Essays in International Portfolio Choice

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Abstract

The last four decades have witnessed a steady increase of foreign equity holdings share. From 1970 to 2004, the share of equities owned by foreign investors increased by five times in the United States, seven times in Japan, and fifteen times in the United Kingdom. In his 2007 Ohlin lecture, Obstfeld stated "it is imperative to understand how investors make asset allocation decisions for different asset classes across countries." This dissertation analyzes international portfolio choice along two dimensions.

In the first chapter, I develop a theory for bilateral asset holdings that takes a gravity form. I discuss how to estimate international financial frictions and conduct a comparative statics analysis within the context of the theory. I also find though that reasonable extensions of the model no longer generate a gravity form. While this does not significantly complicate estimation and comparative statics analysis, it raises questions about the empirical validity of gravity specifications for cross-border financial holdings that need to be addressed in future work.

The second chapter analyzes the impact of the changes in foreign asset holdings on welfare. This chapter adds within-country heterogeneity to the standard open-economy dynamic general equilibrium portfolio choice model. The solution method is illustrated with a preliminary calibration. Decrease of proportional costs is beneficial to the rich agents but harmful to the poor agents whereas decrease of fixed cost is beneficial to the poor agents and has no effect on the rich agents.

To my family

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Contents

Chapter 1

Gravity in International Finance

1.1 Introduction

The past decade has witnessed an explosion of papers estimating gravity equations for cross-border financial holdings. This used to be the territory of the international trade literature, in which there is a long tradition of estimating gravity equations that relate trade flows to country size and various proxies for trade barriers. At least three factors are driving this interest in estimating gravity equations applied to international finance. One is the discovery that gravity equations for international asset trade fit the data at least as well as for goods trade. The contribution by Portes and Rey (2005) is central in this regard. Second, the release of the Coordinated Portfolio Investment Survey by the International Monetary fund, which contains bilateral portfolio holdings for 67 countries since 2001, has been a key driver as well and most of the recent contributions use this data set.[1] Finally, there is a wealth of potential policy questions that can be addressed through the estimation of gravity equations, such as the impact on globalization of harmonization of financial regulations or the formation of monetary or trade unions.

However, this explosion of empirical work on gravity for cross-border financial holdings has taken place without a solid theoretical foundation. As has been well established in the trade literature (e.g. Anderson and van Wincoop (2003)), estimating gravity equations that are not founded in economic theory can lead to biased estimation results due to omitted variables. It also leads to incorrect comparative statics analysis that does not take into account the general equilibrium effects of changes in cross-border barriers.

This chapter is a response to this need for a theoretical foundation of a gravity equation for cross-border asset holdings. I will show that under a certain set of assumptions it is possible to derive a gravity equation for asset trade. I discuss how to estimate cross-border financial frictions in this context and how to conduct proper comparative statics analysis. The empirical work to date is often incon-

[1] A substantial number of papers also use data on external claims by banks from the BIS. Some recent papers that have estimated empirical gravity equations for equity, bond and bank holdings include Ahearne, Griever and Warnock (2004), Aviat and Coeurdacier (2007), Balli (2008), Balli, Louis and Osman (2009), Balta and Delgado (2008), Berkel (2007), Bertaut and Kole (2004), Buch (2000, 2002), Chan, Covrig and Ng (2005), Coeurdacier and Martin (2009), Coeurdacier and Guibaud (2005), Daude and Fratzscher (2008), De Santis and Gérard (2009), Eichengreen and Luengnaruemitchai (2006), Faruqee, Li and Yan (2004), Forbes (2008), García-Herrero, Wooldridge and Yang (2009), Gelos and Wei (2005), Ghosh and Wolf (2000), Jeanneau and Micu (2002), Kim, Lee and Shin (2007), Kim, Lee and Shin (2006), Lane and Milesi-Ferretti (2005), Lane (2005), Lee (2008), Martin and Rey (2004), Pendle (2007), Portes and Rey (2005), Portes, Rey and Oh (2001), Rose and Spiegel (2004), Vlachos (2004), Yu (2009).

sistent with the theory in that either proper source and destination country fixed effects are not included or variables are included in the gravity equation that have no theoretical justification for being there (e.g. asset return correlations).[2]

However, I also show that when relaxing the assumptions of the model in many reasonable directions it is no longer possible to write bilateral asset holdings in a gravity form. It is still possible to estimate international financial frictions in this case and to conduct comparative statics analysis. But this is based on more complex non-linear equations that relate bilateral asset holdings to all bilateral financial frictions, measures of country size and asset return risk.

The paper has several parallels to the contribution by Anderson and van Wincoop (2003) in the trade literature. Just like in this paper, their work was motivated by a large empirical gravity literature without any theoretical foundation. They showed how to derive a simple and intuitive gravity equation from theory and developed the implications for empirical estimation and comparative statics. The gravity equation that I derive for cross-border asset trade is closely analogous to that derived by Anderson and van Wincoop (2003) for goods trade. Bilateral financial positions depend on *relative* barriers: bilateral financial barriers relative to average barriers (multilateral resistance) faced by both source and destination

[2]There are a couple of exceptions though, including Coeurdacier and Martin (2009), Lane (2005) and Vlachos (2004), where estimation is done in a way that is consistent with the theory that I will develop here. It should also be said that while presently there is no justification for many of the existing empirical gravity specifications, I cannot prove that they have no theoretical foundation. All I can say is that currently there is no theory justifying such specifications and it is best for empirical work to be consistent with existing theory.

countries.

As discussed in Anderson and van Wincoop (2004), two key assumptions are needed to generate a gravity specification for trade in goods where bilateral trade is a product of measures of economic size, a bilateral barrier and multilateral resistance indices. The first is trade separability, which says that total production and expenditure are separable from the bilateral allocation of trade across countries. The second condition is that demand depends on a relative price, such as the price of goods from a particular country relative to an overall price index. These conditions are satisfied in a large class of models, including models with product differentiation by country of origin, models with monopolistic competition, the Heckscher-Ohlin model with specialization and even the Ricardian model of Eaton and Kortum (2002).

Such conditions also need to be satisfied to derive a gravity specification for asset trade. A condition analogous to trade separability is that decisions about the overall demand for assets (affected by saving) are separable from the portfolio allocation across assets. This condition is the least problematic and holds in many models. The second condition, that asset demand depends on a relative price, is far less trivial than for goods trade. Asset demand naturally takes a very different form than demand for goods. Optimal portfolio choice leads to asset demand that depends on the inverse of a covariance matrix of all returns times a vector of expected returns of all assets. In that context, it is not trivial to relate demand for individual assets to a relative price. Not surprising therefore, I find that a gravity specification for asset trade is much less robust to changes in model assumptions than in the trade literature.

In order to derive my theoretical gravity equation, I start from a simple static portfolio choice framework. Investors can hold claims on risky assets from a large number of countries. Asset returns are affected both by a country-specific and by a global component. In addition, I allow for trade in a riskfree asset and in an asset whose return is only related to global risk; both are in zero net supply. I introduce international financial frictions in the form of information asymmetries about the country-specific return components.[3] After imposing asset market equilibrium in all markets I show that this leads to a gravity equation where bilateral financial holdings depend on the product of economic size variables (stock market capitalization in the destination country and total investment in stock in the source country) divided by a relative financial friction. The relative friction is equal to the bilateral financial friction divided by the product of multilateral resistance terms from the perspective of source and destination counties.

I consider a variety of generalizations of this benchmark model in which the gravity result falls apart. In particular, I consider the case where there do not exist separate assets that allow agents to hedge factors contributing to cross-border

[3] A substantial literature has documented the relevance of such information asymmetries across countries. See for example Bae, Stulz and Tan (2008), Ahearne et al. (2004), Portes and Rey (2005), Kang and Stulz (1997) any many references in those papers.

return correlations. I also consider different financial frictions that take the form of a tax on foreign returns. And finally, I consider the case of only trade in risky assets, which captures an extreme case of borrowing constraints associated with the riskfree asset. In all these cases, it is no longer possible to write bilateral asset holdings in a gravity form as the product of country-specific variables (economic size, multilateral resistance or any other country-specific variable) and a bilateral friction.

There are two other theories in the literature that generate a gravity specification for asset trade. One approach is that by Martin and Rey (2004) who derive a gravity equation for financial holdings when countries trade claims on Arrow Debreu securities. An extension by Coeurdacier and Martin (2009) shows that this can lead to a gravity equation that is similar to that for goods trade, with bilateral holdings depending both on bilateral frictions and multilateral resistance indices of source and destination countries. The reason for this is that demand for Arrow Debreu securities takes a similar form as the demand for goods under CES preferences. The differentiation of goods by type in the trade literature is now replaced by an analogous differentiation of assets by states in which they have a payoff. Standard constant relative risk-aversion expected utility can then be written as a function of Arrow Debreu asset holdings in a way that is analogous to CES utility as a function of consumption of differentiated goods.

The main limitation of this approach though is that it is not applicable to the types of financial holdings for which we have cross-border data: bilateral equity, bond and bank holdings. The reason is that these assets, on which the empirical gravity literature is based, have non-zero payoffs in multiple states. More precisely, if the asset from one country has a non-zero payoff, assets from other countries generally have a non-zero payoff as well. In the AD framework, if the asset of a country has a positive payoff, the assets of all other countries have a zero payoff.[4] Turning the argument around, it is sometimes argued that any risky asset can be written as a combination of AD securities. But the problem is that these will then be a combination of AD securities from different countries, so that the risky asset is not specific to a particular country.

A second alternative way to derive a theoretical gravity equation, suggested by Milesi-Ferretti and Lane (2004), is a multi-country extension of the model in Obstfeld and Rogoff (2000) that relates barriers in goods trade to portfolio home bias. While theoretically possible, this approach has drawbacks as well. The main problem is that the real exchange rate hedge channel, through which barriers in goods trade affect asset trade in Obstfeld and Rogoff (2000), does not appear to be operative in practice. Using data on equity returns and real exchange rates, van Wincoop and Warnock (2010) show that hedging real exchange rate risk cannot

[4]This also implies that correlations between the returns on Arrow Debreu securities are actually negative. To see this, let r_1 and r_2 be the return on assets that only have a payoff in respectively state 1 and 2 (e.g. $r_1(1) > 0$, $r_1(s) = 0$ for $s \neq 1$). Assuming that states 1 and 2 have non-zero probabilities $\pi(1)$ and $\pi(2)$, I have $cov(r_1, r_2) = Er_1r_2 - Er_1Er_2 = -\pi(1)\pi(2)r_1(1)r_2(2) < 0$. This stands in contrast to the generally positive correlation between asset returns across countries when applied to stocks, bonds or bank earnings.

account for portfolio home bias. Consistent with these findings, Coeurdacier (2009) develops an extension of Obstfeld and Rogoff (2000) to show that for realistic model parameters trade barriers cannot generate a portfolio home bias.

The remainder of the chapter is organized as follows. Section 2 derives a gravity theory for financial holdings from a static multi country portfolio choice framework. It discusses what assumptions are needed to derive such a gravity specification. I also discuss some extensions that preserve the gravity result. Section 3 considers several extensions of the benchmark model where I no longer obtain a gravity specification. Section 4 discusses how to estimate bilateral financial frictions and conduct comparative statics analysis, both when the theory leads to gravity and when it does not. Section 5 concludes.

1.2 A Gravity Theory of Financial Holdings

In this section I develop a gravity model for bilateral asset holdings in a one-good, two-period, N country framework.

1.2.1 The model

The Assets

There are $N+2$ assets. The first N assets are country-specific risky assets. The gravity equation that I will derive applies to these N assets. I will refer to them as equity, although they could also be other risky assets such as corporate bonds, long-term bonds or bank holdings. The supply of the asset in country i is K_i. One can think of this as the capital stock. The equity claim of country i has a real payoff of D_i in period 2, where

$$D_i = 1 + \epsilon_i + \theta_i \epsilon_g \tag{1.1}$$

Here ϵ_i is a country-specific payoff innovation and ϵ_g is a global payoff innovation. The constant term is 1, which is simply a normalization. The country-specific payoff innovations are uncorrelated across countries and with the global innovation. I allow the response to global innovations to be country-specific. I assume that ϵ_g has a mean of 0 and variance σ_g^2. The distribution of the country-specific innovation ϵ_i is discussed below. The price of a country i equity claim in period 1 is Q_i.

The second asset is a riskfree bond that is in zero net supply. The bond pays one unit of the good in period 2 and has a period 1 price of Q_f. Finally, there is an asset whose return is perfectly correlated with the global shock. This asset is also in zero net supply. It has a period 1 price of Q_g and a period 2 payoff of $D_g = 1 + \theta_g \epsilon_g$. This asset allows agents to hedge global risk.

I will write the returns on the $N+2$ assets as

$$
\begin{aligned}
R_i &= \frac{D_i}{Q_i} \qquad i = 1...N && (1.2) \\
R_f &= \frac{1}{Q_f} && (1.3) \\
R_g &= \frac{D_g}{Q_g} && (1.4)
\end{aligned}
$$

These assumptions about the asset market structure are obviously restrictive and I will discuss below how results change when I relax them. At this point, I only briefly comment on the global asset. It allows agents to hedge the global risk factor, so that the only risk that matters for portfolio allocation across the N equity is the country-specific risk. This significantly simplifies the portfolio allocation problem and I will see that it is critical to derive a gravity equation for bilateral asset holdings.

One way to interpret the global asset is as a global equity futures contract, allowing one to buy or sell a claim on the global equity payoff at a futures price of f^g. The payoff on such a contract is

$$1 + \theta_g \epsilon_g + \sum_{i=1}^{N} (K_i/K)\epsilon_i - f^g \tag{1.5}$$

where K is the global capital stock and $\theta_g = \sum_{i=1}^{N}(K_i/K)\theta_i$. The payoff depends on the global shock through the term $\theta_g \epsilon_g$ in exactly the same way as the assumed global asset. Note though that it is not exactly the same as my global asset when the third term that depends on the idiosyncratic shocks is not zero. Because of the law of large numbers, this term will be close to zero when there are many small countries. But with some big countries like the United States and Japan, this is not necessarily the case.

A second, and closely related, possibility is to interpret the global asset as an equity futures contract on a set of multinational firms. For such firms country-specific shocks naturally play less of a role because of their global operations. A third possibility is to interpret the global asset as a derivative whose payoff is specifically connected to shocks that affect the entire world economy, such as an oil price futures contract. Admittedly, though, each of these interpretations of the global asset clearly has their limitations. I will therefore discuss below how results change when I do not allow for such an asset.

Consumption and Portfolio Choice

Agents in country j are born with an endowment of Y_j in period 1 plus a claim on all country j equity. The wealth of country j agents in period 1 after consumption is therefore

$$W_j = Y_j + Q_j K_j - C_j^1$$

where C_j^1 is period 1 consumption.

In period 1, agents decide how much to consume and how to allocate the remainder of the wealth across the $N+2$ assets. The budget constraint is

$$C_j^2 = W_j R_j^p = (Y_j + Q_j K_j - C_j^1) R_j^p \tag{1.6}$$

where the portfolio return is

$$R_j^p = \sum_{i=1}^{N} \alpha_{ij} R_i + \alpha_{gj} R_g + \alpha_{fj} R_f \tag{1.7}$$

Here α_{ij} is the fraction invested in country i equity, α_{gj} the fraction invested in the global asset and α_{fj} the fraction invested in the riskfree asset. These portfolio shares sum to 1.

Agents maximize

$$\frac{\left(C_j^1\right)^{1-\gamma}}{1-\gamma} + \beta \frac{E\left(C_j^2\right)^{1-\gamma}}{1-\gamma} \tag{1.8}$$

The first-order conditions for consumption and portfolio choice are

$$\left(C_j^1\right)^{-\gamma} = \beta E \left(C_j^2\right)^{-\gamma} R_j^p \tag{1.9}$$

$$E\left(C_j^2\right)^{-\gamma} (R_i - R_f) = 0 \quad i = 1...N \tag{1.10}$$

$$E\left(C_j^2\right)^{-\gamma} (R_g - R_f) = 0 \tag{1.11}$$

(1.9) is the standard consumption Euler equation that represents the tradeoff between consumption in periods 1 and 2. (1.10) is a portfolio Euler equation that represents the tradeoff between investment in the equity claim of country i and the riskfree asset. Finally, (1.11) is a portfolio Euler equation that represents the tradeoff between investment in the global and riskfree assets.

The market clearing conditions for country i equity, the global asset and the riskfree asset are

$$\sum_{j=1}^{N} \alpha_{ij} W_j = Q_i K_i \tag{1.12}$$

$$\sum_{j=1}^{N} \alpha_{gj} W_j = 0 \tag{1.13}$$

$$\sum_{j=1}^{N} \alpha_{fj} W_j = 0 \tag{1.14}$$

The period 1 and 2 goods market clearing conditions are

$$\sum_{j=1}^{N} C_j^1 = \sum_{j=1}^{N} Y_j \tag{1.15}$$

$$\sum_{j=1}^{N} C_j^2 = \sum_{j=1}^{N} D_j \tag{1.16}$$

Information Asymmetry

I assume that due to differences in language and regulatory systems, and easier access to local information, domestic agents are more informed than foreigners about the idiosyncratic payoff innovations on domestic equity claims. From the perspective of agents in country j, ϵ_i has a mean of 0 and variance

$$\tau_{ij}\sigma_i^2 \tag{1.17}$$

Information asymmetry is therefore captured by $\tau_{ij} > \tau_{ii}$ when $j \neq i$.[5]

Since this assumption is critical to the derivation of the gravity equation for asset trade, it deserves further discussion. What makes the derivation of a gravity equation for asset trade different from goods trade is that asset trade necessarily involves risk. Without risk there would just be a single riskfree asset that is the same for each country. I know from covered interest rate arbitrage that riskfree returns are indeed equalized across industrialized countries. When introducing financial frictions it is therefore natural to relate them to risk.

There is a substantial body of evidence showing that information asymmetries exist and are relevant in explaining portfolio home bias. Without conducting an extensive survey, I mention just a couple of relevant papers. Bae et al. (2008) find that that the absolute forecast error of annual earnings per share is 7.8% higher for foreign analysts than local analysts. Ahearne et al. (2004) find that home bias of U.S. investors relative to other countries is significantly reduced when the stock of foreign countries is traded on centralized exchanges. This reduces information barriers because of the regulatory and accounting burden imposed on such foreign

[5] While I assume that agents in different countries have different quality signals about ϵ_i, I assume that the expectation of ϵ_i is the same across countries. This can be justified in models with a continuum of agents in each country. See for example Veldkamp and Nieuwerburgh (2009).

firms. Portes and Rey (2005) find that "the geography of information is the main determinant of the pattern of international (financial) transactions", documenting the effect of a variety of information frictions on cross-border equity flows. Kang and Stulz (1997) document that investors tend to invest in foreign firms for which information barriers are lower (large firms with good accounting performance, low unsystematic risk and low leverage).

Information is not exogenous. Investors may acquire more information about countries that they are less informed about. However, this will not necessarily eliminate information asymmetries. Veldkamp and Nieuwerburgh (2009) show that information asymmetries will in fact be amplified when allowing agents to acquire information about different asset payoffs. The reason for this is that it is optimal to acquire more information about assets that have a large weight in the portfolio, which happen to be assets that agents are already relatively well informed about.

Modeling the financial friction τ_{ij} as an information friction differs from the approach in a number of papers that introduce a financial friction simply as a tax or transaction cost that reduces the return on foreign investment. Examples are Tille and van Wincoop (2010a,b), Coeurdacier (2009), Coeurdacier and Guibaud (2005) and Martin and Rey (2004). Many types of capital controls can be thought of as a tax. Danthine, Adjouté, Bottazzi, Fischer, Hamaui, Portes and Wickens (2000) show that transaction costs are larger for cross-border than domestic transactions. I will discuss in Section 4 how results change if instead I model the friction as a tax or transaction cost.[6]

1.2.2 Derivation of Gravity Equation

In solving the model I apply the local approximation solution method developed by Tille and van Wincoop (2010a) and Devereux and Sutherland (2011). I focus on what in a more dynamic model would be called the "deterministic steady state" of asset allocation. In more technical terms, this is the zero-order component. Leaving the algebraic derivations to the Appendix, and omitting the technical order component notation used in the Appendix, I obtain the following intuitive expression for equity portfolio shares:

$$\alpha_{ij} = \frac{1}{\gamma R \sigma_i^2 \tau_{ij}} \left[E(R_i - R_f) - \frac{\theta_i}{\theta_g} E(R_g - R_f) \right] \tag{1.18}$$

where R is the zero-order component of asset returns that is the same for all assets. As is quite standard, portfolio shares depend on the ratio of the expected excess return (second-order component) and the variance of the excess return. As global risk can be separately hedged, both the expected excess return and its variance

[6]Two other explanations for portfolio home bias that have received extensive attention in the literature are associated with a hedge against uncertainty about the return on non-traded assets (e.g. labor income) and a hedge against real exchange rate risk (e.g. non-traded goods or any other source of deviations from PPP). However, empirically these explanations have not fared very well. van Wincoop and Warnock (2010) show that the second explanation can explain virtually no home bias at all. Bottazzi, Pesenti and van Wincoop (1996) and Julliard and Rosa (2009) find that the non-traded asset explanation also does not generate much home bias. It should be said though that there remains on ongoing debate about the role of non-financial wealth (non-traded assets). See Coeurdacier and Gourinchas (2011) for a recent contribution.

remove the global components. The expected excess return therefore subtracts the part that is a compensation for global risk. Analogously, the variance of the excess return only refers to country-specific risk.

Now define

$$\frac{1}{p_i} = \frac{1}{\gamma R \sigma_i^2} \mathrm{E}\left[R_i - R_f - \frac{\theta_i}{\theta_g}(R_g - R_i)\right] \tag{1.19}$$

The variable p_i is proportional to a risk-return ratio: the amount of country-specific risk of asset i as captured by the variance σ_i^2, divided by the expected excess return. The higher p_i, the lower the demand for the asset. The variable p_i is endogenous as it depends on the second-order component of the expected excess return that in equilibrium adjusts to clear equity markets through second-order changes in asset prices. Given the definition of p_i, portfolio allocation (1.18) becomes

$$\alpha_{ij} = \frac{1}{\tau_{ij} p_i} \tag{1.20}$$

I can think of $\tau_{ij} p_i$ as the "price" (risk-return ratio) faced by agents from country j investing in country i.

Write total equity holdings by agents from country j as

$$E_j = \sum_{i=1}^{N} \alpha_{ij} W_j \tag{1.21}$$

Substituting (1.20) yields

$$W_j = E_j P_j \tag{1.22}$$

where

$$\frac{1}{P_j} = \sum_{i=1}^{N} \frac{1}{\tau_{ij} p_i} \tag{1.23}$$

Using this, I can write the total equity claim $X_{ij} = \alpha_{ij} W_j$ by country j on country i as

$$X_{ij} = \frac{P_j}{\tau_{ij} p_i} E_j \tag{1.24}$$

This equation is critical to what follows. Bilateral asset demand depends on a relative price: the "price" (risk-return ratio) of country i equity relative to an overall price index.

Similar to goods trade, I can now derive a gravity specification by combining this demand equation with a set of market clearing equations. The asset market clearing condition for country i equity is

$$\sum_{j=1}^{N} X_{ij} = S_i \tag{1.25}$$

where $S_i = Q_i K_i$ is the country i equity supply. Also define $E = S = \sum_{j=1}^{N} E_j = \sum_{i=1}^{N} S_i$ as the world demand and supply of equity. Then the market clearing condition (1.25) gives the following solution for p_i:

$$p_i = \frac{S}{S_i}\frac{1}{\Pi_i} \tag{1.26}$$

where

$$\frac{1}{\Pi_i} = \sum_{j=1}^{N} \frac{P_j}{\tau_{ij}}\frac{E_j}{E} \tag{1.27}$$

Substituting this solution for p_i back into (1.23) and (1.24), I get the following gravity specification for bilateral asset holdings:

$$X_{ij} = \frac{S_i E_j}{E}\frac{\Pi_i P_j}{\tau_{ij}} \tag{1.28}$$

$$\frac{1}{P_j} = \sum_{i=1}^{N} \frac{\Pi_i}{\tau_{ij}}\frac{S_i}{S} \tag{1.29}$$

$$\frac{1}{\Pi_i} = \sum_{j=1}^{N} \frac{P_j}{\tau_{ij}}\frac{E_j}{E} \tag{1.30}$$

$$P_j E_j = W_j \tag{1.31}$$

For given asset supplies S_i, (zero-order components of) wealth W_j and bilateral frictions τ_{ij}, equations (1.29), (1.30) and (1.31) can be used to jointly solve for P_j, E_j and Π_i for $i = 1, .., N$ and $j = 1, .., N$. Together with (1.28) this determines bilateral asset holdings X_{ij}.

The gravity equation (1.28) implies that bilateral asset holdings X_{ij} are driven by two factors. The first is a size factor: the product of total equity holdings E_j of country j and the supply of equity S_i of country i, divided by the world demand or supply. The second factor is a relative friction. Just as is the case for trade flows, bilateral asset holdings are driven not simply by the bilateral friction τ_{ij}, but rather by the relative friction

$$\frac{\tau_{ij}}{\Pi_i P_j} \tag{1.32}$$

Here Π_i and P_j are so-called multilateral resistance variables that measure the average financial frictions for respectively country i as a destination country and country j as a source country. Given the size factor $S_i E_j / E$, it is this *relative financial friction* that drives the bilateral asset holding X_{ij}.[7]

In order to understand why bilateral asset holdings are driven by this relative financial friction, as opposed to just τ_{ij}, first consider the source country j. Investors from j invest a total of E_j in equity. They will allocate more of this to destination countries for which the bilateral financial friction τ_{ij} is low in comparison to the average financial friction P_j that it faces relative to all destination countries. The relative financial friction (1.32) is also affected by the multilateral resistance Π_i of the destination country. When Π_i is high, country i faces high financial frictions with many source countries. In order to generate equilibrium in the market for country i equity, it will have to offer a low "price" p_i through a

[7]In the goods trade literature the friction is an ad valorem tariff, which has a non-unitary elasticity in the gravity specification that depends on the elasticity of substitution between the goods. Here instead I have an asymmetric information friction. A 1% increase in a bilateral friction raises the country-specific variance by 1%, which gives rise to a 1% drop in the portfolio share invested in that country (holding all else constant) and therefore a unitary elasticity.

high expected return. For a given bilateral barrier τ_{ij} this will raise X_{ij}.

There is one difference relative to the goods trade gravity literature that is worth pointing out. Since the zero-order component of W_j does not depend on financial frictions (see Appendix), (1.31) implies that the total equity investment E_j by country j goes down when its multilateral resistance rises. The reason is that higher financial frictions lead to a shift away from risky assets and towards the riskfree asset. This is not usually the case in gravity models for goods trade where E_j represents the total demand for differentiated goods in country j. However, when introducing a homogeneous good as well as differentiated goods, one can derive an analogous gravity specification for goods trade.[8] As we will see in Section 4, this relationship between total expenditure on risky assets and multilateral resistance has implications for estimation and comparative statics.

1.2.3 Extensions that Retain Gravity

A key question that we need to address is how robust the gravity specification is to the various assumptions that I have made in the benchmark model. I start by discussing some extensions under which the gravity form is retained. In the next section, I discuss a variety of extensions under which gravity no longer applies.

One generalization of the model that leaves the gravity system (1.28)-(1.31) intact is to allow for a more general asset payoff structure, while at the same time

[8] For further discussion of this comparison to the goods trade gravity literature, see the 2010 working paper version of this paper.

assuming that there are separate assets that can hedge uncertainty associated with factors responsible for return co-movements. More precisely, assume that the payoff structure is

$$D_i = 1 + \epsilon_i + \sum_{i=1}^{L} \theta_{il} u_l \tag{1.33}$$

Where all the innovations ϵ_i $(i = 1, .., N)$ and u_l $(l = 1, .., L)$ are uncorrelated. The innovations u_l are common across countries and lead to return co-movement. The benchmark model is a special case of this where $L = 1$ and $u_1 = \epsilon_g$. The extension allows for additional factors generating co-movement, such as for example regional factors. At the same time I assume that there are L assets whose respective payoffs only depend on the common factors u_l. An example is a European equity futures contract when u_l is a European regional factor.

Under this extension it remains the case that any common asset return risk can be separately hedged, so that it is really only the country-specific risk that matters for portfolio allocation among the N equity. While this extension has the advantage that the gravity result can hold under a very general covariance structure of asset returns, obviously the assumption that all common components of returns can be separately hedged is a strong one.[9]

Another extension is to allow for fixed costs associated with investment abroad. If this fixed cost is such that investors only hold claims on a subset of foreign

[9]By far the most important common component is the global component. In the 2010 working paper version of this paper I find that the average absolute value of the covariance between quarterly stock returns among 24 industrialized countries (2000-2007) is reduced by 88% after controlling for the first principal component.

countries, so that some of the X_{ij} are zero, the gravity system (1.28)-(1.31) changes very little. All that needs to be changed is the summation over i in the definition of P_j and the summation over j in the definition of Π_i. For P_j the summation should only be over countries on which country j investors hold positive claims. For Π_i summation should be over countries for which country i has positive liabilities.

A third extension, also related to fixed costs, is perhaps more interesting. It separates agents into two groups. For one group the fixed cost of investing abroad is so large that agents only invest in domestic stocks and bonds. For the other group fixed costs are not large enough to provide a barrier to investment abroad. They behave just like the investors in the benchmark model. This setup is consistent with extensive evidence that many investors only invest in the domestic stock market, as documented by Christelis and Georgarakos (2011), Kyrychenko and Shumb (2009) and many others. The latter paper finds that only about 10% of U.S. investors with directly held stock hold any foreign stock.

This fixed cost is also consistent with a relatively large share of domestic equity (usually well above 50%) held by even very small countries. Without the fixed cost, the benchmark model implies that the share of domestic equity should approach zero when the size of the country becomes small. For example, with N countries of equal size and $\tau_{ij} = \tau > 1$ for $i \neq j$ and $\tau_{ii} = 1$, the equilibrium share held domestically is $\tau/(N + \tau - 1)$, which goes to zero when N becomes big.

It can be shown that this extension again leaves the gravity system (1.28)-

(1.31) unchanged. The only difference is that the information friction τ_{ij} is now multiplied by what may be called a fixed cost friction δ_{ij} where

$$\delta_{ij} = 1 \qquad i \neq j$$

$$\delta_{ii} = W_i^A / W_i$$

Here W_i^A/W_i represents the share of wealth held by diversified agents of country i (A stands for access to foreign markets). For any source country j this equally raises all the cross-border frictions relative to the domestic friction by a magnitude W_j/W_j^A. While gravity is retained, this extension does have some implications for estimation and comparative statics that I discuss in Section 4.

A final extension addresses in a slightly different way the large domestic holdings for even very small countries. In the previous extension, when agents do not have access to foreign markets, they optimally diversify their wealth across domestic stocks and bonds. But some holdings of domestic stock may not be the result of a diversification motive at all, not even between domestic stocks and bonds. One example is insider trading. Kho, Stulz and Warnock (2009) report that as much as 50% of stock is held by insiders in industrialized countries. As a result of agency problems, it is often optimal for an executive to invest in the firm at which the executive is employed. This has nothing to do with diversification motives. The absence of diversification may apply to less wealthy investors as well. First, fixed costs may prevent them from being globally diversified. Second, low collateral may prevent them from borrowing. In that case, all wealth may be allocated to domestic stock (or domestic risky assets in general).

Assume that a fraction μ_i of the wealth of country i is invested exclusively in the domestic stock market for reasons entirely unrelated to diversification. In that case the gravity system (1.28)-(1.31) remains unchanged. All we need to do is to subtract $\mu_i W_i$ from the asset supply S_i, the wealth W_i and domestic holdings X_{ii}. Essentially, we need to take the $\mu_i W_i$ "out of the market".

1.3 Limitations to Gravity

As already emphasized in the introduction, the gravity result derived in the previous section is far from a general one. In this section, I will discuss three quite reasonable extensions of the benchmark model under which the gravity result no longer holds.

1.3.1 General Covariance Structure of Returns

The first extension is to allow for a general covariance structure of asset returns, while assuming that factors generating return co-movement cannot be separately hedged (e.g. global risk cannot be separately hedged through a global asset).

Consider the payoff structure

$$D_i = 1 + \epsilon_i + v_i \tag{1.34}$$

Here ϵ_i is the same country-specific shock as before, with the same variance $\tau_{ij}\sigma_i^2$ from the perspective of agents from country j. But payoffs are now also affected by a shock v_i (uncorrelated with ϵ_i) that is correlated across countries with $var(v_1, .., v_N)' = \mathbf{\Omega}$. Note that in the benchmark specification in the previous section $v_i = \theta_i \epsilon_g$ only captures global shocks. In that case $\mathbf{\Omega} = \theta\theta'\sigma_g^2$, where $\theta = (\theta_1, .., \theta_N)'$. But while I have now further generalized the covariance matrix, the more important assumption is that I no longer allow for assets that hedge the risk associated with the v_i.

In this case portfolio demand becomes quite complex. Defining the vector of portfolio shares for country j investors as $\alpha_j = (\alpha_{1j}, .., \alpha_{Nj})'$, I have

$$\alpha_j = \frac{1}{\gamma}\mathbf{\Phi}_j^{-1}\mathbf{ER} \tag{1.35}$$

where $\mathbf{\Phi}_j = \mathbf{\Omega} + \mathbf{L}_j$, $\mathbf{L}_j$ is a diagonal matrix with $\tau_{ij}\sigma_i^2$ as the i'th diagonal element, and $\mathbf{ER}$ is a vector of expected excess returns defined as

$$\mathbf{ER} = \frac{1}{R}\begin{pmatrix} E(R_1 - R_f) \\ ... \\ E(R_N - R_f) \end{pmatrix}$$

These portfolio shares, together with $X_{ij} = \alpha_{ij} W_j$, imply

$$X_{.,j} = \frac{1}{\gamma} W_j \mathbf{\Phi}_j^{-1} \mathbf{ER} \tag{1.36}$$

where $X_{.,j} = (X_{1j}, .., X_{Nj})'$. Imposing the market clearing conditions $\sum_{j=1}^{N} X_{ij} = S_i$ implies that the vector of expected excess returns is

$$\mathbf{ER} = \gamma \left(\sum_{k=1}^{N} \mathbf{\Phi}_k^{-1} W_k \right)^{-1} \mathbf{S} \tag{1.37}$$

where $\mathbf{S} = (S_1, .., S_N)'$ is the vector of equity supplies. Substituting this solution for $\mathbf{ER}$ back into (1.36) gives

$$X_{.,j} = W_j \mathbf{\Phi}_j^{-1} \left(\sum_{k=1}^{N} \mathbf{\Phi}_k^{-1} W_k \right)^{-1} \mathbf{S} \tag{1.38}$$

This is a complicated non-linear expression. It relates X_{ij} to the entire vectors $(S_1, .., S_N)$ and $(W_1, .., W_N)$ of country size variables, the entire covariance matrix $\mathbf{\Omega}$, all the country-specific payoff variances σ_i^2 as well as all the financial frictions

τ_{ij}.

It can be shown that I can no longer relate X_{ij} to a relative price as in (1.24), no matter how I define the price p_i and price index P_j. This implies that I can no longer derive the system of gravity equations (1.28). Even more generally, I cannot write X_{ij} in any gravity-form, perhaps a different one than derived in the previous section.

In order to see this last point, consider the following very broad definition of a "gravity" specification:

$$X_{ij} = \frac{z}{d_{ij}} Z_i H_j \tag{1.39}$$

Here z is a constant, d_{ij} is a bilateral friction and Z_i and H_j are country specific variables. The term gravity originates from physics, where X_{ij} is the gravitational force between two objects i and j, z is the gravitational constant, d_{ij} is the square of the distance between the objects and Z_i and H_j are their masses.

In economics d_{ij} is often interpreted as distance as well, but more generally as a barrier between i and j (trade barrier for goods trade or financial friction for asset trade). Of course for any specification of bilateral asset trade there are always d_{ij} such that (1.39) holds. In order for (1.39) to have meaning as a gravity equation, d_{ij} must be exclusively related to (financial) frictions between i and j. It should not be related to variables unrelated to such frictions, such as moments of asset returns and country size variables.

In theory-based gravity specifications (such as in the previous section) Z_i and

H_j are products of multilateral resistance and size. However, (1.38) is inconsistent with (1.39) for any specification of Z_i and H_j, no matter the interpretation. In order to illustrate this I focus on the simple case where $N = 2$, where it is possible to analytically invert the various matrices in (1.38).

Start by defining for $i, j = 1, 2$

$$
\begin{array}{lll}
a_{ij} = \tau_{ij}\sigma_i^2 + \mathbf{\Omega}_{ii} & b_j = a_{1j}a_{2j} - \mathbf{\Omega}_{12}^2 & \\
e_1 = \sum_{k=1}^{2} W_k a_{2k}/b_k & e_2 = -\sum_{k=1}^{2} W_k \mathbf{\Omega}_{12}/b_k & e_3 = \sum_{k=1}^{2} W_k a_{1k}/b_k \\
h_1 = e_3 S_1 - e_2 S_2 & h_2 = e_1 S_2 - e_2 S_1 &
\end{array}
$$

I then have

$$
\begin{pmatrix} X_{1j} \\ X_{2j} \end{pmatrix} = \frac{1}{e_1 e_3 - e_2^2} \frac{W_j}{b_j} \begin{pmatrix} a_{2j}h_1 - \mathbf{\Omega}_{12}h_2 \\ a_{1j}h_2 - \mathbf{\Omega}_{12}h_1 \end{pmatrix} \tag{1.40}
$$

The question is whether this takes the general form (1.39), which implies

$$
\frac{X_{12}X_{21}}{X_{11}X_{22}} = \frac{d_{11}d_{22}}{d_{12}d_{21}} \tag{1.41}
$$

It is important to emphasize that d_{ij} is nothing other than a barrier between i and j, which in my application must be either equal to τ_{ij} or some function of that. Importantly, it should not be a function of other variables like variances, covariances and country size variables.

(1.40) implies that

$$\frac{X_{12}X_{21}}{X_{11}X_{22}} = \frac{(a_{22}h_1 - \mathbf{\Omega}_{12}h_2)(a_{11}h_2 - \mathbf{\Omega}_{12}h_1)}{(a_{21}h_1 - \mathbf{\Omega}_{12}h_2)(a_{12}h_2 - \mathbf{\Omega}_{12}h_1)} \tag{1.42}$$

This expression is clearly not just a function of the bilateral barriers τ_{ij}. Even when $\mathbf{\Omega}_{12} = 0$, so that the expression boils down to $a_{22}a_{11}/(a_{21}a_{12})$, it still depends on the variances σ_i^2 and $\mathbf{\Omega}_{ii}$. Only when I set the entire matrix $\mathbf{\Omega}$ equal to zero does this become $\tau_{11}\tau_{22}/(\tau_{12}\tau_{21})$, consistent with (1.41). This confirms that it is simply not possible to express bilateral asset holdings as a gravity form in a general setup.

1.3.2 Financial Friction as Tax or Transaction Cost

As discussed in Section 2, international financial frictions are often modeled in the literature as a tax or transaction cost. One can introduce this in different ways. Consider agents from country j who invest in the assets from country i. In the absence of a tax the return is R_i. One can introduce an additive tax, making the return $R_i - \tau_{ij}$. Alternatively one can introduce a multiplicative tax, making the return $(1 - \tau_{ij})R_i$. One can also tax the price of the asset, making the price $(1 + \tau_{ij})Q_i$ for investors from country j, or tax the dividend. All of these alternative ways of introducing a tax (or transaction cost) lead to fundamentally the same expression once I take a second-order approximation of the first-order conditions.

Introducing a second-order multiplicative tax τ_{ij}, such that the return becomes $(1-\tau_{ij})R_i$, gives

$$\alpha_{ij} = \frac{1}{p_i} - \frac{\tau_{ij}}{\gamma\sigma_i^2} \tag{1.43}$$

with p_i as defined in Section 2. Note that the financial friction now enters in the form of a separate additive term in α_{ij} rather than multiplicative in the first term. The reason is that it subtracts a second-order component from the expected excess return of all assets.

Imposing market equilibrium, I have

$$X_{ij} = \frac{W_j S_i}{W} + \frac{W_j}{\gamma\sigma_i^2}\left(\hat{\tau}_i - \tau_{ij}\right) \tag{1.44}$$

where $W = \sum_{j=1}^{N} W_j$ is world financial wealth and $\hat{\tau}_i = \sum_{j=1}^{N}(W_j/W)\tau_{ij}$ is a weighted average financial friction that destination country i faces with all source countries. It is impossible to write this in the general gravity form (1.39). The reason for this is the additive term on the right hand side of (1.44). As was the case with a general covariance structure, bilateral asset holdings are now a complex non-linear function of country size variables, second moments of asset returns and financial frictions.

1.3.3 Only Trade in Equity

Finally I consider the case in which there is only trade in equity. In the benchmark model, all equity positions are positive while bond holdings are both positive and negative (they aggregate to zero). However, there are no restrictions on borrowing (negative bond holdings). In reality such restrictions can be quite severe and lenders demand collateral from the borrowers. This reduces the extent of the holdings of the riskfree asset, both positive and negative. Rather than explicitly introducing such borrowing constraints based on collateral, here I will only briefly discuss the extreme case that rules out borrowing altogether. In that case, there is only trade in equity. Less severe borrowing constraints, based on collateral, lead to the same qualitative point: gravity falls apart.[10]

Equilibrium bilateral holdings in this case (after imposing market equilibrium) are highly complex. To be precise, I get

$$X_{.,j} = b_j W_j + W_j \mathbf{M}_j \left(\sum_{j=1}^{N} W_j \tilde{\mathbf{M}}_j \right)^{-1} \left(\mathbf{S} - \sum_{j=1}^{N} W_j \tilde{b}_j \right) \qquad (1.45)$$

where b_j is a vector of size N with element i equal to $\frac{1}{\sigma_i^2 \tau_{ij} h_j}$, $h_j = \sum_{i=1}^{N} 1/[\sigma_i^2 \tau_{ij}]$,

[10]This case is also of interest in analogy to the gravity theory for goods trade, where agents usually can buy only differentiated goods.

$\mathbf{S} = (S_2, .., S_N)'$ and $\mathbf{M}_j$ a N by $N-1$ matrix with

$$\mathbf{M}_j[i, k-1] = -\frac{1}{\gamma R \sigma_i^2 \sigma_k^2 \tau_{ij} \tau_{kj} h_j} \quad k \neq i \tag{1.46}$$

$$\mathbf{M}_j[i, i-1] = -\frac{1}{\gamma R \sigma_i^4 \tau_{ij}^2 h_j} + \frac{1}{\gamma R \sigma_i^2 \tau_{ij}} \tag{1.47}$$

$\tilde{b}_j$ and $\tilde{\mathbf{M}}_j$ refer to the last $N-1$ rows of respectively b_j and $\mathbf{M}_j$.

As was the case with the other two extensions, this is a complex expression that relates bilateral asset holdings to measures of country size, second moments of asset returns and financial frictions. It cannot be written in the general gravity form (1.39).

I should finally emphasize that of course the extensions that I have discussed in this section are by no means exhaustive. Others, such as non-financial wealth, may need to be considered as well. But the overall message is that most extensions will not deliver a gravity form.

1.4 Estimation and Comparative Statics

In this section, I will describe how to estimate the size of financial frictions and conduct comparative statics analysis with respect to changes in financial frictions. I will discuss how to do so both for gravity system (1.28) as well as various extensions of it.

1.4.1 Estimation

I first discuss three estimation methods for the bilateral financial frictions based on the gravity system (1.28)-(1.31). The first method is analogous to that commonly used in the trade gravity literature today. I first relate the unobservable bilateral financial frictions to various observables. Specifically, assume that

$$ln(\tau_{ij}) = \sum_{m=1}^{M} \phi_m z_{ij}^m \qquad (1.48)$$

The variables z_{ij}^m need to be such that they can be thought of as affecting financial frictions and particularly information frictions. Examples are language, legal and regulatory similarities. They cannot be things like asset returns or correlations of returns.

Substituting (1.48) into the logarithm of the gravity equation (1.28), and replacing $ln(S_i) + ln(\Pi_i/E)$ and $ln(E_j) + ln(P_j)$ with respectively destination and source dummies η_i and ξ_j, I have

$$ln(X_{ij}) = -\sum_{m=1}^{M} \phi_m z_{ij}^m + \eta_i + \xi_j + \epsilon_{ij} \qquad (1.49)$$

An error term is added that can be interpreted for example as data measurement error of bilateral financial holdings. Regressing the log of bilateral holdings on the z_{ij}^m, as well as source and destination country dummies, provides us with estimates of ϕ_m and therefore the relationship between financial frictions and various observables.[11] Note that when one of the z_{ij}^m variables is a border dummy $Home_{ij}$ that is 1 when $i = j$ and 0 otherwise, it allows us to also estimate the average of all residual cross-border frictions that are not captured by any of the other variables z_{ij}^m.

The second estimation method exploits the fact that when using $W_j = E_j P_j$ I can also write the gravity equation (1.28) as

$$X_{ij} = \frac{W_j S_i}{E} \frac{\Pi_i}{\tau_{ij}} \tag{1.50}$$

Taking logs, defining $\theta_i = ln(S_i) + ln(\Pi_i/E)$ as a destination country dummy, and adding an error term, I have

$$ln(X_{ij}/W_j) = -\sum_{m=1}^{M} \phi_m z_{ij}^m + \theta_i + \epsilon_{ij} \tag{1.51}$$

The difference in comparison to (1.49) is that there is no source country dummy in this regression. This implies that source country specific frictions can now be identified as well: some of the z_{ij}^m may only depend on j. Examples are regulatory quality and financial market sophistication of the source country.

The reason that such source country specific frictions can be identified is as

[11] This method is easily extended to panel data by adding time subscripts to the z_{ij}^m and the source and destination dummies. Note that time-varying financial frictions lead to time-varying multilateral resistance, so that for each period there need to be separate source and destination country dummies.

follows. An increase in source country specific frictions does not change relative financial frictions for that source country because its multilateral resistance rises proportionally. However, the higher multilateral resistance lowers E_j. It causes a general shift out of equity and into bonds by country j. It is this general shift out of equity by a source country that allows us to identify such frictions.

Finally, a third method estimates bilateral frictions directly by using

$$\left(\frac{X_{ij}X_{ji}}{X_{ii}X_{jj}}\right)^{-0.5} = \left(\frac{\tau_{ij}\tau_{ji}}{\tau_{ii}\tau_{jj}}\right)^{0.5} \tag{1.52}$$

or

$$\left(\frac{X_{ij}/W_j}{X_{ii}/W_i}\right)^{-1} = \frac{\tau_{ij}}{\tau_{ii}} \tag{1.53}$$

A drawback of these measures is that they are very sensitive to measurement error of bilateral equity holdings for individual pairs. Such measurement error can be significantly reduced by computing the following weighted harmonic mean of frictions of country i as a destination, which follows from (1.53):

$$\frac{\sum_{j\neq i} W_j}{\sum_{j\neq i} \frac{1}{\tau_{ij}} W_j} = \frac{X_{ii}}{\sum_{j\neq i} X_{ij}} \frac{\sum_{j\neq i} W_j}{W_i} \tag{1.54}$$

where $X_{ii} = S_i - \sum_{j\neq i} X_{ij}$. All that is needed to compute this is the aggregate external equity liabilities of country i, measures of wealth and aggregate stock market capitalization.

So far I have only discussed estimation of (1.28) based on the benchmark model. I now turn to extensions. First consider the fixed cost extensions, which have the advantage that the overall gravity form is retained. If the fixed cost is such that agents invest only in a subset of the destination countries (some of the X_{ij} are zero), all of the estimation methods described above continue to hold when I remove the country pairs for which $X_{ij} = 0$.

Next consider the case where as a result of fixed costs only a fraction of the agents is globally diversified and the other agents invest only in domestic equity and bonds. Defining W_i^A as the wealth of agents that are globally diversified, we have seen that this extension implies that the overall financial friction becomes $\tau_{ij}\delta_{ij}$ with $\delta_{ii} = W_i^A/W_i$ and $\delta_{ij} = 1$ when $i \neq j$. This means that for all $i \neq j$ the friction is still τ_{ij}. One approach is therefore to adopt the first estimation method described above, applied to only cross border holdings ($i \neq j$).[12] Gravity estimation based on cross-border holdings alone (ignoring the X_{ii} observations) is in fact most common in the existing empirical gravity literature.

This has the drawback though that it is impossible to measure the overall magnitude of cross-border information frictions. In particular, we could not identify the coefficient on the residual border dummy $Home_{ij}$, which is zero for all $i \neq j$. We could use the third method described above, based on any of the equations (1.52) through (1.54), to measure overall financial frictions $\tau_{ij}\delta_{ij}$. But it does not

[12] This method can also be applied to the case discussed at the very end of section 2 where some of domestic equity holdings are entirely unrelated to a diversification motive.

allow us to distinguish between information frictions τ_{ij} and the fixed cost friction δ_{ij}.

Another approach is to relate the unobservable W_i^A/W_i to a set of country-specific variables. These would be related to individual-specific variables that have been identified in the literature as affecting whether agents hold any foreign assets. Examples are financial sophistication, resources, education and age, for which it is easy to develop corresponding country-wide measures. So assume

$$ln(\delta_{ii}) = \sum_{l=1}^{L} \mu_l h_i^l \tag{1.55}$$

Let the first variable, h_i^1 be a constant set at 1. Also, let $z_{ij}^1 = Home_{ij}$.

The gravity specification then becomes

$$ln(X_{ij}) = (\psi_1 + \mu_1)Home_{ij} + \sum_{m=2}^{M} \psi_m z_{ij}^m + \sum_{l=2}^{L} \mu_l h_j^l Home_{ij} + \eta_i + \xi_j \tag{1.56}$$

Using data on both cross-border and domestic asset holdings we can estimate the coefficients $\psi_1 + \mu_1$, ψ_m $(m = 2, .., M)$ and μ_l $(l = 2, .., L)$. It is not possible to distinguish the information and fixed cost frictions only to the extent that the former cannot be attributed to variables z_{ij}^m $(m > 1)$ *and* the latter cannot be attributed to the source country variables h_i^l $(l > 1)$.

Finally, consider extensions such as those discussed in Section 3, where we do not get a gravity specification at all. For concreteness, consider the first generalization of Section 3, where I introduced a general covariance structure. Substituting (1.48) into (1.38), taking logs and adding an error term, I get

$$ln(X_{ij}) = f(\phi_1, .., \phi_M; \mathbf{\Omega}, \sigma_k^2, W_k, S_k, z_{kl}^m \;\; k, l = 1, .., N, m = 1, .., M) + \epsilon_{ij} \quad (1.57)$$

This relates bilateral holdings to the unknown parameters $\phi_1, .., \phi_M$ that need to be estimated and a set of data that includes variances and covariances of asset returns, country size variables and the variables impacting the bilateral frictions. This system can then be estimated for example with non-linear least squares.

The same applies to the other extensions discussed in Section 3. While deviations from gravity therefore do not pose any particularly difficult new problems in estimation of international financial frictions, the method obviously stands in stark contrast to the existing empirical gravity literature. An important direction for future empirical work will be to understand whether, and to what extent, such generalizations fit the bilateral asset data better than the gravity specification (1.28).

1.4.2 Comparative Statics

First consider comparative statics analysis in the context of the gravity system (1.28). Consider the impact of a change in τ_{ij} of any magnitude on bilateral asset holdings X_{kl} for any country pair (k, l).[13] Using $E_l = W_l/P_l$, the gravity equation becomes

$$X_{kl} = \frac{S_k W_l}{E} \frac{\Pi_k}{\tau_{kl}} \tag{1.58}$$

The bilateral financial claim X_{kl} is only affected through a change in Π_k/τ_{kl}. All we therefore need to know is the change in Π_k. Substituting $E_l = W_l/P_l$ into (1.30), we have

$$\frac{1}{\Pi_k} = \sum_{s=1}^{N} \frac{1}{\tau_{ks}} \frac{W_s}{E} \tag{1.59}$$

A change in τ_{ij} only affects X_{kl} when $k = i$. Using (1.58) and (1.59), a change from τ_{ij} to τ'_{ij} implies

$$X'_{il} = X_{il} \frac{1}{1 + \frac{X_{ij}}{S_i} \left(\frac{\tau_{ij}}{\tau'_{ij}} - 1 \right)} \frac{\tau_{il}}{\tau'_{il}} \tag{1.60}$$

where the last ratio is 1 when $l \neq j$.

Introducing fixed costs does not change this formula at all, whether it leads to zero cross-border holdings for some country pairs or to a group of agents that does not hold any foreign equity. Note that in the latter case τ_{ij} needs to be replaced by $\tau_{ij}\delta_{ij}$, but when considering only the impact of changes in information frictions $\delta_{ij} = \delta'_{ij}$ and therefore (1.60) still applies.

While a simple analytic comparative statics result such as (1.60) no longer applies under the generalizations considered in Section 3, it is still straightforward to compute the impact of changes in financial frictions even there. Consider the

[13] Of course we could simultaneously change many bilateral frictions, but this simply involves repeating the steps for different i and j, with a multiplicative impact on X_{kl}.

first generalization, a more general covariance structure. For given values of $\mathbf{\Omega}$, σ_k^2, W_k and S_k $(k = 1, .., N)$, which do not depend on bilateral frictions, we can use (1.38) to compute the changes in all bilateral asset holdings resulting from changes in bilateral barriers. The same can be done for the other generalizations.

1.5 Conclusion

The rapidly growing empirical gravity literature on cross-border asset holdings clearly calls out for a theory. I have developed a theory for bilateral asset holdings that takes a gravity form and I discussed how to estimate international financial frictions and conduct comparative statics analysis within the context of the theory. Nonetheless, some strong assumptions needed to be made to derive at such a theory. In contrast to goods trade, where many different types of models generate a gravity structure, reasonable changes in assumptions of my model do not deliver a gravity form for bilateral asset holdings.

This chapter has been entirely theoretical, but it has laid a clear foundation for future empirical work. Even if one accepts the assumptions of my model that lead to a gravity form, existing empirical work often suffers from omitted variables (fixed effects) or the inclusion of variables that do not belong (e.g. return correlations). But perhaps more importantly, future empirical work needs to evaluate the empirical relevance of various extensions such as those I discussed. This is important both to understand what type of model better describes the data and ultimately to estimate the magnitude of cross-border financial frictions.

Chapter 2

Welfare Implications of Financial Globalization

2.1 Introduction

The question of whether globalization and financial deepening are beneficial for the poor is the subject of intense debate among policymakers and researchers. The G20 Financial Inclusion Experts Group (2010) stated that it "reiterates its strong commitment to financial inclusion and recognizes the benefits of universal access to financial services." There is much research on the effect of trade globalization on the poor in the international trade and development economics fields.[1] How-

[1] Some recent examples include Aisbett, Harrison and Zwane (2006), Harrison (2006), Harrison (2006), Ackah, Morrissey and Appleton (2007), Meschi and Vivarelli (2007), Hill and Rapp (2009), Naranpanawa, Bandara and Selvanathan (2011), Carvalho and Teixeira (2011), Castilho, Menndez and Sztulman (2012)

ever, little research that formally treats intra-country wealth distribution has been performed in the international finance field.[2]

The goal of this chapter is to build a model that analyzes the welfare implications of financial globalization across different wealth levels in the open economy. I build a two-country open-economy dynamic general equilibrium portfolio choice model. I follow Krusell and Smith (1998) and introduce wealth heterogeneity within countries. In the model, investors allocate their wealth to a Home asset and a Foreign asset. To participate in the Foreign stock market, investors must pay fixed costs. Because the disutility of paying fixed costs is a decreasing function of wealth, only rich investors choose to participate. The welfare impact of lower fixed costs or financial globalization can differ depending on wealth level.

The international portfolio choice model, or portfolio balance model, has a long history in international finance. Significant early contributions include Black (1974), Branson and Henderson (1985), and Lucas (1982). These early models assume either a static or a complete market. Until the late 2000s, researchers were unable to solve dynamic general equilibrium models with incomplete financial markets without using specific functional form assumptions, such as log utility. Recent work, including that by Tille and van Wincoop (2010a), Evans and Hnatkovska (2005) and Devereux and Sutherland (2010), has solved these models computationally using approximation methods.

[2]For a general discussion regarding the financial globalization and welfare and for the survey, see Kose, Prasad, Rogoff and Wei (2009).

My model is the first to introduce wealth heterogeneity into the open-economy Dynamic Stochastic General Equilibrium portfolio choice model. Campbell (2006) reports that there is considerable heterogeneity in portfolio choice behavior among households within a country. This heterogeneity is particularly important in the international portfolio choice models. as foreign stock market participation is concentrated in wealthy households. The welfare implications of decreased portfolio home bias depend on the wealth. My model also sheds new light on the well-studied welfare implications of the home bias in portfolio holdings. Previous research in this area includes Cole and Obstfeld (1991), Backus, Kehoe and Kydland (1992), van Wincoop (1999), Townsend and Ueda (2007), Evans and Hnatkovska (2007) and Tille (2008).

To illustrate the model and the solution method, I calibrate the model to match data from 1980 to 2010 and extrapolate the model to 2100. This calibration exercise demonstrates that poor, non-foreign asset market participating agents can be benefited by a specific type of financial globalization. To benefit the poor, financial globalization should lower the fixed cost of investing abroad to increase the number of foreign stock market participants. The gain from globalization is greater when the decrease in cost is bilateral. This results from the general equilibrium effect, analogously to the effects discussed in the previous chapter.

The rest of the chapter is structured as follows. Section 2 presents data about the home bias and foreign stock market participation. The model and its solution method are described in Section 3. In Section 4, the calibration is presented. The results are discussed in Section 5, and Section 6 gives the conclusion.

2.2 Data

In this section, I present data regarding international portfolio investment from both aggregated data and microdata.

2.2.1 Macroeconomic Evidence for External Investment

Figure 2.1 is a time-series plot of external liability divided by market capitalization from 1970 through 2004 in the United States, Japan, and the United Kingdom, created using the data assembled by Milesi-Ferretti and Lane (2007). I chose these three countries, because they have the longest data. The share shows an increasing trend. Table 2.1 demonstrates that this trend occurs in all of the G7 countries. On average, the ratio of portfolio liability increased by 17% from 1988 to 2004.

2.2.2 International Investment and Household Wealth

Next, I turn to microdata. I use the triennial Survey of Consumer Finances conducted by the U.S. Federal Reserve Board. This public data set is unique for two reasons. First, it contains detailed information about asset positions. Second, it oversamples wealthy individuals, who are vital for understanding the aggregated

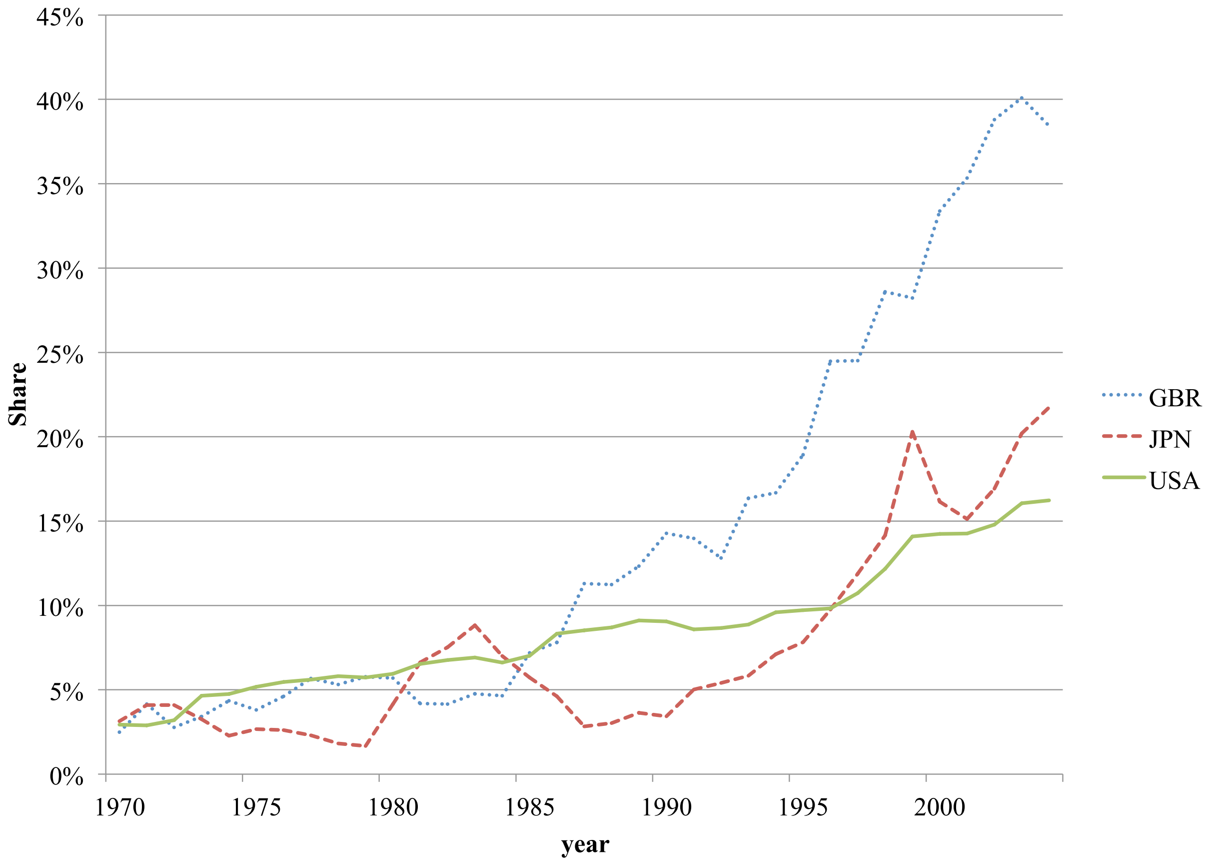

Figure 2.1: Share of portfolio liability relative to GDP
source: Milesi-Ferretti and Lane (2007)

Table 2.1: Changes in gross external liability divided by stock market capitalization in G7 countries

	1988	2004	increase
Canada	12%	22%	10%
Germany	21%	34%	13%
France	13%	38%	25%
United Kingdom	10%	36%	26%
Italy	7%	29%	21%
Japan	3%	20%	17%
United States	8%	13%	5%
Average	11%	27%	17%

source: Milesi-Ferretti and Lane (2007)

behavior of household portfolios.

Table 2.2 shows the significance of wealthy individuals. In 2010, the top 10% of individuals, those with financial wealth greater than 432,030 USD, held 79.2% of the total individual financial wealth in the U.S. In other words, the "average" of the top 10% of individuals held 34 times as much financial wealth as the "average" of the bottom 90%. Wealthy households had a disproportionate influence on the macroeconomic effect of household portfolio choices. Column (2) and column (3) provide the domestic and foreign stock market participation for each wealth level, respectively, with both rates exhibiting increases.

Figure 2.2 is a kernel regression of the log of financial wealth on the indicator variable of foreign stock market participation rate given domestic stock market participation. The figure demonstrates that there is an almost linear relationship between foreign stock market participation and the log of the wealth level. For individuals with awealth level of 100,000 USD, the estimated foreign stock market participation rate was 10%. For individuals with a wealth level of 10,000,000 USD, the estimated foreign stock market participation rate was 31%, more than three times that for the former group. In contrast, the portfolio share of foreign stock depends less on wealth than the participation rates do, given foreign stock market participation. For individuals with wealth levels of 100,000 USD and 10,000,000 USD, the portfolio shares were 23% and 33%, respectively. These percentages did depend on the wealth, but exhibited less elasticity than participation rates did.

Table 2.2: Summary statistics of the Survey of Consumer Finance 2010

Wealth Category	(1) Minimum Financial Wealth	(2) Domestic Participation	(3) Foreign Participation given Domestic Participation	(4) wealth share
Lowest 25%	$0	0.9%	0.0%	0.0%
25% - 50%	$1,410	4.8%	9.7%	0.8%
50% - 75%	$17,000	13.9%	6.5%	5.4%
75% - 90%	$105,620	28.5%	15.5%	14.6%
90% - 99%	$432,030	55.8%	17.6%	43.6%
above 99%	$3,560,000	70.9%	33.4%	35.6%

Source: authors' calculation
Domestic participation is the participation rate in the U.S. market. Foreign Participation given Domestic Participation is the participation rates in the foreign stock market given U. S. stock market participation.

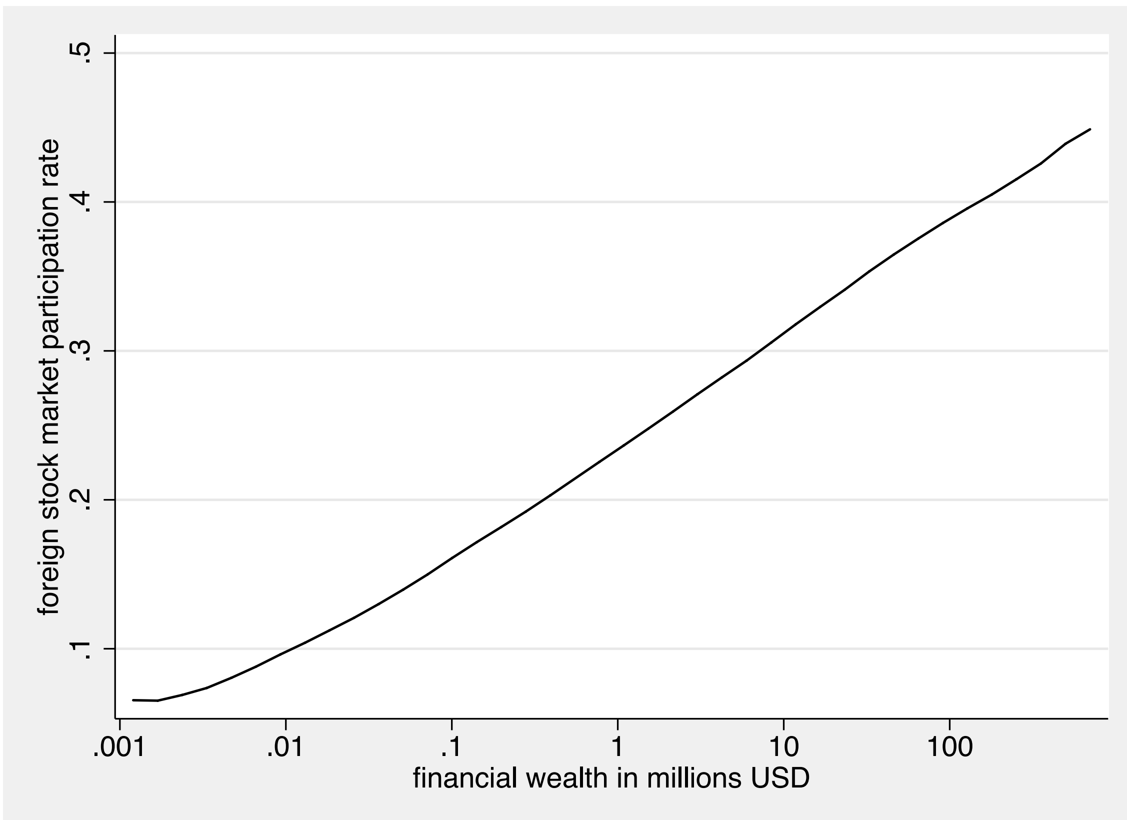

Figure 2.2: Kernel regression of foreign stock market participation on log of financial wealth

Source: authors' calculation

2.3 Model

This section describes a dynamic general equilibrium portfolio choice model in an open economy. This model has two features: an incomplete asset market due to the non participation to the foreign market and heterogeneous wealth distribution within a country. To maintain a focus on the interaction between these new features and portfolio choice, this paper considers a deliberately simplified model.

There are two countries, Home and Foreign, each of which produces a common consumption good using capital and labor. The sizes of these countries are η and $1-\eta$, respectively. The supplies of both capital and labor are fixed. Asset markets are incomplete due to the costs of investing abroad. There are two types of costs. The first type of cost is proportional to the value of the portfolio. This type is quite common in the literature.[3] The second type of cost is fixed, regardless of the investment amount. This cost, combined with heterogeneous wealth, allows us to consider the decision to participate in Foreign asset markets. Because the benefit of participation is an increasing function of wealth, the benefit exceeds the fixed cost only for relatively rich households. There is no cost of participation in the domestic market. Costs decrease over time to capture the long-term trend of Foreign stock market participation rates. In the distant future, costs will reach zero and the asset markets will be complete.

[3] See, for example, Heathcote and Perri (2004), Martin and Rey (2004), Coeurdacier, Kollmann and Martin (2007), and Tille and van Wincoop (2010a)

2.3.1 Firms

Following Coeurdacier et al. (2007) and Tille and van Wincoop (2010a), I adopt a simple production structure. Both countries produce identical products using capital and labor. The production function for Home is

$$Y_t = Z_t K_t^{1-\theta} L_t^{\theta},$$

where Y_t is the output, Z_t is the productivity, and K_t is the capital. L_t is the effective labor:

$$L_t = \sum_i \rho_{i,t} l_{i,t},$$

where $\rho_{i,t}$ is the exogenous labor productivity of agent i and $l_{i,t}$ is the labor input of agent i.

The production function for Foreign is

$$Y_t^* = Z_t^* (K_t^*)^{1-\theta} (L_t^*)^{\theta},$$

where Y_t^* is the output, Z_t^* is the productivity, and K_t^* is the capital. L_t^* is the effective labor:

$$L_t^* = \sum_i \rho_{i,t}^* l_{i,t}^*,$$

where $\rho_{i,t}^*$ is labor productivity of agent i and $l_{i,t}^*$ is labor input of agent i.

A bundle of Home productivity and Foreign productivity $\omega_t = (Z_t, Z_t^*)$ is a discrete Markov process with N^2 possible values, where N is the number of possible values for Z_t and Z_t^*. Following Krusell and Smith (1998), the productivity in each country takes two levels, high and low. Capital supply and effective labor supply are constant and normalized to unity. Therefore, the production in Home and Foreign becomes

$$Y_t = Z_t, \qquad Y_t^* = Z_t^*,$$

respectively.

The labor markets are competitive. The equilibrium effective wages are equal to the marginal product of the effective labor:

$$w_t = \theta Z_t \qquad w_t^* = \theta Z_t^*, \tag{2.1}$$

where w_t and w_t^* are the effective wages in Home and Foreign, respectively.

Stockholders own firms. In each period, stockholders receive dividends, which are equal to total production minus wages paid. Therefore, the dividends D_t and D_t^* in Home and Foreign, respectively, are

$$D_t = (1 - \theta) Z_t \qquad D_t^* = (1 - \theta) Z_t^*. \tag{2.2}$$

Firms pay a dividend to the owner at the beginning of every period based on equation (2.2). The stock returns are defined as

$$R_t = \frac{Q_{t+1} + D_{t+1}}{Q_t}, \qquad R_t^* = \frac{Q_{t+1}^* + D_{t+1}^*}{Q_t^*}$$

where R_t and R_t^* are the stock returns and Q_t and Q_t^* are the stock prices for Home and Foreign, respectively.

2.3.2 Households

Following Tille and van Wincoop (2010a), I adopt the framework of Caballero, Farhi and Gourinchas (2008) to abstract the decisions regarding savings and leisure choices. Namely, agents die with probability χ and same number of new agents are born. Agents consume only during the last period of life, during which they liquidate all of their assets. Agents work only in the first period of life. Their labor supply is inelastic. For periods other than the first and the last, agents make only portfolio choice decisions. Home and Foreign stocks are the only means of savings. Their preference is CRRA, i.e., their utility given consumption c is

$$\frac{c^{1-\gamma}}{1-\gamma}.$$

where γ is the relative risk aversion.

I added heterogeneity in wealth to the framework of Caballero et al. (2008). To generate a nontrivial wealth distribution, I assume that agents differ in their natural-born labor productivity $\rho_{i,t}$. This difference generates a nontrivial wealth distribution in each cohort at age 0 through their labor income. Different portfolio decisions by rich agents and poor agents generate wealth distribution dynamics after that.

We can interpret the labor productivity distributions as the wealth distribution of newly participating investors. For simplicity, I assume that the labor productivity distribution for a cohort born at time t is proportional to a distribution of wealth for the entire country at time t. This assumption implies that the only source of wealth distribution dynamics is the portfolio choices.

The assumptions of working only during the first period of life and of a constant death probability imply that the current wealth is sufficient statistics for agents portfolio choices. That is, the cohort or the age of an agent does not play any role in the model after age 0. The only heterogeneity that matters after that is wealth.

2.3.3 Costs of Investing Abroad

Agents must pay costs when they invest abroad. There are two types of costs, fixed and proportional. A proportional cost is an iceberg type cost. A certain percentage of the foreign asset returns melts. As mentioned previously, a proportional cost for investing abroad is quite common. Agents pay a fixed cost every period when they purchase assets abroad. This cost prevents some agents from investing abroad. There is no cost to invest in domestic assets; therefore, everyone participates in the domestic stock market.

Costs are exogenous and vary with time. They gradually decrease and converge to 0 in the long run, explaining the increasing trend in Foreign stock share. I assume the following functional form for the costs:

$$\psi_t = \begin{cases} \psi_0 \left(\xi_\psi\right)^t & t < T \\ 0 & t \geq T \end{cases} \qquad \tau_t = \begin{cases} \tau_0 \left(\xi_\tau\right)^t & t < T \\ 0 & t \geq T \end{cases} \tag{2.3}$$

where τ_t is the proportional cost, ψ_t is the fixed cost for participation, $0 < \xi_\psi < 1$, and $0 < \xi_\tau < 1$. This assumption ensures the existence of long-run equilibrium. It also allows us to solve the model backward.

The wealth of a Home agent i who participates in the asset market abroad accumulates according to

$$a_{i,t+1} = (a_{i,t} - \psi_t)[k_{i,t}R_t + (1 - k_{i,t})e^{-\tau_t}R_t^*],$$

where $a_{i,t}$ is the asset of agent i at time t, and $k_{i,t}$ is the portfolio share for a Home asset. The wealth of a Home agent who does not participate in the Foreign stock market accumulates according to

$$a_{i,t+1} = a_{i,t}R_t.$$

The wealth of Foreign agent i accumulates according to

$$a^*_{i,t+1} = (a^*_{i,t} - \psi_t)((1 - k^*_{i,t})e^{-\tau_t} R_t + k^*_{i,t} R^*_t)$$

or

$$a^*_{i,t+1} = a^*_{i,t} R^*_t.$$

2.3.4 Bellman Equation

The Bellman equation for a Home agent is

$$v_t(a_{i,t}; \omega_t, \mathbf{\Gamma}_t) = \max_{m_{i,t}, k_{i,t}} \left(\beta(1-\chi)\mathrm{E}\left[v_{t+1}(a_{i,t+1}; \omega_{t+1}, \mathbf{\Gamma}_{t+1})\right] + \beta\chi\mathrm{E}\left[\frac{a^{1-\gamma}_{i,t+1}}{1-\gamma}\right]\right) \tag{2.4}$$

subject to

$$a_{i,t+1} = \begin{cases} (a_{i,t} - \psi_t)(k_{i,t} R_t + (1 - k_{i,t})e^{-\tau} R^*_t) & m_{i,t} = 1 \\ a_{i,t} R_t & m_{i,t} = 0 \end{cases} \tag{2.5}$$

where $\mathbf{\Gamma}_t$ is the distribution of wealth in both countries, β is the discount factor, and $m_{i,t}$ is an indicator for participating in a Foreign asset market. The value function v depends on the wealth distribution as the distribution determines asset prices.

The first-order condition for portfolio choice $k_{i,t}$, given participation, is

$$\beta \mathrm{E}\left([\{1-\chi\}v'_{t+1}(a_{i,t+1}) + \chi a_{i,t+1}^{-\gamma}](R_t - e^{-\tau_t}R_t^*)\right] = 0. \tag{2.6}$$

This condition represents an intratemporal tradeoff between assets. The first part $\{(1-\chi)v'_{t+1}(a_{i,t+1}) + \chi a_{i,t+1}^{-\gamma}\}$ is a pricing kernel, which is the marginal utility of wealth at the time $t+1$. Equation (2.6) states that the covariance between the asset return minus costs and the marginal utility of wealth is equalized in equilibrium. In a complete market without costs, this equalization implies that the ratio of marginal utility is constant. In my model, equalization of marginal utility is not attained, because of the presence of the costs. Even if τ_t is 0, complete risk sharing is not attained, because of the presence of nonparticipation.

2.3.5 Market Clearing

The goods market clearing condition is

$$\eta Z_t + (1-\eta)Z_t^* = \chi(\eta A_t + (1-\eta)A_t^*), \tag{2.7}$$

where A_t and A_t^* are the per capita asset values in each country.

$$A_t = \frac{1}{\eta}\int_i a_{i,t}\mathrm{d}i, \qquad A_t^* = \frac{1}{1-\eta}\int_i a_{i,t}^*\mathrm{d}i$$

For asset markets, I use the following market clearing conditions:

$$\eta Q_t = \int_i k_{i,t} a_{i,t} \mathrm{d}i + \int_{i \in \{m^*_{i,t}=1\}} (1 - k^*_{i,t}) a^*_{i,t} \mathrm{d}i \tag{2.8}$$

$$(1-\eta) Q^*_t = \int_{i \in \{m_{i,t}=1\}} (1 - k_{i,t}) a_{i,t} \mathrm{d}i + \int_i k^*_{i,t} a^*_{i,t} \mathrm{d}i \tag{2.9}$$

.

From (2.8) and (2.9), I have

$$\eta Q_t + (1-\eta) Q^*_t = \eta A_t + (1-\eta) A^*_t$$

Using (2.7),

$$\chi(\eta Q_t + (1-\eta) Q^*_t) = \eta Z_t + (1-\eta) Z^*_t \tag{2.10}$$

2.3.6 Equilibrium

The equilibrium is a bundle of prices (Q_t, Q^*_t), agents' choice variables ($m_{i,t}$, $k_{i,t}$, $m^*_{i,t}$, $k^*_{i,t}$), value functions ($v_t(\cdot)$, $v^*_t(\cdot)$), and law of motion of the wealth distribution $\Lambda_t(\omega_t)$ such that in each time and state, the following conditions apply:

1. The choice variables solve the Bellman equations given the value functions, the prices and the law of motion.

2. The value functions satisfy the Bellman equations given the choice variables.

3. The wags satisfy the labor market clearing condition (2.1).

4. The choice variables satisfy the asset market clearing conditions (2.8) and (2.9).

5. The law of motion is consistent with the choice variables.

2.3.7 Welfare

A certainty equivalent consumption $ce_{i,t}$ for an agent i at time t is defined as a value that satisfies the following relationship:

$$v_t(a_{i,t}) = \sum_{s=1}^{\infty} \frac{(\beta\chi)^{s-t-1}(1-\beta\chi)ce_{i,t}^{1-\gamma}}{1-\gamma} \tag{2.11}$$

where $a_{i,t}$ is an equilibrium wealth.

Since the certainty equivalent consumption at time t depends on the states at time t and before, I introduce another measure for welfare, which does not depend on states. I define an expected certainty equivalent consumption $\nu_{i,t}$ for an agent i at time t as a time 0 expected certainty equivalent consumption:

$$\nu_{i,t} = \mathrm{E}_0(ce_{i,t}). \tag{2.12}$$

2.3.8 Solution Method

The model is solved computationally. Once I can solve the Bellman equation (2.4) given some prices, I can use a standard optimization technique to determine the prices that clear the market. To solve (2.4), I need the value functions, next-period asset prices and wealth distribution of the next period.

I rely on backward recursion for the next-period value function. The key assumption is that after certain periods, the costs converge to 0 and remain at 0 forever. The model becomes stationary in a complete financial market. I can apply the Negishi (1960) method to solve the general equilibrium model, yielding "terminal" value functions and prices. I solve backward from the terminal value function.

I use the approximate aggregation method, proposed by Krusell and Smith (1998) for the wealth distribution and prices. Their method assumes that agents project[4] wealth distributions, which are infinite-dimensional objects, to the space of linear combinations of finite distributional moments. I use the ratio g of aggregate Home wealth relative to aggregate Foreign wealth, as the moment. Using g, I suppose that

$$\log(g_{t+1}) = \phi^g_{0,t}(\omega_{t+1}) + \phi^g_{1,t}(\omega_{t+1})\log(g_t) \tag{2.13}$$

$$\log(Q_t) = \phi^Q_{0,t}(\omega_t) + \phi^Q_{1,t}(\omega_t)\log(g_t) \tag{2.14}$$

where $\phi^j_{i,t}(\omega_t)$ are time-dependent and state-dependent coefficients. I do not need to guess Q^*_t because I can calculate it from equation (2.10).

I determine the equilibrium expectation ϕ iteratively. First, I derive the value

[4]For the interpretation of their method as projection, see Young (2010)

function backwards, using equations (2.13) and (2.14). Then, I simulate the economy forwards, using the value function, and run regressions for equations (2.13) and (2.14) to update ϕ. I repeat this process until ϕ converges. Details of the solution method are provided in the Appendix B.

2.4 Parameters

My benchmark calibration is presented in table 2.3. I calibrate the model using the United States and the rest of the world, which consists of the 24 countries in the OECD definition of "OECD former total", namely, Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, and the United Kingdom. The relative size of Home, η, is set to match the ratio of the U.S. GDP relative to that of the rest of the world: this value was 0.369 in 2011.

The productivity process is calibrated so that the second moments of the model returns match the data. Specifically, the transition matrix and the difference of the aggregate production for High production periods and Low production periods are calibrated to match the excess return variance, the Home return variance, the Goreign return variance and the return correlation. For the Home annual asset returns, I use MSCI USA Standard (Large + Mid Cap) index. For Foreign annual equity returns, I use MSCI ACWI ex USA Standard (Large+Mid Cap). The MSCI ACWI Index is a free float-adjusted market capitalization-weighted index that is designed to measure the equity market performance of developed and emerging markets[5]. These two indices gives 0.9% variance for annual excess return from 2000 to 2012. For the same period, the U. S. stock return variance is 3.6% and the Rest of the World stock return variance is 6.4%. The return correlation is 0.94. The transition matrix is provided in table 2.4. Production for High state and for Low state is set to 1.5553 and 0.4447, respectively.

The parameters regarding the frictions, ψ_0, τ_0, ξ_ψ, and ξ_τ are calibrated to replicate the behavior of the U.S. market. Using these four parameters, I matched the Foreign stock market participation rates in 1992 and in 2010 and the total external portfolio liability divided by market capitalization in 1980 and in 2004. The participation rate data are taken from the Survey of Consumer Finance, and the total external portfolio liability is taken from Milesi-Ferretti and Lane (2007). I assume that the model starts in 1980 and that the friction becomes zero after 2150. I discard the results from the last 50 years.

The initial within-country wealth distribution is calibrated so that the wealth distribution at $t = 31$, or year 2010, matches the Home wealth distribution data from the Survey of Consumer Finance at 2010. The within-country distribution for the Foreign is simply assumed to be same as the Home. The initial value of

[5] For detail of the index, please refer to
http://www.msci.com/products/indices/tools/index.html#ACWI

the ratio of total wealth for Home and Foreign is same as the ratio of GDP.

Following Tille and van Wincoop (2010a), χ, the probability of death, is set to 0.1. This implies an equilibrium wealth consumption ratio of 10. The initial distribution of wealth is taken from the Survey of Consumer Finance 2010 data. I use the total financial wealth.

Other parameters are set to the standard values. The rate of relative risk aversion, γ ,is 2. The labor share of income, θ, is 0.66. The discount factor, β, is set to 0.96.

Table 2.3: Model Parameters

γ, risk aversion	2
β, discount factor	0.96
θ, labor share of income	0.66
η, relative size of Home	0.37
χ, probability of death	0.1
ψ_0, initial fixed cost	0.72%
ξ_ψ, speed of decrease in fixed cost	0.87%
τ_0, initial proportional cost	0.49%
ξ_τ, speed of decrease in proportional cost	1.3%
Aggregate production(High)	1.5553
Aggregate production(Low)	0.4447
T, periods before cost becomes zero	171

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Table 2.4: Transition Matrix of Productivity

0.0141	0.2442	0.7372	0.0045
0.0046	0.7436	0.2378	0.0140
0.0140	0.2378	0.7436	0.0046
0.0045	0.2442	0.7372	0.0141

States are [(High, High), (High, Low), (Low, High), (Low, Low)], respectively

2.5 Results

2.5.1 Simplified Analysis

I first consider the simple exercises to clarify the economics of the model. Agents expect that frictions are positive and do not change over time[6]. I used costs corresponding to the year 2013 in the calibration section: $\tau = 0.32\%$, $\psi = 0.55\%$. At one period, agents are notified that some or all of the frictions are permanently removed. I study the immediate welfare implication of the change.

Result 1. There is a threshold wealth when a fixed cost of foreign stock market participation is introduced, at which only households whose wealth is above the threshold participate in the Foreign stock market.

Result 1 is the basics of all results after this. It states the existence of a threshold wealth for foreign stock market participation. The foreign stock market participation decision is a tradeoff between the fixed cost and the diversification bene-

[6]Due to the computational reasons, frictions becomes zero at the distant future. It does not affect the results.

fit. Because the utility cost of paying the fixed cost is a decreasing function of wealth, relatively rich households participate in the foreign stock market. This phenomenon is true only when the agents' absolute risk aversion is a decreasing function of consumption. The CRRA utility implies decreasing absolute risk aversion. This result will not hold if the utility function is CARA or quadratic.

Result 2. A percentage welfare gain by reducing fixed costs of investing Foreign assets for Home agents is a decreasing function of initial wealth. Rich agents are worse off when the fixed costs are removed.

Result 2 is the first result regarding the welfare implications of foreign investment costs. This result is about the relationship between wealth level and the welfare implication of removing the fixed costs. The relationship is illustrated in figure 2.3. This figure plots the changes in certainty equivalent consumptions from the model with both proportional costs and fixed costs to the models without fixed costs for each Home agent i. Total changes are decomposed to a general equilibrium effect and a partial equilibrium effect. A *partial equilibrium effect* is the effect of a certain change for given prices. A *general equilibrium effect* is defined as the difference between the total effect and the partial equilibrium effect.

The partial equilibrium effect of removing Home fixed costs is a weakly decreasing function of initial wealth. It is positive for poor agents and almost zero for rich agents. Once the fixed cost is removed, poor agents, who do not diversify their portfolio when fixed costs are present, are able to enjoy a diversification benefit. Rich participating agents are also benefited by the lower fixed costs. But the percentage gains for rich agents are smaller.

Next, we study the general equilibrium effect. Removing frictions on a Foreign asset decreases relative demand for a Home asset. The relative price for a Home asset decreases and relative price for a Foreign asset increases. This change has two implications on welfare. First, the change decreases welfare through lower effective wealth of an agent if a portfolio of the agent is biased towards home assets. Second, the change increases welfare through higher dividend yields on Home asset. Note that the dividend solely depends on the productivities. The general equilibrium effects are negative. In this analysis, the first wealth effect dominates the second dividend yield effects. The general equilibrium effect is larger for poor agents because their portfolio is more biased towards Home assets. This bias generates larger wealth effects.

The total effect is a summation of the partial equilibrium effect and the general equilibrium effect. It is a decreasing function of wealth. Since the general equilibrium effect is negative and the partial equilibrium effect is negligible for the rich, the total effect is negative for the rich.

Result 3. Percentage welfare gain by reducing proportional the cost of investing Foreign assets is a increasing function of wealth. Poor agents are worse off by

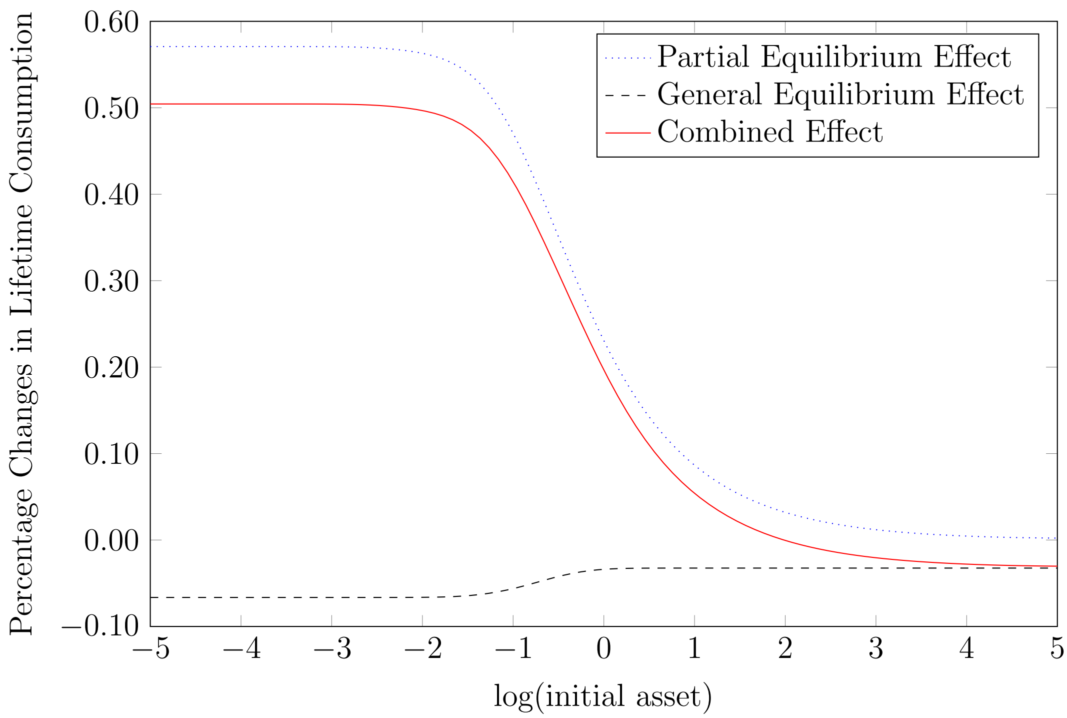

Figure 2.3: Decomposition of the welfare implications of removing fixed costs for Home agents to invest in the Foreign market for different wealth levels. Results are smoothed using the kernel regression

removing proportional costs.

Result 3 is about the welfare implications of removing proportional cost on the Foreign asset market for Home agents. The result is shown in figure 2.4.

The partial equilibrium effect is 0 for the poor, non-participating agents because they do not pay proportional costs. The effect is positive for the rich because they no longer have to pay the proportional costs.

For the general equilibrium effects, removing Home proportional costs decreases relative demand for Home assets. This decreases wealth for Home agents and the reduction is larger for poor agents. For middle-income agents, they stop participating the foreign stock market due to the decrease in their wealth. Welfare loss for middle-income agents is larger than that for poor agents. This middle income agent effect is not present in figure 2.3. In that figure, fixed costs are removed. Therefore, all agents participate in the foreign stock market regardless of their wealth.

The total effect of the partial equilibrium effect and the general equilibrium effect is negative for poor households. It is increasing function of wealth.

Result 4. Reduction of frictions on Home assets for Foreign agents is beneficial to participating Home agents. In particular, bilateral removal of frictions is more beneficial to Home agents than is removal of frictions for Home agents only.

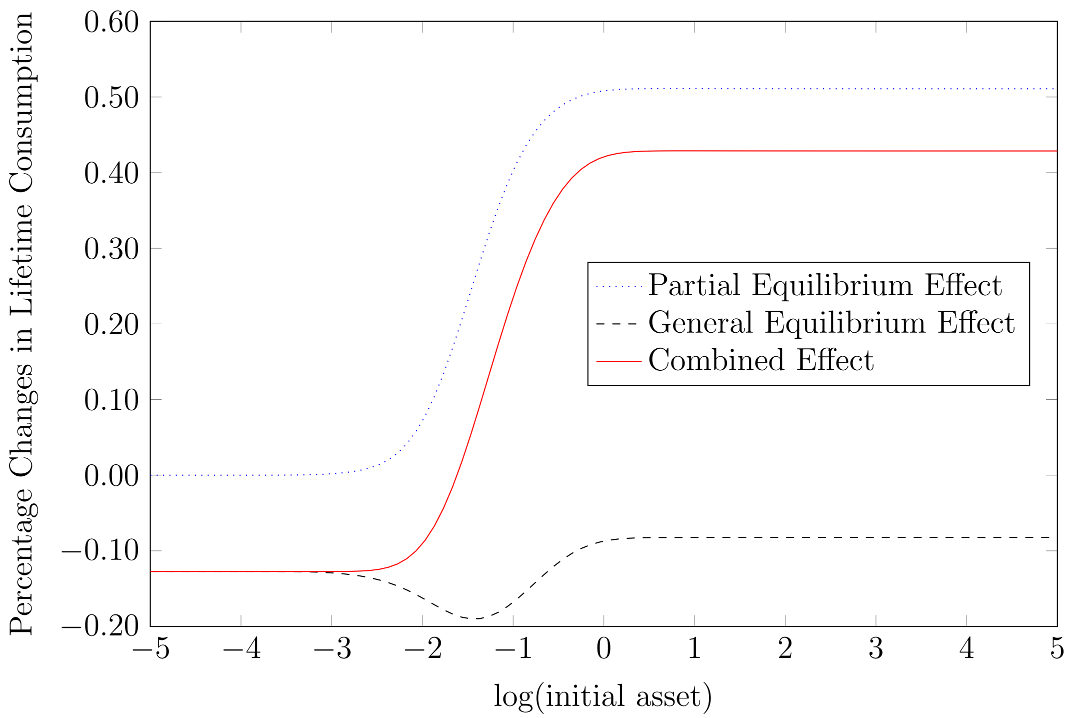

Figure 2.4: Decomposition of the welfare implications of removing proportional costs for Home agents to invest in the Foreign market for different wealth levels. Results are smoothed using the kernel regression.

The previous results are about the reduction of costs paid by Home agents. Result 4 is about the welfare implications of reducing friction paid by Foreign agents. The result is illustrated in Figure 2.5. The dotted, dashed and solid lines correspond to the cases of removing frictions for Home agents, for Foreign agents, and for both, respectively.

The relationship of the effects is not linear. First, let's focus on the dotted line. This is the result of the removal of proportional costs and fixed costs for home agents. However, the combined effect for the poor is lager than a summation of each effects in figure 2.3 and figure 2.4. This comes from nonlinearity of the portfolio decision making. The partial equilibrium effect of removing proportional cost is positive to the poor only when fixed cost is removed.

The dashed line is the effect of removing frictions for Foreign agents. For home agents, this is purely general equilibrium effect. The effect is reverse of the general equilibrium effect in figure 2.4.

The solid line is the result of removing all frictions. For the poor, it is larger than the total of the dashed line and dotted line due to the nonlinearity.

2.5.2 Gradual Cost Change

We now move to the full model. As discussed in section 2.4, frictions gradually decreases. Agents originally expect that the decrease of frictions stops at period

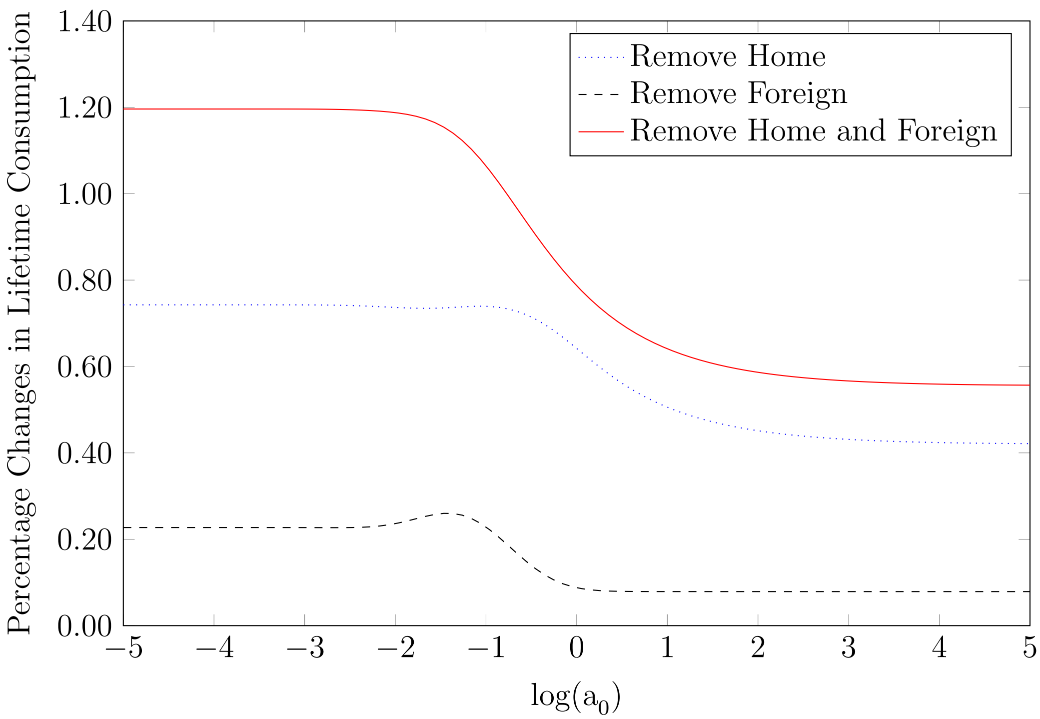

Figure 2.5: Welfare implications of removing frictions for Home agents and Foreign agents. Results are smoothed using the kernel regression.

33 or year 2012 and stays at the level until period 170 or year 2150[7]. However, at the beginning of period 33, agents are notified that some frictions continue to decrease up to year 171.

Result 5. Percentage benefits in welfare due to the decrease of frictions are increasing function of wealth. Bilateral decrease of friction is more beneficial than unilateral decrease for all Home agents.

Figure 2.6 shows the welfare implication of this exercise. Overall shape of the figure is similar to the case with the change in proportional costs in the previous subsection. Poor agents can not participate in the foreign stock market because there still is substantial fixed costs In this exercise as well. The dashed line is the case such that frictions for Foreign agents are decreasing. Due to the general equilibrium effects, the reduction of Foreign frictions is beneficial to all Home agents. The reduction of Foreign frictions is more beneficial to the poor agents than to the rich agents. The dotted line is the case such that frictions for Home agents are decreasing. The reduction of Home costs is beneficial to the rich, participating agents but harmful to poor agents because partial equilibrium effect is almost zero and general equilibrium effect is negative for poor agents. When both frictions are removed, it is more beneficial than the case with unilateral removal. Quantitative changes in welfare is lower than exercises in previous sections because the changes

[7]Frictions become zero after that. Similar to the previous sections, assumptions on period 171 do not affect outcomes at period 33.

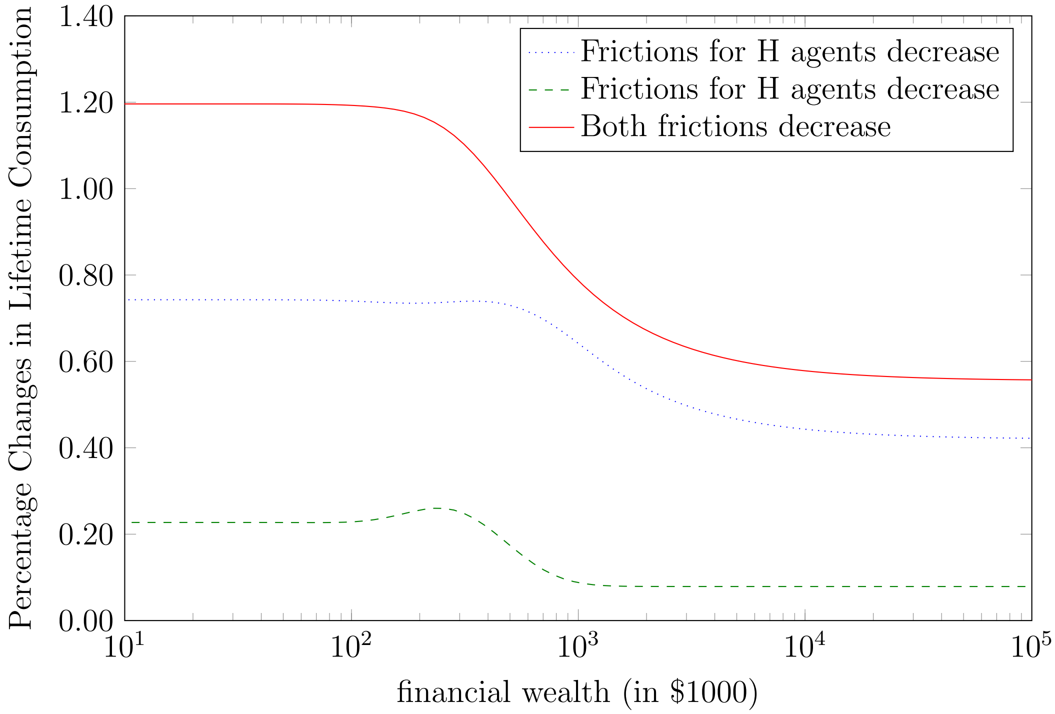

Figure 2.6: Percentage changes in welfare for the Home agents who are living in year 2015 with different wealth level

Wealth is measured by 2010 prices. Results are smoothed using the kernel regression.

in friction is smoother in this exercise.

2.6 Conclusions

I have built an open-economy dynamic general equilibrium portfolio choice model in which agents are endowed with different labor productivities and both the proportional and the fixed costs of investing abroad decreases over time. I have presented the solution method for the model and found that the impact of costs depends on wealth level. I have illustrated the solution method using a calibrated model.

A preliminary calibration exercise demonstrates that poor, non-foreign asset market participating agents can be benefited by a specific type of financial globalization. To benefit the poor, the financial globalization should lower the fixed cost of investing abroad to increase the number of foreign stock market participants. When only the proportional cost is decreased, the gain is limited to rich agents. In fact, decrease of the proportional costs can lower the welfare for the poor agents. The benefit is greater when the decrease in cost is bilateral.

Several extensions could refine our understanding of these aspects. For example, including capital accumulation through the financial market would create another channel for the poor to benefit from financial globalization. Less-than-perfect substitution for home goods and foreign goods may increase the benefits of financial globalization. Refining our understanding of this spillover is a relevant avenue for future work that has major potential policy implications.

Appendix A

Appendices for Chapter 1

A.1 Local Approximation Solution

In this Appendix I apply the local approximation solution method developed by Tille and van Wincoop (2010a) and Devereux and Sutherland (2011) to derive portfolio demand equation (1.20). I decompose the model variables across components of different orders. Any variable x can be written as the sum of its zero, first and higher-order components: $x = x(0) + x(1) + x(2) + \ldots$. The zero-order component, $x(0)$, is the value of x when all standard deviations of model innovations approach zero. The first-order component is proportional to model innovations. The second-order component is proportional to the variance, covariance or product of model innovations, and so on.

There are a total of N^2+5N+4 variables in the model: N^2+N portfolio shares

α_{ij}, α_{gj}; $N+2$ asset prices Q_i, Q_g and Q_f; $N+2$ corresponding asset returns; N period 1 consumption variables $C_{i,1}$; and N period 2 consumption variables $C_{i,2}$. There are N^2+5N+6 equations: N^2+N portfolio Euler equations; N consumption Euler equations; $N+2$ asset market clearing conditions; 2 goods market clearing conditions; $N+2$ definitions of asset returns; and N budget constraints. As there are two periods, I can drop two equations due to Walras' Law. I will drop the market clearing conditions for the riskfree and global assets.

I first need to impose the zero-order components of all equations. This gives:

$$R_i(0) = R_g(0) = R_f(0) \equiv R(0) = \frac{1}{\beta}\left(\frac{Y_w}{D_w}\right)^{-1/\gamma} \tag{A.1}$$

$$Q_i(0) = Q_g(0) = Q_f(0) = \frac{1}{R(0)} \tag{A.2}$$

$$C_{i,1}(0) = \frac{\beta^{-1/\gamma}R(0)^{1-1/\gamma}}{1+\beta^{-1/\gamma}R(0)^{1-1/\gamma}}(Y_i + Q_i(0)K_i) \tag{A.3}$$

$$C_{i,2}(0) = W_i(0)R(0) \tag{A.4}$$

$$\sum_{j=1}^{N} \alpha_{ij}(0)W_j(0) = K_iQ_i(0) \tag{A.5}$$

where $Y_w = \sum_{i=1}^{N} Y_i$, $D_w = \sum_{i=1}^{N} D_i$ and $W_j(0) = Y_j + Q_j(0)K_j - C_{j1}(0)$.

The next step of the solution method involves jointly imposing the second-order component of the difference in portfolio Euler equations across countries together with the first-order component of all equations. This yields a solution to the zero-order component of the difference across countries in portfolio shares together with the first-order component of all other variables. I will follow this method, with one small difference. Rather than just imposing the second-order component of the difference in portfolio Euler equations across countries, I impose the second-order component of all portfolio Euler equations without taking the difference across countries. This will in addition give us a solution to the second-order component of the N equilibrium expected excess returns (which enter in the p_i that are solved from the zero-order component of the market clearing conditions–see the text).

First impose the first-order components of all equations. This gives

$$E(R_i(1)) = E(R_g(1)) = E(R_f(1)) \tag{A.6}$$

$$R_i(1) = R(0)(\epsilon_i + \theta_i \epsilon_g) \tag{A.7}$$

$$R_g(1) = R(0)\theta_g \epsilon_g \tag{A.8}$$

$$R_f(1) = Q_f(1) = Q_i(1) = Q_g(1) = 0 \tag{A.9}$$

$$C_{j1}(1) = 0 \tag{A.10}$$

$$C_{j2}(1) = W_j(0)R_j^p(1) = W_j(0)\left(\sum_{i=1}^{N} \alpha_{ij}(0)R_i(1) + \alpha_{gj}(0)R_g(1)\right) \tag{A.11}$$

Next I impose the second-order component of the portfolio Euler equations. This gives

$$C_{j2}(0)E(R_i(2) - R_f(2)) = \gamma E C_{j2}(1)(R_i(1) - R_f(1)) \tag{A.12}$$

$$C_{j2}(0)E(R_g(2) - R_f(2)) = \gamma E C_{j2}(1)(R_g(1) - R_f(1)) \tag{A.13}$$

Using my result in (A.9) that $R_f(1) = 0$ and the expression for $C_{j2}(1)$ in (A.11), these equations can be rewritten as

$$\begin{aligned}
\frac{1}{R(0)} E(R_i(2) - R_f(2)) &= \gamma \sigma_g^2 \theta_i \left(\sum_{k=1}^{N} \alpha_{kj}(0)\theta_k + \alpha_{gj}(0)\theta_g \right) \\
&\quad + \gamma \alpha_{ij}(0) \sigma_i^2 \tau_{ij} \qquad \text{(A.14)} \\
\frac{1}{R(0)} E(R_g(2) - R_f(2)) &= \gamma \sigma_g^2 \theta_g \left(\sum_{k=1}^{N} \alpha_{kj}(0)\theta_k + \alpha_{gj}(0)\theta_g \right) \qquad \text{(A.15)}
\end{aligned}$$

Substituting (A.15) into (A.14) yields

$$\alpha_{ij}(0) = \frac{1}{\gamma R(0) \sigma_i^2 \tau_{ij}} \left[E(R_i(2) - R_f(2)) - \frac{\theta_i}{\theta_g} E(R_g(2) - R_f(2)) \right] \qquad \text{(A.16)}$$

which is (1.18) in the text.

Appendix B

Appendices for Chapter 2

The appendix explains the solution method for the model.

B.1 Outline

1. Solve an infinite horizon model to get v_T.

2. Run regressions on results from the infinite horizon model to get the initial guess of prices and the law of motion of the ratio of the average wealth $\Lambda(Z_t, Z_t^*)$. Let $g_t = \dfrac{\bar{a}_t + \frac{\theta}{1-\theta}\sum_{j=1}^{N} Q_{j,t}}{\bar{a}_t^* + \frac{\theta}{1-\theta}\sum_{j=1}^{N} Q_{j,t}^*}$, where g_t is a ratio of total wealth, which includes financial wealth and the present value of human capital.

3. Solve the Bellman equation (2.4) for time $T-1$ given v_T and derive v_{T-1}

   (a) Solve a portfolio choice problem first order condition (2.6)

(b) Compare the value function for $m_{i,t} = 1$ and $m_{i,t} = 0$. The larger one is the value function at $t = T - 1$.

(c) Repeat it for all agents and states.

4. Repeat backwards until $t = 1$.

5. Generate samples of average wealth and prices.

   (a) At $t = 1$,find prices and actions that clear the market, given v_2 and the wealth distribution Γ_1

   (b) Generate ω_2 and derive Γ_2.

   (c) Repeat forwards up to $t = T$

   (d) Repeat 5a –5c many times.

6. Calculate residuals of (2.13)–(2.14). If it is not small, update the guess.

B.2 Solving the Infinite Horizon Model

1. Derive policy functions of an infinite horizon planner's problem, given μ, which is the weight of Foreign agents. This is a planner's problem:

$$V(Z, Z^*) = \max_{c,c^*} \left(\mu\eta\chi \frac{c^{1-\gamma}}{1-\gamma} + (1-\mu)(1-\eta)\chi \frac{(c^*)^{1-\gamma}}{1-\gamma} + \beta(1-\chi)\mathrm{E}[V(Z', Z'^*)] \right)$$

$$\text{s.t.} \quad \eta Z + (1-\eta)Z^* = \eta c + (1-\eta)c^*$$

Essentially, this is a static model. There is no capital that generates an intertemporal tradeoff. The policy functions for home consumption and Foreign consumption do not depend on the value functions. The solution is

$$c_t = \frac{\eta Z_t + (1-\eta) Z_t^*}{\eta + \tilde{\mu}(1-\eta)} \tag{B.1}$$

$$c_t^* = \tilde{\mu} \frac{\eta Z_t + (1-\eta) Z_t^*}{\eta + \tilde{\mu}(1-\eta)} \tag{B.2}$$

where $\tilde{\mu} = (\mu/(1-\mu))^{1-\gamma}$. Note that this allocation is attainable if agents in both countries hold assets whose value is proportional to the total output.

A portfolio allocation whose portfolio weight is equal to the current output share can attain this consumption pattern. From (2.10), total price for such a portfolio is proportional to total outputs. The dividends are also proportional to the total outputs.

Therefore, with two equities (claims on capital), the market allocation can attain the planner's solution

2. Derive value functions as a function of wealth. Value functions whose asset equals to world assets satisfy

$$v(\tilde{a}_t, \omega_t) = \beta(1-\chi)\mathrm{E}\left(v(\tilde{a}_{t+1}, \omega_{t+1})\right) + \beta\chi\mathrm{E}\left(\frac{(Z_{t+1}\tilde{a}_{t+1})^{1-\gamma}}{1-\gamma}\right)$$

where $\tilde{a}_t = a_t/Z_t$ is relative wealth.

This gives a system of simultaneous equations of $(v(a_t, \omega_1), v(a_t, \omega_2), v(a_t, \omega_3),$ and $v(a_t, \omega_4))$.

For different wealth level a', we have

$$v(a', \omega_i) = (a'/a_t)^{1-\gamma} v(a_t, \omega_i)$$

because the wealth ratio and consumption ratio are constant across all states and time in the complete market with CRRA utility function. This gives us

$$v(\tilde{a}_{t+1}, \omega_i) = \left(1 + \frac{(1-\chi)(1-\theta)}{(1-\chi)(1-\theta)+\theta}\chi\right)^{1-\gamma} v(\tilde{a}_t, \omega_i)$$

3. Derive a system of prices, given μ, using first order conditions. We have N_ω first order conditions for each current state:

$$\beta\chi \mathrm{E}\left[a^{-\gamma}\left(\frac{Q_{t+1}+(1-\theta)Z_{t+1}}{Q_t} - \frac{Q^*_{t+1}+(1-\theta)Z^*_{t+1}}{Q^*_t}\right)\right] + \beta(1-\chi)\mathrm{E}\left[v'_{t+1}(a)\left(\frac{Q_{t+1}+(1-\theta)Z_{t+1}}{Q_t} - \frac{Q^*_{t+1}+(1-\theta)Z^*_{t+1}}{Q^*_t}\right)\right] = 0$$

From (2.10) we do not need to solve for Q^* once we know Q. Also, $v'_{t+1}(a) = (1-\gamma)v_{t+1}(a)/a$

4. The wealth ratio is equal to consumption ratio, which is $\tilde{\mu}^{-1}$.

5. Run regression to get the initial guess of prices.

B.3 Backward recursion

The algorithm 1 solves the backward recursion of value function, given prices.

Algorithm 1 Derive value functions from $t = T$ to $t = 1$

INPUTS: Law of motions of wealth, prices, and the last period value function v_T, v_T^*.
OUTPUTS: Value functions for each period $(v_t, v_t^*)_{t=1,\dots,T}$
for $t = T - 1 : -1 : 1$ **do**
  Calculate possible values of g_{t+1} using law of motions.
  Calculate Spline interpolation of v_{t+1}, v_{t+1}^* for each g_{t+1} and ω
  /* Solve for market price */
  for $i = 1 : N_g,\ j = 1 : N_\omega$ **do**
    for $ia = 1 : N_a$ **do**
      Solve first order conditions of home
      Solve first order conditions of Foreign
      Compare and accept higher utility. Calculate value functions v_t
      Repeat Foreign
    end for
  end for
end for

B.4 Simulation

The algorithm 2 simulates the economy to obtain the transition of wealth distribution and prices, using the value function derived through the algorithm 1.

Algorithm 2 Simulate the economy and derive residuals

INPUTS: Value functions for each period $(v_t, v_t^*)_{t=1,\dots,T}$
OUTPUTS: Residuals of (2.13)–(2.14).
for $t = 1 : T-1$ **do**
  Calculate possible values of g_{t+1} using law of motions.
  Calculate Spline interpolation of v_{t+1}, v_{t+1}^* for each g_{t+1} and ω
  Guess initial value of a Prices $(\phi^Q, \phi^{Q^*}, \phi^{P_F})$
  /* Solve for market price */
  while $(\phi^Q, \phi^{Q^*}, \phi^{P_F})$ has not converged **do**
    for $i = 1 : N_g,\ j = 1 : N_\omega$ **do**
      for $w = 1 : N_w$ **do**
        Solve first order conditions of home to get utility given participation.
        Compare the utility and utility without participation and accept higher utility.
        Calculate Demand using the weight implied by the wealth distribution.

        Repeat Foreign
      end for
      $r_1(i, j)$ = Total Supply - Total Demand
    end for
    Update $(\phi^Q, \phi^{Q^*}, \phi^{P_F})$ using r_1.
  end while
  Derive residuals in prices in (2.13) – (2.14).
  Derive next period wealth g_{t+1}. Calculate residual in (2.13).
end for

References

Bibliography

Ackah, Charles, Oliver Morrissey, and Simon Appleton, "Who Gains from Trade Protection in Ghana? A Household-Level Analysis," Discussion Papers 07/02, University of Nottingham, CREDIT 2007.

Ahearne, Alan G., William L. Griever, and Francis E. Warnock, "Information Costs and Home Bias: An Analysis of US Holdings of ForeignEquities," *Journal of International Economics*, March 2004, *62* (2), 313–336.

Aisbett, Emma, Ann Harrison, and Alix Zwane, "Globalization and poverty: what is the evidence?," MPRA Paper 36595, University Library of Munich, Germany 2006.

Anderson, James E. and Eric van Wincoop, "Gravity with Gravitas: A Solution to the Border Puzzle," *American Economic Review*, March 2003, *93* (1), 170–192.

_ **and** _ , "Trade Costs," *Journal of Economic Literature*, September 2004, *42* (3), 691–751.

Aviat, Antonin and Nicolas Coeurdacier, "The Geography of Trade in Goods and Asset Holdings," *Journal of International Economics*, March 2007, *71* (1), 22–51.

Backus, David K, Patrick J Kehoe, and Finn E Kydland, "International Real Business Cycles," *Journal of Political Economy*, August 1992, *100* (4), 745–75.

Bae, Kee-Hong, René M Stulz, and Hongping Tan, "Do local analysts know more? A cross-country study of performance of local analysts and foreign analysts.," *Journal of Financial Economics*, 2008, *88(3)*, 581–606.

Balli, Faruk, "New patterns of international portfolio allocation and income smoothing.," 2008. Massey University.

_ , Rosmy J. Louis, and Mohamed A. Osman, "International Portfolio Inflows to GCC Markets: Are There Any General Patterns?," *Review of Middle East Economics and Finance*, 2009, *5* (2), 45–65.

Balta, Narcissa and Juan Delgado, "Home bias and market integration in the EU.," *CESifo Economic Studies*, 2008, *55*, 110–114.

Berkel, Barbara, "Institutional Determinants of International Equity Portfolios - A Country-Level Analysis," *Topics in Macroeconomics, The B.E. Journals in Macroeconomics*, 2007, *7* (1), Article 34.

Bertaut, Carol C. and Linda S. Kole, "What makes investors over or underweight? Explaining international appetites for foreign equities.," 2004. International Finance Discussion Paper 819, Board of Governors of the Federal Reserve System.

Black, Fischer, "International Capital Market Equilibrium with Investment Barriers," *Journal of Financial Economics*, December 1974, *1* (4), 337–352.

Bottazzi, Laura, Paolo Pesenti, and Eric van Wincoop, "Wages, profits and the international portfolio puzzle," *European Economic Review*, 1996, *40*, 219–254.

Branson, William H. and Dale W. Henderson, "The specification and influence of asset markets," in R. W. Jones and P. B. Kenen, eds., *Handbook of International Economics*, Vol. 2 of *Handbook of International Economics*, Elsevier, 00 1985, chapter 15, pp. 749–805.

Buch, Claudia M., "Information or regulation: What is driving the international activities of commercial banks?," 2000. Kiel Working paper 1011.

_ , "Are banks different? Evidence from international data.," *International Finance (1)*, 2002, *5(1)*, 97–114.

Caballero, Ricardo J., Emmanuel Farhi, and Pierre-Olivier Gourinchas, "An Equilibrium Model of "Global Imbalances" and Low Interest Rates," *American Economic Review*, March 2008, *98* (1), 358–93.

Campbell, John Y., "Household Finance," *Journal of Finance*, 8 2006, *61* (4), 1553–1604.

Carvalho, Luis and Aurora A.C. Teixeira, "Where are the poor in International Economics?," FEP Working Papers 425, Universidade do Porto, Faculdade de Economia do Porto September 2011.

Castilho, Marta, Marta Menndez, and Aude Sztulman, "Trade Liberalization, Inequality, and Poverty in Brazilian States," *World Development*, 2012, *40* (4), 821–835.

Chan, Kalok, Vicentiu Covrig, and Lilian Ng, "What Determines the Domestic Bias and Foreign Bias? Evidence from Mutual Fund Equity Allocations Worldwide," *Journal of Finance*, June 2005, *60* (3), 1495–1534.

Christelis, Dimitris and Dimitris Georgarakos, "Investing at home and abroad: different costs, different people?," 2011. Working paper.

Coeurdacier, Nicolas, "Do trade costs in goods markets lead to home bias in equities?," *Journal of International Economics*, 2009, *77*, 86–100.

_ **and Philippe Martin**, “The geography of asset trade and the euro: insiders and outsiders.,” *Journal of Japanese and International Economies*, 2009, *23(2)*, 90–113.

_ **and Pierre-Olivier Gourinchas**, “When bonds matter: home bias in goods and assets,” 2011. National Bureau of Economic Research.

_ **and Stephane Guibaud**, “International equity holdings and stock returns correlations: Does diversification matter at all for portfolio choice?,” 2005. London Business School.

_ **, Robert Kollmann, and Philippe Martin**, “International Portfolios with Supply, Demand and Redistributive Shocks,” NBER Working Papers 13424, National Bureau of Economic Research, Inc September 2007.

Cole, Harold L. and Maurice Obstfeld, “Commodity trade and international risk sharing : How much do financial markets matter?,” *Journal of Monetary Economics*, 1991, *28* (1), 3 – 24.

Danthine, Jean-Pierre, Kpate Adjouté, L. aura Bottazzi, Andreas Fischer, Rony Hamaui, Richard Portes, and Mike Wickens, “EMU and portfolio adjustment,” 2000. CEPR Policy Paper No. 5.

Daude, Christian and Marcel Fratzscher, “The pecking order of crhoss-border investment.,” *Journal of International Economics*, 2008, *74*, 94–119.

Devereux, Michael B. and Alan Sutherland, "Country portfolio dynamics," *Journal of Economic Dynamics and Control*, July 2010, *34* (7), 1325–1342.

_ **and** _ , "Country Portfolios In Open Economy Macro-Models," *Journal of the European Economic Association*, 04 2011, *9* (2), 337–369.

Eaton, Jonathan and Samuel Kortum, "Technology, geography and trade.," *Econometrica*, 2002, *70(5)*, 1741–1779.

Eichengreen, Barry and Pipat Luengnaruemitchai, "Bond markets as conduits for capital flows: How does asia compare?," in T. Ito and A.K. Rose, eds., *International Financial Issues in the Pacific Rim: Global Imbalances, Financial Liberalization, and Exchange Rate Policy*, University of Chicago Press, 2006, pp. 267–313.

Evans, Martin D. and Viktoria V. Hnatkovska, "Financial Integration, Macroeconomic Volatility, and Welfare," *Journal of the European Economic Association*, 04-05 2007, *5* (2-3), 500–508.

Evans, Martin D. D. and Viktoria Hnatkovska, "Solving General Equilibrium Models with Incomplete Markets and Many Assets," October 2005. NBER Technical Working Papers 0318.

Faruqee, Hamid, Shujing Li, and Isabel K. Yan, "The Determinants of International Portfolio Holdings and Home Bias," *IMF Working Papers*, 2004.

Forbes, Kristin J., "Why do Foreigners Invest in the United States?," *Journal of International Economics*, April 2008, *80(1)*, 3–21.

G20 Financial Inclusion Experts Group, "G20 Financial Inclusion Action Plan," 2010.

García-Herrero, Alicia, Philip Wooldridge, and Doo Yong Yang, "Why don't Asians invest in Asia? The determinants of cross-border portfolio holdings.," *Asian Economic Papers*, 2009, *8(3)*, 228–256.

Gelos, R. Gaston and Shang-Jin Wei, "Transparency and International Portfolio Holdings," *Journal of Finance*, December 2005, *60* (6), 2987–3020.

Ghosh, Swati and Holger Wolf, "Is there a curse of location? Spatial determinants of capital flows to emerging markets.," in S. Edwards, ed., *Capital Flows and the Emerging Economies*, University of Chicago Press, Chicago, 2000, pp. 137–158.

Harrison, Ann, "Globalization and Poverty," NBER Working Papers 12347, National Bureau of Economic Research, Inc July 2006.

Heathcote, Jonathan and Fabrizio Perri, "Financial globalization and real regionalization," *Journal of Economic Theory*, November 2004, *119* (1), 207–243.

Hill, Ronald and Justine Rapp, "Globalization and Poverty: Oxymoron or New Possibilities?," *Journal of Business Ethics*, February 2009, *85* (1), 39–47.

Jeanneau, Serge and Marian Micu, "Determinants of international bank lending to emerging market countries.," 2002. BIS Working Paper 112.

Julliard, Christian and Carlo Rosa, "The International Diversication Puzzle is not Worse Than You Think," Technical Report, Working paper 2009.

Kang, Jun-Koo and Rene M. Stulz, "Why Is There a Home Bias? An Analysis of Foreign Portfolio Equity Ownership in Japan," *Journal of Financial Economics*, October 1997, *46* (1), 3–28.

Kho, Bong-Chan, Rene M Stulz, and Francis E Warnock, "Financial globalisation, governance and the evolution of the home bias," Papers 220, Bank for International Settlements December 2009. available at http://ideas.repec.org/p/bis/biswps/220.html.

Kim, Soyoung, Jong-Wha Lee, and Kwanho Shin, "Regional and global financial integration in east asia.," 2006. Working paper, Tufts University.

_ , _ , and _ , "Does corporate governance risk at home affect investment choices abroad?," 2007. NBER working paper 13721.

Kose, M Ayhan, Eswar Prasad, Kenneth Rogoff, and Shang-Jin Wei, "Financial Globalization: A Reappraisal," *IMF Staff Papers*, April 2009, *56* (1), 8–62.

Krusell, Per and Anthony A. Smith Jr., "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, October 1998, *106* (5), 867–896.

Kyrychenko, V. and P. Shumb, "Who holds foreign stocks and bonds? Characteristics of active investors in foreign securities.," *Financial Services Review*, 2009, *18*, 1–21.

Lane, Philip R., "Global bond portfolios and EMU.," 2005. European Central Bank Working Paper 553.

_ **and Gian Maria Milesi-Ferretti**, "The international equity holdings of euro area investors.," 2005. Working paper, Trinity College and IMF.

Lee, Jong-Wha, "Patterns and determinants of cross-border financial asset holdings in east asia.," 2008. ADB Working Paper Series on Regional Economic Integration no. 13.

Lucas, Robert Jr., "Interest rates and currency prices in a two-country world," *Journal of Monetary Economics*, 1982, *10* (3), 335–359.

Martin, Philippe and Helene Rey, "Financial super-markets: size matters for asset trade," *Journal of International Economics*, 2004, *64* (2), 335–361.

Meschi, Elena and Marco Vivarelli, "Globalization and Income Inequality," IZA Discussion Papers 2958, Institute for the Study of Labor (IZA) July 2007.

Milesi-Ferretti, Gian Maria and Philip R. Lane, "International Investment Patterns," *IMF Working Papers*, 2004.

_ and _ , "The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970-2004," *Journal of International Economics*, 2007, *73*, 223–250.

Naranpanawa, Athula, Jayatilleke S. Bandara, and Saroja Selvanathan, "Trade and poverty nexus: A case study of Sri Lanka," *Journal of Policy Modeling*, March 2011, *33* (2), 328–346.

Negishi, Takashi, "Welfare Economics and Existence of an Equilibrium for a Competitive Economy," *Metroeconomica*, 1960, *12*, 92–97.

Obstfeld, Maurice and Kenneth Rogoff, "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?," *NBER Macroeconomics Annual*, 2000, pp. 339–390.

Pendle, Lara, "What determines Australia's foreign equity investment?," 2007. Working paper, University of Sydney.

Portes, Richard and Helene Rey, "The Determinants of Cross-Border Equity Flows," *Journal of International Economics*, March 2005, *65* (2), 269–296.

_ **, Hélène Rey, and Yonghyup Oh**, "Information and capital flows: the determinants of transactions in financial assets.," *European Economic Review*, 2001, *45*, 783–796.

Rose, Andrew K. and Mark M. Spiegel, "A gravity model of sovereign lending: trade, default and credit.," *IMF Staff Papers*, 2004, *51*, 50–63.

Santis, Robert A De and Bruno Gérard, "International portfolio reallocation: diversification benefits and european monetary union.," *European Economic Review*, 2009, *53(8)*, 1010–1027.

Tille, Cedric, "Financial integration and the wealth effect of exchange rate fluctuations," *Journal of International Economics*, 2008, *75* (2), 283 – 294.

_ **and Eric van Wincoop**, "International capital flows," *Journal of International Economics*, 2010, *80* (2), 157 – 175.

_ **and** _ , "A new perspective on the new rule of the current account.," *Journal of International Economics*, 2010, *80(1)*, 89–99.

Townsend, Robert M. and Kenichi Ueda, "Welfare Gains from Financial Liberalization," IMF Working Papers 07/154, International Monetary Fund July 2007.

van Wincoop, Eric, "How big are potential welfare gains from international risksharing?," *Journal of International Economics*, 1999, *47* (1), 109 – 135.

_ **and Francis E. Warnock**, "Can trade costs in goods explain home bias in assets?," *Journal of International Money and Finance*, 2010, *29(6)*, 1108–1123.

Veldkamp, Laura and Stijn Van Nieuwerburgh, "Information Immobility and the Home Bias Puzzle," *Journal of Finance*, 2009, *64(3)*, 11871215.

Vlachos, Jonas, "Do regulatory similarities increase bilateral portfolio holdings?," Working Paper Series 612, Research Institute of Industrial Economics March 2004.

Young, Eric R., "Solving the incomplete markets model with aggregate uncertainty using the KrusellSmith algorithm and non-stochastic simulations," *Journal of Economic Dynamics and Control*, 2010, *34* (1), 36 – 41.

Yu, Gowoon, "Accounting standard and international portfolio holdings: analysis of cross-border holdings following mandatory adoption of IFRS.," 2009. Working paper, University of Michigan Ross School of Business.