

# **A Large-aperture Alumina Composite Lens for Millimeter Wavelength Instruments**

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## Abstract

We introduce a new method for developing a wide-band lens for millimeter wavelengths. This method is largely based on a process known as a vacuum infusion process (VIP) which is commonly used in the field of composites. So far, we have made samples with this method but have not yet succeeded with producing a lens. We provide measurements of the optical properties of materials we have used thus far and outline the theory for producing a functioning lens with this methodology. In the end, we anticipate being able to produce such a lens for  $\leq 1,000$  USD, drastically reducing the cost for such a product.

# A Large-aperture Alumina Composite Lens for Millimeter Wavelength Instruments

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## 1. INTRODUCTION

We are developing a lens for the Millimeter-Wave Kinetic Inductance Detector Camera for Long-Range Imaging Through Optical Obscurants. The motivation for building such a camera is to have an instrument capable of detecting and imaging targets that cannot be seen at optical wavelength in the presence of obscurants like fog, clouds, rain, snow, and so on. Millimeter wavelengths work well for this because there are ‘windows’ in this part of the electromagnetic spectrum through which we can see.[1]

There are two wavebands we intend to cover with our lens. One band is centered on 90 GHz and the other on 150 GHz. We aim to create a single lens with a transmittance as close to 1.0 as possible from 70 GHz to 170 GHz. Datta et al. have created a lens for a similar frequency range (125 GHz to 165 GHz), however, their process requires extreme precision when cutting micron-sized pyramids with a dicing saw. They note that these pyramids sometimes chip. Another downside is the lead time on their lenses (a timescale of a few months). More details on their process can be found in [2].

With our approach to designing such a lens, the lead time is reduced to a timescale on the order of a few days and there is no need for extreme precision. This approach is achievable because of a vacuum infusion process (VIP); see Section A for more detail on this process. Once this process is complete, only slight alterations are required; see Section A.1.

We initially developed slabs of sample material that we intend to use for our lens (like Figure 4) with the VIP. We then cut these slabs into discs with a water jet and smoothed the surfaces of them with a lathe and sand-stones. We then measured the optical properties of these discs and input them into our algorithm to assess the expected transmittance of a lens, consisting of the sample material, within our desired frequency range.

We are in the process of attempting to construct a fully functioning lens with -325 mesh aluminum oxide<sup>1</sup> powder. For this

<sup>1</sup>Aluminum oxide, otherwise known as alumina, is an excellent material for a lens like the one we want to make. It's excellent because alumina has a high refractive index, thus a lens from this material can be made thinner than that of a lens made of a material with a low refractive index. This also results in a lower

process, we used the VIP, but with a custom-made aluminum mold instead of a flat, tempered glass sheet. The first trial run was unsuccessful due to an issue related to the repacking of the alumina powder before infusing.

## 2. METHODS

### A. Vacuum Infusion

The key process to our success so far is the VIP; a common process in the field of composites. The setup begins with applying mold release to the surface that will be touching the infusion material; this step makes it much easier to remove the finished product from whatever surface it is touching while curing. In the case of our samples, the surface is a 33" x 33" x 3/8" sheet of tempered glass. In the case of our lens, the surface is a custom-made aluminum mold. The plumbing for the system (i.e. connecting the vacuum to the catch pot, etc.) is then assembled. From here, we place metal rods with a diameter approximately equal to the thickness of material we want. The separation between the rods is the smaller of the two dimensions we want to have for the sample in the end. After the rods are secure, we fill the space between them with the desired infusion material (in our case, we are using highly-pure alumina grains). We then remove the rods, and place peel ply<sup>2</sup> over the infusion material.

After this step, it is optional to place a flow media over top of the peel ply. If this is done, the flow media needs to be cut so that the resin does not escape over the sides of the infusion material before it seeps all the way through the short dimension (the rod separation). The flow media is only necessary if the infusion material isn't permeable enough for the resin to flow a reasonable distance of the material. The flow media uniformly distributes the resin on top of the infusion material so that the resin only has to soak through the thickness of the material, which is typically much smaller than the width dimension (the rod separation).

After this optional step, a hollow threaded rope with a barbed tee in the center of it is placed on the side opposite to where the resin will be injected. The length of this rope should be slightly less than that of the peel ply dimension to which it is placed parallel to. Then, a spiral tube with a barbed tee in its center is placed on the side of the material where the infusion

loss tangent.[3]

<sup>2</sup>Peel ply is used to keep bonding materials clean until the start of the VIP. It also makes removing the material used for the VIP (like the vacuum bag and tees) much easier.

is introduced. The spiral tube length should be about 2 to 4 inches shorter than the infusion material's dimension to which it is parallel to. A perimeter of vacuum-grade tacky tape<sup>3</sup> is then placed about 3" away from the edges of the peel ply.

Once the vacuum tape is securely in place, a sheet of vacuum bag material, that is at least 6" larger in length and width than the vacuum tape perimeter, is attached to the vacuum tape. In this step, it is important to note the approximate locations of the two barbed tees (which should be placed so that the part of the tee that isn't in the hollow rope or spiral tube is vertical with respect to the glass sheet) because folds of approximately 4" in height, also known as pleats, need to be placed there.

The last step in the setup procedure involves poking a hole in the vacuum bag immediately above the vertical portion of the barbed tees and then attaching the appropriately sized tube to the tees. If done correctly, the setup should look similar to the arrangement in Figures 1 and 2).

Once the setup is complete, it should be tested for any leaks by clamping off the tube that is for introducing the resin and then turning on the vacuum pump. If there are any leaks, the easiest way to get rid of them is to gently press around the suspected leak-point. After all leaks are sealed, the epoxy resin should be prepared.

When preparing the epoxy resin, it is important to note that different resins have different hardener to resin mixing ratios. These ratios vary depending on the type of epoxy being used. Typically, there is a volume ratio and a mass ratio. Either ratio can be used. For example, for our ratio, we used the mass ratio. The brand of epoxy we used, Endurance Technologies' 4281/4284 epoxy resin, calls for a mixing ratio of 22:100 hardener to resin mixing ratio. Therefore, each time we need to produce epoxy resin for an infusion procedure, we determine the volume of epoxy resin<sup>4</sup>,  $V_{ER}$ , we will need for our sample, multiply by the epoxy resin's density,  $\rho_r$ , to get the total mass of the epoxy resin,  $M_{ER}$ , and then multiply that mass by  $\frac{100}{122}$  to get the mass of resin,  $M_{res}$ , needed and  $\frac{22}{122}$  to get the mass of hardener,  $M_h$ , needed. This process is outlined in equations 1 through 3.

$$M_{ER} = V_{ER} * \rho_r \quad (1)$$

$$M_{res} = \frac{100}{122} M_{ER} \quad (2)$$

$$M_h = \frac{22}{122} M_{ER} \quad (3)$$

It is also important to take note of the "pot-life" of the resin because it cannot be effectively used after this duration of time. When choosing a resin, the easiest resins to use for the VIP are the ones with low viscosity because they permeate material the best. In the case of our samples, we also need to pay attention to the index of refraction of the resin since it will effect the refractive index of our finished product.

After the resin is prepared, it can be introduced to the infusion material. To do so, the vacuum pump must be on and the resin-pot must be below the surface on which the infusion material lays. If the pot is above this point, the resin will not be sucked into the material by the VIP. Rather, a "resin-dump" will take place which will result in a non-uniform final product.

<sup>3</sup>FIBREGLAST's Yellow Sealant Tape works well for a vacuum-grade tape.

<sup>4</sup>This volume is determined by knowing the volume of the final product (something infused with resin) and then multiplying that volume by the filling factor (we typically assume a filling factor of 0.5 if we don't already know it).

Before removing the clamp from the resin-infusion-tube, it is necessary to make sure the end of the tube is completely submerged in the resin so that no air is sucked into the vacuum. Then, the clamp is removed from the tube. The resin should flow through the tube and into the infusion material. Once the material is infused, the clamp should be placed back onto the resin-infusion-tube and the vacuum pump should be shut-off. From this step, the cure instructions for the specific resin in use should be followed.

The VIP can be modified, but the underlying, physical concepts are the same. The vacuum sucks the resin through the infusion material and ensures a uniform distribution throughout the material. In the case of our lens, we replaced the tempered glass sheet with an aluminum mold, but all other portions of the VIP procedure are the same.



**Fig. 1.** This is an example of a finished vacuum infusion process set-up without a flow media.

$$\frac{1}{f} = (n_l - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \quad (4)$$

#### A.1. Post Vacuum Infusion: Samples

We produced three measurable samples; an acrylic sample made strictly out of the resin we are using for the infusion process, a sample consisting of 80 mesh aluminum oxide infused with resin via the VIP, and a sample of -325 mesh aluminum oxide that is also infused with the resin. We cut these samples into discs with a diameter of 100 mm by using a Maxi-em water jet. We then used an adjustable insert in a lathe to secure our samples for refining their surfaces. For the acrylic sample, we used a high speed steel tool bit to level off the surface. We then sanded the surface until we achieved a uniformly thick sample (+/- 0.003"). However, the -325 mesh alumina sample required us to use a carbide tool bit to level the surface because our sample is essentially sapphire and, thus, destroys the high speed steel. Also, we used diamond encrusted smoothing stones to "sand" the alumina sample to a uniform thickness (+/- 0.001"). The underside of each sample (the side touching the tempered glass during infusion) does not require any refining because it is already smooth after the VIP. Therefore, we only smoothed out the side of the samples that were in contact with the peel ply during the VIP.

We sent these samples to Jack Sayers and Fabien Defrance at Caltech for measurements of the refractive index and loss



**Fig. 2.** This is a completed vacuum infusion process set-up with a flow media. The alumina powder is the white material under the red grid (the flow media). The peel ply is the translucent, white sheet with masking tape along its perimeter. The yellow substance around the perimeter is vacuum-grade tacky tape. The folds in the vacuum bag (the pink plastic on the surface) are pleats. Pleats are necessary for attaching the two plastic tubes to the barbed tees in the yellow rope and white spiral tube. Without them, an irreparable, large hole would form at the connection point between the tubes and their respective tee.

tangent. Both samples are 100 mm in diameter, but the acrylic is  $4.61 \text{ mm} \pm 0.11 \text{ mm}$  while the 325 mesh alumina sample is  $4.34 \text{ mm} \pm 0.02 \text{ mm}$ .

The samples' optical properties are measured via a set up at the California Institute of Technology at room temperature (293 K). The angle of incidence of the light beam used to measure their properties is  $15^\circ$  with respect to a line normal to the sample's flat face.

We put these samples through 3 cycles of rapid thermal cooling/heating from about 300 K to 80 K. When the samples were approximately at their minimum temperatures, we tested their durability. The only portion of the samples to break were the edges that weren't filleted.

We also calculated the permeability of the -325 alumina sample using equation 5, a form of Darcy's Law. Here,  $v$  is the velocity of the flow front,  $L$  is the distance the resin travels through the powder medium,  $\mu$  is the dynamic viscosity of the



**Fig. 3.** This is an example of what happens if no flow media is used on an infusion material that is not permeable enough. Here, we performed a resin dump to try and force the infusion, rather than letting the vacuum suck the resin through the material. For clarity, a resin dump is performed by raising the bucket of resin above the infusion surface.

resin ( $\mu = 150 \text{ cps}$ <sup>5</sup> in the case of our resin), and  $\delta p$  is the total pressure drop ( $\delta p = \text{atmospheric pressure}$ , unless the VIP is being performed in a pressurized chamber).

$$k = \frac{-vL\mu}{\delta p} \quad (5)$$

We calculated the filling factor<sup>6</sup> via Equation 9, where  $F_F$  is the filling factor,  $V_s$  is the volume of the infusion material, and  $V_{tot}$  is the total volume. The infusion material's volume was measured before the VIP. The total volume is calculated through the use of Equation 8, where  $V_r$  is the volume occupied by the resin which is found with Equations 7 and 6.  $\rho_r$  is the density of the resin ( $1.12 \text{ g/cm}^3$ ),  $M_r$  is the mass of the resin,  $M_s$  is the mass of the infusion material (measured before the VIP), and  $M_{tot}$  is the total mass (measured after the VIP).

$$M_r = M_{tot} - M_s \quad (6)$$

$$V_r = \frac{M_r}{\rho_r} \quad (7)$$

<sup>5</sup>A unit of cps is equal to 0.001 millipascal second

<sup>6</sup>The filling factor is the ratio of the volume occupied by the infusion material (i.e. alumina) to the total volume of the final product (i.e. alumina infused with resin).



**Fig. 4.** This is the side of the -325 mesh alumina sample that was touching the glass during the infusion. It's much smoother than the top side (shown in Figure 5) because it was touching the flat tempered glass.

$$V_{tot} = V_s + V_r \quad (8)$$

$$F_F = \frac{V_s}{V_{tot}} \quad (9)$$

## B. Predicting Lens Performance

To best understand the expected performance of the lens, we developed a script to analyze the transmittance of the lens based on both the wavelength/frequency of incident light and the angle of incident light. The transmittance of the lens is given by equation 11 where  $T$  is the transmittance,  $\hat{n}_t$  is the refractive index of the medium through which light is transmitted,  $\theta_t$  is the angle at which the transmitted light comes out of the lens (this can be found with Snell's Law),  $\hat{n}_i$  is the refractive index of the initial medium,  $\theta_i$  is the angle of incidence,  $t$  is given by equation 10, and  $t^*$  is the complex conjugate of  $t$ . This equation comes from Hecht [4].

$$t = \frac{2Y_0}{Y_0 m_{11} + Y_0 Y_s m_{12} + m_{21} + Y_s m_{22}} \quad (10)$$

$$T = \frac{\hat{n}_t \cos \theta_t}{\hat{n}_i \cos \theta_i} t t^* \quad (11)$$

To find  $t$ , equations 12, 13, 14, and 15 are needed (these equations are also from Hecht)[4]. In these equations,  $\epsilon_0 = 8.85E - 12$



**Fig. 5.** This is the top side of the -325 tabular alumina sample. It's rougher than the smooth side because peel ply and flow media were on top of the powder before the infusion. This resulted in the pattern on the surface. For our sample disc, we smoothed this surface.

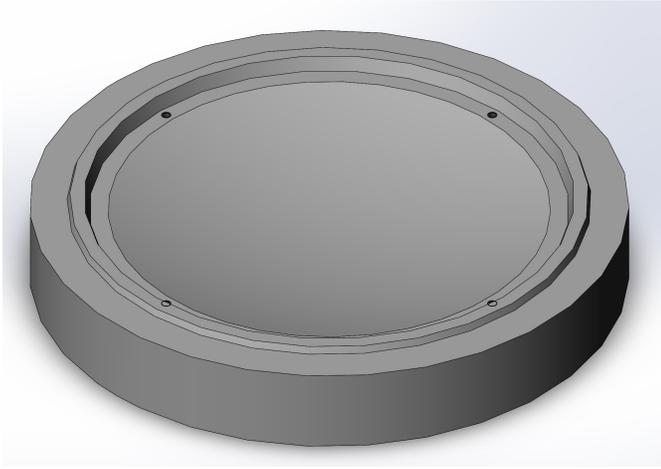
[ $s^2 C^2 m^{-3} kg$ ] and  $\mu_0 = 4\pi E - 7 [m kg C^{-2}]$ ;  $\hat{n}_0$  is the refractive index of the incident medium, and  $\theta_{iI}$  is the angle at which the incident light is refracted. Equation 12 is used when the polarization of the incident electromagnetic wave is perpendicular to the plane-of-incidence and equation 13 is used when the polarization of the electromagnetic wave is parallel to the plane of incidence. When dealing with an incident wave in between these two extremes, they should be weighted according to the angle (i.e. an angle of  $45^\circ$  should be weighted 50/50). Equations 14 and 15 are related to each other in the same way as equations 12 and 13. The only differences here are  $\hat{n}_s$  and  $\theta_{tII}$ ;  $\hat{n}_s$  is the refractive index of the final medium and  $\theta_{tII}$  is the angle at which the light comes out of the final material.

$$Y_0 = \sqrt{\frac{\epsilon_0}{\mu_0}} \hat{n}_0 \cos \theta_{iI} \quad (12)$$

$$Y_0 = \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{\hat{n}_0}{\cos \theta_{iI}} \quad (13)$$

$$Y_s = \sqrt{\frac{\epsilon_0}{\mu_0}} \hat{n}_s \cos \theta_{tII} \quad (14)$$

$$Y_s = \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{\hat{n}_s}{\cos \theta_{tII}} \quad (15)$$



**Fig. 6.** This is the model of the lens mold created with SolidWorks. It will be used to create two halves of a biconvex lens that will then be attached to each other. The overall thickness of the mold is 2.5" (63.5mm). Both radius of curvatures,  $R_1$  and  $R_2$ , of the lens are 600mm (23.622"). The radius of curvature is determined via the Lensmaker's Formula (Equation 4), assuming an index of refraction,  $n$ , of 3.0 and a focal length,  $f$ , of 300mm. The diameter of the lens is 300mm (11.811"). The thickness of one half of the lens is 1.1438" (29.05252mm), therefore the total lens thickness is 2.2876" (58.10504mm). The mold also incorporates a flange for mounting purposes. The diameter of the flange that is in the plane of the lens diameter is 1" (25.4mm) greater in diameter than the lens diameter. The edge of the flange is not perpendicular to the previously mentioned plane. Rather, it is pivoted outward at a  $20^\circ$  angle with respect to the plane orthogonal to the lens plane. The thickness of the flange incorporated into the mold is 10mm (0.393701"), therefore the total flange thickness is 20mm (0.787402") which is 35% of the total lens thickness.

Along with the preceding four equations, the evaluation of  $t$  also requires the matrix equations 16 and 17. The behavior that results from the passing of an electromagnetic wave through a layer of material can be represented by matrix 16 [4]. For this matrix,  $k_0$  is described by equation 18,  $h = 2n_1d \cos \theta_{iII}$  ( $d$  is the material's thickness) and  $Y_1$  is either equation 12 or 13, depending on the polarization of the electromagnetic wave.

$$\mathbf{M}_1 = \begin{bmatrix} \cos k_0 h & i \sin k_0 h / Y_1 \\ Y_1 i \sin k_0 h & \cos k_0 h \end{bmatrix} \quad (16)$$

$$\mathbf{M} = \mathbf{M}_1 \mathbf{M}_2 \dots = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \quad (17)$$

$$k_0 = \frac{2\pi}{\lambda} \quad (18)$$

To clarify, the  $\lambda$  in the equation describing  $k_0$  is given by equation 19; for this,  $c$  is the speed of light and  $\nu$  is the frequency of incident light.

$$\lambda = \frac{c}{\nu} \quad (19)$$

For equations 12 to 15, the complex refractive index of material, equation 20, is needed. It is derived from equation 21 which

is, itself, found through the use of equation 22[5]; equation 22 assumes a low loss tangent. In equation 22,  $n$  is the real part of the material's refractive index and  $\tan \delta$  is the loss tangent of the material. Both of these quantities are measurable.

$$\hat{n} = n \left( 1 - i \frac{\tan \delta}{2} \right) \quad (20)$$

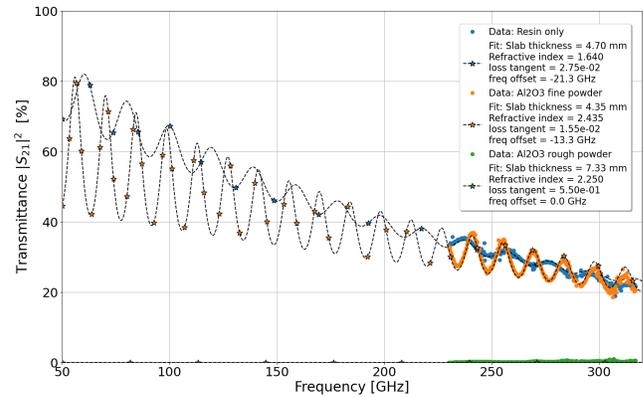
$$\hat{n} = n - i\kappa \quad (21)$$

$$\kappa = \frac{n \tan \delta}{2} \quad (22)$$

### 3. RESULTS

We find the mass of the -325 mesh alumina to be 381 g for the sample shown in Figure 2. The total mass of the sample is 481 g, implying a resin mass of 80 g. The volume of alumina (measured before the VIP) in this sample is  $230.1 \text{ cm}^3$ . Thus, we find the filling factor of the mixture containing 325 mesh aluminum oxide and the resin to be 0.764, assuming  $V_{tot}$  is acquired via the process outlined in the latter portion of Section A.1. In actuality, this is likely not quite the correct total volume because the alumina powder is compacted when the vacuum is induced during the VIP. This compaction is dependent on grain size; the finer the grain, the more compact the powder will be under vacuum. Therefore, the filling factor of 0.764 is likely an under-estimate of the actual filling factor.

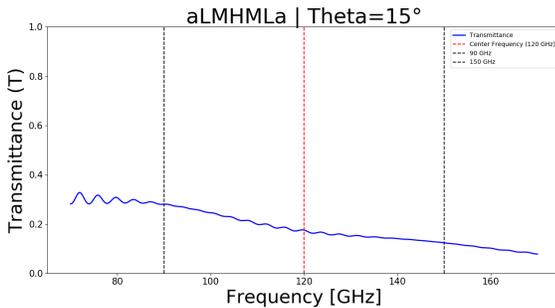
We find the refractive indices to be 1.640 and 2.435 for the resin sample and -325 mesh alumina sample, respectively. The loss tangent of the resin sample is  $2.75\text{E-}02$  and that of the -325 mesh alumina sample is  $1.55\text{E-}02$ . These measurements were taken in a frequency range of about 225 GHz to 325 GHz. These measurements were taken at a temperature of roughly 300 K. We fit this data and interpolated it to our target frequency range of 70 GHz to 170 GHz; this interpolation is shown in Figure 7. The fit begins to deviate at the edges of the frequency range used to measure the samples. We anticipate the transmittance to be closer to the peaks of the interpolation (near 80%) when cooled to cryogenic temperatures.



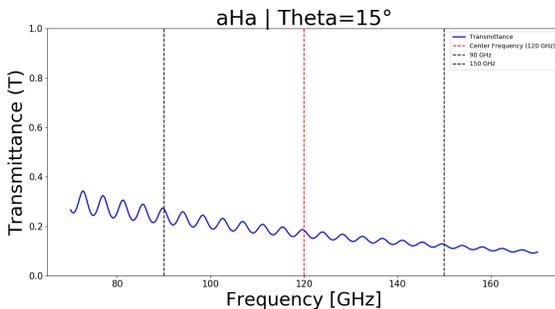
**Fig. 7.** This is the raw data (orange and blue) for the optical properties of our samples along with both a fit to the raw data and an interpolation of it to our target frequency range.

### A. Lens Performance Prediction

Assuming a high index of refraction of 2.435 and a loss of  $1.55E-02$  for the H layer, a medium index of refraction of 2.000 and a loss of  $2.00E-02$  for the M layer, a low index of refraction of 1.640 and a loss of  $2.75E-02$  for the L layer, and an angle of incidence of  $15^\circ$  in the method discussed in Section B, we expect our lens to have the transmittance behavior shown in Figure 8; The M layer properties are ideal properties for our M layer. For strictly the lens (the H layer), we the transmittance behavior should match that of the plot shown in Figure 9. However, the loss of all materials in the lens (the H, M, and L layers) is expected to drop 1 to 2 orders of magnitude when they are cooled to cryogenic temperatures (roughly 80 K). Thus, assuming losses of  $1.55E-04$ ,  $2.00E-04$ , and  $1.640E-04$  for the H, M, and L layers, respectively, we expect the behavior shown in Figures 10 and 11.



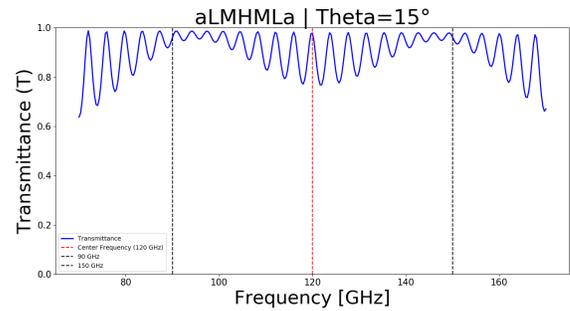
**Fig. 8.** This is the expected transmittance behavior for our range of frequencies for a lens coated with two separate AR coatings. The top AR coating is assumed to consist of the raw epoxy resin and the AR coating underneath it consists of a mix of the resin and the alumina, but with a much lower filling factor than the -325 alumina sample.



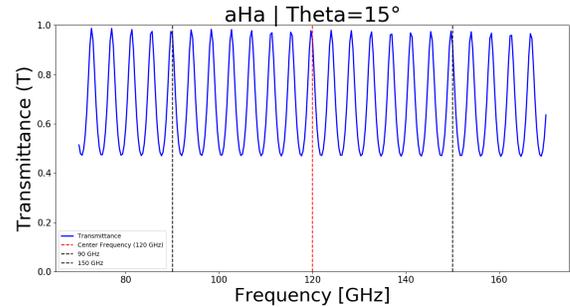
**Fig. 9.** This is the expected transmittance behavior for our range of frequencies for just the lens (no AR coatings).

### B. "Dummy" Lens

We made a "dummy" lens with the lens mold to confirm that it's possible to remove the lens from the mold without damaging it. The lens is shown in Figure 12. The lens is not damaged in any way. We will test the lens for strength and durability in the coming months but expect it to have similar results as the samples. We also plan to thermally cycle the lens.



**Fig. 10.** This is the expected transmittance for our lens with AR coatings when under cryogenic temperatures.



**Fig. 11.** This is the expected transmittance for our lens without AR coatings when under cryogenic temperatures.



**Fig. 12.** This is a view of the "dummy" lens that was produced by mixing resin and -325 mesh alumina powder, rather than using the VIP. The final lens will look similar in shape to this lens and similar in texture to Figure 4.

## 4. CONCLUSION

In conclusion, the most promising materials we have tested so far are the 4281/4284 epoxy resin from Endurance Technology and the -325 mesh aluminum oxide from McMaster-Carr. At room temperature, these materials have relatively high loss tangents, resulting in a transmission no greater than 0.4. We have not yet measured the refractive index and loss tangent of materials at cryogenic temperatures but anticipate them to be much

better for transmission than that of the room temperature values. We plan to do that this coming year. We also plan to measure the properties of 5 other epoxy resins, silicon-carbide powder (of varying grain size) infused with these resins, and mullite (also of varying grain size) infused with these resins. We will measure the properties at both room temperature and cryogenic temperatures.

## REFERENCES

1. J. Sayers, P. K. Day, D. P. Cunnane, B. H. Eom, H. G. LeDuc, R. C. O'Brient, M. C. Runyan, S. A. Bryan, S. B. Gordon, P. D. Mauskopf, B. R. Johnson, H. McCarrick, and T. A. Bhandarkar, "A millimeter-wave kinetic inductance detector camera for long-range imaging through optical obscurants," in *Passive and Active Millimeter-Wave Imaging XXIII*, vol. 11411 D. A. Wikner and D. A. Robertson, eds., International Society for Optics and Photonics (SPIE, 2020), pp. 74 – 90.
2. R. Datta, C. D. Munson, M. D. Niemack, J. J. McMahon, J. Britton, E. J. Wollack, J. Beall, M. J. Devlin, J. Fowler, P. Gallardo, and et al., "Large-aperture wide-bandwidth antireflection-coated silicon lenses for millimeter wavelengths," *Appl. Opt.* **52**, 8747 (2013).
3. T. Nitta, S. Sekiguchi, Y. Sekimoto, K. Mitsui, N. Okada, K. Karatsu, M. Naruse, M. Sekine, H. Matsuo, T. Noguchi, M. Seta, and N. Nakai, "Anti-reflection Coating for Cryogenic Silicon and Alumina Lenses in Millimeter-Wave Bands," *J. Low Temp. Phys.* **176**, 677–683 (2014).
4. E. Hecht, *Optics* (Addison-Wesley, 1998), 5th ed.
5. J. Lamb, "Miscellaneous data on materials for millimetre and submillimetre optics," *Int. J. Infrared Millim. Waves* **17**, 1997–2034 (1996).