A THEORETICAL AND EMPIRICAL ANALYSIS OF THE PRODUCTIVITY OF MONEY

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Abstract of

A Theoretical and Empirical Analysis of

The Productivity of Money

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This paper analyzes the theoretical properties which make money a productive asset. Empirical evidence on the productivity of money is provided by examining whether it is valid to include a real money variable as a factor input in an aggregate production function. This is accomplished by constructing two structural models of the production process based on a Cobb-Douglas production function and a translog production function respectively. These models are empirically estimated to examine the role of money in the production process.

Chapter II presents a theoretical analysis of the characteristics of money which make it a productive asset. An analysis of why these unique characteristics have induced economists to view cash holdings as unproductive is also provided. The general purpose of this chapter is to provide some important insights into the historical development of modern monetary theory.

Chapter III presents a detailed analysis of James Tobin's money growth model. Special attention is given

to his interpretation of the role of money in economic growth. It is shown that the controversial implication derived from this model that the long-run equilibrium rate of capital accumulation is lower in a monetary economy than a non-monetary economy depends upon several restrictive assumptions which do not accurately depict the workings of a monetary economy.

After analyzing the Tobin model, arguments are presented which support the hypothesis that factors which encourage individuals to hold cash balances may promote growth in less-developed economies. These arguments suggest that policies which discourage individuals from holding money may deter rather than stimulate economic growth.

In Chapter IV the money growth models developed by David Levhari and Don Patinkin are reconstructed; one which views money as a consumer's good and one which views money as a producer's good. In constructing these models Patinkin and Levhari retain all of the assumptions made by Tobin except they include the inputed services from real money balances in the definition of disposable income when money is viewed as a consumer's good and they introduce a real money variable into the production function when money is viewed as a producer's good. The implications derived from these models are contrasted with the implications derived from Tobin's money growth model.

The question of whether it is valid to include a real money variable as a factor input in an aggregate production function is examined by constructing two structural models based on a Cobb-Douglas and a translog production function respectively. Drawing from profit maximizing conditions and assuming that markets are perfectly competitive three factor demand equations are derived for each of these functional forms. These equations, together with the production functions, form two four equation structural models which are estimated to determine whether it is valid to include a real money variable as a factor input in an aggregate production function.

Chapter V contains a discussion of previous empirical work done on this question along with a discussion of the empirical tests done for this study. The econometric testing consists of using one- and two-stage least squares on annual time series data. The data cover the period 1929-1967 for the private domestic sector of the United States economy. A technical discussion of the econometric techniques used to estimate the Cobb-Douglas and translog models are included in the Appendix to Chapter V. The results from both models indicate that real money balances are positive and significantly related to real output over this period. Hence they suggest that it is valid to include a real money variable as a factor input in an aggregate production function. In Chapter VI conclusions and implications are presented.

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CHAPTER I

INTRODUCTION

In 1965 James Tobin published a rather controversial article, "Money and Economic Growth."[1] In this article, Tobin constructs a simple money growth model from which he derives the paradoxical conclusion that the long-run equilibrium capital-labor ratio is lower in a monetary economy than in a barter economy. The major policy implication derived from this model is that the rate of capital accumulation can be increased by lowering the yield on money and that the maximum level of capital accumulation is generated when no money is held. Hence, the model implies that inflation can stimulate economic growth. By increasing the opportunity cost of holding money, anticipated inflation induces individuals to economize on their use of real cash balances. They decrease their real cash holdings, substituting into real capital and in so doing increase the level of capital accumulation in the economy.

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[1]James Tobin, "Money and Economic Growth," Econometrica 33 (October, 1965): 671-684.

The implication derived from Tobin's money growth model that inflation can encourage capital accumulation has stimulated a large volume of literature on the relationship between inflation and economic growth. Some economists, following Tobin, maintain that inflation increases the rate of non-monetary savings (savings which contribute to capital accumulation) and investment relative to money savings (savings which do not contribute to capital accumulation) and consumption. Others, however, criticize Tobin for ignoring the productivity gains from utilizing money in a money economy. By arguing that money is a productive asset and that the marginal product of the last unit of real money held is positive, it is possible to conclude that inflation will decrease the total productivity generated from the use of money by encouraging individuals to reduce their real money holdings. This decrease in total product may also cause a reduction in total output. If so, inflation may have a negative effect on economic growth.

To date, the question of whether inflation will stimulate economic growth or retard the process has not been successfully resolved. This is evidenced by the fact that conflicting results concerning the relationship between inflation and economic growth can be derived from the existing money growth model literature.[2] Jerome Stein emphasizes

[2]See Jerome Stein, "Monetary Growth Theory in Perspective," American Economic Review (March, 1970): 85-106.

this point in his article, "Monetary Growth Theory in Perspective."[3] Stein argues that "equally plausible (monetary growth) models yield fundamentally different results."[4] However, following Don Patinkin and David Levhari, we will argue that an attempt to incorporate the productivity gains derived from engaging in monetary exchange should be included in an appropriately specified money growth model.[5] To defend this position, theoretical arguments will be developed and empirical evidence presented which support the hypothesis that money is a productive asset.

In Chapter Two the special characteristics of money which make it a productive asset are discussed. We point out that the use of money as a medium of exchange minimizes the transaction and information costs associated with exchange and consequently provides an efficient instrument for economizing on the services of labor and capital expended in the search-bargain process of exchange. In addition, money is a convenient common denominator which facilitates a comparison of values among commodities and, when

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[3]Ibid.

[4]Ibid., p. 85, 105.

[5]David Levhari and Don Patinkin, "The Role of Money in a Simple Growth Model," American Economic Review 58 (September, 1968): 713-54.

held, money can reduce the risk of capital losses from unforeseen relative price changes, including interest rate changes.

After examining the characteristics of money which make it a productive asset, several reasons why these unique attributes have actually induced economists to view cash holdings as unproductive are discussed. This is accomplished by providing a general overview of some of the influential views concerning the productive and/or unproductive nature of money expressed by several monetary economists. This discussion provides some important insights into the historical development of modern monetary theory. In particular, it enumerates several of the theoretical analyses of the productivity of money which seem to have influenced James Tobin's interpretation of the role of money in economic growth.

In Chapter III a fairly detailed analysis of Tobin's money growth model is presented with particular attention given to the assumptions utilized to derive the implication that the long-run equilibrium capital-labor ratio is lower in a monetary economy than in a non-monetary or barter economy. This controversial implication depends upon several restrictive assumptions. The model assumes that

(1) real capital accumulation is the only substitute for cash holdings

(2) the marginal propensity to save is a constant fraction of disposable income and that it is equivalent in a monetary and non-monetary economy

(3) savers and investors are identical

(4) individuals will perceive their cash holdings as part of their savings but these money savings will not contribute to real capital accumulation

(5) monetary and non-monetary exchange are equally productive.

The major problem with these assumptions is that they ignore several important factors which distinguish a monetary economy from a non-monetary economy. Consequently, a theoretical framework based on these assumptions may not accurately reflect conditions relevant for understanding the workings of a monetary economy. Recognizing this, Tobin's model is critisized for failing to accurately depict a monetary economy. The assumptions upon which the model is based do not really provide a rationale for holding money. Although Tobin allows real money holdings to affect the level of disposable income in the economy, by ignoring the positive impact which financial intermediaries can have on stimulating capital accumulation in a monetary economy and by failing to incorporate the productivity gains derived from engaging in monetary exchange, the model does not provide an accurate framework for analyzing the long-run equilibrium capital market conditions in a monetary economy. Hence, the implications derived from Tobin's money growth model may be invalid.

In addition to the above criticisms, following Ronald McKinnon and Edward Shaw, arguments which support the hypothesis that factors which encourage individuals to hold cash balances may promote growth in less-developed countries are also presented in Chapter Three. In areas where access to external financing is limited and only self-financed investment is feasible, real cash balances may be positively correlated with the level of real capital accumulation rather than being strict substitutes as Tobin's model implies. This implication is derived by assuming that the cost of financing investments with money holdings is generally lower than the cost of financing investments with funds obtained from inventory sales, an alternative option for financing investments when external credit is not available. Hence, inflationary policies which discourage individuals from holding money may inhibit economic growth by increasing the cost of financing capital accumulation and by inhibiting monetary exchange.

In Chapter Four, a reconstruction of the Patinkin and Levhari money growth model, where money is viewed as a producer's good, is presented.[6] In constructing this model Patinkin and Levhari retain all of the assumptions included in Tobin's money growth model but they redefine the production function to include a real money variable as a factor

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[6]Ibid.

input. They do so in order to determine the impact which an attempt to incorporate the productivity gains from holding money into the model has on the comparative static results of the model. The results derived from these two respective money growth models are contrasted. Two structural models are then constructed to enable us to empirically estimate whether it is valid to include a real money variable in an aggregate production function. This is accomplished by offering alternative hypotheses that the correct specification of the production function included in the growth model is the Cobb-Douglas and translog formation respectively. Drawing from profit maximizing conditions and assuming that markets are perfectly competitive three factor demand equations are derived for each of these functional forms. These factor demand equations along with a production function form the two structural models which are estimated to determine whether it is valid to include a real money variable in an aggregate production function.

The results of this estimation are presented in Chapter Five along with a discussion of previous empirical work done on this topic and a description of the data used to estimate the two production function models. The econometric testing consists of using one- and two-stage least squares on time series data (annual). The data cover the period 1929-1967 for the private domestic sector of the United States economy. A technical discussion of the econometric techniques used to estimate the Cobb-Douglas and translog models are included in the Appendix to Chapter Five. In Chapter Six conclusions and implications are presented.

CHAPTER II

AN ANALYSIS OF THE PRODUCTIVE NATURE OF MONEY

II:1: Introduction

Today most economists acknowledge the fact that cash balances provide holders with a non-pecuniary return either in the form of utility or productivity. In particular, Milton Friedman, Harry Johnson, Don Patinkin, David Levhari, Alvin Marty, and Martin Bailey (among others), have made important theoretical contributions clarifying this point.[7] In their analyzes, they apply the same economic principles when analyzing the demand for money as economists use when analyzing the demand for any other capital or consumption good and argue that an optimizing individual will demand cash balances up to the point where the marginal implicit yield from money is equated to the yield from available alternatives. They treat money both as a consumer's capital good yielding a flow of utility services and a producer's

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[7]Milton Friedman, "The Demand for Money: Some Theoretical and Empirical Results," Journal of Political Economy 67 (August, 1959): 327-51 and in the Optimal Quantity of Money and Other Essays (Chicago: Aldine Pub. Co., 1969): 14; Harry Johnson, "Inside Money, Outside Money, Income, Wealth, and Welfare," Journal of Money, Credit, and Banking 1 (February 1969): 30-45. Don Patinkin, Money, Interest, and Prices: An Integration of Monetary and Value Theory, second Edition (New York: Harper & Row Publishers, 1965):

capital good providing services in the production process. This view of money as a productive asset, however, has not been held universally. Although virtually all economists agree that monetary exchange is more efficient than barter exchange, many economists...from Adam Smith to J.M. Keynes ...have interpreted cash holdings as an unproductive form of hoarding.

Contrary to this view, in this chapter the unique characteristics of money which make it a productive asset are examined. Like Friedman, we argue that the quantity of money demanded, like the quantity demanded of any other asset or good, depends upon its marginal implicit yield relative to the yield from available alternatives. In addition, by examining conflicting views concerning the productivity of money, we point out the special characteristics of money which have induced many influential monetary theorists to consider cash balances, and particularly speculative and precautionary balances, an unproductive form of hoarding.

II.2: Why is Money a Productive Asset?

The emergence of a monetary medium of exchange is intimately connected with the growth of market trading areas.

79, 114-115, 469-470, and with David Levhari in "The Role of Money in a Simple One-Sector Growth Model," reprinted in Studies in Monetary Theory by Don Patinkin (New York: Harper & Row Pub., 1972): 205 - 242. Alvin Marty, "Some Notes on Money and Economic Growth," Journal of Money, Credit, and Banking (May, 1969): 258-265; Martin Bailey, National Income and the Price Level (New York: McGraw Hill Book Co., 1971): 54-56.

Both of these institutional innovations are extremely productive since they minimize the transaction and information costs associated with exchange. Money is the one good for which all others exchange and hence its use, as a medium of exchange, obviates the double coincidence of wants characteristic of barter exchange. By releasing labor and capital from the process of distribution to that of production, money provides an efficient instrument for economizing on the services of labor and capital expended in the search-bargain process of exchange. Similarly, by reducing the cost of exchange, money contributes to the expansion of and more efficient operation of the market exchange system.[8] It provides additional opportunities for professional middlemen and specialized traders. It also affects the intertemporal allocation of resources since the availability of a standardized asset which everyone is willing to accept as a means of payment increases the possibility for deferred payments, borrowing, and credit.[9]

For individuals and firms alike, money is a substitute for investment in information needed to complete market

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[8]K. Brunner and A.H. Meltzer, "The Uses of Money: Money in a Theory of an Exchange Economy, American Economic Review 61 (December, 1971): 787.

[9]Ibid., p. 800.

exchanges. By using money for the purpose of exchange, individuals (firms) can both decrease the amount of information they must acquire, process, and store, and reduce the number of transactions needed to complete their desired exchanges.[10] In so doing, monetary exchange increases the level of economic efficiency within a community, enabling individual transactors to enjoy a larger and more diversified basket of consumption goods, including the choice for more leisure.[11] Money, then, is a productive asset both as a resource saving device and as a means for stimulating market activity.

In addition to the productivity gains derived from using money as a medium of exchange, money is also a convenient common denominator which facilitates a comparison of values among various commodities. Without this common denominator it would be much more costly to acquire the necessary pricing information relevant for either profit or utility maximization. Although it is not necessary that the medium of exchange is also the unit of account, it is the logical choice. The number of calculations needed to express and compare exchange values is minimized when the monetary unit also serves as the unit of account.

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[10] Ibid., p. 799.

[11] Ibid., p. 786.

In addition to emphasizing the gains from using money for transaction purposes, most theories of money are also concerned with the role uncertainty plays in determining the demand for money.[12] In a world of uncertainty, individuals may demand money as a form of protection against unforeseen relative price changes (including interest rate changes).[13] Of course, money is not the only asset which will be held for this purpose. Other assets, monetary and non-monetary, will also be selected.[14] However, compared to monetary assets, special information is required to predict the price of a non-monetary asset (assuming that the general price level is relatively stable). This is because the factors which affect relative prices are

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[12]See for example, Sir John Hicks, "A Suggestion for Simplifying the Theory of Money (1935)," Reprinted in Critical Essays in Monetary Theory by J.R. Hicks (Oxford: Clarendon Press, 1967): p. 70 and in Value and Capital: An Inquiry into Some Fundamental Principles of Economic Theory (Oxford, Clarendon Press, 1946): 239-241; J.M. Keynes, The General Theory of Employment, Interest, and Money (New York: Harcourt, Brace, World, Inc., 1964): 170-174; David Laidler, The Demand for Money: Theories and Evidence (Scranton: International Textbook Co., 1969); Milton Friedman, "The Demand for Money: Some Theoretical and Empirical Results," Journal of Political Economy 67 (August, 1959): 327-351.

[13]Reuben A. Kessel and Armen A. Alchian, "Effects of Inflation," Journal of Political Economy 70 (December, 1962): 522.

[14]James Tobin defines monetary assets, including cash, as assets which are marketable, fixed in money value and free of default risk. For further discussion see James Tobin, "Liquidity Preference as Behavior Towards Risk," reprinted in Monetary Theory and Policy: Major Contributions to Contemporary Thought, Edited with Introduction by Richard S. Thorn (New York: Random House): 180.

constantly changing and difficult to specify whereas they have comparatively little impact on the general level of prices; the key variable in determining the value of monetary assets. Since the price of a non-monetary asset is more variable than the price of monetary assets, individuals will generally attempt to reduce the risk of unforeseen price and interest rate changes by holding relatively riskless monetary assets in their portfolios.

Most economists agree that interest-bearing monetary assets can reduce risk. However, they often question why non-interest-bearing money will be held for this purpose.[15] On this point, James Tobin suggests that the inelasticity of expectations about future interest rates on interest-bearing monetary assets and the uncertainty about future interest rates have encouraged individuals to hold money for speculative or precautionary purposes.[16] He points out

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[15]Keynes argues that the demand for money is determined by a precautionary and speculative motive in addition to a transaction motive. He defines the precautionary motive as the desire for security as to the future cash equivalent of a certain proportion of total resources and the speculative motive as the desire of securing profit from knowing better than the market what the future will bring. See J.M. Keynes, The General Theory of Employment, Interest and Money (New York; The Harcourt, Brace, & World, Inc., 1964): 170 - 174. In this dissertation, the precautionary motive will be viewed in the above Keynesian sense and the speculative motive will be viewed as a desire to hold money to be in a position to secure profit, or avoid loss by anticipating market changes.

[16]James Tobin, "Liquidity Preference as Behavior Towards Risk," pp. 178-204.

that the possibility of incurring a capital loss greater than the rate of return from the interest-bearing asset will encourage some individuals to hold cash rather than other relatively riskless yet interest-bearing monetary assets.[17]

The transaction costs of selling an asset to acquire money perhaps offers a more valid theoretical explanation for holding cash balances as a protection against risk and uncertainty.[18] Since money is easily recognized and accepted by market participants, it can generally be exchanged at no discount from the price at which it was obtained.[19] This is not true for any non-monetary asset. When attempting to resell a non-monetary asset it is often both inconvenient and difficult to locate a buyer offering a price as high as that which was originally paid for the asset.[20] Of course, the costs of transforming a relatively risk-free interest-bearing asset are obviously not as formidable as those associated with non-monetary assets. The only

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[17] Ibid. Some economists have argued that this is not a valid theoretical point since savings and loans companies offer interest-bearing assets with fixed capital value.

[18] See for example. William Baumol, "Transactions Demand for Cash: An Inventory Theoretical Approach," Quarterly Journal of Economics 66 (November, 1952): 545-556.

[19] Armen A. Alchian and William R. Allen, University Economics: Elements of Inquiry (Belmont: Wadsworth Publishing Company, Inc., 1972): 572.

[20] Ibid.

difference between these assets and cash balances held for speculative or precautionary purposes is that the interest yield on these assets may be adjusted periodically.[21] However, although interest-bearing monetary assets are close substitutes for speculative and precautionary balances, they are not used as a medium of exchange. The cost of moving from interest-bearing monetary assets into money may outweigh the return from these investments, discouraging individuals from investing money over short periods of time. Since the transaction costs of transforming interest-bearing monetary assets into cash will generally have some effect on the demand for money it should not be ignored when attempting to understand the productive nature of money.[22]

In a frictionless world of perfect certainty, where the transaction costs of executing exchanges and the information costs about market prices and qualities of goods and services are zero, nothing is gained from engaging in monetary exchange or from holding money as a form of protection against risk and uncertainty. In such a world, any good can be used as a medium of exchange, or, more accurately, a medium of exchange is unnecessary. However,

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[21]See footnote 12 for a discussion of precautionary and speculative balances.

[22]James Tobin comments on this point in footnote 4 of his article, "Liquidity Preference as a Behavior Toward Risk," Op. cit., p. 179.

once we acknowledge the positive cost of acquiring information about market arrangements, relative prices, and exchange ratios, then, the unique characteristics which make money a productive asset become more obvious.

Although economists generally agree that the use of money as a medium of exchange is productive as a resource saving device, many have argued that cash holdings are an unproductive form of hoarding which should be discouraged. In effect, economists defend the position that money provides services to its holders when it circulates as a medium of exchange but that it is unproductive during the time it is being held. This distinction between the use of and demand for money can be traced back to the Aristotelean view that money is a "barren asset," and still influences modern monetary theory. In particular, the debate in the money growth literature over whether real money balances should be considered a factor of production stems from differing views concerning the productivity of money. An overview of several of the influential views on the productivity of money may provide some important insights on this controversy and will clarify Knut Wicksell's observation that in the field of monetary theory, "diametrically opposed and sometimes self-contradictory views are defended by most writers."[23]

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[23]Knut Wicksell, Lectures on Political Economy Vol. 2, edited with an introduction by Lionel Robbins (London: Routledge & Kegan Paul Ltd., 1950): 190.

II.3: A Critique of the View That Money is a "Barren Asset"

The theoretical distinction drawn between the use of and demand for money has been perpetuated by the belief -- expressed by most Classical economists -- that money is a "barren" or "sterile" asset. In particular, both Adam Smith and John Locke established a tradition of regarding money as providing exchange value to its holders but not use value. For example, in The Wealth of Nations, Adam Smith suggests that cash holdings are an unproductive form of hoarding even though he was acutely aware of the contribution money makes in distributing goods and services.[24] In his attempt to refute the Mercantilist position that a nation's wealth is determined primarily by its stock of gold and silver, Smith argues that although "money, no doubt, makes always a part of the national capital; ... it generally makes a small part, and always the most unprofitable part of it."[25] (my italics) In addition, he ignores the role money plays in the production process by equating cash holdings with other forms of unemployed capital.

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[24]Adam Smith, An Inquiry into the Nature and Causes of the Wealth of Nations, edited by Edwin Cannan with introduction by Max Lerner (New York: The Modern Library, 1937): 304.

[25]Ibid., p. 406.

> That part of his capital which a dealer is obliged to keep by him unemployed, and in ready money for answering occasional demands, is so much dead stock, which, so long as it remains in this situation, produces nothing either to him or to his country.[26] (my italics)

To the extent that cash holdings are not directly utilized to generate an economy's physical output, Smith is correct to argue that money, itself, does not produce anything. However, this rather narrow view of the production process neglects the qualifying argument that cash balances do indirectly increase the level of output in an economy by freeing scarce resources from the process of distribution to that of physical production.

John Locke also compares unfavorably the role of money and other factor inputs in production processes: "Land produces something new and profitable, and of value to mankind, but money is a barren thing, and produces nothing."[27] (my italics) By viewing the productivity of money in a different manner from that of other factors, both Smith and Locke established a tradition of regarding money as a barren, unproductive asset. This tradition has had an important impact on subsequent work in monetary theory.

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[26] *Ibid.*, p. 304.

[27] John Locke, "Some Considerations of the Consequences of the Lowering of Interest...," 1691. In Locke, *Works*, Vol. X, 1801 edition, pp. 36-37.

When analyzing the nature of money it is important to recognize that an optimizing individual determines the amount of money he is willing to hold in the same manner as he determines his demand for other goods and services. He compares the gains from holding an additional unit of money versus the potential gains from the best alternatives to money. The fact that one is willing to hold money indicates that its marginal yield is at least large enough to compensate for the best available opportunity foregone. W.H. Hutt also makes this point in his article, "The Yield From Money Held."[28] To Hutt, the fact that money units are being held does not mean that they are not being used.[29] He criticizes economists for viewing money as a barren asset and argues that it is fruitless to view cash balances as idle hoards unless it can be shown that, due to mismanagement, some part of one's cash holdings offers no speculative or convenience yield valued above the rate of interest. In this latter situation, Hutt argues that it is possible to view the surplus holdings as "barren" or "idle" just as it is possible to view any other unutilized resources

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[28]William H. Hutt, "The Yield from Money Held," in _On Freedom and Free Enterprise: Essays in Honor of Ludwig Von Mises_, Edited by Mary Senholz (Princeton: D. Van Nostrand Co., Inc., 1956): 196-216.

[29]_Ibid_., p. 214.

as idle.[30] In general, however, cash holdings are not economically idle. Rather, they provide their holders with a continuous flow of non-pecuniary services.[31]

There are two characteristics, unique to money, which may have perpetuated the confusion concerning the yield from money. First, money is different from other assets in that an increase in the purely nominal stock of money within an economic system does not imply that there has been an increase in wealth, welfare, or total utility in that system.[32] This is because the services which money provides do not depend upon the nominal quantity of money in circulation. Alfred Marshall emphasizes this point when he states, "Currency differs from other things in that an increase in its quantity exerts no direct influence on the amount of services it renders."[33]

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[30] Ibid., p. 201. In this passage Hutt is actually incorrect to view the cash holdings as idle balances if they are not valued above the rate of interest. In this case, although the cash holdings are larger than the optimal level, the surplus holdings can only be considered as "barren resources" if the total yield from these balances is zero.

[31] Ibid., p. 214. By continuous flow of productive services I mean the value derived from the stock of cash balances held. This theoretical concept can be applied to any stock, not just money.

[32] Ibid., p. 197.

[33] Alfred Marshall, Money, Credit, and Commerce (New York: Augustus M. Kelley, Bookseller, 1965): 45.

The flow of monetary services, whether for technical or speculative purposes, do however depend upon the real value of money. Hence, an increase in the real value of money, or an increase in the purchasing power of money in an economic system, does imply that the flow of monetary services in that system has increased.[34]

In addition to the confusion between the services rendered by changes in the real and nominal stock of money in an economy, the second reason why economists have viewed money as a barren asset is because, ironically, money is the medium of exchange. Since money is ultimately used to acquire other goods and services, many economists have concluded that money has no utility apart from its exchange value. For example, Wicksteed explicitly adopts this position when interpreting Aristotle's view of money. Wicksteed describes money as

> Something which X, when he has given Y something which Y wants, is willing to receive in exchange though he has *no use* for it himself, because he knows that he can, in his turn, get something which he *does want* in exchange for it. It is a surety which, while not compensating a man, ensures him of compensation.[35] (my italics)

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[34]William H. Hutt, *Op. cit.*, p. 197.

[35]Professor E. Cannan, M.D. Ross, Dr. J. Bonar, and Dr. P.H. Wicksteed, "Who said Barren Metal?" *Economica* 2 (June, 1922): 108-109.

Wicksteed apparently did not recognize the subtle distinction between holding money so that it can be exchanged for other items, and holding money in order to be in a position to acquire things at the most profitable or most convenient time. It is true that money is always held, except perhaps by misers, with the intention of exchanging it for other goods or interest-bearing assets. However, while it is being held, it does provide its holders with a non-pecuniary yield, or "use value."[36] If individuals were unwilling to actually hold money for any length of time, the velocity of money would tend toward infinity.

Unlike Wicksteed, Alfred Marshall and Knut Wicksell did recognize that money provides services which improve one's bargaining position when purchasing either producer or consumer goods.[37] Both authors did not, however, view these services as providing a positive non-pecuniary return. Wicksell argued that "money itself has no marginal utility since it is not intended for consumption."[38] Similarly, Marshall considered the demand for money a decision which

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[36]William H. Hutt, Op. cit., 213.

[37]Alfred Marshall explicitly mentions this point in Op. cit., pp. 44-45. Wicksell also seemed to recognize that money provides a continuous flow of services rather than being concentrated into the moment at which it is spent. See for example, Knut Wicksell, Op. cit., p. 21.

[38]Knut Wicksell, Ibid., pp. 19-20.

locks up resources in a "barren form."

> In every state of society there is some fraction of their income which people find it worthwhile to keep in the form of currency; it may be a fifth, or a tenth, or a twentieth. A large command of resources in the form of currency renders their business easy and smooth, and puts them at an advantage in bargaining; but on the other hand, it locks up in a *barren form*, resources that might yield an income of gratification if invested, say, in extra furniture; or a money income, if invested in extra machinery or cattle.[39] (my italics)

Although Marshall points out that cash holdings can enable an individual to be in a better bargaining position when engaging in exchange, he does not view these monetary services as providing a positive non-pecuniary return. He does, however, argue that furniture provides an individual with an "income of gratification." Hence, he implicitly applies a different criterion when analyzing the demand for money than that used to analyze the demand for other items.

Unlike Marshall, Ludwig von Mises denies that the criterion used to determine the demand for money is different from that of other goods. He argues that although people are always acquiring money in order to purchase other items, this does not imply that one's cash holdings represent an "unintentional remainder" which individuals really do not *want* to hold.[40]

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[39] Alfred Marshall, *Op. cit.*, p. 45.

[40] Ludwig von Mises, *Human Action: A Treatise on Economics* (New Haven: Yale University Press, 1963): 402.

> ...the immense majority of people aim not only to own various vendible goods; they want no less to hold money. Their cash holding is not merely a residuum, an unspent margin of their wealth... . Its amount is determined by a *deliberate demand for cash*. And as with all other goods it is changes in the relation between demand for and supply of money that bring about changes in the exchange ratio between money and the vendible goods.[41] (my italics)

Von Mises also contends that it is incorrect to distinguish between circulating and "idle" or "hoarded" cash balances.[42] "There is no fraction of time in between in which the money is not a part of an individual's or a firm's cash holdings, but just in 'circulation'."[43] Money does not provide services to its holders only when it circulates; rather, individuals acquire money "in order to be ready for the moment in which a purchase may be accomplished."[44]

By viewing cash balances as an unproductive form of hoarding one suggests that the level of cash holdings exceeds that which is considered adequate or normal (i.e., that which is optimal). Disagreeing with this view, von Mises points out that if an individual increases his cash holdings he

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[41] *Ibid*., p. 402.

[42] *Ibid*.

[43] *Ibid*.

[44] *Ibid*., p. 403.

must believe that "some special conditions make it expedient to accumulate a cash holding that exceeds the amount he himself would keep under different conditions... ."[45]

In essence, then, von Mises recognized that the amount of money an individual holds, like the amount of any good demanded, is determined by marginal considerations. Moreover, since barriers to decreasing one's cash holdings are minimal, the amount of money each individual is willing to hold can be considered optimal from his own perspective. Rather than accepting this interpretation of a decision to hold cash balances, John M. Keynes adopted Marshall's view of cash holdings as a barren investment. In fact, Keynes may have done more towards perpetuating the view that money is a sterile asset than his mentor.

In analyzing Keynes' monetary theory, T. Greidanus contends that unlike Marshall, who emphasized the advantages from holding money, Keynes' analysis that money has no utility apart from its exchange value seemed to overlook the gains from holding money.[46] This is not meant to imply that Keynes was unaware of the services money provides its holders. In the General Theory he explicitly argues that "the power of disposal over an asset during a period may offer a potential convenience or security which we shall call

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[45]Ibid., p. 402.

[46]T. Greidanus, The Devélopment of Keynes' Economic Theories, (London: P.S. King and Son, Ltd., 1939): 2-7.

a liquidity-premium."[47] However, like Marshall, Keynes did not consider these services as providing a positive yield. He views the holders of money as refusing to part with "this yield of nil" unless they are compensated with a positive liquidity premium.[48] Here, Keynes was presumably referring to his speculative demand for money since in this passage he refers to the power of disposal over an asset.[49] His writings seem to indicate that speculative and precautionary balances are unproductive, not cash balances held for transaction purposes.[50] This analytical technique of differentiating between a productive transaction demand for money and unproductive speculative and precautionary demand for money has had a significant impact on the development of modern monetary theory. It is this Keynesian distinction between productive and unproductive cash holdings which has led many economists to advocate policies aimed at discouraging individuals from holding money.[51]

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[47] John Maynard Keynes, Op. Cit., p. 226.

[48] Ibid.

[49] As previously mentioned, Keynes defines the speculative motive for liquidity as "the object of securing profit from knowing better than the market what the future will bring forth." Ibid., p. 170.

[50] Keynes makes this point when he states "For the amount of hoarding must be equal to the quantity of money (or -- on some definitions -- to the quantity of money minus what is required to satisfy the transactions-motive),..." Ibid., p. 174.

[51] This idea will be developed more fully in the next chapter of this dissertation.

Although Keynes accepted the view that money is a barren asset, providing its holders with a "yield of nil," he was largely responsible for the incorporation of marginal analysis into modern monetary theory. In the Treatise, he argues that the price-level of investment goods depends upon the relative preference of investors to hold bank-deposits or to hold securities and contends that these preferences are determined by the public's relative desire for liquidity or profit (i.e. the 'bearishness' or 'bullishness' of the public). This analysis provided the foundations upon which J.R. Hicks incorporated the "marginal revolution" into monetary theory. In his 1935 article, "A Suggestion for Simplifying the Theory of Money,"[52] Hicks states,

> We now realize that the marginal utility analysis is nothing else than a general theory of choice, which is applicable whenever the choice is between alternatives that are capable of quantitative expression. Now money is obviously capable of quantitative expression, and therefore the objection that money has no marginal utility must be wrong. People do choose to have money rather than other things, and therefore, in the relevant sense, money must have a marginal utility.[53]

To Hicks, the question of why individuals hold money instead of spending it on consumption goods could easily be explained by the fact that individuals will usually want to set aside some cash to satisfy future consumption wants.[54]

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[52] J.R. Hicks, "A Suggestion for Simplifying the Theory of Money," Reprinted in Critical Essays in Monetary Theory by J.R. Hicks (Oxford: Clarendon Press, 1967): 61-82.

[53] Ibid., p. 63.

[54] Ibid., p. 66.

However, the decision to hold money rather than capital goods posed a much more critical question which Hicks considered the central issue in the pure theory of money.

> What has to be explained is the decision to hold assets in the form of barren money, rather than interest- or profit-yielding securities... . So long as rates of interest are positive, the decision to hold money rather than lend it, or use it to pay off old debts, is *apparently* an unprofitable one.[55] (my italics)

Hicks responds to this question by arguing that the impact of "frictions" on economic behavior, i.e. transactions costs, may discourage an optimizing individual from investing money over short periods of time.[56] The net advantage from investing a given quantity of money will be positive only if the interest or profit earned plus/minus any capital gains or losses, is greater than the cost of investment. With this in mind, Hicks notes that, "...it will be *profitable* to hold assets for short periods, and in relatively small quantities, in monetary form."[57] (my italics)

This analysis provided a sound theoretical framework for the subsequent work done on the demand for money (e.g. that done by Milton Friedman and others).[58] However,

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[55]*Ibid*.

[56]*Ibid*., p. 67.

[57]*Ibid*., pp. 68-69.

[58]See, for example, Milton Friedman, "The Quantity Theory of Money: A Restatement," reprinted in *Monetary Theory* edited by R.W. Clower (Middlesex: Penguin Books, Ltd., 1969): 94-111; see also works cited in footnote 7.

although most economists now accept the view that money provides a flow of utility services to its holders, the argument that money is also a producer's capital good which provides services in the production process is not universally accepted. This debate, which is particularly evident in the money growth literature, stems from a view of money which is still held by some economists that although cash balances provide utility to its holders they are an unproductive form of hoarding which discourages real production and economic growth. This contention will be analyzed in the remaining chapters of this dissertation.

CHAPTER III

A CRITICAL OVERVIEW OF JAMES TOBIN'S MONEY GROWTH MODEL AND ITS RELEVANCE FOR POLICY MAKERS

III.1: Introduction

A major problem with many of the existing theories of money and monetary theory is that the current models often neglect to consider the unique characteristics which distinguish a money economy from a barter system of exchange. Most of the orthodox models (neoclassical and Keynesian) implicitly assume that any good serves equally well as a medium of exchange. Money is viewed as the nth good in a Walrasian tâtonnement system. Robert Clower specifically mentions this problem in his article, "A Reconsideration of the Microfoundations of Monetary Theory."[59]

> Modern attempts to erect a general theory of money and prices on Walrasian foundations have produced a model of economic phenomena that is suspiciously reminiscent of the classical theory of a barter economy.[60]

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[59] Robert W. Clower, "Foundations of Monetary Theory," reprinted in Monetary Theory, edited by Robert Clower (Harmodsworth: Penguin Books, Ltd., 1969): 202-211.

[60] Ibid., p. 202.

Both Clower and Axel Leijonhufvud have attempted to rectify this problem in their work on monetary theory by re-establishing the micro-foundations of macro-theory.[61] Both authors criticize Walrasian models for failing to accurately depict the workings of a monetary economy by distinguishing money from other goods as a source of effective demand. This criticism can also be directed against most of the modern money growth models.[62]

A major purpose of this dissertation is to analyze some of the limitations of the money growth models and to provide some evidence on the appropriate specification of these models. The analysis presented in this chapter will mainly deal with James Tobin's money growth model.[63] However, the criticisms can also be applied to any of the studies which conclude that an increase in the nominal rate of money expansion in an economy will stimulate growth by generating a new, higher level of steady-state capital accumulation.

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[61] See Axel Leijonhufvud, <u>On Keynesian Economics and the Economics of Keynes: A Study in Monetary Theory</u> (London: Oxford University Press, 1968); and in ______________, "The Varieties of Price Theory: What Microfoundations for Macrotheory?" Unpublished Discussion Paper #44 (January, 1974).

[62] We will concentrate on the limitations of James Tobin's money growth model. See James Tobin, "Money and Economic Growth," pp. 671-684.

[63] <u>Ibid</u>.

III.2: Money Growth Models: Are Their Conclusions Valid?

In his 1965 article, "Money and Economic Growth,"[64] Tobin constructs a simple two-asset portfolio behavior model. His work was novel in that he explicitly introduced money into his model whereas most economic growth models prior to Tobin's were nonmonetary. Tobin uses this model to analyze the role which monetary factors play in determining the degree of capital intensity in an economy.

In his analysis Tobin assumes that money serves both as a medium of exchange and as a store of value. It is supplied by and its nominal yield fixed by the central government. He also distinguishes between saving-consumption choices and portfolio choices. The first set of choices determines how much is saved rather than consumed and how much wealth is accumulated. The second set determines the form in which savers will hold their savings i.e. in the form of money savings or physical savings. Tobin follows John M. Keynes and Irving Fisher in making this distinction.[65]

The implications derived from Tobin's model have intrigued many economists and continue to pose some

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[64] Ibid.

[65] Ibid., p. 671.

important, unresolved theoretical questions. To aid the discussion of these implications, two simple growth models will be constructed here. Utilizing assumptions similar to those employed by Tobin, the same conclusions will be generated.

Barter Economy:

Output:

Assume a linear homogeneous production function.

(1) $Q_t = F(K_t, L_t)$

where Q_t = output in time t
L_t = labor in time t
K_t = capital in time t

Reduce both sides of (1) to a per capita basis to determine the output-labor ratio.

$Q_t/L_t = F(K_t/L_t, 1)$

(2) $q_t = f(k_t)$

where $q_t = Q_t/L_t$
$k_t = K_t/L_t$

Labor Market:

(3) $L_t = L_o e^{nt}$

where n = the rate at which the labor force is growing

Take logs of both sides of (3) and differentiate with respect to time.

$$(1/L_t)(dL_t/dt) = n$$

(4) $(\dot{L}_t/L_t) = n$ = natural rate of growth of the supply of labor

Savings Function:

Assume that savings is a constant fraction of income, (s).

(5) $S_t = sF(K_t,L_t)$

Again reduce both sides of (5) to a per capita basis to determine the savings-labor ratio.

(6) $S_t/L_t = sf(k_t)$

Investment Function:

Now assume that savings always equals investment, ex post.

(7) $I_t = sF(K_t,L_t)$

Again reduce both sides of (7) to a per-capita basis.

(8) $I_t/L_t = \dot{K}_t/L_t = sf(k_t)$

Long-run steady-state equilibrium in the capital market is reached when the capital-labor ratio is not growing (i.e. to determine the conditions under which this steady-state equilibrium will hold the following manipulations must be performed):

$$(9) \quad k_t \equiv K_t/L_t$$

Take the logs of both sides and differentiate with respect to time.

$$\dot{k}_t/k_t = \dot{K}_t/K_t - \dot{L}_t/L_t$$

Divide both the numerator and the denominator of $\dot{K}_t/K_t$ by L_t and assume balanced growth.

$$(10) \quad \dot{k}_t/k_t = \frac{\dot{K}_t/L_t}{K_t/L_t} - \frac{\dot{L}_t}{L_t} = \frac{sf(k_t)}{k_t} - n$$

Now, long-run equilibrium in the capital market requires that

$$\dot{k}_t/k_t = 0$$

Hence, the condition necessary for this steady-state is:

$$(11) \quad sf(k_t) = nk_t$$

Equation (11) indicates that the growth in the demand for capital is just equal to the savings available to finance it in a barter economy. However, if money is introduced into our model, and if we assume, as Tobin does, that new money issues are only additions to government debt which will not stimulate any real capital accumulation, equation (11) must be adjusted to exclude savings held in the form of money. In order to facilitate a better understanding of the reasons why Tobin introduces this adjustment into his model, another mathematical illustration will be provided.[66]

The simplest version of a neo-classical money growth model introduces money as wealth but not as income or as a factor of production. Hence, the accumulation of real money balances competes with that of the accumulation of physical investment.

$$(12) \quad I_t = sF(K_t, L_t) - \frac{d(^M/P)}{dt}$$

where $\frac{d(^M/P)}{dt}$ = the change in real money balances

If the demand for real money balances is a constant fraction of income, s_m, and if the money market is in equilibrium, equation (12) can be written as:

$$(13) \quad I_t = sF(K_t, L_t) - s_m \left(\frac{d(F(K_t, L_t)}{dt}\right).$$

Unlike equation (13) where savings only depend upon real output, Tobin assumes that the level of savings is a constant proportion of real output and increases in the real value of money. Introducing this assumption into our model, the investment function becomes:

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[66]A similar example can be found in Edward Shaw, Financial Deepening in Economic Development (London: Oxford University Press, 1973): 34-37.

$$(14) \quad I_t = s[F(K_t,L_t) + s_m \frac{d(F(K_t,L_t)}{dt}] - s_m \frac{[d(F(K_t,L_t)]}{dt}$$

The distinction between the constant propensity (s) to accumulate wealth in both physical and money form, and the physical propensity (s_p) to accumulate physical wealth (real capital accumulation or $I_t/F(K_t,L_t)$) can be drawn by dividing equation (14) by income, $F(K_t,L_t)$. This distinction plays a vital role in Tobin's analysis.

$$(15) \quad s_p = s + s_m n(s - 1)$$

$$s_p = s + s(s_m n) - s_m n$$

A controversial implication can be derived from equation (15). As indicated on the page 36, the condition necessary for steady-state equilibrium in the capital market of a barter economy is: (11) $sf(k_t) = nk_t$. However, by assuming that savings in the form of money <u>will</u> <u>not</u> stimulate any real capital accumulation, we find that in a monetary economy the steady-state equilibrium condition in the capital market is:

$$(16) \quad nk_t = (s + s(s_m n) - s_m n) \cdot f(k_t)$$

Equation (16) implies that although total saving is greater in a monetary than a non-monetary economy since savings depend upon both the level of real output and increases in the real value of money, the amount of physical saving, and hence real capital accumulation will always be lower in a monetary system if any saving is held in the form of money, (i.e. if there is a positive demand for money).[67] This conclusion, however, depends upon five assumptions which fail to capture several of the important distinctions between a monetary and non-monetary economy. These assumptions are respectively:

(1) real capital accumulation is the only substitute for real cash holdings

(2) the marginal propensity to save is a constant fraction of disposable income and that it is equivalent in a monetary and non-monetary economy

(3) savers and investors are identical

(4) individuals will perceive their cash holdings as part of their savings but these money savings will not contribute to real capital accumulation

(5) monetary and non-monetary exchange are equally productive.

One of the major limitations of Tobin's money growth model is that it does not accurately consider the impact which financial intermediaries can have on stimulating capital accumulation in a monetary economy. This is evidenced by both the assumption that the marginal propensity to save is equivalent in a monetary and non-monetary economy and the assumption that money savings will not contribute to real capital accumulation. It is interesting

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[67]This implication holds since $s<1$ then $s(s_m n) < s_m n$ and hence $s_p < s$.

to note that although Tobin borrows the idea to distinguish between saving-consumption choices and portfolio choices from Keynes, Keynes does not assume that money savings will never enter the capital market. In the Treatise on Money, Vol. 1, Keynes maintains that once an individual has decided to save rather than consume he has two options available to him; he may either hold his savings in the form of money or he may hold it in the form of a loan or real capital. Keynes viewed these alternatives as decisions to hoard (choose bank deposits) or to invest (choose securities).[68] Keynes was interested in determining how individuals' decisions to choose between these two alternatives might modify and in some instances dominate the determination of the rate of interest and process of capital determination. Unlike Tobin, however, Keynes explicitly recognized that the banking system can counteract a decision on the part of the public to increase its money savings relative to physical savings.[69] Unless the public does literally hoard their money savings, under their mattresses or elsewhere, the banking system may prevent the price of securities from falling by utilizing bank deposits to purchase the securities which the public

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[68] J.M. Keynes, A Treatise on Money: The Price Theory of Money Vol. 1 (New York: Harcourt, Brace, World, Inc., 1935): 141.

[69] Ibid., pp. 142-3.

is less willing to hold. But as long as some money remains idle, Tobin's conclusion that the level of capital intensity will be lower in a money economy than in a non-monetary system is still valid within the context of his model. If, however, we relax either assumption (2) or (3) specified on page 39, this conclusion can no longer be unambiguously derived.

Following the Keynes-Wicksell approach to money growth models, a fairly strong case can be made against the neo-classical assumption, utilized by Tobin, that savers and investors are identical.[70] By neglecting to consider the fact that individual savers are not always the most efficient investors and that these savers will not voluntarily loan the full amount of their available savings to the most efficient investors, Tobin's model does not give adequate recognition to the productive role which financial institutions can play in both increasing the marginal propensity to save out of disposable income and increasing the efficiency with which the available

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[70]See for example K. Nagatani, "A Monetary Growth Model with Variable Employment," Journal of Money, Credit, and Banking 1 (May, 1969): 188-206, H. Rose, "Unemployment in a Theory of Growth," International Economic Review 7 (September, 1966): 50-58: ____________, "Real and Monetary Factors in the Business Cycle," Journal of Money, Credit and Banking 1 (May, 1969): 153-171; Jerome Stein, "Neo-classical Keynes-Wicksell Monetary Growth Models," Journal of Money, Credit, and Banking 1 (May, 1969): 153-171.

supply of saving is allocated to potential investors. Similarly, the assumption that the marginal propensity to save is constant and equivalent in a monetary and non-monetary economy does not incorporate the positive impact which financial intermediaries can have on the level of capital accumulation and the level of efficiency with which this capital is employed.[71] Tobin's model also ignores the productivity gains which can be derived from holding money and hence does not accurately distinguish between a monetary and non-monetary economy.[72] Don Patinkin and David Levhari have shown that by simply incorporating these productivity gains into Tobin's

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[71]On this point see Raymond Goldsmith, Financial Institutions (New York: Random House, Inc., 1965); __________, Financial Structure and Economic Development (New Haven: Yale University Press, 1969); John Gurley and Edward Shaw, "Financial Aspects of Economic Development," American Economic Review 45 (September, 1955): 515-538; __________ and __________, "Financial Structure and Economic Development," Economic Development and Cultural Change 15 (April, 1969): 257-268; __________ and __________, Money in a Theory of Finance (Washington, D.C.: The Brookings Institution, 1960); Hugh Patrick, "Financial Development and Economic Growth in Underdeveloped Countries," Economic Development and Cultural Change 14 (January, 1966): 174-189.

[72]Don Patinkin and David Levhari, Op. cit., pp. 205-242; Alvin Marty, "Some Notes on Money and Economic Growth," Journal of Money, Credit, and Banking 1 (May, 1969): 252-265.

model, while retaining all of the other assumptions of his model, the conclusion that the rate of growth in the capital stock is negatively related to the level of real money holdings cannot be unambiguously specified on theoretical grounds.[73]

Economic complexities necessitate the use of simplifying assumptions when developing economic models. However, real world applications of the implications derived from empirically invalid assumptions, can be potentially damaging. As Leland Yeager has pointed out,

> No one can properly quarrel with the use of simplifying assumptions; ... One may quarrel, however, when a writer advertises the logical implications of his own freely chosen simplifying assumptions as inexorable conclusions directly applicable to the real world.[74]

This citation becomes quite relevant when considering the policy implications which can be derived from Tobin's money growth model. The model implies that the long-run equilibrium rate of capital accumulation is lower in a monetary economy than in a non-monetary economy because individuals will hold a portion of their savings in the form of money rather than allocating all of their savings into the capital market. Hence, in this model, money and physical capital are viewed as strict substitutes. The

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[73]James Tobin, "Money and Economic Growth," pp. 680-684.

[74]Leland Yeager, "Some Questions about Growth Economics," American Economic Review 44 (March, 1954): 55.

policy implication which can be derived from this model is that by discouraging individuals from holding money the long-run equilibrium rate of capital accumulation in the economy can be increased since individuals will substitute from cash holdings into physical capital.

Inflation increases the cost of holding money and hence will induce individuals to decrease their cash holdings. For this reason, it is frequently argued that inflation can stimulate growth by accelerating the rate of capital accumulation in an economy by encouraging individuals to substitute from cash holdings into physical capital goods. Although in his article Tobin does not explicitly advocate the use of inflation to increase the level of capital intensity within an economy, others, drawing from his analysis, have done so. It is perhaps true that a fully anticipated inflation may increase the level of capital intensity within a financially developed economy by encouraging individuals to substitute physical capital holdings for money holdings.[75] It is not true that this adjustment will lead to a more efficient utilization of scarce resources. Consequently, rather than stimulate economic growth, the inefficiencies generated by inflation may have a detrimental effect on growth.

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[75]Kessel and Alchian, Op. cit., p. 535.

These inefficiencies may be particularly damaging in less-developed countries where actively operating financial institutions have not developed. Under such circumstances, money may play a valuable role in stimulating economic growth. Decisions which generate an increase in the aggregate demand for money in an economy may also stimulate an expansion in the real size of the financial sector in this economy. Theoretical arguments which support this hypothesis are presented in the next section.

III.3: The Role of Real Money Balances in Promoting Growth In Less-Developed Countries

Contrary to the implication derived from Tobin's money growth model that inflation can stimulate capital accumulation by discouraging individuals from holding money, evidence exists which suggests that high and fluctuating rates of inflation may retard the growth process by inhibiting financial intermediaries from financing profitable investments.[76] Theoretical arguments and empirical evidence will

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[76] For evidence on this see Ronald McKinnon, Money and Capital in Economic Development (Washington, D.C.: The Brookings Institution, 1971): 68-88, 100-116; Emprime Eshag and M.A. Kamal, "A Note on the Reform of the Rural Credit System in U.A.R. (Egypt)," in Bulletin of the Oxford University Institute of Economics and Statistics, Vol. 29 (May, 1967); Tom E. Davis, "Eight Decades of Inflation in Chile, 1879-1959: A Political Interpretation," Journal of Political Economy, Vol. 71 (August, 1963): 389-397; Samuel A. Morley, "Inflation and Stagnation in Brazil," Economic Development and Cultural Change, Vol. 19 (January, 1971): 184-203; Herman E. Daly, "A Note on the Pathological Growth of the Uruguayan Banking Sector," Economic Development and Cultural Change, Vol. 16 (October, 1967): 91-96.

be presented which suggest that inflationary policies may have a negative effect on the rate of growth in an economy by encouraging individuals to decrease their real cash holdings. Following Ronald McKinnon and Edward Shaw, arguments will be presented which indicate that real cash holdings and real capital accumulation may be considered complementary assets rather than strict substitutes. This suggests that monetary policies which allow the real rate of return on money to increase may introduce productivity gains which facilitate economic growth.[77]

McKinnon and Shaw point out that in most underdeveloped countries large dispersions in the real rates of return on different investments persist, indicating that capital is being misallocated.[78] Because these conditions have a direct impact on the economy's ability to develop, McKinnon and Shaw argue that when analyzing these economies one must recognize that investments are not homogeneous and that errors in accumulating and allocating resources occur and are costly. In their work they emphasize that the diffusion of technology is slow and expensive, that savers and investors are not identical and that individuals both measure and regard risks differently. The reasons for emphasizing these imperfections are two-fold. First, from

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[77]Ronald McKinnon, Op. cit., pp. 43-47, 57-67, 86-87, 119-21; Edward Shaw, Op. cit., pp. 47-49.

[78]Ronald McKinnon, Op. cit., pp. 5-8, 11, 27-8.

a practical point of view, this approach provides a more appropriate framework for analyzing the major problems which policy makers are concerned with; and second, from a theoretical point of view, it is only by recognizing that markets do not operate perfectly that the productivity gains from using money can be accurately evaluated.

Before successful economic growth can proceed in the LDCs, market boundaries for output, inputs, and financial assets must be expanded to accommodate both the rural and urban areas. At present, the available opportunities to engage activity in trade are quite different in various sectors of these economies. Individuals in the rural areas which are isolated from the urban marketplaces, do not engage actively in monetary exchange. Hence, in the sense that limited monetary exchange takes place in the remote areas of the LDCs, these countries are not yet fully monetized. This condition greatly inhibits the extension of the market system. The scale of common markets in output, wealth, labor, and financial securities is substantially smaller than that which would exist if the domestic monetary system in these economies were more consolidated.[79]

In addition to the problem posed by limited monetary exchange, access to external finance is also difficult to

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[79]Edward Shaw, Op. cit., p. 60.

acquire.[80] There are few organized markets for such primary securities as bonds, mortgages, or common stock in the less developed countries. Many regions do not even have access to the organized banking sector.[81] In areas where there is no access to an operating capital market, or where access is extremely limited, the investment which does occur is primarily self-financed. Under such circumstances money can be used as a conduit through which capital accumulation can occur. This, of course, will only provide a temporary solution to the credit shortage which currently plagues most LDCs. An expansion in the real size of the financial systems in the LDCs is ultimately needed to increase the available supply of external funds in both the urban and rural areas of these countries. However, under present economic and institutional conditions, it is not profitable for the organized banking sectors in the LDCs to satisfy the existing demand for credit, especially in the rural areas.[82]

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[80]On this point see Raymond Goldsmith, Financial Structure and Development, p. 374.

[81]Ronald McKinnon, Op. cit., pp. 37-38; Charles Nisbet, "Interest Rates and Imperfect Competition in the Informal Credit Market of Rural Chile," Economic Development and Cultural Change 16 (October, 1967): 73; U. Tan Wai, "Interest Rates Outside the Organized Money Markets of Underdeveloped Countries," IMF Staff Papers, Vol. 6 (November, 1957): 80-142.

[82]High information and transaction costs pose important economic constraints which inhibit the organized banking sector from expanding into the rural areas of the LDCs. In addition, the institutional usury restrictions imposed on this financial institution further inhibit the organized banking sector from providing credit to high risk areas. See Charles Nisbet, Op. cit., pp. 73-90.

In an environment restricted to self-financed investment, cash holdings may be positively correlated with the level of capital investment. Firms which are limited to self-financed investment generally have two options available to them when deciding how to purchase additional capital. The firm (household) can either store inventories (grain, jewelry, gold, etc.) for eventual sale, or it can accumulate cash balances as a store of value.[83] Since storage costs are generally higher for inventories than for cash balances, the cost of self-financed investment can be reduced and investment stimulated if money holdings are used rather than more costly inventory sales to finance investments. This positive relationship between the level of real cash holdings and real capital accumulation will hold only if the marginal return from holding money is less than the return from additional self-financed investment. Once the yield from these two options is equated, any additional increase in the real return on money will generate a net portfolio substitution out of physical capital into cash balances.[84]

[83]McKinnon points out that a large amount of economic activity which occurs in the LDCs stems from the small household unit. Accordingly, he coined the term firm-household unit to refer to the production generated in these household units.

[84]As previously mentioned, this substitution is not necessarily undesirable, especially from the point of view of the individual who prefers to hold additional cash balances. However, if the yield on money or physical capital has been altered from its market value, one cannot determine, a priori, whether this substitution will make the individual and/or society better off. To make this judgement one would have to assume that utility can be objectively determined. For arguments similar to this see James M. Buchanan, Cost and Choice: An Inquiry in Economic Theory (Chicago: Markham Publishing Co., 1969).

The above analysis suggests that monetary policies can affect the amount of self-financed investment undertaken by altering the real yield on money. The major problem when attempting to actively pursue such policies is that it is obviously difficult to know the relative return (yield) which each investor can earn from self-financed investments and money respectively. Since all the relevant information needed to determine the ultimate impact of a particular policy is difficult to acquire, one should be wary about advocating policies which either actively increase or decrease the yield on a particular asset.[85] Measures aimed at achieving some specified objective may introduce additional distortions into the economy which work in directions opposite to those desired. For example, if the marginal yield on additional self-financed investment is relatively high, inflationary policies may discourage this investment by inhibiting individuals from holding money which can eventually be used to finance this investment. Alternatively, if the marginal yield on additional self-financed investment is greater than, but fairly close to the non-pecuniary yield on money, deflationary policies which increase the yield on money may encourage individuals to permanently increase their real money holdings rather than temporarily hold

[85] My argument here is similar to that one presented in footnote 84.

additional cash balances to eventually finance additional capital expenditures. It is difficult to know, at a moment in time, which of these competing forces will dominate. By actively pursuing either inflationary or deflationary policies, the domestic monetary authorities may introduce undesirable distortions into an economic system.[86] Alternatively, by pursuing policies consistent with a relatively stable price level, the monetary authorities may reduce destabilizing cyclical fluctuations which inhibit growth.[87]

Evidence indicates that many LDCs have pursued inflationary policies which have discouraged individuals from

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[86]See Milton Friedman, A Program for Monetary Stability (New York: Fordham University Press, 1960); ____________, "Monetary and Fiscal Framework for Economic Stability," reprinted in Essays in Positive Economics (Chicago: The University of Chicago Press, 1953): 133-57; ____________, Optimal Quantity of Money and Other Essays (Chicago: Aldine Pub., Co., 1969). Friedman's arguments on the role of the monetary authorities in an economy are based on the assumption that the existence of a monetary authority can increase the monetary stability of an economy. Although I will not criticize this assumption here, it is relevant to point out that this assumption, as opposed to a system of free market banking, is subject to some important criticisms. On this point, see Roger Garrison, "The Gold Standard: Vienna versus Chicago," presented at the Charlottesville Seminar in Austrian Economics (October, 1975).

[87]Again see Milton Friedman, A Program for Monetary Stability. Similarly, Edward Shaw and Ronald McKinnon, among others, also favor this position.

demanding money.[88] Such policies may also have inhibited the growth of actively functioning marketplaces and discouraged small firm-household units from using money as a means of self-financing additional investments. In underdeveloped countries where the real return on money holdings is low or negative due to high rates of inflation, and where there are few financial substitutes for money as a store of value, the rural farmers and small businessmen hold more costly inventories as a form of saving.[89] Similarly, inflation induces more established manufacturers to over-invest in plant capacity or to hold extra stocks of raw materials relative to current operating needs. This substitution of real for monetary assets is inefficient and can be reduced by decreasing inflationary distortions which keep the real yield on money arbitrarily low. By transferring this low productivity capital to more efficient uses, a once-for-all increase in aggregate output can be stimulated.[90] In addition, when coupled with interest regulations, inflation can inhibit the growth of efficiently

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[88] See footnote 76 for citations supporting this point.

[89] Graeme S. Dorrance, "The Effects of Inflation on Economic Development," In Inflation and Growth in Latin America, edited by Werner Baer and Isaac Kertenetzky (Homewood: Richard D. Irwin, Inc., 1964): 37-88.

[90] Ronald McKinnon, Op. cit., p. 63.

operating financial intermediaries by keeping the real return on financial assets low. Such policies restrict the overall level of savings within an economy and prevent scarce capital from being allocated efficiently.

Market rates of interest report the scarcity of savings to investors who plan to draw upon existing resources to accumulate new capital. They warn borrowers that they should invest in capital goods only if this investment will yield a rate of return which is at least equal to the scarcity price of savings. Conversely, policies which actively distort market loan rates (or prices in general) inhibit efficient allocation since prices become less related to actual scarcity values. If the supply and demand for capital funds are not equal at the legally imposed interest ceiling, the regulated intermediaries must rely on arbitrary forms of discrimination or rationing to allocate their capital funds rather than relying on profitability requirements. Incentives to explore new or less certain lending opportunities are inhibited and available funds often flow to relatively safe borrowers whose reputations are known or whose collateral is considered relatively risk-free.

Evidence does indicate that access to bank credit in most LDCs is limited to certain privileged, low-risk enclaves; exclusively licensed import activities, specialized large-scale mineral exporters, and various government agencies.[91] The rural areas in general, and small borrowers (urban and rural) in particular, rarely obtain loans from the organized banking sectors in these countries. For example, from a sample survey of traditional credit markets in rural Chile, Charles Nisbet estimated that only 30 percent of the total rural population are clients of state financial institutions, reform agencies, and private commercial banks. The remaining 70 percent must rely on the informal credit markets where interest rates are considerably higher.[92] The large magnitude of the informal "curb market" operations in South Korea also indicates that the regulated financial markets are unable to provide sufficient funds to satisfy the existing demand for credit.[93] Walter Ness has also provided evidence which suggests that the high rates of inflation experienced in Brazil in the early 1960's, together with effectively imposed interest regulations on long and short-term financial assets, were responsible for the decrease in the size and efficient operations of the financial markets in Brazil.[94]

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[91] Ibid., p. 68.

[92] Charles Nisbet, Op. cit., p. 73.

[93] Ronald McKinnon, Op. cit., pp. 107-108; Edward Shaw, Op. cit., pp. 135-138; and John G. Gurley, Hugh T. Patrick, and Edward S. Shaw, "The Financial Structure of Korea," (Preliminary Draft), U.S. Operations Mission to Korea, July 24, 1965, p. 81.

[94] Walter Ness, "Financial Markets Innovation as a Development Strategy: Initial Results from the Brazilian Experience," Economic Development and Cultural Change 22 (April, 1974): 453-72.

To support the hypothesis that inflation can disrupt financial market operations, particularly in countries where interest regulations are effectively imposed, time series data on the ratio of monetary liabilities to GNP (M_1/GNP,M_2/GNP) for Argentina, Chile, and Brazil are presented at the end of this chapter in Tables III-1 through III-3. (These two ratios as proxies for the real size of the banking systems in these countries.) In addition, to contrast the post-1950 financial developments in these countries with the more successful growth experienced in post-war (World War II) Japan and Germany, data on monetary liabilities to GNP will also be presented on these latter two countries in Tables III-4 and III-5.

The data indicate that during periods of inflation in Argentina, Chile, and Brazil, the ratios of both M_1/GNP and M_2/GNP declined. After 1963, the annual ratio fluctuated, registering both positive and negative percent changes. The M_2/GNP ratio for Chile declined from 0.106 to 0.089 between 1955 and 1957 and did not exceed the 1955 level until 1959. Throughout most of the remaining years between 1960 and 1970, the M_2/GNP ratio increased slowly to reach 0.182. During the 1960's, however, nominal rates of interest greater than 20 percent were paid on some classes of time and savings deposits in Chile.[95] By loosening

[95]Ronald McKinnon, Op. cit., p. 104.

these interest regulations on time and savings deposits, the negative inflationary impact on the demand for these deposits was reduced somewhat even though the overall size of the banking system remained unduly small.

The figures for Brazil and Argentina also suggest that inflation can have a negative impact on the real size of the monetary system in an economy. Between 1950 and 1973, the M_1/GNP ratio declined 36.13 percent in Brazil (from 0.310 to 0.198) and 51.1 percent in Argentina (from 0.363 to 0.100). Similarly, over the same period, the Brazilian M_2/GNP ratio decreased 40.5 percent from 0.385 to 0.229, while the Argentina figure fell from 0.529 to 0.289 (45.4 percent).

Different from the experience in these Latin American countries, the monetary figures for postwar Japan and Germany provide a different picture of financial development. The banking systems in both countries were severely damaged by the Second World War. Rampant inflation severely shook the public's confidence in holding money and wiped out the possibility of carrying over pre-war financial obligations. Consequently, it is possible to view the introduction of the major monetary reforms in 1948-49 as marking a relatively new period of financial development in Japan and Germany even though both countries had a strong banking tradition which would facilitate this redevelopment.[96]

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[96] Ibid., p. 91.

An interesting aspect of the monetary statistics for Germany and Japan is that in both countries the ratio of M_2 to GNP increased quite substantially between 1953-73 while the proportion of M_1 to GNP remained fairly stable. In Germany, M_1/GNP ranged from 17.4 percent of GNP in 1953 to 14.2 percent in 1973 and in Japan it varied from 27.8 percent of GNP to 35.8 percent. Conversely, M_2/GNP doubled during the same period. The proportion of M_2 to GNP increased from 29.7 percent to 56.2 percent in Germany and from 56.7 percent to 111.8 percent in Japan.

This large relative growth in time and savings deposit holdings was a leading factor in the real growth in the banking systems of both of these systems.[97] (See figures presented in Table III-6.) In Japan and Germany time and savings deposit holdings rose from being approximately equal to M_1 in 1953 to being 2.2 and 2.4 times as great as M_1 in 1970. Conversely, time and savings deposit holdings decreased dramatically relative to M_1 holdings during the unstable periods of high inflation prevalent among the Latin American LDCs. For example, John Deaver's data can be used to show that in Chile, between 1928 and 1955, the ratio of time and savings deposits to M_1 fell from about 1.86 to 0.12.[98] After 1955, however, this ratio

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[97] Ibid., pp. 91-96.

[98] John Deaver, "The Chilean Inflation and The Demand for Money," in David Meiselman (ed.), Varieties of Monetary Experience (University of Chicago Press, 1970).

again began to rise since interest ceilings on several classes of time and savings deposits were raised in an attempt to prevent further decreases in the real size of the financial markets.[99]

Figures consistent with the Chilean experience are also available for Argentina and Brazil. Based on a slightly different data series than the IMF uses, Adolfo Diz calculated that in Argentina, the ratio of time and savings deposits to M_1 fell from about 1.36 in 1935 to about 0.29 in 1962.[100] Similarly, in Brazil this ratio fell from a meager 0.177 in 1953 to 0.139 in 1970, with a low 0.029 in 1965. The Brazilian ratio did begin to rise slowly between 1966 and 1970. This increase was probably the result of the financial reforms introduced by the Brazilian military government between 1964-69.[101]

The above figures indicate that time and savings deposits seem to be more volatile, over time, than is the demand for money narrowly defined. They also suggest

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[99]Ronald McKinnon, Op. cit., p. 104.

[100]Adolfo Diz, "Money and Prices in Argentina, 1935-1962," in David Meiselman (ed.), Varieties of Monetary Experience (University of Chicago Press, 1970).

[101]Walter Ness, "Financial Markets Innovation as a Development Strategy: Initial Results from the Brazilian Experience," Economic Development and Cultural Change 22 (April, 1974): 453-72.

that the demand for currency and demand deposits is more inelastic with respect to inflation than is the demand for time and savings deposits. This does not, however, necessarily imply that individuals do not respond to the higher cost of holding cash balances (M_1) in an inflationary and interest regulated environment. A more plausible explanation is that in countries which have experienced prolonged periods of inflation, money holders may have reduced their real cash holdings to a level where the marginal productivity yield from the monetary medium of exchange rises very sharply with each additional unit foregone.

Since the nominal rate of interest on currency and demand deposits is generally zero, the real return on narrowly defined money depends upon the marginal productivity of money (marginal utility from money if held by consumers) and the expected rate of inflation. The inflation inelasticity of demand for M_1 may suggest that individuals have reached the stage where the productivity loss from reducing their demand for money may be greater than the decrease in the real return on money resulting from a higher rate of inflation.[102] Under such

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[102]Herman E. Daly uses this argument to explain why individuals continue to put money in the Uruguay banks during periods of high rates of inflation. Daly cites the high velocity of circulation in Uruguay as an indication that these deposits were kept at a minimum; ($V = 5.43$ in 1961 and varied annually roughly by ± 0.2). See Herman E. Daly, "A Note on the Pathological Growth of Uruguayan Banking Sector," Economic Development and Cultural Change 16 (October, 1967): 91-96.

circumstances, a further increase in the rate of inflation may not induce individuals to further decrease their cash holdings.

Time and savings deposits are not used as a medium of exchange. Hence the implicit yield from the services on these deposits may not be large enough to compensate for the decrease in the real deposit rate resulting from effectively imposed interest regulations in an inflationary environment; ($r_{M2} = g(mpp_{M2}, d - dp/dt)$ where r_{M2} = real rate of return on M_2, mpp_{M2} = marginal productivity of M_2, d = deposit rate on M_2, and dp/dt = rate of change in prices). However, in the absence of interest regulations, the inflationary impact on the real deposit rate may be offset by offering higher nominal rates of interest on various classes on deposits.

Several LDCs have recently introduced such policies in an attempt to reduce the destabilizing effects of inflation.[103] The question still remains, however, as to why policies which keep the real yield on financial assets low or negative have been pursued. On this point, Edward Shaw contends that the historic antipathy to usury which is prevalent in the LDCs has led to the widespread imposition of laws against "high" rates of interest.[104]

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[103] See for example Edward Shaw, Financial Patterns and Policies in Korea, (Seoul): U.S. Operations Mission to Korea, 1967). McKinnon and Shaw also mention several other examples of attempts by LDCs to deregulate interest rates in order to facilitate financial growth. See McKinnon, Op. Cit., pp. 89-117; Shaw, Financial Deepening, pp. 113-147, Walter Ness, Op. Cit., pp. 453-72.

[104] Edward Shaw, Financial Deepening and Economic

In addition to the problem with usury laws, the monetary authorities in most of the LDCs have utilized expansionary monetary policy to finance government deficits.[105]

Economic models which suggest that an expansion in the nominal supply of money is the correct instrument for stimulating capital accumulation have been at least partially responsible for this course of action.[106] These models, however, are only relevant over short periods of time where there is an excess supply of goods and resources at the existing market prices.[107] They implicitly assume that output and savings are extremely elastic with respect to the issue of nominal money. If this were true, we would never be plagued by either the problem of underemployment or underdevelopment. In actuality, a model based on this assumption can be quite detrimental when used as a policy guide since it comes dangerously close to denying the primary economic problem; the problem of economic scarcity. Similarly, models which assume that money substitutes are directly convertible into real capital accumulation and that monetary and non-monetary exchange are equally productive do not provide an

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Development, p. 93; see also J.M. Keynes, The General Theory of Employment, Interest, and Money, p. 351; Rudolph C. Blitz and Millard F. Long, "Economics of Usury Regulation," Journal of Political Economy 73 (Dec., 1965): 608-619.

[105] Edward Shaw, Ibid., p. 94-100.

[106] Ibid., p. 99-100.

[107] F.A. Hayek, A Tiger by the Tail, Compiled and Introduced by Sudha K. Shenoy (The Institute of Economic Affairs, 1972), p. 26.

appropriate analytical framework for investigating the determinants of growth in a monetary economy and hence can be detrimental when used as policy guides.

Table III-1

Financial Structure of Argentina 1950-1973
Monetary and GNP Data in Billions of Current Pesos

	M1 Demand Deposits Plus Currency (1)	Quasi-Money[1] (2)	M2 (1+2) (3)	GNP Gross Nat'l Product (4)	M1/GNP (1÷2) (5)	M2/GNP (3÷4) (6)	Annual %Change M1/GNP (7)	Annual %Change M2/GNP (8)	Annual %Change W.P.I. (9)	Annual %Change Real GNP (10)
1950	25	11	36	68	0.368	0.529	--	--	n.a.	n.a.
1951	30	11	41	95	0.315	0.413	-14.40	-21.93	n.a.	n.a.
1952	34	12	46	112	0.303	0.411	- 3.80	- 0.48	n.a.	n.a.
1953	43	15	58	129	0.333	0.450	9.93	9.49	n.a.	n.a.
1954	52	18	70	145	0.358	0.483	7.51	7.33	n.a.	n.a.
1955	61	20	81	171	0.357	0.474	0.28	- 1.86	n.a.	n.a.
1956	71	29	100	217	0.327	0.461	- 8.40	- 2.74	25.0	n.a.
1957	83	30	113	271	0.306	0.417	- 6.42	- 9.54	31.6	-0.11
1958	119	41	160	385	0.309	0.416	0.98	- 0.24	134.3	8.0
1959	170	46	216	737	0.231	0.293	-25.24	29.57	15.4	-17.9
1960	218	62	280	956	0.228	0.293	1.30	0.0	9.7	12.2
1961	243	77	320	1,140	0.213	0.280	- 6.58	- 4.44	30.4	9.8
1962	250	90	340	1,403	0.178	0.242	-16.43	-13.57	41.8	-5.8
1963	322	131	453	1,725	0.187	0.262	5.06	8.26	26.1	-4.2
1964	459	191	650	2,306	0.194	0.275	3.61	4.96	23.9	6.0
1965	592	263	855	3,604	0.164	0.237	-15.46	-13.86	32.8	26.1
1966	787	337	1,124	4,490	0.175	0.250	6.70	5.49	25.5	3.9
1967	1,092	463	1,557	5,871	0.186	0.265	6.29	6.00	10.0	4.2
1968	1,364	642	2,006	6,832	0.199	0.294	7.00	10.94	5.8	5.8
1969	1,508	795	2,303	8,031	0.225	0.289	13.06	- 1.70	14.1	11.2
1970	1,796	995	2,791	9,348	0.192	0.299	-14.70	3.40	14.4	2.0
1971	2,361	1,394	3,755	13,146	0.179	0.286	- 6.80	- 4.30	40.0	1.3
1972	3,360	2,254	5,614	22,711	0.148	0.247	-17.30	-13.60	75.8	-2.3
1973	6,795	4,133	10,928	37,737	0.180	0.289	21.60	+17.00	50.9	10.1

Source: International Monetary Fund, Financial Statistics Various Issues.

[1] Quasi-Money = Time Deposits at Deposit Money Banks + Quasi-Monetary Liabilities at the Central Bank.

Table III-2

Financial Structure of Chile 1955-1970
Monetary and GNP Data in Undeflated Millions of Pesos

	M1 Demand Deposits Plus Currency (1)	Quasi-Money[1] (2)	M2 Total (1+2) (3)	GNP Gross Nat'l Product (4)	M1/GNP (1÷4) (5)	M2/GNP (3÷4) (6)	Annual %Change M1/GNP (7)	Annual %Change M2/GNP (8)	Annual %Change Wholesale Price Index (9)	Annual %Change Real GNP (10)
1955	.093	.016	.109	1.026	0.091	0.106	--	--	--	--
1956	.130	.025	.155	1.633	0.080	0.095	-12.09	-10.38	+56.25	3.8
1957	.165	.038	.203	2.274	0.073	0.089	- 8.75	- 6.31	+32.0	10.4
1958	.222	.054	.276	2.959	0.075	0.093	2.74	4.49	+24.24	4.8
1959	.294	.162	.456	4.145	0.071	0.110	- 5.33	18.28	+39.02	- 1.1
1960	.384	.211	.595	4.859	0.078	0.122	9.86	10.91	+ 8.77	5.7
1961	.432	.265	.697	5.457	0.079	0.128	1.28	4.92	0.0	4.2
1962	.556	.389	.945	6.595	0.084	0.143	6.33	11.72	+ 9.68	6.8
1963	.747	.459	1.206	9.827	0.076	0.123	- 9.52	-13.99	+47.10	2.8
1964	1.129	.720	1.849	12.493[a]	0.090	0.148	18.42	20.32	+52.00	-12.9
1965	1.867	.997	2.864	17.547[a]	0.106	0.162	17.78	9.46	+32.89	9.1
1966	2.594	1.524	4.418	24.312[a]	0.106	0.169	0.0	4.32	+26.73	12.7
1967	3.241	2.149	5.390	31.814[a]	0.102	0.169	- 3.92	0.0	+19.53	10.7
1968	4.481	3.219	7.700	42.882[a]	0.104	0.180	1.96	6.51	+27.78	6.7
1969	6.100	4.900	10.978	62.400[a]	0.099	0.178	- 4.80	- 1.11	+36.06	11.3
1970	10.000	6.600	16.614	95.100[a]	0.111	0.182	12.12	18.30	+36.65	14.9

Source: International Monetary Fund, Financial Statistics, various issues.

[a]Downward shift in GNP data series.

[1]Quasi-Money = Other deposits at the Central Bank plus Foreign Currency Deposits plus Demand Deposits at Central Bank plus Time deposits at the Central Bank.

Table III-3

Financial Structure of Brazil, 1950-1973
Monetary and GNP Data in Current Billions of Cruzeiras

	M1 Demand Deposits Plus Currency (1)	Quasi-Money[1] (2)	M2 Total (1+2) (3)	GNP Gross Nat'l Product (4)	M1/GNP (1÷4) (5)	M2/GNP (3÷4) (6)	Annual %Change M1/GNP (7)	Annual %Change M2/GNP (8)	Annual %Change Wholesale Price Index (9)	Annual %Change Real GNP (10)
1950	78	19	97	.252	0.310	0.385		--	--	--
1951	91	20	111	.305	0.298	0.364	- 3.87	- 5.45	+25.00	- 3.2
1952	104	21	125	.351	0.296	0.356	- 0.61	- 2.20	20.00	- 4.1
1953	124	22	146	.427	0.290	0.342	- 2.02	- 3.93	16.77	4.2
1954	151	25	176	.554	0.273	0.317	- 5.86	- 7.31	28.57	0.98
1955	178	24	202	.689	0.258	0.293	- 5.49	- 7.57	11.11	11.9
1956	217	25	242	.880	0.246	0.275	- 4.65	- 6.14	20.00	6.4
1957	291	29	320	1.053	0.276	0.303	-12.20	10.18	8.30	11.2
1958	353	33	386	1.304	0.271	0.276	- 1.81	- 2.31	15.38	- 7.3
1959	501	39	540	1.791	0.279	0.302	2.88	2.02	40.00	- 2.0
1960	692	57	749	2.397	0.289	0.312	3.58	3.31	28.57	4.1
1961	1,042	67	1,109	3.475	0.300	0.319	3.81	2.24	40.74	2.9
1962	1,702	71	1,773	5.436	0.313	0.326	4.33	2.19	52.63	2.5
1963	2,792	106	2,898	9.520	0.293	0.304	- 6.71	- 6.75	72.40	1.6
1964	5,191	172	5,363	22.904[a]	0.227	0.234	-22.53	-23.02	91.00	25.9
1965	9,104	265	9,369	36.424[a]	0.250	0.257	10.13	9.83	51.30	5.1
1966	10,483[a]	932[a]	11,415	53.730[a]	0.195	0.212	-22.00	-17.51	37.00	7.7
1967	15,004	1,671	16,675	70.699[a]	0.212	0.236	8.70	11.32	25.25	5.4
1968	21,349	2,702	24,051	98.957[a]	0.215	0.243	1.40	2.97	23.99	13.6
1969	27,410	4,360	31,770	131.900[a]	0.208	0.241	- 3.36	- 0.82	20.16	10.9
1970	34,740	6,030	40,770	204.700[a]	0.170	0.199	-18.27	-17.43	22.06	27.1
1971	45,620	9,360	54,980	271.800[a]	0.168	0.202	- 1.18	1.51	20.50	- 7.1
1972	63,370	14,290	77,660	355.800[a]	0.178	0.218	5.95	7.92	18.40	30.9
1973	93,800	14,630	108,430	473.200[a]	0.198	0.229	11.24	5.04	16.08	14.6

Source: International Monetary Fund, Financial Statistics, various issues.
[1]Quasi-Money = Time deposits at commercial banks + quasi-monetary liabilities at Monetary Authorities.
[a]Upward shift in GNP data series.

Table III-4

Financial Structure of Japan, 1953-1973
Monetary and GNP Data in Billions of Current Yen

	M1 Demand Deposits and Currency (1)	M2 Time and Savings Deposits[1] (2)	Total (1+2) (3)	GNP Gross Nat'l Product (4)	M1/GNP (3÷4) (5)	M2/GNP (3÷4) (6)	Annual %Change M1/GNP (7)	Annual %Change M2/GNP (8)	Annual %Change W.P.I. (1963=100) (9)	Annual %Change in Real GNP
1953	1,937	2,015	3,952	6,965	0.278	0.567	--	--	--	--
1954	2,013	2,543	4,556	7,792	0.258	0.585	-7.19	3.17	-1.0	13.0
1955	2,331	3,064	5,395	8,525	0.273	0.633	5.81	8.21	-2.0	11.7
1956	2,714	3,837	6,551	9,508	0.285	0.689	4.40	8.85	5.2	6.0
1957	2,824	4,767	7,591	11,071	0.255	0.686	-10.52	-0.44	3.0	13.1
1958	3,185	5,870	9,055	11,342	0.281	0.798	6.27	19.16	-6.7	9.8
1959	3,711	7,236	10,947	12,794	0.290	0.856	3.20	7.27	1.0	11.6
1960	4,420	8,937	13,357	15,214	0.290	0.878	0.0	2.57	1.0	17.7
1961	5,258	11,095	16,353	18,487	0.284	0.885	-2.07	0.80	1.0	20.3
1962	5,725	13,360	19,085	21,999[b]	0.260	0.868	-8.45	-1.92	-2.0	21.4
1963	7,702	15,493	23,195	24,404	0.314	0.948	20.77	9.22	2.0	9.0
1964	8,704	17,996	26,700	28,838	0.302	0.926	-3.82	-2.32	0.0	17.9
1965	10,287	20,905[a]	31,192	31,787	0.324	0.981	7.28	5.94	1.0	9.1
1966	11,716	24,601	36,317	36,829	0.321	0.994	0.93	1.13	4.0	11.4
1967	13,369	28,635	42,004	43,585	0.310	0.974	-3.43	-2.01	0.0	18.3
1968	15,155	33,548	48,703	51,677	0.296	0.952	-4.52	-2.26	1.0	17.4
1969	18,282	39,904	58,186	59,689	0.306	0.975	3.28	2.42	2.8	12.3
1970	21,359	47,038	68,395	70,731	0.302	0.969	-1.31	-0.62	3.7	14.3
1971	27,693	56,983	84,676	79,254	0.349	1.068	15.50	16.45	-0.88	21.5
1972	34,526	71,337	105,863	90,612	0.381	1.167	9.17	9.27	0.89	5.4
1973	40,211	85,502	125,813	111,061	0.358	1.118	-6.04	-4.20	15.90	5.7
1974	44,950	97,691	142,641	132,473	0.339	1.076	-5.30	-3.76	31.80	-9.5
1975	49,948	116,207	165,975	144,865	0.344	1.146	1.47	+6.51	2.60	9.6

Source: Internal Monetary Fund, Financial Statistics, Various issues.

[1] Includes time and savings deposits of deposit money banks and "other financial institutions."
[a] Change in data series with a slight downward adjustment in totals.
[b] Discontinuous data with a slight downward adjustment in totals.

Table III-5

Financial Structure of West Germany, 1953-73
Monetary and GNP Data in Billions of Current Deutsche Marks

	M1 Demand Deposits Plus Currency (1)	Time and Savings Dep. (2)	M2 Total (1+2) (3)	GNP Gross Nat'l Product (4)	Ratio of M1/GNP (1÷4) (5)	Ratio of M2/GNP (3÷4) (6)	Annual %Change M1/GNP (7)	Annual %Change M2/GNP (8)	Annual %Change W.P.I. 1963=100 (9)	Annual %Change Real GNP (10)
1953	23.4	16.5	40.1	134.3[a]	0.174	0.297	--	--	--	--
1954	26.5	21.0	47.5	145.4[a]	0.182	0.327	4.59	10.10	-1.0	9.38
1955	29.2	25.3	54.5	180.4	0.162	0.302	-10.99	7.64	1.0	22.80
1956	31.3	29.8	61.1	198.8	0.157	0.307	- 3.09	1.66	2.1	8.00
1957	35.1	37.0	72.1	216.3	0.162	0.333	3.18	8.47	1.0	7.75
1958	39.7	43.7	83.4	231.5	0.171	0.360	5.56	8.11	0.0	6.67
1959	44.4	52.6	97.0	250.9	0.177	0.387	3.51	7.50	-1.0	9.81
1960	47.4	60.4	107.8	296.8	0.160	0.363	- 9.60	-6.67	1.0	17.14
1961	54.5	67.3	121.8	326.2	0.167	0.373	3.95	2.58	2.1	7.82
1962	58.0	76.6	134.6	354.5	0.167	0.380	0.0	1.93	1.0	7.65
1963	62.2	88.6	150.8	377.6	0.165	0.399	- 1.20	5.09	0.0	6.52
1964	67.6	102.5	170.1	413.7	0.163	0.411	- 1.21	3.01	2.0	7.53
1965	72.7	119.8	192.5	460.4	0.157	0.418	- 3.68	1.70	2.0	9.27
1966	74.0	139.7	213.7	490.7	0.151	0.436	- 3.82	4.21	1.9	4.47
1967	81.6	161.6	243.2	494.7	0.165	0.492	9.27	12.84	0.9	1.73
1968	88.4	196.2	284.6	540.0	0.163	0.527	- 1.20	7.10	-5.7[b]	15.39
1969	93.7	223.8	317.5	605.2	0.155	0.525	- 4.90	-0.40	2.0[b]	9.98
1970	102.7	250.3	353.0	685.7	0.149	0.515	- 3.90	-1.90	6.9[b]	6.34
1971	115.8	291.0	406.8	761.9	0.152	0.534	2.00	3.69	4.6[b]	6.53
1972	131.9	337.4	469.3	833.9	0.158	0.562	3.95	5.24	2.6[b]	6.67
1973	132.9	390.0	522.9	927.5	0.143	0.564	- 9.49	0.35	6.6[b]	4.30
1974	149.1	423.4	572.5	997.0	0.150	0.574	4.90	1.77	13.4	-5.19
1975	169.9	480.3	650.2	1,043.6	0.163	0.623	8.67	8.54	4.7[b]	-0.01

Source: International Monetary Fund, Financial Statistics, Various Issues.

[a]Downward biased GNP series due to data discontinuity.
[b]Discontinuity with value added tax excluded from calculations of wholesale price index.

Table III-6

Ratio of Time and Savings Deposits to Demand Deposits and Currency (M1) in Five Countries 1953-1970

	Semi-Industrial Less Developed Countries			Industrial Countries	
Year	Argentina	Brazil	Chile	Japan	Germany
1953	0.348	0.177	n.a.	1.040	0.705
1954	0.346	0.165	n.a.	1.263	0.792
1955	0.327	0.135	0.172	1.314	0.741
1956	0.408	0.115	0.192	1.414	0.952
1957	0.361	0.110	0.230	1.688	1.050
1958	0.345	0.093	0.243	1.843	1.100
1959	0.271	0.077	0.551	1.950	1.180
1960	0.284	0.082	0.549	2.022	1.274
1961	0.317	0.064	0.613	2.110	1.235
1962	0.360	0.042	0.699	2.334	1.321
1963	0.407	0.038	0.614	2.012	1.424
1964	0.416	0.033	0.638	2.068	1.516
1965	0.444	0.029	0.534	2.032	1.648
1966	0.428	0.073	0.588	2.100	1.888
1967	0.426	0.093	0.663	2.142	1.983
1968	0.471	0.127	0.713	2.214	2.219
1969	0.507	0.136	0.809	2.183	2.387
1970	0.554	0.139	0.653	2.202	2.437

Source: International Monetary Fund, Financial Statistics, Various Issues.

CHAPTER IV

A THEORETICAL ANALYSIS OF THE ROLE OF MONEY IN ECONOMIC GROWTH

IV.1: Introduction

In Chapter III, we constructed a money growth model based on the same assumptions James Tobin used to construct his money growth model. The major implication derived from this neo-classical growth model is that the long-run equilibrium rate of capital accumulation in a monetary economy is always below the rate of capital accumulation which can be sustained in a non-monetary economy. This conclusion, however, seems logically inconsistent. If the sole result of introducing money into an economy were to reduce the level of capital accumulation and hence the level of per capita output and consumption, why would it be introduced?[108]

This paradox can be explained by examining the manner in which Tobin introduces money into his growth model.

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[108] Don Patinkin and David Levhari, "The Role of Money in a Simple Growth Model," Reprinted in Don Patinkin, Studies in Monetary Economics (New York: Harper and Row, Pub., 1972): 208.

The major path through which money affects the workings of an economy in Tobin's model is through its effect on real disposable income. Tobin defines real disposable income as

$$Y^d = Y + \frac{d(^M/_P)}{dt}$$

$$= Y + M/P\ (\mu - \pi)$$

where

Y^d = real disposable income

Y = real net national income

M = quantity of money supplied in the economy

P = the price level

$\mu = \frac{\dot{M}}{M}$ = the rate of change of the money supply

$\pi = \frac{\dot{P}}{P}$ = the rate of change of the price level

In defining disposable income in this manner, Tobin allows changes in real money balances (d(M/P)dt) to affect the level of disposable income in the economy. This is his major contribution to monetary growth theory. However, although Tobin does posit a positive demand for money in his model, the model does not really provide a rationale for holding these balances. To do so, one may interpret money balances either as a consumer's good with the non-pecuniary services money provides to its holders in an individual's utility function, or as a producer's good with the services provided by money reflected in a production function.[109] Tobin did not accurately incorporate either of these approaches in his model. The production function included in his model is the same as that used in Robert Solow's barter economy growth model, and the definition of disposable income in Tobin's model only includes the actual increase in the real value of cash balances (d(M/P)/dt) but not the imputed value of their liquidity services.[110] Because of these limitations, Tobin's money growth model does not provide an accurate theoretical framework for analyzing a monetary economy. In this chapter we examine the money growth models developed by Don Patinkin and David Levhari in their article, "The Role of Money in a Simple Growth Model." The theoretical implications derived from these models are compared with those derived from Tobin's money growth model.[111] In addition, two structural production function models are developed in the final section of this chapter which will be utilized to determine whether it is valid to include a real money variable as a factor input in a production function.

IV.2: Money as a Consumer's Good

In their work in monetary growth theory, Don Patinkin and David Levhari attempt to make Tobin's money growth

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[109] Ibid., p. 209.

[110] Robert M. Solow, "A Contribution to the Theory of Economic Growth," Quarterly Journal of Economics 70 (1956): 65-94.

[111] Patinkin and Levhari, Op. cit., pp. 195-242.

model more consistent with the workings of a monetary economy by developing two alternative specifications of his model; one which treats money as a consumer's good and one which introduces money as a producer's good. In developing the first, they merely alter Tobin's model by including the imputed services from real money balances in the definition of disposable income. Hence in their model disposable income is defined as

$$\begin{aligned} Y^d &= Y + M/P(\mu-\pi) + M/P(r+\pi) \\ &= Y + M/P(\mu+r) \end{aligned} \qquad (18)$$

where

Y^d = real disposable income

Y = real net national income

M = the quantity of money supplied in the economy

P = the price level

$\mu = \frac{\dot{M}}{M}$ (the rate of change of the money supply)

$\pi = \frac{\dot{P}}{P}$ (the rate of change of the price level)

$r+\pi$ = the money rate of interest

Different from Tobin's definition of disposable income (equation (17)), Patinkin and Levhari include the term $(M/P(r+\mu))$ in their definition to account for the imputed services money provides consumers. They point out that the decision to hold cash balances implies that, at the margin, the services provided by these cash balances must be valued at the alternative cost of holding these balances (i.e. the opportunity cost). In their model, as in Tobin's, there are only two assets--physical capital and real money balances. The rates of return on these assets are $r = \partial F(K,L)/\partial K$ and, by the assumption of zero storage costs, $-\pi$ respectively, where r is the real rate of interest and $-\pi$ is the rate of decrease in the price level.[112]

An individual will determine the optimum composition of his portfolio by considering the anticipated real rates of return on these two assets. For simplicity, however, Patinkin and Levhari assume that the actual and anticipated rates of return are equal. The anticipated alternative cost of holding cash balances is equal to the difference between what could have been earned from holding a unit of physical capital and what will be earned by holding real money balances. This cost is $r - (-\pi) = r + \pi$, or Fisher's money rate of interest. Accordingly, since the decision to hold cash balances implies that the value of the services derived from these balances must be at least equal to the opportunity cost of holding them, Patinkin and Levhari argue that the imputed value of these services can be depicted as $M/P(r+\pi)$. Hence, in their model, real disposable income is defined as net national income (Y) plus the real value of the increase in the nominal quantity

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[112] Ibid., p. 207.

of money ($\mu(M/P)$) plus the imputed real interest on real money balances ($r(M/P)$). Patinkin and Levhari's definition differs from Tobin's (equation (17)) in that the decrease in the real value of money balances caused by a price increase ($\pi(M/P)$) does not appear in the equation since it is offset by the fact that ($\pi(M/P)$) is also part of the imputed income from holding these cash balances. By introducing the imputed income from holding cash balances into Tobin's model, Patinkin and Levhari show that the conclusion that the rate of capital accumulation in a monetary economy is lower than the rate of capital accumulation in a non-monetary economy no longer holds.

IV.3: Money as a Producer's Good

In addition to developing a model which includes money as a consumer's good, Patinkin and Levhari also attempt to incorporate the productivity gains derived from money into their growth model by including a real money variable as a factor input in the production function. The inclusion of this additional factor input is the only way in which this model differs from Tobin's money growth model. Patinkin and Levhari retain all of the other assumptions made by Tobin when reconstructing his growth model. They do so not because they accept these assumptions as being valid but because they are interested in analyzing the manner in which the inclusion of a real money variable in the production function alters the implications which can be derived from Tobin's money growth model. In the remainder of this chapter we will restrict our attention to this latter model where money is treated as a producer's good.

Money Economy:

Output:

Assume a linear homogeneous production function.

$$(19) \quad Q_t = G(K_t, L_t, (M/P)_t)$$

where

Q_t = output in time t

K_t = capital in time t

L_t = labor in time t

$(M/P)_t$ = real money balances in time t

Reduce both sides of (19) to a per capita basis to determine the output-labor ratio.

$$Q_t/L_t = G(K_t/L_t,\ 1,\ (M/P)_t/L_t)$$

$$(20) \quad q_t = g(k_t, m_t)$$

where

$q_t = Q_t/L_t$

$k_t = K_t/L_t$

$m_t = (M/P)_t/L_t$

Labor Market:

We again assume that the effective supply of labor in the economy grows at a constant rate, n, and that the labor market is always in equilibrium. Mathematically we can express this in the following manner.

(21) $L_t = L_o e^{nt}$

where

n = the rate at which the labor force is growing

Taking logs of both sides of (21) and differentiating with respect to time we get:

(22) $(1/L_t)(dL_t/dt) = n$

$\dot{L}_t/L_t = n =$ natural rate of growth

Savings Function:

Assume that the level of savings is a constant proportion (s) of real output and increases in the real value of money, d(M/P)/dt. To express this mathematically we again assume that the demand for money is a constant fraction of income, (s_m). Hence the savings function can be written

(23) $s[G(K_t,L_t,(M/P)_t) + s_m(d(G(K_t,L_t,(M/P)_t))/dt)]$

Again reduce both sides of (23) to a per-capita basis to determine the savings-labor ratio.

(24) $S_t/L_t = s[g(k_t,m_t) + s_m(d(g(k_t,m_t))/dt)]$

Investment Function:

Assume that savings always equals investment. Also assume, following James Tobin, that savings in the form of real money holdings are only additions to government debt which will not stimulate any real capital accumulation.[113]

$$(25) \quad I_t = s[G(K_t, L_t, (M/P)_t) + s_m(d(G(K_t, L_t, (M/P)_t))/dt)] - s_m[d(G(K_t, L_t, (M/P)_t))/dt]$$

Written in a per-capita basis this becomes

$$(26) \quad I_t/L_t = \dot{K}_t/L_t = s[g(k_t, m_t) + s_m(d(g(k_t, m_t))/dt)] - s_m[d(g(k_t, m_t))/dt]$$

To determine the long-run steady-state equilibrium conditions derived from this model we assume that the price level grows at a constant rate π, and that the nominal quantity of money grows at a constant rate μ, along with our assumption that the effective supply of labor grows at a constant rate, n. We define the steady-state growth path of the system as one in which both the per capita physical capital stock (k_t) and per capita real money balances (m_t) are constant. We must also assume that such a path exists and that the system converges to it.

These assumptions can be expressed mathematically by performing the following manipulations:

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[113] James Tobin, "Money and Economic Growth," Econometrica 33 (October, 1965):671-684.

By definition

$$k_t \equiv K_t/L_t$$
$$m_t \equiv (M/P)_t/L_t$$

Take the logs of both sides of these identies and differentiate with respect to time.

$$(27) \quad a. \quad \dot{k}_t/k_t = \dot{K}_t/K_t - \dot{L}_t/L_t$$

$$b. \quad \dot{m}_t/m_t = \dot{M}_t/M_t - \dot{P}_t/P_t - \dot{L}_t/L_t$$

Divide both the numerator and the denominator of $\dot{K}_t/K_t$ by L_t.

$$(28) \quad a. \quad \dot{k}_t/k_t = \frac{\dot{K}_t/L_t}{K_t/L_t} - \dot{L}_t/L_t$$

$$\dot{k}_t/k_t = \{\frac{s[g(k_t,m_t)+s_m(d(g(k_t,m_t))/dt)]-s_m[d(g(k_t,m_t))/dt]}{k_t}\} - n$$

$$b. \quad \dot{m}_t/m_t = (\mu-\pi-n)$$

Now, long-run equilibrium in the capital and money markets requires that

$$\dot{k}_t/k_t \; \& \; \dot{m}_t/m_t = 0.$$

Hence, the conditions necessary for steady-state equilibrium are

$$(29) \quad a. \quad s[g(k_t,m_t) + s_m(d(g(k_t,m_t))/dt)] - s_m[d(g(k_t,m_t))/dt] = nk_t$$

$$b. \quad \mu - \pi = n$$

Thus, both total capital and total real money balances expand at a constant rate, n, along the growth path.

Of these two equations, (29a and 29b), 29a is particularly interesting since the implications which can be derived from this long-run equilibrium condition are different from the implications derived from the comparable capital market equilibrium conditions generated from James Tobin's money growth model, (equation 16 in Chapter III).

$$(16) \quad (s + s(s_m n) - s_m n) \cdot f(k_t) = nk_t$$

By factoring the production function $g(k_t,m_t)$ from equation (29a) and rearranging terms, equation (29a) can be rewritten as:[114]

$$(29a)' \quad (s + s(s_m n) - s_m n) \cdot g(k_t,m_t) = nk_t$$

The only difference between these two equations (16 and 29a') is the functional form of the production function in

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[114]We obtain this equation (29a)' by making the following mathematical manipulations.

$$(29)a. \quad s[g(k_t,m_t) + s_m(d(g(k_t,m_t))/dt)] - s_m[d(g(k_t,m_t))/dt] = nk_t$$

Multiply the second and third terms on the left hand side of this equation by $g(k_t,m_t)/g(k_t,m_t)$. Then (29)a. can be written

$$s[g(k_t,m_t) + g(k_t,m_t) \cdot s_m(d(g(k_t,m_t))/dt/g(k_t,m_t)]$$

$$- g(k_t,m_t) \cdot s_m[d(g(k_t,m_t))/dt/g(k_t,m_t)] = nk_t$$

$$g(k_t,m_t) \cdot (s + s(s_m n) - s_m n) = nk_t$$

these respective equations.[115] The definition of the physical-saving propensity ($s_p = (s + s(s_m n) - s_m n)$) used in both of these two models is equivalent.[116]

In Chapter III, we pointed out that by assuming that savings in the form of money <u>will</u> <u>not</u> stimulate any real capital accumulation, Tobin derives a capital market

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[115]Patinkin and Levhari constructed their money growth model so that the only difference between their model and Tobin's model would be the functional form of the production function.

[116]This definition of the physical-saving propensity can also be derived by dividing each term in the investment functions of these two money growth models by the production function used in these respective growth models (equations 14 and 25).

$$(14)' \quad I_t/F(K_t,L_t) = s[F(K_t,L_t)/F(K_t,L_t) + s_m(d(F(K_t,L_t))/dt)/F(K_t,L_t)] - [s_m(d(F(K_t,L_t))/dt)/F(K_t,L_t)]$$

$$s_p = s(1 + s_m n) - s_m n$$
$$= (s + s(s_m n) - s_m n)$$

$$(25)' \quad I_t/G(K_t,L_t,(M/P)_t) = s[G(K_t,L_t,(M/P)_t)/G(K_t,L_t,(M/P)_t) + s_m(d(G(K_t,L_t,(M/P)_t))/dt)/G(K_t,L_t,(M/P)_t)] - [s_m(d(G(K_t,L_t,(M/P)_t))/dt)/G(K_t,L_t,(M/P)_t)]$$

$$s_p = s(1 + s_m n) - s_m n$$
$$= (s + s(s_m n) - s_m n)$$

equilibrium condition (represented by equation 16) which indicates that although total saving is greater in a monetary economy than a non-monetary economy since savings depend upon the level of real output and increases in the real value of money, the amount of physical savings will always be lower in a monetary economy if any saving is held in the form of money, (i.e. $s_p < s$). This is because $s < 1$, $s(s_m n) < (s_m n)$ and consequently $(s + s(s_m n) - s_m n) < s$. Hence in Tobin's model, real capital accumulation will always be lower in a monetary economy than a non-monetary economy if there is a positive demand for money (i.e. $s_m > 0$). This implication, however, cannot be unambiguously derived from the Patinkin/Levhari money growth model. By simply altering Tobin's model to include real money balances as a factor input in the production function included in this model, Patinkin and Levhari show that the rather paradoxical implication of Tobin's growth model that real per capita capital accumulation is always lower in a monetary economy than in a non-monetary economy cannot be unambiguously derived.

The capital market equilibrium condition derived from the Patinkin/Levhari model is:

$$(29a)' \quad (s + s(s_m n) - s_m n) \cdot g(k_t, m_t) = nk_t$$

The production function used to derive this equation is G(K,L,M/P) rather than F(K,L). The real money variable is included in the production function to capture the productivity gains derived from engaging in monetary rather than barter exchange. If we assume that the productivity gains from monetary exchange are positive, this functional form implies that for each combination of physical capital and labor, the level of output generated in a monetary economy will be higher than the level of output generated in a non-monetary economy. Similarly, the real rate of interest, real wage rate, and output per capita in a monetary economy are determined on a superior frontier of production possibilities. By including real money as a factor input in the production function the positive income effect from the growth in real money is introduced into the growth model.

Although the definition of the physical-saving propensity ($s_p = s + s(s_m n) - s_m n$) in equation (29a)' is equivalent to the physical-saving propensity in Tobin's model (equation 15), by introducing real money balances as a productive factor input into this money growth model, the implication derived about the long-run equilibrium condition in the capital market is altered. If we assume that the marginal productivity of money is positive ($g'(m)>0$) when examining equation (29a), this implies that each increase in real money per capita increases the flow of physical savings per capita to the capital market. Hence, for each steady-state growth rate (n) in output and the effective supply of labor, a higher capital-labor ratio (k) is possible. The productivity gains derived from using money as a medium of exchange, store of value, and unit of account, raise the capital-labor and consumption-labor ratios in this model along the golden-rule growth path. This does not necessarily imply that the long-run equilibrium capital-labor ratio (k) in the economy will actually be higher in a monetary economy (within the context of the neo-classical money growth model developed in this chapter). The assumption that savings in the form of money will not contribute to real capital accumulation is also included in the Patinkin/Levhari growth model. Real money holdings and real capital accumulation are viewed as strict substitutes rather than complementary assets. Hence any growth in real money will deprive the capital market of savings.

By considering increases in the real value of money as part of disposable income, part of the reduction in physical savings due to this substitution effect is restored. However, as long as the demand for money is positive, the level of real capital accumulation is still lower in a monetary economy than a non-monetary economy if we assume that money affects the workings of the economy only through its effect on real disposable income. It is only by further introducing real money as a factor of production that Patinkin and Levhari alter the conclusion that the flow of savings to the capital market in a monetary economy is lower than the physical savings flow in a non-monetary economy. Once the productivity gains from holding money are taken into consideration, the net result on total savings flows to the capital market is ambiguous. It depends upon the strength of the positive income and savings effects from increases in the real value of money relative to the negative substitution effect from holding money rather than physical capital. If the increase in physical savings from the income and savings effects more than compensates for the reduction in physical savings due to the substitution effect, the capital-labor ratio (k) will be higher in a monetary economy. However, even if the income and savings effects do not completely compensate for the negative substitution effect between real money and physical capital, the productivity gains from using money will raise the best, or golden-rule, capital-labor ratio and consumption-labor ratio in the economy. Finally, even if the steady-state level of capital intensity (k) is lower in a monetary economy, we can argue that the level of per capita output produced in this steady-state equilibrium will always be greater than or equal to the corresponding output in a non-monetary economy. This is because firms always have the option to carrying out production without the use of money.[117]

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[117]Don Patinkin and David Levhari make this point in their article, Op. cit., p. 230.

IV.4: An Extension of the Patinkin/Levhari Money Growth Model

By comparing Tobin's money growth model with the Patinkin/Levhari model, which differs from the former only in its inclusion of real money balances as a variable in the production function, we find that the conclusion that the level of capital intensity in a monetary economy is always lower than that in a non-monetary economy can no longer be unambiguously derived. There is, however, some disagreement over whether it is theoretically valid to include a real money variable as a factor input in a production function. To examine this theoretical question more thoroughly, we develop two structural models which provide a framework for investigating whether it is valid to include a real money variable as a factor input in an aggregate production function. Following Patinkin and Levhari, we assume that all real money balances are held by the business sector of the economy.[118] By this we mean that money is held only because it enables the economic unit in question to acquire or produce a larger quantity of commodities. Hence, for convenience, we assume that money holdings, per se, do not generate any utility. These models are constructed using two different production function specifications,

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[118]Available information of the proportion of gross demand deposits which is held by business indicates that this assumption is not invalid. Data provided by the Federal Reserve on the ownership of gross demand deposits shows that between the end of 1972 and the end of 1976 businesses held an average of 68 percent of outstanding

a Cobb-Douglas and a translog functional form. The models are empirically estimated to determine which of these two functions provides a higher degree of explanatory power.

The Cobb-Douglas Model

Let us hypothesize that the production function, G(K,L,M/P), introduced in Patinkin and Levhari's money growth model has the form

$$(30) \quad q = Ae^{\lambda T}L^{\alpha}K^{B}m^{\gamma}$$

where

q = output

L = labor

K = capital

m = real money balances

T = time trend, proxy for technological change

A = efficiency parameter

λ = a technological parameter

α = elasticity of output w.r.t. labor

β = elasticity of output w.r.t. capital

γ = elasticity of output w.r.t. real money balances

This function can be interpreted as reflecting the assumption that just as the level of output produced in the economy depends upon the quantity of fixed capital and the supply of labor, it also depends upon the quantity of working capital.[119] Since real money balances can be considered

demand deposits. See Federal Reserve Bulletin, December, 1977, p. A-25. The assumption that all real money balances are held by the business sector is carried over to the empirical work. We feel this assumption is valid since even the cash balances held by consumers can be considered part of the production process since these cash holdings facilitate exchange and in this sense they *indirectly* contribute to the level of production in an economy.

[119]Patinkin and Levhari, *Op. cit.*, p. 228.

an important component in the supply of working capital, it can theoretically be considered like any other inventory which enters into the production process. Hence, we want to view the decision to hold money by the economic units operating in the system in the same manner as we view the decision to demand other factor inputs. We accomplish this by assuming that the demand for all three inputs included in the production function (equation (30)) is determined by marginal considerations. We assume that firms employ factor inputs in their production processes up to the point where the cost of employing another unit of this factor input is equal to the marginal product of doing so. These assumptions can be incorporated into our model by differentiating the production function with respect to each input; capital, labor, and real money balances.

$$(31) \qquad \frac{\partial q}{\partial L} = \alpha A e^{\lambda T} L^{\alpha - 1} K^{\beta} m^{\gamma} = \alpha q / L$$

$$(32) \qquad \frac{\partial q}{\partial K} = \beta A e^{\lambda T} L^{\alpha} K^{\beta - 1} m^{\gamma} = \beta q / K$$

$$(33) \qquad \frac{\partial q}{\partial m} = \gamma A e^{\lambda T} L^{\alpha} K^{\beta} m^{\gamma - 1} = \gamma q / m$$

If we assume that markets are competitive, a set of necessary conditions for efficient production is that the price of each input is equal to the value of the marginal product of this input. In our model there are three inputs; X_i = 1,2,3 referring to capital, labor and real money balances respectively. Hence, we can rewrite this assumption as:

$$(34) \quad P_i = (\partial q/\partial X_i)\cdot(P_q)$$

where

$X_i = K, L, m$

P_i = price of the ith input

P_q = price of output

If we further assume that firms are profit maximizers, we can combine equation (34) with equations (31), (32), and (33) respectively to determine the demand for the three factor inputs in the model.

$$(35) \quad P_L = (\alpha q/L)P_q$$

$$(36) \quad P_K = (\beta q/K)P_q$$

$$(37) \quad P_m = (\gamma q/m)P_q$$

Rearranging terms, these three equations can be rewritten as:

$$(35)' \quad (P_L L)/(P_q q) = \alpha$$

$$(36)' \quad (P_K K)/(P_q q) = \beta$$

$$(37)' \quad (P_m m)/(P_q q) = \gamma$$

Written in this form the equations indicate that the cost shares of the inputs in total cost are equal to the elasticity of output with respect to each input. We will refer to these three equations as "decision equations" since if firms are profit maximizers (cost minimizers) they operate as if they have utilized these equilibrium conditions to determine the quantity of capital, labor, and real money balances they will use in their production processes. These three decision equations (equations (35)', (36)', and (37)'), together with the Cobb-Douglas production function (equation (30)) written in log linear form, form one of the production models which we estimate to determine whether it is valid to include a real money balance variable as factor input in a production function. Hence this structural model consists of the following four equations.

(35)' $(P_L L)/(P_q q) = \alpha$

(36)' $(P_K K)/(P_q q) = \beta$

(37)' $(P_m m)/(P_q q) = \gamma$

(38)' $\ln q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln m + \lambda T$

The Translog Model

In addition to the above Cobb-Douglas model, we also hypothesize that the production function, G(K,L,M/P), included in Patinkin and Levhari's money growth model has the functional form of the translog production function.[120]

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[120] The translog production function reduces the Cobb-Douglas production function if $\gamma_{ij} = \gamma_{iT} = \gamma_{TT} = 0$. Hence, the Cobb-Douglas function is a special case of the more generalized translog functional form. In this sense, the Cobb-Douglas production function is more restrictive than the translog function.

The translog production function is written

$$(39) \quad \ln q = \ln\alpha_0 + \alpha_T \ln T + \sum_{i=1}^{n} \alpha_i \ln X_i + 1/2\ (\gamma_{TT}(\ln T)^2) + 1/2 \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln X_i \ln X_j + \sum_{i=1}^{n} \gamma_{iT} \ln X_i \ln T$$

where

q = output

X_i = flow of input services (K,L,(M/P)=m)

T = technology index

$\alpha_0, \alpha_T, \alpha_i, \gamma_{ij}$ are technological parameters and we assume that $\gamma_{ij} = \gamma_{ji}$.

The translog production function is an arbitrary second-order approximation to any continuous production function. In this sense, its functional form is less restrictive than the Cobb-Douglas specification. By including the term $1/2 \sum_i \sum_j \gamma_{ij} \ln X_i \ln X_j$ the translog specification allows for some interaction between the factor inputs included in the production function. The rationale for specifying the interaction terms in this form is not because one expects the inputs to interact in this specific form but merely to allow for some form of interaction. In essence, the translog function is a generalization of the Cobb-Douglas functional form; generalized to allow for factor input interactions. The interaction terms are introduced into the translog function in the same simplistic manner in which the inputs themselves are introduced into the Cobb-Douglas function. In the Cobb-Douglas function the inputs are introduced in log linear form and in the translog function the interaction terms are included as the cross products of the logs of the inputs. This is the simplest way of allowing for some interaction to occur between the factor inputs.

Since the translog specification is a more generalized functional form than the Cobb-Douglas function it seems valuable to also investigate whether it is valid to include a real money variable as a factor input in this alternative specification. However, in order to facilitate empirical estimation of this function form and to enable a valid comparison of the empirical results obtained from the translog function with those generated from the Cobb-Douglas form, certain constraints are imposed on the general form of the translog production function (equation 39).

If we assume that this logarithmic production function exhibits constant returns to scale (CRTS) the following relationship holds.

$$\ln q(\lambda X_1,\ldots,\lambda X_n,T) = \ln q(X_1,\ldots,X_n,T) + \ln\lambda$$

This assumption places the following restrictions on the translog parameters.

$$\text{(A)}_1 \quad \sum_i \alpha_i = 1 \quad \sum_i \gamma_{ij} = 0 \quad \sum_j \gamma_{ij} = 0 \quad \sum_{ij}\sum \gamma_{ij} = 0 \quad \sum_i \gamma_{iT} = 0$$

Proof: (showing that these parameter restrictions $(A)_1$ are sufficient conditions for CRTS)

$$\ln q(\lambda X_1,\ldots,\lambda X_n,T) = [\ln\alpha_0 + \alpha_T \ln T + \sum_i \alpha_i \ln\lambda X_i + 1/2\ \gamma_{TT}(\ln T)^2$$

$$+ 1/2 \sum_{ij}\sum \gamma_{ij}(\ln\lambda X_i)(\ln\lambda X_j) + \sum_i \gamma_{iT}(\ln\lambda X_i)\ln T]$$

$$= [\ln\alpha_0 + \alpha_T \ln T + \sum_i \alpha_i(\ln\lambda + \ln X_i) + 1/2\ \gamma_{TT}(\ln T)^2$$

$$+ 1/2 \sum_{ij}\sum \gamma_{ij}(\ln\lambda + \ln X_i)(\ln\lambda + \ln X_j) + \sum_i \gamma_{iT}\ (\ln\lambda + \ln X_i)(\ln T)]$$

$$= [\ln\alpha_0 + \alpha_T \ln T + \sum_i \alpha_i \ln X_i + \sum_i \alpha_i \ln\lambda + 1/2\ \gamma_{TT}(\ln T)^2$$

$$+ 1/2 \sum_{ij}\sum \gamma_{ij}(\ln X_i \ln X_j) + 1/2 \sum_{ij}\sum \gamma_{ij}\ln X_i \ln\lambda + 1/2 \sum_{ij}\sum \gamma_{ij}\ln X_j \ln\lambda$$

$$+ 1/2 \sum_{ij}\sum \gamma_{ij}\ln\lambda^2 + \sum_i \gamma_{iT}\ln T + \sum_i \gamma_{iT}\ln\lambda \ln T]$$

If $\sum_i \alpha_i = 1$, $\sum_i \gamma_{iT} = 0$, $\sum_{ij}\sum \gamma_{ij} = 0$ this equation reduces to

$$\ln q(\lambda X_1,\ldots,\lambda X_n, T) = \ln\alpha_0 + \alpha_T \ln T + \sum_i \alpha_i \ln X_i + \ln\lambda + 1/2\ \gamma_{TT}(\ln T)^2$$

$$+ 1/2 \sum_{ij}\sum \gamma_{ij}(\ln X_i \ln X_j) + 1/2 \sum_{ij}\sum \gamma_{ij}\ln X_i \ln\lambda$$

$$+ 1/2 \sum_{ij}\sum \gamma_{ij}(\ln X_j \ln\lambda) + \sum_i \gamma_{iT}\ln X_i \ln T$$

Now if we assume that $\gamma_{ij} = \gamma_{ji}$ and further specify that

$$\sum_i \gamma_{ij} = 0, \quad \sum_j \gamma_{ij} = 0 \quad \text{then}$$

$$[1/2 \sum_{ij}\sum \gamma_{ij}\ln X_i \ln\lambda + 1/2 \sum_{ij}\sum \gamma_{ij}\ln X_j] = \ln\lambda \sum_{ij}\sum \gamma_{ij}\ln X_i$$

since $1/2 \sum_{ij}\sum \gamma_{ij}\ln X_j$ can be written $1/2 \sum_{ji}\sum \gamma_{ji}\ln X_i$

$$= 1/2 \sum_{ij}\sum \gamma_{ij}\ln X_i$$

Expanding the term $\ln\lambda \sum_{ij}\sum\gamma_{ij}\ln X_i$ we get

$$(\gamma_{11}\ln X_1 + \gamma_{12}\ln X_2 + \ldots + \gamma_{1n}\ln X_n) + (\gamma_{21}\ln X_1$$

$$+ \ldots + \gamma_{2n}\ln X_n) + \ldots + (\gamma_{n1}\ln X_1$$

$$+ \gamma_{n2}\ln X_2 + \ldots + \gamma_{nn}\ln X_n)$$

$$= \ln X_1 \sum_j\gamma_{ij} + \ln X_2 \sum_j\gamma_{2j} + \ldots + \ln X_n \sum_j\gamma_{nj} = 0$$

Hence, $\ln q(\lambda X_1,\ldots,\lambda X_n,T) = (\ln\alpha_o + \alpha_T\ln T + \sum_i\alpha_i\ln X_i + 1/2\gamma_{TT}(\ln T)^2$

$$+ 1/2 \sum_{ij}\sum\gamma_{ij}(\ln X_i\ln X_j) + \sum_i\gamma_{iT}\ln X_i\ln T) + \ln\lambda$$

$$= \ln q(X_1,\ldots,X_n,T) + \ln\lambda \qquad \text{Q.E.D.}$$

If we further assume that the translog production function is Hicks-neutral with respect to technological change (HNTC) then $\ln q(X_1,\ldots,X_n,T) = \ln q(X_1,\ldots,X_n) + \ln T$. This places three additional parameter restrictions on the translog production function.

$$(A)_2 \quad \alpha_T = 1 \quad \alpha_{TT} = 0 \quad \gamma_{iT} = 0$$

Proof: (showing that these parameter restrictions are sufficient conditions for HNTC)

$$\ln q(X_1,\ldots,X_n,T) = \ln\alpha_o + \alpha_T\ln T + \sum_i\alpha_i\ln X_i + 1/2(\gamma_{TT}(\ln T)^2)$$

$$+ 1/2 \sum_{ij}\sum\gamma_{ij}\ln X_i\ln X_j + \sum_i\gamma_{iT}\ln X_i\ln T$$

With the above restrictions $(A)_2$ this equation reduces to

$$\ln q(X_1,\ldots X_n,T) = (\ln\alpha_o + \sum_i \alpha_i \ln X_i + 1/2 \sum_{ij}\sum \gamma_{ij} \ln X_i \ln X_j)$$

$$+ \ln T = \ln q(X_1,\ldots,X_n) + \ln T \qquad \text{Q.E.D.}$$

Hence, with CRTS and HNTC parameter restrictions imposed, the translog production function can be written in the following, more simplified, form:[121]

$$(40) \qquad \ln q = \ln\alpha_o + \ln T + \sum_{i=1}^{n} \alpha_i \ln X_i + 1/2 \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln X_i \ln X_j$$

where

$$X_i = K, L, M/P{=}m$$

Although equation 40 is the functional form which the translog production function takes when CRTS and HNTC constraints are imposed, in order to provide a valid comparison between this translog production function and a Cobb-Douglas function the manner in which technological change is included in equation 40 must be altered. When devising the translog functional form, E.R. Berndt and L.R. Christensen viewed variable T in equation 40 as a technology index which would measure the rate of technological change and they hypothesized that changes in output would be directly proportion to changes in technology.[122] However, for our empirical work, an index

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[121]This is the functional form E.R. Berndt and L.R. Christersen specified in their article, "The Translog Function and the Substitution of Equipment, Structures, and Labor in U.S. Manufacturing 1928-1968," Journal of Econometrics 1 (March, 1973): 83-84.

[122]Ibid.

which measures the rate of technological change is not available. The variable T in our specification is a time trend used as a proxy for technological change. Although time is positively correlated with technological change it does not provide an <u>exact</u> measurement of this phenomenon. Hence, when estimating the translog function with a time trend included as a proxy for a technological variable, the parameter restriction included in equation 40 which constrains the coefficient of variable T to equal 1 should not be imposed. If this constraint were imposed and a time trend composed of consecutive numbers beginning with the number one were used, the functional form of equation 40 would imply that when moving from year 1 to year 2 output will double. To avoid this bias and to facilitate a valid comparison between empirical results obtained by estimating both a translog and a Cobb-Douglas production function, the parameter constraint on variable T will not be imposed. We will introduce technological change into the translog functional form in the same manner as it is introduced in the Cobb-Douglas form. Hence equation 40 is altered to become

$$(41) \quad \ln q = \ln\alpha_o + \lambda T + \sum_{i=1}^{n} \alpha_i \ln X_i + 1/2 \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln X_i \ln X_j$$

where

q = output

X_i = K,L,M/P=m

T = time trend, proxy for technological change

α_o, α_t, λ, γ_{ij} are technological parameters and we assume that $\gamma_{ij} = \gamma_{ji}$

To develop a more complete production function model using this specification, we again draw from the conditions of profit maximizing behavior in competitive markets to specify our system of equations. The marginal product and logarithmic marginal product of each input in the translog production function are respectively

$$(42) \quad f_i = \partial q/\partial X_i = q/X_i(\alpha_i + \sum_j \gamma_{ij} \ln X_j)$$

$$(43) \quad \partial \ln q/\partial \ln X_i = \alpha_i + \sum_j \gamma_{ij} \ln X_j$$

By multiplying both sides of equation (42) by X_i/q we can also obtain an equation for the output elasticity of the ith input.

$$(44) \quad (\partial q/\partial X_i)\cdot(X_i/q) = (\alpha_i + \sum_j \gamma_{ij} \ln X_j)$$

Again assuming that markets are competitive, a set of necessary conditions for efficient production is

$$(45) \quad P_i = (\partial q/\partial X_i)P_q$$

where

X_i = K,L,M/P=m

P_i = price of the ith input

P_q = price of output

Substituting equation (42) into equation (45) and dividing by P_q we get

$$(46) \quad P_i/P_q = q/X_i(\alpha_i + \sum_j \gamma_{ij} \ln X_j)$$

$$P_i \cdot X_i/P_q \cdot q = (\alpha_i + \sum_j \gamma_{ij} \ln X_j)$$

Hence by assuming profit maximizing behavior in competitive markets we can maintain that the relative cost share of the ith input to total cost (of all three inputs) is equal to the output elasticity of the ith input. From this we can derive the three following profit maximizing equations.

$$(47) \quad M_L = P_L \cdot X_L/P_q \cdot q = \alpha_1 + \gamma_{11} \ln X_L + \gamma_{12} \ln X_K + \gamma_{13} \ln X_m$$

$$(48) \quad M_K = P_K \cdot X_K/P_q \cdot q = \alpha_2 + \gamma_{12} \ln X_L + \gamma_{22} \ln X_K + \gamma_{23} \ln X_m$$

$$(49) \quad M_m = P_m \cdot X_m/P_q \cdot q = \alpha_3 + \gamma_{13} \ln X_L + \gamma_{23} \ln X_K + \gamma_{33} \ln X_m$$

These three equations together with the simplified translog production function

$$(50) \quad \ln q = \ln \alpha_o + \lambda T + \sum_{i=1}^{3} \alpha_i \ln X_i + 1/2 \sum_{i=1}^{3} \sum_{j=1}^{3} \gamma_{ij} \ln X_i \ln X_j$$

make up the alternative, translog production model which will be estimated to determine whether a real money variable belongs in a production function.

CHAPTER V

AN EMPIRICAL INVESTIGATION OF MONEY AS A FACTOR INPUT

V.1: Introduction

In a 1970 article, Jerome Stein stated that "equally plausible (monetary growth) models yield fundamentally different results. ...The crucial question is: which is the correct monetary growth model?"[123] Disagreeing with this view, we have outlined theoretical arguments which favor those models which attempt to account for the productivity gains derived from holding money. In this chapter, empirical evidence on the question of whether it is valid to include a real money variable as a factor input in a production function will be provided. This evidence can be used to determine the appropriate specification of the aggregate production function included in the existing neo-classical money growth models. We will present a brief description of the previous empirical work on this subject and will then analyze the empirical results obtained from estimating the Cobb-Douglas and translog production function models developed in the preceding chapter. A discussion of the analytical techniques utilized to obtain these results will be presented in the appendix to this chapter.

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[123]Jerome L. Stein, "Monetary Growth Theory in Perspective," American Economic Review (March, 1970):85.

V.2: Summary of the Previous Empirical Work on Real Money Balances As a Factor of Production

In August 1972, Allen Sinai and Houston H. Stokes presented the first direct empirical investigation of the theoretical hypothesis that real money balances are a factor of production in their article, "Real Money Balances: An Omitted Variable From the Production Function?"[124] In this article they tested the significance of real money balances as a factor input in a Cobb-Douglas production function. The authors found that over the period 1929-1967, real money balances, regardless of definition (M_1,M_2,M_3), entered significantly in this function fitted to annual data on the private domestic sector of the United States economy. Their results stimulated several other authors to re-estimate this production function using various econometric techniques.[125]

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[124] Allen Sinai and Houston H. Stokes, "Real Money Balances: An Omitted Variable from the Production Function," Review of Economics and Statistics (August, 1972):290-296.

[125] Five notes on the Sinai and Stokes' piece were published in the May, 1975 issue of Review of Economics and Statistics. Alberto Niccoli, "Real Money Balances: An Omitted Variable from the Production Function? A Comment," pp. 241-243; Zmira Prais, "Real Money Balances as a Variable in the Production Function," pp. 243-244; Mohsin S. Khan and Pentti J.K. Kouri, "Real Money Balances as a Factor of Production: A Comment," pp. 244-246; Uri Ben-Zion and Vernon W. Ruttan, "Money in the Production Function: An Interpretation of Empirical Results," pp. 246-247; Allen Sinai and Houston H. Stokes, "Real Money Balances: An Omitted Variable from the Production Function?--A Reply,"* pp. 247-252.

Their work also provided the basis for much of the empirical work presented in this dissertation. For this reason, a detailed review of the Sinai and Stokes' 1972 article will be developed in this section along with a discussion of some of the subsequent empirical work on this topic.

Sinai and Stokes were interested in determining whether a real money variable should be included as a factor input in an aggregate production function.[126] To test this hypothesis, they estimated a Cobb-Douglas production function with non-constant returns to scale and neutral technological change.

$$\text{(V.1)} \qquad Q = Ae^{\lambda T}L^{\alpha}K^{\beta}m^{\gamma}u$$

where

Q = output

L = labor

K = capital

m = real money balances

T = time, proxy for technological change

A = efficiency parameter

u = disturbance term

λ = rate of disembodied technological change

α = elasticity of output w.r.t. labor

β = elasticity of output w.r.t. capital

γ = elasticity of output w.r.t. real money balances

The Cobb-Douglas specification was chosen since Sinai and Stokes felt that in an exploratory study it seemed "more appropriate to begin with a relatively simple and widely used

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[126] Sinai and Stokes, Op. cit., p. 290.

function."[127] They also argued that the relatively simple Cobb-Douglas function was appropriate for their work since the "correct" aggregate production function specification for the U.S. economy was still uncertain despite numerous empirical investigations on this subject.[128] The primary purpose of their study was to examine the potential significance of real money balances in the production function and they did not want to become involved in difficult specification problems.[129]

Sinai and Stokes estimated equation (V.1) in log linear form

$$\text{(V.2)} \quad \ln Q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln m + \lambda T + u'$$

They used the ordinary least squares (OLS) regression technique except in cases where the residuals were serially correlated, in which case an approximate generalized least squares (GLS) technique was used to correct for autocorrelation.[130]

Data for output, labor, and capital were taken from Christensen and Jorgenson's 1970 study, "U.S. Real Product

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[127] Ibid., p. 291.

[128] Ibid.

[129] Ibid.

[130] Ibid., p. 291-292.

and Real Factor Input, 1929-67."[131] In this study Christensen and Jorgenson constructed Divisia quantity indexes of gross private domestic product, labor, and capital services which were adjusted for quality changes and rates of utilization.

Data on nominal money balances were taken from Friedman and Schwartz (1970, pp. 24-72).[132] Conventional definitions of M_1, and M_2 were employed along with a measure of M_3 defined as M_2 plus deposits at Mutual Savings Banks and Postal Savings Systems.[133] These nominal money stock figures were deflated by a Divisia index of factor prices again constructed by Christensen and Jorgenson in 1970.[134] Sinai and Stokes used this factor price index, as opposed to a consumer price index, in order to get a measure of real purchasing power over factor inputs. They were interested in this latter measure since they viewed real money balances as affecting productivity by facilitating the exchange necessary to obtain factor inputs.[135]

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[131] L.R. Christensen and D.W. Jorgenson, "U.S. Real Product and Real Factor Input, 1929-1967," Review of Income and Wealth, series 16 (March, 1970): 19-50.

[132] Milton Friedman and A.J. Schwartz, Monetary Statistics of the United States (New York: Columbia University Press, 1970): 24-72.

[133] Ibid.

[134] L.R. Christensen and D.W. Jorgenson, Op. cit., p. 38.

[135] Sinai and Stokes, Op. cit., p. 292.

In their article, Sinai and Stokes report parameter estimates of the Cobb-Douglas production function both with and without real money balances and a time trend. These estimates are presented in Tables V-1 and V-2 included at the end of this chapter.

The parameter estimates presented in Table V-1 with real money balances (eqs. V.4,5,6) and with a time trend (eq. V.7), indicate that the real balance coefficients along with the coefficients of labor and capital were all positive and significantly different from zero at both a 95 percent and 99 percent confidence interval.[136] Similarly, when the production function was estimated without real money balances but with a time trend as a proxy for technological change, the proxy was also statistically significant.

The sum of the parameter estimates for capital, labor, and real money balances provides an estimate of the returns to scale exhibited by the production function. All of the equations (V.3-V.7) exhibited increasing returns to scale varying from 1.429 (eq. V.7) to 1.817 (eq. V.6). Sinai and Stokes point out that the real money variables account for about 10 to 15 percent of the estimated returns to scale. They also suggest that the high degree of increasing returns to scale shown in equations V.3-V.6, may have been due to the

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[136] Unless otherwise stated, we will be using a 95 percent confidence interval to determine whether the reported coefficients are statistically significant.

omission of an appropriate technology variable which would bias the regression coefficients upward. To account for this, they re-estimated equations V.4-V.6 with a time trend included to serve as a proxy for neutral technological change. (See Table V.2)

The parameter estimates obtained when a time trend was included along with a real money variable have the same sign as those presented in Table V-1. However, the magnitude of the coefficients of capital and real money all fell when estimated with this trend while the labor coefficient increased when estimated with m_1 and m_2 from 0.945 to 0.966 (eqs. V.4 and V.5) and from 1.092 to 1.100 (eqs. V.5 and V.9, respectively) and decreased from 1.195 (eq. V.6) to 1.174 (eq. V.10) when estimated with m_3 and a time trend. The estimated coefficients of labor, capital, m_1, and m_2, reported in Table V-2 were still statistically significant. However, the time trend was not significant when included with m_1 and m_2 (eqs. V.8 and V.9) and, in equation V.10, the m_3 coefficient was not significantly different from zero although the trend was. The production function still exhibited increasing returns to scale when estimated with a time trend although the addition of this trend did result in a decrease in returns to scale. When estimated without a time trend the production function exhibited returns to scale which ranged from 1.7-1.8 (eqs. V.4-V.6). With a trend, this range fell to 1.2-1.6 (eqs. V.8-V.10).

In analyzing these empirical results Sinai and Stokes conclude that real money balances, whether defined as m_1, m_2, m_3 are an important input in the production function, having a significant, independent effect on productivity.[137] This conclusion, however, is somewhat misleading since the coefficient of m_3 was not significantly different from zero when included in the production function along with a trend variable. (Sinai and Stokes do not explicitly mention this point.) The authors also suggest that when estimated without real money balances, the trend variable, included to represent neutral technological progress, may actually have been a proxy for real money balances since the trend was not significantly different from zero when included with a real money variable (eqs. V.8-V.9).[138]

The results presented in the Sinai and Stokes' study provided the stimulus for several other empirical studies.[139] Although their study is somewhat limited in that no attempt is made to correct for simultaneous equation bias, the article did make an important contribution by providing some preliminary evidence on the appropriate specification for neo-classical money growth models. In particular, as Sinai and Stokes point out, their results provide evidence which favors the Patinkin/Levhari money growth model.[140]

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[137] Sinai and Stokes, Op. cit., pp. 294-295.

[138] Ibid., p. 294.

[139] See footnote 130.

[140] Sinai and Stokes, Op. cit., p. 295.

In response to the Sinai and Stokes 1972 article, four notes were published in the May 1975 issue of Review of Economics and Statistics along with a reply by Sinai and Stokes.[141] The criticisms presented in two of these notes, one by Zmira Prais and one by Mohsin S. Khan and Pentti J.K. Kouri, were particularly helpful when formulating the empirical investigation of the productivity of money undertaken in this study. For this reason, a fairly brief review of these comments will be presented before reporting empirical results.

As previously mentioned, the possibility of simultaneous equation bias constituted a potentially important limitation of the Sinai and Stokes' estimation. The authors recognized this problem but did not attempt to correct for it.[142] Zmira Prais notes this and points out that when testing the significance of money in a production function, estimates should be made within a production framework which incorporates other behavioral functions, i.e., a simultaneous system of equations should be estimated.[143] She, however, does not develop this type of model. Instead, in her note and subsequent article, Prais primarily criticizes Sinai and Stokes

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[141]See footnote 130.

[142]Sinai and Stokes, Op. cit., p. 292.

[143]Zmira Prais, "Real Money Balances as a Variable in the Production Function,"* Journal of Money, Credit, and Banking (November, 1976): 540.

for "misusing econometric techniques."[144] She was dismayed by the fact that they followed a "fashionable trend"[145] of reporting results which were obtained after utilizing an autocorrelation correction technique.[146]

Prais is correct to argue that a low Durbin-Watson statistic may indicate that the functional form being estimated is misspecified. However, it can also indicate a serial correlation problem. Consequently, it seems unjustified to criticize Sinai and Stokes for reporting a statistically significant correlation between real money balances and output which was generated after correcting for what may, in fact, be autocorrelation.

In her own estimation, Prais estimates a Cobb-Douglas production function which included a lagged real money variable, (m_{t-1}), in addition to a current real money stock variable, (m_t). The other variables included in her model are the same as those defined in the Sinai and Stokes model (see page 100).

$$\text{(V.11)} \quad \ln Q_t = \ln A + \alpha \ln L_t + \beta \ln K_t + \gamma_1 \ln m_t + \gamma_2 \ln m_{t-1} + u_t$$

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[144] Zmira Prais, Op. cit., p. 243.

[145] Zmira Prais, Op. cit.*, p. 536.

[146] The Durbin Watson statistic indicated that the disturbance terms in the OLS regression equations estimated by Sinai and Stokes were autocorrelated. Sinai and Stokes, Op. cit., pp. 291-292.

The parameter estimates of equation V.11 show that both of these real money variables are significantly correlated with real output (95 percent confidence level); See Table V-3. However, when interpreting these results Prais claims that from a theoretical point of view it is difficult to regard changes in the stock of money as a factor input and argues that as a result this functional form cannot be viewed as a pure production function. Rather, she views it as a "mongrel relationship among macro variables."[147] From this, Prais incorrectly jumps to the conclusion that her results also discredit Sinai and Stokes' findings and claims that her results show that "there is no evidence of the effect of the level of current money stock on output."[148] This conclusion is obviously based on faulty reasoning. Prais is correct to argue that the static Cobb-Douglas production function Sinai and Stokes estimate may be misspecified. She cannot, however, criticize their results by using a model which she, herself, views as a "mongrel relationship" (i.e. she believes that the functional form she has estimated is misspecified). Alternatively, one would gain more useful information by re-estimating the production function using a theoretically reasonable structural framework. Khan and Kouri attempt to do this.[149]

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[147]Zmira Prais, Op. cit.*, p. 539.

[148]Ibid., p. 535.

[149]Mohsin S. Khan and Pentti J.K. Kouri, Op. cit., pp. 244-246.

Khan and Kouri constructed a simultaneous equation model (equations V.12 and V.13) which determines output and real money balances as a function of capital, labor, and the rate of interest. Using this model, they re-estimate a Cobb-Douglas production function along with a simple demand for money function and attempt to correct for the possibility of simultaneous equation bias in the model estimated by Sinai and Stokes.[150]

$$\text{(V.12)} \quad \ln Q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln M + u'$$

$$\text{(V.13)} \quad \ln M = \ln B + a \ln Q + b \ln R + w$$

$$a > 0; \; b < 0$$

where

Q = output   M = real money balances

L = labor   R = long-term rate of interest

K = capital

This model was estimated with annual data for the U.S. economy for the period 1937-1967. Full-information maximum likelihood estimation (FIML) was used. Since they were attempting to correct for any simultaneous equation bias in Sinai and Stokes' model, they used the same Christensen and Jorgenson (1970) and Friedman and Schwartz (1970) data.[151]

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[150] Ibid., p. 245.

[151] For a discussion of the data used to estimate this model see Ibid., p. 245.

They also used a long-term interest rate series from the Historical Statistics of the United States.[152]

The results reported by Khan and Kouri are consistent with those reported by Sinai and Stokes although the Khan and Kouri estimates for labor are lower than those obtained by Sinai and Stokes while their estimates for capital and real money balances are larger. (See Table V-4). In their interpretation of these estimated coefficients, Khan and Kouri conclude that taking the simultaneity bias into account did not invalidate the results obtained by Sinai and Stokes although the estimated size of the elasticity of output with respect to M_1, M_2, and M_3 (0.363, 0.929, 1.896 respectively) seemed too high to justify on theoretical grounds.[153] These three estimates of γ, generated by the FIML method of estimation, are substantially higher than those obtained by Sinai and Stokes. The Sinai and Stokes' estimates of γ are 0.172, 0.214, 0.194 for M_1, M_2, and M_3 respectively in equations V.4-V.6 when a time trend was not included and 0.127, 0.133, 0.087 when a trend variable was included (equations V.8-V.10).

The magnitude of the Khan and Kouri estimates does seem somewhat large to justify on theoretical grounds. However, these results may be biased due to misspecification

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[152] Ibid.

[153] Ibid.

error. Khan and Kouri were correct to assume that real money balances are simultaneously correlated with real output since changes in real output will affect the level of real cash balances in an economy. This assumption is based on the hypothesis that the demand for money is a positive function of changes in the level of real output (income) in the economy. If real output increases, the demand for money will increase causing the value of real money in the economy to increase (other things equal). However, the productivity gains from increases in the real value of money may also have a positive effect on real output. Hence, real money balances are simultaneously correlated with real output. This assumption is carried over to our own empirical work but, unlike Khan and Kouri, we also assume that when estimating an aggregate production capital and labor should also be considered endogenous variables. In their model, Khan and Kouri consider capital and labor to be exogenous variables. However, when using aggregate data, it seems more appropriate to also view the quantity of labor and capital services employed as endogenous variables since producers' decisions concerning stocks and employment utilization rates are often affected by the level of output in the economy.[154] Consequently, rather than developing a

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[154]Sinai and Stokes make this point in Op. cit.*, p. 250. Similarly, Erndt R. Berndt and Laurits R. Christensen also make this point in their article, "The Translog Function and the Substitution of Equipment, Structures, and Labor in U.S. Manufacturing 1928-68," Journal of Econometrics 1 (March, 1973):85.

simultaneous equation model which merely includes a demand for money equation along with the production function, the model should also contain factor demand equations for labor and capital. (Sinai and Stokes also make this point in their reply to Khan and Kouri).[155]

A second, related problem is that Khan and Kouri used full-information maximum likelihood estimation even though they did not use all the information available to them when specifying their model. Finally, Khan and Kouri do not include a time trend in their production function as a proxy for disembodied technological change. The omission of this trend variable and the failure to specify a structural model which incorporates factor demand equations for capital, labor, and real money balances may be responsible for the large estimated coefficients on money balances and the correspondingly high marginal productivities which Khan and Kouri viewed as being "somewhat difficult to accept on economic grounds."[156]

Sinai and Stokes also criticize Khan and Kouri for omitting a trend variable in their production function.[157] Their criticism, however, seems somewhat inappropriate since when

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[155]Sinai and Stokes, Op. cit.*, p. 250.

[156]Khan and Kouri, Op. cit., p. 245.

[157]Sinai and Stokes, Op. cit.*, p. 250.

interpreting their own results, Sinai and Stokes conclude that this trend variable was merely a proxy for real money balances. In the Sinai and Stokes' estimation the trend variable was significant when a real money variable was not included in the production function but statistically insignificant when real balances were included.[158] Hence, Khan and Kouri may have based their production function specification on the results reported by Sinai and Stokes.

V.3: A Re-estimation of a Cobb-Douglas Production Function Including Real Money Balances as a Factor of Production

In order to directly compare our empirical results with those reported by Sinai and Stokes, a Cobb-Douglas production function was re-estimated for the period 1929-1967 using the same annual data as Sinai and Stokes used. A time trend was included to serve as a proxy for technological change. This proxy generally did not appear statistically significant when the production function was estimated as a single equation. It was significant when estimated in the four equation structural model developed in the previous chapter. (These latter results will be presented in Section V-4). The trend was included along with a real money variable in the single equation model in order to reduce the possibility of misspecification

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[158]Sinai and Stokes, _Op. cit_., p. 294.

error.[159] Hence a Cobb-Douglas production function with non-constant returns to scale and neutral technological change was re-estimated in log linear form.

$$\text{(V.2)} \quad \ln Q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln M + \lambda T + u$$

The results from this estimation are reported in Tables V-5 through V-8 at the end of this chapter.

In Table V-5, ordinary least squares parameter estimates are reported. The results indicate that the capital and labor quantity indices are consistently significantly correlated with real output although the technology proxy (T) does not appear significant and only M_1 of the three definitions of real money balances (M_1,M_2,M_3), registers a significant statistical correlation. The R^2 reported for each equation is .9958, .9957, and .9955 respectively. However, the Durbin Watson statistic suggests that the disturbance terms in each regression are autocorrelated. Although the OLS estimates are unbiased, the precise form of the t and F statistics cannot be derived with the least-squares regression technique

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[159]The time trend and real money balances variables are highly colinear. The correlation coefficient between time and M_1,M_2,M_3 respectively is 0.68502, 0.8891, and 0.9146. This multicollinearity problem made it difficult to disentangle the relative influences of the technology proxy and real money balances on output. However, since the trend was significant when estimated in the Cobb-Douglas structural model developed in Chapter 4 (pages 86-89) we felt that it should be included in our single equation estimation even though it did not appear significantly different from zero in these latter regressions. We were concerned that if it were omitted, the coefficient of the real money variable might pick up the effects of technological change on output and hence would be biased upward.

since the sampling variances of the regression coefficients are not valid when the disturbance terms are autocorrelated.[160] To account for this problem, the Cobb-Douglas production function was re-estimated using the Cochrane-Orcutt iterative technique to correct for autocorrelation.[161] These results are reported in Table V-6.

After correcting for autocorrelation, the estimated coefficients for all of the three definitions of real money balances (M_1,M_2,M_3) were positive and significantly correlated with real output. The $\hat{\gamma}$ parameter estimates for M_1, M_2, M_3 respectively rose from 0.099 to 0.145, 0.128 to 0.144, and from 0.082 to 0.109. The elasticity of output with respect to labor, $\hat{\alpha}$, fell from 1.138 to 0.969 when included with M_1 (equations V.14 and V.17), it fell from 1.214 to 1.096 when included with M_2 (equations V.15 and V.18), and from 1.302 to 1.147 when included with M_3, (equations V.16 and V.19). Similarly, the elasticity of output with respect to capital, $\hat{\beta}$, fell from 0.463 to 0.377 when included with M_1, from 0.386 to 0.298 when included with M_2, and from 0.338 to 0.259 when included with M_3. In addition, the trend variable was significantly

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[160] See J. Johnston, Econometric Methods, 2nd edition, (New York: McGraw-Hill Book Co., 1972):246-248.

[161] For a discussion of this regression technique see D. Cochrane and G.H. Orcutt, "Application of Least-Squares Regressions to Relationships Containing Auto-correlated Error Terms," Journal of the American Statistical Association, 44 (1949):32-61.

different from zero when included with M_1 and M_3 (equations V.17 and V.19) although it was insignificant when included with M_2 (equation V.18). The main difference between the OLS estimates and those obtained after correcting for autocorrelation was that the labor and capital coefficients decreased in magnitude while all the real money coefficients increased.

These results can be utilized to obtain an estimate of the marginal product of each of the three inputs included in this production function. Differentiating the Cobb-Douglas production function with respect to capital, labor, and real money balances we get the following three equations:

$$q = Ae^{\lambda T} L^{\alpha} K^{\beta} m^{\gamma}$$

$$\text{(a)} \quad \partial q/\partial L = Ae^{\lambda T} L^{\alpha-1} K^{\beta} m^{\gamma} = \alpha(q/L)$$

$$\text{(b)} \quad \partial q/\partial K = Ae^{\lambda T} L^{\alpha} K^{\beta-1} m^{\gamma} = \beta(q/K)$$

$$\text{(c)} \quad \partial q/\partial m = Ae^{\lambda T} L^{\alpha} K^{\beta} m^{\gamma-1} = \gamma(q/m)$$

Using the estimated coefficients of capital, labor, and real money balances $(\hat{\beta},\hat{\alpha},\hat{\gamma})$ we can impute a value of the marginal physical product of each of these three inputs. Marginal physical products of 0.189, 0.171, and 0.124 are implied by the regression coefficients for M_1, M_2, and M_3 respectively. Similarly, by averaging the estimated coefficients derived in equations V.17, V.18, and V.19, marginal physical products of 0.318 and 0.960 can be imputed for capital and labor. These values are calculated at the mean value of the real gross private domestic product, labor, capital, and real money balances.[162] The marginal product of real money balances is the increased output obtained as a consequence of increases in real balances which release additional labor and capital services for utilization in production instead of distribution. The marginal product of real balances implied by the ml coefficient, 0.145, is 0.189. This means that a dollar's increase in purchasing power over labor and capital is associated with about a two-tenths unit increase in real output for the private sector. This magnitude does not seem unreasonable and although it is small, it seems high enough to suggest a return to holding money balances that is consistent with observed firm behavior.

The results reported in Table V-6 coincide fairly closely with those reported by Sinai and Stokes. Real money balances, regardless of definition, enter significantly in a Cobb-Douglas production function fitted to annual private domestic sector data for the United States over the period 1929-1967. Similarly, the estimated Cobb-Douglas production function exhibited increasing returns to scale ranging from 1.49 to 1.51 (returns to scale reported by Sinai and Stokes ranged from 1.5 to 1.6). However, the relatively large elasticities

[162]Data for real gross private domestic product, labor, and capital are taken from Christensen and Jorgenson (1970). Nominal money balances (M_1,M_2,M_3) are taken from Friedman and

of output reported in Tables V-5 and V-6 may have been the result of a simultaneous equation bias.[163] If capital, labor, and/or real money balances are, themselves, a positive function of the level of real output (income), the estimates of α, β, γ, generated by ordinary least squares and by the one-stage Cochrane-Orcutt iterative technqiue will be biased and inconsistent. To account for this problem, the Cobb-Douglas production function was re-estimated using instrumental variables. These estimates will be biased but consistent. They will not be asympotically efficient.

In order to correct the parameter estimates reported in Tables V-5 and V-6 for the possibility of simultaneous equation bias, we followed Berndt and Christensen in assuming that the following variables are exogenous to the U.S. manufacturing sector: (1) U.S. population, (2) U.S. population of working age, (3) effective rate of sales and excise taxation, (4) effective rate of property taxation, (5) government purchases of durable goods, (6) government purchases of non-durable goods and services, (7) government purchases of labor services, (8) real exports of durable goods, and (9) real exports of non-durable goods and services.[164] In our two-stage

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Schwartz (1970) pp. 24-72. These balances are deflated by Christensen and Jorgenson's factor price index (1970) to obtain a measure of real money balances over the period 1929-1967.

[163]Sinai and Stokes also make this point in a footnote to their article, Op. cit., p. 294.

[164]E.R. Berndt and L.R. Christensen, Op. cit., pp. 93-94.

estimation of the Cobb-Douglas production function we used the above mentioned variables to generate instrumental variables. In addition, since real money balances are included in our estimated production function, we also assumed that the real stock of money, (m_1, m_2, m_3), lagged four periods (annual data) was an exogenous variable which could be used to generate instruments.[165] Hence, we used these twelve variables to estimate the single equation Cobb-Douglas production function using 2SLS.

In Table V-7, two-stage least squares parameter estimates are reported. We assumed that capital, labor, and real money balances are endogenous variables and that the twelve variables described above plus a time trend and constant term are exogenously determined variables. These exogenous variables are used to generate instrumental variables to purge the quantities of capital, labor, and real money balances of

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[165] We assume that the real quantity of money lagged four periods is an exogenous variable for the following reasons. We assume that the nominal supply of money in year t is an exogenous variable determined by the monetary authorities but that the real quantity of money in year t is endogenous. This is because the real quantity of money in year t is determined by the interaction of the demand for and supply of money in year t. Since the demand for money in year t is a positive function of output (income) in year t, the real quantity of money in the year t will also be affected by the level of output (income) in that year. Following the same line of reasoning, we assume that the real quantity of money lagged four periods (t - 4) is affected by the level of output in (t - 4) but is not a function of current output in year t. Hence it seems reasonable to assume that $m_1(-4)$, $m_2(-4)$, and $m_3(-4)$ are exogenous variables.

correlation with the disturbance term in the estimated equations. In essence, this is accomplished by regressing each of the endogenous variables in the production function (capital, labor, and real money balances) against the exogenous variables described above, retrieving the OLS predicted values of the endogenous variables from these regressions and using these predicted values to estimate the Cobb-Douglas production function. Although the Cobb-Douglas production function is over-identified since the number of predetermined variables is greater than the number of endogenous variables, the production function can be estimated.

The results obtained after taking the simultaneity problem into consideration are not significantly different from the OLS and Cochrane-Orcutt results. The parameter estimates before correcting for autocorrelation (Table V-7) are similar to the corresponding coefficients reported in Table V-5. Coefficients of labor and capital are positive and significantly related to real output while the time trend and the real money coefficients are positive but insignificant. It is interesting to note the two-stage estimates of the elasticities of output with respect to labor ($\hat{\alpha}$) are greater in magnitude than those reported in Table V-5 even though one would expect them to fall after correcting for simultaneous equation bias. Conversely, the two-stage estimates of the elasticities of output with respect to capital and real money balances ($\hat{\beta}, \hat{\gamma}$) are smaller than the OLS estimates.

Again, the R^2 statistic for each equation (V.20, V.21, V.22) is high, and the Durbin-Watson statistic indicates that the disturbance term is autocorrelated in each equation.

To correct for autocorrelation we again assumed that the error terms were first-order serially correlated. All the equations (V.20, V.21, and V.22) were re-estimated using a two-stage Cochrane-Orcutt interative technique. The results are reported in Table V-8. The final estimate of $\hat{\rho}$ used to correct these equations was 0.5607, 0.5950, and 0.6173 respectively. After correcting for autocorrelation, the coefficients of labor, capital, and real money balances were all positive and significantly related to real output. The trend variable remained positive but was not statistically significant.

The magnitude of the two-stage least squares estimates reported in Table V-8 did change compared to the one-stage Cochrane-Orcutt estimates reported in Table V-6. The direction of the change was as expected for two of the three labor coefficients; it decreased from 0.969 (equation V.17) to 0.773 (equation V.23) and from 1.097 (equation V.18) to 1.038 (equation V.24) when included with M_1 and M_2 but increased from 1.147 (equation V.19) to 1.174 (equation V.25) when included with m_3. These changes, however, are not statistically significant (i.e. the coefficients reported in Tables V-6 and V-8 respectively are not significantly different from one another.)[166]

Unlike the decrease in magnitude obtained when $\hat{\alpha}$ (elasticity of output with respect to labor) was estimated using a two-stage Cochrane-Orcutt iterative technique, the two-stage estimates of $\hat{\beta}$ and $\hat{\gamma}$ reported in Table V-8 all increased in magnitude compared to the one-stage Cochrane-Orcutt estimates (Table V-6). The elasticity of output with respect to capital ($\hat{\beta}$) increased from 0.377 (equation V.17) to 0.564 (equation V.23), from 0.298 (equation V.18) to 0.430 (equation V.24), and from 0.259 (equation V.19) to 0.364 (equation V.25) when included with m_1, m_2, and m_3 respectively. Similarly, the elasticity of output with respect to m_1, m_2, and m_3 ($\underline{\hat{\gamma}}$) increased from 0.145 (equation V.17) to 0.251 (equation V.23), from 0.144 (equation V.18) to 0.268 (equation V.24), and from 0.109 (equation V.19) to 0.209 (equation V.25). Again, however, none of these coefficients are statistically different from one another.[167]

Our results indicate that the simultaneous equation bias possibly present in the one-stage OLS and Cochrane-Orcutt estimates did not have a significant impact on the estimated parameters. After correcting for this bias, M_1, M_2, and M_3 are still positive and significantly correlated with real output. The magnitude of these two-stage estimates

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[166]We used a t-test to test the hypothesis that the OLS estimate of $\underline{\hat{\alpha}}$ was different from the TSLS estimate of $\underline{\hat{\alpha}}$ and rejected this hypothesis at a 99 percent confidence level.

[167]_Ibid_.

are fairly close to those reported by Sinai and Stokes (Table V-2) although they are significantly smaller than those reported by Khan and Kouri (Table V-4).

Khan and Kouri estimated the elasticity of output with respect to real money balances as 0.363 for M_1, 0.929 for M_2, and 1.896 for M_3. Conversely, our two-stage Cochrane-Orcutt estimates of these three coefficients are respectively 0.251, 0.268, and 0.209. Using these latter estimates, the imputed marginal product of money calculated at the mean value of real output and real money balances (m_1, m_2, m_3) is 0.388, 0.328, 0.245 respectively. Unlike the Khan and Kouri results, these values do not seem difficult to accept on economic grounds. Hence, our re-estimation of the Cobb-Douglas production function supports the hypothesis that real money balances are a productive asset which have a significant impact on the level of real output in the U.S. economy.

Our results also indicate that the Patinkin/Levhari money growth model provides a more accurate depiction of the workings of a monetary economy than do alternative models which fail to account for the productivity of money. These results, however, must be considered preliminary findings since in this single equation production function estimation no attempt is made to incorporate behavioral functions which describe the manner in which firms utilize factor inputs in their production processes. Although the 2SLS estimates discussed in this section do not entirely ignore the structure of production processes, the single equation estimates do not incorporate all of the information available to us. To account for this problem, the more complete structural models developed in Chapter Four are also estimated.

V.4: Estimating the Productivity of Money in a Structural Model Based on a Cobb-Douglas Production Function

In this section empirical results on the productivity of money obtained from estimating a structural model derived from a Cobb-Douglas production function are reported. The model developed in the preceding chapter, consists of a Cobb-Douglas production function along with three behavioral decision equations. These latter three equations attempt to incorporate the manner in which firms utilize factor inputs in their production processes.

Cobb-Douglas Model:

Although the Cobb-Douglas structural model was developed in Chapter Four, it will be briefly reconstructed here in order to facilitate the interpretation of the parameter estimates. The production function is written

$$\text{(V.26)} \qquad q = Ae^{\lambda T}L^{\alpha}K^{\beta}m^{\gamma}e$$

where

q = output

L = labor

K = capital

m = real money balances

T = time (proxy for neutral technological progress)

A = efficiency parameter

λ = a technological parameter

α = elasticity of output w.r.t. labor

β = elasticity of output w.r.t. capital

γ = elasticity of output w.r.t. real money balances

e = disturbance term

Differentiating q with respect to each factor input (K,L,m), assuming that markets are competitive, and rearranging terms, we derive the following three decision equations.

$$\text{(V.27)} \quad (P_L \cdot L/(P_q \cdot q) = \alpha$$

$$\text{(V.28)} \quad (P_K \cdot K)/(P_q \cdot q) = \beta$$

$$\text{(V.29)} \quad (P_m \cdot m)/(P_q \cdot q) = \gamma$$

where

P_L = price of labor

P_K = price of capital

P_m = price of money

P_q = price of output

The marginal relationships depicted in equations V.27, V.28, and V.29 will hold only if all firms are perfect maximizers. Since we expect some errors in cost minimizing behavior these equations should be considered stochastic.

Hence the equations should also include a classical disturbance term.

$$(V.27)' \quad (P_L \cdot L)/(P_q \cdot q) = \alpha + \xi_1$$

$$(V.28)' \quad (P_K \cdot K)/(P_q \cdot q) = \beta + \xi_2$$

$$(V.29)' \quad (P_m \cdot m)/(P_q \cdot q) = \gamma + \xi_3$$

To test whether real money balances are a productive asset which should be included as a factor of production in an aggregate production function, these stochastic equations are estimated along with the Cobb-Douglas production function in log linear form.

$$(V.30)' \quad \ln q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln m + \lambda T + \xi_4$$

Since the parameter estimates $(\hat{\alpha}, \hat{\beta}, \hat{\gamma})$ will generally not be equal when estimating equations V.27' through V.30' separately, the appropriate estimation procedure is to perform a constrained four equation multi-variate regression by minimizing the trace of the residual covariance matrix, i.e. by estimating a 'stacked' regression. This stacking procedure is explained in detail in the appendix to this chapter.

Data:

The data required to estimate this model are the cost of the inputs included in the production function; $((P_L \cdot L)$, $(P_K \cdot K)$, $(P_m \cdot m))$, the value of total output $(P_q \cdot q)$, the quantity of capital (K), labor (L), and real money balances

(M/P) employed, and the quantity of output (q) produced. For all of these variables we use annual data over the period 1929-1967 for the private domestic sector of the U.S. economy.

The real cost of capital services ($P_K \cdot K$) and labor services ($P_L \cdot L$) is imputed by deflating data on current dollar input costs of capital and labor services provided by E.R. Berndt and L.R. Christensen (1973)[168] by Christensen and Jorgenson's Divisia Factor Input Price Index (1970).[169] The real cost of using real money balances (m_1, m_2, m_3) are imputed by deflating nominal balances (M_1, M_2, M_3) provided by Friedman and Schwartz (1970) by Christenson and Jorgenson's Divisia Factor Input Price Index (1970) and multiplying these deflated values by a money market rate of interest, computed at an annual basis (annual rates of four to six months prime commercial paper).[170] Following Patinkin and Levhari, we view this money market rate as a close approximation of the opportunity cost of holding real money balances.[171]

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[168] E.R. Berndt and L.R. Christensen, Op. cit., p. 108.

[169] L.R. Christensen and D.W. Jorgensen, Op. cit., p. 38.

[170] Historical Statistics of the United States: Colonial Times to 1970 (Bicentennial Edition), part 2, (U.S. Department of Commerce Bureau of the Census, 1975):1001.

[171] Patinkin and Levhari, "The Role of Money in a Simple Growth Model," reprinted in Studies in Monetary Economics by Don Patinkin (New York: Harper and Row Pub., 1972):209. It should be noted that it would have been more accurate to use a different price of money for each definition of money included in the estimated equations since the opportunity cost of holding M_1, M_2, and M_3 will differ since interest is paid on time deposits and savings and loan deposits while M_1 is a non-interest bearing asset. We could have created a price of

Finally, following Berndt and Christensen, we assume that the total value of output $(P_q \cdot q)$ is equal to the sum of the input costs.[172] This cost will vary with the definition of real money balances included in the estimation. When using m_1, the total cost of inputs is computed by summing the three input costs $((P_L \cdot L) + (P_K \cdot K) + (P_m \cdot m_1))$. However, when using m_2, or m_3 the real cost of capital and labor remains unchanged but the cost of money becomes $(P_m \cdot m_2)$ and $(P_m \cdot m_3)$ respectively. Hence, the total cost, $(P_q \cdot q) = ((P_L \cdot L) + (P_K \cdot K) + (P_m \cdot m_{i=1,2,3}))$, will vary with the definition of real money balances used in our regressions. In subsequent discussion, we will refer to this total cost as $TOTCST_1 = ((P_L \cdot L) + (P_K \cdot K) + (P_m \cdot m_1))$, $TOTCST_2 = ((P_L \cdot L) + (P_K \cdot K) + (P_m \cdot m_2))$, and $TOTCST_3 = ((P_L \cdot L) + (P_K \cdot K) + (P_m \cdot m_3))$; i.e. $TOTCST_1$ is associated with m_1, $TOTCST_2$ with m_2, and $TOTCST_3$ with m_3.

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money index which would differentiate between the interest-bearing and non-interest-bearing portion of M_2 and M_3. However, although the price of M_2 and M_3 used in our two models does not represent the "true" cost of holding these monetary assets, the bias introduced by this measurement error will be small. Since the correlation among interest rates is very high, we can expect the ratio of the variance in measurement error to the variance in the "true" cost of money to be low and hence the bias introduced will be inconsequential. For a more detailed discussion of the errors in variables problem see J. Johnston, Econometric Methods, 2nd edition, (New York: McGraw-Hill Book Co., 1963): 181-191.

[172] E.R. Berndt and L.R. Christensen, Op. cit., p. 85.

In addition to these cost variables, data on the quantity of capital (K) and labor (L) are Divisia quantity indices taken from E.R. Berndt and L.R. Christensen (1973).[173] Data on the quantity of output (q) is a Divisia production index taken from Christensen and Jorgenson (1970).[174] The reason for using the Divisia quantity indices constructed by Berndt and Christensen is that these indices are constructed to account for changes in quality and different rates of utilization. When constructing their capital stock index, Berndt and Christensen assume that the decline in efficiency of capital goods is geometric and that replacement is proportional to the stock of capital. They also assume that the flows of capital services are proportional to the stocks. The price of total capital services is computed as a ratio of the value of capital equipment and capital structure services to the Divisia quantity index of capital.

The Divisia quantity index of labor measures manhours of production and non-production workers. It is based on data provided by the U.S. Bureau of Labor Statistics. The manhour estimates are adjusted for quality changes. The index is constructed by aggregating the two adjusted manhour series. The price index of labor services is computed as

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[173] Ibid., p. 108.

[174] L.R. Christensen and D.W. Jorgenson, Op. cit., pp. 30-31.

the ratio of labor compensation (including proprietors) to the Divisia quantity index. For a more detailed discussion of the manner in which the Divisia quantity and price indices of capital and labor are constructed see L.R. Christensen and D.W. Jorgenson, "The Measurement of U.S. Real Capital Input, 1929-1967," and in "U.S. Real Product and Real Factor Input, 1929-1967."[175]

The quantity of real balances, (m_1,m_2,m_3), is again derived by deflating nominal balances, (M_1,M_2,M_3), provided by Friedman and Schwartz (1970) by Christensen and Jorgenson's (1970) Gross Private Domestic Factor Input Price Index (1929-1967).[176] In so doing, we obtained an annual measure of the level of real money balances over this period.

Empirical Results Derived From Estimating the Three-Input Cobb-Douglas Function

The parameters appearing in equations V.27' through V.30' are estimated using a 'stacked' regression technique. This estimation procedure enables us to constrain the parameters appearing in two different equations to be equal across equations; (i.e. α in equation V.27' is constrained to equal α in equation V.30'). This is accomplished by

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[175] Ibid., pp. 19-50; L.R. Christensen and D.W. Jorgenson, "The Measurement of U.S. Real Capital Input, 1929-1967," Review of Income and Wealth, Series 15 (December, 1969): 293-320.

[176] Friedman and Schwartz, Op. cit., pp. 24-72; Christensen and Jorgenson, Op. cit., p. 38.

stacking the data from each of the variables in equations (V.27') through (V.30') in a manner which allows us to simultaneously estimate these equations. In essence, we create several new variables, $(Y_i, X_1, X_2, X_3, X_{4i}, X_5)$ by stacking the data associated with the variables in these four equations and use these new variables to estimate a single equation containing all of the parameters appearing in the structural model.

The above variables $(Y_i, X_1, X_2, X_3, X_{4i}, X_5)$ are used to estimate the equation

$$(V.31) \quad Y_i = \ln\hat{A}X_1 + \hat{\alpha}X_2 + \hat{\beta}X_3 + \hat{\gamma}X_{4i} + \hat{\lambda}X_5 + \xi_i$$

We discuss the techniques used to reconstruct this 'stacked' regression model in the Appendix to this chapter and represent the model in matrix notation on page A.6 in the Appendix. Parameter estimates obtained from estimating this stacked model using one and two-stage regression techniques are reported in Table V-9 through Table V-14 at the end of this chapter.

In Table V-9, OLS parameter estimates are reported. These results again indicate that capital, labor, and real money balances (regardless of definition) enter significantly in a Cobb-Douglas production function fitted to annual data over the period 1929-1967 for the private domestic sector of the United States economy. In addition, the estimated coefficient of the technology variable (T) also appears statistically significant when included with each definition of real money balances. The results suggest that the labor input has the strongest impact on the level of output over the period. The elasticity of output with respect to labor, $\hat{\alpha}$, is 0.799, 0.795, and 0.793 when estimated with m_1, m_2, and m_3 respectively. The elasticities of output with respect to capital, $\hat{\beta}$, (0.156, 0.155, and 0.154) and real money balances $\hat{\gamma}$, (0.042, 0.042, 0.042) reported in equations (V.32) through (V.34) are much smaller than the labor elasticity but each of these estimated coefficients is statistically significant at a 99 percent level of confidence.

When estimating this model constant returns to scale is assumed. Hence, the estimated coefficients for $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$ are smaller than those obtained by Sinai and Stokes but these results do support their conclusion that a measure of real money balances should be included as a factor input. Although the impact of real money on the level of real output is small, it is positive and statistically significant, suggesting that money is a productive asset. However, we cannot rely on these results for providing an accurate estimation of a production function since the disturbance terms in regression equations V.32, V.33, and V.34 are autocorrelated and heteroskedastic. Both of these problems were corrected and the results are reported in Tables V-10 and V-11. In Table V-10, parameter estimates are reported which have only been corrected for autocorrelation. The parameter estimates reported in Table V-11 have been corrected for both autocorrelation and heteroskedasticity. A description of the techniques used to eliminate these problems is included in the Appendix to this chapter.

Results Obtained After Correcting for Autocorrelation:

Although the Durbin-Watson statistic reported for regression equations V.32, V.33, V.34 could not be used as an indicator of autocorrelation since when estimating a 'stacked' regression equation this statistic does not provide an accurate measure of serial correlation, we felt that this problem did exist since it was severe in our single equation estimates of the Cobb-Douglas production function (see Tables V-5 and V-7). Sinai and Stokes, Khan and Kouri, and Zmira Prais also reported this problem in their respective production function estimations.[177] To determine whether this statistical problem did exist in our stacked regression, tests were made to determine whether the disturbance terms in equations V.32, V.33, V.34 (ε_t) were significantly correlated with the lagged values of these error terms (ε_{t-1}); i.e. we assumed that first-order autocorrelation existed in these regressions and tested to see whether

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[177] Sinai and Stokes, Op. cit., p. 292; Khan and Kouri, Op. cit., p. 244-245; Zmira Prais, Op. cit., pp. 535-543.

this assumption was valid. We tested for the existence of autocorrelation separately in each of the four blocks of data used to estimate the model and corrected the problem accordingly.

After correcting for autocorrelation, the parameter estimates of $\hat{\hat{\alpha}},\hat{\hat{\beta}},\hat{\hat{\gamma}}$ changed slightly. The estimates of $\hat{\hat{\beta}}$ in equations V.35, V.36 an V.37 all increased in magnitude while the estimates of $\hat{\hat{\alpha}}$, $\hat{\hat{\gamma}}$, and $\hat{\hat{\lambda}}$ all fell. None of the money coefficients, $\hat{\hat{\gamma}}$, are statistically significant at a 95 percent confidence level. The money coefficients did have the correct positive sign, as did all of the other estimated coefficients reported in equations V.35, V.36, and V.37. These latter coefficients are all statistically significant at a 95 percent confidence level. The R^2 statistics for each of these equations are always greater than 0.9 but were consistently lower than those reported in Table V-9. This is to be expected since the variation in $\ln Q_t$, which is quite different from that in $(\ln Q_t - \ln Q_{t-1})$, is easier to explain. The standard error of the regression over the smaller sample space used to estimate these equations (2,...,39,40,...,78,80,...,117,119,...,156) is also slightly lower in each of the regression equations reported in Table V-10.

If autocorrelated disturbances had been the only problem with the OLS regressions reported in Table V-9, the iterative technique used to eliminate this problem would have generated maximum likelihood estimates of the parameters in equation V.31. Serial correlation, however, was not the only statistical problem in our regressions. Two other basic assumptions of the classical normal linear regression model were also violated in these regressions; the assumption of homoskedasticity ($E(\varepsilon_i^2) = \sigma^2$), and possibly the assumption that the dependent variables, X_i, are nonstochastic.

Results Obtained After Correcting for Heteroskedasticity:

The assumption of homoskedasticity for models estimated with time series data is usually valid since the values of the explanatory and dependent variables are typically of a similar order of magnitude at all points of observation. In our model, this assumption is not plausible even though time series data are used to estimate the model since we have stacked data from different variables together to estimate our Cobb-Douglas structural production function model. In so doing, we introduced heteroskedasticity into out model.[178]

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[178]We tested for heteroskedasticity by using the F-statistic to determine whether the standard errors of the regression $S_j = 1,2,3,4$ estimated within each block of data (j=1,2,3,4) were equal across these blocks of data. In essence, then, we assumed that the disturbance terms within each block of data were homoskedastic but tested to determine whether these disturbances were equivalent across the four separate blocks of data used to estimate our stacked regression equations. These statistical tests led us to reject the hypothesis that the variance of the disturbance term was equivalent across these blocks of data.

With heteroskedasticity, the least squares estimators of the regression coefficients are unbiased and consistent but not asymptotically efficient. Thus, if the disturbance term is heteroskedastic, least squares estimators will still have some desirable properties. But when these estimators are used for testing hypotheses or constructing confidence intervals, we require not only that the estimators are unbiased, but also that their estimated variances be unbiased. Otherwise, our hypothesis tests will not be valid.

With heteroskedastic disturbances, the estimated variance of the least squares regression coefficients are biased and the usual tests of significance and confidence limits do not apply. Hence, we cannot rely on the t-statistics reported in Table V-10 to determine whether a real money balance variable belongs in the production function.

To eliminate heteroskedasticity the data already adjusted to eliminate autocorrelation were transformed by a heteroskedasticity correction factor. The method used to detect whether heteroskedasticity had been introduced and the method used to eliminate it is described in the Appendix to this chapter.

As expected, after correcting this statistical problem, the magnitude of these estimates did not vary significantly from those reported in Tables V-9 and V-10. However, the real money balance coefficient for m_1, m_2, m_3 now showed a positive correlation with real output which was statistically significant at a 99 percent confidence level. The estimated coefficients $\hat{\hat{\alpha}}$, $\hat{\hat{\beta}}$, and $\hat{\hat{\lambda}}$ also had the expected signs (all positive) and are statistically significant. The elasticity of output with respect to labor (α) is 0.8, the elasticity of output with respect to capital (β) is 0.16 and the elasticity of output with respect to real money balances (γ) is a relatively small 0.03. Although the real money balance variable had a relatively small impact on the level of output, the coefficient is positive and significant, when estimated in this structural model. The R^2 statistics reported in Table V.11; (0.979, 0.975, and 0.978) with m_1, m_2, and m_3 respectively are slightly larger than those reported in Table V.10; (0.961, 0.950, 0.946), but the standard errors of the regressions estimated over the entire sample space (i.e. over the sample including observations 3,...,39,42,...,78,81,...,117,120,...,156) are slightly higher than those reported in Table V-10.

These results seem fairly reasonable from a theoretical viewpoint. Using the estimates reported in Table V-11 with m_1, (equation V.38), the Cobb-Douglas structural model presented on page 127 in this chapter can be written

(V.27)' $P_L \cdot L/P_q \cdot q = 0.808$

(V.28)' $P_K \cdot K/P_q \cdot q = 0.163$

(V.29)' $P_m \cdot m_1/P_q \cdot q = 0.024$

(V.30)' $\ln q = -0.132 + 0.808 \ln L + 0.163 \ln K + 0.024 \ln m_1 + 0.0014T$

These results seem to provide important additional empirical evidence which supports the hypothesis that money is a productive asset which belongs in the production function. Hence they favor the Patinkin/Levhari money growth model specification over Tobin's model. One final problem with this estimation, however, is the possibility of simultaneous equation bias. To test whether this bias had a significant impact on our results, the Cobb-Douglas model, equations V.27' through V.30', are re-estimated using a two-stage instrumental variables regression technique.

In order to correct the parameter estimates reported in Tables V-9 through V-11 for the possibility of simultaneous equation bias, the procedures suggested by Berndt and Christensen in their 1973 translog article are followed.[179] We assume that capital, labor, and real money balances are endogenous variables, simultaneously correlated with the disturbance term in the estimated equations and considered the twelve variables described on pages 119-120 of this chapter to be exogenous to the U.S. manufacturing sector. These latter variables are used in the two-stage restricted least squares regressions to purge the quantities of capital, labor, and real money balances of correlation with the additive disturbance terms in the regression equations. The results obtained using TSLS, and after correcting for autocorrelation and heteroskedasticity are reported in Tables V-12, V-13, and V-14 respectively. The same iterative technique described

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[179] E.R. Berndt and L.R. Christensen, Op. cit., pp. 81-113.

on pages A-5 through A-12 in the Appendix to the chapter was used to determine the optimal value of ρ_{ij} to correct these regression equations for autocorrelation (i.e. a unique estimate of ρ was estimated for each block of data in the stacked model). Similarly, the iterative technique described in pages A-12 and A-13 of the Appendix was used to eliminate heteroskedasticity.[180] Since these techniques were described in detail for the OLS regressions the technical procedures used to eliminate these statistical problems in the TSLS regressions will not be explained again. However, the empirical results obtained by estimating these two-stage least squares regressions will be examined.

Comparing the OLS and TSLS results reported in Tables V-9 through V-11 and Tables V-12 through V-14 respectively, we find that none of our parameter estimates ($\hat{\hat{\alpha}}, \hat{\hat{\beta}}, \hat{\hat{\gamma}}, \hat{\hat{\lambda}}$) registered a significant change. All of the signs remained unchanged and in many instances the OLS and TSLS parameter estimates were identical. After correcting for autocorrelation and heteroskedasticity, the TSLS parameter estimates of $\hat{\hat{\alpha}}$ were 0.808, 0.805, and 0.804 with m_1, m_2, and m_3 respectively. The comparable OLS estimates reported in Table V-11, were 0.808, 0.804, 0.803. Similarly; the TSLS parameter estimates of $\hat{\hat{\beta}}$, $\hat{\hat{\gamma}}$, and $\hat{\hat{\lambda}}$ showed only minor changes from the OLS

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180 We again used an F-test to determine whether we had heteroskedasticity. See footnote 178 for a more complete discussion of this procedure.

estimates. In fact, the $\hat{\hat{\beta}}$ estimates in equations V.50 and V.52 (with m_1, and m_3) were exactly equivalent to the OLS estimates in equations V.38 and V.40. When estimated with m_2, the $\hat{\hat{\beta}}$ parameter only registered an insignificant 0.001 change in equation V.51 moving from 0.161 (euqation V.39) to 0.162 (equation V.51).

The TSLS estimates of $\hat{\hat{\gamma}}$ were 0.025, 0.027, and 0.028 for m_1, m_2, and m_3 respectively (equations V.50, 51, 52) compared to OLS estimates of 0.024, 0.032, and 0.027 (equations V.38, 39, 40).

These results suggest that simultaneous equation bias did not have a significant impact on the estimated parameters. The results again indicate that real money balances, regardless of definition, appeared positive and significantly correlated with real output. Hence these empirical results support the hypothesis that real money balances are a factor input which should be included in a production function to capture the productivity gains derived from using money as medium of exchange, store of value, and unit of account.

To provide additional evidence to support this hypothesis, a translog production function with a real money variable as a factor input is estimated. The translog production function provides information on the manner in which capital, labor, and real money balances interact with each other in addition to their relationship with the level of real output. A description of the method used to estimate the translog production function and corresponding factor input decision equations is presented in the Appendix to this chapter. An interpretation of the empirical results obtained from estimating this translog model is presented in the next section of this chapter.

V.5: Estimating the Productivity of Money in a Structural Model Based on a Translog Production Function

In this section empirical results are analyzed on the productivity of money obtained from estimating a translog production function along with three factor input decision equations derived from this production function. These results are compared with those obtained from the Cobb-Douglas estimation to determine whether real money balances should be included as a factor input in an aggregate production. To facilitate this analysis the translog model will be briefly redeveloped before presenting and interpreting the empirical results.

Translog Model:

The production function is written

$$\ln q = \ln\alpha_o + \alpha_T \ln T + \sum_{i=1}^{n} \alpha_i \ln X_i + 1/2\gamma_{TT}(\ln T)^2$$

$$+ 1/2 \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln X_i \ln X_j + \sum_{i=1}^{n} \gamma_{iT} \ln X_i \ln T$$

where

q = output

T = technological index

α_o, α_T, α_i, γ_{ij} are technological parameters and we assume $\gamma_{ij} = \gamma_{ji}$.

X_i = flow of input services (K,L,M/P)

If we assume that this logarithmic production function exhibits constant returns to scale (CRTS), that the production function is Hicks neutral with respect to technological change (HNTC), and when using a time trend as a proxy for the technology index, the translog production function can be written in the following more simplified form.[181]

$$\ln q = \ln\alpha_o + \lambda T + \sum_{i=1}^{n} \alpha_i \ln X_i + 1/2 \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln X_i \ln X_j$$

where X_i = K,L,M/P=m

To test the significance of real money balances using this specification we again draw from the conditions of profit-maximizing behavior in competitive markets to specify our system of equations. The marginal product of each input in this production function is

$$(53) \quad f_i = \partial q / \partial X_i = q/X_i(\alpha_i + \sum_j \gamma_{ij} \ln X_j)$$

By multiplying both sides of equation (53) by X_i/q we can

[181]See pages 91-94 in Chapter 4 of this dissertation for a discussion of the mathematical restrictions these assumptions place on the parameters of the translog production function which allow us to write this function in this more simplified form.

obtain an equation for the output elasticity of the ith input.

$$(53)' \quad (\partial q/\partial X_i)(X_i/q) = (\alpha_i + \sum_j \gamma_{ij} \ln X_j)$$

Assuming that markets are competitive, a set of necessary conditions for efficient production is that

$$(54) \quad P_i = (\partial q/\partial X_i)P_q$$

Substituting (53) into (54) and dividing by P_q we get

$$(55) \quad P_i/P_q = q/X_i(\alpha_i + \sum_j \gamma_{ij} \ln X_j)$$

$$(P_i \cdot X_i)/(P_q \cdot q) = (\alpha_i + \sum_j \gamma_{ij} \ln X_j).^{182}$$

Hence by assuming profit maximizing behavior in competitive markets the relative cost share of the ith input to total cost of all three inputs is approximately equal to the output elasticity of the ith input. From this the three following profit maximizing equations can be derived which are assumed to be stochastic.

$$(V.56) \quad M_L = (P_L \cdot X_L)/(P_q \cdot q) = \alpha_1 + \gamma_{11} \ln X_L + \gamma_{12} \ln X_K + \gamma_{13} \ln X_m + \varepsilon_1$$

$$(A) \quad (V.57) \quad M_K = (P_K \cdot X_K)/(P_q \cdot q) = \alpha_2 + \gamma_{12} \ln X_L + \gamma_{22} \ln X_K + \gamma_{23} \ln X_m + \varepsilon_2$$

$$(V.58) \quad M_m = (P_m \cdot X_m)/(P_q \cdot q) = \alpha_3 + \gamma_{13} \ln X_L + \gamma_{23} \ln X_K + \gamma_{33} \ln X_m + \varepsilon_3$$

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[182] It is important to note that if $\sum_j \gamma_{ij} \ln X_j = 0$, equation (55) will reduce to $(P_i X_i)/(P_q q) = \alpha_i$ which is the same equation for the relative cost share of the ith input derived from the Cobb-Douglas production function. See equations (V.26) through (V.29) on page 125 of this chapter.

In specifying equations (V.56, V.57, V.58) it was assumed that the production function exhibited CRTS. Hence, the cost shares M_L, M_K, M_m must sum to unity at each observation. This assumption imposes the following restrictions on the parameter estimates:

$$\text{(B)} \quad \begin{aligned} &\alpha_1 + \alpha_2 + \alpha_3 = 1 \\ &\gamma_{11} + \gamma_{12} + \gamma_{13} = 0 \\ &\gamma_{12} + \gamma_{22} + \gamma_{23} = 0 \\ &\gamma_{13} + \gamma_{23} + \gamma_{33} = 0 \end{aligned}$$

Although equations V.56, V.57, and V.58 contain 12 parameters only 8 are unrestricted due to the assumption of linear homogeneity. Imposing the parameter restrictions implied by linear homogeneity, the three equation system (A) is reduced to two estimating equations; the estimates of the third equation are derived from the parameter estimates of the other two equations using condition (B).

Equations V.57 and V.58 were estimated along with the translog output equation, i.e. the following three equation structural model was estimated.

$$\text{(V.57)} \quad M_K = (P_K \cdot K)/(P_q \cdot q) = \alpha_2 + \gamma_{12}\ln X_L + \gamma_{22}\ln X_K + \gamma_{23}\ln X_m + \varepsilon_2$$

$$\text{(V.58)} \quad M_m = (P_m \cdot m)/(P_q \cdot q) = \alpha_3 + \gamma_{13}\ln X_L + \gamma_{23}\ln X_K + \gamma_{33}\ln X_m + \varepsilon_3$$

$$\text{(V.59)} \quad \ln q = \ln\alpha_o + \lambda T + \sum_{i=1}^{3} \alpha_i \ln X_i + 1/2 \sum_{i=1}^{3} \sum_{j=1}^{3} \gamma_{ij} \ln X_i \ln X_j + \varepsilon_4$$

The restrictions specified in the system of equations (B) imply that the following constraints must be imposed on the above regression equations.

(1) $\alpha_1 = 1 - \alpha_2 - \alpha_3$

(2) $\gamma_{12} = - \gamma_{22} - \gamma_{23}$

(3) $\gamma_{13} = - \gamma_{23} - \gamma_{33}$

(4) $\gamma_{11} = - \gamma_{12} - \gamma_{13} = (\gamma_{22} + 2\gamma_{23} + \gamma_{33})$

To impose these restrictions, equations (V.57), (V.58), and (V.59) must be rewritten in the following manner.

$$\text{(V.57)}' \quad M_K = (P_K \cdot K)/(P_q \cdot q) = \alpha_2 + \gamma_{22}(\ln X_K - \ln X_L) + \gamma_{23}(\ln X_m - \ln X_L) + \varepsilon_2$$

$$\text{(V.58)}' \quad M_m = (P_m \cdot m)/(P_q \cdot q) = \alpha_3 + \gamma_{23}(\ln X_K - \ln X_L) + \gamma_{33}(\ln X_m - \ln X_L) + \varepsilon_3$$

$$\begin{aligned}\text{(V.59)}' \quad \ln q = {} & \ln\alpha_o + \lambda T + (1-\alpha_2-\alpha_3)\ln X_L + \alpha_2 \ln X_K + \alpha_3 \ln X_m \\ & + 1/2(\gamma_{22} + 2\gamma_{23} + \gamma_{33})\ln X_L \ln X_L + 1/2\gamma_{22}\ln X_K \ln X_K \\ & + 1/2\gamma_{33}\ln X_m \ln X_m + (-\gamma_{22} - \gamma_{23})\ln X_L \ln X_K \\ & + (-\gamma_{23}-\gamma_{33})\ln X_L \ln X_m + \gamma_{23}\ln X_K \ln X_m + \varepsilon_4\end{aligned}$$

Rearranging terms the production function (V.59)' can be rewritten:

$$\begin{aligned}\ln q - \ln X_L = {} & \ln\alpha_o + \lambda T + \alpha_2(\ln X_K - \ln X_L) + \alpha_3(\ln X_m - \ln X_L) \\ & + \gamma_{22}(1/2\ln X_K \ln X_K + 1/2\ln X_L \ln X_L - \ln X_L \ln X_K) + \gamma_{23}(\ln X_L \ln X_L \\ & + \ln X_K \ln X_m - \ln X_L \ln X_m - \ln X_L \ln X_K) + \gamma_{33}(1/2\ln X_m \ln X_m \\ & + 1/2\ln X_L \ln X_L - \ln X_L \ln X_m) + \varepsilon_4\end{aligned}$$

The cross-equation CRTS restrictions specified by the system of equations (B) cannot be imposed using equation by equation OLS or TSLS regression techniques. To restrict the parameter estimates to be equal across equations (e.g. to restrict $\gamma_{ij} = \gamma_{ji}$), equations (V.57)', (V.58)', and (V.59)' were estimated using a stacked regression technique. The same data used to estimate the Cobb-Douglas model were used to estimate this translog model. (For a description of this data see pages 126 through 130 of this chapter).

Following the same stacking procedure used to estimate the Cobb-Douglas model (described in the Appendix to this chapter), we created several new variables which were used to simultaneously estimate equations V.57', V.59', and V.59' specified above. After stacking the data we estimated equation (V.60), below, with m_1, m_2, m_3.

$$\text{(V.60)} \quad Y_{i=1,2,3} = \ln\hat{\alpha}_o X1 + \hat{\alpha}_2 X2 + \hat{\alpha}_3 X3_{i=1,2,3} + \hat{\gamma}_{22} X4 + \hat{\gamma}_{23} X5_{i=1,2,3} + \hat{\gamma}_{33} X6_{i=1,2,3} + \lambda X7 + \varepsilon_5$$

where

$i=1,2,3$ indicates that these variables have been created with the three different definitions of real money balances; m_1, m_2, m_3

The stacking procedure utilized to estimate the translog model is presented on page A-15 in the Appendix to this chapter.

Empirical Results Derived from Estimating the Three-Input Translog Production Function

In analyzing the results obtained from estimating the translog production function primary interest is given to the parameter estimates with non-autoregressive and homoskedastic disturbances. However, a brief comparison of the OLS and TSLS parameter estimates which were not corrected for autocorrelation and heteroskedasticity will be made with those estimates which had been corrected for these problems.

In general, the OLS results obtained before correcting for autocorrelation and heteroskedasticity, (Table V-15), are not significantly different in terms of magnitude and signs than those obtained after correcting for these statistical problems (Table V-17). The parameter estimates of α_2 reported in Table V-15 are statistically equivalent to those reported in Table V-17. In equations V.61, V.62, and V.63 (Table V-15) the estimated α_2 parameters with m_1, m_2, m_3 respectively are 0.162, 0.161, and 0.159. After eliminating autocorrelation these estimates remained virtually unchanged. The α_2 parameter estimates reported in equations V.64, V.65, and V.66 in Table V-16 are respectively; 0.161, 0.160, and 0.162. After further correcting for heteroskedasticity the sign of these coefficients remained positive but the magnitude fell slightly. The final estimates of α_2 generated after correcting for both autocorrelation and heteroskedasticity are 0.158, 0.156, and 0.158 in equations V.67, V.68, and V.69 respectively (Table V-17). All of these estimates are statistically significant at a 99 percent confidence level and are close to the corresponding OLS estimates of β for the Cobb-Douglas model (the elasticity of output with respect to capital). These latter coefficients reported in equations V.38, V.39, and V.40 of Table V-11 are 0.163, 0.161, and 0.162 with m_1, m_2, m_3 respectively.

The OLS parameter estimates of the real money balance coefficient, α_3, also remained virtually unchanged after correcting for autocorrelation and heteroskedasticity. The initial $\hat{\alpha}_3$ estimates, reported in Table V-15 are positive and statistically significant at a 99 percent confidence level. In equations V.61, V.62, and V.63 $\hat{\alpha}_3$ is 0.046, 0.049, and 0.050 for m_1, m_2, and m_3. After correcting for autocorrelation these coefficients moved to 0.052, 0.053, and 0.054. All of the parameter estimates are positive and significantly correlated with real output. (See Table V-16.) After correcting for heteroskedasticity, these estimates decreased in magnitude slightly to 0.041, 0.048, and 0.050 respectively in equations V.67, V.68, and V.69 but are still statistically significant at a 99 percent confidence level. (See Table V-17.) These final parameter estimates are close to but slightly larger in magnitude than the corresponding Cobb-Douglas $\hat{\hat{\gamma}}$ estimates (the elasticity of output with respect to real money balances). The Cobb-Douglas estimates reported in Table V-11, after correcting for autocorrelation and heteroskedasticity, are 0.024, 0.032, and 0.027.

The remaining OLS estimates, $\hat{\gamma}_{22}$, $\hat{\gamma}_{23}$, $\hat{\gamma}_{33}$, and λ only changed slightly after correcting for autocorrelation and heteroskedasticity. However, before correcting these statistical problems, the parameter estimates were different depending upon which definition of money was used in the regression. For example, the OLS estimates of $\hat{\gamma}_{22}$ before correcting for autocorrelation and heteroskedasticity (Table V-15) are -0.019, -0.036, and -0.047 when estimated with m_1, m_2, and m_3 respectively. None of these coefficients are statistically significant. After correcting for autocorrelation and heteroskedasticity these parameter estimates moved to -0.014, -0.018, and -0.016 in equations V.67, V.68, and V.69 respectively. Again, none of these estimates are statistically significant at a 95 percent confidence level. Similarly, the final estimates of γ_{23} and γ_{33} are not statistically significant. Finally, the estimates of λ estimated with m_1, m_2, and m_3 respectively are all positive and significantly correlated with real output. The magnitude of this coefficient reported in equations V.67, V.68, and V.69 (Table V-17) is consistently 0.007.

Using these results reported in Table V-17 and the system of equations (B), we can obtain estimates of α_1, γ_{11}, γ_{12}, and γ_{13}.

(B)
$$\alpha_1 + \alpha_2 + \alpha_3 = 1$$
$$\gamma_{11} + \gamma_{12} + \gamma_{13} = 0$$

$$\gamma_{12} + \gamma_{22} + \gamma_{23} = 0$$

$$\gamma_{13} + \gamma_{23} + \gamma_{33} = 0$$

These conditions imply that[183]

(1) $\alpha_1 = 1 - \hat{\hat{\alpha}}_2 - \hat{\hat{\alpha}}_3$
$= 0.797$[184]

(2) $\gamma_{12} = -\hat{\hat{\gamma}}_{22} - \hat{\hat{\gamma}}_{23}$
$= 0.013$

(3) $\gamma_{13} = -\hat{\hat{\gamma}}_{23} - \hat{\hat{\gamma}}_{33}$
$= -0.006$

(4) $\gamma_{11} = -\hat{\hat{\gamma}}_{12} - \hat{\hat{\gamma}}_{13}$
$= -0.007$

Using these parameter estimates along with those reported in Table V-17, the translog structural model developed on pages 128-132 of this chapter can be rewritten in the following manner.

(V.56) $M_L = (P_L \cdot X_L)/(P_q \cdot q) = 0.797 - 0.007 \ln X_L + 0.013 \ln X_K - 0.006 \ln X_m$

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[183]To determine the values of α_1, γ_{12}, and γ_{13} we again use the average values of the parameter estimates α_2, α_3, γ_{22}, γ_{23}, γ_{33} obtained in regression equations V.67, V.68, and V.69 (Table V-17).

[184]The corresponding $\hat{\alpha}$ estimates from the Cobb-Douglas model are 0.808, 0.804, and 0.803 with m_1, m_2, and m_3 respectively. See Table V-11.

(V.57) $M_K = (P_K \cdot X_K)/(P_q \cdot q) = 0.157 + 0.013\ln X_L - 0.016\ln X_K$
$+ 0.003\ln X_m$

(V.58) $M_m = (P_m \cdot X_m)/(P_q \cdot q) = 0.046 - 0.006\ln X_L + 0.003\ln X_K$
$+ 0.003\ln X_m$

(V.59) $\ln q = -0.2134 + 0.797\ln X_L + 0.157\ln X_K + 0.046\ln X_m$
$- 0.003\ln X_L \ln X_L - 0.008\ln X_K \ln X_K + 0.0015\ln X_m \ln X_m$
$+ 0.013 \ln X_L \ln X_K - 0.006\ln X_L \ln X_m + 0.003\ln X_K \ln X_m$
$+ 0.007T$

The TSLS parameter estimates of this translog structural model obtained after correcting for autocorrelation and heteroskedasticity (Table V-20) are consistent, in terms of sign and magnitude, with the OLS parameter estimates (Table V-17). In addition, the results obtained before correcting these statistical problems (Table V-18), are not significantly different from the final TSLS parameter estimates (Table V-20). Hence, we will only briefly consider the results reported in Table V-18 and V-19 and compare them with the final estimates.

The TSLS parameter estimates of $\hat{\alpha}_2$ and $\hat{\alpha}_3$ reported in Table V-18 are basically equivalent to the $\hat{\hat{\alpha}}_2$ and $\hat{\hat{\alpha}}_3$ estimates obtained after correcting for autocorrelation (Table V-19), and the final estimates obtained after further correcting for heteroskedasticity. In equations V.70, V.71 and V.72 (with m_1, m_2 and m_3) $\hat{\alpha}_2$ is 0.162, 0.160 and 0.158 respectively. After correcting for autocorrelation these estimates change slightly to 0.163, 0.162, 0.161 and the final estimates reported in equations V.76, V.77, and V.78 in Table V-20 are 0.160, 0.159, and 0.158 respectively. All of these parameters estimates round to 0.16. They are all positive and significantly correlated with output at a 99 percent confidence level. In addition they are equivalent to the OLS estimates of α_2 in the translog model (Table V-17) as well as to the OLS and TSLS β estimates obtained from the Cobb-Douglas model (Table V-11 and V-14 respectively). Hence again these TSLS parameter estimates indicate that simultaneous equation bias did not have a significant impact on the estimated parameters.

Similar to the $\hat{\alpha}_2$ estimates, the TSLS parameter estimates of $\hat{\alpha}_3$ did not change significantly, in magnitude or in sign, after correcting for autocorrelation and heteroskedasticity. In equations V.70 through V.72 the estimates of $\hat{\alpha}_3$ for m_1, m_2, and m_3 are 0.042, 0.046, and 0.048 respectively. After correcting for autocorrelation each of these estimates increased slightly to 0.047, 0.053, and 0.054 (Table V-19). Both the original OLS results and the nonautoregressive estimates are all statistically significant at a 95 percent confidence level and are consistent with the translog OLS estimates of $\hat{\hat{\alpha}}_3$ obtained after correcting for autocorrelation (Table V-16) but are slightly larger in magnitude than the corresponding OLS and TSLS estimates of $\hat{\hat{\gamma}}$ from the Cobb-Douglas model (Tables V-10 and V-13 respectively). After adjusting the data to account for heteroskedasticity, the

TSLS translog estimates of α_3 decreased slightly to levels consistent with other estimates of α_3. In equations V.76, V.77, and V.78 (Table V-20) estimates of $\hat{\hat{\alpha}}_3$ for m_1, m_2, and m_3 are 0.040, 0.049, 0.052 respectively compared to OLS translog estimates of 0.041, 0.048, and 0.050 (Table V-17). The OLS and TSLS estimates of $\hat{\hat{\gamma}}$ for m_1, m_2, m_3 in the Cobb-Douglas model are respectively 0.024, 0.032, 0.027 (Table V-11) and 0.025, 0.027, 0.028 (Table V-14).

The translog results again support the hypothesis that real money balances are a productive asset. The α_3 parameter estimates are all positive and significantly correlated with the level of output produced in the U.S. economy over the period 1929-1967.

The remaining TSLS estimates, γ_{22}, γ_{23}, and γ_{33} directly estimated in the translog model also remained basically unchanged in both sign and magnitude after correcting for autocorrelation and then for heteroskedasticity. The initial estimates of $\hat{\gamma}_{22}$ in equations V.70, V.71, and V.72 (Table V-18) are negative but insignificant (-0.016, -0.034, and -0.047). After correcting for autocorrelation these estimates decreased to (-0.003, -0.007, -0.012 in equations V.73, V.74, and V.75). After adjusting for heteroskedasticity the signs and magnitude of these estimates did not change significantly. As reported in Table V-20, these estimates are -0.004, -0.005, -0.008 in equations V.76, V.77, and V.78 respectively. None of these estimates are statistically significant. The estimates of γ_{23}, before and after correcting for autocorrelation and heteroskedasticity are consistently positive but the magnitude of these estimates did change. In Table V-18 (before correcting for autocorrelation and heteroskedasticity) these estimates are 0.025, 0.043, and 0.050 with m_1, m_2 and m_3 respectively. The γ_{23} estimate obtained when m_3 was included in the regression equation is statistically significant. However the estimates of γ_{23} obtained with m_1 and m_2 are insignificant. After correcting for autocorrelation these estimates decreased to 0.009, 0.010, and 0.013. None of these latter estimates are statistically significant. After adjusting for heteroskedasticity the estimates decreased further to 0.004, 0.0007 and 0.002 (equations V.76 through V.78). Again, none of these estimates are statistically significant. Finally, the TSLS parameter estimates of γ_{33} changed from -0.015, 0.016, and 0.013 in equations V.70 through V.72 (Table V-18) to -0.009, 0.005, and 0.016 (equations V.73 through V.75) after correcting for autocorrelation. The final estimates of $\hat{\hat{\gamma}}_{33}$, obtained after correcting for both autocorrelation and heteroskedasticity are -0.011, 0.007, 0.020 in equations V.76, V.77, and V.78 reported in Table V-20. The TSLS estimates of λ with m_1, m_2, and m_3 after correcting for autocorrelation and heteroskedasticity (Table V-20) are 0.007, 0.007, and 0.008 respectively. All of these estimates are statistically significant.

Inserting the two stage parameter estimates into the parameter restrictions specified by the system of equations

(B) we can also obtain estimates of α_1, γ_{12}, γ_{13}.[185]

(1) $\alpha_1 = 1 - \hat{\hat{\hat{\alpha}}}_2 - \hat{\hat{\hat{\alpha}}}_3$

$= 0.794$

(2) $\gamma_{12} = -\hat{\hat{\hat{\gamma}}}_{22} - \hat{\hat{\hat{\gamma}}}_{23}$

$= 0.004$

(3) $\gamma_{13} = -\hat{\hat{\hat{\gamma}}}_{23} - \hat{\hat{\hat{\gamma}}}_{33}$

$= -0.007$

(4) $\gamma_{11} = \hat{\hat{\hat{\gamma}}}_{12} - \hat{\hat{\hat{\gamma}}}_{13}$

$= -0.003$

The statistically significant estimates generated by estimating the translog model with TSLS regression techniques are similar to the statistically significant parameter estimates obtained from estimating the translog model with OLS regression techniques. Hence these results again suggest that the potential simultaneous equation bias did not significantly alter the parameter estimates. An F-test was used to determine whether the estimated translog model was equivalent to the Cobb-Douglas model. This test indicated that these two estimated models are not statistically equivalent at a 99 percent confidence level.[186] Although the results obtained by estimating the translog model are consistent with those obtained by estimating the Cobb-Douglas model,

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[185] To determine these values we again use the average values of the parameter estimates α_2, α_3, γ_{22}, γ_{23}, and γ_{33} from regression equations V.76, V.77, and V.78. See Table V-20.

[186] To determine whether the translog production function

the two estimated models are not statistically equivalent. Hence we cannot assume that the translog model reduces down to the Cobb-Douglas model even though the coefficients of the directly estimated interaction terms ($\ln X_K \ln X_K$, $\ln X_K \ln Xm_i$, and $\ln Xm \ln X_m$) are not significantly different from zero. By including these interaction terms into the specification of the structural model, the explanatory power of the model was altered.

In conclusion, the parameter estimates obtained from both the Cobb-Douglas structural model and the translog model indicate that capital, labor, and real money balances are each positive and significantly correlated with real output. In this dissertation, particular attention is

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estimated over the period 1929-1967 with annual data on the private domestic sector of the U.S. manufacturing sector is statistically equivalent to the Cobb-Douglas specification, we compared the sum of the squared residuals for the TSLS Cobb-Douglas estimation (Table V-14) minus the sum of the squared residuals for the TSLS translog estimation (Table V-20) divided by the number of coefficients in the translog model which are set to zero in the Cobb-Douglas function (6) and then divided this figure by the sum of the squared residuals for the translog function (Table V-20) divided by the number of different parameters included in the translog specification (9). This ratio can be written as

$$[\sum_i e_i^2(\text{Cobb-Douglas}) - \sum_i e_i^2(\text{translog})]/6/[\sum_i e_i^2(\text{translog})/114].$$

We compared this ratio to the F-statistic evaluated at (6,114) degrees of freedom (this value is 3.12). For m_1, m_2, m_3 respectively the ratio specified above is 1.444, 2.316, and 2.821. These figures are less than the value of the F-statistic and hence we reject the hypothesis that the translog production function specified over the period 1929-1967 is statistically equivalent to the Cobb-Douglas function estimated over the same period. The values of the sum of the squared residuals for the Cobb-Douglas model with m_1, m_2, m_3 respectively are 0.374075, 0.376988, 0.376582. The values of the sum of the squared residuals for the translog model with m_1, m_2, m_3 respectively are 0.405252, 0.429654, 0.439087.

given to the question of whether real money balances belong as a factor input in the production function. The estimation of both of these production function models consistently indicates that a real money balance variable should be included in the specification.

These results provide additional evidence which supports the Patinkin/Levhari money growth model specification over James Tobin's model. The results indicate that real money balances provided productive services which contributed to the level of output produced in the U.S. economy during the period 1929-1967 and suggest that money is a productive asset. Hence, the empirical evidence presented in this dissertation supports the hypothesis that the decision to hold money, like the decision to demand any asset, productive input, or consumption good, is based on rational, profit maximizing (utility maximizing) considerations. Accordingly, it seems inappropriate to view cash holdings as a barren investment.

Table V-1

Sinai and Stocks' Estimates of the Parameters of the Cobb-Douglas Production Function With and Without Real Money Balances, With a Time Trend, 1929-1967

$$[\ln Q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln M + \lambda T + \nu]^1$$

Equation Number

	(3) without time and real money balances	(4) with M1	(5) with M2	(6) with M3	(7) with time without real money balances
ln A	-3.640 (.250)	-3.022 (.264)	-3.537 (.250)	-3.820 (.276)	-2.032 (.603)
α	1.356 (.087)	.945 (.123)	1.092 (.101)	1.194 (.095)	1.195 (.097)
β	.428 (.050)	.585 (.058)	.470 (.049)	.429 (.050)	.234 (.082)
γ		.172 (.045)	.214 (.061)	.194 (.069)	
$\alpha+\beta+\gamma$	1.784	1.702	1.776	1.817	1.429
λ					.010 (.003)
$\bar{R}^{2}$ [a] (orig.)	.9943	.9951	.9947	.9945	.9951
S.E.E. (orig.)	.0347	.0326	.0338	.0343	.0327
$\bar{R}^2$	.994	.993	.993	.994	.994
S.E.E.	.036	.035	.037	.037	.033
D.W.[c]	1.54	1.43	1.33	1.25	1.44

1/ lnQ = natural log gross private domestic product, quantity index.
lnL = natural log private domestic labor input, quantity index.
lnK = natural log private domestic capital input, quantity index
lnM = natural log real money balances, M1, M2, M3.
T = time trend, 1929 = 0, used as a proxy for technological change
All equations reported were corrected for autocorrelation.

a/ $\bar{R}^2$ (orig.) = adjusted coefficient of determination for equation not corrected for autocorrelation. These equations were not reported.

b/ S.E.E. (orig.) = standard error of estimate for equation not corrected for autocorrelation.

c/ D.W. = Durbin-Watson statistic for corrected equations.

Table V-2

Sinai and Stokes' Estimates of Parameters of the Cobb-Douglas Production Function With Real Money Balances and Time Trend, 1929-1967

$[\ln Q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln M + \lambda T + \nu]$[1]

	Equation Number		
	(8) with M1 Time	(9) with M2 Time	(10) with M3 Time
lnA	-2.273 (.600)	-2.574 (.691)	-2.545 (.759)
α	.996 (.123)	1.100 (.104)	1.174 (.099)
β	.428 (.116)	.323 (.098)	.276 (.090)
γ	.127 (.051)	.133 (.072)	.087 (.079)
$\alpha+\beta+\gamma$	1.221	1.556	1.537
λ	.006 (.004)	.006 (.004)	.008 (.004)
$\bar{R}^{2}$[a] (orig.)	.995	.995	.995
S.E.E.[b] (orig.)	.0325	.0331	.0332
$\bar{R}^{2}$[a]	.995	.995	.995
S.E.E.	.0329	.0332	.0327
D.W.[c]	1.45	1.32	1.31

1/ lnQ = natural log gross private domestic product, quantity index.
lnL = natural log private domestic labor input, quantity index.
lnK = natural log private domestic capital input, quantity index.
lnM = natural log real money balances, M1, M2, M3.
T = time trend, 1929 = 0, used as a proxy for technological change.
All equations reported were corrected for autocorrelation.
Standard errors of regression coefficients are in parentheses.

a/ $\bar{R}^2$ (orig.) = adjusted coefficient of determination for equation not corrected for autocorrelation.

b/ S.E.E. (orig.) = standard error of estimate for equation not corrected for autocorrelation.

c/ D.W. = Durbin-Watson statistic for corrected equation.

Table V-3

Prais' Estimates of the Parameters of the Cobb-Douglas Production Function with Current and Lagged Real Money Balances, 1929-1967[1]

$[\ln Q_t = \ln A + \alpha \ln L_t + \beta \ln K_t + \gamma_1 \ln M_t + \gamma_2 \ln M_{t-1} + \nu_t]$

	M1	M1	M2	M2	M3	M3
lnA	-3.683 (0.301)	-2.911 (0.483)	-3.795 (0.188)	-2.626 (0.786)	-3.783 (0.202)	-2.941 (0.783)
α	1.238 (0.160)	0.940 (0.176)	1.419 (.098)	1.018 (0.161)	1.484 (0.085)	1.072 (0.171)
β	0.484 (0.069)	0.589 (0.085)	0.388 (0.038)	0.515 (0.085)	0.372 (0.037)	0.490 (0.089)
γ_1	0.482* (0.113)	0.278* (0.107)	0.510* (0.099)	0.310* (0.104)	0.502* (0.106)	0.331* (0.111)
γ_2	-0.399* (0.111)	-0.126 (0.166)	-0.510* (0.103)	-0.238* (0.106)	-0.556* (0.133)	-0.233* (0.110)
Alpha 1		0.665 (0.152)		0.775 (0.156)		0.749 (0.154)
R^2	0.9966		0.9971		0.9970	
$\bar{R}^2$	0.9962		0.9967		0.9966	
S.S.Q[2]	0.0008	0.0006	0.0007	0.0005	0.0007	0.0005
D.W.	1.12		1.378		1.376	

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* Monetary coefficients significant at 95 percent level.

1/ Ordinary Least Squares estimates and estimates corrected for first-order autocorrelation. Alpha is the autoregression parameter, obtained by an iterative maximum likelihood procedure.

2/ Estimated residual error variance is adjusted for loss of degrees of freedom.

3/ Source Zmira Prais, "Real Money Balance as a Variable in the Production Function," Journal of Money-Credit, and Banking 7 (November, 1975): 535-543.

Table V-4

Khan and Kouri's FIML Estimates of the Parameters of a Cobb-Douglas Production Function and Demand for Money Function, 1937-1967

M1

(1) $\ln Q = -1.677 + \underset{(0.077)}{0.809} \ln K + \underset{(0.050)}{0.613} \ln L + \underset{(0.107)}{0.363} \ln M1$

$R^2 = 0.9860$ S.E. = 0.028

(2) $\ln M1 = 1.779 + \underset{(0.049)}{0.276} \ln Q - \underset{(0.056)}{0.291} \ln R$

$R^2 = 0.9180$ S.E. = 0.072

M2

(1) $\ln Q = -2.137 + \underset{(0.086)}{0.425} \ln K + \underset{(0.147)}{0.577} \ln L + \underset{(0.308)}{0.929} \ln M2$

$R^2 = 0.959$ S.E. = 0.049

(2) $\ln M2 = 1.508 + \underset{(0.037)}{0.397} \ln Q - \underset{(0.040)}{0.078} \ln R$

$R^2 = 0.8760$ S.E. = 0.053

M3

(1) $\ln Q = -3.614 + \underset{(0.303)}{0.221} \ln K + \underset{(0.431)}{0.311} \ln L + \underset{(1.282)}{1.896} \ln M3$

$R^2 = 0.8210$ S.E. = 0.100

(2) $\ln M3 = 1.696 + \underset{(0.037)}{0.351} \ln Q - \underset{(0.041)}{0.020} \ln R$

$R^2 = 0.7172$ S.E. = 0.053

1/ Standard errors are presented in parentheses below the coefficients.

2/ The R^2's and standard errors of the estimated equations (S.E.) for the individual equations are presented but should be viewed with caution as their properties are not the same as in the case of an OLS regression.

3/ Source: Mohsin S. Khan and Pentti J.K. Kouri, "Real Money Balances as a Factor of Production: A Comment," Review of Economics and Statistics 57 (May, 1975): 244-246.

Table V-5

OLS Parameter Estimates of a Cobb-Douglas Production Function With Real Money Balances and a Time Trend, 1929-67

$$[\ln Q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln M + \lambda T + \nu']^{1}$$

Equation Number

	(14) with M1	(15) with M2	(16) with M3
lnA	-3.474	-4.322	-5.999
	(5.472)	(5.426)	(5.859)
α	1.138*	1.214*	1.302*
	(.140)	(.117)	(.099)
β	.463*	.386*	.338*
	(.108)	(.081)	(.074)
γ	.099*	.128	.082
	(.053)	(.076)	(.079)
$\alpha+\beta+\gamma$	1.700	1.728	1.722
λ	.002	.002	.003
	(.003)	(.003)	(.003)
R^2	.9958	.9957	.9955
S.E.E.	.0316	.0319	.0327
D.W.	.784	.771	.794

1/ lnQ = natural log gross private domestic product, quantity index.
lnL = natural log private domestic labor input, quantity index.
lnK = natural log private domestic capital input, quantity index.
lnM = natural log real money balances, M1, M2, M3.
T = time trend, 1929=1, used as a proxy for technological change.

* Indicates estimated coefficient is statistically significant at a 95% or greater confidence level.

2/ Equations were not corrected for autocorrelation.

3/ Standard errors are presented in parentheses below the coefficients.

Table V-6

Parameter Estimates of a Cobb-Douglas Production Function With Real Money Balances and a Time Trend, 1929-67 (Equations Corrected for Autocorrelation)

$$[\ln Q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln M + \lambda T + \nu']^1$$

	Equation Number		
	(17) with M1	(18) with M2	(19) with M3
lnA	-15.437 (7.995)	-14.811 (9.106)	-16.846 (9.872)
α	.969* (.131)	1.096* (.120)	1.147* (.121)
β	.377* (.121)	.298* (.118)	.259* (.120)
γ	.145* (.055)	.144* (.077)	.109 (.088)
$\alpha+\beta+\gamma$	1.491	1.538	1.515
λ	.008* (.004)	.008 (.005)	.009* (.005)
R^2	.9977	.9975	.9974
S.E.E.	.0234	.0244	.0208
D.W.	1.236	1.132	1.096

1/ lnQ = natural log gross private domestic product, quantity index.
lnL = natural log private domestic labor input, quantity index.
lnK = natural log private domestic capital input, quantity index.
lnM = natural log real money balances, M1, M2, M3.
T = time trend, 1929=1, used as a proxy for technological change.

* Indicates estimated coefficient is statistically significant at a 95% or greater confidence level.

2/ Equation were corrected for autocorrelation.

3/ Standard errors are presented in parentheses below the coefficients.

Table V-7

2SLS Parameter Estimates of a Cobb-Douglas Production Function With Real Money Balances and a Time Trend, 1929-67

$$[\ln Q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln M + \lambda T + \nu']^{1}$$

Equation Number

	(20) with M1	(21) with M2	(22) with M3
lnA	-.814	-2.63	-4.346
	(5.819)	(5.865)	(6.353)
α	1.242*	1.333*	1.410*
	(.163)	(.142)	(.112)
β	.453*	.368*	.325*
	(.120)	(.091)	(.081)
γ	.087	.089	.041
	(.060)	(.091)	(.092)
$\alpha+\beta+\gamma$	1.782	1.790	1.776
λ	.0004	.001	.002
	(.003)	(.003)	(.003)
R^2	.9957	.9956	.9953
S.E.E.	.0321	.0325	.0334
D.W.	.817	.821	.861

1/ lnQ = natural log gross private domestic product, quantity index.
lnL = natural log private domestic labor input, quantity index.
lnK = natural log private domestic capital input, quantity index.
lnM = natural log real money balances, M1, M2, M3.
T = time trend, 1929=1, used as a proxy for technological change.

* Indicates estimated coefficient is statistically significant at a 95% or greater confidence level.

2/ Equations were not corrected for autocorrelation.

3/ Standard errors are presented in parentheses below the coefficients.

Table V-8

2SLS Parameter Estimates of a Cobb-Douglas Production Function With Real Money Balances and a Time Trend, 1929-67
(Equations Corrected for Autocorrelation)

$[\ln Q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln M + \lambda T + \nu']^1$

	Equation Number		
	(23) with M1	(24) with M2	(25) with M3
lnA	-7.418 (9.161)	-3.833 (10.563)	-5.401 (11.463)
α	.773* (.200)	1.038* (.163)	1.174* (.150)
β	.564* (.145)	.430* (.137)	.364* (.137)
γ	.251* (.077)	.268* (.105)	.209* (.114)
$\alpha+\beta+\gamma$	1.588	1.736	1.747
λ	.004 (.005)	.002 (.005)	.003 (.006)
R^2	.997	.997	.997
S.E.E.	.0248	.0254	.0257
D.W.	1.416	1.207	1.200

1/ lnQ = natural log gross private domestic product, quantity index.
lnL = natural log private domestic labor input, quantity index.
lnK = natural log private domestic capital input, quantity index.
lnM = natural log real money balances, M1, M2, M3.

T = time trend, 1929=1, used as a proxy for technological change.

* Indicates estimated coefficient is statistically significant at a 95% or greater confidence level.

2/ Equations were corrected for autocorrelation.

3/ Standard errors are presented in parentheses below the coefficients.

Table V-9

OLS Parameter Estimates of a Structural
Model Based on a Cobb-Douglas Production Function

'Stacked' Regression Equation[1]

$$Y = \ln X1 + \alpha X2 + \beta X3 + \gamma X4 + \lambda X5 + u$$

Equation Number

	(32) with M1	(33) with M2	(34) with M3
lnA	-0.190* (0.016)	-0.196* (0.017)	-0.199*
α	0.799* (0.007)	0.795* (0.008)	0.793* (0.008)
β	0.156* (0.007)	0.155* (0.008)	0.154* (0.008)
γ	0.042* (0.007)	0.042* (0.008)	0.042* (0.005)
$\alpha+\beta+\gamma$	0.997	0.992	0.989
λ	0.006* (0.0007)	0.007* (0.0008)	0.007* (0.0007)
R^2	0.9877	0.9858	0.9850
S.E.E.	0.04496	0.04809	0.04941

1/ The stacking technique used to construct variables Y, X1,...,X5 is described in pages A-1 through A-5 in the appendix to this chapter. In addition, these variables are reconstructed in matrix notation on page A-6 in the appendix.

2/ *Indicates the estimated coefficient is statistically significant at a 95% or greater confidence level.

3/ Equations are not corrected for autocorrelation.

4/ D.W. statistics are not reported since this statistic is not a valid indicator of autocorrelation when a 'stacked' regression technique is used.

5/ Standard errors are presented in parentheses below the coefficients.

Table V-10

OLS Parameter Estimates of a Structural Model
Based on a Cobb-Douglas Production Function
(Estimates Corrected for Autocorrelation)

'Stacked' Regression Equation[1]

$$Y' = \ln A X1' + \alpha X2' + \beta X3' + \gamma X4' + \lambda X5' + \nu'$$

Equation Number

	(35) with M1	(36) with M2	(37) with M3
lnA	-0.137*	-0.142*	-0.146*
	(0.036)	(0.039)	(0.040)
α	0.795*	0.788*	0.785*
	(0.012)	(0.015)	(0.016)
β	0.163*	0.162*	0.161*
	(0.004)	(0.004)	(0.004)
γ	0.034	0.027	0.021
	(0.019)	(0.020)	(0.021)
$\alpha+\beta+\gamma$	0.992	0.997	0.967
λ	0.0015*	0.0015*	0.002*
	(0.0005)	(0.0005)	(0.0005)
R^2	0.9613	0.9505	0.9464
S.E.E.	0.0309	0.0313	0.0316

1/ The variables Y', X1',...,X5' are reconstructed in matrix notation on page A-11 in the appendix to this chapter.

2/ *Indicates the estimated coefficient is statistically significant at a 95% or greater confidence level.

3/ Equations are corrected for autocorrelation but not for heteroskedasticity.

4/ D.W. statistics are not reported since this statistic is not a valid indicator of autocorrelation when a 'stacked' regression technique is utilized.

5/ Standard errors are presented in parentheses below the coefficient.

Table V-11

OLS Parameter Estimates of a Structural Model
Based on a Cobb-Douglas Production Function
(Estimates Corrected for Autocorrelation & Heteroskedasticity)

'Stacked' Regression Equation[1]

$$Y^* = \ln A X1^* + \alpha X2^* + \beta X3^* + \gamma X4^* + \lambda X5^* + \nu^*$$

	Equation Number		
	(38) with M1	(39) with M2	(40) with M3
lnA	-0.132*	-0.128*	-0.130*
	(0.059)	(0.062)	(0.059)
α	0.808*	0.804*	0.803*
	(0.009)	(0.011)	(0.011)
β	0.163*	0.161*	0.162*
	(0.002)	(0.002)	(0.002)
γ	0.024*	0.032*	0.027*
	(0.008)	(0.010)	(0.016)
α+β+γ	0.995	0.997	0.992
λ	0.0014*	0.0013*	0.0013*
	(0.0008)	(0.0008)	(0.00076)
R^2	0.9787	0.9748	0.9778
S.E.E.	0.0537	0.0539	0.0499

1/ A descrpition of the stacking procedure utilized to construct variables Y*, X1*,...,X5* is presented in pages A-12 and A-13 in the appendix to this chapter.

2/ *(next to parameter estimates) indicates that the estimated coefficient is statistically significant at a 95% or greater confidence level.

3/ Equations are corrected for autocorrelation and heteroskedasticity.

4/ D.W. statistics are not reported since this statistic is not a valid indicator of autocorrelation when a 'stacked' regression technique is used.

5/ Standard errors are presented in parentheses below the coefficients.

Table V-12

2SLS Parameter Estimates of a Structural Model
Based on a Cobb-Douglas Production Function

'Stacked' Regression Equation

$$Y = \ln AX1 + \alpha X2 + \beta X3 + \gamma X4 + \lambda X5 + \nu$$

Equation Number

	(44) with M1	(45) with M2	(46) with M3
ln A	-0.173* (0.0159)	-0.178* (0.0171)	-0.181* (0.0175)
α	0.802* (0.0066)	0.798* (0.0071)	0.796* (0.0073)
β	0.157* (0.0065)	0.156* (0.0071)	0.156* (0.0073)
γ	0.041* (0.0065)	0.042* (0.0071)	0.043* (0.0074)
$\alpha+\beta+\gamma$	1.0	0.996	0.995
λ	0.006* (0.0007)	0.006* (0.0008)	0.006* (0.0008)
R^2	0.9895	0.9876	0.9868
S.E.E.	0.04137	0.04468	0.04606

1/ *Indicates the estimated coefficient is statistically significant at a 95% or greater confidence level.

2/ Equations are not corrected for autocorrelation or heteroskedasticity.

3/ D.W. statistics are not reported since this statistic is not a valid indicator of autocorrelation when a 'stacked' regression technique is used.

4/ Standard errors are presented in parentheses below the coefficients.

Table V-13

2SLS Parameter Estimates of a Structural Model
Based on a Cobb-Douglas Production Function
(Autocorrelation Eliminated)

'Stacked' Regression Equation

$$Y' = \ln AX1' + \alpha X2' + \beta X3' + \gamma X4' + \lambda X5' + \nu'$$

Equation Number

	(47) with M1	(48) with M2	(49) with M3
ln A	-0.139* (0.0299)	-0.144* (0.0322)	-0.145* (0.0335)
α	0.799* (0.0118)	0.794* (0.0142)	0.792* (0.0150)
β	0.162* (0.0048)	0.162* (0.0038)	0.161* (0.0038)
γ	0.040* (0.0186)	0.034* (0.0200)	0.032* (0.0208)
$\alpha+\beta+\gamma$	1.00	0.990	0.985
λ	0.0019* (0.0005)	0.0018* (0.0005)	0.0018* (0.0005)
R^2	0.9662	0.9588	0.9551
S.E.E.	0.02951	0.02993	0.03020

1/ *Indicates the estimated coefficient is statistically significant at a 95% or greater confidence level.

2/ Equations are corrected for autocorrelation but not for heteroskedasticity.

3/ D.W. statistics are not reported since this statistic is not a valid indicator of autocorrelation when a 'stacked' regression technique is used.

4/ Standard errors are reported in parentheses below the coefficients.

Table V-14

2SLS Parameter Estimates of a Structural Model Based on a Cobb-Douglas Production Function (Autocorrelation and Heteroskedasticity Corrected)

'Stacked' Regression Equation

$$Y^* = \ln AX1^* + \alpha X2^* + \beta X3^* + \gamma X4^* + \lambda X5^* + \nu^*$$

	Equation Number		
	(50) with M1	(51) with M2	(52) with M3
ln A	-0.139*	-0.138*	-0.138*
	(0.0474)	(0.0503)	(0.0520)
α	0.808*	0.805*	0.804*
	(0.0092)	(0.0113)	(0.01227)
β	0.163*	0.162*	0.162*
	(0.0027)	(0.0020)	(0.0020)
γ	0.025*	0.027*	0.028*
	(0.0078)	(0.0099)	(0.0111)
α+β+γ	0.996	0.994	0.994
λ	0.0018*	0.0017*	0.0016*
	(0.0008)	(0.0008)	(0.0008)
R^2	0.9758	0.9757	0.9746
S.E.E.	0.05044	0.05064	0.0506

1/ *(next to coefficients) indicates the estimated coefficient is statistically significant at a 95% or greater confidence level.

2/ Equations are corrected for autocorrelation and heteroskedasticity.

3/ D.W. statistics are not reported since this statistic is not a valid indicator of autocorrelation when a 'stacked' regression technique is used.

4/ Standard errors are reported in parentheses below the coefficients.

Table V-15

OLS Parameter Estimates of a Structural Model
Based on a Translog Production Function

'Stacked' Regression Equation

$$Y = \ln\alpha_0 X1 + \alpha_2 X2 + \alpha_3 X3 + \gamma_{22} X4 + \gamma_{23} X5 + \gamma_{33} X6 + \lambda X7 + e$$

Equation Number

	(61) with M1	(62) with M2	(63) with M3
$\ln\alpha_0$	-0.1713* (0.0186)	-0.1711* (0.6203)	-0.1727* (0.0207)
α_2	0.162* (0.011)	0.161* (0.011)	0.159* (0.011)
α_3	0.046* (0.012)	0.049* (0.011)	0.050* (0.011)
γ_{22}	-0.019 (0.044)	-0.036 (0.048)	-0.047 (0.050)
γ_{23}	0.023 (0.022)	0.041 (0.028)	0.048 (0.029)
γ_{33}	-0.007 (0.025)	-0.008 (0.033)	-0.006 (0.033)
λ	0.006* (0.0008)	0.006* (0.0009)	0.006* (0.0009)
R^2	0.8079	0.7852	0.7785
S.E.E.	0.0524	0.0558	0.0568

1/ The stacking technique used to construct variables Y, X1,...,X7 are described on page A-14 in the appendix and are reconstructed in matrix notation on page A-15.

2/ *Indicates the estimated coefficient is statistically significant at a 95% or greater confidence level.

3/ Equations are not corrected for autocorrelation or heteroskedasticity.

4/ D.W. statistics are not reported since this statistic is not a valid indicator of autocorrelation when a 'stacked' regression technique is used.

5/ Standard errors are presented in parentheses below the coefficients.

Table V-16

OLS Parameter Estimates of a Structural Model
Based on a Translog Production Function
(Equations Corrected for Autocorrelation)

'Stacked' Regression Equation

$$Y' = \ln\alpha_0 X1' + \alpha_2 X2' + \alpha_3 X3' + \gamma_{22} X4' + \gamma_{23} X5' + \gamma_{33} X6' + \lambda X7' + e'$$

Equation Number

	(64) with M1	(65) with M2	(66) with M3
$\ln\alpha_0$	-0.2069* (0.0521)	-0.2140* (0.0643)	-0.2184* (0.0688)
α_2	0.161* (0.010)	0.160* (0.009)	0.162* (0.012)
α_3	0.052* (0.016)	0.053* (0.015)	0.054* (0.015)
γ_{22}	-0.017 (0.036)	-0.023 (0.039)	-0.027 (0.053)
γ_{23}	0.008 (0.019)	0.014 (0.025)	0.017 (0.032)
γ_{33}	-0.005 (0.033)	0.006 (0.042)	0.011 (0.042)
λ	0.007* (0.002)	0.007* (0.003)	0.007* (0.003)
R^2	0.7969	0.7736	0.6641
S.E.E.	0.0478	0.0513	0.0523

1/ *Indicates the estimated coefficient is statistically significant at a 95% or greater confidence level.

2/ Equations are corrected for autocorrelation but not for heteroskedasticity.

3/ D.W. statistics are not reported since this statistic is not a valid indicator of autocorrelation when a 'stacked' regression technique is used.

4/ Standard errors are reported in parentheses below the estimated coefficients.

Table V-17

OLS Parameter Estimates of a Structural Model
Based on a Translog Production Function
(Equations Corrected for Autocorrelation and Heteroskedasticity)

'Stacked' Regression Equation

$$Y^* = \ln\alpha_0 X1^* + \alpha_2 X2^* + \alpha_3 X3^* + \gamma_{22} X4^* + \gamma_{23} X5^* + \gamma_{33} X6^* + e^*$$

Equation Number

	(67) with M1	(68) with M2	(69) with M3
$\ln\alpha_0$	-0.2053* (0.0688)	-0.2147* (0.0828)	-0.2203* (0.0902)
α_2	0.158* (0.006)	0.156* (0.005)	0.158* (0.006)
α_3	0.041* (0.015)	0.048* (0.017)	0.050* (0.017)
γ_{22}	-0.014 (0.020)	-0.018 (0.021)	-0.016 (0.028)
γ_{23}	0.004 (0.012)	0.001 (0.015)	0.003 (0.020)
γ_{33}	-0.011 (0.032)	0.005 (0.046)	0.014 (0.047)
λ	0.007* (0.003)	0.007* (0.003)	0.007* (0.004)
R^2	0.9151	0.9177	0.8593
S.E.E.	0.0634	0.0665	0.0689

1/ *(next to estimated coefficient) indicates that the estimated coefficient is statistically significant at a 95% or greater confidence level.

2/ Equations are corrected for autocorrelation and heteroskedasticity.

3/ D.W. statistics are not reported since this statistic is not a valid indicator of autocorrelation when a 'stacked' regression technique is used.

4/ Standard errors are presented in parentheses below the coefficients.

Table V-18

2SLS Parameter Estimates of a Structural Model
Based on a Translog Production Function

'Stacked' Regression Equation

$$Y = \ln\alpha_0 X1 + \alpha_2 X2 + \alpha_3 X3 + \gamma_{22} X4 + \gamma_{23} X5 + \gamma_{33} X6 + \lambda X7 + e$$

Equation Number

	(70) with M1	(71) with M2	(72) with M3
$\ln\alpha_0$	-0.1692*	-0.1706*	-0.1724*
	(0.0173)	(0.0187)	(0.019)
α_2	0.162*	0.160*	0.158*
	(0.011)	(0.011)	(0.011)
α_3	0.042*	0.046*	0.048*
	(0.011)	(0.011)	(0.010)
γ_{22}	-0.016	-0.034	-0.047
	(0.043)	(0.047)	(0.049)
γ_{23}	0.025	0.043	0.050*
	(0.021)	(0.028)	(0.030)
γ_{33}	-0.015	-0.016	-0.013
	(0.029)	(0.032)	(0.034)
λ	0.006*	0.006*	0.006*
	(0.0007)	(0.0008)	(0.0008)
R^2	0.8229	0.7989	0.7923
S.E.E.	0.0491	0.0528	0.0537

1/ *Indicates the estimated coefficient is statistically significant at a 95% or greater confidence level.

2/ Equations are not corrected for autocorrelation or heteroskedasticity.

3/ D.W. statistics are not reported since this statistic is not a valid indicator of autocorrelation when a 'stacked' regression technique is used.

4/ Standard errors are reported in parentheses below the estimated coefficients.

Table V-19

2SLS Parameter Estimates of a Structural Model
Based on a Translog Production Function
(Equations Corrected for Autocorrelation)

'Stacked' Regression Equation

$$Y' = \ln\alpha_0 X1' + \alpha_2 X2' + \alpha_3 X3' + \gamma_{22} X4' + \gamma_{23} X5' + \gamma_{33} X6' + \lambda X7' + e'$$

Equation Number

	(73) with M1	(74) with M2	(75) with M3
$\ln\alpha_0$	-0.2136* (0.0524)	-0.2175* (0.0574)	-0.2271* (0.0659)
α_2	0.163* (0.012)	0.162* (0.012)	0.161* (0.012)
α_3	0.047* (0.015)	0.053* (0.016)	0.054* (0.015)
γ_{22}	-0.003 (0.043)	-0.007 (0.049)	-0.012 (0.052)
γ_{23}	0.009 (0.022)	0.010 (0.030)	0.013 (0.033)
γ_{33}	-0.009 (0.031)	-0.005 (0.043)	-0.016 (0.044)
λ	0.007* (0.002)	0.007* (0.002)	0.008* (0.003)
R^2	0.7408	0.6936	0.6753
S.E.E.	0.0457	0.0489	0.0498

1/ *indicates the estimated coefficient is statistically significant at a 95% or greater confidence level.

2/ Equations are corrected for autocorrelation but not for heteroskedasticity.

3/ D.W. statistics are not reported since this statistic is not a valid indicator of autocorrelation when a 'stacked' regression technique is used.

4/ Standard errors are reported in parentheses below the estimated coefficients.

Table V-20

2SLS Parameter Estimates of a Structural Model
Based on a Translog Production Function
(Equations Corrected for Autocorrelation & Heteroskedasticity)

'Stacked' Regression Equation

$$Y^* = \ln\alpha_0 X1^* + \alpha_2 X2^* + \alpha_3 X3^* + \gamma_{22} X4^* + \gamma_{23} X5^* + \gamma_{33} X6^* + \lambda X7^* + e^*$$

	Equation Number		
	(76) with M1	(77) with M2	(78) with M3
$\ln\alpha_0$	-0.2127* (0.0703)	-0.2179* (0.0742)	-0.2277* (0.0846)
α_2	0.160* (0.007)	0.159* (0.007)	0.158* (0.006)
α_3	0.040* (0.016)	0.049* (0.018)	0.052* (0.017)
γ_{22}	-0.004 (0.026)	-0.005 (0.028)	-0.008 (0.030)
γ_{23}	0.004 (0.014)	0.0007 (0.020)	0.002 (0.021)
γ_{33}	-0.011 (0.032)	-0.007 (0.049)	-0.020 (0.050)
λ	0.007* (0.003)	0.007* (0.003)	0.008* (0.003)
R^2	0.8757	0.8628	0.8571
S.E.E.	0.0615	0.0634	0.06406

1/ *indicates the estimated coefficients is statistically significant at a 95% or greater confidence level.

2/ Equations are corrected for autocorrelation and heteroskedasticity.

3/ D.W. statistics are not reported since this statistic is not a valid indicator of autocorrelation when a 'stacked' regression technique is used.

4/ Standard errors are reported in parentheses below the estimated coefficients.

Appendix

I. Technical Description of the Methods Used to Estimate the Four-Equation Cobb-Douglas Structural Model

A. Stacking Procedure

In order to clarify the procedure used to estimate the four equation Cobb-Douglas and translog models a detailed description of the stacking technique introduced to estimate these models will be provided in this Appendix. As indicated in the discussion of the data used to estimate the Cobb-Douglas structural model, for all of the variables contained in equations (V.27)' through (V.30)' we have 39 observations on annual data. Using this data, several new variables $(Y, X_1, X_2, X_3, X_4, X_5)$ are created each having 156 observations which are used to estimate a single equation containing all the parameters in the Cobb-Douglas structural model. These variables are generated by stacking the 39 observations from the variables in each of the four equations in the model. (Actually ten new variables are created since the data on total input costs and the level of real money balances will vary depending upon the definition of money used. Only six of these variables are used at one time to estimate the stacked regression.) This technique should become clear after describing the method used to generate these new variables.

To estimate equations (V.27)' through (V.30)', variable Y_i=1,2,3 (associated with m_1, m_2, m_3) is formed by consecutively stacking the 39 annual observations over the period 1929-1967 on each of the dependent variables in these four equations; i.e. we stack four blocks of data. Hence variable Y_i contains 156 observations; the first block of 39 observations consists of annual data on the relative cost share of labor $((P_L \cdot L)/(P_q \cdot q) = (P_L \cdot L)/TOTCST_{i=1,2,3})$. Observations 40 through 78 are data on the relative cost share of capital $((P_K \cdot K)/TOTCST_{i=1,2,3})$, observations 79 through 117 are data on the relative cost share of money $(P_m \cdot m_{i=1,2,3}/TOTCST_{i=1,2,3})$ and observations 118 through 156 (fourth block of data) are annual observations on the natural log of output (lnq) over this period. For convenience, the relative cost shares of the three inputs will be referred to as $LABSHR_{i=1,2,3}$, $CAPSHR_{i=1,2,3}$, and $MONSHR_{i=1,2,3}$ where i = 1,2,3, indicates whether variables are computed using the definition of money (m_1,m_2,m_3). Using this terminology, $Y_{i=1,2,3}$ can be written in the following matrix form:

$$\underset{\substack{i=1,2,3\\t=1,\dots,156}}{Y_{it}} = [\underset{\substack{i=1,2,3\\t=1,\dots,39\\\text{Block 1}}}{LABSHR_{it}},\quad \underset{\substack{i=1,2,3\\t=40,\dots,78\\\text{Block 2}}}{CAPSHR_{it}},\quad \underset{\substack{i=1,2,3\\t=79,\dots,117\\\text{Block 3}}}{MONSHR_{it}},\quad \underset{\substack{t=118,\dots,156\\\text{Block 4}}}{\ln q_t}]$$

The first three blocks of data will vary depending upon the definition of money being used but the fourth block of data will be the same for Y_1,Y_2,Y_3. Hence, three different dependent variables are created including 156 observations each to estimate the Cobb-Douglas structural model with m_1,m_2,m_3 respectively.

Variable X_1 is created to estimate the constant term, $\hat{\ln A}$, in the Cobb-Douglas production function (equation (V.30)' in the structural model). Since this constant term only appears in equation (V.30)' X_1 is constructed so that this parameter estimate is excluded from the other three equations but included in the production function. This is accomplished by using zeros for the first 117 observations of X_1 (i.e. over the first three blocks of data referring to equations (V.27)' through (V.29)'), and 1's for observations 118 through 156 (i.e. the block of data associated with the production function). This format is needed regardless of the definition of money. Hence, X_1 is included with m_1, m_2, m_3. The estimated coefficient associated with this variable provides an estimate of the constant term in the production function. In matrix form, X_1 can be written

X_{1t}	=	$[0,0,\ldots\ldots,0,$	$1,1,\ldots\ldots,1]$
$t=1,\ldots,156$		$t=1,\ldots,117$ Blocks 1,2,3	$t=118,\ldots,156$ Block 4

Variable X_2 is created for the purpose of estimating the parameter $\hat{\alpha}$, which appears in both equation (V.27)' as the relative cost share of labor and in equation (V.30)' as the elasticity of output with respect to labor. It does not appear in equations (V.28)' or (V.29)'. To account for this, X_2 is constructed to include 1's for its first 39 observations since $\hat{\alpha}$ is really a constant term in equation (V.27), zeros over the next 78 observations (i.e. in blocks 2 and 3 of this stacked variable) and, in the fourth block, annual data on the natural log of a divisia quantity index of labor (lnL) for the period 1929-1967 are included as observations 118,...,156.

X_{2t}	=	[1,.,.....,1,	0,0,........,0,	$\ln L_t$,........]
t=1,...,156		t=1,...,39 Block 1	t=40,...,117 Blocks 2 and 3	t=118,...,156 Block 4

Similarly, variable X_3 is created to estimate the $\hat{\beta}$ parameter appearing in equations (V.28)' and (V.30)' in the structural model and variable $X_{4i=1,2,3}$ is created to estimate $\hat{\gamma}$ appearing in equations (V.29)' and (V.30)'. In matrix form these variables can be written as follows:

X_{3t}	=	[0,0,.....,0,	1,1,......,1,	0,0,........,0,	$\ln K_t$,........]
t=1,...,156		t=1,...,39 Block 1	t=40,...,78 Block 2	t=79,...,117 Block 3	t=118,...,156 Block 4

X_{4it}	=	[0,0,.........,0,	1,1,......,1,	$\ln m_{it}$,........]
i=1,2,3 t=1,...,156		t=1,...,78 Blocks 1 and 2	t=79,...,117 Block 3	i=1,2,3 t=118,...,156 Block 4

Finally, variable X_5 is created for the purpose of estimating $\hat{\lambda}$, the rate of disembodied technological change. Since this parameter only appears in equation (V.30)', X_5 contains zeros over the first 117 observations and a time trend (T=1,...,39) for observations 118 through 156. This trend serves as a proxy for technological change.

$X_{5t} \quad = \quad [0,0,\ldots\ldots,0,1,2,3,\ldots\ldots,39]$

$t=1,\ldots,156$ $\quad$ $t=1,\ldots,117$ $\quad$ $t=118,\ldots,156$
Blocks 1,2,3 Block 4

The above variables $(Y_i, X_1, X_2, X_3, X_{4i}, X_5)$ are used to estimate the equation

$$\text{(V.31)} \quad Y_i = \hat{\ln A}X_1 + \hat{\alpha}X_2 + \hat{\beta}X_3 + \hat{\gamma}X_{4i} + \hat{\lambda}X_5 + \varepsilon_i$$

This 'stacked' regression equation is reconstructed in matrix notation on page A-6 (the following page) of this Appendix.

B. Technique Utilized to Eliminate Autocorrelation from the Cobb-Douglas Structural Model:

For each of the regressions reported in Table V-9, (equations V.32, V.33, V.34), the predicted values of the dependent variables Y_1, Y_2, Y_3 were retrieved; i.e. $\hat{Y}_1$, $\hat{Y}_2$, and $\hat{Y}_3$ were retrieved. This information was utilized to determine the estimated values of the disturbance terms in each of these regressions.

$$\varepsilon_{it} = Y_{it} - \hat{Y}_{it}$$

where

$i=32,33,34$ (referring to equations V.32, V.33, V.34 respectively)

$t=1,\ldots,156$

A-6

$$\underset{\substack{i=1,2,3\\ t=1,\dots,156}}{Y_{it}} = \underset{t=1,\dots,156}{\hat{\ln A}X1_t} + \underset{t=1,\dots,156}{\hat{\alpha}X2_t} + \underset{t=1,\dots,156}{\hat{\beta}X3_t} + \underset{\substack{i=1,2,3\\ t=1,\dots,156}}{\hat{\gamma}X4_{it}} + \underset{t=1,\dots,156}{\hat{\lambda}X5_t} + \underset{\substack{i=1,2,3\\ t=1,\dots,156}}{e_{it}}$$

$$\begin{bmatrix} LABSHR_{it} \\ \vdots \\ i=1,2,3 \\ t=1,\dots,39 \\ \vdots \\ CAPSHR_{it} \\ \vdots \\ i=1,2,3 \\ t=40,\dots,78 \\ \vdots \\ MONSHR_{it} \\ \vdots \\ i=1,2,3 \\ t=79,\dots,117 \\ \vdots \\ \ln q_t \\ \vdots \\ t=118,\dots,156 \\ \vdots \end{bmatrix} = \hat{\ln A} \begin{bmatrix} 0 \\ \vdots \\ t=1,\dots,39 \\ 0 \\ 0 \\ \vdots \\ t=40,\dots,78 \\ 0 \\ 0 \\ \vdots \\ t=79,\dots,117 \\ 0 \\ 1 \\ \vdots \\ t=118,\dots,156 \\ \vdots \end{bmatrix} + \hat{\alpha} \begin{bmatrix} 1 \\ \vdots \\ t=1,\dots,39 \\ 1 \\ 0 \\ \vdots \\ t=40,\dots,78 \\ 0 \\ 0 \\ \vdots \\ t=79,\dots,117 \\ 0 \\ \ln L_t \\ \vdots \\ t=118,\dots,156 \\ \vdots \end{bmatrix} + \hat{\beta} \begin{bmatrix} 0 \\ \vdots \\ t=1,\dots,39 \\ 0 \\ 1 \\ \vdots \\ t=40,\dots,78 \\ 1 \\ 0 \\ \vdots \\ t=79,\dots,117 \\ 0 \\ \ln K_t \\ \vdots \\ t=118,\dots,156 \\ \vdots \end{bmatrix} + \hat{\gamma} \begin{bmatrix} 0 \\ \vdots \\ t=1,\dots,39 \\ 0 \\ 0 \\ \vdots \\ t=40,\dots,78 \\ 0 \\ 1 \\ \vdots \\ t=79,\dots,117 \\ 1 \\ \ln m_{it} \\ \vdots \\ i=1,2,3 \\ t=118,\dots,156 \\ \vdots \end{bmatrix} + \hat{\lambda} \begin{bmatrix} 0 \\ \vdots \\ t=1,\dots,39 \\ 0 \\ 0 \\ \vdots \\ t=40,\dots,78 \\ 0 \\ 0 \\ \vdots \\ t=79,\dots,117 \\ 0 \\ \text{Time trend} \\ \vdots \\ t=118,\dots,156 \\ \vdots \end{bmatrix} + \begin{bmatrix} e_{it} \\ i=1,2,3 \\ t=1,\dots,156 \end{bmatrix}$$

This error term was regressed against the lagged value of the error term (ε_{it-1}) using ordinary least squares regression technique to test for significant correlation between these two terms. However, rather than estimating a single equation, $\varepsilon_{it} = \hat{\rho}\varepsilon_{it-1} + u'$, over the 156 observations available for equations V.32, V.33, and V.34, tests for the existence of autocorrelation were conducted on each of the four blocks of data stacked together to estimate equations V.27' through V.30' in the structural model. Since in estimating the stacked regression equation, the four equations of the structural model were actually simultaneously estimated to constrain the parameters in these equations to be equal across equations, a unique value of $\hat{\rho}$ associated with each of these equations was estimated (i.e. a unique value of $\hat{\rho}$ was estimated over each of the four blocks of data stacked together to estimate the Cobb-Douglas structural model). This was accomplished by redividing the 156 observations on the estimated disturbance term ε_{it} (where i=32,33,34 and t=1,...,156) into the four separate sample groupings corresponding to each of the four blocks of data and then re-estimating $\hat{\rho}_{j=1,2,3,4} = \sum_t \varepsilon_{it}\varepsilon_{it-1} / \sum_t \varepsilon^2_{it-1}$, over each of these sample spaces (i.e. for the samples 2,...,39; 41,...,78; 80,...,117; and 119,...,156 for each of the equations V.32, V.33, and V.34). For example, in order to determine whether the disturbance term from equation 32 (with m_1) is autocorrelated, the following four equations were estimated using OLS regression technique.

(a) $\varepsilon_{it} = c + \hat{\rho}_{i1}\varepsilon_{it-1} + u'$ where $i=32$, $t=2,\ldots,39$

(b) $\varepsilon_{it} = c + \hat{\rho}_{i2}\varepsilon_{it-1} + u'$ where $i=32$, $t=41,\ldots,78$

(c) $\varepsilon_{it} = c + \hat{\rho}_{i3}\varepsilon_{it-1} + u'$ where $i=32$, $t=80,\ldots,117$

(d) $\varepsilon_{it} = c + \hat{\rho}_{i4}\varepsilon_{it-1} + u'$ where $i=32$, $t=119,\ldots,156$

The results obtained are as follows; t-scores are provided in parentheses below the parameter estimates and * indicates that the coefficients are statistically significant at a 95 percent confidence level.[1]

(1) $$\varepsilon_{it} = \underset{(0.8810)}{0.0045} + \underset{(5.238)}{0.6011^*}\ \varepsilon_{it-1}$$

(2) $$\varepsilon_{it} = \underset{(2.445)}{0.0082} - \underset{(-1.505)}{0.2438}\ \varepsilon_{it-1}$$

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[1] To determine whether the disturbance terms in this four equation Cobb-Douglas model were significantly correlated with the lagged values of these disturbance terms we used the t-test. If the estimated $\hat{\rho}_{ij}$ was statistically significant at a 95 percent confidence level we assumed that the disturbance terms in this block of data were autoregressive and transformed the data to account for this statistical problem.

(3) $\varepsilon_{it} = \underset{(-2.095)}{-0.0044} + \underset{(10.248)}{0.7548^*}\ \varepsilon_{it-1}$

(4) $\varepsilon_{it} = \underset{(0.462)}{0.0027} + \underset{(5.137)}{0.6296^*}\ \varepsilon_{it-1}$

Using the same procedure, we also obtained four estimates of ρ for equations V.33 (with m_2) and V.34 (with m_3) respectively. These parameter estimates and corresponding t-scores are

(sample 2,...,39) $\hat{\rho}_{33,1} = \underset{(6.56)}{0.6646^*}$ $\hat{\rho}_{34,1} = \underset{(6.9)}{0.6814^*}$

(sample 41,...,78) $\hat{\rho}_{33,2} = \underset{(-1.69)}{-0.2719^*}$ $\hat{\rho}_{34,2} = \underset{(-1.7)}{-0.2779^*}$

(sample 80,...,117) $\hat{\rho}_{33,3} = \underset{(10.9)}{0.7620^*}$ $\hat{\rho}_{34,3} = \underset{(10.8)}{0.7670^*}$

(sample 119,...,156) $\hat{\rho}_{33,4} = \underset{(6.2)}{0.6999^*}$ $\hat{\rho}_{34,4} = \underset{(6.3)}{0.7062^*}$

The results obtained from estimating the equation $\varepsilon_{it} = c + \hat{\rho}_{ij}\varepsilon_{it-1}$ ($i=32,33,34$ and $j=1,2,3,4$) confirmed the suspicion that first-order serial correlation did pose a serious statistical problem for the OLS regressions reported in Table V-9. To correct this problem we used an iterative OLS estimation technique to determine the best estimates of $\hat{\rho}$ to eliminate serial correlation.[2] The Cochrane-Orcutt iterative regression technique was not utilized to eliminate

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[2]See Jan Kmenta, Elements of Econometrics (New York: The Macmillan Co., 1971):287-288.

autocorrelation since this procedure estimates a single iterated value of ρ using the entire sample space t=2,...,156.

Rather than utilize a single estimate of $\hat{\rho}$ to correct the serial correlation in our estimated regressions, we used the information available on $\hat{\rho}_{ij}$ (i=32,33,34;j=1,2,3,4) in each block of data in the stacked regressions. Hence, each block of data was transformed with a unique value of ρ estimated over the relevant sample space. When estimating $\hat{\rho}_{ij}$ observations 1,40,79,118 were "lost" since they were used to generate the lagged values of the error terms used in the regressions. The technique used to eliminate autocorrelation from equation V.32 (with m_1) will be examined in detail; the same procedure was used to re-estimate equations V.33 and V.34.

Estimates of $\hat{\rho}_{32,j}$ (j=1,...,4) and their corresponding t-statistics reported on pages 186-7 indicate that first-order serial correlation was evident in blocks 1, 3, and 4 of equation V.32; hence $\hat{\rho}_{32,2}$ = 0. To eliminate this autocorrelation problem six new variables were constructed.

$$Y'_{1t} = Y_{1t} - \hat{\rho}_{32,j}Y_{it-1}$$

$$X'_{1t} = X_{1t} - \hat{\rho}_{32,j}X_{1t-1}$$

$$\vdots$$

$$X'_{4t} = X_{4t} - \hat{\rho}_{32,j}X_{4t-1}$$

$$X'_{5t} = X_{5t}$$

With these variables, ordinary least squares estimates of

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$$(35) \quad Y'_{1t} = 1\hat{\hat{n}}AX'_{1t} + \hat{\hat{\alpha}}X'_{2t} + \hat{\hat{\beta}}X'_{3t} + \hat{\hat{\gamma}}X'_{4t} + \hat{\hat{\lambda}}X'_{5t} + u_t'$$

were obtained over the sample space (2,...,39,40,...,78,80, ...,117,119,...,156). These new variables are constructed in matrix notation on the next page of this Appendix.

Using the OLS parameter estimates from V.35 we estimated new "second-round" values of $\rho_{j=1,2,3,4}$. This was accomplished by determining new residuals $\hat{\hat{e}}_t = Y_{1t} - (1\hat{n}AX_{1t} + \hat{\alpha}X_{2t} + \hat{\beta}X_{3t} + \hat{\gamma}X_{4t} + \hat{\lambda}X_{5t})$ where $Y_1, X_1, \ldots, X_5$ are the untransformed variables generated using the original data (raw data). These new residuals were used to generate second-round estimates of $\rho_{j=1,2,3,4}$.

$$\hat{\hat{\rho}}_1 = \sum_t \hat{\hat{\varepsilon}}_t \hat{\hat{\varepsilon}}_{t-1} / \sum_t \hat{\hat{\varepsilon}}^2_{t-1} \qquad t=3,\ldots,39$$

$$\hat{\hat{\rho}}_2 = \sum_t \hat{\hat{\varepsilon}}_t \hat{\hat{\varepsilon}}_{t-1} / \sum_t \hat{\hat{\varepsilon}}^2_{t-1} \qquad t=42,\ldots,78$$

$$\hat{\hat{\rho}}_3 = \sum_t \hat{\hat{\varepsilon}}_t \hat{\hat{\varepsilon}}_{t-1} / \sum_t \hat{\hat{\varepsilon}}^2_{t-1} \qquad t=81,\ldots,117$$

$$\hat{\hat{\rho}}_4 = \sum_t \hat{\hat{\varepsilon}}_t \hat{\hat{\varepsilon}}_{t-1} / \sum_t \hat{\hat{\varepsilon}}^2_{t-1} \qquad t=120,\ldots,156$$

These estimates of $\hat{\rho}_{i=1,2,3,4}$ were equivalent to the $\hat{\rho}_{i=1,2,3,4}$ estimates; i.e. these estimates converged after the second iteration. Similarly, the estimates of $\hat{\hat{\rho}}_{j=1,\ldots,4}$ associated with the equations including m_2 and m_3 respectively also converged after the second iteration. Hence, the estimates of $\hat{\rho}_{ij}$ reported on page 187 were used to obtain nonautoregressive parameter estimates for equations V.35, V.36, and V.37. These results are reported in Table V-10.

(A-12)

$Y1'_t$	$= \ln\hat{\hat{\Lambda}}X1'_t$	$+ \hat{\hat{\alpha}}X2'_t$	$+ \hat{\hat{\beta}}X3'_t$	$+ \hat{\hat{\gamma}}X4'_t$	$+ \hat{\hat{\lambda}}X5'_t$
t=2,...,78, 80,...,117, 119,...,156	t=2,...,78, 80,...,117, 119,...,156	t=2,...,78, 80,...,117, 119,...,156	t=2,...,78, 80,...,117, 119,...,156	t=2,...,78, 80,...,117, 119,...,156	t=2,...,78, 80,...,117, 119,...,156
$LABSHR_t-\rho_1(LABSHR)_{t-1}$	0	$(1-\rho_1)$	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮
t=2,...,39	t=2,...,39	t=2,...,39	t=2,...,39	t=2,...,39	t=2,...,39
⋮	⋮	⋮	⋮	⋮	⋮
	0		0	0	0
$CAPSHR_t$	0	0	1	0	0
⋮	⋮	⋮	⋮	⋮	⋮
t=40,...,78	t=40,...,78	t=40,...,78	t=40,...,78	t=40,...,78	t=40,...,78
⋮	⋮	⋮	⋮	⋮	⋮
	0	0	1	0	0
$MONSHR_t-\rho_3(MONSHR)_{t-1}$	0	0	0	$(1-\rho_3)$	0
⋮	⋮	⋮	⋮	⋮	⋮
t=80,...,117	t=80,...,117	t=80,...,117	t=80,...,117	t=80,...,117	t=80,...,117
⋮	⋮	⋮	⋮	⋮	⋮
	0	0	0		0
$\ln q_t-\rho_4(\ln q)_{t-1}$	$(1-\rho_4)$	$\ln L_t-\rho_4(\ln L)_{t-1}$	$\ln K_t-\rho_4(\ln K)_{t-1}$	$\ln M_{(i=1,2,3)t}-\rho_4(\ln M(i=1,2,3)_{t-1}$	Time Trend
⋮	⋮	⋮	⋮	⋮	⋮
t=119,...,156	t=119,...,156	t=119,...,156	t=119,...,156	t=119,...,156	t=119,...,156
⋮	⋮	⋮	⋮	⋮	⋮

C. Technique Utilized to Eliminate Heteroskedasticity from the Cobb-Douglas Structural Model:

As mentioned in footnote 178 of Chapter five, to test equations V.35, V.36, and V.37 for heteroskedasticity the disturbance terms within each block of data used to estimate these stacked regressions were assumed to be homoskedastic. F-tests were used to determine whether the standard errors of the regression s_j, estimated over each block of data from equations V.35, V.36, and V.37, are equivalent across these blocks of data.[3] The standard errors of the regression estimated over each of the four blocks of stacked data are as follows:

	Equation 35 (With m_1)	Equation 36 (With m_2)	Equation 37 (With m_3)
S_1	0.02263	0.02341	0.02408
S_2	0.01577	0.01546	0.01531
S_3	0.01173	0.01445	0.01580
S_4	0.05336	0.05339	0.05339

These results indicate that the standard errors of the regression estimated over the fourth block of data for equations V.35, V.36, and V.37 are significantly larger than the standard errors of the regression estimated over the

[3]That is, we used the F-test to determine whether $S_1=S_2=S_3=S_4$ in the four blocks of data used to estimate the stacked regressions V.36, V.37, and V.38 respectively.

other three blocks of data. The fourth data block corresponds to the production function in the Cobb-Douglas structural model (equation V.30)' whereas the first three blocks of data correspond to the factor input decision equations: (equations V.27', V.28', and V.29'). The larger standard errors of the production function equation will tend to make the estimates reported in Table V-10 depend more on this latter equation (block four) than is efficient. Consequently, the available information on the input cost shares does not receive the attention it deserves. To eliminate this distortion the data already adjusted to correct for autocorrelation is transformed by a heteroskedasticity correction factor, $k_j = S_4/S_j$ where j=1,2,3,4 corresponds to each block of data used to estimate the stacked regression equations. Hence the following equation is estimated:

$$(V.37)^{*} \quad Y^{*}_{it} = \hat{\hat{\hat{\ln A}}} X^{*}_{1it} + \hat{\hat{\hat{\alpha}}} X^{*}_{2it} + \hat{\hat{\hat{\beta}}} X^{*}_{3it} + \hat{\hat{\hat{\gamma}}} X^{*}_{4it} + \hat{\hat{\hat{\lambda}}} X^{*}_{5it} + e^{*}_{it}$$

where i refers to equations including m_1, m_2, m_3 respectively

t=(3,...,39,42,...,78,81,...,117, 120,...,156)

The variables included in (V.37)* were transformed in the following manner.[4]

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[4]A similar technique is suggested by Jan Kmenta, Op. cit., pp. 264-267.

$$Y^*_{it} = k_{ij}(Y_{it} - \hat{\rho}_{ij}Y_{it-1})$$

$$X^*_{1it} = k_{ij}(X_{1it} - \hat{\rho}_{ij}X_{1it-1})$$

$$\vdots$$

$$X^*_{4it} = k_{ij}(X_{4it} - \hat{\rho}_{ij}X_{4it-1})$$

$$X^*_{5it} = X_{5it}$$

where
i=1,2,3 referring to variables computed with m_1, m_2, m_3
j=1;t=3,...,39
j=2,t=42,...,78
j=3,t=81,...,117
j=4,t=120,...,156

The estimated parameters from equation (V.37)*; ($\hat{\hat{\ln A}}, \hat{\hat{\alpha}}, \hat{\hat{\beta}}, \hat{\hat{\gamma}}, \hat{\hat{\lambda}}$), were used to determine "second-round" estimates of S_j.[5] These values converged after this second round estimation indicating that the heteroskedasticity had been eliminated. The estimated parameters, ($\hat{\hat{\ln A}}, \hat{\hat{\alpha}}, \hat{\hat{\beta}}, \hat{\hat{\gamma}}, \hat{\hat{\lambda}}$) are reported in Table V-11.

II. Technical Description of the Methods Used to Estimate the Four-Equation Translog Structural Model

A. Stacking Procedure

The stacking procedure used to estimate the Cobb-Douglas model was also used to estimate the translog model. To simultaneously estimate equations V.57', V.58', and V.59' presented on pages 131 and 132 of Chapter Five, the data was

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[5]To generate the "second-round" estimates of S_j we determined the estimated value of the disturbance terms in each of the stacked regression equations corrected for autocorrelation (V.35, V.36, and V.37 reported in Table V.10) by creating the variable $e_{it} = Y_{it} - (\hat{\ln A}X1_{it} + \hat{\alpha}X2_{it} + \hat{\beta}X3_{it} + \hat{\gamma}X4_{it} + \hat{\lambda}X5_{it})$. We then regressed this disturbance term against a constant term over the four sample spaces; 3,...,39;42,...,78;81,..., 117;120,...,156 to get a second-round estimate of Sj for each of these four blocks of data: i.e. we used the standard error of the regression reported in these four regression equations.

stacked to simultaneously estimate these three equations in one regression equation. By stacking the data regression equation V.60 was estimated with m_1, m_2, and m_3.

$$(V.60)\quad Y_{i=1,2,3}, = \ln\hat{\alpha}_0 X1 + \hat{\alpha}_2 X2 + \hat{\alpha}_3 X3_{i=1,2,3} + \hat{\gamma}_{22} X4 + \hat{\gamma}_{23} X5_{i=1,2,3} + \hat{\gamma}_{33} X6_{=1,2,3} + \hat{\lambda} X7 + \varepsilon_5$$

where

i = 1,2,3 indicates that these variables have been created with the three different definitions of real money balances; m_1, m_2, m_3.

To describe the manner in which these new variables were generated, equation (V.60) is rewritten in matrix notation on the next page of this Appendix. Variables Yi, X1, X2, $X3_i$, X4, $X5_i$, $X6_i$ and X7 are placed above their corresponding column vectors. When the variable is not affected by changes in the definition of money, the subscript i=1,2,3, is not included.

OLS and TSLS parameter estimates obtained by estimating this stacked regression equation with m_1, m_2, and m_3 respectively are reported in Tables V-15 and V-18 at the end of chapter five. Tests were made on these OLS and TSLS regression equations for the existence of autocorrelation and heteroskedasticity. The disturbance terms in all of these equations, (eqs. V.61, .62, .63 in Table V-15 and eqs. V.70, .71, and .72 in Table V-18), were both autoregressive and

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Y_{it} i=1,2,3 t=1,...,117	= $\ln\hat{\alpha}_0 X_t$ t=1,...,117	+ $\hat{\alpha}_2 X2_t$ t=1,...,117	+ $\hat{\alpha}_3 X3_{it}$ i=1,2,3 t=1,...,117	+ $\hat{\gamma}_{22} X4_t$ t=1,...,117	+ $\hat{\gamma}_{23} X5_{it}$ i=1,2,3 t=1,...,117	+ $\hat{\gamma}_{33} X6_{it}$ i=1,2,3 t=1,...,117	+ $\hat{\lambda} X7$ t=1,...,117
	= $\ln\hat{\alpha}_0$	+ $\hat{\alpha}_2$	+ $\hat{\alpha}_3$	+ $\hat{\gamma}_{22}$	+ $\hat{\gamma}_{23}$	+ $\hat{\gamma}_{23}$	+ λ
$CAPSHR_{it}$	0	1	0	$(\ln X_{Kt} - \ln X_{Lt})$	$(\ln Xm_{it} - \ln X_{Lt})$	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
i=1,2,3 t=1,...,39	t=1,...,39	t=1,...,39	t=1,...,39	t=1,...,39	i=1,2,3 t=1,...,39	t=1,...,39	t=1,...,39
⋮	0	1	0	⋮	⋮	⋮	0
$MONSHR_{it}$	0	0	1	0	$(\ln X_{Kt} - \ln X_{Lt})$	$(\ln X_{mit} - \ln X_{Lt})$	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
i=1,2,3 t=40,...,78	t=40,...,78	t=40,...,78	t=40,...,78	t=40,...,78	t=40,...,78	i=1,2,3 t=40,...,78	t=40,...,78
⋮	0	0	1	0	⋮	⋮	⋮
$\ln q - \ln X_L$	1	$(\ln X_{Kt} - \ln X_{Lt})$	$(\ln X_{mit} - \ln X_{Lt})$	$(1/2 \ln X_{Kt} \ln X_{Kt}$	$(\ln X_{Lt} \ln X_{Lt}$	$(1/2 \ln X_{mit} \ln X_{mit}$	0
.	.	.	.	$+1/2 \ln X_{Lt} \ln X_{LT}$	$+\ln X_{Kt} \ln X_{mit}$	$+1/2 \ln X_{Lt} \ln X_{Lt}$	1
.	.	.	.	$-\ln X_{Lt} \ln X_{Kt})$	$-\ln X_{Lt} \ln X_{mit}$	$-\ln X_{mit} \ln X_{Lt})$	2
.	.	.	.	.	$-\ln X_{Lt} \ln X_{Kt})$	.	3
t=79,...,117	t=79,...,117	t=79,...,117	i=1,2,3 t=79,...,117	t=79,...,117	i=1,2,3 t=79,...,117	i=1,2,3 t=79,...,117	t=79,...,117
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
.	1	.	.	.	.	.	39

heteroskedastic. The same procedure described in the Cobb-Douglas section of this Appendix was followed to eliminate these statistical problems.

B. Technique Utilized to Eliminate Autocorrelation from the Translog Structural Model

To eliminate autocorrelation, an iterative regression technique was used to estimate a unique value of ρ_{ij} in each of the three blocks of data used to esimate the translog model. Hence, the following equation

$$\varepsilon_{it} = c + \rho_{ij}\varepsilon_{it-1} + \mu$$

where
i=61,62,63,70,71, and 72 referring to the regression equations in Table V-15 and V-18.
j=1,2,3 referring to the blocks of data used to estimate these regression equations.

was estimated over each block of data used to estimate the equations reported in Tables V-15 and V-18. For the OLS regression equations (Table V-15) three iterations were needed for equations 61 and 63 before the values of $\hat{\rho}_{ij}$ converged and for equation 62 four estimates were made for each block of data before the estimates converged. The final estimates are as follows:

(sample 2,...,39)	$\hat{\hat{\rho}}_{61,1}=-0.371$	$\hat{\hat{\rho}}_{62,1} = -0.3409$	$\hat{\hat{\rho}}_{63,1}=-0.3207$
(sample 41,...,78)	$\hat{\hat{\rho}}_{61,2}= 0.8035$	$\hat{\hat{\rho}}_{62,2} = 0.8041$	$\hat{\hat{\rho}}_{63,2}= 0.7496$
(sample 80,...,117)	$\hat{\hat{\rho}}_{61,3}= 0.9068$	$\hat{\hat{\rho}}_{62,3} = 0.9064$	$\hat{\hat{\rho}}_{63,3}= 0.9059$

For the TSLS regression equations (Table V-18) three iterations were needed before the values of ρ_{ij} in equation V.70 converged but only two iterations were needed before ρ_{ij} converged for equations V.71, and V.72. The final estimates are:

(sample 2,...,39)	$\hat{\hat{\rho}}_{70,1} = -0.2400$	$\hat{\hat{\rho}}_{71,1} = -0.5245$	$\hat{\hat{\rho}}_{72,1} = -0.5453$
(sample 41,...,78)	$\hat{\hat{\rho}}_{70,2} = 0.8624$	$\hat{\hat{\rho}}_{71,2} = 0.8909$	$\hat{\hat{\rho}}_{72,2} = 0.8496$
(sample 80,...,117)	$\hat{\hat{\rho}}_{70,3} = 0.8616$	$\hat{\hat{\rho}}_{71,3} = 0.8645$	$\hat{\hat{\rho}}_{72,3} = 0.8843$

These above estimates of ρ_{ij} were used to transform variables Y_{it}, $X1_t$, $X2_t$, $X3_{it}$, $X4_t$, $X5_{it}$, $X6_{it}$ and X7 described on page A-15 of this Appendix. To eliminate autocorrelation we created six new variables

$$Y1'_{it} = Y_{it} - \rho_{ij}Y_{it-1}$$
$$X1'_{it} = X1_t - \rho_{ij}X1_{t-1}$$
$$\vdots$$
$$X6'_{it} = X6_{it} - \rho_{ij}X6_{it-1}$$
$$X7'_t = X7_t - \rho_{ij}X7_{t-1}$$

These new variables were used to estimate the stacked regression equations to eliminate autoregressive disturbances. The OLS and TSLS results are reported in Tables V-16 and V-19 respectively at the end of chapter five.

C. Technique Utilized to Eliminate Heteroskedasticity from the Translog Structural Model

The iterative technique described on pages A-13 through A-15 in the Cobb-Douglas section of this Appendix was also used to eliminate heteroskedasticity. For each of the three blocks of data (j=1,2,3) used to estimate equations V.64, V.65, V.66, and equations V.73, V.74, and V.75 the standard error of the regression (S_{ij}) was estimated over each of these blocks of data. An F-test was used to determine whether these standard errors for each of the estimated equations were equal (i.e. an F-test was used to determine whether $S_{i1}=S_{i2}=S_{i3}$ for all i=V.64,V.65,V.66,V.73,V.74,V.75). As with the Cobb-Douglas model, the hypothesis that these standard errors were equal was rejected for each of these equations. The standard errors estimated over the third block of data (i.e. the data for the production function equation) were significantly larger than the standard errors estimated over the other two blocks of data. To eliminate this distortion several new variables were created by transforming the variables already adjusted to eliminate autocorrelation by the correction factor $k_{ij} = S_{i3}/S_{ij}$ (where i=V.64,V.65,V.66, V.73,V.74,V.75 and j=1,2,3). In other words the following new variables were created.

$Y^*_{it} = k_{ij} \cdot Y'_{it}$

$X1^*_{it} = k_{ij} \cdot X1'_t$

.

.

.

$X6^*_{it} = k_{ij} \cdot X6'_t$

$X7^*_t = k_{ij} X7'_t$

where
$j=1, t=2,\ldots,39$
$j=2, t=41,\ldots,78$
$j=3; t=80,\ldots,117$

These transformed variables were used to estimate the following stacked regression equation with OLS and TSLS regression techniques.

$$Y^*_{it} = \ln\hat{\hat{\alpha}}_o X1^*_{it} + \hat{\hat{\alpha}}_2 X2^*_{it} + \hat{\hat{\alpha}}_3 X3^*_{it} + \hat{\hat{\gamma}}_{22} X4^*_{it} + \hat{\hat{\gamma}}_{23} X5^*_{it} + \hat{\hat{\gamma}}_{33} X6^*_{it} + \varepsilon^*$$

where
i=V.73, V.74, V.75 with m_1, m_2, m_3 respectively
$t=2,\ldots,39, 41,\ldots,78, 80,\ldots,117$

The results obtained from estimating this equation with m_1, m_2, and m_3 are reported in Tables V-17 and V-20 respectively.

CHAPTER VI

CONCLUSION

In this dissertation theoretical arguments and empirical evidence is presented which support the notion that money is a productive asset. Economic theory tells us that an optimizing individual will allocate his wealth such that the anticipated utility derived from the last dollar spent on each good is equated. Applying this principle to determine the demand for money we simply argue that an individual determines his cash holdings by comparing the gains from holding an additional unit of money versus the potential gains from holding the best alternative to money. Hence, we follow the tradition established by Ludwig von Mises and formalized by J.R. Hicks that the determinants of the demand for money are no different from the determinants of any other consumer or producer good; both are based upon marginal considerations.

The above analysis suggests that if an individual is willing to hold money, the marginal yield from the last unit held is at least large enough to compensate for the best available opportunity foregone. Since barriers to decreasing or increasing one's cash holdings are minimal, the level of cash balances held must be considered optimal from the individual holder's perspective. If we view money as a flow of utility services, the marginal utility derived from the last unit of money held, which provides the consumer with funds to spend elsewhere, must be at least equivalent to the utility derived from the last dollar spent on any other consumption good. Similarly, if we view cash balances as a producer's good providing services in the production process, the marginal productivity derived from the last unit of money held, which provides the producer with funds to spend elsewhere, is at least equivalent to the marginal productivity derived from the last unit of any other factor of production being used; otherwise it would not be held. Although the above discussion may appear rather obvious to the modern economist since it merely applies basic economic principles to analyze the demand for money, the question of whether money can be analyzed like any other good or factor input has been, and still remains, an important issue in monetary economics. More specifically, the view that money is a productive asset, providing a positive non-pecuniary yield to its holders, has not been universally held. Although virtually all economists agree that monetary exchange is more efficient than barter exchange, many (noted) economists have interpreted cash holdings as an unproductive form of hoarding which provides services to its holders when it circulates as a medium of exchange but which is unproductive during the time it is being held. This position that money has "exchange value" but no "use value" was generally accepted by the Classical economists and has also had a significant impact on modern monetary theory. In this thesis we examine several reasons why this view has been accepted and point out why this theoretical interpretation of money is invalid.

In the second chapter of this dissertation we briefly reiterate the reasons why money is a productive asset. We point out that the use of money as a medium of exchange minimizes the transaction and information costs associated with exchange by obviating the double coincidence of wants characteristic of barter exchange. By providing an efficient instrument for economizing on the resources expended in the search-bargain process of exchange, monetary exchange frees both labor and capital from the process of distribution to the process of production. In addition, since money is a standardized asset which everyone is willing to accept as a means of payment, it increases the possibility of deferred payments, borrowing, and credit and hence affects the intertemporal allocation of resources by enabling an economy to sustain a higher rate of capital accumulation. Monetary exchange also provides additional opportunities for professional middlemen and traders. Hence money is a productive asset both as a resource saving device and as a means for stimulating market activity.

In addition to arguing that money is productive as a medium of exchange, we also support the position that in a world of uncertainty with positive transaction and information costs an optimizing individual may also hold money as a productive form of protection against unforeseen relative price changes even though relatively riskless interest-bearing monetary assets are also available for this purpose. Following J.R. Hicks, we argue that since an interest-bearing monetary asset is not a viable medium of exchange, the transaction costs of moving from interest-bearing monetary assets into money may outweigh the return from investing money over short periods of time.[187] Continuing with this line of reasoning, we argue that it is theoretically invalid to distinguish between a productive transactions demand for money and an unproductive precautionary and speculative demand for money.[188]

Most economists accept the argument that the transactions demand for money is productive. They recognize that an individual's cash receipts and expenditures will in general not be perfectly synchronized and for this reason individuals will want to meet these discrepancies by holding a sufficient

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[187] J.R. Hicks, "A Suggestion for Simplifying the Theory of Money," Reprinted in Critical Essays in Monetary Theory by J.R. Hicks (Oxford: Clarendon Press, 1967): 67.

[188] J.M. Keynes makes this distinction in the General Theory when he argues that cash balances held for reasons other than transaction purposes should be considered an unproductive form of hoarding which does not provide its holders with a positive yield. See The General Theory of Employment, Interest, and Money (New York: Harcourt, Brace, and World, Inc., 1964): 174 and 226.

quantity of money to cover required expenditures.[189] However, since interest-bearing monetary assets provide a close substitute for speculative or precautionary balances, many economists, including J.M. Keynes, have argued that these balances are <u>unproductive</u> and should be discouraged. If we accept the assumption that man is a rational, optimizing being who attempts to allocate his wealth in a manner which will maximize his well-being, it seems rather inconsistent to argue that cash balances held for speculative or precautionary purposes are unproductive "providing a yield of nil" while cash balances held for transaction purposes will provide a positive nonpecuniary yield to its holders.[190] Regardless of the purpose for which these cash balances are held, they must provide a positive marginal yield or they would not be held. Hence to argue that speculative or precautionary balances are unproductive one must be willing to argue that when determining these cash holdings man behaves irrationally and is willing to hold unproductive cash balances, that he is unable to perceive that these cash balances do not provide a positive yield, or that he is completely satiated and hence the opportunity cost of holding the unproductive speculative or precautionary cash balances is zero.

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[189] Don Patinkin uses this arguemnt as an explanation for why individuals hold money in <u>Money, Interest, and Prices: An Integration of Monetary and Value Theory</u>, Second edition, (New York: Harper and Row Publishers, 1965): 14.

[190] Here we are arguing against the position set forth by J.M. Keynes in the <u>General Theory</u>, <u>Op. cit</u>., p. 226.

Since it is difficult to accept any of these arguments as providing a viable explanation for why economists did accept that view that money is an unproductive asset, the question remains as to why this view persisted. On this point we argue that there are two characteristics, unique to money, which have perpetuated the confusion concerning the yield from money. First, money is different from all other assets and goods in that an increase in the nominal stock of money within an economic system does not necessarily imply that an increase in the wealth, welfare, or total utility in that system will follow. On the contrary, an increase in the nominal stock of money may purely result in a rise in the price level in that economy having no effect on the real value of money or the amount of monetary services rendered in the economy. The fact that increasing the supply of money (i.e. the number of units of money) in an economy does not increase the total utility in that economy may have encouraged the Classical economists to believe that cash balances did not provide a positive yield (utility) to its holders. In addition to this, the fact that money is the medium of exchange, used to purchase other goods and services, has also encouraged economists to argue that it has no utility apart from its exchange value i.e. money derives its value from the goods purchased by it. This argument, however, ignores the subtle distinction between holding money so that it can be exchanged for other items and holding money in order to be in a position to acquire things at the most profitable or most convenient time. Once this is recognized, it becomes obvious that money provides services to its holders during the time it is being held, not just when it circulates as a medium of exchange.

Although J.R. Hicks effectively dispelled the notion that money has no marginal utility apart from its exchange value, the Keynesian distinction between the productive transactions demand for money and unproductive speculative and precautionary demand for money has had a significant impact on the development of modern monetary theory. This distinction has encouraged many modern theorists, including James Tobin, to develop theoretical models which suggest that policies aimed at discouraging individuals from holding money can promote economic growth.

In the third chapter we present a critical analysis of the neo-classical money growth model James Tobin developed in his 1965 article "Money and Economic Growth."[191] In this article, Tobin comes to the conclusion that the level of real capital accumulation in a monetary economy will always be lower than the level which would be sustained in a non-monetary economy since individuals will substitute cash holdings for real capital accumulation. This conclusion, however, depends upon several restrictive assumptions upon which his model is based.

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[191]James Tobin, "Money and Economic Growth," Econometrica 33 (October, 1965): 671-684.

Tobin assumes that

(1) savers and investors are identical

(2) the marginal propensity to save is constant and equivalent in a monetary and non-monetary economy

(3) individuals will perceive their cash holdings as a form of savings but these money savings will not contribute to real capital accumulation.

In addition, he implicitly assumes that

(4) monetary and non-monetary exchange are equally productive.

The major problem with these assumptions is that they do not accurately depict a monetary economy. Individuals have no need to hold cash balances as a store of value or as a medium of exchange in Tobin's model. They have the option of holding interest-bearing monetary or non-monetary financial claims on perfectly divisible physical assets which accurately reflect both the real return on capital and the rate of inflation. In addition, the production function included in the model is equivalent to that used in non-monetary growth models such as that developed by Robert Solow. Hence Tobin implicitly assumes that monetary and non-monetary exchange are equally productive. Finally, the model does not accurately consider the impact which financial intermediaries can have on capital accumulation. In essence, the assumptions upon which the model is based do not account for the fact that positive transaction and information costs do exist and that we do not live in a world of perfect certainty. Accordingly, the implication derived from the model that the long-run steady-state rate of capital accumulation is always lower in an economy where individuals hold money, and is maximized when no money is held, is invalid. Similarly, the policy implication that the rate of capital accumulation in an economy can be increased by decreasing the real yield on money through inflation comes under question.

In chapter four we reproduce the two money growth models developed by David Levhari and Don Patinkin in their article "The Role of Money in a Simple Growth Model."[192] One model treats money as a consumer's good and includes the imputed non-pecuniary services money provides its holders in the definition of disposable income included in the model. The second model treats money as a producer's good with the services provided by money reflected in the production function by including a real money variable as a factor input. We contrast the Patinkin/Levhari growth model with Tobin's growth model. By merely introducing the imputed income from holding cash balances or by incorporating the productivity gains derived from money into the production function included in their model, while retaining all of the other restrictive assumptions of Tobin's model, Patinkin and Levhari show that Tobin's conclusion that the long-run equilibrium rate of capital accumulation in a monetary economy can be increased by decreasing the yield on money

[192]Don Patinkin and David Levhari, "The Role of Money in a Simple One-Sector Growth Model," reprinted in Studies in Monetary Theory, by Don Patinkin (New York: Harper & Row Pub., 1972): 205-242.

can no longer be unambiguously derived. By altering Tobin's model in a manner which provides a rational for holding money, Patinkin and Levhari point out that the paradoxical conclusion derived from Tobin's model no longer holds.

This debate concerning the relationship between the yield on money and the rate of capital accumulation in an economy is still unresolved. In part, it is reflected in the disagreement over whether it is theoretically valid to include a real money variable in the production function. Some economists still argue that although money may provide utility to its holders it should not be included as a factor input in the production process. We have already overviewed the theoretical reasons why we disagree with this view. However, in order to provide some empirical evidence on the productivity of money we empirically investigate whether it is reasonable to include a real money variable in a production function.

Following Patinkin and Levhari we assume that all real money balances are held by the business sector and construct two structural models which attempt to capture the manner in which money affects the production process. The models are constructed using two different production functions; the Cobb-Douglas and the translog functional forms. Each structural model consists of four equations; a production function along with three factor demand equations derived from the production function.

In constructing both of these models we assumed that the quantity demanded of all three factor inputs (capital, labor, and real money balances) is determined by marginal considerations; i.e. we assumed that a firm will employ factor inputs in its production process up to the point where the marginal cost of using an extra unit of an input is equal to the marginal product derived from using it. By also assuming that markets are perfectly competitive and that firms are profit maximizers, we equated the price of each input with the value of the marginal product derived from using it. We refer to these factor demand equations as "decision equations."

Based upon the above assumptions the two following models were constructed:

<u>Cobb-Douglas Model</u>:

The Cobb-Douglas production function is written

$$q = Ae^{\lambda T}L^{\alpha}K^{\beta}m^{\gamma}$$

where

q = output	A = efficiency parameter
L = labor	λ = technological parameter
K = capital	α = elasticity of output w.r.t. labor

m = real money balances

T = time trend, proxy for technological change

β = elasticity of output w.r.t. capital

γ = elasticity of output w.r.t. real money balances

Given the above mentioned assumptions, and rearranging terms, we derived three profit maximizing decision equations from this function.

$$(1)' \quad (P_L \cdot L)/(P_q \cdot q) = \alpha$$

$$(2)' \quad (P_K \cdot K)/(P_q \cdot q) = \beta$$

$$(3)' \quad (P_m \cdot m)/(P_q \cdot q) = \gamma$$

Written in this form the equations indicate that the relative cost shares of the inputs to the total cost of production are equal to the elasticity of output with respect to each input. These equations together with the Cobb-Douglas production function written in log linear form represent the Cobb-Douglas structural model.

$$(4)' \quad \ln q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln m + \lambda T$$

Translog Model:

The translog production function can be written as

$$\ln q = \ln \alpha_o + \sum_{i=1}^{3} \alpha_i \ln X_i + 1/2 \sum_{i=1}^{3} \sum_{j=1}^{3} \gamma_{ij} \ln X_i \ln X_j + \lambda T$$

if we assume that it exhibits constant returns to scale and that it is Hicks-neutral with respect to technological change.

where

q = output

X_i = flow of input services (K,L,M/P=m)

T = time trend, proxy for technological change

$\alpha_o, \lambda, \alpha_i, \gamma_{ij}$ = technological parameters and we assume that

$\gamma_{ij} = \gamma_{ji}$

Applying the same assumptions used to derive the Cobb-Douglas model we get the three following decision equations.

$$(1)^* \quad (P_L \cdot X_L)/(P_q \cdot q) = \alpha_1 + \gamma_{11} \ln X_L + \gamma_{12} \ln X_K + \gamma_{13} \ln X_m$$

$$(2)^* \quad (P_K \cdot X_K)/(P_q \cdot q) = \alpha_2 + \gamma_{12} \ln X_L + \gamma_{22} \ln X_K + \gamma_{23} \ln X_m$$

$$(3)^* \quad (P_m \cdot X_m)/(P_q \cdot q) = \alpha_3 + \gamma_{13} \ln X_L + \gamma_{23} \ln X_K + \gamma_{33} \ln X_m$$

These three decision equations together with the simplified translog production function make up our four equation trans-log model.

$$(4)^* \quad \ln q = \ln \alpha_o + \sum_{i=1}^{3} \alpha_i \ln X_i + 1/2 \sum_{i=1}^{3} \sum_{j=1}^{3} \gamma_{ij} \ln X_i \ln X_j + \lambda T$$

In chapter five we estimated the above two models to provide empirical evidence on the productivity of money and to determine whether it seems reasonable to include a real money variable in a production function. In so doing, we extended the empirical work initiated by Allan Sinai and Houston Stokes in their 1972 article, "Real Money Balances:

An Omitted Variable From the Production Function?"[193] In this article Sinai and Stokes provide evidence which supports the notion that a real money variable should be included as a factor input in a production function. To determine whether we could reproduce the results Sinai and Stokes reported in their article we first re-estimated a single equation Cobb-Douglas production function with non-constant returns to scale and neutral technological change in log linear form. This equation was first estimated using the OLS regression technique but a low Durbin-Watson statistic suggested that the disturbance terms were auto-correlated. To correct this problem the Cobb-Douglas production function was re-estimated using the Cochrane-Orcutt iterative technique. The regression results obtained with this latter technique were as follows: (standard errors are presented in parentheses below the coefficients and an * indicates that the estimated coefficient is statistically significant at a 95 percent confidence level).

$$\underset{}{\ln q} = \underset{(8.0)}{-15.4} + \underset{(.131)}{.969^*\ln L} + \underset{(.121)}{.377^*\ln K} + \underset{(.055)}{.145^*\ln m_1} + \underset{(.004)}{.008^*T}$$

$$R^2 = .9977 \quad S.E.E. = .0234 \quad D.W. = 1.236$$

$$\ln q = \underset{(9.1)}{-14.8} + \underset{(.120)}{1.096^*\ln L} + \underset{(.118)}{.298^*\ln K} + \underset{(.077)}{.144^*\ln m_2} + \underset{(.005)}{.008T}$$

$$R^2 = .9975 \quad S.E.E. = .0244 \quad D.W. = 1.132$$

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[193] Allan Sinai and Houston H. Stokes, "Real Money Balances: An Omitted Variable from the Production Function," Review of Economics and Statistics (August, 1972):290-296.

$$\underset{(9.9)}{\ln q = -16.8} + \underset{(.121)}{1.147*\ln L} + \underset{(.120)}{.259*\ln K} + \underset{(.088)}{.109*\ln m_3} + \underset{(.005)}{.009*T}$$

$$R^2 = .9974 \quad S.E.E. = .0208 \quad D.W. = 1.096$$

These results coincide fairly closely with those reported by Sinai and Stokes.[194] The estimated coefficients for all of the three definitions of real money balances (m_1, m_2, m_3) were positive and significantly correlated with real output. Similarly, the labor and capital coefficients were also positive and significant. The technology variable was consistently positive but was not significantly different from zero when estimated with m_2. These results, however, like the parameter estimates obtained by Sinai and Stokes, may be biased due to a simultaneïty bias. When estimating an aggregate production function, one can argue that the amount of labor, capital, and real money balances used in the production process both affects and is affected by the level of real output. Hence one can view the factor inputs included in this production function as endogenous variables.

To correct the parameter estimates for the possibility of simultaneous equation bias, we followed a procedure used by Berndt and Christensen in their article, "The Translog Function and the Substitution of Equipment, Structures, and Labor in U.S. Manufacturing, 1928-68."[195] We assumed that

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[194] See Table V-2 on page 159 in Chapter Five.

[195] Erndt R. Berndt and Laurits R. Christensen, "The Translog Function and the Substitution of Equipment, Structures, and Labor in U.S. Manufacturing 1928-68," Journal of Econometrics 1 (March, 1973): 93-94.

the following variables are exogenous to the U.S. manufacturing sector: U.S. population, U.S. population of working age, effective rate of sales and excise taxation, effective rate of property taxation, government purchases of non-durable goods and services, government purchases of labor services, real exports of durable goods, and real exports of non-durable goods and services. We used these variables together with the real stock of money ($m_{i=1,2,3}$) lagged four periods (annual data) to estimate the Cobb-Douglas production function using the instrumental variables regression technique (TSLS) and, since the disturbance terms were autocorrelated, the two-stage Cochrane-Orcutt iterative regression technique.

After correcting for autocorrelation and the possibility of simultaneity bias the signs of the two stage parameter estimates remained unchanged and they were still statistically significant. These results support the conclusion that real money balances are a productive asset which had a significant effect on the level of real output in the United States over the period 1929-67. However, we considered these results preliminary since in this single equation production function no attempt was made to incorporate behavioral relationships which attempt to describe the manner in which firms utilize factor inputs in their production processes. Although the TSLS estimation does not entirely ignore the structure of production, the single equation estimates do not incorporate all of the information available to us. To account for this problem, we also estimated the more complete Cobb-Douglas and translog structural models developed in this study. Since the parameter estimates (α, β, γ) in the Cobb-Douglas model and the parameters directly estimated for the translog model (α_0, α_2, α_3, γ_{22}, γ_{23}, and γ_{33}), appear in more than one of the four equations in each of these structural models, we used a 'stacked' regression technique to constrain the parameters appearing in separate equations to be equal across equations. This was accomplished by stacking the variables appearing in the four equations in each of these models to enable us to estimate these structural models as single equations.[196]

To estimate the Cobb-Douglas model we created ten new variables (including three real money variables which differ according to the definition of money used i.e. M_1, M_2, M_3) by stacking the data on each of the variables included in this four equation model. By stacking the data in this manner we were able to simultaneously estimate all of the parameters in the model by estimating the following single regression equation

$$(5)' \quad Y_{i=1,2,3} = \hat{\ln A}X1 + \hat{\alpha}X2 + \hat{\beta}X3 + \hat{\gamma}_{i=1,2,3}X4_{i=1,2,3} + \hat{\lambda}X5 + \varepsilon_{i=1,2,3}$$

where i=1,2,3 refers to the three definitions of money used when constructing these variables

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[196]For a detailed description of this stacking procedure see pages A-1 through A-6 and A-15 through A-17 in the Appendix to Chapter Five.

rather than estimating each of the four equations included in the Cobb-Douglas model ((1)', (2)', (3)', (4)') separately. This equation was estimated using three different definitions for the real money variable.

We used both a one and a two stage regression technique to estimate this model.[197] When using either of these regression techniques the disturbance terms were both autocorrelated and heteroskedastic.[198] We used an iterative OLS regression technique to determine a separate estimate of $\hat{\rho}$ to eliminate autocorrelation from each of the four blocks of data used to estimate this stacked regression equation.[199] After adjusting the data to correct for autocorrelation, we used an iterative procedure to determine a correction factor which would make the disturbance terms homoskedastic across each block of data used to estimate the regression equation.[200]

After eliminating both of these statistical problems, the empirical results obtained by using either the OLS or

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[197]OLS and TSLS results for the Cobb-Douglas model are reported on pages 168 through 171 at the end of Chapter Five.

[198]For a description of the method used to detect these statistical problems in our stacked regression equations see pages A-5 through A-9 and pages A-12 and A-13 in the Appendix to Chapter Five.

[199]For a description of the iterative technique used to eliminate autocorrelation from our stacked regression equations see pages A-9 through A-12 in the Appendix to Chapter Five.

[200]For a description of the iterative technique used to eliminate heteroskedasticity see pages A-13 through A-15 in the Appendix to Chapter Five.

the TSLS regression techniques were consistent with our theoretical expectations. All of the estimated coefficients $(\alpha, \beta, \gamma_i, \lambda)$ were positive and statistically significant at a 95 percent confidence level regardless of the definition of real money used in the estimation. Using the TSLS estimates we obtained from the stacked regression equation (5)' with m_1, the Cobb-Douglas structural model can be written in the following manner: (standard errors are reported in parentheses below the coefficients; * indicates statistical significance at a 95 percent or greater confidence level).

$$(1)' \quad (P_L \cdot L)/(P_q \cdot q) = \underset{(0.009)}{0.808^*}$$

$$(2)' \quad (P_K \cdot K)/(P_q \cdot q) = \underset{(0.0027)}{0.163^*}$$

$$(3)' \quad (P_m \cdot m_1)/(P_q \cdot q) = \underset{(0.0078)}{0.025^*}$$

$$(4)' \quad \ln q = \underset{(0.047)}{-0.139^*} + \underset{(0.009)}{0.808^*}\ln L + \underset{(0.0027)}{0.163^*}\ln K + \underset{(0.0078)}{0.025^*}\ln m_1 + \underset{(0.0008)}{0.0018^*}T$$

$R^2 = 0.9758$ S.E.E. = 0.05044

These results, similar to those obtained when the model was estimated using the two other definitions of real money (m_2 and m_3) suggest that the elasticity of output and relative cost share of capital, labor, and real money balances for the U.S. private domestic manufacturing factor estimated with annual data over the period 1929 to 1967 was 0.8, 0.16, and 0.3 respectively. Although the real money variable had a relatively small impact on the level of output, the coefficient was positive and significant and hence supports the hypothesis that money is a productive asset which can be included as a factor input in an aggregate production function. These results also suggest that the Patinkin/Levhari money growth model specification is preferable to the specification Tobin used in his money growth model.

To provide additional evidence on the appropriate specification of a neo-classical money growth model we also estimated the significance of real money balances in a translog production function. In specifying our translog model we assumed that the production function exhibited constant returns to scale (CRTS). Hence the relative cost shares of the three inputs included in the model (L,K,m=M/P) must sum to unity at each observation. This assumption imposed the following restrictions on the parameter estimates:

$$\text{(B)}\quad \begin{aligned} \alpha_1 + \alpha_2 + \alpha_3 &= 1 \\ \gamma_{11} + \gamma_{12} + \gamma_{13} &= 0 \\ \gamma_{12} + \gamma_{22} + \gamma_{23} &= 0 \\ \gamma_{13} + \gamma_{23} + \gamma_{33} &= 0 \end{aligned}$$

In estimating the translog model we imposed these parameter restrictions on the capital and real money decision equations ((2)* and (3)*) and the production function (equation (4)*) described on page 213 in this chapter. The estimates of the labor decision equation were derived from the parameter estimates of the other two decision equations using the above restrictions. We used iterative OLS and TSLS stacked regression techniques to eliminate autocorrelation and heteroskedasticity from the translog structural model.[201]

To impose the parameter restrictions (B) implied by the linear homogeneity assumption equations (2)*, (3)*, and (4)* had to be rewritten as

$$(2)^* \quad (P_K \cdot K)/(P_q \cdot q) = \alpha_2 + \gamma_{22}(\ln X_K - \ln X_L) + \gamma_{23}(\ln X_m - \ln X_L) + \varepsilon_2$$

$$(3)^* \quad (P_m \cdot m)/(P_q \cdot q) = \alpha_3 + \gamma_{23}(\ln X_K - \ln X_L) + \gamma_{33}(\ln X_m - \ln X_L) + \varepsilon_3$$

$$\begin{aligned}(4)^* \quad \ln q - \ln L = {} & \ln\alpha_o + \alpha_2(\ln X_K - \ln X_L) + \alpha_3(\ln X_m - \ln X_L) \\ & + \gamma_{22}(1/2 \ln X_K \ln X_K + 1/2 \ln X_L \ln X_L - \ln X_L \ln X_K) \\ & + \gamma_{23}(\ln X_L \ln X_L + \ln X_K \ln X_m - \ln X_L \ln X_m \\ & - \ln X_L \ln X_K) + \gamma_{33}(1/2 \ln X_m \ln X_m + 1/2 \ln X_L \ln X_L \\ & - \ln X_L \ln X_m) + \lambda T + \varepsilon_4\end{aligned}$$

Using a similar stacking procedure to that used to estimate the Cobb-Douglas model, we created several new variables which were used to estimate equations (2)*, (3)*, and (4)* as a single estimating equation.[202] Hence we

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[201]For a more detailed description of the iterative techniques used to eliminate autocorrelation and heteroskedasticity for the translog regression equations see pages A-18 through A-21 in the Appendix to Chapter Five.

[202]For a more detailed description of this stacking procedure see pages A-15 through A-17 in the Appendix to Chapter Five.

estimated

$$(5)^* \quad Y_{i=1,2,3} = \ln\hat{\alpha}_0 X1 + \hat{\alpha}_2 X2 + \hat{\alpha}_3 X3_{i=1,2,3} + \hat{\gamma}_{22} X4 + \hat{\gamma}_{23} X5_{i=1,2,3} + \hat{\gamma}_{33} X6_{i=1,2,3} + \lambda X7 + \varepsilon_{5_{i=1,2,3}}$$

where i=1,2,3 indicates that these variables have been created using the three different definitions of real money balances; m_1, m_2, m_3

Both OLS and TSLS iterative regression techniques were used to eliminate autocorrelation and heteroskedasticity from the estimating equations.[203]

Similar to our experience with the Cobb-Douglas model, the results we obtained from estimating this translog model were consistent with our theoretical expectations. Since the TSLS parameter estimates obtained after correcting for autocorrelation and heteroskedasticity were not significantly different from those obtained after correcting for these statistical problems using an OLS iterative technique, we will only summarize the TSLS results here, estimated with m_1.[204] Using these results we can rewrite the translog model in the following manner. (Standard errors are presented in parentheses below the estimated coefficients and an * indicates

[203]The OLS and TSLS results for the translog model are reported in Tables V-15 through V-20 on pages 172 through 177 at the end of Chapter Five.

[204]To compare these OLS and TSLS results for the translog model see Tables V-17 and V-20 on pages 174 and 177 respectively.

that the estimated coefficient is statistically significant at a 95 percent or greater confidence level.)[205]

$$(1)^* \quad (P_L \cdot L)/(P_q \cdot q) = 0.80 - 0.003\ln X_L + 0.004\ln X_K - 0.007\ln X_{m_1}$$

$$(2)^* \quad (P_K \cdot K)/(P_q \cdot q) = \underset{(0.007)}{0.160^*} + 0.004\ln X_L - \underset{(0.026)}{0.004\ln X_K} + \underset{(0.014)}{0.004\ln X_{m_1}}$$

$$(3)^* \quad (P_m \cdot m_1)/(P_q \cdot q) = \underset{(0.016)}{0.040^*} - 0.007\ln X_L + \underset{(0.014)}{0.004\ln X_K} - \underset{(0.032)}{0.011\ln X_{m_1}}$$

$$(4)^* \quad \ln q = \underset{(0.070)}{-0.213^*} + 0.80\ln X_L + \underset{(0.007)}{0.160^*}\ln X_K + \underset{(0.016)}{0.040^*}\ln X_{m_1}$$
$$- 0.003\ln X_L \ln X_L - 0.002\ln X_K \ln X_K - 0.005\ln X_{m_1}\ln X_{m_1}$$
$$+ 0.001\ln X_L \ln X_K + 0.007\ln X_L \ln X_{m_1} + 0.004\ln X_K \ln X_{m_1}$$
$$+ \underset{(0.003)}{0.007^*}T$$

$R^2 = 0.8757$ S.E.E. $= 0.0615$

Although at first glance this translog model appears rather complex, these results are consistent with those obtained by estimating the Cobb-Douglas model. The TSLS parameter estimates ($\alpha_1, \alpha_2, \alpha_3$) obtained by estimating the

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[205]Since several of the parameters in the translog model were derived from the linear homogeneity restrictions rather than by directly estimating them, standard errors are not included below each coefficient presented in these equations.

translog model with m_1 are 0.80, 0.16, and 0.04 respectively. These estimates are consistent with the corresponding (α, β, γ) parameter estimates obtained by estimating the Cobb-Douglas model; (0.81, 0.163, 0.25).[206]

In conclusion, the results obtained by estimating both of these production function models indicate that the relationship between real money and output, even after correcting for any simultaneity bias, is positive and statistically significant. This empirical evidence supports the hypothesis that money is a productive asset. It also indicates that a real money variable should be included as a factor input in a production function. Finally the results suggest that it is theoretically invalid to view cash holdings as a barren, unproductive investment.

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[206]To compare the remaining estimates derived using OLS and TSLS regression techniques see Tables V-11 and V-17 on pages 168 and 174 for the OLS parameter estimates of the Cobb-Douglas and translog models respectively and Tables V-14 and V-20 on pages 171 and 177 for the TSLS parameter estimates of these respective models.

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