Essays on Macroeconomics and Labor Markets

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Abstract

The first Chapter of this Dissertation analyzes the general equilibrium welfare effects of implementing a job-search assistance program for unemployed jobseekers. While beneficial to participants, empirical evidence suggests these programs impose costs on non-participants through a crowding out effect on hiring. In the model, agents are heterogeneous with respect to assets, human capital, and the duration of their current unemployment spell. I consider a policy in which all agents out of work one year or more are automatically enrolled in a government-funded assistance program, which increases their job-finding chance by about 48 percent, and I analyze the welfare gains and losses associated with the creation of this policy for all agents. I then decompose these welfare effects into a partial equilibrium channel (individual-level job-search efficiency) and general equilibrium channels (labor market composition, firm hiring behavior, and price and tax effects). I find that the program is welfare-improving by about 0.22 percent. However, general equilibrium effects are negative and non-negligible in magnitude, accounting for a welfare loss of about 0.11 percent. A decline to labor market tightness, which reduces job-finding rates, accounts for most of this welfare loss. Price effects represent a welfare gain of about 0.1 percent, and tax effects are a welfare loss of 0.1 percent. Next, I show that high-skill and asset-poor agents benefit the most from the policy. Finally, I compare the search assistance policy to an unemployment subsidy with the same cost, and show the former delivers a higher welfare gain than the latter.

Chapter 2 studies the optimal scheme of search assistance in an economy populated by heterogeneous agents. By improving the job-finding chances of one agent, all other agents are made worse off, all else equal. This Chapter introduces a planner who takes this fundamental tradeoff into account when choosing the optimal search assistance intensity for all types of agents in the economy in order to maximize the sum of all agents' utility. The model has a two-period horizon and is simple enough to allow an analytical characterization of the planner's choice. For example, I show that the planner's choice of assistance intensity is an increasing function of agents' skill level, as the surplus from re-employment is increasing in skills. I solve three different (but closely related) versions of the model; one in which the planner solves directly for the optimal assistance scheme for all types, and two in which the planner chooses the assistance scheme based on functional rules; these last two allow me to characterize results for a large state space using only a small number of choice variables. After presenting results for these models, I solve an alternative specification that removes all employed agents from the planner's objective function, as these agents pay most of the cost of the program but receive none of the benefit. Now, the planner provides a much higher level of assistance to unemployed agents.

Chapter 3 adds endogenous search effort to the analysis, to study how the creation (and expansion) of a search assistance program affects agents' job-search behavior, and the implications of this behavior for welfare gains and losses. Empirical research has shown that agents' job-search effort is countercyclical; that is, worse labor markets are associated with higher effort. I extend this logic to the context of a job-search assistance program. I define an infinite-horizon model of heterogeneous agents in which the government introduces a search assistance policy that makes jobseekers more likely to find re-employment. I employ two separate matching functions and compare their results. The first function induces agents to reduce their search effort when assistance intensity increases, and the second leads them to increase their effort. For each of these, I analyze the welfare gains from the assistance policy when allowing for this endogenous effort, and compare this to a world where search effort is fixed at a baseline value. I show that the variable-effort specification yields a higher welfare gain for the first matching function, but welfare gains are essentially equal with the second function.

Keywords: Macroeconomics, labor economics, skill erosion, search assistance, heterogeneous agents, labor market policy, matching models, search effort

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All errors are my own.

Contents

Chapter 1

Skill loss and search assistance in general equilibrium

1.1 Introduction

One commonly-utilized policy response to unemployment is search assistance, which refers, broadly speaking, to a set of government- or privately-funded programs that seek to improve a job-seeker's re-employment chances. Approximately 26 percent of unemployed job-seekers in the U.S. report either contacting an employment agency or attending a job-training program within the last four weeks.[1] Such programs have been studied extensively at the *individual* level; i.e., a given program's effectiveness (in terms of re-employment wage, probability of re-employment, or some other relevant outcome variable) for the program participant himself.[2] As one example, Hujer, Thomsen, and Zeiss (2006) use German data to find that participation in a search assistance program increases a person's job-finding rate by about 48 percent.

On the other hand, the literature provides much less evidence of the effects of search assistance on non-participants, and by extension, the entire economy. This chapter augments the numerous individual-level studies of job search assistance by looking at the associated general equilibrium effects. Hujer and Zeiss (2006), which

[1] Source: Author's calculations from the Current Population Survey (CPS). See Appendix D.1.

[2] Numerous surveys exist; for example, see Card, Kluve, and Weber (2010), Heckman, LaLonde, and Smith (1999), and Kluve (2010).

builds on Hujer, Thomsen, and Zeiss (2006), analyzes the aggregate effects of inflows into programs, and find that, overall, inflows into employment increase by less than what their individual-level study indicates.[3] The authors present this as evidence of a substitution effect—as participants' hiring rates increase, firms may simply shift some hiring away from non-participants. Therefore, by helping out one job-seeker, a search assistance program may inadvertently harm another job-seeker.

Therefore, in order to properly analyze such policies, we need to understand the associated tradeoffs. Simply understanding how a search assistance policy affects participants is only part of the story; to get a complete picture, we also need to understand the effect such a policy has on non-participants, and thus the economy as a whole. There are several general equilibrium effects to consider. The first is the composition of the jobseeker pool: by increasing a participant's job-finding chances, the probability that a non-participant finds a job is now relatively lower. The second effect relates to firms' hiring behavior: firms choose, endogenously, how many vacancies to post. These first two effects are closely related; they roughly correspond to the denominator and numerator, respectively, of labor market tightness. Furthermore, we must take into account price effects. For example, since wages are set, according to commonly-used models, via a bargaining game between the firm and the worker based on prevailing labor-market conditions, the existence of search assistance will change wages, which could have important consequences for all workers. Finally, we have wealth effects coming from an income tax: if the government is committed to keeping a constant budget balance, the implementation of search assistance programs will increase taxes.

The contributions of this chapter are threefold. First, I use a structural model to gain an understanding of the welfare effects of the search assistance policy for an economy that is populated by heterogeneous agents. In studying any policy, a natural question to ask is, who benefits the most from this policy? Is anyone harmed, and by how much? How does the benefit to participants compare to the costs imposed (if any) on nonparticipants? Second, to understand the sources of welfare gains and losses, I decompose the welfare effects into various 'channels', separating out the partial

[3] Similar evidence is presented in Gautier et al. (2017). They use Danish data to estimate that non-participants find new jobs at a slower rate after the introduction of a search assistance program.

equilibrium channel (increased individual search efficiency) from general equilibrium, as mentioned above. Finally, I place the search assistance program into context by comparing it to another possible labor market program that a government could implement instead, and compare the welfare changes. I show that if the government takes the money they would spend on search assistance and instead gives it directly to all unemployed agents, the welfare gains are significantly smaller, though still positive.

I use a model similar to Krusell, Mukoyama, and Şahin (2010). The model combines two important features of models in the macroeconomics literature; first, a frictional labor market, in the style of Diamond-Mortensen-Pissarides (DMP); and heterogenous agents, in the style of the Bewley-Huggett-Aiyagari models. Agents are heterogeneous in terms of their asset holdings, their human capital level, their labor market state, and the duration of their unemployment spell (if unemployed). Agents in this economy experience a depreciation of their skills while unemployed and appreciation while employed. After calibrating the model to match U.S. labor market moments, I perform a policy analysis by imposing that all unemployed agents who have been out of work for five or more quarters are enrolled in a government-provided search assistance program, which increases these agents' chances of finding a new job. I look at the heterogeneity in welfare changes according to program participation status along with asset and human capital level. For example, high-skill participants have a higher welfare gain than low-skill participants, as match surplus is increasing in skill level. I then turn to a decomposition of the general equilibrium effects discussed above. For example, I find that general equilibrium effects reduce welfare for the average agent by approximately 0.1 percent. Of these negative general equilibrium effects, labor market composition and firm hiring account for most of it. Tax effects are negative while price effects are actually positive; wages and interest rates both increase when the policy is introduced, which is welfare-improving.

This chapter relates to a few strands of existing literature. One such strand studies the optimal policy response to unemployment. For example, Acemoglu and Shimer (2000) shows that in an environment with risk averse agents, a small positive level of unemployment insurance increases productivity and welfare compared to an environment with no insurance, as it encourages workers to seek higher-paying jobs. Next, Pavoni, Setty, and Violante (2016) consider a wide range of policy options, such as search assistance and welfare benefits, in order to minimize the social planner's cost of delivering a chosen amount of utility to an agent. They show analytically which policy is optimal across a state space that varies over the government's generosity level and the human capital level of the agent. However, their paper does not allow for externalities resulting from one agent's participation in a program. On the other hand, my model allows for these externalities, which are an important source of welfare losses.

Another related strand of literature looks at the aggregate consequences job-finding and job-loss. For example, Ortego-Marti (2015) uses cross-country data and shows that the countries with the lowest rates of job-finding (relative to their job-loss) also have the lowest levels of total factor productivity. The mechanism in his paper is similar to mine: when unemployed workers experience some erosion of their human capital during unemployment, the longer they remain unemployed, the lower their skills are when they finally do become re-employed. Hence, economies with longer average unemployment durations subject jobseekers to a higher degree of skill loss, and thus employ less-productive workers. I find that the introduction of the assistance policy increases TFP by about 0.09 percent, due to a higher-skilled workforce.

The rest of this chapter is organized as follows. Section 1.2 presents the model, followed by a discussion of calibration in Section 1.3. Section 1.4 presents results and Section 1.5 concludes.

1.2 The model

As mentioned in the introduction, the model presented here closely follows Krusell, Mukoyama, and Şahin (2010) in many regards, in that it combines a Diamond-Mortensen-Pissarides frictional matching framework with a Bewley-Huggett-Aiyagari heterogeneous agents framework. Time is discrete. The economy consists of households, firms, and the government. At any point in time, a particular household is either employed or unemployed. As is common in DMP-style matching models, each firm employs exactly one household. Households have concave utility over consumption. Let i denote a household's type.[4]

[4] I use 'household', 'agent', and 'person' interchangeably.

The household's problem is defined by three state variables. First, households hold assets a_t, which they use to save for future consumption. Asset a_t is a composite of two other assets, physical capital and firm ownership equity, which I discuss further below. Households possess skills x_t. With probability δ_e, skills increase for each period a person stays employed, and with probability δ_u, skills decrease for each period he stays unemployed. This process is described in more detail later. The final state variable is the agent's labor market state $s_t \in \mathcal{S} = \{0, 1, 2, ..., \bar{s}\}$. Let $s_t = 0$ denote an employed agent. An agent with $s_t = 1$ has been unemployed for one period, an agent with $s_t = 2$ has been unemployed for two periods, and so on. As a matter of notational convenience, let Ω be the vector of state variables: $\Omega_t = \{a_t, x_t, s_t\}$. Finally, let $f(\Omega)$ denote the economy's density function over the three state variables, which is an endogenous quantity. The total population of the economy is normalized to 1.

1.2.1 Job matching

Firms and jobseekers are matched according to a DMP-style frictional matching process. Jobseekers possess individual search efficiency, denoted by $e(s)$. This parameter, which determines the relative likelihood of particular jobseekers being matched with a firm, is a decreasing function of unemployment duration, and is calibrated so that the model matches US data on unemployment exit rates by duration.[5] Next, define the 'effective' (or efficiency-weighted) mass of unemployment by u, which is defined as

$$u = \sum_{s \geq 1} e(s) f_s(s),$$

where $f_s(s)$ refers to the economy's marginal distribution over state s. Under this specification, the commonly-used variable 'labor market tightness', denoted by θ, is instead defined as the ratio of vacancies v to *effective* unemployment u. That is, $\theta = v/u$.

The number of matches formed in a period is determined by the Cobb-Douglas

[5] In Appendix E.1, I allow e to depend on a worker's skill level in addition to unemployment duration, and show that the main results of the chapter are quantitatively similar.

matching technology $m(u,v)$:

$$m(u,v) = u^{\sigma} v^{1-\sigma}$$

Let $q_F(\theta)$ be the probability that a firm matches with a jobseeker:

$$q_F(\theta) \equiv \frac{m(u,v)}{v} = \theta^{-\sigma}.$$

The probability that a firm matches with a particular *type* of worker is, similarly,

$$\frac{e(s)f(i)}{u}\frac{m(u,v)}{v} = \frac{e(s)f(i)}{u}\theta^{-\sigma},$$

where i denotes a type of jobseeker. Note that while e depends only on unemployment duration, the measure of a type $(f(i))$ also determines match probability. For example, if, among jobseekers with some unemployment duration $\hat{s} \in \mathcal{S}$, there are more high-skilled agents than low-skilled agents. Then, the firm is more likely to be matched with a high-skilled agent than a low-skilled one. Finally, let $q_W(\theta)$ be the probability that a worker of type i finds a job, given by

$$q_W(\theta) \equiv e(s)\frac{m(u,v)}{u} = e(s)\theta^{1-\sigma}. \tag{1.1}$$

Note that I bound $q_W(\theta)$ and $q_F(\theta)$ between zero and one; however, the model never hits these bounds.

1.2.2 Households

Households are infinitely-lived and discount the future at rate $\beta < 1$. As stated above, at any point in time, a household is either employed or unemployed. Let $w(\Omega)$ denote the agent's labor-market earnings; for employed workers, w is their wage, and for unemployed agents, this is unemployment insurance. All agents pay income tax at rate $\tau \in [0,1]$ on their labor market earnings. Furthermore, with exogenous probability ϕ, workers keep their job into next period, and with probability $1-\phi$ their job is destroyed. Households can save using two different assets: physical capital k_t^h, which is an input to production, and equity from ownership of the firms, b_t.[6] Physical capital earns interest rate r_t and depreciates at rate δ_k. The price of one equity share is p_t, and equity pays a per-period dividend of d_t. Since firms are heterogeneous, it is relevant to note that households do not choose which firms' equity to own. Instead, they own shares in a mutual fund comprised of all firms.

The agent's budget constraint is therefore

$$c_t + k^h_{t+1} + p_t b_{t+1} = (1 + r_t - \delta_k)k^h_t + (p_t + d_t)b_t + (1 - \tau)w(\Omega).$$

Since there is no aggregate uncertainty in the economy, physical capital k^h_t and firm equity b_t must earn the same rate of return. That is,

$$1 + r_{t+1} - \delta_k = \frac{d_{t+1} + p_{t+1}}{p_t}. \tag{1.2}$$

Since the return on these two assets is identical, agents are indifferent between them and the portfolio choice is indeterminate. Therefore, we can define a 'composite asset' a_t as the following

$$\begin{aligned} a_t &\equiv k^h_t + p_{t-1}b_t, \qquad \text{and,} \\ a_{t+1} &\equiv k^h_{t+1} + p_t b_{t+1}, \end{aligned} \tag{1.3}$$

and, using Equation (1.2), re-express the budget constraint as

$$c_t + a_{t+1} = (1 + r_t - \delta_k)a_t + (1 - \tau)w(\Omega). \tag{1.4}$$

As the portfolio choice is indeterminate, I assume that all households hold the same *ratio* of physical capital to firm equity. That is, k^h_t/b_t is equal to some constant ξ for all households. This portfolio allocation becomes relevant after the introduction of the policy, which affects the two assets differently. While not modeled explicitly, it is useful to think of a financial intermediary who handles agents' asset portfolios. The intermediary takes asset a_t from agents, rents the physical capital to firms at

[6]The superscript h on k^h_t refers to household capital, as opposed to k^f_t, the amount of capital rented by firms. In equilibrium, the total amount of each is equal.

rental rate r_t, and uses the rest to buy shares of the mutual fund discussed above. The intermediary then pays households a return r_t.

Households choose consumption c_t and tomorrow's asset holdings a_{t+1} to maximize the present discounted sum of lifetime utility. The household's problem is therefore

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(c_t),$$

subject to Equation (1.4) above.

At any point in time, agents possess skills, denoted x_t, which takes on one particular value in a finite set $\mathcal{X}$. That is, $x_t \in \mathcal{X} = \{x_1, x_2, \ldots, x_N\}$. For agents who remain in the same labor market state from one period to the next, skills evolve in the following manner. With an additional period of employment, an agent who currently possesses skill level x_n (where x_n is some element of set $\mathcal{X}$) will see his skills increase to x_{n+1} tomorrow with probability δ_e and remain at x_n with probability $1-\delta_e$. Once an employed agent reaches the highest skill level x_N, his skills remain there with probability one. On the other hand, an additional period of unemployment will cause an agent's skills to deteriorate from x_n to x_{n-1} with probability δ_u and remain at x_n with probability $1-\delta_u$. Upon reaching the lowest skill of x_1, his skills remain there. Furthermore, when an agent transitions from one labor market state to the other, he retains his current skill level for the first period after this transition, after which the above laws of motion take effect.[7] The parameters δ_e and δ_u are calibrated to match wage dynamics.

Turning to a recursive formulation, let $E(\Omega)$ and $U(\Omega)$ be the Bellman equations of employed and unemployed agents, respectively, whose current state vector is Ω. The employed agent's problem can be written recursively as

$$E(\Omega) = \max_{c,a'} \left\{ u(c) + \beta \left(\phi \mathbb{E} \left[E(\Omega'|\Omega) \right] + (1-\phi) \mathbb{E} \left[U(\Omega'|\Omega) \right] \right) \right\},$$

subject to

$$c + a' = (1 + r - \delta_k) a + (1-\tau) w(\Omega), \qquad \text{and,}$$

[7] This scheme is very similar to Ljungqvist and Sargent (1998).

$$a' \geq \underline{a},$$

where $\underline{a}$ is the borrowing limit. Note the expectation operator in the continuation value, which takes into account the possibility (at probability δ_e) that the agent's skills increase one step. On the other hand, the optimization problem for an unemployed worker is

$$U(\Omega) = \max_{c,a'} \left\{ u(c) + \beta \left(q_W(\theta)\mathbb{E}\left[E(\Omega'|\Omega)\right] + (1 - q_W(\theta))\mathbb{E}\left[U(\Omega'|\Omega)\right] \right) \right\},$$

subject to the same two constraints. As with the employed agent, the expectation over tomorrow's state refers to skill depreciation with probability δ_u. Finally, denote agents' decision rules for consumption and savings by $c(\Omega)$ and $a'(\Omega)$.

1.2.3 Firms and production

The economy has a continuum of firms, each of whom employs one person. In this framework, a firm's 'type' is therefore defined by the type of its employee. Hence, the subscript i for firms and for households means the same thing. Production by firm i is given by the Cobb-Douglas function

$$y_i = k_i^{\alpha} x_i^{1-\alpha},$$

where k_i is physical capital, and x_i is the skill level possessed by the firm's employee.[8] The final good y serves as the economy's numeraire and its price is normalized to one. Firms discount the future at rate $1/(1 + r_{t+1} - \delta_k)$. Let $J(\Omega)$ be the value of a filled job and V be the value of a posted vacancy. $J(\Omega)$ represents the (expected) present value of profit streams from the firm. Here, Ω refers to the state vector possessed by the firm's employee. As with households, firms keep their employee into next period

[8] Without loss of generality, the economy-wide Total Factor Productivity (TFP) is set to one, and so I omit this term.

with exogenous probability ϕ. Recursively, the firm's problem is written as

$$J(\Omega) = \max_{k} \left\{ k_i^{\alpha} x_i^{1-\alpha} - r k_i - w(\Omega) + \left(\frac{1}{1 + r' - \delta_k} \right) \left[\phi \mathbb{E} \left[J(\Omega' | \Omega) \right] + (1 - \phi) V \right] \right\}. \tag{1.5}$$

From the firm's first-order conditions, we can derive their demand for capital k_i^*:

$$k_i^* = x_i \left(\frac{r}{\alpha} \right)^{1/(\alpha - 1)}.$$

Every period, entrepreneurs post a vacancy with cost γ. These entrepreneurs do not have any way of 'targeting' specific types of potential employees, so at the time the vacancy is posted, the value of filling this vacancy is actually the expected value of a job match over all types of potential employees. Recall the probability that a firm finds a type-i worker is $e(s)f(i)q_F(\theta)/u$. Therefore, the value of posting a vacancy V is

$$V = -\gamma + \left(\frac{1}{1 + r' - \delta_k} \right) \left[\sum_{s \geq 1} \frac{e(s)f(i)}{u} q_F(\theta) J(\Omega) + (1 - q_F(\theta)) V \right]. \tag{1.6}$$

V can easily be shown to be decreasing in the total number of vacancies posted v. Hence, if V is positive (negative), due to free entry, more firms will enter (leave) the market, which eventually pushes V down (up) to zero in equilibrium.

Every period, households are paid dividend d_t from ownership of firms. Since households own a mutual fund of all firms, as opposed to just one firm, dividend d_t is defined as the sum of per-period profits minus vacancy-posting costs:

$$d_t = \sum_{s=0} \pi_i f(\Omega) - \gamma v_t,$$

where

$$\pi_i = (k_i^*)^{\alpha} x_i^{1-\alpha} - r_t k_i - w(\Omega).$$

1.2.4 Wages

As is common in models with a DMP-style matching structure, wages are set via a generalized Nash bargaining scheme. After forming the match, the firm and the worker bargain over wages in order to maximize the weighted match surplus. Let $\eta \in [0,1]$ be the worker's bargaining share, with the firm having a $1-\eta$ bargaining share. Let $\tilde{E}(\Omega, \tilde{w})$ be the 'auxiliary' problem faced by an employed worker with state vector Ω, *given* a current wage offer of $\tilde{w}$. That is,

$$\tilde{E}(\Omega, \tilde{w}) = \max_{c,a'} \left\{ u(c) + \beta \left(\phi \mathbb{E}\left[E(\Omega'|\Omega)\right] + (1-\phi)\mathbb{E}\left[U(\Omega'|\Omega)\right] \right) \right\},$$

subject to

$$c + a' = (1 + r - \delta_k)a + (1-\tau)\tilde{w}.$$

The firm's auxiliary problem, denoted $\tilde{J}(\Omega, \tilde{w})$, is defined in a similar way. The wage maximizes the Nash product for each type of agent:

$$w(\Omega) = \operatorname*{argmax}_{\tilde{w}} \left(\tilde{E}(\Omega, \tilde{w}) - U(\Omega) \right)^{\eta} \left(\tilde{J}(\Omega, \tilde{w}) - V \right)^{1-\eta}$$

1.2.5 Job search assistance

As used in this chapter, 'search assistance' is a rather broad term for a type of active labor market program aimed at improving a jobseeker's chances of becoming re-employed. Real-life examples include an agency that helps to write a jobseeker's résumé, career counselling to help determine which industries the jobseeker should be targeting, conducting mock interviews, etc. Search assistance enters the model in the following manner. All unemployed agents who meet some eligibility criteria (e.g., those out of work for at least a year, or low-skilled workers), experience an increase in their individual search efficiency to $\tilde{e}_i$ by some factor $\rho > 1$. That is, $\tilde{e}_i = \rho e_i$. These agents' job-finding rates then change to

$$\tilde{q}_W(\theta) = \rho q_W(\theta) = \rho e_i \theta^{1-\sigma}.$$

Search assistance programs therefore indirectly mitigate the amount of skill depreciation an agent experiences by shortening the (expected) duration of an unemployment spell. However, the existence of search assistance may actually *harm* non-recipients, who may have a *lower* job-finding rate. This is because labor market tightness, θ, is determined endogenously. So, if θ decreases when this policy is introduced, non-recipients' job-finding rates will decline, as the matching function is increasing in θ. Intuitively, we can think of search assistance programs as 'polishing' a recipient's résumé, making it more likely to rise to the 'top of the pile'. However, by making one jobseeker's application appear *relatively* better, the policy also makes other jobseekers' applications *relatively* worse.

The government must pay a cost of ψ units of the numeraire for each agent who is enrolled in the assistance program.

1.2.6 Government

The government in this economy exists solely to finance the UI and search assistance systems through collecting income taxes. In reality, unemployed agents receive a particular fraction of their previous wages as UI (the 'replacement rate'). However, defining the model's UI system in this way would require me to know the composition of unemployed agents' state vectors not just at time t but also *at the time they lost their job*, which is some number of periods in the past. Tracking these additional state variables would significantly increase the computational cost of solving the model. Instead, as an approximation, I set person i's UI payment to a fraction λ of the wage of an employed agent with the same level of assets a and skills x as the unemployed agent in question. The fraction λ is then calibrated so that the economy-wide average UI replacement rate matches the data. This system captures the salient features of the true UI system; namely, that UI payments will be an increasing function of the agent's skill level (insofar as wages are increasing in skill level), but the computational costs are negligible.

The government's budget is as follows. On the income side, they collect the fraction τ of all agents' labor market earnings, including wages and UI payments. Recall that, as notation, $w(\Omega)$ denotes both wages and UI payments, depending on the labor market state of the agent in question. On the expenditure side, the government gives out UI payments to some unemployed workers, plus they must pay ψ for every agent who is enrolled in search assistance. Let $\mathcal{A}$ denote the set of agents who recieve UI and let $\mathcal{B}$ denote the set of agents enrolled in search assistance. The government's budget, which is balanced every period, is therefore given by

$$\sum_{s=0} \tau w(\Omega) f(\Omega) + \sum_{\mathcal{A}} \tau w(\Omega) f(\Omega) = \sum_{\mathcal{A}} w(\Omega) f(\Omega) + \sum_{\mathcal{B}} \psi f(\Omega). \tag{1.7}$$

In the model's calibration, $\mathcal{A}$ is equal to those agents unemployed up to 26 weeks, and $\mathcal{B}$ is equal to agents unemployed for at least one year.

1.2.7 Equilibrium

Definition: Competitive equilibrium. A competitive equilibrium in this economy consists of

1. Value functions: $E(\Omega)$, $U(\Omega)$, $J(\Omega)$, and V,
2. Decision rules for consumption and savings: $c(\Omega)$ and $a'(\Omega)$,
3. Prices and other endogenous quantities: $w(\Omega)$, r, τ, p, d, θ, v,
4. The economy's density function f.

Such that

1. Given $w(\Omega)$, r, and τ, the decision rules $c(\Omega)$ and $a'(\Omega)$ solve the household's optimization problem, with associated value functions $E(\Omega)$ and $U(\Omega)$.
2. The government's budget (Equation (1.7)) is balanced.
3. The number of vacancies v is consistent with $V = 0$.
4. The distribution f is invariant.
5. The wage vector $w(\Omega)$ is consistent with Nash bargaining.
6. The interest rate r allows the asset market to clear. That is, the total amount of savings supplied by households equals the total amount of physical capital used by firms plus equity:

$$\sum a'(\Omega) f(\Omega) = \sum k_i^f f(\Omega) + p \sum b_i f(\Omega).$$

   Appendix C.1.3 provides details on the asset market equilibrium.

1.3 Parameters and calibration

This section describes how I calibrate the model's parameters. One period equals one quarter (three months). The state space for skills x is split into five equally-spaced nodes between 0.6 and 1.0. I track unemployment duration for nine periods, so an agent with a duration of 9 (i.e., $s = 9$) has been unemployed for *at least* two years. Turning to parameter values, the borrowing limit $\underline{a}$ is zero. I set α equal to 0.33. The household's discount rate β is equal to 0.99 and the depreciation rate of physical capital, δ_k, is set to 0.012. Together, these two parameters yield an annual real $(r - \delta_k)$ asset return of approximately four percent, and an investment-to-output ratio of approximately 0.20. The investment-to-output ratio is calculated as follows. The law of motion of physical capital K is standard:

$$K_{t+1} = (1 - \delta_k)K_t + I_t,$$

where I_t is investment. From the definition of the composite asset (Eq. (1.3)), $k_{t+1} = a_{t+1} - p_t b_{t+1}$, so,

$$I_t = \sum (a'(\Omega) - p_t b_{t+1}) f(\Omega) - (1 - \delta_k)K_t,$$

where $a'(\Omega)$ is households' decision rule for savings. Note that the total quantity of firm equity and the equity price are not separately identified in the model; as such, without loss of generality, I normalize the total quantity to one. Next, let A'_S be the total quantity of assets supplied by households; that is, the sum of households' savings. Hence, investment in physical capital K_t, is equal to

$$I_t = A'_S - p_t - (1 - \delta_k)K_t.$$

The UI replacement ratio is set to 0.465 for the first two periods of unemployment and zero thereafter. This restriction reflects the fact that, in the absence of extended UI programs often implemented during recessions, unemployed agents are typically only eligible for 26 weeks of benefits. The value of 0.465 is the national average UI replacement rate, averaged over the period 1997:Q1 to 2016:Q1.

I set ϕ, the probability that employed agents stay employed, equal to 0.96. For this, I use updated data from Fallick and Fleischman (2004). They calculate gross worker flows from the Current Population Survey (CPS) starting in January 1994. The monthly average of the *EU* (employed-to-unemployed) flow over the period September 1995 to December 2007 is 0.012. Hence, at a quarterly frequency, the probability of staying employed is $(1 - 0.012)^3 = 0.96$. Next, I calibrate individual match efficiency $e(s)$, which is a function of a jobseeker's unemployment duration. This parameter is set so that the model's exit rates match US data on job-finding rates by duration. Therefore, $e(s)$ is a vector of constants with nine values (recall, I track up to nine quarters of unemployment). The process for calibrating is as follows. Using a matched Current Population Survey (CPS) dataset from the period October 1995 to December 2007, I calculate the unemployment-to-employment (U-E) exit rates by duration of unemployment (which is measured in *months* in the CPS) and average these over the sample period.[9] Just for this section, denote these by $q_{m,t}$. The longest duration reported in the sample is 17 months, so I have 17 observations. To match the model's quarterly frequency, I must convert $q_{m,t}$ to quarterly figures. First, using $q_{m,t}$, I calculate the probability that an unemployed person who has been out of work for t months finds a job within the next *three* months, denoted $q_{q,t}$

$$q_{q,t} = q_{m,t} + (1 - q_{m,t})q_{m,t+1} + (1 - q_{m,t})(1 - q_{m,t+1})q_{m,t+2}.$$

Figure 1 shows $q_{m,t}$ and $q_{q,t}$. Next, I convert these to a quarterly frequency by averaging the monthly values over their corresponding quarters. That is, $\bar{q}_1 = \frac{1}{3}\sum_{t=1}^{3} q_{q,t}$, $\bar{q}_2 = \frac{1}{3}\sum_{t=4}^{6} q_{q,t}, \ldots$, and so on. The $\bar{q}_t$'s are therefore the model's target values for exit rates by quarters of unemployment. I use only five quarters' worth of exit rates from the data, as the sample sizes are very small beyond durations of one year and the data become quite noisy. To remedy this, I regress $\bar{q}_t$ on the log of t, and use the fitted values (denoted as the vector $\hat{q}(d)$) as the target values for $\bar{q}_t$.[10] I extend this line to a total of 9 quarters. Figure 2 shows the results of this regression. The 9

[9] This database was graciously provided by Christopher Nekarda. See Nekarda (2009) for more information.

[10] The equation of this line is $\hat{q}(d) = 0.5837 - 0.1029ln(d)$, where d is duration in quarters. $R^2 = 0.9969$.

values of $e(s)$ are then chosen so the model, in equilibrium, produces exit rates equal to their targets. In other words,

$$\hat{q}(d) = e(s)\theta^{1-\sigma},$$
$$e(s) = \frac{\hat{q}(d)}{\theta^{1-\sigma}}, \qquad \text{for } s = 1, 2, \ldots, 9.$$

As I mentioned above, aggregate match efficiency is not identified separately from individual search efficiency $e(s)$. That is, if I were to scale up aggregate match efficiency by any factor x, then the estimated values for $e(s)$ would all be scaled by factor $1/x$. Hence, without loss of generality, I can set aggregate match efficiency to one.

Following Shimer (2005), I set σ, the elasticity of the matching function with respect to u, to be 0.72. I set η, the worker's bargaining coefficient, equal to 0.67, which delivers a labor share of income of approximately 64 percent. The vacancy cost, γ, is set to approximately 1.005, which yields a vacancy-to-unemployment-rate ratio (the 'traditional' definition of labor market tightness) of about one in equilibrium. This value for γ turns out to be approximately 47 percent of the average quarterly wage. This corresponds to an equilibrium value of θ of 2.4. However, since the model targets specific job-finding and job-loss rates, this choice of θ is completely arbitrary. The per-person cost of search assistance, ψ, is set to a value that is 21 percent of the mean quarterly wage in the baseline economy. This value comes from Hujer, Thomsen, and Zeiss (2006). The assistance program analyzed in their paper, which is also the experiment I use, is estimated to cost approximately €538 per person per month, or €1,614 per quarter. The average quarterly earnings in Germany in 2004 (the year from which the data in Hujer, Thomsen, and Zeiss (2006) come) was €7,518; hence an average cost of 21.46 percent of the wage.

Finally, I calibrate δ_u and δ_e, the probabilities that an unemployed agent loses one step of skills, and an employed agent gains one level of skills, respectively. To pin down δ_u, I use estimates of the wage losses associated with unemployment as a proxy for skill loss. Hence, I set δ_u so that the economy's average wage profile matches the data. Using data from the PSID, Ortego-Marti (2016) estimates that an additional month of unemployment corresponds to a monthly wage loss of 1.22 percent. At a quarterly frequency, this corresponds to a 3.62 percent wage loss rate. I therefore set δ_u equal to 0.2744, so that agents in the economy can expect to lose 3.62 percent in terms of re-employment wages each period they remain unemployed. The value of 3.62 percent is within the range found in other papers as well. For example, Neal (1995) estimates approximately 4.7 percent quarterly wage loss, and Addison and Portugal (1989) find approximately 4.26 percent. Furthermore, Schmieder, Wachter, and Bender (2016) find a quarterly depreciation rate of 2.4 percent. Targeting a wage loss rate anywhere in this range would change quantitative results only slightly. Next, δ_e is calibrated in an equivalent manner. Also using PSID data, Herkenhoff et al. (2015) calculates the median two-year real income growth rate for households aged 25-30 between the years 2005 and 2007 as equal to approximately 1.05 percent at a quarterly frequency. In my model, this corresponds to a value of δ_e equal to 0.0752.

Table 1 summarizes my parameter choices.

1.4 Results and discussion

After calibrating the model according to the moments described in the previous section, I first solve the model without any search assistance programs at all; I denote this the 'baseline' economy. Next, keeping all parameters unchanged, I solve the model again according to the following policy experiment. Suppose that all agents who have been out of work for at least five quarters are automaticaly enrolled in a search assistance program which increases their individual search efficiency by some factor $\rho > 1$. I choose a value for ρ of 1.48; i.e., a 48 percent increase in search efficiency. This number comes from Hujer, Thomsen, and Zeiss (2006). They use data on a short-term job search assistance in Germany to estimate that, controlling for both observable covariates and unobserved heterogeneity, enrollment in the program increases a participant's U-E hazard rate by a factor of 1.48. This assistance program has multiple 'modules', each aimed at increasing the participant's individual job search efficiency. These modules include aptitude tests, taken in order to help the applicant target his search activities appropriately, and training in presentation and job-search skills. The authors point out that while caseworkers have some discretion as to who is eligible to be enrolled in the program, those with a high unemployment duration are given preference, whereas those who face lower barriers to re-employment (e.g. higher-educated or short-term unemployed) are not.

Note that this variable captures the *relative* hazard rate of a participant versus a nonparticipant; it does not take into account any general equilibrium effects. In other words, participants' job-finding rates are now equal to

$$q_W(\theta) = \rho e(s)\theta^{1-\sigma},$$

where $\rho = 1.48$. However, in this new steady-state environment, θ has fallen by approximately 3.8 percent, which, all else equal, reduces *all* jobseeker's re-employment chances by approximately 1.06 percent. Note that the elasticity of $q_W(\theta)$ with respect to θ is equal to $1 - \sigma = 0.28$. Hence, the jobfinding chance of a non-participant is reduced by approximately $0.28 \times 3.8 = 1.06$ percentage point. Similarly, despite the 48 percent increase to participants' search efficiency, their job-finding rate has increased by only 46.9 percent. Tabel 2 summarizes the policy experiment for a selection of key variables.

Why does θ fall when a set of agents receive search assistance? This variable is determined in equilibrium from firms' hiring choices. Solving the vacancy condition (Equation (1.6)) for θ,

$$\theta = \left(\frac{1}{\gamma(1 + r - \delta_k)} \frac{\sum e_i f(i) J(\Omega_i) di}{\sum e_i f(i) di} \right)^{1/\sigma}.$$

There are four endogenous quantities on the right-hand side of this equation: the individual search efficiencies (the e_i terms), the masses of each type of jobseeker (the $f(i)$ terms), the value of each type of job (the $J(\Omega_i)$ terms), and the interest rate r. In the following, I take turns holding all but one of these constant in order to analyze the effect of the term in question, and the analysis is summarized in Table 3.

First, I use the 'policy' values for e_i but hold all other terms constant. In this environment, θ would rise by 0.29 percent. By increasing some agents' search efficiencies, we increase the likelihood that (all else equal) a firm matches with any worker, which increases the marginal benefit of posting one additional vacancy. Hence, in this scenario, vacancies would increase by about 3.2 percent, which is slightly more than the increase to u implied by the increase to e (about 2.9 percent), so the ratio $\theta = v/u$ increases. Next, I use the density function from the policy steady state and hold everything else constant. Now, θ would increase by 0.17 percent. With this scenario, vacancies decline 0.7 percent and u falls 0.87 percent. The change to vacancies is less than (in magnitude) the change to u, so θ rises. The marginal benefit of posting a vacancy decreases slightly now, as firms have a slightly lower (all else equal) probability of matching with a worker. Finally, row 5 of the table shows that changes to J, the value of a filled job, accounts for (more than) the entirety of the change to θ. Holding everything constant except for the J terms, θ would fall by 4.12 percent. Conditional on matching with any agent, the firm's value of this match is lower compared to the baseline. The existence of the search assistance program improves the value of workers' outside options, as unemployment is not quite as bad of a state to be in, so workers are able to command a higher wage. Hence, the value to being matched with a worker falls, which reduces the marginal benefit of posting another vacancy. I omit the effect from a change in r in the analysis as its magnitude is relatively tiny.

Implications for productivity

With the introduction of the policy experiment, the interest rate r increases. This is due to both a decrease in savings from a reduced precautionary motive (unemployment is now not quite as bad), and due to an increased demand for physical capital. The demand curve for physical capital, K^D, is given by

$$K^D = X^E \left(\frac{r}{\alpha}\right)^{1/(\alpha-1)},$$

where X^E is the total amount of employed human capital in the economy, which, unsurprisingly, increases slightly once the economy reaches the 'policy' steady state. Additionally, the *average* skill level of an employed agent increases in the new steady state. This might seem counter-intuitive at first: after all, the search assistance program primarily benefits low-skilled agents, at the expense of high skill agents.[11]

[11] The program does not explicitly target low-skilled agents; however, the mean skill level conditional on unemployment duration is decreasing in duration.

Hence, it would seem to be the case that the average skill level of the workforce would decline, as most new hiring is skewed towards the low-skill end of the distribution. However, the answer lies in the important distinction between the short-run and the long-run. As shown in Figure 3, the average skill level of the workforce (the solid line) indeed does decline in the short-run, as the mean skill level of a newly-hired worker (the dashed line) falls.[12] As time goes on, however, this trend reverses. Since the long-term unemployed now have a higher jobfinding rate (compared to the baseline), they stay unemployed for a much shorter amount of time, which mitigates the amount of skill erosion they experience. As such, the average skill level of the jobseeker pool is now higher relative to the baseline economy. This, in turn, increases the average skill level of *employed* workers.

The implication of this is that firms are now more productive. On one hand, higher-skilled workers obviously produce more output. Furthermore, the complementarity in the production technology between capital and labor leads to more capital being used in production as well. Overall, the economy's aggregate labor productivity (total GDP per employed worker) rises by about 0.15 percent, as the mean duration of an unemployment spell falls by about two weeks. These results are in line with papers such as Ortego-Marti (2015) who uses cross-country data on labor flows and finds a negative relationship between a country's mean duration of unemployment and its TFP. In fact, we can calculate the economy's implied TFP from it's production function. First, recall firms' individual production function: $y_i = k_i^{\alpha} x_i^{1-\alpha}$. Total GDP is therefore $Y = \sum y_i f(i) = \sum k_i^{\alpha} x_i^{1-\alpha} f(i)$. The constant-returns property of the production function means that this summation is also equal to

$$Y = (K^D)^{\alpha} (X^E)^{1-\alpha},$$

where K^D and X^E are, as before, the total amount of physical and employed human capital. We can rewrite this as

$$Y = (K^D)^{\alpha} (\bar{X}^E N)^{1-\alpha},$$

[12]Note that I compute the transition path for 1,000 periods, but I only graph the first 500 periods, as this is where most of the 'movement' is.

where $\bar{X}^E$ is the average skill level of an employed agent and N is the total number of employed agents. Hence, if we re-write the aggregate production function as a 'standard' Cobb-Douglas in physical capital K and employment level N, the TFP term corresponds to $(\bar{X}^E)^{1-\alpha}$. With the introduction of the search assistance policy, this term increases by about 0.09 percent.

1.4.1 Welfare

As is common in the literature, I employ a consumption-based measure to calculate welfare changes. Specifically, I calculate $\varepsilon(\Omega)$ from the following:

$$E_0 \sum_{t=0}^{\infty} \beta^t u((1+\varepsilon(\Omega))\bar{c}_t) = E_0 \sum_{t=0}^{\infty} \beta^t u(\tilde{c}_t),$$

where $\bar{c}_t$ is consumption under the baseline model and $\tilde{c}_t$ is consumption under the Search Assistance policy scheme. In other words, $\varepsilon(\Omega)$ represents the amount of consumption that would need to be given to agents in the baseline economy to be indifferent to the policy economy. Given the assumed log utility function, I can solve directly for $\varepsilon(\Omega)$ as

$$\varepsilon(\Omega) = \exp\Big((\widetilde{V}(\Omega) - \bar{V}(\Omega))(1-\beta)\Big) - 1, \tag{1.8}$$

where $\bar{V}$ is the household's value function for the baseline steady state and $\tilde{V}$ is the value function at the first period of the transition path. Full details of the computational algorithm for the transition path are presented in Appendix C.1.2. The total welfare gain (or loss) for the economy is defined as

$$\sum \varepsilon(\Omega) f(\Omega), \tag{1.9}$$

where $f(\Omega)$ is the mass of each type of agent from the first period of the transition path. Furthermore, the welfare gain for a particular type of agent (e.g., the average unemployed agent), is defined similiarly. For example, the average welfare gain for a long-term unemployed agent (i.e., an agent out of work at least one year at the beginning of the transition) is defined as

$$\frac{\sum_{s\geq 5} \varepsilon(\Omega) f(\Omega)}{\sum_{s\geq 5} f(\Omega)}$$

To begin with, the welfare gain for the economy as a whole (Equation (1.9)) is 0.215 percent of consumption. Furthermore, the welfare changes are positive not just for the economy as a whole, but for *each type of agent*. That is, $\varepsilon(\Omega) > 0$ for all types. As I discuss in more detail below, it turns out that while general equilibrium effects are, by themselves, welfare-reducing, they do not outweigh the welfare gains from the partial equilibrium effects.

There is significant heterogeneity in welfare changes across different types of agents. I start with Column (1) of Table 4, which shows the average welfare gain or loss for different classes of agents. Agents who are program participants (unemployed five quarters or more) at the start of the transition gain about 1.66 percent of welfare. On the other hand, unemployed non-participants gain about 0.36 percent welfare, and employed agents gain about 0.20 percent. The fact that these agents derive a welfare gain from the existence of the search assistance program, even though they are not currently enrolled, is due to the fact that they will become enrolled in the future with a nonzero probability. Put differently, even though these agents are not currently participating, the benefits of enrollment in the program (higher job-finding chances), which these agents might utilize in the future, outweigh the costs of the program.

Next, the welfare gains of the policy show substantial heterogeneity with respect to both assets and skill level. Figures 4 through 6 show the welfare gain of the program as a function of asset level for low- and high-skilled agents. Starting with Figure 4, the highest welfare gains are found among agents with the lowest assets and skills, even though agents in this figure are currently employed. The 'pain' of unemployment is naturally worse for the poorest agents, who have less savings with which to smooth their consumption. On the other hand, the interest rate increases when the policy is introduced, leading to a higher return on savings, the benefit of which is increasing in asset level. This accounts for the small 'peak' in welfare gains at the high end of assets. Next, Figure 5 shows the welfare gain for low- and high-skilled, short-term (i.e., non-participating) unemployed agents. As with the previous figure, note that the poorest agents gain the most welfare. Also, note that high-skilled agents gain more welfare than low-skilled agents. This relationship is due to the fact that wages are an increasing function of an agent's skill level. Therefore, conditional on the same asset level and unemployment duration, the benefit of *becoming re-employed* is greater for a high-skilled agent than a low-skilled agent. This relationship can also be seen in Figure 6, which shows the welfare changes for low- and high-skilled agents by asset level, conditional on being unemployed at least one year (and therefore being enrolled in the program). As before, the highest welfare gains are for the agents with the highest skills and lowest assets. However, the gains are much higher in magnitude relative to the previous figure. Recall, the agents shown in this figure are enrolled in the program, whereas agents in the previous two figures *might* become enrolled in the future. Hence, their welfare gain is much higher.

Another way to look at welfare gains is by duration of unemployment, as shown in Figure 7, which shows the welfare change by duration of unemployment for low- and high-skilled jobseekers. First, look to the left-hand side of the figure; these are agents who are not yet enrolled in the program. Not surprisingly, the welfare gain from the policy increases the closer an agent gets to becoming enrolled, as the likelihood of becoming enrolled in (and therefore benefitting from) the program increase as unemployment duration increases. Then, conditional on enrollment, the welfare gain levels out. Furthermore, at all unemployment durations, high-skilled agents have a higher welfare gain than low-skilled agents. As before, recall that wages are an increasing function of skill level. Hence, the benefit derived from *transitioning* from unemployment to employment is greater for high-skilled agents compared to low-skilled agents.

In the following subsections, I decompose the welfare gains from the implementation of the search assistance policy according to the various channels discussed above. It is important to understand the sources of the welfare gains, and to understand the magnitude of general equilibrium effects. The methodology I employed closely follows Mukoyama (2013). To perform each decomposition, I isolate the variable in question by re-solving the household's problem in an environment where all other variables are held constant at their baseline values, but the variable in question evolves according to the transition path, as calculated above.

1.4.2 Decomposition 1: Partial equilibrium

The first decomposition I perform is to isolate the partial equilibrium effects (higher individual search efficiency) from other general equilibrium effects (lower θ, higher prices and tax rate). Recall, agents unemployed five quarters or more see their $e(s)$ increase by a factor of 1.48. To isolate this channel, I increase $e(s)$ for long-term unemployed agents but hold all other variables constant. Importantly, labor market tightness θ is held constant at its baseline value of 2.4, instead of being allowed to fall approximately 3.8 percent as in the 'full general equilibrium' steady state. This decline to θ would, all else equal, translate to a reduced job-finding rate and therefore be welfare-reducing. Policy participants' job-finding rates are therefore defined as

$$\tilde{q}_i(\theta) = \tilde{e}(s)\bar{\theta}^{1-\sigma},$$

where $\bar{\theta}$ is the baseline value of 2.4. Column (2) of Table 4 shows the welfare changes for this scenario, and Column (3) shows the difference between Columns (1) and (2), and can therefore be interpreted as the general equilibrium effects. In terms of the pure partial equilibrium effects, welfare gains are still the highest for those agents currently participating in the program, with lower gains for those participating in the future. Next, note the values in Column (3). This column shows that general equilibrium effects represent a welfare reduction of between 0.1 and 0.16 percent (in consumption terms). These may seem small, but it is most appropriate to think about the general equilibrium effects in *relative* terms. For example, the welfare gain for the economy as a whole is 0.215 percent, and general equilibrium effects are -0.105 percent. In terms of magnitude, the general equilibrium effects are about 50 percent of the total welfare gain (0.105/0.215=0.488). In other words, ignoring general equilibrium effects and looking only at the partial equilibrium welfare gain of the search assistance policy would overstate these welfare gains by close to 50 percent. These results highlight the importance of understanding general equilibrium effects when analyzing any policy.

In the following subsections, I dig deeper into these general equilibrium effects, starting with labor market tightness.

1.4.3 Decomposition 2: Labor market tightness

As noted above, θ (the ratio of vacancies to the *effective* mass of unemployment) falls when the search assistance policy is introduced. As shown in Figure 8, θ jumps down in the first period, but then rebounds slightly as the economy settles to its new steady state over time. All else equal, this decline to θ leads to lower job-finding rates and is therefore welfare-reducing.

Column (4) of Table 4 shows the welfare effects solely from the fall in θ. According to the table, the fact that θ declines when the policy is introduced represents a welfare loss of between 0.1 and 0.15 percent (of consumption). Unemployed agents lose the most welfare in this case, as they are currently searching for a job, though employed agents also lose welfare, as they will be searching for a job in the future. Short-term unemployed agents (i.e., jobseekers who are not currently program participants) lose approximately 0.13 percent in terms of consumption solely from the reduced job-finding rates they experience as a result of the program. Figure 9 shows the welfare changes for this decomposition for low-skilled employed and unemployed agents. As expected, the poorest agents are hurt the most by the reduction in job-finding rates.

1.4.4 Decomposition 3: Price effects

In this economy, prices play an important role in agents' welfare. First, wages increase for all types of agents. The existence of the search assistance policy improves the value of workers' outside options (unemployment); they are therefore able to command a higher wage. Wages jump up in the first period, then eventually settle to a value higher than their starting point. All else equal, a higher wage is obviously welfare-improving.

Next, as noted previously, the interest rate rises somewhat when the policy is introduced. This comes from two factors. The first is a reduction in supply of savings: as unemployment is now a slightly better state of the world compared to before the introduction of the policy, agents' precautionary savings motive is reduced. Second, there is an increase in demand by firms for physical capital. Since there are no borrowers in this economy, this higher interest rate is necessarily welfare-improving for all agents, though moreso for richer agents.

On the other hand, at time $t = 0$, when the policy is introduced, all agents experience a decrease to the value of their asset portfolio because the equity price jumps down. By itself, this equity price decline is welfare-reducing. The equity price falls because wages jump up at $t = 0$ and hence dividends fall. The precise magnitude of the equity price drop is given below.

Proposition 1.1. For an agent with asset level a_t before the policy is introduced, the value of his portfolio changes by

$$\frac{(\tilde{p}_0 - p_0)a_t}{\bar{A}},$$

where p_0 is the original asset price (i.e., before the policy), $\tilde{p}_0$ is the new asset price, and $\bar{A}$ is the total amount of assets in the economy before the policy introduction.
Proof: See Appendix B.

In my particular policy experiment, the welfare loss from the equity price decline is small enough that it is outweighed by the welfare gains due to higher wages and a higher interest rate, and hence overall, there are welfare gains, not losses, due to price effects. Column (5) of Table 4 summarizes the price effect decomposition. As shown in the table, price effects account for a welfare gain of just under 0.1 percent (in terms of consumption). With respect to asset level, welfare gains exhibit a u-shape pattern, as shown in Figure 10. The less assets an agent has, the more he relies on his wage income, and therefore the higher the welfare gain from higher wages. On the other end, the more assets an agent has, the higher the gain to interest income this agent receives, thus the higher his welfare gain.[13]

[13]The corresponding graph for low- or high-skilled, short- or long-term unemployed agents looks nearly identical.

1.4.5 Decomposition 4: Tax effects

When the policy is introduced, in order to cover the costs of the program, the income tax rate rises slightly. All else equal, this tax increase is obviously welfare-reducing. As shown in Column (6) of Table 4, a higher tax rate accounts for a welfare-reduction of a little under 0.1 percent consumption. As with the other decompositions, the welfare loss is also a function of an agent's asset level, as shown in Figure 11.[14] Since the percent of total income that comes from wages is monotonically decreasing in an agent's wealth level, poorer agents are hurt the most by an increase in taxes.

1.4.6 Alternative specification: unemployment insurance extension

In this section, I consider an alternative policy in which the government does not implement the search assistance program. Instead, they take this money and split it evenly among all unemployed agents as a lump-sum payment. To be clear, I assume the total amount of funds available for this payment is equal to (both in steady-state and every period along the transition path) the exact amount of money the government spent on the search assistance program in the original policy experiment. Let $\tilde{p}_t$ be the payment that every unemployed agents receives at time t, which is equal to

$$\tilde{p}_t = \frac{\psi \sum\limits_{s \geq 5} f_t^0(\Omega)}{\sum\limits_{s \geq 1} f_t^1(\Omega)},$$

where ψ is the per-person cost of search assistance, as before; f_t^0 is the economy's density from the original policy experiment and f_t^1 is the new density. Hence, the summation in the numerator is the number of agents unemployed five quarters or more from the original experiment, and the denominator is the unemployment rate for this new policy experiment. Note that the payment $\tilde{p}_t$ is less than ψ, since it is being split across *all* unemployed agents, not just those unemployed at least five quarters.[15]

[14] See previous footnote.

[15] Even if this payment was split across just these long-term unemployed, it would still be less than ψ: since this new policy experiment does not feature search assistance, there are now about 50 percent more long-term unemployed agents in the jobseeker pool.

Therefore, the size of this payment is extremely small. Recall, ψ is approximately 21 percent of the mean quarterly wage. Now, this payment is only about 1.2 percent of the mean quarterly wage. Unemployed agents' budget constraint is now:

$$c_t + a_{t+1} = (1 + r_t - \delta_k)a_t + (1 - \tau)w_t(\Omega) + \tilde{p}_t.$$

Note the assumption that these new payments are not subject to income tax. Since the payments are so small, this assumption has little effect. Under this new policy experiment, the welfare gains are much smaller, and positive for only about 35 percent of agents. Table 5 compares the welfare changes. Now, long-term unemployed agents only gain 0.09 percent (in consumption terms) with this scheme, versus 1.7 percent under the search assistance policy. That the welfare gains are now much smaller is a reflection of the fact that the payment is so small, compared to the gains from becoming re-employed. Given that $\tilde{p}_t$ is only 1.2 percent of the mean quarterly wage, the consumption gains from becoming employed far outweigh the consumption gains this payment allows. Hence, on a per-dollar basis, the search assistance scheme, which is an investment in a person's re-employment chances, is a significantly more effective way to deliver utility to agents.

One caveat to this conclusion is that the very poorest agents actually prefer the UI payment scheme to the search assistance scheme. For the very poorest agents, the UI payment represents an extremely high fraction of their per-period income (as they have close to zero assets), so the consumption gain from receiving this payment is quite high. This relationship shown in Figure 12. However, I must note that the mass of these agents is extremely small; only 0.0026 percent of unemployed agents in the economy possess this level of assets.

1.5 Conclusion

In this chapter, I develop a structural model of the economy in which workers experience erosion of their skills while unemployed and appreciation of skills while employed. Agents in the economy are heterogeneous according to asset level, skill level, and labor market state. Jobseekers are matched to firms according to a frictional, DMP-style matching technology. I use this model to study the heterogeneous welfare effects derived from the creation of a search assistance policy. This policy increases the job-finding chances of participants (agents out of work for one year or more) but reduces the job-finding chances of non-participants. I find that the existence of this policy is welfare-improving for all agents, even those who are not currently enrolled.

To further understand these welfare gains, I decompose them into the pure partial equilibrium channel (increased individual search efficiency), which is strictly welfare-improving, and general equilibrium channels (lower labor market tightness, higher prices, and higher tax rate). The general equilibrium channels are, overall, welfare-reducing; however, individual components are welfare-improving. For example, with the existence of the search assistance policy, agents earn a higher return on their savings, which is welfare-improving, all else equal. The general equilibrium welfare effects are quantitatively important. For the economy as a whole, there is a welfare gain of about 0.22 percent (in terms of consumption). However, isolating the partial equilibrium channel, the welfare gain is just over 0.32 percent, implying that general equilibrium effects account for about -0.1 percent. Hence, general equilibrium welfare effects represent a significant fraction of the total welfare effects. Put differently, ignoring general equilibrium effects would overstate the welfare gain of the search assistance program by a significant amount. Furthermore, I document heterogeneity in welfare gains across agent types. As with many labor market programs, the poorest agents derive the highest welfare gains overall. Finally, I place this search assistance program into context by comparing it to another program of equal cost. Namely, if the government chose to take the money it would have spent on the search assistance program and divide it across all unemployed agents (*instead of* implementing the program), the welfare gains would be significantly smaller, and only positive for a fraction of the economy. This shows that, on a per-dollar basis, search assistance is a more cost-effective method of improving agents' welfare.

Chapter 2

Optimal search assistance with heterogeneous agents

2.1 Introduction

The previous chapter analyzed the various channels through which a given search assistance scheme affects agents' welfare. While the chosen experiment was shown to be welfare-improving (compared to a world without assistance), and shown to be preferrable to an alternative policy, there is no reason to think this policy is in any way *optimal*. This chapter analyzes the question of optimality by introducing a planner who chooses a search assistance policy scheme for an economy populated by heterogeneous agents in order to maximize the sum of all agents' lifetime utilities.

Recall the discussion from Chapter 1 of the tradeoffs associated with a search assistance program. If one jobseeker is enrolled in an assistance program that improves his jobfinding chances, then any other jobseeker is inherently made worse off, all else equal. In order to analyze the optimal scheme of assistance (given a well-defined objective function), we must employ a model that explicitly allows for this spillover effect. In this Chapter, I define a two-period model populated by agents who are heterogeneous over their wealth level, human capital, and unemployment state. The economy's planner, who is bound by a Diamond-Mortensen-Pissarides-style frictional labor market, then chooses a job-finding rate (and the search assistance intensity that achieves this) for each type of unemployed agent to maximize the population-weighted sum of all agents' lifetime utilities, taking into account the tradeoffs discussed above. Increasing the assistance intensity given to one agent increases the degree of competition in the labor market, which in turn decreases the probability that all other agents find a job, all else equal. Furthermore, assistance is paid for via income taxes; this increased tax burden represents a welfare loss for all agent types.

Given that the model involves certain simplifications relative to the model featured in Chapter 1, it allows an analytical characterization for the planner's decision rules for the assistance scheme and the associated job-finding rates. For example, I show that the optimal search assistance intensity is monotonically increasing in an agent's human capital level, as well as monotonically decreasing in his wealth level. I then solve three different (but closely-related) versions of the basic model. In the first model, the planner directly solves for assistance intensity for each type of agent in the economy's state space. The second model uses a much larger state space, and the planner chooses the optimal job-finding rates according to a rule that is a function of agents' state variables. While clearly a restriction on the planner's choice set, the functional rule represents a simple way to report the planner's choice using very few parameters relative to points in the state space. It is therefore useful to policymakers (for example) wishing to characterize the optimal policy scheme across agent types. The third model is identical to the second, except that the planner chooses assistance intensity, not the job-finding rate, according to a functional rule.

I present results for each model as a function of agents' wealth and human capital level, and unemployment duration. I show that the third model, which chooses assistance intensity according to a functional rule, delivers a higher average welfare gain to unemployed agents than the second model, which chooses job-finding rates. I then solve an alternative specification for the three models by removing employed agents from the planner's objective function. Employed agents make up the vast majority of the economy, but, given the two-period time horizon, never experience any of the benefits of the assistance program. Therefore, the welfare loss they incur through a higher tax rate dominates the social welfare function. By removing them from the planner's objective function, the planner chooses a significantly higher degree of assistance intensity.

As with the previous Chapter, this Chapter builds on the strand of literature analyzing optimal labor market policies, such as Pavoni and Violante (2007), Wunsch (2013), and Pavoni, Setty, and Violante (2016). The key extension this Chapter makes over the existing literature is explicitly allowing for spillover effects across agents, which the previous literature does not. For example, Pavoni, Setty, and Violante (2016) consider a range of policy options and characterize the planner's choice, as a function of a given benefit level and the agent's human capital level. However, their analysis paints an incomplete picture as the assistance scheme given to one type of agent in no way influences the utility of any other agent.

This Chapter is organized as follows. Section 2.2 describes individual agents' optimization problems along with the planner's problem, and Section 2.3 presents an analytical characterization of the model's solution. Section 2.4 discusses parameter calibration, Section 2.5 presents results of all three versions of the model, and Section 2.6 concludes.

2.2 The economy

The economy consists of heterogeneous agents, each of whom solves his own optimization problem. The planner, who is bound by a frictional labor market and other resource constraints, chooses an optimal search assistance scheme (and the implied job-finding rates) to maximize the economy's social welfare function, which is defined as the population-weighted sum of individual agents' lifetime utility functions. Hence, I begin by describing the problem faced by individual agents, then put the pieces together to characterize the planner's choice.

Agents are heterogeneous across three dimensions: wealth level a, human capital ('skill') level x, and labor market state s. State s takes an integer value from 0 to $\bar{s}$, where $s = 0$ refers to the employed state, $s = 1$ means unemployed for one period, $s = 2$ means unemployed for two periods, and so on. Let i denote a particular *type* of agent; this is simply shorthand for the three previously mentioned state variables. Let $f(i)$ be the measure of each type of agent. The total population of the economy is normalized to one; that is, $\sum_i f(i) = 1$.

Agents live for two periods. They have an initial wealth level of a_1 and earn wage income $w_1(i)$ in the first period if they are employed, or $\hat{w}$ if they are unemployed; $w_1(i)$ is increasing in the agent's skill level but $\hat{w}$ is the same across all types. I assume $\hat{w} < w_1(i)^{min}$; that is, the unemployment insurance payment is strictly less than the lowest wage. Agents have the same earnings in the second period; that is, they earn $w_2(i) = w_1(i)$ if employed or $\hat{w}$ if unemployed. Agents earn interest on their savings at rate r, pay income tax at rate τ on their first-period income, and discount the future at rate β. The interest rate is assumed to be exogenous. An agent is employed tomorrow with probability $q(i)$ and unemployed with probability $1 - q(i)$. For currently-employed agents, $q(i) = \phi$, which is exogenous, but for currently-unemployed agents, $q(i)$ is determined through a Diamond-Mortensen-Pissarides matching technology, which I detail below.

Employed agents supply x units of labor to their employer, who produces output according to the production function $y = x(i)^\alpha$, and they earn a wage equal to $\alpha x^{\alpha-1}$ per unit; hence $w_1(i) = w_2(i) = \alpha x(i)^\alpha$. Agents choose today's consumption c_1, tomorrow's asset level a_2, and tomorrow's consumption c_2^e and c_2^u to maximize lifetime utility. Denote the value function for an agent with type i as $V(i)$. The optimization problem for a currently-employed agent with type i is

$$V(i) = \max_{\{c_1, a_2, c_2^e, c_2^u\}} u(c_1) + \beta \left(q(i) u(c_2^e) + (1 - q(i)) u(c_2^u) \right) \tag{2.1}$$

subject to

$$\begin{aligned} c_1 + a_2 &= (1 - \tau) w_1(i) + a_1, \\ c_2^e &= w_2(i) + (1 + r) a_2, \\ c_2^u &= \hat{w} + (1 + r) a_2. \end{aligned}$$

The optimization problem for a currently-unemployed agent is the same, except he earns $\hat{w}$ today instead of $w_1(i)$. By plugging in the budget constraint, the optimization problem can be expressed in terms of just one choice variable, c_1. Letting $u(c) = ln(c)$, the agent's first-order conditions with respect to c_1 imply

$$\frac{1}{c_1^*} = \frac{\beta q(i)(1+r)}{w_2(i) + (1+r)(w_1(i)(1-\tau) + a_1 - c_1^*)} + \frac{\beta(1-q(i))(1+r)}{\hat{w} + (1+r)(w_1(i)(1-\tau) + a_1 - c_1^*)},$$

where c_1^* is this agent's optimal choice of c_1. Note the role of $w_2(i)$ versus $\hat{w}$: the difference in earnings between employment and unemployment is the key driver of the model. If the agent's earnings did not depend on employment state, then the variable q would not matter at all, and the planner would have no incentive to alter it. On the other hand, given the assumption that $w_2(i) > \hat{w}$, the agent's utility is increasing in $q(i)$. Hence, all else equal, a higher employment probability makes agents better off.

2.2.1 Job matching

Workers are matched with jobs according to a frictional Diamond-Mortensen-Pissarides matching technology. As with standard DMP-style models, I define labor market tightness, denoted by θ, as v/u; that is, the ratio of posted vacancies to unemployment. As I am abstracting from the firm side of the labor market, I assume v is fixed at some exogenous value. The term u in this model has a different meaning than in the typical case. Instead of weighting each jobseeker equally, u is weighted by each agent's *individual search efficiency*, denoted by $e(i)$. This parameter, which is heterogeneous across types, governs a jobseeker's likelihood of finding a job relative to others. Therefore, the effective (i.e., efficiency-weighted) measure of unemployment is given by

$$u = \sum_{s \geq 1} e(i) f(i).$$

With this formulation, if one particular worker is inherently twice as likely (for example) to find a job as another jobseeker, it is as though there are two of these workers in the jobseeker pool instead of just one. The number of matches formed is determined by a Cobb-Douglas matching technology $m(u, v)$:

$$m(u, v) = u^{\sigma} v^{1-\sigma}.$$

From this, the probability that a worker of type i finds a job is given by $q(i)$:

$$q(i) = e(i)\frac{m(u,v)}{u} = e(i)\theta^{1-\sigma},$$

which is increasing in his own $e(i)$, but decreasing in $e(j)$ of some other agent $j \neq i$, due to the fact that a higher $e(j)$ will increase u, which is in the denominator of θ.

2.2.2 Search assistance

In this model, search assistance refers to the planner scaling up (or down) an agent's $e(i)$, which makes him more (or less) likely to find a job. Let $e(i) = \rho(i)\bar{e}(i)$. The $\bar{e}(i)$ term is the agent's inherent individual search efficiency, which he possesses in the absense of any intervention by the planner; this parameter is calibrated to be a decreasing function of the agent's unemployment duration s, but does not depend on any other state variable. Let $\bar{q}(i)$ denote the job-finding rate corresponding to an economy without search assistance; that is, $\bar{q}(i) = \bar{e}(i)\theta^{1-\sigma}$ for all types. The planner can then choose to enroll a particular agent in search assistance, which serves to multiply this agent's $\bar{e}(i)$ by some factor $\rho(i)$. Search assistance costs $\psi(\rho(i))$ per jobseeker, where ψ is an increasing and convex cost function. All else equal, $\rho > 1$ makes an agent more likely to find a job next period. However, increasing $e(i)$ for one agent will make $q(j)$ for some $j \neq i$ fall, which is the fundamental tradeoff the planner faces in this economy. Recall, the job-finding probability for agent j is

$$q(j) = \rho(j)\bar{e}(j)\theta^{1-\sigma}$$

Suppose the planner increases $\rho(i)$ for some other agent ($i \neq j$) in the economy. Then, because $u = \sum_i \rho(i)\bar{e}(i)f(i)$, u will decrease, and $q(j)$ decreases. This is precisely the 'crowding-out' or 'substitution effect' discussed in the introduction: by making one agent more likely to find a job, the planner has simultaneously made all other agents worse off.

This tradeoff factors into individual agents' utility functions. Just for now, suppose that the tax rate τ is fixed (i.e., the program is costless) to isolate the effect of changing q on utility. Note that increasing a particular agent's $q(i)$ increases his (expected) lifetime income. Hence, given that c_1 and c_2 are both normal goods, increasing $q(i)$ leads to increasing consumption in both periods, which corresponds to higher utility. Therefore, increasing $q(i)$ will increase lifetime utility for agent i, but will decrease lifetime utility for all other agents, as this corresponds to a decrease to their q's.

Finally, the planner finances the search assistance program through income taxes, at rate τ. The government's budget constraint is

$$\sum_{s\geq 1} \psi(\rho(i))f(i) = \tau \left(\sum_{s=0} w_1(i)f(i) + \hat{w} \sum_{s\geq 1} f(i) \right).$$

The left-hand side is spending on search assistance, and the right-hand side is tax income from wages.

2.2.3 Planner's problem

I now turn to the optimization problem faced by the planner. The planner's objective function is the weighted sum of individual agents' lifetime utilities, given by:

$$\sum_i f(i)V(i),$$

where, as before, $V(i)$ is the value function for each type of agent. The planner chooses $\rho(i)$ for each type (or, the corresponding $q(i)$ for each type) in order to maximize the above objective function, subject to the matching process, the government's budget constraint, and the individual agents' constraints. I solve three different (but closely related) versions of the model, which I detail below.

Model 1: Small state space

First is the "small version"; in this model, the state space is discretized into only a small number of nodes: three in the asset dimension ('poor', 'middle class', and 'rich'), three in the skill dimension ('low-skill', 'middle-skill', and 'high-skill'), and three in the labor market state dimension (employed, short-term unemployed, and long-term unemployed), for a total of 27 nodes. The planner solves for $\rho(i)$ for each of the 18 types of unemployed agents. Formally, the planner's problem is given by:

$$\max_{\rho(i)} \sum_i f(i)V(i),$$

subject to:

$$\rho(i) \geq 1 \ \ \forall \ i,$$
$$q(i) \leq 1 \ \ \forall \ i,$$
$$\sum_{s\geq 1} \psi(\rho(i))f(i) \leq \left(\sum_{s=0} w_1(i)f(i) + \hat{w} \sum_{s\geq 1} f(i) \right).$$

Note the constraint that $\rho(i) \geq 1$. In this version, I assume the planner cannot provide 'negative search assistance' to any agent; i.e., they cannot make any jobseeker worse at finding a job. However, this does not mean that the optimal job-finding rate chosen by the planner, denoted $q^*(i)$, cannot be lower than the no-assistance job-finding rate $\bar{q}(i)$, it just means that agent i's job-finding rate can fall *only* through general equilibrium effects; that is, through a higher degree of competition from other jobseekers. The second constraint is necessary as the matching function is not inherently bounded below 1. Furthermore, the final constraint ensures that the tax rate $\tau \leq 1$.[16]

Model 2: A rule for optimal job-finding rates

The second version of the model features a state space parameterized with many more nodes than the first version, at which point it would be impractical to solve for $\rho(i)$ for each type of agent. Instead, the planner now chooses $q(i)$ (and the $\rho(i)$ implied by this) based on a functional rule, which will depend on agents' state variables but require far fewer choice variables than nodes in the state space. Specifically, let the

[16]Note that $V(i)$ is the solution to the individual agents' optimization problems. As the planner is choosing the optimal $\rho(i)$ for all types, he is also solving each type's optimization problem, given $\rho(i)$, as detailed above. Hence why the model is chosen to be two periods instead of infinite horizon, and with a small number of nodes in the state space. While there is no analytical closed-form decision rule for $c_1^*(i)$, the solution can be found very quickly through numerical methods.

planner's choice of $q(i)$ be given by a Logit function:

$$q(i) = \frac{1}{1+\exp(-t(i))}, \quad \text{where,}$$
$$t(i) = \beta_0 + \beta_1 a(i) + \beta_2 x(i) + \beta_3 s(i) +$$
$$\beta_4 a(i)x(i) + \beta_5 a(i)s(i) + \beta_6 x(i)s(i) + \beta_7 a(i)x(i)s(i).$$

This function is bounded between 0 and 1, and is flexible enough to be either increasing or decreasing in any of the state variables (with interaction terms) depending on the choice of parameters. This function therefore represents a concise rule for stating the planner's optimal choice of job-finding rates as a function of agents' state variables. A functional rule is much more useful to policymakers (for example) than a three-dimensional matrix of numbers, as the solution to Model #1 is. Under this specification, I allow $\rho(i)$ to be less than one, meaning the planner can 'un-assist' a given jobseeker. The convex cost function $\psi(\rho(i))$ is symmetric around $\rho(i) = 1$, so the planner incurs the same cost for reducing $\rho(i)$ as increasing it.

Formally, the planner's problem for this version is given by:

$$\max_{\{\beta_k\}_{k=0}^{7}} \sum_i f(i)V(i),$$

subject to

$$q(i) = \frac{1}{1+\exp(-t(i))},$$
$$t(i) = \beta_0 + \beta_1 a(i) + \beta_2 x(i) + \beta_3 s(i) +$$
$$\beta_4 a(i)x(i) + \beta_5 a(i)s(i) + \beta_6 x(i)s(i) + \beta_7 a(i)x(i)s(i),$$
$$\rho(i) = \frac{q(i)}{\bar{q}(i)} \frac{u_1^{1-\sigma}}{u_0^{1-\sigma}},$$
$$u_1 = \sum_{s\geq 1} \rho(i)\bar{e}(i)f(i),$$

$$\sum_{s\geq 1}\psi(\rho(i))f(i)\leq\left(\sum_{s=0}w_1(i)f(i)+\hat{w}\sum_{s\geq 1}f(i)\right).$$

Note the dependence of $\rho(i)$ on u_1 above, where u_1 is the new value of u after search assistance has been implemented. If u was fixed, then $\rho(i)$ would simply be the ratio of the new job-finding rate $q(i)$ to $\bar{q}(i)$. However, an increase to $\rho(i)$ makes u increase (recall, any particular type has nonzero mass, plus all types may receive some $\rho \neq 1$). This increase to u further increases $\rho(i)$, as the planner must increase the assistance intensity to deliver the chosen q.[17]

Model 3: A rule for optimal search assistance

The third version of the model is similar to the second version, except the planner chooses ρ, not q, according to a functional rule. Specifically, let the planner's choice of ρ be given by a polynomial rule:

$$\rho(i)=\max\{1,\tilde{\rho}(i)\},\ \ \text{where,}$$
$$\tilde{\rho}(i)=\beta_0+\beta_1 a(i)^{\beta_2}+\beta_3 x(i)^{\beta_4}+\beta_5 s(i)^{\beta_6}$$

Depending on the choice of the β's, this polynomial can be increasing or decreasing, and concave or convex, in any of the state variables. Formally, the planner's problem for this version is given by:

$$\max_{\{\beta_k\}_{k=0}^{6}}\sum_i f(i)V(i),$$

subject to:

$$\rho(i)=\max\{1,\tilde{\rho}(i)\},\ \ \text{where,}$$
$$\tilde{\rho}(i)=\beta_0+\beta_1 a(i)^{\beta_2}+\beta_3 x(i)^{\beta_4}+\beta_5 s(i)^{\beta_6},$$
$$u=\sum_{s\geq 1}\rho(i)\bar{e}(i)f(i),$$
$$q(i)=\rho(i)\bar{e}(i)\theta^{1-\sigma}\leq 1,$$

[17] Essentially, finding $\rho(i)\ \forall\ i$ and the corresponding value of u_1 is a fixed point problem, which is described in more detail in Appendix C.2.

$$\sum_{s\geq 1}\psi(\rho(i))f(i) \leq \left(\sum_{s=0}w_1(i)f(i)+\hat{w}\sum_{s\geq 1}f(i)\right).$$

2.3 Characterization of equilibrium

To gain some intuition into the mechanisms present in this economy, consider for now an economy with only two types of agents, i and j, with measures $f(i)$ and $f(j)$. Both of these agents are unemployed. Recall the job-finding rate of an agent with type i

$$q(i) = \rho(i)\bar{e}(i)\theta^{1-\sigma}.$$

The total differential of $q(i)$ with respect to $\rho(i)$ and $\rho(j)$ is

$$dq(i) = \frac{\partial}{\partial \rho(i)}\left[\rho(i)\bar{e}(i)\theta^{1-\sigma}\right]d\rho(i) + \frac{\partial}{\partial \rho(j)}\left[\rho(i)\bar{e}(i)\theta^{1-\sigma}\right]d\rho(j), \tag{2.2}$$

where

$$\frac{\partial}{\partial \rho(i)} = \bar{e}(i)\theta^{1-\sigma} + (\sigma-1)\rho(i)\bar{e}(i)v^{1-\sigma}u^{\sigma-2}\frac{\partial u}{\partial \rho(i)}, \tag{2.3}$$

$$\frac{\partial}{\partial \rho(j)} = (\sigma-1)\rho(i)\bar{e}(i)v^{1-\sigma}u^{\sigma-2}\frac{\partial u}{\partial \rho(j)}. \tag{2.4}$$

Equation (2.3) represents how agent i's job-finding rate changes with respect to changes to his own individual search efficiency (and, importantly, all other agents who share his type; recall there are $f(i)$ of these agents), which has been scaled up (or down) by $\rho(i)$, and Equation (2.4) represents how it changes with respect to another agent's search efficiency. The first term in (2.3), $\bar{e}(i)\theta^{1-\sigma}$, is a *partial equilibrium effect:* this is the change to $q(i)$ as a direct result of a change to $\rho(i)$, holding labor market composition unchanged. The second term in (2.3) and (2.4) represent *general equilibrium effects:* changing ρ for some (or all) agents will increase u, thus acting as an increase in competition from other jobseekers. Given that $\sigma - 1$ is negative and $\partial u/\partial \rho(i)$ is positive, these general equilibrium effects are negative.[18] It can also

[18]It might seem strange there are general equilibrium effects coming from increasing agent i's own ρ; that is, from $\partial q(i)/\partial \rho(i)$. This comes from the fact that there is a nonzero mass of agents in

be shown that Equation (2.3) is strictly positive; that is, when looking at the effect of changing just this agent's ρ, positive partial equilibrium effects outweigh negative general equilibrium effects.[19] Rearranging 2.2 into partial and general equilibrium effects,

$$dq(i) = \underbrace{\bar{e}(i)\theta^{1-\sigma}d\rho(i)}_{\text{Partial equilibrium}} + \underbrace{\rho(i)\bar{e}(i)v^{1-\sigma}(\sigma-1)u^{\sigma-2}\left(\frac{\partial u}{\partial \rho(i)}d\rho(i) + \frac{\partial u}{\partial \rho(j)}d\rho(j)\right)}_{\text{General equilibrium}}.$$

Therefore, increasing $\rho(j)$ will cause $q(i)$ to fall due to general equilibrium effects.

Turning to utility, the first order-condition of the planner's objective function with respect to $q(i)$ is

$$f(i)\frac{\partial V(i)}{\partial q(i)} + f(j)\frac{\partial V(j)}{\partial q(i)} = 0.$$

Hence, the (population-weighted) marginal utility given to agent i is balanced by the (population-weighted) marginal utility taken away from agent j. In the actual model economy, which has more than two types, the planner's first-order condition corresponds to

$$f(i)\frac{\partial V(i)}{\partial q(i)} + \sum_{-i} f(i)\frac{\partial V(-i)}{\partial q(i)} = 0,$$

where the sum over $-i$ refers to all other agents who are not agent i. Increasing $q(i)$ will lead to an increase in c_i^*, since this agent's (expected) lifetime income has increased, which therefore increases his utility. On the other hand, as shown above, increasing $q(i)$ will decrease $q(j)$. Consequently, increasing $q(i)$ *decreases* agent j's (expected) lifetime income, reducing c_j^* and his utility. Therefore, when choosing a search assistance intensity for one agent, the planner must balance the loss of utility

each type. If we were truly changing ρ *just for one particular person*, there indeed would be no general equilibrium effects, as one person is indeed measure-zero. However, the equation is showing the effect of changing not just one person's ρ, but the ρ for *all agents who share this particular type*, of which there are $f(i)$.

[19]See Appendix B.

this implies for all other agents.[20]

This is illustrated in Figure 13. In this Figure, $\rho(i)$ is set to $\rho^*(i)$ for all agents (See the Results section) except for the high-asset, high-skill, short-term unemployed agent (i.e., his $i = (3, 3, 2)$); his $\rho(i)$ varies between 1 and 2. The economy's social welfare function is shown in black; this line peaks at about 1.12. The red, dot-dashed line is $V(i)f(i)$; that is, the value function for just the agent in question, and the blue dashed line is the (weighted) sum of everyone else's value functions.[21] The blue line is strictly decreasing in $\rho(i)$: increasing one agent's search efficiency is strictly welfare-reducing for other agents. On the other hand, $V(i)$ is strictly increasing in $\rho(i)$ *until* about $\rho = 1.8$. This is the value of ρ which would give this agent a job-finding rate equal to 1. After this point, increasing ρ will no longer increase the job-finding rate. In fact, the orange line declines slightly after this point as the tax rate continues to increase.

The effects of search assistance are heterogeneous across the economy's state variables, which imply heterogeneous decision rules for ρ, which I analyze now. To begin with, the utility gain to agent i from increasing $q(i)$ is increasing in this agent's skill level. The intuition is straightforward: since wages are an increasing function of an agent's skill level, the income *gain* (and therefore the consumption gain) from transitioning from unemployment to employment is an increasing function of skill level $x(i)$. This is formalized below.

Proposition 2.1. Consider two unemployed agents i and j with the same asset level but who vary over skill level x, with $x(i) > x(j)$. Then, $dV(i)/dq(i) > dV(j)/dq(j)$.
Proof: See Appendix B.

Proposition 2.1 implies that the planner has more incentive to increase the high-skilled agent's job-finding rate than the low-skilled agent. All else equal, a one-unit increase to $q(i)$ (the high-skilled agent) represents more of a utility gain to the social welfare function than a one-unit increase to $q(j)$.[22] In fact, we can formalize this idea:

[20] The preceding paragraph ignores the effect of the tax rate; a higher tax rate would represent a welfare loss for any agent.

[21] Hence, the blue and red lines sum to the black line.

[22] This statement relies on the measures of agents i and j being equal, as they are in my calibration.

Proposition 2.2. Consider an economy populated by k types of unemployed agents who are identical except for their skill level, with $x(1) < x(2) < \ldots < x(k)$. Then, in terms of the planner's optimal choice of ρ^*, $\rho^*(1) < \rho^*(2) < \ldots < \rho^*(k)$.
Proof: See Appendix B.

We can show a similar result in terms of assets. In this case, $dV(i)/q(i)$ is a decreasing function of assets. Again, the intuition is straightforward: the more assets an agent has, the lower the fraction of his per-period income comes from wages, so the lower the utility gain from becoming employed. This is formalized below.

Proposition 2.3. Consider two unemployed agents i and j with the same skill level x but who vary over asset level a, with $a(i) > a(j)$. Then, $dV(j)/dq(j) > dV(i)dq(i)$.
Proof: See Appendix B.

Similar to Proposition 2.1, Proposition 2.3 implies the planner has more incentive to increase a low-asset agent's job-finding rate than a high-asset agent, since, all else equal, a one-unit increase to $q(j)$ (the low-asset agent) represents more of a utility gain to the social welfare function than a one-unit increase to $q(i)$.[23] As with the previous result, this is formalized below.

Proposition 2.4. Consider an economy populated by k types of unemployed agents who are identical except for their asset level, with $a(1) < a(2) < \ldots < a(k)$. Then, in terms of the planner's optimal choice of ρ^*, $\rho^*(1) > \rho^*(2) > \ldots \rho^*(k)$.
Proof: The proof is essentially identical to the proof of Proposition 2.2 (See Appendix B), so is omitted.

If, for example, $f(j) > f(i)$, then it is not necessarily true that a one-unit increase to $q(i)$ increases the social welfare function more than a one-unit increase to $q(j)$. The statement would still hold for *one particular person* with type i and *one particular person* with type j; however, if there are significantly more type j agents relative to type i agents, then the overall gain to all type j agents may outweigh the overall gain to type i agents.

[23]See previous footnote.

2.4 Calibration

The state space is discretized as follows. For Model #1, there are three nodes for assets, $a(i) \in \{0.01, 0.5, 1\}$, three nodes for skills, $x(i) \in \{1.0, 1.5, 2.0\}$; and three nodes for labor market state, employed, short-term unemployed, and long-term unemployed. The density function $f(i)$ is set as follows: 95% of agents are employed, split evenly across assets and skills (nine nodes), and 5% of agents are unemployed, also split evenly across assets, skills, and labor market state (18 nodes). This implies one-third of the economy are low-asset agents, one-third are medium-asset, and one-third are high-asset, along with one-third low skill, one-third medium skill, and one-third high-skilled.

For Models #2 and #3, there are 30 nodes for assets, split evenly between 1 and 40, and five nodes for skills, $\{0.5, 0.6, 0.7, 0.8, 0.9\}$. There are 10 nodes for labor market state; employed, and unemployed up to nine periods. Hence, there are a total of 1,500 nodes in the state space.

For all versions of the model, one period corresponds to one quarter. Some parameters are common across versions. Households discount the future at rate $\beta = 0.99$, labor's share of income is $\alpha = 0.67$, and the interest rate is $r = 0.00985$, which corresponds to an annual interest rate of 4 percent.

The search assistance cost function is set as follows. From Chapter 1, a search assistance program equivalent to $\rho = 1.48$ costs 0.21 percent of the economy's average wage, denoted $\bar{w}$, per recipient. Let $\bar{\psi} = 0.21\bar{w}$. I assume costs are quadratic in ρ, symmetric around $\psi(1) = 0$, and with $\psi(1.48) = \bar{\psi}$. This implies a cost function of

$$\psi(\rho) = \bar{\psi} \left(\frac{\rho - 1}{0.48} \right)^2$$

Note that this function is the same across all three versions, though $\bar{w}$ (and therefore $\bar{\psi}$) will be different between the small version and the other two. Finally, some parameters are identical to Chapter 1; namely, σ, the matching function elasticity, ϕ, the probability of an employed agent keeping his job, and $\bar{e}(i)$, the unemployed agent's inherent search efficiency. See Section 1.3 for details. Regarding $\bar{e}(i)$, these come from Chapter 1 for Models #2 and #3 only, as this model features the same state space with respect to unemployment duration. For Model #1, I use a weighted average of corresponding values of $\bar{e}(i)$, as follows. Given that there are only two nodes for unemployment duration in this model, I assume $s = 1$ corresponds to unemployed up to one year, and $s = 2$ corresponds to unemployed beyond one year. Therefore, I take the population-weighted average of the first four values of $\bar{e}(i)$ from Chapter 1 for $\bar{e}(s = 1)$ and the average of the remaining values for $\bar{e}(s = 2)$.

Table 7 summarizes my parameter choices.

2.5 Results

2.5.1 Model #1: small state space

I begin by presenting results for the small model. First, Figure 14 shows the planner's decision rules for $\rho(i)$ (left panel) and $q(i)$ implied by $\rho(i)$ (right panel), across asset level, for the three skill levels. The horizontal dotted line in the right panel is $\bar{q}$ for these agents; this is their job-finding rate if there was no search assistance in the economy (i.e., $\rho(i) = 0 \;\forall\; i$). Note that $\rho^*(i)$ and $q^*(i)$ are clearly decreasing in asset level, but also increasing in skill level. A one-unit increase to a high-skilled agent's job-finding rate results in a greater increase to the social welfare function compared to a low-skilled agent, due to the nature of wages in the economy. Recall Proposition 2.1: the consumption *gain* from transitioning from unemployment to employment is given by $w_2 - \hat{w}$, which is increasing in skill level. Hence for a given asset level, the gains to the social welfare function for an increase to q are increasing in x. Furthermore, these results are also consistent with Proposition 2.3, which states that the utility gain from increasing $q(i)$ is decreasing in asset level. The more assets an agent has, the smaller the percent of his total income comes from wages, so the smaller the gain from becoming re-employed.

2.5.2 Model #2: Rule for optimal job-finding rate

Next, I discuss the results from the second model, in which the planner chooses optimal jobfinding rates (and the implied optimal search assistance intensity as well) according to the Logit rule. Coefficient estimates are presented in Table 8. The first column shows results from a version of the model without the interaction terms. Consistent with the first model, q^* is decreasing in an agent's asset level ($\beta_1 = -0.0103$) and increasing in skill level ($\beta_2 = 0.3729$). Furthermore, it is decreasing in the agent's duration of unemployment ($\beta_3 = -0.1105$). This last result is due to the fact that the agent's inherent job-finding rate, $\bar{q}$, is already a decreasing function of unemployment duration, and that search assistance multiplies, as opposed to adds to, an agent's inherent job-finding rate. Hence, a one-unit increase to $q(i)$ becomes more and more expensive to achieve as duration increases.

Next I add in the interaction terms, as shown in the second column of Table 8. The coefficient estimate for β_1 (assets) is now positive, but due to the addition of the interaction terms, this fact is misleading. Note that the derivative of q with respect to asset level is

$$\begin{aligned}\frac{dq(i)}{da(i)} &\propto \beta_1 + \beta_4 x(i) + \beta_5 s(i) + \beta_7 x(i)s(i) \\ &= 0.0047 - 0.0302x(i) + 0.0004s(i) + 0.0010x(i)s(i).\end{aligned}$$

So while β_1 itself is positive, the above derivative is actually negative for any values of x and s in the model's parameterization, so q^* is in fact strictly decreasing in asset level.

Note the negative coefficient for the interaction term between asset and skill level ($\beta_4 = -0.0302$), which states that the slope of q with respect to a gets more negative as skill level increases. The planner's decision rule for job-finding rate by asset level is shown in Figure 15. The left panel shows $q^*(i)$ by asset level for low, medium, and high-skilled agents with an unemployment duration of five quarters, and the right panel shows the corresponding ρ^*'s. The horizontal dotted lines refer to $\bar{q}(i)$ (left panel) and $\rho(i) = 1$ (right panel). As noted above, $q^*(i)$ is decreasing in asset level, and decreasing more steeply for higher-skilled agents relative to lower-skilled agents. The wealthiest agents, those with a wealth greater than about 35, actually see their job-finding rate decline under the planner's scheme. However, note that the intersection of the decision rules with the horizontal lines is slightly further to the left in the left panel than it is in the right panel. This means that there are some values of $\rho(i) > 1$ for which this agent still sees a decrease to his job-finding probability; in this case, negative general equilibrium effects (i.e., decline to q due to *other* agents' increase to ρ) outweigh positive partial equilibrium effects (i.e., increase to q due to increase to $\rho(i)$).

Notice also that q^* is not strictly increasing in skill level: for the very richest agents, it is decreasing in x, which is contrary to the results of Propositions 2.1 and 2.2. However, those Propositions assume the Planner being able to freely choose any value of $\rho(i)$ (and $q(i)$) he wishes; on the other hand, the Logit rule imposes a restriction on $q(i)$ so the planner will almost certainly not be able to achieve the 'true' optimal values. The derivative of q with respect to skill level is

$$\begin{aligned}\frac{dq(i)}{dx(i)} &\propto \beta_2 + \beta_4 a(i) + \beta_6 s(i) + \beta_7 a(i)s(i)\\ &= 1.1310 - 0.0302a(i) - 0.0486s(i) + 0.0010a(i)s(i).\end{aligned}$$

For $s = 5$ (as shown in the figure), this switches from positive to negative at an asset level of about $a = 35$.

The planner's decision for $q^*(i)$ as a function of unemployment duration is shown in Figure 16. The left panel is for a low-skilled agent and the right panel is for a high-skilled agent. The solid black line is $\bar{q}$, which is decreasing in unemployment duration, and is the same across both panels. The dashed blue line is the planner's choice of $q^*(i)$, with the corresponding $\rho^*(i)$ in the dot-dashed red line. Given that the Logit rule is a somewhat restrictive function, the planner chooses $q^*(i)$ that is (almost) linear in duration, which leads to the hump-shape seen in the corresponding choice of $\rho^*(i)$.

2.5.3 Model #3: Rule for optimal search assistance intensity

Turning to the third model, in which the planner chooses $\rho^*(i)$ based on a polynomial rule, coefficient estimates are shown in Table 9. Figure 17 shows the planner's decision rule for ρ (left panel) and q (right panel), by asset level and skill level. As with the other models, the rule is clearly decreasing in asset level and increasing in skill level. However, the planner only gives positive search assistance to the poorest agents;

those with assets less than about 5 to 10, depending on skill level. Agents wealthier than this have $\rho^*(i) = 1$, and so their optimal job-finding rate is less than their $\bar{q}(i)$. Figure 18 shows these results by duration of unemployment for low-skilled (left panel) and high-skilled (right panel) agents. The solid black line is $\bar{q}$, which is the same for each, and the dot-dashed red line is $\rho^*(i)$, which is clearly increasing in skill level, but decreasing in unemployment duration. The reason this is decreasing in unemployment duration comes from the multiplicative nature of ρ. A high-duration agent's job-finding rate is inherently lower than a short-duration agent's; hence, a one-unit increase to ρ for the high-duration agent leads to a smaller gain to q for this agent. Therefore, the marginal benefit of assisting unemployed workers decreases as spell length rises.

2.5.4 Welfare changes

To analyze the welfare gains (or losses) the planner's decision rules imply, I employ a consumption-based measure of welfare. Let $\bar{c}$ be consumption in the state of the world corresponding to the absence of search assistance; that is, $\rho(i) = 1$ and $q(i) = \bar{q}(i)$ for all types, and let $\tilde{c}$ be consumption in the state of the world corresponding to the planner's optimal choice. Then, the welfare gain or loss for type i is the $\omega(i)$ that solves

$$E_0 \sum_{t=0}^{1} \beta^t u((1+\omega(i))\bar{c}_t) = E_0 \sum_{t=0}^{1} \beta^t u(\tilde{c}_t)$$

Given the specification that $u(c) = ln(c)$, we can solve for $\omega(i)$ directly as

$$\omega(i) = \exp\left(\frac{\tilde{V}(i) - \bar{V}(i)}{1+\beta}\right) - 1, \tag{2.5}$$

where $\tilde{V}(i)$ and $\bar{V}(i)$ are agents' value functions corresponding to the planner's optimal choice of ρ and the baseline economy, respectively.[24]

Table 10 shows the average welfare gains for employed, unemployed, and all agents for the three models. Welfare changes for Model #1 are shown in Column (1). Unemployed agents, on average, gain about 0.75 percent (in consumption terms), whereas

[24]See Appendix B.

employed agents lose, on average, just under 0.02 percent. Recall, employed agents receive none of the benefits of the assistance program, due to the model's two-period horizon; however, they pay the tax burden. Welfare changes by asset and skill level are shown in Figure 19. The left panel shows employed agents. The welfare change is negative for all employed agents but is decreasing in magnitude (i.e., becoming less negative) as asset level increases. As wealth level increases, the percentage of an agent's income that comes from wages (and is therefore subject to the income tax) declines, so the welfare loss from a higher tax rate declines. The opposite pattern is seen in the right panel, which shows unemployed agents. The welfare gain is increasing in skill level but decreasing in asset level.

Columns (2) and (3) of Table 10 show the corresponding statistics for Models #2 and #3, respectively.[25] For Model #2, where the planner chooses $q^*(i)$ according to the Logit rule, unemployed agents have an average welfare gain of about 0.08 percent (in consumption terms). On the other hand, the polynomial rule for $\rho^*(i)$ (Model 3), delivers a welfare gain to unemployed agents of about 0.14 percent. The fourth column shows the welfare gains for an alternative specification of Model #3 (Call this Model #3b) where the planner maximizes the social welfare function by choosing just one value of ρ that is shared by all types. In this case, unemployed agents have an average welfare gain of about 0.07 percent. This last result shows that while Model #3b is much simpler and computationally less burdensome than Model #3, the added flexibility of Model #3 delivers approximately double the welfare gain to unemployed agents.

Figures 20 and 21 show the welfare gains by asset and skill level for Models #2 and #3, respectively. In both cases, the basic pattern is identical to Model #1: the welfare losses (for employed agents) or gains (for unemployed agents) are decreasing in asset level but increasing in skill level. As with Model #1, the poorest unemployed agents have the highest welfare gain, for two reasons. First, the welfare gain *per unit* of search assistance is higher for them, as they are less able to smooth their consumption during unemployment and thus gain more from transitioning to employment, and second, the planner gives them a higher degree of search assistance.

[25] Note that the welfare changes for Model #1 (Col. (1)) are not strictly comparable to the other models, as the models feature different state spaces.

2.5.5 Discussion: role of tax rate and employed agents

Since the model is only two periods in length, agents who are employed in the first period will never be enrolled in the search assistance policy. Therefore, they receive none of the benefits of the policy but still incur its costs through the tax rate. Given that employed agents comprise the vast majority—95%—of the economy, their utility dominates the social welfare function. In this subsection, I re-solve all three versions of the model including only unemployed agents in the social welfare function. That is, the planner's objective function is now

$$\sum_{s \geq 1} f(i)V(i),$$

where $s \geq 1$ denotes only unemployed agents. To be clear, I have not removed employed agents from the economy, just from the planner's objective function. Therefore, they still pay (nearly) all of the tax burden for the search assistance program, but the planner no longer cares about maximizing their utility.

Model #1

Removing employed agents from the planner's objective function greatly increases the amount of search assistance provided to unemployed agents, as would be expected. Figure 22 shows the planner's optimal ρ^* and q^* by asset level and skill level. The top two panels are for this new specification, and the bottom panels recreate Figure 14. Note that while the same relationship between skills and assets is preserved (q and ρ are decreasing in assets but increasing in skills), now that the planner no longer cares about employed agents' utility, he provides a significantly higher assistance intensity to unemployed agents. In fact, for low- and middle-wealth, high-skilled agents, the planner chooses a ρ^* that corresponds to a job-finding rate equal to 1.

Model #2

A similar result is shown when the planner chooses q^* according to the Logit rule. The top two panels of Figure 23 show the results for q^* and ρ^* without employed agents, and the bottom two panels recreate Figure 15. Compared to the original specification, the planner chooses higher job-finding rates for low-asset agents but, on the other hand, he chooses lower job-finding rates for high-asset agents. For example, the planner now chooses a job-finding rate of 1 for a low-asset, high-skill agent in the new specification compared to about 0.58 in the old. However, he chooses a job-finding rate of about 0.18 for a low-skilled, high-asset agent in the new specification, compared to about 0.44 in the old. Essentially, the planner's decision rule has become steeper with respect to assets.

Model #3

Finally, Figure 24 shows results for the polynomial rule for ρ. As before, the top two panels show the results for the unemployed-only objective function, and the bottom two panels recreate Figure 17. As with the other models, now that the planner no longer cares about employed agents' utilities, he chooses much higher values for job-finding and search assistance intensities. Comparing the top panels to the bottom panels, a much smaller percentage of the population has $q^*(i) \leq \bar{q}(i)$ and $\rho^*(i) = 1$.

Unsurprisingly, each of these three alternative specifications amplify the welfare gains and losses compared to their corresponding original specifications, as shown in Columns (5) to (7) of Table 10. For example, unemployed agents in Model #3 now gain approximately 0.54 percent (in consumption terms) when the planner omits employed agents from the objective function, compared to a gain of about 0.14 percent previously. On the other hand, employed agents now have a welfare loss of about 0.25 percent, compared to 0.003 percent previously.

2.6 Conclusion

This Chapter introduces a planner to study the optimal scheme of search assistance in an economy populated by heterogeneous agents. By improving the job-finding chances of one agent, all other jobseekers are made worse off, all else equal. The planner, whose objective function is the sum of agents' lifetime utilities, takes this tradeoff into account when choosing the optimal search assistance intensity scheme for each type of agent. The model has a two-period horizon and is simple enough to allow an analytical characterization of the planner's choice. For example, I show that the planner's choice of assistance intensity is an increasing function of agents' skill level but a decreasing function of agents' asset level.

I solve three different (but closely-related) versions of the model; one in which the planner solves directly for the optimal assistance scheme for all types, and two in which the planner chooses the assistance scheme based on functional rules. The latter two models allow me to characterize the optimal policy scheme for a large state space using only a small number of choice variables. I present results for all three versions showing the planner's decision rules as a function of the state space along with the associated welfare gains and losses. I then solve an alternative specification in which I remove employed agents from the planner's objective function, for all three models. Employed agents pay most of the cost of the program but, given the two-period time horizon, enjoy none of the benefits. Removing them from the planner's objective function induces the planner to provide a much higher level of search assistance to unemployed agents, and amplified welfare changes.

Chapter 3

Search assistance with endogenous effort

3.1 Introduction

The matching process presented in Chapter 1 does not explicitly model agents' job search effort. Instead, we can think of that model as a reduced-form technology that represents the net effect on job-finding rates of agents' search effort along with the assistance policy. However, it is reasonable to assume that the creation of a search assistance program will influence jobseekers' search behavior. Indeed, search effort that responds to prevailing labor market conditions is a main result of the standard textbook treatment of frictional labor markets, and has also been shown empirically. This chapter endogenizes search effort and analyzes search behavior and the welfare gains of an assistance policy, comparing worlds where effort is fixed and where effort is endogenous.

Would search effort rise or fall when a search assistance policy is introduced (or expanded)? At a theoretical level, one could easily argue in either direction. On one hand, enrollment in a search assistance program could induce jobseekers to increase their search effort, as the marginal benefit of a one-unit increase to effort has gone up, due to the higher inherent job-finding probability.[26] On the other hand, as with any

[26] Schwartz (2014), who employs a model similar to mine, states that "knocking on doors has a better chance of success when more of those doors have jobs behind them".

labor market insurance program, we can make an argument of moral hazard: given that search is costly (in terms of disutility), workers may have less incentive to search if someone else—in this case, the government—is searching on their behalf.

Therefore, whether search effort increases or decreases after the introduction of search assistance depends critically on modelling assumptions, as we can write down models that lead to either outcome. The choice of modelling assumptions should therefore be disciplined by empirical research, which has shown search effort to be counter-cyclical; that is, when labor markets are 'bad' (by a well-defined metric such as the unemployment rate or labor market tightness), search effort is higher than when labor markets are 'good'. For example, Shimer (2004) measures search effort using the CPS and finds no evidence that effort declined during the 2001 recession, and some evidence it increased. Building on Shimer (2004), a recent paper by Mukoyama et al. (2018) links search data in the CPS to the American Time Use Survey and finds similiar evidence: during economic downturns, non-employed workers are both more likely to search, and they search more conditional on searching at all.

While an imperfect analogy, the empirical findings of counter-cyclical search effort can easily be applied to search assistance. The assistance policy represents an improvment to the labor market—just like any other measure—in the sense that a higher degree of assistance leads to a higher job-finding probability for a given level of effort. In other words, from a jobseeker's perspective, enrollment in a search assistance program is essentially equivalent to living in a world with higher labor market tightness (the ratio of vacancies to unemployment) and therefore this jobseeker's choice of search effort would respond in a similar way.

In this Chapter, I define an infinite-horizon, heterogeneous agents model in which unemployed agents choose their job search intensity, in addition to consumption and savings. Search effort provides increasing and convex disutility, but increases the likelihood of finding a new job. After solving for a baseline, in which there is no search assistance, I introduce the assistance policy by increasing the inherent likelihood of a jobseeker finding reemployment. I calculate the welfare gains of the assistance program, and compare them to the situation where search effort is fixed at the baseline value. I then repeat this experiment for two different matching functions. The first is a CES matching function which induces agents to search with less intensity when the policy is introduced, and the second is a Cobb-Douglas matching function which induces agents to search with more intensity.

I find that with the CES matching function, the welfare gains from the assistance program are much lower when search effort is fixed at the baseline value compared to the case of allowing it to vary. Since search effort falls as assistance intensity increases, agents suffer less disutility from search effort, which outweighs the (relative) welfare loss coming from lower job-finding probabilities. On the other hand, with the Cobb-Douglas matching techology, welfare gains are nearly identical for the fixed- and variable-effort cases: the additional disutility from higher search effort approximately balances out the welfare gain from a higher job-finding rate.

The rest of this Chapter is organized as follows. Section 3.2 discusses the relationship between search effort and the assistance policy and outlines the model. Section 3.3 presents some empirics of job search behavior, Section 3.4 discusses calibration, Section 3.5 presents results of the model, and Section 3.6 concludes.

3.2 A model of search effort and assistance

3.2.1 Search effort and labor market conditions

As mentioned in the introduction, whether an individual agent's search effort increases or decreases with labor market conditions is dependent on modelling assumptions; namely, the relationship between search effort and labor market conditions in the matching functions. This subsection explores this relationship further, before formally presenting the model below in Section 3.2.2.

To fix ideas, consider the simple model described below. Unemployed households choose consumption c, savings s, and search effort e to maximize their lifetime utility (subject to a standard budget constraint):

$$V^U = \max_{c,s,e} \; u(c) - h(e) + \beta(q(e,\chi)V^E + (1 - q(e,\chi))V^U),$$

where V^U and V^E are the Bellman equations associated with unemployment and employment, respectively. Furthermore, u is an increasing and concave function, h is an increasing and convex function, and $q(e, \chi)$ is the probability this agent finds a job, which is increasing in both arguments: search effort e and labor market conditions χ.

In this stylized model, we can think of χ as any factor that governs an agent's job-finding rate for a given level of effort, with a higher χ corresponding to a better labor market. Examples include θ, the commonly-used labor market tightness (i.e., the ratio of posted vacancies to the stock of unemployment), or a search assistance program designed to increase an agent's inherent job-finding chances, or some combination of both

The agent's first-order condition with respect to search effort state

$$h'(e) = q'_e(e, \chi)\beta \left(V^E - V^U\right). \tag{3.1}$$

Hence, the agent chooses search effort such that the marginal cost of one more unit of search, $h'(e)$, is equal to the marginal benefit of search, which is the marginal job-finding rate $q'_e(e, \chi)$ multiplied by the (discounted) value of becoming re-employed, $V^E - V^U$.

Consider the case where search effort e and market conditions χ enter q multiplicatively. For example, let $\chi = \theta$ (labor market tightness). Then, a standard Cobb-Douglas matching function would imply

$$q(e, \theta) = e\theta^{1-\sigma}.$$

In this case, Eq. (3.1) becomes

$$h'(e) = \theta^{1-\sigma}\beta \left(V^E - V^U\right).$$

Given that h is a convex function, an increase to θ will induce a higher level of search effort from this agent. The intuition is straightforward: since search effort multiplies the agent's job-finding probability, a higher value of θ means that a one-unit increase in search effort will lead to an even greater increase in q (for a given match surplus $(V^E - V^U)$. This is shown in Figure 29. The solid black line is $h'(e)$, which is increasing in e. The green, dashed line is $\theta^{1-\sigma}(V^E - V^U)$ for a lower value of θ and the red dot-dashed line is the same for a higher value of θ. Clearly, a higher θ corresponds to a higher e.[27] Therefore, this Cobb-Douglas matching function corresponds to procyclical search effort, which is at odds with recent empirical findings, as discussed in the Introduction.

Next, suppose that χ refers to the combination of government-provided search assistance and labor market tightness. Broadly speaking, this program multiplies the agent's inherent job-finding probability by some factor $\rho > 1$, in a manner identical to previous Chapters. Now, the agent's job-finding rate is

$$q(e, \chi) = e\rho\theta^{1-\sigma}.$$

In this case, the logic is indentical to above, in that a higher degree of search assistance will, all else equal, compel the agent to search *more*, not less.

Instead, consider the following CES matching function:

$$q(e, \chi) = \tilde{q}(\alpha e^{\eta} + (1-\alpha)\chi^{\eta})^{1/\eta},$$

where $\tilde{q}$ is aggregate matching efficiency, α is a share parameter, and as before χ includes both ρ and θ. Now, the agent's first-order conditions with respect to search effort are

$$h'(e) = \tilde{q}\alpha e^{\eta-1}(\alpha e^{\eta} + (1-\alpha)\chi^{\eta})^{(1/\eta)-1}\beta(V^E - V^U). \tag{3.2}$$

Now, if $\eta > 1$, then a higher degree of search assistance (defined within χ) will induce a lower degree of search effort on the part of the agent, and if $\eta < 1$, higher search assistance will lead to higher search effort, as shown in Figure 30. To see this, take the derivative of Equation (3.2) with respect to χ:

$$\frac{\partial^2 q(e, \chi)}{\partial e \partial \chi} = \eta(1/\eta - 1)\Psi, \tag{3.3}$$

[27] Note that the preceding analysis ignores general equilibrium effects. Depending on the precise specification of the matching function, a higher search effort may translate to a lower θ in equilibrium, if e factors into u, which is in the denominator of θ. These general equilibrium effects will mitigate some of the increase to e. Furthermore, the match surplus can change if, for example, wages are set via bargaining. However, an individual agent does not take into account these general equilibrium effects, so it is appropriate to omit them here.

where Ψ is a collection of terms that is strictly positive.[28] Hence, if $\eta > 1$, then Equation (3.3) shifts down when χ increases, implying a lower equilibrium search effort, as shown in the top panel of Figure 30. On the other hand, if $\eta < 1$, the opposite occurs, as shown in the bottom panel.[29]

3.2.2 The model

In light of the discussion above, this subsection presents a formal discussion of the model I use in this chapter. The model is similar to those presented in Chapters 1 and 2, with some key simplifications for tractability.

Households

Households are heterogeneous with respect to their asset level a, their human capital (or 'skill') level x, and their labor market state s, where s is either the employed state or the unemployed state. Households are infinitely lived and discount the future at rate $\beta < 1$. They choose today's consumption c, tomorrow's asset level a', and search effort e to maximize the discounted sum of lifetime utility. As shorthand, let i refer to an agent's *type*; i.e., the agent's particular state vector. Let $f(i)$ be the measure of type i, where f is an endogenous quantity. The total population in the economy is normalized to one, so $\sum_i f(i) = 1$.

Currently-employed households supply x units of labor to their employer, who produces output according to the production function $y = x^{\alpha}$. Agents earn a wage, denoted w, equal to $\alpha x^{\alpha-1}$ per unit of x, or αx^{α} total. Unemployed households earn $\bar{w}$ units of Unemployment Insurance, where $\bar{w} < w^{min}$; that is, the UI payment is strictly less than the lowest wage. Households earn interest on their savings at rate r, where, as in Chapter 2, r is assumed to be exogenous. The probability a currently-unemployed household finds employment next period is given by $q(e, \theta)$, where θ is labor market tightness, which is defined in more detail below. On the other hand, a currently-employed household remains employed with (exogenous) probability ϕ.

Turning to a recursive formulation, the optimization problem for a currently-

[28] As long as $V^E > V^U$, of course.

[29] Note that as $\eta \to 0$, the matching function converges to the Cobb-Douglas case, $q = \tilde{q} e^{\alpha} \chi^{1-\alpha}$.

unemployed household is given by

$$V^U = \max_{c,a',e} \; u(c) - h(e) + \beta \left(q(e,\theta)V^E + (1 - q(e,\theta)V^U \right),$$

subject to

$$c + a' = \bar{w} + (1 + r)a,$$
$$e \leq 1,$$

where u is a strictly increasing and concave utility function, and h is the strictly increasing and convex disutility of search effort. The optimization problem for a currently-employed household is similar except they do not choose search effort, $\bar{w}$ is replaced by w, and q is replaced by ϕ. Note that this Chapter assumes that the search assistance program is costless in order to abstract away from welfare effects coming from the tax rate.

As before, the agent's first-order conditions with respect to search effort e state that

$$h'(e) = \beta q'_e \left(V^E - V^U \right).$$

Furthermore, I assume utility of consumption is given by $u(c) = ln(c)$, and the disutility of search effort is quadratic in e: $h(e) = \psi e^2$.

Job matching

Let u denote the stock of unemployment, which is weighted by jobseekers' search effort. That is,

$$u = \sum_i \rho e(i) f(i).$$

In this Chapter, ρ is assumed to be the same for all types, so we can pull it outside of the above summation. Let v denote the number of posted vacancies, which is assumed to be fixed, and let $\theta = v/u$ be labor market tightness. Next, let $\rho \geq 1$ refer to the intensity of search assistance. This term, which is the same across types of agents, can be thought of simply as an aggregate matching efficiency parameter, as it governs agents' job-finding chances for a given amount of search intensity and labor market tightness. I solve two different versions of the model; one with a Cobb-Douglas matching function similar to Chapter 1, and one with the CES matching function:

$$q(e,\theta) = \tilde{q}e\rho\theta^{1-\sigma}, \quad \text{and,}$$
$$q(e,\theta) = \tilde{q}\left(e^{\eta} + (1-\sigma)(\rho^{1/(1-\sigma)}\theta)^{\eta}\right)^{1/\eta}.$$

where $\eta > 1$. Both of these are clearly increasing in search effort e, search assistance intensity ρ, and labor market tightness θ, and they exhibit the properties discussed in the previous section regarding the relationship between search effort and labor market conditions.[30] Note that the precise functional form of the CES matching function is chosen so that the two functions are equivalent in the limit (with respect to η); that is,

$$\lim_{\eta \to 0} \tilde{q}\left(e^{\eta} + (1-\sigma)(\rho^{1/(1-\sigma)}\theta)^{\eta}\right)^{1/\eta}$$
$$= \tilde{q}e\rho\theta^{1-\sigma}$$

Equilibrium

Definition: Competitive Equilibrium. A competitive equilibrium in this economy consists of

1. Value functions V^E and V^U,
2. Decision rules for consumption, effort, and savings,
3. Various endogenous quantities: θ, $q(e,\theta)$, u,
4. The economy's density function f.

Such that

1. Given ρ and θ, the decision rules for consumption, effort, and savings solve households' optimization problem, with associated value functions V^E and V^U.
2. The distribution f is invariant.

[30] The CES matching function is similar to the matching function in Mukoyama et al. (2018). They estimate their matching function based on their empirical findings that search effort is countercyclical with respect to labor market tightness.

Characterization of equilibrium
An unemployed agent's optimal choice of search effort is increasing in skill level x, which is formalized below.

Proposition 3.1. Holding other state variables constant, an agent's optimal choice of search effort, denoted e^*, is increasing in his skill level x.
Proof: Recall agents' first-order conditions with respect to e:

$$h'(e) = \beta q'_e \left(V^E - V^U\right).$$

Note that the match surplus, $V^E - V^U$, is increasing in skill level since wage is increasing in skill level (and all other state variables are held constant). Then, given that both the left- and right-hand side of the above equation are increasing in e, a higher value of $V^E - V^U$ identifies a higher value of e^* (See Figure 30 for example). ■

3.3 Some empirics of job search

This section builds on the existing empirical literature by documenting some empirical facts regarding job-search behavior. To be considered officially unemployed in the Current Population Survey (CPS), a person must report that they have performed some action towards finding a job within the four weeks prior to the survey date. The CPS gives respondents 12 possible choices of actions, and they can choose up to six of them to report.[31] I use CPS microdata from January 2001 to December 2007. This represents a sample of over 700,000 household-month observations (only counting unemployed observations).[32]

3.3.1 Number of search methods

To begin with, Figure 26 shows the density function of the number of chosen methods for the entire sample, and Figure 27 breaks this down by education group. Over 60

[31] See Table 11 for the definitions of these methods.
[32] Section D.2 provides additional details regarding the data.

percent of jobseekers report using only one or two methods, with fewer than 10 percent reporting using at least five. The mean for the entire sample is 2.28 chosen methods, and this is strictly increasing in education level. A jobseeker with only a high school diploma employs on average 2.12 methods, whereas someone with a college degree employs on average 2.61 methods.[33]

Recall the model's result that optimal search effort is an increasing function of a jobseeker's human capital level; this result is broadly consistent with the above empirical facts, with two caveats. First, education level is a highly imperfect measure of human capital. Furthermore, search effort (or search intensity) is not the same thing as the number of chosen methods: one job seeker could employ six different methods but only spend a total of five minutes per day searching, while another jobseeker could devote three hours per day to just one method. However, recent literature dispels some of the worry regarding this second caveat. Mukoyama et al. (2018) links CPS data to the American Time Use Survey, and finds a strong positive relationship between number of chosen methods (from the CPS) and time spent searching (from the ATUS).[34]

3.3.2 Choice of methods

Next, I analyze jobseekers' choice of different search methods. Figure 28 shows the percent of respondents who include the given method as one of their six reported methods, both for the entire sample (the black bars) and by education groups (the green, red, blue, and gold bars). Methods are sorted in decending order by the overall percent who choose that method. As seen in the Figure, the two most commonly-reported methods are sending out resumes and directly contacting employers, with close to 60 percent of jobseekers reporting using these methods. On the other hand, utilizing employment agencies and attending job-training programs are among the least utilized methods, though public employment agencies are more utilized than private ones.

These results are somewhat different than some international data. For example,

[33] The difference in mean across educational groups is statistically significant at the 1% level, except for the difference between *college* and *beyond college*, which has a p-value of 0.53.

[34] See Figure 1 in their paper.

Weber and Mahringer (2006) conduct a similar analysis using Austrian data from 1997. In their study, the most commonly-used methods were responding to advertisements and asking friends and relatives, with about 60 percent of jobseekers utilizing these methods. In my CPS sample, these are utilized by about 30 and 22 percent, respectively. Furthermore, they find that contacting employers directly is utilized by 35 percent of jobseekers, compared to about 57 percent in my sample. However, there is one major caveat when comparing the results from Weber and Mahringer (2006): their sample consists of 500 *successful* job seekers who found a job in 1997, as opposed to those who are still currently searching. Hence, varying effectiveness of the different methods in terms of finding re-employment will surely account for some of these differences.

To understand some of the factors that influence a jobseeker's choice to employ a particular method, I estimate probit regressions for each of the 12 search methods where the outcome variable is the jobseeker utilizing that method over the last four weeks. For controls, I use education dummies (base category: high school diploma), age, age squared, duration of unemployment in quarters, total number of methods used, dummies for the worker's industry, sex, the current national unemployment rate.

Tables 12 and 13 show the results of these regressions. For each of the 12 methods, the first column omits number of chosen methods as a control variable and the second column includes it. Some interesting patterns emerge. First, without controlling for number of methods, the probability a jobseeker directly contacts an employer is increasing in education level. However, after controlling for number of methods, this pattern reverses. Hence, a college graduate is more likely than a high school graduate to directly contact an employer because a college graduate is more likely to use *any* method (recall Figure 27: the number of chosen methods is increasing in education level); however, given that a jobseeker is utilizing four methods (for example), it is less likely the college graduate includes direct contact as one of these four relative to a high school graduate.

Next, the likelihood of directly contacting an employer decreases with unemployment duration ($\beta = -0.0194$). This suggests perhaps jobseekers exhaust this channel as an unemployment spell drags on, instead trying different methods. The likelihood is also decreasing in the unemployment rate, suggesting that jobseekers view this channel as less useful when there is more slack in the labor market. On the other hand, the likelihood of contacting friends or relatives is an *increasing* function of unemployment duration and the unemployment rate. Taken together, these results suggest how jobsearch behavior changes over the course of a jobless spell and over the business cycle.

Next, jobseekers with a college (or beyond) education are less likely to contact a public employment agency (method 2), but more likely to utilize a private employment agency (method 3), even when controlling for number of chosen methods. While futher research would be needed, these results may speak to the design of employment agencies and what sort of people they target. That is, the results suggest that public employment agencies may be more appropriate for low-skilled jobseekers but private agencies more appropriate for high-skilled jobseekers. Furthermore, the likelihood of attending a job training program (method 11) is increasing in education level and unemployment duration.

3.4 Calibration

The state space is discretized as follows. There are 30 nodes in the asset dimension, equally spaced between 1 and 50, and 5 nodes in the skill dimension, equally spaced between 0.5 and 0.9, and the two nodes for labor market state, employed and unemployed. This represents a total of 500 nodes in the state space. One period in the model corresponds to one quarter. Households discount the future at rate $\beta = 0.99$, and earn interest on their savings at rate $r = 0.00985$; this interest rate corresponds to an annual rate of 4 percent. Labor's share of income, α, is set to 0.67, and the probability an employed worker keeps his job next period is 0.96. This parameter comes from Fallick and Fleischman (2004); see Section 1.3 for details. Next, $\sigma = 0.72$, also consistent with Chapter 1.

I calibrate $\tilde{q}$, the aggregate matching efficiency parameter, so that the model delivers an equilibrium $\bar{q}$ (the mean job-finding rate) equal to 53.21 percent in the $\rho = 1$ baseline. This value corresponds to the $\bar{q}$ in Chapter 1, which was calibrated to match US labor market moments over the period 1995 to 2007. This yields $\tilde{q} =$

1.0871 for the CES matching function and $\tilde{q} = 1.7742$ for the Cobb-Douglas matching function. Next, the disutility of search effort function is $h(e) = \psi e^2$. I calibrate ψ so that average search effort is equal to 0.4 for each matching function specification. This yields $\psi = 0.0560$ for the CES match function and $\psi = 0.0807$ for the Cobb-Douglas function. Finally, I choose $\eta = 1.1$, which represents the elasticity of substitution between search effort and the assistance policy in the CES matching specification.

Table 14 summarizes parameter choices for this Chapter.

3.5 Results

This section discusses results of the experiments I perform, which proceed as follows. I begin by solving the 'baseline' model, in which $\rho = 1$ for all types of agents, corresponding to the state of the world with no search assistance. I then re-solve the model while varying ρ from 1 to 1.25, first allowing agents to choose search effort, and then a second time fixing search effort at its baseline value, and compare the implications for welfare. I then repeat the above for both the CES matching function and the Cobb-Douglas matching function.

As in Chapters 1 and 2, I employ a consumption-based measure of welfare change. The welfare change (gain or loss) for an agent of type i is the value of $\omega(i)$ that solves

$$\sum_{t=0}^{\infty} \beta^t \left[u(\bar{c}(1+\omega(i))) - h(\bar{e})\right] = \sum_{t=0}^{\infty} \beta_t \left[u(\tilde{c}) - h(\tilde{e})\right],$$

where $\bar{c}$ is this agent's consumption in the state of the world where $\rho = 1$, and $\tilde{c}$ is consumption in the state of the world where ρ is some value greater than 1. Therefore, a value of $\omega(i) = 0.03$ (for example) means that increasing ρ from 1 to its new value is equivalent to keeping ρ at 1 but giving this agent 3 percent more consumption. Given that the utility function is assumed to be $u(c) = ln(c)$, we can solve directly for $\omega(i)$:

$$\omega(i) = \exp((\tilde{V} - \bar{V})(1-\beta)) - 1,$$

where $\tilde{V}$ is the agent's value function for the state of the world where $\rho > 1$ and $\bar{V}$ is the agent's value function in the baseline. The average welfare change for a particular group of agents (for example, unemployed agents) is therefore defined as the weighted sum of $\omega(i)$ for this group divided by the population of the given group.

3.5.1 CES matching function

Figure 31 shows how average search effort (top panel) and the average job-finding rate (bottom panel) change as ρ increases from 1 to 1.25, using the CES matching function. As seen in the Figure, average search effort steadily declines as ρ increases, when allowing for variable effort (the solid black line). Given the assumed functional form of the matching technology (and $\eta > 1$) and the convexity of the search disutility function, better labor market conditions (i.e., higher search assistance) induce agents to search for work with less intensity. In this sense, the assistance policy and search effort essentially act as substitutes: the more help the government provides, the less agents choose to search on their own.

Note that average search effort actually increases slightly with ρ even when effort is fixed (the dashed green line). Even though effort for particular agents is fixed at the baseline value, the average level of effort increases due to composition effects. As search assistance intensity increases, the mean skill level of an unemployed agent increases. Since search effort is increasing in skill level, this raises the average search effort. The reason for this composition shift is as follows. Given that $\eta > 1$, the marginal job-finding rate, with respect to a one-unit increase to ρ, is decreasing in search effort; i.e., $\partial^2 q(e,\rho)/\partial\rho\partial e < 0$. Therefore, given that low-skilled agents naturally search with less intensity than high-skilled agents, their job-finding rates increase more than high-skilled agents. Hence, low-skilled agents make up a smaller and smaller share of the jobseeker pool as ρ increases.

The bottom panel shows the corresponding average job-finding rates, which steadily increase under both scenarios. At first, as ρ increases, the fixed-effort job-finding rate increases faster than the variable-effort job-finding rate, as the reduced search intensity clearly dampens some of the increase coming from the assistance policy. However, for higher values of ρ (after about 1.22), composition effects take over and declining average search effort results in a job-finding rate that is increasing faster than the fixed-effort rate. To see this, note that, for a given level of one particular agent's search effort, $q(e, \theta)$ is decreasing in average effort, since this factors into u, which is in the denominator of θ.

Turning to welfare, Figure 32 shows the average welfare changes (relative to the baseline economy with $\rho = 1$ for all agents) for both unemployed (top panel) and employed (bottom panel) agents in the economy, again as a function of search assistance intensity. As before, the solid black line corresponds to variable search effort, and the green dashed line corresponds to fixed effort. First, note that employed agents have a welfare gain from increasing ρ, albeit a smaller amount than unemployed agents. For example, with $\rho = 1.25$, unemployed agents see, on average, a welfare gain of a little over 0.06 percent (in consumption terms) and employed agents gain of about 0.05 percent. The fact that employed agents gain under this policy is due to the infinite horizon of the model: even though these agents are currently employed, they will, with a nonzero probability, lose their job in the future, at which point they will receive the benefit of the policy.

Secondly, note that the welfare gain is significantly lower when search effort is fixed at the baseline value, even though the job-finding rate is higher. Welfare gains for both types of agents are about 50 percent higher when allowing for variable search effort versus fixed search effort. Welfare gains come from two sources. First, clearly the higher ρ leads to, all else equal, higher job-finding rates, and second, when search effort is allowed to fall, agents suffer less disutility from the act of searching itself. Given that the welfare gain in the variable-effort case is higher than the fixed-effort case, it must be the case that the welfare gain from lower search effort outweighs the (relative) welfare loss of a lower job-finding rate. So, even though agents have a lower chance of finding a job when their effort is variable, they still gain in welfare terms because they are spending less time searching.

3.5.2 Cobb-Douglas matching function

Next, I repeat the above analysis using the Cobb-Douglas matching function specification. Figure 33 shows average search effort (top panel) and the average job-finding rate (bottom panel), again as a function of ρ. Now, the results are essentially opposite to the CES case, as expected. Average search effort steadily increases as ρ increases when allowing for variable effort (the solid black line). Furthermore, the average effort level when effort is fixed is declining in ρ, for the same (but opposite) reason as with the CES matching function: now, the mean skill level of the unemployment pool is falling, which reduces the average search effort as low-skilled agents do not search as much as high-skilled agents. Finally, the average job-finding rate increases with ρ; but moreso when search effort is variable.

Figure 34 shows the corresponding welfare gains. As before, employed agents gain some welfare but less than unemployed agents. For example, when $\rho = 1.25$, unemployed agents have an average welfare gain of about 0.045 percent (in consumption terms) but employed agents gain about 0.04 percent. Note that the welfare gains in the variable- and fixed-effort cases are nearly identical, although the fixed effort welfare gains (the dashed green lines) are very slightly higher. In the variable effort case, the welfare gain (relative to the fixed effort case) coming from higher job-finding rates is almost exactly balanced out by a welfare loss (relative to the fixed effort case) coming directly from a higher disutility of search effort.

3.6 Conclusion

This Chapter extends Chapter 1 by endogenizing unemployed workers' search effort, which responds to changing labor market conditions, here given by the search assistance program. I begin with a theoretical discussion of the modelling assumptions that would imply either increasing or decreasing search effort with respect to the creation (or expansion) of search assistance. I then define an infinite-horizon, heterogeneous agents model using two different matching functions; one which induces agents to search less when the assistance program expands and one which induces agents to search more, although, as I discuss in the Introduction, the former matching function is more in line with recent empirical findings than the latter. I then use CPS data to present some empirical facts regarding search activities. I document the frequency at which agents choose the various methods, and I perform regressions to see what factors determine agents' likelihood of choosing any particular method.

I perform the following policy experiment. After solving for a baseline steady state for each matching function, in which there is no assistance policy, I then create and expand the policy by increasing the assistance intensity parameter, similar to the other Chapters. I compare results for the case when individual agents' search effort is chosen endogenously to the case when it is fixed at the baseline value. I show that for the CES matching function (which induces agents to search *less* when the assistance policy increases), fixing search effort leads to a significantly lower welfare gain than in the variable effort case. When search effort is allowed to fall, agents suffer less disutility of search, which more than outweighs a lower job-finding rate. On the other hand, in the Cobb-Douglas case (which induces agents to search *more* when the assistance intensity increases), the fixed- and variable-effort cases deliver nearly identical welfare gains. Now, fixed search effort leads to lower job-finding rates (which is welfare-reducing) but less disutility of search effort (which is welfare-improving); these two effects nearly cancel out.

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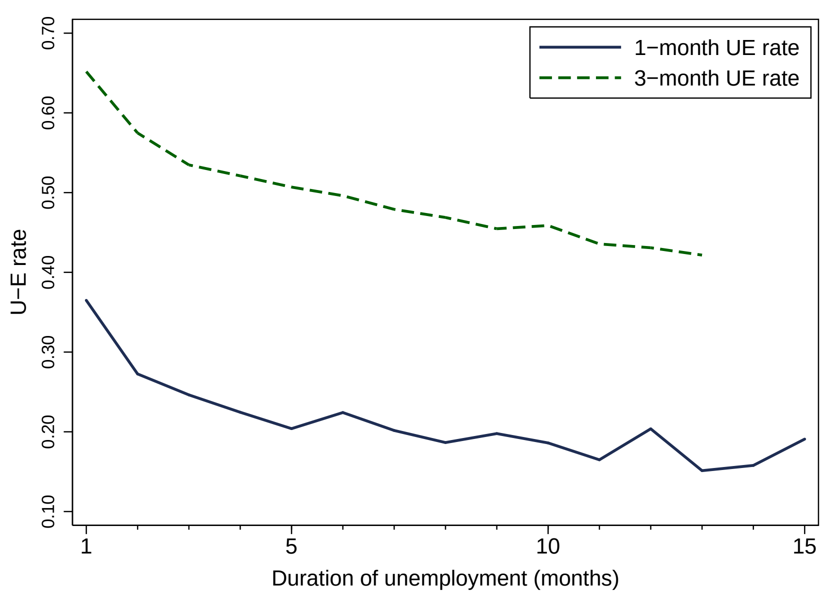
Appendices

Appendix A

Figures and Tables

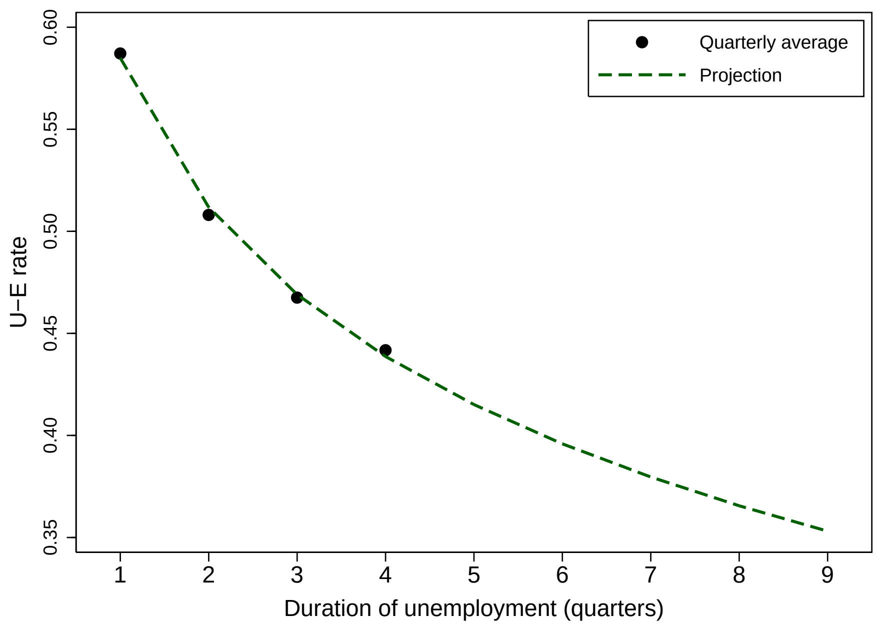
A.1 Chapter 1

Figure 1: Job-finding rate by duration of unemployment, monthly



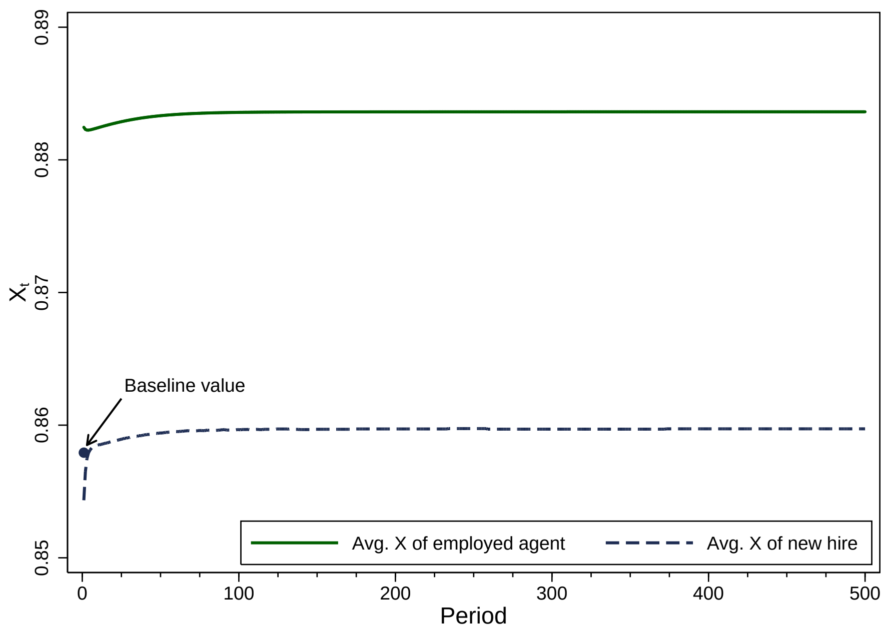
Note: Figure shows the 1-month UE hazard rate ($q_{m,t}$, solid blue line), and 3-month UE hazard rate ($q_{q,t}$, dashed green line). Source: Author's calculations from CPS data.

Figure 2: Job-finding rate by duration of unemployment, quarterly



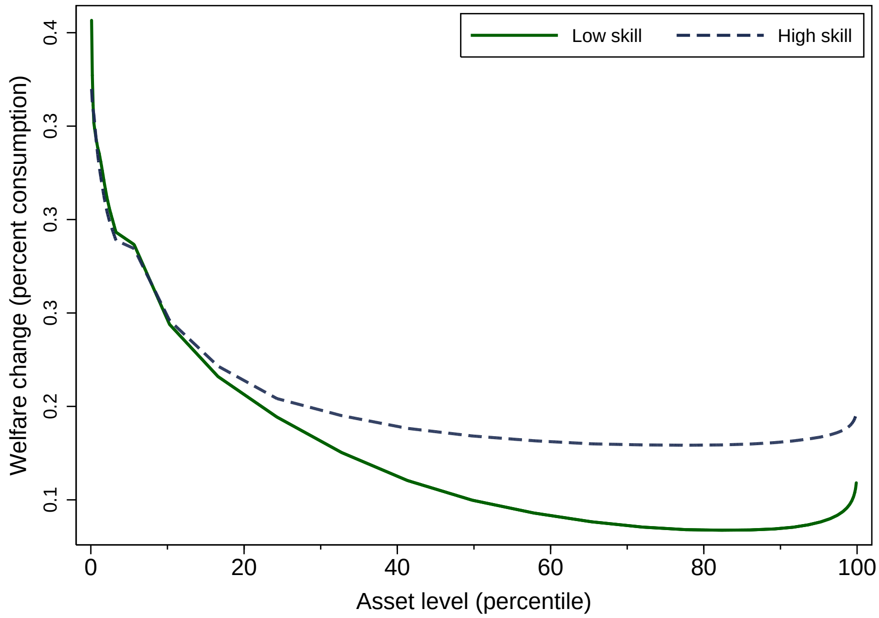
Note: The black circles are the quarterly averages of $q_{q,t}$, denoted $\bar{q}_t$. The dashed green line is a projection from the regression of $\bar{q}_t$ on log(duration), extended to 9 quarters. Source: Author's calculations from CPS data.

Figure 3: Transition path of human capital measures



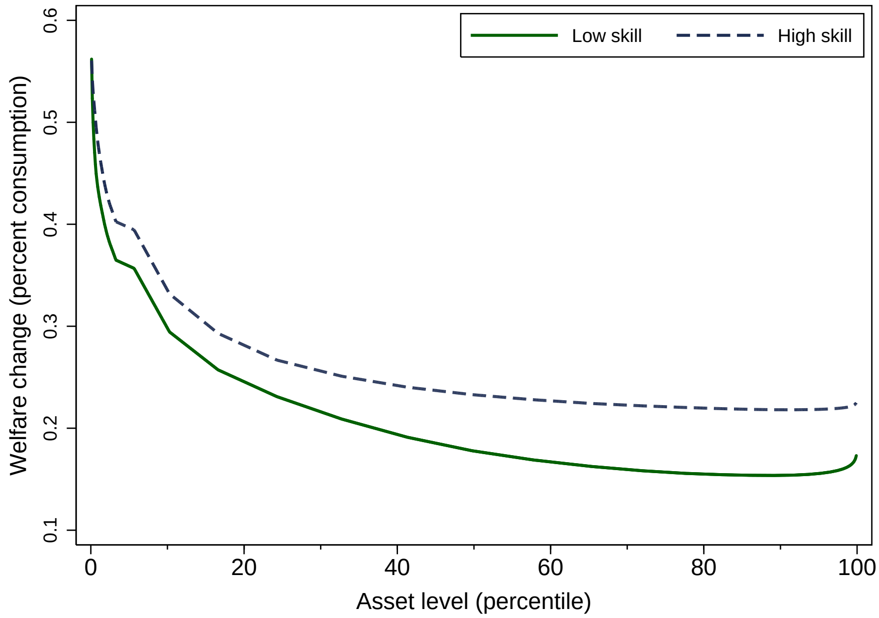
Note: Transition path of the mean human capital level of all employed agents (solid green line) and the mean human capital level of newly-hired agents (i.e., agents in their first period of employment, dashed blue line).

Figure 4: Welfare change by asset and skill level
Full general equilibrium



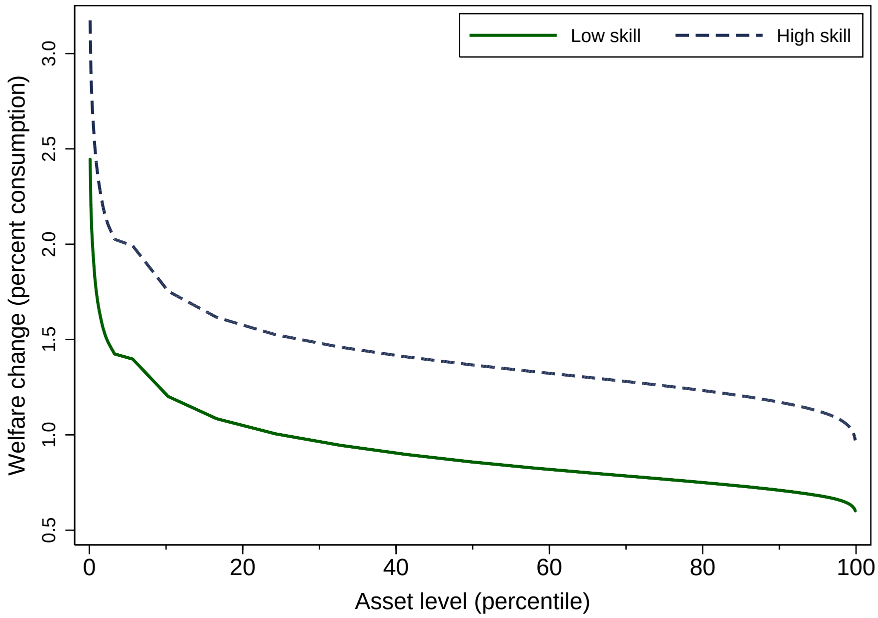
Note: Figure shows the welfare change by asset level (percentile) for low- and high-skilled employed agents.

Figure 5: Welfare change by asset and skill level
Full general equilibrium



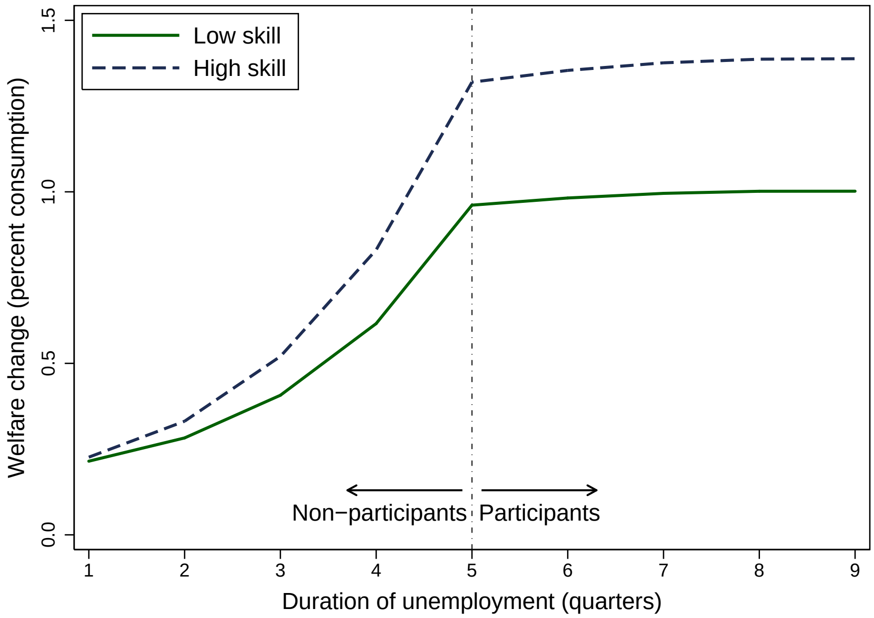
Note: Figure shows the welfare change by asset level (percentile) for low- and high-skilled non-participating unemployed agents.

Figure 6: Welfare change by asset and skill level
Full general equilibrium



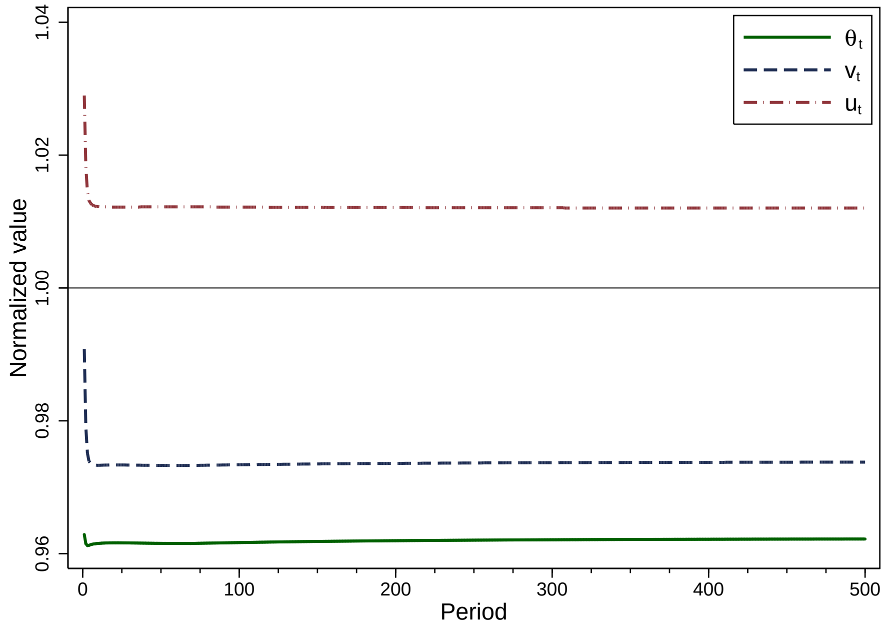
Note: Figure shows the welfare change by asset level (percentile) for low- and high-skilled participants.

Figure 7: Welfare change for unemployed agents
Full general equilibrium



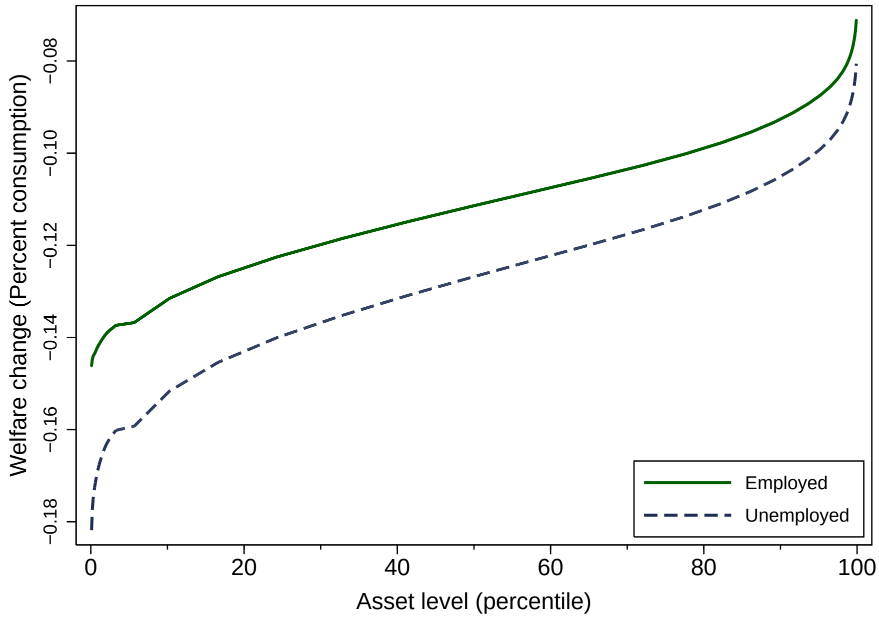
Note: Figures shows the welfare change by duration of unemployment for low- and high-skilled jobseekers. These agents have the median level of assets.

Figure 8: Transition paths for selected labor market variables



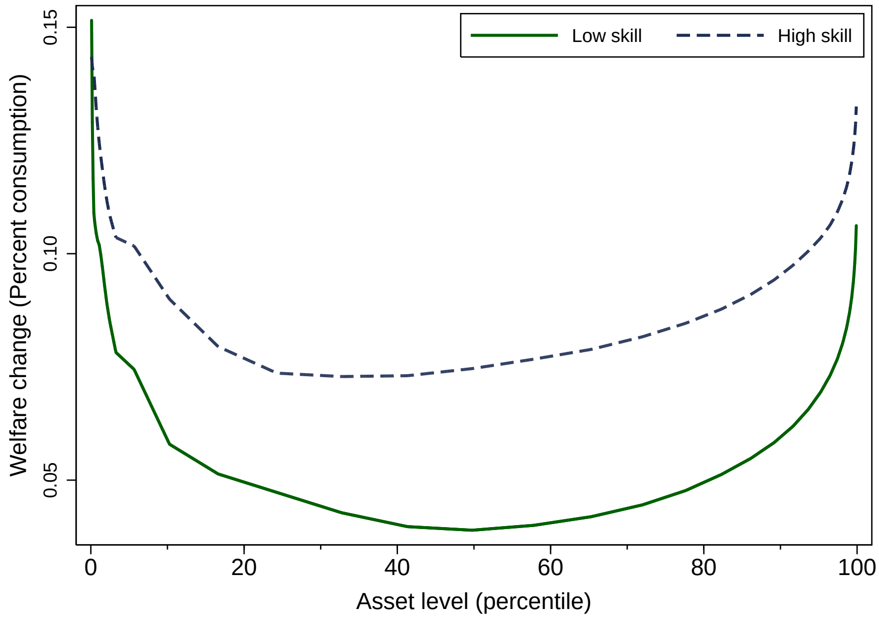
Note: Transition paths for labor market tightness (θ_t; solid green line), vacancies (v_t; dashed blue line), and effective unemployment mass (u_t; dot/dashed maroon line). Each series is normalized so that its value at $t = 0$ (pre-policy) is equal to one.

Figure 9: Welfare change by asset level and employment state
Just θ effect



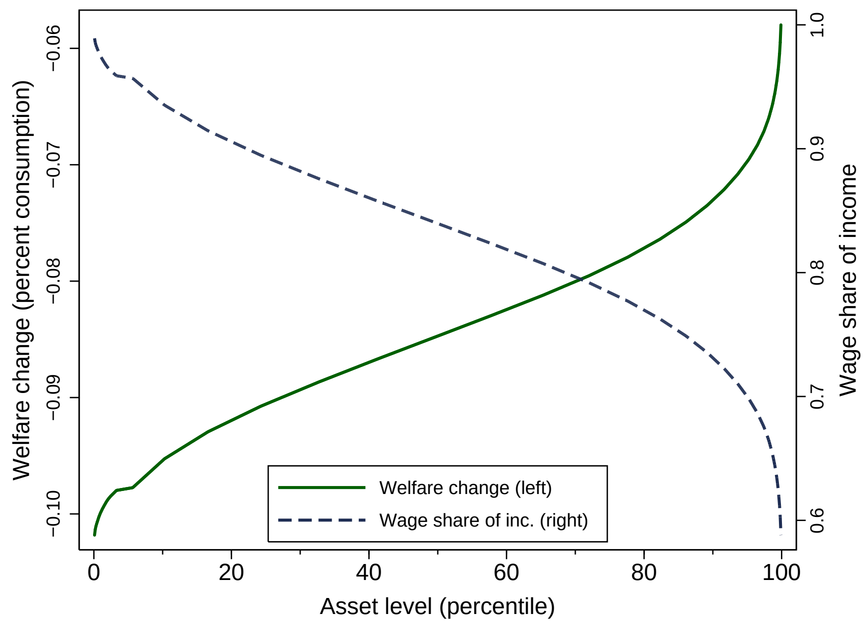
Note: Figure shows the welfare change by asset level (percentile) for low-skilled employed and short-duration unemployed agents; just θ effect.

Figure 10: Welfare change by asset and skill level
Just price effect



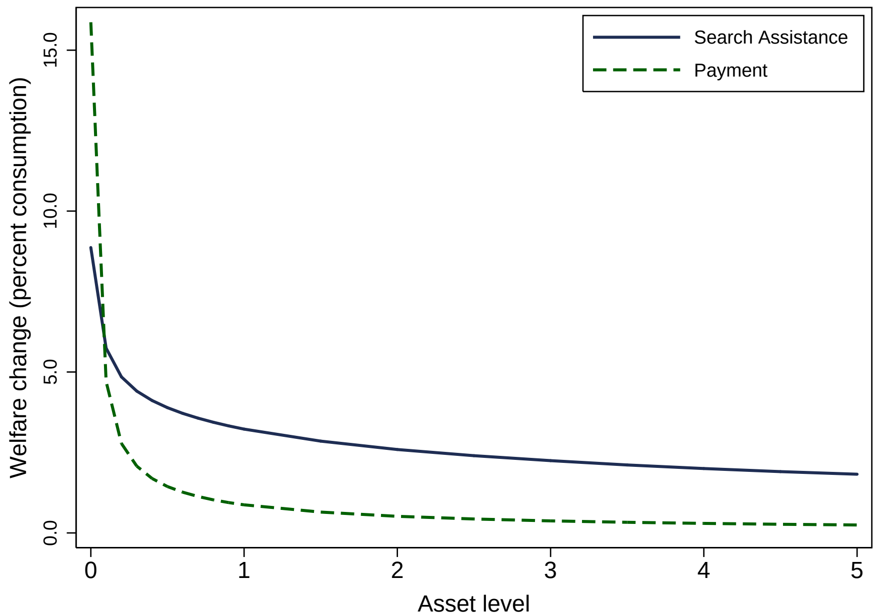
Note: Figure shows the welfare change by asset level (percentile) for low-and high-skilled employed agents; just price effects.

Figure 11: Welfare change by asset level
Just tax effects



Note: Figure shows welfare change by asset level for high-skill employed agents; just tax rate effect (solid green line, left axis), and the wage share of income (dashed blue line, right axis). Wage share of income calculated as $w_t/(w_t + (r_t - \delta_k)a_t)$.

Figure 12: Welfare change by asset level



Note: This figure compares the welfare change among low-skill, high-duration unemployed agents for the search assistance (solid blue line) and UI extension (dashed green line) policies.

Table 1: Baseline model parameters

Variable	Meaning	Value
$\underline{a}$	Borrowing limit	0.0
α	Capital share	0.33
β	Discount rate	0.99
γ	Vacancy cost	1.005
δ_u	Skill loss prob.	0.284
δ_e	Skill increase prob.	0.086
δ_k	Capital depreciation rate	0.012
σ	Match elasticity	0.72
η	Worker's bargaining share	0.67
ϕ	Job-keep prob.	0.96

Table 2: Search assistance policy: selected variables

Variable	Baseline	Policy
θ	2.400	2.309
r	0.0219	0.0219
τ	0.023	0.024
u	0.0291	0.0295
v	0.0699	0.0680

Table 3: Decomposition of change in θ

Scenario	θ	Pct. change
1. Baseline	2.4	
2. Policy	2.309	-3.77 %
3. Just e_i changes	2.407	0.29 %
4. Just $f(i)$ changes	2.404	0.17 %
5. Just $J(\Omega_i)$ changes	2.301	-4.12 %

Table 4: Per-person average welfare changes, percent consumption

	Scenario					
Group	(1)	(2)	(3)	(4)	(5)	(6)
Total	0.215	0.320	-0.105	-0.103	0.082	-0.083
Unemp. $\geq$ 5 quarters	1.659	1.821	-0.162	-0.148	0.072	-0.086
Unemp. $\leq$ 4 quarters	0.359	0.496	-0.137	-0.131	0.077	-0.083
Employed	0.196	0.298	-0.102	-0.101	0.082	-0.083

Note: The columns represent:
(1): Full General Equilibrium specificiation; i.e., no decompositions.
(2): Partial equilibrium effect: only $e(i)$ changes.
(3): General equilibrium effects; i.e., Col (1) minus Col (2).
(4): Matching effect: only θ changes.
(5): Prices effect: only r and w change.
(6): Tax rate effect: only τ changes.

Table 5: Welfare changes (percent consumption)

	Policy	
Group	Search Assist.	UI Payment
All	0.215	0.003
Unemp. $\geq$ 5 qtrs.	1.659	0.091
Unemp. $\leq$ 4 qtrs.	0.359	0.030
Employed	0.196	0.000

Note: Table shows average welfare change (by group) for the search assistance policy vs. a payment given to all unemployed agents.

Table 6: Per-person average welfare changes, percent

	$\mu = 0$ (standard)					
Group	(1)	(2)	(3)	(4)	(5)	(6)
Total	0.215	0.320	-0.105	-0.103	0.082	-0.083
Unemp. $\geq$ 5 quarters	1.659	1.821	-0.162	-0.148	0.072	-0.086
Unemp. $\leq$ 4 quarters	0.359	0.496	-0.137	-0.131	0.077	-0.083
Employed	0.196	0.298	-0.102	-0.101	0.082	-0.083

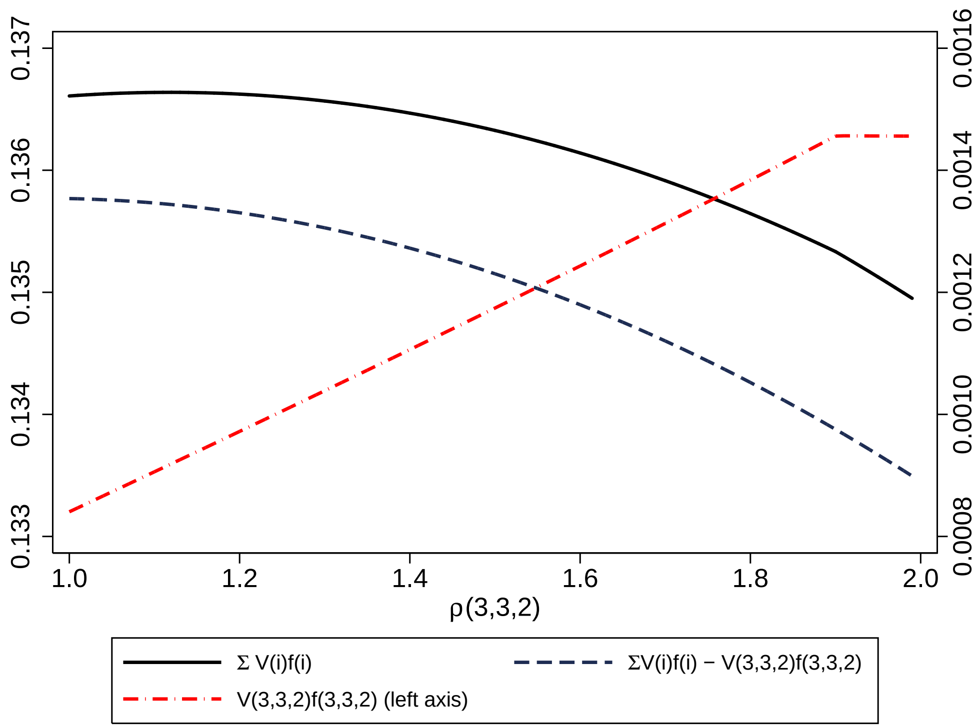
	$\mu = 0.5$					
Group	(1)	(2)	(3)	(4)	(5)	(6)
Total	0.277	0.522	-0.245	-0.120	-0.045	-0.081
Unemp. $\geq$ 5 quarters	1.834	2.288	-0.455	-0.173	-0.198	-0.084
Unemp. $\leq$ 4 quarters	0.439	0.752	-0.313	-0.152	-0.081	-0.080
Employed	0.256	0.495	-0.240	-0.117	-0.042	-0.081

	$\mu = 1.0$					
Group	(1)	(2)	(3)	(4)	(5)	(6)
Total	0.227	0.331	-0.104	-0.144	0.127	-0.087
Unemp. $\geq$ 5 quarters	1.943	2.006	-0.062	-0.208	0.236	-0.091
Unemp. $\leq$ 4 quarters	0.411	0.536	-0.125	-0.181	0.143	-0.087
Employed	0.204	0.307	-0.103	-0.141	0.125	-0.087

Note: The columns represent:
(1): Full General Equilibrium specificiation; i.e., no decompositions.
(2): Partial equilibrium effect: only $e(i)$ changes.
(3): General equilibrium effects; i.e., Col (1) minus Col (2).
(4): Matching effect: only θ changes.
(5): Prices effect: only r and w change.
(6): Tax rate effect: only τ changes.

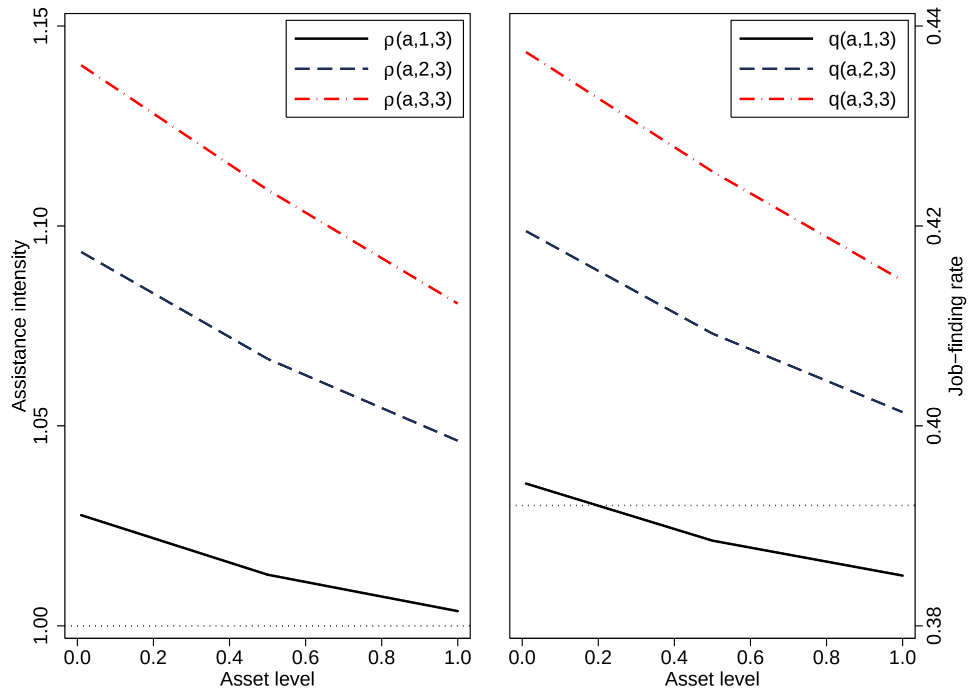
A.2 Chapter 2

Figure 13: Social welfare function versus $\rho(i)$



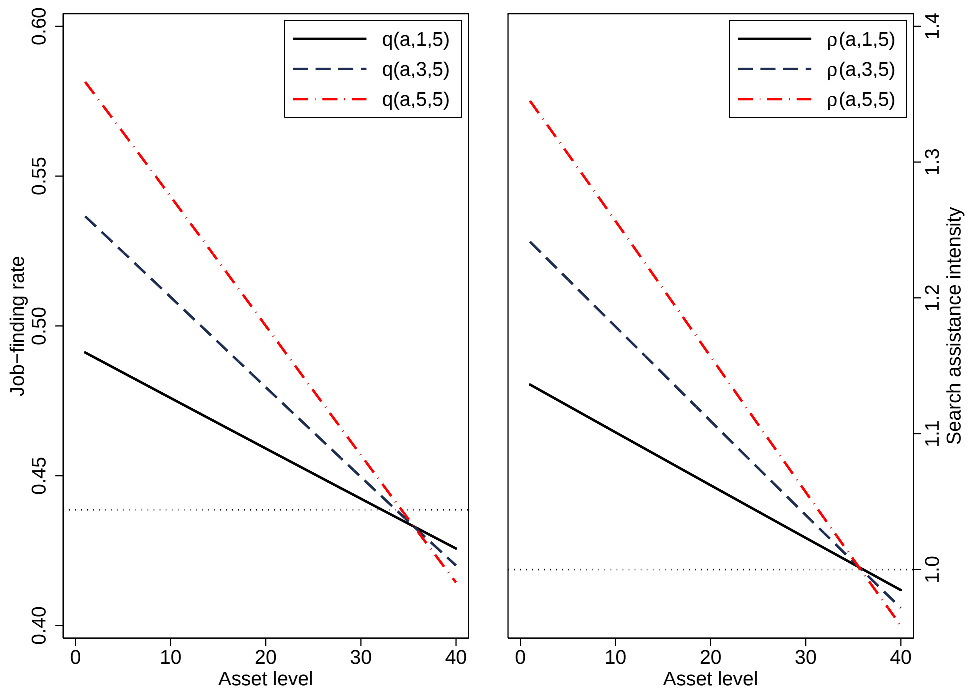
Note: Figure shows the social welfare function versus $\rho(3,3,2)$. The black line is the social welfare function, i.e., $\sum_i V(i)f(i)$; the blue dashed line is the social welfare function *excluding* the high-asset, high-skill, short-duration unemployed agent, and the dot-dashed red line is this agent's value function. Hence, the blue and red lines sum to the black line. $\rho(i)$ for all other types is set at $\rho^*(i)$.

Figure 14: Planner's decision rules for job-finding rate and assistance intensity By asset and skill level, Model #1.



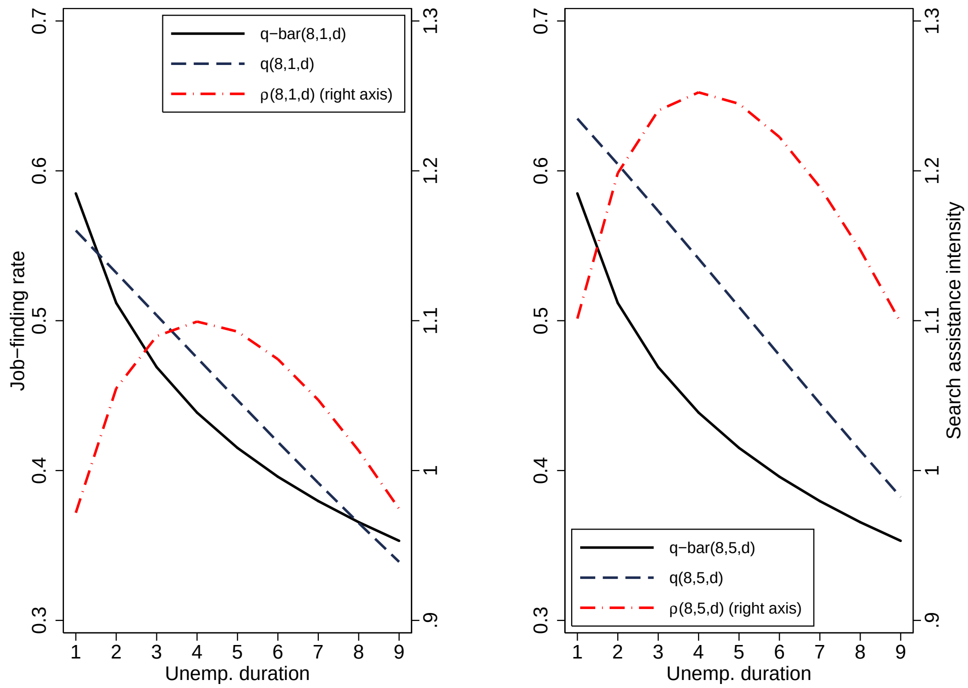
Note: Figure shows the planner's decision rules for search assistance intensity ($\rho^*(i)$, left panel) and the corresponding job-finding rate ($q^*(i)$, right panel), by asset and skill level. The solid black line is low-skill, the dashed blue line is middle-skill, and the dot-dashed red line is high-skill. The horizontal black dotted lines correspond to $\rho = 1$ (left panel) and $\bar{q}(i)$ (right panel).

Figure 15: Planner's decision rules for job-finding rate and assistance intensity By asset and skill level, Model #2.



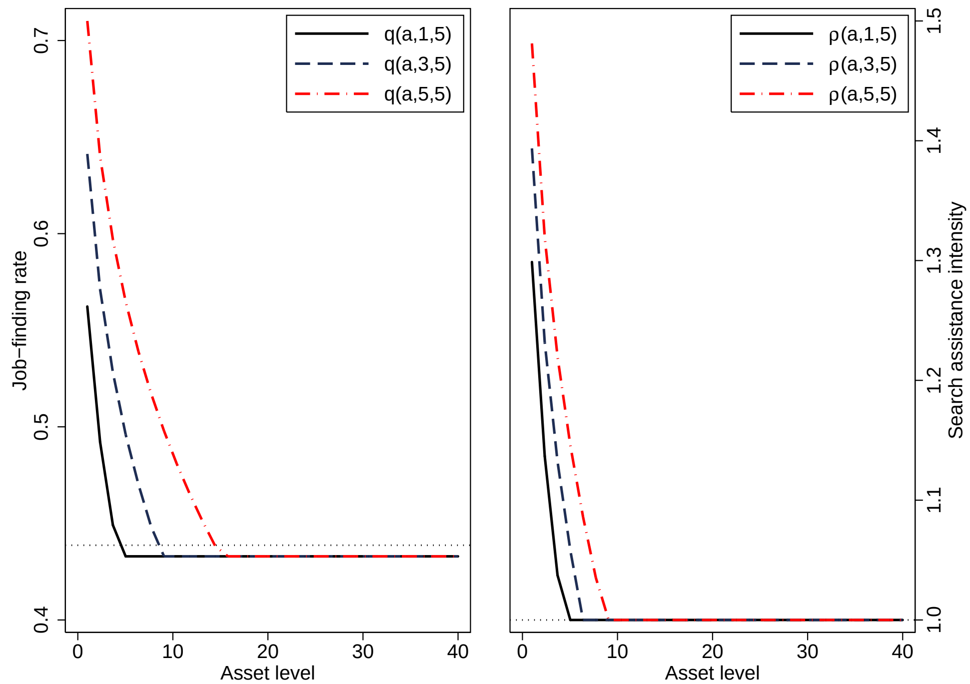
Note: Figure shows the planner's decision rules for job-finding rate ($q^*(i)$, left panel) and the corresponding search assistance intensity ($\rho^*(i)$, right panel), by asset and skill level. The solid black line is low-skill, the dashed blue line is middle-skill, and the dot-dashed red line is high-skill. The horizontal black dotted lines correspond to $\bar{q}(i)$ (left panel) and $\rho = 1$ (right panel).

Figure 16: Planner's decision rules for job-finding rate and assistance intensity By unemployment duration and skill level, Model #2.



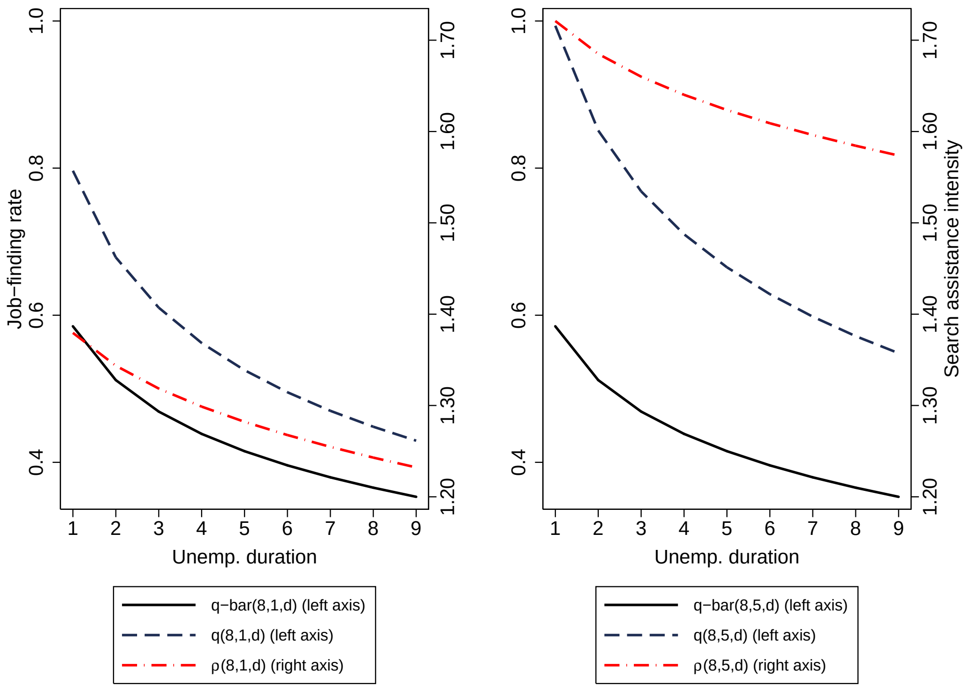
Note: Figure shows the planner's decision rules for job-finding rate ($q^*(i)$) and the corresponding search assistance intensity ($\rho^*(i)$), by unemployment duration and skill level. The left panel shows a low-skill agent and the right panel shows a high-skill agent. The solid black line is $\bar{q}(i)$, the dashed blue line is $q^*(i)$, and the dot-dashed red line is the corresponding $\rho^*(i)$ (right axis).

Figure 17: Planner's decision rules for job-finding rate and assistance intensity
By asset and skill level, Model #3.



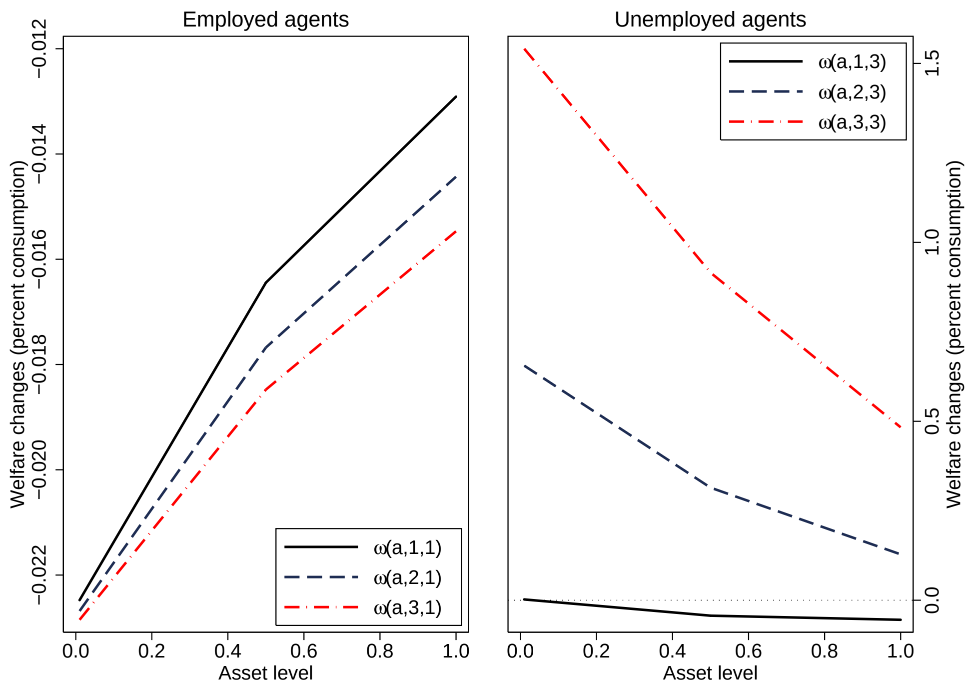
Note: Figure shows the planner's decision rules for search assistance intensity (ρ^*, right panel) and the corresponding job-finding rate (q^*, left panel), by asset and skill level. The solid black line is low-skill, the dashed blue line is middle-skill, and the dot-dashed red line is high skill. The horizontal black dotted lines correspond to $\bar{q}(i)$ (left panel) and $\rho = 1$ (right panel).

Figure 18: Planner's decision rules for job-finding rate and assistance intensity
By unemployment duration and skill level, Model #3.



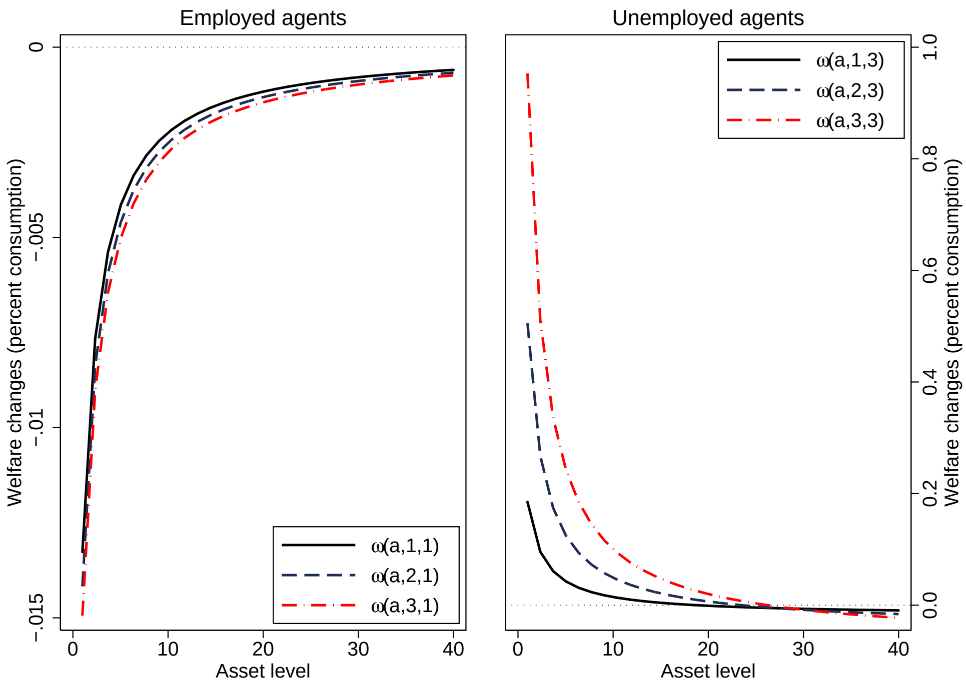
Note: Figure shows the planner's decision rules for job-finding rate ($q^*(i)$) and the corresponding search assistance intensity ($\rho^*(i)$), by unemployment duration and skill level. The left panel shows a low-skill agent and the right panel shows a high-skill agent. The solid black line is $\bar{q}(i)$, the dashed blue line is $q^*(i)$, and the dot-dashed red line is the corresponding $\rho^*(i)$ (right axis).

Figure 19: Welfare changes by asset and skill level, Model #1



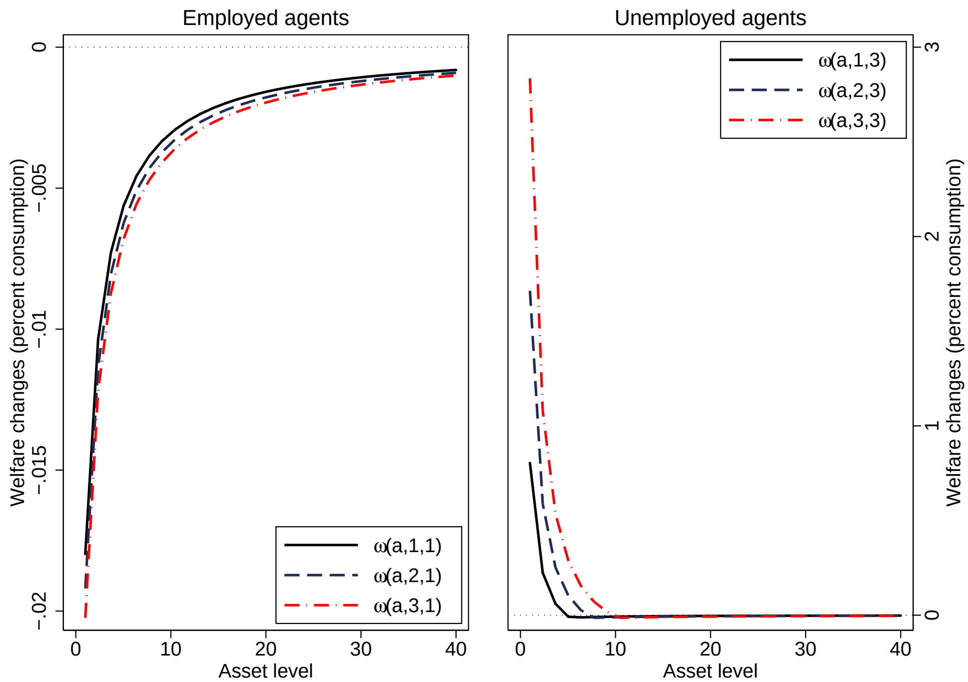
Note: Figure shows welfare changes ($\omega(i)$) as a result of the planner's optimal choice of $\rho(i)$ for Model #1. The left panel is for employed agents and the right panel is for unemployed agents. The solid black line is low-skill, the dashed blue line is middle-skill, and the dot-dashed red line is high skill.

Figure 20: Welfare changes by asset and skill level, Model #2



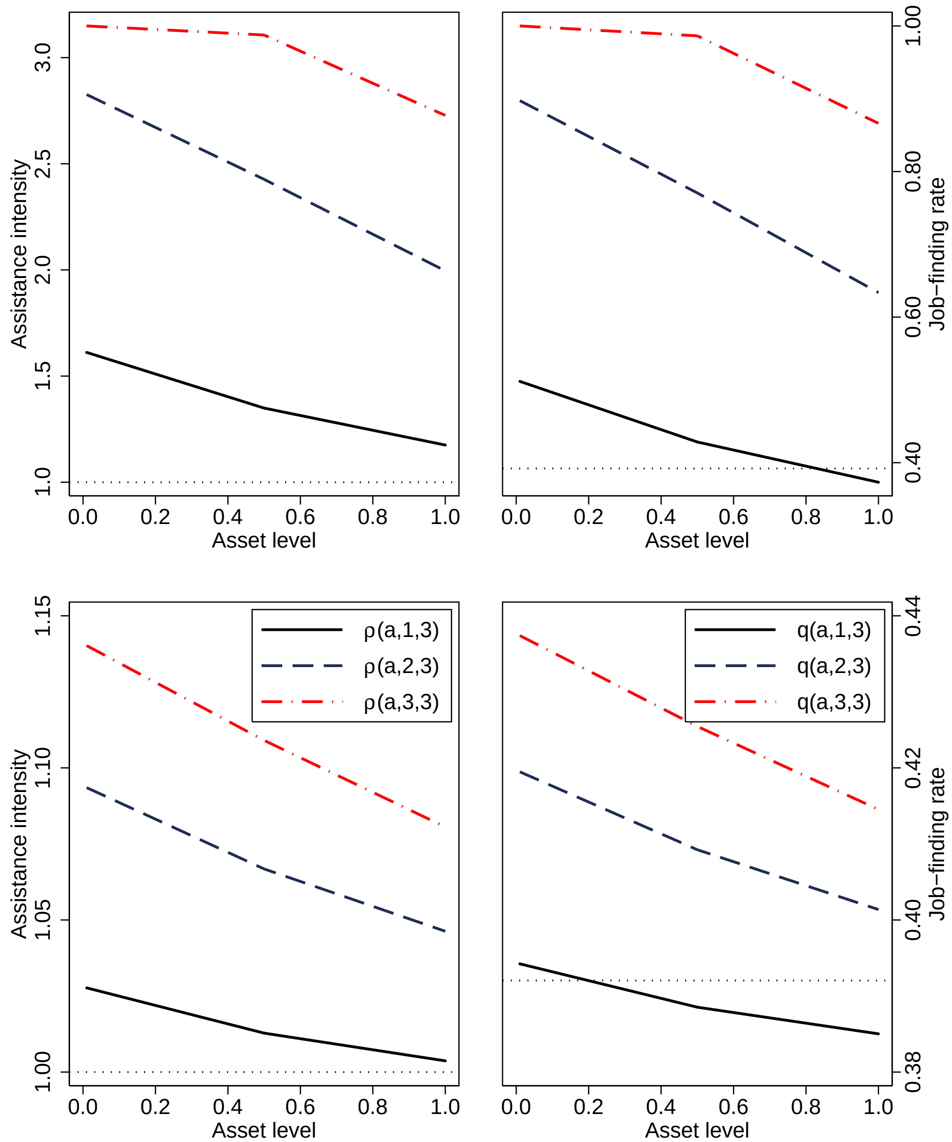
Note: Figure shows welfare changes ($\omega(i)$) as a result of the planner's optimal choice of $\rho(i)$ for Model #2. The left panel is for employed agents and the right panel is for unemployed agents. The solid black line is low-skill, the dashed blue line is middle-skill, and the dot-dashed red line is high skill.

Figure 21: Welfare changes by asset and skill level, Model #3



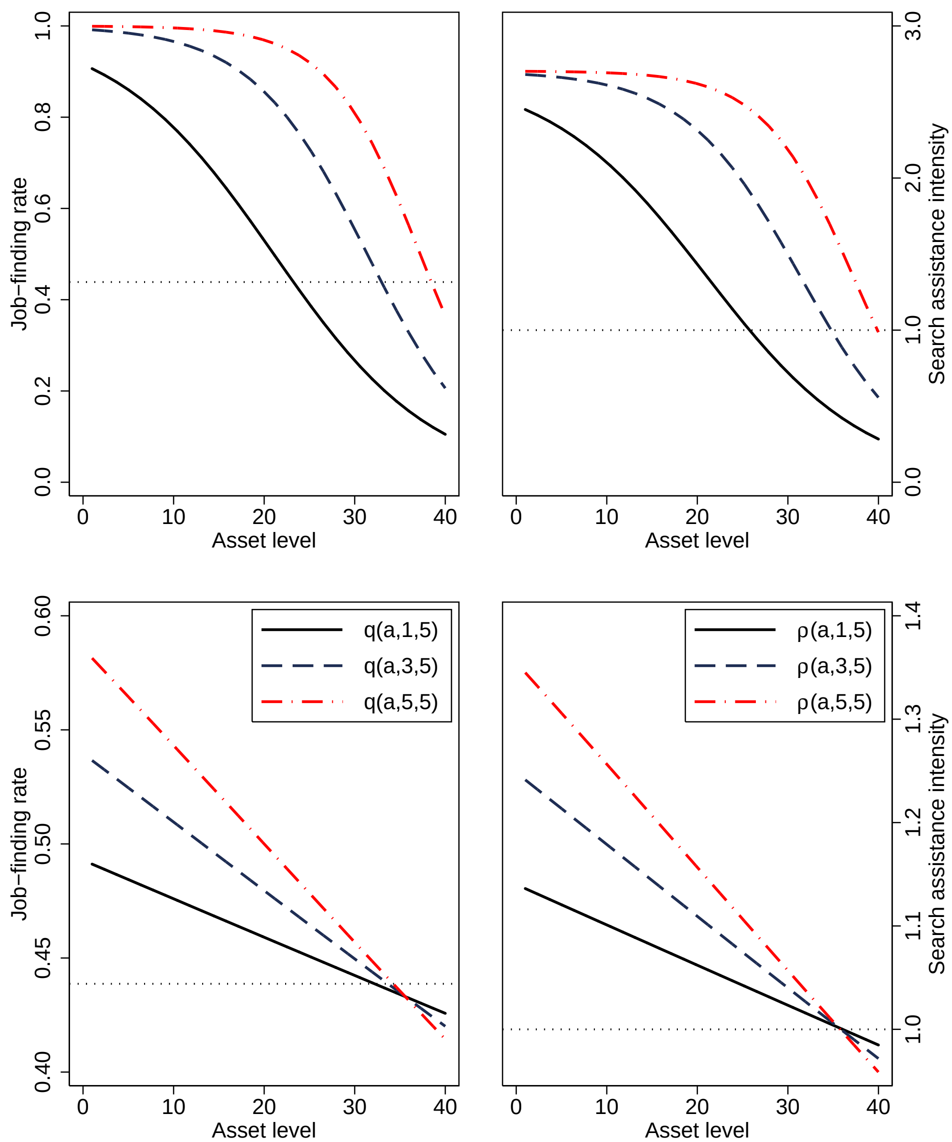
Note: Figure shows welfare changes ($\omega(i)$) as a result of the planner's optimal choice of $\rho(i)$ for Model #3. The left panel is for employed agents and the right panel is for unemployed agents. The solid black line is low-skill, the dashed blue line is middle-skill, and the dot-dashed red line is high skill.

Figure 22: Planner's decision rules for job-finding rate and assistance intensity
By asset and skill level, Model #1.



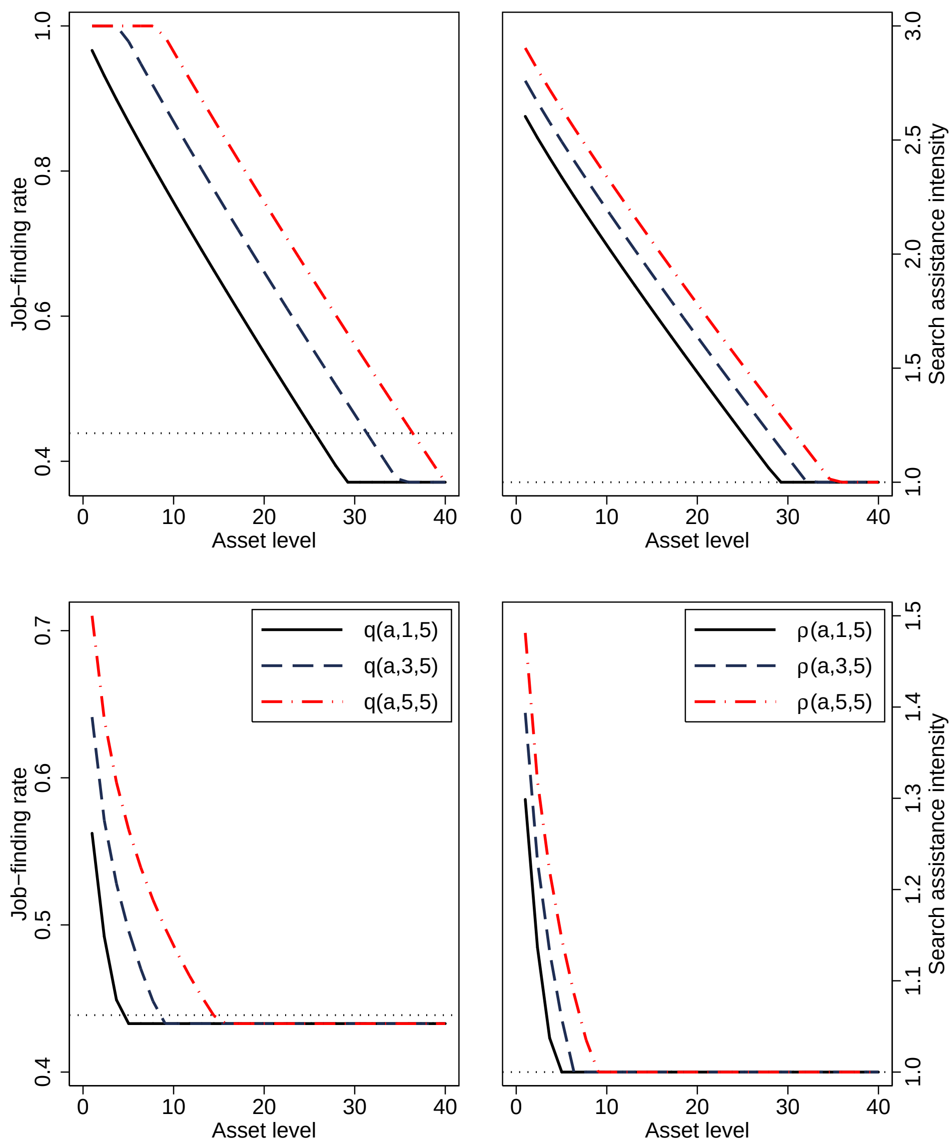
Note: Figure shows the planner's decision rules for search assistance intensity ($\rho^*(i)$, left panels), and the corresponding job-finding rate ($q^*(i)$, right panels), by asset and skill level. The top two panels show the alternative specification with only unemployed agents in the planner's objective function, and the bottom two panels recreate Figure 14. The solid black line is low-skill, the dashed blue line is middle-skill, and the dot-dashed red line is high-skill. The horizontal black dotted lines correspond to $\bar{q}(i)$ (left panel) and $\rho = 1$ (right panel).

Figure 23: Planner's decision rules for job-finding rate and assistance intensity By asset and skill level, Model #2.



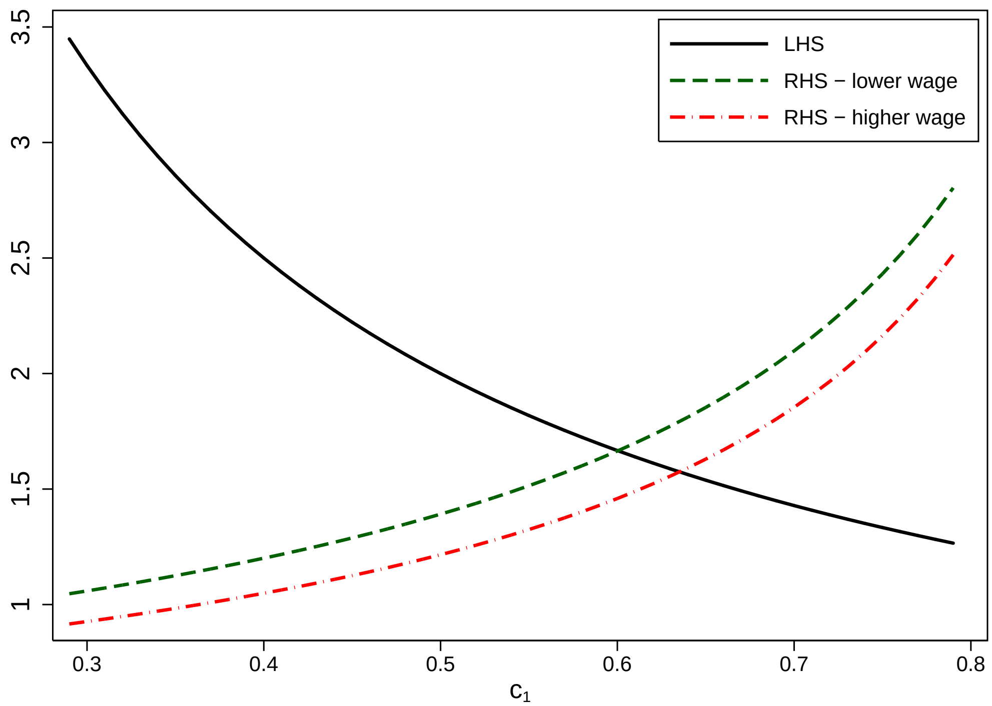
Note: Figure shows the planner's decision rules for job-finding rate ($q^*(i)$, left panels) and the corresponding search assistance intensity ($\rho^*(i)$, right panels), by asset and skill level. The top two panels show the alternative specification with only unemployed agents in the planner's objective function, and the bottom two panels recreate Figure 15. The solid black line is low-skill, the dashed blue line is middle-skill, and the dot-dashed red line is high-skill. The horizontal black dotted lines correspond to $\bar{q}(i)$ (left panel) and $\rho = 1$ (right panel).

Figure 24: Planner's decision rules for job-finding rate and assistance intensity
By asset and skill level, Model #3



Note: Figure shows the planner's decision rules for search assistance intensity (ρ^*, right panels) and the corresponding job-finding rate (q^*, left panels), by asset and skill level. The top two panels show the alternative specification with only unemployed agents in the planner's objective function, and the bottom two panels recreate Figure 17. The solid black line is low-skill, the dashed blue line is middle-skill, and the dot-dashed red line is high skill. The horizontal black dotted lines correspond to $\bar{q}(i)$ (left panel) and $\rho = 1$ (right panel).

Figure 25: Proof of Proposition 2.1



Note: The solid black line is the left-hand side of the first-order conditions; the dashed green line is the right-hand side for a lower wage; the dot-dashed red line is the right-hand side for a higher wage.

Table 7: Baseline model parameters

Variable	Meaning	Value
α	Labor share	0.67
β	Discount rate	0.99
σ	Match elasticity	0.72
ϕ	Job-keep probability	0.96
r	Interest rate	0.00985
$\bar{\psi}$ (Model 1)	Assistance cost function	0.1929
$\bar{\psi}$ (Models 2 & 3)	Assistance cost function	0.1164

Table 8: Logit rule for $q^*(i)$, Model #2

	Coefficient	
Variable	(1)	(2)
β_0: Constant	0.3049	-0.1018
β_1: Assets	-0.0103	0.0047
β_2: Skills	0.3729	1.1310
β_3: Duration	-0.1105	-0.0988
β_4: Assets x Skills		-0.0302
β_5: Assets x Duration		0.0004
β_6: Skills x Duration		-0.0486
β_7: Assets x Skills x Dur		0.0010

Note: Table shows the planner's optimal choice of coefficients for the Logit rule for $q^*(i)$ (Model #2). Column (1) does not include interaction terms but Column (2) does.

Table 9: Polynomial rule for $\rho^*(i)$, Model #3

Variable	Coefficient
β_0: Constant	1.3731
β_1: Assets	-0.7889
β_2: Assets (exponent)	0.2192
β_3: Skills	1.3567
β_4: Skills (exponent)	0.5262
β_5: Duration	-0.1467
β_6: Duration (exponent)	0.3164

Note: Table shows the planner's optimal choice of coefficients for the polynomial rule for $\rho^*(i)$ (Model #3).

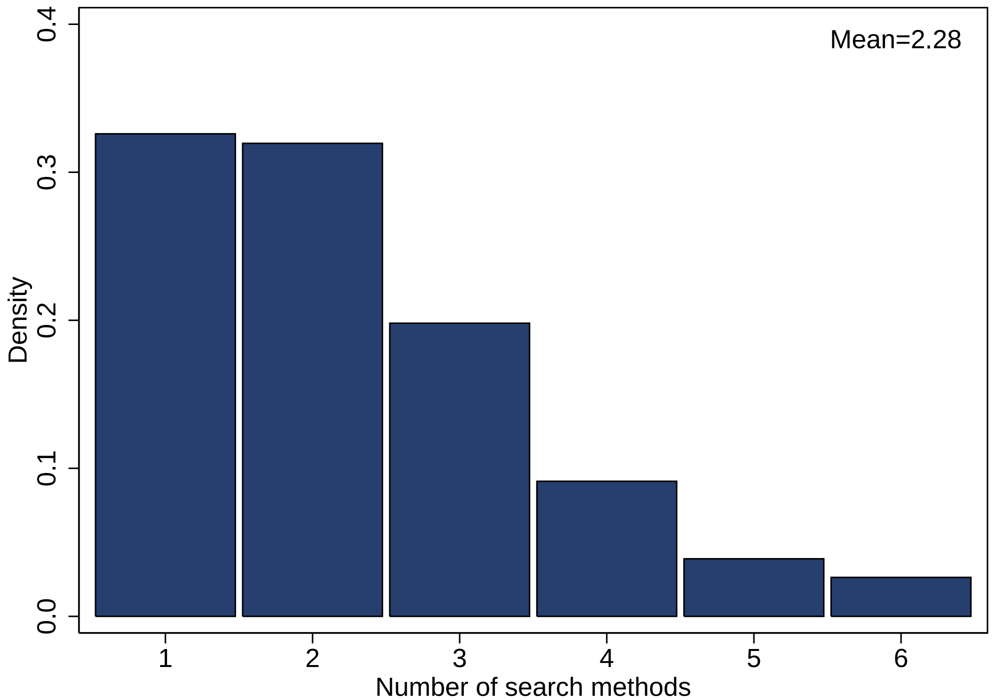
Table 10: Welfare changes, all models

	Model						
Group	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Employed	-0.0190	-0.0025	-0.0034	-0.0019	-3.1554	-0.2976	-0.2465
Unemployed	0.7548	0.0832	0.1356	0.0731	5.3294	0.5421	0.5387
Total	0.0197	0.0017	0.0035	0.0019	-2.7312	-0.2556	-0.2072

Note: Table shows consumption-based welfare changes, averaged by group. The columns represent:
(1): Model #1: small state space
(2): Model #2: Logit rule for $q^*(i)$.
(3): Model #3: Polynomial rule for $\rho^*(i)$.
(4): Model #3b: Choose one value of ρ for all types.
(5): Model #1, with only unemployed agents in the objective function.
(6): Model #2, with only unemployed agents in the objective function.
(7): Model #3, with only unemployed agents in the objective function.

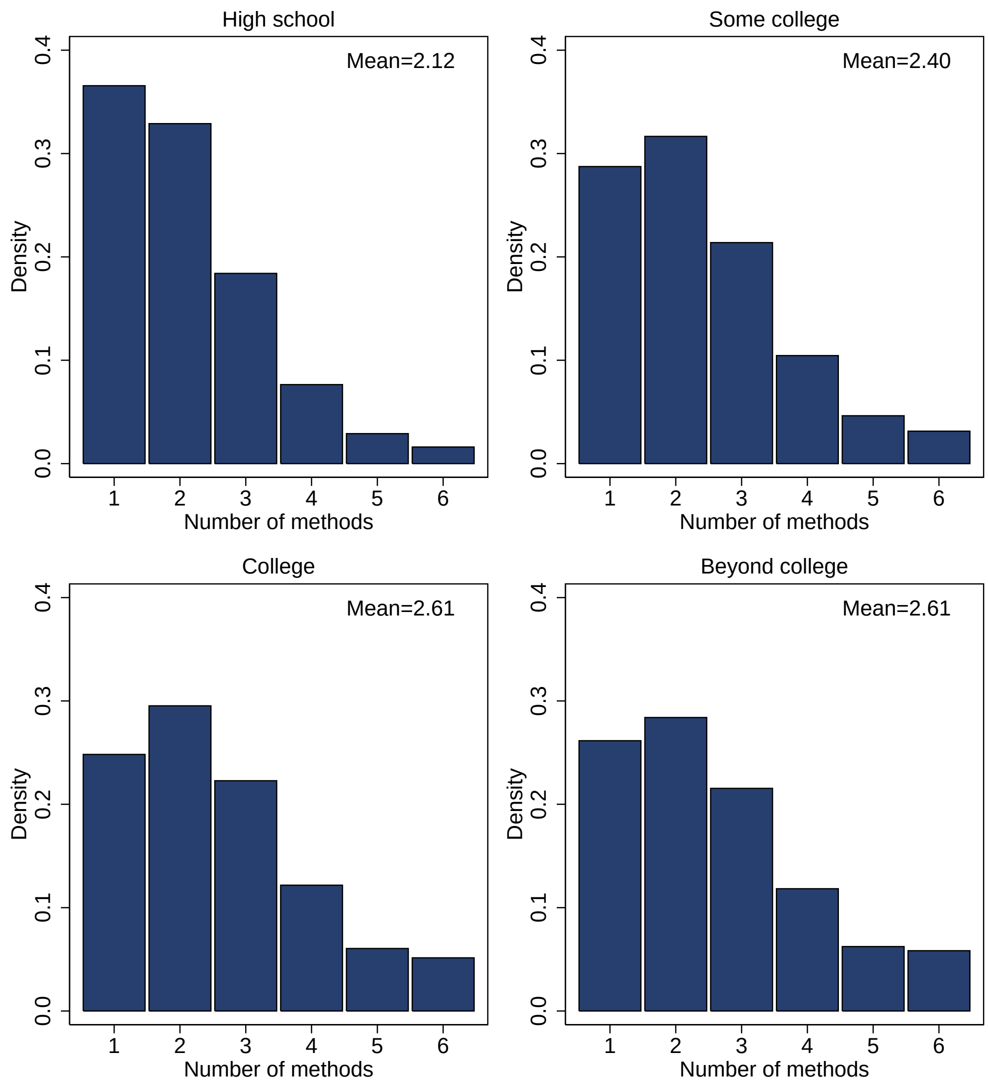
A.3 Chapter 3

Figure 26: Distribution of number of search methods



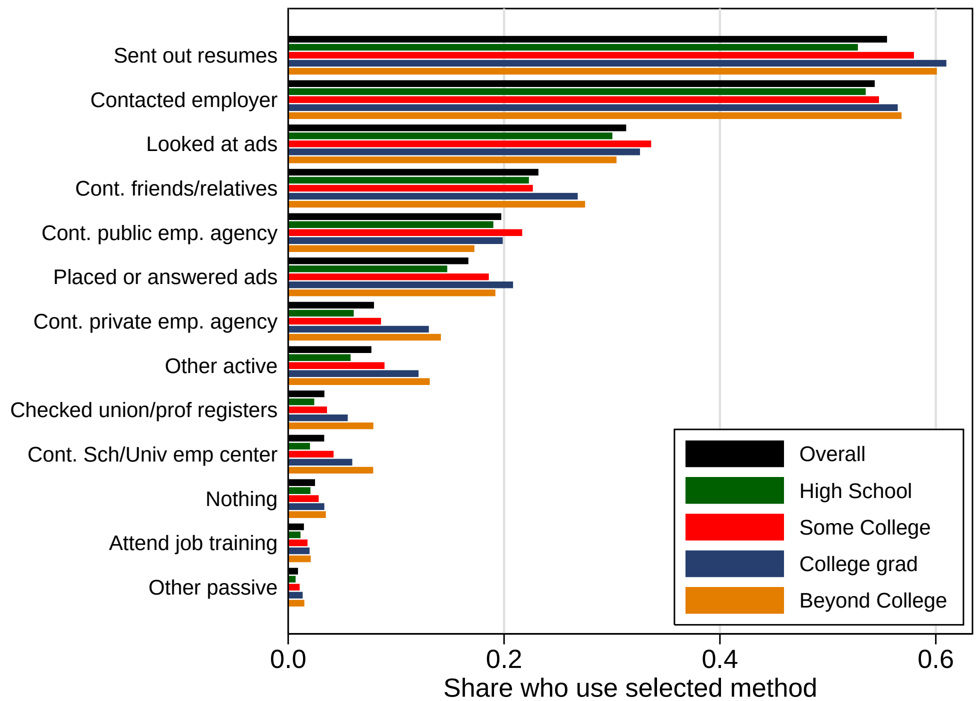
Note: Figure shows the distribution of number of chosen search methods, entire sample.

Figure 27: Distribution of number of search methods
By education level



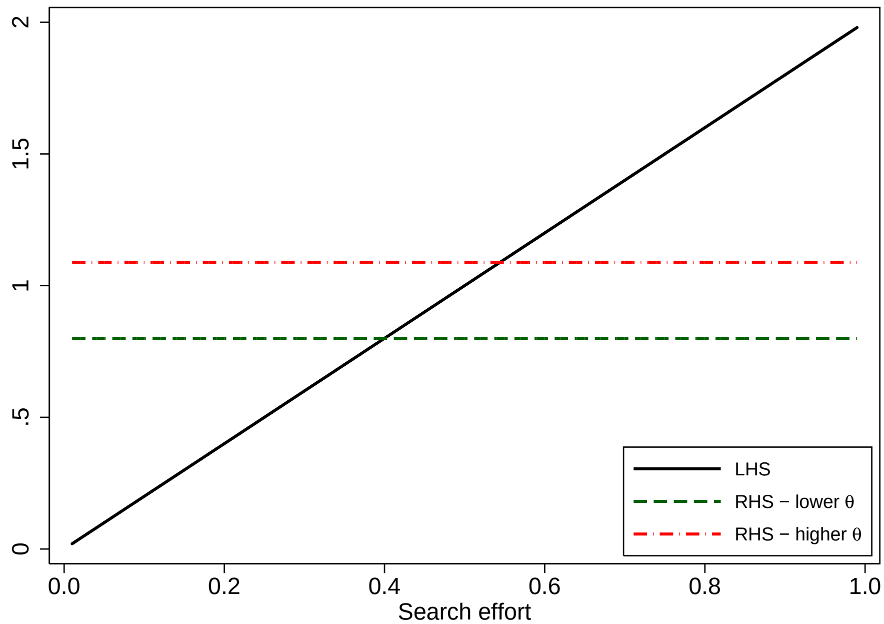
Note: Figure shows the distribution of number of chosen search methods, by education group.

Figure 28: Choice of search methods
By education group



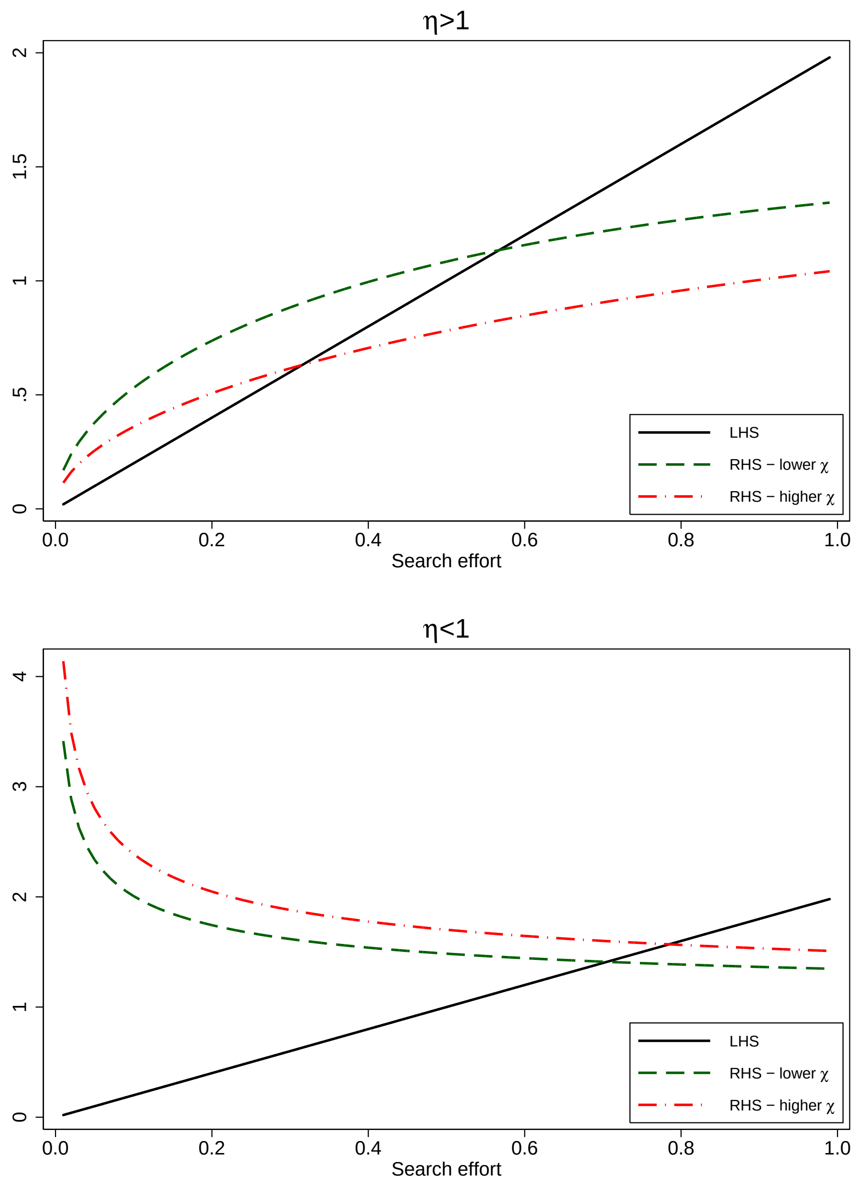
Note: Figure shows percent of jobseekers who report choosing the selected method, overall and by education group.

Figure 29: Optimal choice of search effort



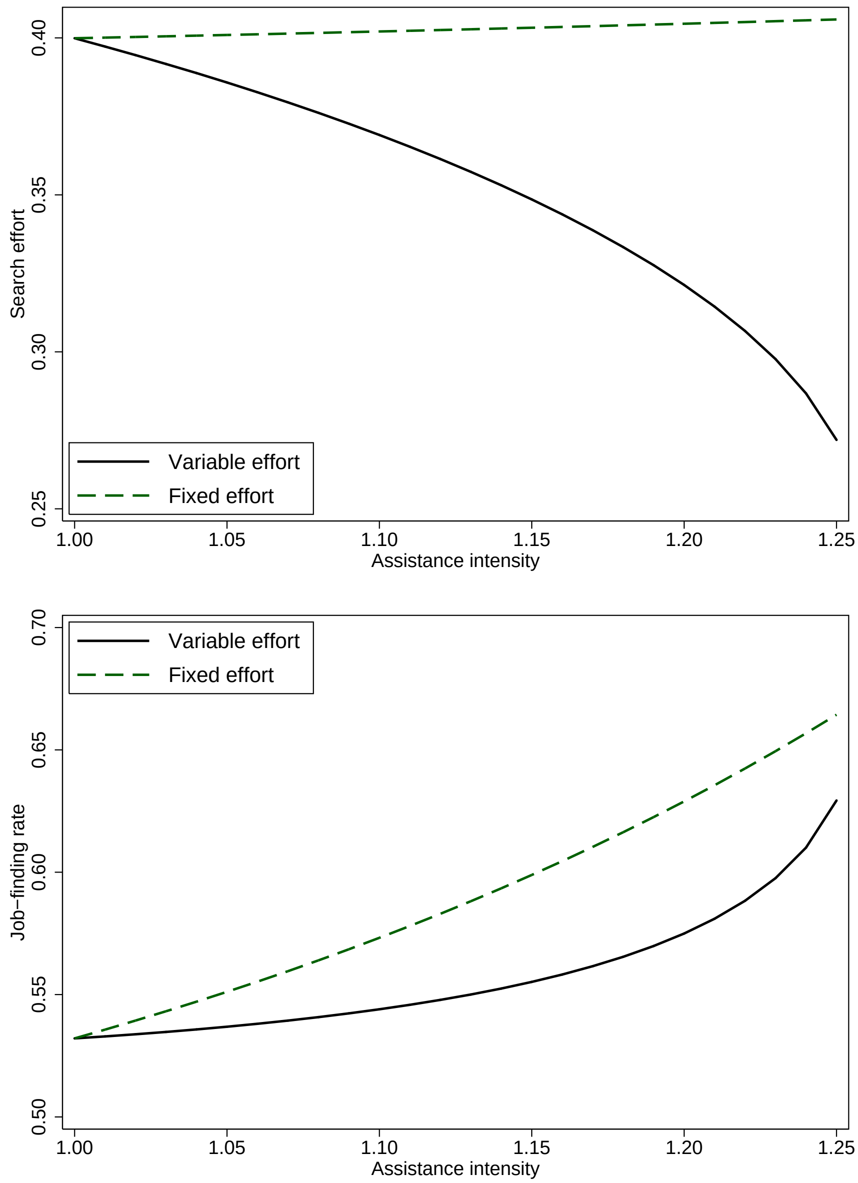
Note: Figure shows agent's choice of search effort for a Cobb-Douglas matching function. The solid black line is the left-hand side of the first-order conditions; the dashed green line is the right-hand size for a lower θ; the dot-dashed red line is the right-hand side for a higher θ.

Figure 30: Optimal choice of search effort



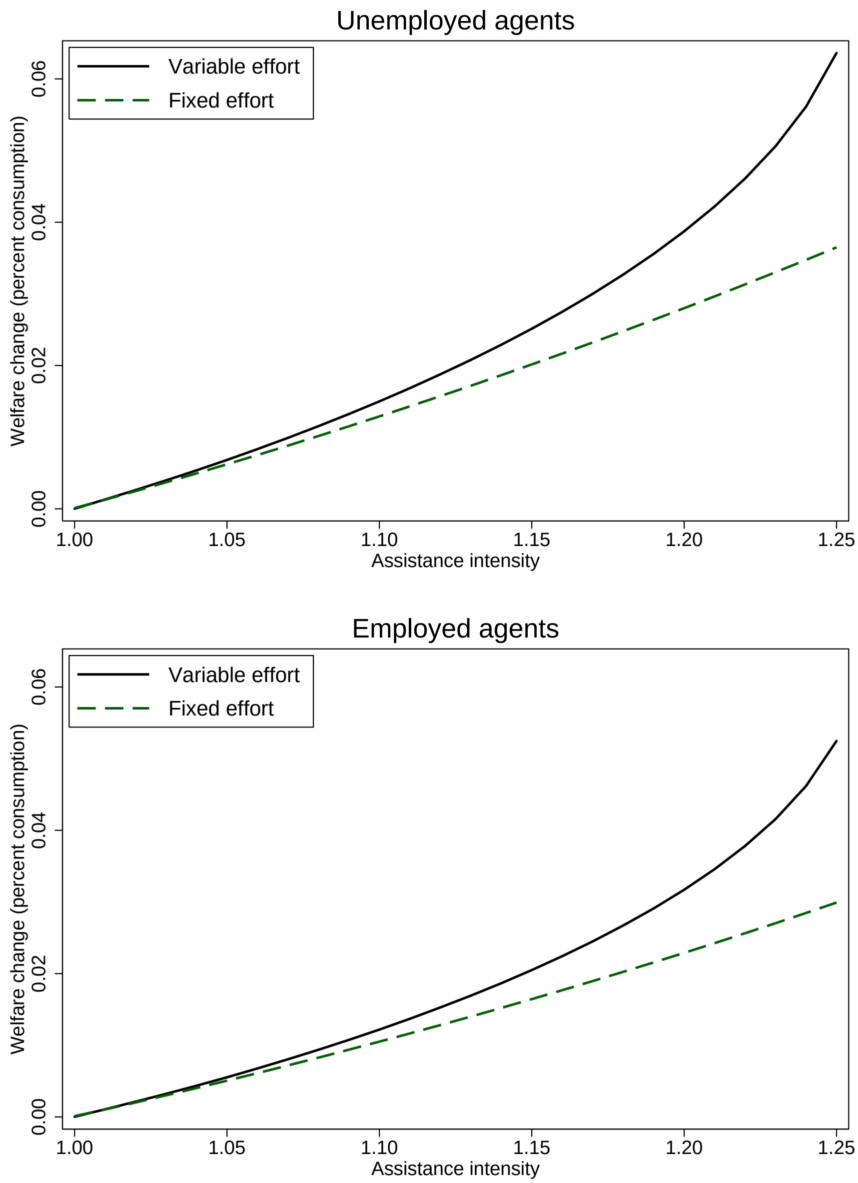
Note: Figure shows agent's choice of search effort for a CES matching function. The top panel is for $\eta > 1$ and the bottom panel is for $\eta < 1$. The solid black line is the left-hand side of the first-order conditions; the dashed green line is the right-hand size for a lower χ; the dot-dashed red line is the right-hand side for a higher χ.

Figure 31: Choice of search effort and resulting job-finding rates
CES matching function



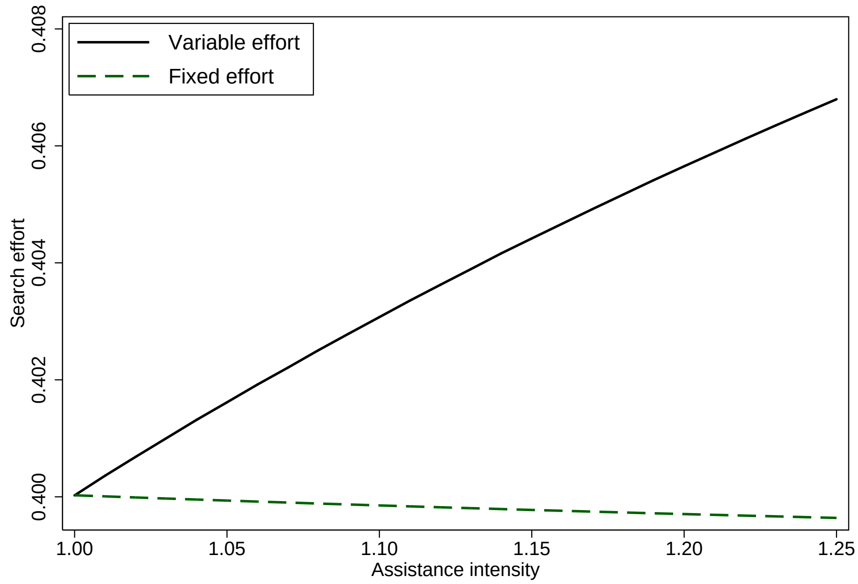
Note: Figure shows the average search effort (top panel) and average job-finding rates (bottom panel) as a function of search assistance intensity for the CES matching function specification. The solid black line allows search effort to be chosen freely by agents and the dashed green line fixes effort at the baseline value.

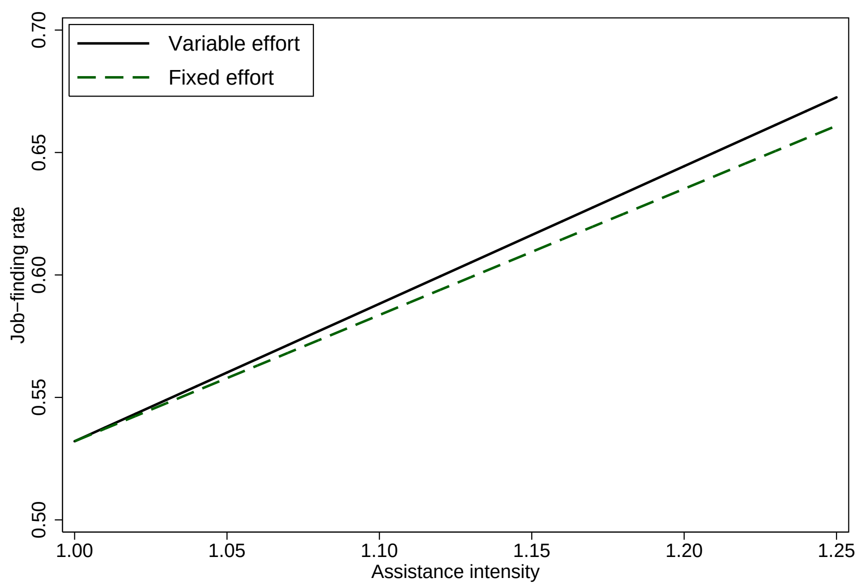
Figure 32: Welfare change as a function of assistance intensity
CES matching function



Note: Figure shows the welfare gain, relative to the baseline of $\rho = 1$, for unemployed agents (top panel) and employed agents (bottom panel) for the CES matching function specification. The solid black line allows search effort to be chosen freely by agents and the dashed green line fixes effort at the baseline value.

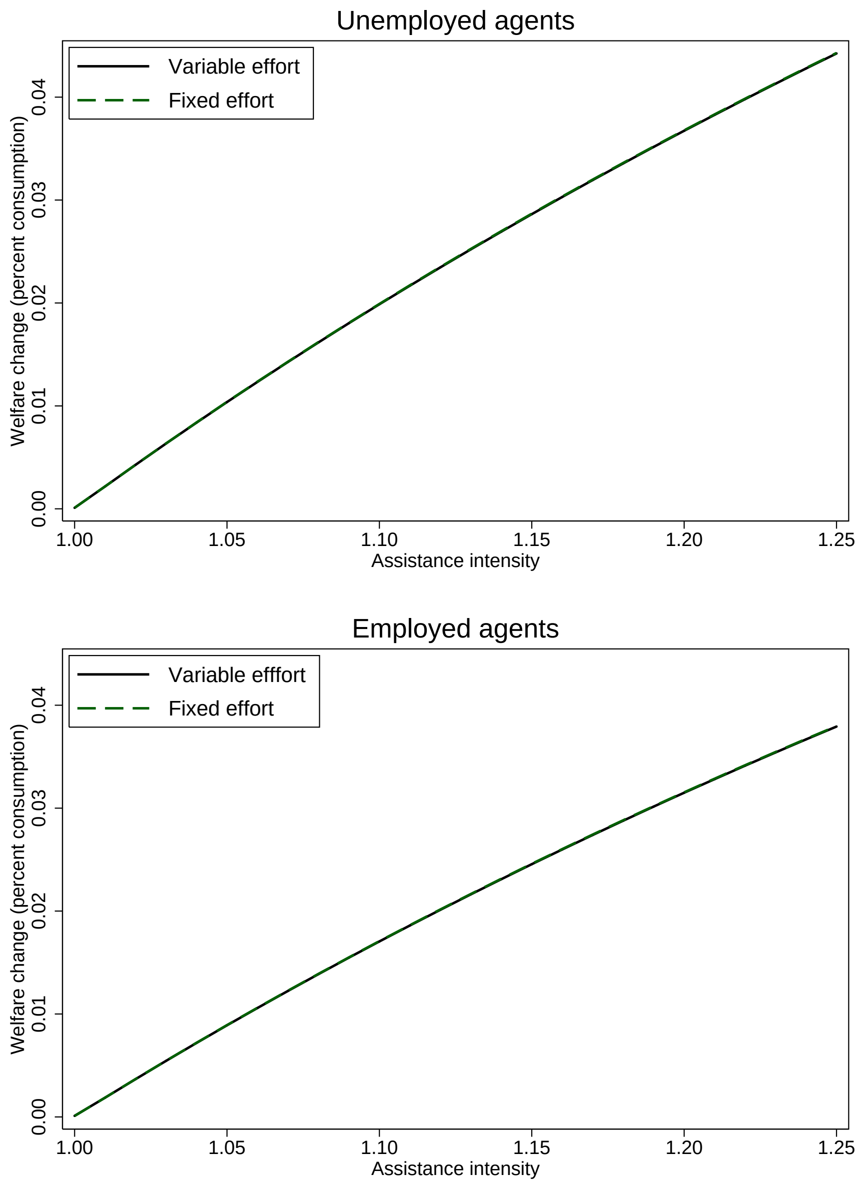
Figure 33: Choice of search effort and resulting job-finding rates
Cobb-Douglas matching function





Note: Figure shows the average search effort (top panel) and average job-finding rates (bottom panel) as a function of search assistance intensity for the Cobb Douglas matching function specification. The solid black line allows search effort to be chosen freely by agents and the dashed green line fixes effort at the baseline value.

Figure 34: Welfare change as a function of assistance intensity
Cobb-Douglas matching function



Note: Figure shows the average search effort (top panel) and average job-finding rates (bottom panel) as a function of search assistance intensity for the Cobb Douglas matching function specification. The solid black line allows search effort to be chosen freely by agents and the dashed green line fixes effort at the baseline value.

Table 11: CPS search methods

Method	Description
1	Contacted employer directly/interview
2	Contacted public employment agency
3	Contacted private employment agency
4	Contacted friends or relatives
5	Contacted school/university employment center
6	Sent out resumes/filled out application
7	Checked union/professional registers
8	Placed or answered ads
9	Other active
10	Looked at ads
11	Attended job training programs/courses
12	Nothing
13	Other passive

Table 12: Probit regression results for search methods

	Method 1		Method 2		Method 3	
Coefficient	(1a)	(1b)	(2a)	(2b)	(3a)	(3b)
Some college	0.0259	-0.0424	0.0162	-0.1008	0.1200	0.0115
	(0.0037)	(0.0039)	(0.0042)	(0.0046)	(0.0055)	(0.0061)
College graduate	0.0669	-0.0507	-0.0930	-0.3327	0.3127	0.1455
	(0.0051)	(0.0053)	(0.0058)	(0.0064)	(0.0067)	(0.0075)
Beyond college	0.0857	-0.0322	-0.2077	-0.4930	0.3548	0.1810
	(0.0077)	(0.0080)	(0.0090)	(0.0101)	(0.0098)	(0.0110)
Unemp. duration	-0.0194	-0.0244	0.0010	-0.0034	0.0023	-0.0023
	(0.0007)	(0.0007)	(0.0008)	(0.0009)	(0.0010)	(0.0011)
Unemp. rate	-0.0228	-0.0381	0.0212	0.0028	0.0085	-0.0129
	(0.0009)	(0.0009)	(0.0010)	(0.0011)	(0.0013)	(0.0014)
No. of methods		0.3596		0.4870		0.4469
		(0.0014)		(0.0016)		(0.0019)

	Method 4		Method 5		Method 6	
Coefficient	(4a)	(4b)	(5a)	(5b)	(6a)	(6b)
Some college	-0.0175	-0.1551	0.3281	0.2782	0.1539	0.0949
	(0.0041)	(0.0046)	(0.0075)	(0.0082)	(0.0037)	(0.0039)
College graduate	0.0941	-0.1164	0.5109	0.3964	0.2475	0.1453
	(0.0055)	(0.0061)	(0.0092)	(0.0101)	(0.0051)	(0.0054)
Beyond college	0.1094	-0.1112	0.6898	0.5931	0.2600	0.1558
	(0.0082)	(0.0092)	(0.0123)	(0.0135)	(0.0077)	(0.0082)
Unemp. duration	0.0202	0.0199	0.0030	-0.0021	0.0035	0.0003
	(0.0008)	(0.0008)	(0.0014)	(0.0016)	(0.0007)	(0.0007)
Unemp. rate	0.0466	0.0325	0.0222	0.0057	0.0084	-0.0052
	(0.0010)	(0.0010)	(0.0017)	(0.0019)	(0.0009)	(0.0009)
No. of methods		0.5286		0.3631		0.3837
		(0.0016)		(0.0024)		(0.0015)

Note: Table shows coefficent estimates for Probit regressions as described in Section 3.3.2. See Table 11 for definitions of the search methods. Standard errors in parentheses.

Table 13: Probit regression results for search methods (continued)

	Method 7		Method 8		Method 9	
Coefficient	(7a)	(7b)	(8a)	(8b)	(9a)	(9b)
Some college	0.1509	0.0955	0.1061	0.0070	0.1926	0.1857
	(0.0074)	(0.0078)	(0.0044)	(0.0048)	(0.0055)	(0.0055)
College graduate	0.3455	0.2433	0.1606	-0.0189	0.3445	0.3324
	(0.0089)	(0.0094)	(0.0058)	(0.0064)	(0.0069)	(0.0069)
Beyond college	0.5115	0.4093	0.1014	-0.1049	0.3816	0.3692
	(0.0120)	(0.0126)	(0.0088)	(0.0098)	(0.0100)	(0.0100)
Unemp. duration	-0.0058	-0.0091	0.0097	0.0079	-0.0014	-0.0016
	(0.0013)	(0.0014)	(0.0008)	(0.0009)	(0.0010)	(0.0010)
Unemp. rate	0.0311	0.0211	0.0206	0.0032	0.0209	0.0197
	(0.0017)	(0.0017)	(0.0010)	(0.0011)	(0.0013)	(0.0013)
No. of methods		0.2563		0.4734		0.0349
		(0.0022)		(0.0016)		(0.0018)

	Method 10		Method 11		Method 13	
Coefficient	(10a)	(10b)	(11a)	(11b)	(13a)	(13b)
Some college	0.0525	-0.0479	0.1389	0.0781	0.1413	0.1197
	(0.0038)	(0.0042)	(0.0095)	(0.0100)	(0.0117)	(0.0119)
College graduate	0.0011	-0.1963	0.1577	0.0485	0.2068	0.1708
	(0.0052)	(0.0057)	(0.0122)	(0.0129)	(0.0147)	(0.0149)
Beyond college	-0.0624	-0.2845	0.1621	0.0406	0.2388	0.2026
	(0.0080)	(0.0087)	(0.0179)	(0.0189)	(0.0209)	(0.0210)
Unemp. duration	0.0145	0.0130	0.0088	0.0075	-0.0167	-0.0170
	(0.0007)	(0.0008)	(0.0017)	(0.0019)	(0.0025)	(0.0025)
Unemp. rate	0.0221	0.0054	0.0012	-0.0107	-0.1315	-0.1329
	(0.0009)	(0.0010)	(0.0022)	(0.0024)	(0.0034)	(0.0034)
No. of methods		0.4928		0.2544		0.0917
		(0.0015)		(0.0029)		(0.0037)

Note: Table shows coefficent estimates for Probit regressions as described in Section 3.3.2. See Table 11 for definitions of the search methods. Standard errors in parentheses.

Table 14: Baseline model parameters

Variable	Meaning	Value
α	Labor share	0.67
β	Discount rate	0.99
η	CES elasticity parameter	1.1
σ	Match elasticity	0.72
ϕ	Job-keep probability	0.96
r	Interest rate	0.00985
ψ (CES)	Search disutility	0.0560
ψ (Cobb)	Search disutility	0.0808
$\tilde{q}$ (CES)	Aggregate match efficiency	1.0871
$\tilde{q}$ (Cobb)	Aggregate match efficiency	1.7742

Appendix B

Proofs

Proof of Proposition 1.1

The value of the agent's assets before the policy is:

$$a_t = k_t^h + p_0 b_t$$

and the value of his assets after the policy is

$$\tilde{a}_t = k_t^h + \tilde{p}_0 b_t,$$

representing a change of

$$\Delta a = \tilde{a}_t - a_t = (\tilde{p}_0 - p_0) b_t. \tag{B.1}$$

Recall that agents hold physical capital to equity in the ratio ξ; that is, $k_t^h / b_t = \xi$, or, $k_t^h = \xi b_t$. Hence, at time $t = 1$, the agent's portfolio allocation is

$$b_t = \frac{a_t}{\xi + p_0}, \qquad k_t^h = \xi \left(\frac{a_t}{\xi + p_0} \right).$$

This means that Equation (B.1) can be written as

$$\Delta a = (\tilde{p}_0 - p_0) \frac{a_t}{\xi + p_0} \tag{B.2}$$

Next, let $\bar{B}$ be the total amount of firm equity in the economy, and let $\bar{A}$ be the total amount of assets (at time $t = 1$). Given that ξ is the same for all agents, ξ must be equal to $\bar{A}/\bar{B} - p_0$. To see this, recall the asset market equilibrium condition:

$$\begin{aligned}\sum a_t &= \sum k_t^h + p_0 \sum b_t \\ \bar{A} &= \sum \xi b_t + p_0 \sum b_t \\ \bar{A} &= (\xi + p_0)\bar{B}. \\ \Rightarrow \xi &= \bar{A}/\bar{B} - p_0.\end{aligned}$$

Then, Equation (B.2) reduces to

$$\Delta a = \frac{(\tilde{p}_0 - p_0)a_t}{\bar{A}/\bar{B}}.$$

This is the equivalent for Proposition 1.1 for some *arbitrary* value for $\bar{B}$ (as discussed before, $\bar{B}$ is not identified in the model). If we assume, without loss of generality, $\bar{B} = 1$, then the above reduces to

$$\Delta a = \frac{(\tilde{p}_0 - p_0)a_t}{\bar{A}}.$$

■

Proof of that Equation 2.3 is positive

Below I show that

$$\frac{\partial}{\partial \rho(i)} = \bar{e}(i)\theta^{1-\sigma} + (\sigma - 1)\rho(i)\bar{e}(i)v^{1-\sigma}u^{\sigma-2}\frac{\partial u}{\partial \rho(i)} > 0.$$

Note that

$$u = \sum_{s \geq 1} \rho(i)\bar{e}(i)f(i), \tag{B.3}$$

hence,

$$\frac{\partial u}{\partial \rho(i)} = \bar{e}(i)f(i)$$

So,

$$\bar{e}(i)\theta^{1-\sigma} > (1-\sigma)\rho(i)\bar{e}(i)v^{1-\sigma}u^{\sigma-2}\bar{e}(i)f(i).$$

Re-arranging and cancelling some terms,

$$u > (1-\sigma)\rho(i)\bar{e}(i)f(i)$$

From Eq. (B.3),

$$u \geq \rho(i)\bar{e}(i)f(i)$$

Multiplying the right-hand side of the above by $(1-\sigma) < 1$ makes it even smaller, hence it holds with strict inequality. ■

Proof of Proposition 2.1

This proposition states that $dV(i)/dq(i)$ is increasing in skill level, or $d^2V(i)/dq(i)dx(i) > 0$. Note that after an application of the envelope theorem, the derivative of the agent's value function with respect to $q(i)$ is

$$\frac{dV(i)}{dq(i)} = \beta\left(u(c_2^{e*}) - u(c_2^{u*})\right)$$

Also note from the budget constraint that $c_2^{e*} = c_2^{u*} + w_2 - \hat{w}$; that is, no matter what c_2^{u*} is, c_2^{e*} is larger by $w_2 - \hat{w}$. Hence, the above can be expressed as

$$\beta\left(u(c_2^{u*} + w_2 - \hat{w}) - u(c_2^{u*})\right).$$

So, I must show the derivative of the above with respect to x is positive. Given that wages are increasing in skill, this is equivalent to showing the derivative with respect to w_2 is positive.[35] As shorthand, let

$$b \equiv u(c_2^{u*} + w_2 - \hat{w}) - u(c_2^{u*}).$$

First, b is clearly increasing in w_2; however, it is decreasing in c_2^{u*}, so I must show

[35] Furthermore, since $\beta > 0$, I can drop this term from the following.

that c_2^{u*} decreases when w_2 increases. In other words:

$$\frac{\partial b}{\partial w_2} = \frac{\partial b}{\partial c_2^{u*}} \frac{dc_2^{u*}}{dw_2} + \frac{db}{dw_2} > 0.$$

The last term, db/dw_2, is straightforward. As w_2 increases, there is a larger 'gap' between c_2^e and c_2^u. Thus for any value of c_2^{u*} (and given that u is increasing), a higher value of w_2 clear indicates a higher value of $u(c_2^{u*} + w_2 - \hat{w}) - u(c_2^{u*})$. So, $db/dw_2 > 0$.

Next, I show that $dc_2^{u*}/dw_2 < 0$. From the agent's first-order conditions,

$$\frac{1}{c_1} = \frac{\beta q(i)(1+r)}{w_2 + (1+r)((1-\tau)\hat{w} + a_1 - c_1)} + \frac{\beta(1-q(i))(1+r)}{\hat{w} + (1+r)((1-\tau)\hat{w} + a_1 - c_1)}$$

As w_2 increases, the value of c_1 that equalizes these first-order conditions must rise. This is more clearly seen in Figure 25. As the right-hand side shifts down, it intersects the left-hand side at a higher value of c_1.[36] Next, note that when c_1 increases, savings must decrease, since first-period earnings have not changed:

$$c_1 + a_2 = (1-\tau)\hat{w} + a_1,$$

which in turns implies c_2^u falls:

$$c_2^u = \hat{w} + (1+r)a_2.$$

So, $dc_2^{u*}/dw_2 < 0$. Furthermore, due to the concavity of the utility function, $db/dc_2^{u*} < 0$. Combining these pieces $\frac{db}{dc_2^{u*}} \frac{dc_2^{u*}}{dw_2} > 0$, and $db/dw_2 > 0$ ■

Proof of Proposition 2.2

This Proposition states that $\rho^*(i)$ is increasing in the agent's skill level, all else equal.[37] The planner's first-order conditions of the objective function with respect to

[36]Note that this proof does not rely on $u(c) = ln(c)$. Any other increasing, concave function will have a similar shape and the result will still hold.

[37]Note that this proof can be used, with minor modifications, to prove Proposition 2.4 as well.

$\rho(1), \rho(2), \ldots \rho(k)$, respectively, are:

$$\frac{\partial V(1)}{\partial q(1)}\frac{dq(1)}{d\rho(1)} + \frac{\partial V(2)}{\partial q(2)}\frac{dq(2)}{d\rho(1)} + \ldots + \frac{\partial V(k)}{\partial q(k)}\frac{dq(k)}{d\rho(1)} = 0,$$
$$\frac{\partial V(1)}{\partial q(1)}\frac{dq(1)}{d\rho(2)} + \frac{\partial V(2)}{\partial q(2)}\frac{dq(2)}{d\rho(2)} + \ldots + \frac{\partial V(k)}{\partial q(k)}\frac{dq(k)}{d\rho(2)} = 0,$$
$$\ldots$$
$$\frac{\partial V(1)}{\partial q(1)}\frac{dq(1)}{d\rho(k)} + \frac{\partial V(2)}{\partial q(2)}\frac{dq(2)}{d\rho(k)} + \ldots + \frac{\partial V(k)}{\partial q(k)}\frac{dq(k)}{d\rho(k)} = 0$$

Therefore, $\rho^*(i)$ is the values of ρ which allow these first-order conditions to hold with equality. Note that as the measure of agent types is identical, I drop the $f(i)$ terms. Setting these equal and re-arranging,

$$\frac{\partial V(1)}{\partial q(1)}\left(\frac{dq(1)}{d\rho(1)} - \sum_{i\neq 1}^{k}\frac{dq(1)}{d\rho(i)}\right)$$
$$=\frac{\partial V(2)}{\partial q(2)}\left(\frac{dq(2)}{d\rho(2)} - \sum_{i\neq 2}^{k}\frac{dq(2)}{d\rho(i)}\right)$$
$$\ldots$$
$$=\frac{\partial V(k)}{\partial q(k)}\left(\frac{dq(k)}{d\rho(k)} - \sum_{i\neq k}^{k}\frac{dq(k)}{d\rho(i)}\right)$$

To be clear, the terms in parentheses are the derivative of $q(i)$ with respect to $\rho(i)$, minus the derivatives of $q(i)$ with respect to the ρ's of all other types. Recall that x is strictly increasing in the type index. Hence, as established in Proposition 2.1, $\partial V(1)/\partial q(1) < \partial V(2)/\partial q(2) < \ldots < \partial V(k)/\partial q(k)$. Therefore, for the above to hold, we need:

$$
\begin{aligned}
&\left(\frac{dq(1)}{d\rho(1)} - \sum_{i\neq 1}^{k} \frac{dq(1)}{d\rho(i)}\right) \\
> &\left(\frac{dq(2)}{d\rho(2)} - \sum_{i\neq 2}^{k} \frac{dq(2)}{d\rho(i)}\right) \\
&\dots \\
> &\left(\frac{dq(k)}{d\rho(k)} - \sum_{i\neq k}^{k} \frac{dq(k)}{d\rho(i)}\right)
\end{aligned}
$$

After re-arranging, this is equivalent to

$$
\frac{d}{d\rho(1)} \sum_{i=1}^{k} q(i) > \frac{d}{d\rho(2)} \sum_{i=1}^{k} q(i) > \ldots > \frac{d}{d\rho(k)} \sum_{i=1}^{k} q(i)
$$

Note that $\sum q(i)$ is increasing and concave with respect to any $\rho(i)$; hence the derivatives in the above expression are decreasing, convex functions. Therefore, for the above inequality to hold in equilibrium, it must be that $\rho^*(k) > \ldots > \rho^*(2) > \rho^*(1)$. ■

Proof of Proposition 2.3

This proposition states that $dV(i)/dq(i)$ is decreasing in asset level. As with Proposition 2.1, this can be expressed as

$$
\frac{dV(i)}{dq(i)} = \beta\left(u(c_2^{e*}) - u(c_2^{u*})\right) = \beta\left(u(c_2^{u*} + w_2 - \hat{w}) - u(c_2^{u*})\right)
$$

Note that as a_1 increases, both c_1 and a_2 increases. That is, the agent consumes some, but not all, of this additional first-period income. Hence, as saving increases, c_2^{u*} also increases. By the same logic as in Proposition 2.1 (concavity of u), higher c_2^{u*} decreases $u(c_2^{u*} + w_2 - \hat{w}) - u(c_2^{u*})$. ■

Calculating welfare change

Equation (2.5) states that welfare change for type i, denoted $\omega(i)$, is

$$\omega(i) = \exp\left(\frac{\tilde{V}(i) - \bar{V}(i)}{1+\beta}\right) - 1.$$

Let $\bar{c}$ be consumption in the absence of search assistance, and let $\tilde{c}$ be consumption in the state of the world corresponding to the planner's optimal choice. Then,

$$E_0 \sum_{t=0}^{1} \beta^t u((1+\omega(i))\bar{c}_t) = E_0 \sum_{t=0}^{1} \beta^t u(\tilde{c}_t)$$

or,

$$\begin{aligned}
&\log((1+\omega(i))\bar{c}_1) + \beta \left(q(i)\log((1+\omega(i))\bar{c}_2^e) + (1-q(i))\log((1+\omega(i))\bar{c}_2^u)\right) \\
=&\log(\tilde{c}_1) + \beta \left(q(i)\log(\tilde{c}_2^e) + (1-q(i))\log(\tilde{c}_2^u)\right) \\
\Rightarrow& \\
&\log(\bar{c}_1) + \beta \left(q(i)\log(\bar{c}_2^e) + (1-q(i))\log(\bar{c}_2^u)\right) + (1+\beta)\log(1+\omega(i)) \\
=&\log(\tilde{c}_1) + \beta \left(q(i)\log(\tilde{c}_2^e) + (1-q(i))\log(\tilde{c}_2^u)\right) \\
\Rightarrow& \\
&\bar{V}(i) + (1+\beta)\log(1+\omega(i)) = \tilde{V}(i) \\
\Rightarrow& \\
\omega(i) =& \exp\left(\frac{\tilde{V}(i) - \bar{V}(i)}{1+\beta}\right) - 1.
\end{aligned}$$

Appendix C

Computational Methods

C.1 Computational Methods: Chapter 1

This section describes in detail the computational methods I use to solve the model. Note: Ω is the three dimensional state vector over assets, skill level, and labor market state. First, a note on the state space. The grid for assets a is split into 120 unequally-spaced nodes between 0.0001 and 500. Most of the nodes are concentrated at the lower-end of the grid to adequately capture the curvature of the value function for the poorest agents.

C.1.1 Equilibrium

To solve for the model's equilibrium,

1. Start with initial guesses for

   - Household and firm value functions: $E(\Omega)$, $U(\Omega)$, and $J(\Omega)$
   - Decision rules $c(\Omega)$, and $a'(\Omega)$.
   - The invariant distribution f.
   - All of the endogenous variables that households and firms need to solve their respective problems: the income tax rate τ, the interest rate r, labor market tightness θ, and the vector of wages $w(\Omega)$. From θ, calculate $q_W(\theta)$ and $q_F(\theta)$.

2. Given these guesses, solve the household's problem at each cell in the state space via value function iteration. By substituting in the budget constraint, the household's Bellman equations can be rewritten as optimization problems with only one choice variable, savings. I then use Brent's method on the first-order condition of the Bellman equation with respect to assets. The continuation value is interpolated using monotonic cubic Hermite splines with respect to assets, and by linear splines in the other two directions. Monotone cubic Hermite splines are shape-preserving while also allowing for smooth derivatives. Repeat this until the value functions converge to within a chosen level of tolerance. This process yields new value functions and decision rules.

3. Solve the firm's problem. Using the current guess for r, the firm i's optimal choice of capital is

$$k_i^* = x_i \left(\frac{r}{\alpha}\right)^{1/(\alpha-1)}.$$

   Plugging this in to the firm's value, I iterate on the equation:

$$J(\Omega) = (k_i^*)^\alpha x_i^{1-\alpha} - rk_i^* - w(\Omega) + \left(\frac{1}{1+r'-\delta_k}\right)\phi\mathbb{E}(J(\Omega'))$$

   Note that the expectation operator is over two dimensions in the state space: with probability δ_e, the employee's skills will increase from level x_j today to level x_{j+1} tomorrow. Also, the employee's assets will be different tomorrow, according to his own policy function, which affects tomorrow's wages. As this step involves iteration only, not optimization, it is relatively quick. Since $V = 0$ in equilibrium, we can ignore this term of the firm's problem (hence why we do not need an initial guess for V).

4. Calculate the economy's invariant distribution. The procedure for this follows Young (2010). In short, I am re-allocating mass in today's distribution to tomorrow's distribution using the transition probabilities among the various states, repeating until f converges.

   To fix ideas, consider an agent with assets a_j, skill level x_j, and who is employed today, where a_j is some value in the grid for assets, and x_j is some value in the grid for skills. Let $\tilde{f}_t$ denote this agent's probability mass today, $\tilde{f}_t = f_t(a_j, x_j, 0)$. This agent's mass is allocated to tomorrow's distribution in the following manner. First, note that since assets can take on any Real value, the value of his assets tomorrow, a_{t+1}, is *almost surely* not one of the grid points. Instead, write a_{t+1} as a linear combination of its two surrounding grid points: $a_{t+1} = ma_0 + (1-m)a_1$, where $m \in [0,1]$ and $a_{t+1} \in [a_0, a_1]$. Next, recall the probability this agent's skills increase (conditional on staying employed) is δ_e, and the probability that he stays employed is ϕ. Hence, $\tilde{f}_t$ is allocated to the following cells in tomorrow's state space:

$$\begin{aligned}
f_{t+1}(a_0, x_j, 0) &= m(1-\delta_e)\phi\tilde{f}_t, \\
f_{t+1}(a_1, x_j, 0) &= (1-m)(1-\delta_e)\phi\tilde{f}_t, \\
f_{t+1}(a_0, x_{j+1}, 0) &= m\delta_e\phi\tilde{f}_t, \\
f_{t+1}(a_1, x_{j+1}, 0) &= (1-m)\delta_e\phi\tilde{f}_t, \\
f_{t+1}(a_0, x_j, 1) &= m(1-\phi)\tilde{f}_t, \\
f_{t+1}(a_1, x_j, 1) &= (1-m)(1-\phi)\tilde{f}_t.
\end{aligned}$$

This process is repeated for all agents.[38] Given certain conditions, f will eventually converge to the economy's invariant distribution.

5. Use market clearing conditions and the new guess for f to compute new values for the above endogenous variables $(\tau, r, \theta, w(\Omega))$.

   Labor market tightness

   For θ, I use the value of a posted vacancy V (Equation (1.6)), which, using the fact that $V = 0$ in equilibrium, can be solved analytically for θ. Rearranging the vacancy condition,

$$\gamma = \left(\frac{1}{1+r-\delta_k}\right) \sum \frac{e(s)f(\Omega)}{u} q_F(\theta) J(\Omega').$$

   Note the Ω' in the continuation value: the match begins next period, so I must

[38] Note that in this particular example, x_j is assumed to be below the upper bound on skills; otherwise, they could not increase.

take the expecation over the worker's type tomorrow. With a Cobb-Douglas matching function, this becomes

$$\gamma = \left(\frac{1}{1+r-\delta_k}\right) \sum \frac{e(s)f(\Omega)}{u} \theta^{-\sigma} J(\Omega'),$$

$$\gamma(1+r-\delta_k) = \frac{\theta^{-\sigma}}{u} \sum e(s)f(\Omega)J(\Omega').$$

Letting $\widehat{J}$ denote the summation term, θ can therefore be solved as

$$\theta = \left(\frac{\widehat{J}}{\gamma u(1+r-\delta_k)}\right)^{1/\sigma}.$$

Note that in this step, we have already calculated a new value for u, using the new value for f, from Step 4

$$u = \sum_{s \geq 1} e(s)f(s).$$

Intuitively, if $V > 0$, firms will enter the market, so the current guess for θ is too low, and if $V < 0$, firms will leave the market, so the current guess for θ is too high. From the new guess for θ, calculate new guesses for $q_W(\theta)$ and $q_F(\theta)$.

Wages

The wage vector $w(\Omega)$ is updated for every element in the state space using Nash bargaining, as shown in Section 1.2.4. Define the 'auxiliary' problem faced by an employed worker with state vector Ω, *given* a current wage offer of $\tilde{w}$, as:

$$\widetilde{E}(\Omega, \tilde{w}) = \max_{c_t, a_{t+1}} \left\{ u(c_t) + \beta[\phi E(\Omega') + (1-\phi)U(\Omega')] \right\}$$

subject to

$$c_t + a_{t+1} = (1+r_t-\delta_k)a_t + (1-\tau)\tilde{w},$$

Let the solution (policy function) to this problem be $a_{t+1} = \tilde{\psi}(\Omega, \tilde{w})$.

Next, define the auxiliary problem for the firm, given a current wage of $\tilde{w}$, as

$$\widetilde{J}(\Omega, \tilde{w}) = (k_i^*)^\alpha x_i^{1-\alpha} - r_t k_i^* - \tilde{w} + \left(\frac{1}{1 + r_{t+1} - \delta_k}\right)\left[\phi J(\widetilde{\Omega}') + (1-\phi)V\right],$$

where $\widetilde{\Omega}'$ is the state vector implied by the solution to the employee's auxiliary problem; that is, using $a_{t+1} = \tilde{\psi}(\Omega, \tilde{w})$.

So, the wage $w(\Omega)$ is the value that maximizes the Nash product $P(\Omega, \tilde{w})$:

$$w(\Omega) = \underset{\tilde{w}}{\text{argmax}}\left\{P(\Omega, \tilde{w}) = \left(\widetilde{E}(\Omega, \tilde{w}) - U(\Omega)\right)^\eta \widetilde{J}(\Omega, \tilde{w})^{1-\eta}\right\}.$$

Note that I am using the fact that $V = 0$ in equilibrium to omit this term. To guarantee that each side's match surplus is positive, I impose upper and lower bounds on the wage. For the upper bound, I impose that the firm will never pay a wage that makes their period profit negative. Hence,

$$w^{UB}(\Omega) = (k_i^*)^\alpha x_i^{1-\alpha} - r k_i^*$$

This condition is sufficient to guarantee that J is nonnegative, since J is simply the discounted sum of per-period profits.

The lower bound on wage is calculated to ensure that the worker's match surplus, $E(\Omega) - U(\Omega)$, is nonnegative. That is, $w^{LB}(\Omega)$ is such that the condition

$$\widetilde{E}(\Omega, w^{LB}) - U(\Omega) = 0$$

is satisfied. Note that w^{LB} may be less than the UI payment this agent would receive. This is due to two factors: first, the fact that wages increase (in expected value) over the life of the match whereas the UI payment decreases (to zero), and second, the fact that, under the model's parameterization, the chance of keeping a job (the E-E hazard) is higher than the probability of finding a job (the U-E hazard). Intuitively, an employed worker would be willing to give up some consumption *in the current period* for the privilege of being employed, which is still a better state to be in compared to unemployment, when considered over the lifetime of that state.

I use the Golden Section method on the Nash product $P(\Omega, \tilde{w})$.

Income tax rate

Recall the government's budget constraint:

$$\sum_{s=0} \tau w(\Omega) f(\Omega) + \sum_{\mathcal{A}} \tau w(\Omega) f(\Omega) = \sum_{\mathcal{A}} w(\Omega) f(\Omega) + \sum_{\mathcal{B}} \psi f(\Omega).$$

The new guess for τ is therefore

$$\tau = \frac{\sum_{\mathcal{A}} w(\Omega) f(\Omega) + \sum_{\mathcal{B}} \psi f(\Omega)}{\sum_{s=0} w(\Omega) f(\Omega) + \sum_{\mathcal{A}} w(\Omega) f(\Omega)}$$

Interest rate

Finally, I compute a new guess for r according to the asset market equilibrium condition. If the total supply of assets (household savings, the left-hand side) exceeds the demand (firm demand for capital plus equity, the right-hand side), then the interest rate is too high, and vice-versa. Formally, I solve the following equation:

$$A'_S = X_E \left(\frac{r}{\alpha}\right)^{1/(\alpha-1)} + b/(r - \delta_k).$$

This equation is derived in more detail below, in Appendix C.1.3. I use Brent's Method to find the root of this equation.

6. We have now obtained new guesses for $(\tau, r, \theta, w(\Omega))$. Using these, repeat from Step 2 until these variables converge to within a chosen level of tolerance.

C.1.2 Transition path

In order to compute the welfare effects of the search assistance policy scheme, I compute the full transition path from one steady state to the other. At time $t = 0$, the economy is at its initial steady state, denoted $SS0$, with no search assistance programs. At time $t = 1$, the government announces that *from that point onward*, all unemployed agents who have been out of work for at least five quarters will be enrolled in the search assistance program. Eventually, at time $t = T$, the economy settles to a new steady state, denoted $SS1$.

The algorithm to compute the transition path is as follows.

1. Choose a value of T that is large enough to allow the economy to converge to the second steady-state. I use $T = 1,000$.

2. Solve for the two steady states individually. $SS0$ corresponds to $t = 0$ and $SS1$ corresponds to time $t = T$.

3. Guess a path for all of the variables that households and firms need to solve their respective problems; namely, r_t, τ_t, θ_t, and $w_t(\Omega)$. From θ_t, we can calculate $q_W(\theta_t)$ and $q_F(\theta_t)$.

4. Starting at period $T - 1$, solve the household's problem *backwards*. We solve backwards since we already have the value function at time T. Solve the household's problem all the way back to period 1. This step yields updated values for the value and policy functions for every period.

5. In a manner identical to Step 4, solve the firm's problem. This yields updated values for the firm's value function.

6. Starting at time $t = 2$, use the transition probabilities among the various states to calculate the economy's density from $t = 2$ to $t = T$. Note that the density function at time $t = 1$ must be adjusted to reflect the change in asset holdings due to the change in equity price the instant the policy is announced, as explained above. This adjustment only affects the asset dimension of the density function; i.e., the *marginal* distributions for skills, duration, and labor market state are unchanged.

7. Using the various market clearing conditions, calculate new values for r_t, τ_t, θ_t, and $w_t(\Omega)$. For τ_t, θ_t, and w_t, computation follows in the exact same manner as with the steady state, being careful with time subscripts. For example, the equation for θ_t involves r_{t+1}, not r_t. To save time, I perform Nash bargaining

only for the first 100 periods. For periods 101 to T, I interpolate based on the revenue-sharing rules implied by the Nash bargaining solutions at $t = 100$ and $t = T$.

For the interest rate and equity price, the algorithm is as follows. First, take p_T and r_T (the values at the endpoint) as given. Next, calculate d_t for the entire path. Then, recalculate p_t, K_t, and r_t by looping *backwards* from $t = T - 1$ to $t = 1$:

(a) $p_t = \dfrac{p_{t+1} + d_{t+1}}{1 + r_{t+1} - \delta_k}$,

(b) $K_{t+1} = A_{t+1} - p_t$,

(c) $r_{t+1} = \alpha \left(\dfrac{K_{t+1}}{X_{t+1}^E} \right)^{\alpha - 1}$,

where K_t is the total amount of physical capital at time t, X_t^E is the total employed human capital at time t, and A_t is total assets at time t. Note that we will get a new estimate for p_0 (i.e., the period before the policy change):

$$p_0 = \frac{p_1 + d_1}{1 + r_1 - \delta_k}$$

Let $\tilde{p}_0$ be the asset price from the original equilibrium, and $\tilde{A}_1$ be total assets from the original equilibrium:

$$\tilde{A}_1 = K_1 + \tilde{p}_0.$$

To ensure asset market equilibrium in every period, A_1 will need to be adjusted by the amount $p_0 - \tilde{p}$:

$$\begin{aligned} \Delta A_1 &= A_1 - \tilde{A}_1 \\ &= K_1 + p_0 - K_1 - \tilde{p}_0 \\ &= p_0 - \tilde{p}_0. \end{aligned}$$

Note that this relies on the fact that K_1 is fixed. In my case, this adjustment is a decrease, as the new updated value for p_0 is less than $\tilde{p}_0$. As noted before, this adjustment only affects the economy's distribution in the asset dimension.

8. Repeat until variables converge in all periods.

C.1.3 Asset market equilibrium

Recall the definition of the composite asset a_t, where k_t^h is physical capital, b_t is the quantity of firm equity, and p_t is the price of equity:

$$a_t \equiv k_t^h + p_{t-1} b_t$$

Summing over all households, equilibrium in the asset market is characterized by:

$$\begin{aligned} \sum a'(\Omega) f(\Omega) &= \sum k^h f(\Omega) + \sum p b f(\Omega) \\ &= \sum k^f f(\Omega) + p \sum b f(\Omega) \end{aligned}$$

The second line uses the fact that $\sum k^h = \sum k^f$; that is, the total amount of capital owned by households is equal to the total amount used by firms. Also, I am able to drop time subscripts because, in steady state, variables are constant. The left-hand side is the total quantity of assets supplied by households, and the right-hand side is the total quantity of assets demanded by firms, split between physical capital and equity. The total quantity of firm equity is not identified in the model, so without loss of generality we can normalize this to one. Doing so, the above becomes

$$\sum a'(\Omega) f(\Omega) = \sum k^f f(\Omega) + p$$

Next, recall the no-arbitrage condition (Equation (1.2)). Because p_t is constant in steady state, the price of equity can be written as

$$p = \frac{d}{r - \delta_k},$$

where d is the per-period firm dividend, which is defined as per-period profits π minus vacancy costs γv. So, the asset market equilibrium condition is

$$\sum a'(\Omega)f(\Omega) = \sum k^f f(\Omega) + d/(r - \delta_k)$$

Recall that firm i's demand for physical capital is

$$(k_i^f)^* = x_i \left(\frac{r}{\alpha}\right)^{1/(\alpha-1)},$$

Summing over all firms yields the total amount of capital stock demanded by all firms in operation:

$$\sum_{s=0}(k_i^f)^* f(\Omega) = \sum_{s=0} x_i f(\Omega) \left(\frac{r}{\alpha}\right)^{1/(\alpha-1)}$$

For notational simplicity, let K_D and X_E represent the two summation terms in the above equations. Then, re-write as:

$$K_D = X_E \left(\frac{r}{\alpha}\right)^{1/(\alpha-1)},$$

Next, let A'_S be the total quantity of assets supplied by households; that is, the sum of households' savings:

$$A'_S = \sum a'(\Omega)f(\Omega$$

Hence, the asset market equilibrium condition can therefore be written as

$$A'_S = X_E \left(\frac{r}{\alpha}\right)^{1/(\alpha-1)} + d/(r - \delta_k). \tag{C.1}$$

The right-hand side is strictly decreasing in r, so r can be solved for using numerical methods.

C.2 Computational Methods: Chapter 2

This section describes the computational methods I use to solve the model in Chapter 2. Broadly, the solution involves two layers; first, the choice of optimal search intensity or job-finding rate (depending on version), and within this, the individual households' optimization problems. The solution method is as follows.

First, initialize variables. Given the assumptions of the state space and the density function f, we know $w(i)$ and $\bar{\psi} = 0.21\bar{w}$, along with the baseline tax rate, and the baseline value for u. Furthermore, choose baseline values for θ and $v = u\theta$. The choice of θ is arbitrary; however, given that $\bar{e}(i)$ comes from the model in Chapter 1, it is most appropriate to choose 2.4 for the baseline value of θ, as this is the baseline value from Chapter 1. Note that v is fixed.

Next, call the optimizer routine, which chooses ρ^* or q^*. I use the *FFSQP* optimizer routine in FORTRAN. Within this, the procedure depends on which model we are solving:

1. **Model #1**

   The optimizer guesses values of $\rho(i)$ for each of the 18 unemployed agent types. At each guess, find the u implied by these:

   $$u = \sum_{s \geq 1} \rho(i)\bar{e}(i)f(i).$$

   And the job-finding probability:

   $$q(i) = \rho(i)\bar{e}(i)v^{1-\sigma}u^{\sigma-1}$$

   bounding below 1 if necessary.

2. **Model #2**

   The optimizer guesses values for the Logit rule coefficients. The jobfinding rates implied by these are

   $$q(i) = \frac{\exp(t(i))}{1 + \exp(t(i))}, \quad \text{where}$$

   $$t(i) = \beta_0 + \beta_1 a(i) + \beta_2 x(i) + \beta_3 s(i) + \\ \beta_4 a(i)x(i) + \beta_5 a(i)s(i) + \beta_6 x(i)s(i) + \beta_7 a(i)x(i)s(i).$$

Determine the ρ's implied by these q's:

$$\rho(i) = \frac{q(i)}{\bar{q}(i)} \frac{u_1^{1-\sigma}}{u_0^{1-\sigma}}, \text{ where}$$

$$u_1 = \sum_{s \geq 1} \rho(i) \bar{e}(i) f(i).$$

Starting from an initial guess of each $\rho(i)$, iterate on the above until the $\rho(i)$'s and u_1 converge.

3. **Model #3**

   The optimizer guesses values for the polynomial rule coefficients. Search assistance intensity is

$$\rho(i) = max\{1, \tilde{\rho}(i)\}, \text{ where}$$

$$\tilde{\rho}(i) = \beta_0 + \beta_1 a(i)^{\beta_2} + \beta_3 x(i)^{\beta_4} + \beta_5 s(i)^{\beta_6}.$$

   As with Model #1, these imply a value of u and $q(i)$; see Step 1 above.

Regardless of model version, proceed with solving the household's problem. First, update the tax rate τ:

$$\tau = \frac{\sum_{s \geq 1} \psi(\rho(i)) f(i)}{\sum_{s=0} w_1(i) f(i) + \hat{w} \sum_{s \geq 1} f(i)}$$

Finally, given the current guess of $\rho(i)$, $q(i)$, and τ, solve the household's optimization problem. I solve for c_1^* by using Brent's method on the first-order conditions with respect to c_1. This implies values for c_2^{e*} and c_2^{u*}. The planner's objective function is the sum of agents' value functions:

$$\sum_i V(i) f(i)$$

Repeat until the solver converges.

C.3 Computational Methods: Chapter 3

This section describes the computational methods I use to solve the model in Chapter 3. These methods are broadly similar to the other Chapters, so I omit many details. The solution proceeds as follows.

1. For each matching function (CES and Cobb-Douglas), solve for the baseline steady-state. The baseline has no search assistance ($\rho = 1$). In this step, I calibrate ψ and $\tilde{q}$ so that model moments match target moments. To solve for the baseline:

   (a) Start with initial guesses for value functions and all endogenous quantities needed to solve the household's problem.

   (b) At each point in the state space, solve the household problem, which can be expressed in terms of just two choice variables, savings a' and search effort e. I use the $FFSQP$ solver routine in FORTRAN. Repeat until the value functions converge to within a chosen level of tolerance.

   (c) Calculate the invariant distribution. The procedure is identical to Chapter 1; see Section C.1.

   (d) Given the decision rules for search effort and the new guess for f, update guesses for u, θ, and q

   (e) Repeat until convergence.

   As noted, in this step we are calibrating ψ so that $\bar{e}$ (mean search effort) equals 0.4 and calibrating $\tilde{q}$ so that $\bar{q}$ (mean job-finding rate) equals 0.5321.

2. Next, increase ρ in increments of 0.01 from 1 to 1.25, solving four versions of the model:
   (1a) CES matching function, with effort $e(i)$ chosen endogenously,
   (1b) CES matching function, but $e(i)$ is fixed at the baseline value,
   (2a) Cobb-Douglas matching function, with effort $e(i)$ chosen endogenously,
   (2b) Cobb-Douglas matching function, but $e(i)$ is fixed at the baseline value.

Within each of these, the procedure is the same as above, *except* we are keeping ψ and $\tilde{q}$ fixed at their values calibrated from the baseline. Note that versions (1b) and (2b) are significantly quicker to solve relative to (1a) and (2a), as they only involve one choice variable instead of two.

3. From these, we calculate welfare gains (or losses) relative to the baseline economy. Recall, welfare change $\omega(i)$ is given by

$$\omega(i) = \exp((\tilde{V} - \bar{V})(1 - \beta)) - 1,$$

where $\bar{V}$ is agent i's value function at the baseline economy ($\rho = 1$) and $\tilde{V}$ is this agent's value function when $\rho > 1$.

Appendix D

Data Sources and Construction

D.1 Data sources: Chapter 1

This Appendix section gives more detail on data used in the paper.

- The **Percent of jobseekers who utilize some form of search assistance** (Introduction) is calculated from Current Population Survey (CPS) microdata. The survey asks whether a non-employed respondent has done anything to find work during the last 4 weeks, and if so, which methods. The respondent is allowed to choose up to 6 methods out of 13 choices. I count any non-employed respondent who says they have have utilized any one (or more) of the following four methods:
  - "Contacted public employment agency" (Choice 2)
  - "Contacted private employment agency" (Choice 3)
  - "Contacted school/university employment center" (Choice 5)
  - "Attended job training programs/courses" (Choice 11)

  Over the sample period of January 2001 to July 2017, 26.6 percent of non-employed respondents report using at least one of these methods.

- **Labor share of income** is defined as Total Compensation of Employees (Line 2 of NIPA Table 1.12) as a fraction of National Income (Line 1 of NIPA Table

1.12). Both series are nominal. Source: U.S. Dept. of Commerce, Bureau of Economic Analysis (BEA). I use quarterly data and average over 1947:Q1 to 2016:Q1.

- The **UI replacement rate** is the percent of a recipient's previous earnings that he receives as UI. I use the national average replacement rate at a quarterly frequency, and average over the period 1997:Q1 to 2016:Q1. This series is fairly constant over this range, so changing the endpoints has no substantive effect. Source: U.S. Dept. of Labor, Employment and Training Administration. See `http://www.oui.doleta.gov/unemploy/ui_replacement_rates.asp`.

- The **job retention rate** (ϕ) comes from Fallick and Fleischman (2004). The authors use CPS microdata to calculate gross worker flows starting in January 1994, and continue to update the data to the present. Their data can be found at `https://www.federalreserve.gov/pubs/feds/2004/200434/200434abs.html`. I take the average of their $EUhaz$ (employed-to-unemployed) variable over the period September 1995 to December 2007.

- The **per-person search assistance cost** (ψ) comes from Hujer, Thomsen, and Zeiss (2006), and from the OECD. First, the authors estimate the German search assistance program costs €538 per person per month, or €1,614 per person per quarter. Next, the average annual earnings for Germany come from the OECD statistical database `https://stats.oecd.org/Index.aspx?DataSetCode=AV_AN_WAGE`. For the year 2004, the average annual wages for Germany were €30,072, or €7,518 per quarter. Hence, 1614/7518 = 0.2147 of the average quarterly wage.

D.2 Data sources: Chapter 3

I use CPS public-use monthly microdata files from January 2001 to December 2017, downloaded from `https://thedataweb.rm.census.gov/ftp/cps_ftp.html`. I only keep observations who report being unemployed in the interview month (`pemlr` $\in \{3, 4\}$). In addition to variables already in the data, I construct a variable indicating duration of unemployment in quarters, and recode education into four categories: 1) high school graduate or equivalent, 2) some college but no degree or Associate's degree, 3) Bachelor's degree, 4) Master's, Doctorate, or other professional degree. I then construct a variable indicating whether each of the 13 search method choices was reported as any of the person's 6 chosen methods. After merging across all months, the data set contains over 700,000 household-month observations.

Appendix E

Alternative Calibrations and Specifications

E.1 Alternative calibrations: Chapter 1

In this section, I consider an alternate specification for e, individuals' matching efficiency, so that it depends on skill level x in addition to unemployment duration. Denote this $e(x,s)$:

$$e(x,s) = x^{\mu}\tilde{e}(s).$$

Note that if $\mu = 0$, this corresponds to the baseline case, and $\tilde{e}(s)$ is equal to $e(s)$ from the main text. Now that an individual jobseeker's search efficiency depends on both his skill level and his unemployment duration, it becomes natural to think of the concept of duration dependence, of which $\tilde{e}(s)$ is an estimate. Papers such as Kroft et al. (2013) find that, even when controlling for a job applicant's qualifications, there is evidence of negative duration dependence in jobfinding rates. Specifically, these authors send out many fake resumes to potential employers. They find that, conditional on having the same observable qualifications (in terms of education and experience, etc.), the callback rate is a decreasing function of the applicant's unemployment duration. To see that $\tilde{e}(s)$ is a measure of duration dependence, consider an unemployed agent with skill level $\hat{x}$ and unemployment duration $\hat{s}$. Even if he experiences no skill loss this period, he can expect his job-finding chance to be lower next period by the amount

$$\begin{aligned}\Delta q_W(x,s) &= q_W(x,s+1) - q_W(x,s) \\ &= \theta^{1-\sigma} x^{\mu} (\tilde{e}(s+1) - \tilde{e}(s))\end{aligned}$$

In this section, I consider two values for μ: 0.5 and 1. For each of these, I re-calibrate the model, solving for a new vector of $\tilde{e}(s)$ so that the model delivers exit rates equal to the data in equilibrium. Broadly speaking, the results are very similar, both qualitatively and quantitatively. The first panel of Table 6 shows the per-person average welfare effect for the full GE policy (Column 1) and the four decompositions (Columns 2 to 6) for the 'standard' calibration, i.e., with $\mu = 0$. The next two panels show the exact same statistics for $\mu = 0.5$ and $\mu = 1$, respectively. As shown in the table, the differences in results between the three values of μ are relatively small. Note that the welfare effects shown in the table are expressed as percents, so differences between the panels are on the order of tenths and hundredths of a percent. For example, the pure partial equilibrium effect for policy participants ranges between 1.8 percent and 2.3 percent. As such, regardless of the choice of μ, the qualitative story of the results is unchanged.