Using the Concrete-Representational-Abstract Instructional Framework to Support Students with Disabilities and Mathematics Difficulties in Inclusive Settings: An Investigation of Teacher Knowledge and Practice

Susan Aigotti, Ph.D. Candidate

University of Virginia School of Education & Human Development

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“For those I have loved, or who have been so lenient and gracious as to have loved me, I have not words enough here, and I remember with gratitude how they have made me speechless in return.” – Christopher Hitchens

Abstract

The persistent achievement gap in mathematics between students with disabilities (SWD) and their non-disabled peers underscores the need for effective instructional strategies in inclusive classrooms. This dissertation investigates how third- through fifth-grade general education teachers conceptualize and enact the Concrete-Representational-Abstract (CRA) instructional framework, an evidence-based practice (EBP) widely endorsed for improving mathematical outcomes for SWD and students with mathematical difficulties (MD). Despite the empirical support for CRA, it remains unclear how general education teachers implement this strategy and whether modifications are made that might compromise its effectiveness. Through a qualitative study involving interviews and artifact analysis, this research explores teachers' understanding and reported use of CRA in inclusive settings. The findings reveal variations in teachers' conceptualization of CRA and their application of the framework in practice, potentially influenced by factors such as professional learning, confidence, and access to resources. The study highlights the gap between research and practice, offering insights into the barriers that hinder the adoption and faithful implementation of EBPs like CRA in general education classrooms. These results have implications for the design of professional development, teacher education, and policy, aiming to enhance the instructional quality and mathematical proficiency of all students, particularly those with disabilities.

Table of Contents

Abbreviations

ADHD	Attention Deficit Hyperactivity Disorder
CCSS	Common Core State Standards
CEC	Council for Exceptional Children
CRA	Concrete-Representational-Abstract
CRA-I	Concrete-Representational-Abstract-Integrated
EBD	Emotional Behavioral Disorder
EBP	Evidence-Based Practice
IES	Institute for Educational Sciences
MD	Mathematical Difficulty
NAEP	National Assessment for Educational Progress
NCES	National Center for Educational Statistics
NCII	National Center for Intensive Interventions
NCTM	National Council for Teachers of Mathematics
SLD	Specific Learning Disability
SWD	Student With Disability
US DoE	United States Department of Education
VDOE	Virginia Department of Education
VRA	Virtual-Representational-Abstract

List of Tables & Figures

Chapter 1: Introduction

Statement of the Problem

In the United States, at least 65% of students with disabilities (SWD) spend the majority of the school day in general education classrooms (National Center for Educational Statistics) [NCES], 2022a). Several federal policies, such as the Individuals with Disabilities Act (IDEA; 2004) and the Every Student Succeeds Act (ESSA; 2015), require educating SWD in general education classrooms whenever possible. IDEA mandates a free and appropriate public education in the least restrictive environment. Policymakers attempted to address the growing achievement gap between SWD and their non-disabled peers by including a provision in ESSA requiring states to include SWD in statewide assessments and report the test data annually (National Council on Disability, 2018).

However, neither access to a general education setting nor mandating SWD take common assessments guarantees better academic outcomes. The National Center for Education Statistics (NCES) has released 2022 assessment data for the National Assessment for Educational Progress (NAEP). Unfortunately, the results highlight that fourth- and eighth-grade students' mathematics achievement fails to meet proficiency benchmarks (NCES, 2022b). Only 37% of fourth and 27% of eighth graders scored proficient or higher in mathematics (NCES, 2022b). The percentage of SWD, defined as having either an Individualized Education Program (IEP) or protection under Section 504 of the Rehabilitation Act of 1973 (NCES, 2022b), who scored at the proficient level is even smaller. Among SWD, only 16% of fourth and 7% of eighth graders scored proficient or higher. The most current results are similar to those in years past. Mathematics achievement scores for SWD have been significantly lower than their non-disabled peers for the past 20 years (NCES, 2022a), and the overall low percentage of non-disabled students meeting proficiency points to ongoing concern over the number of students experiencing mathematical difficulty (MD), even in the absence of a disability. Poor mathematics achievement levels are concerning because mathematics achievement is associated with better post-graduation opportunities (Moses & Cobb, 2001). Adults lacking mathematical competency struggle to find employment and complete daily tasks (Geary, 2011). We must increase the percentage of students meeting proficiency benchmarks in mathematics.

A plausible reason for this could be the shift in mathematics educators' methods and philosophy of instruction. Publications such as *The Final Report of the National Mathematics Advisory Panel* (NMAP, 2008) and *Adding it Up* (2001) by the National Research Council (NRC) urged for reforms to instructional practices and began a shift from focusing on procedural fluency to helping students develop conceptual understanding and reasoning skills (NMAP, 2008; NRC, 2001). Mathematics educators are encouraged to view their role as facilitators and provide opportunities to develop problem-solving strategies and struggle productively with mathematics problems. Unfortunately, students with learning disabilities are often unable to grapple with problem-solving without support. Many students with learning disabilities (LD) struggle unproductively because of cognitive overload from tasks that overtax their working memory, long-term memory, and executive functioning (Jitendra, 2013).

Due to its abstract nature, the general education mathematics curriculum can be challenging for students with disabilities (SWD) or those at risk for mathematical difficulties (MD) to access (Witzel & Little, 2016). Special education researchers have investigated strategies and interventions designed to minimize the barriers SWD or MD encounter in their general education classrooms (Doabler et al., 2015; Gersten et al., 2005; Jitendra & Star, 2011). The design of math interventions considers common challenges for SWD and those at risk for mathematical difficulties, such as working memory and executive functioning limitations. By accommodating these limitations, interventions support SWD need to engage in mathematical learning successfully (Geary, 2004; Passolunghi & Siegel, 2004; Peng et al., 2018). One intervention widely accepted in the special education community as an evidence-based practice (EBP) that benefits SWD and those at risk is the concrete-representational-abstract (CRA) framework (Agrawal & Morin, 2016; Bouck, Satsangi, et al., 2018; Lafay et al., 2019).

Mathematics education teachers (MET) and special education teachers (SET) share a common goal: providing evidence-based instruction that leads to a deep understanding of mathematical concepts and supports mathematical problem-solving. As noted, CRA is an EBP for mathematics instruction for SWD. However, despite the abundant empirical evidence and recommendations from the research community, professional teaching organizations, and governing bodies supporting the use of CRA to improve mathematical outcomes for SWD or MD, it is unknown the extent to which general education teachers choose to implement CRA or whether they are making modifications to critical elements of the intervention that could impact its efficacy. Teachers' knowledge of CRA or ineffective modifications could potentially explain why the results of recent national mathematics assessments indicate that many students in the US are not meeting proficiency benchmarks (NCES, 2022b). A recent study by Hott et al. (2019) reinforces this possibility. They conducted interviews and surveys to determine teachers' perceptions of algebra strategies and interventions. They found that many teachers were not using or were unfamiliar with appropriate EBPs to meet the needs of SWD or MD (Hott et al., 2019).

Research related to the research-to-practice gap and the curriculum enactment process also provides insight into why general education teachers might not provide the most effective instruction for SWD or MD. Teachers' adoption and implementation of EBPs are hindered by a negative perception and lack of accessibility of research (Cook & Cook, 2011; Foster, 2014; Hornby et al., 2013), ineffective professional learning (Koutselini, 2008), and a perception that EBPs do not align with knowledge, beliefs, or school context might hinder the decision to adopt an EBP or impact a teacher's enactment of an EBP (Cook et al., 2013; Foster, 2014; Greenwood & Abbot, 2001; Klingner et al., 2013; Vaughn et al., 2000). Furthermore, what teachers intend to teach and the enactment of their lessons are influenced by 1) their knowledge of the content, students, and practice (Hill et al., 2008; Remillard & Heck, 2014); 2) their confidence with the content, students, and practice (Fixsen et al., 2005; Remillard & Heck, 2014); 3) policies and the educational environment (Foster, 2014) and 4) access to resources and support (Remillard & Heck, 2014; Stein et al., 2007).

Purpose

In this chapter, I introduced a nationwide problem that impacts many students and teachers. General education teachers are responsible for providing mathematical instruction for SWD or MD. Despite the extensive research examining how to improve access to the general education curriculum, mathematics achievement scores continue to show little improvement. One explanation is that general education teachers may lack awareness of EBPs for mathematics instruction for SWD like CRA, or that general education teachers make modifications to CRA that reduce its effectiveness. Evidence suggests that a combination of barriers and factors influences teachers' decision to adopt EBPs or use them with fidelity.

This qualitative study explores how third- through fifth-grade general education teachers conceptualize CRA (an EBP) and how they report enacting CRA in their inclusive classrooms. I combined data collected from interviews and artifacts to understand the phenomenon better. My findings inform future attempts to design research that is effective and applicable to the teaching context general education teachers experience. Acknowledging the significance of the general education teacher's role in the educational outcomes of SWD or MD, I investigated the following research questions:

1. How do third- through fifth-grade general education teachers conceptualize the CRA strategy?
2. How do third- through fifth-grade general education teachers report enacting CRA to teach mathematics to SWD or MD in inclusive settings?

Significance

Exploring how teachers conceptualize and report enacting the CRA strategy will benefit students, teachers, school administrators, teacher educators, and the research community. From a top-down perspective, the research community will benefit from answers to my questions because they will have information that can guide the development of interventions more readily translated into authentic classroom experiences. They will also be able to make more informed decisions regarding translating their work to practitioners by designing dissemination plans that take into account the factors and barriers that surface during my analysis. Teacher educators will also benefit from this investigation because answers related to teacher conceptualization can guide their design of course instruction so that it focuses on key elements and decision-making that allow for adaptation without negatively impacting the intervention's effectiveness.

Both district and building-level administrators will have information that could assist them in addressing barriers and factors within the context of policies and environmental factors. My study results will impact teachers by funneling changes from researchers, teacher educators, and school administrators. They will gain knowledge and confidence from improved learning experiences. Adopting EBPs with fidelity will be easier with practices designed with their needs in mind and reducing negative influencing factors and barriers. Finally, the results from my study will have the greatest impact on all students, especially SWD or MD, by improving their access to high-quality instructional practices, such as CRA.

Summary

In this chapter, I introduce evidence justifying the need for a study investigating how general education elementary teachers conceptualize and report using and why they modify an EBP, such as CRA, to teach mathematics to SWD or MD. I also provide a clear problem statement and the research questions necessary to respond to the problem. In the next chapter, I review the body of literature concerning SWD or MD, EBPs, and CRA, as well as what we know about barriers to using EBPs and the enacted curriculum. I also provide a conceptual framework that guides the design and analysis of this study. In the third chapter, I describe the methods used for this investigation. It includes my research design, a description of the research context and participants, my plan for collecting data, a plan to analyze the data, and validity and reliability. I present the results from this study in the fourth and fifth chapters. The final chapter discusses my main findings in relation to other research as well as implications for future research.

Chapter 2: Literature Review

In this chapter, I first present present an overview and synthesis of the research relevant to my study. There are four main sections: 1) understanding evidence-based practices (EBPs) and making the case that the concrete-representational-abstract (CRA) strategy is an EBP, 2) a description of CRA and a research synthesis, 3) a description of the research-to-practice gap, and 4) a synthesis of the literature on how teachers enact curriculum. Second, I present the conceptual framework supporting my study of how third- through fifth-grade elementary teachers conceptualize CRA, how they modify CRA in their inclusive setting, and why they make decisions about modifying CRA.

Evidence-Based Practices

We must provide all students with access to high-quality mathematics instruction. Students who struggle with mathematics may or may not have disabilities impacting their executive functioning (Nelson & Powell, 2018). These challenges can make it difficult for them to access the general education mathematics curriculum. Students with disabilities (SWD) or mathematical difficulties (MD) benefit from EBPs and specialized instruction (Jitendra & Star, 2011; Nelson & Powell, 2018;) For a practice to be considered evidence-based, the research community considers multiple factors. Unfortunately, not all disciplines use the same criteria to determine what makes a practice evidence-based. Even within a discipline, competing organizations publish guides for making decisions. However, a practice backed by many high-quality studies demonstrating a significant effect on student learning is generally considered evidence-based (Cook & Cook, 2011).

CRA: An Evidence-Based Practice

There is a substantial body of research on CRA. Appendix A contains summaries for traditional CRA studies (Table 1), integrated CRA studies (Table 2), and traditional virtual-representational-abstract studies (VRA; Table 3) Special education researchers have conducted several systematic literature reviews to answer questions about EBPs for improving mathematical outcomes for SWD. For example, Jitendra et al. (2016) evaluated 25 studies to determine if using representations was an EBP for students with MD. After evaluating the quality of these studies, they determined that only 13 met the criteria for the high-quality or acceptable-quality category. About half of these studies included interventions involving both concrete and pictorial representations. Based on the quality of the study and significant effect sizes, the team concluded that using representations in mathematical interventions (like the CRA sequence) was an EBP. Bouck, Satsangi, et al. (2018) published their synthesis of 20 studies investigating the effectiveness of the CRA intervention for SWD. After reviewing all studies for methodological quality, they determined that the CRA intervention was an EBP when teaching computation or basic facts to elementary and middle school-aged students.

Lafay et al. (2019) reviewed 38 studies involving manipulatives to improve immediate learning, maintenance, and transfer for students with or at risk for a mathematical disability. They determined that there was enough evidence to tentatively support the claim that interventions utilizing manipulatives are an EBP for students struggling to learn mathematics or with a mathematical disability. Extending on the work of Lafay and colleagues, Peltier et al. (2020) conducted a meta-analysis of single-case research to determine for whom and under what conditions interventions involving manipulatives are effective. They examined 53 studies and found that interventions using manipulatives, such as CRA, were useful when implemented by researchers and teachers across various participant characteristics. Using manipulatives also improved outcomes across mathematics domains regardless of whether students used concrete or virtual manipulatives.

Support for using CRA as an instructional practice comes from several sources. To disseminate information from research into the hands of school leaders and teachers, researchers write practitioner-friendly papers to provide information about EBPs, recommendations for using them with certain populations of students, and implementation support. Professional organizations, such as the National Council for Teachers of Mathematics (NCTM) and the Council for Exceptional Children (CEC), have published recommendations to improve mathematical outcomes for students that draw heavily on components of the CRA sequence (McLeskey et al., 2017; Williams et al., 2018). Practice guides and online training from government-funded research centers such as the Institute for Educational Sciences (IES) and the National Center for Intensive Interventions (NCII) also incorporate elements of the CRA intervention into their recommendations for improving mathematics instruction. Support for CRA also comes directly from the United States Department of Education (US DoE), the federally endorsed Common Core State Standards (CCSS), and state education agencies (Berry et al., 2020b; CCSS, 2010; Fuchs et al., 2021).

Practitioner articles target teachers as their main audience. These articles aim to make recommendations and guide the implementation of EBPs in the classroom. Several published practitioner articles advise on implementing CRA for specific populations. Witzel et al. (2008) provided secondary teachers with clear and concise justification for using CRA and how it benefits students with learning disabilities. They present step-by-step procedures and resources to support implementation. While acknowledging the lack of research investigating mathematical strategies to meet the needs of students with emotional behavioral disorders (EBDs), Riccomini et al. (2008) and Peltier and Vannest (2018) provide teachers with research-based arguments for how CRA supports both the educational and emotional needs of this population of students. Like Witzel et al. (2008), the authors of both articles provide teachers with detailed examples, recommendations for adjusting the intervention, and resources to support using CRA in their classrooms. Agrawal and Morin (2016) provide detailed recommendations for using CRA to support the learning of students with mathematical disabilities across various mathematics topics (e.g., place value, computations, fractions, algebra). They also provide teachers with helpful tips for each intervention phase and conclude with a table of resources to support implementation.

Professional organizations make recommendations for teaching mathematics through publications such as Principles to Actions (NCTM, 2014) and High Leverage Practices (CEC; McLeskey et al., 2017). In Principles to Action, NCTM (2014) recommends seven effective practices: using and making connections between multiple representations (e.g., pictures, concrete materials, tables, graphs, numbers, and symbols). Furthermore, in 2018, NCTM published the work of Williams et al. (2018), The Mathematics Lesson-Planning Handbook, Grades 6-8, in which they advise teachers to use the CRA sequence to support the development of mathematical understanding. CEC published High Leverage Practices in 2017 (McLeskey et al., 2017). These practices covered collaboration, assessment, social/emotional/behavioral, and instruction. While not recommended by name, the High Leverage Practices include three elements of the CRA intervention: scaffolded supports, explicit instruction, and intensive instruction (including using visual aids and introducing mnemonic strategies).

The US DoE provides funding to support research leading to improved student educational outcomes. In addition, IES and NCII publish practice guides condensing research findings into clear recommendations or offer training in designing and implementing evidence-based interventions. For example, NCII (n.d.) provides training for implementing intensive interventions in mathematics. In Part 2 of Module 4, How Should Multiple Representations be Used within Intensive Intervention, the teachers watch videos and complete activities to improve their ability to identify appropriate concrete and virtual manipulatives and design CRA interventions to meet the needs of individual students (NCII, n.d). IES recently published Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades (Fuchs et al., 2021). Their recommendations also included key elements of the CRA sequence, such as providing students with systematic instruction that combines concrete and semi-concrete representations to support conceptual and procedural understanding.

The US DoE published a 2008 report titled Foundations for Success: The Final Report of the National Mathematics Advisory Panel (NMAP). The panel made research-based recommendations for improving mathematics education for all students. These recommendations include key elements of the CRA intervention, such as designing instruction that supports the mutually supportive development of conceptual and procedural understanding and combining visual representations with explicit instruction. Responding to the growing national concern over student achievement and the No Child Left Behind Act of 2001 (NCLB, 2002; mandating states establish and assess ambitious standards for all students), a collective of state governors, school board officials, and teachers created a universal set of rigorous English and mathematics standards known as the Common Core State Standards (CCSS; 2010). Currently, 41 states use the CCSS to guide teachers' instruction.

According to the CCSS, mathematically proficient students use multiple representations to demonstrate a conceptual understanding of mathematics and support procedural fluency for problem-solving (both key elements of CRA). Many mathematics standards throughout all grade levels embed this practice. Several states, like Virginia, drafted their standards in response to No Child Left Behind. The VDOE published a resource guide, Evidence-Based Specially Designed Instruction in Mathematics (Berry et al., 2020a), and its companion document, Students with Disabilities in Mathematics Frequently Asked Questions (Berry et al., 2020b). These documents strongly endorse using the CRA sequence to teach mathematics across all grade levels. Specifically, the VDOE recommends that teachers use concrete materials when introducing new skills or concepts and connect those concrete materials to pictorial and abstract representations. The VDOE also provides teachers with examples comparing CRA to the standard algorithm, links to video tutorials, and links to free virtual manipulatives.

CRA Explained

The CRA intervention is often presented in the literature as a gradual release model, meaning that the instructional sequence provides intensive support that is systematically removed to support student mastery and independence (Flores, 2009). It is a three-phase sequence in which students learn a new skill or concept first using concrete objects (manipulatives), then with representations (pictures), and finally using written computation of abstract problems (involving only numbers and symbols) without the assistance of either the manipulatives or pictures (Agrawal & Morin, 2016; Peltier & Vannest, 2018; Witzel et al., 2008). Typically, students need to demonstrate mastery, usually with criteria of at least 80% accuracy on a learning measure, to move from one intervention phase to the next. Explicit instruction is embedded in each phase and is considered an effective approach to mathematics instruction for students with learning disabilities (Archer & Hughes, 2010). It involves providing students with an advanced organizer, teacher modeling with think-alouds, guided practice with feedback, and independent practice to assess mastery (Archer & Hughes, 2010; Gersten et al., 2009; Hudson & Miller, 2006). It is common to embed strategy instruction between CRA's representational and abstract phases. Research suggests that explicitly teaching a cognitive strategy before the abstract phase facilitates students' ability to solve problems without the assistance of manipulatives or representations (Mancl et al., 2012). Other times, the teacher might embed a self-regulation strategy or teach problem types using schema-based instruction (Flores et al., 2016) before introducing CRA.

Theoretical Foundation for CRA

Cognitive and developmental psychology theories provide evidence of how CRA's key elements contribute to its effectiveness. The theory that learning progresses from concrete to abstract supports the design of CRA (Kolb & Kolb, 2017). It suggests that children draw on their experiences interacting with the physical world to support understanding abstract concepts (Bruner, 1966; Piaget, 1952). The CRA intervention also reinforces the mastery of both conceptual and procedural knowledge (Agrawal & Morin, 2016; Witzel, 2005). Conceptual knowledge is a deep understanding of a topic that allows students to connect prior knowledge and new information (Miller & Hudson, 2007; Schneider et al., 2011). Procedural knowledge is knowing the steps necessary to solve a problem or complete a task (Miller & Hudson, 2007). In sequential CRA instruction, the concrete and representation phases target conceptual understanding (Peltier & Vannest, 2018) by helping students make connections between something familiar (e.g., the manipulative or drawing) and the new skill (e.g., adding or comparing fractions). The lessons that bridge the representational and abstract phases target procedural knowledge.

A misconception about conceptual and procedural knowledge development is that they develop independently. It is more accurate to think of procedural and conceptual knowledge as a symbiotic relationship in which experiences supporting growth in one domain will strengthen the other (Rittle-Johnson et al., 2015). When implemented with fidelity, teachers make explicit connections between the familiar (concrete models) procedure and the unfamiliar (abstract) mathematical concept or skill to support SWD and MD (Agrawal & Morin, 2016; Miller et al., 2011).

Synthesis of the Research

Learning Disabilities

According to the National Center for Educational Statistics (NCES; 2022a), students with specific learning disabilities account for 32 percent of all students with disabilities. According to Part B, Sec. 300.8 (c) (10) of the Individuals with Disabilities Act (2004), a specific learning disability (SLD) is a learning processing disorder that affects the ability of a student to "listen, think, speak, read, write, spell, or do mathematical calculations" There is substantial research suggesting that the CRA sequence is effective for students with specific learning disabilities. For example, Maccini and Hughes (2000) used a multiple probe across participants design to determine the effectiveness of the concrete-semi concrete-abstract sequence for solving integer word problems. The intervention also included an instructional strategy called STAR ("Search, Translate, Answer, Review") to support problem-solving performance. Six high school students with documented learning disabilities participated in the study. A research team member implemented the intervention in a resource room with small groups of students. The manipulative used during the concrete phase of instruction was Algebra tiles. In Maccini and Hughes' (2000) study, participants used drawings of algebra tiles during the semi-concrete phase after demonstrating mastery with manipulatives. Assessment of student mastery occurred at least four times. The researchers collected follow-up data up to ten weeks after the intervention to assess the maintenance and transfer of skills to new tasks. Key findings from this study demonstrated that participants' problem-solving skills improved following each phase and that their strategy use also improved. In addition, the researchers found that students could generalize the skill to novel integer problems.

Similarly, Flores and Milton (2020) used a multiple probe across participants design to investigate the effects of the CRA sequence on improving students with learning disabilities' ability to use the partial products algorithm and solve word problems involving addition, subtraction, or multiplication. The participants were three students in fifth or sixth grade identified with either attention deficit hyperactivity disorder (ADHD) or SLD. A special education teacher met with students one-on-one to deliver 15 lessons three days per week: the first four lessons involved the teacher and students manipulating base-ten blocks to solve multiplication problems. During lessons five through eight, the base-ten blocks were replaced by drawings of tallies, lines, and squares. Before beginning the abstract phase, the teacher taught the students two strategies: FAST ("Find what you are looking for, Ask yourself, 'What are the parts of the problem,' Set up the numbers, and Tie down the sign") and RENAME ("Read the problem, Address the ones column, "Note the ones column, Address the tens column, Mark the tens, and Examine the columns and check") to help them solve equations and to help them determine the appropriate operation needed to solve a word problem.

In Flores and Milton's (2020) study, students completed problems without the assistance of manipulatives or representations for the remaining five lessons. Students were assessed three times throughout the intervention. The researchers also collected follow-up data up to three weeks after that intervention and again a year later for at least two participants. Flores and Milton found that instruction using the CRA sequence combined with the strategies FAST and RENAME was related to improved multiplication fluency, conceptual understanding, and procedural knowledge. Over time, the students maintained their learning.

Intellectual Disabilities and Autism Spectrum Disorder

Researchers have also investigated the use of CRA for students with other disabilities that impact a student's educational achievement. These disabilities include intellectual disabilities (ID), developmental delay (DD), autism spectrum disorder (ASD) and emotional behavioral disorders (EBD). Using a multiple probe across behavior study, Strozier and colleagues (2015) investigated the effects of CRA sequence combined with the DRAW ("Discover the sign, Read the problem, Answer with a conceptual representation, and Write the answer") and RENAME strategies for students with ASD. The participants were three elementary students in third or fourth grade. This study took place in an extended school year program sponsored by a university. A teacher with several years of experience delivered the intervention to all students in a small group over 20 days (about three weeks). Participants worked on mastering addition with regrouping, subtraction with regrouping, and multiplication facts 0 to 5 using base-ten blocks, drawing lines and tallies, and finally, only numbers and symbols. All students progressed at their own pace but had to master one skill before moving to the next. Students completed daily assessments during the intervention. The researchers found that the CRA sequence led to student mastery of computational skills and the ability to demonstrate a conceptual understanding of place value.

Extending this research, Hinton et al. (2020) conducted a study to investigate the effects of the CRA sequence on improving counting skills for students with and without disabilities in an inclusive setting. Participants included 24 students with and without disabilities in pre-kindergarten through second grade. About half of the students were diagnosed with ASD, ID, or DD. A researcher and several graduate students who were certified teachers implemented the intervention during a three-week summer camp sponsored by a research university. Students participated in CRA instruction for about 15 minutes daily, using colored bears during the concrete phase and ten frame flashcards during the representational phase. A key finding from this study is that students with and without disabilities showed significant improvement post-intervention, demonstrating that students with disabilities benefit from explicit CRA instruction and that their non-disabled peers benefit as well.

Mathematical Difficulties

Some students experience difficulties with mathematics that are not related to a disability. The literature defines students with MD as at-risk for a learning disability (Miller & Mercer, 1993), a student from a low socio-economic background performing below average according to teacher reports and state-wide assessment scores (Witzel et al., 2003), currently failing their math class (Flores, 2009), or receiving tier 2 (Flores et al., 2018) or tier 3 (Flores & Hinton, 2019) instruction in a failure prevention program. Several notable studies have examined the effectiveness of the CRA sequence for students with MD. Witzel et al. (2003) compared students' achievement with learning difficulties or disabilities in solving algebra transformations in two experimental conditions: instruction using the CRA model and instruction using repeated abstract lessons. SWD and MD struggle with mastering algebra content because of the abstract nature of this domain (Witzel & Little, 2016).

In Witzel et al.'s (2003) study, the participants were 34 pairs of secondary students labeled at-risk for difficulties or with a learning disability. A general education teacher provided the instruction for both members of the pair. However, one student received the treatment (instruction using the CRA sequence), and the other student received the control (typical math instruction). The intervention occurred in the general education classroom and consisted of 19 lessons designed to teach students to transform equations. Witzel and colleagues found that students in the treatment group performed better than students in the control group on algebra transformations and made fewer procedural errors when solving problems. Participants also demonstrated the ability to generalize the skill to related tasks and maintain the skill over time. However, it is not clear how long they waited to do follow-up testing.

Flores et al. (2018) continued to investigate the use of CRA for students with MD. Using a pretest-posttest design, they sought to determine the effectiveness of using CRA as a tier-two intervention to teach several related skills. Participants were 17 fifth-grade students who demonstrated weakness in solving computation problems with fractions. The researchers implemented the intervention over five weeks, meeting with students for 20 minutes four days per week. Three to five students worked in small groups in an empty classroom during an assigned intervention time. By incorporating real-world application problems into the intervention, the researchers elevated the rigor of the content. Flores et al. used a variety of manipulatives (e.g., fraction tiles, number lines, coins, base ten blocks and squares, and fraction blocks) and representations (e.g., area models, shading in squares, and pictures of coins on number lines). Participants made significant progress after instruction in their overall performance, problem-solving skills, and strategy use. Unfortunately, they did not report follow-up data to determine if the participants could maintain the skill post-intervention.

Integrated Approach

Maccini and Hughes (2000) recommended examining the CRA strategy to determine if changes in the order of phase delivery or elements of the treatment improve the strategy's effectiveness. They raised concerns over the efficiency of CRA interventions with a prescribed number of lessons. Several researchers have conducted studies investigating the effectiveness of an integrated approach to the CRA sequence (CRA-I; Bundock et al., 2021; Flores & Hinton, 2022; Morano et al., 2020; Scheuermann et al., 2009; Strickland & Maccini, 2013; Yakubova et al., 2016). The hypothesis is that CRA-I addresses concerns related to the amount of time traditional CRA requires, the lack of differentiation for individual progress and needs, and the fact that teaching each stage to mastery in isolation could limit conceptual and procedural knowledge development. Teachers or interventionists implementing CRA-I make explicit for students the connections between the concrete, representational, and abstract versions of a problem by presenting them simultaneously.

CRA-I is effective at improving mathematical outcomes for mathematical concepts and skills thought to be more abstract (e.g., rate of change, Bundock et al., 2021; solving equations, Scheuermann et al., 2009; multiplying linear expressions, Strickland & Maccini, 2013). At the same time, it is also effective at improving the outcomes in the domain of number sense and computation (Flores & Hinton, 2022; Morano et al., 2020; Yakubova et al., 2020). While still a growing line of research, the results of studies investigating the effects of CRA-I suggest that it is at least as effective for SWD as traditional CRA (Morano et al., 2020). Participants involved in CRA-I studies range from kindergarten (Yakubova et al., 2016) to ninth grade and have MD (Flores & Hinton, 2022), SLDs (Morano et al., 2020; Scheuermann et al., 2009; Strickland & Maccini, 2013), or ASD (Bundock et al., 2021; Morano et al., 2020; Yakubova et al., 2016).

Virtual Approach

While the VRA sequence research lacks the breadth of traditional CRA, it is still widely accepted as an EBP (Park et al., 2022). Typically, the VRA intervention uses the same procedures as the CRA sequence. The key difference between the two interventions is that in place of concrete manipulatives, the students work with virtual manipulatives through websites or applications like BrainingCamp (2020). Current research supports using virtual manipulatives in and outside the context of VRA interventions because they are as effective as concrete manipulatives for improving mathematical outcomes for SWD or MD (Bouck, Chamberlain et al., 2017; Peltier et al., 2020; Satsangi et al., 2016). There are several practical benefits to using virtual manipulatives instead of traditional manipulatives. One benefit is that students report feeling less stigmatized using virtual manipulatives, especially in secondary grades (Bouck, Bassette, et al.,2017; Bouck, Park et al., 2018; Satsangi et al., 2016).

Another benefit of using virtual manipulatives is access to a much larger range of options than are likely to be available in the traditional concrete form in any given classroom. The large variety of virtual manipulatives also makes the VRA sequence compatible with more complex math problems or skills (Bouck & Flanagan, 2009; Satsangi & Miller, 2017). The VRA strategy is known to be effective at improving outcomes in the domain of number sense and computation for both SWD and MD, including students with specific learning disabilities, attention deficit hyperactivity disorder, and intellectual disabilities (Bouck, Bassette et al., 2017; Bouck, Park, et al., 2018; Bouck et al., 2021). Participants in VRA studies range in age from second grade (Bouck et al., 2021) to eighth grade (Bouck, Bassette, et al., 2017; Park et al., 2020).

Limitations

There are gaps in the CRA literature related to the noticeable absence of a subset of participants. A noticeable exclusion is participants diagnosed with an Emotional and Behavioral Disorder (EBD). A systematic search of peer-reviewed literature yielded only two studies, including participants with an EBD diagnosis (Harris et al., 1995; Sealander et al., 2012). Across all studies, only three participants had a diagnosed EBD. The lack of representation in the literature is alarming due to the challenges in identifying EBDs in school-aged children (Forness et al., 2012), the co-occurrence of EBDs and ADHD (Landrum, 2011), and the growing number of students exposed to known risk factors associated with EBDs since the beginning of the COVID-19 pandemic (Hirsch et al., 2022).

Another noticeable gap limiting the generalizability of findings is the need for more interventions implemented in authentic settings. Many SWD spend 80% or more of their day in general education classrooms, receiving much of their instruction from a general education teacher (NCES, 2022b). However, much of the literature on CRA indicates the intervention takes place in non-inclusive spaces such as hallways (Bouck et al., 2017b), resource rooms (Flores, Hinton, & Schweck, 2014; Maccini & Hughes, 2000; Root et al., 2021), offices (Yakubova et al., 2016), conference rooms (Maccini & Ruhl, 2000), and university-sponsored programs (Flores, Hinton, Strozier, & Terry, 2014; Strozier et al., 2015). A systematic review of the literature resulted in only four studies conducted in general education settings (Flores & Milton, 2022; Harris et al., 1995; Witzel, 2005; Witzel et al., 2003). It is unknown if the presence of peers engaging in similar or alternate instructional activities moderates the effects of CRA on student outcomes.

Expanding on concerns over limitations related to the replicability of results to authentic classroom experiences, the lack of studies including teachers as implementors of the intervention is also concerning. Teachers are responsible for delivering high-quality instructions and interventions to all students. Nevertheless, general education teachers implemented the intervention in only four studies (Flores & Hinton, 2022; Harris et al., 1995; Witzel, 2005; Witzel et al., 2003), and special education teachers implemented the intervention in only five studies (Flores & Milton, 2020; Mancl et al., 2012; Miller & Mercer, 1993; Milton et al., 2019; Morano et al., 2020). Due to the few studies that have included teachers as implementors, it is difficult to determine if general education teachers think the intervention is useful for inclusive settings. Teachers' opinions regarding the efficacy and practicality of intervention procedures and components should inform the design of effective interventions that are perceived as useful (Hott et al., 2019).

Only Flores and Hinton (2022) asked general education teachers questions about the social validity of the intervention. The general education teachers involved in this study reported that the intervention was easy to use and effective and that they would recommend the intervention to other teachers and students. Special education teachers implementing the intervention were more likely to be asked questions related to the social validity of the intervention. Like the general education teachers, many had positive opinions of the intervention (Flores & Milton, 2020; Milton et al., 2019; Morano et al., 2020). The teacher in Flores and Milton (2020) reported that she would keep everything about the intervention the same. The special education teachers in Morano et al. (2020) thought the intervention was easy to implement, but some thought the lessons needed to be shorter.

Several studies examined social validity regarding the efficacy and positive opinion of the CRA intervention from participants' teachers. Several teachers reported that the intervention improved students' performance or confidence in mathematics class (Flores & Hinton, 2019; Flores, Hinton, & Strozier, 2014; Root et al., 2021). Others added that they would recommend using the intervention to other teachers or students (Flores, 2009; Flores, Hinton, & Schweck, 2014) and believed the intervention targeted important skills that benefited students (Root et al., 2021; Yakubova et al., 2016). Teachers of participants in the study conducted by Maccini and Hughes (2000) reported that they would be interested in learning more about the implementation if the researchers would follow up with training and support.

Despite the limitations, they do not discredit the previously reported benefits of the CRA intervention for both SWD and MD. Due to its abstract nature, the general education mathematics curriculum can be difficult for SWD or those at risk for MD to access (Witzel & Little, 2016). The CRA intervention considers common challenges for SWD or MD, such as limitations in working memory and executive functioning (Geary, 2004; Passolunghi & Siegel, 2004; Peng et al., 2018). Providing the support necessary to eliminate barriers to accessing the general education mathematics curriculum and allowing students to engage in mathematical learning successfully is a strength of the CRA intervention.

Research-to-Practice Gap

NCES has released 2022 assessment data for the National Assessment for Educational Progress (NAEP), and results indicate that fourth- and eighth-grade mathematics achievement is below basic proficiency (NCES, 2022b). The achievement scores for SWD were even more concerning. This trend has remained unchanged for the last twenty years (NCES, 2022a).

Implementation Fidelity

Despite the substantial body of research investigating EBP for improving mathematical outcomes for SWD and MD, the results of the most recent NAEP indicate that these practices and interventions are not translating into achievement. Research suggests that teachers are not implementing EBPs (like the CRA intervention) in their classrooms or do not implement them with fidelity (Biesta, 2007; Cook & Cook, 2011; Hott et al., 2019). Implementation fidelity is important across many fields of academic research. Researchers study the effectiveness of interventions intended to solve an existing problem. To ensure the validity and reliability of their findings, they designed implementation procedures and demonstrated that participants received the same treatment. The extent to which practitioners follow the procedures for delivering an EBP is known as implementation fidelity (Ledford & Gast, 2018). Without quality mathematical instruction, SWD or MD are unlikely to become proficient in the knowledge and skills necessary for advanced mathematics courses. Failure to master algebra limits opportunities post-graduation for future academic and career success (Moses & Cobb, 2001). The research-to-practice gap has been the topic of several articles advocating for changes that will lead to improved academic outcomes for SWD or MD (Cook et al., 2013; Foster, 2014; Greenwood & Abbot, 2001; Klingner et al., 2013; Vaughn et al., 2000).

Barriers to Implementation

One of the challenges associated with implementing an EBP is the nature of research. According to Greenwood and Abbot (2001), teachers often have a negative perception of research and feel that researchers often neglect to include them in developing interventions. Recommended practice for special education research suggests that the social validity of the intervention is an important part of the investigation (Ledford & Gast, 2018). Assessing social validity involves questioning participants and stakeholders to determine the need for the intervention and receiving input on the intervention procedures (Wolf, 1978). The nature of research also poses a barrier to implementing EBPs. Research conducted in controlled environments is less likely to be viewed as replicable in everyday practice (Foster, 2014). Greenwood and Abbot (2001) propose that one problem is that researchers are not creating interventions that work in authentic classroom settings. Accessibility of the research is another concern. Articles published in peer-reviewed journals are often locked behind a paywall, preventing teachers from accessing quality research. Research is often written for an academic audience. Practitioners struggle to translate vague research lacking clear procedures (Greenwood & Abbot, 2001).

Another barrier is the ineffectiveness of the professional development model adopted by most school districts (Foster, 2014). Due to the struggles with accessing and translating research, teachers depend on professional development opportunities to learn about new EBPs. Cook et al. (2013) advocate for the dissemination of information to be clear, interesting, and relevant to real life. Teachers often need clarification on professional learning (Foster, 2014). Professional development fails at providing effective professional learning if the delivery of instruction is too brief for teachers to learn a practice well enough to implement with fidelity (Klingner et al., 2013). They also need opportunities to plan and collaborate with their colleagues on assimilating the intervention into their current instructional practices (Foster, 2014). Even when schools make efforts to provide professional learning that is clear, ongoing, and includes opportunities to collaborate with colleagues to plan instruction, successful implementation of EBPs relies on teachers receiving feedback from a coach or expert on implementing what they have learned (Cook et al., 2013; Foster, 2014).

A final barrier to implementing an EBP with fidelity is a lack of alignment with teacher knowledge, teacher experience, or the teaching and learning context. Teachers are more likely to implement EBP when it aligns with the current philosophy of teaching and learning held by most teachers and administrators (Foster, 2014). According to Vaughn et al. (2000), teachers believe their teaching methods are already effective. Their article also points out that it is important to demonstrate how a practice aligns with the opposing knowledge and beliefs of general and special education teachers. Teachers who receive support in adapting practices to align with their teaching and learning context are likelier to adopt an EBP (Klingner et al., 2013).

Impact on CRA

Limitations of previous CRA research potentially contribute to teachers perceiving barriers to implementing the intervention with fidelity in their classroom. The perception that CRA is not an EBP for all students is reinforced by the volume of research studies focusing on improving outcomes for SWD or MD. Most CRA studies are conducted in controlled environments using highly trained researchers to implement the intervention procedures, a noted limitation that is a barrier to implementation if it reinforces the teachers' perceptions that CRA is only useful in ideal conditions. While many teachers interviewed by researchers reported positive perceptions of the effectiveness of CRA and believed the intervention was useful and important, very few of those teachers had hands-on exposure to the intervention. Researchers documented teachers struggling with the CRA intervention in at least two instances (Bouck, Park, et al., 2018; Morano et al., 2020). Bouck, Satsangi, et al. (2018) noted that many general education teachers struggle to implement the CRA sequence correctly, failing to systematically transition students through the phases. In another study, Morano et al. (2020) received feedback from teachers that the intervention was sometimes frustrating to implement because the lessons were too long, and students felt held back by the time required for teacher modeling and guided practice embedded into the procedures.

Enacted Curriculum

Research on the barriers to implementing EBPs provides information about why teachers may not adopt or use a practice with fidelity. However, it does not explain how teachers interpret and translate the practice for their students. This includes definitions of intended and enacted curriculum, conceptualizing the process for enacting mathematics curriculum, and research investigating how teachers modify EBPs and curriculum.

According to Stein et al. (2007), curriculum, or the content taught in classrooms, has three different versions: the published curriculum provided to teachers, the intended curriculum, and the enacted curriculum. The intended curriculum is what teachers plan to teach in their classrooms (Remillard & Heck, 2014). It includes their interpretations of the official curriculum, instructional materials, and professional decisions. The enacted curriculum is what students ultimately experience in the classroom and has the greatest impact on their learning (Remillard & Heck, 2014). The enacted curriculum is what happens in the classroom. It includes the mathematics content and the practices used to deliver it; the instructional interactions between teachers, students, and materials; teacher pedagogical moves; and tools and resources.

Transforming Curriculum

It is important that teachers enact a curriculum using evidence-based practices for all students, but especially for SWD and MD. Although extensive research supports the use of EBPs, many factors influence classroom enactment, which can reduce the effects on the academic outcomes of SWD and MD.

Remillard and Heck (2014) conceptualized the curriculum enactment process for mathematics education. They present a framework illustrating the interactions between the different curriculum domains and how different factors, directly and indirectly, influence what happens in classrooms. This research is interested in the operational domain of curriculum and what happens in the classroom. The operational curriculum includes the intended and enacted curricula and student outcomes. The intended curriculum is designed for a specific context (students, time, environment). The designated curriculum, student outcomes, and instructional materials directly influence it. Other contextual factors influencing what teachers intend to teach are teacher knowledge of, beliefs about, and practices with mathematics, pedagogy, learning, and curriculum resources; teachers' understanding of students' needs; expectations of school and community; and access to resources. The intended curriculum, student outcomes, and instructional materials influence the enacted curriculum. The contextual factors influencing the enacted curriculum are teachers' and students' knowledge, beliefs, and practices; access to resources; and contextual opportunities and constraints. According to Stein et al. (2007) and Remillard and Heck (2014), curriculum changes throughout planning and implementing instruction.

Teacher Enactment

Research findings from several qualitative investigations confirm that teachers modify curriculum and EBPs intentionally in response to factors and barriers (Gelmez-Burakgazi, 2020; Holstein, 2012; Moyer, 2003; Wooley et al., 2013). These studies support the claim that there is a research-to-practice gap influencing teachers' use of curriculum and practices supported by research. Moreover, their findings support the development of my conceptual framework and the propositions and rival explanations I present in the next section.

Moyer (2003) interviewed and observed ten middle school math teachers over a year to investigate how they used manipulatives in their classrooms after completing a 2-week summer training program. Her findings indicated that despite teachers verbalizing in interviews that they believed manipulatives improved mathematics education, they were not demonstrating that belief in their enacted practices. Patterns from the analysis supported several conclusions. First, teachers can conceptually understand that manipulatives support learning but feel incapable of incorporating them into the procedure-driven curriculum that many schools use. Second, teachers who attempted to use manipulatives struggled to use them as intended. Instead of embracing their full potential, they relied on replicating the general principles of manipulative use. Third, due to the pressure caused by high-stakes assessments their students were required to take every year, teachers expressed concern that diverging from methods students could rely on for the test was not a good use of instructional time. Finally, teachers did not implement manipulatives in their classrooms because they lacked knowledge of conceptually representing mathematics using manipulatives.

Holstein (2012) conducted a case study investigating how teachers transformed a mathematics curriculum. After collecting data from six teachers, she found that teachers were mostly faithful to the mathematics curriculum they were using, but there was a range of implementation fidelity. Changes to the math curriculum related to the content (the math content covered or the types of math problems used) or the procedures (delivery of instruction). Based on findings and the theoretical research on fidelity, Holstein proposes four types of implementers: 1) thorough piloting, 2) adopting but adapting content, 3) adopting but adapting pedagogy, and 4) partial piloting teachers. Patterns emerged in the analysis connecting the level of fidelity with teachers' beliefs and knowledge. High content fidelity is associated with teachers with positive beliefs about the curriculum and low content knowledge. Teachers with low content knowledge indicated they did not feel confident seeking outside curriculum. Low content fidelity is associated with teachers with strong beliefs about mathematics that do not align with the curriculum. Teachers with positive beliefs about the curriculum and teaching had high procedural fidelity. Low procedural fidelity was associated with teachers' low content knowledge and negative beliefs about students.

The work of Wooley et al. (2012) provides a deeper understanding of the factors and barriers that influence teachers enacting an intervention. To conduct this investigation, they selected 30 seventh-grade teachers for focus groups. Factors related to adaptability and compatibility influenced teachers' implementation. Teachers modified the intervention when they experienced time constraints incompatible with their teaching schedule or did not know how to align the intervention to a lesson. Teachers modified the intervention when they did not believe it was compatible with their students' needs or academic level. Another factor influencing intervention implementation also emerged from the focus group. Teachers modified implementation if they did not believe the intervention was useful or effective. Finally, teachers indicated that factors related to the school environment impacted the implementation of an intervention. Some schools mandate a specific intervention, while others allow more flexibility in choosing which intervention to adopt.

Twenty first-through fourth-grade teachers participated in Gelmez-Burakgazi's (2020) investigation of fidelity to instructional practices. She conducted a multiple-case study, including semi-structured interviews and documents. The study aimed to develop a deeper understanding of why teachers modify instruction. Gelmez-Burakgazi found that teacher, student, and context-based characteristics were directly related to the decision to modify and the type of modification.

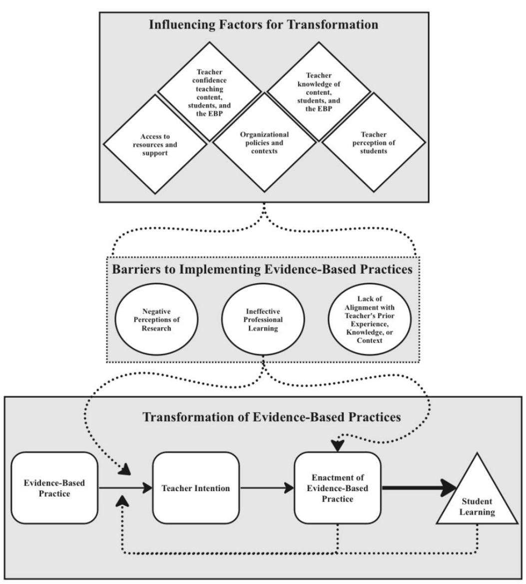
An important key finding is that teachers modified the curriculum intentionally. These changes were also not the result of some misconception but because they disagreed with the curriculum. These disagreements were often related to teacher beliefs about pedagogy or the usefulness of the activities. Teachers also expressed that modifications were often necessary in response to challenges related to limited resources, lack of time, and large class sizes.

Conceptual Framework

This study explores how third- through fifth-grade general education teachers conceptualize CRA, modify CRA to teach SWD or MD, and why teachers modify CRA. The following framework (see Fig. 1) details how I conceptualize how teachers transform a practice from one supported by research evidence to one ultimately implemented by teachers in classrooms and experienced by students.

Figure 1.

Conceptual Framework for Transformation of Evidence-Based Practices



Note. Adapted from the works of Foster (2014), Remillard and Heck (2014), Stein et al. (2007).

The conceptual framework supporting this study draws on the work of Stein et al. (2007), Remillard and Heck (2014), and Foster (2014). In 2007, Stein and colleagues published a framework to support researchers investigating how curriculum changes from the intent of the author to the delivery by teachers in classrooms. Remillard and Heck (2014) developed a more detailed framework illustrating the interactions between the different curriculum domains and how different factors directly and indirectly influence classroom events. Foster (2014) examined the enablers and barriers to implementing EBPs. These barriers affect teachers' ability or decision-making around instructional choices.

Teachers transform evidence-based practices during two phases. (Stein et al., 2007). Evidence-based practices go through a transformation as teachers plan their intended lessons. Another transformation occurs during the enactment of the lesson. While transformations appear to occur in a chronological sequence, the transformation process of evidence-based practices is not linear. It can be transformed multiple times in response to teachers' perception of real-time implementation and their assessment of student learning. Teachers make decisions about transforming evidence-based practices in response to several influencing factors (Remillard & Heck, 2014) that are filtered through barriers to implementation (Foster, 2014).

My conceptual framework begins with an EBP transformed through the planning and enactment of teaching. An arrow from Evidence-Based Practice to Teacher Intention represents the transformative process as teachers plan their intended instruction. An arrow from Teacher Intention to Enactment of Evidence-Based Practice represents the second transformation. This transformation happens when teachers enact the EBP as part of their lesson. A large, bolded arrow from Enactment of Evidence-Based Practice to Student Learning highlights the impact of the enacted curriculum on student learning. The transformation process is not linear. Teachers often transform their intentions regarding an EBP in response to their perception of the implementation in real-time and their assessment of its effectiveness in impacting student learning. Throughout the transformation process, various factors impact teachers, such as their understanding of the subject matter, confidence in teaching and utilizing evidence-based practices, perceptions of their students, and the teaching environment's policies and contextual factors. Additionally, access to resources and support is crucial to their development. The rectangle at the top of the figure contains the factors.

Dotted arrows connect the Influencing Factors for Transformations to the two points in time the transformation of an EBP occurs. The Barriers to Implementing Evidence-Based Practices are a comprehensive filter for the factors involved. Several barriers to implementing EBPs also help explain the transformation from a practice supported by research to the practices teachers implement in their classrooms. Teachers will implement an EBP with fidelity only if they positively perceive the practice or if the research is inaccessible. They must also receive effective professional learning that supports the deep understanding and maintenance of the practice. The EBP must also align with the teacher's prior knowledge, experience, and the teaching and learning context. The conceptual framework supports the following research propositions:

Proposition 1

The first research question guiding this investigation seeks to understand how third- through fifth-grade elementary teachers conceptualize the CRA strategy. Drawing on my conceptual framework, I propose that ineffective professional learning is a barrier to implementing EBP. According to Foster (2014), teachers develop misconceptions when professional learning is brief and shallow, without making a concerted effort to connect the learning with prior knowledge in an ongoing manner. This framework supports the proposition that the conceptualization of CRA will vary based on their professional learning.

Proposition 2

The second research question guiding this investigation seeks to understand how third- through fifth-grade elementary teachers report their enactment of the CRA strategy to teach SWD or MD. Drawing on my conceptual framework, teachers transform EBP in response to the barriers, individual factors, and external factors such as the teaching and learning context (Foster, 2014; Remillard & Heck, 2014). EBPs are transformed in real-time as teachers process, interpret, plan, implement, and respond to student learning. This framework supports the proposition that teachers modify one or more elements of CRA to teach mathematics to SWD or MD in their inclusive classrooms.

Rival Propositions

Throughout the investigation, I examined potential rival explanations. According to Yin (2018), examining rival explanations improves the internal validity of a study. A potential rival explanation for the first research question is that conceptualization may vary based on other factors unrelated to the quality of professional learning. A rival explanation for the second research question is that teachers do not modify their enactment of CRA to teach mathematics to SWD or MD.

Summary

This chapter I reviewed the relevant literature related to my topic and presented the conceptual framework supporting the study of how third- through fifth-grade general education teachers conceptualize CRA and how they modify CRA to teach mathematics to SWD or MD in inclusive classrooms. First, I defined EBP and presented the relevant research on EBP that supports mathematics instruction for SWD and MD. Next, I presented the literature on the EBP of interest for this study, CRA. That section included definitions and procedures, theoretical foundation, empirical support, recommendations, a synthesis of research investigating its effectiveness, and limitations. The next section addressed those limitations further, reviewing the research-to-practice gap and implementation fidelity. I also presented a review of the literature on the intended and enacted curriculum. That section included definitions of the terms, frameworks used to study curriculum enactment, and a synthesis of the research applying these frameworks to studying how teachers enact curriculum. Finally, the chapter ended with an explanation of the conceptual framework supporting the investigation of the phenomenon.

Chapter 3: Methods

Using qualitative methods, I explored how third- through fifth-grade general education teachers conceptualize the concrete-representational-abstract (CRA) strategy, how they report enacting CRA to teach mathematics to students with disabilities (SWD) or mathematics difficulties (MD), and which barriers influence their enactment of the CRA strategy to teach mathematics in inclusive settings. My conceptual framework supported the investigation of this phenomenon. I entered this investigation relying on research suggesting that teachers transform evidence-based practices (EBPs) during two phases. (Stein et al., 2007). One transformation occurs as teachers plan their intended lesson. Another transformation occurs during the enactment of the lesson. EBPs are transformed multiple times in response to teachers' perception of implementation in real-time and their assessment of student learning. Teachers make decisions about transforming evidence-based practices in response to several influencing factors (Remillard & Heck, 2014) that are filtered through barriers to implementation (Foster, 2014).

I also approached my investigation from a constructivist theoretical perspective. A *constructivist theoretical perspective* is one in which individuals create meaning from their lived experiences (Creswell & Creswell, 2018). Crotty (1998) identified three assumptions about constructivism. First, because people construct meaning from the world, open-ended questions allow researchers to investigate individuals' views to understand a phenomenon. Second, because historical and social perspectives of our world influence how we make sense of it, researchers should be aware of their interpretation of information gathered throughout the study based on their background. Third, the interaction between the researcher and participant potentially influenced the conclusions because meaning is generated through human interaction.

This study methodology aligns with my purpose, conceptual framework, and theoretical perspective to answer the following research questions:

1. How do third- through fifth-grade general education teachers conceptualize the CRA strategy?
2. How do third- through fifth-grade general education teachers report enacting CRA to teach mathematics to SWD or MD in inclusive settings?

This chapter provides a proposed methodology to meet this objective. It covers the following topics: 1) research design and rationale, 2) research context, 3) population and sample, 4) data sources and collection, 5) plan for analysis, and 6) validity and reliability.

Research Design

Researchers design studies to ensure the careful alignment of theoretical perspectives (orientation guiding the researcher), conceptual or theoretical frameworks, the purpose of the research, questions guiding the inquiry, research methodology, methods, and plans for analysis (Bhattacharya, 2017; Creswell & Creswell, 2018). Qualitative research is a method of inquiry aimed at understanding a phenomenon by investigating how people interpret and make meaning of their experiences with that phenomenon (Merriam & Tisdell, 2015). This method stands in contrast to the aims of quantitative methods of inquiry that seek universal truths about the world by testing theories in controlled experiments (Creswell & Creswell, 2018). Qualitative research is often conducted from a constructivist epistemological stance (Bhattacharya, 2017). The purpose of this study was to explore how third- through fifth-grade education teachers conceptualize CRA, report enacting CRA to teach mathematics in an inclusive setting, and why they modify the CRA strategy, both when planning and again as they enact the practice with students in the classroom by investigating their lived experiences and how they make meaning of those lived experiences. A qualitative method of inquiry drawing from a constructivist theoretical perspective was best suited for this purpose.

According to Creswell and Creswell (2018), there are five basic categories of qualitative research: narrative research, phenomenology, grounded theory, ethnographies, and case studies. A case-study design was most appropriate for my study. The narrative research design was not appropriate because my research purpose was not to craft a chronological story about the lives of teachers. I chose not to use a phenomenology research design because my research purpose went beyond understanding the shared experiences of teachers using CRA to teach mathematics to SWD or MD. While I was open to the possibility that teachers might respond in ways that did not align with my propositions, I was not seeking to generate new theories about how teachers use CRA, how and why they make decisions to modify CRA or their perception of barriers to using CRA.

Rationale

To meet the purpose of this study, I selected a multiple-case research design. A case study is an "in-depth description and analysis of a bounded system" (Merriam & Tisdell, 2015, p. 39). The description and analysis are supported by collecting data from multiple sources, such as interviews, observations, and documents (Creswell et al., 2007; Merriam & Tisdell, 2015). According to Yin (2018), an exploratory case study research design should answer how or why questions seeking to explain real-life, contemporary phenomena over which the investigator has little or no control. Using multiple cases rather than a single case improves the robustness of findings because it involves a replication logic that helps demonstrate external validity (Yin, 2018). Replication logic involves using individual cases that allow for literal replication (propositions will be true or modified in similar ways for individual cases) and theoretical replications (propositions will not be true for all cases, but the differences were predictable and explainable using theory). The sampling method discussed later demonstrates the purposive selection of cases that allowed for literal and theoretical replication.

Case Selection

The ideal population of interest were third- through fifth-grade teachers who reported using CRA to teach mathematics, had a full license for their teaching assignment, had more than five years of teaching experience, and taught mathematics to students with and without disabilities at public schools in Virginia. To gain access to this population, I distributed both physical and digital fliers sharing details about the study and requesting interested participants complete a survey. This population was ideal because of its size and homogeneity. I needed a large pool of potential participants to sample from who had heard of CRA and reported using it at least occasionally to provide instruction or remediation. I decided to focus on teachers with more than five years of teaching experience based on research suggesting a connection between the number of years of teaching experience and teacher quality. The effects of teaching experience level off after the first five years of teaching (Goe, 2007). One interpretation is that lack of experience in the first five years has a meaningful effect on overall teacher quality, but beyond that, years of experience is no longer a significant factor.

Using a non-random purposive sample from this population allowed me to conduct a deep investigation of how third- through fifth-grade teachers conceptualize CRA, report implementing CRA, and identify the factors and barriers that influence the way they intend to implement CRA to teach mathematics to SWD or MD. I used purposive sampling to ensure that the sample included individuals with specific characteristics crucial to addressing my research questions. Given the focus on third- through fifth-grade general educators teaching math in inclusive settings, selecting participants from different education levels, years of experience, and school contexts was essential.

The following inclusion criteria were necessary for participation: 1) held a current license for the grade level they teach, 2) employed in a public school in the state of Virginia teaching third, fourth, or fifth grade, 3) taught mathematics, 4) taught in inclusive settings (presence of students with and without disabilities), and 5) had more than five years of teaching experience. I excluded participants if they did not meet all the inclusion criteria. My ideal sample size was approximately six to eight participants (Creswell, 2013). Due to my sample's homogenous nature and the richness of data I collected, Creswell recommends four to five cases. The cases of interest for this study are third- through fifth-grade teachers who report using CRA to teach mathematics to students with and without disabilities at public schools in Virginia and have more than five years of teaching experience. The level of exposure to specialized mathematics education coursework, years of teaching experience, school context, and access to resources and support further bounded the cases.

This multiple-case study included three cases bounded by years of teaching experience, the school context, specialized mathematics education coursework, and access to resources and support. I selected six participants with a range of teaching experience, education, school context, and access to specialized training via mathematics coaches or leadership roles they hold in the school. Data saturation is the point in an investigation where it is possible to make conclusions, and further data collection will not produce additional insights (Glaser & Strauss, 1967). A carefully selected sample of six participants representing a range of experience and environmental factors provided me with enough data to make conclusions within and across cases.

Recruitment

First, I sent a brief letter of introduction and overview of the research study (Appendix D) with a link to a survey (Appendix E) to the emails of graduates from a teacher preparation program at a large public university in Virginia. The Teacher Education office provided a file with the contact information for recent graduates from their Master of Teaching program. I also distributed physical and digital fliers to recruit volunteer participants. After talking to managers, I hung physical copies of the flier in coffee shops, grocery stores, teacher supply stores, bookstores, and gyms. I shared a digital version of the flier through professional organizations on social media sites and emailed teacher contacts across the state. Like the email, the flier included information about the purpose of the study, the population of interest, the researcher's contact information, and a link to the survey.

The survey included questions used to select eligible participants. I sorted and reviewed the surveys regularly, enabling me to reach out to potential participants quickly. Appendix D has examples of communication with participants during and after recruitment. Eligible participants received an email requesting they schedule time on Zoom to go over the study and provide an opportunity to ask questions, review the informed consent (Appendix F), and conduct the first interview. The informed consent reviewed what teachers could expect from participation in the study, the estimated time commitment, confidentiality procedures, any potential benefits and risks, and the process for removing themselves from the study.

This qualitative multiple-case study investigated how six third- through fifth-grade general education teachers with more than five years of teaching experience, currently teaching mathematics to students with and without disabilities at a public school in the state of Virginia conceptualize CRA, report implementing CRA in their classroom, and why they modify CRA to teach mathematics to SWD or MD. Through this investigation, I sought an understanding of how and why teachers transform a practice supported by evidence in the research into the practice that is ultimately implemented by teachers in the classroom and experienced by students. The investigation required exploring teachers' knowledge and conceptualization of CRA, their reported implementation of CRA, and the factors and barriers that influenced how they use CRA in their inclusive classroom. For a full research protocol, see Appendix B.

Research Context

This study took place within the context of Virginia public elementary schools. Virginia is a large and diverse mid-Atlantic state. This state has over 130 school districts serving an estimated 1.3 million students. According to the latest NAEP results, the average mathematics achievement level of fourth graders in this state was on par with the national average (NCES, 2022). Through data collection at the time of testing, NCES estimated that 15% of fourth graders have an identified disability. The average scale score for fourth-grade SWD on the mathematics test is similar to the national scale score for this same group of students. Table 4 (see Appendix G) includes relevant school contextual factors for the elementary schools where the participants are employed. I collected data on each school from the Virginia Department of Education website and used the NCES locale classification to determine the best description of the community for each school.

Bluebird Elementary is in a small city in Region 5 of Virginia. It is a virtual school that enrolls students from 11 elementary schools, three middle schools, and two high schools. They have 48 students enrolled in their K-5 elementary program. They do not report to the state as a separate school. In response to COVID-19, this school district created a virtual program to allow families to make choices that fit their needs and circumstances. The teachers teach all subjects and occasionally have in-person days for testing or community building. The teachers and students at Bluebird Elementary have access to technological tools, such as BrainingCamp and virtual whiteboards. The students also have hands-on materials for use at home. There is a math coach who provides support and training. A special education teacher is also available and serves the entire K-12 program. There is only one teacher per grade level at the elementary school. The third-grade teacher has 60-85 minutes for math instruction, and the fourth-grade teacher is allotted 75 minutes for math instruction.

Cardinal Elementary is in a small city in Region 1 of Virginia. Just under 400 students attend this school. Nearly 80% of the students are black. The school serves an economically disadvantaged population, with about 78% of students meeting the criteria for that classification. Only about 1% of the student population receives ELL support. SWD make up about 10% of the school's student body. The pass rate for SWD on the Spring 2023 third through fifth-grade math SOL is 12%. There are three fourth-grade classrooms at this school, with teachers departmentalized. Only one of these classrooms is inclusive. The school provides two math interventionists, one who works with the primary grades and one who works with the upper elementary grades. There is a special education co-teacher, but they do not collaboratively teach. The special education services typically occur in a separate classroom. The school allows for 90 minutes of mathematics instruction per day. The first half is for whole-group instruction, and the second half is for small-group instruction.

Oriole Elementary is in a midsize city in Region 2 of Virginia. There are about 750 students enrolled. Of these students, over 70% are white. Black and Hispanic students also comprise about 16% of the student population. Only 10% of the students meet the criteria for ELL classification. Just over 20% of the population is considered economically disadvantaged.

SWD make up 10% of the student body. The pass rate for SWD on the Spring 2023 third through fifth-grade math SOL is 45%. This school has five fifth-grade classrooms. The teachers provide instruction for all subjects. There is only one inclusion classroom in the fifth grade, and a special education co-teacher is present to help provide math instruction. They do not collaborate often; typically, the teacher pulls students out for special education services. The school schedule allows for 70 minutes of mathematics instruction per day. A math coach is available to support teachers. *Math* is the first academic content taught that day and occurs immediately after PE.

Pelican Elementary is in a rural fringe school district in Region 5 of Virginia. It is an accredited school with just over 500 students. Students at this school are predominantly white (about 58%). Hispanic students are the next largest racial group, and 14% of the student population meets the criteria for ELL classification. Just under 40% of students qualify as economically disadvantaged. SWD make up about 12% of the student body. The pass rate for SWD on the Spring 2023 third through fifth-grade math SOL was 23%. There are four fifth-grade classrooms at this school. Teachers provide instruction for all subjects. Two fifth-grade classrooms are inclusion classrooms, co-taught with a special education teacher. No math coach or specialist is available for the teachers to use as a resource. Pelican Elementary is also part of a public-school consortium that collaborates to share resources, assessments, pacing, and data to improve student performance. At Pelican Elementary, teachers have 70 minutes of their instructional day to teach mathematics. The teachers have an additional 30 minutes set aside for remediation. Math or reading intervention takes place at this time. Math instruction is after lunch. The co-teacher spends about 30-45 minutes with each classroom. He also takes students to his room during remediation rather than work with them in the inclusion classroom.

Participants

Six teachers participated in this study. I created an online survey using Qualtrics and collected information from them about their education, teaching experience, licensing, grade level, and usage of CRA (see Table 5 in Appendix G). Participants in this study taught at schools from four different districts in Virginia. Two participants were employed in a rural fringe school district and taught at Pelican Elementary. Three participants taught in small city districts; two taught at Bluebird Elementary, and one taught at Cardinal Elementary. One participant taught in a midsize city school district and taught at Oriole Elementary.

Ms. Aspen has a master's degree from a private liberal arts school in Virginia. She is in her tenth year as an educator. She was an assistant teacher at a preschool in a nearby city for the first two years. She has taught fifth grade at Pelican Elementary for the last eight years but has only taught math for the last four years. She serves as the fifth-grade team leader. There are 18 students in Ms. Aspen's class, and five receive special education services. She describes these students as far below grade level. She also has four students who receive support from an ESL teacher.

Ms. Birch has a bachelor's degree from a public university in Alabama. She serves as the Mentor Coordinator at her school. She is in her tenth year of teaching. She taught third grade in a Midwest state for the first five years. For the past five years, she has taught in the same district she teaches now, but she previously spent two years teaching fourth grade in person. Currently, there are seven students in her third-grade classroom at Bluebird Elementary. None of her students receive support from a special education teacher, but she is aware that some of her students have ADHD. More than half of her students qualify for additional academic support. They receive an additional 30 minutes of tutoring every day.

Mr. Chestnut has completed some graduate work from a private university in Virginia. He is in his 24th year as an educator. For the first three years, he taught fourth through sixth grade at a private school in a nearby city. He then spent 14 years in a neighboring county teaching fifth grade. He mostly taught collaborative math but has occasionally taught other subjects when necessary. Mr. Chestnut has taught fifth grade at Pelican Elementary for the last seven years. There are 18 students in his classroom, and three receive special education services. Typically, the students who receive special education services in his classroom have ADHD, specific learning disabilities, or processing disorders.

Ms. Elm has an EdD from a large public university in Virginia. She is also currently returning to school to complete a master's degree that will allow her to gain endorsement as a math specialist in Virginia. She has extensive knowledge of evidence-based practices and mathematics content. Ms. Elm is in her ninth year of teaching. She spent the first seven years teaching in a different state. For five years, she taught third and fourth graders math, science, and social studies at two different Title I schools. One school closed for not making academic progress. These schools also served large populations of non-native English speakers (about a third of students were immigrants and refugees). For the remaining two years, she served as a math specialist. She is in her second year teaching fifth grade at Oriole Elementary. There are 22 students in Ms. Elm's class this year. Six of these students receive special education services. On average, the students she teaches are academically further behind than the other fifth-grade classrooms. She is the only inclusion teacher in her grade. She has a special education co-teacher.

Ms. Hickory has earned two master's degrees as an educator. Both degrees are from large public universities in Virginia. She recently completed the degree necessary to apply for endorsement as a math specialist. Ms. Hickory is in her 17th year as an educator. For 16 years, she taught at the same school that served an inner-city population of students. For the first nine years, she taught second through fifth grade. For the last seven years, she served as an instructional coach for her school. This year is Ms. Hickory's first year at Cardinal Elementary, serving as a math interventionist. However, during this study, she stepped in as a fourth-grade math teacher after a teacher suddenly resigned. Ms. Hickory did not report how many students are in her classes, but she teaches math to all fourth-grade students at Cardinal Elementary.

Ms. Walnut has a bachelor's degree from a private liberal arts school in Virginia. She serves as the elementary lead for her school. Ms. Walnut is in her 12th year of teaching and has taught second through fourth grade. For the first eight years, she taught second and third grade at a Title 1 School in a rural county neighboring the district where she currently teaches. She primarily taught reading. After COVID-19, she spent one year teaching third grade virtually for the same rural school district. When the county ended the virtual school, she moved to her current district, and this is her third year teaching all subjects to fourth graders virtually at Bluebird Elementary. Ms. Walnut has 14 students. None of her students receive support from a special education teacher, but she is aware that some of her students have ADHD, and she has one student in child study.

Data Sources & Collection

Yin (2018) describes six sources of evidence collected in case studies: documents, archival records, interviews, direct observations, participant observation, and physical artifacts. As mentioned earlier, case studies typically incorporate data from multiple sources (Creswell et al., 2007). Case study researchers using multiple data sources can triangulate their findings, strengthening the construct validity of the study (Merriam & Grenier, 2019; Yin, 2018).

Therefore, this study's primary data collection sources are interviews and physical artifacts. Data collected from interviews can provide insight into what a participant thinks or feels. The interviews were semi-structured, meaning a set of pre-determined open-ended questions supports the line of inquiry. However, questions can deviate if needed to gain more clarity from participant responses (Bhattacharya, 2017). Physical documents such as instructional materials and student artifacts supported the investigation by corroborating the teacher's description of their intended teaching practice. See Appendix H for the data collection timeline (Table 6) and alignment of the research questions, components of my framework (Table 7) addressed by the research questions, and data collection methods for investigating each research question.

Following the research protocol, I used a systematic and organized approach to collect data for this multiple case study from multiple sources. Each case included data from at least two participants, including semi-structured interviews and artifacts. I organized the data by participant in a secure digital drive called DropBox. Files included audio and visual recordings of semi-structured interviews, original transcripts, clean transcripts, and teacher-submitted artifacts of instructional materials. I also stored the reflective journals conducted after each semi-structured interview in DropBox.

Interviews

Interviews with individual study participants occurred between December 2023 and January 2024. The semi-structured interviews lasted approximately 60 minutes each. Participants selected a convenient time to schedule their interview. Each participant completed two interview sessions for a total of 120 minutes. All interviews took place privately to ensure confidentiality and put the participants at ease. Interview sessions were conducted over Zoom, allowing for recording, documentation using transcription, and video corroboration of researcher notes. As participants answered questions, I took notes on observations, clarification needed for questions, topics to probe or investigate more, and preliminary theories.

I conducted a pilot interview with three teachers who have experience teaching mathematics to SWD or MD. They gave me feedback on the questions' wording, order, and understanding. Based on their feedback, I adjusted the questions and split the interview into two sessions instead of one. The pilot interview sessions were estimated to last two hours, but the longest lasted 90 minutes.

At the beginning of each interview, I reminded participants that I was recording the session and that their participation was voluntary. I then reviewed the purpose of the study and encouraged them to ask me to clarify or repeat questions if needed. The interview protocol is in Appendix I. At the end of each interview, I thanked the teachers for taking the time to participate in the study. I reminded them of upcoming tasks (submitting documents) and the time and date of any upcoming interviews. The first semi-structured interview focused on their knowledge and conceptualization of CRA and explored how they describe implementing CRA in their inclusive classroom. The second semi-structured interview focused on discussing the artifacts they submitted and describing factors and barriers that influenced their reported enactment of CRA. After each interview, I reviewed the recording and produced a clean transcript to ensure I accurately captured all data.

Artifacts

After the first interview session, I asked teachers to submit copies of supporting documents by scanning and uploading copies to a secure electronic drive, DropBox. Data collection was conducted from December 2023 to January 2024. Artifacts included in the study are instructional materials provided by the teacher demonstrating their use of CRA. Potential instructional materials are lesson plans, videos shown to students, notes provided to students, presentation materials (i.e., slides), examples of student practice opportunities, and photographs of teacher-generated examples using manipulatives. At the beginning of the second interview, I asked the participants to explain how and when they used each artifact in the lesson. These artifacts yielded data about how teachers plan to use CRA and their reported enactment of CRA.

Confidentiality

All interviews, field notes, and supporting documents were handled confidentially. The data was assigned a code name, and the list connecting the participants' names with the data was kept in a secure, locked file. Upon completion of the study, this list, along with all recordings, documents, and transcriptions, will be destroyed.

Analytic Strategies

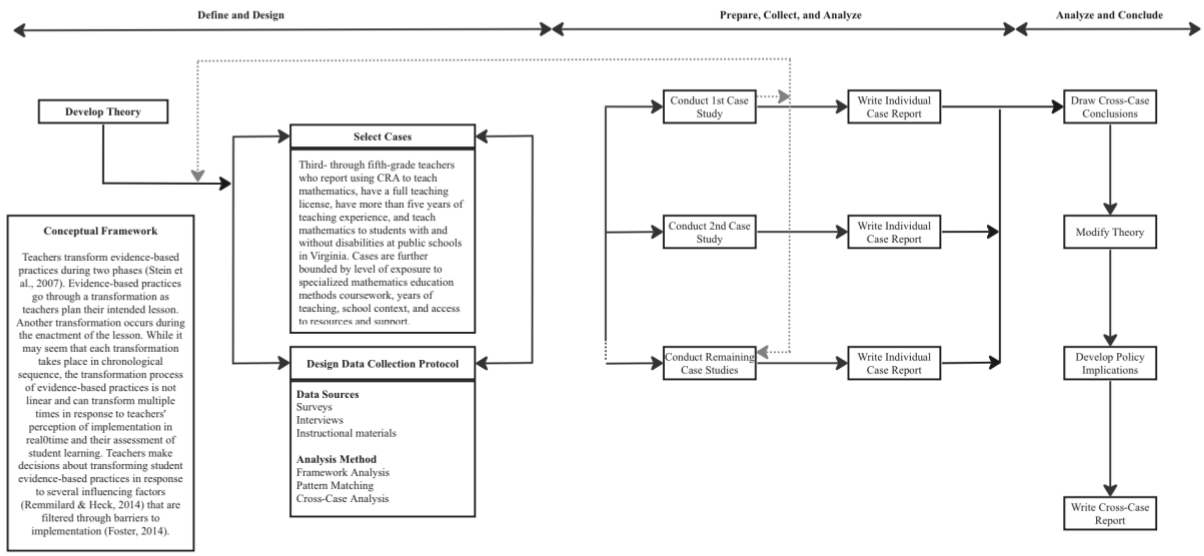
The analysis included data collected from interviews and artifacts. I sorted and organized data by participant and type. Analysis and data collection occurred simultaneously (See Figure 2). A detailed preliminary timeline of data collection, analysis, and writing is available in Appendix J. A multiple-case study requires both within-case and cross-case analysis (Merriam & Tisdell, 2015; Yin, 2014). I used a blended approach of deductive and inductive coding at each stage of analysis. This blended approach to coding and analyzing the interviews and documents allows me to perform framework and thematic analyses.

Coding

Deductive coding matches data collected from each case to pre-determined codes based on a conceptual framework. Analysis of case studies is typically an inductive process, with the

Figure 2.

Multiple-Case Study Procedure



Note. (Yin, 2018)

researcher examining the data and looking for emergent patterns and themes (Bhattacharya, 2017). This analysis process is iterative, occurring during and after data collection (Bhattacharya, 2017; Merriam & Tisdell, 2015). After member checking was complete, I began data analysis using a blended deductive and inductive coding approach. The coding approach is detailed in Appendix K. Inductive analysis allowed me to identify themes and subthemes. This process was accomplished by repeatedly reading the interview transcripts (Bhattacharya, 2017; Merriam & Tisdell, 2015). Deductive analysis facilitated a framework analysis within and across cases. Deductive codes were rooted in the conceptual framework that informed the design and methods for this multiple case study.

Within-Case Analysis

During the first cycle of coding data, I used a combination of holistic and hypothesis coding to code data from the interview transcripts deductively. This process involves pattern matching to compare observed and predicted patterns based on propositions and rival explanations (Yin, 2018). According to Miles et al. (2020), researchers with a general idea of what they are looking for in their data can use holistic coding. Using holistic codes, the transcripts from the first interview included the following sections: "teacher background," "school context," "confidence," and "reported enactment." Holistic codes from the second interview included "alignment," "intention," "enactment vs. planning," "teacher knowledge," “teacher confidence,” "organizational policies," "access to resources and support," "perception of students," and "perception of research." Next, I used hypothesis codes to sort data based on my research questions. I began by sorting data into units that address each research question. I coded excerpts from both interview transcripts for each participant as "conceptualization," "reported enactment," and “barriers.”

I further refined each of those codes. From "conceptualization," the codes "knowledge" and "definition" emerged. Isolating the data related to "knowledge" led to the subcodes "name and describe," "decision making," and "effectiveness." After isolating the data related to "definition," the subcodes "framework," "gradual release," and "sequence" emerged. From “reported enactment,” the codes “planning,” “implementation,” and “modifications” emerged. After isolating the data related to “planning,” I identified subcodes “design” and “planning support.” The subcodes “actions” and “planning vs enacting” emerged after isolating the data related to “implementation.” Finally, the data related to “modifications” resulted in the additional subcodes “minimal,” “moderate,” and “extensive.”

In the second coding cycle, I used an inductive approach to identify pattern codes to reconfigure the first cycle codes. I scanned, read, and re-read the interview transcripts for each participant. This process allowed me to identify emerging patterns and insights from interview data related to my purpose and rooted in my conceptual framework (Merriam & Tisdell, 2015). After sorting all the data for a single case, I familiarized myself with it by examining it for patterns and observations that became categories or themes (Bhattacharya, 2017; Merriam & Tisdell, 2015). The inductive coding process added clarity, details, and opportunities for rival explanations to emerge. Three patterns emerged from data related to "conceptualization" and "reported enactment." Participants had "advanced," "emerging," or "developing" conceptualizations and reported enactment of CRA.

Next, I used a constant comparative method for analyzing information for each participant in the case, looking for similarities within cases to demonstrate literal replication (Bhattacharya, 2017; Yin, 2018). I then wrote individual case reports with preliminary results. These results informed the analysis of additional cases. After completing the individual case reports for the first and second cases, I used a constant comparative method to analyze them. This process involved looking for similarities across cases to demonstrate literal replications and predictable differences across cases to demonstrate theoretical replications (Yin, 2018).

Validity and Reliability

The quality of exploratory qualitative research is judged based on three tests: construct validity, external validity, and reliability (Yin, 2018). The following paragraphs describe the methods employed to ensure this study's validity and reliability.

Construct Validity

A study with construct validity demonstrates that the concept or phenomenon of interest is measured accurately. Yin (2018) recommends using multiple data sources and allowing for member checking. In this study, I collected data for each case from a minimum of two participants in the form of surveys, interviews, and instructional materials. Collecting data from multiple sources is a form of data triangulation, allowing me to demonstrate evidence convergence (Yin, 2018). Finally, I sent participants drafts of my initial reports on their cases to ensure construct validity. All participants had an opportunity to comment, correct, or clarify. I have demonstrated construct validity for this qualitative study.

External Validity

External validity refers to the generalizability of findings to other contexts. In qualitative research, this means demonstrating that your findings support analytic generalizations – comparing the results to an established theory. In this study, I draw on two theoretical frameworks to explore the factors influencing how teachers transform EBPs (Remmilard & Heck, 2014) and the barriers to implementing an EBP (Foster, 2014). During the deductive analysis phase, I used codes generated from my conceptual framework to look for patterns and insights related to my purpose (Merriam & Tisdell, 2015). The results presented in the next two chapters provide deeper insight into the theoretical frameworks that informed the design of this multiple-case study.

Reliability

A study is reliable if the procedures outlined in the methods allow for replication (Yin, 2018). Although it is challenging to replicate a multiple-case study exactly, Yin (2018) advises qualitative researchers to use a case study protocol, develop a case study database, and maintain a chain of evidence to ensure reliability. Before beginning this study, I developed a case study protocol (See Appendix B) providing an overview of the case study, data collection procedures, protocol questions, and a preliminary outline for the case study report. I also created a case study database. Using a secure digital storage web application, Box, I systematically collected and organized all data and research reports. The case study protocol and database work together to establish a chain of evidence that supports the connection between research questions, methods, data sources, and my framework.

Reflexivity

As a former elementary and middle school general education mathematics teacher, I co-taught in classrooms with a special education teacher. My classrooms were typically evenly split between SWD and students who experienced MD (students more than two years below grade level in mathematics). I have experienced struggles with implementing EBPs, such as CRA, within my classroom setting. I cannot separate who I am as an observer from who I am as a researcher.

My identity and my experiences may influence the study in multiple ways. I believe that my experience as a math educator who taught SWD or MD made me more aware of the challenges those educators face in making math accessible to all students. I constantly reflected on my observations to check whether I was finding the patterns and themes because I was looking for them and considered what I might be missing if I focused only on my expectations. I need to work hard to listen and interact with an open mind when interviewing my participants. My identity as a researcher and doctoral student may make it challenging for teachers to feel comfortable confiding in me or admitting a lack of knowledge, confidence, or experience. I must build trust with them to gain the access I need for this research.

Summary

In this chapter, I reviewed the methods for conducting my study to explore how third- through fifth-grade general education teachers conceptualize CRA, report using CRA in their inclusive classrooms, and why they modify CRA to teach mathematics in their inclusive classrooms. I provided a detailed explanation of my method of inquiry and the rationale supporting the decision to use a qualitative multiple-case design to investigate the phenomenon. This chapter also included a description of the research context, participant background, and experience that supported the bounding of my cases and the population of interest. I reviewed my sampling methods and explained my recruitment plan and steps to protect their human rights and confidentiality. This chapter also included a plan for collecting, organizing, storing, and analyzing participant data and steps taken to demonstrate validity and reliability.

Chapter 4: Conceptualization

This chapter explores how third- through fifth-grade teachers conceptualize the concrete-representational-abstract (CRA) strategy. During the second coding cycle, two themes emerged from the interview data. Conceptualization included how participants defined CRA and their knowledge of the strategy. Participants defined CRA in three ways: a framework, a gradual release model, and a sequence. Knowledge included naming and describing the phases and procedure, explaining the criteria for phase progression, and explaining why CRA was effective. Analysis of the interview data also yielded three distinct levels of conceptualization among participants (advanced, emerging, and developing). Conceptualization varied with participants' exposure to specialized mathematics education coursework and years of experience teaching mathematics. Table 8 (see Appendix L) summarizes the three levels of conceptualization.

The remainder of this chapter presents the results for the first research question from multiple data sources, including the survey and individual semi-structured interviews for six participants. In the following sections, I will report on three pairs of teachers representing three conceptualization levels. An advanced conceptualization includes defining CRA as a framework in which teachers explicitly model for students the mathematics content, procedures, and connections between the phases and extensive knowledge of the strategy. An emerging conceptualization of CRA includes defining CRA as a gradual release model in which teachers provide scaffolded instruction but do not make connections between the phases of CRA and general knowledge of the strategy. A developing conceptualization of CRA includes defining CRA as a sequence with no connection between the three phases and either inaccurate or vague knowledge of the strategy.

Advanced Conceptualization

Ms. Elm and Ms. Hickory both have exposure to advanced courses in mathematics education and are either currently in school or have finished school in the last two years. Ms. Elm has an EdD in Curriculum and Instruction and is currently completing graduate-level coursework to qualify for endorsement as a math specialist. She teaches fifth grade at Oriole Elementary, a school located in a midsize city, and she describes the community zoned for her school as affluent. Ms. Hickory holds two master's degrees, one of which enables her to receive endorsement as a math specialist. She is a math interventionist, but she taught 4th grade in a small city at the time of the study. The community zoned for her school, Cardinal Elementary, is urban. Ms. Hickory describes it as one in which many households experience economic hardship. Both indicate that they use CRA often for both instruction and remediation.

The teachers in this study, who have the most advanced mathematics education coursework and extensive experience teaching mathematics, have an advanced conceptualization of CRA. They both define CRA as a framework in which teachers model for students the math content, procedures, and how the phases are connected. Helping students make connections is an important strategy component. Both participants in this case also demonstrate extensive knowledge of the phases and procedures for teaching the strategy. They accurately name and provide rich, detailed descriptions of all three phases of CRA. When describing the procedure, they frequently mention modeling as a method to help students make connections. Another finding from this study is that participants, in this case, explain that teachers should rely on a combination of student data (a set mastery criterion) and professional knowledge or experience to determine student readiness to transition from one phase to the next. Both participants explain that the strategy improves student learning by simultaneously building math content knowledge and procedural understanding through a learning progression.

Ms. Elm

Ms. Elm has comprehensive knowledge of CRA. She has extensive experience teaching mathematics in the upper elementary grades and taking advanced coursework in mathematics education. Her goal is to become a math specialist eventually. Ms. Elm expressed that while she feels very confident using CRA to teach mathematics to students with disabilities (SWD) or mathematics difficulties (MD), she wishes she understood the source of her students' learning difficulties better. Her teacher preparation program did not include instruction on the CRA strategy, but she remembers learning the importance of using manipulatives and drawings to teach mathematics. The first time she remembers hearing about CRA was from a friend who attended training on the Singapore Method. She explains that she is always curious about how or why something works and enjoys taking time to unpack common strategies used to teach mathematics so that she can understand them better. After hearing about CRA from her friend, Ms. Elm stated she wanted to learn more so she read independently to learn how to use it.

She defines the CRA strategy as a framework in which teachers model mathematics content, procedures, problem-solving, and decision-making. Her description stands out among other participants because she highlights the importance of making connections between phases. In her words, "I think something that sometimes gets overlooked when people use CRA is really making sure that the connections are made between each step so that it's not like you're doing three different things."

When asked to name and describe the phases of CRA, Ms. Elm accurately names the phases of CRA and provides a rich and detailed description of each phase. In her description of the concrete phase, she emphasizes that students work with hands-on objects to show their thinking and make sense of mathematical content and procedures. The manipulatives are tools, but they could also be everyday materials. While she does not think the concrete phase is less confusing or easier than the other phases, she believes that students are less likely to make careless errors that typically occur in the representation or abstract phase (e.g., forgetting a tally mark or missing a step in a procedure). Ms. Elm further justifies her response by adding, "I know research says the manipulatives are the best step to start with."

She describes the representation phase as an opportunity for students to transition into drawing the math content. During this phase, students build connections between the concrete and the abstract. The drawings may not always be exact replications of the manipulatives used during the concrete phase. For example, she notes that sometimes students can use a part-part whole diagram to represent word problems. When asked, Ms. Elm initially claims it is important for students to draw their representations. However, she acknowledges that she prefers premade drawings for some mathematics content, such as fractions. According to her, student learning advances when they problem-solve how to visualize math content. Compared to other participants, Ms. Elm’s description of the abstract phase is more detailed. She explains that the abstract phase is not always computational. Furthermore, she describes the abstract phase as the stage of learning when students are automatic with the math content or skill.

Ms. Elm explains that it is best practice to use a mastery criterion to determine when students are ready to transition between the phases of CRA. However, she believes that using representations is important for students, and even if they have mastered math content, she continues to require representations all year. Ms. Elm blends knowledge of the appropriate method (student data) for making decisions about student readiness to transition with her professional knowledge and experience.

Ms. Elm is not only able to explain how CRA improves student learning, but she is also able to explain why it is effective. She explains that CRA successfully improves student learning because it helps students understand why the procedures work the way they do. According to Ms. Elm, explicit teaching is essential to the success of CRA. She uses explicit instruction to help students learn what the manipulatives show them about the math content, how to use them in various contexts, and how the phases are connected. Although Ms. Elm believes that all students across K-5 mathematics can benefit from CRA, she argues that some students may not need all phases of CRA depending on their mastery level of the content. She also worries CRA may be more challenging for upper elementary and secondary mathematics content because it is hard to find manipulatives and pictures related to the skills.

My analysis revealed that Ms. Elm has an advanced conceptualization of CRA. She defines CRA as a framework model in which students make connections between the phases to build content and procedural knowledge of mathematics. Furthermore, she demonstrates an extensive knowledge of CRA. She provides an accurate and thorough description of the phases and procedures, appropriate methods for identifying students who are ready to transition from one phase of CRA to the next, and an understanding of why CRA is effective.

Ms. Hickory

Ms. Hickory has a thorough knowledge of CRA. She officially holds a position at her school as a math coach, but she has had to teach math for all three fourth-grade classrooms due to vacancies. She has substantial teaching experience and coursework, having taught math for the last 17 years and earning two master's degrees with additional math education coursework. Her experience helps her feel confident using CRA to teach mathematics to SWD or MD. However, Ms. Hickory admits that if she sees that SWD or MD continue to struggle with the content, she will resort to using "tricks" to teach a procedure. CRA was taught in the master's program she completed to become a math specialist. She also credits learning about CRA from other math coaches and the math specialist in her district. Despite her extensive education, Ms. Hickory states that she never feels done learning and her avid desire to learn leads her to ask questions, read articles, and explore resources on the VDOE website.

Ms. Hickory also defines CRA as a framework in which teachers use explicit instruction to model mathematics content and procedures and provide practice opportunities with immediate feedback. Like Ms. Elm, Ms. Hickory's description of the procedure stands out because she also highlights the importance of making explicit connections between phases. She explains, "Sometimes there is a gap between the phases, but the goal is that students will make the connections between the manipulative, the drawings, and the numbers and symbols."

She can accurately name the phases of CRA and provides a rich description of the phases and the procedure for implementation. During the concrete phase, she explains, teachers should introduce a new math skill or topic with a manipulative without focusing on abstract numbers or symbols. According to her, it is all about touching and feeling and hands-on learning with manipulatives. She explains that moving around the objects is how students connect the concrete and the abstract, and starting with manipulatives is important because students cannot build math content knowledge and procedural understanding without it.

According to Ms. Hickory, students should create a visual image of the manipulatives used during concrete instruction in the representation phase. This phase supports student learning by building connections between the concrete and abstract phases. She explains that teachers can help students make this connection by prompting them to remember how they used the manipulatives and showing them how to do the same thing with a drawing. The drawing component also helps her understand what students are thinking and have deep conversations with students about their process. It is important to Ms. Hickory that students draw the images themselves rather than use premade images. She sees the representation as the bridge between concrete and abstract. After using representations, students learn the math content with just numbers and symbols during the abstract phase.

When asked what criteria teachers should use to determine when a student is ready to transition to a new phase of CRA, Ms. Hickory explains that as a veteran teacher, it is a skill she has internalized. Using a blended approach, Ms. Hickory relies on her professional knowledge and student data to help her make decisions. She draws on her professional knowledge, observing how quickly students answer questions and how well they can explain their work. Her school system collects student data daily in the form of exit tickets and sets mastery at 70%. While their curriculum recommends a set number of days to provide instruction with manipulatives before moving on to representations, Ms. Hickory explains that she uses her judgment to determine that. She does not think students need a minimum or maximum number of lessons at each phase before they are ready to move on. Instead, individual student progress should guide how long they need instruction at each phase.

Ms. Hickory explains that CRA is effective because it helps to build content knowledge and procedural understanding. She explains that students develop strong content knowledge by building connections between all three phases. She likes the concrete phase, but it would be the representation phase if she had to pick one element to identify as key to CRA's effectiveness. She justifies her choice: "It starts the processing in their brains of understanding. And then as they move into the next phase, it just generally connects the bridge back to concrete manipulatives or conceptual understanding of why we're doing what we're doing." When asked, Ms. Hickory reports that she thinks all students across grade levels benefit from CRA instruction and that CRA is appropriate for teaching all math content.

My analysis revealed that Ms. Hickory has an advanced conceptualization of CRA. According to her interview responses, she defines CRA as a framework in which teachers explain the content, model how to perform a procedure, and explicitly show students how the phases of CRA are connected. Ms. Hickory has extensive knowledge of CRA, and her description of the phases and procedures is rich with detail and insight. She identifies appropriate methods for determining student mastery and can explain why CRA is effective.

Summary

In this study, I found that participants with specialized mathematics education coursework and extensive experience teaching mathematics have an advanced conceptualization of CRA. They define the strategy as a framework in which teachers model mathematics, making it explicit how each phase of CRA is connected. Both participants demonstrate a deep knowledge of CRA by using accurate and rich descriptions for each phase of CRA. They also demonstrate their depth of knowledge by the detailed process they describe to identify student readiness to transition through the phases of CRA. Further, participants with an advanced conceptualization understand that CRA is effective because it helps students learn math content through a systematic learning progression in which students benefit from teachers modeling their thinking and decision-making, explaining the math content, and demonstrating multiple procedures.

Emerging Conceptualization

Mr. Chestnut and Ms. Birch have similar levels of education and teaching experience. Both have been out of school for ten or more years and have taught math at the upper elementary level for most of their career. Mr. Chestnut has 24 years of teaching experience and teaches fifth grade at Pelican Elementary, a school located in a rural fringe community. According to the Virginia Department of Education, about 35% of students attending Pelican Elementary experience an economic disadvantage. Ms. Birch has ten years of teaching experience and teaches third grade at Bluebird Elementary, a virtual school serving a small city. Information about the demographics of the community is difficult to determine because students from all over the city attend her school, and the school system reports data on students based on the school they would attend in person. Both report that they use CRA often to provide instruction and remediation to students in inclusive settings.

The teachers in this study with extensive experience teaching mathematics but lack specialized mathematics education coursework, have an emerging conceptualization of CRA. Both participants define CRA as a gradual release model in which teachers model math content or skills to support student learning without explaining how the phases are connected. Furthermore, both demonstrate general knowledge of the phases and procedures for teaching CRA. For example, they can accurately name and provide a general description of all three phases of CRA. When describing the procedure, both participants mention that modeling provides students with a foundation to construct their knowledge. They also think teachers can determine student readiness to transition to the next phase of CRA by looking for student cues. Both participants explain that the strategy improves student learning by teaching abstract math content with familiar objects and visuals.

Mr. Chestnut

Mr. Chestnut has accurate but general knowledge of CRA. He has the most teaching experience compared to all other participants. He has been teaching for 24 years, 21 of those years spent teaching fifth-grade math. While he has extensive experience teaching upper elementary mathematics, he has not had recent formal college coursework in math methods. His teacher preparation program did not include a specific math methods course, opting for a general methods course. Overall, Mr. Chestnut comes across as knowledgeable and confident in using CRA to teach mathematics to SWD or MD. He states that he would feel more confident if he better understood how to support the diverse needs of SWD. Though it was not called CRA, Mr. Chestnut explains that he first learned about CRA from a math specialist in a previous school district. He adopted CRA because it aligned with what he was taught in his teacher preparation program; to use manipulative and representations. In recent years, he took it upon himself to read education-focused websites and blogs to help him improve his teaching with CRA.

He defines the CRA strategy as a gradual release model in which teachers provide scaffolded instruction to support student learning. Mr. Chestnut states that teachers should start with concrete, and as students demonstrate understanding, they can work their way up to the abstract. His definition of CRA does not go as far as those with advanced conceptualization of CRA. Though he mentions several times that the phases help students make connections, he does not explain the importance of explicitly modeling those connections for students. Instead, Mr. Chestnut focuses on modeling to "give them some parameters and give you a chance to model why you're doing it."

After accurately naming the phases of CRA as concrete, representational, and abstract, Mr. Chestnut then briefly describes the phases and procedure. His descriptions are less rich and detailed than those with more advanced conceptualizations of CRA. For example, he states that students interact with familiar and tangible objects during the concrete phase. As students move objects around, they can see what happens to them, and this helps them develop their understanding of math content. Mr. Chestnut explains that after students have had a chance to model with a physical object, teachers should have students represent the mathematics content with a 2D drawing rather than a 3D object. This is a transition step, and depending on the content, students should both draw the images and use pre-made images. He makes the case that students build better connections when they draw the image but concedes that students also need to know how to interpret images for benchmarks and the SOL. When students demonstrate proficiency working with drawings, Mr. Chestnut explains that teachers can introduce students to the abstract phase. In this phase, students work with numbers and algorithms without the 2D or 3D representations.

When asked how he knows students are ready to transition from one phase to the next, Mr. Chestnut responds that students are ready if "they can explain to you what they're doing and why they're doing it and how that's helping them solve the problem." Unlike those with advanced conceptualizations of CRA, Mr. Chestnut has more of an emerging conceptualization because he relies solely on his professional knowledge to guide his decision-making. He draws on his prior professional experience to identify observable cues from his students.

Mr. Chestnut hypothesized that CRA is effective because it helps students generalize their learning through an improved understanding of the math content and procedure. The concrete phase is key to this effectiveness because students learn more than a set of steps to memorize. Again, Mr. Chestnut stops short of explaining how students should connect at the concrete phase, stating that what they learned during the concrete phase "should carry over to the representation and abstract phase." When asked, he shares that he believes CRA benefits all students at all grade levels but using CRA to teach more complex mathematics at the secondary level would be difficult.

My analysis revealed that Mr. Chestnut has an emerging conceptualization of CRA. He defines CRA as a gradual release model, focusing on supporting student learning by introducing the math content or skill with familiar objects and visuals. However, he does not highlight the importance of helping students make connections between each phase. He demonstrates that he has general knowledge of CRA, providing brief but accurate descriptions of the phases and procedure, using professional experience to determine student mastery, and offering additional details explaining how the individual elements impact student learning.

Ms. Birch

While Ms. Birch's description of CRA is accurate, she demonstrates only general knowledge of the strategy. She has ten years of teaching experience but lacks advanced or recent formal college coursework on mathematics methods. However, Ms. Birch did receive CRA training from a special education researcher at the start of her career. Her classroom was part of a study, and the researcher provided professional development training to the teachers on using CRA. She also learned about CRA from her school district's math specialist. This teacher would come into her class and model how to use CRA and how to implement the strategy in her classroom context. Ms. Birch is very confident using CRA to teach mathematics to SWD or MD. Though SWD need more support than non-disabled peers, she feels comfortable providing that support because she transferred what she learned about CRA to apply to the virtual environment.

Ms. Birch's definition of CRA falls short of participants with an advanced conceptualization of CRA. Like Mr. Chestnut, she defines the strategy as a gradual release model in which teachers provide scaffolded student support. Her definition also fails to highlight the importance of helping students make connections between each phase. However, Ms. Birch's description of modeling is slightly different because she explains that modeling provides a scaffold so students know what to do. Her understanding of modeling is largely procedural.

Ms. Birch correctly identifies the three phases of CRA as concrete, representational, and abstract. However, she expresses uncertainty with her accuracy. She then provides a brief overview of each phase. Starting with the concrete phase, she explains it is usually a manipulative right before you. Due to her virtual context, she includes digital tools in her description of the concrete phase. Ms. Birch states that the concrete phase helps students develop solutions and understand why. However, unlike Mr. Chestnut, she does not elaborate on how manipulatives support students in building any connection to the abstract.

According to Ms. Birch, students draw pictures of the manipulatives during the representation phase. She explains, "I would have my kids literally draw out the base-10 blocks on a piece of paper." Ms. Birch also thinks this stage benefits teachers by allowing them to see how students are thinking about the content and identify the strategies they are using. Like Mr. Chestnut, she argues that students gain more from the act of drawing the representation than from looking at a pre-made image. She continues her description of CRA by explaining that the abstract phase is when students use symbols, like equations, to solve math.

When asked, Ms. Birch explains that it is difficult to set a rule for when students are ready to transition from one phase of CRA to the next because every student and standard differs. Like Mr. Chestnut, she relies solely on her professional knowledge and experience to help her identify when students are ready to transition to a new phase of CRA. She claims, "I think you just have to read your students and know and understand and see the frustration level." Ms. Birch briefly acknowledges that this method is not always accurate, adding "and also know that it's okay if you move and they need to go back."

Unlike participants with an advanced conceptualization of CRA, Ms. Birch cannot provide a clear explanation for why CRA is effective. In alignment with her belief that CRA is a tool teachers use to scaffold student learning, Ms. Birch explains that CRA is effective because it allows students to pick a strategy that works for them and builds their confidence in mathematics. She adds that modeling is key to this effectiveness because students would not know where to start if the teacher did not walk them through the correct method. She is only concerned that CRA would not benefit students who are embarrassed by using manipulatives or drawings. Ms. Birch states that good teachers prevent students from feeling singled out by making sure all students use manipulatives and pictures at some point.

My analysis revealed that Ms. Birch has an emerging understanding of CRA. She defines CRA as a gradual release model, supporting students with scaffolded support by first modeling a procedure using manipulatives, then with drawings, and finally with numbers and symbols. Like Mr. Chestnut, she also does not highlight the importance of helping students make connections between each phase of CRA. Ms. Birch has an accurate but general understanding of the phases and procedures for implementing the strategy to teach mathematics to SWD or MD. Like Mr. Chestnut, she provides clear descriptions of the phases but only provides brief explanations for how the individual elements lead to student learning.

Summary

In this study, I found that participants with extensive teaching experience who lack specialized mathematics education coursework have an emerging conceptualization of CRA. They define the strategy as a gradual release model in which teachers provide scaffolded instruction. While they do not explicitly explain the need to model how the phases are connected, they understand that representations bridge the concrete and abstract phases. They demonstrate basic knowledge of CRA by providing accurate and general descriptions for each phase of CRA. Their basic knowledge of CRA includes an incomplete understanding of how teachers should make decisions about student readiness to transition from one phase to the next. According to participants with an emerging conceptualization of CRA, the strategy is effective because students learn abstract math content using familiar visuals.

Developing Conceptualization

Ms. Walnut and Ms. Aspen have only been teaching mathematics for the last four years due to school policies implemented at the beginning of the COVID-19 pandemic. Ms. Walnut has 12 years of teaching experience and has taught fourth grade at Bluebird Elementary, a virtual school in a small city school system, for the last four years. Though she has a bachelor's degree, she is currently enrolled in a master's degree for school leadership. Like Ms. Birch, her students' data is reported to the state based on the school they would attend in person. She reported on her survey that she always uses CRA to provide new instruction and often uses CRA to provide remediation. Ms. Aspen has taught fifth grade at Pelican Elementary, a school located in a rural fringe school district, for ten years. According to the Virginia Department of Education, about a third of students attending Pelican Elementary experience an economic disadvantage. She already holds a master's degree but does not have advanced coursework in math methods or content. Ms. Aspen reported on her survey that she rarely uses CRA to provide instruction and sometimes uses it to remediate.

The teachers in this study with limited experience teaching mathematics and who lack specialized mathematics methods coursework have a developing understanding of CRA. Both participants define CRA as a sequence of teaching in which students learn math content or skills using manipulatives or hands-on objects, drawings or pictures, and numbers. Like those with an emerging conceptualization of CRA, they also do not mention helping students make connections between the phases. Both participants have limited knowledge of CRA. They demonstrate limited knowledge by their inability to accurately name the phases of CRA and their vague descriptions of the phases. When describing the procedure, participants in this case respond with little elaboration other than presenting math concepts or skills in three different ways. Neither participant knows how to determine when a student is ready to transition from one phase of CRA to the next. They also do not clearly explain why CRA is effective for teaching mathematics.

Ms. Walnut

Ms. Walnut demonstrates that she has an inadequate knowledge of CRA. Though she has more than ten years of teaching experience, she has only been teaching math for the last four years. While she is currently enrolled in a master's degree program, none of her classes include math methods. Her limited knowledge of math instructional practices and brief experience teaching mathematics affects her confidence in teaching mathematics to SWD or MD with CRA. She laments, "It's still so new to me. On top of everything else, I just haven't had the chance to learn more about how to use it better." When answering questions, Ms. Walnut frequently confuses CRA with another math strategy known as Building Fact Fluency. For example, she reports she learned to engage students in number talks using real-life objects and pictures.

What stands out about Ms. Walnut's definition of the CRA strategy is that she focuses on the sequence, teaching first with manipulatives, then images, and finally numbers. Like Ms. Birch, she focuses mostly on modeling a procedure, explaining that she thinks it is important to model multiple ways to solve a problem. However, her response does not rise to the emerging conceptualization category because she does not explain how modeling is a way to scaffold instruction for students. Her definition is also different from those with advanced conceptualization because she does not describe the importance of helping students make connections between the phases of CRA.

When asked to name and describe the phases of CRA, Ms. Walnut cannot name the phases accurately. After being provided with accurate names, she attempts to describe each phase. Her descriptions are vague and occasionally inaccurate. During the concrete phase, she explains that students discuss what they see and know after looking at a photograph of everyday objects. As a virtual teacher, Ms. Walnut also utilizes digital manipulatives through a web-based app, Braining Camp. She is unsure how manipulatives support students' learning but believes it is helpful for students to see a math concept physically. Ms. Walnut's description of the representation phase is also vague. She explains that students make pictures of the objects used during the concrete phase. According to her, seeing a shaded picture of a number helps make math a little more solid for students. Though it is difficult in her virtual context, she asks students to draw their representations using a virtual whiteboard. She states, "I feel like it is helpful to them, the ones that actually do it. It's kind of like a connection to them." When describing the abstract phase, Ms. Walnut inaccurately explains that it is an opportunity to push students into describing what they see.

Like those with an emerging conceptualization of CRA, Ms. Walnut explains the importance of using observations of her students to determine their readiness to transition from one phase of CRA to the next. However, her professional knowledge and experience have led her to use inappropriate student cues. Ms. Walnut explains, "It's almost like they refuse to do it. They say, 'I don't want to do this.' I ask them, ' Well, can you do the standard algorithm?' You just have to ask them."

Like other components of CRA knowledge, Ms. Walnut cannot provide a clear and in-depth explanation for how or why CRA improves student learning. She responds that it is important for them to know. Ms. Walnut also does not think one element of CRA is key to its effectiveness, stating that all the elements play an important role. She does not elaborate further. Like Ms. Birch, Ms. Walnut also expresses concerns that CRA might be too "babyish" for older students whose teachers have to find tricks, like using Legos, to get students to use manipulatives.

My analysis revealed that Ms. Walnut has a developing conceptualization of CRA. She defines CRA as a sequence that progresses from concrete to abstract with no connections between the phases. Ms. Walnut demonstrates that she has an inaccurate or vague knowledge of CRA and is unable to provide accurate descriptions of the phases. When describing the procedure, Ms. Walnut focuses on describing CRA as a strategy for teaching multiple methods for solving a problem. She is also unable to explain why CRA is effective for teaching mathematics to SWD or MD.

Ms. Aspen

Ms. Aspen has limited knowledge of CRA. She also lacks confidence, often unsure if she answers questions correctly. She has only four years of experience teaching mathematics in fifth grade and has not taught math at other grade levels. Her lack of knowledge and experience also impacts her confidence in teaching SWD or MD. Ms. Aspen admits that she struggles to teach students who do not get a math concept or skill the first time she teaches it because she does not know how to teach a skill in more than one way. She also does not know which math content students should have mastered by the fifth grade. According to Ms. Aspen, CRA was not part of her teacher preparation program, and she only learned about CRA during a brief professional development course offered by her school system two years ago.

Like Ms. Walnut, Ms. Aspen's definition of the CRA procedure stands out because she describes it vaguely as a sequence. Ms. Aspen does not think the CRA strategy is designed for authentic classroom settings, explaining that to teach it, teachers "start with the concrete or the manipulatives, then move to the drawing, and then the numbers. It doesn't always happen. But that's in an ideal world." This definition does not align with an emerging conceptualization of CRA because she does not describe how the strategy provides scaffolded support. Unlike those with advanced conceptualization of CRA, Ms. Aspen fails to explain the importance of helping students make connections between the phases.

Though Ms. Aspen can only name the concrete phase of the CRA strategy accurately before giving up, she continues describing the phases and procedures after being reminded. Compared to participants with an emerging conceptualization of CRA, Ms. Aspen's descriptions of the phases are vague and lack rich detail or insight. She describes the concrete phase of CRA as the manipulative hands-on piece. The phase improves students' problem-solving ability because they learn more than memorizing steps. However, unlike Mr. Chestnut, she does not elaborate on what happens in the concrete phase to help students learn. Ms. Aspen provides a similarly vague description of the representation phase, explaining that after using hands-on manipulatives, teachers should introduce students to drawing. Here, she references learning modalities, explaining that students who are visual learners can process information at the representation phase and gain a better understanding of what they are doing. She expresses uncertainty over whether it matters if an image is generated by the student or provided by the teacher. Ultimately, she decides that it depends on the content. Ms. Aspen does not provide an example or explain her reasoning for why some content needs to be drawn by the student and some content can be pre-made. Her description of the abstract phase is also vague. After describing the first two phases, Ms. Aspen briefly mentions that the abstract phase is just numbers.

When asked which criteria teachers should use to determine whether a student is ready to transition from one phase to the next, Ms. Aspen responds that she does not know. After some clarification from the researcher, Ms. Aspen again states that the CRA strategy is unrealistic in her context. She expresses her skepticism and responds, "In an ideal world, I'd like to say mastery. But that's just so unrealistic because you might have some students who will take all year." Unlike those with emerging conceptualization, Ms. Aspen ultimately provides an inaccurate response, stating that she would use a set number of lessons to determine when students are ready to transition. Further probing led Ms. Aspen to guess that students would need a minimum of two lessons at each phase to be successful. This decision is not rooted in any professional learning or experience.

On the surface, Ms. Aspen is aware that CRA supports deeper learning in a way that strictly procedural instruction does not provide. However, her elaboration on how or why the concrete and representational phases support student learning is weak. She reiterates that it helps students learn more than just the steps. Ms. Aspen thinks the concrete phase is key to student learning because it helps students visualize the math concept or skill. Though Ms. Aspen thinks CRA benefits many students across grade levels, she is skeptical that the strategy is useful with more difficult math skills, like the order of operations and converting metric units.

My analysis revealed that Ms. Aspen has a developing conceptualization of CRA. She defines CRA as a sequence in which teachers show students how to solve a problem using manipulatives, drawings, and numbers. Though Ms. Aspen's responses are more accurate than Ms. Walnut's, she still demonstrates a limited understanding of CRA because her descriptions of the phases and procedures are vague and brief. There are occasional inaccuracies, such as when she suggests an inappropriate method to determine student readiness to transition between the phases. Ms. Aspen describes CRA as an effective strategy for teaching mathematics but does not understand why.

Summary

In this study, I found that participants with limited experience teaching math and who also lack specialized mathematics education coursework have a developing conceptualization of CRA. The two participants that fit this description define the strategy as a sequence that progresses from concrete to abstract and neglect to explain how the phases are connected. Both participants demonstrate a limited understanding of CRA by providing inaccurate or vague descriptions of each phase and the procedures of CRA. Another example of their limited knowledge of CRA is their inappropriate understanding of how teachers should make decisions regarding student readiness to transition from one phase to the next. When asked, neither participant can clearly explain how CRA improves student learning.

Comparing Conceptualization

The first research question's purpose is to explore how well third- through fifth-grade teachers know CRA and how they conceptualize the strategy. By conducting a multiple-case study, I explore similarities that persist regardless of exposure to specialized mathematics education courses, years of teaching, school context, or access to a math coach. This approach also made identifying meaningful, predictable differences possible when considering my conceptual framework.

Through two coding cycles, data analysis uncovered patterns influencing three distinct categories of teachers' conceptualizations of CRA. I proposed that teachers' conceptualization of CRA would vary based on the quality of their professional learning. This proposition was rooted in my conceptual framework. Ineffective professional learning is a barrier to implementing an EBP such as CRA. Teachers develop misconceptions when professional learning is brief and shallow, without making a concerted effort to connect learning with prior knowledge. I also looked for disconfirming evidence that would disprove my proposition. One rival proposition is that teachers' conceptualization of CRA would vary based on some other factor. It appears that teaching experience may also explain variations in teachers’ conceptualization of CRA.

The pair of teachers whose conceptualization of CRA is advanced took specialized coursework in the field of mathematics education and had extensive experience teaching mathematics in upper elementary grades. Alternatively, the pair of teachers whose conceptualization of CRA is emerging had limited formal education in mathematics methods and described their professional development as ineffective. However, like the first pair of teachers, they also had extensive experience teaching mathematics. Their extended professional experience teaching mathematics in upper elementary school could explain their emerging understanding. Finally, the pair of teachers whose conceptualization of CRA is developing also do not remember learning about CRA in their math methods course and describe ineffective professional development experiences. Both teachers are relatively new to teaching mathematics at the upper elementary level. It appears that experience teaching may explain why teachers demonstrate an emerging conceptualization of CRA despite their limited professional learning.

Experience also seems to be a key factor explaining why teachers who have minimal experience teaching mathematics are still developing their conceptualization of CRA.

Chapter 5: Reported Enactment

This chapter explores how third- through fifth-grade teachers report enacting the concrete-representational-abstract (CRA) strategy to teach students with disabilities (SWD) or mathematical difficulty (MD). During the second coding cycle, three themes emerged from the interview data. The reported enactment included the participants' plan for using CRA to teach a mathematical skill, the reported implementation, and intentional modifications. Participants' plans included the lesson design (skill or topic, materials used, and planned pedagogical steps) and planning support (the resources participants accessed while preparing the lesson). Reported implementation included participants describing their teaching actions and changes made to the lesson plan at the time of implementation. Intentional modifications were either minimal, moderate, or extensive. Analysis of the interview data also yielded three distinct levels of reported enactment among participants (advanced, emerging, and developing). Reported enactment varied according to participants' knowledge and confidence with the CRA strategy and factors related to the teaching and learning context. Table 9 (see Appendix L) summarizes the three levels of reported enactment.

The subsequent sections of this chapter present the results for the second research question, drawing from multiple data sources, including individual semi-structured interviews and instructional materials for six participants. The focus is on three pairs of teachers, each representing a different level of reported enactment. Clear and detailed lesson plan descriptions and materials, explicit connections between the phases, and minimal modifications to the strategy characterize an advanced reported enactment of CRA.

I define minimal modifications as any change to the strategy that does not eliminate any phase or the components of explicit instruction. An emerging reported enactment of CRA includes clear but general lesson plan descriptions and materials, implicit connections between the phases, and moderate modifications to the strategy. I define moderate modifications as any change to the procedure while keeping the strategy structure intact. A developing reported enactment of CRA includes vague or incomplete lesson plan descriptions and materials, an absence of connections between the phases, and extensive modifications to the strategy. I define extensive modifications to the strategy as changes to the structure of the phases and the procedure for implementing the strategy. This chapter also includes a section dedicated to results from cross-case analysis, highlighting the significant impact of the CRA strategy on students' learning.

Advanced Reported Enactment

As mentioned in the previous chapter, Ms. Elm and Ms. Hickory completed specialized mathematics education courses during their graduate studies. They also both report feeling very confident in their ability to use CRA to teach mathematics in their inclusive classrooms and help other teachers learn how to use CRA. According to Ms. Elm, Oriole Elementary provides teachers with abundant support and resources aligned to CRA, reducing barriers to using CRA in her fifth-grade inclusive classroom. Ms. Hickory also reports that her school, Cardinal Elementary, provides extensive support and resources to help them design high-quality lesson plans.

The teachers in this study who have in-depth knowledge of CRA and report having high confidence in their ability to use CRA to teach mathematics in inclusive settings and train other teachers how to use CRA have a reported enactment that is characterized as advanced. Access to extensive resources and support may also factor in their reported enactment. They provide clear and detailed lesson plan descriptions and materials, vividly depicting their intended lesson. When reporting their implementation, they describe providing students with explicit instruction regarding how the phases are connected and use explicit instruction to make their thinking visible to support student learning.

Additionally, teachers whose reported enactment is advanced describe making minimal modifications to the CRA strategy during implementation. While they admit they do not always wait for students to demonstrate mastery at one phase before introducing another, they describe intentional remediation for those students who are pushed forward before demonstrating readiness. Additionally, participants whose reported enactment is advanced describe minimizing the abstract phase during their instruction. Instead, they expose students to abstract mathematical notation throughout the concrete and representation phases while making explicit connections. I define these modifications as minimal because students still receive all components of the CRA strategy, including explicit instruction, support at their level of mastery in the form of remediation, and exposure to all three phases.

Ms. Elm

Ms. Elm's confidence in using the CRA strategy to provide mathematics instruction in an inclusive setting is palpable. She trusts her ability to select appropriate manipulatives to teach most math skills and model how to use them for students. While she has her preferred manipulatives, she is also open to using others in her instruction. Ms. Elm's confidence extends to her ability to explain the CRA strategy to a colleague, which she has successfully done to help other teachers on her grade level implement it in their classrooms.

Her school, Oriole Elementary, provides teachers with a wealth of materials to aid in designing comprehensive lesson plans, including a curriculum aligned to CRA with teacher-facing materials about misconceptions, skill progression, and guidance for using CRA to teach

each unit. She also benefits from ample resources and support from key individuals in her school, such as administrators, math coaches, and specialists. Oriole Elementary dedicates about 70 minutes of mathematics instruction daily for fifth graders.

For this study, Ms. Elm describes and provides lesson materials for a fifth-grade unit covering single and multi-step word problems. Within this skill, she plans to teach lessons on division and multiplication problems. Her school's pacing guide allows for about two weeks to teach this skill, and she explains that during this time, she wants to spend a week targeting division and another week targeting multiplication. She makes time for whole and small-group instruction when planning her lesson. To support student learning, she selects base-10 blocks for the concrete instruction and drawings of base-10 blocks for representation instruction.

Ms. Elm thoroughly describes her intended lesson and implementation using CRA for ten days of instruction on single multistep word problems focusing on multiplication and division. While Ms. Elm uses CRA regularly, she uses it most in differentiated small-group instruction. She uses CRA to help students build foundational learning to progress to more complex skills. Ms. Elm explains that she typically provides concrete instruction to the whole group for one or two days. The abstract mathematical notation is present alongside her concrete modeling. She then introduces the representation phase, and while most of her class will continue instruction at the representational level, she continues to use manipulatives in small groups for students who need additional support. Ms. Elm also describes using manipulatives and representations nearly every day. Though she introduces the concrete and representation phases separately, she embeds the abstract alongside each phase, a common practice for teachers conceptualizing CRA as a framework model. As a result, she explains, students spend little time working exclusively with the algorithm.

On this first day of instruction for this skill, Ms. Elm describes using manipulatives during a whole group lesson. During subsequent whole-group instruction, she uses representations. Still, she explains that when she pulls students for small-group instruction, she will continue to provide concrete instruction for about half of her students. She supports their progress by emphasizing the connection between the concrete, representation, and abstract. Ms. Elm's lesson materials demonstrate that after introducing students to concrete and representation, she prompts them to connect their pictures to the algorithm or partial quotients. Using explicit instruction, she explains to students how to use the manipulatives to solve problems and employs think-alouds to help draw their attention to important content features. In Ms. Elm's words, "Embedded in the explicit instruction, I also have my think-alouds of what I'm thinking when I'm doing this and trying to help students make strategic decisions about the manipulatives they're using or the representation they're using. Even within a representation, what might be more effective, drawing ten lines or writing the number ten?"

Though Ms. Elm admits to intentionally modifying the CRA strategy, her changes are minimal because she still exposes students to all components of the CRA strategy, including explicit instruction, support at their level of mastery, and exposure to all three phases. Ms. Elm discloses that she moves quickly from the concrete to the representation phase. The abstract phase is not her priority, and she may refrain from introducing procedural methods or algorithms to allow students to build a deeper understanding. As a result of this quick transition, Ms. Elm realizes she intentionally introduces representations before all students are ready. She explains that this decision is made for practicality because she does not think there is enough time in the schedule. However, students continue to receive additional differentiated small-group instruction and a blended approach with manipulatives and drawings to speed up student progress. Having a plan for continuing to provide appropriate instruction for students at their level of mastery makes Ms. Elm's modifications minimal.

Ms. Hickory

Ms. Hickory also feels confident using CRA to teach mathematics instruction in an inclusive setting. She describes learning how to select and use a variety of manipulatives to teach mathematics across all grades. However, if unsure which manipulative to select, she looks for answers by checking reputable education websites and the Virginia Department of Education. Ms. Hickory also describes feeling very confident in explaining CRA to her colleagues. She points out that over the summer, she was asked to develop and lead a professional development on CRA for elementary teachers across the district.

According to Ms. Hickory, her school provides teachers with materials to help them design high-quality lesson plans. Teachers and students have access to an extensive mathematics curriculum focused on universal design for learning, equity, and ambitious instruction aligned to the state standards. This curriculum highlights multiple representations and aligns well with the CRA strategy. Ms. Hickory also states she can access ample resources and support from key individuals like her administrator and other math coaches in her school and district. Cardinal Elementary provides 90 minutes of mathematics instruction every day for fourth graders.

For this study, Ms. Hickory describes a fourth-grade unit covering simplifying fractions and provides relevant instructional materials. As part of this larger unit, she plans to teach equivalent fractions. Her school's pacing guide allows for about two and a half weeks to teach simplifying fractions, and she wants to spend at least four days on equivalent fractions. Ms. Hickory plans to use small group instruction throughout all lessons. To support student learning, she selects circle pieces and fraction tiles for the concrete instruction and drawings of fraction pieces for representation instruction.

Ms. Hickory also thoroughly describes her intended lesson using CRA for four days of instruction on making and identifying equivalent fractions. She reports using CRA to teach all her students and prefers to conduct her teaching in a small-group setting. Though she does not emphasize the abstract phase, like students in Ms. Elm's class, her students receive exposure to mathematical notation alongside the concrete and representational phases. Explicit instruction is a key component of Ms. Hickory's CRA instruction. She says, "We follow the 'I do, we do, you do' method where I'm doing it first, and you're watching. You might have questions and I answer those questions. Then, we do it together where I could provide immediate feedback to them. And then they're trying it and practicing it on their own." She explains that allowing students to move back and forth between the phases is important to helping students gain confidence even if they have previously demonstrated mastery.

For the first three days of instruction for this skill, Ms. Hickory provides concrete small-group instruction alongside the abstract mathematical notation. She introduces students to the skill in her small groups using fraction circles and fraction tiles. Students have access to the manipulatives every day. Ms. Hickory provides explicit instruction for students, modeling how to use the manipulatives to create equivalent fractions. On day three, she begins introducing representations but continues to provide access to manipulatives even if they demonstrate mastery. She describes using a combination of teacher discussions and exit tickets to help her identify when it was time to introduce the next phase. With her students who continue to struggle, Ms. Hickory showed them how to use the multiplication chart to make equivalent fractions and determine the factors of a number. She admits she uses this "trick" to help them in addition to using CRA in hopes that they will not feel stuck, and it will further support the connections she is trying to help them build.

Ms. Hickory does not describe making any intentional changes to the CRA strategy. However, she remembers making them in the past, before she was formally trained. For example, she said she would often skip the concrete phase because she didn't understand how to use manipulatives to model a concept and didn't want to teach her students misinformation. As a math coach, she notices the teachers she supports often make the same decisions. They are usually too embarrassed to admit they don't understand, so they skip that step. She also disclosed that, in the past, she would move students to the next phase before they were ready because she didn't know how to help them, and she hoped they would eventually catch on.

Summary

In this study, I found that participants whose reported enactment is advanced describe feeling very confident using CRA to teach mathematics in inclusive settings and in their ability to train other teachers to use CRA. They also have in-depth knowledge of the CRA strategy. An advanced reported enactment is characterized by lesson plan descriptions and materials that are detailed and extensive, making it easy to understand what they intended to teach during their lesson. Furthermore, highlighting the use of explicit instruction to make their thinking visible to students and to help students understand how the phases of CRA are connected is characteristic of advanced reported enactment. While all participants admitted intentionally modifying CRA during planning and implementation, teachers whose reported enactment is advanced describe making minimal modifications to the phases or procedure.

Emerging Reported Enactment

Unlike the participants whose reported enactment is characterized as advanced, Mr. Chestnut and Ms. Birch have not taken specialized mathematics education courses during their teacher training. Still, they have extensive experience teaching mathematics at the upper elementary level. They both report feeling confident using CRA to teach mathematics in their inclusive classrooms, but they express diminished confidence in explaining CRA to another teacher. Mr. Chestnut discloses that he feels limited by his access to resources and support at Pelican Elementary, creating a barrier to how he implements CRA in his fifth-grade inclusive classroom. Ms. Birch, on the other hand, describes ample access to resources and support at Bluebird Elementary, making it easier for her to implement CRA in her third-grade inclusive virtual classroom.

The teachers in this study who report having high confidence in their ability to use CRA to teach mathematics in inclusive settings but feel diminished confidence in their ability to train someone to use the strategy report enactment that is characterized as emerging. For this pair of teachers, access to extensive resources and support appears to be a minor factor in their reported enactment. Despite varying differences in access to resources and support, they provide clear lesson plan descriptions and materials, allowing the reader to develop a general idea of what would occur during the lesson. Though they do not describe making explicit connections between the phases of CRA for students, both participants report making pedagogical decisions intended to support students in recognizing the connections. However, their method for helping students see the connection between phases relies on exposure rather than direct instruction.

Lastly, teachers whose reported enactment is emerging describe making intentional but moderate modifications to the strategy. Like teachers whose reported enactment is advanced, they admit to removing a mastery criterion for phase progression. However, teachers whose reported enactment is emerging do not describe a specific plan for helping struggling students make progress. They also neglect to use components of explicit instruction like think-alouds intended to draw student attention to important content features. I define these modifications as moderate because they leave the structure of the CRA strategy intact, making only procedural modifications.

Mr. Chestnut

Mr. Chestnut expresses confidence in his ability to provide effective CRA instruction to students in his inclusive classroom. He describes feeling capable of selecting the appropriate manipulatives to support student learning and recognizing student readiness to transition from one phase of CRA to the next. Despite his confidence, Mr. Chestnut says he is only partially confident in explaining CRA to other teachers. This diminished confidence is due in part to his lack of recent formal training. In the time that has passed since the professional development provided by his previous school district, he has lost track of which components are considered "official" and which parts are modifications he has made to fit the needs of his classroom.

Pelican Elementary, where Mr. Chestnut teaches fifth grade, does not provide teachers with the same support and resources as the schools where Ms. Elm and Ms. Hickory are employed. Though he does have access to a district-wide online drive folder, it is used as a space for teachers to share resources rather than a place for the district to offer guidance or suggestions. The teachers in his district are strongly encouraged to use resources from the VDOE. Still, Mr. Chestnut often prefers to use materials he finds on education websites that he can modify and adjust for his objectives. Access to materials and support from key individuals is also limited. Often, he does not have enough materials for his students to conduct whole-group instruction with manipulatives. Other schools in his district have access to a math coach, but his school does not. Despite this absence, he reports that his building administrator supports his choice to use CRA to teach mathematics. Pelican Elementary allows for about 70 minutes of mathematics instruction, but Mr. Chestnut estimates students receive about 65 minutes due to transitions.

For this study, Mr. Chestnut describes and provides materials for a fifth-grade lesson on equivalent fractions. His school's pacing guide allows for about ten days to teach the related standard on fractions, which includes making and identifying equivalent fractions and decimals and comparing fractions, decimals, and mixed numbers. Four of those days, he plans to teach this skill. All instruction for this skill took place in small groups. To support students in the concrete phase, Mr. Chestnut selects fraction bars. At the representation phase, students are exposed to both rectangles in place of fraction bars. While planning this lesson, Mr. Chestnut sought out engaging activities to supplement his instruction that he thought would appeal to students, such as incorporating pop culture references and sports.

Mr. Chestnut clearly and concisely describes his lessons using CRA to teach equivalent fractions over four days. He neglects to submit a daily lesson plan with his materials but includes a summary in writing of what took place at each phase of CRA. The materials also include student practice activities but not materials used during teacher modeling or guided practice. Though it is possible to understand his lesson implementation generally, his description is not as detailed or specific as that of teachers with more advanced reported enactments. This lack of detail makes it slightly more difficult to envision his reported enactment. In general, Mr. Chestnut describes a gradual release model for implementing CRA. He explains that he starts every day with the fraction tiles to reactivate prior learning, but as the lessons progressed, he introduced the representation phase and then the abstract phase.

On the first day of instruction, Mr. Chestnut says he teaches students how to make equivalent fractions using fraction tiles. It is unclear if abstract mathematical notation (aside from the numbers written on fraction tiles) is present during this phase of instruction. The next day, he introduced representations with a scaffolded approach in which students had an example of two equivalent fractions shaded for them, a part of an equivalent fraction shaded for them. Then, he had students shade in and identify only the numerator of the equivalent fraction. Mr. Chestnut's lesson includes an activity in which pairs of students made equivalent fractions, but one student used manipulatives, and the other student shaded in rectangles. This activity, along with starting the lesson with a review of the skill with manipulatives, was an implicit attempt to help students make connections between the phases of CRA. Unfortunately, as Mr. Chestnut notes, "I'm making the assumption they're making that connection on their own. And they're not."

Mr. Chestnut's reported enactment includes moderate modifications impacting the procedure for implementing CRA. As defined in this study, moderate modifications are any change that affects the strategy's implementation procedure while keeping the phases' overall structure intact. While describing his lesson, Mr. Chestnut references that he sometimes introduces a phase of CRA prior to student readiness. He says he does this to "get them ready for testing and be able to do the problems that they're going to see on the test because they won't have manipulatives." Furthermore, while describing his lesson, Mr. Chestnut leaves out an essential component of explicit instruction: teacher think-aloud. Unlike teachers with advanced reported enactment, he does not describe drawing students' attention to important features of the content. Despite this oversight, his lessons provide other elements of explicit instruction, such as scaffolded instruction, a systematic plan for fading of supports, opportunities for feedback, and purposeful practice opportunities.

Ms. Birch

Like Mr. Chestnut, Ms. Birch feels confident in her ability to use CRA in her virtual classroom to meet the needs of all students. This confidence includes selecting appropriate manipulatives and modeling how to use them to work through math problems. She admits she feels slightly less comfortable deciding when students are ready to transition, but this has more to do with keeping up with the pacing. Ms. Birch also shares that while she knows how to use CRA, it's been so long since she's done it in a traditional classroom that she wouldn't feel as confident explaining it to a colleague.

Ms. Birch's school, Bluebird Elementary, provides teachers with more resources and support than Mr. Chestnut's school. Teachers at Bluebird Elementary can access curricula and materials through a web-based application called Braining Camp and Virtual Virginia. In addition to virtual manipulatives, the students are provided with a home set of concrete manipulatives and virtual whiteboards to support CRA instruction. Despite the small size of her school, she also has access to a math coach and believes her building administrator supports her choice to use CRA to teach mathematics. Ms. Birch describes teaching mathematics for at least 60 minutes a day to her third-grade students, but she can provide an additional 25 minutes of instruction a few times a week to allow for remediation.

For this study, Ms. Birch describes and provides materials for a third-grade lesson on multiplication and division. The district recommends that teachers spend about five weeks teaching this skill, and Ms. Birch estimates that she spends at least 20 of those days teaching with some CRA component. To support student learning at the concrete phase of instruction, Ms.

Birch used Braining Camp to select a variety of virtual manipulatives, ensuring her students see multiplication and division modeled using groups, arrays, area models, and number lines. The line between a virtual manipulative and a virtual representation can be difficult to distinguish in a virtual setting. During the manipulative phase, it is key that students interact with the virtual manipulative in some way. Ms. Birch uses resources available to her on Braining Camp to help her plan this lesson.

Ms. Birch provides a clear but general description of her intended lessons using CRA to teach multiplication and division over five weeks. Though her reported lesson description is brief, she provides copies of all materials she uses to teach her lessons, including notes for modeling, guided practice, and independent practice. Compared to teachers with advanced reported enactment, her materials do not include a specific lesson plan or brief written explanation for the steps she followed day by day, which makes it slightly more difficult to visualize what took place. Ms. Birch's lesson description and materials suggest she uses CRA as a gradual release model to guide students toward independent mastery without the support of visuals or representations.

At the beginning of a new skill, Ms. Birch provides students with copies of a slide show from Braining Camp called a "pre-made." For the first two to three days of the unit, Ms. Birch uses direct instruction to teach students at the concrete phase. Students have examples to complete that support the lesson objective. Over the next two to three days, she uses direct instruction at the representation phase. Students complete similar problems during this phase that support the lesson objective. Based on her understanding of the standard, she must teach students multiple ways to represent multiplication and division with manipulatives and pictures. Finally, she presents abstract problems to students and asks them to use any model they like to show their work.

Though students are exposed to opportunities to build connections between the phases of CRA, Ms. Birch uses implicit strategies to support this goal. Like Mr. Chestnut, she pairs students she knows prefer different strategies so they can see how to complete the work using multiple representations. Once students have moved through all three phases of CRA, they can select the representation they like best when completing assignments. Ms. Birch says that ideally, students will choose the more efficient abstract phase, but some are not ready for it. She acknowledges that using a set number of lessons at each phase of CRA leads some students to struggle occasionally. Ms. Birch explains, “If I can tell they are struggling, I just say, ‘Hey, you can use the strategy from yesterday, too.’ They have access to Braining Camp, and in their at-home learning kit, they have the red and yellow counters they can use.” This difference separates her reported enactment from those whose reported enactment is advanced because she does not describe a plan for helping struggling students make progress.

The modifications Ms. Birch makes to the CRA strategy do not impact the overall structure of the phases. Students are still exposed to teacher modeling, guided practice, and independent practice for all three phases of CRA. However, she makes changes to CRA that impact the procedure for implementing the strategy. Ms. Birch recognizes that students sometimes struggle after she introduces a new phase of CRA before they are ready. She justifies this by explaining, “Simply because the real talk of it – like SOLs and everything else – you have to at least show it to them, even if they don’t fully understand it. You’re doing them a disservice if you don’t at least show it. Knowing that they are going to struggle or get it wrong.” Like Mr. Chestnut, she also neglects to include a description of teacher think-aloud as part of her explicit instruction. However, it is clear from her materials and descriptions that all other elements of explicit instruction are present.

Summary

In this study, I found that teachers whose reported enactment is emerging describe feeling confident in their ability to use CRA to teach mathematics in inclusive settings but feel less satisfied in their ability to train others to use the strategy. They also have an accurate but general understanding of the CRA strategy. Unlike the pair of teachers whose reported enactment is characterized as advanced, this pair of teachers have dissimilar access to resources and support, suggesting that the quality of support and resources may not influence the reported enactment of teachers who have substantial experience teaching mathematics. Lesson plan description and materials that are clear but lacking in explicit details are characteristic of an emerging reported enactment. Another characterization of emerging reported enactment is teachers making implicit connections between the phases, relying on exposure rather than direct instruction to support students in building connections between the concrete, representational, and abstract phases. Lastly, making moderate modifications to the procedure for implementing CRA is characteristic of reported enactment that is characterized as emerging.

Developing Reported Enactment

As noted previously, Ms. Walnut and Ms. Aspen, despite their ten years of teaching experience, have only taught math at the upper elementary level for the last four years. Compared to the other pairs of teachers in this study, they report low confidence levels in using CRA to teach mathematics in their inclusive classrooms and their ability to explain CRA to a colleague. Ms. Walnut, teaching at Bluebird Elementary, is fortunate to have access to a wealth of resources and support. However, Ms. Aspen, who teaches at Pelican Elementary, faces a different reality with fewer resources and support. This limitation concerns Ms. Aspen, who feels this impacts her ability to plan quality CRA lessons.

The teachers in this study who report having the lowest level of confidence in their ability to use CRA to teach mathematics in inclusive settings and explain it to other teachers report enactment of CRA that is developing. Access to extensive resources and support does not seem to influence their implementation of CRA. They have varying access compared to each other and similar access to the other teachers whose reported enactment is emerging. The lesson plans and materials this pair of teachers provide are vague and incomplete, making it difficult for the reader to understand the intended lesson clearly. Furthermore, their description of how they implement CRA does not include making explicit or implicit connections between two or more phases.

Extensive modifications to CRA's phases or implementation procedures are also characteristic of a developing reported enactment. For example, reported enactment is developing when teachers expose students to phases of CRA before they demonstrate readiness and do not have a plan for providing specific remediation to support their progress. Moreover, several components of explicit instruction are absent from their reported implementation, disclosing the removal of a phase of CRA from their instruction and leaving out multiple steps in the explicit instruction process. I define these modifications as extensive because they change the structure of the phases and the strategy implementation procedure.

Ms. Walnut

Ms. Walnut describes lower confidence levels in her ability to provide effective CRA instruction to students in her virtual inclusive classroom. She explains that she would feel more confident in her abilities with more time to learn about the strategy. Her low confidence translates to her ability to explain CRA to another teacher. Ms. Walnut shares that due to the competing and overwhelming requirements for her job, she hasn't had time to learn how to use the new strategy as well as she'd like.

Like Ms. Birch, Ms. Walnut also teaches at Bluebird Elementary. She describes abundant access to resources and supports through their Braining Camp subscription, access to Virtual Virginia materials, and the home learning kits provided to her students. These kits include concrete manipulatives as well as virtual whiteboards. Ms. Walnut expresses sincere gratitude for the support from key individuals at her school who support mathematics instruction. She describes meeting with the math coach at least once a month, and her building administrator has taken steps to ensure the teachers have access to resources that support their ability to use CRA in a virtual setting. Bluebird Elementary includes 75 minutes in the schedule for fourth-grade mathematics instruction, though Ms. Walnut admits she sometimes teaches math longer. She wishes she had an additional 30 minutes to teach her students mathematics.

For this study, Ms. Walnut describes and provides materials for a fourth-grade lesson on equivalent fractions. Her school's pacing guide allows for about two weeks to teach skills related to the fourth-grade fraction standard. Ms. Walnut estimates that she will spend about five days, or half that time, teaching students how to make and identify equivalent fractions. All instruction took place in a whole group setting. Ms. Walnut selects virtual fraction tiles on Braining Camp to support instruction at the concrete and representational phases. As previously mentioned, the key difference between a virtual manipulative and a virtual representation is whether students interact with the image during instruction or practice. Ms. Walnut explains that she recycled this lesson from last year. She also shares that she has received support from the previous fourth-grade teacher. Despite having access to a math coach, Ms. Walnut said that she only reaches out to her if there is a problem with a student or a tricky situation. She doesn't reach out to her for support with her lesson planning.

Ms. Walnut provides a vague and incomplete description of her lesson using CRA to teach equivalent fractions over five days. In her materials, she does not include a daily lesson plan or a summary explaining how she uses her materials for these lessons. The materials include nine pages of guided notes, student practice, and an independent practice activity. One page of the student notes provides the learning objectives. Four pages review vocabulary and activate prior knowledge. Three additional pages direct students to explore making equivalent fractions with virtual manipulatives, identifying equivalent fractions from a picture, and matching equivalent fractions provided in numerical form.

Without a clearer description of what took place, it is impossible to identify Ms. Walnut's daily steps throughout this lesson accurately. However, in response to follow-up questions, Ms. Walnut describes using most of these materials as guided practice. It is unclear if she provides any direct modeling for students because, throughout her description, she neglects to describe using any support for students through think-alouds. She clarifies that students only use manipulatives on the first day of instruction. She justifies this choice, stating, "I feel like they're a little more advanced, and they could easily look at the pictures or the numbers. I didn't spend a lot of time on it. If I had younger students, I probably would use more time for it." During their limited time receiving concrete instruction, students used the physical fraction tiles from their home learning kits. Ms. Walnut also describes introducing representations on the first day after a brief introduction with the fraction tiles. She also does not refer to providing explicit or implicit instruction regarding the connections between each phase. She typically presents the concrete, representation, and abstract phases in isolation. However, she does provide practice for students to identify the number of parts out of a whole for a particular fraction using virtual fraction tiles on the Braining Cramp.

Ms. Walnut's reported enactment includes extensive modifications impacting both the structure of the strategy's phases and the procedure for implementing CRA. Though Ms. Walnut describes using manipulatives to review parts of a fraction and claims that students have access to manipulatives if they want them throughout the lessons, she does not describe using manipulatives to provide, modeling, guided practice, or assess students on their ability to represent equivalent fractions independently. Ms. Walnut also describes moving forward with a new phase of instruction without checking for student readiness and does not attempt to combine phases to support remediation. She admits, "I feel like that part of not taking the time to measure if they're ready to go forward or not is probably one of my personal downfalls." Finally, as alluded to above, Ms. Walnut does not describe using explicit instruction to implement her lessons for this skill. Her materials suggest that students were likely asked to explore the skill before direct instruction or teacher modeling.

Ms. Aspen

Like Ms. Walnut, Ms. Aspen also describes low or non-existent confidence in her ability to provide effective CRA instruction to students in her inclusive classroom and in her ability to explain CRA to a colleague. She says she would feel more confident if she could have more in-depth training focusing on using CRA with application experience. She confides she is unsure how to select manipulatives or representations that best align with her lesson objectives or identify when students are ready to transition from one phase to the next. She exposes students to instruction before they are ready, and she struggles with handling this.

Ms. Aspen, like Mr. Chestnut, teaches at Pelican Elementary and has more limited access to resources and support than Ms. Walnut. She does not mention accessing her district's online drive folders for teachers to share resources. However, she does mention using the mathematics instructional plans provided by the Virginia Department of Education. Ms. Aspen also feels limited by her access to resources and support. She explains that while, in general, the school has limited resources, teachers who have been teaching math longer than her have collected more resources over the years. Furthermore, unlike Mr. Chestnut, who generally feels their building administrator provides support, Ms. Aspen discloses that she thinks her principal could do more to support teachers using CRA by providing more effective professional development opportunities. Pelican Elementary allots 70 minutes for mathematics instruction for fifth grade. However, Ms. Aspen explains that due to transitions time necessary for her students to calm down after lunch, she usually only has 60 minutes of math instruction time.

For this study, Ms. Aspen describes and provides materials for a fifth-grade measurement lesson on area, perimeter, and volume. Her school's pacing guide allows for about four weeks to teach both fifth-grade measurement standards. Ms. Aspen estimates she will spend about two weeks instructing students on solving practical problems involving area, perimeter, and volume and differentiating when to use the three measurements. Instruction took place in both whole and small groups. To support student instruction at the concrete phase, Ms. Aspen selects Unifix cubes. She makes this choice because she has access to a lot of these. For the representation phase, Ms. Aspen selects whiteboards and dry-erase markers. Depending on the activity, she may also use grid paper. Ms. Aspen explains that she usually checks for resources available through the Virginia Department of Education website. According to her, they do not always align with CRA, but she appreciates they give her a starting point for her lessons.

Like Ms. Walnut, Ms. Aspen provides a vague and incomplete description of her lesson using CRA to teach area, perimeter, and volume over two weeks. Though she does not include a lesson plan or a summary describing the steps she follows daily, Ms. Aspen briefly describes using a sequential approach for this measurement skill. The materials she submitted include vocabulary notes, a coloring activity sorting when it is appropriate to use area, perimeter, and volume as a measurement, and practical problems using area, perimeter, and volume. The lack of clarity makes it difficult to accurately visualize the lessons she intends to use to teach this skill.

Ms. Aspen plans to use CRA instruction for about three or four of the ten days in the pacing guide. Students are exposed to hands-on manipulatives during whole-group instruction during the first two days. Ms. Aspen reports, "For area, perimeter, and volume, we always start by building some sort of rectangular structure and discussing how big it is and how you could describe it." Once students move into small groups, she introduces them to drawings on the third day. The remaining days are all abstract. The only description Ms. Aspen provides about her abstract instruction is that they work in small groups for about five of the remaining days. Though there is little evidence of explicit instruction throughout her description or in her materials, Ms. Aspen mentions that she models solving word problems, suggesting that Ms. Aspen may feel more confident providing procedural modeling using algorithms than providing direct instruction with manipulatives or drawings. In a departure from Ms. Walnut, Ms. Aspen provides remediated instruction for students who struggle the most and incorporates instruction using manipulatives and pictures together. Unlike teachers whose reported enactment is advanced, Ms. Aspen explains making this decision to speed students along and not because she knows this will help them build connections between the phases.

Ms. Aspen's reported enactment also includes extensive modification impacting both the structure of the strategy's phases and the implementation procedure for CRA. Like Ms. Walnut, Ms. Aspen describes providing students access to Unifix cubes in a whole group setting on the first two days. However, it appears that students are using the manipulatives as part of an exploration activity and not as a tool for understanding the meaning of area, perimeter, and volume. Furthermore, Ms. Aspen admits that she often removes instruction using manipulatives because she doesn't know how to use them. When time is limited, she may skip the representation phase. She also modifies elements of the procedure for implementing CRA. For example, Ms. Aspen discloses that she doesn't check for student mastery, explaining, "I move on regardless. I have to get them ready for an SOL. I need to expose them to the type of questions they'll see." Finally, multiple components of explicit instruction are missing from her lesson description and materials. She neglects to describe her use of think-alouds to focus student attention on important features, a systematic plan for successfully fading supports, an opportunity for students to respond and receive feedback, and clear, purposeful practice opportunities at each phase of the CRA strategy.

Summary

In this study, I found that teachers whose reported enactment is developing with a describe feeling low confidence in using CRA to teach mathematics in inclusive settings and even less confidence in explaining the strategy to a colleague. Their knowledge of CRA is inaccurate or vague. Like the teachers whose reported enactment is best characterized as emerging, this pair of teachers do not share the same level of access to resources and support, suggesting that the enactment of CRA by educators who have limited experience teaching mathematics at the upper elementary level may not be influenced by the presence of resources alone. Less plan descriptions and materials that are vague or incomplete are characteristic of a developing reported enactment. Characteristic of developing reported enactment, this pair of teachers did not describe providing students with explicit or implicit opportunities to understand how the phases of CRA are connected. A final characterization of developing reported enactment is making extensive modifications to both the phases and the procedure for implementing the strategy.

Comparing Enactment

The second research question's purpose is to explore how third- through fifth-grade teachers report enacting the CRA strategy in their inclusive classrooms. Using a multiple-case research design, I examined similarities that persist across differences in exposure to specialized mathematics education courses, years of teaching experience, and the teaching and learning context. I also explored meaningful, predictable differences informed by my conceptual framework.

After thorough data analysis, I identified patterns influencing three levels of reported enactment: advanced, emerging, and developing. I proposed that teachers would modify the CRA strategy in response to barriers, individual factors, or external factors. Teachers transform a practice when they lack understanding or access to necessary resources and support. I also looked for alternative explanations that would disprove my proposition. One rival proposition is that teachers do not make modifications to the CRA strategy in the presence of barriers or other influencing factors. It appears that teachers do make modifications to the CRA strategy, but those modifications may be influenced more by teacher knowledge and comfort-level. It also appears that individual factors may have more influence on reported enactment than external factors, such as access to resources and support or instructional time.

The pair of teachers whose reported enactment of CRA is characterized as advanced have extensive knowledge of the CRA strategy, including how components of the strategy impact student learning. They also report high levels of confidence using CRA to support SWD or MD. Additionally, they would feel very comfortable explaining CRA to another teacher. Alternatively, the pair of teachers whose reported enactment of CRA is characterized as advanced demonstrate a general understanding of the CRA strategy. They can accurately name components and explain the implementation procedure. However, they only feel comfortable using CRA to teach math in their inclusive setting, expressing doubt in their ability to explain the strategy to other teachers. Finally, the pair of teachers whose reported enactment is characterized as developing have limited knowledge of CRA. They describe the strategy vaguely or inaccurately, expressing uncertainty regarding how it impacts student learning. They also report low levels of comfort using CRA to teach math to SWD or MD and would not want to try and explain CRA to another teacher. It appears that knowledge of the EBP and confidence-level are intertwined, having similar influences on how teachers report enacting CRA in their classroom.

The teachers whose reported enactment is advanced work at schools that provide them with ample access to high-quality resources and support is available from key individuals like math coaches or specialists. However, the pair of teachers whose reported enactment is emerging have different access to resources and support at their schools. One describes his access as limited while the other describes having access to materials, tools, and support that makes implementing CRA less challenging. Similarly, teachers whose reported enactment is developing describe differences in access to resources and support. One teacher thinks that her access to resources and support is a major barrier to how she uses CRA and the other describes having sufficient access to resources. It appears that access to resources and support cannot overcome the barriers created by other factors that may influence how teachers report enacting CRA in inclusive settings.

Chapter 6: Discussion and Conclusion

According to recently collected educational data, about 65% of students with disabilities (SWD) spend most of their day in general education classrooms (NCES, 2022a). Although both the Individuals with Disabilities Education Act (2004) and Every Student Succeeds Act (2015) mandate the use of evidence-based practices (EBP), mathematics achievement results on the most recent National Assessment for Educational Progress demonstrate that both SWD and mathematical difficulties (MD) are not benefiting from the instruction they receive in their general education classrooms. A possible explanation for this can be found in the literature related to the research-to-practice gap, implementation fidelity, and the curriculum enactment process.

Several publications document the barriers that influence the implementation of EBP. These barriers include negative perception or inaccessibility of research (Cook & Cook, 2011; Foster, 2014; Hornby et al., 2013), ineffective professional learning (Koustelini, 2008), and lack of alignment between the EBP and teachers' knowledge, beliefs, or school context (Cook et al., 2013; Foster, 2014; Greenwood & Abbot, 2001; Klingner et al., 2013; Vaughn et al., 2000). Teachers' enactment of the curriculum is also influenced by 1) their knowledge of the content, students, and practice (Hill et al., 2008; Remillard & Heck, 2014); 2) their confidence with the content, students, and practice (Fixsen et al., 2005; Remillard & Heck, 2014); 3) policies and the educational environment (Foster, 2014) and 4) access to resources and support (Remillard & Heck, 2014; Stein et al., 2007). My study seeks to expand on this literature by exploring how general education teachers conceptualize and enact the concrete-representational-abstract (CRA) strategy, an EBP recommended for teaching mathematics to SWD (Bouck, Satsangi, et al., 2018; Jitendra et al., 2016; Lafay et al., 2019; Peltier et al., 2020).

This qualitative study explores the knowledge and practice of an EBP by third- through fifth-grade general education teachers. Specifically, I sought to answer the following research questions: 1) How do third- through fifth-grade general education teachers conceptualize the CRA strategy and 2) How do third- through fifth-grade general education teachers report enacting the CRA strategy in their inclusive classrooms to meet the needs of SWD or MD?

The recruitment sample consisted of six general education classroom teachers from four public schools in different school districts across the state of Virginia. Three participants teach fifth, two teach fourth, and one teaches third grade. All the teachers are fully licensed for their teaching assignments. Their teaching experience ranges from 9 to 24 years, with an average of 13 years of full-time teaching experience.

I collected data from two semi-structured interviews and artifacts such as lesson plans, notes provided to students, presentation materials, practice opportunities, and assessment materials. The data was then coded deductively using a combination of holistic and hypothesis codes, allowing for pattern-matching to compare observed and predicted patterns based on propositions and rival propositions. I used an inductive approach to identify categories and subcategories during second-cycle coding.

In chapters four and five, I presented the findings of each research question. The themes that emerged during the analysis provide insight into what teachers know about CRA, how they use CRA to teach mathematics in their inclusive settings, and possible factors and barriers that influence their conceptualizations and reported enactment. From the data, I identified three main findings: 1) conceptualization of CRA seems to be influenced by teachers' professional learning and experience, 2) enactment of CRA seems to be influenced by teachers' knowledge and comfort level, and 3) access to resources and support may not be enough to overcome contextual challenges. These findings make significant contributions to the field. The results not only fill a gap in the existing literature but also hold the potential to improve the methods for preparing general education teachers to teach mathematics in inclusive settings and the design of instructional strategies to meet the needs of practitioners and the students they are intended to support.

This chapter discusses those findings and their contribution to the field's understanding of the research-to-practice gap, implementation fidelity, and curriculum enactment process. In the following sections, I will discuss the findings in relation to prior research and highlight implications for the field of education research, teacher education programs, and school districts. I will also present limitations and recommend future research to continue exploring questions about how teachers interpret and use CRA to teach mathematics in inclusive settings.

How Professional Learning and Experience Shape Conceptualization

In the literature on the research-to-practice gap, ineffective professional learning is one barrier that impacts implementation fidelity. As Foster (2014) notes, teachers develop misconceptions when they engage in brief and shallow professional learning that does not support them in connecting their learning with prior knowledge over time. My findings also suggest that teachers exposed to ineffective professional learning are likely to develop misconceptions. However, my findings also suggest that experience may help teachers overcome the potentially negative impact of ineffective professional learning. Consequently, this study expands what is known about the research-to-practice gap by incorporating years of teaching experience as a factor that may influence teacher conceptualizations of EBP such as CRA.

Analysis of teacher interview data revealed that general education teachers define the CRA strategy in one of three ways. Teachers who have a developing conceptualization of CRA define the strategy as a sequence in which teachers first use manipulatives, then images, and finally numbers to teach mathematics. This is a traditional description of CRA (Hinton & Flores, 2018; Mancl et al., 2012). Initially, the intervention was designed sequentially, and intervention research described methods for implementation that supported this definition. Other teachers who had an emerging conceptualization of CRA defined the strategy as a gradual release model in which teachers scaffold mathematics instruction. This alternative description emphasizes the importance of the non-linear shift of cognitive work from teacher to student (Flores, 2009; Maccini & Ruhl, 2000). Teachers who had advanced conceptualization of CRA defined CRA as a framework in which teachers explicitly model for students how the phases are connected using an integrated approach to instruction. This more contemporary description emphasizes how conceptual and procedural learning are bi-directional, and instruction is more effective and efficient when it integrates multiple phases (Bundock et al., 2021; Morano et al., 2020; Strickland & Maccini, 2013).

Despite these various ways of defining CRA, there is consensus in special education research on how to teach mathematics using the CRA strategy. Across the board, all studies embed explicit instruction at every phase of the strategy. This includes using advanced organizers, teacher modeling with think-alouds, guided practice with feedback, and independent practice to assess mastery (Archer & Hughes, 2010; Gersten et al., 2009; Hudson & Miller, 2006). Moreover, CRA strategy instruction is more effective when teachers make explicit for students the connections between familiar procedures and unfamiliar mathematical concepts or skills (Argawal & Morin, 2016; Miller et al., 2011). Based on findings from this study, it appears that teachers who have advanced conceptualizations incorporate explicit instruction and emphasize the importance of making explicit connections between the phases, while teachers who have emerging and developing conceptualizations neglect to include this important element in their descriptions.

There are several research syntheses that explore how to improve teacher knowledge and practice through professional learning or coaching (Desimone, 2009; Kennedy, 2016; Kraft et al., 2018). According to Desimone (2009), professional learning is most effective when it focuses on specific content, incorporates active learning opportunities, is consistent with the teacher's prior knowledge and experience, provides support over an extended period of time, and provides opportunities for teachers to interact with other teachers about the new information. Kennedy (2018) explored the effectiveness of four methods (prescription, strategies, insight, and body of knowledge) used by professional development programs to support teacher enactment. His results suggest that teacher enactment improves when they engage in intellectually engaging professional development. Furthermore, enactment improved when teachers collaborated with a coach on their lesson planning. Kraft (2018) echoed the findings of Desimone and Kennedy in his investigation of how coaching can impact teacher practice. He found that teachers benefit from group training followed by context-specific and focused coaching, suggesting that teacher knowledge and practice are interconnected.

These findings could lead one to speculate that professional learning is key to teachers' conceptualization of the CRA strategy. The general education teachers who participated in my study learned about CRA in various ways: some more effective than others. Teachers with extensive knowledge and an advanced conceptualization of CRA also had access to advanced mathematics methods courses in their graduate education. The CRA strategy was a focal point of some of their coursework, and they participated in meaningful discussions on how the elements impact student learning. Both teachers had either completed or were in the process of completing graduate degrees necessary in the state of Virginia to apply for a math specialist endorsement. In contrast, teachers who learned about CRA as part of professional development or from teacher-focused websites did not demonstrate the same depth of knowledge. In some instances, these teachers had a basic level of knowledge of the strategy and how it improves student learning, but other teachers had only inaccurate or vague knowledge.

Unfortunately, effective professional learning experience was not typical for teachers participating in this study. They expressed feeling unprepared by their teacher education program to teach mathematics to SWD or MD. This is unsurprising given that prior research suggests teacher education programs did not prepare pre-service teachers to teach mathematics effectively (Ball et al., 2005). Across the board, participants in this study reported learning about manipulatives in their methods courses, but instruction stopped short of including CRA as a strategy. This was true even of teachers with graduate degrees in elementary education, suggesting that graduate-level math methods courses do not provide more in-depth coverage of instructional strategies than undergraduate versions. Some teachers also expressed disappointment in the quality of their professional development opportunities. Teachers who felt dissatisfied with their training described it as shallow, irrelevant, and too brief. Of interest, several teachers indicated they wished their professional development showed them how to use the strategy with grade-level content and how to identify appropriate manipulatives for each skill. Ultimately, it appears that teachers' conceptualization of CRA was influenced by professional learning that lacked clarity and was irrelevant to their lived experience as educators.

Ineffective professional learning did not seem to affect all teachers the same. As noted, teachers' conceptualizations were either advanced, emerging, or developing. Teachers whose conceptualizations were emerging or developing had similar perceptions of their math methods courses and professional development experiences. Years of experience teaching mathematics may be the factor that creates the divide between these two conceptualizations.

Prior research suggests that experience impacts teacher knowledge and practice. For example, Gilmour and Henry (2018) found that teacher practice is influenced by years of experience because they learn skills and knowledge on the job. However, the effects of teaching experience level off after the first few years (Clotfelter et al., 2007; Goe, 2007; Ladd & Sorensen, 2017). Furthermore, Kennedy (2016) describes the way teachers incorporate new practices into their teaching as a slow and incremental process. In a related study in the field of elementary science education, researchers found evidence suggesting that science teachers reassigned to a new grade level may engage in teaching practices similar to the practices of novice teachers because they lack relevant content and pedagogical knowledge (Hanuscin et al., 2020).

My study builds on this prior research, providing further insight into how teacher experience shapes how teachers conceptualize an EBP like CRA. The pair of teachers whose conceptualization of CRA was emerging had ten or more years of experience teaching math in inclusive settings. Expanding on the work of Gilmour and Henry (2018) and Hanuscin et al. (2020), it is possible that they learned the knowledge and skills because of on-the-job experience. Moreover, the pair of teachers whose conceptualization of CRA was developing had only been teaching mathematics in inclusive settings for four years. It also appears that the type of teaching experience may influence conceptualization. The pair of teachers whose conceptualizations were advanced had experience teaching math in schools they identified as “struggling.” It is possible that teachers with experience working in low-achieving schools may have more advanced conceptualizations of EBP because they are more motivated to seek out instructional strategies to support their students outside of formal professional learning.

Experience seems to diminish the negative effects of ineffective professional learning. Teachers' conceptualizations benefited from access to specialized math methods courses. These courses provided ongoing targeted instruction that goes beyond teaching the basics of evidence-based practices. In contrast, teachers who described their professional learning as brief and shallow in nature had either emerging or developing conceptualizations. Experience varied between these pairs of teachers. Teachers with less than five years of experience teaching math were still developing their conceptualization of CRA. Ultimately, this finding demonstrates a need to improve professional learning for both pre-service and in-service teachers to minimize misconceptions about EBPs such as CRA.

How Teacher Knowledge and Confidence Shape Enactment

In the literature on the curriculum enactment process for mathematics education, Remillard and Heck (2014) identify teacher knowledge and beliefs about mathematics, pedagogy, learning, and curriculum resources as factors influencing the intended curriculum. Furthermore, qualitative research on teacher modifications to curriculum and EBP intentionally in response to factors and barriers (Gelmez-Burakgazi, 2020; Holstein, 2012; Moyer, 2003; Wooley et al., 2013). The findings from my study align with this prior research. It appears that how teachers' knowledge of CRA may have influenced their reported enactment. My findings expand on the prior research by exploring the interconnectedness of teachers' confidence and knowledge. Like conceptualization, three categories of reported enactment emerged during analysis: advanced, emerging, and developing. The reported enactment seemed to vary according to the teachers' level of knowledge and confidence.

According to Gilmour and Henry (2018), general education teachers typically do not have knowledge of the EBPs that support the learning of SWDs. Other research highlights the importance of teacher knowledge and how it affects their instructional practices. A study from 2005 also suggests that many elementary teachers have weak mathematical knowledge, which makes it difficult for them to effectively use instructional materials (Ball et al., 2005). Additional research led to the identification of components of mathematical knowledge for teaching (Ball et al., 2008). Ball and colleagues (2008) theorize that effective teaching of mathematics requires an understanding of 1) general mathematics content, 2) specialized content knowledge needed to unpack learning for students, 3) how students learn math (including common misconceptions), and 4) pedagogical practices.

According to a study conducted by Hill et al. (2005), knowledge of the curriculum may be an essential component of teachers' pedagogical content knowledge. This study explored how teacher knowledge impacts their instruction. Teacher knowledge was related to the quality of modifications teachers made to instructional materials and that practice suffers when teachers lack strong knowledge because of the demand required to carefully sequence mathematical tasks and activities. Moyer (2003) also found evidence to suggest that teachers did not implement manipulatives in their classrooms because they lacked the knowledge to represent mathematics using manipulatives conceptually.

Teachers in my study are not unlike other teachers who have been observed implementing CRA. In one intervention study, researchers documented teachers' struggle to implement the CRA sequence correctly, noting they frequently failed to systematically transition students through the phases (Bouck, Satsangi, et al., 2018). Analysis of interview data suggests that teachers in my study also failed to systematically transition students through the phases of

CRA. This failure persists regardless of the teacher's knowledge or conceptualization of CRA. In fact, even when teachers recognize the importance of transitioning students through the phases based on student mastery, they admit that they do not in practice.

Though this similarity stands out, how teachers in my study handled the decision varied. Based on the data, it appeared some teachers intentionally made this decision, while others seem to have made the decision out of necessity in response to other factors. Teachers who intentionally made this decision indicated that they addressed this shortcoming by adjusting their small-group instruction to continue remediating students. For example, teachers whose reported enactment was advanced described remediating students by working with students using two phases simultaneously. Teachers whose reported enactment was developing indicated that their decision was made without prior planning. As a result, they did not know how to help their students and admitted they pushed them along in hopes that students would eventually understand.

My findings also suggest that teachers' knowledge and comfort level with the CRA strategy might be related to the modifications they made during enactment. I found that teachers made minimal, moderate, or extensive modifications to the strategy. Teachers who made minimal modifications to CRA had in-depth knowledge of the strategy and expressed confidence in their ability to use CRA to teach SWD or MDs. They also had experience explaining CRA to other teachers. In comparison, teachers who made moderate modifications to CRA demonstrated they had basic knowledge of the strategy. Their comfort level with CRA was mixed, expressing confidence in using CRA to teach mathematics but doubting their ability to explain CRA to another teacher. Teachers who made extensive modifications to CRA demonstrated they had inaccurate or vague knowledge of CRA and expressed doubt in their abilities to use CRA or explain CRA to another teacher.

It is also possible that teachers' beliefs about knowledge deficits impact their enactment. Teachers whose reported enactment is advanced appeared to have a more positive outlook when encountering something they did not know or understand. They saw knowledge deficits as an opportunity to learn, not as something to be ashamed of. A positive outlook and a drive to find answers and understanding may explain why these teachers had more in-depth knowledge, heightened confidence, and advanced reported enactment. In contrast, teachers whose reported enactment is developing had a more negative outlook regarding knowledge deficits. They felt like they could not overcome the barrier on their own when encountering unfamiliar information. Perceiving a lack of knowledge as a weakness may explain why these teachers had vague or inaccurate knowledge, low confidence, and developing reported enactment.

There is evidence to suggest the inseparable nature of knowledge and confidence and their impact on the instructional practices of mathematics teachers. For example, Sani and Burges (2022) conducted a longitudinal study examining the effectiveness of a learning program to support re-training teachers with experience in other subjects to enter mathematics classrooms. Their findings suggest that their participants' confidence in teaching mathematics was tied to their subject knowledge. Teacher confidence improved as a result of providing instruction in the mathematics content and immersive experiences teaching the subject. Furthermore, they found that teachers with increased confidence levels were able to focus on developing their mathematics teaching practices. In a related study, Holstein (2012) found that teachers with positive beliefs about both curriculum and teaching had high procedural fidelity, while low procedural fidelity was associated with low content knowledge and negative beliefs about students.

My study provides additional insight into how teacher confidence impacts their enactment of CRA, an EBP to teach mathematics in inclusive settings. Like the work of previous researchers, it appears that teachers' knowledge and comfort levels were inseparable from each other. Teachers in my study appeared to be aware of how well they understood the CRA strategy, which impacted their confidence. Unfortunately, low confidence seemed to prevent teachers from seeking resources that may help them build a stronger foundation of knowledge. Low confidence also seemed to lead some teachers to avoid using CRA because they did not understand how to teach the skill using manipulatives or when students did not make progress quickly enough. Efforts to improve implementation fidelity must address knowledge deficits and confidence levels. Information about CRA needs to be presented in a clear and interesting manner that is relevant to the teachers' context (Cook et al., 2013). Additionally, teachers need exposure to practice opportunities with targeted coaching. This approach will help teachers adjust their practice in their classroom context, thus building confidence in their ability to use CRA to teach in an inclusive classroom and strengthening their knowledge of the strategy.

Comfort level and knowledge of the CRA strategy seem to have an interconnected influence on teachers' reported enactment. Teachers who expressed feeling very confident and had in-depth knowledge of CRA reported making minimal modifications to the strategy and focused their instruction on explicitly modeling the connections between each phase. Alternatively, teachers who expressed little or no confidence and had vague or inaccurate knowledge of CRA reported making more extensive modifications to the strategy. Moreover, their instruction did not provide opportunities for students to make connections between the phases. Comfort level and knowledge varied simultaneously with teachers' reported enactment. This finding indicates a need for teacher education programs to incorporate meaningful practice opportunities for pre-service teachers, incorporating theoretical learning with coaching experiences. In doing so, teachers will enter the classroom with the knowledge and confidence to provide high-quality mathematics instruction for SWDs or MDs.

How Access to Resources and Other Contextual Factors Shape Enactment

According to the literature on the curriculum enactment process, resources and support influence what teachers intend to teach and what happens in the classroom (Remillard & Heck, 2014). Moreover, teachers transform the curriculum in response to the teaching and learning context (Foster, 2014; Remillard & Heck, 2014). In a qualitative study by Gelmez-Burakgazi (2020), findings suggest that teachers modified EBPs in response to challenges related to limited resources, lack of time, and large class sizes. The findings from this study are similar: Participants modified CRA in response to barriers and individual factors. However, findings from this study expand on the prior literature, suggesting that access to resources and support may matter less than other factors. There did not appear to be a relationship between participants' access and their conceptualization or enactment. Conceptualization and enactment were more sensitive to individual than contextual factors.

Analysis of teacher interview data suggests that teachers can only use available resources if they know how to access them or use them to enhance their instruction. Indeed, teachers' knowledge of curriculum resources influences the intended curriculum (Remmilard & Heck, 2014). The pair of teachers at Pelican Elementary provided different responses when asked to describe their access to resources and support. One teacher described limited access to resources and support and felt her administrator was not doing enough to support her. Unlike the other teacher, she did not mention access to a digital drive folder where teachers shared resources. While the other teacher also noted that access to resources and support was a barrier, he could identify more resources. This indicates a need for schools to ensure that teachers know about the available resources and supports and how to access them.

Additionally, how teachers use resources influences the quality of their instruction (Hill et al., 2008). Results from my study also suggest that even when teachers are aware of the resources and support available, they do not always know how to use them. For example, the pair teachers at Bluebird Elementary both describe ample access to resources and support, including curriculum materials, virtual manipulatives, and hand-held manipulatives. However, one teacher did not use all the resources available to support her CRA instruction. Though both described access to a math coach, neither reached out for support or feedback on their lesson planning or enactment of the CRA strategy.

Based on the findings I have shared with you, one could speculate that time-related challenges have a bigger influence on teachers' reported enactment. Prior research supports this speculation. Wooley (2012) found that teachers modified interventions when they experienced time constraints incompatible with the practice. Teachers in my study spoke about several time-related barriers. The teachers who conducted whole-group instruction using concrete manipulatives expressed frustration with the time it takes to prepare sets of materials for each student. All participants in my study justified their decision to transition students before mastery because they prioritized exposing students to all phases of CRA. Teachers were driven to make this decision by the pacing mandated by their school districts. Pacing varied across school districts, but those variations were related to the order of introduction for each math unit.

It is possible that teachers need support to align CRA within their teaching and learning context. Teachers who receive support in adapting practices to align with their teaching and learning context are likelier to adopt EBPs (Klingner et al., 2013). It appears that teachers with access to curriculum and resources well aligned with CRA report enacting the strategy with minimal modifications. For example, one teacher's school provides detailed information for teachers explaining how to use CRA within each unit. Another teacher's school provides an evidence-based curriculum incorporating manipulatives and representations into every lesson. It also seems teachers struggle to implement CRA with fidelity because of the competing challenges of high-stakes testing.

Moyer (2003) found that due to the pressure caused by high-stakes assessments, teachers expressed concern that diverging from methods students could rely on for the test was not a good use of instructional time. Teachers in my study report similar concerns. The pressure of preparing students for the end-of-year assessment always weighs on them. Even teachers whose reported enactment is advanced indicate they emphasize the representational phase because even if students can't master the abstract skill, they can always rely on a drawing or image to help them. Other teachers are focused on representations because the high-stakes test cannot assess students using manipulatives, but they are often asked to answer questions involving representations. Student mastery is evaluated using numerous representations, and he feels pressured to familiarize his students with as many as possible.

Teachers in my study described access to a range of resources to support their lesson planning and instruction. However, having access to more resources and support from a math coach does not appear to influence teachers' conceptualization or reported enactment of CRA. Instead, it seems that in the presence of contextual challenges, teachers will still make at least moderate modifications to the strategy. A possible implication for school districts may be to consider investing in providing ample access to resources as well as training aimed at helping teachers understand how to find and use those resources effectively. Furthermore, teachers may need training to adapt the practice to their specific teaching and learning context.

Implications

The findings of my study may have broad implications for research, practice, and policy. Educational researchers may benefit from understanding how teachers conceptualize CRA to improve their dissemination of findings to the practitioner community. They may also benefit from understanding how teachers report enacting CRA to help them identify key information that could improve implementation. Teacher education programs and school districts may benefit from understanding how teachers conceptualize and report enacting CRA to inform the design of math methods courses and professional learning experiences to strengthen teacher knowledge and confidence. School districts may also consider helping teachers identify and understand how to use available resources.

Educational Research

Educational researchers seek information that will improve the learning experience for all students by conducting studies that improve understanding of educational issues, investigating instructional practices, and testing their effectiveness. Findings from research studies must be disseminated to key audiences to translate results into practice. Unfortunately, writing practitioner-friendly articles in academic journals may not effectively steer teachers toward an advanced conceptualization of CRA. Many participants in my study indicated that they searched online for resources. Educational researchers focused on improving mathematics instruction for

SWD or MD may need to use less traditional methods to reach practitioners with essential information. Some examples include podcasts, social media, webinars, and blogs.

In addition to circulating information where practitioners are likely to see it, educational researchers may also want to focus on key information that could improve the implementation of CRA. First, using the findings from this study, researchers could address the barriers and challenges of using CRA by advising on how to modify the strategy without impacting its effectiveness. Also, the research community no longer characterizes CRA as a sequence, preferring to describe CRA as a framework instead. However, teachers searching independently may still encounter outdated information or sources that do not clearly explain the framework model. Finally, educational researchers could also target the needs of teachers who need support using CRA to teach a range of mathematics content, including selecting manipulatives, explaining how the manipulatives work, seeing sample lesson plans, and tips for identifying student readiness to transition through the phases.

Educational researchers may have a bigger impact on teachers' practice if they provide resources and information using communication methods that teachers are more likely to access. Furthermore, teachers need access to high-quality information to support their conceptualization and enactment of EBPs such as CRA. Educational researchers can help by creating accessible resources that target how teachers define CRA and providing a database with advice on using CRA in their classrooms.

Teacher Education

Elementary teacher education programs prepare pre-service teachers with the knowledge and skills necessary to be highly effective teachers for all content through sixth grade. This wide range of knowledge may result in shallow learning in math methods courses, focusing on brief exposure to many EBPs. An implication for teacher education programs could be improving the design of elementary math methods courses to allow the focus of instruction to move beyond surface-level exposure to EBP. Teacher education programs may improve how teachers enact EBPs, like CRA, by ensuring pre-service teachers are taught how each phase supports student learning, the importance of systematically transitioning students through the phases, how to explicitly model how the phases are connected, and how to adjust to the presence of common classroom challenges.

Providing pre-service teachers with multiple practice opportunities to implement CRA in inclusive settings may also help them build confidence in teaching math, teaching SWD or MDs, and enacting the strategy when they enter their own classrooms. Teacher education programs could embed practical application activities in their math methods courses and field placements, using targeted feedback on the implementation to help pre-service teachers improve their practice. Using an observation tool like COACH would allow pre-service teachers to record themselves implementing CRA multiple times in their field placement. After each recording, the pre-service teacher would receive targeted feedback on their implementation fidelity and coaching for improvement.

Teacher education programs may strengthen preservice teachers' knowledge by emphasizing in math methods courses 1) why the CRA strategy is effective, 2) how the individual components of CRA impact student learning, 3) the importance of helping students build connections between the phases, and 4) how to adjust to the presence of common challenges. By providing meaningful practice opportunities with targeted feedback, teacher education programs may also improve pre-service teachers' confidence in their ability to teach math, use CRA, and teach SWD or MDs. Teachers with strong knowledge and high confidence levels may be better equipped to overcome barriers and factors that make using EBPs such as CRA challenging.

School District Policy

As noted previously, teachers need ongoing professional learning and feedback from a coach or expert on implementing an EBP (Cook et al., 2013; Foster, 2014; Klingner et al., 2013). A possible implication from my findings could influence the professional development strategies adopted by school districts. They can support their teachers by providing regular and effective professional learning experiences and feedback from a coach or expert on their implementation. Professional learning is effective when teachers are prompted to discuss and practice what they learned with other teachers and is relevant to the teacher's context. Furthermore, these learning experiences are more effective when schools revisit the topics throughout the year. School districts may consider allowing teachers multiple opportunities to engage in problem-solving, bring up questions, and co-plan instruction using CRA. Finally, implementation improves when teachers are given timely feedback on their practice. A mathematics coach or teacher with advanced knowledge and experience using CRA can provide valuable non-evaluative feedback to teachers to help them design and implement high-quality lessons.

Improving access to resources and support that improve the implementation of CRA is another possible implication for school districts. This includes ensuring that teachers have sufficient materials, a high-quality curriculum, and access to expert advice and feedback. It is also helpful for school districts to provide specific training about the available resources and supports, including how to access and use them. Though teachers new to the school district are likely trained on the resources and support available, it may also be helpful to reintroduce this beneficial information to teachers assigned to new teaching roles.

School districts may reduce barriers and challenges to implementing EBPs, such as CRA, by providing effective professional learning opportunities that 1) help teachers connect the practice to their current teaching, 2) provide time for teachers to work together to plan relevant lessons, and 3) includes observation with timely feedback on their implementation. Furthermore, school districts may improve how teachers implement CRA by 1) providing access to resources and 2) designing training that increases the likelihood that teachers know how to use available resources.

Limitations and Future Research

My study explored how six third- through fifth-grade teachers conceptualize and report enacting CRA in their inclusive settings. If future research attends to the following limitations, more insights could be gained about how teachers use CRA to teach mathematics to SWD or MD.

Observations of Teachers

The current study is limited because the collected data did not include observations of teachers enacting the CRA strategy. Due to challenges with obtaining consent from school districts to allow for research on school grounds, I decided to rely on teacher reporting of their enactment. Instructional materials submitted by participants provided additional insight and helped confirm what teachers reported on a basic level. Future research incorporating observations of teachers enacting CRA in inclusive settings for an entire instructional unit could lead to a better understanding of how teachers interpret and enact EBPs such as CRA. Observations of teachers could also enhance our understanding of the types of transformations teachers make to an EBP. Researchers may identify potential modifications that teachers are unaware they are making. Finally, collecting observation data on teachers' implementation may also help researchers determine the relationship between the types of modifications teachers make to CRA and the presence of specific barriers and factors.

Evaluation of Modifications

This dissertation research is also limited because it does not assess the quality of the modifications teachers make in their enactment of CRA. This was an exploratory study aimed at understanding teachers' understanding and practice. Though I categorize conceptualization and enactment as advanced, emerging, and developing, I avoided making evaluative statements because that was not the scope of this study. Future research could seek to answer questions related to whether the ways in which teachers conceptualize or transform CRA in the enactment of lessons impact the strategy's effectiveness on student outcomes. Answering this question might include using a fidelity checklist to determine the quality of the lessons and enactment. Collecting data from students throughout the school year would also provide additional insight into the efficacy of teachers' enactment in terms of acquisition, maintenance, and generalizability.

Sample Size

While purposive sampling offers rich, context-specific data, it is important to acknowledge that this method may limit the generalizability of the findings to the broader population. Though my participants represented a range of education levels, years of teaching experience, and school contexts, the findings are limited because my study focuses on the conceptualizations and reported enactment of six general educators who teach mathematics to upper elementary students in inclusive settings in Virginia. Including the perspectives and experiences of a wider range of teaching experiences would be informative. For example, further research projects could compare the conceptualizations and enactment of the following groups of teachers: 1) teachers who have taken advanced math methods coursework but have less than five years of classroom teaching experience, 2) special education teachers who co-teach mathematics in inclusive settings, and 3) teachers who experience different school contexts (e.g., grade levels and states).

Conclusion

This multiple-case qualitative study explored how third- through fifth-grade general education teachers conceptualize and report enacting the CRA strategy in inclusive settings. One finding suggests that a combination of professional learning and teaching experience influences how teachers conceptualize the CRA strategy. Another finding suggests that teachers' reported enactment varied according to their knowledge and comfort level. Furthermore, the study findings suggest that providing access to resources and support is not enough to overcome other barriers to implementation.

Exploring how teachers conceptualize and report enacting the CRA strategy to teach math in their inclusive classrooms is necessary to understand the factors and barriers that lead to modifications that may impact the strategy's efficacy. This understanding will improve how researchers disseminate findings, teacher education programs design math methods courses, and school districts design training and implement policies for allocating resources and support. These improvements will equip teachers with the knowledge, confidence, and support necessary to implement CRA effectively in their inclusive classrooms, ensuring that SWDs or MDs are provided the support necessary to eliminate barriers to accessing the general education curriculum.

References

Agrawal, J., & Morin, L. L. (2016). Evidence-based practices: Applications of concrete representational abstract framework across math concepts for students with mathematics disabilities. *Learning Disabilities Research & Practice, 31*(1), 34-44. https://doi.org/10.1111/ldrp.12093

Archer, A. L., & Hughes, C. A. (2010). *Explicit instruction: Effective and efficient teaching*. Guilford Publications.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 29*(1), 20-22, 43-46. http://hdl.handle.net/2027.42/65072

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Berry, K., Powell, S., Virginia Department of Education (2020a). Evidence-based specially designed instruction in mathematics. Virginia Department of Education. https://www.doe.virginia.gov/home/showpublisheddocument/28625/638090424862930000

Berry, K., Powell, S., Virginia Department of Education (2020b). Students with disabilities in mathematics frequently asked questions https://www.doe.virginia.gov/home/showpublisheddocument/28607/638090425150300000

Bhattacharya, K. (2017). *Fundamentals of qualitative research: A practical guide*. Routledge.

Biesta, G. (2007). Why "what works" won't work: Evidence-based practice and the democratic deficit in educational research. *Educational theory*, *57*(1), 1-22. https://doi.org/10.1111/j.1741-5446.2006.00241.x

Bouck, E. C., Bassette, L., Shurr, J., Park, J., Kerr, J., & Whorley, A. (2017). Teaching equivalent fractions to secondary students with disabilities via the virtual–representational–abstract instructional sequence. *Journal of Special Education Technology*, *32*(4), 220-231. https://doi.org/10.1177/0162643417727291

Bouck, E. C., Chamberlain, C., & Park, J. (2017). Concrete and app-based manipulatives to support students with disabilities with subtraction. *Education and Training in autism and Developmental Disabilities*, *52*(3), 317-331. https://www.jstor.org/stable/26420403

Bouck, E. C., & Flanagan, S. (2009). Assistive technology and mathematics: What is there and where can we go in special education. *Journal of Special Education Technology*, *24*(2), 17-30. https://journals.sagepub.com/doi/abs/10.1177/016264340902400202

Bouck, E., Long, H., & Jakubow, L. (2021). Teaching students to solve subtraction problems online via the virtual-representational-abstract instructional sequence. *Investigations in Mathematics Learning*, *13*(3), 197-213. https://doi.org/10.1080/19477503.2021.1963157

Bouck, E. C., Park, J., Cwiakala, K., & Whorley, A. (2020). Learning fraction concepts through the virtual-abstract instructional sequence. *Journal of Behavioral Education*, *29*, 519-542. https://link.springer.com/article/10.1007/s10864-019-09334-9

Bouck, E. C., Park, J., Satsangi, R., Cwiakala, K., & Levy, K. (2019). Using the virtual-abstract instructional sequence to support acquisition of algebra. *Journal of Special Education Technology*, *34*(4), 253-268. https://doi.org/10.1177/0162643419833022

Bouck, E. C., Park, J., Shurr, J., Bassette, L., & Whorley, A. (2018). Using the virtual–representational–abstract approach to support students with intellectual disability in mathematics. *Focus on Autism and Other Developmental Disabilities*, *33*(4), 237-248. https://doi.org/10.1177/1088357618755696

Bouck, E. C., Satsangi, R., & Park, J. (2018). The concrete–representational–abstract approach for students with learning disabilities: An evidence-based practice synthesis. *Remedial and Special Education*, *39*(4), 211-228. https://doi.org/10.1177/0741932517721712

Bruner, J. S. (1966). *Toward a theory of instruction* (Vol. 59). Harvard University Press.

Bundock, K., Hawken, L. S., Kiuhara, S. A., O'Keeffe, B. V., O'Neill, R. E., & Cummings, M. B. (2021). Teaching rate of change and problem solving to high school students with high incidence disabilities at tier 3. *Learning Disability Quarterly*, *44*(1), 35-49. https://doi.org/10.1177/0731948719887341

Clotfelter, C. T., Ladd, H., & Vigdor, J. L. (2007). How and why do teacher credentials matter for student achievement? *National Bureau of Economic Research.* https://doi.org/10.3386/w12828

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. Retrieved from https://learning.ccsso.org/wp-content/uploads/2022/11/Math_Standards1.pdf

Cook, B. G., & Cook, S. C. (2013). Unraveling evidence-based practices in special education. *The Journal of Special Education*, *47*(2), 71-82. https://doi.org/10.1177/0022466911420877

Cook, B. G., Cook, L., & Landrum, T. J. (2013). Moving research into practice: Can we make dissemination stick? *Exceptional Children*, *79*(2), 163-180. https://doi.org/10.1177/001440291307900203

Cook, B. G., & Cook, S. C. (2013). Unraveling evidence-based practices in special education. *The Journal of Special Education*, *47*(2), 71-82. https://doi.org/10.1177/0022466911420877

Creswell, J.W. (2013). *Qualitative inquiry and research design: Choosing among five approaches* (3rd ed.). SAGE Publications.

Creswell, J.W., & Creswell, J. D. (2018). *Research design: Qualitative, quantitative, and mixed methods approaches* (5th ed.). SAGE Publications.

Creswell, J. W., Hanson, W. E., Clark Plano, V. L., & Morales, A. (2007). Qualitative research designs: Selection and implementation. *The Counseling Psychologist*, *35*(2), 236-264. https://doi.org/10.1177/0011000006287390

Crotty, M. J. (1998). *The foundations of social research: Meaning and perspective in the research process*. SAGE Publications.

Doabler, C. T., Baker, S. K., Kosty, D. B., Smolkowski, K., Clarke, B., Miller, S. J., & Fien, H. (2015). Examining the association between explicit mathematics instruction and student mathematics achievement. *The Elementary School Journal*, *115*(3), 303-333. https://doi.org/10.1086/679969

Every Student Succeeds Act, 20 U.S.C. § 6301 (2015).

Fixsen, D. L., Naoom, S.F., Blasé, K.A., Friedman, R.M., & Wallace, F. (2005). Implementation research: A synthesis of the literature. Tampa Fl: University of South Florida, National Implementation Research Network.

Flores, M. M. (2009). Teaching subtraction with regrouping to students experiencing difficulty in mathematics. *Preventing School Failure: Alternative Education for Children and Youth*, *53*(3), 145-152. https://doi.org/10.3200/PSFL.53.3.145-152

Flores, M. M., & Hinton, V. M. (2019). Improvement in elementary students' multiplication skills and understanding after learning through the combination of the concrete-representational-abstract sequence and strategic instruction. *Education and Treatment of Children*, *42*(1), 73-100. https://www.jstor.org/stable/26623024

Flores, M. M., & Hinton, V. M. (2022). The effects of a CRA-I intervention on students' number sense and understanding of addition. *Remedial and Special Education*, *43*(3), 183-194. https://doi.org/10.1177/07419325211038009

Flores, M. M., Hinton, V. M., & Burton, M. E. (2016). Teaching problem solving to students receiving tiered interventions using the concrete-representational-abstract sequence and schema-based instruction. Preventing School Failure: Alternative Education for Children and Youth, 60(4), 345-355. https://doi.org/10.1080/1045988X.2016.1164117

Flores, M. M., Hinton, V. M., & Schweck, K. B. (2014). Teaching multiplication with regrouping to students with learning disabilities. *Learning Disabilities Research & Practice, 29*(4), 171-183. https://doi.org/10.1111/ldrp.12043

Flores, M. M., Hinton, V., & Strozier, S. D. (2014). Teaching subtraction and multiplication with regrouping using the concrete-representational-abstract sequence and strategic instruction model. *Learning Disabilities Research & Practice*, *29*(2), 75-88. https://doi.org/10.1111/ldrp.12032

Flores, M. M., Hinton, V. M., Strozier, S. D., & Terry, S. L. (2014). Using the concrete-representational-abstract sequence and the strategic instruction model to teach computation to students with autism spectrum disorders and developmental disabilities. *Education and Training in Autism and Developmental Disabilities, 49*(4), 547-554. https://www.jstor.org/stable/24582350

Flores, M. M., Hinton, V. M., & Taylor, J. L. J. (2018). CRA fraction intervention for fifth-grade students receiving tier two interventions. *Preventing School Failure: Alternative Education for Children and Youth*, *62*(3), 198-213. https://doi.org/10.1080/1045988X.2017.1414027

Flores, M. M., & Milton, J. H. (2020). Teaching the partial products algorithm using the concrete-representational-abstract sequence. *Exceptionality, 28*(2), 142-160. https://doi.org/10.1080/09362835.2020.1772070

Forness, S. R., Freeman, S. F., Paparella, T., Kauffman, J. M., & Walker, H. M. (2012). Special education implications of point and cumulative prevalence for children with emotional or behavioral disorders. *Journal of Emotional and Behavioral Disorders*, *20*(1), 4-18. https://doi.org/10.1177/1063426611401624

Foster, R. (2014). Barriers and enablers to evidence-based practices. *Kairaranga*, *15*(1), 50-58. https://doi.org/10.54322/kairaranga.v15i1.181

Fuchs, L. S., Bucka, N., Clarke, B., Dougherty, B., Jordan, N. C., Karp, K. S., Woodward, J., Madhavi, J., Gersten, R., Newman-Gonchar, R., Schumacher, R., Haymond, K., Lyskawa, J., Keating, B., & Morgan, S. (2021). *Assisting students struggling with mathematics: Intervention in the elementary grades* (WWC 2021006). National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, US Department of Education. *http://whatworks.ed.gov*.

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities*, *37*(1), 4-15. https://doi.org/10.1177/00222194040370010201

Gelmez-Burakgazi, S. (2020). Curriculum adaptation and fidelity: A qualitative study on elementary teachers' classroom practices. *Issues in Educational Research, 30*(3), 920–942.

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research*, *79*(3), 1202-1242. https://psycnet.apa.org/doi/10.3102/0034654309334431

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38*(4), 293–304. https://doi.org/10.1177/00222194050380040301

Gilmour, A. F., & Henry, G. T. (2018). A comparison of teacher quality in math for late elementary and middle school students with and without disabilities. *The Elementary School Journal*, *118*(3), 426-451. https://doi.org/10.1086/696140

Glaser, B.G & Strauss, A.L., (1967). *The discovery of grounded theory: Strategies for qualitative research*. Aldine Publishing Company.

Goe., L. (2007). The link between teacher quality and student outcomes: A research synthesis. *National Comprehensive Center for Teacher Quality*. https://files.eric.ed.gov/fulltext/ED521219.pdf

Greenwood, C. R., & Abbott, M. (2001). The research to practice gap in special education. *Teacher Education and Special Education*, *24*(4), 276-289. https://doi.org/10.1177/088840640102400403

Hanuscin, D. L., de Araujo, Z., Cisterna, D., Lipsitz, K., & van Garderen, D. (2020). The re-novicing of elementary teachers in science? Grade level reassignment and teacher PCK. *Journal of Science Teacher Education*, *31*(7), 780-801.

Harris, C. A., Miller, S. P., & Mercer, C. D. (1995). Teaching initial multiplication skills to students with disabilities in general education classrooms. *Learning Disabilities Research & Practice, 10*(3), 180-195. https://doi.org/10.1080/1046560X.2020.1778845

Hinton, V., Gibbs, A., & Franklin, T. (2020, Winter). The effects of CRA/CSA explicit instruction for students with and without disabilities taught in an inclusive setting. *Journal of the American Academy of Special Education Professionals*, 46-57.

Hirsch, S. E., Bruhn, A. L., McDaniel, S., & Mathews, H. M. (2022). A survey of educators serving students with emotional and behavioral disorders during the COVID-19 pandemic.

Behavioral Disorders, *47*(2), 95-107. https://doi.org/10.1177/01987429211016780

Holstein, K. A. (2012). A characterization of teachers' implementations of a mathematical decision-making curriculum [ProQuest LLC]. In ProQuest LLC.

Hornby, G., Gable, R. A., & Evans, W. (2013). Implementing evidence-based practice in education: What international literature reviews tell us and what they don't. *Preventing School Failure: Alternative Education for Children and Youth, 57*(3), 119-123. https://doi.org/10.1080/1045988X.2013.794326

Hott, B. L., Dibbs, R. A., Naizer, G., Raymond, L., Reid, C. C., & Martin, A. (2019). Practitioner perceptions of algebra strategy and intervention use to support students with mathematics difficulty or disability in rural Texas. *Rural Special Education Quarterly*, *38*(1), 3-14. https://doi.org/10.1177/8756870518795494

Hudson, P., Miller, S. P., & Butler, F. (2006). Adapting and merging explicit instruction within reform based mathematics classrooms. *American Secondary Education*, 19-32. https://www.jstor.org/stable/41219810

Individuals with Disabilities Education Act, 20 U.S.C. § 1400 (2004).

Jitendra, A. K. (2013). Understanding and accessing standards-based mathematics for students with mathematics difficulties. *Learning Disability Quarterly*, *36*(1), 4-8.

Jitendra, A. K., Nelson, G., Pulles, S. M., Kiss, A. J., & Houseworth, J. (2016). Is mathematical representation of problems an evidence-based strategy for students with mathematics difficulties? *Exceptional Children*, *83*(1), 8-25. https://doi.org/10.1177/0014402915625062

Jitendra, A., & Star, J. (2011). Meeting the needs of students with learning disabilities in inclusive mathematics classrooms: The role of schema-based instruction on mathematical problem-solving. *Theory Into Practice*, *50*(1), 12–19. https://doi.org/10.1080/00405841.2011.534912

Kennedy, M. M. (2016). How does professional development improve teaching? *Review of Educational Research*, *86*(4), 945–980. https://doi.org/10.3102/0034654315626800

Klingner, J. K., Boardman, A. G., & McMaster, K. L. (2013). What does it take to scale up and sustain evidence-based practices? *Exceptional Children*, *79*(2), 195-211. https://doi.org/10.1177/001440291307900205

Kolb, A. Y., & Kolb, D. A. (2017). Experiential learning theory as a guide for experiential educators in higher education. *Experiential Learning & Teaching in Higher Education*, *1*(1), 7-44. https://nsuworks.nova.edu/elthe/vol1/iss1/7

Koutselini, M. (2008). Teacher misconceptions and understanding of cooperative learning: An intervention study. *Journal of Classroom Interaction*, *43*(2), 34-44. http://www.jstor.org/stable/23869676

Ladd, H. F., & Sorensen, L. C. (2017). Returns to teacher experience: Student achievement and motivation in middle school. *Education Finance and Policy*, *12*(2), 241-279. https://doi.org/10.1162/EDFP_a_00194

Lafay, A., Osana, H. P., & Valat, M. (2019). Effects of interventions with manipulatives on immediate learning, maintenance, and transfer in children with mathematics learning disabilities: A systematic review. *Education Research International*, 1-21. https://doi.org/10.1155/2019/2142948

Landrum, T.J. (2011). Emotional and behavioral disorders. In J.M Kauffman & D.P. Hallahan (Ed.), *Handbook of special education* (pp. 209-220). Routledge. https://doi.org/10.4324/9780203837306

Ledford, J. R., Lane, J. D., & Gast, D. L. (2018). *Single case research methodology*. Routledge.

McLeskey, J., Barringer, M., Billingsley, B., Brownell, M., Jackson, D., Kennedy, M., Lewis, T., Maheady, L., Rodriguez, J., Scheeler, M., Winn, J., Ziegler, D. (2017). *High leverage practices in special education: The final report of the HLP Writing Team.* Council for Exceptional

Children, Collaboration for Effective Educator Development, Accountability and Reform. Retrieved from http://ceedar.education.ufl.edu/portfolio/ccsc-2017-high- leverage-practices/

Maccini, P., & Hughes, C. A. (2000). Effects of a problem-solving strategy on the introductory algebra performance of secondary students with learning disabilities. *Learning Disabilities Research & Practice*, *15*(1), 10–21. https://doi.org/10.1207/SLDRP1501_2

Maccini, P., & Ruhl, K. L. (2000). Effects of a graduated instructional sequence on the algebraic subtraction of integers by secondary students with learning disabilities. *Education and Treatment of Children*, *23*(4), 465–489. https://www.jstor.org/stable/42899634

Mancl, D. B., Miller, S. P., & Kennedy, M. (2012). Using the concrete-representational-abstract sequence with integrated strategy instruction to teach subtraction with regrouping to students with learning disabilities. *Learning Disabilities Research & Practice*, *27*(4), 152-166. https://doi.org/10.1111/j.1540-5826.2012.00363.x

Merriam, S. B., & Grenier, R. S. (Eds.). (2019). *Qualitative research in practice: Examples for discussion and analysis*. John Wiley & Sons.

Merriam S.B. & Tisdell, E.J. (2015). *Qualitative research: A guide to design and implementation.* John Wiley & Sons.

Miller, S. P., & Hudson, P. J. (2007). Using evidence-based practices to build mathematics competence related to conceptual, procedural, and declarative knowledge. *Learning Disabilities Research & Practice*, *22*(1), 47-57. https://doi.org/10.1111/j.1540-5826.2007.00230.x

Miller, S. P., & Mercer, C. D. (1993). Using data to learn concrete-semiconcrete-abstract instruction for students with math disabilities. *Learning Disabilities Research & Practice 8*(2), 89-96.

Miller, S. P., Stringfellow, J. L., Kaffar, B. J., Ferreira, D., & Mancl, D. B. (2011). Developing computation competence among students who struggle with mathematics. *Teaching Exceptional Children*, *44*(2), 38-46.

Morano, S., Flores, M. M., Hinton, V., & Meyer, J. (2020). A comparison of concrete-representational-abstract and concrete-representational-abstract-integrated fraction interventions for students with disabilities. *Exceptionality*, *28*(2), 77-91. https://doi.org/10.1080/09362835.2020.1727328

Moses, R. P., & Cobb Jr., C. (2001). Organizing algebra: The need to voice a demand. *Social Policy*, *31*(4), 4.

Moyer, P. S. (2001). Are We Having Fun Yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47*(2), 175–197. https://doi.org/10.1023/A:1014596316942

National Center for Education Statistics. (2022a). *Students with disabilities. Condition of Education*. U.S. Department of Education, Institute of Education Sciences. Retrieved July 6, 2022, from https://nces.ed.gov/programs/coe/indicator/cgg.

National Center for Educational Statistics (2022b). *The Nation's Report Card: Mathematics.* http://nces.ed.gov/nationsreportcard/

National Center on Intensive Intervention at American Institutes for Research (n.d.). *Instructional delivery (Intensive intervention in mathematics: Module 4)*. https://intensiveintervention.org/instructional-delivery-math-course#Module4Part2

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematics success for all.* National Council of Teachers of Mathematics.

National Council on Disability. (2018). IDEA series: Every student succeeds act and students with disabilities. https://ncd.gov/sites/default/files/NCD_ESSA-SWD_Accessible.pdf

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. U.S. Department of Education. https://files.eric.ed.gov/fulltext/ED500486.pdf

National Research Council, & Mathematics Learning Study Committee. (2001). *Adding it up: Helping children learn mathematics*. National Academies Press.

Nelson, G. & Powell, S.R. (2018). A systematic review of longitudinal studies of mathematics difficulty. *Journal of Learning Disabilities, 51*(6), 523-539. https://doi.org/10.1177/0022219417714773

No Child Left Behind Act, P.L. 107-110, 20 U.S.C. § 6319 (2002).

Park, J., Bouck, E. C., & Smith, J. P. (2020). Using a virtual manipulative intervention package to support maintenance in teaching subtraction with regrouping to students with developmental disabilities. *Journal of Autism and Developmental Disorders*, *50*, 63-75. https://doi.org/10.1007/s10803-019-04225-4

Park, J., Bouck, E. C., & Fisher, M. H. (2021). Using the virtual–representational–abstract with overlearning instructional sequence to students with disabilities in mathematics. *The Journal of Special Education*, *54*(4), 228-238. https://doi.org/10.1177/0022466920912527

Park, J., Bryant, D. P., & Shin, M. (2022). Effects of interventions using virtual manipulatives for students with learning disabilities: A synthesis of single-case research. *Journal of Learning Disabilities, 55*(4), 325-337. https://doi.org/10.1177/00222194211006336

Passolunghi, M. C., & Siegel, L. S. (2004). Working memory and access to numerical information in children with disability in mathematics. *Journal of Experimental Child Psychology*, *88*(4), 348-367. https://doi.org/10.1016/j.jecp.2004.04.002

Patton, M. Q. (2015). Sampling, qualitative (purposeful). In G. Ritzer (Ed.) *The Blackwell encyclopedia of sociology*. https://doi.org/10.1002/9781405165518.wbeoss012.pub2

Peltier, C., & Vannest, K. J. (2018). Using the concrete representational abstract (CRA) instructional framework for mathematics with students with emotional and behavioral disorders. *Preventing School Failure: Alternative Education for Children and Youth*, *62*(2), 73-82. https://doi.org/10.1080/1045988X.2017.1354809

Peltier, C., Morin, K. L., Bouck, E. C., Lingo, M. E., Pulos, J. M., Scheffler, F. A., Suk, A., Mathews, L.A., Sinclair, T.E., Deardorff, M. E. (2020). A meta-analysis of single-case research using mathematics manipulatives with students at risk or identified with a disability. *The Journal of Special Education*, *54*(1), 3-15. https://doi.org/10.1177/0022466919844516

Peng, P., Wang, C., & Namkung, J. (2018). Understanding the cognition related to mathematics difficulties: A meta-analysis on the cognitive deficit profiles and the bottleneck theory. *Review of Educational Research*, *88*(3), 434-476. https://psycnet.apa.org/doi/10.3102/0034654317753350

Piaget, J. (1952). The fourth stage: The coordination of the secondary schemata and their application to new situations. In J. Piaget & M. Cook (Trans.), *The origins of intelligence in children* (pp. 210–262). W W Norton & Co. https://doi.org/10.1037/11494-005

Remillard, J.T., & Heck, D.J. (2014) Conceptualizing the curriculum enactment process in mathematics education. *ZDM Mathematics Education* **46**, 705–718 https://doi.org/10.1007/s11858-014-0600-4

Riccomini, P. J., Witzel, B., & Robbins, K. (2008). Improving the mathematics instruction for students with emotional and behavioral disorders: Two evidenced-based instructional approaches. *Beyond Behavior*, *17*(2), 24-30.

Rittle-Johnson, B., Schneider, M., & Star, J. R. (2015). Not a one-way street: Bidirectional relations between procedural and conceptual knowledge of mathematics. *Educational Psychology Review, 27*, 587-597. https://doi.org/10.1007/s10648-015-9302-x

Root, J. R., Cox, S. K., Gilley, D., & Wade, T. (2021). Using a virtual-representational-abstract integrated framework to teach multiplicative problem solving to middle school students with developmental disabilities. *Journal of Autism and Developmental Disorders*, *51*, 2284-2296. https://doi.org/10.1007/s10803-020-04674-2

Sani, N., & Burghes, D. (2022). Longitudinal study of ‘retraining’ non-maths specialist teachers to become capable, confident teachers of mathematics. *International Journal of Mathematical Education in Science and Technology*, *53*(9), 2438-2464. https://doi.org/10.1080/0020739X.2021.1890248

Satsangi, R., & Miller, B. (2017). The case for adopting virtual manipulatives in mathematics education for students with disabilities. *Preventing School Failure: Alternative Education for Children and Youth*, *61*(4), 303-310. https://doi.org/10.1080/1045988X.2016.1275505

Satsangi, R., Bouck, E. C., Taber-Doughty, T., Bofferding, L., & Roberts, C. A. (2016). Comparing the effectiveness of virtual and concrete manipulatives to teach algebra to secondary students with learning disabilities. *Learning Disability Quarterly*, *39*(4), 240-253. https://psycnet.apa.org/doi/10.1177/0731948716649754

Scheuermann, A. M., Deshler, D. D., & Schumaker, J. B. (2009). The effects of the explicit inquiry routine on the performance of students with learning disabilities on one-variable equations. *Learning Disability Quarterly*, *32*(2), 103-120. https://doi.org/10.2307/27740360

Schneider, M., Rittle-Johnson, B., & Star, J. R. (2011). Relations among conceptual knowledge, procedural knowledge, and procedural flexibility in two samples differing in prior knowledge. *Developmental Psychology*, *47*(6), 1525-1538. https://psycnet.apa.org/doi/10.1037/a0024997

Sealander, K. A., Johnson, G. R., Lockwood, A. B., & Medina, C. M. (2012). Concrete–semiconcrete–abstract (CSA) instruction: A decision rule for improving instructional efficacy. *Assessment for Effective Intervention*, *38*(1), 53-65. https://doi.org/10.1177/1534508412453164

Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. *Second handbook of research on mathematics teaching and learning*, *1*(1), 319-370.

Strickland, T. K., & Maccini, P. (2013). The effects of the concrete–representational–abstract integration strategy on the ability of students with learning disabilities to multiply linear expressions within area problems. *Remedial and Special Education*, *34*(3), 142–153. https://doi.org/10.1177/0741932512441712

Strozier, S., Hinton, V., Flores, M., & Terry, L. (2015). An investigation of the effects of CRA instruction and students with autism spectrum disorder. *Education and Training in Autism and Developmental Disabilities*, *50*, 223-236. https://www.jstor.org/stable/24827537

Vaughn, S., Klingner, J., & Hughes, M. (2000). Sustainability of research-based practices. *Exceptional Children*, *66*(2), 163-171.

Williams, L. A., Kobett, B. M., & Miles, R. H. (2018). *The mathematics lesson-planning handbook, grades 6-8: Your blueprint for building cohesive lessons*. Corwin Press.

Witzel, B. S. (2005). Using CRA to teach algebra to students with math difficulties in inclusive settings. *Learning Disabilities: A Contemporary Journal, 3*(2), 49-60. https://files.eric.ed.gov/fulltext/EJ797683.pdf

Witzel, B. S., & Little, M. E. (2016). *Teaching elementary mathematics to struggling learners*. Guilford Publications.

Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice*, *18*(2), 121–131. https://doi.org/10.1111/1540-5826.00068

Witzel, B. S., Riccomini, P. J., & Schneider, E. (2008). Implementing CRA with secondary students with learning disabilities in mathematics. *Intervention in School and Clinic*, *43*(5), 270-276. https://doi.org/10.1177/1053451208314734

Wolf, M. M. (1978). Social validity: The case for subjective measurement or how applied behavior analysis is finding its heart. *Journal of Applied Behavior Analysis*, *11*(2), 203-214. https://doi.org/10.1901/jaba.1978.11-203

Woolley, M. E., Rose, R. A., Mercado, M., & Orthner, D. K. (2013). Teachers teaching differently: A qualitative study of implementation fidelity to professional development. *Journal of Education and Training Studies*, *1*(1), 55–68.

Yakubova, G., Hughes, E. M., & Shinaberry, M. (2016). Learning with technology: Video modeling with concrete–representational–abstract sequencing for students with autism spectrum disorder. *Journal of Autism and Developmental Disorders*, *46*, 2349-2362. https://doi.org/10.1007/s10803-016-2768-7

Yin, R. K. (2018). *Case study research and applications: Design and methods*. SAGE Books.

Appendix A

Summary of CRA Studies

Table 1.

Summary of Traditional CRA Studies

Study	Participants	Design	Mathematical Skills	Setting & Interventionist	Maintenance Data	Social Validity
Bouck, Park, et al. (2017)	*n* = 4 6th grade ID; LD	multiple probe across participants	Change making with coins word problems	Hallway, research team member	2 weeks after	Students & Teacher
Flores (2009)	*n* = 6 3rd MD; LD	multiple probe across groups	Two-digit subtraction w/regrouping	Researcher	2-4 weeks after intervention	Students & Teacher
Flores & Hinton (2019)	*n* = 3 3rd MD	multiple baseline across participants	Multiplication w/regrouping word problems	Classroom; afterschool; researcher	2 weeks after	Students & Teacher
Flores & Milton (2020)	*n* = 3 5th-6th ADHD; LD	multiple probe across behaviors	Two-digit multiplication word problems	Resource room; sped teacher	Up to 8 weeks after	Teacher
Flores, Hinton, & Schweck. (2014)	*n* = 4 4th-5th LD	multiple probe across participants	Multiplication w/regrouping two-digit numbers	Resource room; Researcher	2 weeks after	Students & Teacher
Flores, Hinton, & Strozier (2014)	*n* = 3 3rd MD	multiple probe across behaviors	Subtraction w/regrouping ones place; subtractions w/regrouping ones & tens place; multiplication w/regrouping one-digit multiplier; multiplication w/regrouping two-digit multiplier	Researcher, rural school	4 weeks after instruction	Teacher
Flores, Hinton, Strozier, & Terry (2014)	*n* = 11 K-7th ASD; ID	paired sample t-test	Addition & subtraction facts	Extended school year program ran by university; students from university	None	None
Flores et al. (2016)	*n* = 3 3rd MD	multiple probe across participants	Word problems	Classroom, after school, researcher	4 weeks after	Students & Teacher
Flores et al. (2018)	*n* = 17 5th MD	pre-post	Fraction and decimals (understanding, representing,	Researcher, spare classroom	None	None

Study	Participants	Design	Mathematical Skills	Setting & Interventionist	Maintenance Data	Social Validity
			computing, word problems)			
Harris et al. (1995)	*n* = 13 2nd EBD; LD	multiple baseline across groups	Multiplication facts 0-81; word problems	Classroom; gen ed teacher	None	None
Hinton & Flores (2019)	*n* = 2 3rd MD	multiple probe across behaviors	Rounding three-digit numbers; subtraction w/regrouping ones place; subtraction w/regrouping ones & tens place; comparison of fractions	Classroom; researcher	3 weeks after intervention	Students
Hinton et al. (2020)	*n* = 24 PreK-2nd ASD; ID; LD	paired sample t-test	Counting skills	3-week summer camp at university; certified teachers	None	None
Maccini & Hughes (2000)	*n* = 6 9th-12th LD	multiple probe across participants	Adding, subtracting, multiplying, & dividing integers word problems	Resource room; researcher	Up to 10 weeks following	Teachers
Maccini & Ruhl (2000)	*n* = 3 8th LD	multiple probe across participants	Subtraction of integers & related word problems	Conference room; researcher	Up to 6 weeks after	Students
Mancl et al. (2012)	*n* = 5 4th-5th LD	multiple probe across participants	Subtraction w/regrouping computation; subtraction w/regrouping word problems	Resource room, sped teacher	None	None
Miller & Mercer (1993)	*n* = 9 1st-5th ID; LD; MD	multiple baseline across participants	Addition facts w/sums ranging from 10-18; division facts w/quotients ranging from 0-9; coin sums up to 50 cents	Gen ed classroom and resource room; special education teacher	1 week after	None
Milton et al. (2019)	*n* = 5 4th-6th LD; OHI	multiple probe across participants	Multiplication and division	Resource room; special education teacher	Every 2 weeks, up to 8 weeks after	Students & Teacher
Strozier et al. (2015)	*n* = 3 3rd-4th ASD	multiple baseline across behaviors	addition w/regrouping; subtraction w/regrouping; multiplication facts 0-5	Extended school year program sponsored by university; teacher	None	Teacher

Study	Participants	Design	Mathematical Skills	Setting & Interventionist	Maintenance Data	Social Validity
Witzel (2005)	*n* = 231 6th-7th MD	pre-post	Solving linear functions	Gen ed, classroom	3 weeks after	None
Witzel et al. (2003)	*n* = 68 6th-7th LD	matched pairs	Algebraic transformation equations	Gen ed teacher, classroom	Yes, and generalization	None

Note. ADHD = attention deficit hyperactivity disorder; ASD = autism spectrum disorder; EBD = emotional behavioral disorder; ID = intellectual disabilities; LD = learning disability; OHI = other health impairment; MD = mathematics difficulty

Table 2.

Summary of Integrated CRA Studies

Study	Participants	Design	Mathematical Skills	Setting & Interventionist	Maintenance	Social Validity
Bundock et al. (2005) CRA-I + Strategy	*n* = 9^{th} ASD; LD; OHI	multiple probe across participants	Rate of change	Researcher	1-7 weeks after	None
Flores & Hinton (2022) CRA-I	*n* = 2^{nd} MD	multiple probe across participants	Commutative property; magnitude; derived facts; relationship between addition and subtraction; missing addend problems, single-digit equations; addition fluency	Classroom, gen ed teacher	2-4 weeks after	Students & Teacher
Morano et al. (2020) CRA vs CRA-I	*n* = 5^{th}-6^{th} ASD; LD; OHI	pre-post	Unit fraction and fraction equivalence	Resource room; SPED teacher		Students & Teacher
Root et al. (2021) VRA-I	*n* = 9^{th}-12^{th} ASD; ID	multiple probe across participants	Multiplicative comparison word problems	Resource room, researcher	After completing intervention	Students & Teacher
Scheuermann et al. (2009) EIR	*n* = 6^{th}-8^{th} LD	multiple probe across participants	One variable equations embedded in word problems	Classroom, researcher	11 weeks after	None
Strickland & Maccini (2013) CRA-I	*n* = 8^{th}-9^{th} LD	multiple probe across participants	Multiplying linear expressions embedded in area word problems	Office; researcher	3-6 weeks after intervention	Students
Yakubova et al. (2016) CRA-I + Strategy	*n* = K-1^{st} ASD	multiple baseline across behaviors	Addition of single-digit numbers; subtraction of single-digit numbers; comparison of 2 or 3 single-digit numbers	Office, researcher	3 weeks after	Students & Teacher

Note. ADHD = attention deficit hyperactivity disorder; ASD = autism spectrum disorder; EBD = emotional behavioral disorder; ID = intellectual disabilities; LD = learning disability; OHI = other health impairment; MD = mathematics difficulty

Table 3.

Summary of Traditional VRA Studies

Study	Participants	Design	Mathematical Skills	Setting & Interventionist	Maintenance	Social Validity
Bouck, Bassette, et al. (2017)	*n* = 3 7^{th}-8^{th} ID; LD; OHI	multiple probe across participants	Equivalent fractions	Hallway; researcher and 2 doc students	2 weeks after intervention	Students
Bouck, Park, et al. (2018)	*n* = 2 7^{th} ID; LD	multiple probe across behaviors	Place value; single-digit addition; two-digit by one-digit subtraction w/regrouping	Hallway; researcher	2 weeks after intervention	Students
Bouck et al. (2021)	*n* = 3 2^{nd}-4^{th} ADHD; MD	multiple probe across participants	Double-digit subtraction w/regrouping	Online, researcher	1 week after intervention	Students & Parents

Note. ADHD = attention deficit hyperactivity disorder; ASD = autism spectrum disorder; EBD = emotional behavioral disorder; ID = intellectual disabilities; LD = learning disability; OHI = other health impairment; MD = mathematics difficulty

Appendix B

Research Protocol

Section A: Overview of the Case Study	
Mission & Goals	The purpose of this study is to explore how third- through fifth-grade general education teachers conceptualize CRA, how they report using CRA in their inclusive classrooms, and what factors and barriers influence the way they intend to use CRA in their inclusive classrooms.
Study Questions	**RQ1:** How do third- through fifth-grade general education teachers conceptualize the CRA strategy? **RQ2:** How do third- through fifth-grade general education teachers report implementing CRA to teach mathematics to SWD or MD in inclusive settings?
Propositions and Rival Explanations	**Proposition 1:** Teachers' conceptualizations of CRA will vary based on their professional learning. • **Rival Proposition 1:** Teachers' conceptualization of CRA will vary based on some other factor. • **Alignment to Conceptual Framework:** Ineffective professional learning is a barrier to implementing EBP. According to Foster (2014), teachers develop misconceptions when professional learning is brief and shallow in nature, without making a concerted effort to connect learning with prior knowledge in an ongoing manner. **Proposition 2:** Teachers respond to barriers and factors by making modifications to the phase of CRA and/or the procedure for using CRA to teach mathematics to SWD or MD in their inclusive classrooms. • **Rival Proposition:** Teachers do not make modifications to their enactment of CRA to teach SWD or MD. • **Alignment to Conceptual Framework:** Teachers transform EBP in response to the barriers, individual factors, and external factors such as the teaching and learning context (Foster, 2014; Remillard & Heck 2014). EBP are transformed in real-time as teachers process, interpret, plan, implement, and respond to student learning.
Conceptual Framework Key Readings	**Conceptual Framework for Teacher Enactment of Evidence-Based Practices** Adapted from: • Foster's (2014) examination of barriers to implementing evidence-based practices • Remillard and Heck's (2014) curriculum enactment process which describes the influences and contextual factors that determine the enacted curriculum. • Stein et al.'s, (2007) temporal phases of curriculum use examining the transformation from written curriculum to the enacted curriculum. (See Appendix C for an enlarged figure) Teachers transform evidence-based practices during two phases. (Stein et al., 2007). Evidence-based practices go through a transformation as teachers plan their intended lesson. Another transformation occurs during the enactment of the lesson. While it may seem that each transformation takes place in a chronological sequence, the transformation process of evidence-based practices is not linear and can transform multiple times in response to teachers' perception of implementation in real-time and

	their assessment of student learning. Teachers make decisions about transforming evidence-based practices in response to several influencing factors (Remillard & Heck, 2014) that are filtered through barriers to implementation (Foster, 2014).
Role of Protocol	This protocol's role is to serve as an agenda for the researcher's line of inquiry.

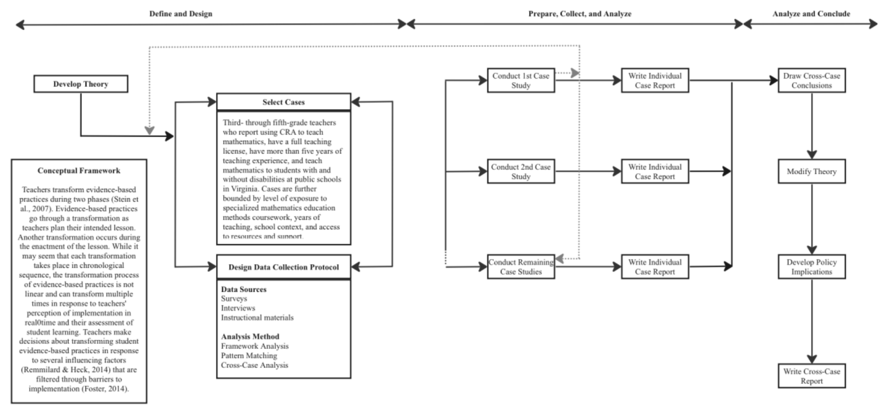
Section B: Data Collection Procedures

Gaining Access to Participants	To see recruitment Emails, flyers, and survey, see Appendices D and E) • Once IRB approval is received, email the list of teachers on record, upload flyer to social media and spread through word of mouth. • Send reminder email • Review responses and isolate data from respondents willing to participate. Look up pass rates, enrollment, region, minority percentage, and economically disadvantaged percentage SES of each school. • Select the respondents that best represent each case • Reach out via email to selected respondents thanking them for their willingness to participate and with information about scheduling a session to review informed consent and conduct first interview.
Protecting Human Subjects	• Obtain IRB approval (Will be included in Appendices when obtained) • Obtain informed consent (see Appendix F) • The information collected in this study will be handled confidentially. The data will be assigned a code number. The list connecting participant's name to this code will be kept in a locked file. When the study is completed and the data have been analyzed, this list will be destroyed. Participant names or identifying information will not be used in any report. All recordings of interviews will be stored on a secure network through the university and destroyed after the study has been completed.

Data Sources	• Survey
	• Video recordings of interviews
	• Instructional materials
	    ○ Lesson plans
	    ○ Slides used for instruction
	    ○ Notes

Resources Needed	• Charged computer with Wi-Fi, camera, and microphone. • Zoom software installed. • Notepad, pens • Private office space. • Scanning and printing hardware • Interview protocol
Data Collection Schedule	December 2023 – January 2023 conduct interviews and collect documents for analysis (See Study Timeline in Appendix J)

Diagram Fig. 2
Multiple-Case Study Procedure (Yin, 2018)

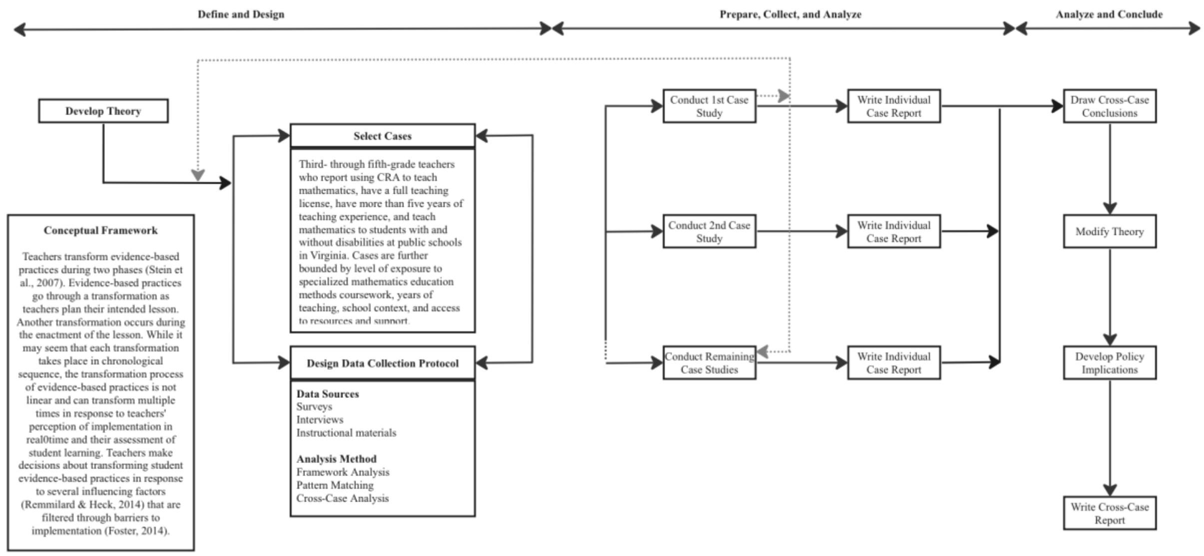


Section C: Protocol Questions

Level 1: questions verbalized to specific interviewees	See <u>Appendix I</u> for interview protocol questions for Session 1 and Session 2.
Level 2: questions about each case, which represent your line of inquiry	Does a teacher's reported comfort level and knowledge of teaching mathematics to SWD or MD impact how they intend to use CRA? How well do third- through fifth-grade teachers understand CRA? Which sources do third- through fifth-grade teachers report provided them with their understanding of CRA? Do third- through fifth-grade teachers report using CRA as it is intended? Are there elements of CRA teachers are more likely to modify? Do third- through fifth-grade teachers intentionally modify CRA? Which barriers to implementing CRA do teachers identify as being a factor in decisions to modify? Which barriers to implementing CRA do teachers identify as being the most challenging to overcome?
Level 3: questions asked of the pattern of findings across multiple cases	Are there similarities that persist regardless of education level, years of teaching, school context, or presence to a math coach? Are there meaningful differences based on years of teaching, school context or access to a math coach?
Level 4: questions asked of an entire study	Is there sufficient evidence to suggest that the results are transferable to a larger population of third- through fifth-grade teachers who use CRA to teach mathematics to SWD or MD?
Level 5: normative questions about policy recommendations	Do we need to improve the way we train elementary teachers to teach mathematics to SWD or MD? Do we need to improve the design of CRA?

Figure 2.

Multiple-Case Study Procedure



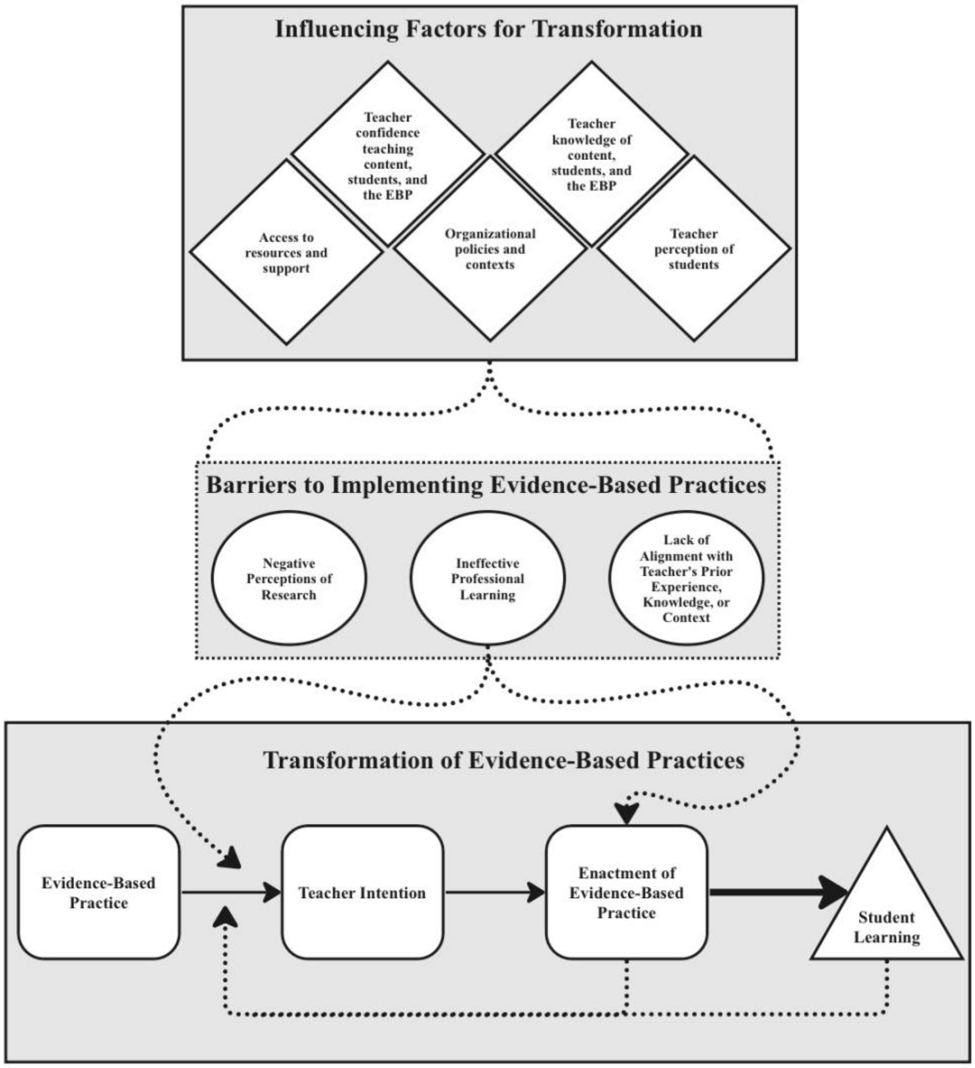
Note. (Yin, 2018)

Appendix C

Conceptual Framework

Figure 1.

Conceptual Framework for Transformation of Evidence-Based Practices



Note. Adapted from the works of Foster (2014), Remillard and Heck (2014), Stein et al. (2007).

Appendix D

Recruitment

Flyer

We are recruiting 3rd-5th grade teachers in Virginia for a research study.

Purpose:

The purpose of the study is to explore what teachers know about a math strategy involving manipulatives and how they use it in their inclusive classrooms.

Eligibility:

- Teach 3rd - 5th grade at a public school in Virginia.
- Teach mathematics in an inclusive classroom.
- Use manipulatives to teach mathematics.

Participants will:

- Take part in two interviews.
- Submit lesson plans and instructional materials.
- Receive $100 gift card for full participation.

Deadline to join:

December 1, 2023

Scan for Survey

UNIVERSITY of VIRGINIA

Interested in participating?
Contact Susan Aigotti at
sma2kf@virginia.edu
IRB #6127

Initial E-mail

To: Elementary Teachers

RE: Research Recruitment Opportunity

Using the Concrete-Representational-Abstract Instructional Framework to Support Students with Disabilities and Mathematics Difficulties in Inclusive Settings: An Investigation of Teacher Knowledge, Practice, Perceptions, and Decision Making (UVA IRB-SBS #6127).

Hello!

My name is Susan Aigotti. I plan to conduct the study listed above in partial fulfillment of my doctoral work in Special Education at the University of Virginia. My advisor is Dr. Stephanie Morano. The Institutional Review Board approved my research plan. Now, I seek to recruit volunteers to participate in the study.

I am seeking participation from general educators teaching mathematics in a Virginia public school to students with and without disabilities in grades 3 – 5.

The study's purpose is to explore what general education teachers know about CRA, how they use CRA in their inclusive classrooms, how they make instructional decisions about using CRA in their inclusive classrooms, and what barriers influence their use of CRA in their classrooms.

Please complete a short survey by clicking here. Results from this study may help us understand how to design mathematics interventions that meet the real needs of teachers and students with disabilities and mathematical difficulties.

If you have any questions or concerns, please contact me at sma2kf@virginia.edu. I appreciate your assistance and look forward to hearing back from you.

Sincerely,

Susan Aigotti, M.S.Ed. (she/her)
Ph.D. Candidate, Special Education
University of Virginia
School of Education and Human Development

Reminder E-mail

To: Elementary Teachers

RE: Reminder of Research Recruitment Opportunity

Using the Concrete-Representational-Abstract Instructional Framework to Support Students with Disabilities and Mathematics Difficulties in Inclusive Settings: An Investigation of Teacher Knowledge, Practice, Perceptions, and Decision Making (UVA IRB-SBS #6127).

Hello!

I would like to ask you again about your potential participation in my research study. The recruitment window is still open! If you have not already completed the survey, I am hoping to recruit general educators who are currently teaching mathematics in a Virginia public school to students with and without disabilities in grades 3 – 5.

The study's purpose is to explore what general education teachers know about CRA, how they use CRA in their inclusive classrooms, how they make instructional decisions about using CRA in their inclusive classrooms, and what barriers influence their use of CRA in their classrooms.

Please complete a short survey by clicking here. Results from this study may help us understand how to design mathematics interventions that meet the real needs of teachers and students with disabilities and mathematical difficulties.

If you have any questions or concerns, please contact me at sma2kf@virginia.edu. I appreciate your assistance and look forward to hearing back from you.

Sincerely,

Susan Aigotti, M.S.Ed. (she/her)
Ph.D. Candidate, Special Education
University of Virginia
School of Education and Human Development

Selection E-Mail

To: Elementary Teachers

RE: Thank You

Using the Concrete-Representational-Abstract Instructional Framework to Support Students with Disabilities and Mathematics Difficulties in Inclusive Settings: An Investigation of Teacher Knowledge, Practice, Perceptions, and Decision Making (UVA IRB-SBS #6127).

Hello

I wanted to thank you for completing the survey for my study. After reviewing your responses, I would like to extend an invitation to participate in the full study.

I want to schedule a meeting with you over Zoom. This first meeting may take longer than an hour to review the study requirements, and you can ask me questions before we get started. Please click **here** to sign up between Dec. 4th and Dec. 17th. If these days and times are inconvenient, please get in touch with me. I am happy to work with your schedule.

I appreciate your willingness to meet with me to discuss your participation further. If you have any questions or concerns, please contact me at sma2kf@virginia.edu. I appreciate your assistance and look forward to meeting you.

Sincerely,

Susan Aigotti, M.S.Ed. (she/her)
Ph.D. Candidate, Special Education
University of Virginia
School of Education and Human Development

Reminder to Schedule Interview and Consent

To: Elementary Teachers

RE: Schedule and Consent

Using the Concrete-Representational-Abstract Instructional Framework to Support Students with Disabilities and Mathematics Difficulties in Inclusive Settings: An Investigation of Teacher Knowledge, Practice, Perceptions, and Decision Making (UVA IRB-SBS #6127).

Hello ______

I know that teachers are very busy at this time of the semester. I have extended my availability for the first interview. To help keep the time down, I am attaching the consent form so that you can look it over prior to our meeting. This will keep the interview time down to one hour. Please let me know if you are no longer interested in participating in the study.

Click **here** to sign up for a time. If these days and times are not convenient for you, please contact me. I am happy to accommodate your schedule.

I appreciate your willingness to meet with me to discuss your participation further. If you have any questions or concerns, please contact me at sma2kf@virginia.edu. I appreciate your assistance and look forward to meeting you.

Sincerely,

Susan Aigotti, M.S.Ed. (she/her)
Ph.D. Candidate, Special Education
University of Virginia
School of Education and Human Development

Consent E-Mail

To: Elementary Teachers

RE: Reminder & Consent Form

Using the Concrete-Representational-Abstract Instructional Framework to Support Students with Disabilities and Mathematics Difficulties in Inclusive Settings: An Investigation of Teacher Knowledge, Practice, Perceptions, and Decision Making (UVA IRB-SBS #6127).

Hello ________,

I'm looking forward to our first meeting tomorrow! Before the meeting, please review the consent form, which outlines the purpose of the study, your rights, and safeguards for your confidentiality.

At the start of the interview, we will go over it together, and you will have an opportunity to ask me any questions. If you verbally consent to participation, we will begin the interview. You must sign and return the consent form to me as soon as possible after the initial interview. I will review different options for returning it to me at our first session.

I am also including the Zoom link and my cell phone number for our meeting. If you need help accessing the link, please contact me, and I can help you troubleshoot.

https://virginia.zoom.us/j/4267007398
434-661-7656

Thank you so much for your time, and I will see you soon!

Sincerely,

Susan Aigotti, M.S.Ed. (she/her)
Ph.D. Candidate, Special Education
University of Virginia
School of Education and Human Development

DropBox Email

To: Elementary Teachers

RE: DropBox & Next Interview

Using the Concrete-Representational-Abstract Instructional Framework to Support Students with Disabilities and Mathematics Difficulties in Inclusive Settings: An Investigation of Teacher Knowledge, Practice, Perceptions, and Decision Making (UVA IRB-SBS #6127).

Hello ______,

Thank you again for the time you spent with me yesterday, allowing me to interview you. As we discussed, I will need you to submit instructional materials demonstrating how you use CRA in your classroom. Please select one unit or mathematics concept that you have taught or plan to teach using CRA and submit anything you think could be useful. Examples include, but are not limited to, the following:

- Notes
- Lesson plans
- Photographs of teacher-generated models (or screenshots of virtual manipulatives)
- Activities
- Assignments
- Examples of problems used
- Slideshows or power points
- Videos shown to the class (can submit links on a Word document)
- Screenshots of virtual manipulatives

Please do not feel pressured to create anything new for this study portion. These materials will help me clearly understand how you use CRA in your classroom. I am not evaluating how you use CRA; I am only looking for evidence of how teachers make it work for them in real classrooms.

You should have already received an invitation from Box to your individual folder, Instructional Materials _______. I have selected this pseudonym to maintain your anonymity.

Please submit these documents as soon as possible to the following link: ___________

As we discussed, I plan to get a summary of your responses to you by next __________.

Also, if you have not already done so, please schedule a follow-up second interview at the following link:

https://calendar.app.google/JY46uJePBQSqSMkw5

If these days and times do not work for you, please let me know when you are available, and I will do my best to make that work.

Sincerely,

Susan Aigotti, M.S.Ed. (she/her)
Ph.D. Candidate, Special Education
University of Virginia
School of Education and Human Development

Summary and Reminder Email

To: Elementary Teachers

RE: Summary and Reminder

Using the Concrete-Representational-Abstract Instructional Framework to Support Students with Disabilities and Mathematics Difficulties in Inclusive Settings: An Investigation of Teacher Knowledge, Practice, Perceptions, and Decision Making (UVA IRB-SBS #6127).

Hello ________,

I am looking forward to seeing you tomorrow for our second interview! I have attached a summary of your responses from our first interview. Before we start the second interview, we will quickly review this summary together.

Please let me know if you have any questions or concerns.

Sincerely,

Susan Aigotti, M.S.Ed. (she/her)
Ph.D. Candidate, Special Education
University of Virginia
School of Education and Human Development

Summary

To: Elementary Teachers

RE: Summary

Using the Concrete-Representational-Abstract Instructional Framework to Support Students with Disabilities and Mathematics Difficulties in Inclusive Settings: An Investigation of Teacher Knowledge, Practice, Perceptions, and Decision Making (UVA IRB-SBS #6127).

Hello ________,

Thank you again for your time and effort in helping me complete this study. I have completed your summary for Interview 2. If you have any corrections or clarifications, please let me know. I will start the analysis next week.

Please let me know if you have any questions or concerns. I hope the rest of your year goes smoothly!

Sincerely,

Susan Aigotti, M.S.Ed. (she/her)
Ph.D. Candidate, Special Education
University of Virginia
School of Education and Human Development

Appendix E

Survey

Using the Concrete-Representational-Abstract Instructional Framework to Support Students with Disabilities and Mathematics Difficulties in Inclusive Settings: An Investigation of Teacher Knowledge, Practice, Perceptions, and Decision Making (UVA IRB-SBS #6127).

1. What is your preferred name?

2. What is your preferred email for any future contact?

3. What is your highest level of education?
   - Bachelor's Degree
   - Some graduate coursework
   - Master's Degree

4. Where did you complete your teacher education?
   - Virginia college or university
   - An out-of-state college or university
   - An alternative teacher education program
   - Other

5. Please give the name of the school where you completed your teacher education program or the name of the alternative teacher program (if you selected other, describe).

6. Are you licensed to teach?
   - Yes
   - No, but I have a provisional license
   - No

7. What is the name of your school district?

8. What is the name of your school?

9. Please indicate the grade level you teach (select more than one if you teach multiple grades).
   - K
   - 1st
   - 2nd
   - 3rd
   - 4th
   - 5th
   - 6th

10. An inclusive classroom means that there are students with disabilities (e.g., learning disabilities, ADHD, autism spectrum disorder, or emotional-behavioral disabilities) taught alongside non-disabled students (who may or may not need additional support or interventions).

11. Given this definition, would you consider your classroom to be an inclusive classroom?
    - Yes
    - No

12. Do you teach mathematics to students with and without disabilities in your classroom?
    - Yes
    - No

13. Is there a math coach in your school district?
    - Yes
    - No
    - Unsure

14. If you answered yes, have you received support or training from the math coach?
    - Yes
    - No

15. Do you serve in a leadership role for your school (e.g., grade level lead, department chair)?
    - Yes
    - No

16. If you answered yes, what is it called, and how would you describe your role?

17. Have you heard of a mathematics instructional practice called the Concrete-Representational-Abstract (CRA) sequence (a strategy involving a combination of manipulatives, pictures, and written mathematics)?
    - Yes
    - No

18. How frequently would you say you teach new mathematics content using the Concrete-Representational-Abstract (CRA) sequence to teach mathematics to students with and without disabilities in your classroom?
    - Never
    - Rarely (2-3 lessons per year)
    - Sometimes (4-5 lessons per year)
    - Often (6-10 lessons per year)
    - Always

19. How frequently do you provide interventions for mathematics content using the Concrete-Representational-Abstract (CRA) sequence to teach mathematics to students with and without disabilities in your classroom?
    - Never
    - Rarely (2-3 lessons per year)
    - Sometimes (4-5 lessons per year)
    - Often (6-10 lessons per year)
    - Always

20. To your knowledge, does your school district recommend a specific instructional practice for teaching mathematics (e.g., inquiry-based instruction, explicit or direct instruction, problem-based learning, CRA)
    - Yes
    - No

21. If you answered yes, what do they recommend (please list as many as you can think of)?

22. Has the school district you work for provided you with training (professional development) for any mathematical instructional practices?
    - Yes
    - No

23. If you answered yes, which ones (please list as many as you can think of)?

24. Participation in this study requires two semi-structured interviews, each lasting 45 to 60 minutes. You will also be asked to provide documentation of instructional practice related to the use of CRA (e.g., lesson plans, notes, PowerPoint presentations, assignments, and photos of teacher-generated models). The total time anticipated for participation is 4 hours. Compensation includes a $100 gift card mailed to you upon completion of interviews and submission of instruction documentation.

25. Would you be interested in participating in this study?
    - Yes
    - No

Appendix F

Informed Consent Agreement

Please read this consent agreement carefully before you decide to participate in the study.

Consent Form Key Information:

- Participate in at least two interviews about your teaching experience, confidence, and knowledge of teaching mathematics to students with and without disabilities, knowledge of concrete-representational-abstract (CRA) strategy, use of CRA, and barriers to using CRA
- Provide copies of lesson plans and instructional materials demonstrating your use of CRA
- No information collected that will connect identity with responses

Purpose of the research study: The purpose of the study is to explore what general education teachers know about CRA, how general education teachers use CRA in their inclusive classrooms, how general education teachers make instructional decisions about using CRA in their inclusive classrooms, and what barriers influence the way general education teachers use CRA in their classrooms.

What you will do in the study: You will take part in at least two interviews with the possibility of being asked for a follow-up interview to clarify responses to questions. Questions will relate to current and past teaching experiences; comfort and knowledge of teaching mathematics; comfort and knowledge of teaching students with disabilities; knowledge of an instructional practice called concrete-representational-abstract strategy (CRA); experiences using CRA to teach mathematics; and challenges with using CRA. You will also be asked to provide lesson plans and instructional materials demonstrating your use of CRA in the classroom. You can skip any questions that make you uncomfortable, and you can stop participating at any time.

Permission to record: In order to ensure that all sessions are conducted in the same manner and that there is no interference in the study, all sessions will be recorded over Zoom. These videos will only be viewed by official members of the research team.

Time required: The interviews will require about **2 hours** of your time. Collecting and submitting documents and materials will require about **2 hours** of your time.

Risks: There are no anticipated risks in this study.

Benefits: There are no direct benefits to you for participating in this research study. The study may help us understand how to design mathematics interventions that meet the real needs of teachers and students with disabilities and mathematical difficulties.

Confidentiality: The information that you give in this study will be handled confidentially. Your data will be assigned a code number. The list connecting your name to this code will be kept in a locked file. When the study is completed and the data has been analyzed, this list will be destroyed. Your name will not be used in any report. All recordings of interviews will be stored on a secure network through the university and destroyed after the study has been completed.

Voluntary participation: Your participation in the study is completely voluntary.

Right to withdraw from the study: You have the right to withdraw from the study at any time, but you will not receive compensation.

How to withdraw from the study: If you want to withdraw from the study, tell the interviewer to stop the interview. There is no penalty for withdrawing. If you would like to withdraw after your materials have been submitted, please contact sma2kf@virginia.edu.

Payment: You will receive a $100 gift card as compensation for the time you invest in participating in this study.

If you have questions about the study, contact:
Susan Aigotti
Department of Curriculum, Instruction and Special Education, Bavaro Hall 225 PO Box 400273
University of Virginia, Charlottesville, VA 22903.
(434) 924-5404
sma2kf@virginia.edu

Faculty Advisor
Stephanie Morano
Department of Curriculum, Instruction, and Special Education, Bavaro Hall 225 PO Box 400273
University of Virginia, Charlottesville, VA 22903.
(434) 924-5404
Sm6rk@virginia.edu

To obtain more information about the study, ask questions about the research procedures, express concerns about your participation, or report illness, injury or other problems, please contact:
Tonya R. Moon, Ph.D.
Chair, Institutional Review Board for the Social and Behavioral Sciences
One Morton Dr Suite 400
University of Virginia, P.O. Box 800392
Charlottesville, VA 22908-0392
Telephone: (434) 924-5999
Email: irbsbshelp@virginia.edu
Website: https://research.virginia.edu/irb-sbs
Website for Research Participants: https://research.virginia.edu/research-participants
UVA IRB-SBS # 6127
Agreement:
I agree to participate in the research study described above.

Print Name: ____ Date: _____________

Signature: ____
You will receive a copy of this form for your records.

Appendix G

School Context and Participant Summary

Table 4.

School Contextual Factors

School	Locale[a]	Region	Enrollment	SWD	Economically Disadvantaged	Accreditation Status	Math Pass Rate SWD[c]	Presence of Math Coach[d]
Bluebird Elementary	Small City	5	48[b]					Yes, shared
Cardinal Elementary	Small City	1	388	10.3%	77.8%	Accredited	12%	Yes, two
Oriole Elementary	Midsize City	2	754	9.2%	21.8%	Accredited	45%	Yes, one
Pelican Elementary	Rural Fringe	5	503	11.7%	35.8%	Accredited	23%	No

Note: Unless otherwise noted, information found on the Virginia Department of Education Website under School Quality Profiles updated Fall 2023; FRL = free or reduced lunch.

[a] The Locale was determined using NCES Locale classification and Criteria.

[b] Enrollment was determined by asking the school for their K-5 enrollment

[c] Pass rate does not include growth.

[d] Presence of Math Coach determined from Survey

Table 5.

Participants' Background and Experience

Participant	Gender	Degree	License	Years Teaching*	Grade	Years Teaching Math*	CRA Instruction	CRA Remediation	School
Ms. Aspen	F	Master's	PreK-6	10	5th	4	Rarely	Sometimes	Pelican
Ms. Birch	F	Bachelor's	PreK-6	10	3rd	10	Often	Often	Bluebird
Mr. Chestnut	M	Bachelor's	K-8 Biology	24	5th	17	Often	Often	Pelican
Ms. Elm	F	Doctorate	PreK-6 ESL PreK-12	9	5th	9	Often	Often	Oriole
Ms. Hickory	F	Master's	PreK-6	17	4th	17	Often	Often	Cardinal
Ms. Walnut	F	Bachelor's	PreK-6	12	4th	4	Always	Often	Bluebird

Appendix H

Data Collection

Table 6.

Data Collection Timeline

Data Source	Collection Date	Type of Documentation
Survey	Nov. 9 – Dec. 1	Survey via Qualtrics
Informed Consent	Dec. 4 – Dec. 31	Signed Consent Forms
1st Interview	Dec. 4 – Dec. 31	Zoom recording & transcriptions of Interview Protocol 1
Document Collection	Starting Dec. 5	Instructional Materials
2nd Interview	Dec. 11 – Jan. 31	Zoom recording & transcriptions of Interview Protocol 1

Table 7.

Alignment of Research Question, Framework, and Data Collection

Research Question	Conceptual Framework	Data Collected
How do 3rd- through 5th-grade general education teachers conceptualizing the CRA strategy?	Ineffective professional learning is a barrier to implementing EBP. According to Foster (2014) teachers develop misconceptions when professional learning is brief and shallow in nature, without making a concerted effort to connect the learning with prior knowledge in an ongoing manner	Interview Data • Responses to questions exploring knowledge of CRA that may influence the intended and enacted use of CRA
How do third- through fifth-grade general education teachers report enacting CRA to teach mathematics to SWD or MD in inclusive settings?	Teachers transform EBPs in response to the barriers, individual factors, and external factors such as the teaching and learning context (Foster, 2014; Remillard & Heck 2014). EBP are transformed in real-time as teachers process, interpret, plan, implement, and respond to student learning.	Interview Data • Responses to questions exploring intended use of CRA Document Data • Instructional materials and student artifacts demonstrating intended and enacted use of CRA.
Why do third- through fifth-grade teachers modify CRA to teach mathematics to SWD or MD in inclusive settings?	Teachers will not implement an EBP with fidelity if they do not have a positive perception of the practice or if the research is inaccessible. They must also be provided with effective professional learning that supports the deep understanding and maintenance of the practice. The EBP must also align with the teacher's prior knowledge, experience, and the teaching and learning context.	Interview Data • Responses to questions exploring the factors and barriers influencing the intended and enacted use of CRA.

Appendix I

Interview Protocol

Session 1

Introduction

Thank you so much for taking the time to meet with me today. I need to record this part to document that I reviewed the consent form with you. Is that ok?

BEGIN RECORDING

Let's start by reviewing the purpose of this study.

Your participation in this study will help me explore what general education teachers know about CRA, how they use CRA in their inclusive classrooms, how they make instructional decisions about using CRA in their inclusive classrooms, and what barriers influence their use of CRA in their classrooms.

Participation in this study will involve completing two interviews and submitting instructional materials. These instructional materials can be lesson plans, notes, activities, photographs of teacher-generated models, slide shows, or videos. Anything that you used to plan or execute the lesson or lessons.

I also want to remind you that your participation in this interview is completely voluntary. You have the right to refuse to answer any question. You may end the interview anytime and ask me to remove you from the study.

Do you have any questions about the information contained in the consent form?

After our interview, please sign the consent form and email me a copy. You can sign it digitally or print it out and sign it. If you print it out and sign it, you can scan it or take a photo to return to me.

Do you consent to participation in this study?

(*If yes, proceed*)

(*If no, end interview*)

We are now transitioning to the interview portion of this meeting.

Throughout the interview, please feel free to ask me to clarify or repeat any questions. I will continue to record the interview.

Questions

RQ3: Why do third- through fifth-grade general education teachers modify CRA to teach mathematics to SWD or MD in

Transformative Process:

- The influence of barriers and factors on the decision to modify an EBP

Barriers	**Factors**
• Lack of alignment with teacher's prior experience and the teaching and learning context • Negative perceptions and inaccessibility of research • Ineffective professional learning	• Teacher knowledge • Teacher self-efficacy • Organizational policies and contexts • Teacher perception of students • Access to resources and support

I'd like to get to know you as a teacher and collect some background information about you. This first set of questions is related to your teaching experience.

1. Tell me about your teaching experiences.
   a. How many years total?
   b. Grades
   c. Subjects
   d. Locations/settings

2. How many years have you been in your current position?

I'd like to get to know you as a teacher and collect some background information about you. This first set of questions is related to your teaching experience.

Thinking about your current position only, I want to ask you some questions about your mathematics instructional setting and the students you teach.

3. How many minutes are you allotted to teach mathematics in the schedule?
   a. Do you typically teach mathematics for that long?
   b. What would you change about the time you are allotted for teaching mathematics?
      i. Time of day? Amount of time? Why?

4. Describe the students in your classroom. I'm curious about your students' range of academic achievement and demographics.
   a. What are some of the disabilities and difficulties of students you work with?
   b. Are your students representative of the entire grade? School?

5. Describe the support you or your students receive from special education while you teach mathematics.
   a. Is there a co-teacher, TA, pull-out, push-in, or planning support?

6. How confident do you feel teaching the mathematics content at your grade level?
   a. Is there anything that could help you feel more confident?

7. How confident do you feel teaching mathematics to students with disabilities or mathematics difficulties?
   a. Is there anything that could help you feel more confident?

8. What strategies do you use to support teaching mathematics to students with disabilities or difficulties?

I'd like to get to know you as a teacher and collect some background information about you. This first set of questions is related to your teaching experience.

9. Tell me about your teaching experiences.
   e. How many years total?
   f. Grades
   g. Subjects
   h. Locations/settings

10. How many years have you been in your current position?

Thinking about your current position only, I want to ask you some questions about your mathematics instructional setting and the students you teach.

11. How many minutes are you allotted to teach mathematics in the schedule?
    c. Do you typically teach mathematics for that long?
    d. What would you change about the time you are allotted for teaching mathematics?
       i. Time of day? Amount of time? Why?

12. Describe the students in your classroom. I'm curious about your students' range of academic achievement and demographics.
    c. What are some of the disabilities and difficulties of students you work with?
    d. Are your students representative of the entire grade? School?

13. Describe the support you or your students receive from special education while you teach mathematics.
    b. Is there a co-teacher, TA, pull-out, push-in, or planning support?

14. How confident do you feel teaching the mathematics content at your grade level?
    b. Is there anything that could help you feel more confident?

15. How confident do you feel teaching mathematics to students with disabilities or mathematics difficulties?
    b. Is there anything that could help you feel more confident?

16. What strategies do you use to support teaching mathematics to students with disabilities or difficulties?

RQ1: How do third- through fifth-grade general education teachers conceptualize the CRA strategy?
Transformation Process:

- The transformation of EBP into what teachers intend to teach.
- The transformation of EBP from what teachers intend to teach to the enactment of the practice.

Barriers	Factors

- Inaccessibility of research
- Ineffective professional learning
- Lack of alignment with teacher prior knowledge, experience, or context.
- Teacher knowledge
- Teacher self-efficacy
- Organizational policies and contexts
- Access to Resources and Supports

Now, I'm going to ask you some questions about your knowledge of CRA and, later, about how you use it. I realize that how you use it may be different from what you know about the strategy.

17. Describe the phases of CRA and the steps for implementing it.

18. How does the concrete phase support student learning?

19. How does the representation phase support student learning?
    a. How important is it for students and teachers to draw the images used at this phase?

20. How does teacher modeling support student learning?

21. What criteria should a teacher use to determine when a student is ready to transition from C to R to A?
    a. How often should you check?
    b. Do you think a student needs a minimum number of lessons at each phase of CRA?

22. Why do you think CRA is effective for teaching students?
    a. Which element of CRA do you think is key to its effectiveness?

23. Is there any situation (grade level, specific student groups, content) in which you think CRA would not benefit students?

After answering about your knowledge of CRA, I'm curious about the sources of your knowledge. You may not remember specifically, but give me as much information as you can.

24. Which sources do you remember learning about CRA from?
    a. Possible responses include – teacher preparation, websites, video tutorials, articles, conferences, professional development, and conversations with colleagues.
    b. Is there one source that stands out as having the most impact on your understanding of CRA?

25. How confident do you feel explaining CRA to other teachers?
    a. Is there anything that could help you feel more confident?

RQ2: How do third- through fifth-grade general education teachers report enacting CRA to teach mathematics to SWD or MD in inclusive settings?
RQ3: Why do third- through fifth-grade general education teachers modify CRA to teach mathematics to SWD or MD in inclusive settings?
Transformative Process:

- The transformation of EBP into what teachers intend to teach.

- The transformation of EBP from what teachers intend to teach to the enactment of the practice.
- The influence of barriers and factors on the decision to modify an EBP

Barriers	**Factors**
• Inaccessibility of research	• Teacher knowledge
• Ineffective professional learning	• Teacher self-efficacy
• Lack of alignment with teaching and learning context	• Teacher perception of students
	• Organizational policies and support
	• Access to resources and support

This next set of questions concerns how you use CRA in your current classroom. If you have to think back to prior years, that is ok, but please clarify if you are drawing on previous experience rather than current experience.

26. Which mathematics content have you used, or plan to use, CRA to teach?

27. Now, I want you to answer these questions using a specific unit or mathematics topic you have taught using CRA. Which mathematics content are you thinking of?

28. About how many days did your pacing guide allow for this particular content?
    a. About how many days did you spend using CRA?

29. Describe how you used CRA.
    a. Grouping
    b. Which manipulatives did you use?
        i. How frequently did students use the manipulatives?
    c. How did you use representations?
    d. Explain how you progressed through the phases.
        i. How did you decide when to transition from one phase to the next?
        ii. Estimate how many days you spent at each phase.
    e. Did you combine CRA with any other strategies?
    f. How do you help students make the connection between C, R, and A?

30. Do you use CRA more as a whole or small group, or would you say equally?

31. How do you decide when to use CRA instruction?

32. How do you decide who receives CRA instruction?

33. Have you ever intentionally combined the phases?
    a. Describe what you did.
    b. Why did you do this?

34. Have you ever intentionally skipped a phase?
    a. Describe what you did.
    b. Why did you do this?

35. Have you ever moved students to another phase even if you weren't sure they were ready?

a. Describe what you did.
b. Why did you do this?

36. Have you ever intentionally made other changes to the phases or the implementation of CRA?
    a. Describe what you did.
    b. Why did you do this?

37. How confident do you feel using the CRA strategy to teach mathematics in an inclusive setting?
    a. Do you feel confident selecting appropriate manipulatives or representations to align with the skill?
    b. Do you feel confident with modeling and helping students make connections?
    c. Do you feel confident deciding how long students need instruction at each phase?
    d. Is there anything that would help you feel more confident?

38. Is there anything that influenced your decision to use CRA in the classroom as you described?

Conclusions and Follow-Up

That concludes the first interview. Are there any questions I didn't ask you today that you wish I had asked?

Do you have any questions for me about anything we talked about today?

I appreciate you taking time out of your busy schedule to meet with me today. Before you go, we must discuss the next steps and schedule your second interview.

Tomorrow, I will send you an invite to a secure Dropbox folder with your pseudonym name. Please submit examples of any instructional materials that demonstrate your use of CRA. Within a week of this meeting, I will send you a follow-up reminder and a summary of your responses. Please submit the instructional materials several days before we meet so I can review them before our second interview. If I have questions, I will ask you about them then.

I will put a link in the chat to let you know my availability for our second interview. Do any of these dates and times work for you?

https://calendar.app.google/JY46uJePBQSqSMkw5

If anything changes, please reach out to me by email to reschedule.

I hope you have a great rest of your day, and I look forward to meeting with you again!

Session 2

Introduction

Hello, thanks for taking the time to meet with me again today. Like last time, I am going to record this interview, but it will be kept secure and then destroyed after the study is completed.

BEGIN RECORDING

I also want to remind you that your participation in this interview is completely voluntary. You have the right to refuse to answer any question. You may end the interview anytime and ask me to remove you from the study.

Do you understand?
(*If yes, proceed*)
(*If no, end interview*)

I sent you the summary or responses from our last discussion. We talked about your teaching experiences and knowledge of the CRA strategy, and you described a typical lesson using CRA. That information has been helpful to my study. Before we start the next set of questions, I want to allow you to clarify anything we previously discussed.

As we begin this interview, please feel free to ask me to clarify or repeat any questions.

RQ2: How do third- through fifth-grade general education teachers report enacting CRA to teach mathematics to SWD or MD in inclusive settings?

Transformative Process:

- The transformation of EBP into what teachers intend to teach.
- The transformation of EBP from what teachers intend to teach to the enactment of the practice.
- The influence of barriers and factors on the decision to modify an EBP

Barriers

- Negative perceptions and inaccessibility of research
- Ineffective professional learning
- Lack of alignment to teacher prior knowledge, experience, and the teaching and learning context.

Factors

- Teacher knowledge
- Teacher self-efficacy
- Teacher perception of students
- Organizational policies and support
- Access to resources and support

I looked over your instructional materials and reviewed your responses to how you use CRA for a typical lesson.

I have a few clarifying questions: *questions are written in my notebook for individual teachers.*

Let's look together at the documents you submitted. Please walk me through how you used each document.

Thank you. I'll now describe the typical procedures for using CRA and ask you some questions.

I am not passing any judgment about your methods for implementing CRA. My research study investigates *how* teachers use CRA in real classrooms, how they make decisions, and why they make those decisions.

"CRA is a three-phase sequence in which students are taught a new skill or concept using concrete objects (manipulatives), representations (pictures), and written computation of abstract problems (involving only numbers and symbols) without the assistance of either the manipulatives or pictures. Students are expected to demonstrate mastery, usually a criterion of at least 80% accuracy on the learning measure, to move from one phase of the intervention to the next. Researchers have also used a set number of lessons at each phase. Explicit instruction is embedded in each phase. This involves providing students with an advanced organizer, teacher modeling with think-aloud, guided practice with feedback, and independent practice to assess mastery."

**Copy and paste it into the Zoom chat so teachers can read and think.*

1. Do you feel that your lesson aligns with this description of CRA? In what ways?

2. If it does not align, did you make any changes intentionally?

3. Did you seek advice from any source when planning how you would implement CRA in the way you have (e.g., colleagues, websites, specialists, curriculum, articles, etc.)?
    a. Which ones?
    b. Describe the advice.

4. How do these changes improve the strategy for students and teachers?

5. Describe any changes you made while enacting your lesson that are absent in your submitted instructional materials.

6. Do you believe students in your classroom made academic progress due to your CRA instruction?
    a. How do you know?

RQ3: Why do third- through fifth-grade general education teachers modify CRA to teach mathematics to SWD or MD in inclusive settings?

Transformative Process:

- The transformation of EBP into what teachers intend to teach.
- The transformation of EBP from what teachers intend to teach to the enactment of the practice.
- The influence of barriers and factors on the decision to modify an EBP

Barriers

- Negative perceptions and inaccessibility of research
- Ineffective professional learning
- Lack of alignment to teacher prior knowledge, experience, and the teaching and learning context.

Factors

- Teacher knowledge
- Teacher self-efficacy
- Teacher perception of students
- Organizational policies and support
- Access to resources and support

For the next set of questions, I am interested in exploring the factors that may influence how you use CRA in your classroom.

Tell me about any barriers or challenges you face when planning or implementing the CRA strategy. This includes obstacles related to your training, knowledge of mathematics, school or district-level policies, school factors, the students you teach, access to resources and support, or confidence.

**Copy and paste it into the Zoom chat so teachers can read and think.*

Tell me about any support that has improved your ability to plan for or implement the CRA strategy. Again, this could include your training, knowledge of mathematics, school or district-level policies, school factors, the students you teach, access to resources and support, or confidence.

**Copy and paste it into the Zoom chat so teachers can read and think.*

Now, I will ask you some follow-up questions about specific factors. You may have already mentioned some of these. I may ask you to think about them slightly differently or answer a particular question about them.

Training Experience

7. What, if anything, would you change about how you were taught to use CRA?
8. Has your training prepared you to modify CRA to work in your context?

Teacher Knowledge

9. Do you feel that you have sufficient knowledge of math, strategy, and identifying students' needs to use CRA?

Organizational Policies

10. Does your district or school-level administrator implement policies that make CRA easier or more difficult?
11. In what ways does your planning time influence the way you use CRA?
12. In what ways does instructional time for math support or create a barrier to using CRA?
13. In what ways does the pacing guide support or create a barrier to using CRA?
14. In what ways does the curriculum support or create a barrier to using CRA?
15. In what ways do the standards support or create a barrier to using CRA?

Access to Resources and Support

16. Do you feel like you have sufficient access to resources to use CRA?
17. In what ways does your building administrator support or create a barrier to using CRA?
18. In what ways does a math coach support or create a barrier to using CRA?
19. In what ways does your math lead support or create a barrier to using CRA?
20. In what ways does your grade level team support or create a barrier to using CRA?

Teacher Self-Efficacy

21. In what ways does your confidence influence the way you use CRA?

Teacher Perception of Students

22. Do you feel like your students have challenges that make it difficult to use CRA?
23. How does student behavior influence how you use CRA?

24. How does your students' academic level influence how you use CRA?

Negative Perceptions of Research

25. How does the preparation time influence the way you use CRA (i.e., passing out manipulative or other materials)
26. Is there anything about the procedures for implementing CRA that you find impractical for an authentic inclusion classroom?
27. How do you think your students feel about using CRA?
28. Based on our conversation, is there one challenge that stands out to you as the biggest challenge you must address when using CRA in your classroom?
29. What could a researcher like me, trying to understand teachers' needs, do to improve the CRA strategy?

30. Is there anything else positive or negative you'd like to say about CRA?

Conclusion and Follow-Up

That concludes our second interview. Are there any questions I didn't ask you today that you wish I had asked?

Do you have any questions for me about anything we talked about today?

Are you open to being contacted if I need to clarify something or have follow-up questions? What is the best way to contact you?

You will receive a summary of today's responses. If there is anything you would like to add, change, or clarify, please let me know by email. You can also email me if you have any questions or concerns about our conversation today or your inclusion in the study.

Have a great rest of your day.

Appendix J

Study Timeline – Data Collection, Analysis, and Writing

Date	Administrative Tasks	Data Collection	Analysis
Aug 24	Proposal Approved		
Oct 29	IRB Approval #6127		
Nov 1	Distribute recruitment fliers	• ***Inclusion survey***	• Sort based on years of teaching, SES of school, presence of math support • Review responses and isolate data from respondents willing to participate.
Dec 2023	• Collect and organize consent forms. • Transcribe and review **Interview 1 Data.** • Write up summary of **Interview 1 Data** responses. • Send summary of **Interview 1 Data** for member checking. • Send email with dropbox link to submit **Document Data**	• **Interview 1 Data** • **Document Data**	• Conduct first pass for emerging themes of **Interview 1 Data** • Apply themes and subthemes to **Interview 1 Data**
January 2024	• Transcribe and review **Interview 2 Data** • Write up summary of **Interview 2 Data** responses. • Send summary of **Interview 2 Data** for member checking.	• **Interview 2 Data** • **Document Data**	• Conduct first pass for emerging themes of **Interview 2 Data** • Apply themes and subthemes to **Interview 2 Data**

Date	Administrative Tasks	Data Collection	Analysis
Feb 2024	• Touch base with advisor and committee. • Use themes and subthemes to draft individual case report for **Case 1, RQ 1**		• Conduct second pass of **Interview 1 and 2 Data** • Adjust themes and subthemes for **Interview 1 and 2 Data**
Mar 2024	• Touch base with advisor and committee • Begin biweekly check-in's • Use themes and subthemes to draft individual case report for **Case 1-3, RQ 1**		• Conduct third pass of **Interview 1 and 2 Data** • Adjust themes and subthemes for **Interview 1 and 2 Data**
Apr 2024	• Touch base with advisor and committee • Biweekly check-in's • Use themes and subthemes to draft individual case report for **Case 1-3, RQ 1** • Draft cross-case report **RQ1**		• Within-case analysis of **Case 1, RQ1** • Within-case analysis of **Case 2, RQ1** • Within-case analysis of **Case 3, RQ1** • Cross-case analysis **RQ1**

Date	Administrative Tasks	Data Collection	Analysis
May 2024	• Touch base with advisor and committee. • Weekly check-in's • Use themes and subthemes to draft individual case report for **Case 1-3, RQ 2 & 3** • Draft cross-case report **RQ 2 & 3** • Modify theory. • Develop policy implications. • Edit Ch 1-3 to include topics that surface in analysis. • Write draft of Ch 4 • Write draft of Ch 5 • Write draft of Ch 6		• Within-case analysis of **Case 1, RQ 2 & 3** • Within-case analysis of **Case 2, RQ 2 & 3** • Within-case analysis of **Case 3, RQ 2 & 3** • Cross-case analysis **RQ 2 & 3**
Jun 2024	• Edit and Revise Ch 1 – 5 for consistency and clarity • Submit to Dissertation Committee **June 1** • Prepare Defense		
~~~~~~~~~~ DEFENSE June 15th ~~~~~~~~			

Appendix K

Coding

First Cycle Coding

The analysis unit for this first coding cycle was each participant's entire interview transcript. The purpose of the first coding round is to summarize data segments. I used two preliminary coding techniques (holistic and hypothesis) to assign codes based on what I deductively assumed the data would contain. I used a deductive approach called holistic coding during the first analysis phase. Holistic coding is useful when the researcher knows general topics are present in the data. I generated these codes by clustering data with similar themes. These topics became headings and subheadings used to organize the transcripts.

Holistic Codes	Definition
Teacher Background	Participant discusses their teacher preparation, grade levels taught, subjects taught, or years of experience.
School Context	Participant discusses school or community demographics, class size, instructional time, or inclusive model.
Confidence	Participant discusses confidence or comfort teaching mathematics, SWD or MD, or using the CRA strategy.
Conceptualizing CRA	Participant discusses the definition, name of phases, description of phases, procedure, or effectiveness of CRA.
Source of Knowledge	Participant discusses how or where they learned about CRA.
Reported Enactment	Participant discusses how they have used or plan to use CRA.
Alignment	Participant discusses how their reported enactment or understanding of CRA is similar or different from the definition provided by the researcher.
Intention	Participant discusses whether they made intentional or unintentional changes to CRA.
Enactment vs. Planning	Participant discusses how their reported enactment was different from their planned lesson.

Knowledge	Participant discusses what they know about mathematics, or SWD or MD.
Organizational Policies	Participant discusses the standards, pacing guide, assessment schedule, curriculum, professional development, or planning time.
Access to Resources and Support	Participant discusses their access to tools, curriculum, support from specialists, training, and coaching with feedback.
Perception of Students	Participant discusses their negative or positive perception of the students they teach currently or in the past. These perceptions include behaviors, abilities, or identities.
Perception of Research	Participant discusses their perception of research(ers), the CRA strategy, or accessibility of research.

During the first analysis phase, I used another deductive approach: hypothesis coding. Hypothesis codes are created from predictions about what might appear in the data before analysis. These codes allowed me to separate the interview data into content that would help me answer each research question.

Hypothesis Codes	**Definition**
Conceptualization	Participant discusses their knowledge of CRA to include a definition, the phases, the procedures, or how it supports student learning.
Definition	Participant discusses how they define CRA.
Framework	Participant defines CRA as a framework in which teachers model mathematics, making explicit for students how the phases are connected.
Gradual Release	Participant defines CRA as a gradual release model in which teachers scaffold mathematics instruction without making explicit for students how the phases are connected.
Sequence	Participant defines CRA as a sequence in which teachers first use manipulatives, then images, and finally numbers to teach mathematics without making explicit for students how the phases are connected.
Knowledge	Participant discusses what they know about CRA.

Name and Describe	Participant discusses the names of the phases or describes the phases or procedure.
Decision Making	Participant discusses how students should progress through the phases of CRA.
Effectiveness	Participant describes why individual elements of the CRA strategy are effective at improving student learning.
Reported Enactment	Participant describes how they enact CRA in their classroom. Could be this school year or past school years.
Planning	Participant describes their plan for teaching CRA, including the design and their planning resources.
Design	Participant describes the design of the lesson for teaching CRA, including the topic or skill, materials used, instructional methods, and planned pedagogical steps.
Planning Support	Participant describes the resources and tools they used to support their planning.
Implementation	Participant describes their lesson implementation. This includes describing their actions, changes made to the plan during enactment, and perception of the enactment.
Actions	Participant describes the pedagogical steps that took place while implementing the lesson.
Planning vs. Enactment	Participants discuss how their implementations differed from their plan.
Modifications	Participant describes intentional modifications made to the phases, procedure, or mastery criterion. Modifications were minimal, moderate, or extensive.
Minimal	Participant describes intentionally modifying the strategy by removing a mastery criterion for phase progression with an explicit plan for remediation and skipping the abstract phase but combing it with another phase.
Moderate	Participant describes intentionally modifying the strategy by removing a mastery criterion for phase progression without an explicit plan for remediation and neglecting to draw student attention to important features of the content through think-alouds.
Extensive	Participant describes intentionally modifying the strategy by removing a mastery criterion for phase progression without an explicit plan for remediation, skipping multiple steps of explicit

	instruction, and removing a phase without combining it with another phase.

Second Cycle Coding

The data segments for each hypothesis code were the analysis unit for this second coding cycle. The second coding round involves grouping the summaries from the first round into categories, themes, or concepts. I used inductive analysis to identify pattern codes.

Pattern Codes for Conceptualization	**Definition**
Advanced	Participant describes CRA as a framework and has extensive knowledge of CRA.
Emerging	Participant describes CRA as a gradual release model and has general knowledge of CRA.
Developing	Participant describes CRA as a sequence and has limited or inaccurate knowledge of CRA.

Pattern Codes for Reported Enactment	**Definition**
Advanced	Participant describes making explicit connections between two or more phases of CRA and makes minimal modifications to the phases or the procedure for implementing CRA.
Emerging	Participant describes making implicit connections between two or more phases of CRA and makes moderate modifications to the procedure for implementing CRA, but not the phases.
Developing	Participant does not describe making connections between two or more phases of CRA and makes extensive modifications to the phases and procedure for implementing CRA.

Appendix L

Results

Table 8.

Conceptualization of CRA

Category	Definition	Naming and Describing	Decision Making	Effectiveness
Advanced	CRA is a framework in which teachers model mathematics, making explicit for students how the phases are connected.	Accurate naming of phases and rich detailed descriptions of the phases and procedures.	A blend of assessment data and professional knowledge should drive student transition through the phases of CRA.	CRA is effective because it builds conceptual understanding by using a strategic learning progression that connects all phases.
Emerging	CRA is a gradual release model in which teachers scaffold mathematics instruction without making explicit for students how the phases are connected.	Accurate names of phases and general descriptions of the phases and procedures.	Observed student cues (e.g., frustration, ability to explain a concept or process, and independence) should drive student transition through the phases of CRA.	CRA is effective because it gives students tools to communicate their knowledge and build conceptual understanding by making connections between the abstract and a familiar visual.
Developing	CRA is a sequence in which teachers first use manipulatives, then images, and finally numbers to teach mathematics without making explicit for students how the phases are connected.	Inaccurate naming of phases and vague description of phases and procedures.	Uncertain or inappropriate method should drive student transition through the phases of CRA.	Incomplete or vague explanations for how CRA improves student learning.

Note: The categories emerged from the data during the second analysis phase.

Table 9.

Reported Enactment of CRA

Category	Planning	Reported Implementation	Intentional Modifications
Advanced	Lesson plan description and materials are clear and elaborate, providing the reader with a mental image of what would take place during the lesson.	Reported implementation of CRA includes making explicit connections between two or more phases.	Intentional minimal modifications, including removing a mastery criterion for phase progression with an explicit plan for remediation and skipping the abstract phase but combining it with another phase.
Emerging	Lesson plan description and materials are clear and provide the reader with a general idea of what would take place during the lesson.	Reported implementation of CRA includes making implicit connections between two or more phases.	Intentional moderate modifications, including removing a mastery criterion for phase progression without an explicit plan for remediation and neglecting to draw student attention to important features of the content through think-alouds.
Developing	Lesson plan description and materials are vague, incomplete, and do not provide the reader with a clear idea of what would take place during the lesson.	Reported implementation of CRA does not include making connections between two or more phases.	Intentional extensive modifications, including removing a mastery criterion for phase progression without an explicit plan for remediation, skipping multiple steps of explicit instruction, and removing a phase without combining it with another phase.

Note: The categories emerged from the data during the second analysis phase.