Atmospheric Thermal Tides of Exoplanets

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Abstract

Gravitational interactions between a planet and its host star may have important effects on the orbit and rotation rates of the two bodies. Gravitational tides, through tidal friction, change the rotation period of a planet, ultimately synchronizing it with its orbital period and tidally locking it when the orbit is nearly circular. While the theory of gravitational tidal friction (Darwin 1898) has been invoked for some time now to understand circularization of orbits and synchronization of spins, the possibility of additional physical effects which may compete with tidal friction is relatively less explored.

In this thesis the mechanism of "thermal tides" will be explored through hydrodynamic simulations, and applied to understand the expected rotation rates of planets orbiting very close to their host star. Thermal tides describe fluid flows in planetary atmospheres driven by time-dependent stellar irradiation as experienced on asynchronously rotating planets. This time-dependent heating may also generate mass quadrupoles which could then be torqued by the stellar tidal force. These "thermal tide torques" may either reinforce or counteract torques from gravitational tidal friction, either trying to accelerate or reverse a planet's rotation rate. A balance of opposing torques could occur at a rotation rate distinct from synchronous rotation, and therefore the addition of thermal tide effects may allow a theoretical understanding of exoplanet rotation rates, which are as yet unmeasured.

We conducted four simulations varying the orbital period of a hypothetical planet. We found that the thermal tide torque increases toward shorter orbital periods. Additionally, we found that the time scale for torque equilibrium increases with orbital period. These results can contribute to further understanding of the effects of stellar irradiation on atmospheric heating and planetary rotation rates, and can shed light on expected rotation rates of planets orbiting very near their star, where tidal effects are important.

Keywords: atmospheric thermal tides, thermal forcing, tidal evolution, exoplanets, Athena++

1 Introduction

Exoplanets, or planets that orbit around other stars, were first discovered in the 1990s (Sager and Lissauer 2011). Exoplanets are most easily detected very near their star, using the radial velocity and transit methods, and as a result the detected population is quite different from our own solar system, where the planets are much more distant from the star. The closest exoplanet orbital separations are just outside the star, more than 10 times closer than Mercury is from the Sun. The small population of "directly imaged" exoplanets are much more distant from their star, well outside the orbit of Uranus in our own solar system. In addition, while gas giants in our solar system are distant from the Sun, "hot Jupiter" exoplanets can be found orbiting at just a few stellar radii, in seeming contradiction to pre-1990's planet formation theories. Lastly, the full range of exoplanets from sub-Earth sized, to the super-Earth and mini-Neptune's not found in our solar system, to planets 10 times more massive than Jupiter, are found around other stars. The most common type of solar system found in our galaxy is now thought to be systems of multiple small planets, in nearly circular orbits, orbiting very near their star, where the star will usually be much smaller than our Sun (Lissauer and de Pater 2001).

In addition to differences in planet sizes and orbital separations, the rotation rate of planets close to their parent star may be quite different from rotation rates in our own solar system. To make a connection with our experience here on Earth, a planet's day is the length of time for a planet to make a single rotation around its axis, and a planet's year is the length of time for a planet to make one full revolution around its star. For Earth, a year is 365.25 days, each 24 hours long. However, some planets have years that are equal to their day–that is, the length of one rotation is the same as that of one orbit. This is called "tidal locking". These interactions are due to friction acting on motions induced by gravitational tides, or "tidal friction" (Goldreich and Soter 1966). Gravitational tides are the variation in gravitational acceleration over the body, which tends to stretch or compress it. If there is some friction acting on these motions (e.g. viscosity) then there is a lag between the force and the response. This "lagging tidal bulge" will be misaligned with the line joining the two bodies, and the tidal force can then apply a non-zero torque to alter the rotation period. The end state of this process (at least for nearly circular orbits) is a rotation period synchronized to the orbital period of the planet (Correia and Laskar 2011).

Two examples of gravitational tidal friction effects can be seen for the Earth-Moon system. First, the Moon's orbital period and rotational period are equal to each other, due to tides exerted on the Moon by the Earth, and tidal friction in the Moon. It is because the Moon's rotation is tidally locked to its orbit that we on Earth always see the same face of the moon. The second effect of tidal friction is for the tide raised in the Earth by the Moon. Soon after it's formation, the Moon is thought to have had a much smaller semimajor axis than it does now (Goldreich and Soter 1966), and the orbit has expanded over time. At the same time, the Earth is believed to have been rotating much faster in the past, and to have slowed its rotation over time, with the angular momentum transferred to the Moon's orbit, causing it to move away over time (Goldreich and Soter 1966; Correia and Laskar 2011).

We should see similar effects in other binary systems (star-star, planet-planet, planet-moon, etc.). Yet, we don't see this same phenomenon occur with Venus, even though it is closer to the Sun than Earth. The rotation period of Venus is extremely slow in comparison to Earth, with a single rotation lasting over 200 days–over a month longer than its revolution (Lissauer and de Pater 2001). In addition to rotating slowly Venus is rotating in the opposite direction that it is orbiting, counter to expectations from tidal friction torques. While Venus' rotation may have been slowed due to tidal friction, it does not have a synchronous rotation with the Sun that would be expected, given the Earth-Moon system. One possible explanation for this slow and retrograde rotation is that there must be another torque altering Venus' rotation, competing with tidal friction torques.

Mercury, too, also experiences the effects of gravitational tidal friction, and these effects are larger because of its close proximity to the Sun. The influence of solar tides has pushed Mercury into a spin-orbital resonance of 3:2 and an orbital period to rotation period ratio of 88 to 59 days (Goldreich and Soter 1966). This makes Mercury's year much closer to the length of Mercury's day when compared to a planet like Earth. As noted by Goldreich and Soter (1966), Mercury is particularly interesting due to its non-synchronous rotation and its lack of tidal locking, which should be the ultimate goal of exclusive gravitational tidal systems.

Thermal tides are the result of uneven heating in a planet's atmosphere that drive fluid motions (Arras and Socrates 2010). As with all tidal phenomena, this effect is more important for planets closer to their host star. As the planet rotates, the dayside is heated and the nightside cools. The atmosphere on the dayside becomes much hotter and less dense than on the nightside. Because hot air is less dense than cooler air, the heated air on the dayside will begin to rise. This creates a density gradient and fluid motion, with heated air rising as it faces the star, cooling as the air migrates to face outwards due to the planet's rotation and sinking, and rising again once it is heated by the star. The peak of these thermal tides appear when the planet is facing "noon" and are at their lowest when the planet is facing "midnight," with equilibrium points at dawn and dusk (Arras and Socrates 2010; Showman et al. 2008). Like gravitational tides, thermal tide torques can influence the speed of the planet's rotation,, either accelerating or decelerating the rotation.

This can help explain the rotation of Venus. Heat radiating from its solid surface, combined with its daily uneven heating, result in convection within the atmosphere (Ingersoll and Dobrovolskis 1978). It is believed that the thermal atmospheric tides caused by this uneven heating–combined with resonance with Earth–have worked to somewhat cease its synchronous tidal evolution and have worked to cause retrograde rotation (Ingersoll and Dobrovolskis 1978; Goldreich and Soter 1966; Gold and Soter 1969). The friction caused by the density gradients and layering within the atmosphere has contributed to this phenomenon enormously.

The same logic can be applied to exoplanets with atmospheres. Gravitational tidal friction tries to synchronize the planet's rotation rate to that of the orbit. Similarly, time-dependent irradiation in non-synchronized planets drives fluid motion which created quadrupole moments in the atmosphere, and the stellar tidal force can torque these quadrupole moments, possibly creating an opposing torque. Once a balance of torques is reached, the spin rate is at an equilibrium, which may deviate from the synchronous rate, so that the same side of the planet is not always facing the star.

Simulations modeling atmospheric thermal tides and gravitational tides working in tandem to alter the rotation period of a planet have received less attention in the literature. While previous work has been conducted to understand the relationships between thermal tides, gravitational tides, and the effects of day/night Newtonian cooling, they have rarely been implemented into a single model. We aim to use the Athena++ software to simulate an exoplanet heated as it rotates to observe its quadrupole moments, torque, and temperature along its radius to determine whether its rotation is affected by thermal tides.

2 Methods

2.1 Gravity and Pressure forces in Hydrostatic Balance

For the initial condition of the atmosphere, we assume that the exoplanet exists in hydrostatic balance - that is, pressure gradient force produced by the planet is outward and is equal to the gravity pushing inward. This gravity, which can be described as $-GM\rho/r^2$, must be equal to the pressure gradient dP/dr (Vallis 2017).

2.2 Fluid Equations

There are three fluid equations that govern our simulated exoplanet: the mass continuity equation, the conservation of momentum, and the energy equation.

Mass cannot be created nor destroyed. For fluids, this mass can only be transferred in and out of a given volume. Because our planet's atmosphere is constantly in a state of motion, there is an uneven density distribution. This means that the mass of the air parcels being heated and cooled are not simply disappearing in a given space–they are simply being transported to different positions in the planet.

The gas satisfies Newton's equation of motion $F = ma$, as applied to a fluid. The mass in this instance is representative of multiple particles within the fluid, which can be translated to ρa, or the mass per volume of a fluid multiplied by its acceleration. This sum consists of the gravity of the planet (ρG) and the gas pressure gradient per volume. This accounts for three out of the five necessary equations used. The gravitational tides of the star can seize onto the planet's quadrupole moments and apply the torque to change the planet's spin. While we do not directly include the stellar tidal acceleration in the momentum equations, we compute this torque in a post-processing of the simulations, given the measured quadrupole moments.

Lastly, one must account for the conservation of energy. While the internal energy of the planet is transformed into different forms, the total internal energy is conserved in the absence of explicit heating or cooling processes. This total internal energy can be broken down into two different types: the bulk motion of the fluid and its internal (thermal) motions. This internal motions consists of individual particles that will move at a faster speed when the fluid is heated, and will move at a slower speed when it is cooled. This is expected in adiabatic motion. However, our simulated planet is not purely adiabatic since we include an explicit heating/cooling term to represent heating by stellar radiation.

2.3 Exoplanet parameters

Our simulation aims to examine the torque on a planet caused by atmospheric thermal tides, which can ultimately lead to a change in its rotation period. Our exoplanet will have an angular frequency of rotation of $\Omega_{\rm spin} = 2\pi/P_{\rm spin}$, where $P_{\rm spin}$ is the rotation period, and will be heated and cooled by a host star. If the temperature of the planet extends above the chosen "goal" temperature, then it will be cooled. These interactions should trigger fluid motion which will redistribute the mass and cause differences in gas density. Ideally, this should result time-changing quadrupole moments which should allow for a torque to be generated to change the planet's rotational period. We should expect to see a non-zero torque that would result in a rotation period dissimilar to its orbital period. Our model, however, does not consider the frictional force that would arise from currents in the atmosphere.

To better understand thermal tides and their resulting quadrupole moments and torque, we modeled thermal tides on a rotating Hot Jupiter using the Athena++ software (Princeton University, Version 24.0; Stone et al.). Our chosen exoplanet atmosphere is assumed to be completely gaseous with an inner computational boundary of $R_1 = 7.0 \times 10^9$ cm, and an outer boundary of $R_2 = 7.1 \times 10^9$ cm. The mass was assumed to be $M_p = 2 \times 10^{30}$ g, with a GM_p value of $1.3 \times 10^{23} g$.

Our simulations are carried out in a reference frame rotating at $P_{\rm spin} = 1$ day and hence Coriolis and centrifugal forces are included. The latter causes a slight bulge at the equator. The boundary conditions at the inner and outer radial boundary are that mass is not allowed to move through the boundaries, so that the total mass of the atmosphere remained constant. We used an adiabatic index of $\gamma = 1.4$, which is appropriate for translational and rotational motion of H_2 molecules. Initially, our simulation starts with a uniform temperature of $T = 10^3$ K and in hydrostatic balance. All simulations were conducted with a duration of $t_{\rm lim} = 4 \times 10^6$ s, with 128, 32, and 32 grid points in the r, θ, and ϕ directions.

The thermal forcing of the atmosphere was included by a Newtonian cooling term in the energy equation of the form

$$\dot{e} = -\frac{\rho C_p (T - T_{\rm goal})}{\tau_{\rm time}}, \tag{1}$$

where ρ is the mass density, $C_p = (5k_bT/2\mu m_p)$ is the specific heat per unit mass, T is the temperature, $T_{\rm goal}$ is a position and time-dependent reference temperature, and $\tau_{\rm time}$ is the thermal time. This thermal forcing tries to force the fluid to $T = T_{\rm goal}$ over a timescale τ. The specific choices for $T_{\rm goal}$ and τ will be discussed below. This Newtonian heating/cooling function causes time-dependent changes in temperature, and is the driver of the fluid motions.

All simulations were conducted with initial base pressures of $P_{\rm b} = 1$ bar at $r = R_1$. Orbital frequencies were varied to assess their effects ($\Omega_{\rm orb} = 2\pi/P_{\rm orb}$.)

2.4 Simulation equations

Implementing the rotating reference frame and integrating the effects of the centrifugal force and Coriolis force, density at any point on the grid was solved using

$$\rho = \rho_{\rm b} e^{(GM_p/a^2)(R_1/r-1)+\Omega_{\rm spin}^2(\varpi^2-R_1^2)/(2a^2)} \tag{2}$$

where the cylindrical radius is calculated as $\varpi = r\sin(\theta)$.

The gravitational acceleration felt by the planet due to a star of mass M_s at orbital distance D can be written as $\mathbf{g} = -\nabla\Phi$, where Φ is the gravitational potential. The potential can be broken down into different components, with the quadrupole piece representing the leading order effect of tides. This one piece of the potential can be written

$$\Phi = -\frac{GM_s}{D^3}\left(\frac{3}{2}\left(\frac{xx_s + yy_s}{D}\right)^2 - \frac{1}{2}r^2\right), \tag{3}$$

where (x, y) are Cartesian coordinates centered on the planet, and (x_s, y_s) are the position of the star in that coordinate system, in the orbital plane.

The thermal forcing generates quadrupole moments defined by

$$Q_{ij} = \int d^3x \rho(\vec{x}) x_i x_j \tag{4}$$

for $i, j = x, y, z$. The torque on the planet can then be calculated as an integral of the tangential tidal acceleration against the mass density, leading to an expression for the torque τ to be

$$\tau = \frac{3}{2}\Omega_{\rm orb}^2 \left((Q_{xx} - Q_{yy})\sin(2[\Omega_{\rm orb} - \Omega_{\rm spin}]t) - 2Q_{xy}\cos(2[\Omega_{orb} - \Omega_{spin}]t)\right) \tag{5}$$

To simulate day-night heating and cooling, the Newtonian Cooling function was implemented. $\phi_{\rm star} = (\Omega_{\rm orb} - \Omega_{\rm spin})t$ was used to describe the azimuthal position of the star in the rotating reference frame. We defined a set temperature to return to once the planet is heated by the equation

$$T_{\rm goal} \quad = \quad T_0 + \delta T_2 \cos\left(2[\phi - \phi_{star}]\right), \tag{6}$$

where $\delta T_2 = 100\,\mathrm{K}$, allowing for a fluctuation of 100 K in either direction, and $T_0 = 10^3\,\mathrm{K}$. The actual calculation of temperature caused by day/night heating and cooling can be calculated from simulation variables as $T = \mu m_{amu} P/(k_b \rho)$, where $\mu = 2.3$, $m_{amu} = 1.67 \times 10^{-24}\,\mathrm{g}$, and $k_b = 1.38 \times 10^{-16} \mathrm{erg\,g^{-1}}$. Thermal time τ is defined as $\tau = \tau_0(1 + P/P_0)$ where $\tau_0 = 10^3\,\mathrm{s}$ and $P_0 = 1\,\mathrm{mbar}$.

2.5 Experiments

We were interested in examining how the effect of orbital period $P_{\rm orb}$ may effect the torque of the simulated planet. Therefore, we conducted four simulations, each altering the orbital period.

For all simulations, $P_{\rm spin} = 1\,\mathrm{day}$. Simulations 1-4 (sim1, sim2, sim3, and sim4) altered the orbital periods of each simulation as $P_{\rm orb} = \infty, 4\,\mathrm{days}, 2\,\mathrm{days}, 0.5\mathrm{days}$. Thermal forcing is dependent on the difference in the angular velocity of the planet's spin and orbit.

3 Results

The torque $\tau(t)$ is found to exhibit significant oscillations. However, these oscillations do not lead to long-term changes as they would integrate to zero. We time averaged the torque to find the piece which will cause long-term secular changes in the rotation rate of the planet.

We found that the oscillation frequencies and average torques varied in magnitude and direction depending on the orbital period. At an orbital period of $\mathrm{P_{orb}} = \infty$, the average torque was exactly zero, and there were no oscillations of the planet. This was because the prefactor $\Omega_{\rm orb} = 0$. In contrast, simulations of 0.5 days, two days, and four days saw variations in their average torques (Figures 1.2, 1.3, and 1.4). The four day simulation had the smallest torque in both magnitude and direction, with an average torque $\tau = -1.35 \times 10^{29}\,\mathrm{dyne\,cm}$. Unlike the other simulations, the average torque was in the opposite direction of spin. The average torques for $\mathrm{P_{orb}} = 0.5$ days and $\mathrm{P_{orb}} = 2$ days are both positive (Figures 1.4 and 1.3). However, the two day simulation had a smaller average torque $\tau = 1.38 \times 10^{29}\,\mathrm{dyne\,cm}$, while the 0.5 day simulation gave $\tau = 1.93 \times 10^{29}\,\mathrm{dyne\,cm}$. Larger fluctuations at the beginning of each simulation can largely be attributed to initial transients. These torques were within a factor of two in magnitude to one another, despite the four day simulation's difference in spin direction and the larger differences in orbital period (four times larger for the two day simulation and eight times larger for the four day simulation).

We find that the mean torque, averaged over all the time steps in the output file, has a value much smaller than the oscillating torque. This may imply

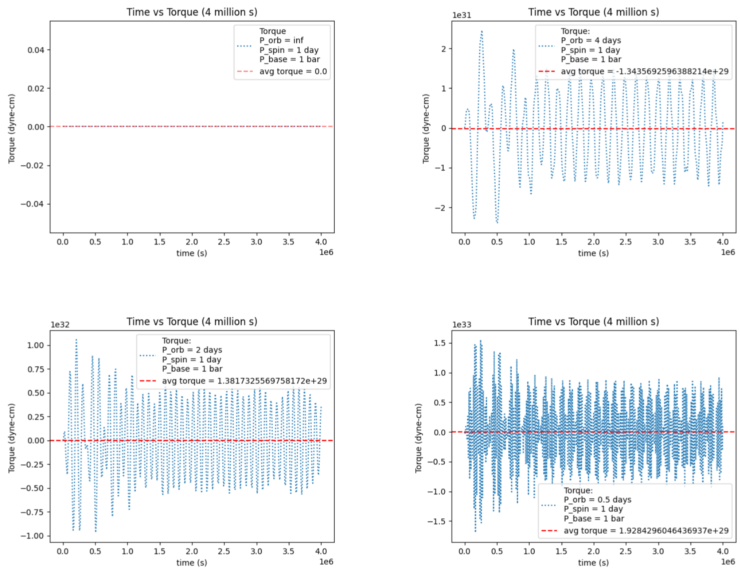

Figure 1: Orbital period simulations conducted for 4 million seconds. *Figure 1.1* (top left): $P_{orb} = \infty$. *Figure 1.2* (top right): P_{orb} = four days. *Figure 1.3* (bottom left): P_{orb} =two days. *Figure 1.4* (bottom right): P_{orb} = 0.5 days.

that the results could be dependent on the time interval used for the averaging (e.g. if the initial transient phase was ignored). While beyond the scope of this thesis, future studies should investigate the convergence of the average torque with both simulation time, number of grid points used, and amplitude of the temperature variation used in the forcing.

4 Discussion

Altering the orbital period seems to have an effect on the calculated average torque for our exoplanet. Alternatively, an orbital period that was relatively longer than its spin period (Figure 1.2) resulted in a negative average torque ($\tau = -1.35 \times 10^{29}$ dyne cm). This result is similar to one of the findings in Arras and Socrates (2010), where retrograde rotation was associated with longer orbital periods in relation to rotational periods in hot Jupiters. The longer orbital period resulted in an increased rotation period that eventually induced retrograde rotation (Arras and Socrates 2010)–though, Arras and Socrates (2010) also considered eccentricity, which was not included in our simulation. These average torques were also much smaller than their maximum oscillation amplitudes.

For orbital periods that are closer in magnitude and direction to the one day rotational period, average torques remained in the original direction and increased in magnitude with a decrease in orbital period (Figures 1.3 and 1.4). A decreased orbital period would be generally associated with a decreased orbital angular velocity, since it can be assumed that there is a smaller distance that a planet has to travel. If the rotation period stays the same in all simulations, the planet's rotational angular velocity must also stay constant. A smaller difference between these two angular velocities would result in a larger torque, which could be why the 0.5 day orbital period simulation (Figure 1.4) has the largest average torque magnitude. Smaller orbital periods are also associated with being closer to the host star, allowing for the planet to be more affected by heating than for planets further away (longer orbital periods) (Correia and Laskar 2011). However, a close proximity to the host star would also increase the effects of gravitational tides (Arras and Socrates 2010).

Despite the exclusion of friction in our model, and the shortened timescale of the simulations conducted, these experiments can provide valuable insight for the timescales that tidally locked systems operate on (Arras and Socrates 2010). Estimates for the formation of tidally locked systems have been calculated to be much shorter than the age of tidally evolved exoplanets . A negligible torque generated by tidal interactions would allow for the rotation period of a planet to be unaltered, therefore allowing for gravitational tides to force the planet into a tidally locked system. One can relate torque and this timescale by a few general equations. The spin-up timescale to achieve torque equilibrium, or t_{spin}, can be calculated using

$$t_{spin} = \frac{I\Omega_{spin}}{\tau}$$

where I represents the moment of inertia. This can be estimated roughly using

$$I = 0.1MR_1^2$$

. If we define the mass as $M_p = 2 \times 10^{30}$g and our radius as 7.0×10^9cm, we can then calculate an estimated I and t_{spin} value for our simulations, which we have outlined below:

$P_{\rm orb}$	$\Omega_{\rm spin}(\frac{\rm rad}{\rm s})$	Calculated moment of inertia (I)	Torque (τ), dyne-cm)	$t_{\rm spin}$ (s)
0.5 days	7.272×10^{-5}	9.8×10^{48}	1.92×10^{29}	3.70×10^{15}
2 days	7.272×10^{-5}	9.8×10^{48}	1.38×10^{29}	5.16×10^{15}
4 days	7.272×10^{-5}	9.8×10^{48}	-1.34×10^{29}	5.30×10^{15}

Table 1: Calculated t_{spin}

As the orbital period increases, the timescale for the exoplanet to reach torque equilibrium also increases (Table 1). Since the time scale for a tidally evolved rotational period would be inversely proportional to the torque generated from thermal forcing, a smaller torque would result in a larger time scale. Larger orbital periods often indicate a larger semimajor axis, so planets further from their host star would not experience as much thermal forcing to trigger strong responses to atmospheric thermal tides. At the same time, they are far enough away from their host stars to withstand the gravitational forcing that would trigger a shorter t_{spin} time scale. The differences in the torque equilibrium time scale between different orbital periods seems to diminish with a larger orbital period. We recommend the analysis of the density, velocity, and temperature profiles in the atmosphere in order to exhibit the fluid motions for future studies.

In summary, the duration of the orbital period directly impacts the magnitude and direction of a generated torque in hypothetical solid-surface like exoplanets via differences in angular velocity. A smaller orbital period is most commonly associated with a larger torque, which can combat the synchronous tidal locking tendencies of gravitational tides that work to slow a planet's rotation. In turn, the torques produced by atmospheric thermal tides may work to decrease the time scales to change the rotation rate of a planet seen in tidal evolutions. Density, velocity, and temperature profiles are recommended to further understand the physical effects of atmospheric thermal tides.

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