Decision Models in Consumer Lending in the Context of Economic Uncertainty

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Abstract

Credit scores are the primary vehicle for assessing the risk of loan applicants. Scores are mapped to the likelihood of an applicant defaulting or becoming seriously delinquent on the loan within a pre-determined period. The default probabilities are used to determine the profitability and the capital requirement for each borrower. Scorecards are built on historical data that are aggregated across many years and hence, possibly across many economic cycles. However, there is evidence in literature that default rates should be considered conditional on current and future economic conditions. This research focusses on improving decision making in retail credit through consideration of future economic conditions. The fundamental issue that we address is that the performance of a acquisition decision policy may be dependent on prevailing economic conditions during the loan period, and yet the policy must be specified and implemented before the loan period and hence before the economic environment is known with certainty.

We addressed this research opportunity in four ways. Firstly, we develop methods for incorporating forecasts of future economic conditions into acquisition decisions for scored retail credit and loan portfolios. We suppose that a portfolio manager is faced with two possible future economic scenarios, each characterized by a known probability of occurrence and by known performance functions that give expected profit and volume. We show that, despite the uncertainty of performance induced by economic conditions, every efficient policy consists of a single cutoff score, provided the expected profit and volume performance curves in each scenario are concave.

Secondly, we prove that misestimating regulatory capital requirements in either direction results in a negative impact on profit. A source of misestimation is due to errors in forecasts of future economic scenarios, resulting in differences between the reserve amount and the amount required under the realized economic condition. Thirdly, we develop methods for incorporating forecasts of future economic conditions into acquisition decisions for a portfolio manager faced with capital constraints and costs.

Finally, we give consideration to decisions by borrowers faced with a sequence of credit offers. From the definition of adverse selection in static lending models, we show that homogenous borrowers take-up offers at different instances of time when faced with a sequence of loan offers. We postulate that bounded rationality and diverse decision heuristics used by consumers drive the decisions they make about credit offers. Under that postulate, we show how observation of early decisions in a sequence can be informative about later decisions and can, when coupled with a type of adverse selection, also inform credit risk during the period of account performance.

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Background

In many nations, risk scores are the primary vehicle for assessing the risk of applicants for consumer or retail credit. Risk scores are produced using scorecards, which are statistically-constructed models that map behavioral or financial data on an applicant, such as payment history on other loans or home ownership, to a real-valued output. An applicant's score can be interpreted as an assessment, relative to others in the population, of the likelihood that he or she will default or become seriously delinquent on the loan. The core decision problem in portfolio acquisition is to set core cutoffs for one or more scorecards to define a policy for accepting or rejecting individual accounts. Cutoffs are chosen to achieve an acceptable tradeoff between conflicting business objectives, such as maximizing profit, maximizing market share, and minimizing risk.

Typically, the scores are mapped to default rates independent of the prevailing economic condition during account performance. However, there is evidence that default rates should be considered conditional on current and future economic conditions [13, 48]. An opportunity for improved decision making in retail credit can be found in consideration of future economic conditions on loan acquisition decisions. Suppose a portfolio manager has access to scorecard performance in each of the multiple possible future economic conditions and forecasts on the occurrence of future economic scenarios. We consider methods to incorporate such conditional scorecard performance and forecast information in credit lending decisions by the portfolio manager.

In analyzing a portfolio manager's decision, we propose a simplified system of models to capture the interrelationships between a consumer bank's business decisions with respect to consumer credit portfolios, its capital requirements, and the borrowers' decision to take-up a credit offer. The framework of our system is illustrated in Figure 1.1. In our simplified system, there are four major stake holders; the portfolio manager representing the bank, the consumer, the regulator, and the share holders. A consumer bank's acquisition model drives the portfolio creation process that creates a consumer credit portfolio. An important aspect of the acquisition model is the decision whether a bank will offer credit to an applicant and the offer rate. When an offer is made, each consumer in the offer population makes a decision whether to take-up the offer or not. The loan portfolio, consisting of accounts opened by those borrowers accepting credit offers, in turn drives the bank's objective through the bank model. The bank model governs objectives of owning such a portfolio, such as maximizing economic profits, maximizing market share, and minimizing risk. Furthermore, the characteristics of the portfolio and the risk models such as the capital adequacy models determine the capital requirement

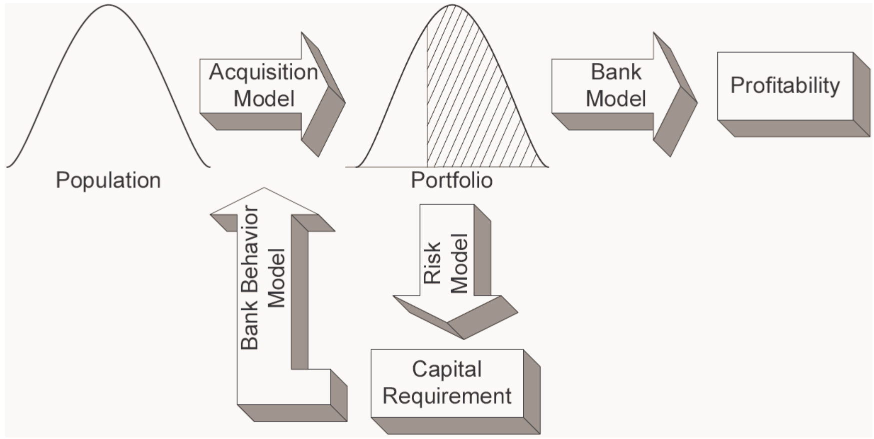

Figure 1.1: Interaction of stake holder decisions in a consumer lending system.

(equity)[1] needed to cover the unexpected losses. The regulatory capital model is set by the regulators in order to govern capital allocation in banks. The bank behavior model describes how a bank operates in terms of risk tolerance and how it raises equity capital from the share holders. The capital requirement, the bank behavior model and potential borrowers' decisions in turn affect loan acquisition decisions. The bank will alter its acquisition policies commensurate with the amount of equity capital that it can raise based on its behavior model.

In this dissertation, we incorporate forecasts of economic conditions into a portfolio manager's decision models with consideration given to decisions by consumers and to regulatory models. For example, suppose a portfolio manager is faced with multiple possible future economic scenarios, each characterized by a known probability of occurrence

[1]For banks, capital refers specifically to equity whereas to non-depository firms, capital includes all funding sources, i.e. debt, equity and quasi-equity. Capital requirement refers to equity requirement.

and by known performance functions that give expected profit and volume. Suppose account performance is dependent on the prevailing economic scenario. In making an offer decision, portfolio managers face a trade-off maximizing profit and market share while minimizing risk. Decision making in consumer lending can be improved when considering future economic conditions in acquisition decisions.

A related problem in consumer lending is the amount of unencumbered capital to retain for regulatory capital purposes. While it is the regulator's intention to perfectly model the credit risk to the bank, there are many potential sources of error that may result in a misestimation of true capital requirements. Sources of error include lack of model fidelity, errors in forecasts of inputs into the regulatory models, parameter misestimation, failure to incorporate bank specific inputs, and non-stationarity of consumer behavior. Basel I set the capital requirement at 8 percent of risk assets, while the standard approach of Basel II sets it at 6 percent of risk assets. Neither of these requirements is tailored to the risk profile of the bank's portfolio. The IRB approach of Basel II, however, takes a bank-specific view of the capital requirement. Figure 1.2 shows the capital requirement per unit of loan for different levels of portfolio risk, assuming loss given default of one and homogeneous scores for the population.

The IRB approach of Basel II is generally recognized as being an improvement over Basel I's one-size-fits-all approach of 8 percent of risk assets, but for the moment let us go beyond that and assume that IRB of Basel II is actually a perfectly accurate representation of true capital requirements. From Figure 1.2, we observe that for portfolios

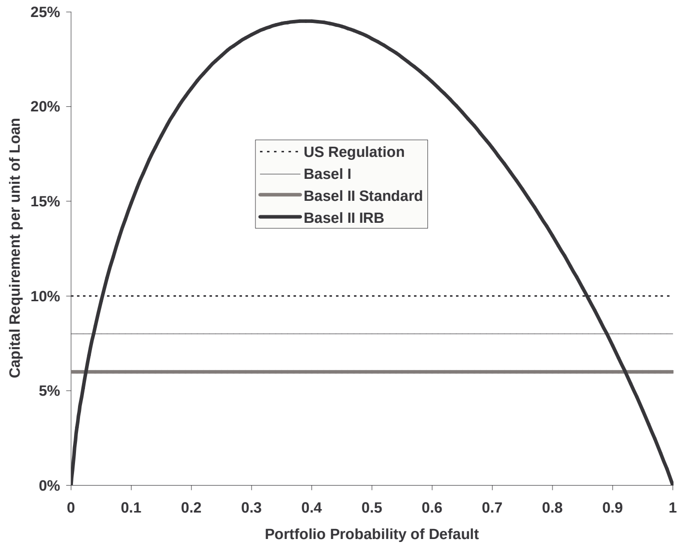

Figure 1.2: Capital requirement per unit of loan under different regulatory functions with loss given default of one.

with a probability of default below 0.037 or above 0.89, the capital requirements are higher for Basel I than for Basel II IRB. One would conclude that Basel I overestimates the true capital requirement for this range of portfolio probabilities of default. In contrast, capital requirements for Basel II IRB are higher than those of Basel I for portfolio probabilities of default between 0.037 and 0.89. One would conclude that Basel I underestimates the true capital requirement in these cases. Based on these observations, we propose to study the impact on profit due to misestimation of regulatory capital.

Suppose misestimating regulatory capital results in negative profit impact. Since default probabilities, which are input into the Basel formula, are conditional on future economic scenarios, it follows that an opportunity for improved decision making in consumer lending can be found in consideration of future economic conditions on regulatory capital decisions.

A further improvement in credit decisions is giving consideration to borrowers' decision in take-up or rejecting a credit offer. Oliver and Thaker [28] show there is a relationship between take-up rates and default rates. This indicates, consumers' decision to take-up credit offers during acquisition period may be informative on default rates under future economic conditions. Hence, there may be opportunity for improved decision making by considering consumer decision to take-up or reject a credit offer.

1.2 Organization of the Thesis

The rest of the thesis is organized as follows. In Chapter 2, we review the literature on portfolio manager's credit scoring decisions, regulatory models and borrowers' decision heuristics when considering a credit offer. This is followed by an introduction to basic notation for credit scoring and decision making, along with concepts for the evaluation of the business worth of a portfolio of credit accounts in Chapter 3.

In Chapter 4, we develop methods for incorporating forecasts of future economic conditions into acquisition decisions for scored retail credit and loan portfolios. We suppose that a portfolio manager is faced with two possible future economic scenarios, each characterized by a known probability of occurrence and by known performance functions that give expected profit and volume. We suppose further that he must choose in advance the scoring strategy and score cutoffs to optimize performance. Our goal is to map every efficient decision for a portfolio manager maximizing expected profit and market share.

In Chapter 5, we theoretically examine the performance implications of misestimating the regulatory capital requirement for a portfolio manager in a stylized consumer bank. Our aim is to show the impact on profits due to misestimation of regulatory capital requirements.

In Chapter 6, we extend the decision problem studied in Chapter 4 by incorporating the cost of regulatory capital in to the business metrics. We assume a portfolio manager operates under capital constraints when making portfolio acquisition decisions. Given Basel II capital requirement and the negative impact on economic profit of misestimating the regulatory capital as shown in Chapter 5, the risk-neutral portfolio manager must choose both a cutoff score and its associated level of capitalization at acquisition stage, i.e., prior to account performance. We construct the set of efficient operating points in the market-share and profit space for a portfolio manager operating under the assumption account performance is independent of the prevailing economic condition. We extend this to develop methods in constructing the efficient frontier for a portfolio manager operating under the assumption of multiple future economics scenarios.

Chapter 7, we give consideration to the impact on acquisition decision due to borrowers' take-up decisions. From the definition of adverse selection in static lending models, we show that homogenous borrowers take-up offers at different instances of time when faced with a sequence of loan offers. We postulate that bounded rationality and diverse decision heuristics used by consumers drive the decisions they make about credit offers. Under that postulate, we show how observation of early decisions in a sequence can be informative about later decisions and can, when coupled with a type of adverse selection, also inform credit risk during the period of account performance (regardless of future economic conditions). We show through two examples how lenders may use such information in setting their offer rates.

Conclusions and implications for future work are drawn in Chapter 8.

Chapter 2

Literature Review

In this chapter, we review the literature on credit scoring, Basel requirements, and consumers' decisions in a credit lending setting.

2.1 Credit Scoring Decisions

The decision problem of setting cutoff scores has been extensively studied in the literature. Early work has its focus on cutoff policies for a single objective, typically maximizing expected profit, under the assumption that the decision maker has access to only one scorecard (see, e.g., Capon [8], Hoadley and Oliver [20], Lewis [22], Thomas et al. [41]). More recently, Oliver and Wells have extended the treatment of cutoff policy decision making to include notions of efficiency associated with competing business objectives, with particular focus on the tradeoffs between expected profit, market share, and risk [30]. The theory of cutoff policies for multiple scorecards has developed in parallel. Zhu, Beling, and Overstreet have considered scenarios in which the decision maker has access to a training dataset that contains, for each individual in the training population, a performance outcome and scores from each scorecard [49, 50]. Multiple scorecards may be combined or fused into a single model in such cases, and if the combination is accomplished using the Bayesian method in Zhu et al. [49] the resulting scorecard dominates the originals in statistical and business performance.

There are many potential situations in which multiple risk scorecards may be available to the decision maker and yet there is no training data set that would allow the combination of the collection in the sense of Zhu et al. [49, 50]. In such situations, Beling, Covaliu, and Oliver prescribe a method for choosing a scorecard and an efficient cutoff under the assumption that only one of the available scorecards may be employed [4]. Scott et al. [37] suggest constructing the convex hull of the Receiver Operating Characteristic (ROC) curves of a collection of scorecards, a method which can be interpreted as randomizing over the choice of model. Cutoff policies for multiple scorecards may be increasingly relevant to decision makers, as the flat maximum effect suggests that there is little performance difference between the various types of statistical models that can be used to assess applicant risk, given the roughly the same data as input [31]. The clear corollary to this observation is that competitive advantage is more likely to be found by exploiting untapped sources of predictive data than it is to be found by refining the algorithms and statistical methodologies that are used on today's data. The trend will be toward the construction of scorecards that incorporate novel characteristics that do not correspond to data elements that have been in long-term and broad collection, and the use of these new models in conjunction with legacy scorecards [12].

A recent line of research has been to forecast the influences of economic conditions on the performance of loan portfolios. There is considerable evidence that default rates should be considered conditional on current and future economic conditions [13, 48]. Most lenders have structured the account acquisition process to allow for easy adaptation of decision policies to account for changing estimates of the *priori* quality of applicant pools [22, 30, 40]. In conjunction with a host of market factors, prevailing economic conditions also influence the revenues and losses to be expected from an individual credit account. It is especially important to account for economic variability in revenue and loss streams in models of the long-term evolution of customer behavior and profitability, such as the Markov models proposed by Thomas et al. [42]. The statistical performance of scorecards in rank ordering applicants by risk also may vary with economic conditions, and suggestions have appeared in the literature that it might be desirable to develop a suite of scorecards, each tailored to a particular economic condition [1, 40]. Numerous proposals have been made for incorporating economic information into the design of scorecards and other risk prediction models. Zandi [48], for instance, has developed a model in which a score based on economic indicators conditioned on geographic area and employment type is added onto the normal credit score. Bellotti and Crook take a survival analysis approach to melding macroeconomic variables with traditional behavioral and application variables [5].

A related opportunity for improved decision making in retail credit can be found in consideration of future economic conditions on loan acquisition decisions. We consider methods for incorporating forecasts of future economic conditions into acquisition decisions for scored retail credit and loan portfolios. Suppose scorecards are built specifically for each possible future economic conditions. However, the decision maker is required to set an accept/reject policy prior to account performance and hence, the prevailing economic condition during account performance is known. We assume the decision maker knows the performance to expect from each scorecard in terms of profit and volume given the realization of any particular economic scenario. We also assume that the decision maker has prior beliefs about the occurrence of each economic scenario that can be quantified as probabilities for the purposes of computing expected value.

In addition to the accept/reject policy, the decision maker is required to retain capital to cover unexpected losses. Since account performance is dependent on prevailing economic condition, unexpected loss is dependent on the prevailing economic scenario during account performance. In the next section, we review Basel capital requirements.

2.2 Basel Capital Requirements

In 1974, The Bank of International Settlements (BIS) established the Basel Committee on Banking Supervision to formulate broad standards, guidelines, and best practices to the international community. Capital adequacy became a major focus. In 1988, the committee took a major step by classifying the riskiness of different types of credit and of imposing minimum amounts of capital against such risks. The Accord, known as Basel I, was appealingly simple and imposed for the first time a capital requirement targeted to a standard of capital to weighted risk assets of eight percent. This crude attempt to create a level field for banks had unintended consequences, causing distortions and irrational lending which actually heightened systemic risk. This led to the Second Basel Accord, or Basel II, initiated in 1998 and now at long last in the early stages of its implementation. Basel II attempts to assure risk sensitive capital allocation in addition to the segregation and quantification of operational and credit risk. This regulation rests on the three integrated pillars: minimum capital requirements, a supervisory review process and market discipline.

The core issue of establishing a minimum capital requirement requires the measurement of credit risk and the modeling of unexpected loss. Modeling unexpected loss is done by estimation of portfolio loss distributions, and for this purpose a cottage industry with several well developed commercial risk models has evolved. The underlying model in Basel II is based on the one factor value risk model developed from Merton's theory [24, 46]. With a theoretical foundation based in financial economics, this approach is used for all types of credit exposure, including retail or consumer credit. In contrast, consumer banking theory has evolved over the past three decades from the theory of statistics and the perspective of raw empiricism inherent in consumer credit scoring, an approach that has proven effective in modeling large diversified portfolios of small consumer loans [41].

Despite the topic's societal importance and progress in bank research since 1988, the year of the first Basel Accord, a lack of consensus on the optimal design of bank capital regulation still exists [35]. Indeed, enforcing minimum standards enhances bank stability but also serves as a potential source of incremental costs [35]. Along similar lines, VanHoose's review of the literature [45] concludes that the intellectual underpinnings for the proposed Basel II system are not particularly strong and emphasizes "the need for further research efforts modeling diverse financial institutions". We trust that our work here will add to that context.

Berger's early empirical analysis of the relationship between bank capital and relative profitability presents a mixed picture of hypothetical relationships between bank capital and profits which run counter to theoretical expectations [6]. In contrast, VanHoose finds widespread agreement in the theoretical academic literature that constraining capital standards will likely lead to a reduction in total lending and accompanying increases in market loan rates with "substitution away from lending to holding alternative assets" [45]. There is also agreement in "a longer term increase in capital ratios which may or may not be accompanied by a rise in capital ratios with less agreement on the direction of total lending" [45]. How risk based capital regulation "influences choices banks make on the margin is central to whether risk-based capital regulation makes banks and the banking system safer" [45]. After all, capital cushions can evaporate rapidly if banks choose to make riskier asset choices or fail to exert their resources in evaluating adverse selection and moral hazard risks [45].

Diamond and Rajan's theoretical treatment of bank capital stresses that "optimal bank capital structure trades off the effects on liquidity creation, costs of bank distress, and the ability to force borrower repayment." [15]. They emphasis that diversification and risk management are substitutes for bank capital. Without a theory of the effects of bank capital, it has not been possible to analyze the trade-offs between bank responses to uncertainty [15].

Clearly, Basel II's evolution from a well established financial economic theory has led to regulatory capital requirements that are risk based and far closer to real economic risk than that the one-size-fits-all standard model of Basel I. Yet, it too is far from flawless (see, e.g., Gup [18]). The Basel II risk model does not consider the external environment such as interest rate changes nor portfolio size, and suffers on a micro basis the regulatory risk that arises from capital requirement misestimation, i.e., the difference between the retain capital amount and the required amount after accounts have defaulted. Sources of misestimation include conceptual errors in the underlying model, errors in parameter estimation, political influence on the choice of parameter values, and overrides [32]. An important parameter input to the risk-based Basel II advanced approach is the forecasted probability of default for each borrower. An error in forecasting default probabilities results in capital requirement misestimation. Given multiple possible future economic conditions, a portfolio manager's aim is to be adequately capitalized for the realized economic condition. In such a case, the portfolio manager faces a trade-off between the cost of regulatory capital and the risk of portfolio default.

In the next section, we review the literature on borrowers' decision making heuristics and adverse selection. We postulate that inference about the decision heuristics used by consumers when accepting or rejecting a loan offer at acquisition stage may provide a new source of information for lenders about performance at a latter period regardless of the prevailing economic condition.

2.3 Borrowers' Decisions

In setting loan prices (or loan interest rates), loan portfolio managers face a trade-off between response and risk. Consumers prefer lower loan rates and hence lower loan rates results in higher take-up of the products, but lower profits for each account. Loan pricing is further complicated by the phenomenon of *adverse selection* in which the default rates of individuals who accept a loan offer may be higher than that of those who decline the offer, all other factors being equal [33]. Adverse selection is thought to be the result of information asymmetry. Credit bureau reports and public records, which lenders use as input for credit risk and response models, may not reflect the circumstances and immediate financial needs of the borrower. Additionally, there may be subtle relationships between price elasticity and adverse selection [28].

As a result of information asymmetry, portfolio managers may view a group of consumers as homogenous, but private information held by the consumers differentiate their risk profiles. Portfolio managers do not have access to such private information. It would then seem obvious that reducing the information asymmetry between borrowers and lenders will result in more targeted marketing of credit products with appropriate rates. Typically, public information used by portfolio managers are those that are input into scorecards, i.e., financial, demographic and other personal information. Given flat maximum effect [31], new data sources or new variables need to be found in order to improve scorecard performance.

Consumer make decisions on taking up an offer prior to the period of account performance. We postulate that inference about the decision heuristics used by consumers when accepting or rejecting a loan offer at acquisition stage may provide a new source of information for lenders about performance at a latter period. Humans use heuristics to make decisions, which are simple rules, but often lead to decision errors. Kahneman and Tversky were among the first to establish cognitive basis for errors arising from decision heuristics (see Tversky and Kahneman [43] and Tversky and Kahneman [44]). Limited cognitive ability and incomplete information are some of the reasons for errors in decision making by human subjects. Many experiments have been conducted to reveal the decision heuristics used by human subjects in various classes of decision problems (see Winkler and Murphy [47] for more).

One particular class of decision problems is the sequential decision problem in which agents are required to make a sequence of binary decisions. Sequential decision problems are of particular interest to consumer lending because consumers are often faced with a sequence of loan offers for which they make take/no take decisions. Suppose inference about decision heuristics could provide added information on borrower take/no-take behavior, then such information may reduce the information asymmetry between lenders and borrowers. Hence, it is possible observation of early decisions in a sequence can be informative about later decisions and can, when coupled with a type of adverse selection, also inform credit risk.

Chapter 3

Notation, Assumptions, and Basic Models

In this chapter, we introduce basic notation for credit scoring and decision making, along with concepts for the evaluation of the business worth of a portfolio of credit accounts and formula to determine regulatory capital.

3.1 Credit Scores

As part of the decision processes involved in assembling a consumer credit portfolio from a population of potential customers seeking credit, applicants are scored on a statistical scorecard built on historical data. The inputs of the scorecards are a vector of behavioral information for each customer, which we denote by $\bar{x}$, while the outcome of the scorecard is a real-valued risk score $s(\bar{x})$. The output score of each applicant is used to predict the customer's performance, over a given period, in repaying money borrowed on the credit account. Let Z denote the random variable account performance. A common notion in theory and practice is to consider two mutually exclusive and exhaustive performance outcomes, G and B. The event G is associated with a *good* customer and implies that the customer does not default or become seriously delinquent during the performance period, whereas the event B is associated with a *bad* customer and implies the converse.

We use $p(G|\bar{x})$ and $p(B|\bar{x})$ to denote the posterior conditional probabilities of the outcomes G and B, respectively, for a customer with behavioral data $\bar{x}$. Further we make the common assumption that each risk score $s(\bar{x})$ computed using the data $\bar{x}$ has the property that $p(G|s(\bar{x})) = p(G|\bar{x})$ and $p(B|s(\bar{x})) = 1 - p(G|\bar{x})$, which implies that risk assessment can be done using only scores, and that once these are computed there is no need to reference the underlying behavioral data. Consequently, we suppress references to $\bar{x}$ in what follows, writing s for the score of an individual rather than $s(\bar{x})$. We denote the random variable score by S. See Bellotti and Crook [5], Hand and Henley [19] for more details on credit scoring.

In reference to a population of applicants, we use p_G and p_B to denote our prior belief that an applicant is good or bad, respectively. Let $f(s|G)$ and $f(s|B)$ denote the likelihood of score s given the performance outcomes G and B, respectively. Then the posterior probability that an individual with score s is good or bad is, respectively,

$$
\begin{aligned}
p(G|s) &= f(s|G)p_G/f(s) \\
p(B|s) &= f(s|B)p_B/f(s),
\end{aligned}
$$

where $f(s)$ is the density function of score for the population of applicants. We use F to denote the cumulative score distribution; that is, $F(s) = \int_{-\infty}^{s} f(u)du$.

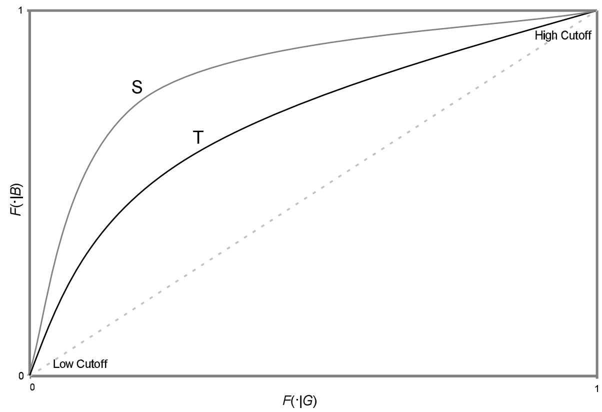

Figure 3.1: ROC curves for two scorecards, S and T.

3.1.1 ROC Curves

A common statistical performance measure is the Receiver Operating Characteristic (ROC) curve, which is a plot of the cumulative score distribution of the bad applicants rejected $F(s|B)$ versus the cumulative score distribution of the good applicants rejected $F(s|G)$.

There are two ROC curves of theoretical interest. The ROC curve of a non-discriminating predictor is the line segment connecting the origin $(0, 0)$ to the point $(1, 1)$. Conversely, the ROC curve of a perfect scorecard or predictor is piecewise linear with two segments, one connecting $(0, 0)$ to $(0, 1)$, where all bad population is perfectly identified, and one connecting $(0, 1)$ and $(1, 1)$, corresponding to the good population. Credit scores are generally constructed so that risk decreases as score increases. An important consequence of this property is that the ROC curve for a scorecard is always concave in theory, though in practice it may not be because of data noise.

The concept of dominance is important in the context of multiple ROC curves, which we consider in later chapters. Scorecard S is said to *dominate* scorecard T, if the ROC curve of S is everywhere on or above that of scorecard T. Figure 3.1 illustrates the relationship between the ROC curves of scorecards S and T in a case where S dominates T.

3.2 Business Objectives, Acquisition Models and EPV Curves

In practice, a portfolio manager would set a cutoff score to enable the realization of a bank's objectives, such as maximizing profit, maximizing volume, or minimizing default losses. The portfolio's manager's primary objective is to maximize expected profit. We define profit in terms of a simplified model of bank operations similar to that proposed by Oliver and Thomas [29]. For each unit of loan given out to the applicants, the bank earns $1 + r_L$ from each good account where r_L is the return on loans. It recovers $C(1 - f_D)$ from each bad account, where f_D is the fractional loss given default and C is the exposure at default. We make the simplifying assumption that $C = 1$, which ensures that the amount recovered from a bad account with a unit loan, $C(1 - f_D)$, is less than than the loan amount. In order to extend a unit of credit, the bank needs to fund the loan volume including both good and bad accounts. We assume loan volume is funded entirely with debt at an interest rate of r_B, with $r_B < r_L$. Following Oliver and Wells [30], the portfolio manager can maximize the expected operating income from a fixed population of applicant by including in the portfolio every applicant who has a nonnegative expected operating income (revenue less loss), that is a score s satisfying

$$E_Z[I(s)] = (1 + r_L)p(G|s) + (1 - f_D)p(B|s) - (1 + r_B) \geq 0.$$

Since $p(G|s) = 1 - p(B|s)$, it follows that,

$$E_Z[I(s)] = (r_L - r_B)p(G|s) - (f_D + r_B)p(B|s) \geq 0.$$

The concavity nature of ROC curves for scorecards implies that the manager can achieve this by setting a *cutoff score* and adopting the policy that all credit applicants who score above the cutoff are accepted and all who score below the cutoff are rejected.

Let $V(s)$ denote the fraction of the applicant population that is accepted given a cutoff score s. Following language in Oliver and Wells [30], we term this quantity the portfolio *volume* realized with the cutoff s. The expected volume is $E_S[V(s)] = 1 - F(s)$. Since each good account provides a revenue of $r_L - r_B$, and that the lender incurs a loss of $f_D + r_B$ on every bad account. The expected portfolio operating income is then,

$$\begin{aligned}
E_S[I_N(s)] &= \int_s^\infty [(r_L - r_B)p(G|u) - (f_D + r_B)p(B|u)]\, f(u)du \\
&= \int_s^\infty (r_L - r_B)p_G f(u|G)du - \int_s^\infty (f_D + r_B)p_B f(u|B)du \\
&= (r_L - r_B)p_G[1 - F(s|G)] - (f_D + r_B)p_B[1 - F(s|B)].
\end{aligned}$$

We assume operating expenses are fixed and, hence, have no effect on the derivation of optimal policies. Thus, we exclude them from our model. Due to the regulatory requirements, the bank covers unexpected losses by raising equity capital, which is determined using the formula for capital requirement k_i. There is an opportunity cost of equity r_Q or the return on equity required by the shareholders. Given a one period model, the expected economic profit for an account is calculated by taking the operating income less the economic cost of equity $r_Q k_i(s)$, i.e.

$$E_Z[P(s, k_i(s))] = E_Z[I(s)] - r_Q k_i(s). \tag{3.1}$$

We assume $E_Z[P(-\infty, k_i(s))] < 0$ and $E_Z[P(\infty, k_i(s))] > 0$.

The expected profit for a portfolio is,

$$\begin{aligned} E_S[E_Z[P(s, Q_i(s'))]] &= \int_s^\infty [(1 + r_L)p(G|u) + (1 - f_D)p(B|u) - (1 + r_B)]\, f(u)du - r_Q Q_i(s') \\ &= E_S[I_N(s)] - r_Q Q_i(s'), \end{aligned} \tag{3.2}$$

where

$$Q_i(s') = \int_{s'}^\infty k_i(u) f(u) du, \tag{3.3}$$

For notational convenience, we express $E_S[P(s, Q_i(s'))] \equiv E_S[E_Z[P(s, Q_i(s'))]]$.

We define the following notation with superscript s for portfolio metrics when a portfolio manager does not apply a cutoff score policy but instead accepts scores in the set $\omega \subseteq S$ and applies capitalization function $k_i(s)$. Let

$$E_S[E_Z[P^s(\omega, Q_i^s(\omega))]] = \int_{s \in \omega} E_Z[P(s, k_i(s))] f(s) ds$$

be the expected profit, where

$$Q_i^s(\omega) = \int_{s \in \omega} k_i(u) f(u) du. \tag{3.4}$$

For notational convenience, we express $E_S[P^s(\omega, Q_i^s(\omega))] \equiv E_S[E_Z[P^s(\omega, Q_i^s(\omega))]]$. The corresponding expected portfolio volume is $E_S[V^s(\omega)] = \int_{s\in\omega} f(s)ds$. We refer to ω and $\omega^c = \{s|s \notin \omega\}$ as the *accept set* and the *non-accept set*, respectively.

Oliver and Wells [30] consider cutoff policies in the context of the competing business objectives of maximizing volume (which is a surrogate for market share), minimizing loss, and maximizing profit, with zero capital requirement, i.e., $k_i(s) = 0\ \forall s$. The fact that tradeoffs exist between the business objectives makes it important for decision makers to operate on the *efficient frontier*, which is the maximal set of operating points that are not dominated by other operating points.

Suppose we restrict attention to the metrics of expected profit and expected volume, with $k_i(s) = 0\ \forall s$. Of special importance are plots of expected profit versus expected volume parameterized by score cutoff. We term the collection of points $(E_S[P(s,0)], E_S[V(s)])$ for all s an Expected-Profit-Volume (EPV) curve. An EPV point with cutoff score s is *efficient* if there exist no other cutoff $\hat{s}$ with $E_S[P(\hat{s},0)] = E_S[P(s,0)]$ and $E_S[V(\hat{s})] > E_S[V(s)]$. Figure 3.2 shows an example EPV curve. The points on the curve to the right of point A form the efficient frontier. It should be noted that if the ROC curve for a scorecard is concave then the EPV curve for that scorecard must also be concave.

For $Q_i(s) \neq 0$, a portfolio manager may follow the regulatory capital requirements in setting the amount of capital to retain. In the next section, we introduce the regulatory capital formula.

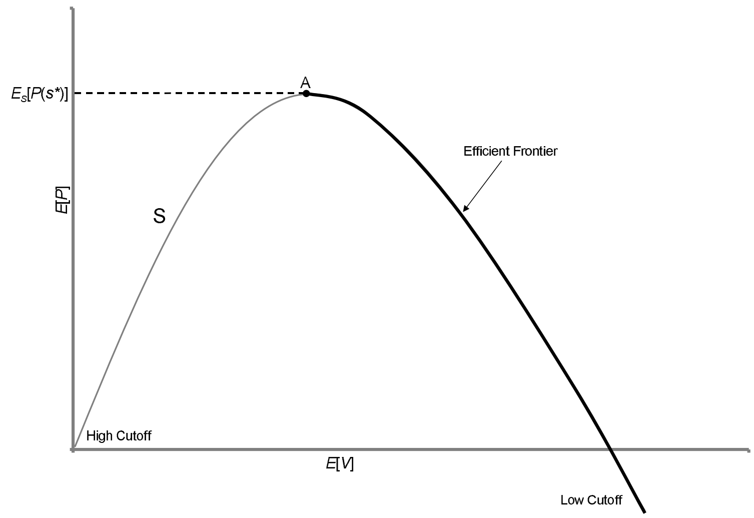

Figure 3.2: $E_S[P]$-$E_S[V]$ plot depicting the Efficient Frontier with $k_i(s) = 0\ \forall s$.

3.3 Regulatory capital requirement

Currently, capital requirement is based on the Basel II Accord. Capital requirements may be calculated in one of two ways. The standard approach requires banks to hold 6% of risk assets. The capital requirement using the Internal Rating Based (IRB) method is determined using a formula specified by the Basel II Accord. The capital amount under this method covers unexpected losses at a confidence level of 99.9%. Unlike the standard approach, the capital requirement calculated in the advanced approach of the IRB is risk depended (see Figure 3.3). Using the advanced approach, the capital requirement for a unit loan to a customer with score s is

$$k_B(p(B|s)) = f_D\left[\Phi\left(\sqrt{\frac{1}{1-\rho}}\Phi^{-1}(p(B|s)) + \sqrt{\frac{\rho}{1-\rho}}\Phi^{-1}(0.999)\right)\right] - f_D p(B|s), \quad (3.5)$$

where ρ is a correlation coefficient specific to loan portfolio type (see Basel Committee on Banking Supervision [3]), and $\Phi()$ is the cumulative density function for the standard normal distribution. The first term on the right-hand side of Equation 3.6 is the capital required to cover total losses at a confidence level of 99.9%. The second term is equal to the expected loss and is subtracted from forecasts of total losses at 99.9% confidence level since expected losses are expected to be priced for from the beginning [32]. See Perli and Nayda [32], Schönbucher [36] for the derivation of the regulatory requirement formulae.

Scores are constructed such that probability of default, $p(B|s)$ and score, s have a one-to-one relationship. It follows that the capital requirement function can be expressed as a function of score,

$$k_R(s) = f_D \left[\Phi \left(\sqrt{\frac{1}{1-\rho}} \Phi^{-1}(p(B|s)) + \sqrt{\frac{\rho}{1-\rho}} \Phi^{-1}(0.999) \right) - p(B|s) \right]. \tag{3.6}$$

The correlation coefficient for "qualifying revolving products" is 0.04 [3]. This results in the shape of the plot observed in Figure 3.3.

The capital requirement curve $k_R(s)$ is concave with respect to probability of default, $p(B|s)$ for mortgages and qualifying revolving portfolios [7]. However for "other retail portfolios", the capital requirement curve is concave everywhere, except for a region of local convexity that exists approximately between $p(B|s) \in (4.903\%, 15.184\%)$ [7]. The capital requirement monotonically increases with respect to probability of default in the regional of convexity. These properties are important when discussing profit dominance

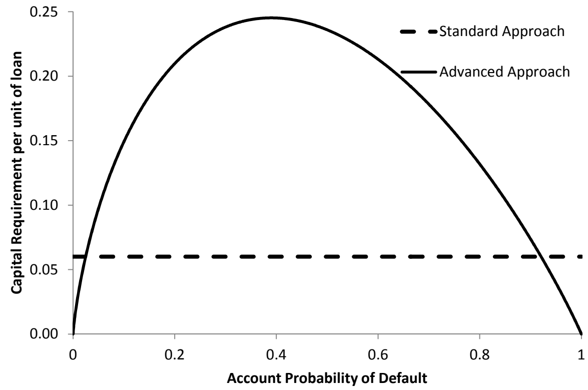

Figure 3.3: Regulatory capital requirement per unit of ‘qualified revolving’ loan under the Basel II Accord.

in Chapter 6.

Chapter 4

Scoring Decisions in the Context of Economic Uncertainty

4.1 Introduction

In this chapter, we develop methods for incorporating forecasts of future economic conditions into acquisition decisions for scored retail credit and loan portfolios. Our focus is the impact of future economic conditions on scoring decisions and hence following Oliver and Wells [30], we set economic capital requirement to zero. The fundamental issue that we address is that the performance of a cutoff policy may be dependent on prevailing economic conditions during the loan period, and yet the policy must be specified and implemented before the loan period and hence before the economic environment is known with certainty. Our focus is on decision making rather than predictive modeling. We assume that the scorecards available to the decision maker are fixed, and thus the decision of interest is how to define a policy that maps the available scores for an applicant into an accept or reject decision. In our framework, the decision maker knows the performance to expect from each scorecard in terms of profit and volume given the realization of any particular economic scenario. We also assume that the decision maker has prior beliefs about the occurrence of each economic scenario that can be quantified as probabilities for the purposes of computing expected value.

The remainder of the chapter is organized as follows. In Section 4.2, we study the decision faced by a portfolio manager who is cognizant that the future will bring one of two possible economic scenarios, each of which is characterized by a probability of occurrence and by performance functions that give expected profit and volume for each cutoff score. We show that, despite the uncertainty of performance induced by economic conditions, any efficient policy consists of a single cutoff, provided the expected profit and volume performance curves in each scenario are concave. If these curves are not concave, efficient operating points can be characterized as cutoffs on a redefined score. In Section 4.3, we study cases where two scorecards are available to the portfolio manager. We show that it may be advantageous to randomly choose the scorecard to be employed, and we provide methods for selecting efficient operating points under randomization. It should be noted that our randomization scheme differs from that proposed in Scott et al. [37] in that our method cannot be readily interpreted as an action on ROC curves. Discussion in Section 4.3 is limited to cases with two scorecards and two economic scenarios, but our approach and results generalize to more scorecards and more economic scenarios. Finally, in Section 4.4, we summarize our findings.

4.2 Single Scorecard in Multiple Economic Scenarios

In this section we consider the case of a portfolio manager who is facing the task of deciding which applicants to accept into the portfolio, given a single scorecard and a model for future economic conditions and the impact of these conditions on the business performance metrics associated with the portfolio. The primary aim of the portfolio manager is to adopt an acquisition policy that is efficient in the sense that it achieves maximum expected volume for a given expected profit. Our goal is to understand the structure that is shared by every efficient acquisition policy.

Let us label by S the scorecard that is available to the manager. Each applicant has an associated score produced by S that we model as a random variable S. (Note that the scorecard is denoted using roman type, while the associated score random variable is denoted in italics). The portfolio manager is cognisant that the future will bring one of two possible economic scenarios, and this scenario will prevail during the period of performance for the loans in the portfolio. We model the uncertainty of economic conditions using a binary random variable K, interpreting $K = 0$ as being the realization of one economic scenario and $K = 1$ as being the realization of the other. Assume the portfolio manager has a prior belief about the probability of each scenario being realized, and define $\gamma = p(K = 0)$ (hence $1 - \gamma = p(K = 1)$).

In a multiple economic scenario case, we assume the portfolio manager has access to an EPV curve for each scenario that gives the performance (i.e., profit and volume) of the accepted population in expectation, conditional on the scenario. We call each such curve a *conditional* EPV curve. Since the decision to accept or reject applicants must be made prior to the realization of an economic scenario, the portfolio manager would base the acquisition decision on an EPV curve that is *unconditional* on the economic scenario. In this chapter, for notational convenience we write $E_S[P(s)] \equiv E_S[P(s,0)]$. Let $E_S[P(s)|K]$ and $E_S[V(s)|K]$ be the expected profit and expected volume conditioned on economic scenario K. It follows that

$$E_S[P(s)] = \gamma E_S[P(s)|K=0] + (1-\gamma)E_S[P(s)|K=1]$$

and,

$$E_S[V(s)] = \gamma E_S[V(s)|K=0] + (1-\gamma)E_S[V(s)|K=1].$$

The cutoff score decision is made prior to the economic scenario occurring and for any given score s, the volume remains constant for all scenarios; i.e., $E_S[V(s)|K=0] = E_S[V(s)|K=1]$. Since $0 \leq \gamma \leq 1$, it follows that

$$E_S[V(s)] = E_S[V(s)|K=0] = E_S[V(s)|K=1].$$

The unconditional EPV curve is then the collection of points $(E_S[P(s)], E_S[V(s)])$. Figure 4.1 provides examples of conditional EPV curves for two economic scenarios and the corresponding unconditional EPV curve.

The unconditional EPV curve is concave if the EPV curve for each scenario is concave. Concavity of the unconditional EPV curve, in turn, ensures that the portfolio manager

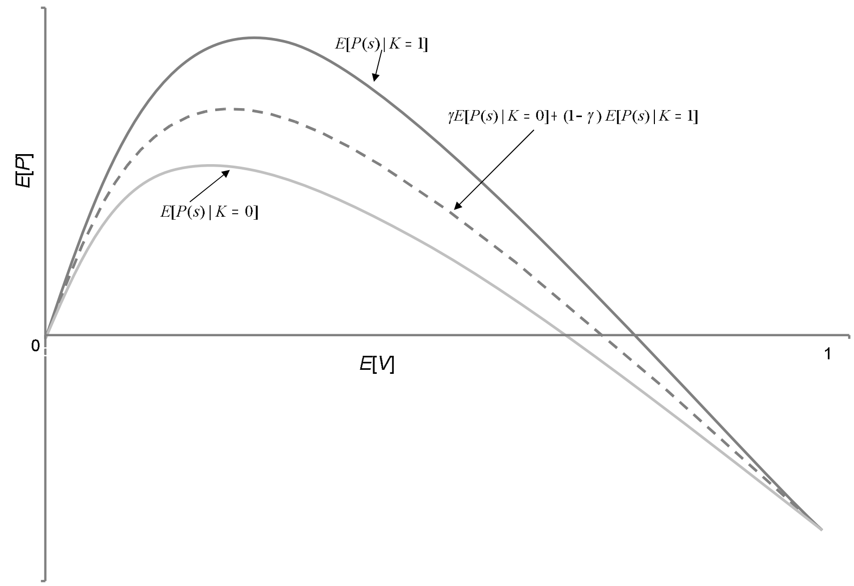

Figure 4.1: $E[P] - E[V]$ curve.

can achieve an efficient operating point by following a single score cutoff policy, which is common practice in industry (cf. Oliver and Wells [30]).

It may be that the scorecard was designed to predict the probability of an applicant defaulting in a given economic scenario. In that case, the EPV curve for that economic scenario would be concave. The EPV curve for the alternate scenario, however, might not be concave, which in turn might result in the unconditional EPV curve also not being concave. The difficulty with a non-concave EPV curve is that the the portfolio manager might not be able to achieve an efficient operating point by following a single score cutoff policy. In such a case, however, a new random variable S', may be defined such that the EPV curve constructed using this score is concave. We define the new score in terms of the gradient of the EPV curve. In particular, if $S = s$ then we set $S' = s'(s)$, where

$$s'(s) = \frac{\partial E_S[P(s)]}{\partial E_S[V(s)]}.$$

The expected profit for a cutoff score s' is

$$E_{S'}[P(s')] = \int_{-\infty}^{\infty} P^N(s, s')ds$$

such that

$$P^N(s, s') = \begin{cases} \frac{\partial E_S[P(s)]}{\partial s} & \text{if } \frac{\partial E_S[P(s)]}{\partial E_S[V(s)]} \geq s' \\ 0 & \text{otherwise.} \end{cases} \tag{4.1}$$

The expected volume for a cutoff score s' is

$$E_{S'}[V(s')] = \int_{-\infty}^{\infty} V^N(s, s')ds,$$

where

$$V^N(s, s') = \begin{cases} \frac{\partial E_S[V(s)]}{\partial s} & \text{if } \frac{\partial E_S[P(s)]}{\partial E_S[V(s)]} \geq s' \\ 0 & \text{otherwise.} \end{cases} \tag{4.2}$$

Note that as s' decreases to zero both expected profit and expected volume increase, with $E_{S'}[P(s')]$ reaching a maximum at $s' = 0$. As s' decreases below zero expected volume increases but expected profit decreases. In follows that the only efficient cutoffs are non-positive values of s'.

Following Oliver and Wells [30], a single score cutoff decision may be made using the new score.

Now let us consider a different structure to the decision problem. Assume the portfolio manager can randomly assign each individual in the applicant population to one of two sub-populations, say sub-populations $\mathcal{C}$ and $\mathcal{D}$. Every individual is scored on the same scorecard, but the portfolio manager has the ability to set differing cutoff scores for each sub-population. Note this decision structure would allow the portfolio manager to implement a policy in which the acquisition decision is optimized for one economic scenario on sub-population $\mathcal{C}$ and for the other scenario on sub-population $\mathcal{D}$.

Let $E[P(\alpha, s_{\mathcal{C}}, s_{\mathcal{D}})]$ and $E[V(\alpha, s_{\mathcal{C}}, s_{\mathcal{D}})]$ be the expected profit and expected volume where α is the probability that any individual is assigned to subpopulation $\mathcal{C}$ and $s_{\mathcal{J}}$ is the cutoff score for subpopulation $\mathcal{J}$, where $\mathcal{J} \in \{\text{C, D}\}$. Then, the expected profit is

$$E[P(\alpha, s_{\mathcal{C}}, s_{\mathcal{D}})] = \alpha E[P(s_{\mathcal{C}})] + (1-\alpha)E[P(s_{\mathcal{D}})],$$

where, $E[P(s_{\mathcal{J}})] = \gamma E[P(s_{\mathcal{J}})|K=0] + (1-\gamma)E[P(s_{\mathcal{J}})|K=1]$ for $\mathcal{J} \in \{\text{C,D}\}$. The expected volume is

$$E[V(\alpha, s_{\mathcal{C}}, s_{\mathcal{D}})] = \alpha E[V(s_{\mathcal{C}})] + (1-\alpha)E[V(s_{\mathcal{D}})].$$

Suppose the portfolio manager desires to maximize expected profit while creating an expected portfolio size of desired volume V_0. The portfolio manager's optimization problem can be written as

$$\begin{array}{ll} \max_{\alpha, s_{\mathcal{C}}, s_{\mathcal{D}}} & \alpha E[P(s_{\mathcal{C}})] + (1-\alpha)E[P(s_{\mathcal{D}})] \\ \text{s.t.} & \alpha E[V(s_{\mathcal{C}})] + (1-\alpha)E[V(s_{\mathcal{D}})] = V_0 \\ & 0 \leq \alpha \leq 1 \end{array} \tag{4.3}$$

We defined a score $s_{\mathcal{M}}$ such that $E[V(s_{\mathcal{M}})] = V_0$. Since the unconditional EPV curve is concave,

$$E[V(s_{\mathcal{M}})] \geq \alpha E[P(s_{\mathcal{C}})] + (1-\alpha)E[P(s_{\mathcal{D}})] \quad \forall s_{\mathcal{C}}, s_{\mathcal{D}}.$$

For a desired volume V_0, the maximum expected profit is achieved when $s_{\mathcal{M}} = s_{\mathcal{C}} = s_{\mathcal{D}}$. This implies that, to be efficient, the portfolio manager should use a single cutoff score policy for the entire population.

We summarize the portfolio manager's action as follows. Suppose the portfolio manager wishes to maximize expected profit with a constraint ensuring a lower bound on expected volume of V_0. That is, the portfolio manager wishes to solve the constrained optimization problem

$$\begin{array}{ll} \max_s & E[P(s)] \\ \text{s.t.} & 1 - F(s) \geq V_0. \end{array} \tag{4.4}$$

Let s^* denote the cutoff score that solves $\max_s E[P(s)]$, the unconstrained problem of maximizing expected profit. If $V_0 \leq E[V(s^*)]$ then s^* is also the solution to the manager's problem. If $V_0 \geq E[V(s^*)]$, however, the manager should use $F^{-1}(1 - V_0)$ as the cutoff score.

We have shown that, despite the uncertainty of performance induced by economic conditions, every efficient policy consists of a single cutoff, provided the expected profit and volume performance curves in each scenario are concave. If these curves are not concave, efficient operating points can be characterized as a single cutoff on a redefined score.

4.3 Multiple Scorecards in Multiple Economic Scenarios

The focus of Section 4.2 is portfolio decisions in which the portfolio manager has access to one scorecard. Here we extend consideration, making the assumption that the decision maker has access to two scorecards, S and T, and that the performance of the population is a function of which of two possible economic scenarios is realized in the future. The discussion in Section 4.2 shows that there is no loss of generality in assuming that each scorecard is described only by an EPV curve that is concave and unconditional with respect to economic scenario. In particular, then, we assume the manager has the curves $(E_S[P(s)], E_S(V(s)])$ and $(E_T[P(t)], E_T(V(t)])$. It should be noted that the approach and results generalist easily to more scorecards and more economic scenarios.

Of chief interest are cases where neither of the two EPV curves is dominant, and we restrict the discussion accordingly. We further assume that the manager lacks access to a database consisting of performance outcomes and scores for both scorecards, implying that the scorecards may not be fused in the sense of Zhu et al. [49, 50]. Beling, Covaliu, and Oliver prescribe a method for choosing a scorecard and an efficient cutoff under the assumption that only one of the available scorecards may be employed [4]. Figure 4.2 illustrates a case in which neither of the EPV curves is dominant. Under the assumption in Beling et al. [4] that only one scorecard is used, the efficient frontier in EPV space is the union of segments of the EPV curves of the scorecards. In Figure 4.2, the efficient frontier consists of the union of the portion of the curve for scorecard T between points

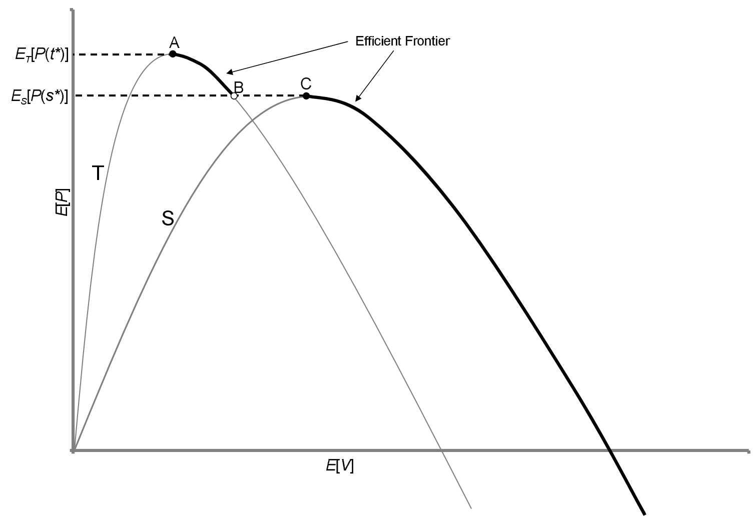

Figure 4.2: $E[P]$-$E[V]$ Multiple scorecards with no dominant scorecards with discontinuous efficient frontier.

A and B (non-inclusive) and the portion of the curve for scorecard S to the right of the point C. Note that this frontier includes a discontinues jump in expected volume. Below we propose two methods for making use of both scorecards in a manner not considered in Beling et al. [4]. The efficient frontiers corresponding to these methods dominate those in Beling et al. [4], and include points that are not on the EPV curve of either of the scorecards.

4.3.1 Fixed Allocation of Applicants

In the case where neither scorecard dominates in expected performance across economic scenarios, we propose that the manager consider making use of both scorecards through

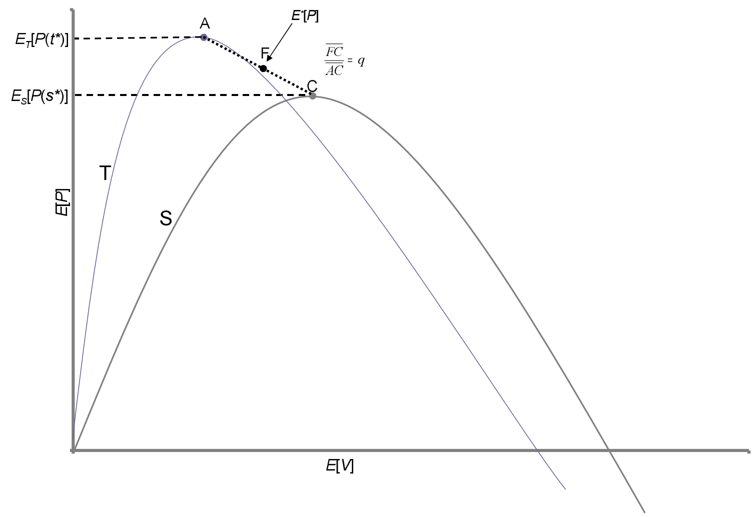

Figure 4.3: The maximum achievable expected profit in the fixed allocation strategy is at point F.

a randomization scheme. In this scheme, an applicant is scored on scorecard T with probability q and scorecard S with probability $(1 - q)$. This is akin to tossing, for each applicant, a coin that lands heads with probability q. If the coin lands heads, then scorecard T is applied, and otherwise scorecard S is applied. In expectation, then, a fraction q of the population is allocated to scorecard T and a fraction $(1-q)$ is allocated to S. We call this the *fixed allocation* case. The only controls that are available for choosing an operating point are the score cutoffs s, to be applied to applicants scored on scorecard S, and t to be applied to applicants scored on scorecard T.

In Figure 4.3, the maximum expected profit that can be achieved in this case is the point F on the chord AC.

Given cutoff scores s and t, the expected total profit and volume are

$$E[P(s,t)] = qE_T[P(t)] + (1-q)E_S[P(s)] \tag{4.5}$$

$$E[V(s,t)] = qE_T[V(t)] + (1-q)E_S[V(s)], \tag{4.6}$$

where $E_S[P(s)]$ and $E_T[P(t)]$ are the expected profit for scorecards S and T at cutoff score s and t, respectively, and $E_S[V_(s)]$ and $E_T[V(t)]$ are the expected volume for S and T at cutoff score s and t, respectively. Let s^* and t^* denote the profit maximizing cutoff scores for the individual scorecards; that is, $s^* = \arg\max_s E_S[P(s)]$ and $t^* = \arg\max_t E_T[P(t)]$. The maximum expected total profit is then

$$E[P(s^*,t^*)] = qE_T[P(t^*)] + (1-q)E_S[P(s^*)]$$

and the expected volume at maximum expected total profit is total profit is $E[V(s^*,t^*)]$.

If the portfolio manager wishes to operate at a higher expected volume than the profit maximizing volume, the cutoff scores s and t should be decreased in a manner that yields the smallest decrease in expected profit for the desired increase in expected volume. The way to achieve a marginally higher volume efficiently is to decrease the cutoff of the scorecard that has the shallower EPV curve at the current operating point. Repeated application of this notion, starting at the maximum profit point, defines a constructive method for tracing out the efficient frontier for the fixed allocation case. The efficient frontier is illustrated in Figure 4.4.

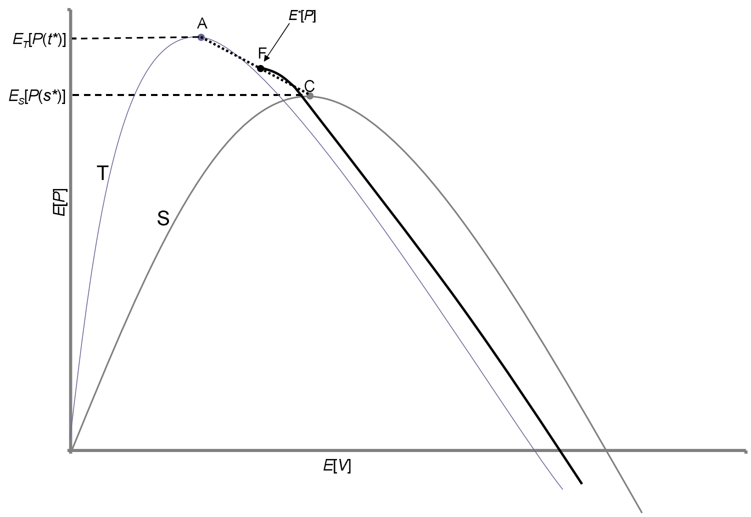

Figure 4.4: The efficient frontier for a fixed allocation strategy.

4.3.2 Variable Allocation of Applicants

In fixed allocation, the probability q that the coin comes up heads is fixed and outside the control of the decision maker. Suppose instead that the decision maker has the ability to set q. Consider the convex hull of the EPV curves for the two scorecards, which is constructed by drawing the unique line segment that is tangent to both curves, as illustrated in Figure 4.5.

The efficient frontier in Figure 4.5 consists of the union of the segment AHJ with all points on the EPV curve for S that are to the right of J. If the decision maker wishes to operate on the curve segment AH, then scorecard T would be used exclusively by setting $q = 1$. Likewise, if the decision maker wishes to operate the right of point J, then scorecard S would be used exclusively by setting $q = 0$. If, however, the decision

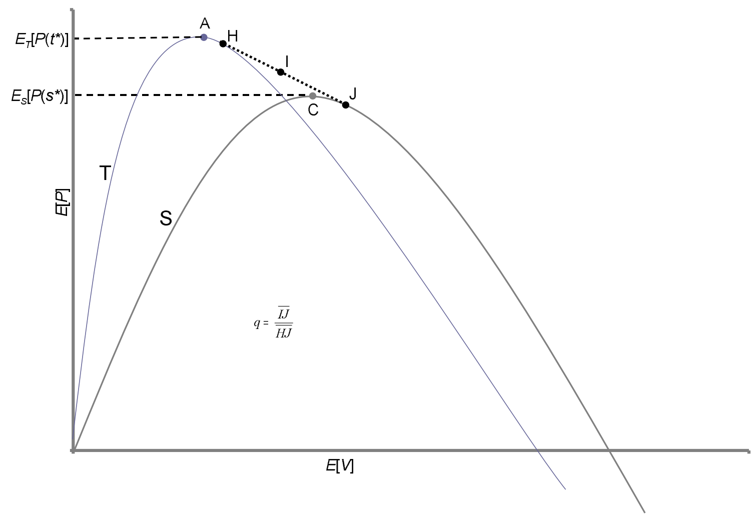

Figure 4.5: Construction of the convex hull.

maker wishes to operate at a point I between H and J, as illustrated in Figure 4.5, then this can be achieved by a randomization strategy in which $q = \overline{\mathrm{IJ}}/\overline{\mathrm{HJ}}$, the ratio of the lengths of the line segments in the figure, and in which the cutoffs for scorecards S and T are chosen to correspond with the points J and H, respectively.

The efficient frontier of the variable allocation strategy dominates the efficient frontier of the fixed allocation strategy, as illustrated in Figure 4.6.

Figure 4.7 illustrates another case in which neither of two EPV curves is dominant. The efficient frontier consists of curve AB and all points on the EPV curve of scorecard S to the right and below point B. We illustrated our method for variable allocation on the curves with a discontinuous efficient frontier(Figure 4.2) but our findings are applicable to the curves such as in Figure 4.7.

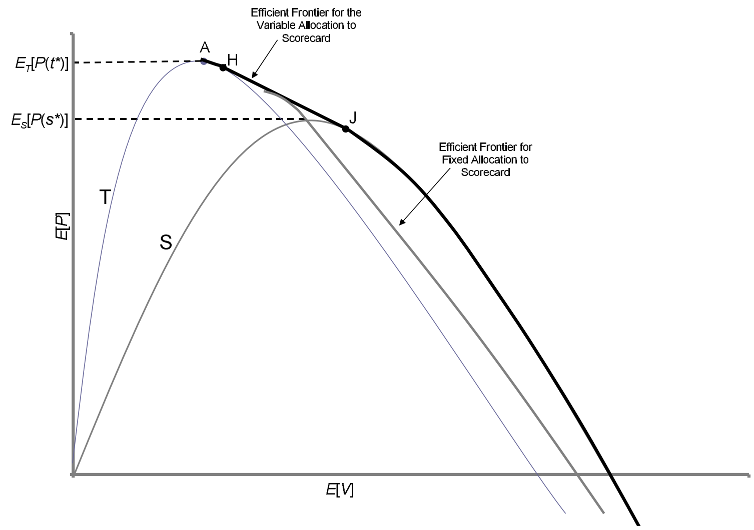

Figure 4.6: Dominance of variable allocation strategy over fixed allocation strategy.

4.3.3 Implementation of Randomization Strategies

Here we consider two methods of implementing the variable allocation strategy. In the first method, the decision maker chooses the scorecard to apply to each applicant randomly with a pre-determined probability q based on the desired operating point. One might imagine the decision maker tossing a biased coin to determine which scorecard to use on each client, and hence we call this implementation method the *multiple-toss strategy*. It is possible under the multiple toss strategy that two customers with identical credit scores receive differing decision outcomes, one customer being accepted into the portfolio and the other being rejected. Such disparity in acquisition decisions may not be palatable from a business perspective, and so we are motivated to consider a second method that has the property that applicants with identical scores are treated identically.

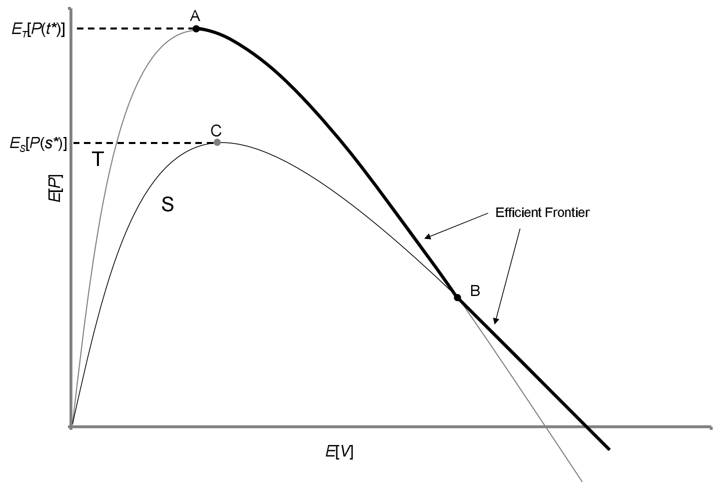

Figure 4.7: $E[P]$-$E[V]$ Multiple scorecards with no dominant scorecards with continuous efficient frontier.

This second method, which we call the *one-toss strategy*, is inherently simpler in implementation. The decision maker randomly chooses a scorecard to apply to the entire applicant pool. This strategy takes its name from the fact that it can be implemented with a single toss of a biased coin. Consider again Figure 4.5. The decision maker may elect to choose q and appropriate score cutoffs so as to achieve the point I in expected profit and expected volume, but once the coin has been tossed it will be either point H or point J that is actually expected. The expected profit for the one-toss strategy is,

$$E[P] \quad = \quad qE[P|T] + (1-q)E[P|S].$$

In the multiple-toss strategy, by contrast, any of the continuum of operating points between H and J may be expected after the conclusion of the coin tossing. We proceed to determine the expected profit for the multiple-toss strategy. Let M be the random variable that describes the number of individuals that are scored on scorecard T, where $M \in (0, n)$. Suppose m individuals are scored on scorecard T, then

$$E[P|M = m] = \frac{m}{n}E[P|T] + \frac{n-m}{n}E[P|S].$$

The probability of the event that m individuals are scored on scorecard T is

$$p(M = m) = \left(\begin{array}{c} n \\ m \end{array} \right) q^m(1-q)^{(n-m)}.$$

It follows that

$$\begin{aligned}
E[P] &= \sum_{m=0}^{n} E[P|M = m]p(M = m) \\
&= \sum_{m=0}^{n} [(\frac{m}{n}E[P|T] + \frac{n-m}{n}E[P|S])(\left(\begin{array}{c} n \\ m \end{array} \right) q^m(1-q)^{(n-m)})] \\
&= \frac{1}{n}E[P|T]\sum_{m=0}^{n}(m)(\left(\begin{array}{c} n \\ m \end{array} \right) q^m(1-q)^{(n-m)}) \\
&\quad +E[P|S]\sum_{m=0}^{n}(\left(\begin{array}{c} n \\ m \end{array} \right) q^m(1-q)^{(n-m)}) \\
&\quad -\frac{1}{n}E[P|S]\sum_{m=0}^{n}(m)(\left(\begin{array}{c} n \\ m \end{array} \right) q^m(1-q)^{(n-m)}).
\end{aligned}$$

From the binomial distribution we know that

$$\sum_{m=0}^{n} \left(\begin{array}{c} n \\ m \end{array} \right) q^m(1-q)^{(n-m)} = 1,$$

which is the sum of all probabilities in the binomial distribution and

$$\sum_{m=0}^{n} (m) \left(\begin{array}{c} n \\ m \end{array} \right) q^m(1-q)^{(n-m)} = nq,$$

which is the expected value in a binomial distribution. Therefore, we have

$$
\begin{aligned}
E[P] &= \frac{1}{n}E[P|T]nq + E[P|S] - \frac{1}{n}E[P|S]nq && (4.7)\\
&= qE[P|T] + (1-q)E[P|S]. && (4.8)
\end{aligned}
$$

We note that the expressions $E[P]$ are equivalent for both strategies.

4.4 Summary

This chapter extends the line of research in the creation of efficient consumer credit portfolios. Clearly, consumer credit portfolio managers consider future economic conditions within this context. Here, we specify the case of explicit probabilistic economic scenarios and the related performance of scorecards. In case of a single scorecard, we reach the rather intuitive conclusion that all efficient portfolio acquisition decisions consist of cutoff policies, though a score transformation may be involved. In the case of multiple scorecards, we show that randomized strategies yield efficient frontiers that dominate deterministic strategies. We further show that randomization can be implemented in a fashion that causes all applicants with same score to receive the same accept or reject decision.

Chapter 5

Estimation Error in Regulatory Capital Requirements

5.1 Introduction

In this chapter, we stylistically model the impact of capital regulatory misestimation on a consumer banks economic profit assuming that consumer banks risk management abilities are significantly better than their regulatory counterparts. We refer to our banks ability to determine economic capital requirements more accurately than its regulatory counterpart as *omniscience*. This is not meant to suggest visionary powers but simply the ability of our consumer banks risk managers to assess and process more complete and accurate information into economic capital with greater accuracy than the regulatory community. Clearly, this is within the spirit of the Second Basel Accord and its evolution. Let us emphasize that our intent here is not to provide yet another simulated test of Basel II but to formally set forth the theory underlying the value impact to banks of regulatory misestimation of capital requirements. Convenient to this limited theoretical objective, historic minimal regulatory capital requirements include a rather broad set of changes from one size fits all in Basel I to the individualized risk management bank inputs in advanced versions of Basel II. Within the stylized context of our models, we examine the obvious and historical likely case of regulatory capital overestimation. We then turn to its historically less likely counterpart, the case of underestimation.

In Section 5.2, we describe our stylized bank's behavior and controls available to the bank. In Section 5.3, we model the impact of misestimating capital requirements on consumer bank profitability. Here, we create two stylized cases of overestimating and underestimating the regulatory capital requirements under an assumption of constant cost of capital. We then extend this analysis to cover the variable cost of equity case. Following this, in Section 5.4, we illustrate the model with a numerical example of a credit card portfolio. In the last section, we draw conclusions and provide suggestions for further research.

5.2 Bank Behavior Model and Controls

In Chapter 1, we proposed a simplified system of models to capture the interrelationships between a consumer bank's business decisions with respect to consumer credit portfolios and its capital requirements (see Figure 1.1). In Chapter 3, we introduced a single cutoff score policy, which is an input to determine the economic profitability of a consumer loan portfolio. Economic profitability is, in turn, influenced by the risk model that determines the equity capital requirement and also influences the acquisition model. The bank behavior model is a set of rules that governs the behavior of our bank. We assume the bank operates under the single objective of maximizing expected economic profit. In addition, the bank always follows regulatory requirements, and so maintains a capital reserve that is no less than the regulatory capital requirement. The equity capital raised will then affect the acquisition decision as illustrated in Figure 1.1. Given an applicant population, the acquisition model produces a cutoff score that determines which of the applicants are granted credit. The cutoff score is determined to maximize expected economic profit; that is, the cutoff solves the problem $\max_s E_S[P(s, Q_R(s))]$, where

$$E_S[P(s, Q_R(s))] = (r_L - r_B)p_G(1 - F(S|G)) - (f_D + r_B)p_B(1 - F(S|B)) - r_Q Q_R(s) \quad (5.1)$$

Later we consider situations in which the bank is not able raise the equity capital required to operate at maximum expected economic profit. In such cases, the cutoff is determined as the solution of a constrained optimization problem.

We assume that the bank follows a strict risk management policy. In particular, the bank maintains equity sufficient to cover unexpected losses at a fixed or predetermined level (such the 99.9% coverage that is the target of Basel II). Our fundamental premise is that the regulatory requirements may be misestimated, however, with the level of error expressed relative to a notion of the *true capital requirement*. We define the true capital requirement to be the level of equity capital that covers the unexpected loss at the fixed confidence level adopted by the bank (e.g., 99.9%). Let $k_T(s)$ denote the true capital requirement per unit loan as a function of score and $Q_T(\bar{s})$ denote the total portfolio capital requirement given a score cutoff $\bar{s}$. Then $Q_T(\bar{s}) = \int_{\bar{s}}^{\infty} k_T(s)f(s)ds$. We make the assumption that the bank is omniscient in the sense that it learns the true capital requirement a short while after the start of the loan period. For example, such an assumption is justified in mortgage portfolios where early delinquencies and early Loss Given Defaults will provide for an improved forecasts of future defaults and the loss time series for the portfolio respectively. We have chosen to model the impact of misestimation in this method rather than compare the impact of misestimation on two banks, one with true capital regulatory formula and the other with the misestimated formula. In a two bank model, the impact of underestimation is not immediate and may result in the bank with the underestimated formula having higher profits due to lower total cost of economic capital. This bank with the underestimated formula will operate under higher risk of bankruptcy. We have instead chosen to model the impact of misestimation on profits under similar capitalization. Below we discuss the limited control options available to the bank in pursuing its risk management and economic profit maximizing strategy in the event that it discovers that the true capital level is different from the regulatory capital level.

5.2.1 Controls available to the bank

We envision that after setting a cutoff and thereby determining the ultimate composition of the portfolio of loans, the bank engages in the process of of raising the corresponding equity. Our assumption is that the bank bases this raise on the regulatory capital requirement, which represents the best estimate of the true capital requirement prior to the start of the loan period. After a short period of operation and observation, the bank determines the true capital requirement, which could be higher or lower than the regulatory requirement. We assume that the bank does not have the option to engage in a second equity raise or acquire new accounts at this point. In contrast, the bank does have the option to decrease its loan portfolio (and thereby reduce risk) by selling off accounts. We assume this is a zero profit transaction.

5.3 Implications of Misestimation

In this section we show that misestimation in either direction reduces the ability of the bank to maximize profits. Much of the interest in this theoretical exercise involves the bank's use of credit scoring technology and optimal economic profit levels with that input. For simplicity, we first assume a constant opportunity cost for equity in proving our proposition that misestimation results in economic profit and, hence, relative value declines for shareholders. We then extend these results by considering a cost structure for equity that is based on Modigliani and Miller's classical theoretical proof that capital structure or the mix of debt and equity has no impact on firm value [11, 26, 27, 25].

5.3.1 Impact of overestimation

When acquiring the loan portfolio, the portfolio manager will initially maximize the expected economic profit based on the regulatory capital requirement function $k_R(s)$. The bank also raises equity capital commensurate with the profit maximizing cutoff score. After a brief period upon acquiring the portfolio, the portfolio manager realizes the true capital requirement function $k_T(s)$, where $k_T(s) < k_R(s)$, $\forall s$. The higher regulatory requirement results in higher equity capital, i.e. $k_T(s) < k_R(s)$, $\forall s$ results in $Q_T(\bar{s}) < Q_R(\bar{s})$, $\forall \bar{s}$.

In order to determine the impact of overestimation of regulatory capital requirement, we need to establish the relationship between capital requirements, profit maximizing cutoff scores and maximum profit. Suppose $k_1(s)$ and $k_2(s)$ are two capital requirement functions and s_1^* and s_2^* be the cutoff scores that maximize expected profit when using $k_1(s)$ and $k_2(s)$, respectively.

Proposition 5.1. *If* $k_1(s) < k_2(s) \forall s$*, then* $s_1^* \le s_2^*$ *and* $E_S[P({s^*}_2, Q_2(s_2^*))] < E_S[P({s^*}_1, Q_1(s_1^*))]$.

Proof. Since s_1^* is the optimal cutoff score when $k_1(s)$ is the capital requirement function,

$$E_S[P(s_2^*, Q_1(s_2^*))] \le E_S[P(s_1^*, Q_1(s_1^*))]. \tag{5.2}$$

Since $k_1(s) < k_2(s)$, it follows that $E_S[P(s_2^*, Q_2(s_2^*))] < E_S[P(s_2^*, Q_1(s_2^*))]$. It follows that $E_S[P(s_2^*, Q_2(s_2^*))] < E_S[P(s_1^*, Q_1(s_1^*))]$. Since s_2^* is the optimal cutoff score when $k_2(s)$ is the capital requirement function, $E_S[P(s_1^*, Q_2(s_1^*))] \le E_S[P(s_2^*, Q_2(s_2^*))]$. It follows that $E_S[I_N(s_1^*)] - r_Q Q_2(s_1^*) \le E_S[I_N(s_2^*)] - r_Q Q_2(s_2^*)$. We can rewrite this inequality as

$$E_S[I_N(s_1^*)] - r_Q Q_2(s_1^*) + r_Q Q_1(s_1^*) - r_Q Q_1(s_1^*) \le E_S[I_N(s_2^*)] - r_Q Q_2(s_2^*) + r_Q Q_1(s_2^*) - r_Q Q_1(s_2^*),$$

which is equivalent to

$$E_S[P(s_1^*, Q_1(s_1^*))] - r_Q Q_3(s_1^*) \le E_S[P(s_2^*, Q_1(s_2^*))] - r_Q Q_3(s_2^*),$$

where $Q_3(s) = Q_2(s) - Q_1(s) = \int_s^\infty [k_2(u) - k_1(u)]du$. Using expression (5.2), it follows that $Q_3(s_2^*) \leq Q_3(s_1^*)$. Since, $k_2(s) - k_1(s) > 0$, $\forall s$, it follows thats $s_1^* \leq s_2^*$. $\square$

Proposition 5.1 demonstrates that with higher capital requirements $k_2(s) > k_1(s)$, the profit maximizing cutoff score is higher $s_2^* \geq s_1^*$ and optimal profit is lower $E_S[P(s^*{}_2, Q_2(s_2^*))] < E_S[P(s^*{}_1, Q_1(s_1^*))]$.

Suppose the regulatory formula is $k_R(s)$ and the true capital requirement formula is $k_T(s)$ with $k_T(s) < k_R(s)$, $\forall s$. Hence the capital requirement is overestimated by the regulation. When using regulatory capital requirement, the optimal cutoff score is $s_R^* = \arg\max_s E_S[P(s, Q_R(s))]$. When using the true capital requirement function, the true optimal cutoff score is $s_T^* = \arg\max_s E_S[P(s, Q_T(s))]$. Since $k_T(s) < k_R(s)$, we know by Proposition 5.1 that $s_T^* \leq s_R^*$. Note that the profit maximizing cutoff score under the true requirement is lower than the cutoff score under the regulatory requirement and that both the regulatory capital reserve amount and true capital reserve amount are met. Hence, the portfolio manager takes no action. Therefore, the profit drag, $\Delta E_S[P]$, is,

$$\Delta E_S[P] = E_S[P(s_T^*, Q_T(s_T^*))] - E_S[P(s_R^*, Q_R(s_R^*))]. \tag{5.3}$$

Since $k_R(s) > k_T(s)$, it follows from Proposition 5.1 that $E_S[P(s_T^*, Q_T(s_T^*))] > E_S[P(s_R^*, Q_R(s_R^*))]$, and hence $\Delta E_S[P] > 0$. Note that there are two factors underlying the profit drag. The first is the income change due to different cutoff scores, and the second is due to the difference in the economic cost of equity. In some cases, when s_T^* and s_R^* are approximately equal, the difference in the loan portfolios between the two requirements is *de minimis*. In this situation, the impact on net income is much less than that of the economic cost of equity ($\Delta E_S[I_N] << r_Q \Delta Q$), and the drag on the expected profit ($\Delta E_S[P]$) is mainly due to the change in the capital requirement:

$$\Delta E_S[P] \approx r_Q \int_{\bar{s}}^{\infty} \left(k_R(s) - k_T(s)\right) f(s) ds, \tag{5.4}$$

where $\bar{s} = s_T^*$ or $\bar{s} = s_R^*$.

Figure 5.1 shows the relationship between expected profit and cutoff scores. Figure 5.1 also illustrates the relationship between the capital requirement and cutoff score. The capital requirement function is a monotonic decreasing function with respect to cutoff score. When acquiring the portfolio, the portfolio manager will regard the regulatory capital as the true equity capital and will operate on point R with an expected profit of $E_S[P(s_R, Q_R(s_R))]$ and a capital reserve of $Q_R(s_R)$. After a short period, the true requirement will be realised but the portfolio manager will not be able to decrease the cutoff score to take advantage of the true capital requirement as the bank always operates at the regulator's level. When the regulatory capital is overestimated, the expected profit using regulatory capital will be lower than the true level, and the cutoff score will be higher. This results in profit drag.

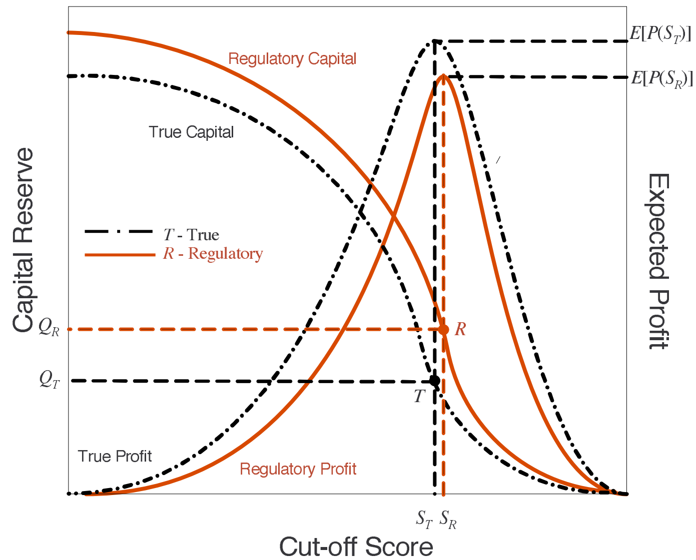

Figure 5.1: Impact on expected economic profit due to overestimation of capital requirement.

Impact of overestimation with limited equity capital

In the above analysis, we assumed there was unlimited equity capital available at the time of decision. In this section, we analyze the special case where there is a constraint on how much the investors are willing to provide. We assume the portfolio manager and the investors know both the true capital requirement as well as the higher regulatory needs. The portfolio manager will like to fulfill both the true capital needs as well as the regulatory needs. However, the investors are only willing to provide the true capital needs of the bank. Hence, the portfolio manager is limited by the capital provided by the investors. At the point of acquisition, the optimization problem is that of determining the cutoff score in order to maximize expected profit using the regulatory capital requirement function,

$$\max \quad E_S[P(\bar{s}, Q_R(\bar{s}))] \tag{5.5}$$

$$\text{s.t.} \quad Q_R(\bar{s}) \leq Q_s \tag{5.6}$$

where Q_s is the maximum equity capital that the shareholders will provide.

If $Q_s \geq Q_R(s_R^*)$, then the portfolio manager will apply the cutoff score s_R^* to the population and keep a capital reserve of $Q_R(s_R^*)$. This would result in a profit drag of $\Delta E_S[P] = E_S[P(s_T^*, Q_T(s_T^*))] - E_S[P(s_R^*, Q_R(s_R^*))]$. Since the regulatory capital requirement is overestimated, we know from Proposition 5.1 that $\Delta E_S[P] > 0$.

If $Q_s < Q_R(s_R^*)$, the portfolio manager will apply a cutoff score, s_A, such that $Q_R(s_A) = Q_s$. This results in a profit drag of $\Delta E_S[P] = E_S[P(s_T^*, Q_T(s_T^*))] - E_S[P(s_A, Q_R(s_A))]$. Since the regulatory capital requirement function $k_R(s)$ is greater than the true requirement function $k_T(s)$, we know from Proposition 5.1 that $E_S[P(s_T^*, Q_T(s_T^*))] > E_S[P(s_R^*, Q_R(s_R^*))]$. Since s_R^* is the profit maximizing cutoff score under regulatory capital requirement, $E_S[P(s_R^*, Q_R(s_R^*))] \geq E_S[P(s_A, Q_R(s_A))]$. Therefore, $E_S[P(s_T^*, Q_T(s_T^*))] > E_S[P(s_A, Q_R(s_A))]$. This implies that $\Delta E_S[P] > 0$.

In Figure 5.2, we illustrate the special case where the shareholders are only willing to provide equity capital commensurate with the profit maximizing cutoff score under the true capital requirement, i.e. $Q_s = Q_T(s_T^*)$. At the time of acquiring the portfolio, the portfolio manager wishes to maximize profit under his best knowledge of capital requirement. At this point, the regulatory capital requirement is considered to be true

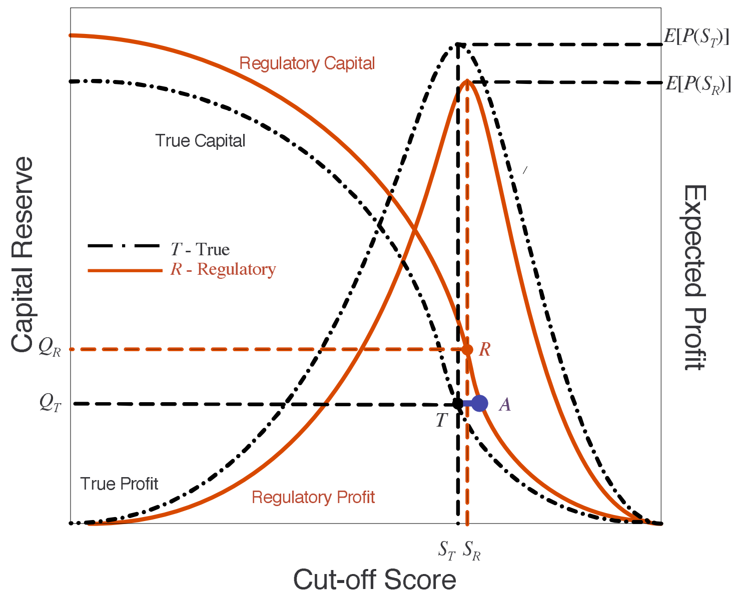

Figure 5.2: Impact on expected economic profit due to overestimation of capital requirement under limited capital.

capital requirement. The portfolio manager will then wish to operate at point R but due to the the equity capital constraint is forced to operate on point A with cutoff score s_A. This results in a profit drag of $\Delta E_S[P] = E_S[P(s_T^*, Q_T(s_T^*))] - E_S[P(s_A, Q_R(s_A))]$.

5.3.2 Impact of underestimation

We now turn to what most bank practitioners might regard as the highly unlikely case of regulatory requirement underestimation, but what we have demonstrated in Figure 1.2 to be a highly relevant case under Basel II.

Suppose that the regulatory function is $k_R(s)$ and the true capital requirement function is $k_T(s)$, with $k_R(s) < k_T(s)$, i.e. the capital requirement is underestimated by the regulation. Initially the portfolio manager will maximize profit using the regulatory capital requirement function. The cutoff score using the regulatory capital requirement is $s_R^* = \arg\max_s E_S[P(s, Q_R(s))]$ and the capital requirement is $Q_R(s_R^*)$. Under the true capital requirement formula, the optimal cutoff score is $s_T^* = \arg\max_s E_S[P(s, Q_T(s))]$. Note that by Proposition 5.1 $s_T^* \geq s_R^*$. The corresponding true capital requirement is $Q_T(s_T^*)$.

After creating the portfolio using the cutoff score of s_R^*, the bank discovers that the true capital requirement function should be $k_T(s)$ with $k_T(s) > k_R(s)$, $\forall s$. Consider the bank's options assuming first that the bank can raise incremental equity capital instantaneously and at a negligible cost. It can then operate at its true capital requirement and maximize profit. This results in a negligible profit drag, i.e. $\Delta E_S[P] \approx 0$. Assuming the costless and readily available incremental equity, regulators could rest easy having underestimated the regulatory capital requirements. This is a trivial and unlikely case. Given that this is not likely, we turn to the more interesting case of the portfolio manager shrinking portfolio to fit the capital reserve to the true capital requirement.

In this case, the bank will shrink its loan portfolio with an objective to maximize the expected profit under the true capital requirement. The equity required is constrained by the equity capital raised earlier under the regulatory capital requirement. The portfolio manager will reduce the portfolio size in a manner such that the economic capital at hand will be sufficient for the smaller portfolio based on the true capital requirement and this is achieved by selling a portion of the portfolio. The accounts sold are chosen randomly from the portfolio. The expected profit for the smaller portfolio is $E_S[P(s_R^*, Q_T(s_R^*))]\,(Q_R(s_R^*)/Q_T(s_R^*))$.

$$E_S[I_N(s_R^*)] - r_Q Q_T(s_R^*) \leq E_S[P(s_T^*, Q_T(s_T^*))], \tag{5.7}$$

since s_T^* is profit maximizing cutoff score under the true capital requirement. It follows that,

$$(E_S[I_N(s_R^*)] - r_Q Q_T(s_R^*))\,[Q_R(s_R^*)/Q_T(s_R^*)] < E_S[P(s_T^*, Q_T(s_T^*))], \tag{5.8}$$

since $Q_R(s_R^*) < Q_T(s_R^*)$. Hence, $\Delta P > 0$.

It should be noted that the results hold true even in the conservative case of the portfolio manager selling off the least profitable accounts. In such a case, the profit drag would be smaller.

As shown in Figure 5.3, the portfolio manager initially operates at point R believing that the regulatory capital requirement is the true capital requirement. After operating for a brief period, the portfolio manager determines that point T with cutoff score s_T^* is the optimal operating point under the true capital requirement. However, under the cutoff score s_R^*, the true capital requirement is $Q_T(S_R^*)$ indicated by point A. So the portfolio manager will reduce the size of the portfolio by $Q_R(S_R^*)/Q_T(S_R^*)$, which results in a profit of $E_S[P(s_R^*, Q_T(s_R^*))]\,(Q_R(s_R^*)/Q_T(s_R^*))$.

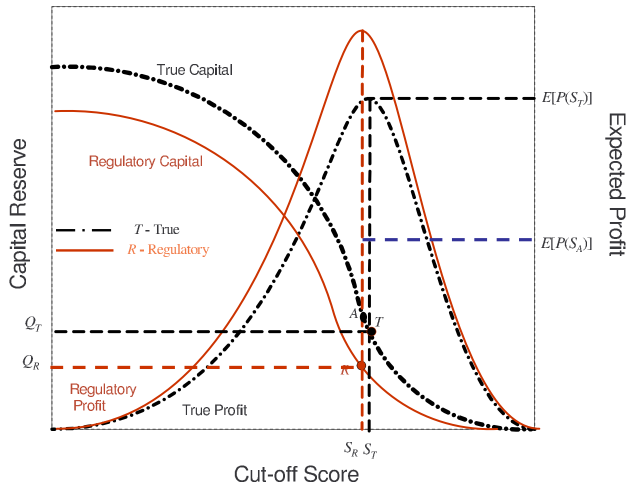

Figure 5.3: Impact on expected economic profit due to underestimation of capital requirement.

5.3.3 Impact of Modigliani-Miller Theorem

Up to this point, we have assumed a fixed cost of equity. We now extend our proof of profit drag to account for Modigliani-Miller theorem, which assumes a more conservative assumption for the opportunity cost of equity. We adapt our work to the neo-classical economic assumptions of Franco Modigliani and Merton Miller's work on capital structure. They stated the following Modigliani and Miller [26]:

1. Proposition I - The market value of a company is not affected by the capital structure of the company.

2. Proposition II - The cost of equity is a linear function of the company's debt to

equity ratio.

This leads to the weighted average cost of capital r_A required to fund both the equity and debt as follows,

$$r_A = \frac{D(s_1)}{Q_i(s_2) + D(s_1)} r_B + \frac{Q_i(s_2)}{Q_i(s_2) + D(s_1)} r_Q(D(s_1), Q_i(s_2)), \tag{5.9}$$

where $D(s_1) = 1 - F(s_1)$ is the amount loaned to the customers which is equal to the debt of our stylized bank and $Q_i(s_2)$ is the equity requirement using the formula $k_i(s)$ and score s_2. Note, we purposefully denote the score that determines the equity capital differently than the score that determine the debt. This implies that the if $s_1 \neq s_2$, the equity held by the bank is not equal to the equity required for the debt level under the capital requirement function $k_i(s)$. By Proposition 5.1, r_A is not affected by any change in the capital structure (i.e. $\frac{D(s_1)}{Q_i(s_2)}$). The cost of equity, $r_Q(D(s_1), Q_i(s_2))$, is determined by both the debt and the equity of the bank. Hence, the cost of equity is

$$r_Q(D(s_1), Q(s_2)) \quad = \quad r_A + \frac{D(s_1)}{Q_i(s_2)}(r_A - r_B) \tag{5.10}$$

It should be noted that the Modigliani and Miller framework is based on rigid assumptions that capital markets are perfect and incomplete. In addition, a critical assumption is that there is no risk of default and the default costs are zero.

Overestimation case

Proposition 5.1 holds true under Modigliani-Miller theorem, i.e. $k_1(s) < k_2(s) \forall s$, then $s_1^* \leq s_2^*$ and $E_S[P(s^*{}_2, Q_2(s_2^*))] < E_S[P(s^*{}_1, Q_1(s_1^*))]$, where $k_i(s)$ denotes a capital requirement function, s_i^* denotes the profit maximizing cutoff score using capital requirement function $k_i(s)$ and $E_S[P({s^*}_i, Q_i(s_i^*))]$ denotes the expected economic profit with cutoff score s_i^* and capital equity of $Q_i(s_i^*)$ which was determined using the capital requirement function $k_i(s)$.

Suppose s_T^* and s_R^* are the profit maximizing cutoff score under true and regulatory requirement functions. Since, the regulatory requirement is overestimated, by Proposition 5.1 $s_T^* \leq s_R^*$. The portfolio manager will initially apply cutoff score s_R^* believing the regulatory capital requirement is at the true level. However, the portfolio manager realizes the true requirement function a short period after acquiring the portfolio. Since $s_T^* \leq s_R^*$ and equity at hand covers the regulatory required equity, the portfolio manager takes no action. This results in a profit drag of

$$\Delta E_S[P] = E_S[P(s_T^*, Q_T(s_T^*))] - E_S[P(s_R^*, Q_R(s_R^*))]. \tag{5.11}$$

Hence by Proposition 5.1 $\Delta E_S[P] > 0$.

Underestimation case

As in the constant cost of equity scenario, upon realising the true requirement level the portfolio manager may adjust the portfolio volume with the constraint that the capital reserve required for the adjusted portfolio is equal to the capital reserve raised earlier under the regulatory capital requirement. The portfolio size is reduced by selling off accounts chosen randomly. The expected profit is then,

$$(E_S[I_N(s_R^*)] - r_Q(D(s), Q_T(s_R^*))Q_T(S_R^*))\,(Q_R(S_R^*)/Q_T(S_R^*))$$

. This results in a profit drag of

$$\Delta E_S[P] = E_S[P(s_T^*, Q_T(s_T^*))] - E_S[P(s_R^*, Q_T(s_R^*))]\,(Q_R(S_R^*)/Q_T(S_R^*))\,. \tag{5.12}$$

$E_S[P(s_R^*, Q_T(s_R^*))] \leq E_S[P(s_T^*, Q_T(s_T^*))]$ since s_T^* is the profit maximizing cutoff score under the true capital requirement. Since $Q_R(S_R^*) < Q_T(S_R^*)$, $\Delta E_S[P] > 0$. Note, the debt to equity ratio under the true capital requirement and cutoff score s_R^* equals the debt to equity ratio for the portfolio after the volume adjustment.

5.4 Numerical Example

Suppose the regulatory capital requirement function is $k_R(s)$ and the true capital requirement function is represented as $k_T(s)$ such that $k_T(s) = \alpha k_R(s)$, where $\alpha \in (0, \infty)$. If $\alpha \in (0, 1)$, then $k_T(s) = \alpha k_R(s) < k_R(s)$ and the regulatory requirement is overestimated. Whereas, if $\alpha \in (1, \infty)$, then $k_T(s) = \alpha k_R(s) > k_R(s)$ and the regulatory requirement is underestimated. Though simple and analytically convenient, this multiplier model can be used to characterise many realistic misestimation scenarios. Assume, for example, the regulatory requirement follows the Basel II IRB approach. This approach to estimating the capital reserve required for the unexpected loss uses a parameter value ρ that is a correlation coefficient. Using Brazilian consumer credit data, De Andrade and

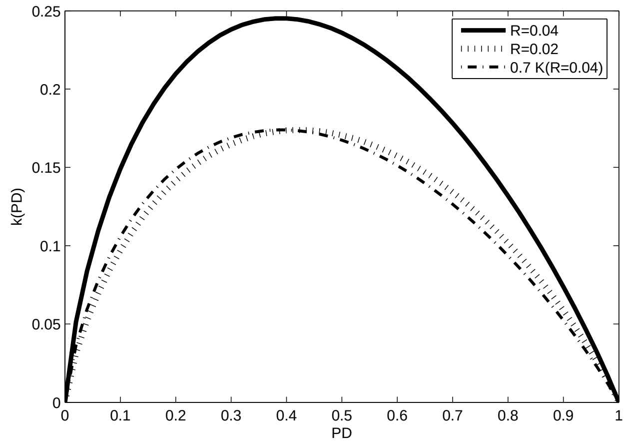

Figure 5.4: Basel II Accord capital requirement misestimation.

Thomas [14] determine the correlation coefficient for the Basel category of "other retail exposures" to be 2.28%, whereas Basel II stipulates it to be 3%. For the Basel category of "qualifying revolving credit exposures", the correlation is set to be 4%. However, some industry experts believe that the best empirical estimate should be 2%. Figure 5.4 shows the Basel II stipulated capital requirement at various levels of probability of default for correlation coefficient (ρ) values of 2% and 4%. The figure illustrates that with $\rho = 2\%$, the equity capital requirement is approximately 70% of the equity capital requirement with $\rho = 4\%$. This misestimation example represents an overestimation case with $\alpha = 0.70$.

While ρ is one reason why the regulatory formula could be misestimated, there are other reasons for misestimation of regulatory capital, such as misestimation of loss given default and hence, the motivation to keep misestimation general. Another example for the occurrence of misestimation of capital requirements is due monotonic population drift. Monotonic population drift will result in misestimation of probability of default. However, misestimation of probability of default may result in one subpopulation of account in the portfolio having underestimated its true capital requirements while another having overestimated its true capital requirement. We restrict ourselves to the case where the whole portfolio is misestimated on one direction.

We now numerically illustrate the impact of misestimation using the following example for a qualifying revolving retail credit (i.e. credit card) portfolio considered in Oliver and Wells [30], Oliver and Thomas [29]. Using a constant opportunity cost of equity and the other parameters in Table 5.1, we first consider the profit drag that would arise if Basel II's standard 6% minimum regulatory capital were misestimated as Basel I's 8% and its related U.S. level of 10%. Figure 5.5 illustrates the historic misestimation assuming the Basel II standard approach is true.

Table 5.1: Parameter values for numerical experiments.

Parameter	p_B	p_G	ρ	$f(s\|G)$	$f(s\|B)$	f_D	C	r_L	r_B	r_Q
Value	0.088	0.912	0.04	$N(\frac{s-3.985}{1.815}, 1)$	$N(\frac{s-0.691}{1.815}, 1)$	0.5	1	0.1	0.05	0.20

For these historic regulatory benchmarks the alpha factors are 0.75 and 0.60, respectively. Assuming a true equity level of 6%, the profit drag as a percentage of maximum

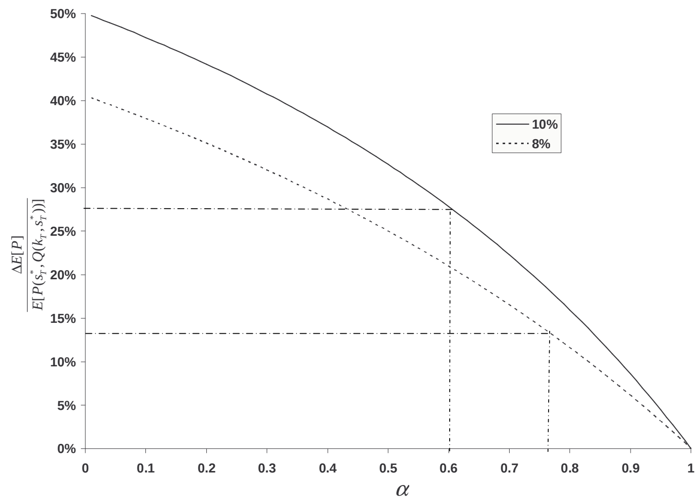

Figure 5.5: Profit Drag due to overestimation of historical regulatory requirements.

economic profit is 14.1% for $\alpha = 0.75$ and 27.8% for $\alpha = 0.60$. Figure 5.5 shows profit drag as a percentage of maximum economic profit as a function of α. For our data set, then, Basel II's standard capital requirements appear to represent a significant reduction in potential economic profit drag on consumer banks assuming its lower standard capital requirement is closer to true economic capital than its predecessors.

Now let us consider a numerical illustration of misestimation potential imbedded in the Basel II IRB approach. Most observers would agree that this is the most sophisticated regulatory approach to capital requirements to date. Again, we use the parameters in Table 5.1, a constant opportunity cost of equity, and illustrate a range of misestimation levels as in Figure 5.6 below. In Figure 5.6, we illustrate the impact of profit drag due to misestimation of the regulatory formula as well as the misestimation of the correlation coefficient ρ, where ρ_T and ρ_R are the true and regulatory specified correlation

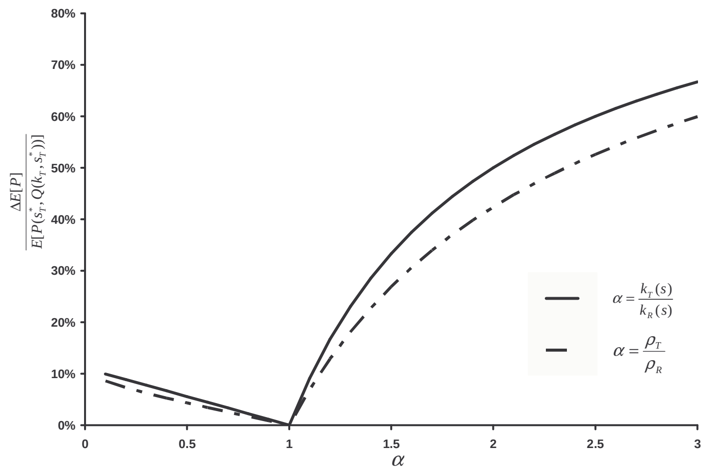

Figure 5.6: Profit drag on expected profit assuming Basel II IRB as regulatory requirement with constant cost of equity.

coefficient respectively. For the case of $\alpha = 0.70$, there is a profit drag of 3.4% of maximum expected profit realizable under the true capital requirement function. It should be noted that 3.4% represents a lower bound in the sense that including operating costs for our stylized bank would only raise the percentage.

To complete our comparison for the case, we present in Figure 5.7 the profit drag as a fraction of the capital reserve determined at the maximum profit under the true capital requirement. At $\alpha = 0.70$, the profit drag as a percent of the capital reserve is 8.4% under the case where the regulatory function is misestimated.

The numerical results presented thus far are based on a constant opportunity cost of equity of 20%. As considered earlier, a variable of cost of equity model can be obtained within the spirit of Modigliani and Miller's seminal work [11, 26, 27, 25]. Under variable

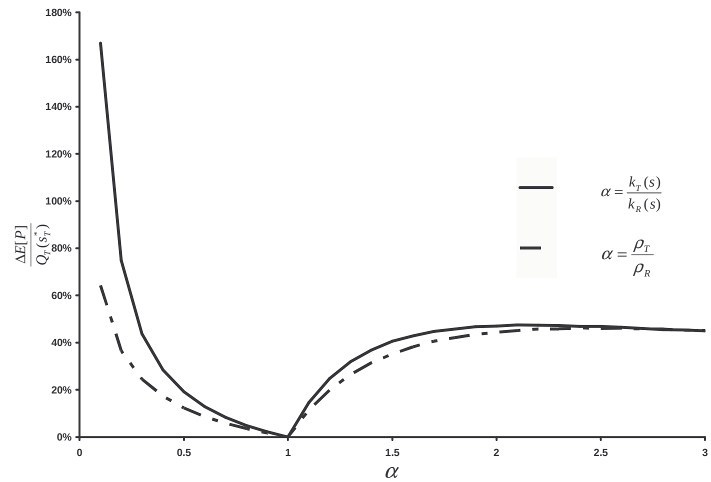

Figure 5.7: The drag on expected profit as a fraction of capital reserve with constant cost of equity.

cost of equity, the profit drag as a percent of maximum profits and as a percent of capital reserve is 2.5 % and 3.4%, respectively, for $\alpha = 0.70$. Thus even under the most restricted assumptions – a more enlightened regulatory regime and an opportunity cost of equity linearly rising with the debt to equity ratio – we find positive profit drag associated with regulatory misestimation. This is in accord with our theoretical results.

5.5 Summary

The bank we study maintains two invariants. First, it always follows regulatory requirements by maintaining at least as much equity as prescribed by the regulations. Second, irrespective of regulation it maintains a constant risk of portfolio default; that is, the bank constrains operations relative to available equity so as to bound the probability that unexpected losses exceed equity. When regulation differs from true capital requirements, our bank may adjust the size or volume of its portfolio to maintain both invariants. Our results show that, regardless of direction, misestimation of capital requirements creates a drag on economic profit or, equivalently, a dead-weight loss of shareholder value.

A portfolio manager is required to set aside unencumbered capital for regulatory purposes at acquisition stage and hence, before the economic condition during account performance is revealed. Given there is a cost to mistestimating regulatory capital, a portfolio manager has opportunity to improve decision making by incorporating forecasts of future economic conditions at acquisition stage. In the next chapter, we incorporate forecasts of future economic conditions into the portfolio creation decision.

Chapter 6

Scoring Decisions with Regulatory Constraints

6.1 Introduction

In Chapter 4, we considered the case of a portfolio who is required to make accept/reject decision given forecasts and scorecard performance for multiple future economic scenarios. In Chapter 5, we showed there is a negative profit impact on a portfolio manager misestimating the regulatory capital requirement. In this chapter, we incorporate cost of regulatory capital into a portfolio manager's decision. We consider the case of a risk-neutral decision maker whose primary decision is the accept/reject decision for each loan application. In addition, the portfolio manager retains capital in order to be compliant with regulatory capital requirements, and is constrained by the amount of unencumbered capital that may be used for regulatory purposes. The portfolio manager has access to a single scorecard to forecast credit risk of borrowers. In the first instance, we assume credit risk is independent of the prevailing economic condition during account performance and show methods to construct the EPV efficient frontier. This is followed by the case of a portfolio manager with access to scorecard performance data conditional on prevailing economic conditions and forecasts of each possible economic condition during account performance period. We show methods to construct the EPV efficient frontier for this multiple economic scenario case.

The most relevant work to this chapter are the following: Oliver and Wells (2001); Beling, Covaliu and Oliver (2005) as well as Chapters 4 and 5 from this dissertation. Efficient frontier and profit maximization calculation as illustrated by Oliver and Wells (2001); Beling, Covaliu and Oliver (2005); and the decision model in Chapter 4 do not consider economic or regulatory capital in the decision making process. In Chapter 5, we included cost of regulatory capital in the decision making process, where the impact of misestimation of the regulatory requirement amounts is analyzed. However in Chapter 5, we assumed the portfolio manager's objective is to maximize profit. In contrast, this chapter deals with the trade-offs between multiple objectives faced by the portfolio manager, including the case of a manager operating under capital constraints. This chapter provides the theoretical framework for constructing the efficient frontier when capital costs and constraints are considered in the decision making process. This chapter's main contributions are the following: (1) this work includes the cost of regulatory capital in the profitability model and incorporates capital constraints in the decision making process; (2) this work establishes a disjoint accept population is possible when regulatory capital is considered in the decision making process; and (3) this work establishes the unexpected result of creating different heterogeneous portfolios with different portfolio risk profile that have exactly the same portfolio regulatory requirement.

We organize the chapter in the following manner; In Section 6.2, we introduce the set of rules that govern the bank's behavior. In Section 6.3, we consider the decision faced by a portfolio manager who must make both a capital and accept/reject decision in creating the consumer loan portfolio. This section is restricted to a single economic scenario. We study the cases of a manager with or without capital constraints, and construct the efficient frontier for both cases. In Section 6.4, we show through an example the problem of a decision maker operating under capital constraints. Finally, in Section 6.5, we summarize and discuss our findings. The proofs for the propositions developed in this chapter are found in the chapter appendix.

6.2 Bank behavior policies and controls

The bank behavior policies is the set of rules that govern the bank's behavior. The bank is required to fund the consumer loans. These are funded through debt such as from commercial banks. In addition, the bank raises equity capital to act as reserves for regulatory purposes. We assume the bank always complies with the regulatory requirement, and hence maintains capital reserve, $k(s)$ no less than the amount stipulated by regulation for each account, i.e, $k(s) \geq k_R(s)\ \forall s$. The bank has certain controls available to it. The portfolio manager decides the set of scores to accept and the complementary set of scores to reject. In addition, the portfolio manager sets the capitalization level, $k(s)$, under the constraint $k(s) \geq k_R(s)$ for all accepted score s.

We assume the expected loss is priced for in the expected revenue, i.e., $f_D p(B|s) \leq$

$r_L(s)p(G|s)$. It follows that $r_L(s) \geq f_D p(B|s)/p(G|s)$. For model tractability, we assume the loan rate is set such that the expected operating income, $E_Z[I(s)]$ is monotonically decreasing and linear with respect to $p(B|s)$ with $\delta E_Z[I(s)]/\delta p(B|s) \leq -r_Q f_D$. For the case of constant loan rate, i.e., $r_L \equiv r_L(s)$ and no cost of capital included in the profit function, e.g., $r_Q = 0$, Equation 3.1 reduces to the profit function defined by Oliver and Wells [30].

The capital requirement formulae is derived for a portfolio consisting of accounts with the same probability of default (see Perli and Nayda [32]). Due to the law of large numbers, as the portfolio size tends to infinity, the average portfolio default rate will equal to the account probability of default [32]. For a portfolio consisting of multiple risk segments, we follow Botha and van Vuuren [7] when determining the regulatory capital requirement. The capital requirement for each constituent loan of a portfolio is calculated and then aggregated to determine the capital requirements for the total portfolio [7]. It follows the portfolio regulatory capital requirement is $Q_R^s(\omega) = \int_{s \in \omega} k_R(s) f(s) ds$, where applicants with scores in the set ω are granted credit.

6.3 Single Economic Scenario

In this section, we consider the case of a portfolio manager who is faced with the task of creating a portfolio under the assumption account performance is independent of the prevailing economic conditions, i.e., single economic scenario. The portfolio manager has access to a scorecard in order to forecast the default risk of applicants, an opportunity to raise capital, and models to evaluate business metrics associated with the bank's objectives. The portfolio manager sets both the accept/reject score decision and the capital amount when determining the operating point on the EPV space. We construct the efficient frontier in the expected profit-expect volume (EPV) space.

The portfolio manager sets capitalization function, $k(s)$ with the constraint $k(s) \geq k_R(s)\ \forall s$. Since $r_Q \geq 0$ and $k(s) \geq k_R(s)$, it follows from Equation (3.1) that $E_Z[P(s, k(s))] \leq E_Z[P(s, k_R(s))]$ for any function $k(s)$ used to calculate capital requirements. Therefore, the portfolio profit $E_S[P^s(\omega, Q^s(\omega))] \leq E_S[P^s(\omega, Q^s_R(\omega))]$. Since $E_S[V^s(\omega)]$ is independent of the capitalization function $k(s)$, it follows that the portfolio manager always applies the regulatory formula $k_R(s)$. Hence, the problem simplifies to determining the score accept set ω.

An operating point may be constructed by multiple accept sets, i.e., different accept sets that result in the same expected portfolio profit and expected portfolio volume. However, we are only interested in finding at least one accept set for each feasible operating point on the efficient frontier. For ease of analysis, we define ω as a union of separated score intervals on the score domain $s \in (-\infty, \infty)$. Two sets are separated sets if each is disjoint from the other's closure. Proposition 6.1 provides the basis for a simplifying view of accept sets.

Proposition 6.1. *For each feasible operating point, there exists an accept set such that each of the separated subsets are of non-zero measure.*

On the basis of Proposition 6.1, we assume all accept sets are constructed as a union of separated score intervals each with non-zero measure. Similarly, all non-accept sets are a union of separated score intervals each with non-zero measure.

In this section, we construct the efficient frontier for two cases: the case where there is no capital constraint, and the case of capital constraint.

6.3.1 No capital constraint

In order to construct the efficient frontier, we show the expected profit of an account for a customer is monotonically increasing in s. We use this monotonic property to construct the efficient frontier.

From Equation (3.6), let the total loss (both expected and unexpected loss),

$$k_U(s) = \Phi\left(\sqrt{\frac{1}{1-\rho}}\Phi^{-1}(p(B|s)) + \sqrt{\frac{\rho}{1-\rho}}\Phi^{-1}(0.999)\right).$$

Therefore, $k_R(s) = f_D\left[k_U(s) - p(B|s)\right]$. Rearranging the terms in the expected profit Equation (3.1),

$$E_Z[P(s, k_R(s))] = E_Z[I(s)] + f_D r_Q p(B|s) - r_Q f_D k_U(s). \tag{6.1}$$

The left terms on the right hand side, $E_Z[I(s)] + f_D r_Q p(B|s)$ increases with respect to s. Since default risk, $p(B|s)$, decreases with respect to score, it follows that $k_U(s)$, and hence the third term in Equation (6.1) is strictly decreasing in s. Therefore $E_Z[P(s, k_R(s))]$ is monotonically increasing with respect to s.

Suppose a portfolio manager wishes to maximise portfolio profit while achieving an expected volume of V_0, i.e., determine the accept set ω that solves $\max_{\{\omega|E_S[V^s(\omega)]=V_0\}} E_S[P^s(\omega, Q_R^s(\omega))]$. We show through Proposition 6.2 this may be achieved by applying a single cutoff score s_a, and accepting all scores $s \geq s_a$ while declining scores in the range $s < s_a$. This is called a single-cutoff score strategy.

Proposition 6.2. *Suppose ω_0 solves $max_{\{\omega|E_S[V^s(\omega)]=V_0\}} E_S[P^s(\omega, Q_R^s(\omega))]$. There exists a score s_a such that $E_S[P^s(\omega_0, Q_R^s(\omega))] = E_S[P^s(\{s|s \geq s_a\}, Q_R^s(\omega))]$ and $E_S[V^s(\omega_0)] = E_S[V^s(\{s|s \geq s_a\})]$.*

By Proposition 6.2, a single-cutoff score strategy results in a dominating operating point for a given expected volume. For ease of notation, we refer to the expected portfolio profit with cutoff score s_a and regulatory formula $k_R(s)$ as $E_S[P(s_a, Q_R^s(s_a))]$ where $E_S[P(s_a, Q_R^s(s_a))] = E_S[P^s(\{s|s \geq s_a\}, Q_R^s(\{s|s \geq s_a\}))]$. Note, we drop the subscript s when indicating portfolio metric under a single-cutoff score strategy. Similarly, the portfolio volume $E_S[V(s_a)] = E_S[V^s(\{s|s \geq s_a\})]$ and the portfolio regulatory amount $Q_R(s_a) = Q_R^s(\{s|s \geq s_a\})$.

Since $E_Z[P(s, k_R(s))]$ is monotonically increasing with respect to s, maximum profit is achieved by accepting all scores with non-negative expected account profit, i.e., $E_Z[P(s, k_R(s))] \geq 0$. It follows that the profit maximizing cutoff score s^* is the score that solves $E_Z[P(s^*, k_R(s))] = 0$.

The slope of the EPV curve,

$$\frac{\partial E_S[P(s, Q_R(s))]}{\partial E_S[V(s)]} = -E_Z[P(s, k_R(s))].$$

Since $E_Z[P(s, k_R(s))]$ is monotonically increasing in s, the slope of the EPV curve is strictly concave. Therefore, the efficient frontier consists of all operating points constructed by applying cutoff scores in the set $\{s_a : s_a \leq s^*\}$.

6.3.2 Constrained capital decision

Thus far, we assumed the portfolio manager has the opportunity to raise unlimited capital. Under such a case, we showed an efficient operating point is attained through a single-cutoff score policy. Suppose instead, the portfolio manager is restricted by the amount of capital that may be raised. The portfolio manager must decide both the set of scores to accept and the amount of capital to retain for capitalization purposes under a capital constrained case. We motivate this constrained capital study through the following example. Suppose the portfolio manager's goal is to maximise portfolio volume, regardless of expected profit. In such a case, this volume maximizing point on the efficient frontier is achieved by accepting those applicants with scores requiring the least amount of regulatory capital, i.e., at the extreme ends of the score domain (see figure 3.3). Clearly, this volume maximizing point on the the efficient frontier can not be constructed with a single-cutoff score policy as in Section 6.3.1. In this section, we determine those policies that result in an efficient portfolio.

Suppose $\Omega(K)$ is the set of accept sets that result in operating points on the EPV curve for the constrained problem, where K is the maximum capital that may be raised. We assume the maximum allowable capital amount to be $K < Q_R(-\infty)$ else the problem is that of the unconstrained case (see Section 6.3.1). Suppose s_K is the cutoff score that solves $Q_R(s_K) = K$ and the set $\Omega_0(K) = \{[s_c, \infty)|s_c \geq s_K\}$. Since $\Omega_0(K)$ is the set of accept sets forming the unconstrained EPV curve with $Q_R(s_c) \leq K$, it follows that $\Omega_0(K) \subseteq \Omega(K)$. The problem simplifies to determining the operating points for the set $\Omega_1(K) = \Omega(K) \setminus \Omega_0(K)$. Since $Q_R(s) > Q_R(s_K)$ for all $s < s_K$, it follows that no operating points formed by a single cutoff-score strategy is an element of the set $\Omega_1(K)$.

Suppose the bank follows the simplified approach of Basel II. In such a scenario, the capital requirement is constant 6 percent of risk assets. We may express Equation (3.1) as,

$$\begin{aligned} E_Z[P(s, k_R(s))] &= E_Z[I(s)] - r_Q k_R(s) \\ &= E_Z[I(s)] - r_Q\,[6\%]\,. \end{aligned}$$

Since $E_Z[I(s)]$ is a linear decreasing function of $p(B|s)$, it follows that the expected profit $E_Z[P(s, k_R(s))]$ is a linear decreasing function of $p(B|s)$. The EPV curve may then be constructed as in Section 6.3.1 with $s \in [s_K, \infty)$, where s_K solves $Q_R(s_K) = K$ and $\Omega_1(K) = \emptyset$.

Suppose instead the bank follows the advanced approach of Basel II. In order to construct the EPV curve, we establish the definition of profit-dominance.

Definition. *We say an accept-set ω_1 is a profit-dominated set if there exists a set ω_2 such that $E_S[P^s(\omega_2, Q^s_R(\omega))] > E_S[P^s(\omega_1, Q^s_R(\omega))]$, $E_S[V^s(\omega_2)] = E_S[V^s(\omega_1)]$, and $Q^s_R(\omega_2) \leq Q^s_R(\omega_1)$. We denote the profit dominance of ω_1 by ω_2 as $\omega_2 \overset{\mathcal{P}}{\succ} \omega_1$ or $\omega_1 \overset{\mathcal{P}}{\prec} \omega_2$. In such a case, we also say the operating point formed by ω_1 is a profit-dominated operating point.*

We note that no operating point on the EPV curve is a profit-dominated operating point. The following result relates profit dominance of individual sets to supersets.

Proposition 6.3. *Suppose there exist sets ω_1, ω_2 and ω_3 such that $\omega_1 \cap \omega_2 = \emptyset$, $\omega_1 \cap \omega_3 = \emptyset$, and set $\omega_2 \overset{\mathcal{P}}{\succ} \omega_3$. Then, $\{\omega_1 \cup \omega_2\} \overset{\mathcal{P}}{\succ} \{\omega_1 \cup \omega_3\}$.*

The capital requirement curve $k_B\left(p(B|s)\right)$ is concave with respect to probability of default, $p(B|s)$ for all three consumer asset classes, except for a region of local convexity for "other retail portfolios" [7]. In order to establish the characteristics of profit-dominated operating points, we define it over multiple steps. Firstly, we use the concave property of the capital requirement curve to show that all profit-dominated operating points must be a result of accept sets at the either end of the risk spectrum within the concave region (see Proposition 6.4). This result implies all profit-dominated operating points for mortgages and qualifying revolving portfolios are formed through a single cutoff-score or double cutoff-score strategy. In the region of local convexity for "other retail portfolios", the expected account profit $(E_Z[P(s, k(s))])$ and the regulatory requirement $(k_R(s))$ are monotonically increasing and decreasing with respect to respect to score s respectively. We use these properties in Proposition 6.5 to show profit-dominated operating points for "other retail portfolios" are also formed through a single cutoff-score or double cutoff-score strategy.

Proposition 6.4. *Suppose the capitalization function, $k_B\left(p(B|s)\right)$, is concave with respect to $p(B|s)$ in the score range $\omega = [s_1, s_2]$. Suppose there exists an accept set $\omega_1 \subset \omega$ with $\omega_1 \neq \{[s_1, s_3] \cup [s_4, s_2]\}$ for some $s_1 \leq s_3 < s_4 \leq s_2$. There exists a set $\omega_2 = \{[s_1, s_3] \cup [s_4, s_2]\}$ for some $s_1 \leq s_3 < s_4 \leq s_2$ such that $\omega_2 \succ^{\mathcal{P}} \omega_1$.*

Now, we characterize the accept set in the region of convexity for "other retail portfolios".

Proposition 6.5. *Suppose the capitalization function, $k_B\left(p(B|s)\right)$, is increasing with respect to $p(B|s)$ in the score range $[s_1, s_2]$. Suppose set $\omega_1 \subset [s_1, s_2]$ but $\omega_1 \neq [s_1, s_3]$ for some $s_1 < s_3 < s_2$. There exists a set $\omega_2 = [s_1, s_3]$ for some $s_1 < s_3 < s_2$ such that $\omega_2 \succ^{\mathcal{P}} \omega_1$.*

Let $\Gamma_1 = \{(-\infty, s_i] \cup [s_j, \infty) | s_i < s_j\}$. By Propositions 6.3, 6.4 and 6.5, all elements not in Γ_1 are profit-dominated operating points. Therefore, $\Omega_1(K) \subseteq \Gamma_1$. Suppose score s^* solves $\max_s k_R(s)$. For each score $s_i \in (-\infty, s^*)$, there exists a corresponding score s_i' such that $k_R(s_i) = k_R(s_i')$ with $s_i' \in (s^*, \infty)$. We use this property in Proposition 6.6 further characterizes efficient accept sets.

Proposition 6.6. *Suppose score s^* solves $\max_s k_R(s)$. For each score $s_i \in (-\infty, s^*)$, there exists a corresponding score s_i' such that $k_R(s_i) = k_R(s_i')$ with $s_i' \in (s^*, \infty)$. Any accept set $\omega_1 \subseteq [s_1, s_2]$ with either exclusively $f(s_i) \leq f(s_i')$ $\forall s_i \in [s_1, s_2]$ or exclusively*

$f(s_i) > f(s_i') \; \forall s_i \in [s_1, s_2]$ is a profit dominated set.

By Propositions 6.3 and 6.6, all elements of $\Gamma_1 = \{(-\infty, s_i] \cup [s_j, \infty) | s_i < s_j\}$ with $k_R(s_i) > k_R(s_j)$ are profit-dominated operating points. It follows that $\Omega_1(K) \subseteq \Gamma_2$, where $\Gamma_2 = \{(-\infty, s_i] \cup [s_j, \infty) | s_i < s_j, k_R(s_i) \leq k_R(s_j)\}$. We show through Proposition 6.7 that any element in Γ_2 that is not binding in capital requirement is not an element of $\Omega_1(K)$.

Proposition 6.7. *Suppose $\omega_1 \in \Gamma_2$ with $Q_R^s(\omega_1) < K$, where K is the maximum available unencumbered capital, then $\omega_1 \notin \Omega_1(K)$.*

Let $\Gamma_3 = \{(-\infty, s_i] \cup [s_j, \infty) | s_i < s_j, k_R(s_i) \leq k_R(s_j), Q_R^s((-\infty, s_i] \cup [s_j, \infty)) = K\}$. It follows from Propositions 6.7, $\Omega_1(K) \subseteq \Gamma_3$. We show through Proposition 6.8 that no two operating points in Γ_3 result in the same expected volume. It follows that no elements of Γ_3 profit-dominate another element of Γ_3.

Proposition 6.8. *Suppose $\omega_1, \omega_2 \in \Gamma_3$ then $E_S[V^s(\omega_1)] \neq E_S[V^s(\omega_2)]$.*

It follows from Proposition 6.8, all elements in Γ_3 result in different expected volume. Therefore, no elements of Γ_3 profit-dominate another element of Γ_3. Therefore, $\Gamma_3 \subseteq \Omega_1(K)$. Since $\Omega_1(K) \subseteq \Gamma_3$, it follows that $\Gamma_3 = \Omega_1(K)$. Therefore, the operating points formed by elements of $\Gamma_3 \cup \Omega_0(K)$ is the EPV curve.

Proposition 6.9. *The EPV curve is concave.*

By Proposition 6.9 and assuming $E_S[P(-\infty, Q_R(-\infty))] < 0$, it follows that the operating points formed by the elements of Ω_1 in the EPV curve decreases in expected profit as expected volume increases. Hence, all elements of $\Omega_1(K)$ are part of the efficient frontier.

To summarize, the EPV curve under the capital constraint case is constructed as follows. Let K be the maximum capital that may be raised, where $K \leq Q_R(-\infty)$ and let s_K be the cutoff score that solves $Q_R(s_K) = K$. We construct the set of operating points constructed through a single-cutoff score strategy for all cutoff-scores $s_i \geq s_K$. Then, we extend the EPV curve by successively increasing the score s_i and accepting scores with the least expected profit in a manner that capital constraint is binding. The maximum volume point is the one constructed by the set $\omega = (-\infty, s_j] \cup [s_i, \infty)$, where $Q_R^s(\omega) = K$, $k_R(s_j) = k_R(s_i)$ and $s_j < s_i$. Suppose s^* is the unconstrained profit maximizing score. If $s^* \leq s_K$, the efficient frontier consists of all operating points formed by $\Omega_1(K)$. If $s^* > s_K$, the efficient frontier consists of all operating points formed by $\Omega_1(K) \cup [s_K, s^*]$.

Figure 6.1 illustrates an example of the efficient frontier for the constrained problem.

6.4 Multiple Economic Scenario

In the previous section, we constructed the efficient frontier for a single economic scenario. In this section, we assume account performance is dependent on the prevailing economic condition during account performance. We consider the case of a portfolio

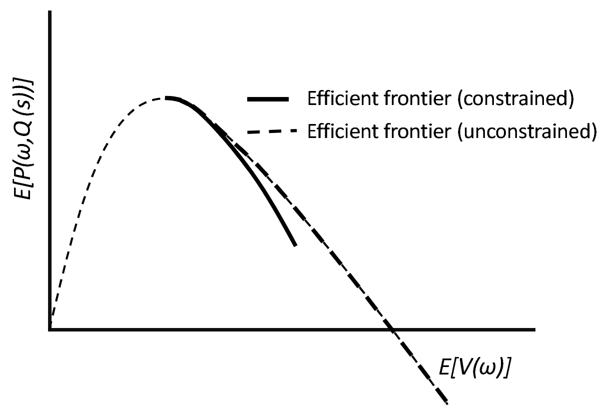

Figure 6.1: The efficient frontier for the constraint problem.

manager whose acquisition and capitalization decision is made with consideration to each of the possible economic condition. The portfolio manager has access to forecasts of probability of realization for each economic condition. The portfolio manager must set the accept/reject policy and the level of capitalization prior to account performance. We assume accept/reject decisions are made such that the portfolio manager is always compliant with regulatory requirements. We restrict our study to the case of two economic scenarios but the methodology may be extended to more scenarios. The goal of this section is to show methods to construct the efficient frontier for a portfolio manager operating under the case of multiple economic scenarios.

To keep this analysis simple, we model this as a process with two time steps. At time $t = 0$, the portfolio manager has access to unencumbered capital K_T. At this time, the portfolio manager also has access to forecasts on the probability of realization for the two economic scenarios and the expected performance of accounts under each scenario. The portfolio manager sets aside capital amount $K \leq K_T$ and invests the rest $K_T - K$ in a one-off investment opportunity with return r_Q. The capital amount K that is set aside is then used as regulatory capital for the loan portfolio. At time $t = 1$, one of two economic scenarios is realized and the portfolio manager sets the accept/reject decision to create a portfolio. At this time, the portfolio manager is cognizant of the prevailing economic scenario and the available unencumbered capital, and the accept/reject decision is set in a manner that results in an efficient portfolio. At time $t = 1$, the capital amount is no longer a decision variable as in Section 6.3.2. However, the regulatory capital requirement for the portfolio is a constraint and may not exceed K, i.e., $Q^s_R(\omega) \leq K$, where ω is the accept set.

Let economic scenario be a random variable U. We denote the conditional probability of default for an account with score s under economic scenario u by $p(B|s, u)$. Similarly, the conditional probability of good is $p(G|s, u)$. It follows that the expected portfolio operating income with accept set ω_u under economic scenario $u \in U$ is

$$\begin{aligned}
E_S[P^s(\omega_u, 0)|U = u] &= \int_{s\in\omega_u} [(1 + r_L(s))p(G|s, u) + (1 - f_D)p(B|s, u) - (1 + r_B)]\, f(s)ds \\
&= \int_{s\in\omega_u} [E_S[P^s(s, 0)|U = u]]\, f(s)ds.
\end{aligned}$$

The accept/reject decision is made such that $Q_u^s(\omega_u) \leq K$, where $Q_u^s(\omega_u)$ is the regulatory capital requirement with accept set ω_u under economic scenario u. Since the decision to retain capital amount K was made at time $t = 0$ and the accept/reject decision is at time $t = 1$, this may result in excess capital, $K - Q_u^s(\omega_u)$. However, the total cost of capital inclusive of the excess amount is $r_Q K$. Therefore, the expected profit of a portfolio with accept set ω_u under economic scenario $u \in U$ is $E_S[P^s(\omega_u, 0)|U = u] - r_Q K$.

The expected volume

$$E_S[V^s(\omega_u)] = \int_{s \in \omega_u} f(s) ds.$$

The EPV curve under economic scenario u with a capital amount, K is constructed as in Section 6.3.2. The portfolio manager may now set the accept/reject decision on the efficient frontier.

Suppose at time $t = 1$, economic scenario $U = 1$ is realized. The portfolio manager creates a portfolio with accept set ω_1, where $Q_1^s(\omega_1) \leq K$. Similarly, had economic scenario $U = 2$ been realized, a portfolio is created with acceptance set ω_2, where $Q_2^s(\omega_2) \leq K$. The unconditional expected profit at time $t = 0$ is determined by three variables, i.e., an accept set for each economic scenario (ω_1 and ω_2) and the retained capital amount (K). For notational convenience, we define $E_U[P^s(\omega_1, \omega_2, K)]$ as the unconditional expected profit at time $t = 0$. Hence,

$$
\begin{aligned}
E_U[P^s(\omega_1, \omega_2, K)] &= q\,(E_S[P^s(\omega_1, 0)|U = 1] - r_Q K) + (1 - q)\,(E_S[P^s(\omega_2, 0)|U = 2] - r_Q K) \\
&= qE_S[P^s(\omega_1, 0)|U = 1] + (1 - q)E_S[P^s(\omega_2, 0)|U = 2] - r_Q K,
\end{aligned}
$$

where q (and $(1-q)$) is the forecasted probability for the realization of economic scenario $U = 1$ (and $U = 2$). Similarly, the unconditional expected volume at time $t = 0$ is,

$$
E_U[V^s(\omega_1, \omega_2)] = qE_S[V^s(\omega_1)] + (1 - q)E_S[V^s(\omega_2)]. \tag{6.2}
$$

Our goal in this section is to determine the maximal set of operating points that are not dominated by other operating points at time $t = 0$ (i.e., on the unconditional EPV space), for a portfolio manager with access to unencumbered capital K_T. The feasible region may be approximated by simulating operating points on the unconditional EPV space. For each operating point on the EPV space, accept sets ω_1 and ω_2 are simulated such that $Q^s_u(\omega_u) \leq K_T \; \forall u \in \{1, 2\}$. The unconditional expected profit is,

$$
E_U[P^s(\omega_1, \omega_2, K)] = qE_S[P^s(\omega_1, 0)|U = 1] + (1 - q)E_S[P^s(\omega_2, 0)|U = 2] - r_Q \max[Q^s_1(\omega_1), Q^s_2(\omega_2)],
$$

where $\max[Q^s_1(\omega_1), Q^s_2(\omega_2)] \leq K_T$ is the retained capital amount at time $t = 0$. The respective unconditional portfolio volume is determined using Equation 6.2.

This is then repeated multiple times to simulate for the feasible region and hence, the efficient frontier may be approximated. We show through Proposition 6.10 below, if at least one of the accept set ω_u is a dominated operating point on the EPV space conditioned on the respective economic scenario, then the resulting operating point at time $t = 0$ will be dominated on the unconditional EPV space, and hence could be eliminated from the simulation approximating the efficient frontier.

Proposition 6.10. *Let $\Gamma_u(K)$ be the set of accept sets resulting in efficient operating points under economic scenario u and available capital K at $t = 1$. Suppose the portfolio manager retains capital amount of K at time $t = 0$. Without loss of generality, suppose $\omega_1 \in \Gamma_1(K)$, and suppose ω_2 is a feasible operating point with $Q_2^s(\omega_2) \leq K$ but $\omega_2 \notin \Gamma_2(K)$, then the resulting operating point with ω_1, ω_2 and K at time $t = 0$ is a dominated operating point in the unconditional EPV space.*

By Proposition 6.10, all efficient operating points on the unconditional EPV space is constructed by efficient accept sets on the conditional EPV space. These efficient accepts sets are determined as in Section 6.3.2. As a result of Proposition 6.10, the efficiency of a simulation to approximate the unconditional efficient frontier is increased.

6.5 Summary

We extend the literature in creating efficient consumer loan portfolios. Oliver and Wells [30] constructed the efficient frontier in the expected profit-loss space and the expected profit-volume space, for a decision maker creating a consumer loan portfolio. In Chapter 4, we extended the literature to include economic forecasts when constructing the efficient frontier. In both these work, the decision maker was required to determine the accept/reject score set. In this chapter, the decision maker is required to set the accept/reject decision incorporating regulatory capital reserves in the decision making process.

In Section 6.3, we consider the case of a single economic scenario and construct the efficient frontier for two cases; one without capital constraint, and the other with capital constraint. We showed that under the capital constraint scenario, a purely single-cutoff score strategy is no longer applicable. The efficient frontier is constructed through a combination of single-cutoff score and double-cutoff score strategies. A portfolio manager operating under the advanced approach of Basel II and wanting to increase expected volume beyond the maximum volume achievable with a single cutoff-score strategy, is required to accept customers at the riskiest end of the score spectrum. There are two contributing reasons for the disjoint accept set in a capital constraint environment: the concave property of regulatory formulae and the monotonic decreasing, non-concave relationship between expected account profit and score. We showed disjoint accept sets result in binding capital constraint. Hence, every disjoint accept set resulting in an efficient operating point on the EPV space require equal regulatory capital requirement. When the EPV curve is plotted parameterized by disjoint accept sets described in this chapter, an increase in expected volume does not result in higher capital requirement despite the increase in average default risk in the portfolio.

In Section 6.4, we considered the case of multiple economic scenarios. We specified the process to simulate for the efficient frontier in the case of a portfolio manager who is faced with a capital retention decision prior to the accept/reject decision. We showed the efficient frontier may be constructed by restricting the simulation space to efficient accept sets on the conditional EPV space.

In this chapter, we considered a once-off investment decision by the portfolio manager. An important extension of this work is to consider multiple successive investment decisions.

Appendix

Proposition 6.1. *For each feasible operating point, there exists an accept set such that each of the separated subsets are of non-zero measure.*

Proof. Suppose an operating point is constructed by accepting scores in the set $\omega_1 = \{s|s \in (\cup_{i=1}^{M}\alpha_i) \cup (\cup_{j=1}^{N}\beta_j)\}$, where α_i's are score intervals with zero measure, β_j's are score intervals with non-zero measure, and α_i's and β_j's are separated subsets. Let $\omega_0 = \{s|s \in (\cup_{i=1}^{M}\alpha_i)\}$ and $\omega_1 = \{s|s \in (\cup_{j=1}^{N}\beta_j)\}$. Since $E_S[P^s(\omega_0, Q_R^s(s))] = 0$, $E_S[V^s(\omega_0)] = 0$ and $Q_R^s(\omega_0) = 0$, it follows that $E_S[P^s(\omega, Q_R^s(\omega))] = E_S[P_(\omega_1, Q_R^s(\omega))]$, $E_S[V^s(\omega)] = E_S[V^s(\omega_1)]$ and $Q_R^s(\omega) = Q_R^s(\omega_1)$. $\square$

Proposition 6.2. *Suppose ω_0 solves $max_{\{\omega|E_S[V^s(\omega)]=V_0\}}E_S[P^s(\omega, Q_R^s(\omega))]$. There exists a score s_a such that $E_S[P^s(\omega_0, Q_R^s(\omega))] = E_S[P^s(\{s|s \geq s_a\}, Q_R^s(\{s|s \geq s_a\}))]$ and $E_S[V^s(\omega_0)] = E_S[V^s(\{s|s \geq s_a\})]$.*

Proof. Let $\omega_a = \{s|s \geq s_a\}$ with $E_S[V^s(\omega_a)] = V_0$. Since ω_0 solves $\max_{\{\omega, E_S[V^s(\omega)]=V_0\}}E_S[P^s(\omega, Q_R^s(\omega))]$, it follows that $E_S[P^s(\omega_0 \setminus \omega_a, Q_R^s(\omega_0 \setminus \omega_a))] \geq E_S[P^s(\omega_a \setminus \omega_0, Q_R^s(\omega_a \setminus \omega_0))]$. Since $E_Z[P(s, k_r(s))]$ is strictly increasing in s, it follows that $E_S[P^s(\omega_a \setminus \omega_0, Q_R^s(\omega_a \setminus \omega_0))] \geq E_S[P^s(\omega_0 \setminus \omega_a, Q_R^s(\omega_0 \setminus \omega_a))]$. Therefore, $E_S[P^s(\omega_0, Q_R^s(\omega_0))] = E_S[P^s(\omega_a, Q_R^s(\omega_a))]$. □

Proposition 6.3. *Suppose there exist sets* ω_1, ω_2 *and* ω_3 *such that* $\omega_1 \cap \omega_2 = \emptyset$, $\omega_1 \cap \omega_3 = \emptyset$, *and set* $\omega_2 \succ^{\wp} \omega_3$. *Then,* $\{\omega_1 \cup \omega_2\} \succ^{\wp} \{\omega_1 \cup \omega_3\}$.

Proof. Since $\omega_2 \succ^{\wp} \omega_3$, it follows that $E_S[P^s(\omega_2, Q_R^s(\omega_2))] > E_S[P^s(\omega_3, Q_R^s(\omega_3))]$, $E_S[V^s(\omega_2)] = E_S[V^s(\omega_3)]$, and $Q_R^s(\omega_2) \leq Q_R^s(\omega_3)$. Therefore, $E_S[P^s(\omega_1 \cup \omega_2, Q_R^s(\omega_1 \cup \omega_2))] > E_S[P^s(\omega_1 \cup \omega_3, Q_R^s(\omega_1 \cup \omega_3))]$, $E_S[V^s(\omega_1 \cup \omega_2)] = E_S[V^s(\omega_1 \cup \omega_3)]$, and $Q_R^s(\omega_1 \cup \omega_2) \leq Q_R^s(\omega_1 \cup \omega_3)$. □

Proposition 6.4. *Suppose the capitalization function,* $k_B(p(B|s))$, *is concave with respect to* $p(B|s)$ *in the score range* $\omega = [s_1, s_2]$. *Suppose there exists an accept set* $\omega_1 \subset \omega$ *with* $\omega_1 \neq \{[s_1, s_3] \cup [s_4, s_2]\}$ *for some* $s_1 \leq s_3 < s_4 \leq s_2$. *There exists a set* $\omega_2 = \{[s_1, s_3] \cup [s_4, s_2]\}$ *for some* $s_1 \leq s_3 < s_4 \leq s_2$ *such that* $\omega_2 \succ^{\wp} \omega_1$.

Proof. For any s, let $r(s) = p(B|s)$. Since S is a random variable with density $f(s)$ and there exists a bijective relationship between s and $p(B|s)$, we can view $R = r(S)$ as a random variable with density $g(r(s))$. Note, $g(r(s)) = f(s)$. For notational convenience, we refer to $r(s) \equiv r \in R$. Let $r_i = p(B|s_i)$. Therefore, $k_B(r)$ is concave with respect to r in the range $[r_2, r_1]$. We define set $\gamma_1 = \{r(s)|s \in \omega_1\}$. Let $V_0 = \int_{\omega_1} f(s)ds$. It follows that $\int_{\gamma_1} g(r)dr = V_0$.

There exists scores r_5, $r_6 \in (r_2, r_1)$ such that $\int_{[r_2, r_5]} g(r)dr = \int_{[r_6, r_1]} g(r)dr = V_0$.

For each score $r_i \in [r_2, r_5]$, there exists a unique score r_i' with $r_6 < r_i' < r_1$, such that $\int_{[r_2,r_i]\cup[r_i',r_1]} g(r)dr = V_0$. Let $\gamma(r) \equiv \{[r_2, r] \cup [r', r_1]\}$ and $E_R[V^s(\gamma(r))] = \int_{\gamma(r)} g(u)du$

Let $J(r, w(r)) = \int_{\{\gamma(r)|E_R[V^s(\gamma(r))]=V_0\}} w(u)g(u)du$ for any continuous function $w(r)$.

We define a linear function of r, $h(r) = m_1 r + m_0$ such that $h(r) \geq 0$ for $r \in [0, 1]$, where $m_1 < 0$ is the slope and $m_0 = h(0)$. We use Weierstrass definition to show that $J(r_i, h(r))$ is continuous with respect to $r_i \in [r_2, r_5]$ (see [2]).

$$\begin{aligned}
|J(r, h(r)) - J(a, h(r))| &= \left| \int_{\gamma(r)} h(u)g(u)du - \int_{\gamma(a)} h(u)g(u)du \right| \\
&= \left| \int_{[r,a]} h(u)g(u)du - \int_{[r',a']} h(u)g(u)du \right|.
\end{aligned}$$

Let $L = \max_{\{r\}} g(r)$. Since, $\int_{\gamma(r)} g(u)du = \int_{\gamma(a)} g(u)du = V_0$, and $h(r) \geq 0$ is a decreasing function of $r \in [r_2, r_1]$, it follows that $\left|\int_{[r,a]} h(u)g(u)du\right| > \left|\int_{[r',a']} h(u)g(u)du\right| > 0$. Therefore,

$$\begin{aligned}
|J(r, h(r)) - J(a, h(r))| &< \left| \int_{[r,a]} h(u)g(u)du \right| \\
&< \left| \int_{[r,a]} m_0 L du \right| \\
&= m_0 L \left| r - a \right|.
\end{aligned}$$

Fix $\epsilon > 0$. Let $|r - a| < \delta$ and let $\delta < \frac{\epsilon}{m_0 L}$. It follows that $|J(r, h(r)) - J(a, h(r))| < \epsilon$. Therefore, $J(r, h(r))$ is a continuous function of $r \in [r_2, r_5]$.

Since $J(r, h(r))$ is a continuous function of $r \in [r_2, r_5]$, there exists a score $r_7 \in [r_2, r_5]$ such that $J(r_7, h(r)) = \int_{\gamma_1} h(u)g(u)du$. Since, $h(r)$ is a linear function of r and $\int_{\gamma(r_7)} g(u)du = \int_{\gamma_1} g(u)du = V_0$, it follows that $\int_{\gamma(r_7)} ug(u)du = \int_{\gamma_1} ug(u)du$. Therefore,

$J(r_7, h(r)) = \int_{\gamma_1} h(u)g(u)du$ for any linear function $h(r) > 0$ with $r \in [0,1]$ and $m_1 \in (-\infty, \infty)$.

Since $\{\gamma_1 \setminus \gamma(r_7)\} \subseteq (r_7, r_7')$, we construct a linear function $h(r)$ such that it intersects $k_B(r)$ at two points in the region $r \in (r_7, r_7')$ with $\int_{\{\gamma_1\setminus\gamma(r_7)\}} h(u)g(u)du = \int_{\{\gamma_1\setminus\gamma(r_7)\}} k_B(u)g(u)du$. Since $\int_{\{\gamma(r_7)\setminus\gamma_1\}} h(u)g(u)du = \int_{\{\gamma_1\setminus\gamma(r_7)\}} h(u)g(u)du$, it follows that $\int_{\{\gamma(r_7)\setminus\gamma_1\}} h(u)g(u)du = \int_{\{\gamma_1\setminus\gamma(r_7)\}} k_B(u)g(u)du$. However, since $h(r)$ intersects $k_B(r)$ at two points in the region $r \in (r_7, r_7')$, it follows that $h(r) > k_B(r)$ for $r \in \gamma(r_7)$. Hence, $\int_{\{\gamma(r_7)\setminus\gamma_1\}} k_B(u)g(u)du < \int_{\{\gamma(r_7)\setminus\gamma_1\}} h(u)g(u)du$. Therefore, $\int_{\gamma(r_7)} k_B(u)g(u)du < \int_{\gamma_1} k_B(u)g(u)du$.

Let s_3 be such that $p(B|s_3) = r_7'$. Let $\omega_2 = \{s|r(s) \in \gamma(r_7)\} = [s_1, s_3] \cup [s_7, s_2]$. From Equation (3.1), $E_Z[P(s, k_r(s))] = (r_L(s) - c_D) - (r_L(s) + f_D)r(s) - r_Q k_B\,(r(s))$. Since $(r_L(s) - c_D) - (r_L(s) + f_D)r(s)$ is a linear function of $r(s)$, it follows that $E_S[P^s(\omega_2, Q_R^s(\omega_1))] > E_S[P^s(\omega_1, Q_R^s(omega_1))]$. Therefore, set $\omega_2 \succsim^{p} \omega_1$. □

Proposition 6.5. *Suppose the capitalization function, $k_B\,(p(B|s))$, is increasing with respect to $p(B|s)$ in the score range $[s_1, s_2]$. Suppose set $\omega_1 \subset [s_1, s_2]$ but $\omega_1 \neq [s_1, s_3]$ for some $s_1 < s_3 < s_2$. There exists a set $\omega_2 = [s_1, s_3]$ for some $s_1 < s_3 < s_2$ such that $\omega_2 \succsim^{p} \omega_1$.*

Proof. Since $k_B\,(p(B|s))$ is increasing with respect to $p(B|s)$ in the score range $[s_1, s_2]$, it follows from Equation (3.1), the expected profit $E_Z[P(s, k_r(s))]$ is decreasing in the same range. There exists a score s_3 such that $\omega_2 = [s_1, s_3]$ with $E_S[V^s(\omega_2)] = E_S[V^s(\omega_1)]$, and

$E_S[P^s(\omega_2, Q_R^s(\omega_2))] > E_S[P^s(\omega_1, Q_R^s(\omega_1))]$. Since $k_B\,(p(B|s))$ is increasing with respect to $p(B|s)$ in the score range $[s_1, s_2]$, it follows that $Q_R^s(\omega_2) < Q_R^s(\omega_1)$. □

Proposition 6.6. *Suppose score s^* solves $\max_s k_r(s)$. For each score $s_i \in (-\infty, s^*)$, there exists a corresponding score s'_i such that $k_r(s_i) = k_r(s'_i)$ with $s'_i \in (s^*, \infty)$. Any accept set $\omega_1 \subseteq [s_1, s_2]$ with either exclusively $f(s_i) \leq f(s'_i)\ \forall s_i \in [s_1, s_2]$ or exclusively $f(s_i) > f(s'_i)\ \forall s_i \in [s_1, s_2]$ is a profit dominated set.*

Proof. Since $E_Z[P(s, k_r(s))]$ is an increasing function of s and s_i is defined such that $s_i < s'_i$, it follows that $E_Z[P(s_i, k_r(s))] < E_Z[P(s'_i, k_r(s))]\ \forall s_i \in [s_1, s_2]$.

Suppose $f(s_i) \leq f(s'_i)\ \forall s_i \in [s_1, s_2]$. Since $E_Z[P(s_i, k_r(s))] < E_Z[P(s'_i, k_r(s))]$, there exists a set $\omega_2 \subseteq [s'_3, s'_1]$ for some score $s'_3 \in [s'_2, s'_1]$ such that $E_S[P^s(\omega_2, Q_R^s(\omega_2))] > E_S[P^s(\omega_1, Q_R^s(\omega_1))]$, $E_S[V^s(\omega_2)] = E_S[V^s(\omega_1)]$, and $Q_R^s(\omega_2) \leq Q_R^s(\omega_1)$.

Suppose instead, $f(s_i) > f(s'_i)\ \forall s_i \in [s_1, s_2]$. Since $E_Z[P(s_i, k_r(s))] < E_Z[P(s'_i, k_r(s))]$, there exists sets $\omega_2 = [s'_2, s'_1]$ and $\omega_3 \subset [s_1, s_2]$ such that $E_S[P^s(\omega_2 \cup \omega_3, Q_R^s(\omega_2 \cup \omega_3))] > E_S[P^s(\omega_1, Q_R^s(\omega_1))]$, $E_S[V^s(\omega_2 \cup \omega_3)] = E_S[V^s(\omega_1)]$, and $Q_R^s(\omega_2 \cup \omega_3) = Q_R^s(\omega_1)$.

Therefore, ω_1 is a profit dominated set. □

Proposition 6.7. *Suppose $\omega_1 \in \Gamma_2$ with $Q(\omega_1) < K$, where K is the maximum available unencumbered capital, then $\omega_1 \notin \Omega_1(K)$.*

Proof. Let $\omega_1 = (-\infty, s_1] \cup [s_2, \infty)$. Let score s_3 be such that $Q_R^s([s_3, s_2]) = K - Q_R^s(\omega_1)$. Since $K < Q(-\infty)$, it follows that $s_1 < s_3$. Let score s_4 be such that $E_S(V^s([s_4, s_1])) =$

$E_S(V^s([s_3, s_2]))$. Since the expected profit is an increasing function of score s, it follows that $E_Z[P(s_i, k_r(s_i))] < E_Z[P(s_j, k_r(s_j))]$ $\forall s_i \in [s_4, s_1]$ and $s_j \in [s_3, s_2]$. Let $\omega_2 = \{s | s \in \{(-\infty, s_4] \cup [s_3, \infty)\}\}$. It follows that $E_S[P^s(\omega_2, Q_R^s(\omega_2))] > E_S[P^s(\omega_1, Q_R^s(\omega_1))]$ and $E_S[V^s(\omega_2)] = E_S[V^s(\omega_1)]$. Therefore, $\omega_1 \notin \Omega_1(K)$. □

Proposition 6.8. *Suppose* $\omega_1, \omega_2 \in \Gamma_3$ *then* $E_S[V^s(\omega_1)] \neq E_S[V^s(\omega_2)]$.

Proof. Without loss of generality, suppose $\omega_1 \setminus \omega_2 = [s_1, s_2)$ and $\omega_2 \setminus \omega_1 = (s_3, s_4]$ with $s_1 < s_2 < s_3 < s_4$. Since, $\omega_1, \omega_2 \in \Gamma_3$, it follows that $Q_R^s(\omega_1) = Q_R^s(\omega_2) = K$. Therefore, $\int_{[s_1,s_2)} k_r(s) f(s) ds = \int_{(s_3,s_4]} k_r(s) f(s) ds$. However, by Proposition 6.6, $k_r(s_i) > k_r(s_j)$ $\forall s_i \in [s_1, s_2)$ and $\forall s_j \in (s_3, s_4]$. Therefore, $\int_{[s_1,s_2)} f(s) ds < \int_{(s_3,s_4]} f(s) ds$. It follows that $E_S[V^s(\omega_1)] \neq E_S[V^s(\omega_2)]$. □

Proposition 6.9. *The EPV curve is concave.*

Proof. Suppose the EPV curve is strictly convex between $E_S[V^s(\omega_1)]$ and $E_S[V^s(\omega_2)]$. Let $\omega_1 = (-\infty, s_1] \cup [s_2, \infty)$ and $\omega_2 = (-\infty, s_3] \cup [s_4, \infty)$. Without the loss of generality, let $s_1 < s_3 < s_2 < s_4$. The convexity implies there exists a set $\omega_3 \in \Omega_1(K)$ such that $E_S[P^s(\omega_3, Q_R^s(\omega_3))] < x E_S[P^s(\omega_1, Q_R^s(\omega_1))] + (1 + x) E_S[P^s(\omega_2, Q_R^s(\omega_2))]$ and $E_S[V^s(\omega_3)] = x E_S[V^s(\omega_1)] + (1 - x) E_S[V^s(\omega_2)]$ for some $0 < x < 1$. There exists a set $\omega_4 \subseteq \omega_1 \cup \omega_2$ such that $\omega_4 \nsucc^p \omega_3$. However, by Propositions 6.4, 6.5, and 6.6, it follows that $\omega_4 \in \Omega_1(K)$. Since $E_S[V^s(\omega_3)] = E_S[V^s(\omega_4)]$, it follows by Proposition 6.8 that $\omega_3 \equiv \omega_4$.Therefore, EPV curve cannot be strictly convex in any region. □

Proposition 6.10. *Let $\Gamma_u(K)$ be the set of accept sets resulting in efficient operating points under economic scenario u and available capital K at $t = 1$. Suppose the portfolio manager retains capital amount of K at time $t = 0$. Without loss of generality, suppose $\omega_1 \in \Gamma_1(K)$, and suppose ω_2 is a feasible operating point with $Q_2^s(\omega_2) \leq K$ but $\omega_2 \notin \Gamma_2(K)$, then the resulting operating point with ω_1, ω_2 and K at time $t = 0$ is a dominated operating point in the unconditional EPV space.*

Proof. Since $\omega_2 \notin \Gamma_2(K)$, there exists an accept set $\omega_3 \in \Gamma_2(K)$, such that $E_S[P_2(\omega_3, 0)] > E_S[P_2(\omega_2, 0)]$ and $E_S[V_2(\omega_3)] \geq E_S[V_2(\omega_2)]$ with $Q_2(\omega_3) \leq K$. Therefore, $E_U[P^s(\omega_1, \omega_3, K)] > E_U[P^s(\omega_1, \omega_2, K)]$ and $E_U[V^s(\omega_1, \omega_3)] \geq E_U[V^s(\omega_1, \omega_2)]$. It follows the resulting operating point with ω_1, ω_2 and K is a dominated operating point. $\square$

Chapter 7

Consumer Decision Heuristics

7.1 Introduction

In Chapter 4, we considered the case of a portfolio manager with access to multiple scorecards operating under multiple economic conditions. In Chapter 5, we proved that misestimating regulatory capital requirements results in a negative profit impact. In the following chapter, we incorporated cost of regulatory capital into acquisition decisions. We showed methods to construct the efficient frontier for a portfolio manager operating under multiple possible future economic scenarios. In this chapter, we turn attention to the impact of consumer decision in taking up or rejecting credit offers on portfolio manager's offer decisions. From the definition of adverse selection in static lending models, we show that homogenous borrowers take-up offers at different instances of time when faced with a sequence of loan offers. We postulate that bounded rationality and diverse decision heuristics used by consumers drive the decisions they make about credit offers. Under that postulate, we show how observation of early decisions, regardless of the prevailing economic conditions, in a sequence can be informative about later decisions and can, when coupled with a type of adverse selection, also inform credit risk. We show through two examples how lenders may use such information in setting their offer rates.

The chapter is organized as follows. Section 7.2 extends the definition of adverse selection from Oliver and Thaker [28] to a sequential offer setting that will serve as the basis for further study of the borrower's decision process. Section 7.3 discusses bounded rationality in human decision making and reviews literature on categorizing agents by their sequential decision making behavior. Section 7.4 introduces two sequential decision problems in the consumer lending space. The first decision problem relates to auction mechanisms for peer-to-peer lending. Lenders cognizant of decision heuristics in the context of consumer lending may offer a lower bid rate and, hence, win the bidding process. We derive policy implications for a marketplace desirous of increasing borrowers' utility through lower interest rates. The second decision problem, set in the context of direct mail, is that of a lender required to choose when to market offers relative to the competition. We show how the lender may incorporate information learned about the decision heuristics of individual consumers. Section 7.5 offers concluding remarks and suggestions for further research.

7.2 Adverse Selection

In this section, we introduce the mathematical definition of adverse selection. We then extend notions of adverse selection to timing adverse selection (TAS) and provide motivation for the study of consumers' decision making process in the consumer lending space.

Suppose a portfolio manager has access to a homogenous population to which a credit product is marketed. We say a population is homogenous when members of the population have no observable differences between them. Suppose the portfolio manager makes an offer of credit with rate r. Once an offer is made, some subset of the population, the *Take* population, will accept the offer and open an account. Let T denote the event that an individual takes up an offer; so T^c is the event the individual declines the offer. We denote the probability of default for a borrower with characteristic vector and offer rate r as $p(B|\bar{x}, r)$. The conditional probability of default for the Take population is then written as $p(B|T, \bar{x}, r)$.

Oliver and Thaker [28] define adverse selection as

$$p(B|T, \bar{x}, r) \quad > \quad p(B|\bar{x}, r). \tag{7.1}$$

Equation 7.1 states the probability of a member in the Take population defaulting is higher than the probability of default in the general population, i.e., both the Take and Non-Take population.

We use the total probability theorem to obtain the Bads among the Non-Takes, i.e.,

$$p(B|\bar{x}, r) \quad = \quad p(B|T, \bar{x}, r)p(T|\bar{x}, r) + p(B|T^c, \bar{x}, r)p(T^c|\bar{x}, r). \tag{7.2}$$

Following Oliver and Thaker [28], Bayes' Rule can relate the conditional probability of

Bad of a Take to the conditional probability of Take by a Bad, i.e.,

$$\frac{p(B|T,\bar{x},r)}{p(B|\bar{x},r)} = \frac{p(T|B,\bar{x},r)}{p(T|\bar{x},r)}. \tag{7.3}$$

Since, $p(T|\bar{x},r)+p(T^c|\bar{x},r) = 1$, combining Equations 7.1 and 7.2 results in the following inequality:

$$p(B|\bar{x},r) \quad > \quad p(B|T^c,\bar{x},r). \tag{7.4}$$

Equation 7.4 indicates the Non-Take population have a higher credit quality than the total population.

In defining Equation 7.4, we assumed the portfolio manager makes a one-time offer of credit. Suppose instead of a one-time offer strategy, the portfolio manager markets repeatedly. At each of a finite number of epochs, the manager has the option to market to individuals who have not previously taken an offer. Below we show that, due to adverse selection, the credit quality of those not-taking up any prior offers improve after every marketing instance.

From Equations 7.1 and 7.3, it follows that,

$$\frac{p(B|T_1,\bar{x},r_1)}{p(B|\bar{x},r_1)} = \frac{p(T_1|B,\bar{x},r_1)}{p(T_1|\bar{x},r_1)} > 1, \tag{7.5}$$

where T_i is the random variable indicating take-up at the i^{th} offer and r_i is the offer-rate in the i^{th} marketing instance. It follows from Equations 7.4 and 7.5, that the credit quality of the non-take population after the first marketing instance is higher than the credit quality of the population prior to the first marketing instance, i.e.,

$p(B|T_1^c, \bar{x}, r_1) < p(B|\bar{x}, r_1)$, where T_i^c indicates the event a borrower declines the i^{th} offer. Suppose the portfolio manager markets a second time to those who did not take up the offer in the first marketing instance. It follows from Equations 7.1 and 7.3 that,

$$\frac{p(B|T_2, T_1^c, \bar{x}, \bar{r}_2)}{p(B|T_1^c, \bar{x}, \bar{r}_2)} = \frac{p(T_2|B, T_1^c, \bar{x}, \bar{r}_2)}{p(T_2|T_1^c, \bar{x}, \bar{r}_2)} > 1, \tag{7.6}$$

where $\bar{r}_i$ is a vector of all past and current offers, i.e., $\bar{r}_i = \{r_1, r_2, ..., r_i\}$.

Equations 7.4 and 7.6 can both be generalized for the i^{th} marketing instance, i.e.,

$$p(B|T_1^c, ..., T_{i-1}^c, \bar{x}, \bar{r}_i) \ > \ p(B|T_1^c, ..., T_{i-1}^c, T_i^c, \bar{x}, \bar{r}_i) \tag{7.7}$$

and

$$\frac{p(B|T_i, T_1^c, ..., T_{i-1}^c, \bar{x}, \bar{r}_i)}{p(B|T_1^c, ..., T_{i-1}^c, \bar{x}, \bar{r}_i)} = \frac{p(T_i|B, T_1^c, ..., T_{i-1}^c, \bar{x}, \bar{r}_i)}{p(T_i|T_1^c, ..., T_{i-1}^c, \bar{x}, \bar{r}_i)} > 1. \tag{7.8}$$

Note that equations 7.7 and 7.8 are extensions of Equations 7.1 and 7.3. Equation 7.7 implies that due to adverse selection, the credit quality of successive non-take population improves after each marketing instance. This is due to the higher probability of Take among Bads than the general population at each marketing instance. Furthermore, Equation 7.8 indicates there is a time component to adverse selection. We call this time dependent characteristic of adverse selection, *timing adverse selection* (or TAS). Note, thus far the vector of offer rates in $\bar{r}_i$ has not been specified.

Suppose some members of a marketed population decline all prior offers, it follows from Equation 7.8,

$$p(T_i|B, T_1^c, ..., T_{i-1}^c, \bar{x}, \bar{r}_i) > p(T_i|T_1^c, ..., T_{i-1}^c, \bar{x}, \bar{r}_i).$$

It follows since $p(T_i|T_1^c, ..., T_{i-1}^c, \bar{x}, \bar{r}_i) \geq 0$, the probability of take-up for Bads is strictly positive and is greater than the probability of take-up among the general population. In such a scenario where some members of a marketed population decline all prior offers, there is a positive probability at each marketing instance of a Bad declining all prior offers and taking up the latest offer, , i.e., $p(T_i|B, T_1^c, ..., T_{i-1}^c, \bar{x}, \bar{r}_i) > 0$ for all i. Observations of real-life subjects faced with sequential decision making problems, have shown homogenous subjects taking up offers at different instances of a sequence. In the next section, we take a borrower's view of receiving a sequence of offers and discuss the decision heuristic observed in similar sequential decision making problems.

7.3 Bounded Rationality

In Section 7.2, we considered the case of a portfolio manager repeatedly marketing a credit product to a non-take population. The non-take population is updated after every offer is made. We showed under Oliver and Thaker's definition of adverse selection (see Oliver and Thaker [28]), the credit quality of the non-take population improves monotonically with marketing instance and that there is a timing aspect to adverse selection.

Suppose now we take a borrower's view. A borrower receives a sequence of offers from a lender. We assume the offer expires before the next offer arrives. As each offer arrives, the borrower is required to make a decision on whether to take the offer. If an offer not taken, the borrower waits for the next offer. When rejecting an offer, the borrower risks the chance of receiving only lower quality offers in the future. A broad set of literature indicates that humans do not necessarily make rational decisions when faced with sequential decision problems, in part because of our bounded ability to take in information and limited cognitive abilities. Such limitations are known as *bounded rationality*. Bounded rationality may explain why homogenous borrowers accept credit offers at different point of time when faced with a sequence of offers. We provide an example, in the form of a well-studied problem known as the secretary problem, of how human subjects make decisions in a sequential decision problem setting.

7.3.1 Decision Heuristics

The secretary problem, also known as the dowry problem, is a well-studied sequential decision problem involving optimal stopping theory. The secretary problem in its simplest form is as follows [16]. Suppose a manager wishes to fill a secretarial position. There is only one such position available, for which there are N applicants. The manager is aware of the number of applicants. We assume the applicants can be rank-ordered from best to worst candidates without ties. The applicants are then interviewed sequentially and in a random fashion. Once an applicant is interviewed, the manager is required to make a decision to hire the applicant or not. If the applicant is hired, no further interviews takes place. However if the applicant is not hired, the decision maker interviews the next candidate. Rejected applicants cannot be recalled. The objective of the manager is to hire the best possible applicant. After each interview, the manager faces a trade-off, i.e., the manager could hire the current interviewee and risk the chance that a better applicant would have arrived later on in the interview process, or not hire the current interviewee but no higher quality applicant arrives later.

The optimal solution can be described using the idea of a *candidate*. An applicant is a candidate if he or she is the best applicant interviewed thus far. The optimal solution is then for the manager to reject the first $h-1$ applicants, some integer $h \geq 1$, and then choose the next candidate [16]. Let N denote the number of applicants. For $N > 1$, the probability of selecting the best applicant is,

$$\begin{aligned}
\phi_N(h) &= \sum_{j=h}^{N} p(\text{j-th applicant is the best applicant and is selected}) \\
&= \sum_{j=h}^{N} \left(\frac{1}{N}\right)\left(\frac{h-1}{j-1}\right) \\
&= \left(\frac{h-1}{N}\right)\sum_{j=h}^{N} \frac{1}{j-1}.
\end{aligned}$$

The optimal solution is $h^* = \text{argmax}_t \phi_N(t)$. This is easily solved for small values of N. As $N \to \infty$, $h^* = N/e$ [16]. It follows that for large values of N, it is approximately optimal for the manager to interview 36.8% of the applicants and then select the next applicant better than all previously interviewed applicants. The probability of successfully choosing the best candidate is approximately 36.8% (see Ferguson et al. [16], and Gilbert and Mosteller [17] for more). Stewart [39] extended the secretary problem to one where the number of options is unknown. Under the assumption arrival times of each option is independent and identically distributed exponential random variable, the probability of choosing the best candidate with such a policy is $1/e$, which is the asymptotic optimal probability value for when the length is known [39].

Because of bounded rationality and behavioral biases, humans do not necessarily make decisions in a rational manner. Experiments in decision making with real-life subjects have shown diverse decision making heuristics. A field experiment by Seale and Rapoport [38] is particularly important because it demonstrates that when people were presented with the secretary problem, they did not generally behave optimally but rather in fashions that could be explained as mixtures of three decision heuristics, each with a parameter. The decision making strategies reported by Seale and Rapoport [38] are:

1. *Cutoff rule* - Reject the first $h-1$ applicants and then hire the next candidate.
2. *Successive non-candidate rule* - Hire the first candidate who follows h successive non-candidate applicants since the last candidate.
3. *Candidate counting rule* - Hire the h^{th} candidate.

Note that of the three decision rules, only the cutoff rule is optimal, and then only if the correct parameter is chosen. Seale and Rapoport [38] observed that human subjects seemed to follow a mixture of rules, with mixture weights and parameter values varying across individuals.

We speculate that multiple decision heuristics are in use by individuals responding to sequential credit offers. Such decision heuristics found among borrowers might explain why timing adverse selection occurs in practice. Furthermore, values for heuristics parameters might correlate with notions of patience on the part of the borrow, an idea explored below.

7.3.2 Credit Hunger

In a field experiment with low to moderate income households, Meier and Sprenger [23] tested whether time preferences can explain credit behavior. They measured time preferences of individuals through choice experiments. The choice experiment outcomes were then matched to credit report and tax return data. After controlling for disposable income and other characteristics, less patient individuals were found to have lower credit scores and higher default rates. While Meier and Sprenger's field experiment did not control for credit score, we posit that even when individuals do not have any observable differences, impatient consumer behaviors lead to higher default risk. We call this *credit hunger*.

If, as we speculate, credit hunger exists in consumer credit populations, there would be value in recognizing individuals with that characteristic. Methods for learning decision strategies from the observation of actions could provide such an ability. In the next section, we introduce recent work in machine learning that addresses related problems.

7.3.3 Behavior-based Agent Recognition

Suppose there exists a set of decision heuristics, similar to those observed by Seale and Rapoport [38], governing consumers' decision making processes. Any lender that could gain the ability identify the decision heuristics being used by individual borrowers might then be able to achieve an advantage in lending strategy, relative to competitors without that ability. Recent work in machine learning has addressed a class of problems called Behavior-based Agent Recognition (BAR), which center on the recognition of decision strategies (or the identity of agents) based on observation of decisions made by agents in sequential problems.

Qiao and Beling [34] address the BAR problem by modeling the decision problem faced by agents as a Markov decision process (MDP). They use inverse reinforcement learning (IRL) to the learn the reward vector of the MDP from the observed actions of the agents. The reward vector is, in turn, used as the feature space for supervised and unsupervised learning of decision agent identities. On several problems, feature spaces constructed from rewards learned from IRL outperform those constructed directly from observed actions [34]. For the secretary problem, Qiao and Beling [34]conduct a simulation experiment in which a distinct base parameter value was applied to each heuristics rule from Seale and Rapoport [38]. In addition, random noise was added to actions of the decision agents. The feature space learned from IRL resulted in clusters with high-accuracy relative to ground truth. The method did not require inputs on any description of the decision heuristics as a basis for recognition.

Suppose historical data of consumers' accept/reject decisions for a sequence of offers was available, including related historical account performance. In such a scenario, using Qiao and Beling's IRL model-based method, it might be possible to cluster consumers based on their decision heuristics. In addition, using the historical account performance, one could relate risk and response behavior to individual decision heuristics as well as the historical proportion of the borrower population using each decision heuristics. Furthermore, in identifying decision heuristics of historical population, distribution of parameter values for each decision heuristics could be estimated. A portfolio manager with access to such information might then incorporate his knowledge of the borrowers' decision heuristics in the consumer loan offer strategy.

7.4 Problems involving Timing Adverse Selection

In this section, we introduce two sequential decision problems found in consumer loan settings. In the first problem, we introduce the lending process in a social lending platform, where lenders offer loans to borrowers through a bidding process. We model the offer policies of portfolio managers cognizant of notions of credit hunger and the resulting impact on the final-rate offered to the borrower. Whereas in the first problem, lenders were merely cognizant of credit hunger, in the second problem we assume lenders have access to greater information such as the distribution of decision heuristics and the distribution of heuristics parameter values found in a borrower population. The portfolio manager is required to decide whether to market a credit product to a homogenous population. We assume each member of the marketable population receives a sequence of offers until an offer is taken-up by that member. The portfolio manager is required to decide whether to market in one of the instances of the offer sequence, and is required to decide on the offer rate if the product is marketed.

7.4.1 Social Lending

Social lending offers an avenue for consumers to borrow money outside of the traditional banking system, where money is borrowed from lenders wanting to earn higher rates on their investment than through other accessible investment vehicles. Typically, these lenders are consumers. An internet marketplace provides the platform to bring together borrowers and lenders in order to benefit both parties. Such lending practices have been growing in many markets. In the United States, Peer-to-peer lending crossed the $1 billion in outstanding loan amount in 2012[1]. In China, regulatory tightening of bank credit has resulted in the growth of peer-to-peer social lending[2]. Generally, potential borrowers register on a social lending site, and list both their details and loan requirement. Lenders, then compete to provide the loan at a competitive rate. The marketplace specifies the mechanism from which the rates are set with different rate

[1] http://techcrunch.com/2012/05/29/peer-to-peer-lending-crosses-1-billion-in-loans-issued/

[2] http://www.bloomberg.com/news/2012-07-23/china-shadow-bankers-go-online-as-peer-to-peer-sites-boom.html

setting mechanisms used by different marketplaces. Until recently, an auction mechanism for rate setting was used by the largest marketplace in the United States, Prosper.com. In this section, we model the impact of credit hunger on the offer rates in a social lending setting with an auction mechanism.

Prosper is the first peer-to-peer lending marketplace in the United States, currently with over 2 million members and $692 million of funded loans[3]. Prosper offers unsecured loans with fixed rates. The loans are fully amortized over the lending periods of 3 or 5 years. Prosper's current mechanism works as follows. A borrower creates a loan request, specifying the purpose of loan and the loan amount. A customers specific interest rate is calculated using Prosper's internal models and listed for potential lenders to view. Lenders then compete to provide portions of the loan on first-come basis. This is known as a posted-price mechanism [4]. Prior to December 20, 2010, Prosper followed the auction model in setting the lending rate. In this mechanism, the borrower lists an amount and a reserve rate. The reserve rate is the maximum rate, she is willing to take on for the loan. Lenders then bid on both the loan amount and an offer rate. At the close of the bidding process, the loan application is considered successful if the total loan amount bid by the lenders is no less than the requested amount. Only lenders bidding lower offer rates than the requested reserve rate are considered in determining the total loan amount. This mechanism is a uniform price mechanism where each winning lender receives the same

[3] http://www.prosper.com/about/. Data accessed on the 14th of February, 2014.

[4] http://www.lendacademy.com/prosper-com-ending-their-auction-process-dec-19th/. Accessed on the 28th of February, 2014.

rate.

In Prosper's auction mechanism, the lending process has a two-week bidding period. However, the borrower has the option to either close the bidding process once the total loan amount bid by the lenders reaches the requested amount level, or wait until the end of the official bidding period. We note that there is a de-facto signalling process that occurs when a borrower does not close their position immediately after the requested loan amount level has been reached. The observation that a borrower has not closed the position allows for lenders to lower their bid in recognition of possible lower default risk of the borrower.

The Prosper auction environment provides a natural environment to study timing adverse selection, particularly credit hunger. We show credit hunger has policy implications for a social lending marketplace wishing to provide greater benefits to the borrower. Chen et al. [10] analyzed Prosper's mechanism as a game of complete information that fully characterizes the Nash equilibria found in such mechanisms. In contrast, we demonstrate that the provision of providing the lender with incremental information relating to credit hunger may result in lower rates for lower risk borrowers.

Notation, bidding process and Nash equilibrium

In this section, we define notation and the basic model required to understand the Nash equilibrium rates when lenders are cognizant of credit hunger.

We follow Ceyhan et al. [9] in modeling the bidding process as a three time step process. Suppose a borrower wants to borrow an amount D and specifies a reserve interest rate of R. Both D and R are publicly listed on the borrower's listing at time $t = 0$. Each competing lender, L_i, specifies the amount she is willing to lend, a_i and her bid rate, b_i. Once the total loan amount bid by the lenders with $b_i \leq R$, exceeds the requested amount, the borrower has the option to stop the bidding process. We consider this time $t = 1$. All winners are announced at this point. If the borrower does not stop the bidding process, the lending process carries on until time $t = 2$, which is the maximum allowable time specified by the marketplace. In our model, we restrict each lender to bid at most once between time step 0 and 1. Note, time length between $t = 0$ and $t = 1$ varies for each lending process. Between time $t = 1$ and $t = 2$, any lenders not in a winning position may lower their rate bid in order gain a winning position. As each lender bids a lower rate, the latest leading lenders are announced. We assume each lender, L_i has a private rate, r_i which is the lowest rate she is willing to bid based on the characteristics of the borrower. Between time $t = 1$ and $t = 2$, lenders may only lower their bid if each bid increases their utility $u_i = x_i(p - r_i)$, where x_i is the loan amount bid by lender L_i and p is the winning rate. In this section, we drop all notions of a characteristics vector since we talk of one borrower. We assume each lender's private rate is a function of the credit risk of the borrower and the lender's forecast of future economic conditions. Once a lender is declared a winner any time during the process, she may not pull out of the bidding process. The goal of each lender is to increase their utility. Our goal is to characterize the Nash equilibrium price when lenders are cognizant of credit hunger.

For completeness, we provide the definition of Nash equilibrium.

Definition. *(Nash equilibrium) [10] A bid profile $b = (b_1, ..., b_n)$ is a Nash equilibrium if no lender can increase her utility by unilaterally changing her bid, that is keeping the bids of other lenders fixed.*

In the bidding process, the following allocation rules specifies the amount of loan allocated to each lender.

Definition. *(Allocation rules, last winner and first loser) [10]. Given a bid profile $b = (b_1, ..., b_n)$, order lenders such that $b_i \leq b_{i+1}$ for all i. Let $k = min\{j | \sum_{i=1}^{j} a_i \geq D, j = 1, ..., n\}$. The allocation is defined as $x_i = a_i$ for $i < k$, $x_k = D - \sum_{i=1}^{k-1} a_i$ and $x_i = 0$ for $i > k$, where x_i is the amount borrowed from lender L_i. We refer to L_k as the last winner and L_{k+1} as the first loser. Let Δ be the list of winners when all lenders bid their true rate. We denote α and $\alpha + 1$ as the index of the last winner and first loser in Δ.*

Note that $\sum_i x_i = D$. In Prosper's mechanism, the lending rate, $p = r_k$ when the last winner L_k does not fully utilize her budget (i.e., $x_k < a_k$), and $p = r_{k+1}$ when the last winner L_{k+1} fully utilizes her budget.

In addition, we require the following definition in order to define the price p in a

Nash equilibrium.

Definition. *(β) [10]. Suppose we order all lenders indexed in a non-decreasing order of their true interest rates. For each $L_j \in \Delta$, let L_β be the last winner in Δ when the set of lenders is restricted to $\{L_1, ..., L_{j-1}, L_{j+1}, ..., L_n\}$, i.e., it is the smallest index k such that $\sum_{i=1, i \neq j}^{k} a_i \geq D$. Define $\beta = max_{L_j \in \Delta} \beta_j$.*

Chen et al. [10] provide bounds for the final price p.

Lemma 1. *[10] The price p in any Nash equilibrium b satisfies $r_{\alpha+1} \leq p \leq r_\beta$. Furthermore, $p = r_j$ for some L_j with $r_{\alpha+1} \leq r_j \leq r_\beta$.*

Note that in a Nash equilibrium, there is a finite set of prices which p can hold.

Suppose Nash equilibrium is reached at time $t = 1$. Assuming no lender may pull out of a winning bid, a borrower's winning rate cannot worsen between time $t = 1$ and $t = 2$. Since the winning rate may not worsen, the borrower does not face any trade-off in terms of the loan-rate. It would seem obvious that a borrower should remain in the bidding process in the hope of a lower loan-rate. However, credit hungry borrowers may close the bidding process at $t = 1$ and the winning rate at $t = 1$ is the final loan rate. On the other hand, a borrower who does not close the bidding process provides an opportunity for the lenders to bid a lower rate.

Nash equilibrium rates

Suppose some of the participating lenders are cognizant of credit hunger. We assume such lenders perceive the borrower's action as a signal of a lower borrower risk profile and hence, revise their bidding rates.

Now, we consider the case of one participating lender who is cognizant of notions of credit hunger. Suppose lender L_k is aware of credit hunger and has an initial rate of r_k^0 at time $t = 0$. Suppose at time $t = 1$, the borrower does not close the bidding process. This signals to the lender L_k that the borrower is less risky than previously thought and hence, the cognizant lender revises her private rate downwards. Let r_k^1 denote the new rate at time $t = 1$ with $r_k^1 < r_k^0$. Let r_1^W and r_2^W be the winning rate at time $t = 1$ and $t = 2$ respectively. We consider the following cases:

Suppose $r_k^1 < r_1^W < r_k^0$. Since $r_1^W < r_k^0$, the cognizant lender is not in a winning position at time $t = 1$. However, she may revise her rate down to $r_1^W - \epsilon$ for some $\epsilon > 0$ in order to move into a winning position and hence increase her utility. This re-bidding process dislodges the last winner at time $t = 1$. If the previously winning lender had bid her true rate between time $t = 0$ and $t = 1$, the process stops and $r_2^W = r_1^W - \epsilon$. If the winning lender at time $t = 1$ had not bid her true rate, then the auction process continues between those who have not bid their true rate and the cognizant lender, resulting in lower final rate for the borrower, i.e., $r_2^W < r_1^W$.

Suppose $r_1^W = r_k^0$, then lender L_k is the last winner. She will then lower her bid only if it results in greater utility through greater allocation of the loan to her.

Suppose $r_1^W \geq r_k^0$. In this scenario, the cognizant lender will not revise her rate as any revision will have no impact on her utility as the winning rate is set by another lender.

Suppose instead, $r_1^W \leq r_k^1$. Since $r_k^1 < r_k^0$, the cognizant lender is in a winning position at time $t = 1$, but is not the lender setting the final rate. Any lowering of her bidding rate will not result in an allocation change. It follows that the winning lenders at time $t = 1$ remain the winning lenders at time $t = 2$.

In the above example, only one lender was cognizant of credit hunger. In such a case, notions of credit hunger will only affect the final rate if the cognizant lender gains by lowering her rate and increase her utility in the process. This has policy implications for marketplaces wanting to benefit borrowers through lower final rates. For example, such marketplaces can set training policies to increase awareness of credit hunger. If more lenders are cognizant of credit hunger, those lenders may revise their bidding rates down for a patient borrower. This will result in greater situations where $r_i^1 < r_1^W \leq r_i^0$ and thereby increasing the probability of lowering the final lending rates between time $t = 1$ and $t = 2$.

7.4.2 Direct Mail Lender's Decision Problem

In Section 7.4.1, we showed if a lender is cognizant of credit hunger, this may result in a lower winning loan rate. Suppose instead, lenders had access to greater information such as access to historical data on credit offer decisions for consumers who were required to make decisions on a sequence of credit offers as well as their respective account performance data. The lender may then forecast the distribution of decision heuristics found in a potential borrower population and incorporate such information in the lending decisions. In this section, we discuss the impact of such forecasts on a lender's decision in a direct mail setting.

Problem definition

We setup the lender's decision problem as follows. Suppose a lender has access to a homogenous population and wishes to market credit offers. Since the population is a homogenous population, we do not include the characteristic vector $\bar{x}$ in our analysis. The lender is required to decide whether to market or not to the population on the i^{th} instance in a sequence of offers. If the lender decides to market on the i^{th} instance then the lender is required to set the i^{th} offer rate with a single objective of maximizing expected profit. The lender also has access to a historical database of similar homogenous consumer population with information on sequence of past offers and the respective consumers' decisions on those credit offers. In addition, the lender has access to related historical account performance data.

A lender with access to borrowers' historical offer decision data may apply a BAR method on the data and categorize borrowers into groups, with each group associated with a single decision heuristics. Suppose all decision heuristics are ordered in some manner. Let $\psi_j = \psi_j(h_j)$ denote decision heuristics j with heuristics parameter h_j. In order to simplify our model, we define a candidate offer. A candidate offer is one that consumers prefer to rates on past offers. Since consumers prefer lower loan offer rates to higher rates, candidate offers are lower rates than all past offers' rates. We assume under each decision heuristics, only candidate offers are taken up by borrowers. Given historical data, the portfolio manager is able to estimate in a population, the probability of a borrower in a population using decision rule j, i.e., $p(\psi_j)$; and the conditional probability of a borrower using parameter value $h_j = k$, i.e., $p(h_j = k|\psi_j)$. Similarly, the portfolio manager may estimate default probability of a borrower conditioned on decision heuristic ψ_j, parameter h_j, and Take, i.e., $p(B|\psi_j, h_j, T)$, where the random variable T indicates the event an account was opened. Note, the default probability is conditioned on an account being opened rather than the timing of the Take, T_i.

The decision to take up an offer by a member of the population is dependent on the decision heuristics, the associated parameter, and a history of past and current offer rates. The decision heuristics and the heuristics parameter drives the cognitive process, while the sequence of offers determines the offer experience of the decision maker. In order to estimate the take rates, a sequence of offers must be specified. Let

$\bar{r}_i$ indicate a sequence of i offers with r_i the last offer. Hence, the probability of a Take conditioned on decision heuristics, the associated parameter, and a history of past and current offer rates may be estimated from the historical database, i.e., $p(T_i|\psi_j, h_j, \bar{r}_i)$. Suppose the last offer r_i is not a candidate offer, i.e., $r_i \geq \min_i[r_1, r_2, ..., r_{i-1}]$. Since all offers accepted by a consumer are candidates, it follows that $p(T_i|\bar{r}_i, \psi_i, h_i) = 0$ $\forall \psi_i, h_i$ and $r_i \geq \min_i[r_1, r_2, ..., r_{i-1}]$.

Lender's decision

The lender is required to make a decision on whether to make an i^{th} offer in a sequence of offers. We assume the lender is only able to market on the i^{th} offer. If the lender makes an offer, it then is required to set the offer rate. We assume the cost of marketing to a single borrower is C_M. It follows that the expected profit conditional on take is,

$$\begin{aligned} E[P|\bar{r}_i, T_i, \psi_j, h_j] &= (1 + r_i)\left[1 - p(B|\psi_j, h_j, T_i)\right] \\ &+ (1 - f_D)\left[p(B|\psi_j, h_j, T_i)\right] - (1 + r_B) - C_M. \end{aligned}$$

Therefore,

$$\begin{aligned} E[P|\bar{r}_i, T_i, \psi_j, h_j] &= (r_i - r_B - C_M)\left[1 - p(B|\psi_j, h_j, T_i)\right] \\ &- (r_B + f_D + C_M)\left[p(B|\psi_j, h_j, T_i)\right]. \end{aligned}$$

Since the cost of marketing to a consumer who does not take-up the offer is C_M, it follows that the expected profit conditioned on a decision heuristics, ψ_j and parameter

h_j is,

$$E[P|\bar{r}_i, \psi_j, h_j] = E[P|\bar{r}_i, T_i, \psi_i, h_i]p(T_i|\psi_j, h_j, \bar{r}_i) - C_M\left[1 - p(T_i|\psi_j, h_j, \bar{r}_i)\right].$$

It follows that, the expected profit condition on a sequence of offer $\bar{r}_i$ is,

$$E[P|\bar{r}_i] = E[P|\bar{r}_i, \psi_i(h_i), h_i]p(h_j = k|\psi_j)p(\psi_j),$$

where $p(h_j = k|\psi_j)$ and $p(\psi_j)$ are estimated from the historical database. Note, borrowers only take-up candidate offers. Since the lender's objective is to maximize expected profit and since all offers that are taken-up are candidate offers, the lender solves the following maximization problem,

$$\begin{array}{ll} \max_{r_i} & E[P|\bar{r}_i] \\ \text{s.t.} & r_i < \min_i[r_1, r_2, ..., r_{i-1}]. \end{array} \tag{7.9}$$

Suppose $r_i = r^*$ is the profit maximizing rate, i.e., $r^* = \text{argmax}_{r_i}\left[E[P|\bar{r}_i]\right]$. The lender markets in the i^{th} instance if $E[P|\bar{r}_{i-1}, r^*] \geq 0$. By Proposition 7.1, $r^* = \min_{r_j, j\in[1,i-1]}[r_j] - \epsilon$. We may think of ϵ as the marginal improvement in offers.

Proposition 7.1. $r^* = min_{r_j, j\in[1,i-1]}[r_j] - \epsilon$.

Proof. Suppose r' and r'' are two possible candidate offers with $r' < r''$. Since $E[P|\bar{r}_i]$ increases monotonically with respect to all candidate offers r_i, it follows that $E[P|\bar{r}_{i-1}, r_i = r'] < E[P|\bar{r}_{i-1}, r_i = r'']$ for $r' < r''$, i.e., the lender prefers borrowers to accept a product with a higher offer rate given all else equal. Given borrowers will reject a non-candidate offer then the portfolio manager will offer a rate $r_i = \min_i[r_1, r_2, ..., r_{i-1}] - \epsilon$ for some $\epsilon > 0$. □

Note, since borrowers only take-up candidate offers, the rate set by the lender maximizing on expected profit is a candidate offer. We did not relate this to the quality of the candidate offer, i.e., the model presented in this section seems to indicate a lower candidate offer rates does not attract more customers as would be expected in practice. The scenario presented here assumes the lender is cognizant of the order in which his offer lies within a sequence of offers. We assumed all consumers had received the same past offers prior to the current offer. In practice, when an offer is made, a lower offer rate attracts a higher take-up rate, i.e., the quality of the candidate offer affects the response rate. This may be attributed to each borrower receiving multiple offers from different lenders and hence each borrower is at a different point on sequences of offers.

7.5 Conclusion

This chapter extends the line of research in consumer lending to include the notion of sequential decision making. We use the definition of adverse selection presented by Oliver and Thaker [28] to introduce the notion of timing adverse selection. We explain this phenomenon through bounded rationality resulting in diverse decision heuristics used by consumers. Along with decision heuristics in consumer lending space, we introduced the notion of credit hunger. This was followed by introducing a method used to cluster agents based on their decision heuristics–Behavior-based Agent Recognition. Finally, the chapter illustrated the impact of credit hunger and decision heuristics on two decision problems in the consumer lending space.

Chapter 8

Conclusions and Future Work

Given the finding by De Andrade and Silva [13] and Zandi [48] that default rates should be considered conditional on current and future economic conditions, decision making in lending can be improved by giving consideration to future economic conditions on loan acquisition decisions. In addition, since default rates are input into Basel II capital requirement formula, decision making on regulatory capital can be improved with similar consideration to future economic scenarios. In this dissertation, we considered four decisions in the consumer lending space.

In Chapter 4, we studied the scoring decision for a portfolio manager with access to scorecards specifically built for each possible future economic scenarios. We assumed the portfolio manager had access to both performance data for each scorecard under each economic condition as well as the forecast of occurrence of the future economic condition. Under the assumption that the portfolio manager's objectives are to maximize expected profits and market share, we developed methods to construct the set of efficient operating points on the expected profit-expected market share decision space.

In Chapter 5, we proved that misestimation of regulatory capital requirement results in a negative impact on profit. The underestimation costs that we describe are entirely due to our assumption that the bank is constrained in raising equity on short time scales. The ability to raise costless, incremental capital would serve to offset the dead weight loss, assuming the bank has the behavioral motivation to operate at an optimal economic capital point. In the neo classical world which we consider, shareholder value is maximized by operating at the true economic capital level. Given a population of omniscient and well behaved banks in frictionless capital markets, no regulatory requirement would be an optimal policy. If capital is constrained, however, even here regulatory misestimation affords societal costs despite what might be seen as benevolent regulations for capital requirements. Therefore, banks should be somewhat skeptical regarding the net benefit of even the more enlightened regulatory capital requirements imbedded in Basel II type regulation. This assumes that they can replicate our 'omniscient' consumer bank's ability to model true economic capital better than their regulatory counterparts. Large banks initially expected that the capital relief they would obtain from improved models and tailored regulatory models would offset the large investment in and maintenance of staff and information technology systems, but have become more doubtful in this regard over time [21].

In Chapter 6, we showed a portfolio manager constrained by the amount of available unencumbered capital for regulatory capital purposes, may operate under a double-cutoff score strategy. The contribution of this chapter to the literature is as follows. Firstly, this work incorporates capital constraints and cost of regulatory capital. Secondly, the work establishes conditions under which a single cutoff-score is not an efficient decision. Thirdly, this work establishes the unexpected result of portfolios with different portfolio-risk profiles that have the same portfolio regulatory requirement. Lastly, we show that for a multiple-economic scenario case, the efficient frontier may be determined by combinations of operating points on the economic scenario specific efficient frontier.

In Chapter 7, from the definition of adverse selection in static lending models, we showed that homogenous borrowers take-up offers at different instances of time when faced with a sequence of loan offers. We showed examples in literature where bounded rationality and diverse decision heuristics used by consumers drive the decisions they make in sequential decision problems. Assuming similar consumer behavior when faced with a sequence of credit offer, we show how observation of early decisions in a sequence can be informative about later decisions and can, when coupled with a type of adverse selection, also inform credit risk during the period of account performance. We show through two examples how lenders may use such information in setting their offer rates.

In summary, we proposed improvement to consumer lending decisions by incorporating performance information under each possible future economic conditions. We incorporated into our decision models capital requirement as specified by Bank of International Settlement, and notions of bounded rationality and decision heuristics of potential borrowers.

8.1 Future Work

Decision models presented in this dissertation are extensions of theoretical models found in literature. Opportunities for future work lies in empirical demonstration of our models. For example, given multiple scorecards built for each possible future economic condition, empirical work is required to determine the benefit of our multiple economic condition decision model. In Chapter 5, we demonstrated numerically the impact on profit of misestimating regulatory capital, an opportunity for future research lies in determining the sensitivity in a system of banks of misestimating regulatory capital to total profit drag. This provides an indication to regulators on the effort and cost associated with improving the regulatory capital models.

Chapter 7 provides the greatest opportunity for future research. As with adverse selection, timing adverse selection is not easy to measure in practice (see Oliver and Thaker [28] for discussion on measuring adverse selection in practice). However, sequential decision making experiments in other settings provide evidence that humans employ diverse decision heuristics, and this in turn suggests the existence of timing adverse selection. In order to categorize historical borrowers into clusters of decision heuristics, both take and non-take decision information is required. While offers taken-up by borrowers are found in credit bureau records, to our knowledge lenders do not share information on past declined offers. In addition, determining consumer lending decision heuristics requires an audit of offers, take behavior for all offers, and account performance for every offer accepted by a consumer. While we have shown examples of the impact of timing adverse selection on a lender's decision, field experiments and further research is required in order to understand the phenomena described in this chapter. In testing timing adverse selection and in determining consumers' decision heuristics, a sequence of offers need to made and consumers' decisions recorded. Such a sequence of offers may be disrupted by other lenders marketing their own products. Joint marketing strategy is required between competing lenders in order to understand both adverse and timing adverse selection.

There are two further important extensions to this research. We assumed a portfolio manager's objectives are to maximize expected profit and expected market share. An important extension is the consideration of variance of the objectives due to uncertainty in future economic conditions. Additionally in this dissertation, we assumed a single bank model. An important extension is the consideration of multiple-banks in competition. This is especially pertinent for extension to Chapter 7, where a bank may make an offer later in the hope of attracting borrowers with lower risk.

We look forward to working on these extensions.

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