the two equations and rearranging yields [12]:

$$\exp(\rho) = \frac{(i - i^e)(4 - 2r^k \exp(r))}{2i - ir^k \exp(r) + i^e r^k (1 + r)}$$

Now that I have obtained an expression for ρ in terms of μ, I plug this into Equation (7). Also, using the equation for the function $g(\bar{\omega})$ from Appendix A and

[12] See Appendix C for details

equation (6) to substitute for $\bar{\omega}$, obtain :

$$\exp(\mu) = \frac{2i\ \exp(r) - ir^k\ \exp(2r) + i^e r^k\ \exp(2r)}{4ij - 2ijr^k(1+r)}$$

6.5.2 Data and Additional Parameters

Historical data on loan amounts and interest rates is readily available. I use the total per-capita value of business loans outstanding (the value of business loans outstanding divided by population) from the Federal Reserve Economic Data database. The quarterly real interest rate is calculated from the bank prime rate and ex post quarterly inflation (both from the Federal Reserve Economic Data database). The rental rate of capital is taken from Gomme and Rupert's (2008) calculation of the U.S. return to capital.

This still leaves us with 5 unknowns (j, ρ, μ, n, and $\bar{\omega}$) and only three equations from the optimal contract problem. To overcome this difficulty, I assign values to two unknowns. Ability, j, equals one, which is the mean of the distribution of j. The amount of internal investment, n, is derived by setting the business's share of income to 0.39[13]. This implies that 39% of the funding comes from the business, while 61% is borrowed from outside sources. This implies that $n = 0.39i$. Since the amount of the loan is known ($i - n$), we are able to solve for i and n. Now there are three remaining unknowns (ρ, μ, and $\bar{\omega}$) and three equations.

[13]This value comes from the steady state solution to the Carlstrom and Fuerst (1997) agency cost model

6.5.3 Financing Costs and Default Threshold Values

The parameter μ represents the percent of capital seized from defaulters that is destroyed in the process of monitoring borrowers who attempt to default. Figure 2 shows the quarterly values for this parameter between 1964 and 2000. Figure 4 plots both monitoring costs and real interest rates. The monitoring cost estimates range from 24% to 30%. There is a noticeable drop in the magnitude of the monitoring cost in 1980. Between 1980 and 2000, monitoring costs are significantly lower in magnitude than they were before 1980. Note that μ moves in the opposite direction of the real interest rate. When the real interest rate rises in the early 1980's, the monitoring cost drops. The drop in monitoring costs beginning in the 1980's is intuitively sensible - information sharing techniques are constantly improving, and the 1980's marked the beginning of the widespread use of personal computers. In fact, the use of personal computers in homes and offices more than doubled from 2 million in 1981 to 5.5 million in 1982 (website reference, see bibliography). This may account for increased ease of record-keeping and loan-tracking, which could have reduced the costs associated with monitoring borrowers.

The costs associated with borrowing money are measured by ρ in the model. This parameter represents the percentage of total capital that is destroyed in the process of finding funding for a capital project. Figure 3 shows the estimated quarterly value of ρ between 1964 and 2000, while Figure 5 shows both ρ and real interest rates. Search costs range from almost 13% to 17%. Unlike monitoring costs, search costs increase slightly in 1980 and remain at slightly higher levels through 2000. The search cost moves in parallel with the real interest rate. When the overall level of the real interest rate rises in the early 1980's, the search costs also rise slightly. This is contrary to intuitive expectations - it could be surmised that an increase in technological availability would make it easier and cheaper to search for a loan. However, improved use of technology may actually have made it more *difficult* to obtain a loan beginning in the 1980's. Guru and Horne (2000) find that consolidation in the U.S. banking industry since approximately 1985 has significantly reduced the number of small, local banks and has had a significant impact on small-business lending. Avery and Samolyk (2000) also observe reduced small-business loan availability in markets affected by bank mergers in the 1990's. Rural markets are more affected than urban markets. This is consistent that small-business lending is done mostly by local institutions, which have expert knowledge of the prospects in the local economy (Avery and Samolyk (2000)). Such judgements are much more difficult for a large, national bank to make. The sum of this literature thus indicates that, while financial frictions in the overall economy may have been reduced by bank consolidation, it may have caused increased loan search costs for small businesses.

Estimating values for ρ and μ allows us to estimate $\bar{\omega}$. This is the cutoff value of the capital production shock below which the borrower will default. Figure 6 shows the estimate of $\bar{\omega}$ between 1964 and 2000. The range of this variable is small; it takes on values between 1.59 and 1.89. The shock, ω, is uniformly distributed on $[0, 2]$, so the values for $\bar{\omega}$ are a bit above the median of the distribution of ω. The value of $\bar{\omega}$ implies (approximately) a 88% probability of default for a small business owner with ability j of 1, which is the median ability. The interpretation of this "default probability" is not entirely obvious. The first important observation is that a person of average ability ($j = 1$) is unlikely to start a small business. Hipply (2010) estimates that the rate of self-employment in the U.S. is about 10%. Thus, if individuals selfselect correctly, only people in the top 10% of ability would start a small business. This would translate to a j value of approximately 1.4. Thus, an entrepreneur with a lower ability would have a higher probably of default than someone with an ability more in line with estimated self-employment rates.

In the model, a business defaults if they make less than the amount they owe the borrower (principal plus interest). It might be reasonable to interpret the model default probability as the survival probability of a small businesses. Kirchhoff and Philips (1989) find that about 40 % of small businesses survive for six years, which is in line with the model default probability of 40 %. In reality, a business might have reserves from which to pull in a period where they make less than the loan repayment amount. Because reserves are not an element in this model, we could consider the default in the model to be simply losing money. A study by the National Federation of Independent Business indicates that about 30 % of business lose money over the life of the business (Klein (1999)). This is certainly lower than the model default probability, but the failed businesses are more difficult to poll and therefore may be underrepresented in the study. In addition, as previously mentioned, the estimated default rate is based on an average ability, which may not be representative of the ability of people who actually start small businesses in the data. Finally, the model does not account for credit scoring, so that a default in this period does not have an effect on the loan terms available in the next period. This may translate to a greater probability of default in the model because there is no mechanism to penalize the small business for defaulting.

6.5.4 Stability of Costs

For both monitoring and search costs, volatility increases significantly in the late 1970's and early 1980's (see Figure 7 and Figure 8). The standard deviation of the monitoring cost, μ, starts to drop in the third quarter of 1984. The standard deviation of the search cost, ρ, begins trending down around 1982. The standard deviation of the monitoring cost, μ, seems to vary a little more - its range is from 0.001 to 0.013. The standard deviation of the search cost, however, varies from 0.001 to 0.006. There does seem to be significant variability within the standard deviations.

6.5.5 Financial Deregulation and Borrowing Costs

In the static model, monitoring costs decrease dramatically in the early 1980's, while search costs increase around the same time. Real interest rates also increase at approximately the same time. The time series of monitoring costs roughly mirrors the pattern of real interest rates (Figure 4). Monitoring costs decrease when real interest rates increase. Note that the gap between real interest rates and monitoring costs prior to 1980 shrinks (and in some cases disappears) after 1980. The time series of search costs matches the graph of real interest rates almost exactly (Figure 5). When interest rates increase in the early 1980's, so do borrowing search costs. In a way, this is intuitive - increased interest rates may cause the potential borrower to do more research and compare institutions in order to find the best possible rate. In addition, a higher interest rate signals a reduced availability and/or a higher demand for loans, which could also cause loan search costs to increase.

In addition to a decrease in the magnitude of costs around 1980, volatility also begins to decrease during this time. However, volatility settles at or slightly above its pre-1975 level. This differs from the path of macroeconomic volatility, in which post-1984 volatility is significantly lower than all pre-1984 volatility levels (see Figure 1).

The period during which monitoring costs fall and search costs rise corresponds with a time of deregulation in the financial industry. The Depository Institutions Deregulation and Monetary Control Act (DIDMCA) was passed on March 31, 1980. This lines up almost exactly with the observed change in magnitude of the borrowing costs. Although deregulation increased the flexibility of banking institutions overall, there is evidence that deregulation actually decreased the availability of small business loans, while increasing the ease with which standard corporate loans could be obtained. This may explain the drop in monitoring costs and the coincident rise in loan search costs. Regardless of the explanation, the timing suggests that there may be a relationship between financial deregulation and borrowing costs.

7 Conclusion

In summary, I construct a Dynamic Stochastic General Equilibrium model with some standard shocks. I also add two unique shocks to capture costs associated with borrowing. These shocks, a cost incurred in searching for funding and a cost incurred by lenders who must monitor borrowers, are introduced via a static optimal contract model. The shocks explicitly capture the more generic investment cost shocks that have been discussed in previous work. I formulate two version of the model: one in which the variance of the shocks is fixed over time and one in which the variance of the shocks may change over time.

I estimate the parameters of the models using two methods. First, I find a standard first-order estimate using Dynare. I also implement a particle filter process in Fortran to obtain second-order accurate parameter estimates for each version of the model. The particle filter, which does not have the linearity assumption of the Kalman filter, allows me to account for both first and second order effects.

After obtaining the parameter estimates, I analyze the dynamics of the baseline time-invariant model. I find that the model does a fair job of capturing the variability of various economic data series that are not used in model estimation. The model also captures a decrease in the volatility of borrowing costs in the 1980's, during a time of financial deregulation in the United States. I also find that the second order component of the model is critical in a few cases. In particular, it changes the direction of the impulse response functions for the household's capital in response to a search cost shock. In addition, the second-order estimates of the steady state values of the borrowing costs are much more reasonable than the first-order estimates.

Although the particle filter required significant coding and substantial processing time, the second order effects are critical to developing a full understanding of the model dynamics. In the second order, I am able to estimate reasonable values for the borrowing costs, as well as capture a decrease in borrowing cost variability in the 1980's. I also capture the fluctuations of other economic variables in a reasonable manner. Thus, the major innovations of my research - introducing explicit borrowing costs and implementing the particle filter to obtain second-order parameter estimates - are both relevant and provide information about the effect of borrowing costs and financial deregulation on the macroeconomy.

8 Tables

Parameter	Symbol	Value
Capital Share of Production	α	0.33
Depreciation Rate	δ	0.1
Discount Rate	β	0.99
Firm Share of Capital	γ	0.5
Steady State of Labor Disutil Shock	μ_ψ	1.0
Steady State of Cap Adj Shock	μ_ζ	1.0

Table 1: Fixed Parameters

		Time-Invariant							
		Prior Distribution			Posterior Distribution				
Param	Description	Density	Mean	Std	Mode	Median	Std	[10, 90]	
υ	Inverse Frisch Labor	G	2.00	0.75	4.5418	2.7322	1.2767	1.6463	4.8839
ρ_z	Technology Shock AR Param	B	0.60	0.20	0.2425	0.0792	0.1046	0.007	0.2324
ρ_ρ	Search Cost AR Param	B	0.60	0.20	0.0498	0.6792	0.2945	0.0889	0.9164
ρ_μ	Monitoring Cost AR Param	B	0.60	0.20	0.9133	0.9589	0.0611	0.8510	0.9982
ρ_ψ	Labor Disutility AR Param	B	0.60	0.20	0.9830	0.9783	0.0502	0.8892	0.9995
ρ_ζ	Firm Cap Adj Cost AR Param	B	0.60	0.20	0.9999	0.8759	0.1186	0.7081	0.9848
θ_z	Technology Shock Var	I	0.05	0.01	0.0139	0.0240	0.0091	0.0139	0.0376
θ_μ	Monitoring Cost Var	I	0.10	0.10	0.0219	0.0391	0.0331	0.0200	0.0863
θ_ρ	Search Cost Var	I	0.06	0.10	0.1376	0.0581	0.0650	0.0261	0.1658
θ_ψ	Labor Disutility Shock Var	I	0.01	0.01	0.0260	0.0192	0.0072	0.0128	0.0291
θ_ζ	Cap Adj Shock Var	I	0.01	0.01	0.0271	0.0494	0.0523	0.0236	0.1179
μ_ρ	SS Search Cost	N	-1.00	1.00	-0.7732	-0.5117	0.4615	-0.1883	-1.2878
μ_μ	SS Monitoring Cost	N	-4.00	3.00	-7.4917	-2.0462	1.9495	-0.6865	-5.7555

Table 2: Second Order Time Invariant Prior Densities and Posterior Distributions

Shock	Implied STD	Estimated Process STD
μ	0.0308	0.0391
ψ	< 0.0001	0.0192
z	0.0007	0.0240
ζ	0.0128	0.0494
ρ	0.0093	0.0581

Table 3: Second Order Time Invariant
Recovered Shock STD Compared to Estimated Shock STD

Data	Source	Shocks	Mean Unfilter	Std	Output Corr
Relative Price of Investment	Data	All	0.7806	0.0084	-0.0916
	Sim	All	0.8193	0.1943	-0.4109
Rental Price of Capital	Data	All	0.09	0.071	0.523
	Sim	All	0.07	0.074	0.637
Interest Rate	Data	All	0.084	0.050	-0.176
	Sim	All	0.091	0.611	-0.4258

Table 4: Second Order Time Invariant
Simulations of Selected Variables not used in Estimation

Data	Source	Shocks	Std	Output Corr
Production	Data	All	0.0130	1.00
	Sim	All	0.0220	1.00
Investment	Data	All	0.0468	0.170
	Sim	All	0.1928	0.509
Consumption	Data	All	0.0074	0.121
	Sim	All	0.0438	0.773

Table 5: Second Order Time Invariant
Simulations of Selected Variables Used in Estimation

Data	Model Variable	Shocks	Unfiltered Mean
Ability Cutoff	j	All	1.0411
Default Threshold	ω	All	0.5972
Search Cost	ρ	All	0.9500
Monitoring Cost	ρ	All	0.8200

Table 6: Second Order Time Invariant
Mean of Other Selected Simulated Variables

		Time-Invariant							
		Prior Distribution			Posterior Distribution				
Param	Description	Density	Mean	Std	Mode	Median	Std	[10, 90]	
υ	Inverse Frisch Labor	G	2.00	0.75	1.7188	1.8238	0.6953	1.0814	2.9926
ρ_z	Technology Shock AR Param	B	0.60	0.20	0.9836	0.9856	0.0051	0.9785	0.9912
ρ_ρ	Search Cost AR Param	B	0.60	0.20	0.9996	0.9993	0.0010	0.9985	0.9997
ρ_μ	Monitoring Cost AR Param	B	0.60	0.20	0.9809	0.9841	0.0045	0.9787	0.9890
ρ_ψ	Labor Disutility AR Param	B	0.60	0.20	0.6667	0.6265	0.2722	0.3180	0.8590
ρ_ζ	Firm Cap Adj Cost AR Param	B	0.60	0.20	0.9651	0.9540	0.0334	0.8985	0.9853
θ_z	Technology Shock Var	I	0.05	0.01	0.0398	0.0383	0.002	0.0360	0.0411
θ_μ	Monitoring Cost Var	I	0.10	0.10	0.0876	0.1043	0.0050	0.0973	0.1122
θ_ρ	Search Cost Var	I	0.06	0.10	0.0245	0.0225	0.0013	0.0210	0.0242
θ_ψ	Labor Disutility Shock Var	I	0.01	0.01	0.0057	0.0080	0.0021	0.0046	0.0166
θ_ζ	Cap Adj Shock Var	I	0.01	0.01	0.0038	0.0040	0.0009	0.0030	0.0052
μ_ρ	SS Search Cost	N	−1.00	1.00	−1.00	0.2557	1.00	0.1092	0.3470
μ_μ	SS Monitoring Cost	N	−4.00	3.00	−3.8739	−8.1329	0.1220	−8.5006	−7.5428

Table 7: First Order Time Invariant Prior Densities and Posterior Estimates

Data	Source	Shocks	Mean Unfilter	Std	Output Corr
Relative Price of Investment	Data	All	0.7806	$8.38\text{x}10^{-3}$	$-9.16\text{x}10^{-2}$
	Sim	All	0.7815	$7.19\text{x}10^{-2}$	$-4.55\text{x}10^{-1}$
Rental Price of Capital	Data	All	0.09	0.071	0.523
	Sim	All	0.06	0.079	-0.708
Interest Rate	Data	All	0.084	0.050	-0.176
	Sim	All	0.05	0.388	0.105

Table 8: First Order Time Invariant
Simulations of Selected Variables Not Used in Estimation

Data	Source	Shocks	Std	Output Corr
Production	Data	All	0.0130	1.00
	Sim	All	0.0207	1.00
Investment	Data	All	0.0468	0.170
	Sim	All	0.1424	0.476
Consumption	Data	All	0.0074	0.121
	Sim	All	0.0231	0.883

Table 9: First Order Time Invariant
Simulations of Selected Variables Used in Estimation

Data	Model Variable	Shocks	Unfiltered Mean
Ability Cutoff	j	All	1.1006
Default Threshold	ω	All	0.6256
Search Cost	ρ	All	0.9567
Monitoring Cost	ρ	All	0.4324

Table 10: First Order Time Invariant
Mean of Other Selected Simulated Variables

		Time-Invariant								
		Prior Distribution			Posterior Distribution					
Param	Description	Density	Mean	Std	Mode	Median	Std	[10, 90]	
μ_ρ	SS Search Cost	N	−1.00	1.00	−1.0000	−0.0494	1.0000	−0.2576	0.1701
μ_μ	SS Monitor Cost	N	−3.00	2.00	−3.0000	−6.7991	0.1206	−7.5900	−6.0721
ρ_{ez}	Tech Process AR Param	B	0.80	0.10	0.8461	0.8120	0.1001	0.6678	0.9191
$\rho_{e\rho}$	Monitoring Cost Process AR Param	B	0.20	0.10	0.1539	0.1886	0.1001	0.0835	0.3340
$\rho_{e\mu}$	Search Cost Process AR Param	B	0.60	0.10	0.6096	0.5996	0.1064	0.4710	0.7241
$\rho_{e\psi}$	Labor Disutility Process AR Param	B	0.40	0.10	0.3903	0.3952	0.1064	0.2718	0.5244
$\rho_{e\zeta}$	Cap Adj Cost Process AR Param	B	0.80	0.10	0.8461	0.8198	0.1001	0.6734	0.9212
μ_{ez}	SS Tech Shock Variance	I	0.04	0.001	0.0387	0.0391	0.0021	0.0364	0.0419
$\mu_{e\rho}$	SS Search Cost Variance	I	0.025	0.001	0.0244	0.0225	0.0011	0.0212	0.0238
$\mu_{e\mu}$	SS Monitor Cost Variance	I	0.10	0.005	0.1042	0.0982	0.0056	0.0921	0.1050
$\mu_{e\psi}$	SS Labor Disutility Shock Variance	I	0.03	0.005	0.0278	0.0291	0.0046	0.0236	0.0369
$\mu_{e\zeta}$	SS Cap Adj Cost Variance	I	0.06	0.005	0.0364	0.0348	0.0025	0.0326	0.0379
ν_z	Tech Shock Var	I	0.05	0.05	0.0286	0.0402	0.0107	0.0233	0.0841
ν_ζ	FirmCap Adj Shock Var	I	0.01	0.1	0.0046	0.0071	0.0019	0.0037	0.0242
ν_μ	Monitoring Cost Shock Var	I	0.03	0.05	0.0153	0.0227	0.0060	0.0125	0.0619
ν_ρ	Search Cost Shock Var	I	0.03	0.05	0.0153	0.0237	0.0060	0.0126	0.0555
ν_ψ	Labor Disutility Shock Var	I	0.02	0.05	0.0097	0.0128	0.0039	0.0075	0.0265

Table 11: First Order Time Varying Prior Densities and Posterior Estimates

			Time-Invariant		
			Posterior Distribution		
Param	Description	Model	Mode	Median	Std
μ_ρ	Search Cost SS	Time-Invariant	-1.00	0.256	1.00
μ_ρ		Time-Varying	-1.00	-0.050	1.00
μ_μ	Monitoring Cost SS	Time-Invariant	-3.87	-8.133	0.120
μ_μ		Time-Varying	-3.00	-6.799	0.120
θ_z	Tech Shock Var	Time-Invariant	0.040	0.038	0.002
μ_{ez}		Time-Varying	0.039	0.039	0.002
θ_ρ	Monitoring Cost Var	Time-Invariant	0.024	0.022	0.001
$\mu_{e\rho}$		Time-Varying	0.024	0.022	0.001
θ_μ	Search Cost Var	Time-Invariant	0.088	0.104	0.005
$\mu_{e\mu}$		Time-Varying	0.104	0.098	0.006
θ_ψ	Labor Disutil Var	Time-Invariant	0.006	0.008	0.002
$\mu_{e\psi}$		Time-Varying	0.028	0.029	0.005
θ_ζ	Cap Adj Cost Var	Time-Invariant	0.004	0.004	0.001
$\mu_{e\zeta}$		Time-Invariant	0.036	0.035	0.002

Table 12: First Order Time-Invariant and Time-Varying Comparison

		Time-Invariant							
		Prior Distribution			Posterior Distribution				
Param	Description	Density	Mean	Std	Mode	Median	Std	[10, 90]	
μ_ρ	SS Search Cost	N	1.00	1.00	0.5389	1.8487	0.8963	0.5389	0.2.7545
μ_μ	SS Monitor Cost	N	3.00	2.00	0.0101	1.1280	0.6967	0.0255	2.0214
ρ_{ez}	Tech Process AR Param	B	0.80	0.10	0.9588	0.4035	0.3025	0.1137	0.9588
$\rho_{e\rho}$	Monitoring Cost Process AR Param	B	0.20	0.10	1.00	0.4870	0.3030	0.1772	1.00
$\rho_{e\mu}$	Search Cost Process AR Param	B	0.60	0.10	0.8951	0.8179	0.1469	0.5963	0.9971
$\rho_{e\psi}$	Labor Disutility Process AR Param	B	0.40	0.10	0.3386	0.6096	0.2880	0.2747	0.9988
$\rho_{e\zeta}$	Cap Adj Cost Process AR Param	B	0.80	0.10	0.8422	0.9459	0.1006	0.7438	0.9999
μ_{ez}	SS Tech Shock Variance	I	0.04	0.001	0.0427	0.0391	0.0625	0.0321	0.1904
$\mu_{e\rho}$	SS Search Cost Variance	I	0.025	0.001	0.0053	0.0987	0.0143	0.0014	0.0255
$\mu_{e\mu}$	SS Monitor Cost Variance	I	0.10	0.005	0.0526	0.0066	0.0923	0.0026	0.2076
$\mu_{e\psi}$	SS Labor Disutility Shock Variance	I	0.03	0.005	0.0405	0.0373	0.0216	0.0285	0.0839
$\mu_{e\zeta}$	SS Cap Adj Cost Variance	I	0.06	0.005	0.0702	0.0519	0.0242	0.0226	0.0892
ν_z	Tech Shock Var	I	0.05	0.05	0.0286	0.0564	0.0118	0.0220	0.0543
ν_ζ	FirmCap Adj Shock Var	I	0.01	0.1	0.1371	0.1344	0.0343	0.0921	0.1812
ν_μ	Monitoring Cost Shock Var	I	0.03	0.05	0.0672	0.0637	0.0108	0.0484	0.0746
ν_ρ	Search Cost Shock Var	I	0.03	0.05	0.3116	0.1256	0.1148	0.0255	0.3138
ν_ψ	Labor Disutility Shock Var	I	0.02	0.05	0.0461	0.0349	0.0103	0.0249	0.0496

Table 13: Second Order Time Varying Prior Densities and Posterior Estimates

9 Figures

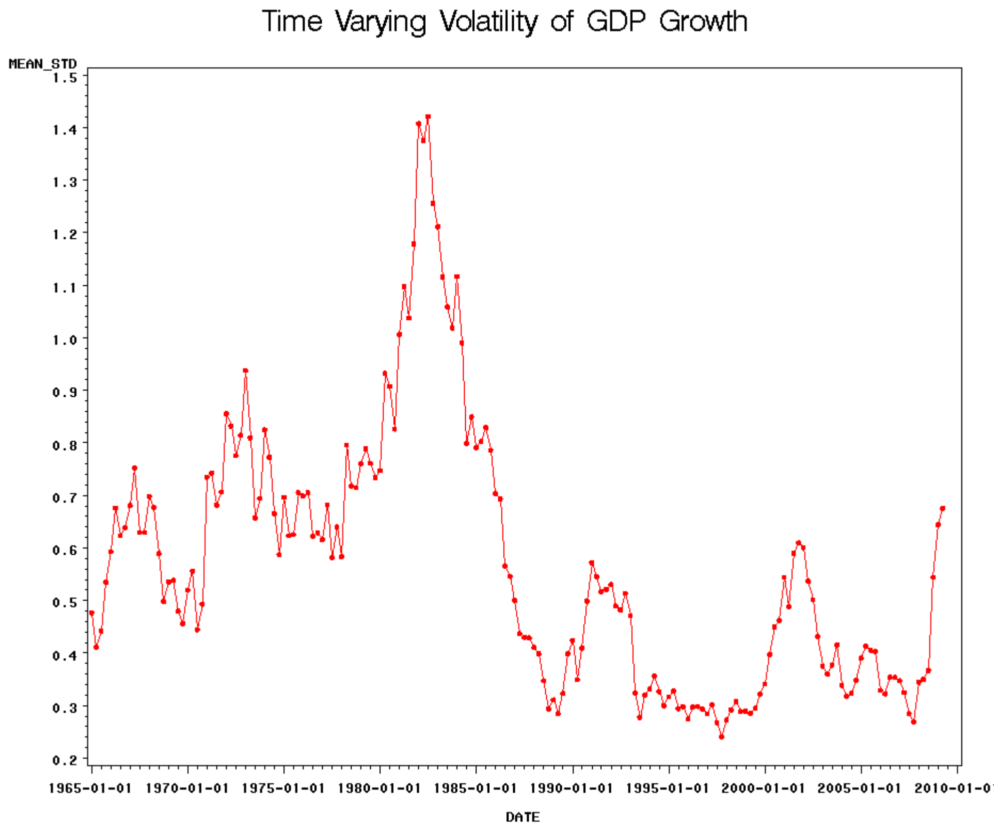


Figure 1: Time-Varying Volatility of GDP Growth: 1964 to 2009

Figure 2: Model Monitoring Costs: 1964 to 2000
Optimal Contract Model

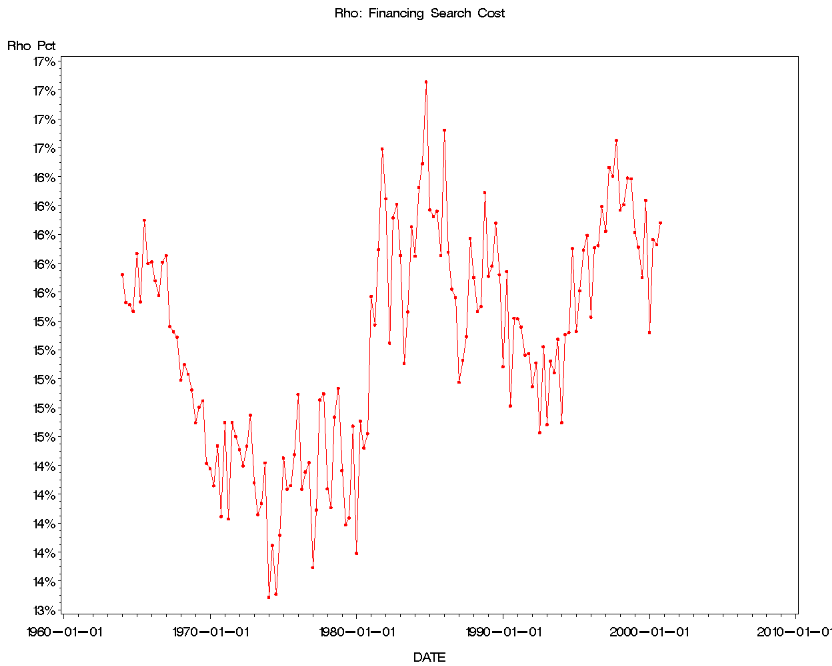


Figure 3: Model Financing Search Costs: 1964 to 2000
Optimal Contract Model

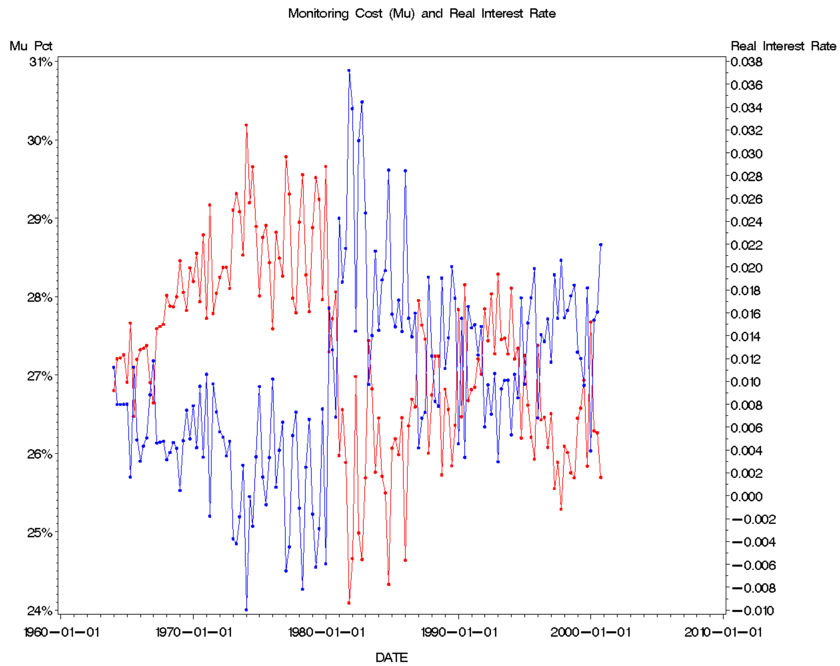


Figure 4: Model Monitoring Costs and Quarterly Real Interest Rates: 1964 to 2000 Optimal Contract Model

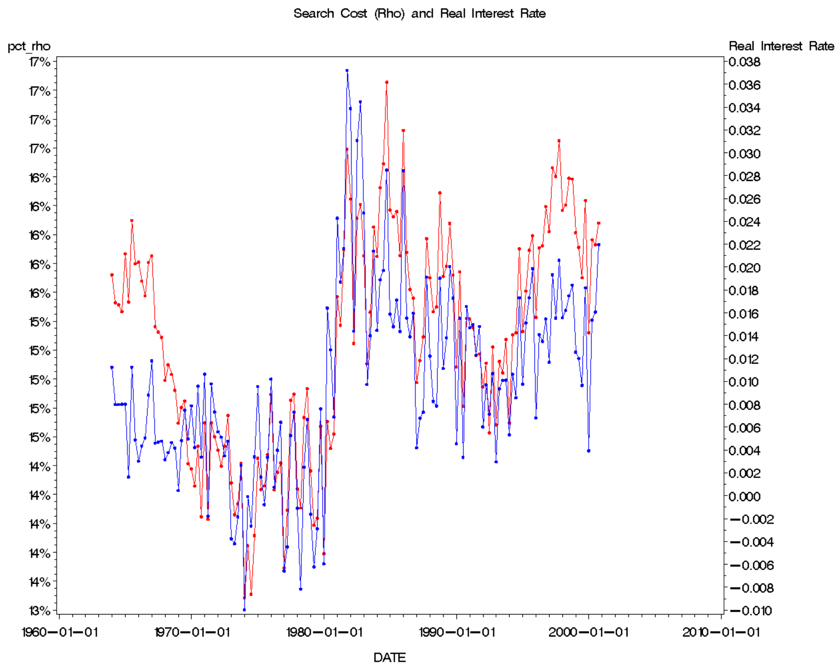


Figure 5: Model Search Costs and Quarterly Real Interest Rates: 1964 to 2000
Optimal Contract Model

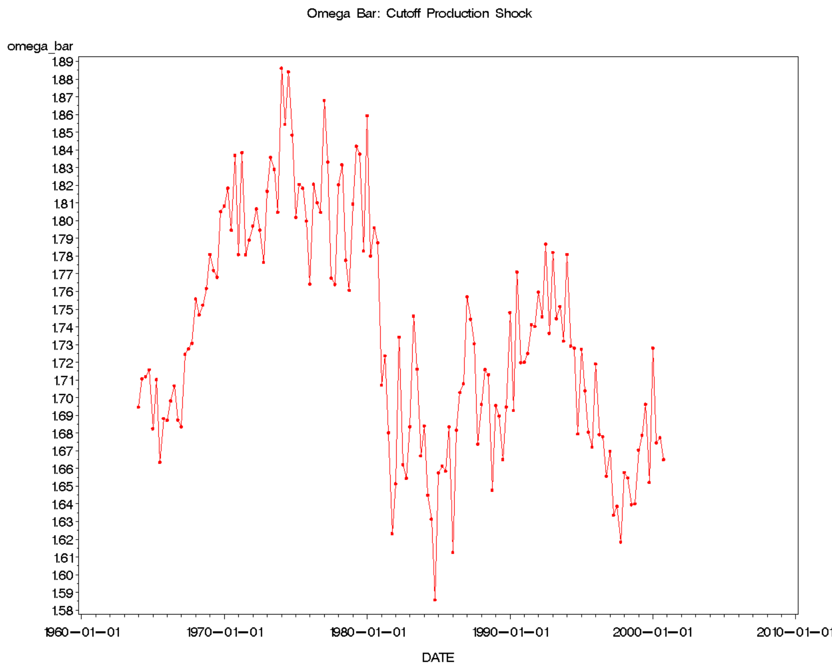


Figure 6: Model Default Threshold Values: 1964 to 2000
Optimal Contract Model

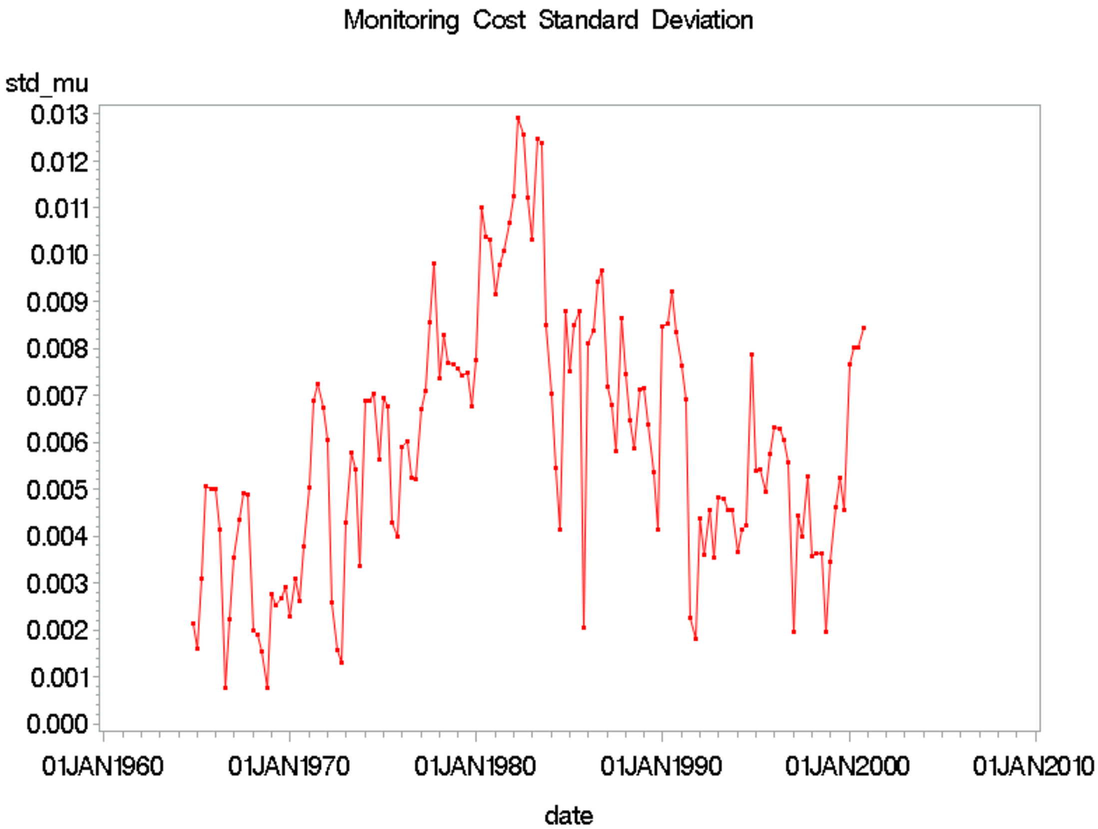


Figure 7: Time-Varying Volatility of Model Monitoring Costs: 1964 to 2000
Optimal Contract Model

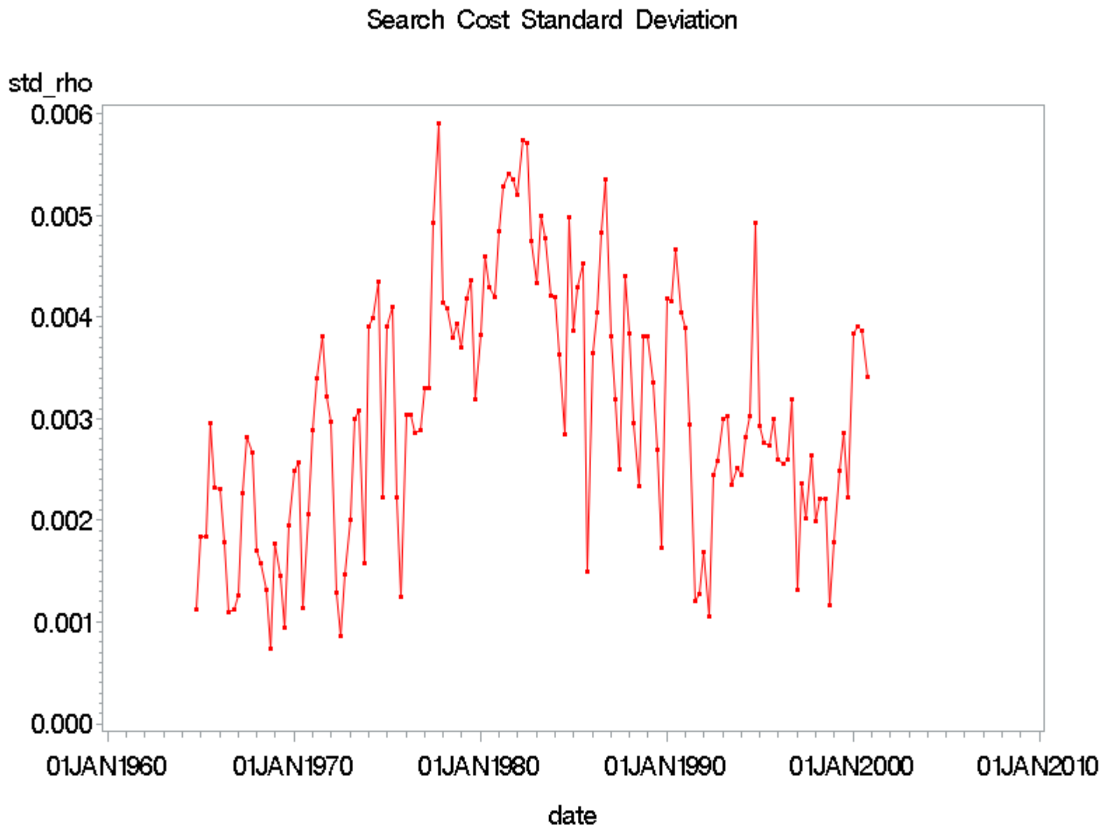


Figure 8: Time-Varying Volatility of Model Search Costs: 1964 to 2000
Optimal Contract Model

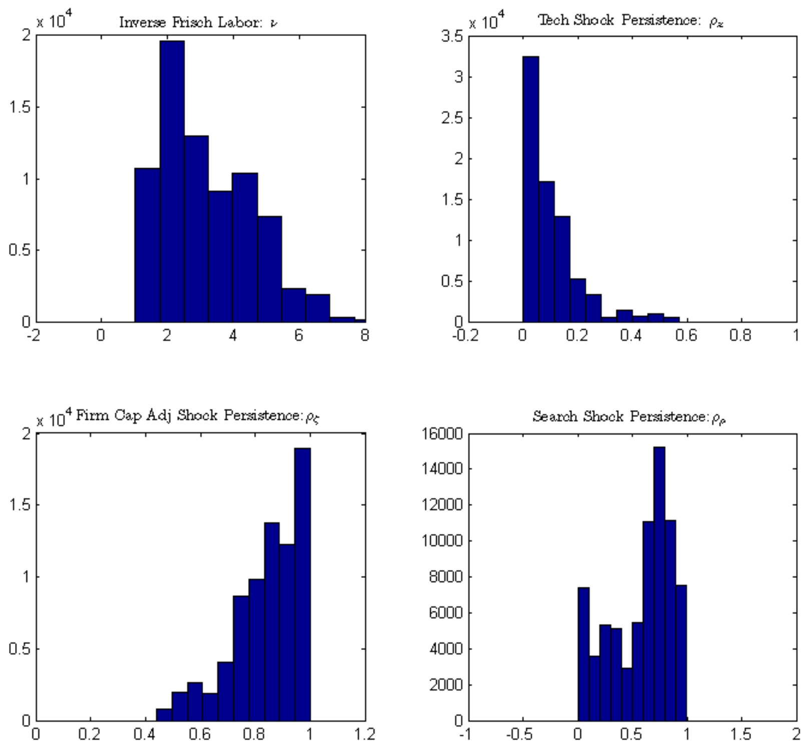


Figure 9: Second Order Time Invariant Posterior Distributions

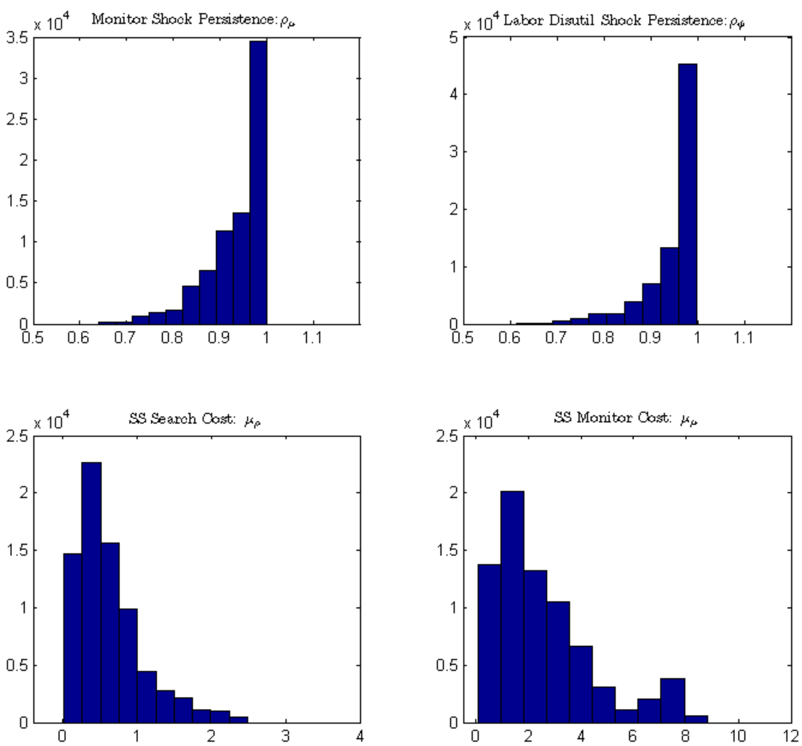


Figure 10: Second Order Time Invariant Posterior Distributions

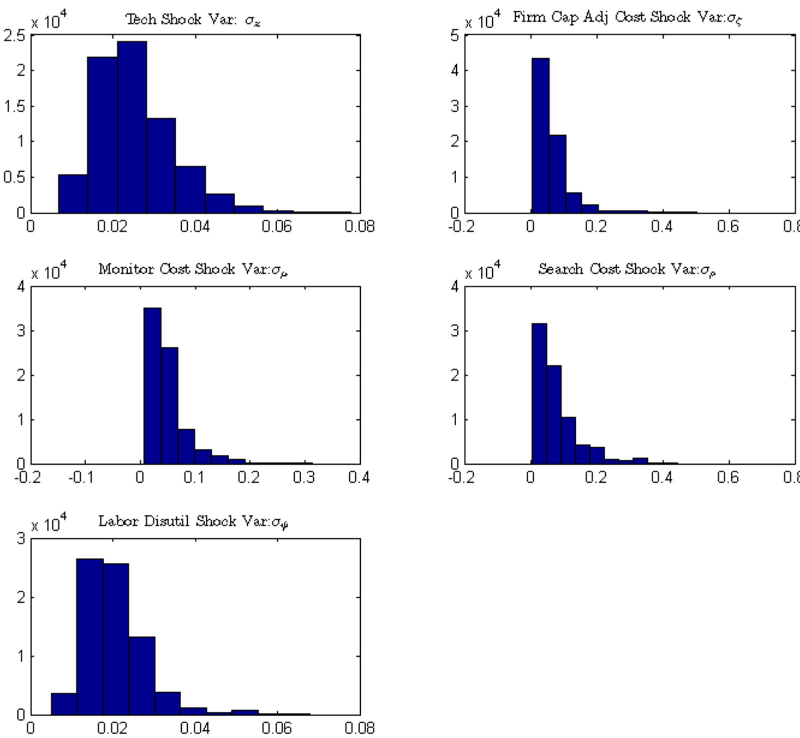


Figure 11: Second Order Time Invariant Posterior Distributions

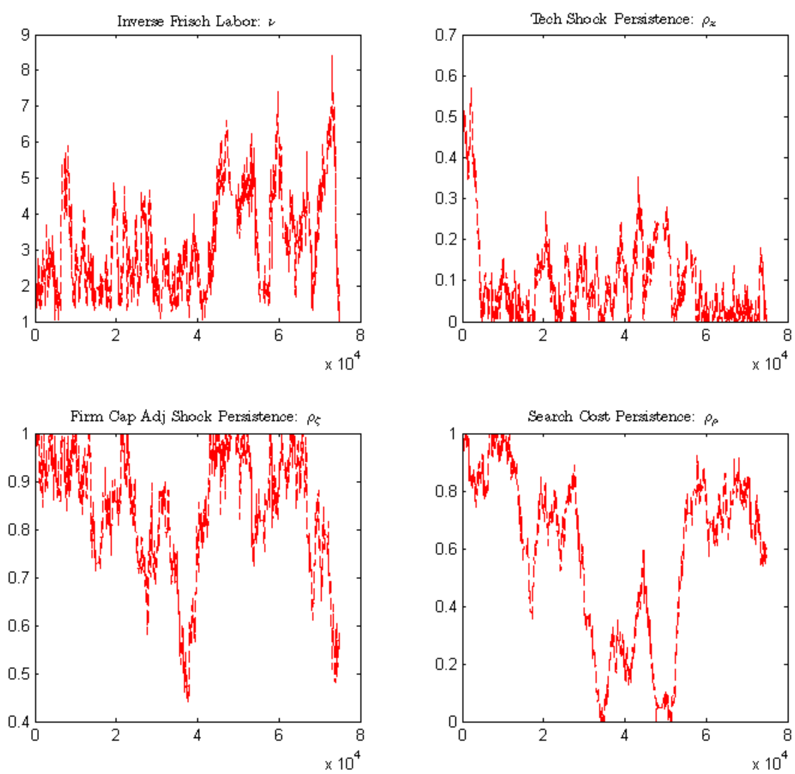


Figure 12: Second Order Time Invariant MH Sample Chain

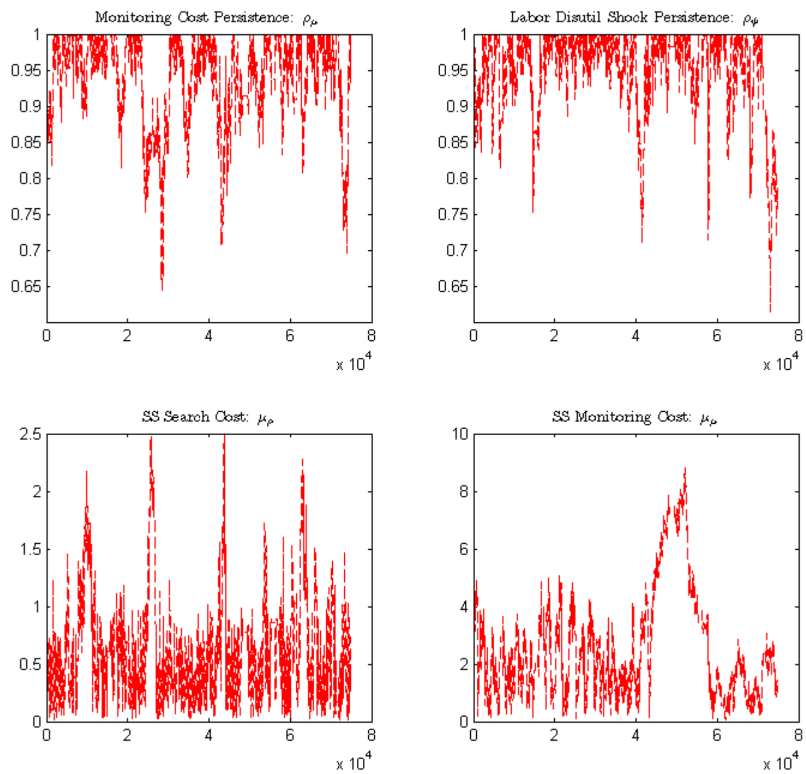


Figure 13: Second Order Time Invariant MH Sample Chain

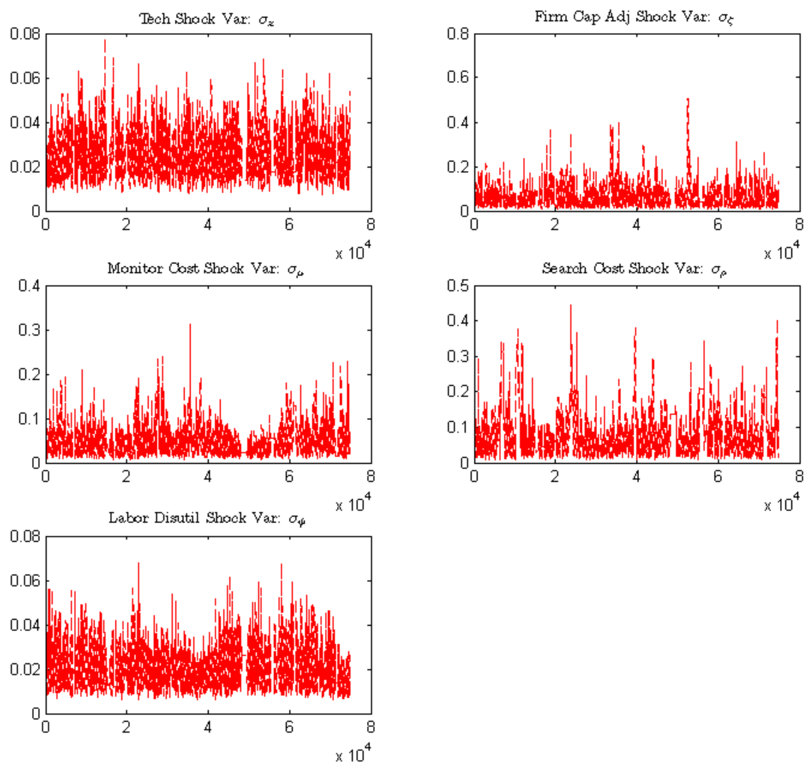


Figure 14: Second Order Time Invariant MH Sample Chain

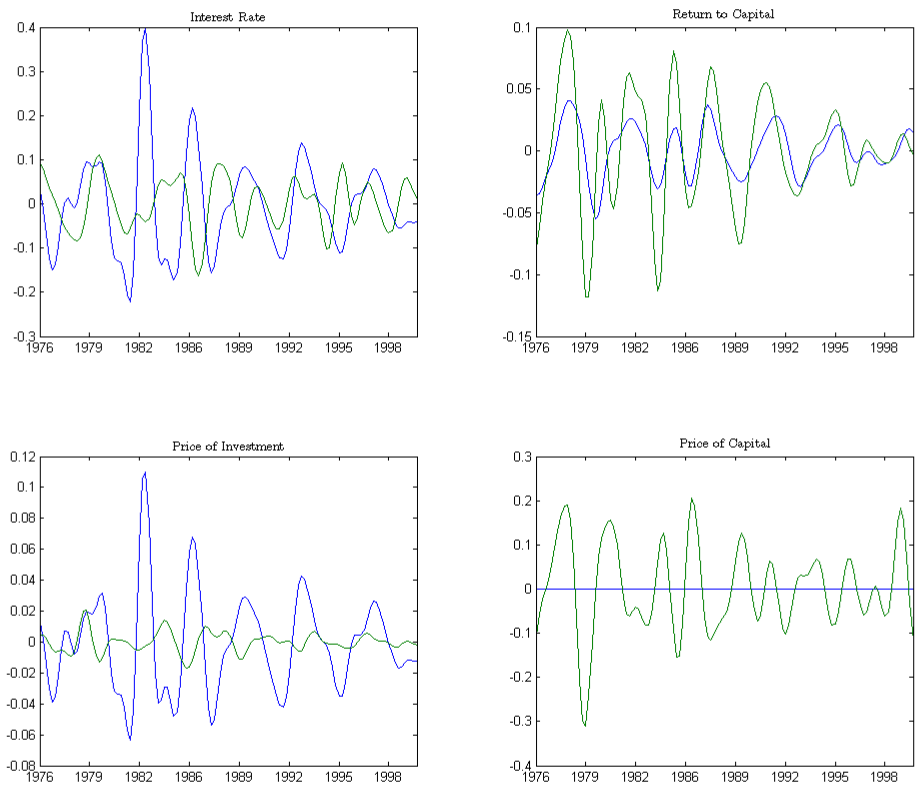


Figure 15: First Order Solution, Second Order Parameter Est, Time Invariant
Implied Variable Series: Not used in Estimation
Green: Data, Blue:Model

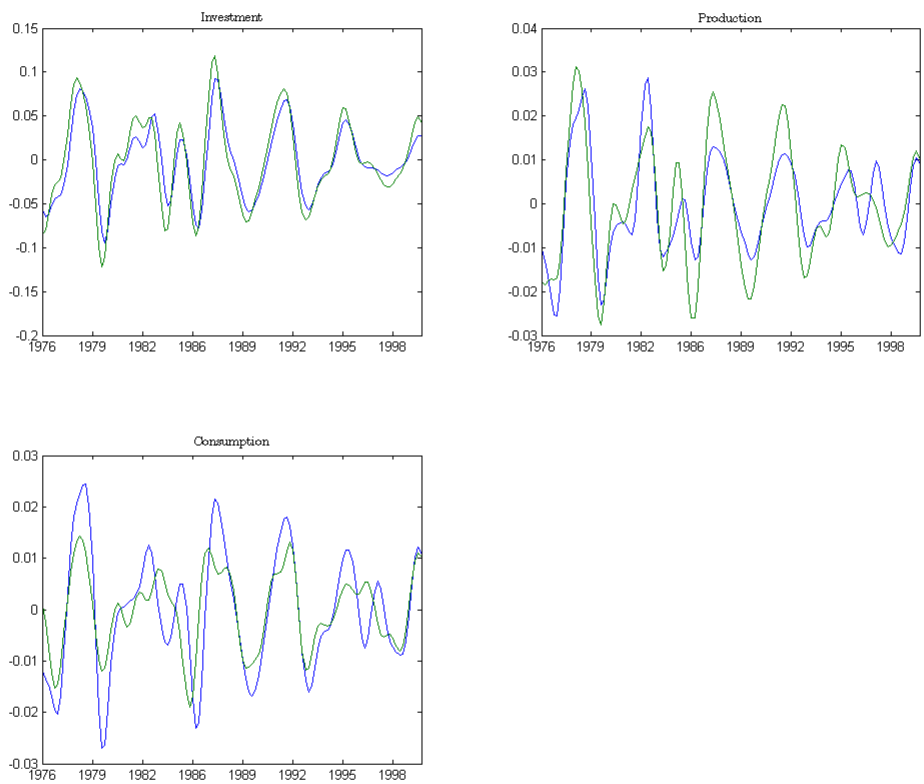


Figure 16: First Order Solution, Second Order Parameter Est, Time Invariant
Implied Variable Series: Used in Estimation
Green: Data, Blue:Model

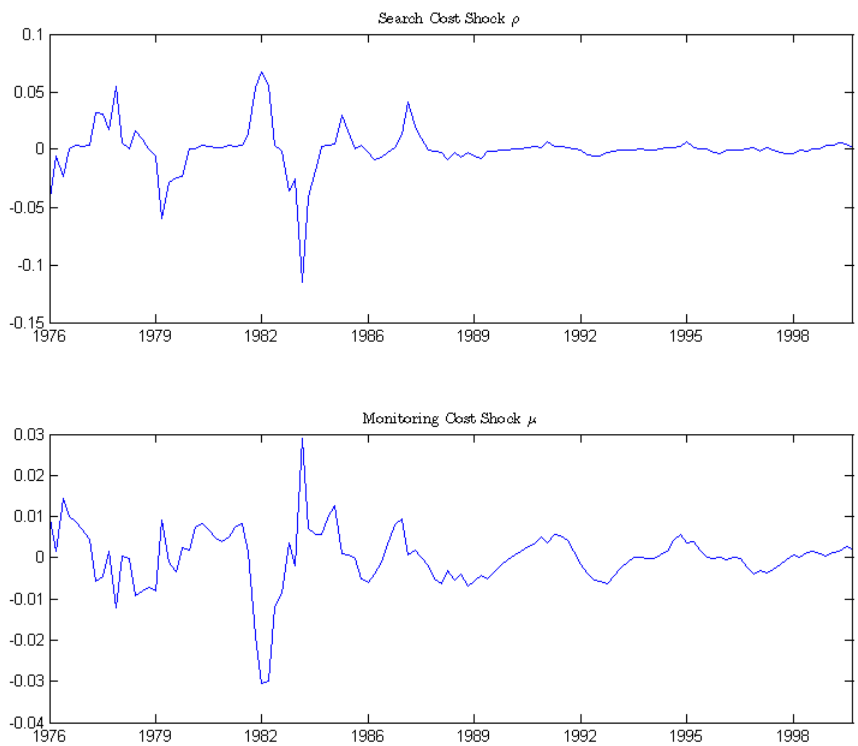


Figure 17: First Order Solution, Second Order Parameter Est, Time Invariant Recovered Shock Series
Green: Data, Blue:Model

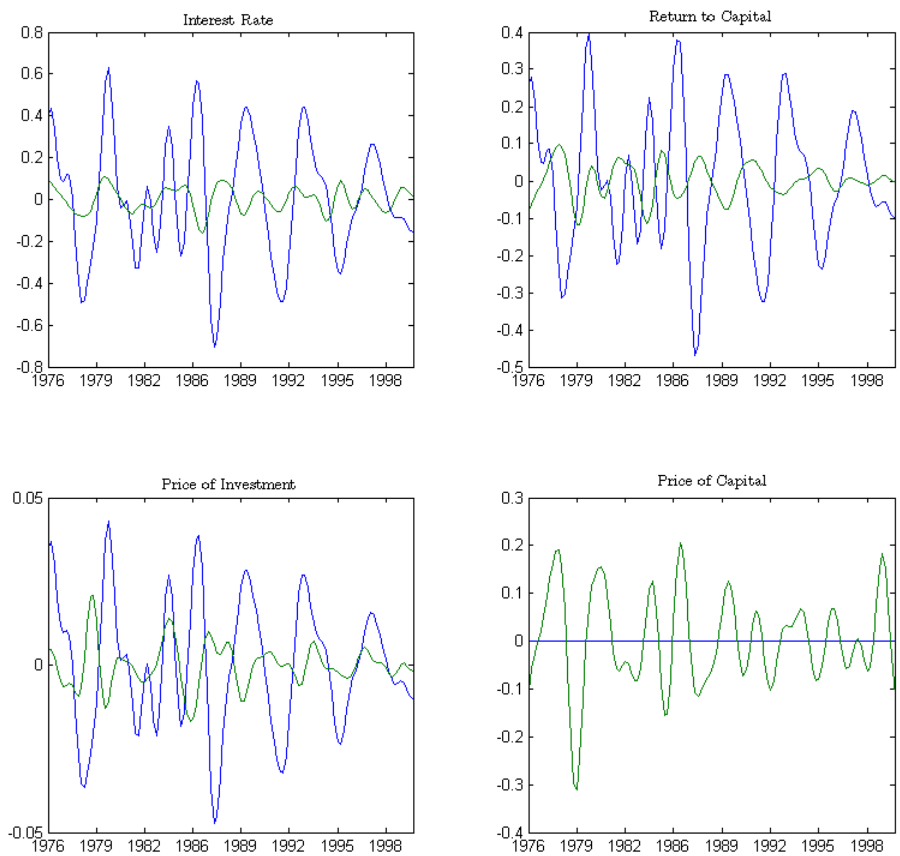


Figure 18: Second Order Solution, Second Order Parameter Est, Time Invariant
Implied Variable Series: Not used in Estimation
Green: Data, Blue:Model

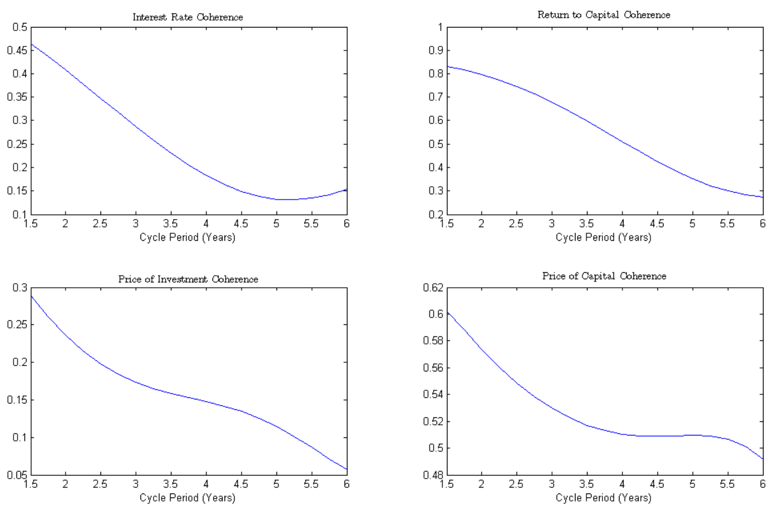


Figure 19: Second Order Solution, Second Order Parameter Est, Time Invariant Implied Variable Series Coherence: Not used in Estimation

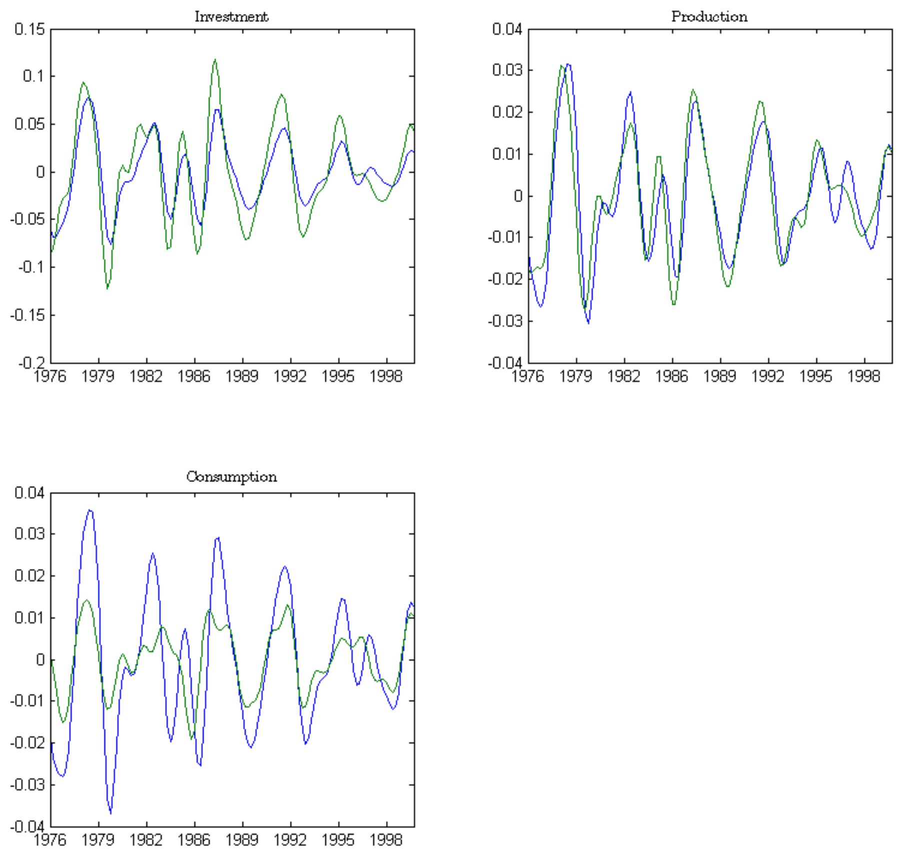


Figure 20: Second Order Solution, Second Order Parameter Est, Time Invariant
Implied Variable Series: Used in Estimation
Green: Data, Blue:Model

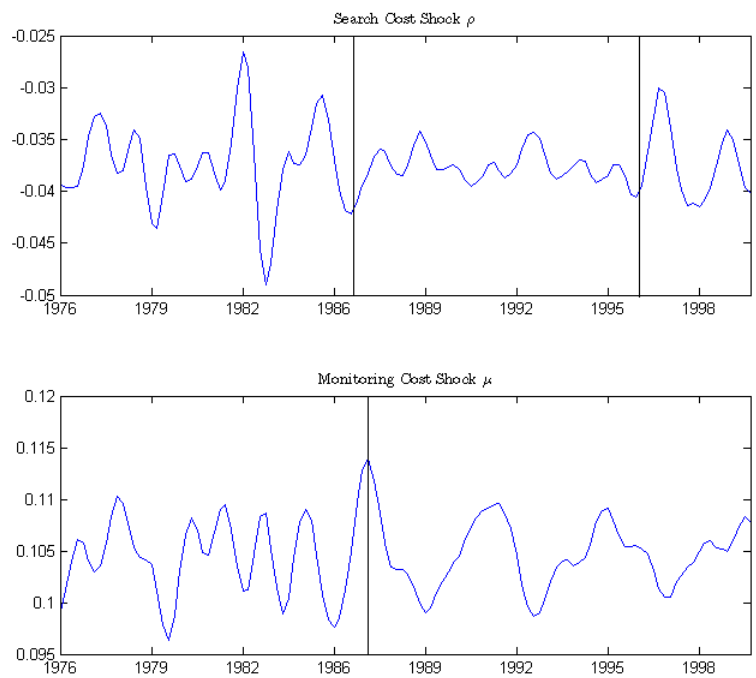


Figure 21: Second Order Solution, Second Order Parameter Est, Time Invariant
Recovered Shock Series
Green: Data, Blue:Model

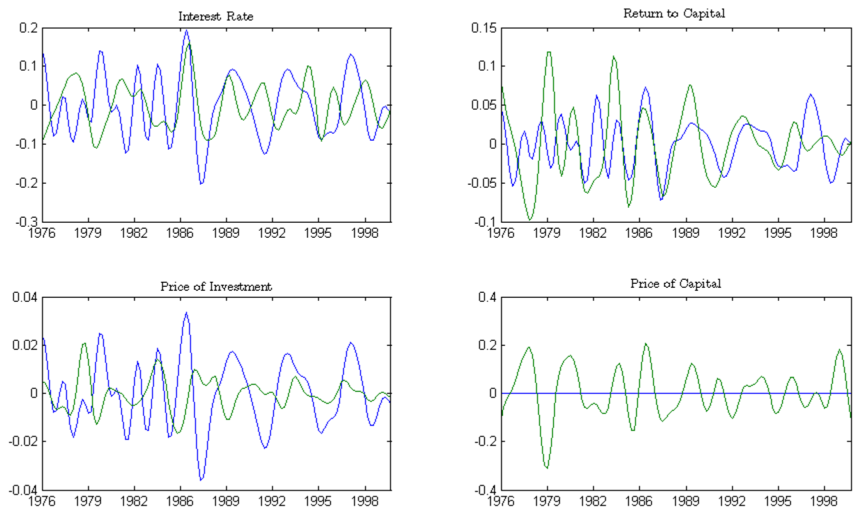


Figure 22: Second Order Solution, Second Order Parameter Est, Time Invariant
No Capital Adjustment Cost
Implied Variable Series: Not used in Estimation
Green: Data, Blue:Model

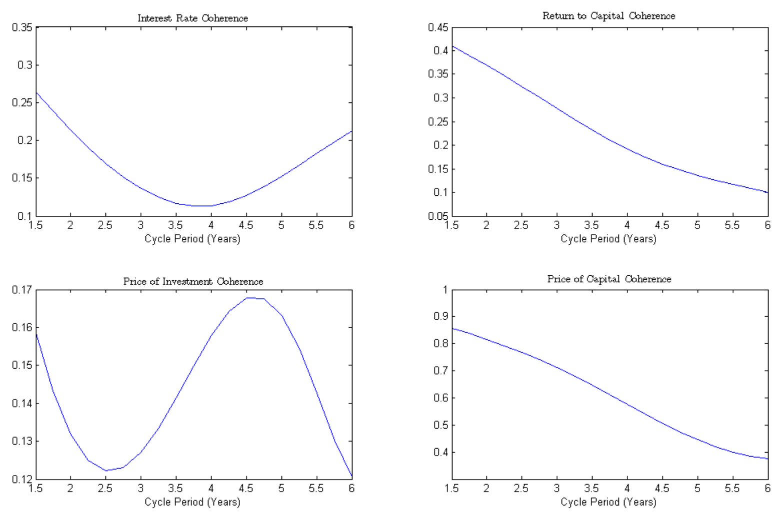


Figure 23: Second Order Solution, Second Order Parameter Est, Time Invariant
No Capital Adjustment Cost
Implied Variable Series Coherence: Not used in Estimation

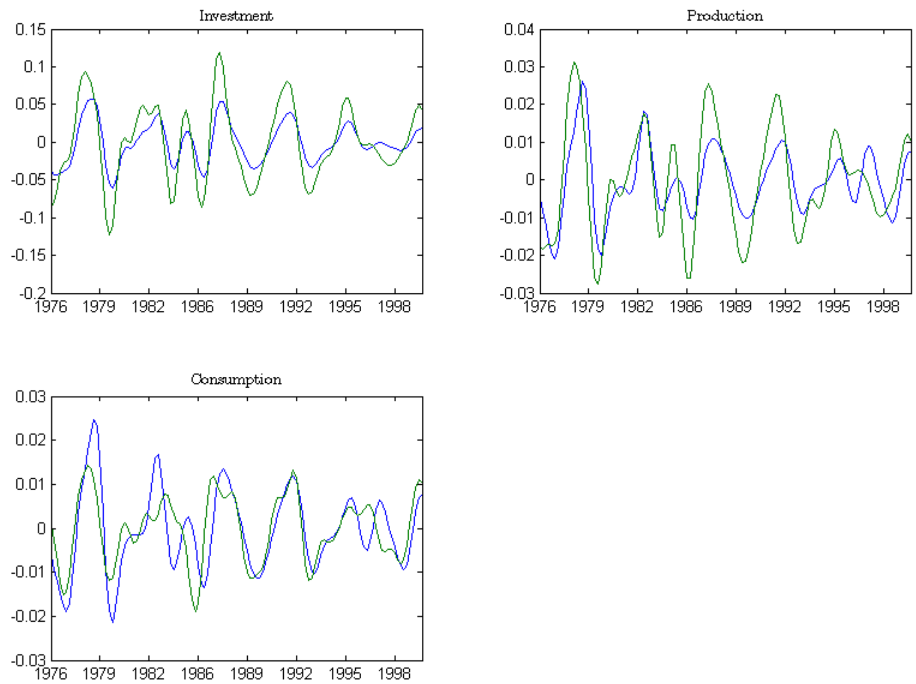


Figure 24: Second Order Solution, Second Order Parameter Est, Time Invariant
No Capital Adjustment Cost
Implied Variable Series: Used in Estimation
Green: Data, Blue:Model

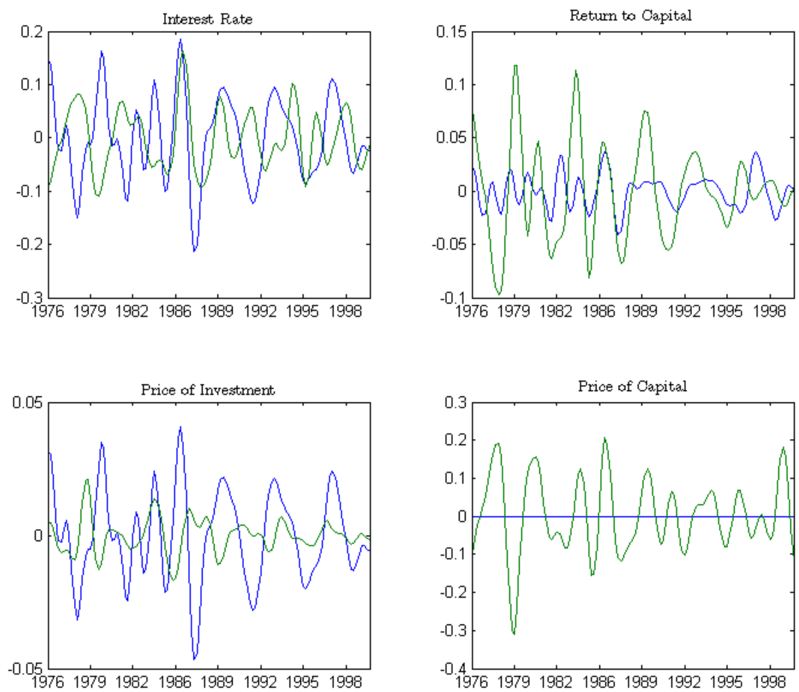


Figure 25: Second Order Solution, Second Order Parameter Est, Time Invariant
Modified Capital Adjustment Cost
Implied Variable Series: Not used in Estimation
Green: Data, Blue:Model

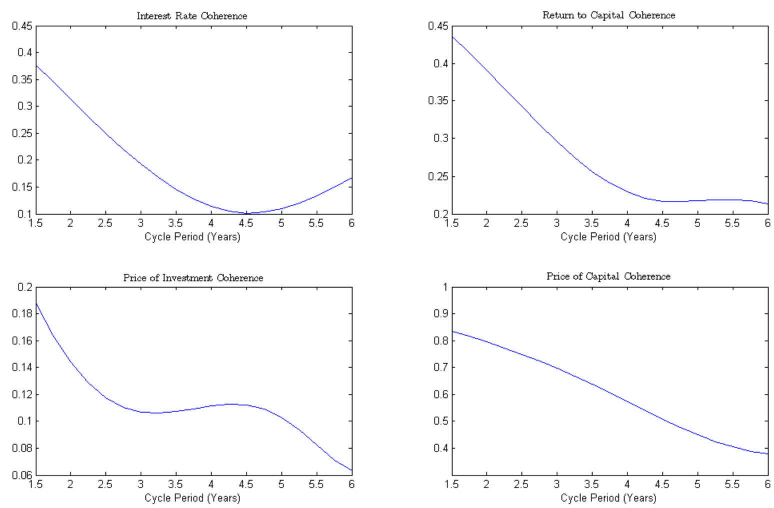


Figure 26: Second Order Solution, Second Order Parameter Est, Time Invariant Modified Capital Adjustment Cost
Implied Variable Series Coherence: Not used in Estimation

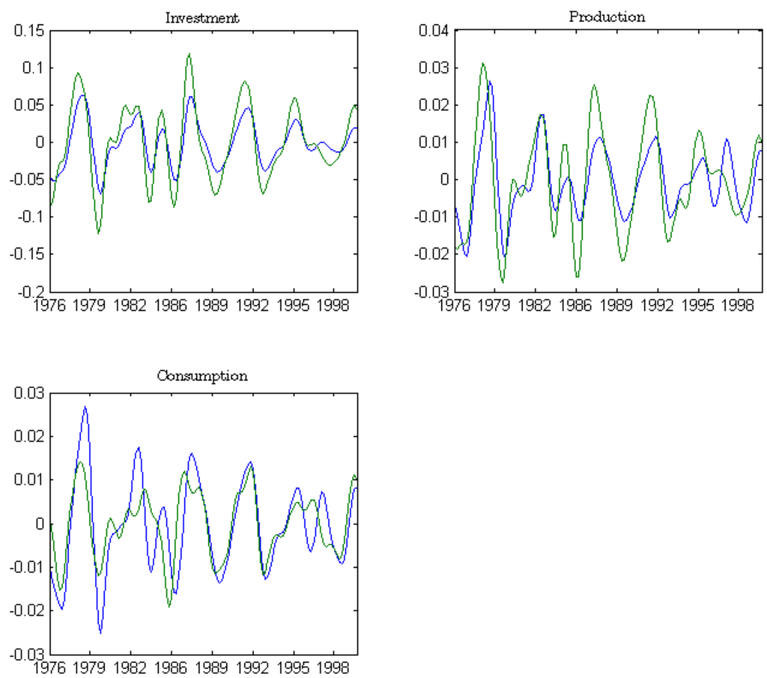


Figure 27: Second Order Solution, Second Order Parameter Est, Time Invariant
Modified Capital Adjustment Cost
Implied Variable Series: Used in Estimation
Green: Data, Blue:Model

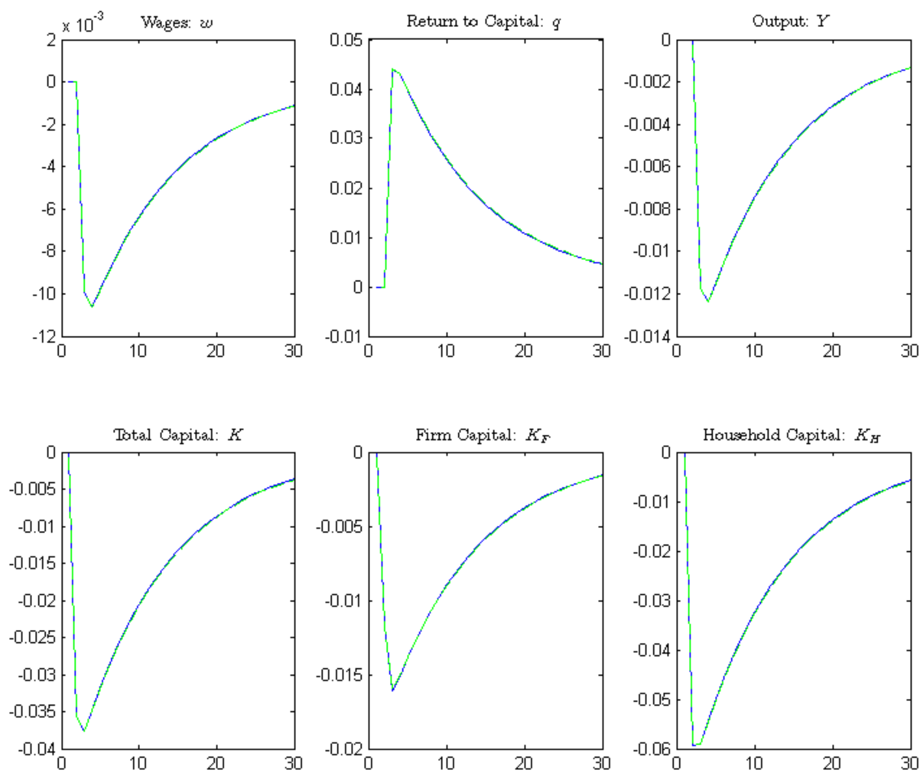


Figure 28: Second Order Time Invariant Param Est 1st and 2nd order IRF
Technology Shock: z
Green: 1st Order, Blue: 2nd order

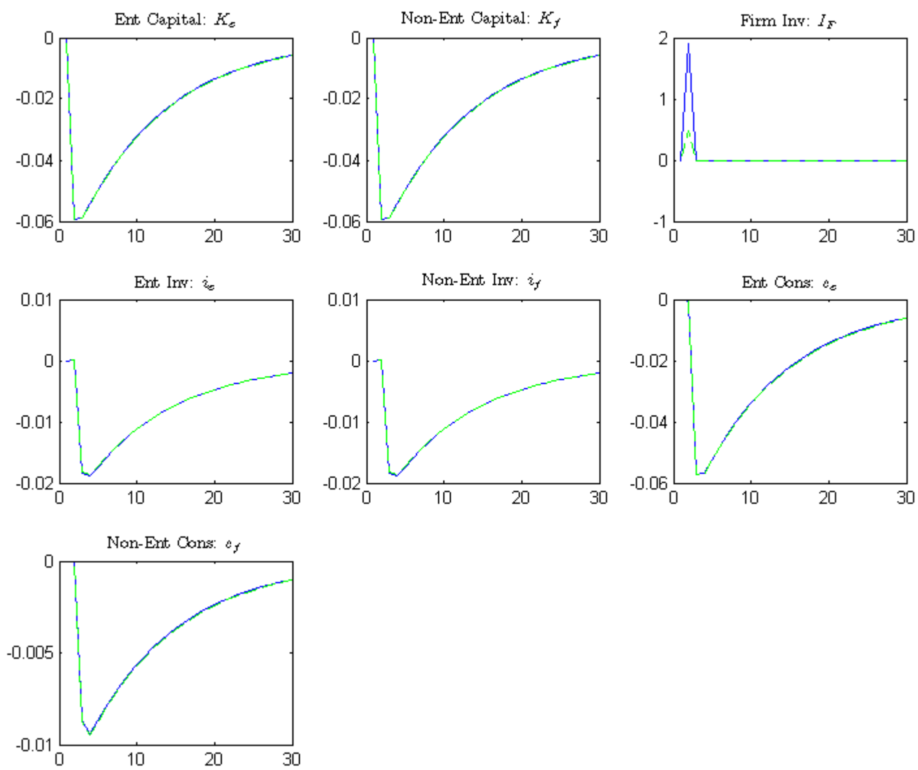


Figure 29: Second Order Time Invariant Param Est 1st and 2nd order IRF
Technology Shock: z
Green: 1st Order, Blue: 2nd order

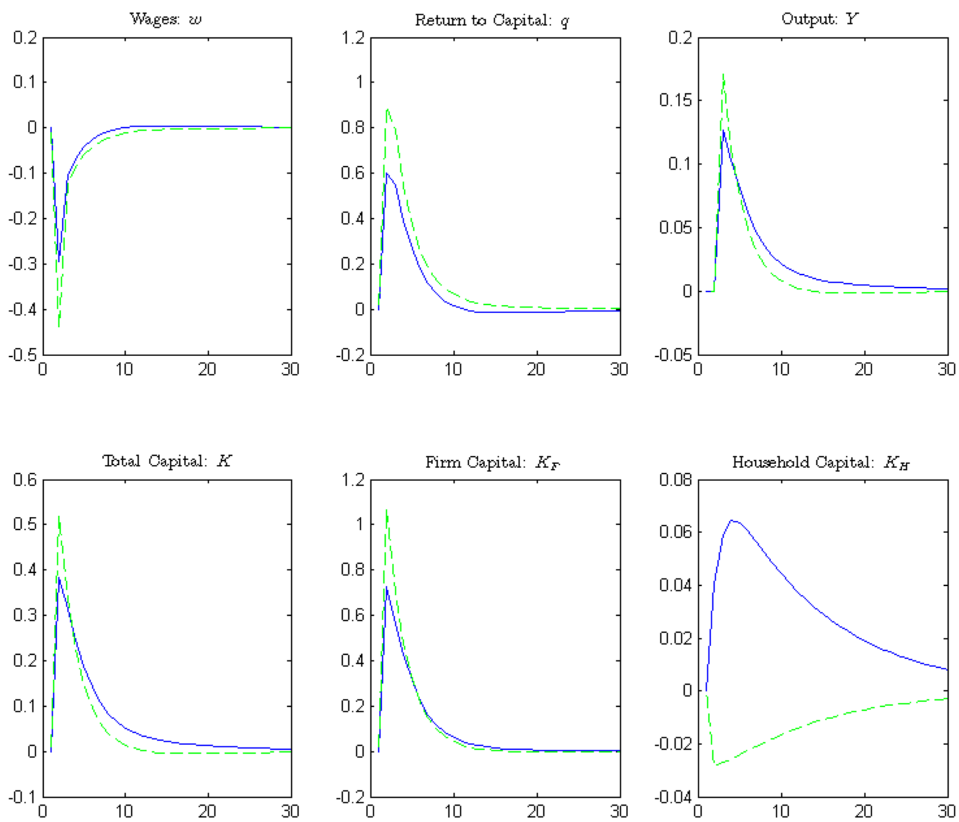


Figure 30: Second Order Time Invariant Param Est 1st and 2nd order IRF
Search Cost Shock: ρ
Green: 1st Order, Blue: 2nd order

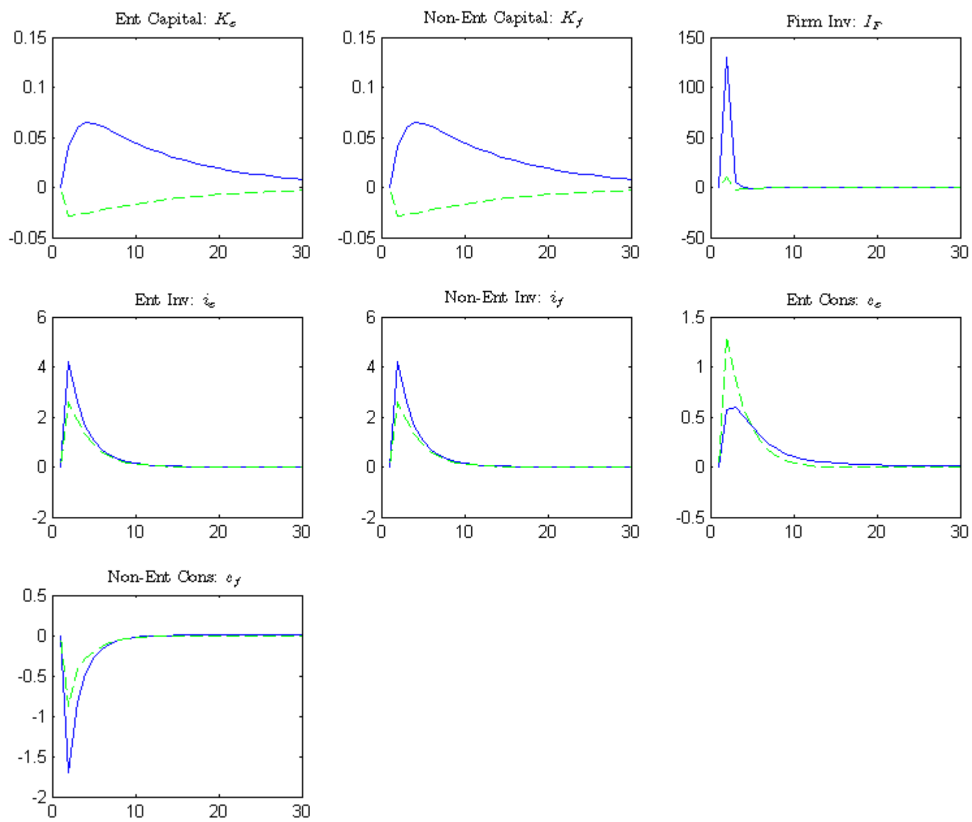


Figure 31: Second Order Time Invariant Param Est 1st and 2nd order IRF
Search Cost Shock: ρ
Green: 1st Order, Blue: 2nd order

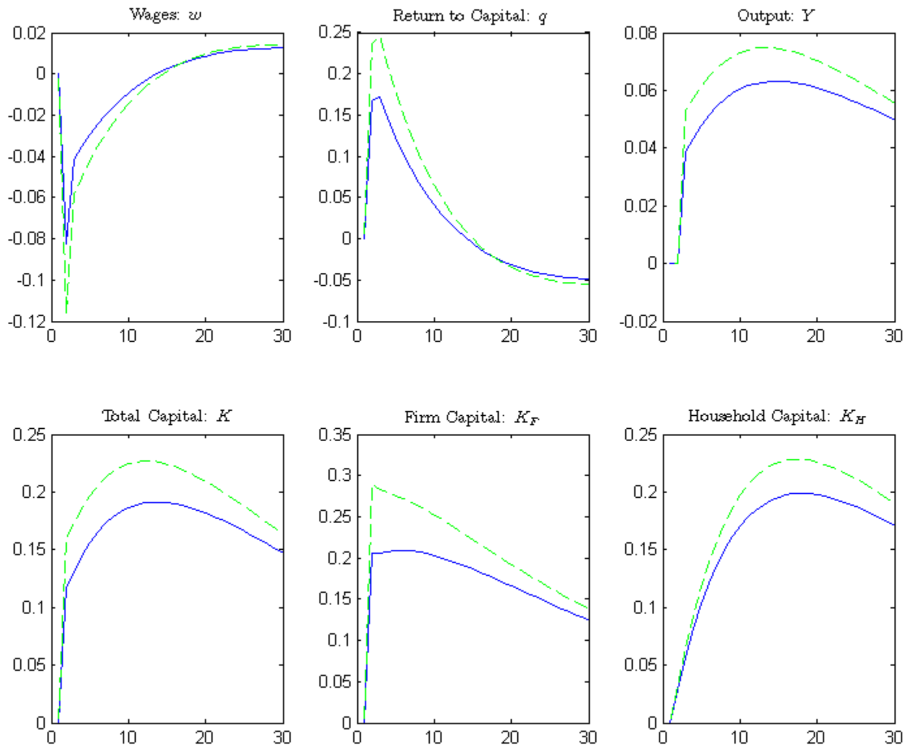


Figure 32: Second Order Time Invariant Param Est 1st and 2nd order IRF
Monitoring Cost Shock: μ
Green: 1st Order, Blue: 2nd order

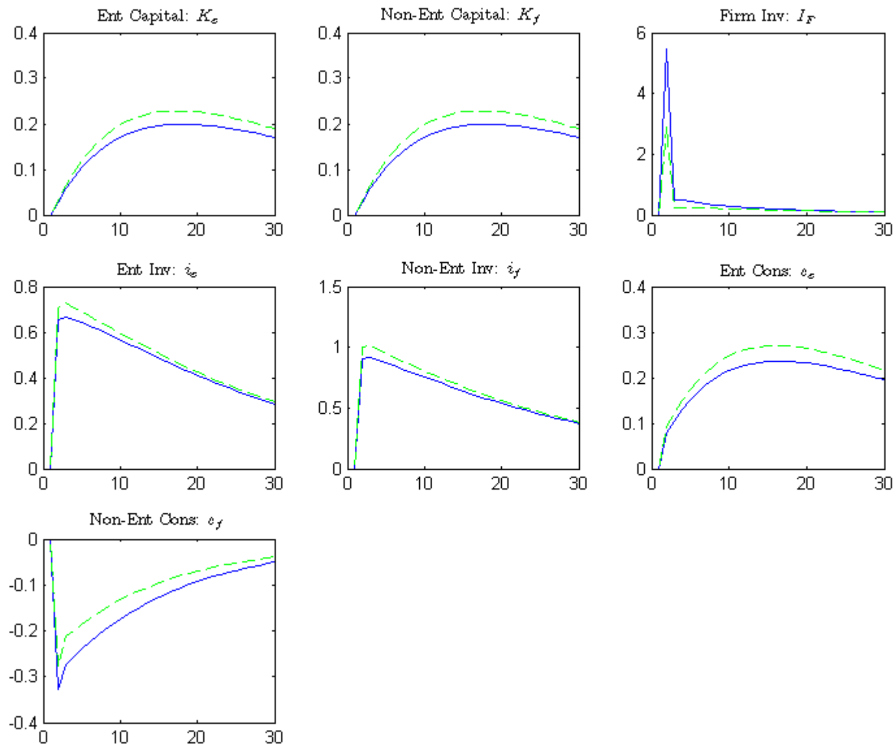


Figure 33: Second Order Time Invariant Param Est 1st and 2nd order IRF
Monitoring Cost Shock: μ
Green: 1st Order, Blue: 2nd order

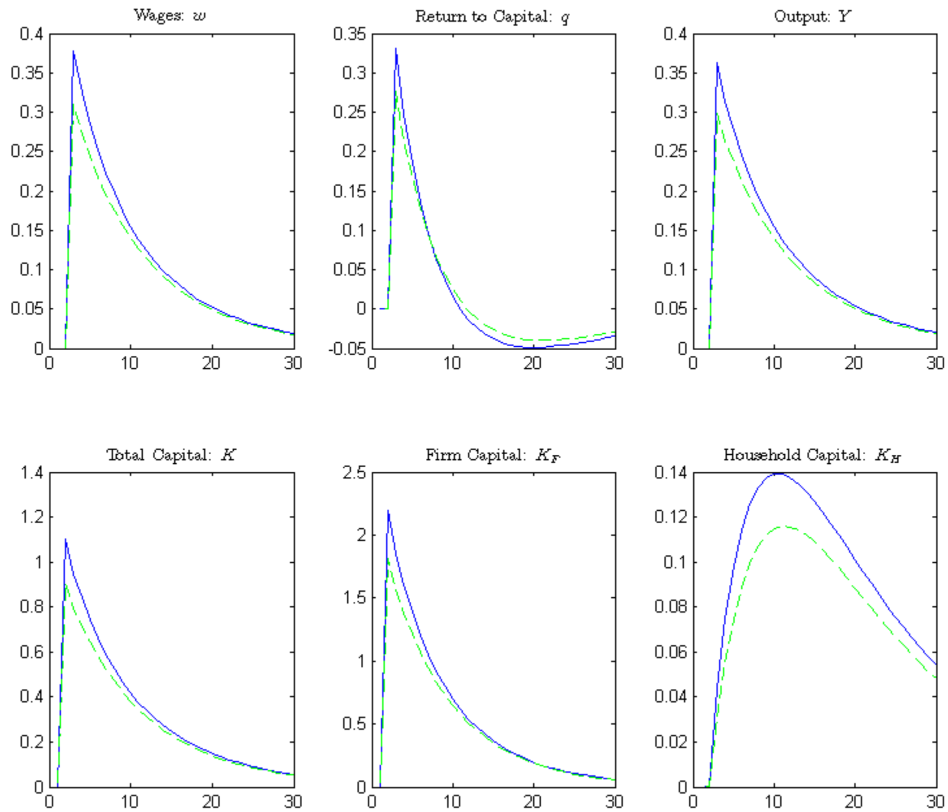


Figure 34: Second Order Time Invariant Param Est 1st and 2nd order IRF
Capital Adjustment Cost Shock: ζ
Green: 1st Order, Blue: 2nd order

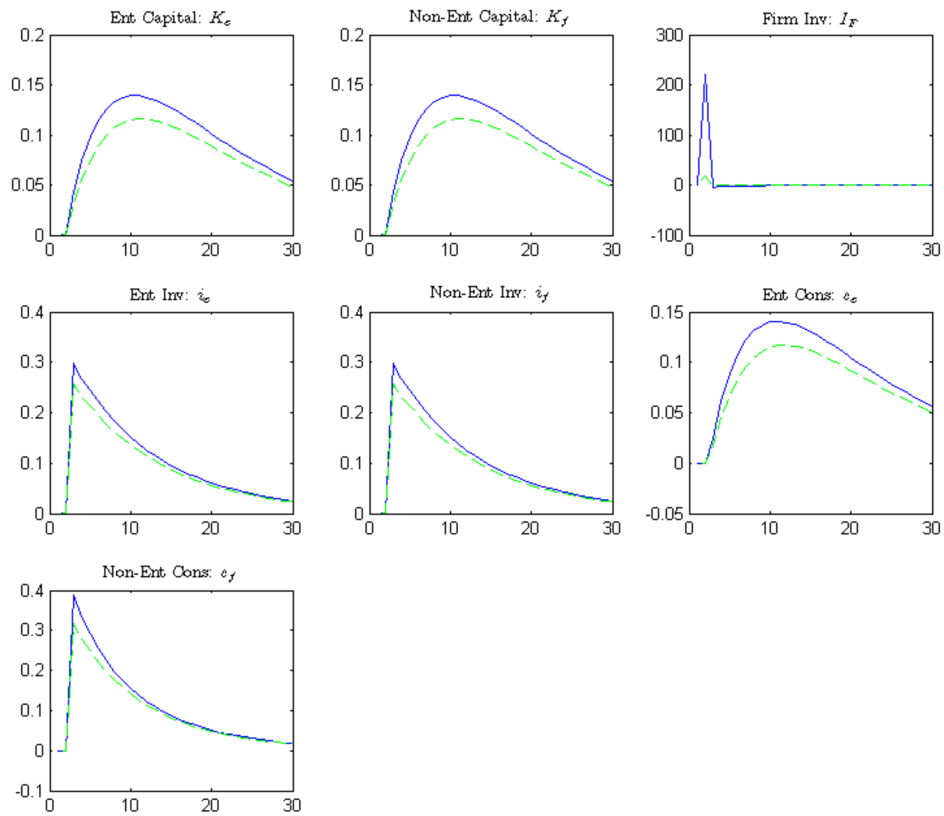


Figure 35: Second Order Time Invariant Param Est 1st and 2nd order IRF
Capital Adjustment Cost Shock: ζ
Green: 1st Order, Blue: 2nd order

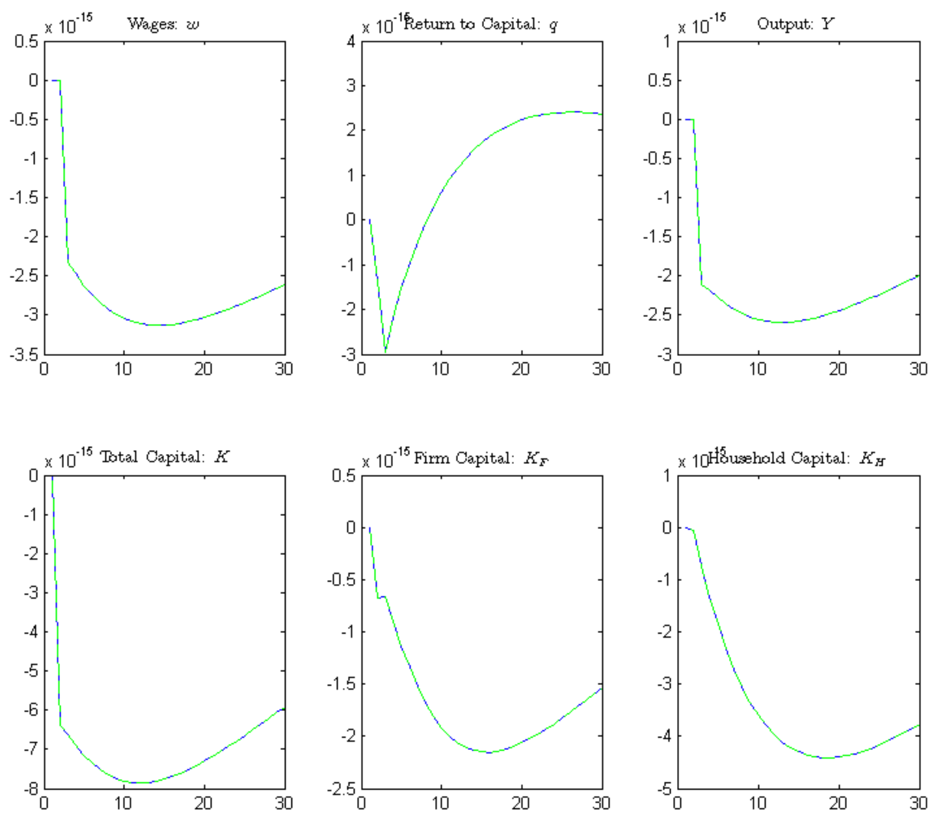


Figure 36: Second Order Time Invariant Param Est 1st and 2nd order IRF
Labor Disutility Cost Shock: ψ
Green: 1st Order, Blue: 2nd order

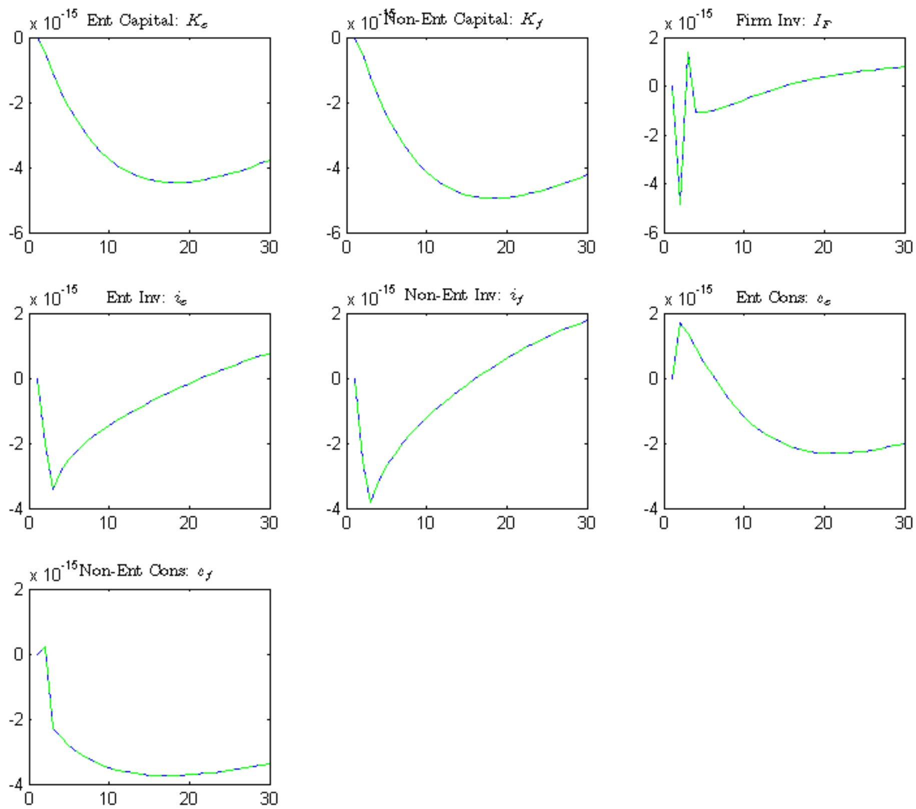


Figure 37: Second Order Time Invariant Param Est 1st and 2nd order IRF
Labor Disutility Cost Shock: ψ
Green: 1st Order, Blue: 2nd order

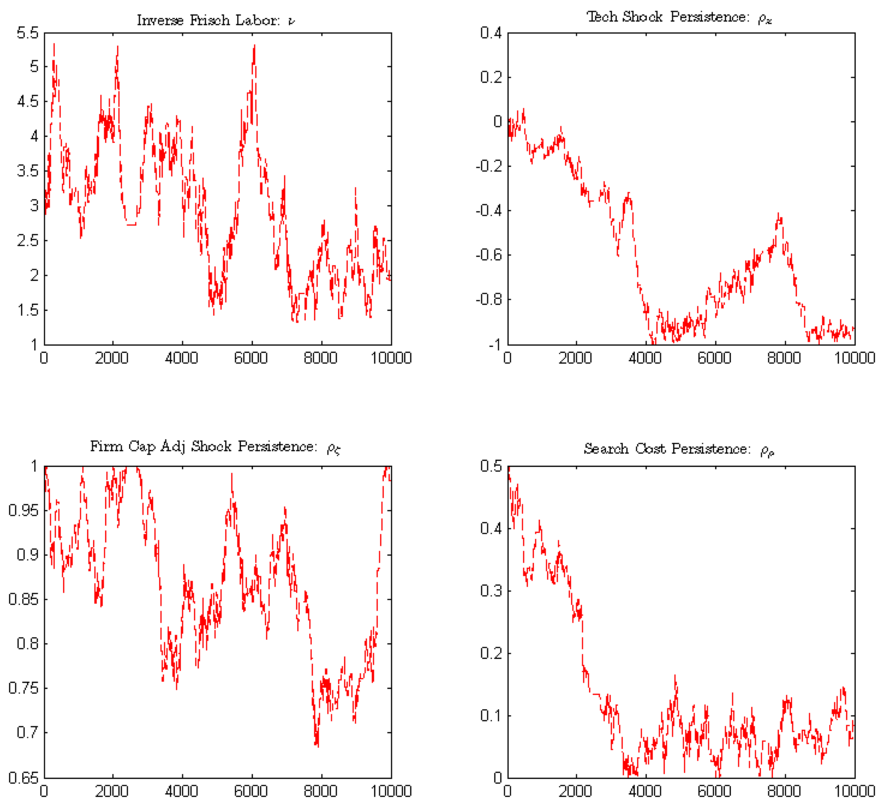


Figure 38: Second Order Time Invariant Unbounded Test
MH Sample Chain

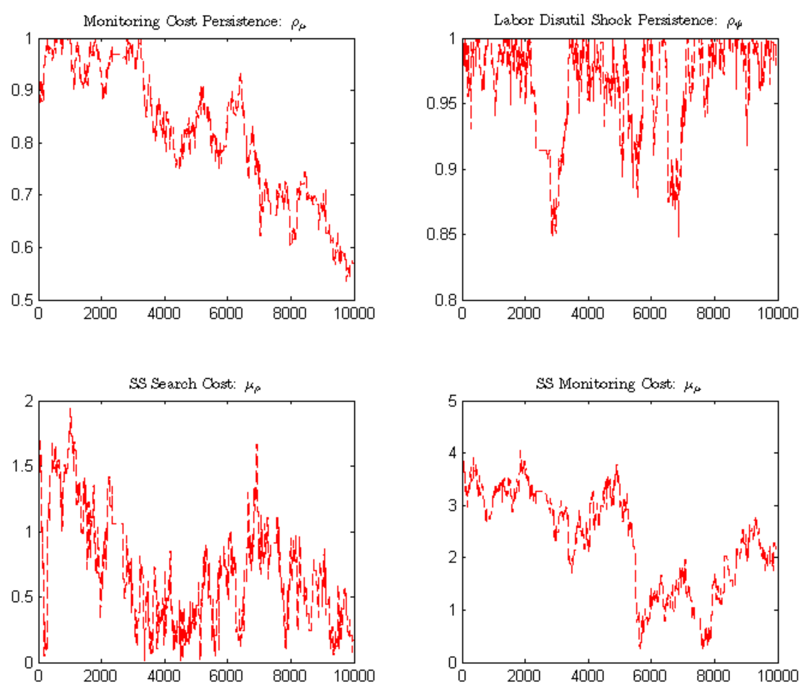


Figure 39: Second Order Time Invariant Unbounded Test MH Sample Chain

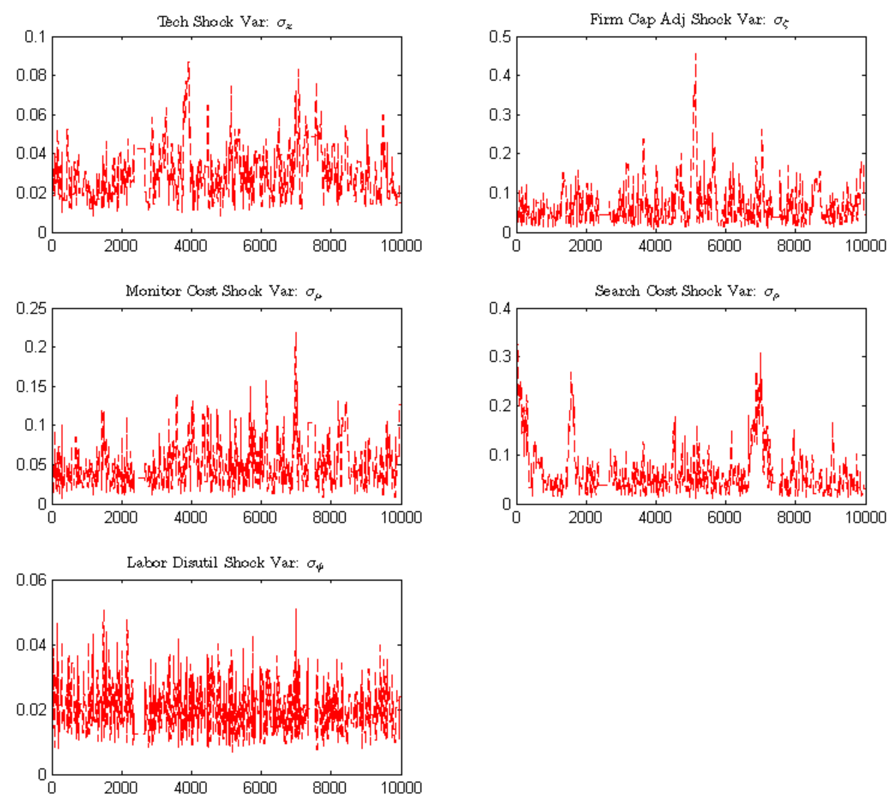


Figure 40: Second Order Time Invariant Unbounded Test
MH Sample Chain

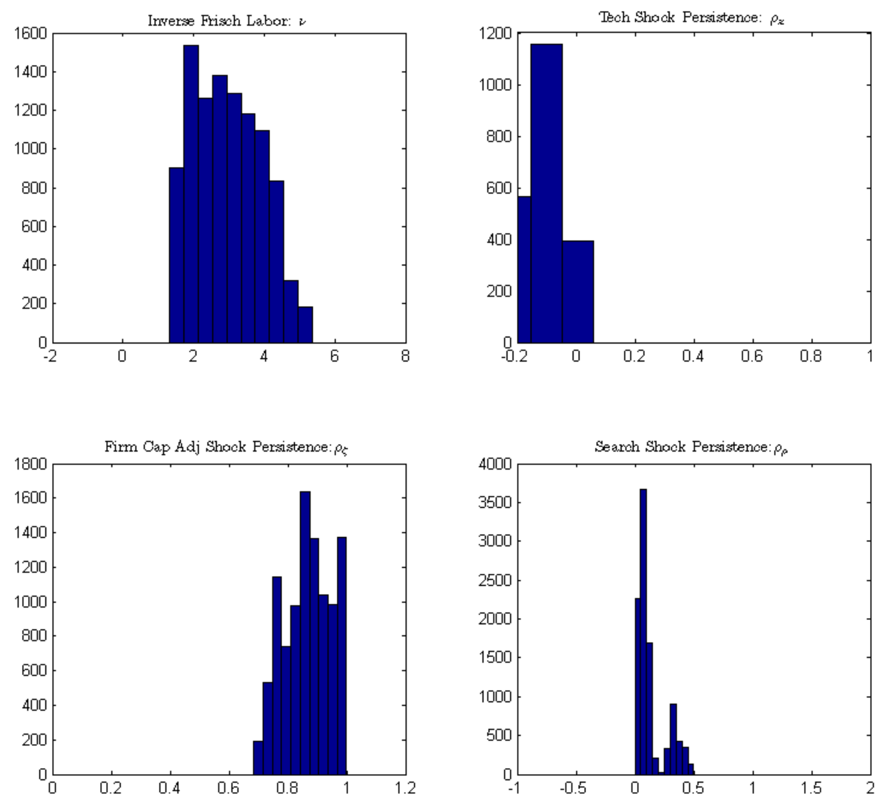


Figure 41: Second Order Time Invariant Unbounded Test Posterior Distributions

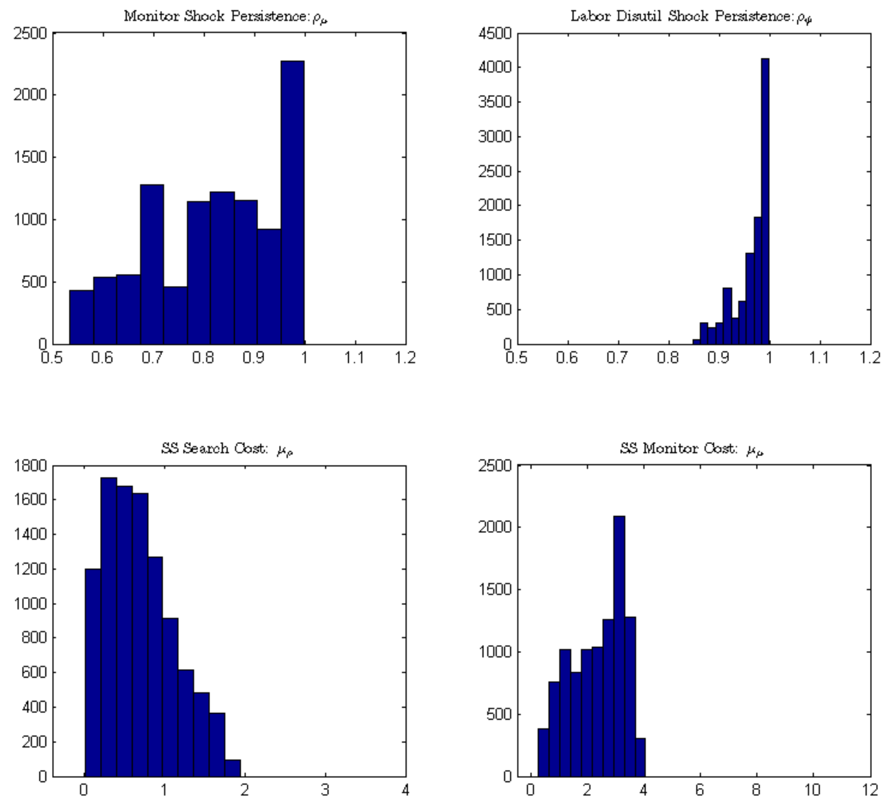


Figure 42: Second Order Time Invariant Unbounded Test Posterior Distributions

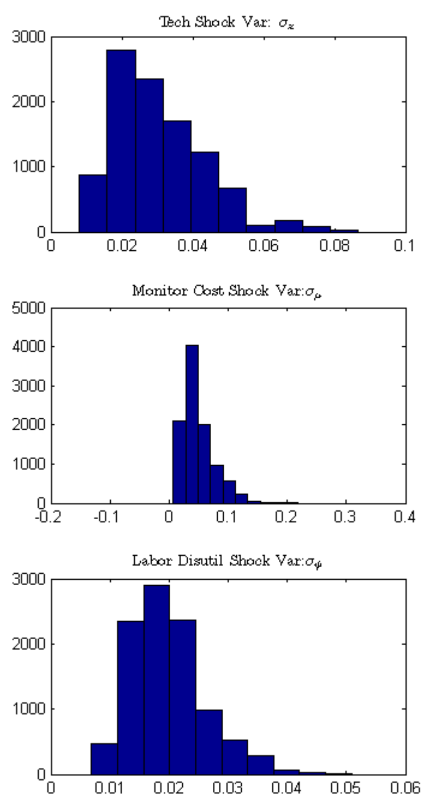


Figure 43: Second Order Time Invariant Unbounded Test Posterior Distributions

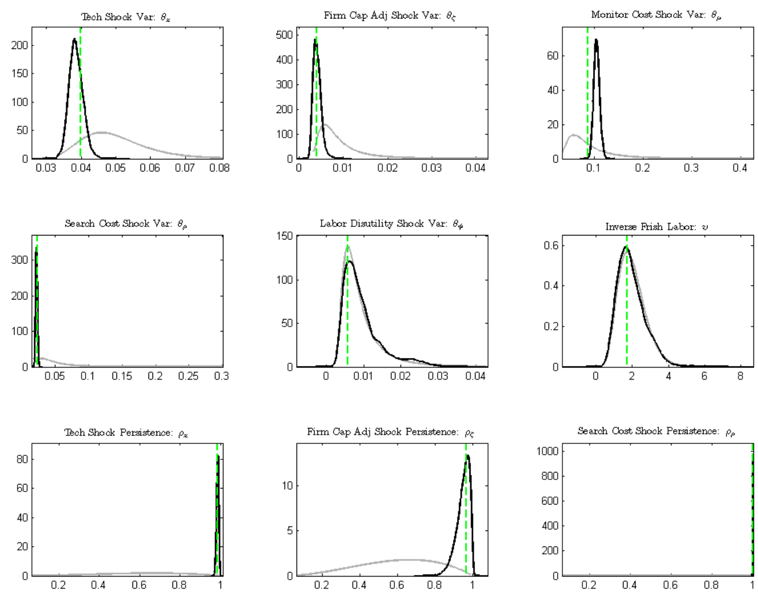


Figure 44: First Order Time Invariant Posterior Distributions

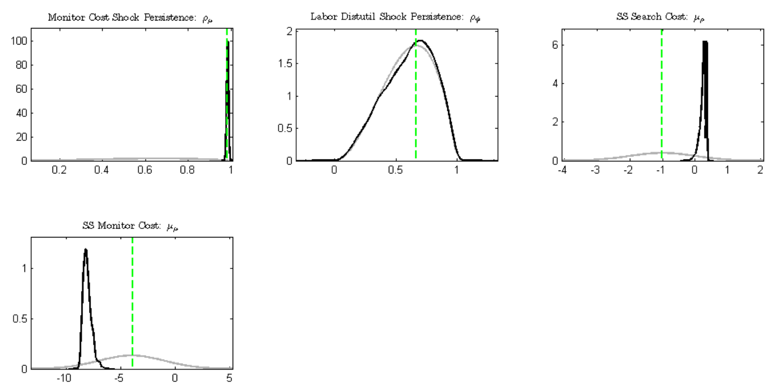


Figure 45: First Order Time Invariant Posterior Distributions

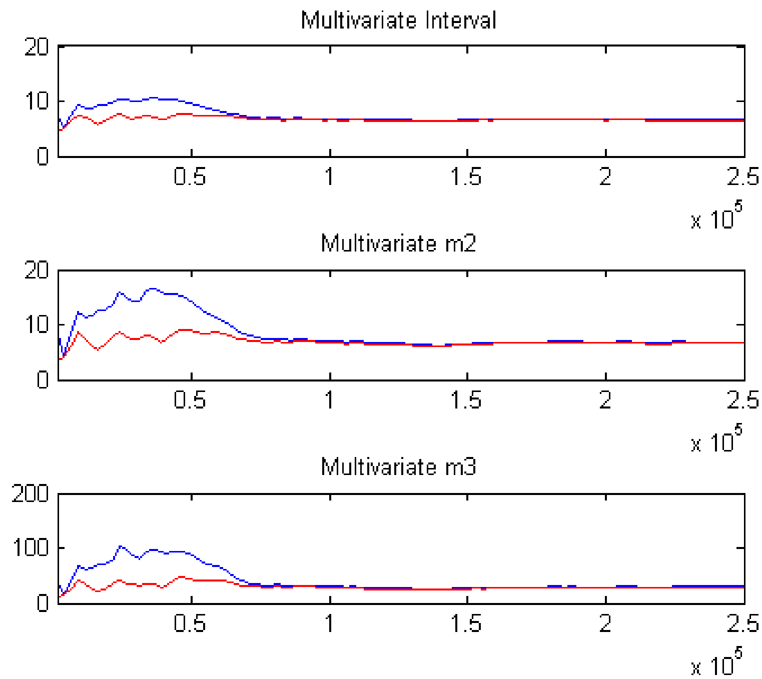


Figure 46: First Order Time Invariant Multivariate Convergence Diagnostics

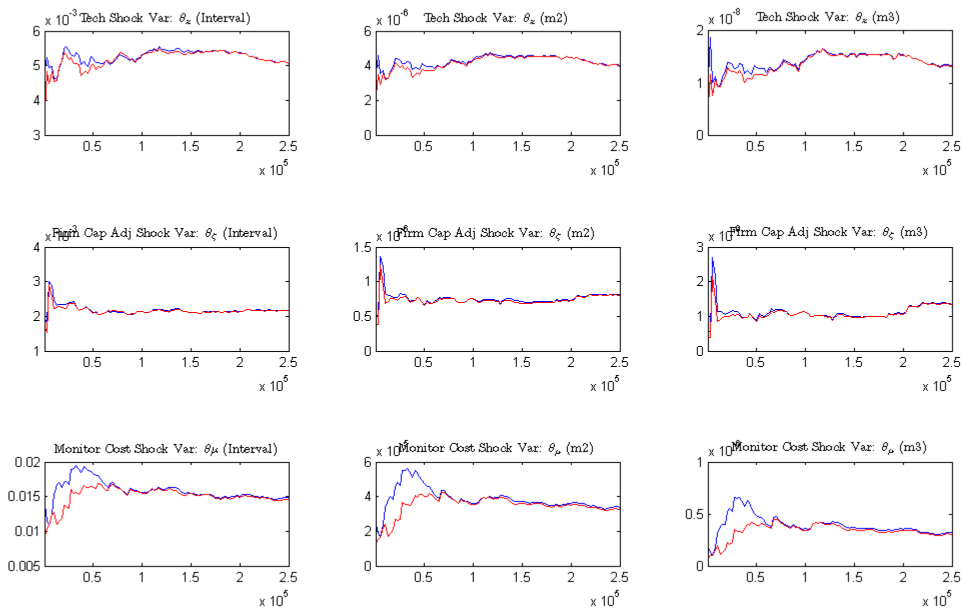


Figure 47: First Order Time Invariant Univariate Convergence Diagnostics

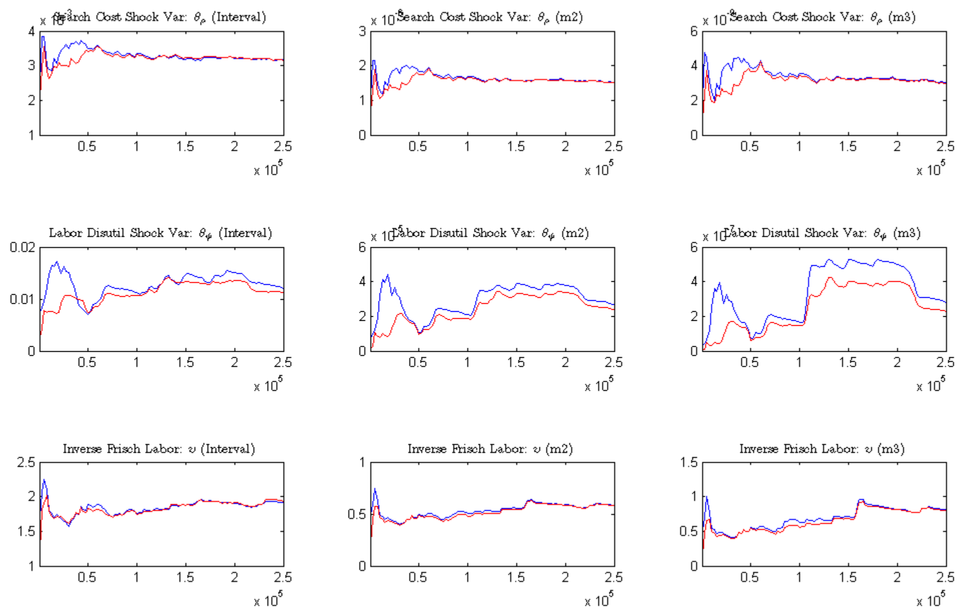


Figure 48: First Order Time Invariant Univariate Convergence Diagnostics

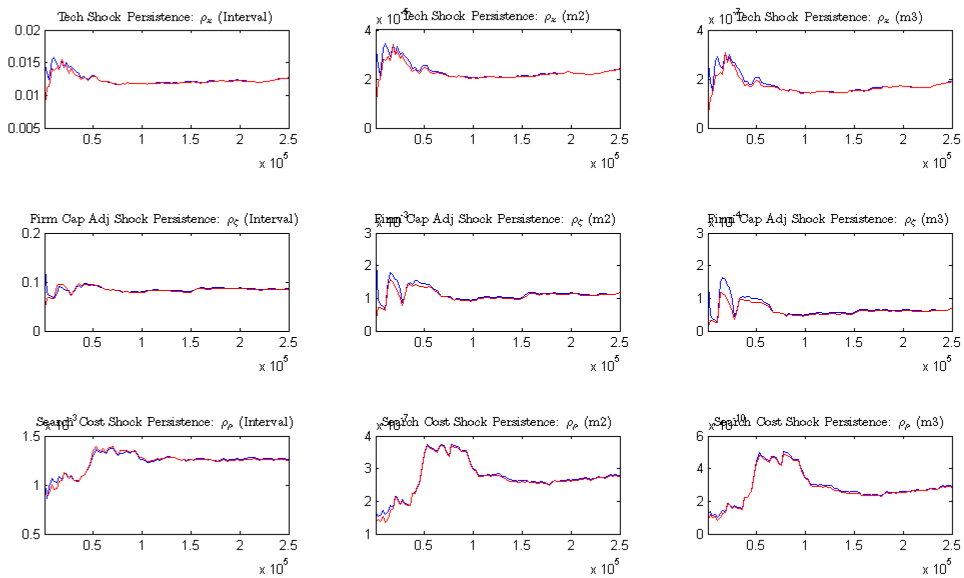


Figure 49: First Order Time Invariant Univariate Convergence Diagnostics

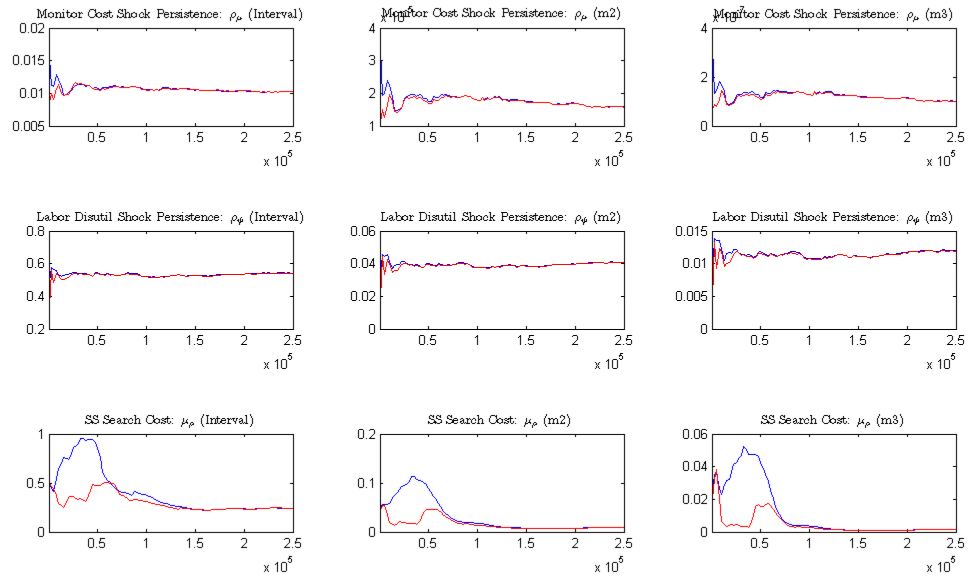


Figure 50: First Order Time Invariant Univariate Convergence Diagnostics

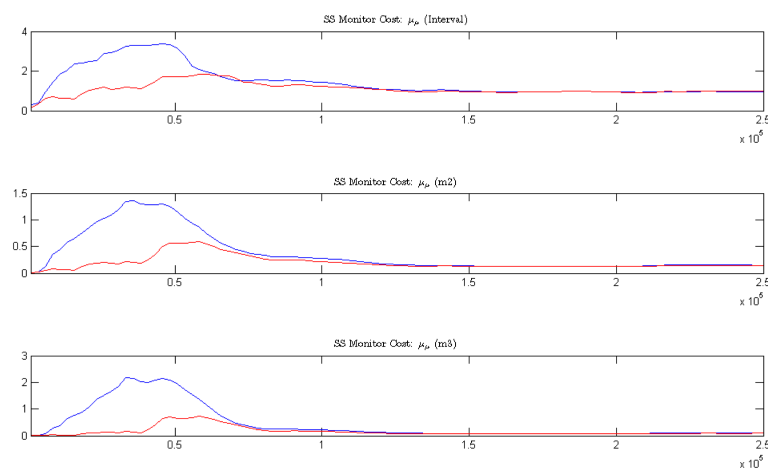


Figure 51: First Order Time Invariant Univariate Convergence Diagnostics

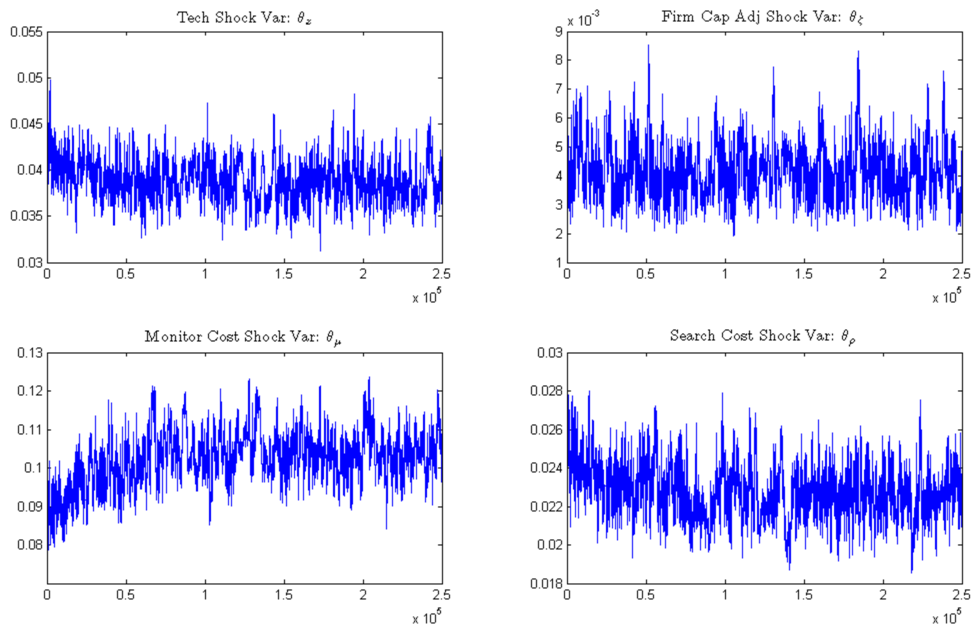
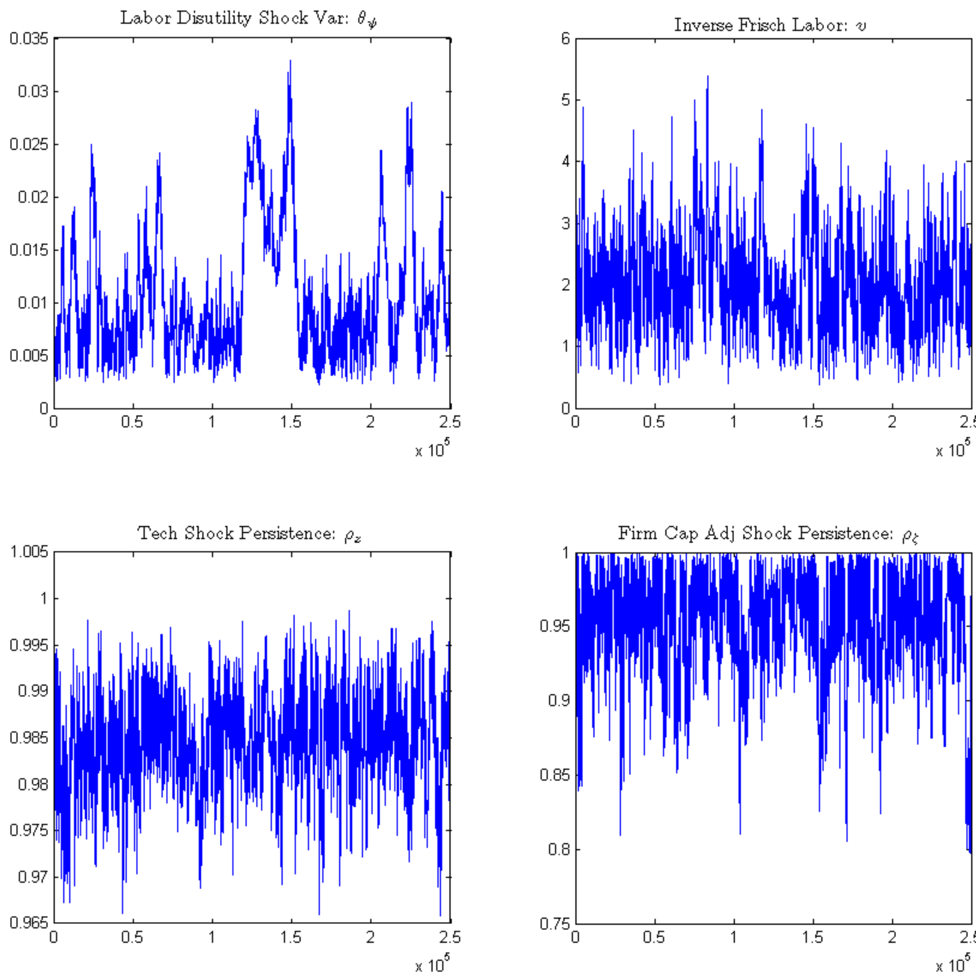


Figure 52: First Order Time Invariant MH Sample Chain



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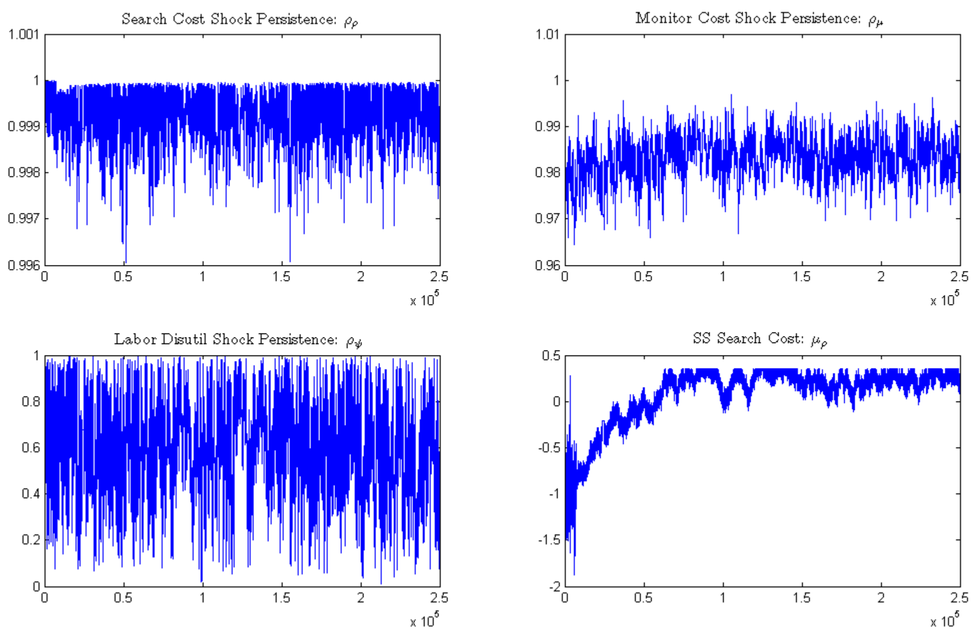

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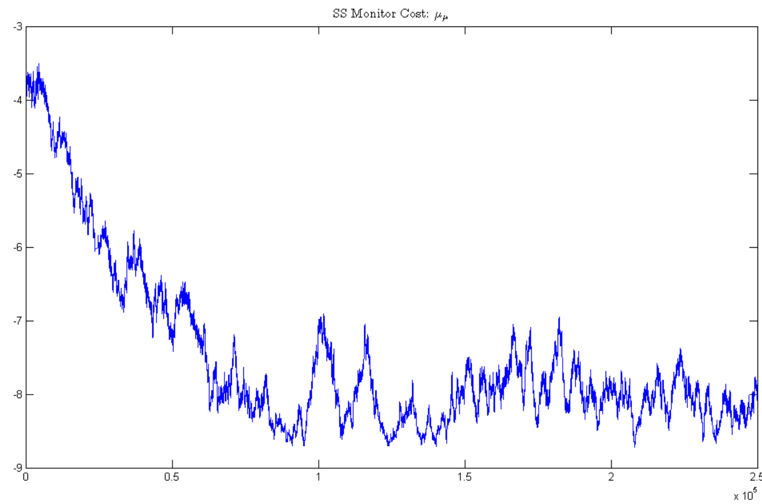

Figure 55: First Order Time Invariant MH Sample Chain

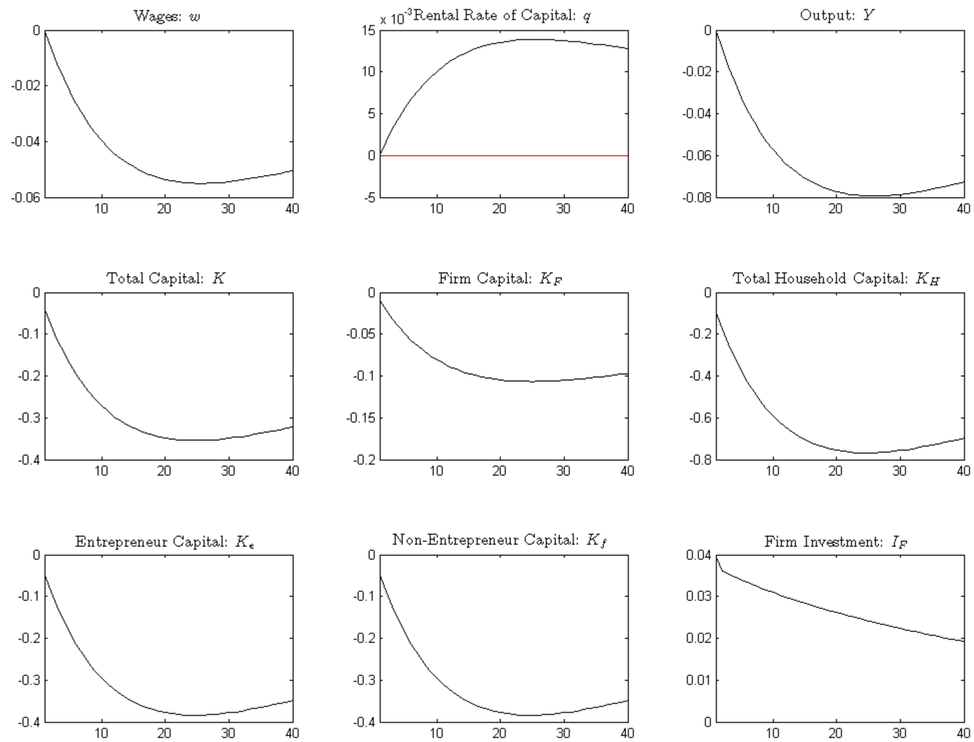

Figure 56: First Order Time Invariant Impulse Response Functions
Technology Shock: z

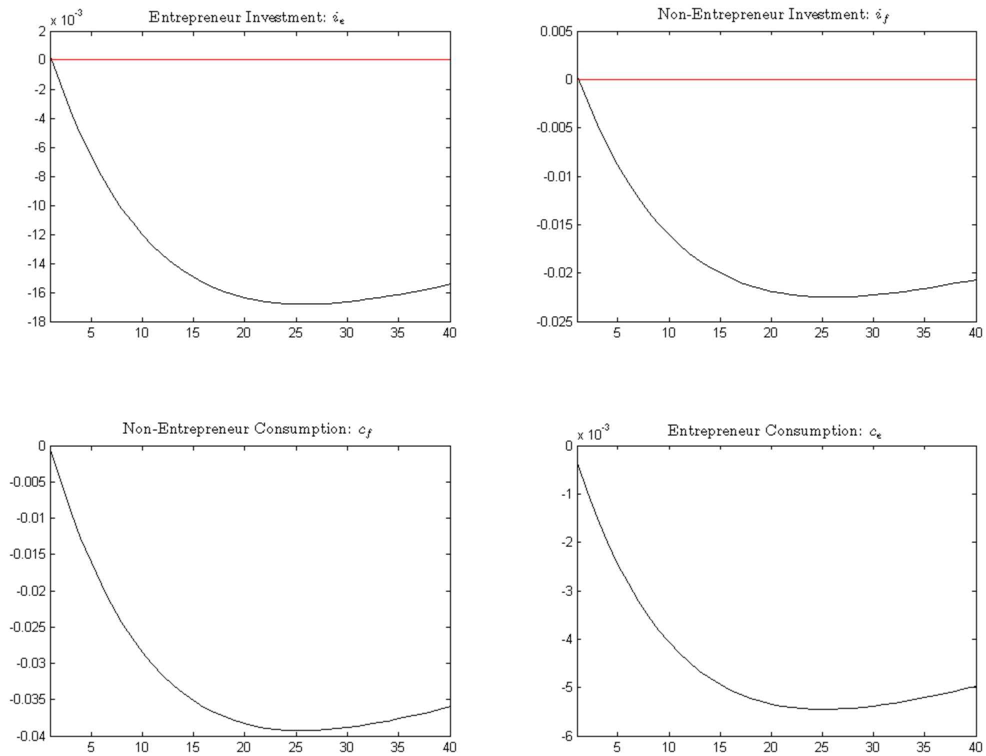

Figure 57: First Order Time Invariant Impulse Response Functions
Technology Shock: z

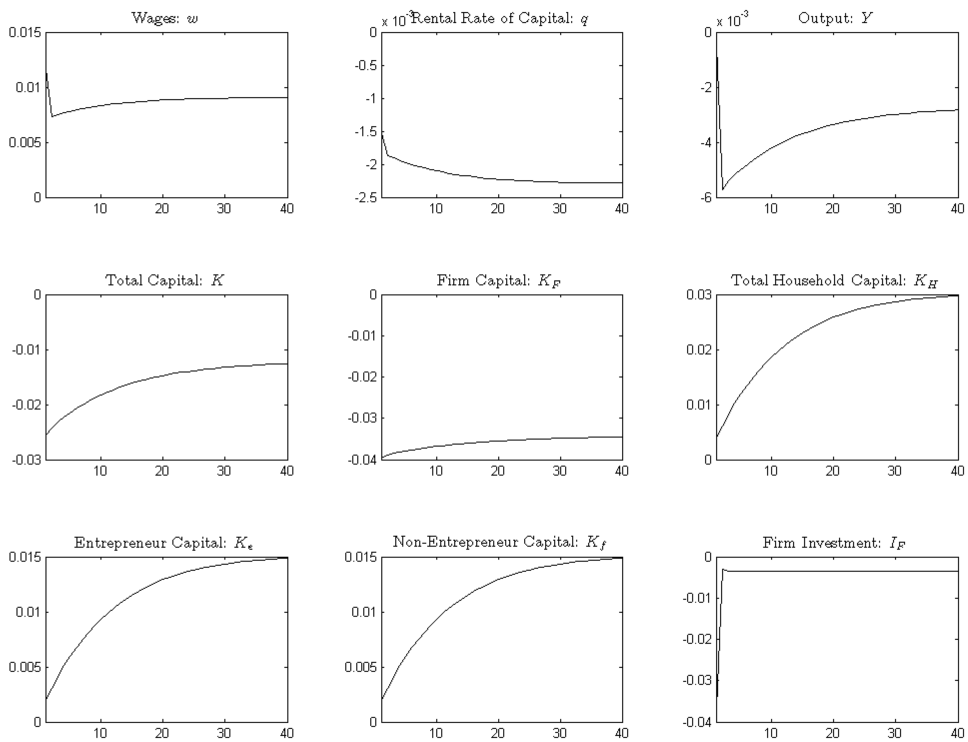

Figure 58: First Order Time Invariant Impulse Response Functions
Search Cost Shock: ρ

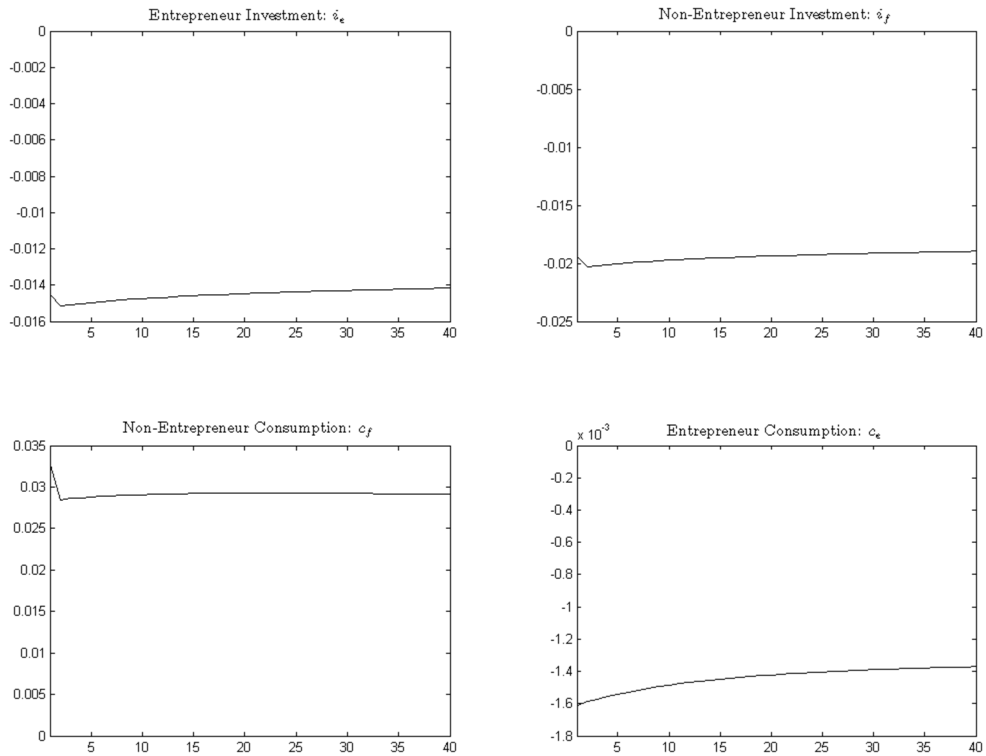

Figure 59: First Order Time Invariant Impulse Response Functions
Search Cost Shock: ρ

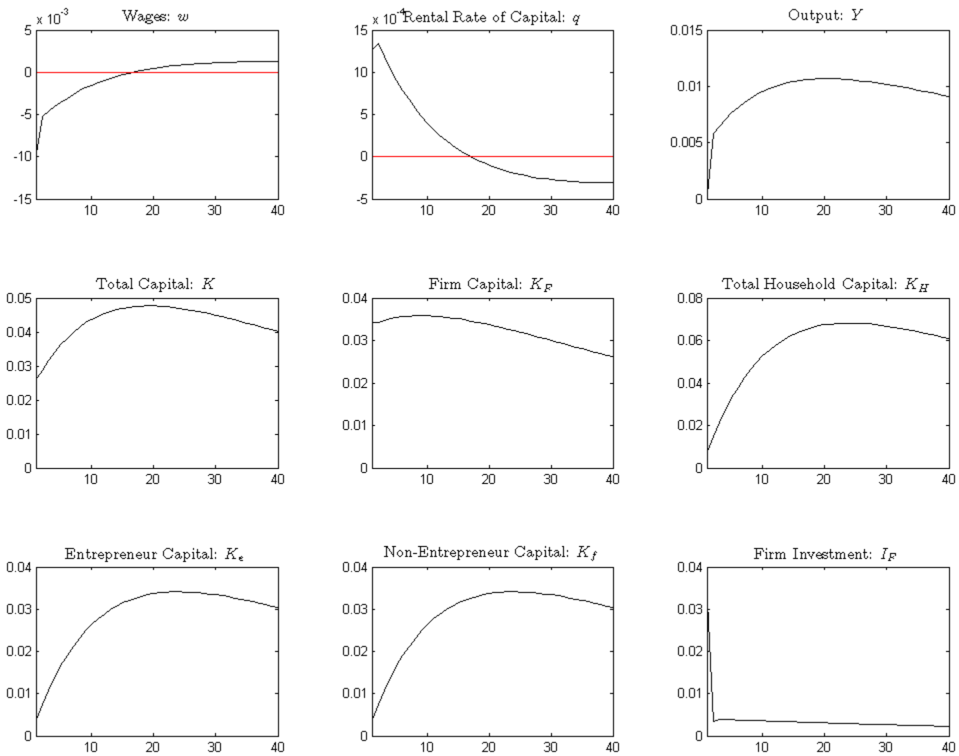

Figure 60: First Order Time Invariant Impulse Response Functions
Monitoring Cost Shock: μ

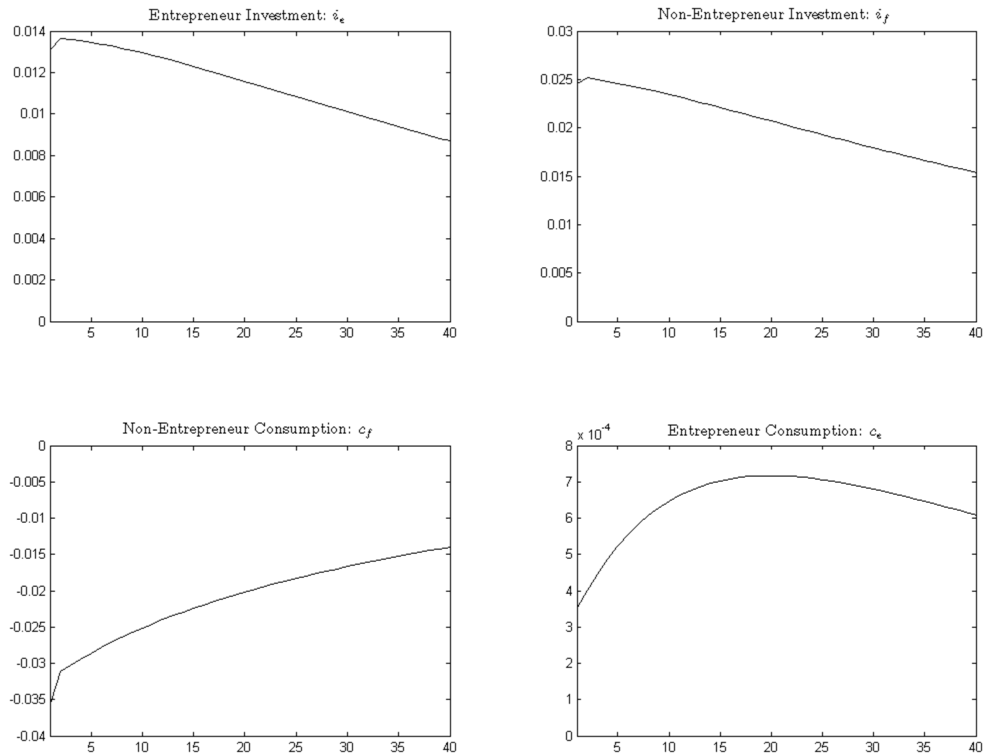

Figure 61: First Order Time Invariant Impulse Response Functions
Monitoring Cost Shock: μ

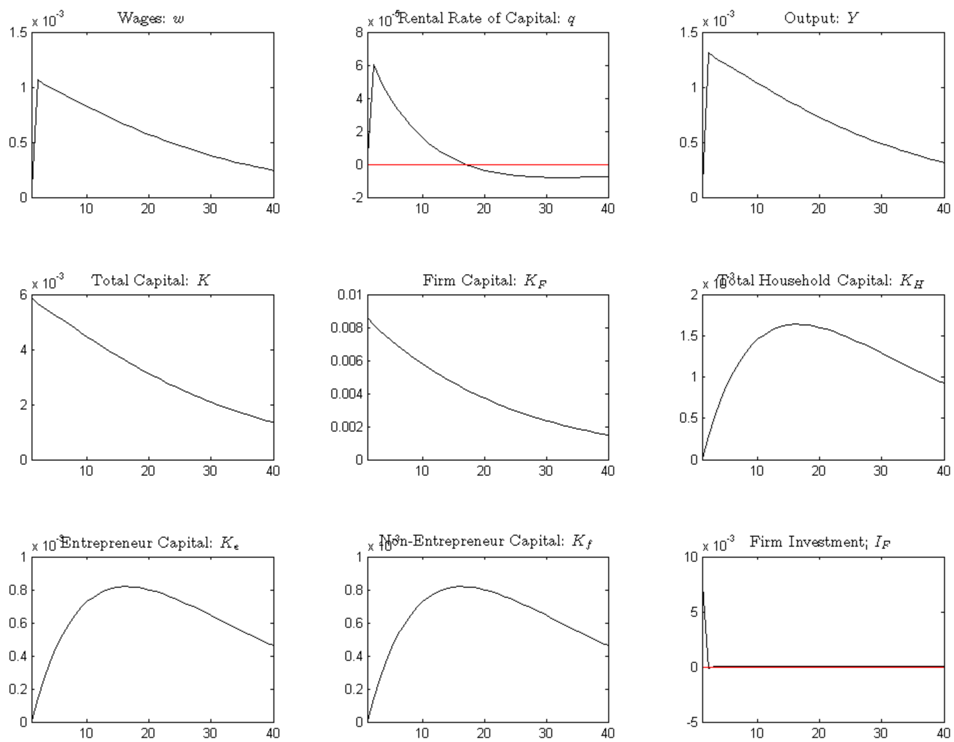

Figure 62: First Order Time Invariant Impulse Response Functions
Capital Adjustment Cost Shock: ζ

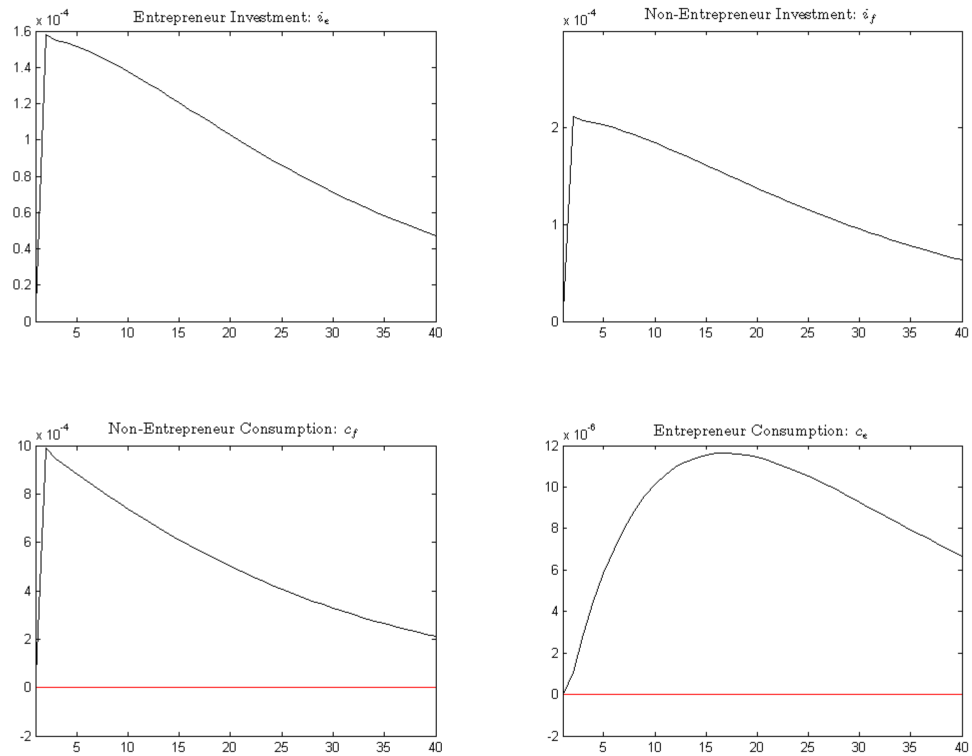

Figure 63: First Order Time Invariant Impulse Response Functions
Capital Adjustment Cost Shock: ζ

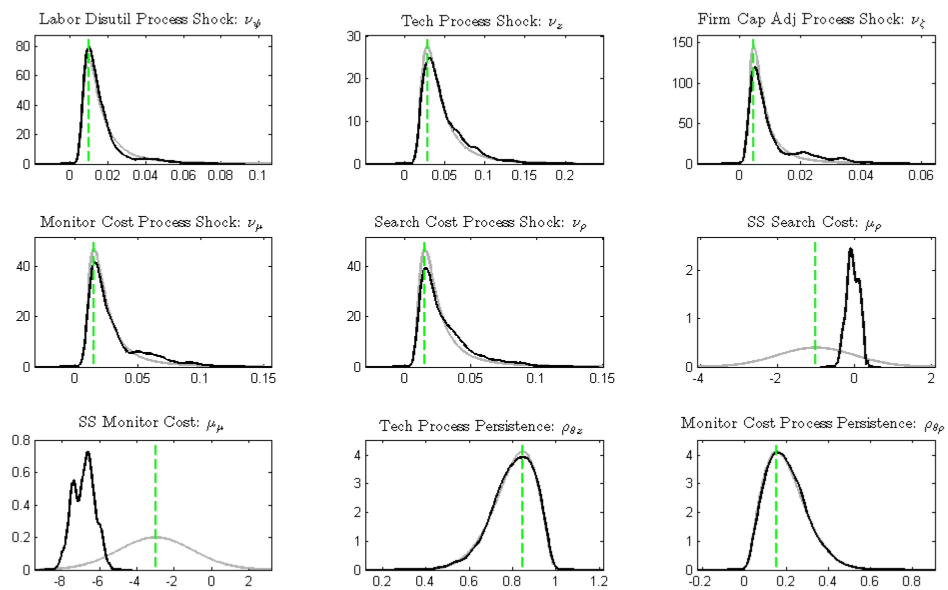

Figure 64: First Order Time Varying Posterior Distributions

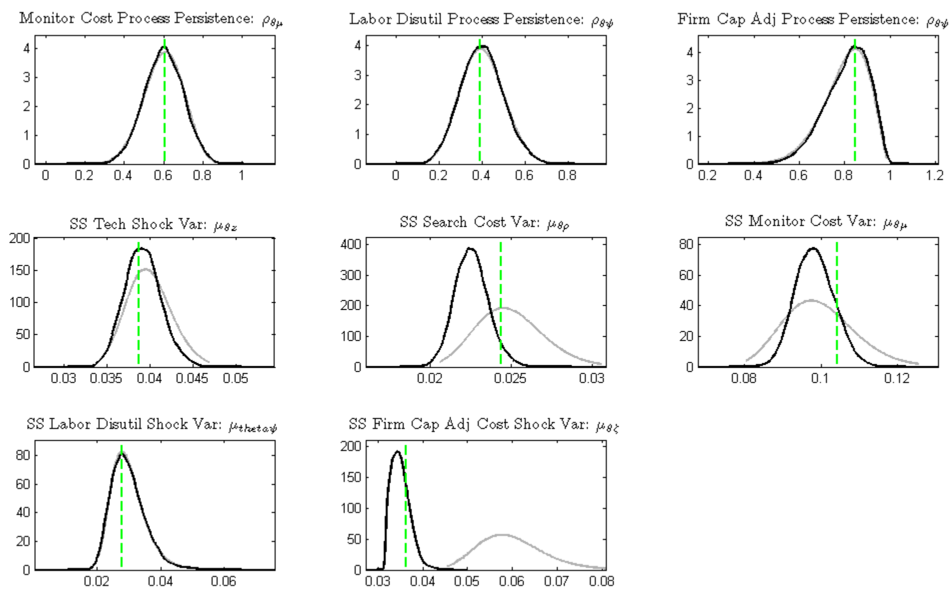

Figure 65: First Order Time Varying Posterior Distributions

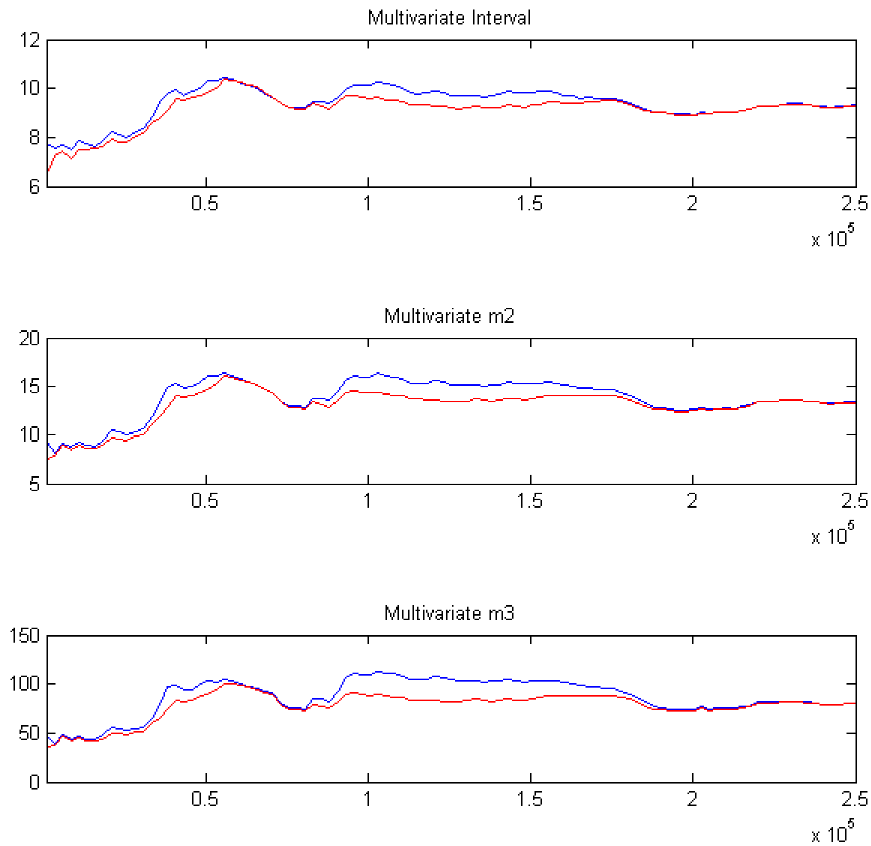

Figure 66: First Order Time Varying Multivariate Convergence Diagnostics

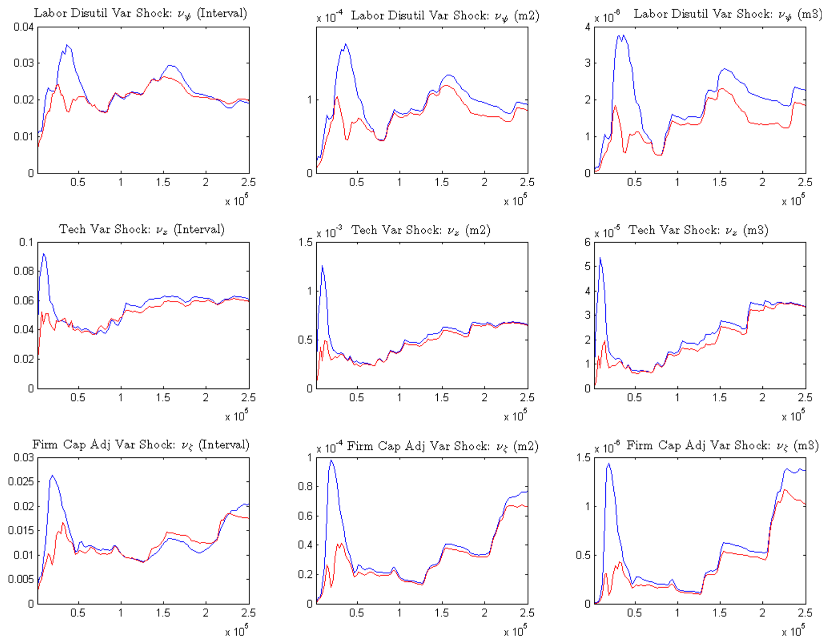

Figure 67: First Order Time Varying Univariate Convergence Diagnostics

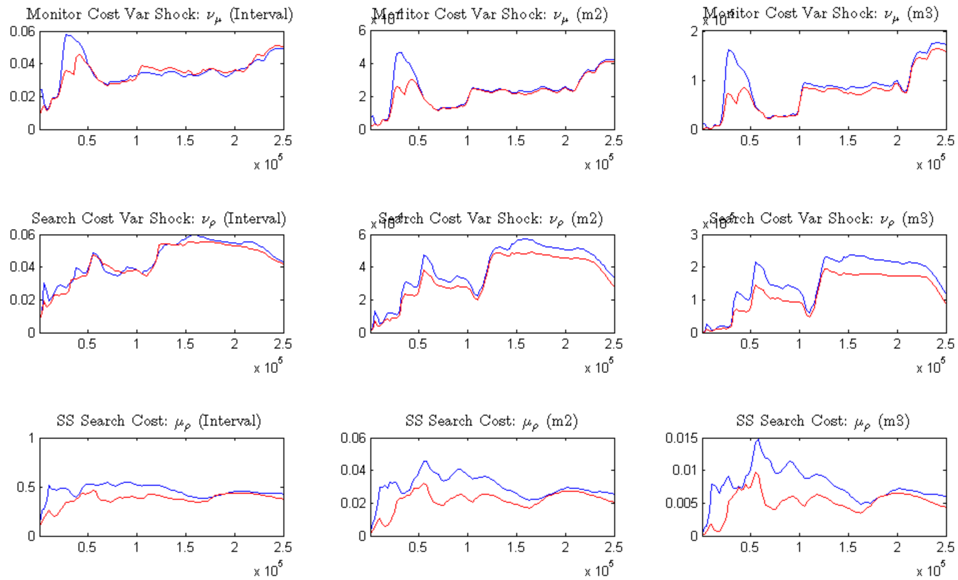

Figure 68: First Order Time Varying Univariate Convergence Diagnostics

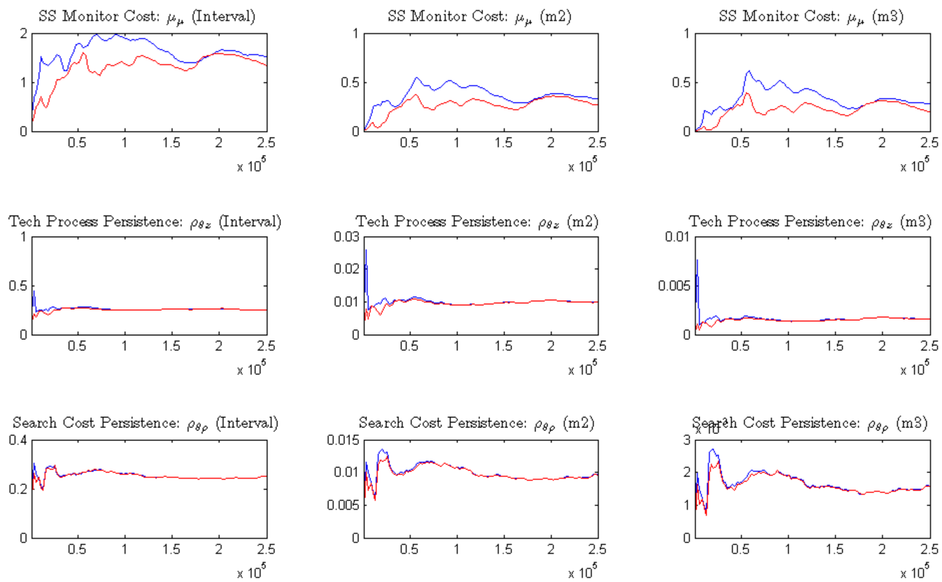

Figure 69: First Order Time Varying Univariate Convergence Diagnostics

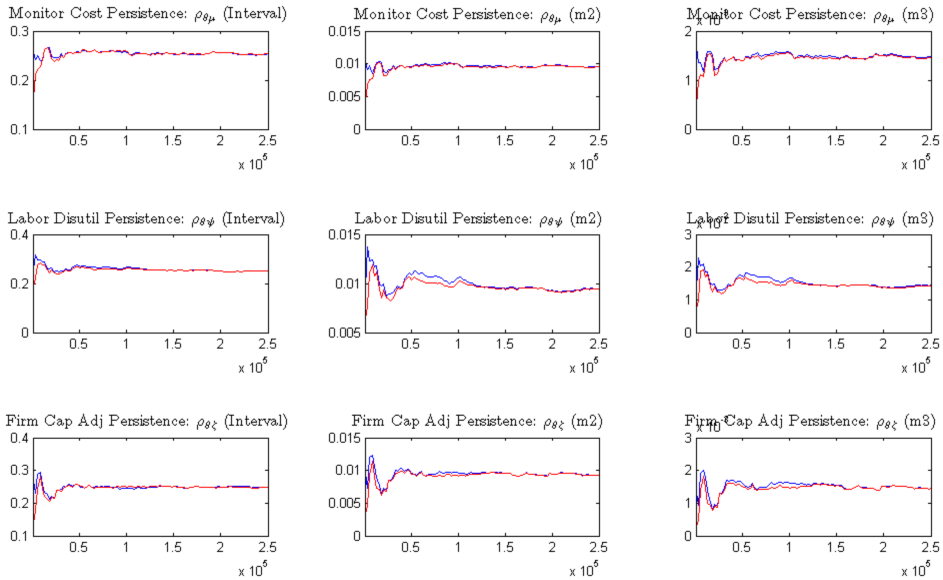

Figure 70: First Order Time Varying Univariate Convergence Diagnostics

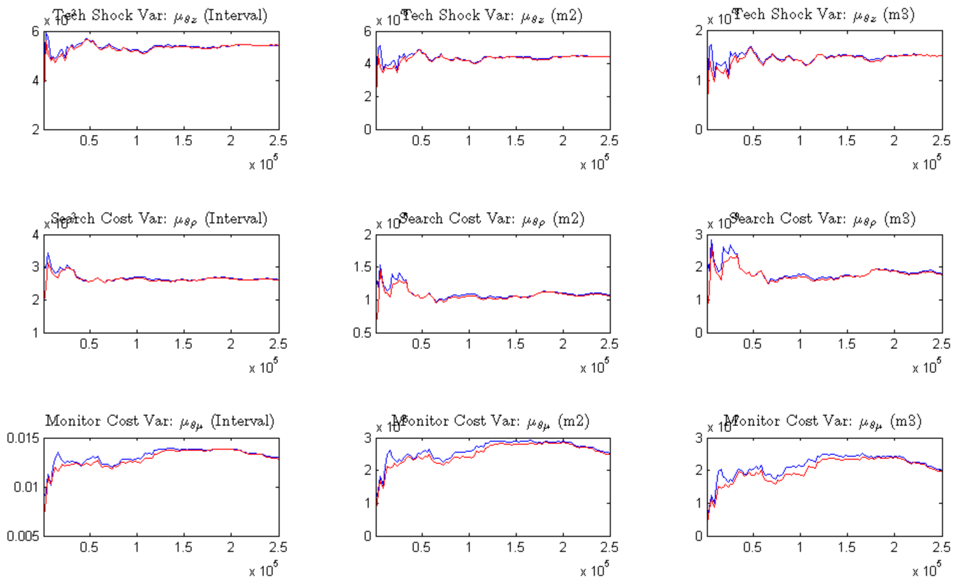

Figure 71: First Order Time Varying Univariate Convergence Diagnostics

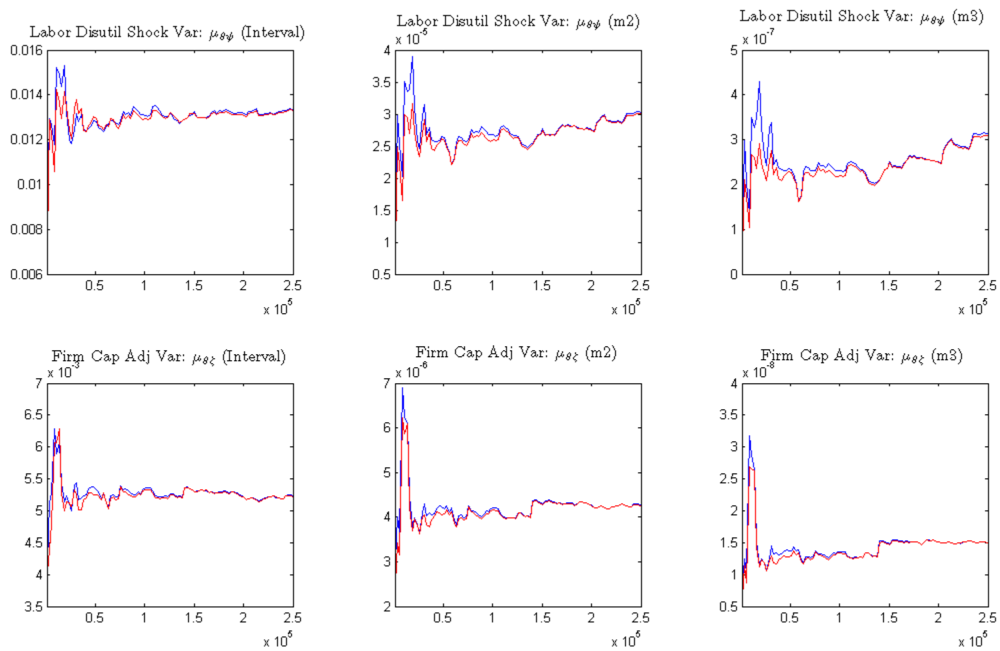

Figure 72: First Order Time Varying Univariate Convergence Diagnostics

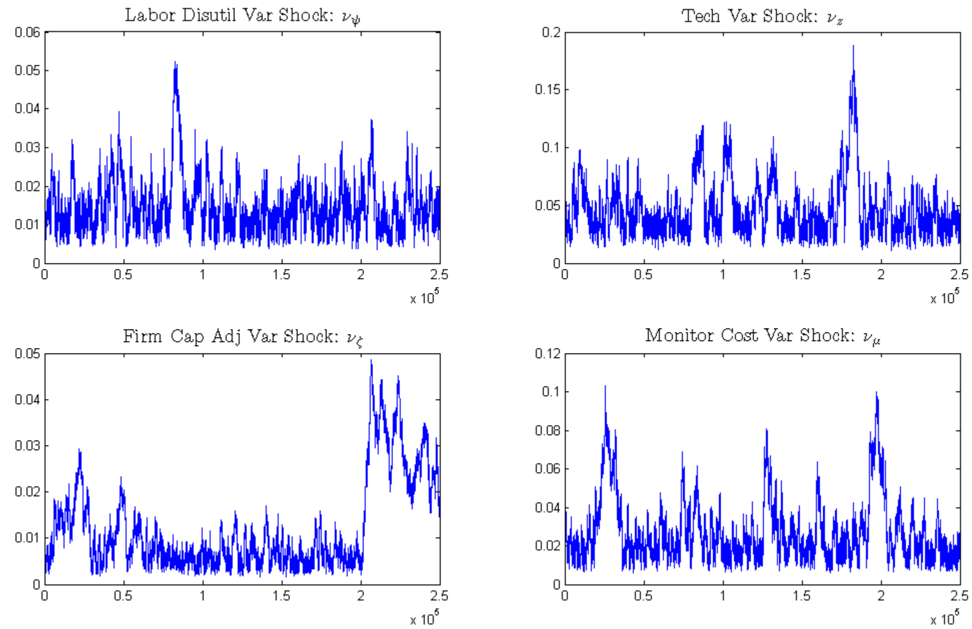

Figure 73: First Order Time Varying MH Sample Chain

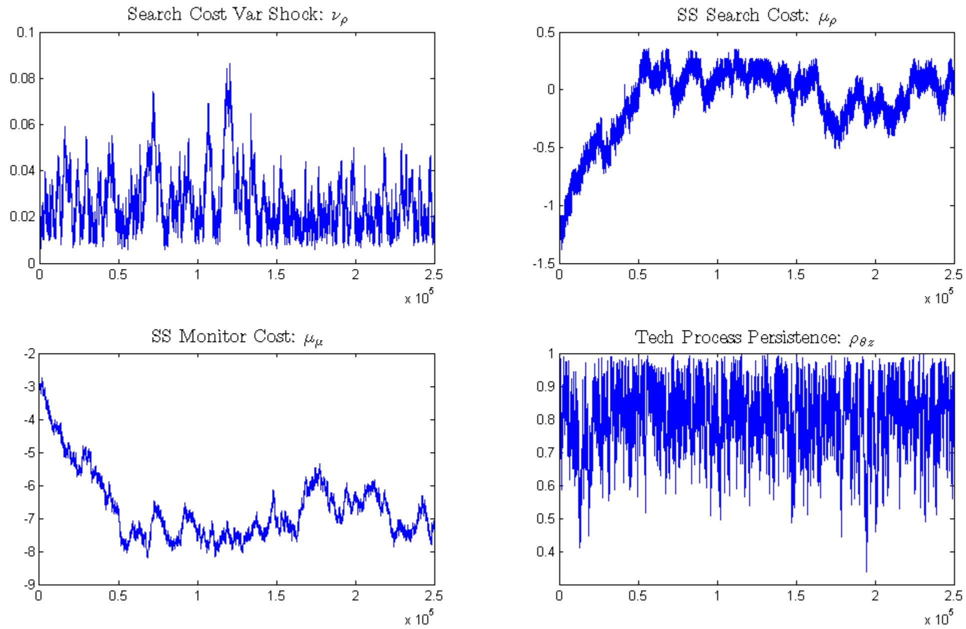

Figure 74: First Order Time Varying MH Sample Chain

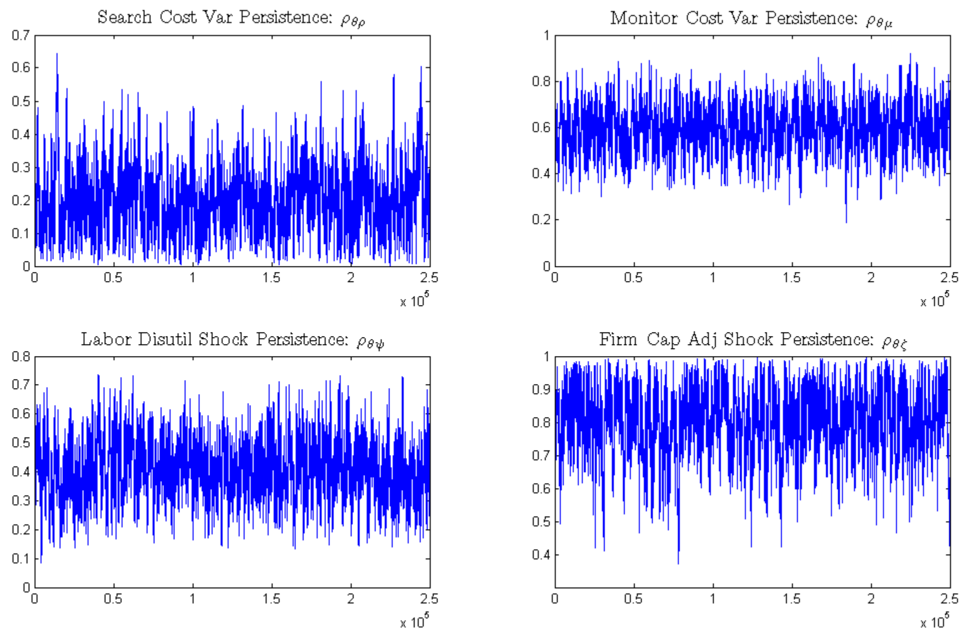

Figure 75: First Order Time Varying MH Sample Chain

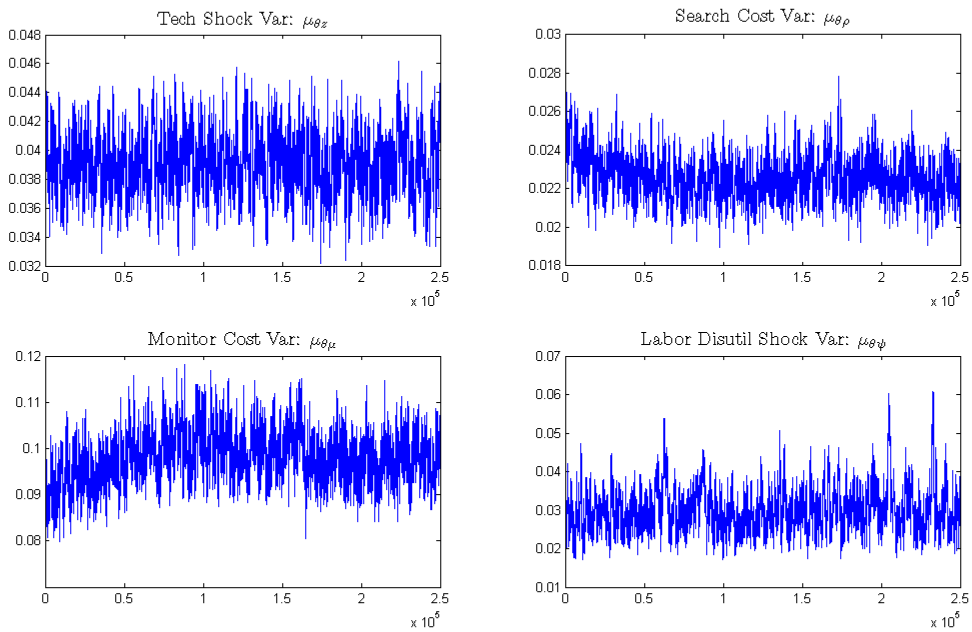

Figure 76: First Order Time Varying MH Sample Chain

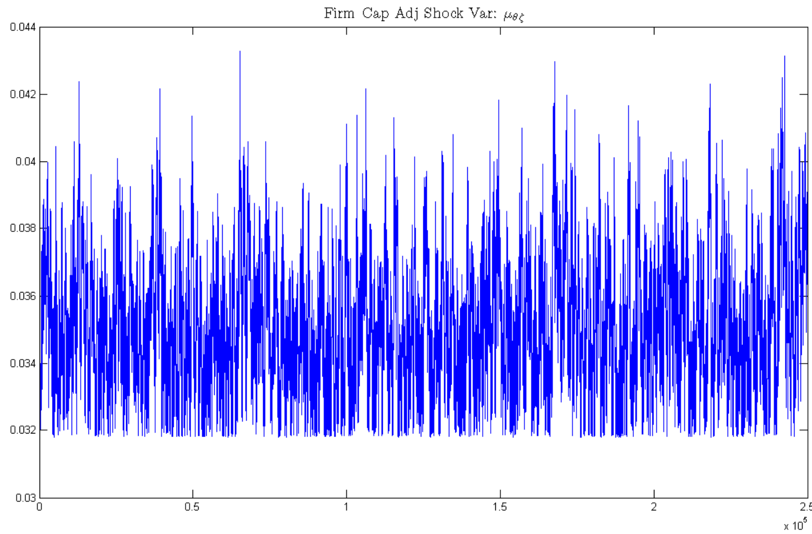

Figure 77: First Order Time Varying MH Sample Chain

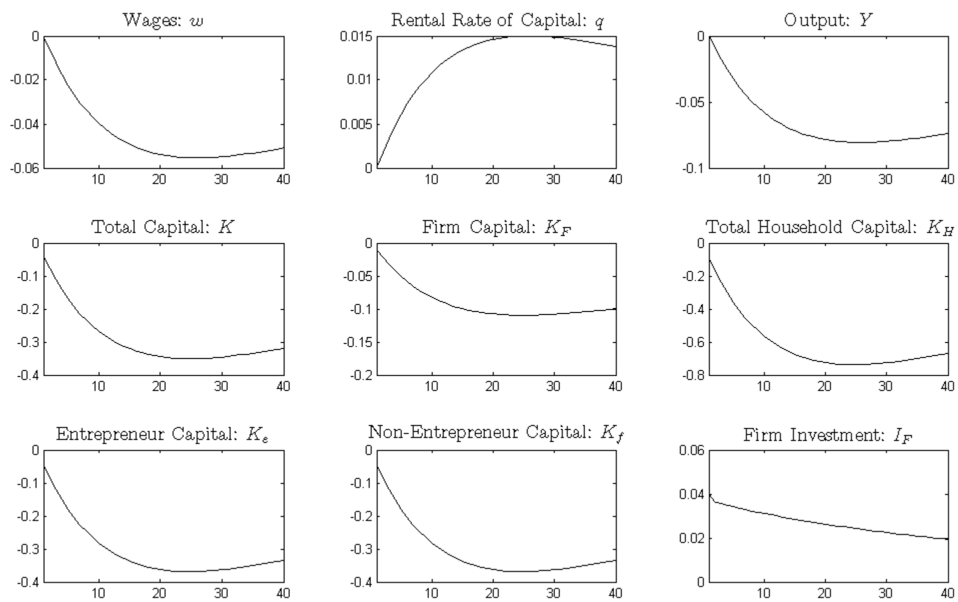

Figure 78: First Order Time Varying Impulse Response Functions
Technology Shock: z

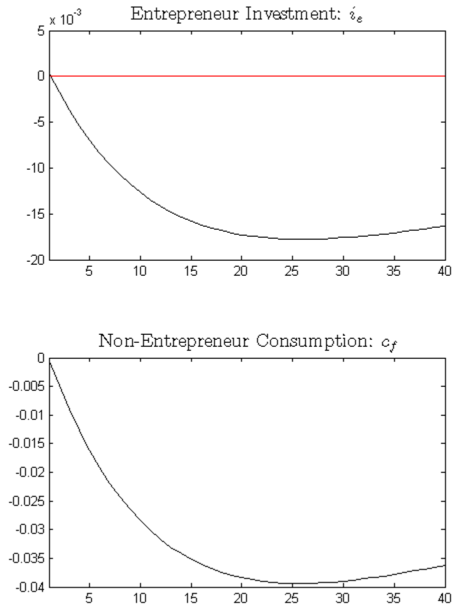

Figure 79: First Order Time Varying Impulse Response Functions
Technology Shock: z

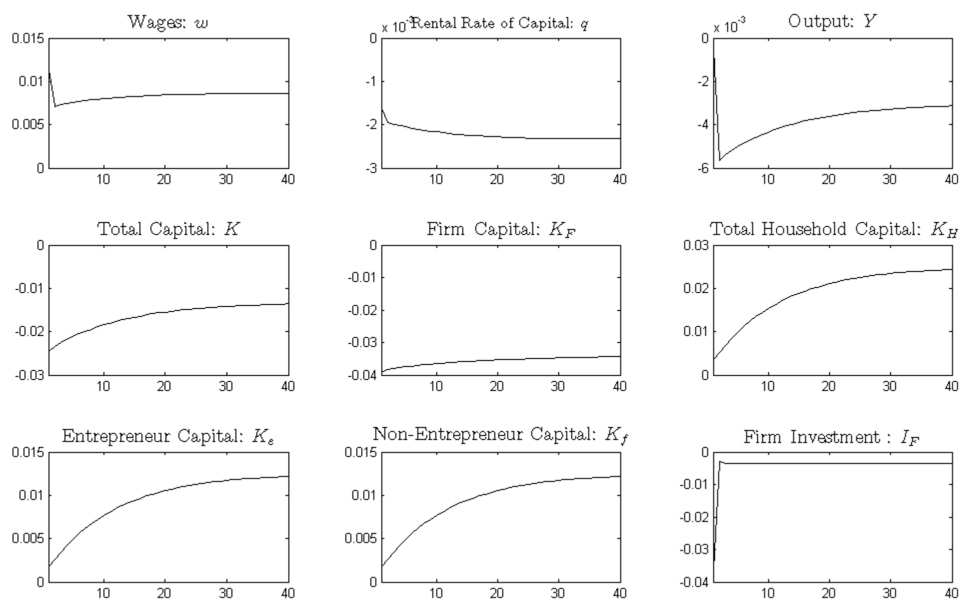

Figure 80: First Order Time Varying Impulse Response Functions
Search Cost Shock: ρ

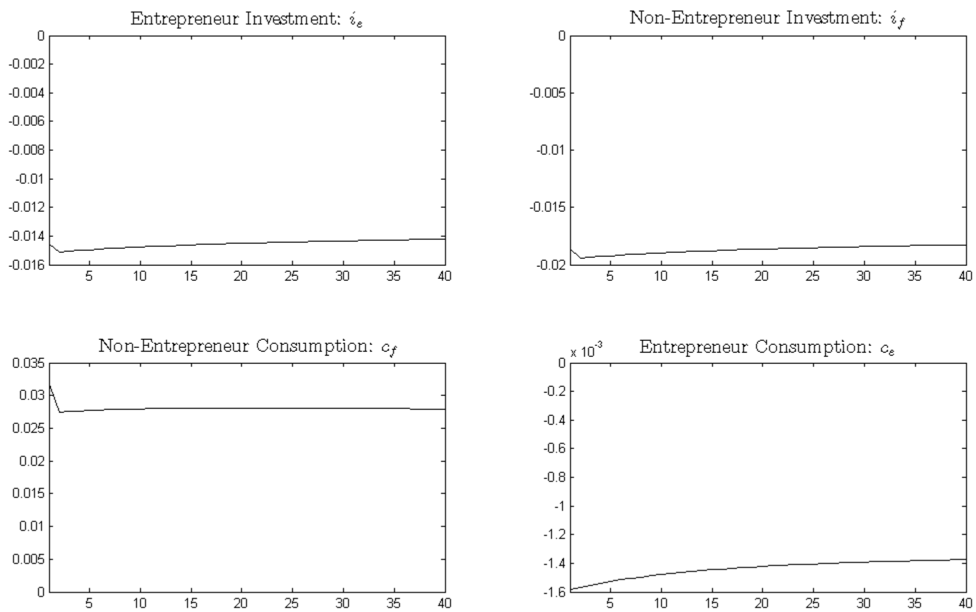

Figure 81: First Order Time Varying Impulse Response Functions
Search Cost Shock: ρ

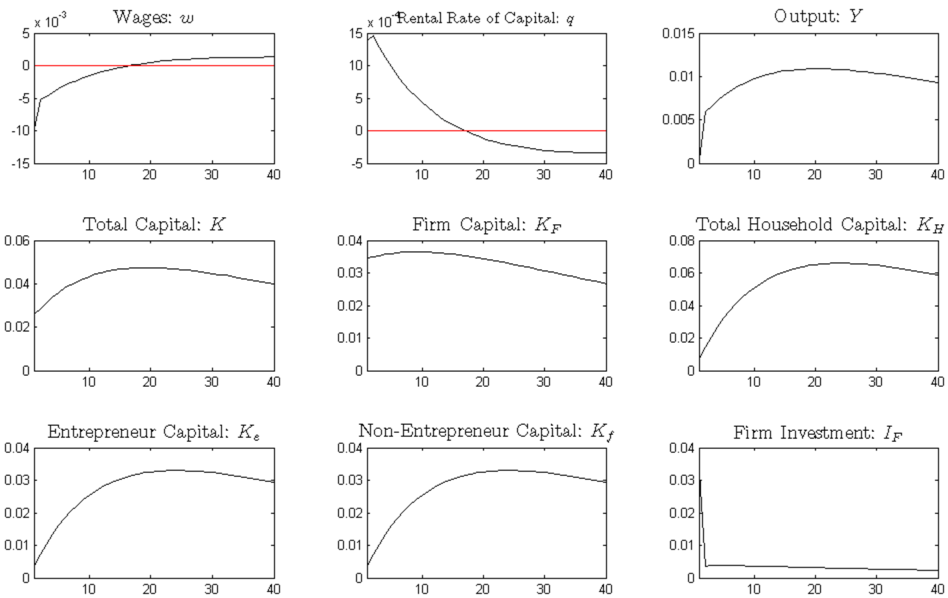

Figure 82: First Order Time Varying Impulse Response Functions
Monitoring Cost Shock: μ

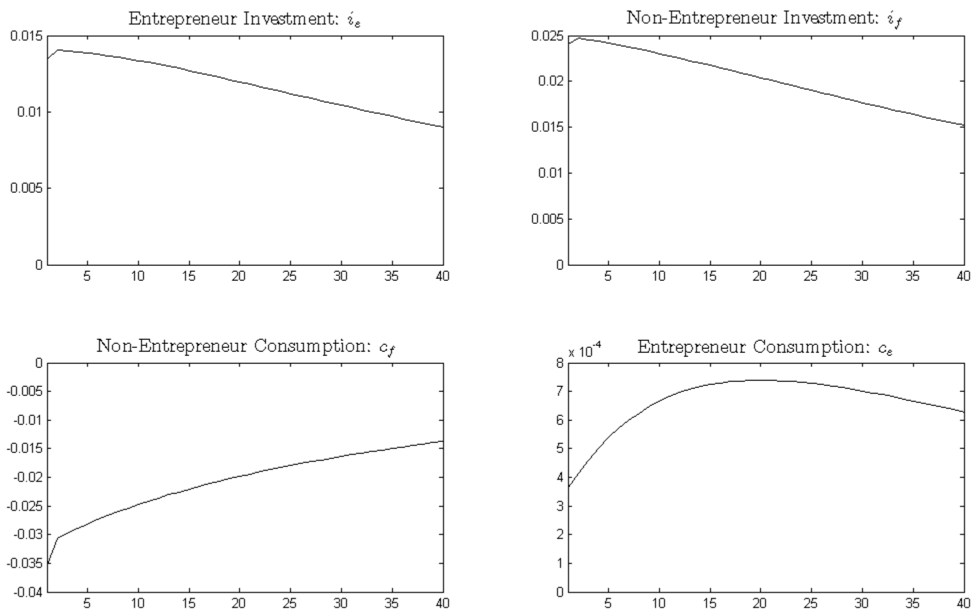

Figure 83: First Order Time Varying Impulse Response Functions
Monitoring Cost Shock: μ

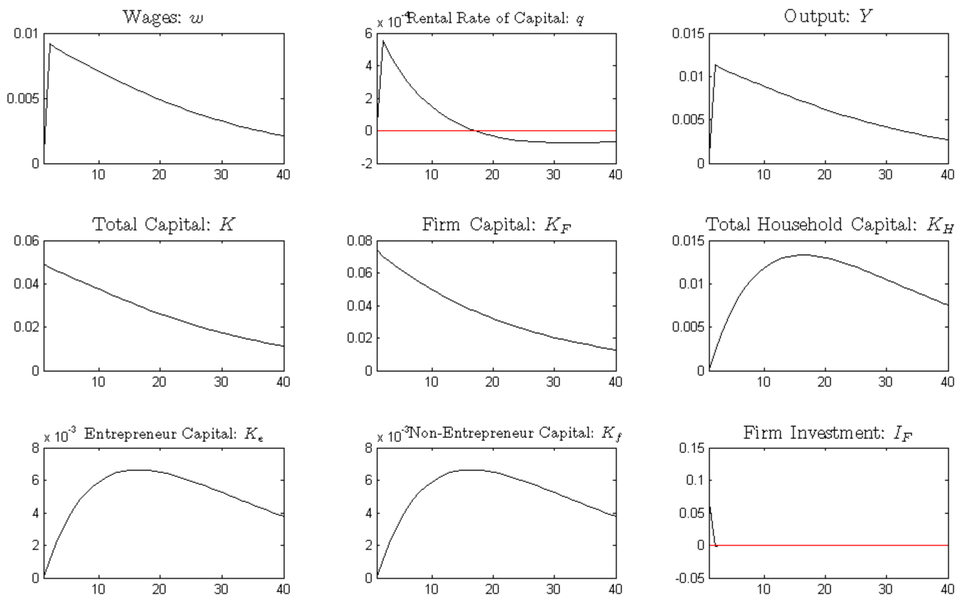

Figure 84: First Order Time Varying Impulse Response Functions
Capital Adjustment Cost Shock: ζ

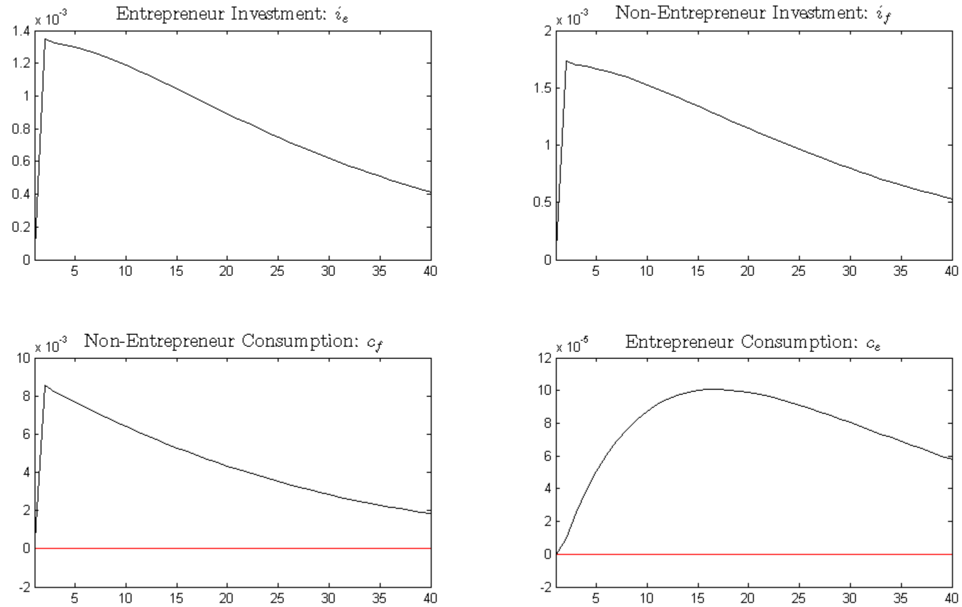

Figure 85: First Order Time Varying Impulse Response Functions
Capital Adjustment Cost Shock: ζ

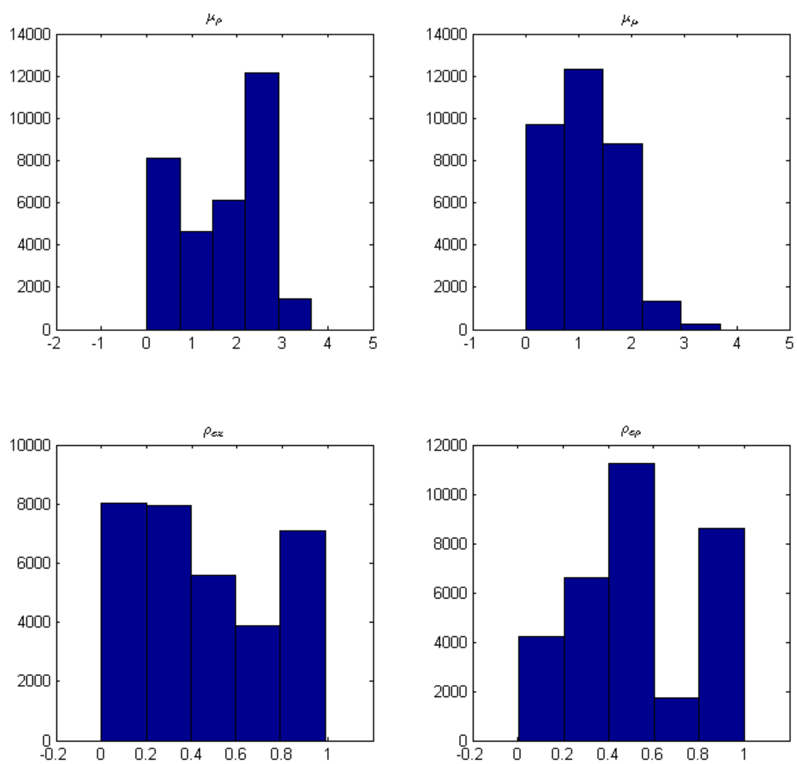

Figure 86: Second Order Time Varying Posterior Distributions

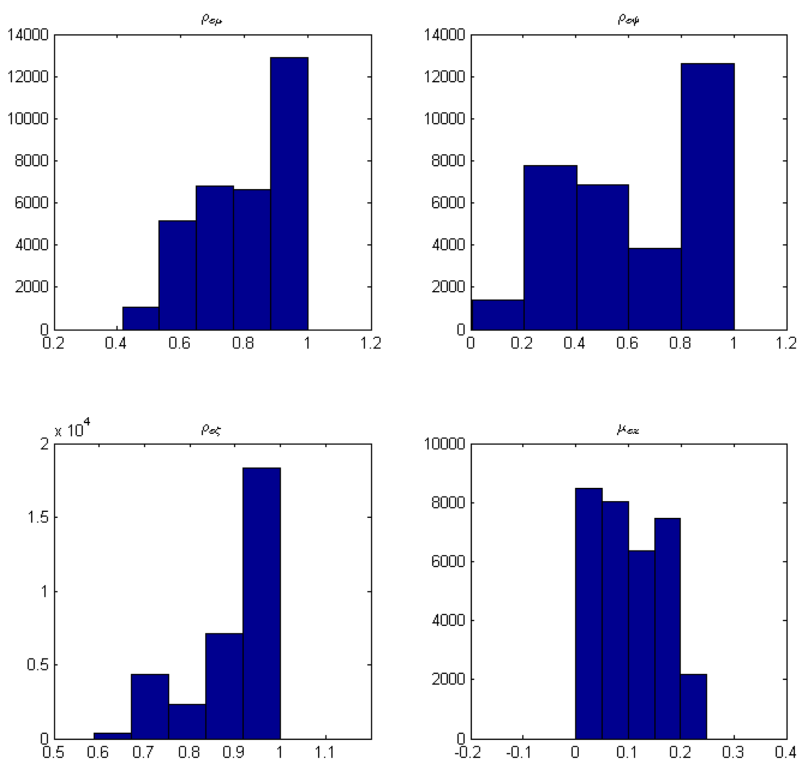

Figure 87: Second Order Time Varying Posterior Distributions

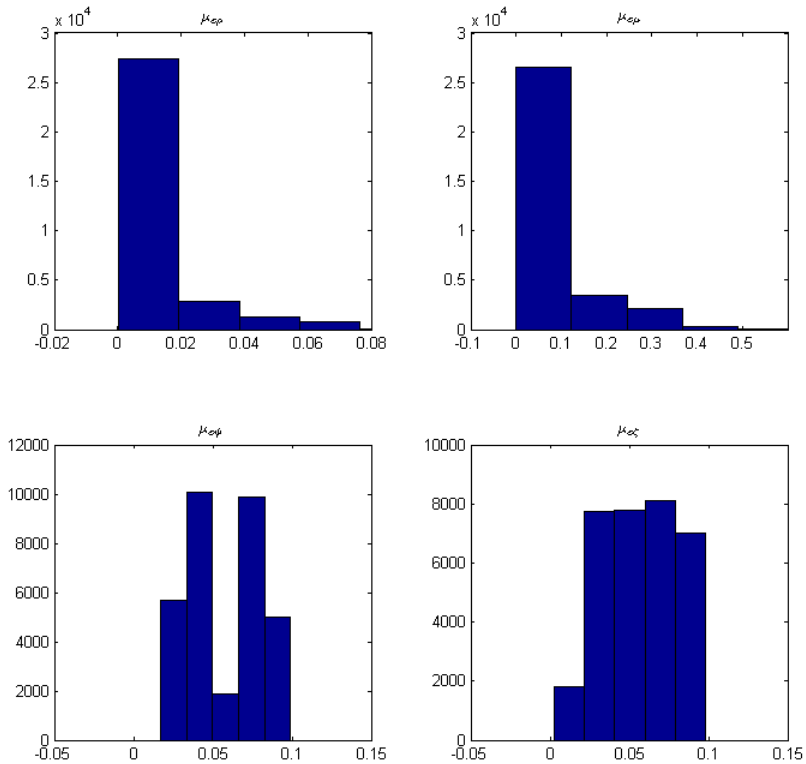

Figure 88: Second Order Time Varying Posterior Distributions

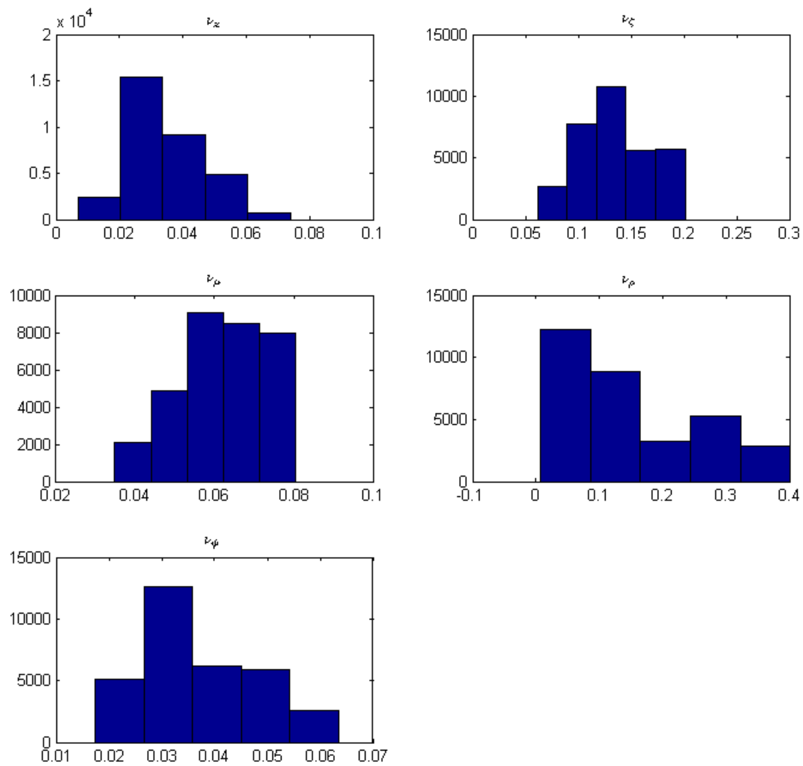

Figure 89: Second Order Time Varying Posterior Distributions

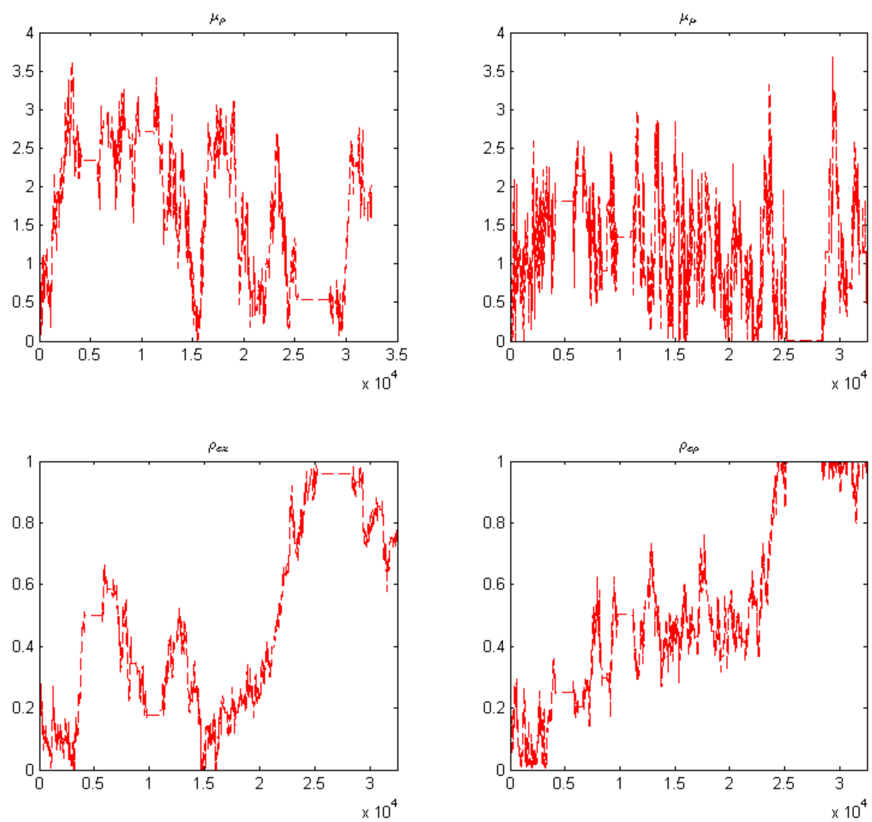

Figure 90: Second Order Time Varying MH Sample Chain

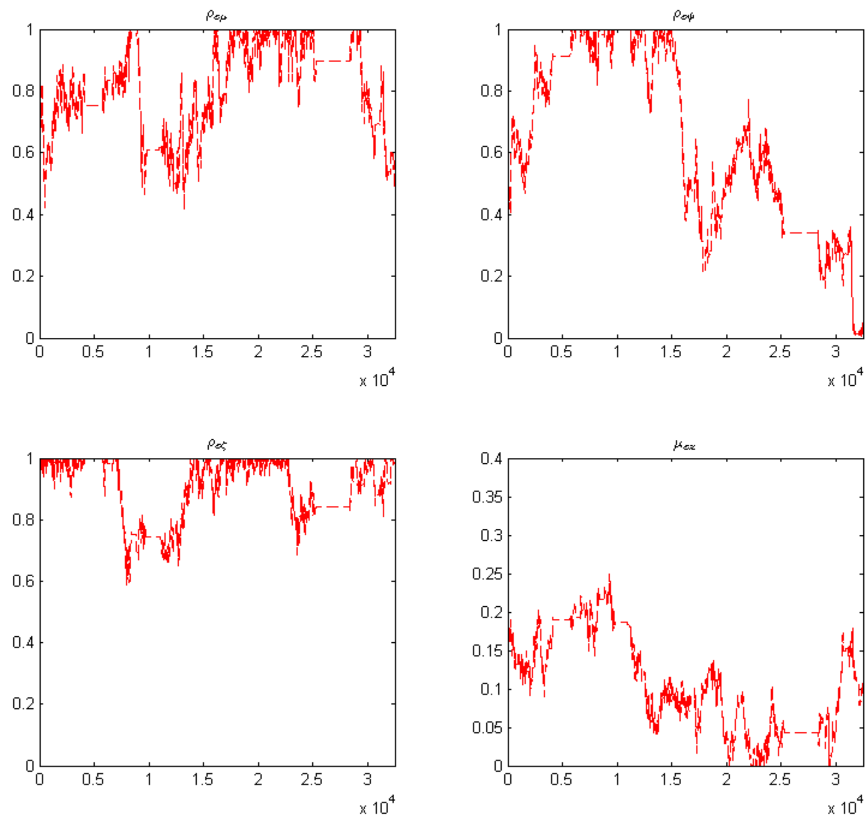

Figure 91: Second Order Time Varying MH Sample Chain

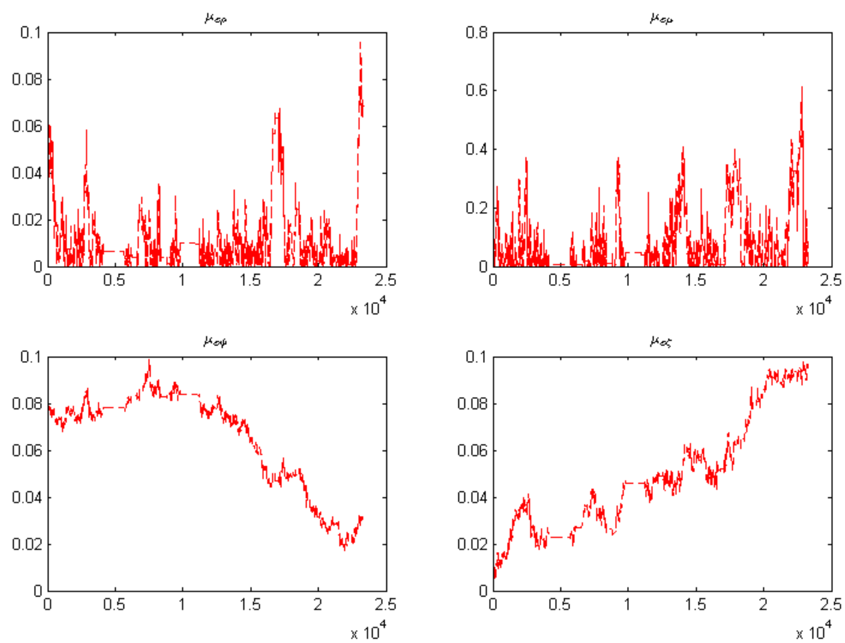

Figure 92: Second Order Time Varying MH Sample Chain

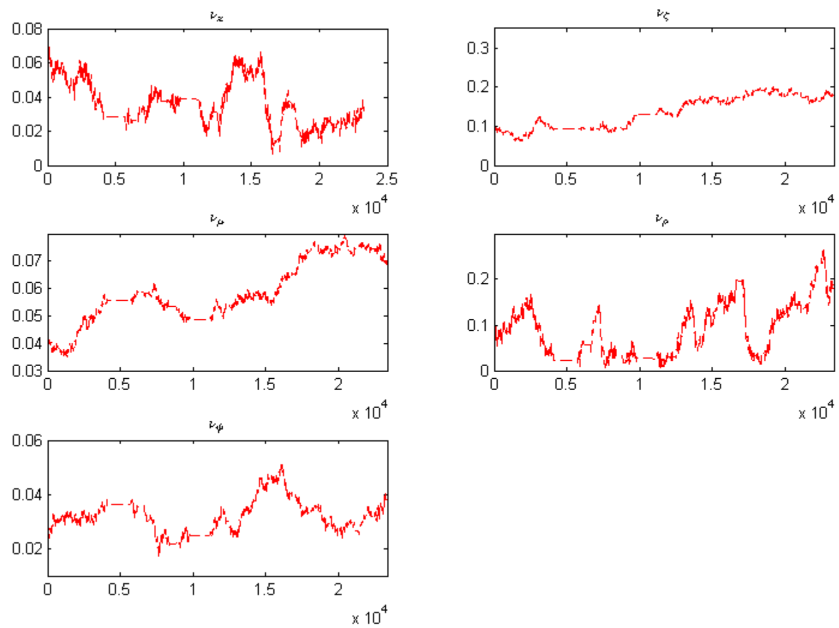

Figure 93: Second Order Time Varying MH Sample Chain

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A Appendix A: Optimal Contract Useful Relations

The functions $f(\bar{\omega}, j)$ and $g(\bar{\omega}, j)$ can be simplified, given the uniform distribution of ω on $[0, 2]$:

$$f(\bar{\omega}, j) = \frac{1}{2}\int_{\bar{\omega}}^{2} \omega d\omega - (1 - \frac{\bar{\omega}}{2})\bar{\omega} = \frac{1}{4}\bar{\omega}^2 - \bar{\omega} + 1$$

and:

$$g(\bar{\omega}, h) = \frac{\exp(\mu)}{2}\int_{0}^{\bar{\omega}} \omega d\omega + (1 - \frac{\bar{\omega}}{2})\bar{\omega} = \frac{\exp(\mu)}{4}\bar{\omega}^2 + \bar{\omega} - \frac{1}{2}\bar{\omega}^2$$

The derivatives of the functions $f(\bar{\omega}, j)$ and $g(\bar{\omega}, j)$ with respect to $\bar{\omega}$ are given by:

$$f'(\bar{\omega}, j) = \frac{1}{2}\bar{\omega} - 1$$

and

$$g'(\bar{\omega}, h) = \frac{\exp(\mu)}{2}\bar{\omega} + 1 - \bar{\omega}$$

The sum of $f'(\bar{\omega}, j)$ and $g'(\bar{\omega}, j)$ is:

$$f'(\bar{\omega}, j) + g'(\bar{\omega}, j) = \frac{\exp(\mu)}{2}\bar{\omega} - \frac{1}{2}\bar{\omega}$$

When solving for the optimal contract, the following relation is also useful:

$$f(\bar{\omega}, j) + g(\bar{\omega}, j) = 1 + \frac{\exp(\mu)}{4}\bar{\omega}^2 - \frac{1}{4}\bar{\omega}^2$$

Finally, note that:

$$f'(\bar{\omega}, j)^2 = \left[\frac{1}{2}\bar{\omega} - 1\right]^2 = \frac{1}{4}\bar{\omega}^2 - \bar{\omega} + 1 = f(\bar{\omega}, j)$$

B Appendix B: Solving for the Optimal Contract

The non-entrepreneur's participation constraint must hold with equality, so Equation 15 can be used to solve for i as a function of $\bar{\omega}$:

$$i = \frac{i^f}{1 - r^k j \ \exp(\rho) g(\bar{\omega})}$$

Plugging this into the objective function, the maximization problem becomes:

$$\max_{\bar{\omega}, r^k} r^k j \ \exp(\rho) f(\bar{\omega}) \frac{i^e}{1 - r^k j \ \exp(\rho) g(\bar{\omega})}$$

$$s.t. \quad \bar{\omega} i j \ \exp(\rho) = (1 + r)(i^f)$$

In addition, note that Equation (8) defines r in terms of i and $\bar{\omega}$:

$$r = \frac{\bar{\omega} i j \ \exp(\rho)}{i^f} - 1$$

So the problem reduces to:

$$\max_{\bar{\omega}} q j \ \exp(\rho) f(\bar{\omega}) \frac{i^e}{1 - q j \ \exp(\rho) g(\bar{\omega})}$$

The first order condition with respect to $\bar{\omega}$ i:

$$r^k j \ \exp(\rho) f'(\bar{\omega}) \frac{i^e}{1 - q j \ \exp(\rho) g(\bar{\omega})} + (q j \ \exp(\rho))^2 f(\bar{\omega}) g'(\bar{\omega}) \frac{i^e}{(1 - q j \ \exp(\rho) g(\bar{\omega}))^2} = 0$$

Canceling $\frac{i^e r^k j \ \exp(\rho)}{1-qj \ \exp(\rho)g(\bar{\omega})}$, obtain:

$$f'(\bar{\omega}) + qj \ \exp(\rho)f(\bar{\omega})g'(\bar{\omega}) \left[\frac{1}{1 - qj \ \exp(\rho)g(\bar{\omega})}\right] = 0$$

Multiplying both sides by $1 - r^k j \ \exp(\rho)g(\bar{\omega})$ obtain:

$$\left[1 - r^k j \ \exp(\rho)g(\bar{\omega})\right] f'(\bar{\omega}) + qj \ \exp(\rho)f(\bar{\omega})g'(\bar{\omega}) = 0$$

Recall that $[f'(\bar{\omega})]^2 = f(\bar{\omega})$. Then the equation becomes:

$$\left[1 - r^k j \ \exp(\rho)g(\bar{\omega})\right] f'(\bar{\omega}) + qj \ \exp(\rho) \left[f'(\bar{\omega})\right]^2 g'(\bar{\omega}) = 0$$

This can be divided by $f'(\bar{\omega})$ to obtain:

$$\left[1 - r^k j \ \exp(\rho)g(\bar{\omega})\right] + qj \ \exp(\rho)f'(\bar{\omega})g'(\bar{\omega}) = 0$$

The solution to the optimal contract problem is given by three equations. The first solution equation is the equation above. The second is the borrower's participation constraint, and the third is the definition of $\bar{\omega}$. Thus, these three equations form the solution the problem:

$$\begin{aligned} \left[1 - r^k j \ \exp(\rho)g(\bar{\omega})\right] + qj \ \exp(\rho)f'(\bar{\omega})g'(\bar{\omega}) = 0 \\ i = \frac{i^f}{1 - r^k j \ \exp(\rho)g(\bar{\omega})} \\ r = \frac{\bar{\omega} i j \ \exp(\rho)}{i^f} - 1 \end{aligned}$$

These equations can be solved for the optimal values of total investment, i, interest rate, r, and the failure rate, $\bar{\omega}$.

C Appendix C: Solving the Entrepreneurial Family's Optimization Problem

The entrepreneurial family's optimization problem is:

$$\max_{c_t^e, i_t^e, K_{t+1}^e} \sum_{t=0}^{\infty} N\beta_t^t(1.5-\bar{j}_t)\left(ln(c_t^e)-\frac{1}{1+\nu}\right)$$

$$s.t. \qquad N(c_t^e+i_t^e)(1.5-\bar{j}_t)=q_tK_t^e$$

$$K_{t+1}^e=(1-\delta)K_t^e+N\ \exp(\rho_t)\int_{\bar{j}_t}^{1.5} i_t j f(\bar{\omega}_t^*(j),j)dj$$

$$i_t=\frac{i_t^e}{1-q_{t+1}\ \exp(\rho_t) j g(\bar{\omega}_t^*(j),j)}$$

Combining the second and third constraints, obtain the following law of motion of capital:

$$K_{t+1}^e=(1-\delta)K_t^e+N\ \exp(\rho_t)i_t^e\int_{\bar{j}_t}^{1.5}\frac{jf(\bar{\omega}_t^*(j),j)}{1-q_{t+1}\ \exp(\rho_t)jg(\bar{\omega}_t^*(j),j)}dj$$

Note that the expression:

$$\int_{\bar{j}_t}^{1.5}\frac{jf(\bar{\omega}_t^*(j),j)}{1-q_{t+1}\ \exp(\rho_t)jg(\bar{\omega}_t^*(j),j)}dj$$

is independent of all of the entrepreneur's decision variables, so it is functionally constant in the entrepreneur's maximization problem. To simplify notation for now, let:

$$\gamma_t^e = \int_{\bar{j}_t}^{1.5} \frac{j f(\bar{\omega}_t^*(j), j)}{1 - q_{t+1}\ \exp(\rho_t) j g(\bar{\omega}_t^*(j), j)} dj$$

The Lagrangian for the maximization problem is given by:

$$L = \sum_{t=0}^{\infty} \beta^t \Bigg[N\left(1.5 - j_t\right) \left(ln\left(c_t^e\right) - \frac{1}{1+\nu} \right) \\ + \lambda_{1t}\left[q_t K_t^e - N\left(1.5 - \bar{j}_t\right)\left(c_t^e + i_t^e\right)\right] \\ + \lambda_{2t}\left[(1-\delta)K_t^e + N\ \exp(\rho_t) i_t^e \gamma_t^e - K_{t+1}^e\right] \Bigg] \quad (11)$$

The first order condition with respect to c_t^e is given by:

$$\lambda_{1t} = \frac{1}{c_t^e}$$

The first order condition with respect to i_t^e is given by:

$$\lambda_{2t} = \frac{(1.5 - \bar{j}_t)}{\exp(\rho_t)\gamma_t^e c_t^e}$$

Finally, the first order condition with respect to K_{t+1}^e is given by:

$$-\beta_t^t \lambda_{2t} + \beta_{t+1}^{t+1}\left[\lambda_{1t+1} q_{t+1} + \lambda_{2t+1}(1-\delta)\right] = 0$$

Plugging in for λ_{1t} and λ_{2t}, obtain the Euler Equation:

$$\frac{\beta_t^t (1.5 - \bar{j}_t)}{\exp(\rho_t)\gamma_t^e c_t^e} = \beta_{t+1}^{t+1}\left[\frac{q_{t+1}}{c_{t+1}^e} + \frac{(1.5 - \bar{j}_t)(1-\delta)}{\exp(\rho_{t+1})\gamma_{t+1} c_{t+1}^e}\right]$$

D Appendix D: Solving the Non-Entrepreneurial Family's Optimization Problem

The non-entrepreneurial family's optimization problem is given by:

$$\max_{c_t^f, l_t} \sum_{t=0}^{\infty} N\beta_t^t(\bar{j} - 0.5)\left(ln(c_t^f) - \psi_t \frac{l_t^{1+\nu}}{1+\nu}\right) \tag{12}$$

$$s.t. \quad N(c_t^f + i_t^f)(\bar{j}_t - 0.5) = q_t K_t^f + N(\bar{j}_t - 0.5)w_t l_t$$

$$l_t \leq 1$$

The Lagrangian for this problem is simple:

$$\begin{aligned} L = \sum_{t=0}^{\infty} \beta_t \Bigg[& N(\bar{j}_t - 0.5)\left(ln(c_t^f) - \psi_t \frac{l_t^{1+\nu}}{1+\nu}\right) \\ & + \lambda_{1t}\left[q_t K_t^f + N(\bar{j}_t - 0.5)w_t l_t - N(c_t^f + i_t^f)(\bar{j}_t - 0.5)\right] \\ & + \lambda_{2t}\left[1 - l_t\right]\Bigg] \end{aligned} \tag{13}$$

The resulting first order condition on c_t^f is:

$$\lambda_{1t} = \frac{1}{c_t^f}$$

and the first order condition on l_t is:

$$N(\bar{j}_t - 0.5)\psi_t l_t^{\nu} - \lambda_{2t} + \lambda_{1t} N(\bar{j}_t - 0.5)w_t = 0$$

Finally, the transversality condition on l_t is:

$$\lambda_{2t}(1 - l_t) = 0$$

Substituting for λ_{1t}, the first order condition on l_t becomes:

$$N(\bar{j}_t - 0.5)\psi_t l_t^{\nu} + N(\bar{j}_t - 0.5)\frac{w_t}{c_t^f} = \lambda_{2t}$$

E Appendix E: Function Approximation

A function may be approximated using a Taylor series about a real or complex number, a, which takes the form:

$$f(x) \sim f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + ...$$

I will obtain a second order accurate model solution and parameter estimation, so I will use the second order Taylor series approximation.

E.1 The Entrepreneurial Family

The Euler Equation contains an integral that cannot be solved analytically. The integral takes the following form:

$$\int_{\bar{j}}^{1.5} \frac{jf(\bar{\omega}_t^*)}{1 - r_t^k \exp(\rho_t) jg(\bar{\omega}_t^*)} dj$$

The expression inside the integral cannot be integrated analytically, so I obtain a linear approximation of this expression. Let Γ_t^e be the expression to be integrated:

$$\Gamma_t^e = \frac{jf(\bar{\omega}_t^*)}{1 - r_t^k \exp(\rho_t) jg(\bar{\omega}_t^*)}$$

The expression Γ_t^e will be integrated with respect to j, so the Taylor approximation will be taken with respect to j. The approximation will be about the median value of j, which equals 1. Let j_{mid} represent the median value of j such that $j_{mid} = 1$. The Taylor series approximation about j_{mid} is given by:

$$\Gamma_t^e \sim \Gamma_t^e(j_{mid}) + \frac{\frac{\delta \Gamma_t^e(j_{mid})}{\delta j}}{1!}(j - j_{mid}) + \frac{\frac{\delta^2 \Gamma_t^e(j_{mid})}{(\delta j)^2}}{2!}(j - j_{mid})^2$$

Recall from Appendix A that $f(\omega)$ and $g(\omega)$ are functions of ω. Using the expressions for $f(\omega)$ and $g(\omega)$ from Appendix A, $\Gamma_t^e(j_{mid})$ is given by:

$$\Gamma_t^e(j_{mid}) = \frac{\frac{1}{4}\omega^2 - \omega + 1}{1 - r^k \exp(\rho)\left(\omega + \frac{\exp(\mu)}{4}\omega^2 - \frac{1}{2}\omega^2\right)}$$

The first derivative of Γ_t^e with respect to j and evaluated at j_{mid} is given by:

$$\frac{\delta \Gamma_t^e(j_{mid})}{\delta j} = \frac{4\left(\omega - 2\right)^2}{\left(4\omega r^k \exp(\rho) - 2\omega^2 r^k \exp(\rho) + \omega^2 r^k \exp(\mu) \exp(\rho) - 4\right)^2}$$

The second derivative of Γ_t^e with respect to j and evaluated at j_{mid} is given by:

$$\frac{\delta^2 \Gamma_t^e(j_{mid})}{(\delta j)^2} = \frac{-\left[8r^k \exp(\rho)(\omega - 2)^2\left(4\omega + \exp(\mu)\omega^2 - 2\omega^2\right)\right]}{\left(4\omega r^k \exp(\rho) - 2\omega^2 r^k \exp(\rho) + \omega^2 r^k \exp(\mu) \exp(\rho) - 4\right)^3}$$

To simplify the Γ_t^e expression, let the above expressions be represented by the following:

$$\Gamma_{0,t}^e = \Gamma_t^e(j_{mid})$$
$$\Gamma_{1,t}^e = \frac{\delta\Gamma_t^e(j_{mid})}{\delta j}$$
$$\Gamma_{2,t}^e = \frac{\delta^2\Gamma_t^e(j_{mid})}{(\delta j)^2}$$

Then the Taylor series for Γ_t^e is given by:

$$\Gamma_t^e \sim \Gamma_{0,t}^e + \Gamma_{1,t}^e(j - j_{mid}) + \frac{1}{2}\Gamma_{2,t}^e(j - j_{mid})^2$$

This can be expanded to obtain the following equation, which is quadratic in j:

$$\begin{aligned}\Gamma_t^e &\sim \Gamma_{0,t}^e + \Gamma_{1,t}^e(j - j_{mid}) + \tfrac{1}{2}\Gamma_{2,t}^e(j^2 - 2j_{mid}j + j_{mid}^2) \\ &= \Gamma_{2,t}^e j^2 + (\Gamma_{1,t}^e - 2j_{mid}\Gamma_{2,t}^e)j + (\Gamma_{0,t}^e - \Gamma_{1,t}^e j_{mid} + \Gamma_{2,t}^e j_{mid}^2)\end{aligned}$$

This can then be integrated to obtain the approximation for γ_e^t:

$$\begin{aligned}\gamma_e^t &= \textstyle\int_{\bar{j}}^{1.5} \Gamma_t^e dj \\ &\sim \textstyle\int_{\bar{j}}^{1.5} \Gamma_{2,t}^e j^2 + (\Gamma_{1,t}^e - 2j_{mid}\Gamma_{2,t}^e)j + (\Gamma_{0,t}^e - \Gamma_{1,t}^e j_{mid} + \Gamma_{2,t}^e j_{mid}^2)dj \\ &\sim \tfrac{1}{3}\Gamma_{2,t}^e j^3 + \tfrac{1}{2}(\Gamma_{1,t}^e - 2j_{mid}\Gamma_{2,t}^e)j^2 + (\Gamma_{0,t}^e - \Gamma_{1,t}^e j_{mid} + \Gamma_{2,t}^e j_{mid}^2)j \;|_{\bar{j}}^{1.5}\end{aligned}$$

The second order approximation for the expression γ_t^e is given by:

$$\gamma_t^e \sim \frac{1}{3}\Gamma_{2,t}^e(1.5^3 - \bar{j}^3) + \frac{1}{2}(\Gamma_{1,t}^e - 2j_{mid}\Gamma_{2,t}^e)(1.5^2 - \bar{j}^2) + (\Gamma_{0,t}^e - \Gamma_{1,t}^e j_{mid} + \Gamma_{2,t}^e j_{mid}^2)(1.5 - \bar{j})$$

E.2 The Non-Entrepreneurial Family

The Euler Equation contains an integral that cannot be solved analytically. The integral takes the following form:

$$\int_{\bar{j}}^{1.5} \frac{jg(\bar{\omega}_t^*)}{1 - r_t^k \exp(\rho_t) jg(\bar{\omega}_t^*)} dj$$

The expression inside the integral cannot be integrated analytically, so I obtain a linear approximation of this expression. Let Γ_t^f be the expression to be integrated:

$$\Gamma_t^f = \frac{jg(\bar{\omega}_t^*)}{1 - r_t^k \exp(\rho_t) jg(\bar{\omega}_t^*)}$$

The expression Γ_t^f will be integrated with respect to j, so the Taylor approximation will be taken with respect to j. The approximation will be about the median value of j, which equals 1. Let j_{mid} represent the median value of j such that $j_{mid} = 1$. The Taylor series approximation about j_{mid} is given by:

$$\Gamma_t^f \sim \Gamma_t^f(j_{mid}) + \frac{\frac{\delta \Gamma_t^f(j_{mid})}{\delta j}}{1!}(j - j_{mid}) + \frac{\frac{\delta^2 \Gamma_t^f(j_{mid})}{(\delta j)^2}}{2!}(j - j_{mid})^2$$

Recall from Appendix A that $g(\omega)$ is a function of ω. Using the expression for $g(\omega)$ from Appendix A, $\Gamma_t^f(j_{mid})$ is given by:

$$\Gamma_t^f(j_{mid}) = \frac{\omega + \frac{\exp(\mu)}{4}\omega^2 - \frac{1}{2}\omega^2}{1 - r^k \exp(\rho)\left(\omega + \frac{\exp(\mu)}{4}\omega^2 - \frac{1}{2}\omega^2\right)}$$

The first derivative of Γ_t^f with respect to j and evaluated at j_{mid} is given by:

$$\frac{\delta\Gamma_t^f(j_{mid})}{\delta j} = \frac{(4\ \exp(\mu) - 8)\omega^2 + 16\omega}{(4\omega r^k\ \exp(\rho) - 2\omega^2 r^k\ \exp(\rho) + omega^2 r^k\ \exp(\mu)\ \exp(\rho) - 4)^2}$$

The second derivative of Γ_t^f with respect to j and evaluated at j_{mid} is given by:

$$\frac{\delta^2\Gamma_t^f(j_{mid})}{(\delta j)^2} = \frac{-\left(2r^k\ \exp(\rho)((4\ \exp(\mu) - 8)\omega^2 + 16\omega)(4\omega + \omega^2\ \exp(\mu) - 2\omega^2)\right)}{(4\omega r^k\ \exp(\rho) - 2\omega^2 r^k\ \exp(\rho) + \omega^2 r^k\ \exp(\rho) - 4)^3}$$

To simplify the Γ_t^f expression, let the above expressions be represented by the following:

$$\begin{aligned}
\Gamma_{0,t}^f &= \Gamma_t^f(j_{mid}) \\
\Gamma_{1,t}^f &= \frac{\delta\Gamma_t^f(j_{mid})}{\delta j} \\
\Gamma_{2,t}^f &= \frac{\delta^2\Gamma_t^f(j_{mid})}{(\delta j)^2}
\end{aligned}$$

Then the Taylor series for Γ_t^f is given by:

$$\Gamma_t^f \sim \Gamma_{0,t}^f + \Gamma_{1,t}^f(j - j_{mid}) + \frac{1}{2}\Gamma_{2,t}^f(j - j_{mid})^2$$

This can be expanded to obtain the following equation, which is quadratic in j:

$$\begin{aligned}
\Gamma_t^f &\sim \Gamma_{0,t}^f + \Gamma_{1,t}^f(j - j_{mid}) + \tfrac{1}{2}\Gamma_{2,t}^f(j^2 - 2j_{mid}j + j_{mid}^2) \\
&= \Gamma_{2,t}^f j^2 + (\Gamma_{1,t}^f - 2j_{mid}\Gamma_{2,t}^f)j + (\Gamma_{0,t}^f - \Gamma_{1,t}^f j_{mid} + \Gamma_{2,t}^f j_{mid}^2)
\end{aligned}$$

This can then be integrated to obtain the approximation for γ_f^t:

$$\begin{aligned}
\gamma_f^t & = \int_{\bar{j}}^{1.5} \Gamma_t^f dj \\
& \sim \int_{\bar{j}}^{1.5} \Gamma_{2,t}^f j^2 + (\Gamma_{1,t}^f - 2 j_{mid} \Gamma_{2,t}^f) j + (\Gamma_{0,t}^f - \Gamma_{1,t}^f j_{mid} + \Gamma_{2,t}^f j_{mid}^2) dj \\
& \sim \tfrac{1}{3} \Gamma_{2,t}^f j^3 + \tfrac{1}{2} (\Gamma_{1,t}^f - 2 j_{mid} \Gamma_{2,t}^f) j^2 + (\Gamma_{0,t}^f - \Gamma_{1,t}^f j_{mid} + \Gamma_{2,t}^f j_{mid}^2) j \ |_{\bar{j}}^{1.5}
\end{aligned}$$

The second order approximation for the expression γ_t^f is given by:

$$\gamma_t^f \sim \frac{1}{3} \Gamma_{2,t}^f (1.5^3 - \bar{j}^3) + \frac{1}{2} (\Gamma_{1,t}^f - 2 j_{mid} \Gamma_{2,t}^f)(1.5^2 - \bar{j}^2) + (\Gamma_{0,t}^f - \Gamma_{1,t}^f j_{mid} + \Gamma_{2,t}^f j_{mid}^2)(1.5 - \bar{j})$$

F Appendix F: The Particle Filter Code

F.1 The Steady State

The steady state cannot generally be found analytically. After simplifying my model equations, I use the Fortran non-linear solver, DNEQNF, which is available via the IMSL computing library. I solve for the remaining variables analytically (9 in the time-invariant case and 14 in the time-varying case). Of these, the majority are AR(1) shocks, which can easily be solved for the steady state value.

F.2 The Second-Order Model Solution

The model solution is given by the function g, which determines the time t control variables given the time t state variables:

$$y_t = g(x_t)$$

and the function h, which describes the evolution of the state variables:

$$x_{t+1} = h(x_t) + \nu\sigma\eta_{t+1}$$

Let n_y be the number of decision variables and n_x be the number of state variables.

The large scale of the DSGE model means that the policy functions cannot be solved for analytically. Thus, I use the second order perturbation method described by Schmitt-Grohe (2005). The second-order approximations of g and h take the following form:

$$\begin{aligned}[g(x,\sigma)]^i = \;& [g(\bar{x},0)]^i + [g_x(\bar{x},0)]^i_a[(x-\bar{x})]_a + [g_\sigma(\bar{x},0)]^i[\sigma] \\ & +\tfrac{1}{2}[g_{xx}(\bar{x},0)]^i_{ab}[(x-\bar{x})]_a[(x-\bar{x})]_b \\ & +\tfrac{1}{2}[g_{\sigma\sigma}(\bar{x},0)]^i[\sigma][\sigma]\end{aligned}$$

and

$$\begin{aligned}[h(x,\sigma)]^j = \;& [h(\bar{x},0)]^j + [h_x(\bar{x},0)]^j_a[(x-\bar{x})]_a + [h_\sigma(\bar{x},0)]^j[\sigma] \\ & +\tfrac{1}{2}[h_{xx}(\bar{x},0)]^j_{ab}[(x-\bar{x})]_a[(x-\bar{x})]_b \\ & +\tfrac{1}{2}[h_{\sigma\sigma}(\bar{x},0)]^j[\sigma][\sigma]\end{aligned}$$

where $i = 1, ..., n_y$, $a, b = 1, ..., n_x$ and $j = 1, ..., n_x$. Note that n_y is the number of control variables and n_x is the number of state variables. The superscript on the function derivative describes the matrix row, and the subscript describes the column. For example, the expression $[g_x(\bar{x},0)]^i_a$ refers to the (i,a) element of the matrix $[g_x(\bar{x},0)]$.

Schmitt-Grohe (2005) provides Matlab code to estimate each component of the h and g functions described above. I have converted this code to Fortran, and it is used in the particle filter to estimate the second-order accurate model solution for each set of parameter values. The solution functions are then used by the particle filter to calculate the likelihood function.

F.3 The Particle Filter

The particle filter assumes a continuous decision rule, h, by which the state variables of the model evolve, given a vector of shocks, u_t and the time t state variable values, x_t. The shocks are i.i.d. with means of zero. The evolution of the state variables is described by the following equation:

$$x_{t+1} = h(x_t, u_t)$$

The control variables are determined by the policy function, g, given the shocks u_t and the time t values of the state variables:

$$y_t = g(x_t, u_t)$$

Before describing the particle filter, the following definitions will be useful. A particle refers to a draw from the distribution of state variables. A swarm is a collection of many particles. Thus, a swarm drawn from the distribution of state variables should itself closely represent the distribution of these state variables.

We want to choose the set of parameters, μ, which maximize the likelihood function of the observed data conditional on the parameter values:

$$L(X^T|\mu) = \pi_{t=1}^T \int p(\upsilon_t(X^t, s_0))p(s_)|X^t)ds_0$$

To calculate the probabilities, the particle filter follows the following steps:

1. Initialize: Each particle in the first swarm is drawn from a normal distribution with a mean of each state variable's steady state value and a variance calculated from the simulated standard deviations from the first order simulation

2. For each particle, x_{it}, in the swarm, draw an i.i.d. exogenous shock, u_{it}

3. Construct the period t control variables and the period $t+$ decision variables according to the decision rules:

$$x_{t+1} = h(x_t, u_t)$$

$$y_t = g(x_t, u_t)$$

4. Assign a likelihood to each particle based on the assumption that forecast error is normally distributed:

$$y_{it} - y_t = e_{it} \sim N(0, \sigma_e)$$

5. Use importance sampling with the likelihoods calculated in the previous step to draw particles from the period t swarm to construct the period $t+1$ swarm, using the policy function:

$$x_{it+1} = h(x_{it}, u_{it})$$

6. Continue the process starting at step 2 for each period of observed data

After the process has completed, the average likelihood of all particles will be the estimate for $L(X^T|\mu)$

F.4 Global Optimization

The Tibor Csendes' GLOBAL routine[14] is used to maximize the likelihood function calculated by the particle filter. The parameter values at the global maximum likelihood are used as the starting point for the Metropolis-Hastings chain. The global routine is time-consuming and takes approximately 24 hours to complete for the time-invariant model with 20,000 particles and approximately 36 hours to complete for the time-varying model with 20,000 particles.

F.5 Metropolis-Hastings

The Metropolis-Hastings chain uses the parameter values that maximize the likelihood as its starting point. For subsequent draws, the candidate vector of parameter values is drawn using the last draw and the jump density. The jump density for each parameter was originally the posterior variance from the first order parameter

[14] The GLOBAL fortran code is available on Csendes' website, $http://www.inf.u-szeged.hu/\ csendes/linkek_en.html$

estimation. However, some jump densities were altered to allow the chain to move more quickly. After constructing the candidate draw, its likelihood is evaluated using the particle filter. Again,the particle filter uses 20,000 particles in each period to calculate the likelihood. The ratio of the likelihood of the candidate to the likelihood of the last period's draw is then multiplied by the ratio of the proposal density in both directions. Let the product of the two expressions be referred to as a. Then, the chain accepts the candidate draw with a probably of a. It stays with last period's draw with a probability $(1 - a)$.