Market Segmentation and Non-Parametric Asset Pricing

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Abstract

I examine market segmentation between equity and bond markets. Although under the no arbitrage principle equity and bond markets should prove integrated, real-world frictions may induce some degree of market segmentation. To assess the extent of this segmentation, I use non-parametric estimators of the stochastic discount factor (SDF). I make four contributions in this work. First, the non-parametric methods I use surmount several econometric limitations of previous such investigations. Second, I propose a novel machine learning-based SDF estimator. Third, I examine time variation in the extent of segmentation between equity and bond markets, which previous work has not empirically tested. Fourth, I use dual-asset-class SDF estimates to examine cross-asset-class trading signals, which have immediate practical applications. I find evidence of integration between equity and bond markets in the full sample. However, cross-asset-class information proves difficult to exploit out of sample in cross-sectional pricing and trading applications.

1 Introduction

Asset prices equal expected discounted future payoffs. All of asset pricing stems from this principle. In theory, under the no arbitrage condition, there exists a stochastic discount factor (SDF) - a strictly positive random variable that discounts random future payoffs - that prices all assets (Hansen & Richard, 1987). Such an SDF, if correctly estimated, should explain the variation in expected returns across any cross-section of assets. Thus, in theory, a single SDF should price the cross-section of equities and bonds. Of course, in theory there is no difference between theory and reality. In reality, on the other hand, it is possible that equity and bond markets are "segmented." Market segmentation arises when investors do not necessarily trade across asset classes, and can lead to different pricing properties. I seek to investigate the extent of market segmentation between equity and bond markets by answering the following question: Do bond yields contain information that can help explain the cross-section of equity returns and vice versa? Specifically, I use non-parametric estimates of the SDF to determine if an SDF estimated from equities and bonds possesses greater explanatory power than one derived from a single asset class.

My examination of this question interacts with three veins of the existing asset pricing literature. First is previous research on market segmentation in a variety of settings. Most prior work in market segmentation has focused on segmentation in international equity markets and segmentation across the term structure. Second is research on cross-sectional stock and bond pricing. Stocks command a risk premium because they often perform poorly in times of high marginal utility for investors, such as recessions or other negative economic events (Cochrane, 2017). Thus, to the extent bond yields capture investor expectations of future economic activity, they should serve as proxies for priced risk factors in the cross-section of equity returns (Fama & French, 1993; Koijen et al., 2017). Third is prior work on SDF estimation. Extracting the SDF from asset returns proves difficult in general due to uncertainty regarding the correct parametric form of the SDF. However, non-parametric methods, such as recent work by Ghosh et al. (2016) and Galpin et al. (2017), obviate the need to specify a functional form for the SDF, and thus avoid model misspecification.

My analysis employs modern econometric techniques to assess the extent of equity and bond market segmentation in the United States. Specifically, I apply the non-parametric SDF estimation methods of Ghosh et al. (2016) and Galpin et al. (2017), as well as a machine learning-based SDF estimation method motivated by Ghosh et al. (2016), to daily United States equity and bond data. If an SDF estimated from stock and bond returns explains more of the cross-sectional variance in stock and bond returns than does an SDF estimated from stock or bond returns alone, then information in bond yields helps price equities and vice versa. This conclusion would constitute evidence against market segmentation.

My research makes several contributions to the existing literature. First, the non-parametric methods I use can price larger cross-sections of test assets than was previously possible and preclude the need for a benchmark model. Much previous work on jointly pricing equities and bonds has focused on developing structural models of the SDF that provide economic intuition for market integration (Bakshi & Chen, 2005; Campbell et al., 2009; Lettau & Wachter, 2011). Comparatively less work has examined market segmentation from a more direct empirical standpoint (Fama & French, 1993; Koijen et al., 2017). Most of these empirical studies jointly price only a relatively small cross section of assets. Yet it is possible that models that successfully price a small cross section will fail to price a larger set of assets. Moreover, these works usually assess the marginal explanatory power of bond factors beyond a specific benchmark model (e.g. the Fama-French 3 factor model). However, with the "zoo" of factors discovered in the past twenty years, it is possible that newly discovered equity factors subsume the information provided by bond factors (Cochrane, 2011). The methods I use surmount these problems. Second, I propose a novel machine learning-based SDF estimation technique that can potentially overcome some econometric issues faced by existing non-parametric methods. Third, I examine the extent to which equity and bond markets become more segmented during stressful periods. Although some theoretical work suggests the possibility of time-varying segmentation, previous work has not empirically tested this notion. Fourth, I examine cross-asset-class trading signals based on dual-asset-class SDF estimates (e.g. a signal derived from stock and bond momentum) that have immediate practical applications. Thus, my research provides a thorough assessment of market segmentation between equity and bond markets currently missing from the existing literature.

The remainder of this proposal proceeds as follows. In Section 2 I review the existing literature on market segmentation, joint stock and bond pricing, and SDF estimation. In Section 3 I detail the methods I use and the empirical tests I conduct. In Section 4 I discuss the data I use. In Section 5 I outline my hypotheses for the tests I run. Section 7 concludes the proposal.

2 Literature Review

In this section I review the existing literature on market segmentation, joint stock and bond pricing, and SDF estimation.

2.1 Market Segmentation

Market segmentation arises when investors are limited in their ability to share risks in a particular market (Cochrane, 2011). The existing literature highlights market frictions and heterogeneous preferences as the major reasons for limited risk sharing. Previous work on market segmentation mostly focuses two specific cases: segmentation in international equity markets and segmentation in bond markets across the term structure.

2.1.1 International Equity Market Segmentation

Early work on segmentation in international equity markets postulated that market frictions due to structural barriers to the free flow of capital induced "segmentation premia" in national equity markets where global investors could not effectively arbitrage price differences away. Researchers often appealed to differences in currency areas, political regimes, trade barriers, and capital controls as drivers of segmentation (Solnik, 1974). Furthermore, empirical work found evidence of "mildly segmentation" in international equity markets. For example, Errunza & Losq (1985) found that securities with restricted access due to the above frictions earned "super risk premia." Nevertheless, as economic liberalization has removed barriers to the free flow of capital, empirical support for segmented international markets has weakened. As a result, theories of segmented markets have been replaced with equilibrium models of capital flows (Errunza & Miller, 2000; Duffie & Strulovici, 2012).

2.1.2 Segmentation Across the Term Structure

Most market segmentation research has focused on segmentation across the term structure. The earliest work in this vein dates back to Modiglini and Sutch's proposal of the preferred habitat model in the 1960's (Modigliani & Sutch, 1966, 1967). The preferred habitat model represents one of the many attempts to extend the expectations hypothesis to account for the empirical fact that the yield curve slopes upward (Gürkaynak & Wright, 2012). Under the expectations hypothesis, long run interest rates reflect future expectations of short term interest rates, so risk neutral investors should prove indifferent as to what maturity they lend at. The preferred habitat model, however, holds that many investors are risk averse and often have obligations at fixed maturities in the future (e.g. pension funds). These investors seek certainty in their ability to finance these future obligations, and hence have definite preferences over what maturities to invest at (Modigliani & Sutch, 1967). As a result, "the interest rate is determined by the supply and demand of bonds of that particular maturity" (Gürkaynak & Wright, 2012). That is, bond market segmentation arises along the yield curve due to heterogeneous preferences.

The preferred habitat model has not received much serious attention until recently due to its inability to explain why arbitrageurs don't enter the market and flatten the yield curve. Modigliani & Sutch (1967) only address this issue on the surface by appealing to transaction costs as an obstacle to such arbitrage. However, interest in the preferred habitat model has reemerged after the 2008 financial crisis due to clear empirical evidence of market segmentation that emerged in that period. Greenwood & Vayanos (2008) and Vayanos & Vila (2009) have developed a preferred habitat model in which arbitrageurs are themselves risk-averse. Others such as Fontaine & Garcia (2011) have focused on the frictions fixed income arbitrageurs face, such as liquidity constraints. Gürkaynak & Wright (2012) hypothesize that in times when transactions costs or risk aversion spike, arbitrageurs may prove insufficiently able to integrate different portions of the yield curve. For example, in late 2008, the U.S. Treasury yield curve exhibited clear segmentation, with short term notes yielding less than longer term bonds with the same maturity dates.

Thus, market segmentation can manifest even in in liquid cross-sections. No obvious frictions inhibit fixed income investors from also participating in equity markets, so one would not expect equity and bond markets to be segmented. As discussed in the Section 2.2, previous work suggests that equity and bond markets are quite integrated. Yet it is conceivable that heterogeneous investor preferences over these two asset classes paired with spikes in arbitrageur risk aversion and other frictions might induce segmentation in extreme periods, such as financial crises.

2.2 The Cross-Section of Bonds and Equities

As discussed above, under the no arbitrage principle there exists a single SDF that prices all assets, including stocks and bonds. As a result, previous theoretical and empirical work has sought to unify equity and bond pricing.

2.2.1 Theoretical and Empirical Links between Bonds and the SDF

In this particular application, factors derived from bond yields should help price the cross-section of equity returns for economically intuitive reasons. Under a time-separable utility function, the SDF represents the growth rate of an investor's marginal utility (Campbell, 2000). By construction, an asset's expected excess return is proportional to the negative of it's covariance with the SDF. Since stocks are negatively correlated with the SDF, they perform poorly when the SDF is high, or equivalently, when marginal utility is high. As a result, equities command a risk premium because they often perform poorly in times of high marginal utility for investors, such as recessions or other negative economic events (Cochrane, 2017). Thus, to the extent that factors derived from bond yields can predict future economic activity, they should inform the SDF.

A large body of empirical work finds that bond factors do forecast future economic activity. Macroeconomic shocks impact investor expectations about economic fundamentals (e.g. inflation, growth) and expectations about policy responses to these fundamentals, and thus contemporaneously affect the term structure (Campbell et al., 2009). At the same time, bond factors also predict future economic activity. Harvey (1988) establishes that the real term structure contains predictive information of consumption growth. Stock & Watson (1989) find that long-term/short-term Treasury yield spreads serve as useful leading indicators of a variety of macroeconomic variables. Many authors link term structure information to future GDP growth (Estrella & Hardouvelis, 1991; Plosser & Rouwenhorst, 1994; Haubrich & Dombrosky, 1996). In more recent work, Brooks (2011) finds that "tent factor" of Cochrane & Piazzesi (2005), a linear combination of forward rates, forecasts unemployment at quarterly frequencies. Gilchrist & Zakrajšek (2012) find that corporate bond credit spreads forecast payroll employment, unemployment, and industrial production at horizons of up to a year. Koijen et al. (2017) demonstrate that many bond factors (e.g. the Cochrane-Piazzesi tent factor, the slope factor of Litterman & Scheinkman (1991), and others) forecast future economic activity, measured by the Chicago Fed National Activity Index and GDP, at business cycle horizons of two to three years.

2.2.2 Cross-Sectional Bond and Stock Pricing

These theoretical and empirical results have motivated work on cross-sectional bond and stock pricing. This body of research provides evidence that bond factors do help explain equity returns and vice versa. I divide this work into two categories. The first is empirical investigations that conduct regressions to estimate the empirical prices of risk of various factors. Fama & French (1989) find that dividend yields forecast bond returns, while the "default-premium," the credit spread between AAA bonds and the market portfolio of corporate bonds, and the "term-premium," the spread between AAA yields and one-month Treasury yields, forecast excess equity returns. Fama & French (1993) demonstrate that the default and term premia factors add significant explanatory power to the cross-section of equity returns beyond the market, SMB, and HML factors. They also find that these equity factors help explain the cross-section of low-grade corporate bond returns. Cochrane & Piazzesi (2005) show that their tent factor predicts not only excess returns of one to five year Treasuries, but also excess stock returns at one-year horizons. Koijen et al. (2017) use bond factors to empirically explain the value premium. They demonstrate that the cash flows of value stocks have greater exposure than those of growth stocks to bond factors that predict economic activity at business cycle horizons. As a result, value stocks command a premium because their cash flows are more significantly impacted in times when investor marginal utility is already high. Thus, this body of work finds that empirically bond factors do help explain the equity returns.

The second category of work in cross-sectional stock and bond pricing focuses on structural models of equity and bond returns. These authors propose models, usually affine term structure models, of the SDF that they calibrate to observed data. They then compare the moments yielded by return series generated under the model to observed moments to assess model's realism. Compared to the purely empirical investigations discussed above, this vein of literature places more emphasis on theoretical justification for the model structure and input variables chosen. Bakshi & Chen (2005) propose a stock valuation model based on a single factor term structure model of the SDF and obtain relatively small pricing errors. Wachter (2006) proposes a consumption based model that produces realistic bond and stock volatility and a high equity premium. Bekaert et al. (2009) propose an affine term structure model of the SDF based on fundamental macroeconomic variables such as inflation and consumption growth. They find that this model matches the empirical volatilities of dividend and consumption growth, and generates a large equity premium and low risk free rate. Lettau & Wachter (2011) achieve similar results with a model parametrized to dividend growth, inflation, and short-term real rates, and in particular have some success in matching the value premium. Departing from the macroeconomic basis of most of these models, Gabaix (2012) finds that a structural model of the SDF based on rare disasters explains a variety of asset pricing puzzles, including jointly pricing stocks and bonds. Campbell et al. (2009) employ a multifactor term structure model based on real interest rates, inflation, and a state variable to explain the time-varying covariance of stock and bond returns.

The cross-sectional stock and bond pricing literature suggests Deep theoretical underpinnings for why bond factors should help price equities and vice versa. Empirical results corroborate this theory. As discussed in Section 2.3, modern econometric techniques have the ability to better quantify the extent of this cross-asset-class pricing ability.

2.3 Empirical SDF Estimation

The SDF is a fundamentally important object in asset pricing. Indeed, if one could perfectly characterize the SDF and the stochastic payoff process of a given asset, he or she could perfectly price that asset. As a result, many researchers have sought methods to estimate the SDF. The ideal SDF model would directly link asset returns to to underlying macroeconomic variables, describing the observed data well and providing theoretical insight (Campbell, 2000). Unfortunately, structurally motivated SDF models, such as those discussed in Section 2.2, can usually at best only qualitatively match certain aspects of the observed data. For example, many structural models do yield a sufficiently volatile SDF as determined by the Hansen-Jagannathan lower bound (Hansen & Jagannathan, 1991). These models cannot, however, generate return patterns that fit observed asset returns well (Cochrane, 2017). As a result, many researchers have developed methods to extract the the SDF directly from observed asset prices. Even if these methods cannot necessarily illuminate the underlying macroeconomic fundamentals of the SDF, they often prove practically useful (e.g. for risk management, derivatives pricing, and asset allocation). Moreover, these methods can help identify which systematic risk factors empirically impact asset returns.

2.3.1 SDF Estimation from Options Data

Most previous work on extracting the SDF directly from asset prices has used options data. Options prove theoretically appealing for this application because they contractually specify payoffs in different states of nature and trade on exchanges. Early work in this area designed parametric techniques to extract the state price density (closely related to SDF) from option prices (Ross, 1976; Banz & Miller, 1978; Breeden & Litzenberger, 1978). These approaches impose strict assumptions on the distribution of the SDF, and hence fall victim to the dangers of model misspecification (Hansen, 2014).

Thus, subsequent work focused on developing *non-parametric* methods that more accurately describe observed data due to their lack of unrealistic assumptions. For example, Jackwerth & Rubinstein (1996) extract the risk-neutral probability distribution, which is closely related to the SDF, from daily S&P 500 index options data. Aït-Sahalia & Lo (1998) and Aït-Sahalia & Lo (2000) develop non-parametric methods to extract the state price density, also closely related to the SDF, from options prices. Rosenberg & Engle (2002) present a non-parametric method to estimate the SDF using S&P 500 index options data and a stochastic volatility model of the S&P 500. The limitation of these models is their reliance on options data. Options are not nearly as liquid as stocks. Indeed at strike prices far from the spot price, options can be quite illiquid. Thus, data quality can significantly limit the performance of these methods. To cope with this issue, these papers use S&P 500 index options, which are perhaps the most liquid options. As a result, one cannot easily apply these methods to other stocks with less liquid options chains, and certainly not to other asset classes such as bonds, which do not have exchange traded options.

2.3.2 General Non-Parametric SDF Estimation

Fortunately, recent work has presented methods to extract the SDF from general asset return data, thereby allowing researchers to estimate the SDF directly from, for example, equity return data. Ghosh et al. (2016) provide an information-theoretic approach to estimating the SDF. Given a cross-section of assets (the authors focus on equity portfolios), the proposed method minimizes a particular entropy-based loss function (the Kullback-Leibler information Criterion), subject to the constraint that the SDF pricing equation holds. The resulting optimization problem has a solution that expresses the SDF as a nonlinear function of the Lagrange multipliers of the optimization problem and the observed asset returns. The authors construct a rolling out-of-sample estimate of the SDF by computing the Lagrange multipliers from previous data, and then calculating the SDF over the evaluation period from those out-of-sample estimates and the contemporaneous cross-sectional returns. They then use this "Information SDF" (I-SDF) and its linear projection onto the return space, called the "Information Portfolio" (I-P), as single factor pricing models. The I-SDF and I-P series allow one examine how well the SDF extracted from a given set of portfolios prices those portfolios out of sample. Ghosh et al. (2016) find that both the I-SDF and I-P cross-sectionally price the test assets better than Fama-French three-factor and Carhart four-factor models. The I-SDF provides a flexible and computationally tractable way to extract the SDF from any set of test assets. Thus, one can easily extend this analysis to other classes by, for example, including bonds in the cross-section of test assets.

Galpin et al. (2017) conduct a similar but slightly different analysis. These authors use the I-SDF of Ghosh et al. (2016), which they call the exponential tilting estimator, and another non-parametric SDF estimator motivated by the continuously-updated estimator of Hansen et al. (1996) to estimate the SDF directly from a set of systematic risk factors, not from a set of test assets. In this way, Galpin et al. (2017) obtain the Lagrange multipliers from the pricing constraints for the set of test factors as in Ghosh et al. (2016). They then test the significance of these Lagrange multipliers in the SDF via tests developed by Newey & Smith

(2004). These tests allow the authors to determine which factors add significant information to the SDF when conditioned on the other test factors. The authors argue that adding in auxiliary test assets "muddles inference," whereas their method can directly determine which factors prove economically significant. They find that four factors suffice to characterize the SDF: the market, profitability, investment, and value-profitability-momentum. This method also proves easily extensible to other asset classes.

3 Methodology

In this section I detail the methods I use and the empirical tests I conduct. To ensure robustness, I test for equity and bond market segmentation in three different ways.

3.1 Testing for Segmentation via SDF Estimation

I first detail two non-parametric SDF estimation techniques and then outline the statistical tests I use to test for market segmentation.

3.1.1 SDF Estimation Method of Ghosh et al. (2016)

Ghosh et al. (2016) use an information-theoretic approach to non-parametrically extract the SDF from a cross section of test assets. Specifically, they derive the "least-informative" SDF that prices all of the assets in the sample period. Least-informative in this context means the SDF corresponding to the risk-neutral probability measure that deviates least from the hypothetical physical probability measure, while still pricing all assets. Precisely, Ghosh et al. (2016) present the following non-parametric, maximum-likelihood estimate of the risk-neutral probability measure $\mathbb{Q}$ given the physical probability measure $\mathbb{P}$:

$$\underset{\mathbb{Q}}{\operatorname{argmin}}\, D(\mathbb{Q}|\mathbb{P}) \text{ s.t. } \mathbb{E}_{\mathbb{Q}}[\mathbf{R}_t^e] = \mathbf{0} \in \mathbb{R}^n, \tag{3.1}$$

where $\mathbf{R}_t^e \in \mathbb{R}^n$ is the vector of excess returns of our n test assets in period t, and $D(\mathbb{Q}|\mathbb{P})$ is the Kullback-Leibler Information Criterion (KLIC). The KLIC is a measure of the extent to which one probability distribution deviates from another. Thus, $\mathbb{Q}$ as given by (3.1) is the risk-neutral measure that requires the least additional information over the physical probability measure but that still prices all test assets. Some measure-theoretic simplifications to (3.1) yields the following equivalent optimization problem:

$$\underset{\{M_t\}_{t=1}^T}{\text{argmin}} \frac{1}{T}\sum_{t=1}^{T} M_t \ln(M_t) \text{ s.t. } \frac{1}{T}\sum_{t=1}^{T} M_t \mathbf{R}_t^e = \mathbf{0}, \tag{3.2}$$

where M_t is the value of the SDF at time t. As Ghosh et al. (2016) note, (3.2) has the following solution:

$$M_t^{Info} = \frac{e^{\boldsymbol{\lambda}_T^\top \mathbf{R}_t^e}}{\sum_{t=1}^T e^{\boldsymbol{\lambda}_T^\top \mathbf{R}_t^e}}, \quad \boldsymbol{\lambda}_T = \underset{\boldsymbol{\theta}}{\text{argmin}} \frac{1}{T}\sum_{t=1}^{T} e^{\boldsymbol{\theta}^\top \mathbf{R}_t^e}, \tag{3.3}$$

where $\boldsymbol{\lambda}_T \in \mathbb{R}^n$ is the vector of Lagrange multipliers required for the pricing constraint in (3.2) to hold. The notation M_t^{Info} follows from Ghosh et al. (2016) naming their estimator the "Information SDF." To summarize, M_t^{Info} as given by (3.3) corresponds to essentially the least-complex risk-neutral probability measure that still prices all assets. Note that (3.3) gives a way to estimate the SDF out-of-sample: extract $\boldsymbol{\lambda}_T$ from a previous sub-sample $\{t_0 - s, t_0 - s + 1, \ldots, T - s\}$, $s > T - t_0$, and calculate each M_t based on $\mathbf{R}_t^e$ $\forall t \in \{t_0, \ldots, T\}$. In the next section, I propose a similar non-parametric SDF method.

3.1.2 Neural Network-Based SDF Estimation

The non-parametric SDF estimation method of Ghosh et al. (2016) motivates a neural network-based SDF estimation procedure. Neural networks, also known as "deep nets," are a machine learning method that uses a set of training examples to non-parametrically learn a continuous function between, in this setting, a vector-valued input variable and a vector-valued output variable. Note that (3.3) specifies the SDF M_t as a non-linear function of $\mathbf{R}_t^e$ and the parameter vector $\boldsymbol{\lambda}_T$ obtained from the pricing constraint. In the same way, a neural network learns a parameter vector $\boldsymbol{\omega}$ that yields the "best" mapping between the input and output variables according to some loss function.

Precisely, let $\sigma(x) = 1/(1+e^{-x})$ be the logistic function. Note that σ maps x continuously into the range $(0,1)$, exactly the domain of the SDF. As above, let $\mathbf{R}_t^e \in \mathbb{R}^n$ be the vector of excess asset returns in period t. Motivated by the pricing constraint in (3.2), let

$$\mathcal{L}(\boldsymbol{\omega}; \mathbf{R}_t^e) = \frac{1}{n}\|\mathbf{R}_t^e \sigma(\boldsymbol{\omega}^\top \mathbf{R}_t^e)\|_2^2 = \frac{1}{n}\sum_{i=1}^{n}(R_{t,i}^e \sigma(\boldsymbol{\omega}^\top \mathbf{R}_t^e))^2, \tag{3.4}$$

be the least-squares loss function, where $R_{t,i}^e$ is the excess return of the i-th asset in period t. Note that if we let $M_t = \sigma(\boldsymbol{\omega}^\top \mathbf{R}_t^e)$, then minimizing $\mathcal{L}(\boldsymbol{\omega}; \mathbf{R}_t^e)$ corresponds to finding an SDF that forces the pricing constraint to (almost) hold in period t. Thus, let

$$M_t^{Deep} = \sigma(-\boldsymbol{\omega}^\top \mathbf{R}_t^e), \quad \boldsymbol{\omega} = \operatorname*{argmin}_{\boldsymbol{\theta}} \frac{1}{T}\sum_{t=1}^{T}\mathcal{L}(\boldsymbol{\theta}; \mathbf{R}_t^e), \tag{3.5}$$

so M_t^{Deep} is the SDF that forces the pricing constraint to hold in the full sample. (3.5) is typically solved in the Deep learning literature via stochastic gradient descent. For brevity, I omit the details of stochastic gradient descent and refer the reader to Heaton et al. (2016), a review of the uses of Deep learning in finance, for more details. Note that M_t^{Deep} can be estimated in a rolling out-of-sample fashion in exactly the same fashion described above for M_t^{Info}. Figure 3.1.2 illustrates the neural network described in this section graphically.

This SDF estimator bears several structural similarities to that of Ghosh et al. (2016). Both estimators model the SDF as a nonlinear function based on a parameter vector learned from the pricing constraint. Indeed, the specific nonlinear mappings in both cases are functionally similar. The difference between the two methods is the manner in which the parameter vectors $\boldsymbol{\lambda}_T$ and $\boldsymbol{\omega}$ are learned. Yet comparing the equation for $\boldsymbol{\lambda}_T$ in (3.2) to the equation for $\boldsymbol{\omega}$ in (3.5), we see that the only difference in the ways $\boldsymbol{\lambda}_T$ and $\boldsymbol{\omega}$ are learned is the particular loss function chosen. Thus, this neural network SDF estimation technique is no more complex than the estimation method of Ghosh et al. (2016). However, the neural network approach offers several potential advantages in this setting. First, whereas Ghosh et al. (2016) acknowledge that their method requires a large time series dimension relative to the cross-sectional dimension, neural networks have proven useful in many high-dimensional applications (e.g image recognition), and thus might offer superior performance in this setting as we expand the cross-section of assets. Second, since I seek to model the SDF in an out-of-sample fashion, overfitting of the parameter vector to a previous subsample represents a serious concern. Deep learning offers several methods to prevent overfitting (e.g. regularization, dropout layers) that manifest as slight modifications to the optimization problem in (3.5). Thus, this neural network estimator may provide a more informative SDF than that yielded by the method of Ghosh et al. (2016). At the very least, this method provides an additional robustness check for the results yielded by the other tests I use.

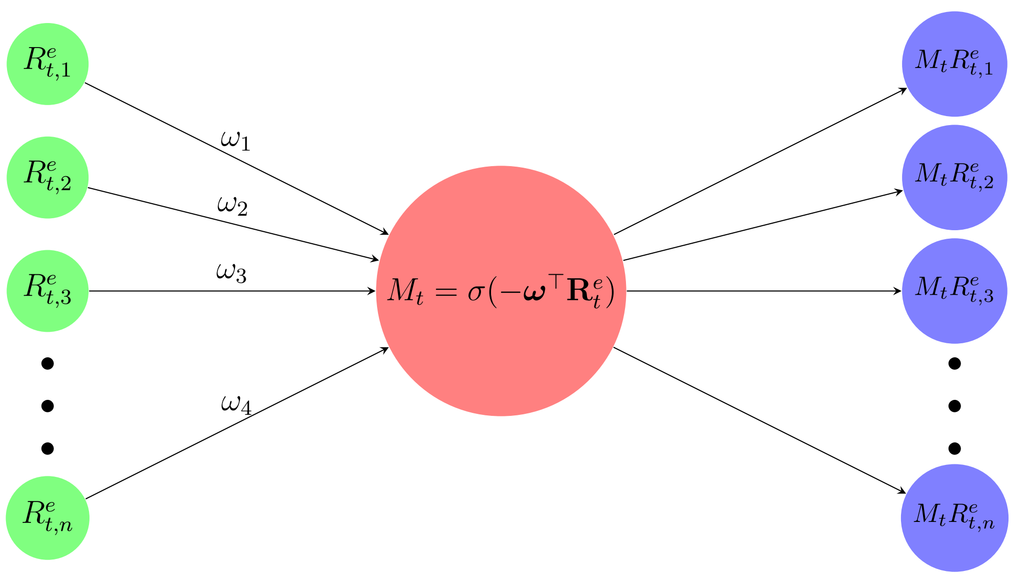


Figure 1: Graphical representation of neural network used to estimate the SDF.

3.1.3 Tests of Market Segmentation

I estimate the SDF via (3.3) and (3.5) in a rolling out-of-sample fashion from three sets of assets: a set of equity test assets (S_{Equity}), a set of bond test assets (S_{Bond}), and the union of these two sets (S_{All}). Denote these six SDFs as $M^{Equity,j}$, $M^{Bond,j}$, and $M^{Dual,j}$, $j \in \{Info, Deep\}$, respectively. Now in a similar manner to Ghosh et al. (2016), $\forall i \in \{Equity, Bond, Dual\}$, $j \in \{Info, Deep\}$ construct tradeable factor-mimicking portfolios by linearly projecting each SDF onto the return space:

$$\mathbf{w}^{i,j} = -\frac{\widehat{\boldsymbol{b}}}{|\widehat{\boldsymbol{b}}^\top \eta|}, \quad \left(\widehat{a}, \widehat{\boldsymbol{b}}\right) = \underset{a,\boldsymbol{b}}{\operatorname{argmin}} \frac{1}{T} \sum_{t=1}^{T} \left(M_t^{i,j} - a - \boldsymbol{b}^\top \mathbf{R}_t^e\right)^2 + \gamma \|(a, \boldsymbol{b})\|_2^2, \tag{3.6}$$

where $\mathbf{w}^{i,j}$ is the vector of portfolio weights and η is a vector of conformable ones. Here the coefficient γ is the regularization parameter. The second term in the optimization problem penalizes large coefficients a and $\boldsymbol{b}$, thereby constraining the positions weights of the mimicking portfolios. Ghosh et al. (2016) omit this regularization term and construct the mimicking portfolio via simple ordinary least squares (OLS) regression. However, when working with a wider cross section of assets, the OLS portfolio weights take on unreasonably large weights, and regularization can yield portfolios that perform better out of sample. This technique of penalizing the usual OLS problem by the L_2 norm of the coefficient vector is known as ridge regression (Hoerl & Kennard, 1970).

Let $\mathbf{R}_t^{e,i}, i \in \{Equity, Bond, Dual\}$ be the vector of excess returns for each set of test assets. Then $R_t^{e,i,j} = \mathbf{w}^{i,j\top} \mathbf{R}_t^{e,i} \in \{Equity, Bond, Dual\}, j \in \{Info, Deep\}$ is the factor-mimicking portfolio excess return for period t, where the weight vector $\mathbf{w}^{i,j\top}$ has been calculated out-of-sample from a previous subsample. Thus, $R_t^{Equity,j}, R_t^{Bond,j}, R_t^{Dual,j}, j \in \{Info, Deep\}$, represent six pricing factors.

I use these six pricing factors to test for market segmentation in two ways.

Benchmark-free tests of market segmentation

My first test of market segmentation involves two benchmark-free cross-sectional Fama-

MacBeth regressions. By obviating the need to specify a benchmark model, I strip out the potential statistical biases that arise from using the wrong benchmark model (e.g. omitted variable bias). The first pass of this test is to obtain factor loadings for each asset on its corresponding single-asset-class SDF and on the dual-asset-class SDF. For $j \in \{Info, Deep\}, i \in \{Equity, Bond\}$, regress the excess returns for asset $k \in S_i$ on $R_t^{e,i,j}$ and $R_t^{e,Dual,j}$:

$$R_{t,k}^e = \alpha + \beta_k^{i,j} R_t^{e,i,j} + \epsilon_t,$$
$$R_{t,k}^e = \alpha + \beta_k^{Dual,j} R_t^{e,Dual,j} + \epsilon_t.$$

Now cross-sectionally regress the sample expected returns for each assets $k \in S_i$, denoted $\overline{R_k^{e,i}}$, on the vectors of factor loadings:

$$\overline{R_k^{e,i}} = \alpha + \theta^{i\top} \beta_k^{i,j} + \epsilon_k, \tag{3.7}$$
$$\overline{R_k^{e,i}} = \alpha + \theta^{Dual\top} \beta_k^{Dual,j} + \epsilon_k. \tag{3.8}$$

Here, $\theta^{i\top}$ and $\theta^{Dual\top}$ represent the risk premia earned by exposure to the factor-mimicking portfolios for $M^{i,j}$ and $M^{Dual,j}$. If the full model (3.8) has significantly better fit than the nested model (3.7), then one would conclude that the dual-asset-class SDF explains significantly more of the variance in cross-sectional stock or bond returns than does a single-asset-class SDF. This conclusion would provide evidence against market segmentation. I conduct this test both in the full sample and in stressful subsamples.

I use two tests to determine if the full model (3.8) has better fit than the nested model (3.7). First, I use the Vuong closeness test for non-nested models (Vuong, 1989), which computes a normally-distributed z-statistic based on the Kullback-Leibler divergence between each of two non-nested models model and the hypothetical true model, to determine which better fits the data. Second, I evaluate the difference in alphas between the full model (3.8) and the nested model (3.7). Since these cross-sectional alphas represent pricing errors, the full model yielding a significantly smaller alpha than the nested model would constitute evidence of market integration. I use seemingly unrelated regression (SUR) to estimate the standard error of the alpha difference. In SUR, one stacks the vectors for the response variables used in two separate regressions and forms a block-diagonal matrix from the two individual design matrices, as shown in (3.9). One then regresses the combined response vector on the block-diagonal design matrix to obtain a stacked vector of regression coefficients and a covariance matrix for the regression coefficients, from which one can obtain a standard error for the alpha difference. This regression takes the following form:

$$\begin{pmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{pmatrix} = \begin{pmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \end{pmatrix} \begin{pmatrix} \mathbf{X}_1 & 0 \\ 0 & \mathbf{X}_2 \end{pmatrix} + \begin{pmatrix} \boldsymbol{\epsilon}_1 \\ \boldsymbol{\epsilon}_2 \end{pmatrix}, \tag{3.9}$$

where

$$\boldsymbol{y}_j, \epsilon_j \in \mathbb{R}^n, \boldsymbol{\beta}_j \in \mathbb{R}^d, \mathbf{X}_j \in \mathbb{R}^{n \times d}, j \in \{1, 2\},$$

for n the number of observations and d the number of parameters.

Comparison to equity and bond benchmarks

My second test of market segmentation compares the average alphas for all test assets in each asset class under a benchmark model and under that same benchmark model augmented with the single and dual-asset-class SDF-mimicking portfolios. I use the Fama-French five factor model with momentum as the equity benchmark (Fama & French, 2016), and the level-slope-curvature three factor model as the bond benchmark (Litterman & Scheinkman, 1991).

For each asset $k \in S_{Equity}$, conduct the following three time series regressions for all

$j \in \{Info, Deep\}$:

$$R^e_{t,k} = \alpha_k + b_k R^e_{M,t} + s_k SMB_t + h_k HML_t + r_k RMW_t + c_k CMA_t + m_k MOM_t + \epsilon_{k,t}, \tag{3.10}$$

$$R^e_{t,k} = \alpha^{j,0}_k + b^{j,0}_k R^e_{M,t} + s^{j,0}_k SMB_t + h^{j,0}_k HML_t + r^{j,0}_k RMW_t \tag{3.11}$$

$$+ c^{j,0}_k CMA_t + m_k MOM_t + \beta^{j,0}_k R^{Equity,j}_k + \epsilon_{k,t}, \tag{3.12}$$

$$R^e_{t,k} = \alpha^{j,1}_k + b^{j,1}_k R^e_{M,t} + s^{j,1}_k SMB_t + h^{j,1}_k SMB_t + r^{j,1}_k RMW_t \tag{3.13}$$

$$+ c^{j,1}_k CMA_t + m_k MOM_t + \beta^{j,1}_k R^{Dual,j}_k + \epsilon_{k,t}. \tag{3.14}$$

Here $R^e_{M,t}$ is the excess return of the market, SMB_t is the size factor, HML_t is the value factor, RMW_t is the profitability factor, CMA_t is the investment factor, all as described in Fama & French (2016), and MOM_t is the momentum factor.

For each asset $k \in S_{Bond}$, conduct the following three time series regressions for all $j \in \{Info, Deep\}$:

$$R^e_{t,k} = \alpha_k + l_k Level_t + s_k Slope_t + c_k Curvature_t + \epsilon_{k,t}, \tag{3.15}$$

$$R^e_{t,k} = \alpha^{j,0}_k + l^{j,0}_k Level_t + s^{j,0}_k Slope_t + c^{j,0}_k Curvature_t + \beta^{j,0}_k R^{Bond,j}_k + \epsilon_{k,t}, \tag{3.16}$$

$$R^e_{t,k} = \alpha^{j,1}_k + l^{j,1}_k Level_t + s^{j,1}_k Slope_t + c^{j,1}_k Curvature_t + \beta^{j,1}_k R^{Dual,j}_k + \epsilon_{k,t}. \tag{3.17}$$

Here $Level_t$ is the 1-year field, $Slope_t$ is the 10-year yield minus the 2-year yield, and $Curvature_t$ is the 5-year yield minus the average of the 10 and 2 year yields.

For $j \in \{Info, Deep\}, i \in \{Equity, Bond\}$, let

$$\overline{\alpha}_i = \frac{1}{|S_i|} \sum_{k \in S_i} |\alpha_k|, \quad \overline{\alpha}^{j,0}_i = \frac{1}{|S_i|} \sum_{k \in S_i} |\alpha^{j,0}_k|, \quad \overline{\alpha}^{j,1}_i = \frac{1}{|S_i|} \sum_{k \in S_i} |\alpha^{j,1}_k|. \tag{3.18}$$

If $\overline{\alpha}^{j,0}_i$ is significantly less than $\overline{\alpha}_i$, as measured by the reduction in average absolute t-statistic, then the single-asset-class SDF adds significant additional pricing power over the benchmark model. If $\overline{\alpha}_i^{j,1}$ is significantly less than $\overline{\alpha}_i^{j,0}$, then the dual-asset-class SDF adds adds significant additional pricing power over the single-asset-class SDF. For example, if $\overline{\alpha}_{Equity}^{j,1} << \overline{\alpha}_{Equity}^{j,0}$, then bond information helps price equities, which is evidence against market segmentation.

The benefit of this test is that, even though it requires specifying benchmark models, it can impart economic intuition about the extent of market integration in the form of the basis point reduction in average alpha due to each of the SDFs.

3.2 Testing for Segmentation via SDF Information

In this section, I detail the method of Galpin et al. (2017), which tests which assets add significant information to the SDF. Galpin et al. (2017) establish that in this setting,

$$\boldsymbol{\lambda}_T \xrightarrow{D} \mathcal{N}(0, \mathbf{P}), \quad \mathbf{P} = \frac{1}{T}\sum_{t=1}^{T} M_t^{info}\mathbf{R}_t^e\mathbf{R}_t^{e\top} \in \mathbb{R}^{n\times n}, \tag{3.19}$$

where n is the number of test assets and $\boldsymbol{\lambda}_T$ is the vector of Lagrange multipliers used in the SDF estimation method of Ghosh et al. (2016) defined in (3.3). Thus, a test of $H_0 : \lambda_{T,k} = 0$ is a test of if the pricing constraint for asset k binds, and so is a test of if asset k adds a significant amount of information to the SDF. Thus, I first test, in the full sample and in stressful sub periods which individual equity portfolios and bonds add a significant amount of information to the SDF.

Now let $\boldsymbol{\lambda}_T^i$ represent the $\boldsymbol{\lambda}_T$ vector derived from the set of test assets $S_i, i \in \{Equity, Bond, All\}$. I next test, in the full sample and in stressful subsamples, the following joint null hypothesis for $i \in \{Equity, Bond\}$:

$$H_0^i : \lambda_{T,k}^{All} = 0, \forall k \in S_i.$$

Rejecting H_0^{Equity} would imply that the equity test assets collectively add a significant amount of information to the SDF. Similarly, rejecting H_0^{Bond} would imply that the bonds collectively add a significant amount of information to the SDF. Galpin et al. (2017) establish the following convergence result that provides a simple means to conduct this hypothesis test:

$$\sum_{t=1}^{T} M_t^f \ln(M_t^f) - M_t^r \ln(M_t^r) \xrightarrow{D} \chi^2(d),$$

where M_t^f is the SDF estimated from the full cross section and M_t^r is the Information SDF estimated from the restricted cross section, and d is the difference in the number of assets between the full and restricted cross sections.

Thus, this method provides another benchmark-free test of market segmentation.

3.3 Dual-Asset-Class Trading Signals

The third way I test for market segmentation is by examining the performance of dual-asset-class trading strategies. I compare the performance of common factor strategies and variants of those strategies informed by a dual-asset-class SDF. If the dual-asset-class SDF significantly improves performance, then bonds help price equities, and vice versa. This conclusion would suggest market integration.

I examine trading signals for the value and momentum anomalies based on sorted equity and bond portfolios. I use the standard Fama-French decile sorts as the equity portfolios. Following the setup of Brooks & Moskowitz (2017), I use the 1, 3, 5, 7, and 10 year U.S. Treasury bonds sorted on real yields as the bond value assets, and these same bonds sorted on trailing 12-month returns as the momentum assets. For each set of trading assets (either the ten equity or five bond portfolios), consider the following three strategies based on these assets:

1. **Control strategy**: Long the greatest anomaly portfolio (tenth equity decile for if trading equities, fifth bond quintile if trading bonds) short the least anomaly portfolio.

2. **Single-SDF strategy**: Extract the SDF out-of-sample from the trading assets by (3.3) and (3.5). Project each SDF onto the return space of these portfolios via (3.6)

to create two tradeable anomaly portfolios. As noted by Ghosh et al. (2016), these portfolios should represent the tangency portfolios of these ten assets.

3. **Dual-SDF strategy**: Extract the SDF out-of-sample from the trading assets and the set of non-trading assets by (3.3) and (3.5). Project each SDF onto the return space of these equity portfolios via (3.6) to create two tradeable anomaly portfolios.

If the single-SDF strategy performs significantly better than the control strategy (on the bases of Sharpe ratio and alpha to a benchmark factor model), then the SDF we estimate does uncover a better asset allocation, and perhaps identifies the tangency portfolio. If the dual-SDF strategy performs significantly better than the single-SDF strategy, then information in bond yields (stock returns) helps price and trade equities (bonds), which would be evidence against market segmentation. In addition to providing another robustness check to my other tests of market segmentation, testing these dual-asset-class trading signals also highlights the practical value of the SDF estimation techniques I use.

4 Data

In this section I discuss the data I use. I use daily U.S. stock and bond returns. To ensure robustness, I use a wide cross section of test equity portfolios. Specifically, I use daily decile sorts on size, value, momentum, investment, and profitability, obtained from Kenneth French's data library. This data set covers the period from July 1963 to September 2017. In order to capture information from the full yield curve, I use U.S. Treasury zero-coupon bonds with maturities of 1, 3, 5, 7, and 10 years. The CRSP Fixed Term Indexes dataset has daily data on these maturities from June 1961 to December 2016.

I use the NBER recession periods as the stressful periods for my subsample analyses. For the real yield proxies used to form the bond value assets, I use the data provided by Chernov & Mueller (2012), who model quarterly real yields, from 1971 to 2002 for maturities of 1, 3, 5, 7, and 10 years. From 2003 to 2016, I use daily TIPS yields for maturities of 5, 7, and 10 years. For the 1 and 3-year yields during this period, I estimate real yields by subtracting the nominal yield from expected maturity-matched CPI inflation forecasts obtained from the Federal Reserve Bank of Philadelphia. This dataset covers provides inflation forecasts at a quarterly frequency for 1 and 10 year time horizons. I obtain 3-year expected inflation via linear interpolation.

The full data set I use for the tests outlined in Sections 3.1 and 3.2 covers the period from July 1963 to December 2015. I use a ten-year rolling window to construct the Information and Deep SDFs, so the out-of-sample period covers September 1973 to December 2015. I evaluate dual-asset-class trading strategies discussed in Section 3.3 on the period November 1981 to December 2016.

5 Hypotheses

In this section, I summarize the empirical tests I conduct and outline my priors on the results of these tests. Overall, based on the success of previous joint bond and stock pricing efforts, as discussed in Section 2.2, I expect to find evidence of market integration. No arbitrage is a fairly weak assumption, especially in liquid cross sections, so theory suggests a single SDF will price stocks and bonds. Furthermore, previous work, such as Fama & French (1993), has found bond factors to be informative in the cross-section of equity returns and vice versa. Even though, as discussed in section 2.1.2, the preferred habitat hypothesis and work on term structure segmentation suggest that equity and bond markets may display signs of segmentation in stressful periods, I maintain the same prior here due to the results of previous full-sample analyses. Table 1 exhibits the empirical tests I run and my priors on the results of each of these tests.

Table 1: Summary of empirical tests and associated priors.

Test	Statistical H_0	Prior
F-test/likelihood ratio test of single vs. dual-asset-class SDF in cross-sectional regression	$\forall j \in \{Info, Deep\}$ H_0^j : Full model does not have better fit than nested model	$\forall j \in \{Info, Deep\}$ H_0^j will be rejected
Comparison of average alphas under benchmark and SDF-augmented benchmarks	$\forall i \in \{Equity, Bond\}$, $j \in \{Info, Deep\}$ $H_0^{1,i,j} : \overline{\alpha}_i = \overline{\alpha}_i^{j,0}$ $H_0^{2,i,j} : \overline{\alpha}_i^{j,0} = \overline{\alpha}_i^{j,1}$	$\forall i \in \{Equity, Bond\}$, $j \in \{Info, Deep\}$ $H_0^{1,i,j}$ and $H_0^{2,i,j}$ will be rejected.
Significance tests of equity and bond Lagrange multipliers in SDF estimate	$\forall i \in \{Equity, Bond\}$: $H_0^i : \lambda_{T,k}^{All} = 0, \forall k \in S_i$	$\forall i \in \{Equity, Bond\}$ H_0^i will be rejected
Comparison of control, single-SDF, and dual-SDF trading strategies	$\forall a \in \{value, momentum\}$, $j \in \{Info, Deep\}$ $H_0^{a,j,1}$: Single-SDF strategy does not outperform dual-SDF strategy $H_0^{a,j,2}$: Dual-SDF strategy does not outperform single-SDF strategy	$\forall a \in \{value, momentum\}$, $j \in \{Info, Deep\}$ $H_0^{a,j,1}$ and $H_0^{a,j,2}$ will be rejected

6 Results

In this section I detail the results of the empirical tests I conduct.

6.1 SDF and Mimicking Portfolio Summary Statistics

Table 2 displays summary statistics for the Information and Deep SDFs, and their corresponding mimicking portfolios (MPs). The Information SDFs are normalized to have mean of 1 by construction, while the Deep SDFs happen to be centered at .5, which isn't surprising given their construction via the logistic function. The equity and full cross-section Information SDFs are extremely volatile ($\sigma > 4$), and all three Information SDFs have extraordinarily positive skewness (> 40) and kurtosis (> 2000), as illustrated by the histogram in Panel A of Figure 2. The Deep SDFs are much less volatile, with insignificant skewness (about 1 at most), and significantly lower, yet still very high, kurtosis (up to 19). The histogram in Panel B of Figure 2 displays the roughly normal distribution (on a log scale) of the full cross-section Deep SDF.

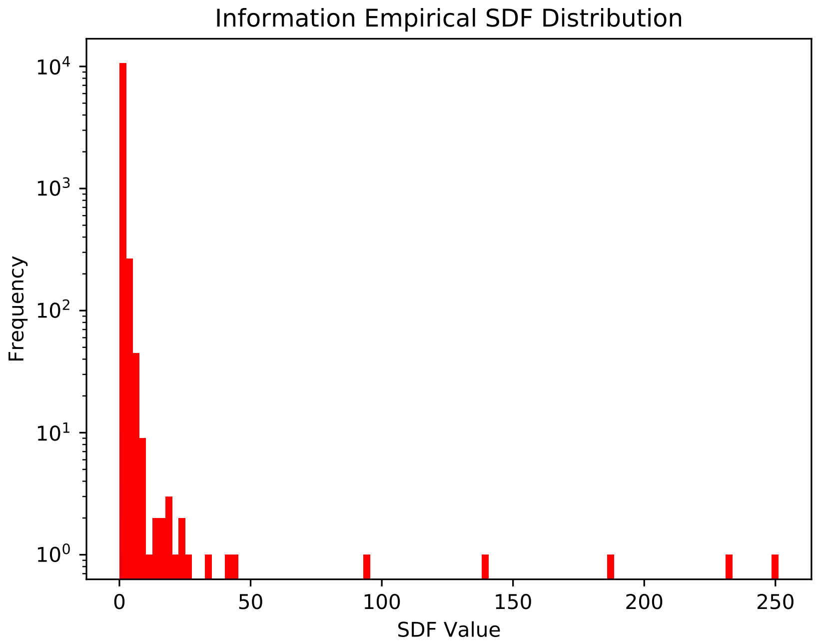
Panel A: Information SDF						
Statistic	Eq. SDF	Bonds SDF	Both SDF	Eq. MP	Bonds MP	Both MP
Mean	1.00	1.00	1.00	6.01e-4	9.64e-5	5.63e-4
Std Dev	4.20	0.119	4.25	0.0103	2.76e-3	9.17e-3
Skewness	46.7	43.2	46.4	-0.591	0.0294	-0.744
Kurtosis	2.45e+3	3.28e+3	2.40e+3	12.1	4.70	14.2
Sharpe	-	-	-	0.0585	0.0349	0.0615
Panel B: Deep SDF						
Statistic	Eq. SDF	Bonds SDF	Both SDF	Eq. MP	Bonds MP	Both MP
Mean	0.500	0.500	0.500	3.91e-4	8.14e-5	4.10e-4
Std Dev	0.0217	3.88e-3	0.0219	0.0122	2.48e-3	0.0138
Skewness	1.01	0.407	1.00	-0.409	0.125	-0.347
Kurtosis	19.0	10.8	19.1	6.92	8.05	7.90
Sharpe	-	-	-	0.0320	0.0329	0.0297

Table 2: Daily SDF and mimicking portfolio (MP) summary statistics.

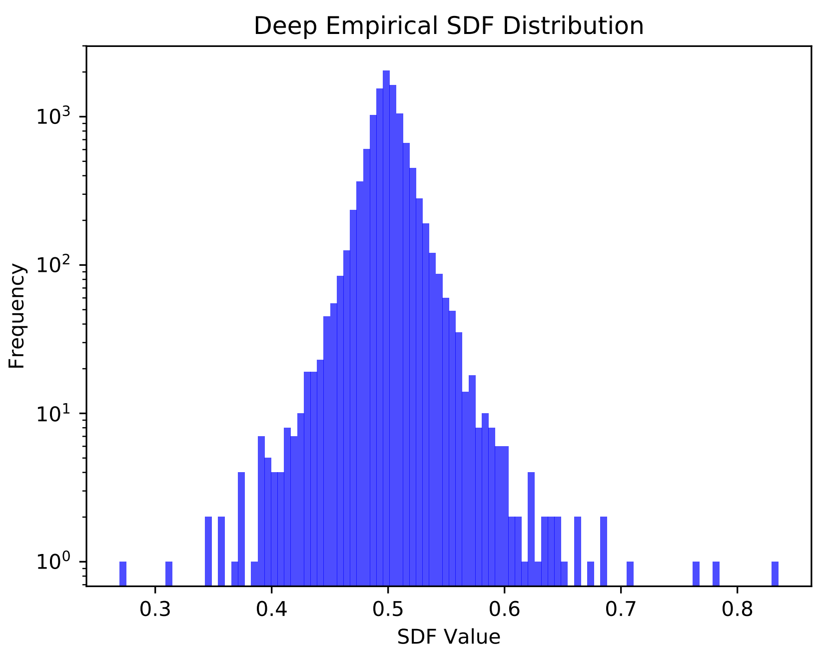
The MPs perform well in terms of Sharpe ratio after annualizing the numbers in Table 2 by multiplying by $\sqrt{252}$. For the Information method, the equity and full cross-section MP Sharpe's (.0585 and .0615) are significantly higher than the bond MP Sharpe (.0349), and the full cross-section MP Sharpe is a fair bit higher than the equity MP Sharpe. For Deep method, the bond MP Sharpe (.0329) is slightly higher than the equity MP Sharpe (.0320), which is slightly higher than the full cross section MP Sharpe (.0297). Figure 3 exhibits the cumulative growth of \$1 in each of the MPs and demonstrates that the Information equity MP achieves the highest cumulative return over the sample period.

Figure 4 exhibits the MP weights for the Information and Deep SDFs extracted from the full cross section. The weights appear to conform with a reasonable ex-ante expectation about which portfolios the SDF should load strongly on (e.g. high value and high momentum portfolios have large, positive weights). Moreover, the MP weights demonstrate significant time variation, which suggests that the factor structure of asset returns is not stable over time.

Table 3 displays the correlations between the Information and Deep SDFs extracted from



(a) Information SDF



(b) Deep SDF

Figure 2: Empirical distributions of Information and Deep SDFs extracted from the full cross section. The y-axes have been log-scaled to highlight tail behavior.

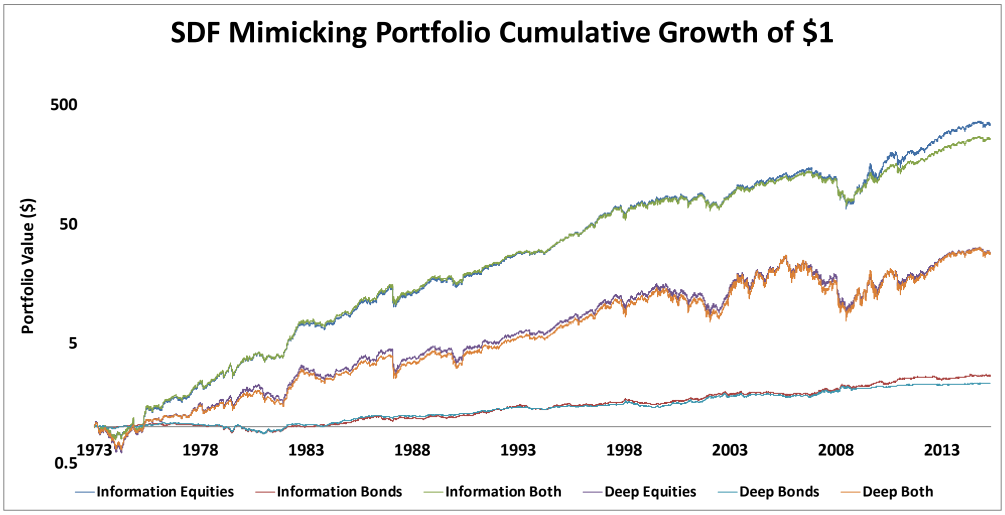
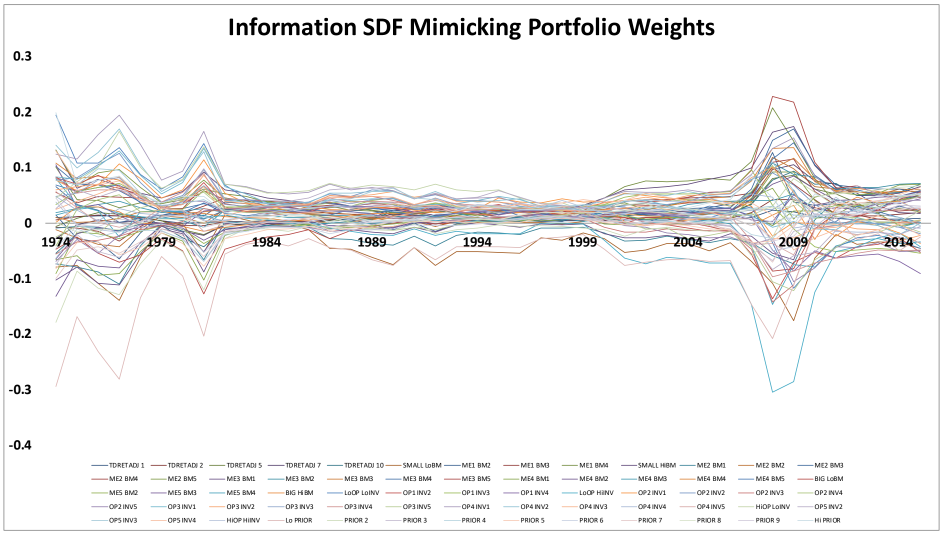


Figure 3: Cumulative returns to each of the SDF-mimicking portfolios for the equity, bond, and combined ("Both") cross sections, extracted via the Information and Deep SDF methods.

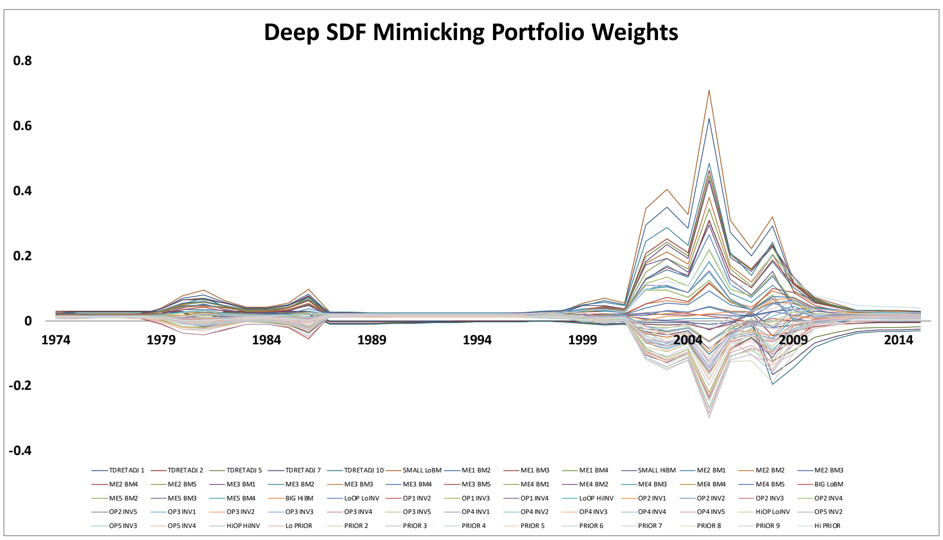
the equity, bond, and full cross sections, and for the corresponding MPs. The SDFs are almost always negatively correlated with their corresponding MPs, as one would expect given the construction of the MP weights. The equity and full cross-section SDFs are almost perfectly correlated (close to 1.0) for both methods, as are the equity and full cross-section MPs for both methods. On the other hand, the bond SDFs are very weakly, and usually slightly negatively, correlated with the equity and both SDFs, as are the corresponding MPs. Thus, it appears that adding equities to the bond cross section introduces some new information, but not necessarily vice versa.

Statistic	Info Eq. SDF	Info Bonds SDF	Info Both SDF	Info Eq. MP	Info Bonds MP	Info Both MP	Deep Eq. SDF	Deep Bonds SDF	Deep Both SDF	Deep Eq. MP	Deep Bonds MP	Deep Both MP
Info Eq. SDF	1.0	-0.01	1.0	-0.11	0.02	-0.13	-0.04	0.02	-0.04	-0.07	0.02	-0.06
Info Bonds SDF	-0.01	1.0	0.01	0.04	-0.26	0.03	-0.03	-0.0	-0.08	0.03	-0.23	0.03
Info Both SDF	1.0	0.01	1.0	-0.11	0.01	-0.13	-0.04	0.01	-0.04	-0.07	0.02	-0.06
Info Eq. MP	-0.11	0.04	-0.11	1.0	-0.12	0.99	-0.22	0.16	-0.21	0.9	-0.09	0.87
Info Bonds MP	0.02	-0.26	0.01	-0.12	1.0	-0.07	0.07	0.34	0.23	-0.14	0.89	-0.16
Info Both MP	-0.13	0.03	-0.13	0.99	-0.07	1.0	-0.21	0.19	-0.2	0.88	-0.07	0.83
Deep Eq. SDF	-0.04	-0.03	-0.04	-0.22	0.07	-0.21	1.0	0.02	0.98	-0.36	0.11	-0.36
Deep Bonds SDF	0.02	-0.0	0.01	0.16	0.34	0.19	0.02	1.0	0.11	0.13	0.22	0.11
Deep Both SDF	-0.04	-0.08	-0.04	-0.21	0.23	-0.2	0.98	0.11	1.0	-0.35	0.25	-0.36
Deep Eq. MP	-0.07	0.03	-0.07	0.9	-0.14	0.88	-0.36	0.13	-0.35	1.0	-0.12	0.99
Deep Bonds MP	0.02	-0.23	0.02	-0.09	0.89	-0.07	0.11	0.22	0.25	-0.12	1.0	-0.13
Deep Both MP	-0.06	0.03	-0.06	0.87	-0.16	0.83	-0.36	0.11	-0.36	0.99	-0.13	1.0

Table 3: Daily SDF and mimicking portfolio (MP) correlations.



(a) Information SDF-mimicking portfolio weights



(b) Deep SDF-mimicking portfolio weights

Figure 4: SDF-mimicking portfolio weights for full cross section demonstrate clear time-variation.

6.2 Tests via SDF Estimation

Cross-sectional benchmark-free tests of market segmentation

Table 4 displays the full-sample Fama-MacBeth regression results for pricing the equity (Panel A) and bond (Panel B) cross sections using the single and dual-asset-class SDFs estimated via the Information and Deep methods, and using their corresponding MPs. The SDFs and MPs do a fair job at pricing the the equity cross section. The R^2s are mostly in the double digits digits, and all but one of the prices of risk are significant. The SDF prices of risk all have the correct sign (negative), but two of the MP risk prices have the wrong sign (negative when should be positive). Of course, these factors do not achieve the greater than 90% R^2s reported by Fama & French (2016) for the Fama-French five-factor model, but at the same time the equity cross-section being priced here is much larger than that used in Fama & French (2016), where the authors separately conduct Fama-MacBeth regressions for sets of 25 size-value, size-profitability, and size-investment sorted portfolios. Here, I price 60 size-value, profitability-investment, and momentum sorted portfolios simultaneously.

The SDFs and MPs price the bond cross-section very well. Most of the R^2s are over 80% and all of the prices of risk are significant. Two of the SDF risk prices have the correct sign (negative) while three of the MP risk prices have the correct sign (positive). The superior pricing ability in the bond cross section likely yields from the fact that the bond cross section is much smaller than the equity cross section (5 assets vs. 60 assets).

The stressful sub period Fama-MacBeth regression results displayed in Table 5 do not differ in any meaningful way from the full-sample results except that fewer of the t-statistics are positive due to the reduced power of the smaller sample.

Table 6 presents the results for the Vuong closeness test comparing the model fits between the single and dual-asset-class SDFs and MPs, for both the equity (Panel A) and bond (Panel B) cross-sections. Positive test statistics mean the single-asset class factor provides a better model fit (i.e. prices the cross section better, which is evidence of segmentation) than the dual-asset class factor, and vice versa for negative test statistics (evidence of integration). For

Panel A: Equity Cross-Sectional Regressions										
Row	const.	$\theta_{SDF}^{Eq.,Info}$	$\theta_{MP}^{Eq.,Info}$	$\theta_{SDF}^{Both,Info}$	$\theta_{MP}^{Both,Info}$	$\theta_{SDF}^{Eq.,Deep}$	$\theta_{MP}^{Eq.,Deep}$	$\theta_{SDF}^{Both,Deep}$	$\theta_{MP}^{Both,Deep}$	$\overline{R}^2(\%)$
(1)	1.604e-4	-0.7701								9.1
	(2.132)	(-2.627)								
(2)	9.282e-4		-5.993e-4							13.1
	(5.059)		(-3.141)							
(3)	1.616e-4			-0.7744						8.8
	(2.124)			(-2.581)						
(4)	9.385e-4				-5.409e-4					14.5
	(5.300)				(-3.314)					
(5)	4.000e-4					4.607e-4				2.8
	(12.42)					(1.649)				
(6)	7.569e-4						-5.332e-4			13.9
	(6.038)						(-3.239)			
(7)	3.993e-4							4.642e-4		2.9
	(12.65)							(1.668)		
(8)	6.977e-4								-5.448e-4	10.7
	(5.710)								(-2.836)	

Panel B: Bond Cross-Sectional Regressions										
Row	const.	$\theta_{SDF}^{Bond,Info}$	$\theta_{MP}^{Bond,Info}$	$\theta_{SDF}^{Both,Info}$	$\theta_{MP}^{Both,Info}$	$\theta_{SDF}^{Bond,Deep}$	$\theta_{MP}^{Bond,Deep}$	$\theta_{SDF}^{Both,Deep}$	$\theta_{MP}^{Both,Deep}$	$\overline{R}^2(\%)$
(1)	-8.702e-6	-0.02273								64.3
	(-0.2428)	(-2.863)								
(2)	3.886e-5		5.947e-5							87.5
	(3.504)		(5.378)							
(3)	2.615e-5			7.279						9.1
	(0.4650)			(1.183)						
(4)	6.035e-5				-9.573e-3					84.9
	(6.918)				(-4.855)					
(5)	3.726e-5					1.486e-4				83.8
	(2.850)					(4.651)				
(6)	3.840e-5						5.252e-5			85.6
	(3.189)						(4.975)			
(7)	3.644e-5							1.758e-3		87.3
	(3.155)							(5.345)		
(8)	5.037e-5								-2.034e-3	80.0
	(4.196)								(-4.124)	

Table 4: Full-sample benchmark-free Fama-MacBeth regressions following the setup in (3.7) and (3.8)

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Panel A: Equity Cross-Sectional Regressions										
Row	const.	$\theta_{SDF}^{Eq.,Info}$	$\theta_{MP}^{Eq.,Info}$	$\theta_{SDF}^{Both,Info}$	$\theta_{MP}^{Both,Info}$	$\theta_{SDF}^{Eq.,Deep}$	$\theta_{MP}^{Eq.,Deep}$	$\theta_{SDF}^{Both,Deep}$	$\theta_{MP}^{Both,Deep}$	$\overline{R}^2(\%)$
(1)	-4.195e-4	0.3324								-0.3
	(-12.29)	(0.9061)								
(2)	2.845e-4		-6.322e-4							17.1
	(1.417)		(-3.626)							
(3)	-4.204e-4			0.3339						-0.4
	(-12.34)			(0.8672)						
(4)	3.287e-4				-6.324e-4					18.3
	(1.604)				(-3.770)					
(5)	-5.159e-4					6.060e-4				1.9
	(-8.672)					(1.466)				
(6)	3.181e-4						-8.589e-4			26.9
	(1.981)						(-4.761)			
(7)	-5.207e-4							6.570e-4		2.4
	(-8.764)							(1.558)		
(8)	2.840e-4								-8.157e-4	26.3
	(1.827)								(-4.700)	

Panel B: Bond Cross-Sectional Regressions										
Row	const.	$\theta_{SDF}^{Bond,Info}$	$\theta_{MP}^{Bond,Info}$	$\theta_{SDF}^{Both,Info}$	$\theta_{MP}^{Both,Info}$	$\theta_{SDF}^{Bond,Deep}$	$\theta_{MP}^{Bond,Deep}$	$\theta_{SDF}^{Both,Deep}$	$\theta_{MP}^{Both,Deep}$	$\overline{R}^2(\%)$
(1)	1.777e-4	-5.137e-3								-17.1
	(2.818)	(-0.6453)								
(2)	1.154e-4		9.966e-5							86.3
	(5.317)		(5.123)							
(3)	2.010e-4			4.824						-4.6
	(6.419)			(0.9078)						
(4)	1.342e-4				-4.985e-3					76.9
	(5.415)				(-3.782)					
(5)	1.068e-4					3.474e-4				80.0
	(3.722)					(4.121)				
(6)	1.207e-4						9.961e-5			81.8
	(4.969)						(4.362)			
(7)	1.024e-4							2.588e-3		89.6
	(4.966)							(5.965)		
(8)	1.340e-4								-4.626e-3	78.8
	(5.666)								(-3.983)	

Table 5: Stressful sub period benchmark-free Fama-MacBeth regressions following the setup in (3.7) and (3.8).

example, the top left box compares the cross-sectional fit of the Fama-MacBeth regression when regressing the entire eqity cross section on the Information equity SDF versus on the Information dual SDF. The full-sample (Panel A) results overall are inconclusive. Three out of eight tests indicate segmentation, one indicates integration, and four are inconclusive. The stressful sub period results (Panel B) are equivalently inconclusive: two out of eight tests show integration and six are inconclusive.

Panel A: Full Sample				
	Equities		Bonds	
SDF Method	SDF	MP	SDF	MP
Info	1.727 (0.0421)	-1.194 (0.884)	1.304 (0.0961)	0.4562 (0.324)
Deep	-0.4141 (0.661)	2.159 (0.0154)	-5.668 (1.00)	2.985 (1.42e-3)
Panel B: Stressful Sub Periods				
	Equities		Bonds	
SDF Method	SDF	MP	SDF	MP
Info	0.7234 (0.235)	-2.319 (0.990)	-0.8677 (0.807)	1.310 (0.0951)
Deep	-1.351 (0.912)	1.535 (0.0624)	-2.289 (0.989)	0.6723 (0.251)

Table 6: Cross-sectional Vuong closeness test results for comparing the cross-sectional fit of the second stage of the Fama-MacBeth regressions of each cross section (equities or bonds) on single versus dual-asset-class SDFs and MPs. The top value in each cell is the normally-distributed z-statistic yielded by the Vuong closeness test, while (right-tail) p-values are provided in parentheses. Positive values (significance highlighted in red) indicate segmentation while negative values (significance highlighted in green) indicate integration.

Table 7 compares the Fama-MacBeth alphas between the single and dual-asset-class factors. This table displays the difference in alphas between the two models, and the corresponding t-statistic for the alpha difference derived from SUR. The values in Table 7 take the form $\alpha_{\text{dual asset class}} - \alpha_{\text{single asset class}}$, so positive differences indicate a reduction in cross-sectional pricing error when incorporating dual-asset-class information (evidence of integration). For example, the top left box compares the cross-sectional alpha of the second stage of the Fama-MacBeth regression when regressing the entire equity cross section on the Information equity SDF versus on the Information dual SDF. In the full sample (Panel A), the results are inconclusive: two out of eight tests show integration, two show segmentation, and four tests are inconclusive. The stressful sub period results (Panel B) prove similarly inconclusive: three out of eight tests indicate integration, two tests show segmentation, and three are inconclusive.

Panel A: Full Sample				
	Equities		Bonds	
SDF Method	SDF	MP	SDF	MP
Info	-1.205e-6	-1.026e-5	-3.485e-5	-2.149e-5
	(-1.76)	(-0.856)	(-1.04)	(-4.05)
Deep	6.916e-7	5.924e-5	8.225e-7	-1.197e-5
	(2.93)	(8.10)	(0.670)	(-6.86)
Panel B: Stressful Sub Periods				
	Equities		Bonds	
SDF Method	SDF	MP	SDF	MP
Info	9.183e-7	-4.422e-5	-2.329e-5	-1.886e-5
	(4.94)	(-6.89)	(-1.32)	(-1.57)
Deep	4.821e-6	3.413e-5	4.338e-6	-1.327e-5
	(5.98)	(8.60)	(0.502)	(-1.97)

Table 7: Cross-sectional alpha comparison results for comparing the cross-sectional alphas of the second stage of the Fama-MacBeth regressions of each cross section (equities or bonds) on single versus dual-asset-class SDFs and MPs. The top value in each cell is the $\alpha_{\text{dual asset class}} - \alpha_{\text{single asset class}}$ difference, while t-statistics are provided in parentheses. Negative values (significance highlighted in red) indicate segmentation while positive values (significance highlighted in green) indicate integration.

Thus, the benchmark-free, cross-sectional pricing tests fail to provide strong evidence for either market segmentation or integration.

Time-series benchmarked tests of market segmentation

Tables 8 (full sample) and 9 (stressful sub periods) compare the average, absolute time-series alphas yielded by regressing each asset in each cross section on the asset class benchmark model (Fama French five factor model with momentum for equities and level-slope-curvature for bonds) versus the benchmark model augmented with a single-asset-class factor versus the benchmark model augmented with a dual-asset-class factor. For example, the top left cell in Panel A states that the average absolute alpha across the entire equity cross section under the benchmark model is 8.510e-5 (t = 1.928). When we add the Information equity SDF to that model, the average absolute alpha becomes 8.748e-5 (t = 1.717). While the average absolute pricing error increases in magnitude, it decreases in significance, although the increase in magnitude is economically insignificant (0.6 basis points annually) and the decrease in significance is statistically insignificant ($\delta t = -.211$). These tables display differences in the form $\alpha_{\text{left model}} - \alpha_{\text{right model}}$. A positive difference (i.e. the augmented model yields a smaller absolute average pricing error) in the second row of Panels A and B means that the new factor adds pricing information beyond the benchmark. A positive difference in the fourth row means that the dual asset class factor adds pricing information beyond what the single asset class factor contains. I don't have a particularly sharp test here, but we can still see if the change in significance is large (highlighted are changes in average absolute t-stat of greater than 2).

This test does not provide strong evidence that the new pricing factors add significant time-series pricing information above the benchmark models. In the full sample (Table 8), the second row in Panels A and B demonstrate that the single asset class pricing factors don't appear to add any new pricing information about equities beyond the equity benchmark, since none to these reductions in average absolute alpha are positive and significant. However, the single asset class pricing factors do provide bond pricing information beyond the bond benchmark, since three out of four of the reductions in average absolute alpha t-statistic are positive and significant. For two of these three tests, the reduction in alpha by including the new single-asset-class factor is also economically significant (over 1.2% annually). In stressful sub periods (Table 9), none of the changes in average absolute alpha in the second row of Panels A and B, likely due to the reduced power we have in the smaller sample (10549 observations in the full sample vs. 1492 in stressful sub periods).

This test also fails to provide strong evidence either in favor or against market segmentation. In the full sample (Table 8), only three of eight fourth row tests using both the Information and Deep SDF methods are significant, and all suggest segmentation. Five of these eight tests are inconclusive. For two of these three tests, the reduction in alpha by

Panel A: Info SDF				
	Equities		Bonds	
SDF Method	SDF	MP	SDF	MP
Benchmark, Single Alphas	8.510e-5 , 8.784e-5 (1.928 , 1.717)	8.510e-5 , 8.019e-5 (1.928 , 1.845)	7.417e-5 , 1.791e-3 (6.794 , 2.585)	7.417e-5 , 2.280e-5 (6.794 , 3.995)
Benchmark, Single Difference	-2.739e-6 (0.2106)	4.911e-6 (0.08273)	-1.717e-3 (4.209)	5.137e-5 (2.799)
Single, Dual Alphas	8.784e-5 , 8.755e-5 (1.717 , 1.718)	8.019e-5 , 7.833e-5 (1.845 , 1.806)	1.791e-3 , 7.118e-5 (2.585 , 6.548)	2.280e-5 , 7.852e-5 (3.995 , 6.813)
Single, Dual Difference	2.883e-7 (-4.589e-4)	1.864e-6 (0.03909)	1.720e-3 (-3.964)	-5.572e-5 (-2.818)

Panel B: Deep SDF				
	Equities		Bonds	
SDF Method	SDF	MP	SDF	MP
Benchmark, Single Alphas	8.510e-5 , 5.261e-3 (1.928 , 1.821)	8.510e-5 , 8.572e-5 (1.928 , 1.945)	7.417e-5 , 0.09696 (6.794 , 5.021)	7.417e-5 , 2.366e-5 (6.794 , 4.395)
Benchmark, Single Difference	-5.176e-3 (0.1064)	-6.189e-7 (-0.01745)	-0.09688 (1.773)	5.052e-5 (2.398)
Single, Dual Alphas	5.261e-3 , 5.349e-3 (1.821 , 1.864)	8.572e-5 , 8.571e-5 (1.945 , 1.944)	0.09696 , 7.402e-3 (5.021 , 6.381)	2.366e-5 , 8.027e-5 (4.395 , 7.060)
Single, Dual Difference	-8.872e-5 (-0.04239)	5.431e-9 (1.583e-3)	0.08955 (-1.360)	-5.661e-5 (-2.665)

Table 8: Full-sample comparison of time series alphas. Rows 1 and 3 of each panel display the average, absolute, time-series alpha for each of the two models being compared with OLS t-statistics in parentheses. Rows 2 and 4 of each panel display the difference in average, absolute, time-series alphas with differences in average, absolute t-statistics provided in parentheses. These differences are in the form $\alpha_{\text{left model}} - \alpha_{\text{right model}}$, so positive differences mean the right model yields a smaller average absolute pricing error than the left model. Significance is highlighted in green (for positive changes in average absolute t-statistic) and red (for negative changes).

replacing the single-asset-class factor with a dual-asset-class factor is economically significant (over 1.2% annually). In stressful sub periods (Table 9), none of the fourth row tests are significant, so the results are even more inconclusive.

Panel A: Info SDF				
	Equities		Bonds	
SDF Method	SDF	MP	SDF	MP
Benchmark, Single Alphas	1.397e-4 , 1.303e-4 (1.112 , 1.013)	1.397e-4 , 1.378e-4 (1.112 , 1.097)	2.903e-5 , 1.742e-3 (1.222 , 1.409)	2.903e-5 , 2.227e-5 (1.222 , 1.197)
Benchmark, Single Difference	9.340e-6 (0.09894)	1.850e-6 (0.01520)	-1.713e-3 (-0.1877)	6.754e-6 (0.02435)
Single, Dual Alphas	1.303e-4 , 1.304e-4 (1.013 , 1.013)	1.378e-4 , 1.360e-4 (1.097 , 1.081)	1.742e-3 , 2.609e-5 (1.409 , 1.208)	2.227e-5 , 2.857e-5 (1.197 , 1.183)
Single, Dual Difference	-1.297e-7 (3.763e-5)	1.829e-6 (0.01591)	1.716e-3 (0.2017)	-6.300e-6 (0.01451)

Panel B: Deep SDF				
	Equities		Bonds	
SDF Method	SDF	MP	SDF	MP
Benchmark, Single Alphas	1.397e-4 , 9.062e-3 (1.112 , 1.540)	1.397e-4 , 1.387e-4 (1.112 , 1.102)	2.903e-5 , 0.06337 (1.222 , 1.979)	2.903e-5 , 2.268e-5 (1.222 , 1.208)
Benchmark, Single Difference	-8.922e-3 (-0.4278)	9.348e-7 (0.01016)	-0.06335 (-0.7578)	6.353e-6 (0.01311)
Single, Dual Alphas	9.062e-3 , 8.553e-3 (1.540 , 1.508)	1.387e-4 , 1.389e-4 (1.102 , 1.104)	0.06337 , 7.384e-3 (1.979 , 2.617)	2.268e-5 , 2.782e-5 (1.208 , 1.166)
Single, Dual Difference	5.083e-4 (0.03184)	-2.145e-7 (-1.895e-3)	0.05599 (-0.6379)	-5.144e-6 (0.04202)

Table 9: Stressful sub period comparison of time series alphas. Rows 1 and 3 of each panel display the average, absolute, time-series alpha for each of the two models being compared with OLS t-statistics in parentheses. Rows 2 and 4 of each panel display the difference in average, absolute, time-series alphas with differences in average, absolute t-statistics provided in parentheses. These differences are in the form $\alpha_{\text{left model}} - \alpha_{\text{right model}}$, so positive differences mean the right model yields a smaller average absolute pricing error than the left model. Significance is highlighted in green (for positive changes in average absolute t-statistic) and red (for negative changes).

Why do the tests of market segmentation via SDF estimation fail to produce strong evidence for or against market segmentation? Because pricing wide cross sections of assets proves very difficult, especially in the out-of-sample manner done here where each year's SDF and MP are constructed using only the previous 10 years of data. As we see in the histograms in Figure 2 in Section 6.1, the SDF series prove very noisy with very fat tails, especially under the Information method. Thus, the fact that these factors have as much explanatory power as they do (see Tables 4 and 5) is in an of itself slightly surprising. In the next section, I discuss the tests of segmentation via SDF information, which considers the entire time series of returns unconditionally, and yields stronger results.

6.3 Tests via SDF Info

The method of Galpin et al. (2017) provides strong evidence of market integration. Table 10, exhibits the (normally-distributed) Lagrange multipliers and accompanying t-statistics from (3.3) for each asset in the combined equity and bond cross section, when the SDF is estimated from the full sample and from just the subsample of stressful sub periods. The first row demonstrates the return on the one-year Treasury proves a significant determinant of the SDF in both the full sample and in stressful sub periods. Table 11 presents the results of the asset deletion test from Galpin et al. (2017). The first row illustrates that when bonds are removed from the set of assets used to estimate the SDF, a significant amount of information is lost, in both the full sample and in stressful sub periods. Likewise, the second row demonstrates that a significant amount of information is lost when equities are removed from the cross-section, in the full sample and in stressful sub periods.

The decisiveness of these results in contrast to the inconclusiveness of the regression results in Section 6.4 likely derives from the unconditional, in-sample nature of the results in this section. Whereas the tests in 6.4 try to price assets in a true out-of-sample fashion, the tests in this section examine the composition of the SDF in the entire sample. Given the time-varying nature of the SDF composition exhibited in Section 6.1, one shouldn't find the comparatively poor performance of the out-of-sample pricing tests surprising.

6.4 Dual-Asset-Class Trading Strategies

In this section, I examine the performance of the dual-asset-class trading strategies described in Section 3.3. Overall, I do not find evidence that incorporating dual-asset-class information

	Full Sample		Stressful Sub Periods	
Asset	λ_{asset}	T-Stat	λ_{asset}	T-Stat
TREASURY 1 YEAR	-69.4	-3.40	-92.2	-2.49
TREASURY 2 YEAR	3.86	0.278	-9.66	-0.406
TREASURY 5 YEAR	-3.68	-0.395	6.70	0.422
TREASURY 7 YEAR	-11.3	-1.52	-9.28	-0.761
TREASURY 10 YEAR	7.87	1.44	3.85	0.420
SMALL LoBM	27.0	9.56	20.4	2.93
ME1 BM2	-8.41	-2.57	-1.49	-0.178
ME1 BM3	4.99	1.26	-6.04	-0.644
ME1 BM4	-12.3	-3.04	-12.6	-1.22
SMALL HiBM	-20.1	-4.98	19.9	1.98
ME2 BM1	0.794	0.261	0.514	0.0686
ME2 BM2	0.391	0.113	-17.6	-2.26
ME2 BM3	4.19	1.10	6.25	0.700
ME2 BM4	1.66	0.450	4.91	0.557
ME2 BM5	8.01	2.83	0.723	0.117
ME3 BM1	3.86	1.24	2.32	0.291
ME3 BM2	-1.20	-0.330	4.34	0.475
ME3 BM3	3.67	0.987	9.16	1.04
ME3 BM4	-0.0795	-0.0228	6.24	0.825
ME3 BM5	-2.51	-1.03	-7.56	-1.51
ME4 BM1	4.04	1.26	18.2	2.20
ME4 BM2	7.54	2.08	15.3	1.77
ME4 BM3	9.14	2.67	7.04	0.988
ME4 BM4	2.82	0.869	-4.83	-0.640
ME4 BM5	1.22	0.575	10.1	2.26
BIG LoBM	52.8	10.7	87.9	7.08
ME5 BM2	24.1	6.53	32.9	3.82
ME5 BM3	14.7	4.69	26.3	3.80
ME5 BM4	18.5	7.04	16.2	2.71
BIG HiBM	4.37	2.66	2.54	0.749
LoOP LoINV	-5.32	-2.58	-17.6	-3.48
OP1 INV2	-1.04	-0.548	-5.43	-1.18
OP1 INV3	-0.422	-0.239	-1.82	-0.496
OP1 INV4	-4.06	-2.41	-1.90	-0.541
LoOP HiINV	-2.57	-1.37	-4.33	-0.941
OP2 INV1	-4.59	-2.36	-2.30	-0.498
OP2 INV2	-5.50	-2.58	-5.23	-1.09
OP2 INV3	-6.27	-3.30	-7.07	-1.90
OP2 INV4	-9.97	-4.51	-12.3	-2.52
OP2 INV5	-5.51	-2.94	-5.51	-1.29
OP3 INV1	-7.06	-4.25	-8.06	-1.99
OP3 INV2	-8.92	-4.14	-11.3	-1.93
OP3 INV3	-12.6	-4.80	-29.3	-4.59
OP3 INV4	-8.55	-3.87	-9.76	-1.98
OP3 INV5	-6.40	-3.31	-9.28	-1.90
OP4 INV1	-11.6	-6.55	-15.6	-3.40
OP4 INV2	-13.0	-6.03	-20.3	-3.99
OP4 INV3	-12.1	-4.98	-20.0	-3.37
OP4 INV4	-14.2	-5.56	-33.6	-5.28
OP4 INV5	-11.4	-5.52	-22.8	-4.15
HiOP LoINV	-11.3	-6.38	-15.0	-3.81
OP5 INV2	-12.4	-5.96	-30.9	-6.28
OP5 INV3	-16.0	-6.74	-30.2	-5.47
OP5 INV4	-14.6	-6.21	-36.3	-5.86
HiOP HiINV	-16.3	-8.21	-15.2	-3.17
Lo PRIOR	5.27	3.69	2.00	0.784
PRIOR 2	1.41	0.631	1.62	0.368
PRIOR 3	-0.0623	-0.0230	11.1	1.94
PRIOR 4	3.22	1.11	16.7	2.86
PRIOR 5	8.39	2.89	8.19	1.29
PRIOR 6	8.23	2.59	7.48	1.03
PRIOR 7	13.5	4.04	15.1	1.96
PRIOR 8	5.90	1.79	15.8	1.88
PRIOR 9	12.7	4.21	42.7	5.05
Hi PRIOR	1.23	0.526	9.90	1.71

Table 10: Galpin et al. (2017) significance test results. Significant t-statistics are highlighted in green.

	Full Sample			Stressful Sub Periods		
Asset Class Removed	χ^2_{df}	P-Value	DF	χ^2_{df}	P-Value	DF
Bonds	20.8	8.98e-4	5	203	0.00	5
Equities	269	0.00	60	279	0.00	60

Table 11: Galpin et al. (2017) asset deletion test results.

improves single-asset-class trading strategy performance.

Tables 12 and 13 display the summary statistics for the equity and bond, respectively, value and momentum strategies. Panel A in Table 12, demonstrates that all four equity value strategies outperform the benchmark in terms of Sharpe ratio. Yet comparing the Information Single and Information Dual columns, we see that for the Information method, incorporating bonds into SDF estimation does not change the performance of the equity value strategy at all. For the Deep method, incorporating bond Info raise Sharpe insignificantly (.0297 to .0333).

Panel B in Table 12 exhibits similar results for the equity momentum strategies. Adding bonds to the cross section does not change the Information momentum strategy performance at all, while it lowers the Deep strategy Sharpe ratio insignificantly (.0309 to .0305). Unlike with the value equity strategies, however, none of the equity momentum strategies outperform the benchmark in terms of Sharpe, and the Information momentum strategy actually yields a negative expected daily return.

Table 13 presents summary statistics for the bond trading strategies that fit the pattern established by the equity results. The value strategy results in Panel A demonstrate that the single and dual-asset-class Information strategies achieve identical performance metrics, while the the dual-asset-class Deep strategy achieves an insignificantly lower Sharpe ratio than the single-asset-class Deep strategy (.0449 vs. .0452). The Information strategies achieve lower Sharpe ratios than the benchmark, while the Deep strategies attain higher Sharpe ratios.

Panel B of table 13 shows the bond Information momentum strategies fail to outperform the benchmark in terms of Sharpe ratio, while both Deep strategies do achieve higher Sharpe

Panel A: Value Strategy Results					
Statistic	Info Single	Deep Single	Info Dual	Deep Dual	Benchmark
Mean	3.92e-4	3.70e-4	3.92e-4	3.76e-4	1.87e-4
Std Dev	0.0115	0.0124	0.0115	0.0113	9.73e-3
Skewness	-0.579	-0.597	-0.579	-0.709	0.113
Kurtosis	18.2	27.3	18.2	17.3	7.72
Sharpe	0.0340	0.0297	0.0340	0.0333	0.0192
Panel B: Momentum Strategy Results					
Statistic	Info Single	Deep Single	Info Dual	Deep Dual	Benchmark
Mean	-6.42e-4	3.39e-4	-6.42e-4	3.42e-4	5.25e-4
Std Dev	0.0605	0.0110	0.0605	0.0112	0.0154
Skewness	-4.48	-0.529	-4.48	-0.499	-1.04
Kurtosis	127	15.4	127	14.2	18.6
Sharpe	-0.0106	0.0309	-0.0106	0.0305	0.0340

Table 12: Daily equity trading strategy summary stats.

ratios than the benchmark. Here again, incorporating equities in the cross section does not alter the Information strategy performance at all, but actually significantly decreases the Deep strategy Sharpe by almost doubling volatility while barely changing expected return as compared to the single-asset-class Deep strategy. Incorporating equity information appears to significantly increase the volatility of the SDF and its mimicking portfolio.

Panel A: Value Strategy Results					
Statistic	Info Single	Deep Single	Info Dual	Deep Dual	Benchmark
Mean	9.61e-5	1.17e-4	9.61e-5	1.07e-4	7.62e-5
Std Dev	0.162	2.58e-3	0.162	2.39e-3	0.00419
Skewness	2.01	0.269	2.01	0.250	-0.0779
Kurtosis	168	6.91	168	6.63	4.22
Sharpe	5.94e-4	0.0452	5.94e-4	0.0449	0.0182
Panel B: Momentum Strategy Results					
Statistic	Info Single	Deep Single	Info Dual	Deep Dual	Benchmark
Mean	4.85e-6	1.18e-4	4.85e-6	1.31e-4	5.54e-5
Std Dev	0.0148	2.64e-3	0.0148	4.79e-3	0.00396
Skewness	-2.98	0.277	-2.98	0.366	-0.0317
Kurtosis	150	6.56	150	33.5	4.73
Sharpe	3.28e-4	0.0447	3.28e-4	0.0273	0.0140

Table 13: Daily bond trading strategy summary stats.

Thus, incorporating dual-asset-class information does not significantly increase the Sharpe ratios of these equity and bond value and momentum trading strategies. Examining the factor regressions in Tables 14 and 15, we see that the dual-asset-class informed trading strategies also do not achieve significantly higher time-series alphas than the single-asset-class informed trading strategies.

In Panel A of Table 14, all of the equity value trading strategy alphas are negative, more negative than the benchmark alpha, and three of the four alphas are significantly negative. All four strategies and the benchmark load significantly positively on HML. The dual-asset-class Information strategy achieves the same factor loadings as the single-asset-class Information strategy. The dual-asset-class Deep strategy achieves an alpha that is greater, but more negative in t-statistic, as compared to the single-asset-class Deep strategy.

In Panel B of Table 14, all of the equity momentum trading strategy alphas are negative, the benchmark alpha is positive, and all five of these alphas are insignificant. Three of the four strategies load significantly positively on MOM. Here again, the dual-asset-class Information strategy achieves the same factor loadings as the single-asset-class Information strategy. The dual-asset-class Deep strategy achieves an alpha that is slightly more negative, but slightly less negative in t-statistic, as compared to the single-asset-class Deep strategy.

Table 15 displays similar regression results for the bonds strategies. Panel A exhibits negative insignificant alphas for the Information value strategies, positive significant alphas for the Deep strategies, and a positive insignificant benchmark alpha. The single and dual-asset-class Information strategies have identical factor loadings, while the single-asset-class Deep strategy has a slightly greater and more significant alpha than then dual-asset-class Deep strategy.

Panel B exhibits negative insignificant alphas for the Information momentum strategies, positive significant alphas for the Deep strategies, and a positive insignificant benchmark alpha. Here again, the single and dual-asset-class Information strategies have identical factor loadings. However, the dual-asset-class Deep strategy alpha is slightly higher than that for the single-asset-class strategy, but has a significantly smaller t-statistic. Referring to Table 13, it appears that the increased volatility of the dual-asset-class SDF mimicking portfolio

Panel A: Value Strategy Results

	Info Single	Deep Single	Info Dual	Deep Dual	Benchmark
const	-3.897e-5	-4.397e-5	-3.897e-5	-3.313e-5	-3.025e-5
	(-2.205)	(-1.558)	(-2.205)	(-2.575)	(-0.5519)
Mkt-RF	1.060	1.082	1.060	1.048	0.1386
	(169.6)	(65.08)	(169.6)	(265.9)	(12.03)
SMB	0.09419	0.07169	0.09419	0.06618	0.3482
	(11.26)	(4.737)	(11.26)	(10.24)	(13.75)
HML	0.5038	0.6180	0.5038	0.3736	1.312
	(18.49)	(4.765)	(18.49)	(16.43)	(35.71)
RMW	-1.777e-3	-0.02581	-1.777e-3	0.04055	-0.3287
	(-0.1376)	(-0.5647)	(-0.1376)	(2.667)	(-9.688)
CMA	1.033e-3	-0.1222	1.033e-3	0.02063	0.1313
	(0.04807)	(-1.279)	(0.04807)	(0.9105)	(2.843)
MOM	-0.01860	-0.07177	-0.01860	-0.03583	-0.04281
	(-1.664)	(-2.800)	(-1.664)	(-3.747)	(-2.092)
$\overline{R}^2$	98.3	92.8	98.3	98.9	71.7

Panel B: Momentum Strategy Results

	Info Single	Deep Single	Info Dual	Deep Dual	Benchmark
const	-1.348e-3	-4.321e-5	-1.348e-3	-4.441e-5	8.841e-5
	(-1.585)	(-1.592)	(-1.585)	(-1.148)	(1.286)
Mkt-RF	0.8955	1.015	0.8955	1.008	-0.01808
	(5.023)	(139.4)	(5.023)	(92.35)	(-0.8011)
SMB	-0.2050	0.08814	-0.2050	0.05856	-0.1791
	(-1.455)	(5.467)	(-1.455)	(2.556)	(-5.790)
HML	-1.698	0.1129	-1.698	0.1193	-0.2776
	(-3.614)	(5.520)	(-3.614)	(4.611)	(-2.884)
RMW	-0.2747	0.1264	-0.2747	0.09203	0.1448
	(-0.7896)	(2.965)	(-0.7896)	(1.874)	(2.879)
CMA	1.548	-0.03460	1.548	-0.08050	0.07906
	(3.908)	(-0.9774)	(3.908)	(-1.987)	(0.8665)
MOM	1.888	0.02005	1.888	0.08478	1.671
	(4.474)	(0.8007)	(4.474)	(2.233)	(56.36)
$\overline{R}^2$	11.4	96.3	11.4	91.6	81.5

Table 14: Equity trading strategy factor regressions. HAC-adjusted t-statistics (maximal Newey-West lag of 30 days) are presented in parentheses.

bleeds into a higher standard error for the alpha term.

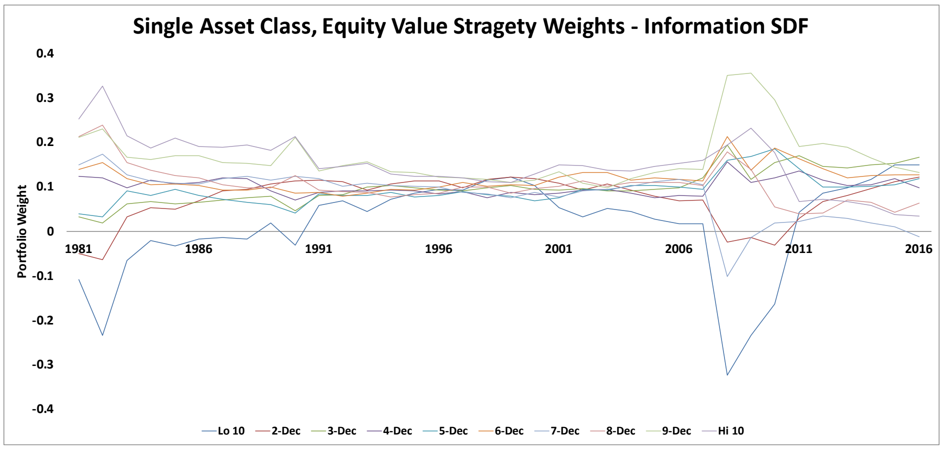
Panel A: Value Strategy Results					
	Info Single	Deep Single	Info Dual	Deep Dual	Benchmark
const	-1.538e-4	8.380e-5	-1.538e-4	7.581e-5	6.024e-3
	(-0.07716)	(5.321)	(-0.07716)	(5.298)	(1.482)
Level	-21.50	-2.742	-21.50	-2.613	-136.8
	(-2.735)	(-12.44)	(-2.735)	(-13.22)	(-3.431)
Slope	-30.75	-1.403	-30.75	-1.301	-212.9
	(-2.537)	(-8.961)	(-2.537)	(-9.747)	(-4.497)
Curvature	-11.98	-1.202	-11.98	-0.9745	-145.1
	(-1.500)	(-5.457)	(-1.500)	(-5.124)	(-3.392)
$\overline{R^2}$	1.4	51.4	1.4	53.4	11.0

Panel B: Momentum Strategy Results					
	Info Single	Deep Single	Info Dual	Deep Dual	Benchmark
const	-1.278e-5	8.477e-5	-1.278e-5	8.884e-5	4.230e-3
	(-0.06583)	(5.331)	(-0.06583)	(2.505)	(1.065)
Level	-1.435	-2.779	-1.435	-3.476	-111.2
	(-1.891)	(-12.25)	(-1.891)	(-7.292)	(-3.569)
Slope	0.6755	-1.502	0.6755	-1.456	-117.0
	(0.5668)	(-8.564)	(0.5668)	(-7.015)	(-3.179)
Curvature	0.04476	-1.238	0.04476	-1.729	-58.63
	(0.06715)	(-4.998)	(0.06715)	(-4.341)	(-1.531)
$\overline{R^2}$	0.5	50.8	0.5	23.7	4.9

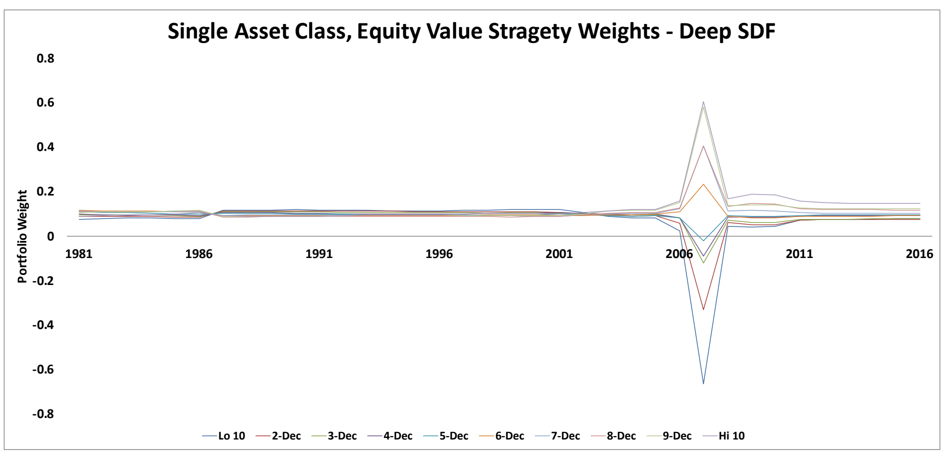
Table 15: Bond trading strategy factor regressions. HAC-adjusted t-statistics (maximal Newey-West lag of 30 days) are presented in parentheses.

As an interesting side note, all eight of the Information and Deep, single and dual-asset-class, value and momentum strategies have significantly positive market betas of close to one. While the SDF-mimicking portfolio weights are not constrained to be positive, both the Information and Deep SDF-mimicking portfolio weights do happen to be positive on average, as illustrated by Figure 5 for the single-asset-class equity value strategies.

Why are the performance metrics so similar between the single and dual-asset-class informed strategies? As illustrated in Table 16, the single and dual-asset-class informed strategies prove highly correlated ($> .95$) across the set of trading assets (equities vs. bonds) and SDF estimation method (Information vs. Deep) used. The SDF correlations in Table 17 show that the single and dual-asset-class SDFs are also highly-correlated, but less so than the factor-mimicking portfolios. Thus, bonds do appear to add some new information to the equity SDF, but this information cannot be expressed through the linear span of the equity



(a) Information SDF



(b) Deep SDF

Figure 5: MP weights for the single-asset-class equity value strategies constructed via the Information and Deep SDF methods.

returns, and vice versa. Hence, any unique information found in the dual-asset-class cross-section does not improve performance of the single-asset-class trading strategies. Figure 6 displays the growth of $1 in each of the trading strategies and benchmarks, and illustrates the co-movement in the returns to these strategies. Importantly, the return correlation doesn't break down during the stressful sub periods highlighted in gray.

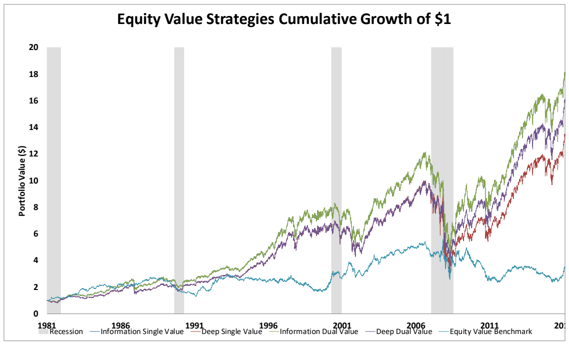
Panel A: Equity Strategy Return Correlations

	Info Single Value	Deep Single Value	Info Dual Value	Deep Dual Value	Info Single Mom	Deep Single Mom	Info Dual Mom	Deep Dual Mom	Bench. Value	Bench. Mom
Info Single Value	1.0	0.95	1.0	0.99	0.04	0.96	0.04	0.93	0.28	-0.26
Deep Single Value	0.95	1.0	0.95	0.97	0.07	0.92	0.07	0.89	0.3	-0.32
Info Dual Value	1.0	0.95	1.0	0.99	0.04	0.96	0.04	0.93	0.28	-0.26
Deep Dual Value	0.99	0.97	0.99	1.0	0.07	0.97	0.07	0.94	0.23	-0.25
Info Single Mom	0.04	0.07	0.04	0.07	1.0	0.1	1.0	0.09	-0.14	0.27
Deep Single Mom	0.96	0.92	0.96	0.97	0.1	1.0	0.1	0.99	0.09	-0.16
Info Dual Mom	0.04	0.07	0.04	0.07	1.0	0.1	1.0	0.09	-0.14	0.27
Deep Dual Mom	0.93	0.89	0.93	0.94	0.09	0.99	0.09	1.0	0.06	-0.11
Bench. Value	0.28	0.3	0.28	0.23	-0.14	0.09	-0.14	0.06	1.0	-0.36
Bench. Mom	-0.26	-0.32	-0.26	-0.25	0.27	-0.16	0.27	-0.11	-0.36	1.0

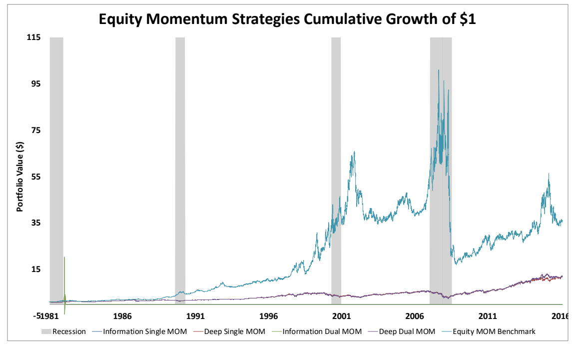
Panel B: Bond Strategy Return Correlations

	Info Single Value	Deep Single Value	Info Dual Value	Deep Dual Value	Info Single Mom	Deep Single Mom	Info Dual Mom	Deep Dual Mom	Bench. Value	Bench. Mom
Info Single Value	1.0	0.15	1.0	0.17	-0.82	0.14	-0.82	0.08	-0.19	-0.15
Deep Single Value	0.15	1.0	0.15	0.96	0.08	0.98	0.08	0.69	0.52	0.39
Info Dual Value	1.0	0.15	1.0	0.17	-0.82	0.14	-0.82	0.08	-0.19	-0.15
Deep Dual Value	0.17	0.96	0.17	1.0	0.07	0.96	0.07	0.64	0.46	0.34
Info Single Mom	-0.82	0.08	-0.82	0.07	1.0	0.11	1.0	0.07	0.24	0.29
Deep Single Mom	0.14	0.98	0.14	0.96	0.11	1.0	0.11	0.7	0.51	0.43
Info Dual Mom	-0.82	0.08	-0.82	0.07	1.0	0.11	1.0	0.07	0.24	0.29
Deep Dual Mom	0.08	0.69	0.08	0.64	0.07	0.7	0.07	1.0	0.39	0.23
Bench. Value	-0.19	0.52	-0.19	0.46	0.24	0.51	0.24	0.39	1.0	0.34
Bench. Mom	-0.15	0.39	-0.15	0.34	0.29	0.43	0.29	0.23	0.34	1.0

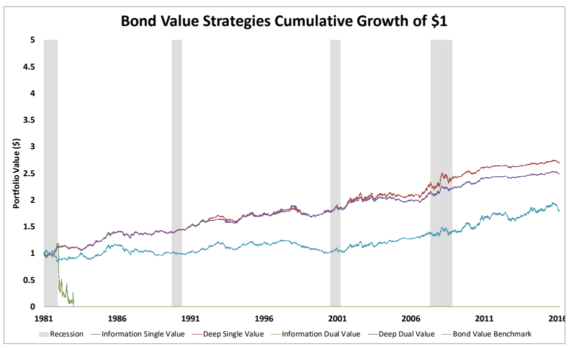
Table 16: Trading strategy return correlations.



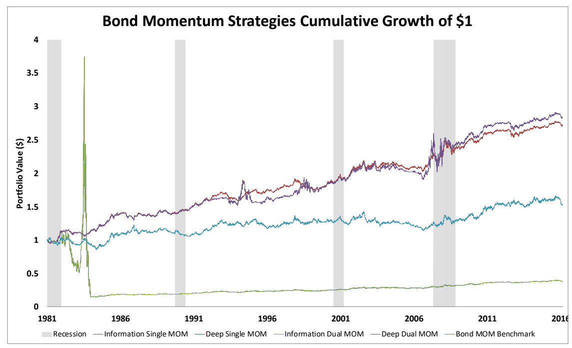
(a) Equity Value Cumulative Returns



(b) Equity Momentum Cumulative Returns



(c) Bond Value Cumulative Returns



(d) Bond Momentum Cumulative Returns

Figure 6: Cumulative growth of $1 in each of the equity and bond value and momentum strategies and benchmarks. NBER recession periods are highlighted in gray.

Panel A: Equity Strategy SDF Correlations

	Info Single Value	Deep Single Value	Info Dual Value	Deep Dual Value	Info Single Mom	Deep Single Mom	Info Dual Mom	Deep Dual Mom
Ghosh Single Value	1.0	0.23	0.83	0.26	0.16	0.02	0.12	0.05
Deep Single Value	0.23	1.0	0.17	0.95	0.21	0.77	0.19	0.75
Ghosh Dual Value	0.83	0.17	1.0	0.1	0.12	0.01	0.18	-0.06
Deep Dual Value	0.26	0.95	0.1	1.0	0.2	0.69	0.11	0.77
Ghosh Single Mom	0.16	0.21	0.12	0.2	1.0	0.19	0.9	0.18
Deep Single Mom	0.02	0.77	0.01	0.69	0.19	1.0	0.18	0.95
Ghosh Dual Mom	0.12	0.19	0.18	0.11	0.9	0.18	1.0	0.1
Deep Dual Mom	0.05	0.75	-0.06	0.77	0.18	0.95	0.1	1.0

Panel B: Bond Strategy SDF Correlations

	Info Single Value	Deep Single Value	Info Dual Value	Deep Dual Value	Info Single Mom	Deep Single Mom	Info Dual Mom	Deep Dual Mom
Ghosh Single Value	1.0	-0.16	0.49	-0.23	0.49	-0.12	0.18	-0.12
Deep Single Value	-0.16	1.0	-0.07	0.05	-0.14	0.98	-0.09	0.12
Info Dual Value	0.49	-0.07	1.0	0.1	0.21	-0.05	0.18	-0.06
Deep Dual Value	-0.23	0.05	0.1	1.0	-0.18	0.05	0.11	0.77
Info Single Mom	0.49	-0.14	0.21	-0.18	1.0	-0.19	0.43	-0.12
Deep Single Mom	-0.12	0.98	-0.05	0.05	-0.19	1.0	-0.12	0.12
Info Dual Mom	0.18	-0.09	0.18	0.11	0.43	-0.12	1.0	0.1
Deep Dual Mom	-0.12	0.12	-0.06	0.77	-0.12	0.12	0.1	1.0

Table 17: Trading strategy SDF correlations.

7 Conclusion

Overall, I find evidence of integration between equity and bond markets in the full sample. I also find no evidence of increased segmentation in stressful sub periods, a result that does not corroborate the preferred habitat hypothesis in the context of equity and bond markets. Exploiting this integration, however, to price and trade equities and bonds out of sample proves difficult due to the inherent challenges in working with wide cross sections of assets. Nevertheless, the non-parametric SDF estimation procedures considered in this work do show some promise in being able to tackle higher-dimensional cross sections than have traditionally been considered in the literature.

Going forward, this line of research lends itself to several extensions. First, the Deep SDF estimator used in this work employs a simple neural network architecture (a single layer). More sophisticated architectures (e.g. deep long short memory networks) may uncover an SDF that better prices the cross section out of sample. Second, if more sophisticated non-parametric SDF estimators perform better out of sample, extending the analysis in this work to include other asset classes (e.g. currencies, commodities, corporate bonds, etc.) might lead to evidence of broad market integration, and point the way to a unified theory of asset pricing. For example, non-parametric construction of cross-asset class systematic risk factors that price a wide cross section of assets might not only "tame the factor zoo," but would also have significant portfolio management implications.

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