

# **A STRIVE TO NET-ZERO: INSULATION IN RESIDENTIAL HOUSING**

A Research Paper submitted to the Department of Mechanical and Aerospace Engineering  
In Partial Fulfillment of the Requirements for the Degree  
Bachelor of Science in Mechanical Engineering

By

Jack Pazin

May 6, 2022

On my honor as a University student, I have neither given nor received unauthorized aid on this assignment as defined by the Honor Guidelines for Thesis-Related Assignments.

ADVISOR

Harsha Cheliah, Department of Mechanical and Aerospace Engineering

# **A Strive to Net-Zero: Insulation in Residential Housing**

A Technical Report submitted to the Department of Mechanical and Aerospace Engineering

Presented to the Faculty of the School of Engineering and Applied Science  
University of Virginia · Charlottesville, VA

In Partial Fulfillment of the Requirements for the Degree  
Bachelor of Science, School of Engineering

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Spring, 2022

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On my honor as a University Student, I have neither given nor received  
unauthorized aid on this assignment as defined by the Honor Guidelines  
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# A Strive to Net-Zero: Insulation in Residential Housing

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## **1. Objective**

The initial objective of this project was to design and retrofit an existing building to have a net-zero energy footprint. The focus was for the design to follow the guidelines set out by the Race to Zero design competition (*U.S. Department of Energy*, 2018). Due to budget and logistical constraints, the scope of the project was limited to the design of three systems: energy generation, heating and cooling, and insulation. Our team's project focused on the development of the insulation system for the house to maximize energy savings and minimize the work needed to heat the house from the solar panels that the energy generation team would be incorporating into the design. In addition to creating the most efficient system possible for the house, goals of sustainability, safety, and affordability were incorporated into design considerations. Due to obstacles encountered throughout the project, the focus changed from physically renovating to suggesting the renovations and designs that could be installed and implemented in the future. The objective, however, remained to determine the best insulation system according to our design goals. Collaboration with the other groups also became less active, but we still wanted to consider the effects of our design on the other two categories.

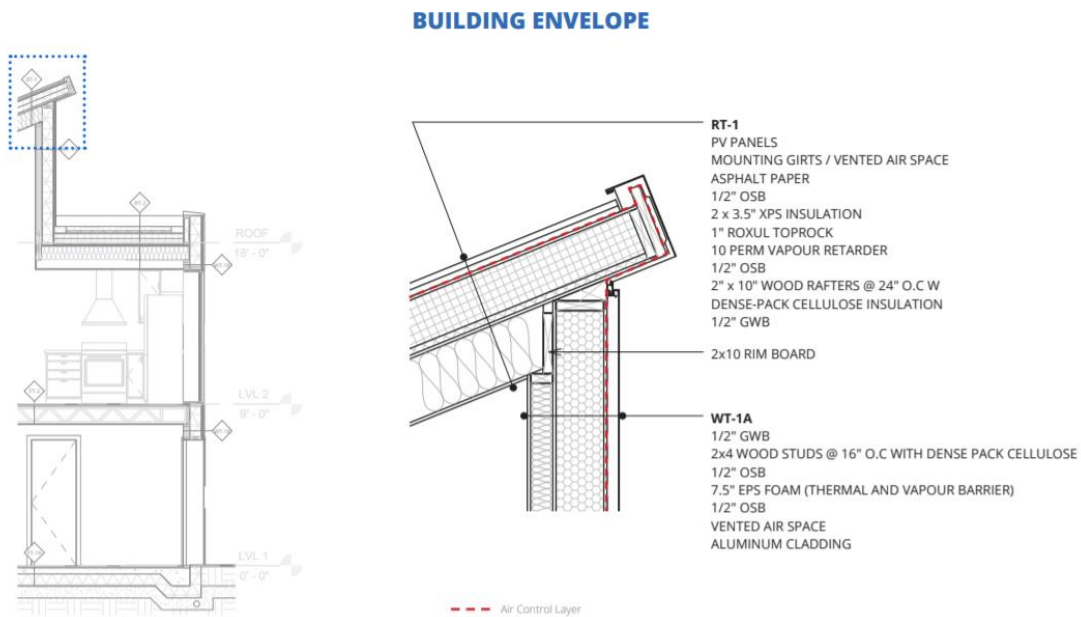
## **2. Background**

### **2.1. Existing Systems**

The purpose of insulating a space is to reduce unwanted thermal transport into or out of a system. Insulation performance is quantified by a property called R-value. A higher R-value corresponds to higher thermal resistance, thereby reducing thermal transport. The total thermal resistance through a material consists of conduction through the solid wall, convection through the air, and radiation exchange with the surroundings. Gas conduction has by far the largest

impact on thermal conductivity. More porous materials are better for insulation because they have a lower thermal conductivity due to a greater overall presence of air in the volume, and thus prevent heat transfer through the material. However, moisture can affect how well a material insulates. In contrast, thermal masses, like mud bricks, while useful for specific applications, absorb heat and release it later on. High density thermal masses are not good insulators for this reason (Gangassaeter, et al., 2017).

Typical existing systems utilize a layered building envelope to minimize the thermal transport and heat losses or gains from any gaps. Figure 1 presents an example building envelope from the 2017 Race to Zero Competition. As shown, both cellulose and EPS foams are used as insulation. These are very porous and lightweight materials, creating a highly thermally-resistant system.



**Figure 1.** Example building envelope showing layers of two types of insulation. (U.S. Department of Energy, 2017)

### 2.1.1. Comparison of Efficiencies

Based on the R-values for different materials, the heat loss in Watts (Joules per second) was calculated for a few selected materials across a temperature gradient of 30°C. From this heat loss analysis, it can be seen clearly that aerogels and vacuum insulated panels result in far lower losses when compared with the other materials. Using materials with a lower thermal conductivity will greatly increase the efficiency of the system by lessening the demands imposed upon the heating and cooling system of the building. Using the best thermal performing material would help the net-zero building to perform much more efficiently in warmer parts of the year.

**Table 1.** Heat Loss Per Hour for Selected Materials

<b>Material</b>	<b>Rate Energy Loss (W per sq. meter)</b>
Fiberglass batts	44.826 - 58.737
Cellulose	44.826 - 54.948
Mineral Wool batts	51.618 - 56.78
Aerogel	4.259
Vacuum Insulated Panels	3.407
Extruded Polystyrene	34.068 - 37.853
Expanded Polystyrene	47.316
Polyurethane	29.293 - 30.417

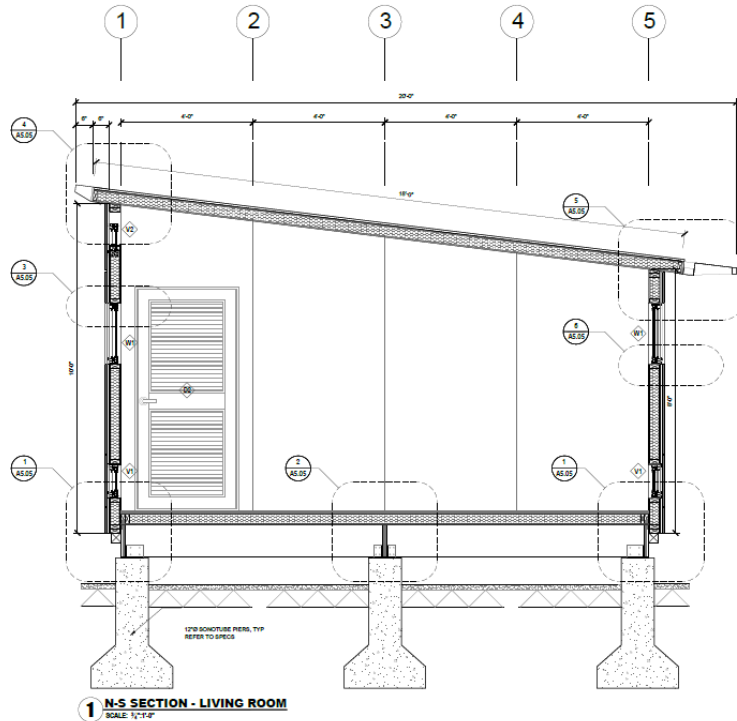
Polyisocyanurate	29.293 - 30.417
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Ideally, all of these materials would have been tested and the heat losses compared to these literature values, but many supply chain and logistical issues led to this not being possible. The materials to note here are fiberglass, extruded polystyrene (XPS), expanded polystyrene (EPS), and mineral wool. These are the materials that we ended up testing, and according to this chart, XPS should behave most efficiently, leading to the least heat loss per unit area for any temperature gradient.

**2.2. Design Constraints**

Building a house from the ground up would be time-consuming and costly, so our team found an existing building that could be retrofitted into a net-zero house. A small UVA-owned building, named Milton Airfield reCOVER building, was chosen for the project. It is located just outside Charlottesville and had previously been built as a prototype design for housing in Haiti. Because we started with a structure that was already built, certain constraints were considered for our design. For our team, which was designing the insulation for the house, the most important constraint was the current wall thickness, which from the prototype drawings was determined to be 4.5”, with two ½” pieces of drywall on each side and 3.5” of expanded polystyrene insulation in between.





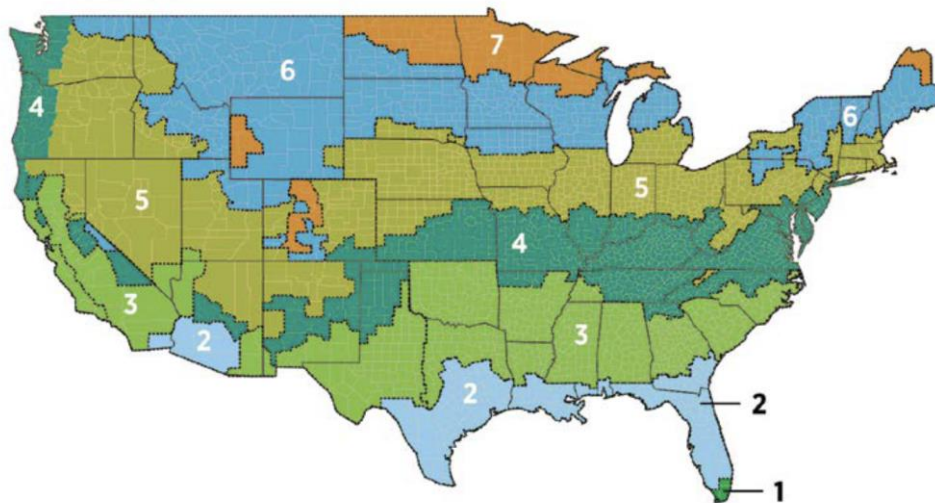
**Figure 2.** Cross Section of House Drawing with Dimensions. (*ReCOVER TDRH Prototype 2\_Partial Set [PDF]*).

Due to logistical constraints, our team did not obtain permission to physically renovate the building. The scope of the design was modified to be built in-lab, where various insulation materials would be tested. Based on our findings, we were able to recommend the best insulation for future installation in the reCOVER house. Other factors such as budget and timing imposed significant constraints on the project. Our class as a whole did receive funding for our design, but we did not request materials that were unreasonably expensive. Because of this, money was certainly an important consideration in the design of our insulation system. We were limited to a time constraint of two semesters to design, test, and analyze our model. While the initial goal was to renovate the reCOVER house itself, after running into challenges gaining permission to alter the reCOVER building, the scope of our project changed dramatically. In the future, it may

become feasible for a Capstone group to use our findings to perform a full-scale renovation at the Milton Airfield property.

### 2.3. Standards and Codes

Insulation standards and R-value requirements vary regionally across the United States. The Office of Energy Saver (OES) divides the country into 7 climate zones (Figure 3), each with different insulation expectations for walls, attics and floors. The reCOVER building at Milton Airfield in Charlottesville, VA falls in region four of the OES' climate zones. This region expects R-38 for ceilings, R-19 for floors, and R-13 for walls of residential buildings (*IECC*, 2009).



**Figure 3.** Climate Zones in the United States (OES, n.d.)

Just as different materials vary in thermal performance, they also provide different levels of safety. Certain materials are noncombustible, flame resistant and will only ignite at very high temperatures, while others such as cellulose need to be treated with fire-retardant chemicals prior to installation (*Insulation Institute*, n.d.). Materials with facings like foil and kraft paper cannot be left exposed and must be installed with a code-approved barrier. The chemicals and dust from certain materials can also raise health concerns. Fiberglass and mineral wool are the most

thoroughly tested materials, while a large portion of spray foams still need to undergo extensive testing for an appropriate safety evaluation. The final safety qualities of insulation are the material's resistance to mold and corrosion. Inorganic materials such as fiberglass, mineral wool and spray foams are effective against mold as they do not promote mold to spread. Non-corrosive materials help protect pipes and wires that would otherwise be subject to corrosion from certain chemicals in insulation. In addition to thermal performance, insulation systems must consider fire and health safety as well as mold and corrosion resistance.

Those are the standards for any residential building, but buildings aiming for a net-zero energy footprint go even further. From an environmental approach, this entails using materials with recycled content. From a performance perspective, this means thicker walls or materials with significant thermal resistance. Modern day net-zero energy buildings in mild climates, such as region four in Figure 3, are reported to range from R-19 and higher (*Zero Energy Project*, n.d.). Since the reCOVER building was initially designed for the 1.3 million people left homeless after Haiti's 2010 Earthquake (Ford, 2010), it is equipped to handle a tropical climate and not Virginia's diverse seasonal weather. As of now, the reCOVER building does not meet industry standards and falls far short of the standard thermal resistance of net-zero residential buildings.

### **3. Organizational Timeframe**

#### **August 21 - September 9, 2021**

- Initial brainstorming on Capstone topic of interest, decision made to focus on design of Net-Zero Energy house, following Race to Zero Design Competition guidelines

#### **September 14 - October 5, 2021**

- Switched focus to insulation design instead of appliances, did background research on where to insulate in a home, typically used materials (with a focus on sustainable

materials), how to calculate 1D heat transfer through a wall while accounting for radiation, conduction, and convection

**October 7 - 28, 2021**

- More detailed research on existing systems (including state-of-the-art materials), performed more detailed heat transfer analysis in MATLAB

**October 28 - November 18, 2021**

- More background research on reflective Raman SkyCool technology, more detailed SolidWorks analysis performed to model 3D heat transfer through house, detailed cost benefit analysis performed to determine materials needed for experimental testing

**November 30 - December 14, 2021**

- Finish up preliminary analysis and necessary research/planning for experimental design, come up with initial plan for where to order materials from

**January 25 - February 10, 2022**

- Designing a heat transfer experiment that would successfully determine the R-value of the insulation materials using the difference in temperature on either surface
- During this time we learned that we would not be able to actually renovate the reCOVER house due to restrictions set in place by facilities
- Timeline shifted due to our inability to develop full-scale tests

**February 10 - March 4, 2022**

- Assembly of the testing apparatus became the main focus of our design project
- Ordering materials, including the different insulation materials for testing as well as all the different parts required for testing

**March 14 - April 2, 2022**

- Preliminary testing of the experimental setup
- Adjusted box design by adding extra insulation to the other sides for heat loss reduction

**April 3 - April 25, 2022**

- Experimental testing of each material, 5 in total including the control group, at two different temperatures
- Completed online simulations to support the data

**April 25-May 3**

- Data analysis, mainly comparison to literature values for thermal conductivity
- Calculations to account for heat loss through all sides of the box using 1D heat transfer models
- Drawing conclusions and discussing recommendations for future work
- Performed convective current analysis through SolidWorks by modeling a heating coil in air under the effects of gravity
- Performed similar analysis through ANSYS Fluent by modeling a 2x2 plexiglass box under certain temperature boundary conditions on each face

Task	August	September	October	November	December	January	February	March	April	May
Brainstorming	■	■								
Background		■	■							
1D Analysis		■	■	■						
Existing Systems			■	■						
Cost-Benefit			■	■						
3D Modeling			■	■	■	■	■	■		
Design Phase					■	■	■	■		
Gathering Materials						■	■	■		
Assembly						■	■	■		
Preliminary Testing								■	■	
Online Simulations								■	■	■
Full Testing									■	■
Report Findings										■
Analyze Data										■

**Figure 4.** GANTT Chart

## **4. Design Specifications**

While the design must meet the industry standard of R13 (described in 2.3. Standards and Codes) for the reCOVER building's walls, an even higher R-value would advance the goal of every other team and help the reCOVER building achieve a net-zero energy footprint. It is estimated that a building in a cold climate, like Virginia, with R40-rated insulation will essentially reach a net-zero footprint (*Zero Energy Project*, n.d.). While this would not be possible without making the insulation thicker and taking away from the interior space, any R-value above the industry standard will help get the building closer to its goal.

Even though we did not plan to renovate the reCOVER building in the end, certain specifications stayed the same for our design project. The specification for R-value became less relevant, but if we were to recommend one of the testing materials for an actual residential building, these same R-values would be necessary. Another important specification involves fire resistance and safety. If an insulation is faced and installed in a concealed space, the flame spread index cannot exceed 25, and the smoke developed index cannot exceed 450 (*Henrico County Virginia*, 2022). This project did not attempt to test these fire safety qualifications, but these values can be found in literature to back up any recommendations made with any of the test materials.

## **5. Concept Developmental Details**

### **5.1. Alternate Approaches Considered**

Some state-of-the-art materials were considered for the design. Specifically, we looked into aerogels and vacuum insulation panels (VIP's). Aerogels are a type of silica gel with a complex drying process which creates a very porous structure. The resulting material has a

nanoporous structure with one of the most promising thermal conductivities of any other insulation. Pores sizes can range from 5-70 nm if correctly produced, with a resulting thermal conductivity of 12-20 mW/mK. Aerogels are 85-99.99% air by volume because of the presence of these pores (Gangassaeter, 2017). Due to the structure of the aerogels, they are very brittle and difficult for use in most industrial applications.

Vacuum insulation panels (VIP's) were also considered for use in the insulation system because of their promising thermal conductivity. VIP's consist of an open porous core and have thermal conductivities ranging from 3 to 4 mW/mK in fresh condition, and about 8 mW/mK after 25 years of use. Typically this is due to moisture diffusion or air diffusion into the vacuum envelope (Jelle et al., 2011). This is a known disadvantage of VIP's, as well as the lack of flexibility to adapt a VIP to a certain project. VIP's must be ordered in advance in a certain size, and they cannot be cut to fit the size if there is a mistake.

Due to the drawbacks and challenges associated with both Aerogels and VIP's, it was determined that, despite their promising efficiencies, these materials would not be feasible for insulation applications in a residential setting.

## 5.2. Selected System

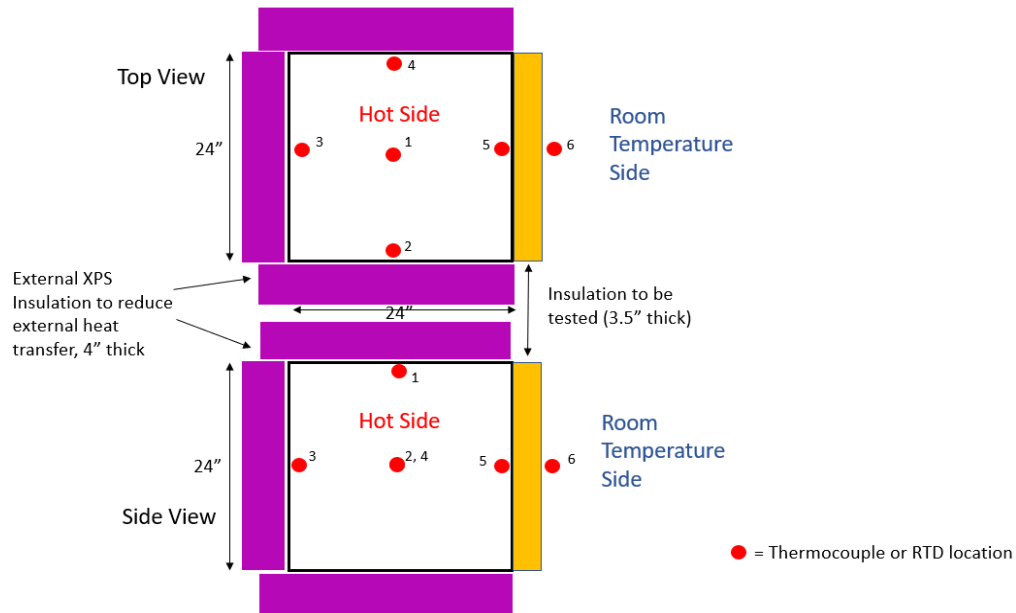


**Figures 5 & 6.** Above shown is the model with the locations of the Variac, tape heater, and wooden frame for the test wall labeled.

For our selected design system, we tested four types of insulation and compared the effectiveness of those insulations with a control, which had no insulation. We tested air (the control), mineral wool, fiberglass, extruded polystyrene, and expanded polystyrene. Based on the results from our cost benefit analysis (discussed later in the report) these materials were deemed the best for the given application (residential insulation) as opposed to the aerogels and VIPs. Due to cost constraints and material availability, we were unable to test other potentially promising materials, including denim insulation, cellulose, and polyisocyanate. These materials were tested in the lab using a three-dimensional heat transfer system. This was built using a five-sided acrylic cube measuring approximately 2'x 2'x 2', with one open side to be the designated test section for the design. At this open side, we had a test wall composed of a 2'x 2' wooden



frame built from two by fours with drywall pieces nailed to the two exterior sides, creating a 3.5” gap between the drywall faces. It was in this gap that we were able to place and remove insulation for testing. The insulation could be up to 3.5” thick. As visualized in Figure 7, additional insulation was added to the other sides of the box to minimize the rate of heat transfer occurring.



**Figure 7.** Diagram showing the thermocouple locations (red) from both top and side views.

In the inside of the box, a small tape heater increased the ambient temperature as well as the surface temperatures of the insulation. The heat was used to create a temperature gradient across the model wall containing the insulation and drywall. In order to control the temperature better, the tape heater was plugged into a Variac variable transformer. The Variac was able to control the current going into the heating tape. The six thermocouples were used to measure the temperature at different places within the testing apparatus (see Figure 7 for specific locations). Thermocouples 1-4 were used to inform us when the box had reached the desired interior temperature and thermocouples 5-6 were used to measure the temperature gradient across the

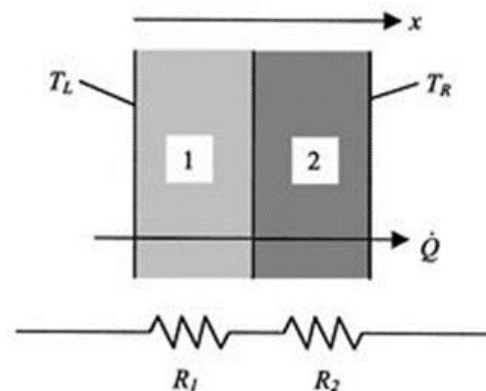
insulation. By measuring the temperature gradient, and knowing the material properties of the insulation to be tested, the rate of heat transfer going through the test wall could be determined. The tests were taken over a period of two hours in order to allow the system to reach equilibrium. Equilibrium was defined as a point where the thermocouples stayed within  $\pm 0.1^\circ\text{C}$  with the Variac set to a constant voltage.

## 6. Detailed Analysis

### 6.1. Model Development

#### 6.1.1. One-Dimensional Model

Modeling insulation inside a wall is easiest when the wall is considered an infinite plane and therefore heat is transferred only in one dimension through the wall. The model then becomes a thermal resistance network where the thermal conductivities of the drywall and various insulation materials can be added together in series. Figure 8 shows a diagram of an example thermal resistance network.



**Figure 8.** Heat transfer across a composite slab (series thermal resistance). (MIT, 2006).

The heat flowing through the insulation in this configuration is analogous to electric current flowing through two different resistors in series. Because this model can be solved with

fairly simple equations, a MATLAB code was developed to analyze the 1D system. The MATLAB code used simply involved plugging variables into the following derived equation.

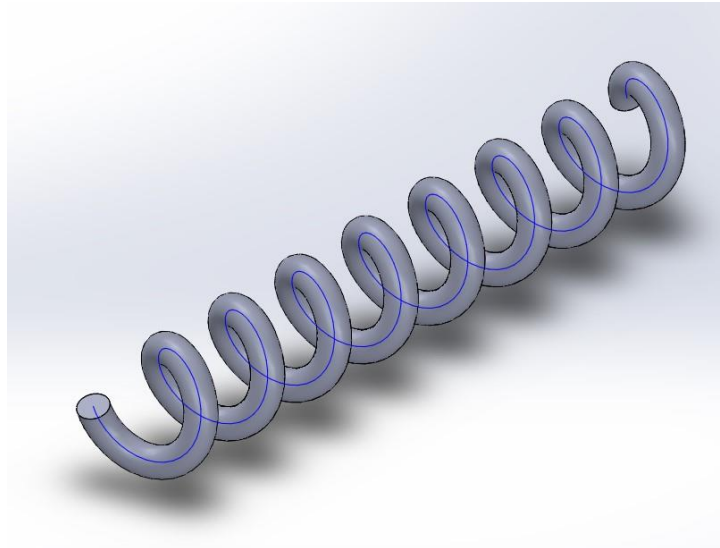
$$R_{val} = \frac{1}{\epsilon_{in}\delta (T_{s,in} + T_{\infty,in}) (T_{s,in}^2 + T_{\infty,in}^2) + h_{in}} + \frac{1}{\epsilon_{out}\delta (T_{s,out} + T_{\infty,out}) (T_{s,out}^2 + T_{\infty,out}^2) + h_{out}} + \sum_{n=1}^x \frac{L_i}{k_i}$$

In this equation,  $\epsilon_{in}$  (unitless) is the inner surface's emissivity coefficient,  $\epsilon_{out}$  (unitless) is the outer surface's emissivity coefficient,  $T_{s,in}$  (K) is the inner surface temperature,  $T_{\infty,in}$  (K) is the indoor air temperature infinitely far away from the insulation,  $T_{s,out}$  (K) is the outer surface temperature,  $T_{\infty,out}$  (K) is the outdoor air temperature infinitely far away from the insulation. Also,  $h_{in}$  (W/m<sup>2</sup>K) is the convection coefficient of the air indoors,  $h_{out}$  (W/m<sup>2</sup>K) is the convection coefficient of the air outdoors,  $L_i$  (m) is the thickness of a given panel, and  $k_i$  (W/mK) is the given thermal conductivity of a given panel.  $\delta$  is the Stefan-Boltzmann constant (5.67\*10<sup>-8</sup>W/m<sup>2</sup>K<sup>4</sup>). Overall, the equation was derived by assuming the convection and radiation components worked in parallel on each side while everything else was in series (including the results of the combined radiation and convection on each side). This derivation can be seen as analogous to electrical resistors and how their resistances add together while in series while their reciprocals add up to the reciprocal of the total resistance while in parallel.

### 6.1.2. Computational Fluid Dynamics (CFD) Simulation

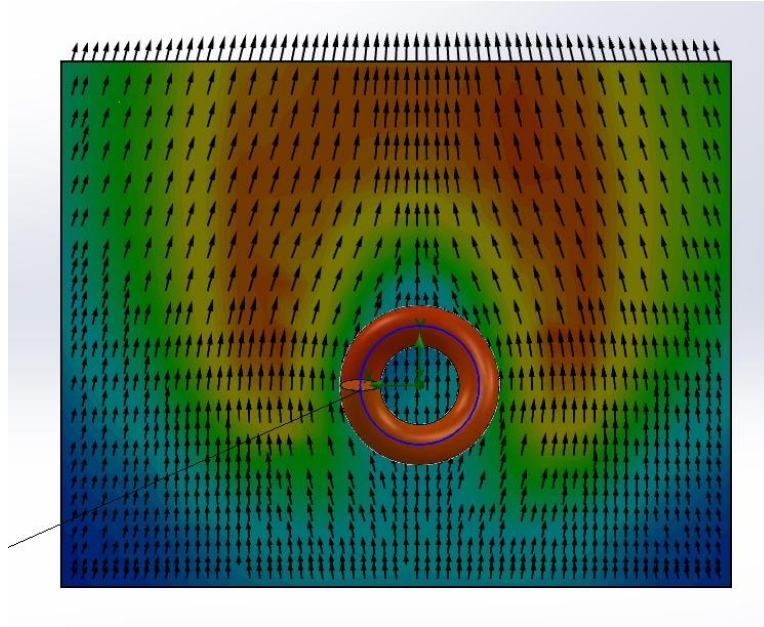
To anticipate whether convection would be present inside the heated box of the design, a preliminary CFD simulation was conducted using SolidWorks Flow Simulation. The horizontal helix coil seen in Figure 9 was assembled in SolidWorks to resemble the tape-heater. Due to limitations concerning the exact position, the number of turns, the material and the temperature of the tape-heater, this digital model was solely used to determine whether convection was

present and was not used for any quantitative results. Arbitrary dimensions, materials and temperatures were selected.



**Figure 9.** Helix Coil Assembled in SolidWorks to Resemble the Tape-Heater

The helix coil was used to run an external analysis with heat conduction in solids applied. To introduce convective currents, gravity was included in the system with the correct orientation. After the helix coil was selected as the heat source, the simulation was run and the following cut plot was generated. The contour and vectors in Figure 10 indicate that convection was present and should be expected in the physical design.



**Figure 10.** Cut Plot Showing the Presence of Convection

## 6.2. Assumptions

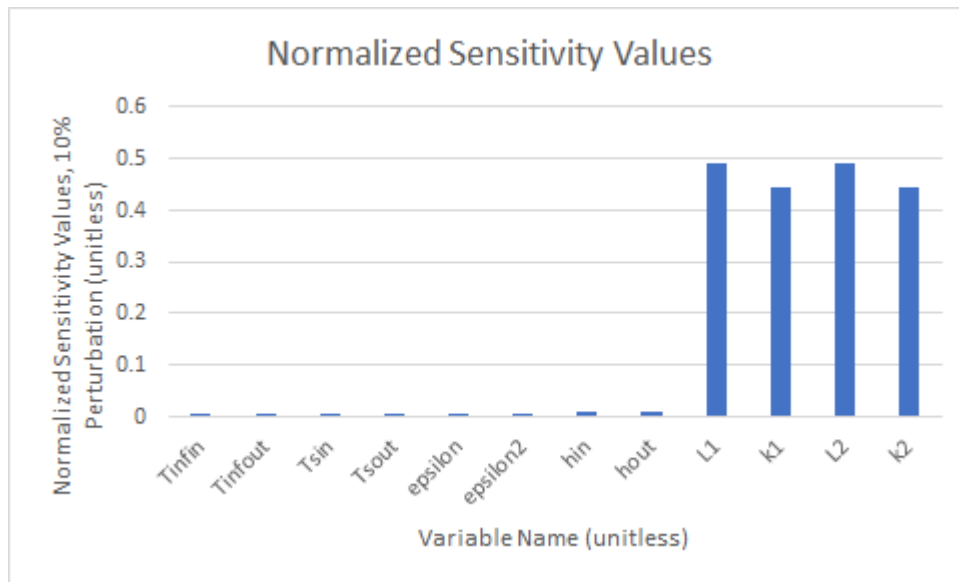
The model has many assumptions, including that there are no leaks in the materials, that each wall of the system can be treated as an infinite plane, and that the system is in a steady state, meaning it does not change with time. Using a one-dimensional assumption is reasonable given the fact that the thickness of the wall is much smaller than the length of each wall in the house, therefore allowing for each wall to be accurately modeled as an infinite plane. A steady state assumption is also reasonable considering the relatively slow rate at which temperature changes inside and outside of the house. Assuming that there are no leaks, however, may be proven incorrect when testing real prototype systems. Another assumption is that there is perfect contact between the layers of insulation, which results in no temperature drop at the interface between the materials (*Engineering Notes*, 2017). In a controlled experiment, our hope is to reach as close to perfect contact as possible between layers. Also, it is assumed that the air temperatures labeled  $T_{\infty, x}$  are measured infinitely far away from the insulation. The test space used will attempt to

make this assumption accurate by adding temperature probes relatively far away from the insulation. Another important assumption is that the existing system is composed of expanded polystyrene, and that the standard drywall we ordered is equivalent to the plywood boards that served as drywall in the reCOVER building.

### 6.3. Sensitivity Analysis

When the sensitivity analysis was performed on the one-dimensional heat transfer model, it was found that increasing the thickness of an individual panel had the largest effect on the system's R-value. This effect was the same regardless of which panel was made thicker. Changing the thermal conductivity of a panel had the second largest effect on the system's R-value and the effect was also the same regardless of which panel had its thermal conductivity changed. This effect was almost as large as the effect resulting from increasing a panel's thickness. Changing the convection coefficient of either side had the next largest effect but was very small when compared to changing a panel's thickness or thermal conductivity. Finally, changing the emissivity or temperature at any location resulted in very small effects on the system's R-value. For the sensitivity analysis, a 10% perturbation was used and values were normalized. The variables on the bar graph correspond as follows:  $T_{\text{infin}}$  (nominal value: 298K) is the indoor air temperature far away from the wall,  $T_{\text{infout}}$  (nominal value: 258K) is the outdoor air temperature far away from the wall,  $T_{\text{sin}}$  (nominal value: 293K) is the indoor surface temperature of the wall, and  $T_{\text{sout}}$  (nominal value: 263K) is the outdoor surface temperature of the wall.  $\text{Epsilon}_1$  is the emissivity (unitless, nominal value: 0.9) of the outdoor surface of the wall,  $\text{epsilon}_2$  is the emissivity (unitless, nominal value: 0.9) of the indoor surface of the wall,  $h_{\text{in}}$  (nominal value: 100W/m<sup>2</sup>K) is the indoor convection coefficient, and  $h_{\text{out}}$  (nominal value: 100W/m<sup>2</sup>K) is the outdoor convection coefficient.  $L_1$  (nominal value: 0.05m) is the thickness of

the first layer of insulation,  $L_2$  (nominal value: 0.05m) is the thickness of the second layer of insulation,  $k_1$  (nominal value: 0.1W/mK) is the thermal conductivity of the first layer of insulation, and  $k_2$  (nominal value: 0.1W/mK) is the thermal conductivity of the second layer of insulation. From this analysis it can be concluded that focusing on changing the thickness and thermal conductivity of the insulation panels will have by far the greatest impact on the insulation's R-value if the assumptions governing the one-dimensional heat transfer analysis hold true.



**Figure 11.** Normalized Sensitivity Values

## 6.4. Cost Analysis

In addition to the \$4,000 provided by the University of Virginia's Mechanical Engineering Department, \$8,000 of funding was granted from the Experiential Learning Fund (ELF) in the fall of 2021. The energy generation, heating and cooling and insulation teams requested \$5,460, \$4,250 and \$2,050 from the ELF, respectively. Our portion of the funding will be allocated towards 4 types of testing materials as well as any additional materials or tools

required to construct the design. The tape-heater, 10 thermocouples, PASCO Capstone software, and plexiglass panels will be borrowed from UVA for the design.

Before proceeding with a traditional cost-benefit analysis based solely on performance and cost, a ranking system was designed to account for the sustainability of each insulation material. The value of each material was evaluated with the equation in Figure 12, where a higher material value indicates better performance, cost and sustainability. All three inputs to the equation were found through online purchase listings at Lowe’s, Home Depot or a similar website. Each material’s R-value per inch and recycled content were explicitly stated on the listing, whereas the cost per cubic inch was found by dividing the purchase cost by the total cubic inch coverage (length x width x thickness).

$$\text{Material value} = \frac{R\text{-value per inch}}{100 \times (\text{Cost per cubic inch})} + \frac{\text{Recycled content}}{10}$$

**Figure 12.** Equation Used to Calculate Material Value

The values of each material are summarized below in Table 2, where denim is the best material and cellulose is the worst. Most materials did not consist of any recycled material or did not have the percentage listed online. It is also worth noting that spray foams were not considered in this analysis due to their incompatibility with layered insulation systems. Due to supply chain limitations, not all materials below were able to be attained during the project.

**Table 2.** Material Performance, Cost and Sustainability Analysis

Material	R-value per inch $\left(\frac{R\text{-value}}{\text{in}}\right)$	Cost per cubic inch $\left(\frac{\$}{\text{in}^3}\right)$	Recycled Content (%)	Material Value
Cellulose	1.3	0.014	n/a	0.9
Denim	3.7	0.002	80	32.1
Extruded Polystyrene	5.0	0.004	20	13.8

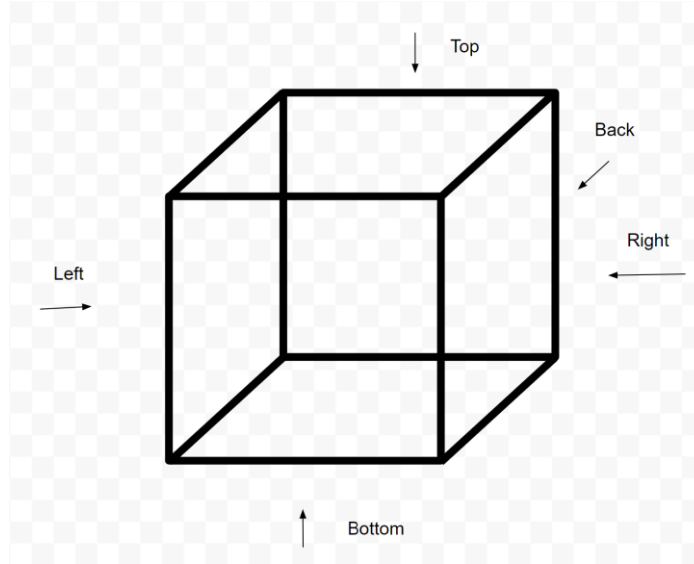


Fiberglass	3.7	0.013	n/a	2.9
Mineral Wool	4.3	0.011	n/a	4.0
Polyisocyanurate	6.0	0.007	n/a	8.6
Reflective roll	3.0	0.010	n/a	2.9

## 6.5 Experimental Results & Analysis

### 6.5.1 Estimating Heat Loss Through the Sides

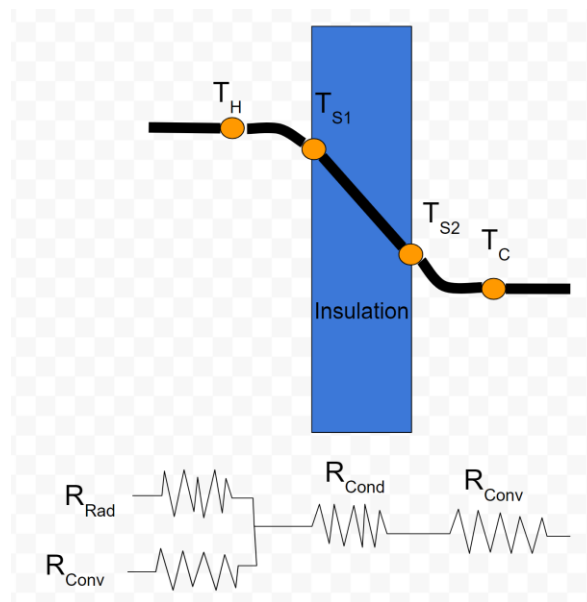
While estimating the rate of heat loss from the sides of the box, the box can be treated as a set of thermal resistance networks. The rate of heat loss can be summed up as  $dQ/dt_{Total} = dQ/dt_{Top} + dQ/dt_{Bottom} + dQ/dt_{Left} + dQ/dt_{Right} + dQ/dt_{Front} + dQ/dt_{Back}$  where  $dQ/dt_{Front}$  is the side with the insulation being tested and  $dQ/dt_{Bottom}$  is neglected due to the difficulty of calculating the rate of heat loss through the floor and the relatively small contribution shown in previous studies on full-scale homes (*Where am I losing heat in my home?*, 2016, para. 5). Figure 13 shows how the sides of the box are arranged. Calculations assumed all 1D criteria including assuming the system is in steady state and that each thermal resistance network is an infinite plane. Therefore, the equation for the rate of heat loss through the remaining sides can be written as  $dQ/dt_{Total} = dQ/dt_{Back} + dQ/dt_{Top} + dQ/dt_{Left} + dQ/dt_{Right}$ .



**Figure 13.** Diagram of the model box, with each side labeled.

The equation for calculating the rate of heat loss on each side can be written as  $dQ/dt = \Delta T/R_{Total}$ . Note that the R in this equation is NOT the same thing as a material's R-value since R-values are normalized for surface area while R within thermal resistance equations is not. This can be further written as  $dQ/dt = (T_H - T_C)/(((R_{conv}R_{Rad})/(R_{conv} + R_{Rad})) + R_{cond} + R_{conv})$  where  $R_{Rad} = 1/(\epsilon\sigma A_s(T_{s1}^2 + T_H^2)(T_{s1} + T_H))$ ,  $R_{conv} = 1/(h_{conv}A_s)$ , and  $R_{cond} = \sum_i L_i/(k_i A_{s_i})$ . This is due to a radiation and a convection process occurring in parallel on the inside. These combined processes are in series with the conduction through the insulation panels and the convection outside the box. Figure 14 below shows the locations for  $T_H$ ,  $T_{s1}$ ,  $T_{s2}$ , and  $T_C$ .  $\epsilon$  is the emissivity constant and ranges from 0-1 depending on the material's ability to emit energy as thermal radiation.  $\sigma$  is the Stefan Boltzmann constant and has a value of  $5.6704 \cdot 10^{-8} \text{W/m}^2\text{K}^4$ .  $L$  is the thickness of each insulation panel in meters,  $k$  is the thermal conductivity of each panel in W/mK, and  $A_s$  is the surface area of each panel in  $\text{m}^2$ . All temperatures were

converted to kelvin before calculating heat loss rates. Further calculations with values substituted in can be found in Appendix A.



**Figure 14.** Diagram of steady state heat loss through an individual panel. Note which forms of heat transfer are in series vs in parallel.

$dQ/dt_{sides,40C}$  was found to be about 8.5W while  $dQ/dt_{sides,50C}$  was found to be about 13W.

The heat loss calculations give reasonable values and show that the majority of the heat loss was traveling through the insulations being tested since most tests required about 40-80W to maintain steady state conditions. That being said, these calculations are not the most accurate way to find the heat loss through the remaining panels and could be improved upon by running simulations instead of relying on hand calculations with lots of assumptions that are not technically true.

### 6.5.2 Heat Loss for Insulation

Following a similar procedure to what was discussed in 6.5.1, the heat loss was calculated for each insulation material through the test wall of the model. This was done at both

temperatures tested, 40°C and 50°C, with the results for each temperature summarized in Tables 3 and 4 respectively.

**Table 3.** Experimental Rate of Heat Loss, 40°C

<b>Material</b>	<b>Temperature Gradient (°C)</b>	<b>R-value per inch (Theoretical)</b>	<b>Calculated Rate Heat Loss (W)</b>	<b>Estimated Rate Heat Loss through other 5 sides (W)</b>	<b>Variac Power (W)</b>
XPS	20.17	5	2.0418	8.483	41.34
Mineral Wool	18.518	3.0	2.6205	8.483	38.22
Fiberglass	17.606	3.71	2.0711	8.483	33.6
EPS	16.424	3.6	3.0546	8.483	37.44
Control	12.326	3.6	10.0914	8.483	44.1

**Table 4.** Experimental Rate of Heat Loss, 50°C

<b>Material</b>	<b>Temperature Gradient (°C)</b>	<b>R-value per inch (Theoretical)</b>	<b>Calculated Rate Heat Loss (W)</b>	<b>Estimated Rate Heat Loss through other 5 sides (W)</b>	<b>Power from Variac (W)</b>
XPS	28.05	5	2.8396	13.01	53.7
Mineral Wool	27.512	3	3.8933	13.01	53.7

Fiberglass	26.39	3.71	3.1044	13.01	51.3
EPS	25.53	3.6	4.7484	13.01	53.7
Control	20.054	3.6	16.4184	13.01	72.36

The first column in Tables 3 and 4 indicates the material tested, with the control defined as no insulation in the wooden frame. The second column is the temperature gradient measured across the test wall, which is the difference in measured temperature between thermocouples 5 and 6 (see Figure 7). The R-value per inch was included in the third column as a reference. The fourth column contains the experimental rate of heat loss in Watts through the test wall, which was calculated using the following formula:

$$dQ/dt = \frac{\Delta T [K] * A [m^2]}{(R/5.678) [m^2 * K/W]}$$

In this formula,  $\Delta T$  is the temperature gradient,  $A$  is the area of the wall,  $R$  is the system R-value (or the total R-value of the test wall), and 5.678 is a constant that converts R-value to RSI, which is in metric units (R-values in the United States are in imperial units (ft<sup>2</sup>·°F·h/BTU), while metric R-values are known as RSI values (m<sup>2</sup>·K/W)). For these calculations, the R-value of the system took into account the R-value of the insulation used and how thick that insulation was, the R-value of the drywall on each side, and convection currents that would be present on both sides of the wall or inside any gaps. See section 6.5.1 for a more thorough explanation of this calculation. This experimental rate of heat loss was used to calculate the experimental thermal conductivity for each material, which is discussed later in section 6.5.3.

The fifth and sixth columns of Tables 3 and 4 contain the estimated rate of heat loss in Watts through the other five sides of the box (discussed in detail in section 6.5.1) and the approximate power used by the Variac to maintain equilibrium temperature, also in Watts. The Variac power was calculated by measuring the current and the voltage that the Variac was outputting at the end of each trial and multiplying those values together. This calculation assumes that all of that energy was going into the box to heat it and does not account for losses due to resistance or inaccuracy of the measurement. For this reason, the Variac power was a secondary piece of data used for comparison; it was not used for calculations. It is, however, interesting to note that columns 4 and 5 do not sum to equal the power in column 6 in Tables 3 and 4. This could imply that more losses are occurring that have not yet been accounted for.

### **6.5.3 Thermal Conductivities for Insulation Materials**

The results for the experimental thermal conductivities of each material at each temperature are summarized in Table 5. These experimental thermal conductivity values are compared with the theoretical values found in literature. The experimental thermal conductivities were calculated using the following formula:

$$K = \frac{Q_{dot} [W] * l [m]}{A [m^2] * \Delta T [K]}$$

In this formula,  $Q_{dot}$  is the experimental rate of heat loss discussed in section 6.5.2,  $l$  is the insulation thickness used in testing,  $A$  is the area of the test wall and  $\Delta T$  is the temperature gradient. Looking at Table 5, the experimental thermal conductivities were found for each material at 40°C and 50°C. For all materials, the thermal conductivities were exactly the same out to four decimal places at each respective temperature.

Additionally, the experimental R-value for each material was calculated using the experimental thermal conductivity values substituted into the formula below:

$$R_{val} = \frac{l [m]}{k [W/(m * K)]} * 5.678$$

In this formula,  $l$  is the thickness of the insulation in meters,  $k$  is the experimental thermal conductivity in  $W/(m * K)$ , and that is all multiplied by a constant 5.678 in order to convert the metric units to imperial.

**Table 5.** Experimental Thermal Conductivities and R-Values

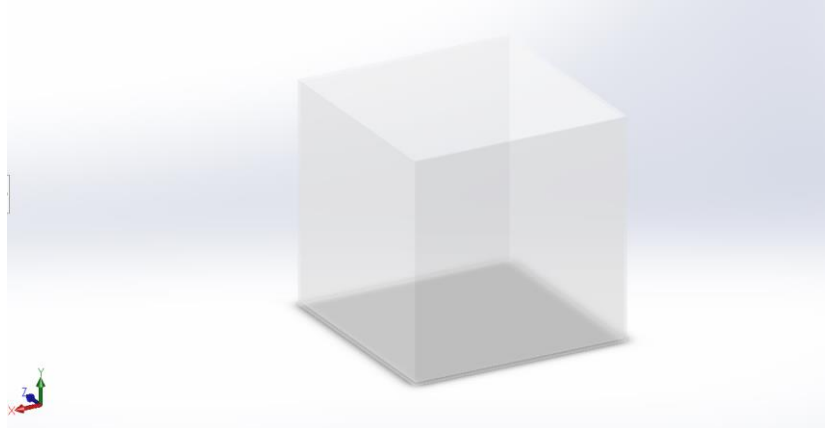
Material	Experimental Thermal Conductivity, 40°C (W/m*K)	Experimental Thermal Conductivity, 50°C (W/m*K)	Theoretical Thermal Conductivity (W/m*K)	Experimental R-value per inch
XPS	0.0252	0.0252	0.034	5.017
Mineral Wool	0.0412	0.0412	0.035	3.002
Fiberglass	0.0343	0.0343	0.043	3.706
EPS	0.0309	0.0309	0.0313	3.609
Control	0.2385	0.2385	0.024	1.94e-5

Looking at the results summarized in Table 5, we were able to verify that all four materials tested behaved the same at different temperatures by proving that they had consistent thermal conductivities. Additionally, we were able to verify the R-value per inch of each material, allowing us to conclude that these materials perform to the level at which they were advertised. Most materials performed relatively close to the thermal conductivities that were published in literature. The control did not perform close to the predicted thermal conductivity because that assumes that the air inside the test wall remains perfectly still, which is certainly not the case. Based on the simulations performed in section 6.1.2, it can be concluded that there are likely convection currents developing inside the wall when there is no insulation. This results in a much higher rate of heat loss than if the air remained perfectly still, which explains the poor agreement between the R-value of still air and the experimental R-value of the control found in testing. The extruded polystyrene (XPS) insulation performed the best, with a thermal conductivity of  $0.0252 \text{ W/(m}\cdot\text{K)}$ . This aligns with what was expected because the XPS insulation had the highest R-value going into testing at 5 per inch.

## **6.6 ANSYS Fluent Simulation**

The goal for creating 3D models and simulations for this design project was to confirm and observe possible effects of convection in air currents on our system and analysis. One way that we observed these effects was with ANSYS Fluent software. First, in SolidWorks, a model was created simply of a 2x2 cube made of  $\frac{1}{4}$  in thick plexiglass (Figure 15).



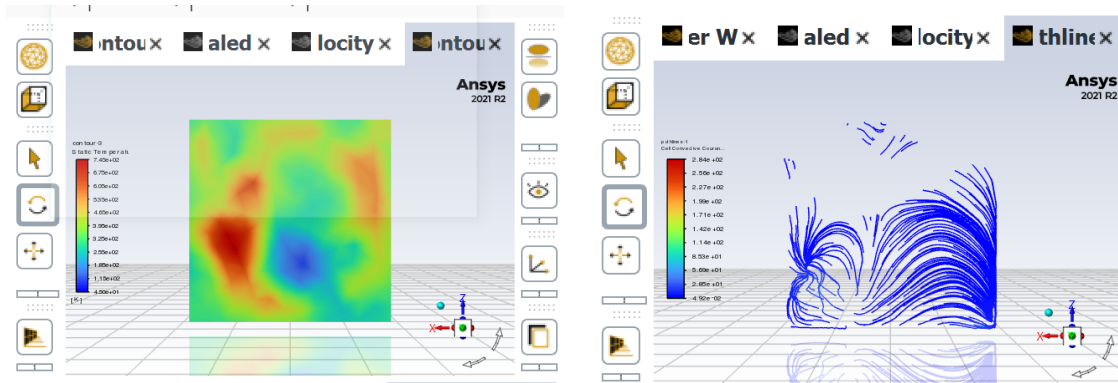


**Figure 15.** 3D SolidWorks Model

This box ended up being a simplified version of our testing apparatus, which consisted of a plexiglass box with five sides, one side being the insulation during testing, with several small holes drilled for temperature probes. The goal of the fluent simulation was to determine how air currents may have affected our results, most importantly the readings of the surface temperature on the inside surface of the insulation.

The basic steps for running a simulation in Fluent were as follows: uploading geometry, meshing, setting boundary conditions and selecting modes, and calculating results. First, the SolidWorks model was uploaded to the Workbench space in Fluent. Next, the model was meshed in order to perform the simulation. The mesh chosen was fine enough to get accurate results but coarse enough not to take extended periods of time to run calculations. Also in the mesh window, each side of the box was given a name so that it would become its own zone where boundary conditions could be added. In order to see the results of convection inside of the testing box, fixed temperature boundary conditions were added to each wall except where the insulation would go based on values found experimentally during one of the tests. This was possible because each side of the box had a temperature probe stuck through it. Although those values were not technically surface measurements, they were assumed to be close enough to see

meaningful results. Gravity was also added in the correct direction, which ended up being the negative z-axis in this case based on the labels of each side. Finally, a very small velocity in the z direction was added to simulate air flow inside the box. The Energy mode was used and the flow was set to laminar so that temperature and convective currents could be studied, the results are shown in the following figure.



**Figure 16.** ANSYS Fluent Simulation Results

The picture on the left shows the temperature gradient over the surface of the box that represents the insulation side, and the picture on the right represents the convective currents flowing against this same side of the box. Looking at both of these visual representations of convection, it can be concluded that the surface temperature of the insulation likely varied during testing, so a more accurate result could be obtained by taking measurements in multiple places on the surface. These graphs essentially show that convection is occurring, and that our results may be slightly different from literature values because we could not easily account for this in our calculations of thermal conductivity and R-value.

## 6.7 Energy Savings Analysis

Based on the experimental R-values found as results of our experiment, it is possible to estimate the total savings one would expect if the insulating material inside the reCOVER house

was changed to a better performing one. This calculation is important because one of the objectives of our design experiment was to design a system that would minimize energy usage and cost. Many assumptions were required to make this calculation, but it is still a useful way to visualize the effectiveness of upgrading insulating materials. The first assumption was that someone could actually live in the reCOVER house and use the average amount of energy that a Virginia home uses, which is not a good assumption in reality but is applicable for this calculation. The next assumption, gathered from literature on the topic, is that 50% of the energy cost for the house is due to heat escaping from the roof, walls, and foundation (Pierce, 2020). Finally, it was assumed that the average energy bill in Virginia was \$132.50 per month (SaveOnEnergy, 2022). Table 6 summarizes the calculations made to determine total energy savings in a year by changing the insulating material from EPS to various test materials.

**Table 6.** Experimental Energy Savings Analysis

Material	Temp. Gradient (C)	Experimental R-value per inch (m <sup>2</sup> *K/W*in)	Heat Transfer per unit area (W/m <sup>2</sup> )	Difference in Heat Loss from Control (%)	Energy Bill Savings per month	Energy Bill Savings per year
XPS	20.17	5.017	16.69580569	27.13352398	\$17.98	\$215.71
Mineral Wool	18.518	3.002	27.7451223	21.08965128	\$13.97	\$167.66
Fiberglass	17.606	3.706	22.40428649	2.219669157	\$1.47	\$17.65
EPS (control)	16.424	3.609	22.91287654			

The following equation was used to calculate the heat transfer per unit area through the walls of the house:  $\dot{q} = \Delta T / R_{val}$ . The change in temperature,  $\Delta T$ , was found experimentally, as well as the R-value,  $R_{val}$ . Because we would be comparing the same area of walls, floor, and ceiling in the house between the different materials, we determined that it was acceptable to use

the heat transfer per unit area for the calculation. Because any heat transfer through the walls of the house would mean heat loss, the difference between each material and the control, EPS, was calculated in the form of a percentage. Taking the control as being 50% of the energy bill cost, which was another assumption, the difference from control percentage value was multiplied by half of the energy bill. This value represented the difference in cost between the two materials.

From these values, it can be concluded that XPS will offer the most energy cost savings if installed in the reCOVER house. This result made sense because XPS had the highest experimental R-value and therefore should have been the best-performing material. In addition, XPS scored well in terms of sustainability and cost, which further solidifies the conclusion that XPS would be the best design choice in the reCOVER house, and theoretically in any other home as well.

## **7. Summary/Conclusions**

### **7.1. Evaluation**

The objective of the net-zero design project was to create an energy efficient residential building capable of producing as much energy as it uses in a year. Creating an efficient, cost-effective, sustainable combination of materials for insulation in the net-zero home would help alleviate the current impact residences have on the environment as well as save homeowners money on their energy bills. Existing insulation systems certainly do the job of minimizing energy loss due to significant amounts of heat transfer through exterior walls and attic spaces, but many of the materials are not environmentally sustainable or not capable of establishing a

thermal resistance high enough for a net-zero energy footprint. The lack of thermal performance from the existing insulation in the reCOVER building clearly showed a new system was necessary if it were to ever reach net-zero status.

Overall, a significant amount of research was done to better understand how insulation works and how different materials' success is measured in the industry. More state-of-the-art materials, like aerogels and vacuum insulation panels, were ruled out due to the logistics of implementing these materials in a residential setting. The goal was to design a model to test other insulation materials that could realistically be used to replace the existing insulation at the reCOVER house. Once the basic design for the model was known, an analysis was done. This analysis consisted of creating a 1D heat transfer model using radiation, conduction, and convection to predict rates of heat loss, creating an insulation testing apparatus, and creating SOLIDWORKS and Fluent simulations to analyze convective currents within the testing apparatus.

Ultimately throughout this project, we found a way to essentially add an inch of insulation to the reCOVER building without changing the thickness of the wall. This was done by constructing a design that accurately identified the thermal performance of materials based on experimental R-value. XPS insulation proved to be the best option given the available wall space, but future work is definitely needed to help this building reach net-zero status.

## **7.2 Future Work**

Future work could be done to further improve the findings and accomplishments of this project. Most obviously, the design could be used to perform more tests with different materials and temperatures. Since only 40°C and 50°C were tested, it would be beneficial to have results from colder temperatures to simulate winter conditions. To improve the material testing

procedure, a custom temperature controller (possibly using PID or LQR control) would allow for easier operation of the variac. Additional research could be done to test the effectiveness of SkyCool, a paint that reflects thermal energy that would otherwise be absorbed by the roof as thermal mass. Future work could look for more connections between other aspects of the reCOVER building to ultimately help it reach net-zero status.

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## 9. Appendix

### Appendix A: Finding heat loss through the sides

Calculations for approximating heat loss through sides at 40°C:

Left, right, and middle sections, 40°C:

$$R_{Rad} = 1/(\epsilon\sigma A_s(T_{s1}^2 + T_H^2)(T_{s1} + T_H))$$

$T_H = 40^\circ\text{C} = 313\text{K}$ ,  $T_C = 21^\circ\text{C} = 294\text{K}$ ,  $T_{s1} \sim 38^\circ\text{C}$  (estimated since surface temperatures on insulations being tested were about 2°C cooler than the inside ambient temperature),  $T_{s2} \sim 23^\circ\text{C}$

(estimated since surface temperatures on insulations being tested were about 2°C cooler than the inside ambient temperature),  $k_{\text{XPS}} = 0.03 \text{ W/mK}$ ,  $k_{\text{Acrylic}} = 0.2 \text{ W/mK}$ ,  $h_{\text{conv}} \sim 50 \text{ W/m}^2\text{K}$  (values in literature range from 10-100  $\text{W/m}^2\text{K}$  for natural convection),  $A_s = 3 \cdot 24^2 / 39.37^2 \text{ m}^2$  (due to left, right, and back sides being combined), and  $\varepsilon \sim 0.95$  (values for XPS not found, plastics range from 0.9-0.97). Note that the 39.37 being mentioned in the surface area value represents inches being converted into meters.

$$R_{\text{Rad}} = 1 / (0.95 * 5.6704E - 8 * 3 * (24^2 / 39.37^2) (311^2 + 313^2) (311 + 313)) = 0.1371$$

$$R_{\text{conv}} = 1 / (50 * 3 * 24^2 / 39.37^2) = 0.01794$$

4 inches of XPS, 0.25 inches of acrylic

$$R_{\text{cond}} = ((4/39.37) / (0.03 * (3 * 24^2 / 39.37^2))) + ((0.25/39.37) / (0.2 * 3 * 24^2 / 39.37^2)) = 3.0663$$

$$R_{\text{Total,sides}} = ((0.1371 * 0.01794) / (0.1371 + 0.01794)) + 3.0663 + 0.1371 = 3.2192 \text{ K/W}$$

$$dQ/dt_{\text{Total,sides}} = (313 - 294) / 3.2192 = 5.9021 \text{ W}$$

Top, 40°C:

$T_H = 45^\circ \text{C}$ ,  $T_{S1} \sim 43^\circ \text{C}$  (estimated), and  $A_s = 24^2 / 39.37^2$ . All other values are the same.

Convection results in a hotter top even after steady state conditions are reached.

$$R_{\text{Rad}} = 1 / (0.95 * 5.6704E - 8 * 3 * (24^2 / 39.37^2) (316^2 + 318^2) (316 + 318)) = 0.3921$$

$$R_{\text{conv}} = 1 / (50 * 24^2 / 39.37^2) = 0.05382$$

$$R_{\text{cond}} = ((4/39.37) / (0.03 * (24^2 / 39.37^2))) + ((0.25/39.37) / (0.2 * 24^2 / 39.37^2)) = 9.1988$$

$$R_{\text{Total,top}} = ((0.3921 * 0.05382) / (0.3921 + 0.05382)) + 9.1988 + 0.05382 = 9.300 \text{ K/W}$$

$$dQ/dt_{\text{Total,top}} = (318 - 294) / 9.300 = 2.5806 \text{ W}$$

Total, 40°C:

$$dQ/dt_{\text{Total,40C}} = 2.5806 + 9.300 = 8.4827 \text{ W} *$$

\*Rounding error



Estimated heat loss at 50°C:

Use same process,  $T_H = 50^\circ C$ ,  $T_{sI} \sim 48^\circ C$  (estimated), and  $A_s = 3 * 24^2 / 39.37^2$ . All other values are the same.

Left, right, and back, 50°C:

$$R_{Rad} = 1 / (0.95 * 5.6704E - 8 * 3 * (24^2 / 39.37^2) (321^2 + 323^2) (321 + 323)) = 0.1247$$

$$R_{conv} = 1 / (50 * 3 * 24^2 / 39.37^2) = 0.01794$$

$$R_{cond} = ((4 / 39.37) / (0.03 * (3 * 24^2 / 39.37^2))) + ((0.25 / 39.37) / (0.2 * 3 * 24^2 / 39.37^2)) = 3.0663$$

$$R_{Total,sides} = ((0.1247 * 0.01794) / (0.1247 + 0.01794)) + 3.0663 + 0.1247 = 3.0999 K/W$$

$$dQ/dt_{Total,sides} = (323 - 294) / 3.099 = 9.355 W$$

Top, 50°C:

$T_H = 55^\circ C$ ,  $T_{sI} \sim 53^\circ C$  (estimated), and  $A_s = 24^2 / 39.37^2$ . All other values are the same.

$$R_{Rad} = 1 / (0.95 * 5.6704E - 8 * 3 * (24^2 / 39.37^2) (326^2 + 328^2) (326 + 328)) = 0.3572$$

$$R_{conv} = 1 / (50 * 24^2 / 39.37^2) = 0.05382$$

$$R_{cond} = ((4 / 39.37) / (0.03 * (24^2 / 39.37^2))) + ((0.25 / 39.37) / (0.2 * 24^2 / 39.37^2)) = 9.2990$$

$$R_{Total,top} = ((0.3572 * 0.05382) / (0.3572 + 0.05382)) + 9.1988 + 0.05382 = 9.2990 K/W$$

$$dQ/dt_{Total,top} = (328 - 294) / 9.2990 = 3.656 W$$

Total, 50°C:

$$dQ/dt_{Total,50C} = 9.355 + 3.656 = 13.011 W$$